Assignment 4
Uncertainty

Due on **TBA**

Name ____________________

Matriculation Number ____________________

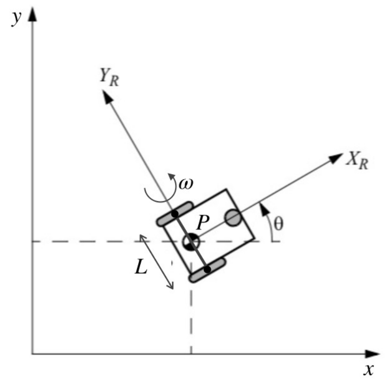
Instructions

1. Attach this **cover sheet** to the work that you hand in.
2. Please complete the feedback questions and submit separately.
3. Type up your solutions or write **neatly**. You may write the mathematical formulas and draw pictures by hand.

Grading

Problem	Max Points	Points
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	**100**	

1. A *differential-drive* robot uses two independently powered wheels mounted on a common axis. It controls the motion by varying the angular velocities of the two wheels, ω_l and ω_r. The radius of each wheel is r. The distance between the two wheels is L. Let $P = (x_P, y_P)$ be the midpoint on the common axis between the two wheels and let θ be the orientation of the robot with respect to the horizontal axis. See the figure below. The robot's configuration is then (x_P, y_P, θ). If $\omega_\mathrm{l} = \omega_\mathrm{r}$, then the robot translates only, with linear velocity $v_P = \omega_\mathrm{l} r = \omega_\mathrm{r} r$. If $\omega_\mathrm{l} = -\omega_\mathrm{r}$, then the robot rotates only, with angular velocity $\omega_P = 2\omega_\mathrm{l} r/L = -2\omega_\mathrm{r} r/L$.



(*a*) In general, given ω_l and ω_r, what is the linear velocity v_P and angular velocity ω_P at P?
Hint. P is the midpoint along a rigid common axis between the two wheels.

(*b*) Let $\mathbf{x} = (x, y, \theta)^T$ and $\mathbf{u} = (\omega_\mathrm{l}, \omega_\mathrm{r})^T$. If the robot has current configuration $\mathbf{x}$ and applies $\mathbf{u}$ for time duration Δt, what is the new configuration $\mathbf{x}'$? Write down the discrete-time state-transition equation $\mathbf{x}' = f(\mathbf{x}, \mathbf{u})$.

2. Unmanned aerial vehicles (UAVs) have gained popularity in many applications, including photography, surveillance, and delivery. One basic UAV capability is hovering, *i.e.*, staying in a fixed 3-D pose for an extended duration. For this, the UAV needs a sensor looking downwards to measure the distance to the ground. Choose among the options below a suitable distance sensor for a light-weight indoor UAV and justify your choice:

- camera
- LIDAR
- ultrasonic sensor

The primary considerations include sensor accuracy, weight, cost, ...

3. Consider the problem of finding a minimum-cost path in a weighted, directed graph.

(*a*) Given a graph along with start and goal nodes, describe *precisely* how to solve the minimum-cost path problem as an MDP. Specify the states, actions, state-transition function, and reward function for the MDP. If a finite-horizon MDP is used, specify the horizon. If an infinite-horizon MDP is used, specify the discount factor.

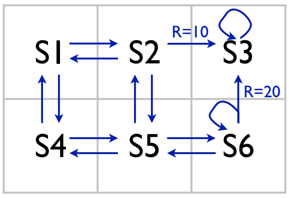
(*b*) Suppose that value iteration is used to solve the MDP model. Explain how to extract the minimum-cost path from the resulting value function.

4. The reward function of a task has the form $R(s, a, s')$. In other words, it depends on both the start state s and the end state s', as well as the action a.

(*a*) How would you model this task using the standard MDP model, whose reward function has the form $R(s, a)$? Describe your model carefully.

(*b*) If value iteration is applied to the MDP model, would it generate an optimal policy for the original problem in the limit? If so, explain why. If not, give a counter-example.

5. The grid-world MDP shown below consists of six states: $S = \{s_1, s_2, \ldots, s_6\}$. The actions for each state are indicated by arrows in the diagram below. If the reward for an action is non-zero, it is indicated right next to the corresponding arrow. Each action brings the robot deterministically from one state to a neighboring state, except for the actions at s_3 and s_6, The action for s_3 is a self-loop and keeps the robot stationary at s_3. If the robot tries to move from s_6 to s_3, it succeeds with probability $p = 0.5$ and receives a reward of 20; it stays in s_6 with probability $1 - p = 0.5$ and receives no reward. The discount factor is 0.8.



(*a*) Apply value iteration to this modified grid-world MDP for 3 iterations. Assume that $V_0(s) = 0$ for each $s \in S$. Calculate the value $V_3(s)$ for each state after iteration $t = 3$.

(*b*) Let V^* be the optimal value function, *i.e.*, the value function for an optimal policy. What is the value $V^*(s_6)$?

(*c*) Indicate in the diagram above the optimal action for each state.

(*d*) If the probability of success p decreases below a threshold p_0, the optimal action at s_6 will change. What is the threshold p_0? Justify your answer.

6. Consider the *Whack-the-Mole* example discussed in the class. Suppose that at time $t = 0$, the probability distribution for the mole's initial location is $(1, 0, 0)$. Answer the following questions using the state-transition function and the observation function given in the lecture slides. Show the steps of your calculation.

(*a*) Before receiving any observations, what can we say about the probability distribution on the mole's location at time $t = 3$?

(*b*) After receiving an observation z_2 at time $t = 1$, what is the probability that the mole is located at state s_2?

7. Particle filtering uses a finite set of samples to approximate a continuous probability distribution. This may lead to various difficulties in practice. In this problem, we explore a common one called *particle collapse*. Consider the extreme situation in which a robot stays stationary and does not move at all. It also receives no observations. The probability distribution on the initial robot state is uniform. Suppose that we use 2 particles sampled uniformly at random from the initial probability distribution. Naturally the weights for the two particles are $(1/2, 1/2)$.

(*a*) After one step of particle filtering, what is the probability that we have 2 particles representing 2 distinct locations? What is the probability that we have 2 particles representing the same location?
Hint. Consider the resampling step carefully.

(*b*) Answer the same questions above, after N steps of particle filtering.

(*c*) The outcome in the previous parts is highly undesirable. Why?

(*d*) Suppose that we use K particles with K much larger than 2. Would it help improve the situation? Why or why not?

(*e*) What would you to alleviate the issue of particle collapse? Explain your idea.

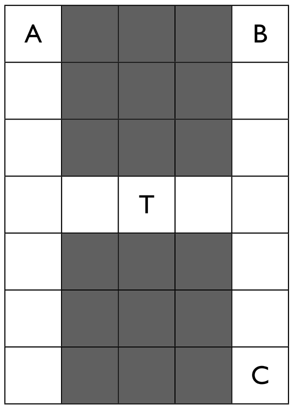
8. In the class, we have covered three filtering algorithms: histogram filter, particle filter, and Kalman filter. Choose a suitable filtering algorithm for the tasks below and justify your choices.

(*a*) Before a robot vehicle enters an intersection, it monitors an incoming human-driven vehicle in order to decide on suitable actions. Specifically, the robot assumes that the human driver may be *aggressive*, *normal*, or *conservative* and tries to infer the driver's type according to its observed behavior.

(*b*) A targeting system tracks an incoming enemy aircraft far away, using RADAR. The objective is to determine the exact 3-D position of the aircraft in order to launch a missle attack.

9. Consider the *Girl-and-Tiger* example discussed in the class. Initially the tiger is behind the left or the right door with equal probabilities. Write down the formal POMDP model for this example according to the description given in the lecture slides. Specify clearly all the elements of the POMDP: the states, actions, observations, state-transition function, observation function, and reward function.

10. Consider the grid environment below. The robot is initially located at $A, B,$ or C with equal probabilities, and the robot's goal is to reach the destination T and stay there. In each time step, the robot may stay put or move from its current grid cell to a neighboring cell deterministically. If the robot tries to move to a neighboring cell occupied by obstacles, it stays unmoved in its current cell. There are no observations.



    (*a*) Of the three probabilistic models that we have discussed in the class, MDP, HMM, and POMDP, which one is most suitable for this task and why?

    (*b*) Describe a policy for the robot to reach T with probability 1.