BRAC University
Department of CSE
Semester Final Exam : Summer 2018
CSE230 : Discrete Mathematics
Time: 2 hours
Total Marks: 100

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You have to answer **any 10** out of the following 12 questions. Answers without any supporting work will not be given credit. You have a maximum of 2 hours to complete the exam.

Name: ID: Section:

1. Four people, A, B, C and D, come to an old bridge in the middle of the night. The bridge is rickety and can only support 2 people at a time. The people have only one flashlight, which needs to be held by any group crossing the bridge because of how dark it is. Moreover, each person crosses the bridge at a different speed:

   - A takes 1 minute
   - B takes 2 minutes
   - C takes 5 minutes
   - D takes 10 minutes

   If two people are crossing the bridge together, it will take both of them the time that it takes the slower person to cross.
   Unfortunately, there are only **17 minutes** worth of batteries left in the flashlight. How can the four travellers cross the bridge before time runs out?
   Hint: You want the two slower people to be crossing together.   [10]

2. (a) Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.
   Hint: You can prove the statement either using truth table or by developing a series of logical equivalences.   [5]
   (b) Suppose, a is an integer. Prove that, if $a^2 - 2a + 7$ is even, then a is odd.   [5]

3. (a) In a certain Residential Semester at BRACU, a total of 50 students are enrolled who are advised to exercise regularly. On the first day, the students choose the exercise suitable for them:

   - 25 decided to jog
   - 20 decided to play basketball
   - 15 decided to swim
   - 10 decided to play basketball and jog
   - 5 decided to play basketball and swim
   - 7 decided to jog and swim
   - 2 extremely athletic students decided to do all three of the exercises

   How many students decided not to do any of these activities?   [5]
   (b) For the given sets $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5, 6\}$ and $C = \{5, 6, 7, 8\}$, verify that $A \cup (B \cup C) = (A \cup B) \cup C$. After that, verify your proof using Venn Diagram.   [5]

4. (a) Let, $f(x) = x^2 - 3x + 5$ be a function where, $f : A \to B$. Given that, $A = \{x : x \in \mathbb{N} \text{ and } x < 10\}$ and $B = \{y : y \in \mathbb{R} \text{ and } 0 < y < 60\}$. Check whether the function f is onto or one-to-one or both. Here, $\mathbb{N}$ stands for all natural numbers and $\mathbb{R}$ for all real numbers. **[5]**

   (b) What is the value of $\sum_{k=4}^{8} (-1)^k$? Assume that, the usual laws for arithmetic apply to this summation. **[5]**

5. In Manitoba, the format of the license plates follow a specific pattern: XXY - NNN.

   - Here, X denotes an uppercase letter that can take any values from A to Z.
   - N denotes a digit that can take any of the values 2 through 9
   - Y denotes EITHER a digit that can take the values 0 or 1 OR an uppercase letter that can take any values from A to Z.

   [10]

6. (a) In a Discrete Mathematics course, students are evaluated using a total of nine possible letter grades: $A, A-, B, B-, C, C-, D, D-$ and F. What is the minimum number of students required to be sure that at least six will receive the same grade? **[5]**

   (b) How many binary bit strings of length 10 neither starts with 1 nor ends with 00 ? **[5]**

7. BRACU Admission Test sample question paper consists of ten (10) questions, which are divided into two parts A and B. Usually, the question setter proceeds with the following structure:

   - Part A contains four (4) questions
   - Part B contains of six (6) questions

   A candidate is required to attempt eight (8) questions out of those ten (10), maintaining the following criteria:

   - At least 3 should be answered from Part A
   - At least 3 should be answered from Part B.

   Find out in how many ways the candidate can select the questions if he can answer all questions equally well. **[10]**

8. Assuming all the variables are non negative integers, find the co-efficient of x^{47} from the expansion of $\left(\frac{2x}{3} + x^2\right)^{27}$
   Hint: You can use the Binomial Theorem to expand the expression. **[10]**

9. (a) A couple has three children, the older being a boy. What is the probability that they have at least one girl? **[5]**

   (b) A couple has three children, one of which is a boy. What is the probability that they have two girls? **[5]**

10. In Bangladesh, 51% of the adults are males. One adult is randomly selected for a survey involving health insurance.

    (a) Find the probability that the selected person is a male. **[2]**

    (b) It is later found that the selected person has a Ph.D degree. Also, 9.5% of the males have a Ph.D degree, whereas 1.7% of the females have a Ph.D degree (based on data from the Ministry of Education). Use this additional information to find the probability that the selected person is a male given that the person had a Ph.D degree. The data table from the Ministry of Education is given below: **[8]**

	Ph.D Holder (D)	Not a Ph.D Holder ($\overline{D}$)	Total
Male (M)	4845	46155	**51000**
Female ($\overline{M}$)	833	48167	**49000**
Total	**5678**	**94322**	**100000**

11. The following figure is a graphical representation of a directed graph.
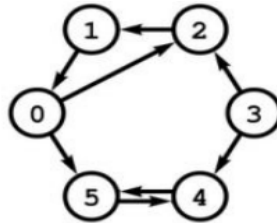
Represent the graph using -

(a) Adjacency Matrix [**3**]

(b) Adjacency List [**3**]

(c) Incidence Matrix [**4**]

12. The following is a graphical representation of an undirected graph G. Find out whether G has the following. If the answer is yes, mention the respective path or circuit in the answer script.

(a) an Euler path [**5**]

(b) an Euler circuit [**5**]

The End.