

# GNN Explainability

Shichang Zhang

11.02.2021

# Roadmap

- Model Explainability
  - Motivating Examples for Images and Tabular Data
- GNN Explainability (Graph Data Explainability)
  - Graphs vs. Images vs. Tabular Data
  - SubgraphX (ICML 2021)

# Model Explainability

- Goal: understand black-box models, e.g. NNs.
- Existing approaches
  - Instance-level
    - Example-specific understanding, why an input data is mapped to a certain output
  - Model-level
    - High-level generic understanding, how the model mechanism leads to a certain output

# Motivating Examples: Images

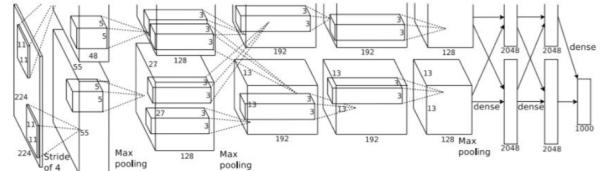
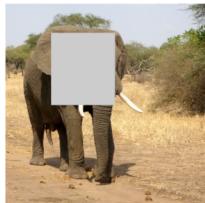
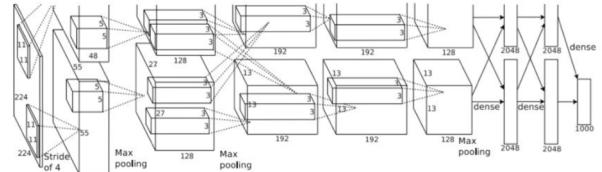
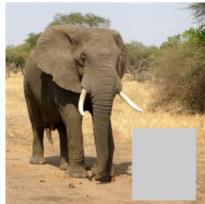
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Which pixels matter:  
Saliency via Occlusion

Mask part of the image before feeding to CNN,  
check how much predicted probabilities change

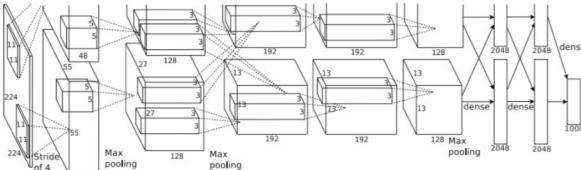
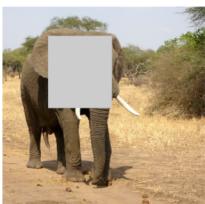
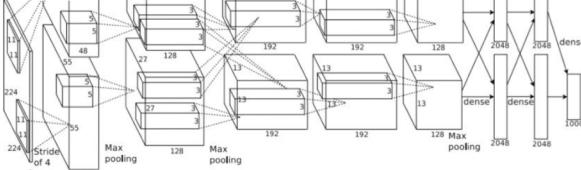


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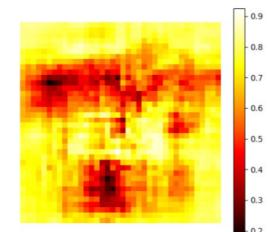
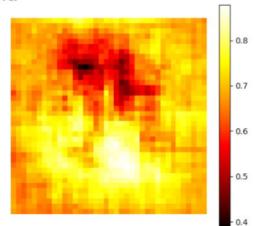
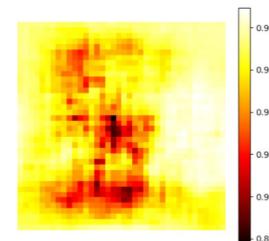
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## Which pixels matter: Saliency via Occlusion

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Zeiler and Fergus, "Visualizing and Understanding Convolutional Networks", ECCV 2014



Boat image is CC0 public domain  
Elephant image is CC0 public domain  
Go-Karts image is CC0 public domain

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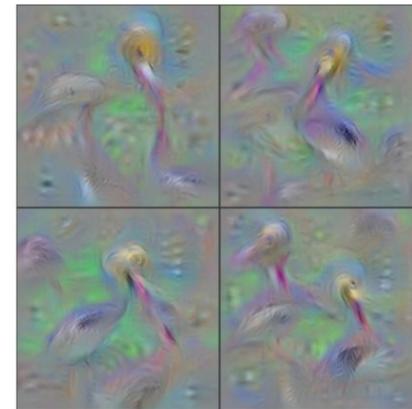
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Random Initialization



Flamingo



Pelican

Synthesized

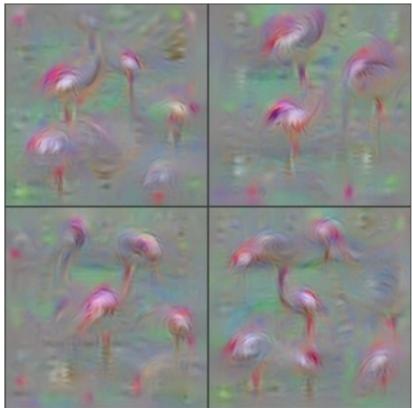
Yosinski et al, "Understanding Neural Networks Through Deep Visualization", ICML DL Workshop 2014.  
Figure copyright Jason Yosinski, Jeff Clune, Anh Nguyen, Thomas Fuchs, and Hod Lipson, 2014.

Nguyen et al, "Multifaceted Feature Visualization: Uncovering the Different Types of Features Learned By Each Neuron in Deep Neural Networks", ICML Visualization for Deep Learning Workshop 2016. Figures copyright Anh Nguyen, Jason Yosinski, and Jeff Clune, 2016;

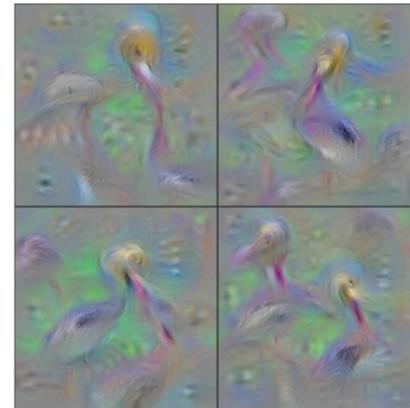
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Smart Initialization considering multimodality.  
The “grocery store” class



Synthesized



Ground Truth

Yosinski et al, "Understanding Neural Networks Through Deep Visualization", ICML DL Workshop 2014.  
Figure copyright Jason Yosinski, Jeff Clune, Anh Nguyen, Thomas Fuchs, and Hod Lipson, 2014.

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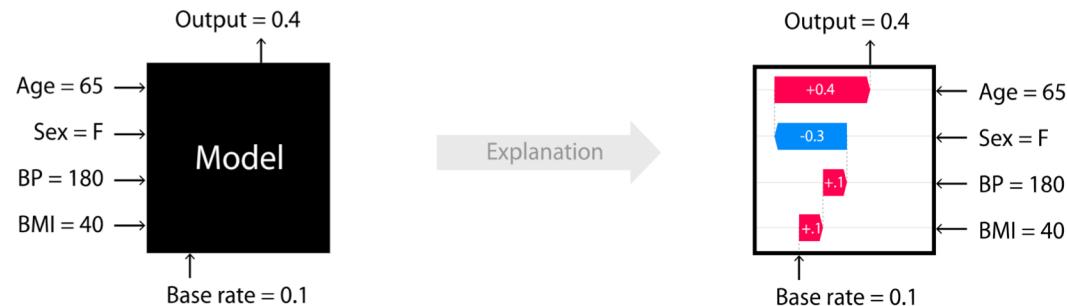
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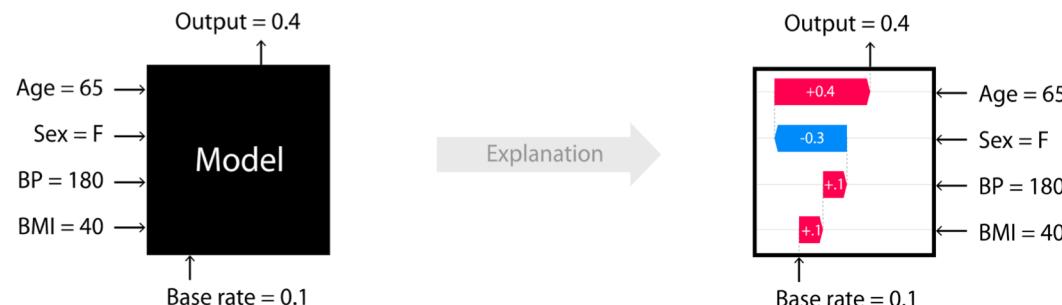
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- Model-level
  - Ex. a simple linear model



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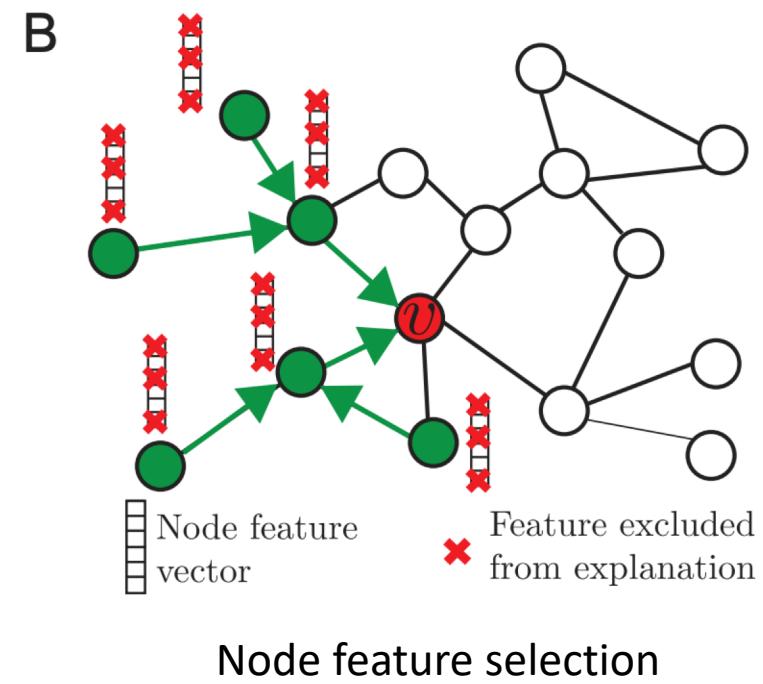
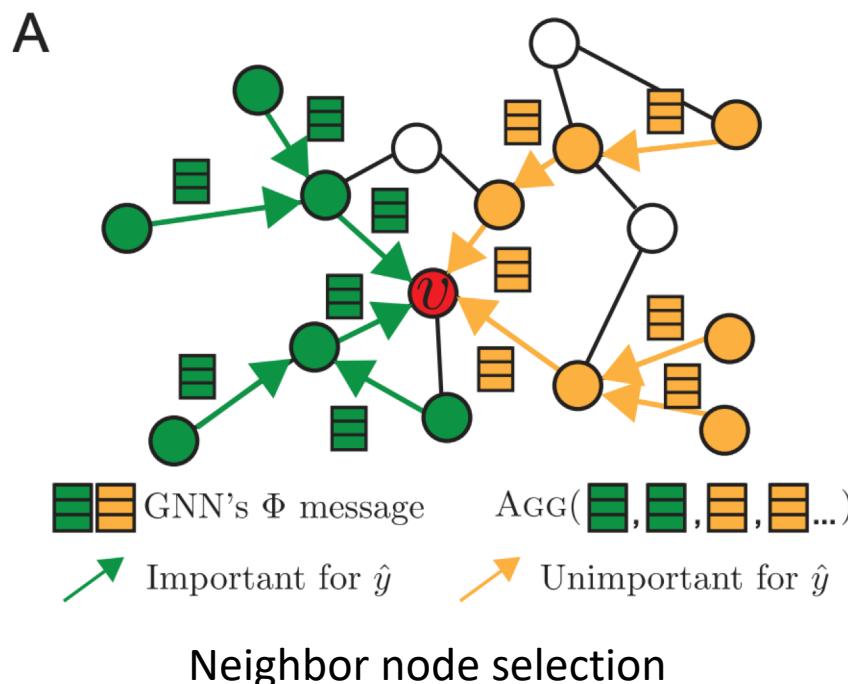
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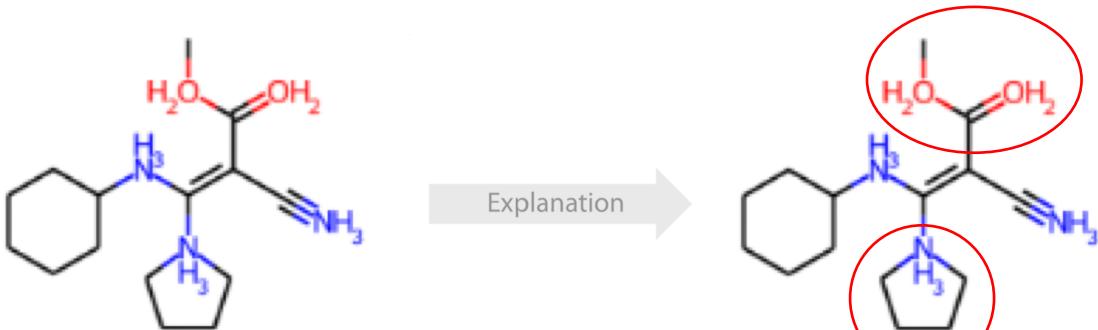


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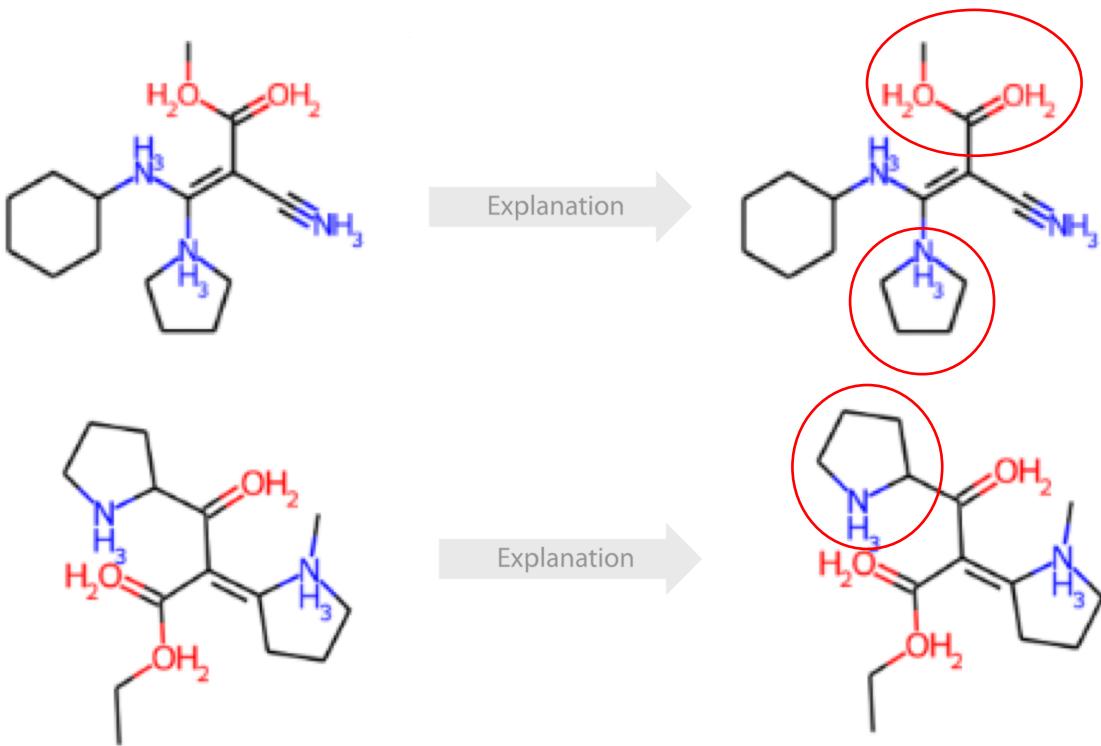
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Important subgraph selection

Model-level  
(data-level)



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- Explanation of graphs is more meaningful than images
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  - Graph explanation may reveal useful knowledge
    - Data-level graph classification explanation is similar to frequent pattern mining.
- Explanation of graphs is more challenging than tabular data
  - Graphs as tabular data with structure information

# SubgraphX (ICML21)

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## **On Explainability of Graph Neural Networks via Subgraph Explorations**

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**Hao Yuan<sup>1</sup> Haiyang Yu<sup>1</sup> Jie Wang<sup>2</sup> Kang Li<sup>3</sup> Shuiwang Ji<sup>1</sup>**

- Instance level
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# Problem Formulation

- Goal: Identify the most important subgraph for classifying a graph

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$f(\cdot)$  GNN to be explained

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$\mathcal{G}^*$  The most important subgraph

- Objective

$$\mathcal{G}^* = \operatorname*{argmax}_{|\mathcal{G}_i| \leq N_{\min}} \text{Score}(f(\cdot), \mathcal{G}, \mathcal{G}_i)$$

# Challenges

- There are too many subgraphs. How can we explore them?
- What is a reasonable score function?

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- Variables needed for the MCTS algorithm
  - $C(\mathcal{N}_i, a_j)$  denotes the number of counts for selecting action  $a_j$  for node  $\mathcal{N}_i$ .
  - $W(\mathcal{N}_i, a_j)$  is the total reward for all  $(\mathcal{N}_i, a_j)$  visits.
  - $Q(\mathcal{N}_i, a_j) = W(\mathcal{N}_i, a_j)/C(\mathcal{N}_i, a_j)$  and denotes the averaged reward for multiple visits.
  - $R(\mathcal{N}_i, a_j)$  is the immediate reward for selecting  $a_j$  on  $\mathcal{N}_i$ ,  
$$R(\mathcal{N}_i, a_j) = \text{Score}(f(\cdot), \mathcal{G}, (\mathcal{N}_i, a_j))$$

# Subgraph Exploration via Monte Carlo Tree Search (MCTS)

- Each MCTS iteration selects a path to a leaf node  $\mathcal{G}_\ell$

$$a^* = \underset{a_j}{\operatorname{argmax}} Q(\mathcal{N}_i, a_j) + U(\mathcal{N}_i, a_j),$$

$$U(\mathcal{N}_i, a_j) = \lambda R(\mathcal{N}_i, a_j) \frac{\sqrt{\sum_k C(\mathcal{N}_i, a_k)}}{1 + C(\mathcal{N}_i, a_j)},$$

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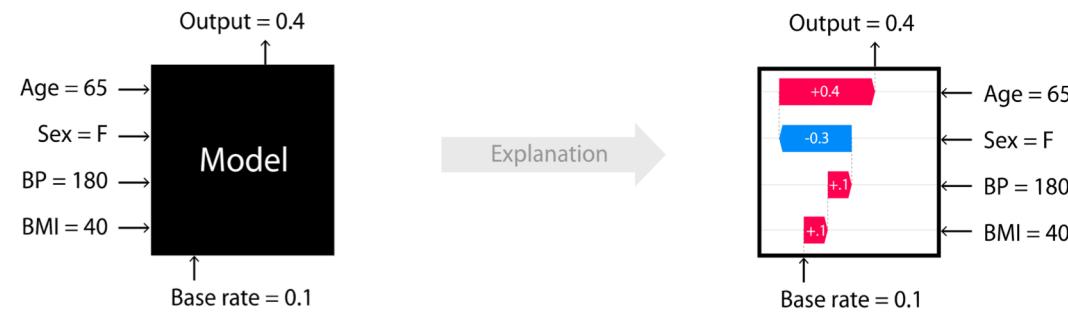
$$W(\mathcal{N}_i, a_j) = W(\mathcal{N}_i, a_j) + \text{Score}(f(\cdot), \mathcal{G}, \mathcal{G}_\ell).$$

- Finally, select the subgraph with the highest reward from the leaf level

# Score Functions

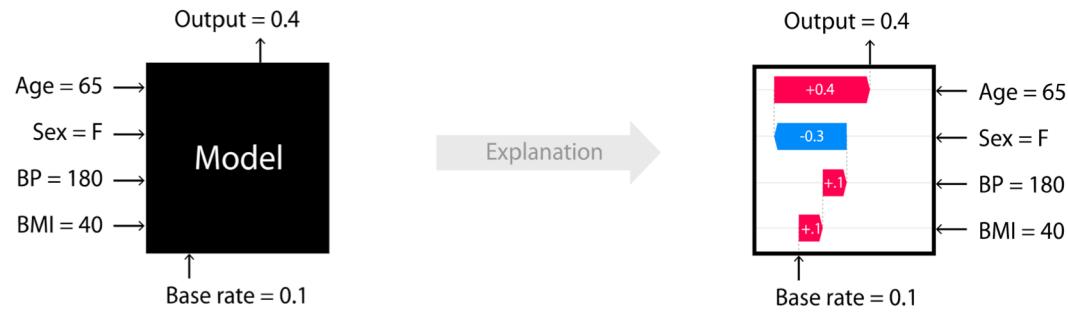
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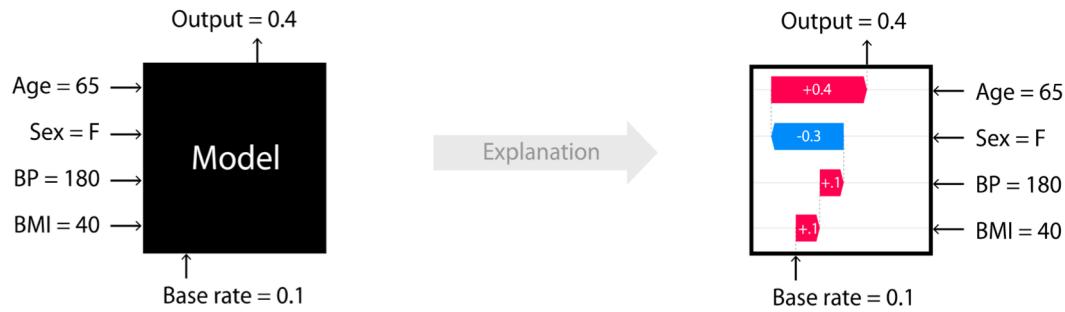


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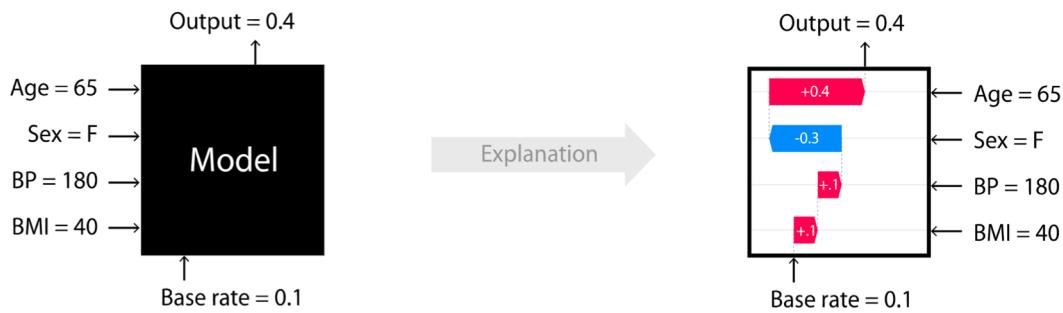
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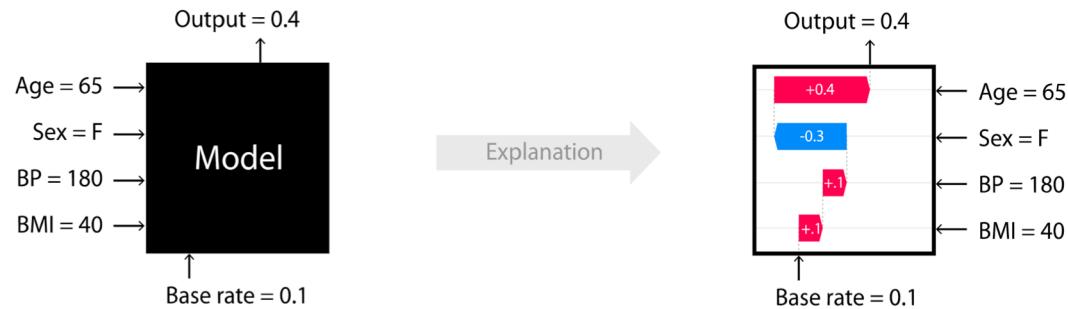
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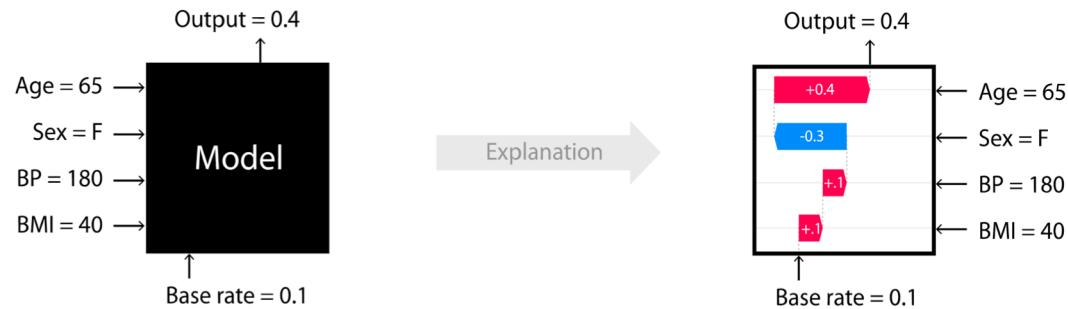
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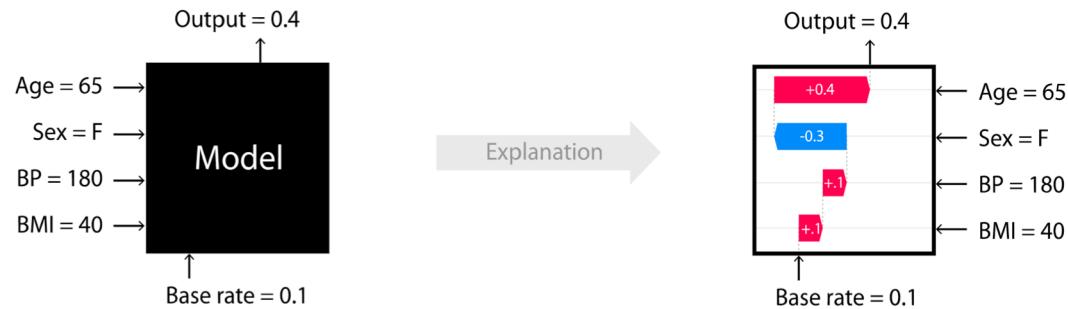
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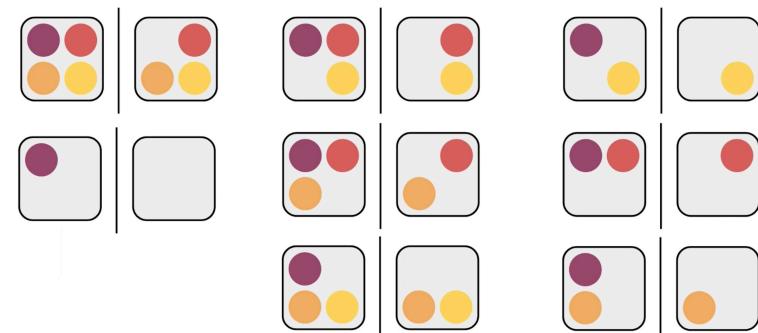


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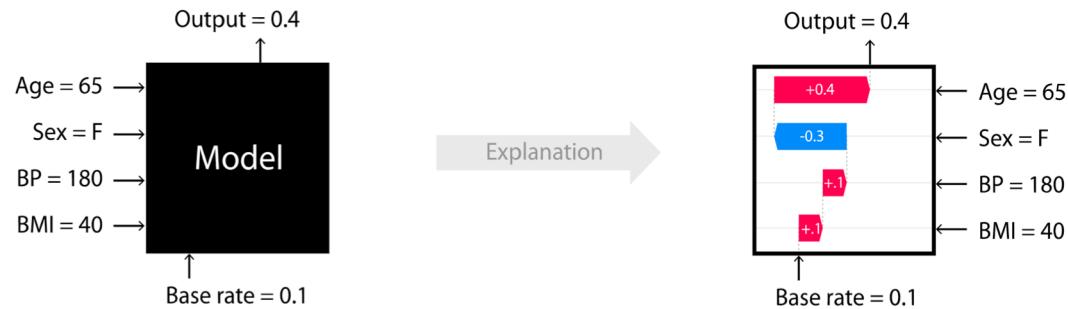
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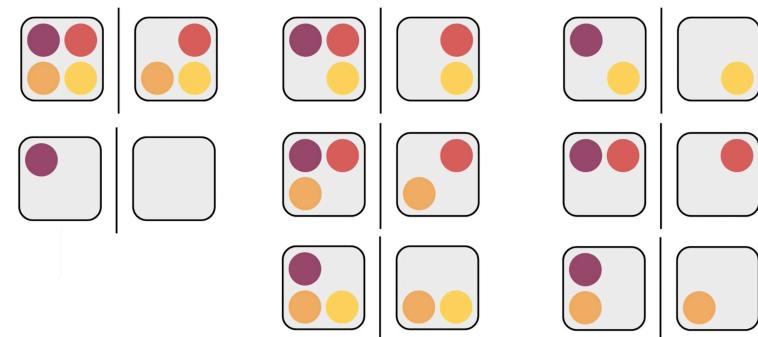


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  - Shapley value:  $I_\nu(f) = \frac{1}{|F|} \sum_{S \subseteq F \setminus \{f\}} \frac{1}{\binom{|F|-1}{|S|}} \Delta(f, S, \nu)$ 
$$\Delta(f, S, \nu) = \nu(S \cup \{f\}) - \nu(S)$$

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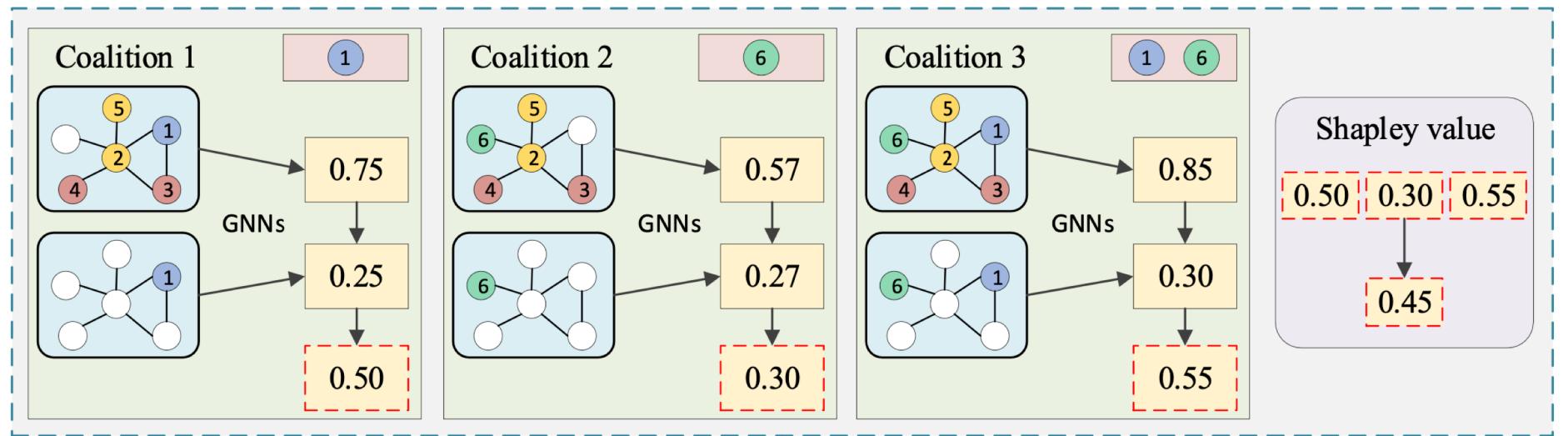
$$\nu = \text{GNN}$$

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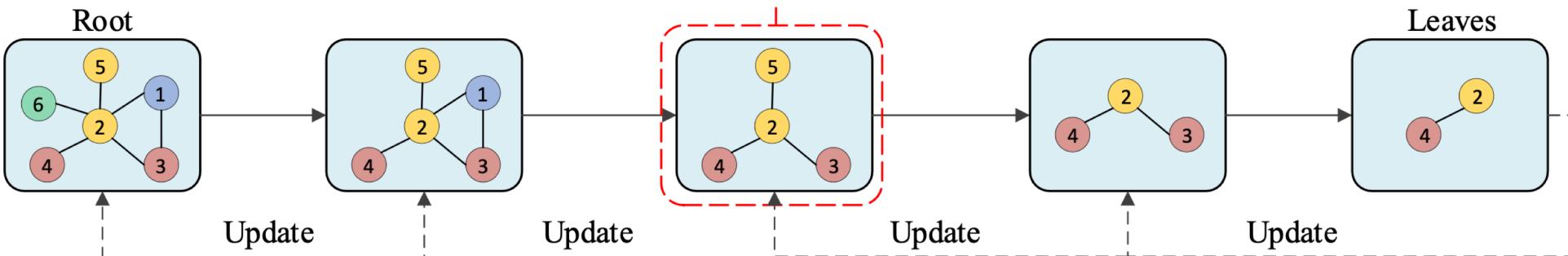
$$\Delta(f, S, \nu) = \nu(\bar{S \cup \{f\}}) - \nu(\bar{S})$$

# SubgraphX Framework

Shapley Value Scoring

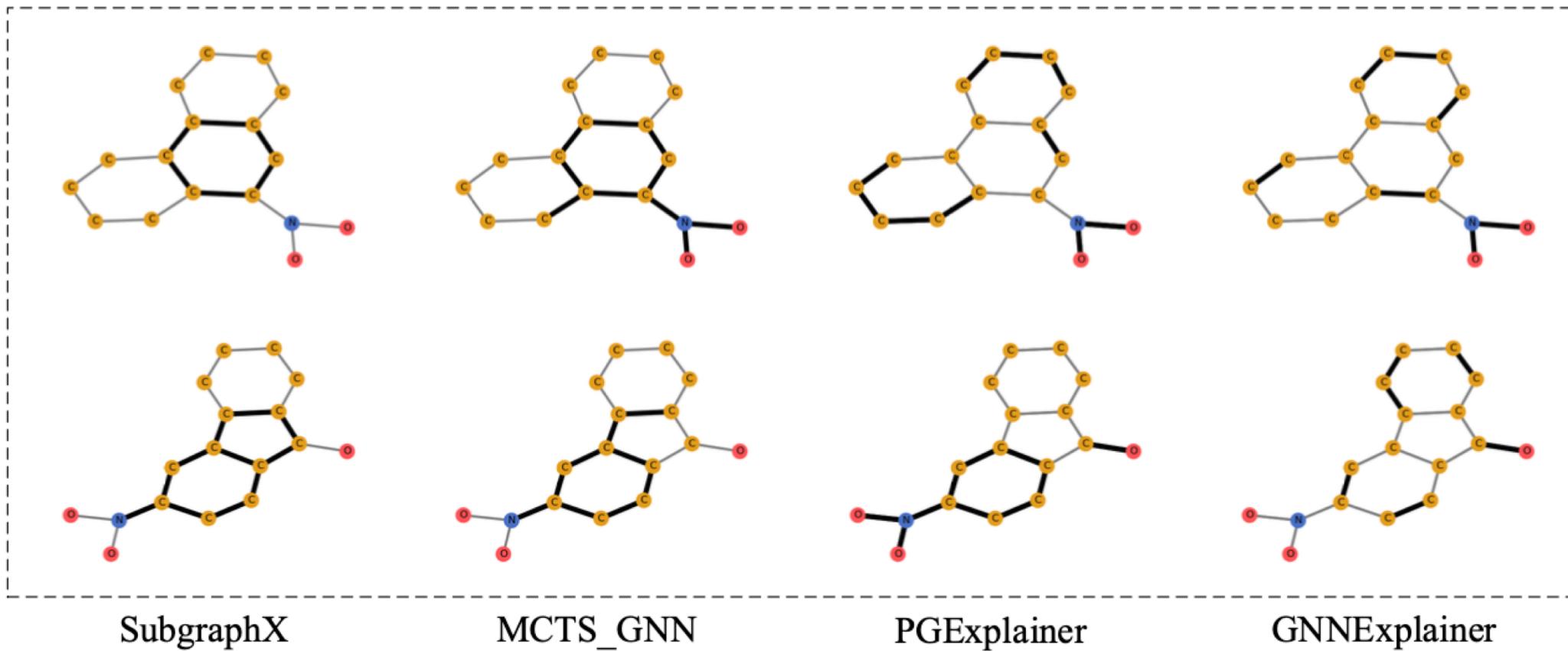


MCTS



# Result Visualization

- MUTAG dataset for molecule classification



# Reference

- SubgraphX: Yuan, H., Yu, H., Wang, J., Li, K., & Ji, S. (2021). On explainability of graph neural networks via subgraph explorations:  
<https://arxiv.org/pdf/2102.05152.pdf>
- Shapley value:  
<https://proceedings.neurips.cc/paper/2017/file/8a20a8621978632d76c43dfd28b67767-Paper.pdf>

# Appendix