Homework 01
CSE 230 : Discrete Mathematics

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Problem A. Let p, q, and r be the propositions
p : Grizzly bears have been seen in the area.
q : Hiking is safe on the trail.
r : Berries are ripe along the trail.
Write these propositions using p, q, and r and logical connectives.

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

e) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.

f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

Problem B. Write each ofthese statements in the form "if p, then q" in English.

a) I will remember to send you the address only if you send me an e-mail message. .

b) To be a citizen ofthis country, it is sufficient that you were born in the United States.

c) If you keep your textbook, it will be a useful reference in your future courses.

d) The Red Wings will win the Stanley Cup iftheir goalie plays well.

e) That you get the job implies that you had the best credentials.

f) The beach erodes whenever there is a storm.

g) It is necessary to have a valid password to log on to the server.

h) You will reach the summit unless you begin your climb too late.

Problem C. State the converse, contrapositive, and inverse of each of these conditional statements and write using propositional variables with logical connectives.

a) If it snows tonight, then I will stay at home.

b) I go to the beach whenever it is a sunny summer day.

c) When I stay up late, it is necessary that I sleep until noon.

Problem D. Construct truth tables for each of these compound propositions.

a) $(p \vee q) \rightarrow (p \wedge q)$

b) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

c) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

d) $(p \vee q) \rightarrow (p \oplus q)$

e) $(p \oplus q) \leftrightarrow (p \rightarrow q)$

Problem E. Find out whether the following compound propositions are tautology or contradiction or contingency.

a) $\neg p \rightarrow (p \rightarrow q)$

b) $(p \wedge q) \rightarrow (p \rightarrow q)$

c) $(\neg p \wedge (p \vee q)) \rightarrow q$

d) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow q)$

e) $((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r$

Problem F. a) Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are equivalent.

b) Show that $\neg(p \oplus q)$ and $(p \leftrightarrow q)$ are logically equivalent.

c) Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not equivalent.

d) Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.

e) Show that $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent.

Problem G. a) Let P(x) denote the statement "$x \leq 4$" What are the truth values? - P(0) , P(4), P(6)

b) Let P(x) be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English. - i) $\exists x P(x)$, ii) $\forall x P(x)$, iii) $\exists x \neg P(x)$, iv) $\forall x \neg P(x)$

c) Translate these statements into English, where C(x) is "x is a comedian" and F(x) is "x is funny" and the domain consists of all people. - i) $\forall x(C(x) \rightarrow F(x))$, ii) $\forall x(C(x) \wedge F(x))$, iii) $\exists x(C(x) \rightarrow F(x))$, iv) $\exists x(C(x) \wedge F(x))$

Problem H. Let C(x) be the statement "x has a cat," let D(x) be the statement "x has a dog," and let F(x) be the statement "x has a ferret." Express each of these statements in terms ofC(x), D(x), F(x), quantifiers, and logical connectives. Let the domain consist of all students in your class.

a) A student in your class has a cat, a dog, and a ferret.

b) All students in your class have a cat, a dog, or a ferret.

c) Some student in your class has a cat and a ferret, but not a dog.

d) No student in your class has a cat, a dog, and a ferret.

e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has one of these animals as a pet.

Problem I. Let Q(x) be the statement "$x + 1 > 2x$" If the domain consists of all integers, what are these truth values? a) $Q(O)$, b) $Q(-1)$, e) $Q(l)$, d) $\exists x Q(x)$, e) $\forall x Q(x)$, f) $\exists x \neg Q(x)$, g) $\forall x \neg Q(x)$