Standardized Incidence Ratio and Confidence Limits

Various authors discuss the standardized mortality ratio (SMR) and provide exact and approximate confidence limits for the true SMR. These results are also applicable to the standardized incidence ratio (SIR). The following sections provide a brief outline of the results and give references to more detailed discussions.

Definition of the SIR

Suppose the person-time from the study group (i.e. cohort) is allocated among *M* cells defined by the cross-classification of various adjustment variables such as gender, race, attained age group, and attained calendar year group. Let t_k represent the person-time and D_k represent the observed events that the cohort subjects contribute to the *k*th cell, and let λ_k^* represent the standard rate for the *k*th cell, where *k* = 1, 2, ..., *M*. Given this notation, the SIR is defined as

$$\mathrm{SIR} = \frac{\sum_{k=1}^{M} D_k}{\sum_{k=1}^{M} t_k \lambda_k^*} = \frac{D}{E^*}$$

where the total number of events observed in the cohort is $D = \sum_{k=1}^{M} D_k$, and the total number of expected events is $E^* = \sum_{k=1}^{M} E_k^* = \sum_{k=1}^{M} t_k \lambda_k^*$ (Breslow and Day, 1987; Sahai and Khurshid, 1996).

Exact Confidence Limits for the True SIR

Sahai and Khurshid (1993, 1996) discuss the following method for obtaining the exact confidence limits, SIR_L and SIR_U, for the true SIR, ϕ. Assuming *D* is Poisson distributed with mean μ = *E(D)*, confidence limits for μ are obtained using the relationship between the Poisson distribution and the chi-square distribution. Then these limits are divided by the total number of expected events, *E**, to obtain the limits

$$\mathrm{SIR}_L = \frac{\chi^2_{2D,\alpha/2}}{2E^*} \quad \text{and} \quad \mathrm{SIR}_U = \frac{\chi^2_{2(D+1),1-\alpha/2}}{2E^*}$$

where $\chi^2_{v,\alpha}$ is the 100α percentile of the chi-square distribution with *v* degrees of freedom.

Approximate Confidence Limits for the True SIR

The exact limits, SIR_L and SIR_U, can be approximated by applying the Wilson and Hilferty (1931) approximation for chi-square percentiles, which is

$$\chi^2_{v,\alpha} = v\left(1 - \frac{2}{9v} + Z_\alpha\sqrt{\frac{2}{9v}}\right)^3.$$

The resulting approximate limits for the true SIR, ϕ, are

$$\mathrm{SIR}_{\hat{L}} = \frac{D}{E^*}\left(1 - \frac{1}{9D} + \frac{Z_{\alpha/2}}{3\sqrt{D}}\right)^3$$ and

$$\mathrm{SIR}_{\hat{U}} = \frac{D+1}{E^*}\left(1 - \frac{1}{9(D+1)} + \frac{Z_{1-\alpha/2}}{3\sqrt{D+1}}\right)^3.$$

where Z_α is the 100α percentile of the standard normal distribution (Rothman and Boice, 1979, 1982; Breslow and Day, 1987; Sahai and Khurshid, 1993, 1996). Rothman and Boice (1979, 1982) mention that these limits were first proposed by Byar (unpublished).

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