CSE 230 Sec-2 & Sec-7

Practice sheet on **Counting & Probability**

This practice sheet contains practice problems covering the following topics:

- Basic Rules of Counting
- Permutations & Combinations
- Binomial Theorems
- Discrete Probability
- Bayes' Theorem

Instructions:

- No soft copies allowed. **You have to submit hard copy of your work to me**. There will be a folder placed for submitting your work outside my room **UB50301.**

- **Deadline: Sunday, December 2nd at 5 PM**.

Question Codes: (**B)** - Beginner, (**I)** - Intermediate, (**A)** - Advanced

Practice Problems:

1. The chairs of your classroom are to be labeled with a letter and a positive integer not exceeding 150. What is the largest number of chairs that can be labeled differently? (B)

2. Someone have 4 pair of pants, 6 shirts, 8 pairs of socks, and 3 pairs of shoes. Ignoring the fact that some of the combinations may look ridiculous, in how many ways can he get dressed? (I)

3. Twelve people belong to a club. How many ways can they pick a president, vice president, secretary, and treasurer? (I)

4. Suppose we flip 5 coins. Compute the probability that we get 0, 1, or 2 heads. (I)

5. A student takes a test with 16 multiple-choice questions. Since he/she has never been to class he/she has to choose at random from the 4 possible answers. What is the probability that he/she will get exactly 3 right? (A)

6. What is the minimum number of students required in a discrete mathematics class to be sure that at least eight will receive the same grade, if there are five possible grades, A, B, C, D,E and F? (B)

7. Show that at any party there are two people who have the same number of friends at the party (assume that all friendships are mutual). (I)

8. Calculate the number of signals that can be sent by 6 flags of different colours taking one or more at a time. (I)

9. What is the coefficient of x^{101} y^{99} in the expansion of $(2x - 3y)^{200}$? (B)

10. In how many different ways can the letters A, A, B, B, B, C, D, E be arranged if the letter C must be to the right of the letter D? (I)

11. How many distinct four-digit numbers can be formed by the digits 1, 2, 3, 4, 5, 5, 6, 6? (A)

12. What is the probability that a positive integer selected at random from the set of positive integers not exceeding 100 is divisible by either 3 or 7? (B)

13. Suppose **E** is the event that a randomly generated bit string of length four begins with a 1 and **F** is the event that this bit string contains an odd number of 1 s. Are E and F independent, if the 16 bit strings of length four are equally likely? (B)

14. In Dhaka, it's rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability 1/2, and given that it is not rainy, there will be heavy traffic with probability 1/4. If it's rainy and there is heavy traffic, I arrive late for work with probability 1/2. On the other hand, the probability of being late is reduced to 1/8 if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 1/4. You pick a random day.

- **a.** What is the probability that it's not raining and there is heavy traffic and I am not late? (I)
- **b.** What is the probability that I am late? (I)
- **c.** Given that I arrived late at work, what is the probability that it rained that day? (A)

15. A diagnostic test has a probability 0.95 of giving a positive result when applied to a person suffering from a certain disease, and a probability0.10 of giving a (false) positive when applied to a non-sufferer. It is estimated that 0.5% of the population are sufferers. Suppose that the test is now administered to a person about whom we have no relevant information relating to the disease (apart from the fact that he/she comes from this population). Calculate the following probabilities:

- **a.** The test result will be positive; (B)
- **b.** Given a positive result, the person is a sufferer; (I)
- **c.** Given a negative result, the person is a non-sufferer; (I)
- **d.** The person will be misclassified. (A)