



CSE 230 : DISCRETE MATHEMATICS
FINAL EXAMINATION : SPRING 2018
TIME: 2 HOURS MARKS:100

ANSWER ANY 05 (FIVE) OF THE FOLLOWING 06 (SIX) QUESTIONS
[N.B.: TO UNDERSTAND THE QUESTIONS IS A PART OF EXAMINATION]

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1. a) Your sock drawer contains **ten pairs of white socks** and **ten pairs of black socks**. [5]
If you're only allowed to take one sock from the drawer at a time and you can't see what color sock you're taking until you've taken it, **how many socks do you have to take** before you're guaranteed to have at least one matching pair?
- b) A farmer is taking **a fox, a chicken, and a bag of grain** home. To get there, he must cross a river, **but he's only allowed to take one item across the bridge with during each trip**. If the fox is left alone with the chicken, the fox will eat the chicken. If the chicken is left alone with the grain, the chicken will eat the grain. How can the farmer cross the river without any of his possessions being eaten? Please note that while making return trips he can also carry one animal/item back with him as well. [5]
- c) Prove whether $(p \rightarrow r) \vee (q \rightarrow r)$ is logically equivalent to $((p \wedge q) \rightarrow r)$. [10]
2. a) A question paper consists of 10 questions divided into two parts A and B. [10]
Each part contains five questions. A candidate is required to attempt six questions in all of which at least 2 should be from part A and at least 2 from part B. In how many ways can the candidate select the questions if he can answer all questions equally well?
- b) What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F? Explain your answer using the pigeonhole principle. [5]
- c) How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 75 different people who have entered a contest? [5]

3. a) Let $G=(V,E)$ be a directed graph with v_1, v_2, \dots, v_n vertices and e_1, e_2, \dots, e_m edges. Then the incidence matrix with respect to this ordering of V and E is the $n \times m$ matrix $M=[A_{ij}]$, Where [10]

$$A_{ij} = \begin{cases} -1 & \text{if edge } j \text{ leaves vertex } i \\ 1 & \text{if edge } j \text{ enters vertex } i \\ 0 & \text{otherwise} \end{cases}$$

According to the above definition draw a directed graph from the following incidence matrix Fig. 3(a).

$$A = \begin{bmatrix} -1 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Fig. 3(a)

- b) Considering the following graphs find out if it contains the Euler Circuit, Euler Path or both. [10]

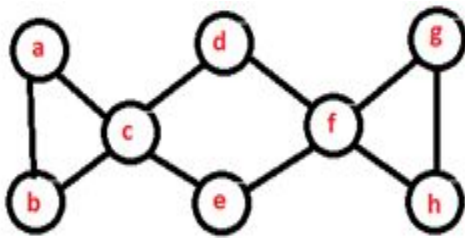


Fig. a

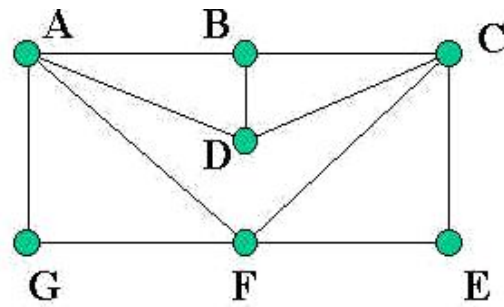


Fig. b

4. a) Suppose, Warner Bros. Pictures has decided to pick 3 superheroes for their next movie from Marvel and DC superheroes. They will consider Superman, Ironman, Batman and Wonder Woman. Suppose, You are a fan of both Batman and Superman. Find out the probability of picking Superman in the team where Batman is already in the team. [10]
- b) A box of chocolates contains 10 Dairy Milk, 8 Cadbury and 5 Toblerone. You have to pick any chocolate randomly 3(three) times such that - [10]
- What you pick you will keep it in the box before next pick
 - What you pick you will keep it in your pocket

What is the key difference between these two events? Find the probability of picking 3(three) Dairy Milks in both cases.

5. a) The number of students who attend a school could be divided among **10, 12, or 16** buses, such that each bus transports an equal number of students. What is the **minimum number of students** that could attend the school? [10]

- b) Determine whether the integers of the set $A = \{12, 17, 31, 37\}$ are **pairwise relatively prime or not**? Justify your answer. [5]

- c) Which memory locations are assigned by the hashing function $h(k) = k \bmod 101$ to the records of insurance company customers with the following social security numbers:

i) 372201919

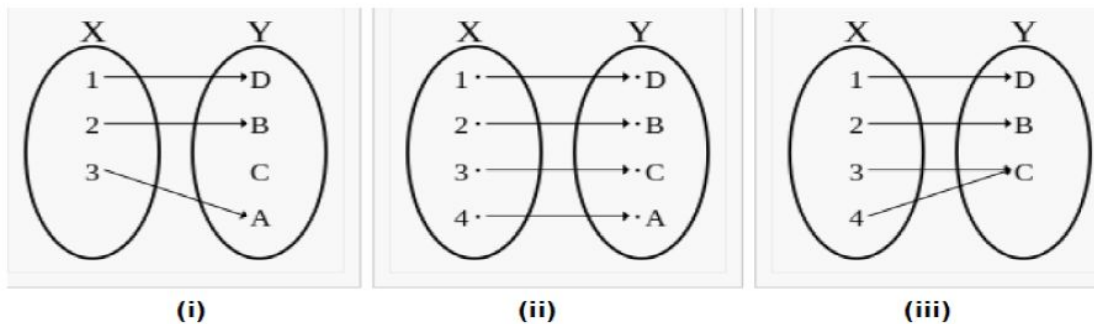
ii) 501338753

6. a) Find out if the following function is **one-to-one**? [5]

$$f(x) = x^2 \text{ where } f: R \rightarrow R$$

- b) From the following figures, determine if they are **one to one, onto or both**. [5]

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- c) 1000 students went to a school carnival. 30 had a hamburger, soft drink and ice-cream. 240 had hamburgers. 50 had a hamburger and a soft drink. 330 had soft drinks. 100 had a soft drink and ice-cream. 380 had ice-cream. 80 had a hamburger and ice-cream. **How many of them had nothing?** [10]