

Principles of Data Mining

Ensemble Techniques

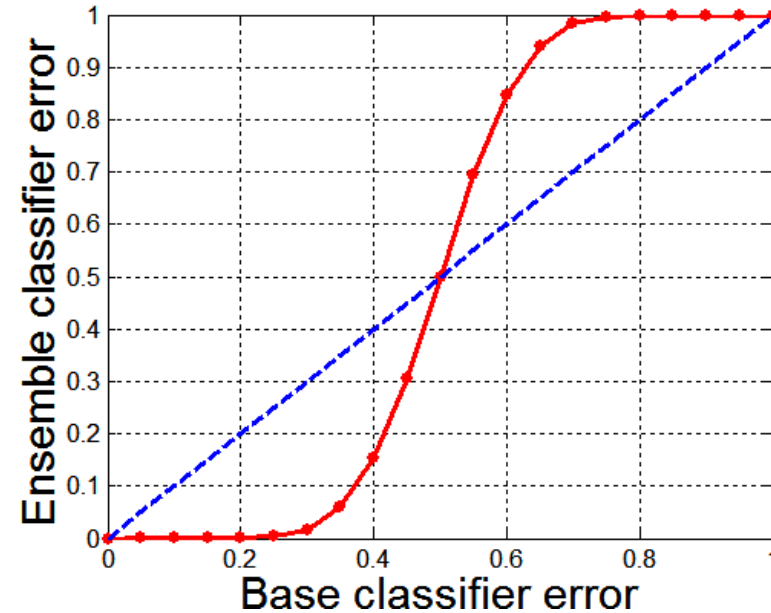
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Ensemble Methods

- Construct a set of classifiers from the training data
- Predict class label of test records by combining the predictions made by multiple classifiers

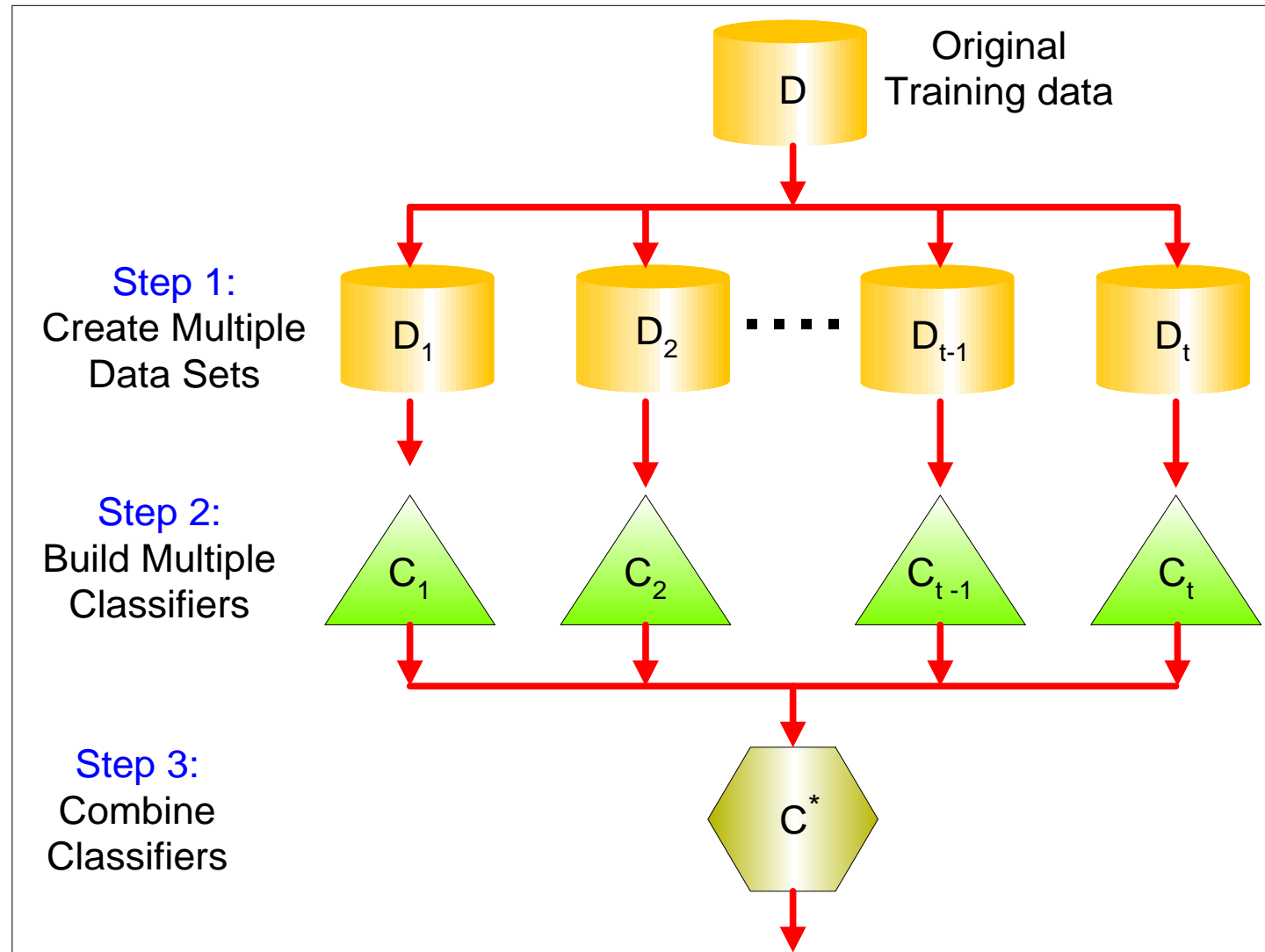
Why Ensemble Methods work?

- Suppose there are 25 base classifiers
 - Each classifier has error rate, $\varepsilon = 0.35$
 - Assume errors made by classifiers are uncorrelated
 - Probability that the ensemble classifier makes a wrong prediction:



$$P(X \geq 13) = \sum_{i=13}^{25} \binom{25}{i} \varepsilon^i (1 - \varepsilon)^{25-i} = 0.06$$

General Approach



Types of Ensemble Methods

- Manipulate data distribution
 - Example: bagging, boosting
- Manipulate input features
 - Example: random forests
- Manipulate class labels
 - Example: error-correcting output coding

Bagging

- Sampling with replacement

Original Data	1	2	3	4	5	6	7	8	9	10
Bagging (Round 1)	7	8	10	8	2	5	10	10	5	9
Bagging (Round 2)	1	4	9	1	2	3	2	7	3	2
Bagging (Round 3)	1	8	5	10	5	5	9	6	3	7

- Build classifier on each bootstrap sample
- Each data instance has probability $1 - (1 - 1/n)^n$ of being selected as part of the bootstrap sample

Bagging Algorithm

Algorithm 5.6 Bagging Algorithm

- 1: Let k be the number of bootstrap samples.
 - 2: for $i = 1$ to k do
 - 3: Create a bootstrap sample of size n , D_i .
 - 4: Train a base classifier C_i on the bootstrap sample D_i .
 - 5: end for
 - 6: $C^*(x) = \arg \max_y \sum_i \delta(C_i(x) = y)$, $\{\delta(\cdot) = 1$ if its argument is true, and 0 otherwise. $\}$
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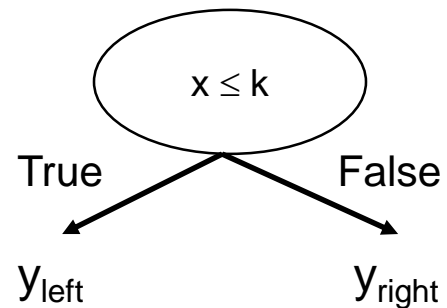
Bagging Example

- Consider 1-dimensional data set:

Original Data:

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
y	1	1	1	-1	-1	-1	-1	1	1	1

- Classifier is a decision stump
 - Decision rule: $x \leq k$ versus $x > k$
 - Split point k is chosen based on entropy



Bagging Example

Bagging Round 1:

x	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.6	0.9	0.9
y	1	1	1	1	-1	-1	-1	-1	1	1

$x \leq 0.35 \rightarrow y = 1$

$x > 0.35 \rightarrow y = -1$

Bagging Example

Bagging Round 1:

x	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.6	0.9	0.9
y	1	1	1	1	-1	-1	-1	-1	1	1

$x \leq 0.35 \rightarrow y = 1$
 $x > 0.35 \rightarrow y = -1$

Bagging Round 2:

x	0.1	0.2	0.3	0.4	0.5	0.5	0.9	1	1	1
y	1	1	1	-1	-1	-1	1	1	1	1

$x \leq 0.7 \rightarrow y = 1$
 $x > 0.7 \rightarrow y = 1$

Bagging Round 3:

x	0.1	0.2	0.3	0.4	0.4	0.5	0.7	0.7	0.8	0.9
y	1	1	1	-1	-1	-1	-1	-1	1	1

$x \leq 0.35 \rightarrow y = 1$
 $x > 0.35 \rightarrow y = -1$

Bagging Round 4:

x	0.1	0.1	0.2	0.4	0.4	0.5	0.5	0.7	0.8	0.9
y	1	1	1	-1	-1	-1	-1	-1	1	1

$x \leq 0.3 \rightarrow y = 1$
 $x > 0.3 \rightarrow y = -1$

Bagging Round 5:

x	0.1	0.1	0.2	0.5	0.6	0.6	0.6	1	1	1
y	1	1	1	-1	-1	-1	-1	1	1	1

$x \leq 0.35 \rightarrow y = 1$
 $x > 0.35 \rightarrow y = -1$

Bagging Example

Bagging Round 6:

x	0.2	0.4	0.5	0.6	0.7	0.7	0.7	0.8	0.9	1
y	1	-1	-1	-1	-1	-1	-1	1	1	1

$x \leq 0.75 \rightarrow y = -1$
 $x > 0.75 \rightarrow y = 1$

Bagging Round 7:

x	0.1	0.4	0.4	0.6	0.7	0.8	0.9	0.9	0.9	1
y	1	-1	-1	-1	-1	1	1	1	1	1

$x \leq 0.75 \rightarrow y = -1$
 $x > 0.75 \rightarrow y = 1$

Bagging Round 8:

x	0.1	0.2	0.5	0.5	0.5	0.7	0.7	0.8	0.9	1
y	1	1	-1	-1	-1	-1	-1	1	1	1

$x \leq 0.75 \rightarrow y = -1$
 $x > 0.75 \rightarrow y = 1$

Bagging Round 9:

x	0.1	0.3	0.4	0.4	0.6	0.7	0.7	0.8	1	1
y	1	1	-1	-1	-1	-1	-1	1	1	1

$x \leq 0.75 \rightarrow y = -1$
 $x > 0.75 \rightarrow y = 1$

Bagging Round 10:

x	0.1	0.1	0.1	0.1	0.3	0.3	0.8	0.8	0.9	0.9
y	1	1	1	1	1	1	1	1	1	1

$x \leq 0.05 \rightarrow y = 1$
 $x > 0.05 \rightarrow y = 1$

Bagging Example

- Summary of Training sets:

Round	Split Point	Left Class	Right Class
1	0.35	1	-1
2	0.7	1	1
3	0.35	1	-1
4	0.3	1	-1
5	0.35	1	-1
6	0.75	-1	1
7	0.75	-1	1
8	0.75	-1	1
9	0.75	-1	1
10	0.05	1	1

Bagging Example

- Assume test set is the same as the original data
- Use majority vote to determine class of ensemble classifier

Round	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1.0
1	1	1	1	-1	-1	-1	-1	-1	-1	-1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	-1	-1	-1	-1	-1	-1	-1
4	1	1	1	-1	-1	-1	-1	-1	-1	-1
5	1	1	1	-1	-1	-1	-1	-1	-1	-1
6	-1	-1	-1	-1	-1	-1	-1	1	1	1
7	-1	-1	-1	-1	-1	-1	-1	1	1	1
8	-1	-1	-1	-1	-1	-1	-1	1	1	1
9	-1	-1	-1	-1	-1	-1	-1	1	1	1
10	1	1	1	1	1	1	1	1	1	1
Sum	2	2	2	-6	-6	-6	-6	2	2	2
Sign	1	1	1	-1	-1	-1	-1	1	1	1

Predicted
Class

Boosting

- An iterative procedure to adaptively change distribution of training data by focusing more on previously misclassified records
 - Initially, all N records are assigned equal weights
 - Unlike bagging, weights may change at the end of each boosting round

Boosting

- Records that are wrongly classified will have their weights increased
- Records that are classified correctly will have their weights decreased

Original Data	1	2	3	4	5	6	7	8	9	10
Boosting (Round 1)	7	3	2	8	7	9	4	10	6	3
Boosting (Round 2)	5	4	9	4	2	5	1	7	4	2
Boosting (Round 3)	4	4	8	10	4	5	4	6	3	4

- Example 4 is hard to classify
- Its weight is increased, therefore it is more likely to be chosen again in subsequent rounds

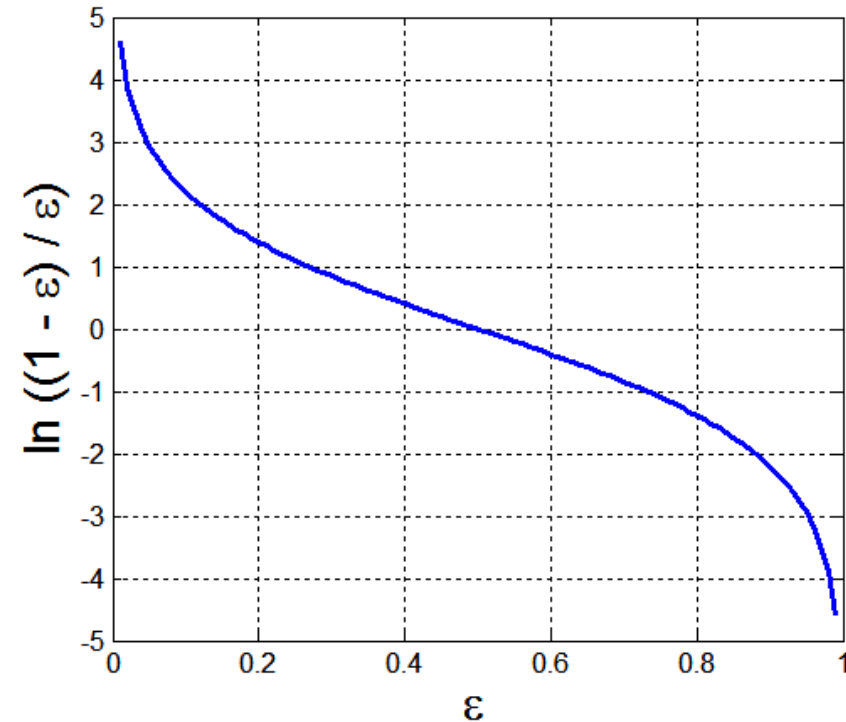
AdaBoost

- Base classifiers: C_1, C_2, \dots, C_T
- Error rate:

$$\varepsilon_i = \frac{1}{N} \sum_{j=1}^N w_j \delta(C_i(x_j) \neq y_j)$$

- Importance of a classifier:

$$\alpha_i = \frac{1}{2} \ln \left(\frac{1 - \varepsilon_i}{\varepsilon_i} \right)$$



AdaBoost Algorithm

- Weight update:

$$w_j^{(t+1)} = \frac{w_j^{(t)}}{Z_i} \begin{cases} \exp^{-\alpha_i} & \text{if } C_i(x_j) = y_j \\ \exp^{\alpha_i} & \text{if } C_i(x_j) \neq y_j \end{cases}$$

where Z_i is the normalization factor

- If any intermediate rounds produce error rate higher than 50%, the weights are reverted back to $1/n$ and the resampling procedure is repeated

- Classification:
$$C^*(x) = \operatorname{argmax}_y \sum_{i=1}^T \alpha_i \delta(C_i(x) = y)$$

AdaBoost Algorithm

Algorithm 5.7 AdaBoost Algorithm

- 1: $\mathbf{w} = \{w_j = 1/n \mid j = 1, 2, \dots, n\}$. {Initialize the weights for all n instances.}
 - 2: Let k be the number of boosting rounds.
 - 3: **for** $i = 1$ to k **do**
 - 4: Create training set D_i by sampling (with replacement) from D according to \mathbf{w} .
 - 5: Train a base classifier C_i on D_i .
 - 6: Apply C_i to all instances in the original training set, D .
 - 7: $\epsilon_i = \frac{1}{n} [\sum_j w_j \delta(C_i(x_j) \neq y_j)]$ {Calculate the weighted error}
 - 8: **if** $\epsilon_i > 0.5$ **then**
 - 9: $\mathbf{w} = \{w_j = 1/n \mid j = 1, 2, \dots, n\}$. {Reset the weights for all n instances.}
 - 10: Go back to Step 4.
 - 11: **end if**
 - 12: $\alpha_i = \frac{1}{2} \ln \frac{1-\epsilon_i}{\epsilon_i}$.
 - 13: Update the weight of each instance according to equation (5.88).
 - 14: **end for**
 - 15: $C^*(\mathbf{x}) = \arg \max_y \sum_{j=1}^T \alpha_j \delta(C_j(\mathbf{x}) = y)$.
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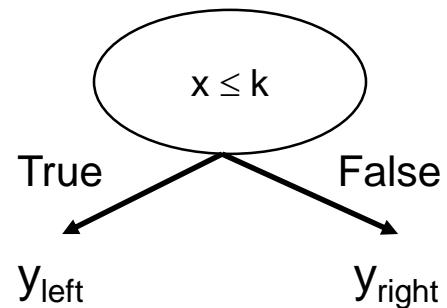
AdaBoost Example

- Consider 1-dimensional data set:

Original Data:

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
y	1	1	1	-1	-1	-1	-1	1	1	1

- Classifier is a decision stump
 - Decision rule: $x \leq k$ versus $x > k$
 - Split point k is chosen based on entropy



AdaBoost Example

- Training sets for the first 3 boosting rounds:

Boosting Round 1:

x	0.1	0.4	0.5	0.6	0.6	0.7	0.7	0.7	0.8	1
y	1	-1	-1	-1	-1	-1	-1	-1	1	1

Boosting Round 2:

x	0.1	0.1	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3
y	1	1	1	1	1	1	1	1	1	1

Boosting Round 3:

x	0.2	0.2	0.4	0.4	0.4	0.4	0.5	0.6	0.6	0.7
y	1	1	-1	-1	-1	-1	-1	-1	-1	-1

- Summary:

Round	Split Point	Left Class	Right Class	alpha
1	0.75	-1	1	1.738
2	0.05	1	1	2.7784
3	0.3	1	-1	4.1195

AdaBoost Example

- Weights

Round	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1.0
1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
2	0.311	0.311	0.311	0.01	0.01	0.01	0.01	0.01	0.01	0.01
3	0.029	0.029	0.029	0.228	0.228	0.228	0.228	0.009	0.009	0.009

- Classification

Round	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1.0
1	-1	-1	-1	-1	-1	-1	-1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	-1	-1	-1	-1	-1	-1	-1
Sum	5.16	5.16	5.16	-3.08	-3.08	-3.08	-3.08	0.397	0.397	0.397
Sign	1	1	1	-1	-1	-1	-1	1	1	1

Predicted
Class