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Leaders-driven particle swarm optimizer

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ABSTRACT

In particle swarm optimization (PSO), the traditional learning strategy is often criticized for its inadequate exploiting information from top-performing particles, which leads to low efficiency and premature convergence. To address these drawbacks, this paper proposes a leaders-driven particle swarm optimizer (LDPSO), where the search process is guided by superior members. In LDPSO, particles with superior performance are designated as leaders, while the remaining individuals form a group of followers. Leaders are responsible for identifying and guiding populations to focus on promising areas, while the followers work under leaders' guidance to conduct detailed exploitation. A bidirectional search strategy is tailored for leaders to leverage their superior search experience, thereby facilitating high-quality and diverse global exploration. A unidirectional search strategy is designed for followers, enabling them to focus on intensively exploiting promising regions based on guidance derived from leaders. Additionally, a leader-follower collaborative jump-out strategy is introduced to help stagnant particles—those failing to improve over several generations—escape from local optima. These particles are reactivated by cooperating with both leaders and followers to explore previously unvisited regions. Furthermore, the number of leaders gradually decreases, shifting the search from diversified global exploration to focused local exploitation. Experimental results demonstrate that LDPSO outperforms 22 PSO and non-PSO algorithms in terms of search accuracy, stability, and robustness. LDPSO converges rapidly on unimodal functions, maintains diversity in multimodal problems, ranks highest across various dimensions, and proves effective in solving flexible job shop scheduling problems through numerical simulations.

1. Introduction

Optimization problems are prevalent in various scientific research and engineering applications. Early studies preferred deducing the optimal solutions through the rigorous mathematical deduction or the development of exact solving algorithms. While these algorithms are effective for problems with smaller sizes or simpler structures, they are not suitable for addressing large-scale and complex real-world optimization challenges. Consequently, research on various heuristic algorithms has flourished, with swarm intelligence algorithms have attracted persistent attention from scholars, including artificial bee colony (ABC) (Karaboga et al., 2005), particle swarm optimization (PSO) (Kennedy & Eberhart, 1995), artificial protozoa optimizer (APO) (Wang et al., 2024), and so on. The swarm intelligence algorithms are random search methods inspired by the collective behavior of natural groups, such as bird flocks and fish schools. They are equipped with powerful searching capabilities without knowing resolvable structural information, allowing them to arrive at satisfactory solutions efficiently.

Particle swarm optimization (PSO), proposed by Kennedy and Eberhart (1995), has long been a prominent technique in the field of swarm intelligence. In PSO, each solution is regarded as a massless particle in the search space. To guide the population's search toward optimal solutions, all particles adopt a same velocity updating rule. This rule dynamically adjusts the velocity of particle i based on its past experiences (represented as $Pbest_i$) and the population's wisdom (denoted as $Gbest$). In this flexible and efficient mechanism, each particle continuously refines its search direction, thereby improving the solution quality. Moreover, the interaction mechanism between individuals and the collective enables the method to achieve a trade-off between global and local search. Consequently, PSO has obtained significant attention across various fields, leading to its applications in areas such as numerical optimization (Zhan, Zhang, Li, & Chung, 2009a), parameter estimation (Wang, Zhao, Chen, & Liu, 2021), bin-packing problem (Sun, Li, Wang, & Xie, 2025), job shop scheduling (Xu, Wang, Zhang, Yang, & Liang, 2025), and so on.

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Despite the notable success of PSO, it often struggles to find the global optimal solutions when tackling complex optimization problems. This limitation is primarily attributed to premature convergence, wherein particles are trapped in local optima during the early stage. This significantly reduces population diversity and impairs the algorithm's ability to perform effective global exploration (Tang, Huang, Tan, Fang, & Huang, 2024; Zheng et al., 2022). The main reasons for these defects include: (a). inadequate utilization of high-quality search information; (b) the adoption of the unified learning strategy. For example, except for the best-performing particle—whose personal and social learning exemplars are both $Gbest$—each particle's best experience can only be utilized by itself. Moreover, canonical PSO restricts all particles to learn from the same $Gbest$, which limits population diversity and increases the risk of premature convergence and search stagnation.

During the last decades, increasing attention has been devoted to the development of learning strategies that utilize search experience shared across particles to improve search efficiency. Some studies have shown that utilizing search experience from high-quality solutions can improve local exploitation performance. Inspired by orthogonal experimental design, Zhan, Zhang, and Liu (2009b) proposed an orthogonal learning strategy, which utilizes previous search experience from multiple individuals to construct learning exemplars. In the work by Zhao and Wang (2022), particles with superior performance were used to find more promising regions. Similarly, Xia et al. (2019) utilized the search experiences of elites with better fitness values and profiteers with higher improvement rates to construct learning exemplars. Meanwhile, three learning strategies were proposed to allow individuals to select the most suitable exemplar for learning. Xu et al. (2019) proposed a dimensional learning strategy to discover and integrate promising information from the population. Another research direction attempts to enhance population diversity and subsequently improve search performance by leveraging diverse search experience. Liang, Qin, Suganthan, and Baskar (2006) proposed the comprehensive learning particle swarm optimizer, in which each particle updates its velocity by learning from the historical best positions of other particles across different dimensions, thereby promoting search diversity. Differently from Liu, Li, Ren, and Pang (2022), particles used the search experience of those that performed better and worse than themselves as attractors and repellers to improve the search diversity. Further, as demonstrated in Li, Jing, Chen, and Chen (2023), the elite-common co-evolution mechanism was devised, which allows particles to simultaneously learn the evolutionary experience from elite and common particles. Tian et al. (2024) introduced a diversity-guided PSO variant, where two learning strategies are proposed to maintain population diversity. Subsequently, a heterogeneous $Pbest$-guided comprehensive learning PSO was proposed by Meng and Li (2024) to improve global diversification and local intensification abilities. While these methods improve diversity by using poorly performing individuals, excessive emphasis on their influence may lead to over-diversification.

The PSO variants mentioned above provide effective learning mechanisms and outperform the canonical PSO. However, these studies have mainly focused on either improving global diversification or enhancing local intensification. Few have considered leveraging the experience from top-performing particles to simultaneously enhance both global diversification and local intensification. To fully utilize promising search information, this paper proposes a leaders-driven particle swarm optimizer (LDPSO). According to the performance of individuals, LDPSO divides the population into leaders with superior performance and followers with inferior performance. A bidirectional search strategy is designed for leaders to enhance search diversity while avoiding blindness. Followers employ a unidirectional search strategy and to thoroughly explore the promising regions discovered by the leaders. A particle is considered to be in evolutionary stagnation if its performance remains unimproved over successive generations. To address this issue, a leader-follower collaborative jump-out strategy is implemented. Moreover, as the search progresses, the number of leaders gradually decreases while the proportion of followers increases. Consequently, the population's search pattern transitions from leaders-driven global exploration to followers-dominated local exploitation.

In the experiments, the search behavior is first investigated, followed by a comprehensive parameter sensitivity analysis. Subsequently, the effectiveness of the components of LDPSO is verified. The analyses confirm that each component positively contributes to the performance improvement, and collectively, they provide the most significant enhancement. Next, LDPSO's competitiveness is demonstrated by comparing it with 11 state-of-the-art PSO variants and 11 recently proposed non-PSO methods on CEC2017 benchmark problems. The results indicate that LDPSO either outperforms or is highly competitive with these competitors. Moreover, population diversity is analyzed to reveal the potential reasons for its effectiveness. Finally, LDPSO is applied to solve flexible job shop scheduling problem, and the comparison demonstrates its capability to address complex scheduling challenges.

The remainder of this paper is organized as follows. In Section 2, we first introduce the canonical PSO algorithm and its development trends. Subsequently, Section 3 describes the proposed method. Extensive experimental results are presented in Section 4. Next, Section 5 employs the LDPSO to address a standard flexible job shop scheduling problem. Finally, Section 6 provides conclusions and outlines future research directions.

2. Related work

2.1. Canonical particle swarm optimization

As a population-based metaheuristic algorithm, PSO is inspired by the swarm behavior observed in nature. Particles in the population fly through a D-dimensional search space and collaborate to find the global optimal position. Each particle is associated with two vectors, i.e., the velocity vector (V) and the position vector (X). Mathematically, the velocity and position update rules are defined as follows (Ratnaweera, Halgamuge, & Watson, 2004; Shi & Eberhart, 1998):

$$\begin{aligned} &V_{i,d}^{t+1} = \omega \cdot V_{i,d}^{t} + c_1 \cdot r_1 \cdot (Pbest_{i,d}^{t} - X_{i,d}^{t}) + c_2 \cdot r_2 \cdot (Gbest_{d}^{t} - X_{i,d}^{t}), \\ &X_{i,d}^{t+1} = X_{i,d}^{t} + V_{i,d}^{t+1}; \end{aligned} \tag{1}$$

where V_i^t and X_i^t denote velocity and position of particle i in the $t - th$ generation; ω represents the inertial weight; c_1 and c_2 are two acceleration factors; d represents the $d - th$ dimension of the search space; r_1 and r_2 are two random numbers uniformly distributed within the range of $[0, 1]$; $Pbest_i$ is the best position experienced by particle i; $Gbest$ is the best position found by the entire population.

The indices and variables used in this paper are summarized in Table 1.

2.2. Literature review

Particle swarm optimization (PSO) is a swarm intelligence optimization technique that has been continuously improved by researchers. Based on the focus, the improved PSO variants can be broadly classified into three categories, i.e., variants based on parameter control, information sharing, and learning strategies.

Parameter control is the most common method for enhancing optimization performance. In pioneering work, Shi and Eberhart (1998) first introduced the inertia weight (ω) into PSO. Since then, various inertia weight strategies have been proposed to improve the search performance, including linearly decreasing inertia weight (Shi & Eberhart, 1998), chaos-based inertia weight (Chen et al., 2018), random-based inertia weight (Chen, Wang, Chen, & Qiu, 2020), and other designs (Jiyue, Liu, & Wan, 2023). Ratnaweera et al. (2004) proposed a time-varying acceleration strategy to balance global and local search capability by controlling the coefficients c_1 and c_2. Later, the adaptive PSO was introduced by Zhan et al. (2009a), where the search process was divided

Table 1
Indices and variables used in this paper.

Symbol	Description
i, k, j	indices for particles
α	eliminate factor
d	index for dimensions
K	the number of leaders
N	total number of particles
D	dimension of problems
ω	inertial weight
c_1,c_2	acceleration factors
r_1,r_2,r_3,r_4	random numbers within the range of [0,1]
$Pbest_i$	the best position searched by particle i
$Gbest$	the best position found by whole particles
$F(X)$	the fitness of X
$X^t_{i,d}$	position of d-th demension of the i-th particle at t-th generation
$V^t_{i,d}$	velocity of d-th demension of the i-th particle at t-th generation
$NFEs$	current consumed number of fitness evaluations
$MaxNFEs$	the maximum number of fitness evaluations
$R_{\max}/R_{\min}$	the maximum/minimum ratio of leaders in the population

into four states, allowing for the adaptive adjustment of the parameters ω, c_1, and c_2. Furthermore, a randomized particle swarm optimizer was proposed by Liu et al. (2021), where Gaussian white noise was adopted to randomly perturb the c_1 and c_2.

It is worth noting that diverse information-sharing mechanisms, particularly underlying topological structures, exert a significant impact on the performance of PSO. Consequently, the incorporation of appropriate topological structures has emerged as a mainstream research direction. A dynamic neighborhood structure was developed by Suganthan (1999), where particles exchange search information with only a limited number of individuals. A fully informed PSO was proposed by Mendes, Kennedy, and Neves (2004), where each one utilizes the search information from all other particles to update its velocity. Gong and Zhang (2013) incorporated the small-world network model into PSO by assigning each dimension with a randomized small-world topology, thereby changing the dynamics of information propagation. As introduced by Lin, Sun, Yu, Wu, and Tang (2019), a ring topology structure was adopted, allowing particles to utilize the search information of their neighbors to construct learning exemplars. Ren, Xu, Meng, and Pan (2024) proposed a fully informed search scheme to enhance the algorithm's exploitation capacity. Additionally, Zhao, Wu, Pang, and Zhong (2025) integrated PSO with low-discrepancy sequences to improve global search capabilities and conjugate gradient methods to enhance local search efficiency.

Moreover, designing effective learning strategies (evolutionary mechanisms) is a promising research direction (Liang et al., 2006; Zhan et al., 2009b). Zhang (2023) established three elite archives to preserve high-performing individuals, which in turn facilitated the development of six distinct learning strategies. Similarly, Zhang and Lin (2022) designed three learning strategies for different particles to enhance search performance and efficiency. Additionally, a dimensional learning strategy was proposed by Xu et al. (2019), aimed at discovering and integrating promising search information from the population. Jiyue et al. (2023) divided the population into multiple subswarms, with three learning strategies were designed for individuals with different roles. Among the existing studies, strategies that consider multiple exemplars or learning modes have garnered significant interest. Xia et al. (2019) associated each particle with three potential learning exemplars, allowing them to select the appropriate evolutionary mechanism based on the their qualities. Furthermore, Li et al. (2023) introduced an elite-common co-evolution mechanism, which leverages the search experience of elite and common particles to construct diversification and intensification exemplars, respectively. Jiyue et al. (2023) divided the swarm into multiple subswarms, and the particles in each subswarm have three different social learning exemplars. Additionally, Lynn and Suganthan (2017) divided the population into small and large subswarms to enhance search diversity. The comprehensive learning strategy (Liang et al., 2006) was applied in the small subswarm, while in the larger subpopulation, the search strategies from (Peram, Veeramachaneni, & Mohan, 2003), (Qu, Suganthan, & Das, 2012), (Shi & Eberhart, 1998) and (Ratnaweera et al., 2004) were hybridized together as an ensemble approach. Guo et al. (2025) introduced an adaptive difference mutation-based learning strategy to maintain population diversity. Recently, Zhu et al. (2024) integrated center-of-mass traction strategy and forgetting mechanism into the PSO framework, resulting in enhanced evolutionary efficiency.

These algorithms demonstrate the ongoing efforts to improve the performance of PSO and underscore the significance of parameter-tuning strategies, information-interaction mechanisms, and learning strategies. Most information-interaction mechanisms are implemented based on topology structures, often neglecting the quality of information. Meanwhile, the majority of algorithms utilize relatively poor-performing search information to enhance search diversity, which may introduce negative effects, thereby weakening convergence ability. Different from these methods, LDPSO seeks to construct learning strategies using information derived from superior particles, aiming to balance exploration diversity and convergence efficiency.

3. The proposed method: LDPSO

Particles with superior performance are consistently prioritized in the construction of learning strategies, as they play a pivotal role in guiding search behavior and influencing optimization results. Most efforts concentrated on utilizing excellent search experience to enhance local search, with little emphasis for diversified exploration. Therefore, these studies have flaws, such as reduced global diversification, rapid premature convergence, and diminished population diversity. Thus, it is necessary to utilize the advantageous information in the population to enhance the global exploration ability, especially for multimodal problems. As shown in Fig. 1, particles in LDPSO are categorized into leaders and followers. Leaders, equipped with superior search capabilities, are tasked with exhibiting diverse global exploration behaviors and guiding the population toward convergence in promising regions. Each leader learns from two peers who perform equally well, thereby balancing search quality with diversity. In contrast, followers with inferior performance require more effective guidance instead of persisting in unproductive search directions. Consequently, followers should concentrate on providing search support rather than acting as explorers, which is beneficial for their self-improvement. Once leaders identify promising regions, they should immediately fly toward these areas and conduct local searches to find more promising solutions. In this mechanism, excellent search insights are gradually transaction from the leaders to the followers. During the search process, particles that fail to improve their performance over several consecutive generations are considered stagnated. In such cases, a leader-follower collaborative jump-out strategy is activated. Leaders provide guidance by sharing superior information, while followers contribute diversified experience, pulling the stagnated particles to escape from local optimal areas. Moreover, as the search progresses, the number of leaders gradually decreases, and the global exploration initially led by them is progressively transitioned to local exploitation dominated by followers.

This section details the LDPSO framework from five perspectives: leader-follower division, a bidirectional search strategy for leaders, a unidirectional search strategy for followers, a cooperative jump-out mechanism between leaders and followers, and a dynamic adjustment scheme for the number of leaders.

3.1. The division of leaders and followers

During the evolutionary process, LDPSO categorizes particles into leaders and followers. Specifically, individuals with superior fitness values are designated as leaders, while the others are classified as followers. For a minimization problem, better performance is represented by a lower value of personal best experience ($Pbest$). Therefore, at each

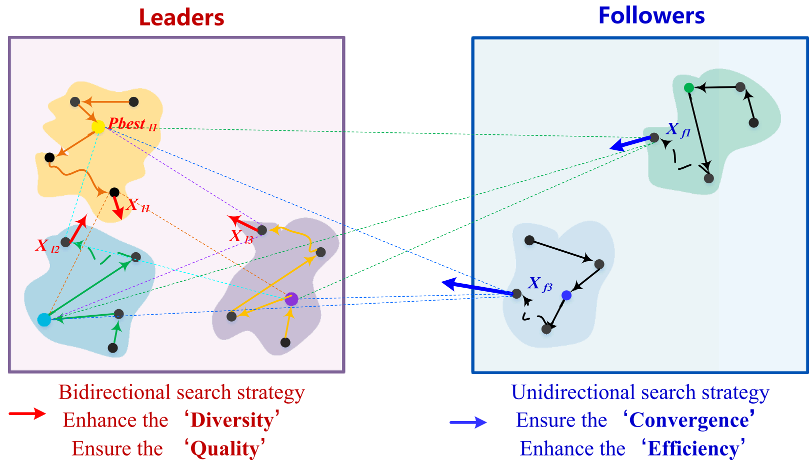


Fig. 1. The interaction between leaders and followers.

generation, particles are sorted according to their $Pbest$, *i.e.*:

$$\{X_1, X_2, \dots, X_j, \dots, X_N | F(Pbest_1) \le F(Pbest_2) \le \dots \le F(Pbest_j) \le \dots \le F(Pbest_N)\} \tag{2}$$

where $F(Pbest_j)$ represents the fitness value of $Pbest_j$. The top K individuals, X_1, X_2, ..., and X_K, are designated as leaders, while the remaining individuals are classified as followers.

3.2. Bidirectional search strategy for leaders

The leaders demonstrate exceptional performance and play a crucial role in guiding the evolution of the swarm. Specifically, they identify promising areas via a bidirectional search strategy, which allows for high-quality exploration. Moreover, by guiding followers toward these high-quality regions, the algorithm's exploitation performance is enhanced. Therefore, leaders must bear the crucial responsibility of maintaining population diversity and acting as explorers to discover additional high-quality regions. In human society, individuals often learn from multiple models who excel in various areas or possess specific skills. Drawing on the concept of community learning, a bidirectional search strategy is proposed for leaders. As presented in Fig. 2 and formulated in Fig. 3, this strategy effectively leverages the search information provided by these leaders and avoids blind search behaviors.

$$V_{i,d}^{t+1} = \omega \cdot V_{i,d}^t + c_1 \cdot r_1 \cdot (Pbest_{k,d}^t - X_{i,d}^t) + c_2 \cdot r_2 \cdot (Pbest_{j,d}^t - X_{i,d}^t). \tag{3}$$

As shown in Fig. 2, each leader i has two guidance directions. The first direction involves searching towards leader k, who performs worse than leader i, to enhance search diversity. Meanwhile, leader k maintains a relatively high performance, thereby ensuring search quality. The second direction derived from leader j, who performs better than leader i, with the aim of boosting local intensification. Compared to Eq. 1, the first learning exemplar is modified from $Pbest_{i,d}^t$ to $Pbest_{k,d}^t$. This modification helps particle i decrease the influence of its personal best position, facilitating the exploration of unvisited areas. In classical PSO, all particles are attracted to $Gbest$, leading to rapid convergence towards temporary local optima and resulting in a crowded population distribution. In LDPSO, social learning becomes more diverse, ensuring both learning efficiency and quality. Consequently, the search diversity is maintained, thereby enhancing global exploration capabilities.

3.3. Unidirectional search strategy for followers

The performance of followers is relatively poor, making them unsuitable for conducting high-quality exploration. Thus, being a local exploiter is more appropriate for them. Once the leaders identify promising regions, the followers should immediately fly toward these areas to engage in local intensification behavior. Consequently, a unidirectional search strategy is designed for followers. Unlike the bidirectional search strategy, which aims for both diversity and quality, the followers focus solely on achieving promising convergence.

$$V_{i,d}^{t+1} = \alpha \cdot \omega \cdot V_{i,d}^t + c_1 \cdot r_1 \cdot (Pbest_{j,d}^t - X_{i,d}^t) + c_2 \cdot r_2 \cdot \left(\sum_{k=1}^{K} \left(\frac{\frac{1}{k}}{\sum_{g=1}^{K} \frac{1}{g}} \cdot Pbest_{k,d}^t \right) - X_{i,d}^t \right). \tag{4}$$

where j is a randomly selected leader, $Pbest_k$ represents the best position from the $k-th$ leader. Additionally, α is employed to reduce the impact of the previous velocity. This adjustment enables followers to rapidly respond to the changes of leaders, facilitating a more rapid localized search of promising regions.

As illustrated in Eq. 4, each follower acquires valuable knowledge from all the leaders. Note that, the learning weight $\left(\frac{\frac{1}{k}}{\sum_g^K \frac{1}{g}} \right)$ of leader k depends on its performance. The better a leader performs, the greater its contribution.

3.4. Leader-follower collaborative jump-out strategy

When tackling multimodal optimization problems, particles are prone to becoming trapped in local optima. This often leads to a continuous decline in velocity, thereby impairing the exploration capability. In the absence of additional search information for guidance, they may stop updating for several generations, thereby losing the ability to escape from local optima. Considering that leaders may be located near the optimal solutions, while followers carry more diverse information, a leader-follower collaborative jump-out strategy is proposed. Mathematically, this strategy can be formulated as follows.

$$\hat{X}_{i,d} = Pbest_{i,d} + r_3 \cdot (Pbest_{k,d} - Pbest_{i,d}) + r_4 \cdot (Pbest_{j,d} - Pbest_{i,d}); \tag{5}$$

where $\hat{X}_{i,d}$ represents the new jumped-out position of particle i, d denotes the $d-th$ search dimension, $Pbest_k$ and $Pbest_j$ represent the best positions found by leader k and follower j, respectively. The r_3 and r_4 are two random numbers uniformly distributed between 0 and 1. Through the collaboration between leader k and follower j, some components of $\hat{X}_{i,d}$ can explore broader territories within the search space. Meanwhile, this strategy depends on the interaction of three high-quality exemplars, thereby enhancing robustness and solution quality.

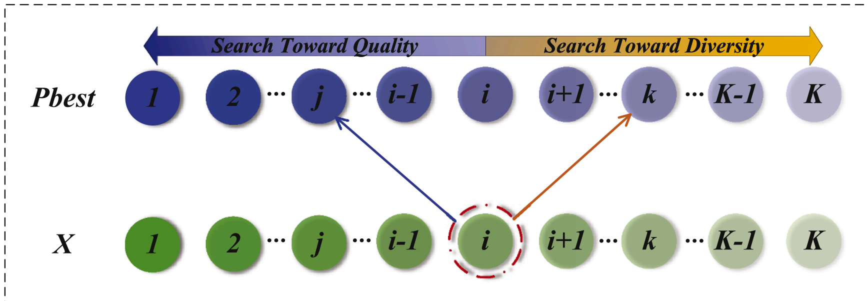


Fig. 2. Diagram of the bidirectional search strategy.

3.5. Number adjustment of leaders

As previously discussed, leaders are responsible for broadly exploring the search space, enhancing global exploration capabilities and maintaining population diversity. In contrast, followers contribute to the search by concentrating on promising regions. To ensure that global diversification gradually gives way to local intensification, a larger number of leaders is preferred in the early stage, as extensive exploration is crucial for identifying diverse and high-potential areas. As the search progresses, promising regions are identified, and increasing the number of followers to enhance local exploitation is required. Therefore, the number of leaders (K) is determined by Eq. 6:

$$K = \left\lceil R_{max} - (R_{max} - R_{min}) \cdot N \cdot \frac{NFEs}{MaxNFEs} \right\rceil; \tag{6}$$

where N refers to the population size, and the notation $\lceil a \rceil$, known as the "ceiling" of a, denotes the smallest integer that is greater than or equal to a. Additionally, $R_{\max}$ and $R_{\min}$ represent the maximum and minimum ratios of leaders in the population, respectively. Moreover, *NFEs* and *MaxNFEs* denote the current number of fitness evaluations consumed and the maximum number of fitness evaluations allowed, respectively. As indicated by Eq. 6, the number of leaders decreases from $\lceil R_{\max} \cdot N \rceil$ to $\lceil R_{\min} \cdot N \rceil$. The larger the $R_{\max}$ or the smaller the $R_{\min}$, the stronger the diversification or intensification provided by algorithm. This approach ensures that the population thoroughly explores the search space during the early stage and subsequently converges towards promising areas to enhance local search performance.

3.6. The framework of LDPSO and time complexity analysis

The pseudocode and framework of the LDPSO are presented in Algorithm 1 and Fig. 3, respectively[1] It can be observed that the LDPSO retains the simplicity of the classical PSO while introducing additional strategies. Initially, a population with N particles is randomly generated, and subsequently, the $Pbest$ and $Gbest$ are identified. Before each generation begins its search, particles are sorted according to their performance. The top K individuals are selected as leaders, while the remaining particles serve as followers. The leaders employ a bidirectional search strategy, utilizing their superior information to conduct a high-quality and globally diversified search. The followers take full advantage from leaders, quickly converging on the high-quality areas. Furthermore, if a particle fails to improve G consecutive generations, it is deemed to be trapped in a deep local optimum. In response, leaders and followers collaborate to transfer high-quality and diverse search momentum to the stagnant particle, thereby facilitating its escape from local optima.

As illustrated in Algorithm 1 and Fig. 3, the computational complexity mainly arises from the following aspects: population initialization, population sorting, velocity updates, position updates, and the jump-out strategy. The population initialization consumes $O(N \cdot D)$ time. Population sorting involves a standard sequence sorting procedure with a time complexity of $O(N \cdot \log(N))$. The division operation scans the population only once, which takes $O(N)$ time. The velocity and position updates together require $O(2 \cdot N \cdot D)$ time. Additionally, the time complexity for the jump-out strategy is $O(N \cdot D)$. It is important to note that the jump-out strategy and velocity updates are incompatible. Consequently, the time complexity of the position update is primarily governed by the velocity update process, resulting in an overall complexity of $O(2 \cdot N \cdot D)$. Consequently, the total time complexity of LDPSO is $O(N \cdot D + T(N \cdot \log(N) + N + 2 \cdot N \cdot D))$ (T is the maximum number of iterations). In comparison, the classical PSO algorithm has a slightly lower time complexity ($O(N \cdot D + T \cdot (2 \cdot N \cdot D))$), as

Algorithm 1 Pseudocode of the LDPSO.

Input: N, *MaxNFEs*, $R_{\max}$, $R_{\min}$, and G
Output: $Gbest$ and $F(Gbest)$
for $i = 1 : N$ **do**
  Randomly initialize X_i and V_i;
  Evaluate $F(X_i)$;
  $Pbest_i = X_i$;
end for
Set $Gbest$ to the current best position of particles;
$NFEs = N$;
while *NFEs*<*MaxNFEs* **do**
  Sort according to the performance of *Pbest*;
  Determine the number of leaders (K) based on Eq. 6;
  Divide the population into leaders and followers according to Section 3.1;
  for $i = 1 : N$ **do**
    if Particle i does not stagnate (ceases improving for G generations) **then**
      if particle i belongs to leaders **then**
        Execute bidirectional search strategy according to Eq. 3 to update V_i;
      else
        Execute unidirectional search strategy according to Eq. 4 to update V_i;
      end if
      $X_i = X_i + V_i$;
    else
      Execute the jump out strategy according to Eq. 5;
    end if
    Evaluate $F(X_i)$;
    $NFEs = NFEs + 1$;
    Update $Pbest_i$ and $Gbest$;
  end for
end while
return Gbest and F(Gbest)

[1] The code is publicly available at https://github.com/ShicunZhao/LDPSO.

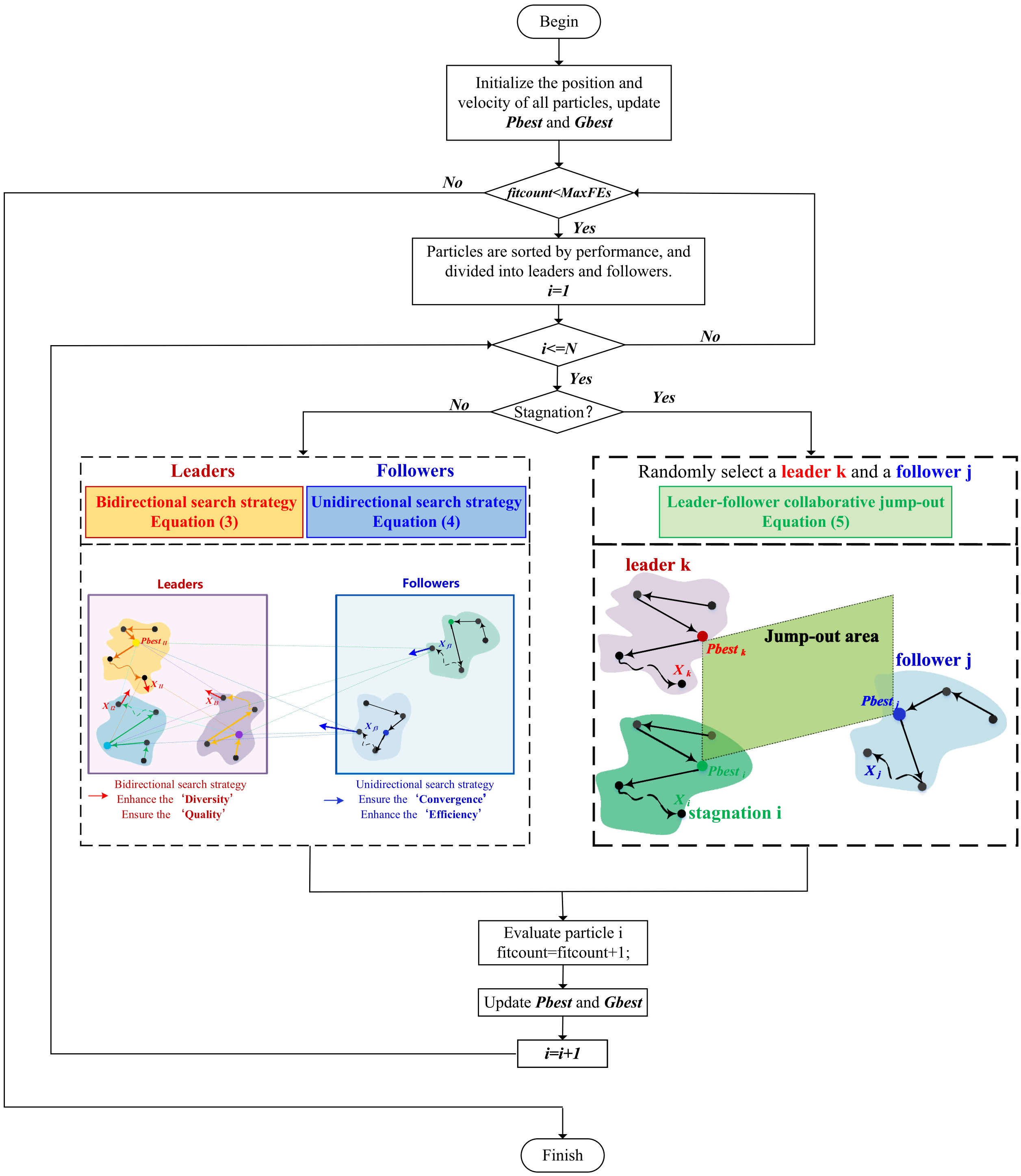


Fig. 3. The framework of the LDPSO.

LDPSO requires additional sorting and division operations, which contributes $O(N \cdot \log(N))$ and $O(N)$, respectively, to the time complexity of LDPSO.

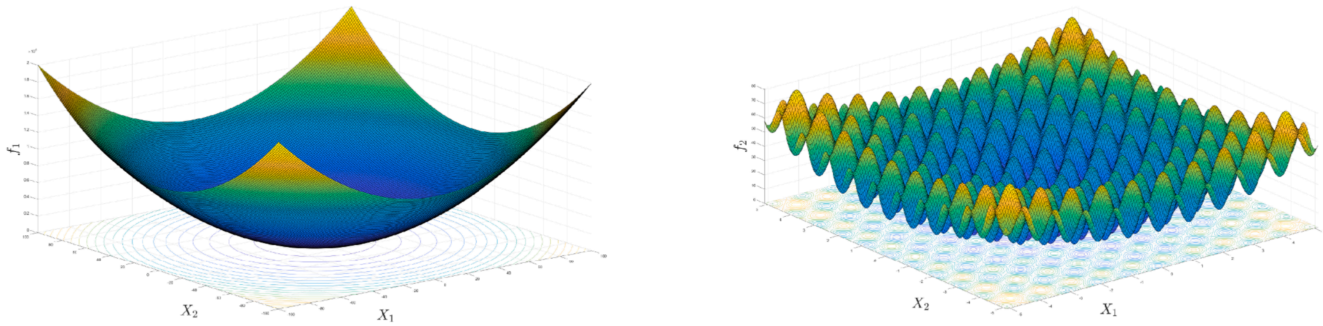
4. Experiments

Numerical experiments are conducted to validate the performance of the LDPSO. Firstly, in Section 4.1, the search behavior of the proposed LDPSO is examined. Subsequently, the benchmark functions and experimental settings are introduced in Section 4.2. Then, the parameter collaboration is performed in Section 4.3. In Section 4.4, the effectiveness of each component in LDPSO is assessed. Next, in Section 4.5, the LDPSO is compared with 11 well-known PSO variants, followed by a comparison with 11 state-of-the-art non-PSO methods in Section 4.6. Finally, the population diversity is analyzed in Section 4.7.

All experiments are implemented in MATLAB R2020a and run on a laptop equipped with an AMD Ryzen 7 5800H CPU @ 3.2GHz, 32 GB memory, under the Windows 10 operating system.

4.1. Investigation on search behavior

In this subsection, the search behavior of LDPSO is investigated via two classical benchmark problems. The first problem is Sphere function, while the second is Rastrigin function. The Sphere function is classified



(a) Sphere function. (b) Rastrigin function.

Fig. 4. The 3-D map and contour map for Sphere function and Rastrigin function. a. Sphere function. b. Rastrigin function.

as a unimodal problem, and its formulation is presented in Eq. 7. As shown in Fig. 4a, the search range is limited as $[-100, 100]^D$, with the optimal position located at $[0]^D$ and an optimal value of 0.

$$f_1(X) = \sum_{d=1}^{D} x_d^2 \tag{7}$$

The Rastrigin function, shown in Fig. 4b, has several local minima and is highly multimodal. Its mathematical formulation is provided in Eq. 8. The search range is limited as $[-5.12, 5.12]^D$, with the optimal position located at $[0]^D$ and an optimal value of 0.

$$f_2(X) = \sum_{d=1}^{D} (x_d^2 - 10 \cdot \cos(2\pi x_d) + 10) \tag{8}$$

This experiment uses the maximum number of fitness evaluations ($MaxNFEs$) as the stopping criterion, set at $10000 * D$. In LDPSO, 50 search agents are employed. Figs. 5 and 6 present the search trajectories and convergence plots on the Sphere and Rastrigin functions, respectively.

From Figs. 5 and 6, it can be observed that particles can identify potential optimal regions in early stage. Subsequently, under the guidance of leaders, particles gradually fly towards the optimal position, indicating that LDPSO achieves an promising balance between global diversification and local intensification. In addition, compared to unimodal Sphere functions, particles converge slower on multimodal Rastrigin functions. This slower convergence benefits the leaders in fully exploring the search space, thereby preserving population diversity and facilitating local optima avoidance. For the Sphere function, the interaction between leaders and followers enables the LDPSO to demonstrate a commendable convergence speed. When addressing multimodal function, LDPSO occasionally gets stuck in local optima. However, driven by the jump-out strategy, it can effectively escape these local optima and achieve high precise solutions.

4.2. Benchmark functions and experimental setting

The CEC2017 test suite (Wu, Mallipeddi, & Suganthan, 2017) is employed for numerical simulation. As presented in Table 2, benchmark functions can be classified into four categories: unimodal functions (F_1 and F_3[2]), multimodal functions (F_4~F_{10}), hybrid functions (F_{11}~F_{20}), and composition functions (F_{21}~F_{30}).

The unimodal functions (F_1 and F_3) only have one local optimum, which is suitable for evaluating exploitation ability. In contrast, multimodal functions ($F_4 - F_{10}$) have multiple local optima and are employed to measure exploration capabilities. Hybrid functions ($F_{11} - F_{20}$) are characterized by their non-separable and complex properties, and they are used to evaluate both exploitation and exploration abilities. Composition functions ($F_{21} - F_{30}$), full of challenging problems, have the characteristics of asymmetrical and numerous local optima, which are utilized to evaluate comprehensive performance.

The experiment uses the maximum number of fitness evaluations ($MaxNFEs$) as the stopping criterion, which is set to $10000 * D$ (D is the number of dimensions) (Wu et al., 2017). Each algorithm utilizes 50 search agents, running independently 30 times on each function to reduce statistical error. In each run, the best results are recorded for comparison. The mean values ($Mean$) from the 30 independent runs are provided, while the standard deviation (Std) is employed to measure the algorithm's stability. Meanwhile, the ranking information ($Rank$) is calculated based on the mean values. Moreover, to make a more convincing comparison, a non-parametric statistical test, $Wilcoxon$ rank-sum test ($W - test$) with a significance level $\alpha = 0.05$, is adopted. The

Table 2
CEC2017 benchmark functions.

Modality	$No.$	$Function\ Name$	$f_{\min}$
$Unimodal$	F_1	Shifted and Rotated Bent Cigar Function	100
	F_3	Shifted and Rotated Zakharov Function	300
$Multimodal$	F_4	Shifted and Rotated Rosenbrocks Function	400
	F_5	Shifted and Rotated Rastrigins Function	500
	F_6	Shifted and Rotated Expanded Scaffers F6 Function	600
	F_7	Shifted and Rotated Lunacek Bi-Rastrigin Function	700
	F_8	Shifted and Rotated Non-Continuous Rastrigins Function	800
	F_9	Shifted and Rotated Levy Function	900
	F_{10}	Shifted and Rotated Schwefels Function	1000
$Hybrid$	F_{11}	Hybrid Function 1 (N = 3)	1100
	F_{12}	Hybrid Function 2 (N = 3)	1200
	F_{13}	Hybrid Function 3 (N = 3)	1300
	F_{14}	Hybrid Function 4 (N = 4)	1400
	F_{15}	Hybrid Function 5 (N = 4)	1500
	F_{16}	Hybrid Function 6 (N = 4)	1600
	F_{17}	Hybrid Function 6 (N = 5)	1700
	F_{18}	Hybrid Function 6 (N = 5)	1800
	F_{19}	Hybrid Function 6 (N = 5)	1900
	F_{20}	Hybrid Function 6 (N = 6)	2000
$Composition$	F_{21}	Composition Function 1 (N = 3)	2100
	F_{22}	Composition Function 2 (N = 3)	2200
	F_{23}	Composition Function 3 (N = 4)	2300
	F_{24}	Composition Function 4 (N = 4)	2400
	F_{25}	Composition Function 5 (N = 5)	2500
	F_{26}	Composition Function 6 (N = 5)	2600
	F_{27}	Composition Function 7 (N = 6)	2700
	F_{28}	Composition Function 8 (N = 6)	2800
	F_{29}	Composition Function 9 (N = 3)	2900
	F_{30}	Composition Function 10 (N = 3)	3000

[2] The F_2 is deleted from CEC2017 due to its unstable behavior in higher-dimensional cases.

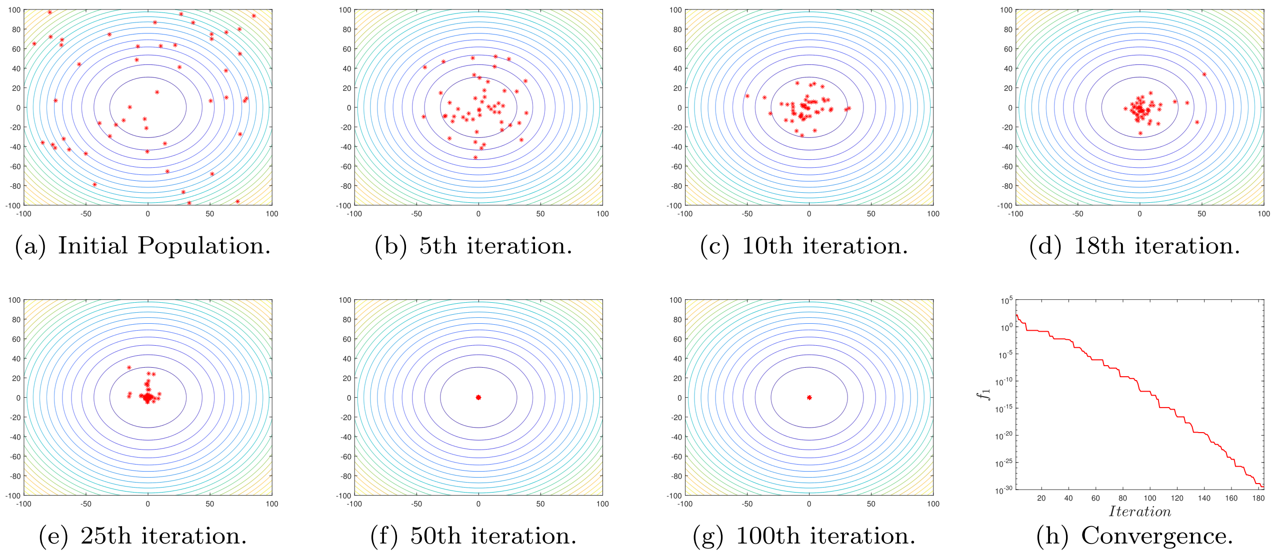


Fig. 5. Search behavior on unimodal Sphere function. a. Initial Population. b. 5th iteration. c. 10th iteration. d. 18th iteration. e. 25th iteration. f. 50th iteration. g. 100th iteration. h. Convergence.

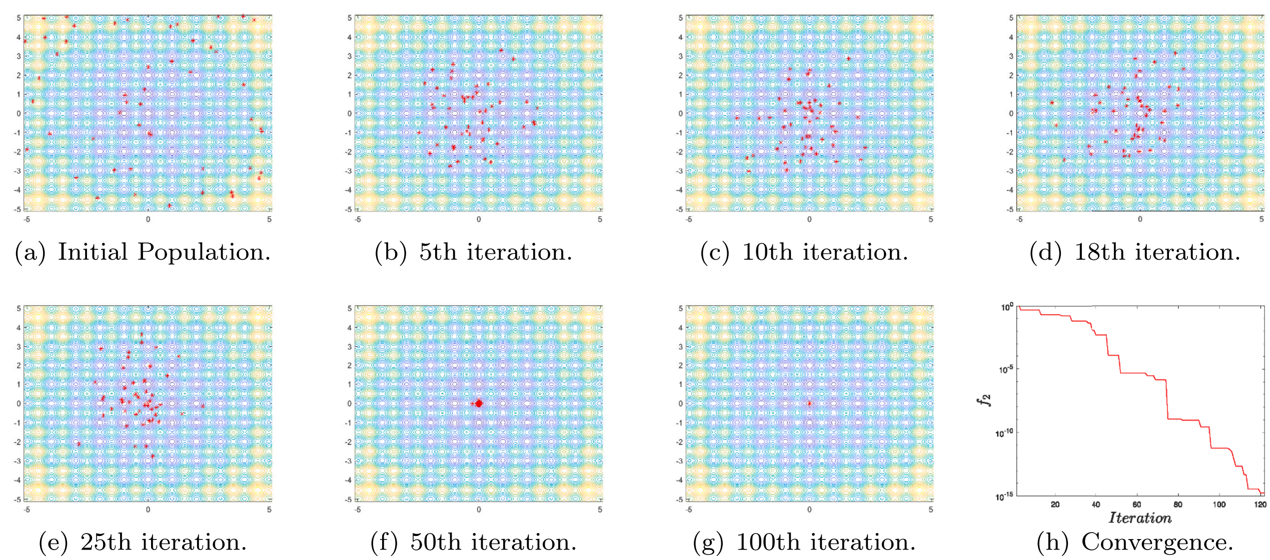


Fig. 6. Search behavior on multimodal Rastrigin function. a. Initial Population. b. 5th iteration. c. 10th iteration. d. 18th iteration. e. 25th iteration. f. 50th iteration. g. 100th iteration. h. Convergence.

symbols " + ", " = ", and " - " represent that LDPSO performs "*better*", "*insignificant*", and "*worse*" than the compared method, respectively. For each algorithm, the indicator "#*Best*" represents the number of times it achieves the best performance across all functions, while the symbols "#*Defeated*", "#*Equal*", and "#*Win*" represent the number of times the corresponding peer method is defeated by, ties with, and outperforms LDPSO, respectively. Furthermore, the Friedman rank-and-sum test, also with a significance level of $\alpha = 0.05$, is conducted. If the p-value is less than 0.05, it indicates a statistically significant difference between the results. The best cases are highlighted with black letters on a gray background.

4.3. Parameter collaboration

The parameter sensitivity experiment is conducted to achieve the best performance of LDPSO. There are four critical parameters in LDPSO, $R_{\max}$, $R_{\min}$, G, and α. The parameters $R_{\max}$ and $R_{\min}$ determine the number of leaders and directly affect the optimization performance. Considering that the number of leaders should remain constant or gradually decrease to balance early global diversification and later intensification capabilities. Therefore, the candidate combinations for ([$R_{\max}$, $R_{\min}$]) are set to {[0.5,0.5], [0.5, 0.4], ..., [0.5, 0.1], [0.4,0.4], ...,[0.1, 0.1]} (15 cases). The parameter G determines the frequency at which particles execute the jump-out strategy. Smaller values may make particles sensitive to the search process, while larger values may cause particles to sink deeper into local optima. The value of α determines the degree to which followers identify with the leader, where lower values correspond to higher identification and higher values to lower identification. The candidate values for G and α are presented as $G \in \{10, 15, 20, 25\}$ and $\alpha \in \{0.1, 0.2, 0.3, 0.4\}$, respectively. Consequently, a total of 240 ($15 \times 4 \times 4$) different parameter combinations need to be investigated. Each combination is evaluated using LDPSO for 30 independent runs. Finally, the mean score is calculated (final mean score = 241 - final mean rank). The final results are presented in Fig. 7.

As shown in Fig. 7, different parameter combinations exhibit various search performances. When the parameters $R_{\max}$ and $R_{\min}$ are set

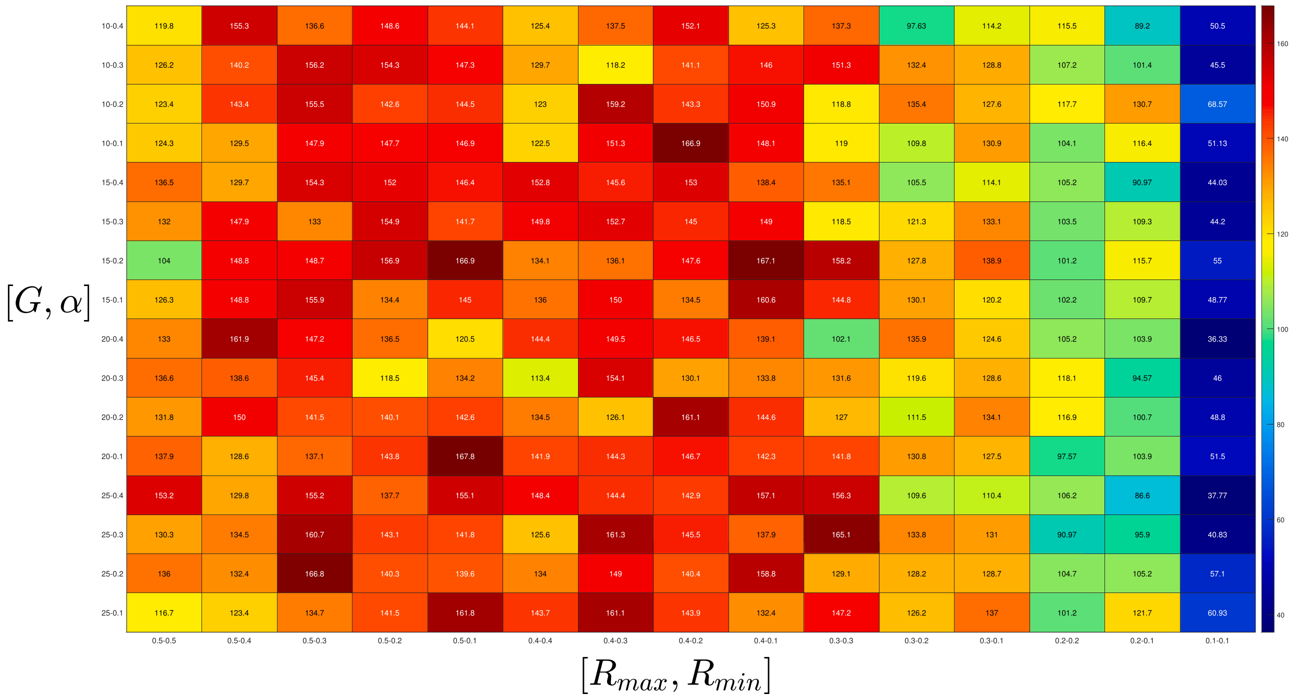


Fig. 7. Performance presentation of different parameter combinations (The horizontal axis represents the combination of parameters $R_{\max}$ and $R_{\min}$, and the vertical axis represents the combination of G and α).

too small (such as in the combinations [0.2,0.2], [0.2,0.1], [0.1,0.1]), the performance is difficult to satisfy. Additionally, the combination of parameters G and α significantly impacts the LDPSO's performance, highlighting the necessity of the parameter sensitivity analysis. As shown in the Fig. 7, two combinations exhibit the best performance ([0.5,0.1,15,0.2] and [0.4,0.2,10,0.1]). After evaluation, the optimal parameters for LDPSO are finally determined as $R_{\max}=0.5$, $R_{\min}=0.1$, $G=15$, and $\alpha=0.2$.

4.4. Ablation experiment

This subsection is specifically designed for conducting ablation experiments to examine the effectiveness of the bidirectional learning strategy, unidirectional learning strategy, leader-follower collaborative jump-out strategy, and number adjustment strategy. The following variants are developed:

- LDPSO1: the leaders abandon the bidirectional search operation and adopt the classical learning strategy (learn from $Pbest$ and $Gbest$);
- LDPSO2: the followers replace the unidirectional learning strategy with the classical learning strategy;
- LDPSO3: the leader-follower collaborative jump-out strategy is removed from LDPSO;
- LDPSO4: the quantity adjustment strategy is deleted.

The parameters in LDPSO1~LDPSO4 are consistent with those in LDPSO. Each algorithm is independently executed 30 times. The Friedman rank-and-sum statistical results are presented in Table 3.

As shown in Table 3, the p-values across all dimensions are less than 0.05, indicating statistically differences among the results produced by all algorithms. Additionally, LDPSO1~LDPSO4 rank lower than that of LDPSO, suggesting that LDPSO generally performs the best. The performance of LDPSO3 is significantly lower than that of LDPSO, indicating that the leader-follower collaborative jump-out strategy plays a crucial role. Furthermore, the results demonstrate that each improvement strategy delivers the positive effect, and their organic combination yields the best performance.

Table 3
The Friedman rank-and-sum results for ablation results.

Algorithm	30-D		50-D		100-D	
	rank	p-value	rank	p-value	rank	p-value
LDPSO1	2.17		2.03		2.23	
LDPSO2	3.23		3.47		3.30	
LDPSO3	4.17	2.15E-09	4.20	1.67E-09	4.50	4.01E-11
LDPSO4	3.57		3.33		3.17	
LDPSO	**1.87**		**1.97**		**1.80**	

As shown in Eq. 9, the position-based population diversity reflects the distribution of individuals during the search process, which helps to further analyze the characteristics of different strategies (Zhan, Zhang, & Shi, 2010).

$$Diversity = \frac{1}{N}\sum_{i=1}^{N}\sqrt{\sum_{d=1}^{D}(X_{i,d}-\bar{X}_d)^2}; \tag{9}$$

where $\bar{X}_d$ is the mean value of $d-th$ dimensional position.

The diversity curves on four representative functions are shown in Fig. 8. Fig. 8a indicates that for unimodal function, LDPSO1 exhibits a rapid convergence trend. Meanwhile, the diversity of LDPSO decreases more slowly. This suggests that the bidirectional learning strategy improves exploration performance, enabling LDPSO to thoroughly explore the search space. Fig. 8b–d demonstrate that the diversity of LDPSO rapidly decreases in the early stage, stabilizes in the middle, and ultimately converges to the optimal region in the later stage. Notably, the diversity of LDPSO is lower than that of LDPSO3 during the early stage but higher in the later stage. This may be attributed to the leader-follower collaborative jump-out strategy, which helps LDPSO avoid local optima in the early stage and quickly identify promising regions. LDPSO3, lacking the jump-out strategy, gradually gets trapped in local optima and loses diversity. In contrast, with the support of the jump-out strategy, LDPSO can effectively escape from local optima in the later stage and continue searching for the global optimum.

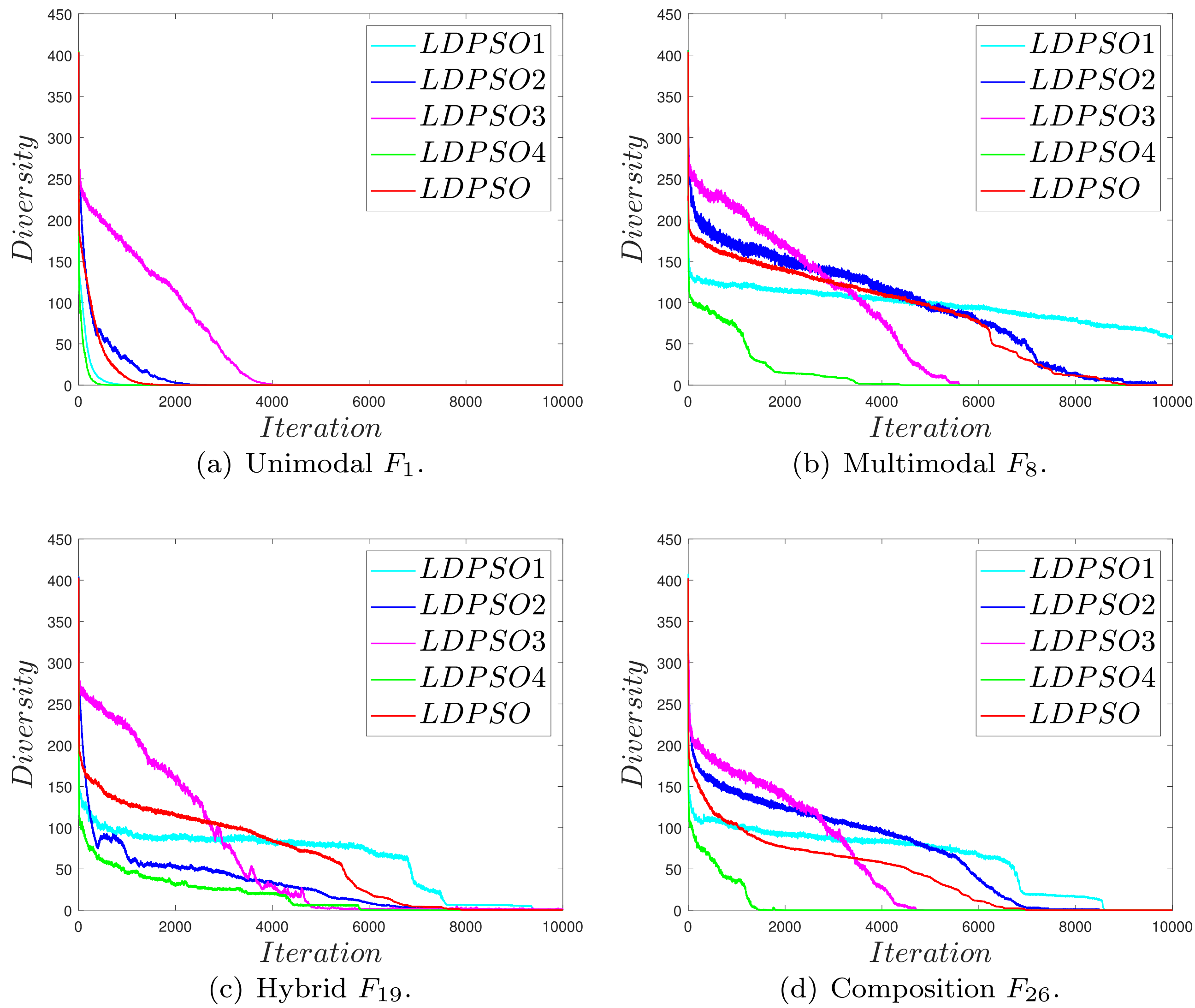


Fig. 8. Population diversity on different functions under 50-D. a. Unimodal F_1. b. Multimodal F_8. c. Hybrid F_{19}. d. Composition F_{26}.

The comparison among LDPSO and its variants confirms the effectiveness of its components. To further examine the performance of LDPSO, it will be compared with state-of-the-art PSO variants and non-PSO variants in the following context.

4.5. Comparison between LDPSO and PSO variants

In this subsection, experiments are performed to examine the performance of the LDPSO by comparing it with eleven widely used or recently proposed PSO variants. The competitors included in the comparison are:

- GPSO: PSO with linearly decreasing inertial weight (ω) (Shi & Eberhart, 1998);
- TVACPSO: PSO with time-varying acceleration factors ($c_{1,2}$) (Ratnaweera et al., 2004);
- SLPSO: social learning PSO (Cheng & Jin, 2015);
- MPSO: modified PSO method using adaptive strategy (Liu, Zhang, & Tu, 2020);
- TAPSO: triple archives PSO (Xia et al., 2019);
- AWPSO: sigmoid-function-based adaptive weighted PSO (Liu et al., 2019);
- APDPSO: PSO variant with superior particles pulling plus inferior particles pushing mechanism;
- AMSEPSO: PSO with adaptive multi-strategy ensemble (Yang, Yu, & Huang, 2022);
- PSOsono: PSO with hybrid search paradigms (Meng, Zhong, Mao, & Liang, 2022);
- CDLPSO: cooperative PSO with difference learning strategy (Li et al., 2023);
- EAPSO: elite archives-driven particle swarm optimizer (Zhang, 2023).

The parameter settings recommended in the original paper, summarized in Table 4, will be directly employed for each competitor. Tables 5–7 present the comparison results of LDPSO and 11 compared PSO variants across 30 dimensions, 50 dimensions, and 100 dimensions, respectively.

From Table 5, it is evident that LDPSO exhibits the best performance. It ranks first on 14 functions, second on eight, and third on four. CDLPSO, a strong competitor, ranks first on 4 functions, followed by EAPSO (3 functions), SLPSO (3 functions), and TAPSO (2 functions). Additionally, APDPSO, AMSEPSO, and PSOsono each attain the best performance on 1 function.

Specifically, PSOsono performs best on F_1, followed closely by LDPSO. Additionally, six algorithms (TVACPSO, MPSO, TAPSO, PSOsono, EAPSO, and LDPSO) acquire the optimal solution on F_3, with TAPSO has the smallest variance. The promising results obtained by

Table 4
Parameter settings of comparison PSO algorithms.

Algorithm	Parameter Setting	Ref
GPSO	$\omega = 0.9 \sim 0.4$, $c_1 = c_2 = 2$	(Shi & Eberhart, 1998)
TVACPSO	$\omega = 0.9 \sim 0.4$, $c_1 = 2.5 \sim 0.5$, $c_2 = 0.5 \sim 2.5$	(Ratnaweera et al., 2004)
SLPSO	$\omega = 0.9 \sim 0.4$, $\gamma = 0.01$, $\eta = 1.496$	(Cheng & Jin, 2015)
MPSO	$\omega = 0.9 \sim 0.4$, $c_1 = c_2 = 2$	(Liu et al., 2020)
TAPSO	$\omega = 0.7298$, $p_c = 0.5$, $p_m = 0.02$, $M = N/4$	(Xia et al., 2019)
AWPSO	$a = 0.000035 \cdot search\ range$, $b = 0.5$, $c = 0$, $d = 1.5$	(Liu et al., 2019)
APDPSO	$\omega = 0.7298$, $c = 1.49$	(Liu et al., 2022)
AMSEPSO	Singer chaotic $\omega = 0.9 \rightarrow 0.4$, $c_1 = 2.5 \rightarrow 1.5$, $c_2 = 1.5 \rightarrow 2.5$, $\sigma = 0.3 \cdot N$, $Stag_{\max} = 5$	(Yang et al., 2022)
PSOsono	$\omega = 0.9 \sim 0.4$, $r = 0.5$, $\epsilon = \frac{D}{N} \cdot 0.01$	(Meng et al., 2022)
CDLPSO	$\omega = 1 \sim 0$, $c_2 = 2 \cdot \omega$, $c_1 = 2 - c_2$, $c_p = 0.75$, $c_s = 5 \cdot \omega$	(Li et al., 2023)
EAPSO	$\omega \in [0,1]$, $\lambda_1 \in [0,1]$, $\lambda_2 \in [0,1]$	(Zhang, 2023)
LDPSO	$\omega = 0.9 \sim 0.4$, $c_1 = 2.5 \sim 0.5$, $c_2 = 0.5 \sim 2.5$, $R_{\max} = 0.5$, $R_{\min} = 0.1$, $G = 15$, $\alpha = 0.2$	-

LDPSO exhibit its convincing exploitation ability. This success can be attributed to follower's unidirectional search mechanism. Under this strategy, followers engage in enhanced local search behavior within the promising areas identified by the leaders, thereby avoiding aimless searching.

The multimodal functions ($F_4 \sim F_{10}$) are utilized to assess the exploration capabilities. The LDPSO acquires the best results on 2 functions (F_5 and F_6) and ranks in the top three on the remaining functions, suggesting that LDPSO has promising exploration ability. Additionally, TAPSO ranks the first on F_4, while SLPSO slightly surpasses the LDPSO on F_7, F_8, and F_{10}, achieving the best rank. From these results, it can be inferred that the bidirectional learning strategy effectively utilizes advantageous search experience to enhance search performance. Guided by high-quality and diverse exemplars, the leaders fly toward multiple promising areas for global search, ensuring high-quality and diverse behavior.

The hybrid functions ($F_{11} \sim F_{20}$) present greater challenges than multimodal functions due to their non-separability subcomponents. The LDPSO acquires the best results on 3 functions (F_{11}, F_{15}, and F_{19}) and ranks second on F_{12}, F_{13}, F_{16}, and F_{17}. CDLPSO outperforms its competitors on 3 functions (F_{16}, F_{17}, and F_{20}), while EAPSO obtains the optimal performance on F_{12} and F_{18}. Additionally, AMSEPSO and APDPSO excel on F_{13} and F_{14}, respectively. In LDPSO, the number of leaders decreases as the search progresses. The global diversification gradually gives way to localized intensification, guiding the population to converge near the globally optima and carry out improved local search behavior.

The composition functions ($F_{21} \sim F_{30}$) present significant challenges due to their properties, including multi-modality, non-separability, asymmetry, and varying characteristics around different local optima. Thus, appropriate information interaction between leaders and followers is essential. The LDPSO demonstrates exceptional advantages in this group. It acquires the best results on 9 functions while slightly underperforms EAPSO on F_{25}. In LDPSO, the search experience of the leaders is effectively transferred to the followers, who serve as search assistants by conducting detailed search activities in the identified promising regions. Additionally, the jump-out strategy, which involves the cooperation between leaders and followers, fully utilizes their information interaction. This collaboration increases the likelihood of stagnated particles escaping from local optima.

Based on the Wilcoxon test results, LDPSO achieves statistically superior performance than GPSO, TVACPSO, SLPSO, and other variants on most benchmark functions. Meanwhile, LDPSO ranks within the top three on 26 functions, demonstrating both robustness and superiority across diverse problems.

Table 6 presents the comparative results under 50 dimensions. It can be seen that the LDPSO yields the best results on 18 functions and ranks in the top three on 25 functions. CDLPSO and EAPSO are the second-best methods since they acquire the best performance on 4 functions. TAPSO, AMSEPSO, and PSOsono each attain the optimal result on 1 function.

Specifically, LDPSO acquires the best results on the two unimodal functions (F_1 and F_3). For multimodal functions ($F_6 \sim F_{10}$), LDPSO demonstrates the best performance on 6 functions ($F_5 \sim F_{10}$), and ranks third on F_4, slightly underperforms EAPSO. For hybrid functions ($F_{11} \sim F_{20}$), the LDPSO ranks in the top three on 7 functions and acquires the best results on F_{13} and F_{20}. CDLPSO excels in solving hybrid functions since it obtains the best results on 3 functions (F_{11}, F_{16}, and F_{17}). Following closely behind is EAPSO, which performs best on F_{14} and F_{18}. Additionally, PSOsono and AMSEPSO obtain the best results on F_{15} and F_{19}, respectively. For composition functions ($F_{21} \sim F_{30}$), LDPSO ranks first on 8 out of 10 functions (except for F_{25} and F_{30}). Additionally, CDLPSO and TAPSO respectively performs best on F_{25} and F_{30}.

The Wilcoxon comparison results indicate that LDPSO outperforms or matches the performance of other competitors for most of the functions tested. It "wins over", "equals to", and "under-performs" the compared PSO variants in 271, 26, and 22 out of 319 (29×11) cases, respectively. This further confirms the effectiveness of the learning strategies in LDPSO.

Table 7 reports the comparison results under 100-dimension. It can be observed that the LDPSO demonstrates exceptional performance, achieving the best results on 16 functions and ranking in the top three across all problems. The second-best method, EAPSO, achieves the best results on 4 functions (F_4, F_{12}, F_{14}, and F_{28}), while CDLPSO yields the best results on 3 functions (F_{16}, F_{20}, and F_{29}). AMSEPSO obtains the best results on 3 functions (F_{15}, F_{19}, and F_{21}) and TAPSO ranks first on 2 functions (F_{18} and F_{25}). Notably, the worst ranking of LDPSO is third, indicating that its performance is more robust. Furthermore, the Wilcoxon test results indicate that LDPSO outperforms, ties with, and underperforms the other variants in 289, 19, and 11 out of 319 cases, respectively.

Fig. 9 depicts the convergence curves on 9 representative functions under 50 dimensions. Fig. 9a indicates that LDPSO is the most stable algorithm, and it demonstrates strong robustness and better solution accuracy. Fig. 9b,c reveal that LDPSO effectively avoids the attraction of local optima and achieves superior performance, despite its slower convergence in the initial stage. As shown in Fig. 9d, LDPSO converges more rapidly than most of PSO variants, and consistently approaches the global optimum. The comparisons in Fig. 9e,f indicate that LDPSO can easily escape from local optima and exhibits stable search behavior, which benefits from the organic coupling between leaders and followers. Fig. 9g–i demonstrate that while none of the methods can find the theoretical optimal value, but LDPSO provides a stable convergence process and yields the best results.

Moreover, the Friedman rank-and-sum results are listed in Table 8. It is evident that LDPSO achieves the best performance across various dimensions, demonstrating its unparalleled advantages. Meanwhile, the performance of LDPSO becomes increasingly competitive as the problem dimension rises, highlighting its potential to optimize complex problems. Additionally, all p-values are below 0.05, indicating significant differences among the results provided by different algorithms.

Furthermore, Fig. 10 provides the time consumption of different algorithms on the functions. It can be seen that GPSO has the shortest execution time, as it does not involve any additional operations. In contrast, the use of multiple elite archives and several learning strategies increases EAPSO's computational consumption. As previously discussed, LDPSO only requires additional sorting and division operations, resulting in a lower execution time than EAPSO, which places it in third position. However, its performance is highly effective, offering superior so-

Table 5
Comparisons between LDPSO and PSO variants under 30-D.

Func	Criteria	GPSO	TVACPSO	SLPSO	MPSO	TAPSO	AWPSO	APDPSO	AMSEPSO	PSOsono	CDLPSO	EAPSO	LDPSO
F_1	*Mean*	2.87E+09	2.35E+08	6.08E+03	3.45E+07	3.52E+03	2.04E+09	2.66E+09	3.33E+03	**1.20E+03**	3.42E+07	4.69E+03	1.57E+03
	Std	2.70E+09	4.89E+08	7.57E+03	6.44E+07	4.34E+03	1.90E+09	2.85E+09	3.82E+03	1.52E+03	1.87E+08	4.92E+03	1.60E+03
	Rank(W)	12(+)	9(+)	6(=)	8(+)	4(=)	10(+)	11(+)	3(=)	1(=)	7(+)	5(=)	2
F_3	*Mean*	8.48E+03	3.00E+02	4.12E+03	3.00E+02	**3.00E+02**	4.44E+03	3.60E+04	8.59E+02	3.00E+02	5.26E+02	3.00E+02	3.00E+02
	Std	1.06E+04	2.42E-11	4.77E+03	1.16E+00	2.00E-13	4.42E+03	5.64E+03	9.79E+02	3.90E-01	1.45E+02	3.93E-12	3.66E-10
	Rank(W)	11(+)	3(-)	9(+)	6(+)	1(-)	10(+)	12(+)	8(+)	5(+)	7(+)	2(-)	4
F_4	*Mean*	7.69E+02	5.12E+02	4.98E+02	4.70E+02	**4.26E+02**	7.64E+02	7.73E+02	5.27E+02	5.00E+02	5.11E+02	4.32E+02	4.57E+02
	Std	2.83E+02	4.03E+01	1.58E+01	3.31E+01	3.12E+01	2.64E+02	3.00E+02	1.29E+01	5.85E+01	2.51E+01	3.13E+01	1.53E+01
	Rank(W)	11(+)	8(+)	5(+)	4(+)	1(-)	10(+)	12(+)	9(+)	6(+)	7(+)	2(=)	3
F_5	*Mean*	5.84E+02	5.67E+02	5.34E+02	5.62E+02	5.57E+02	5.87E+02	6.00E+02	5.47E+02	5.55E+02	6.63E+02	5.87E+02	**5.20E+02**
	Std	2.41E+01	1.72E+01	9.91E+00	1.40E+01	9.26E+00	1.99E+01	3.00E+01	4.25E+01	1.89E+01	1.67E+00	2.22E+01	2.62E+01
	Rank(W)	8(+)	7(+)	2(+)	6(+)	5(+)	10(+)	11(+)	3(=)	4(+)	12(+)	9(+)	1
F_6	*Mean*	6.03E+02	6.01E+02	6.00E+02	6.00E+02	6.00E+02	6.08E+02	6.19E+02	6.00E+02	6.02E+02	6.00E+02	6.01E+02	**6.00E+02**
	Std	2.11E+00	8.64E-01	1.35E-01	9.43E-01	1.15E-02	2.05E+00	2.39E+00	4.48E-04	1.23E+00	1.15E-01	2.32E+00	0.00E+00
	Rank(W)	10(+)	8(+)	5(+)	6(+)	3(+)	11(+)	12(+)	2(+)	9(+)	4(=)	7(=)	1
F_7	*Mean*	8.18E+02	8.04E+02	**7.61E+02**	7.98E+02	7.85E+02	8.26E+02	8.91E+02	8.12E+02	8.26E+02	8.78E+02	8.23E+02	7.62E+02
	Std	3.32E+01	1.80E+01	1.04E+01	2.11E+01	1.26E+01	3.60E+01	6.60E+01	5.14E+01	7.55E+01	1.58E+01	3.72E+01	4.45E+01
	Rank(W)	7(+)	5(+)	1(+)	4(+)	3(+)	10(+)	12(+)	6(+)	9(+)	11(+)	8(+)	2
F_8	*Mean*	8.77E+02	8.58E+02	**8.39E+02**	8.64E+02	8.53E+02	8.79E+02	8.85E+02	8.54E+02	8.39E+02	9.26E+02	8.79E+02	8.46E+02
	Std	1.62E+01	1.84E+01	1.23E+01	2.12E+01	1.26E+01	2.52E+00	1.86E+01	4.52E+01	1.18E+01	4.80E+01	2.53E+01	5.85E+01
	Rank(W)	8(+)	6(+)	1(+)	7(+)	4(+)	9(+)	11(+)	5(=)	2(+)	12(+)	10(+)	3
F_9	*Mean*	1.25E+03	9.45E+02	9.02E+02	1.05E+03	9.70E+02	1.58E+03	2.36E+03	9.01E+02	9.34E+02	**9.00E+02**	2.00E+03	9.00E+02
	Std	3.53E+02	5.30E+01	3.40E+00	1.13E+02	7.24E+01	9.82E+02	6.00E+02	1.13E+00	5.28E+01	0.00E+00	1.78E+03	1.39E-01
	Rank(W)	9(+)	6(+)	4(+)	8(+)	7(+)	10(+)	12(+)	3(+)	5(+)	1(=)	11(+)	2
F_{10}	*Mean*	4.06E+03	4.03E+03	**3.07E+03**	4.13E+03	3.45E+03	4.26E+03	5.87E+03	4.68E+03	4.68E+03	6.21E+03	4.19E+03	3.13E+03
	Std	7.83E+02	4.62E+02	6.01E+02	5.97E+02	5.51E+02	8.06E+02	1.37E+03	5.96E+02	5.14E+02	2.20E+03	7.49E+02	1.31E+03
	Rank(W)	5(+)	4(+)	1(=)	6(+)	3(+)	8(+)	11(+)	9(+)	10(+)	12(+)	7(+)	2
F_{11}	*Mean*	1.35E+03	1.26E+03	1.18E+03	1.21E+03	1.19E+03	1.34E+03	1.43E+03	1.15E+03	1.22E+03	1.14E+03	1.18E+03	**1.13E+03**
	Std	1.14E+02	5.46E+01	4.57E+01	5.16E+01	2.78E+01	2.43E+01	6.80E+01	4.08E+01	6.90E+01	2.84E+01	4.00E+01	2.70E+01
	Rank(W)	11(+)	9(+)	5(+)	7(+)	6(+)	10(+)	12(+)	3(+)	8(+)	2(=)	4(+)	1
F_{12}	*Mean*	7.17E+07	9.05E+06	3.48E+05	8.06E+05	2.12E+04	1.62E+07	2.40E+07	7.67E+04	4.75E+05	6.71E+06	**1.71E+04**	1.88E+04
	Std	1.45E+08	1.05E+07	4.09E+05	8.09E+05	1.66E+04	5.59E+06	4.88E+07	7.96E+04	6.62E+05	3.65E+07	1.18E+04	6.38E+03
	Rank(W)	12(+)	9(+)	5(+)	7(+)	3(=)	10(+)	11(+)	4(+)	6(+)	8(+)	1(=)	2
F_{13}	*Mean*	6.51E+06	2.42E+06	1.13E+04	1.63E+04	1.31E+04	8.03E+05	1.33E+04	**5.82E+03**	1.45E+04	6.01E+04	2.24E+04	9.85E+03
	Std	1.79E+07	1.31E+07	9.60E+03	7.47E+03	1.31E+04	1.71E+06	2.44E+03	4.34E+03	1.30E+04	9.92E+03	2.28E+04	6.29E+03
	Rank(W)	12(+)	11(+)	3(=)	7(+)	4(=)	10(+)	5(+)	1(-)	6(=)	9(+)	8(=)	2
F_{14}	*Mean*	4.68E+04	4.10E+03	2.95E+04	2.09E+03	3.48E+03	**1.97E+04**	1.97E+03	5.95E+03	2.30E+03	4.75E+03	2.54E+03	3.30E+03
	Std	6.49E+04	2.89E+03	4.55E+04	9.93E+02	3.56E+03	1.37E+04	1.97E+03	5.64E+03	2.29E+03	1.93E+03	1.07E+03	1.26E+03
	Rank(W)	12(+)	7(=)	11(+)	2(-)	6(-)	10(+)	1(-)	9(=)	3(-)	8(+)	4(-)	5
F_{15}	*Mean*	3.87E+04	1.62E+04	3.67E+03	3.09E+03	4.31E+03	6.23E+04	4.16E+03	7.96E+03	5.31E+03	3.78E+04	1.48E+04	**2.52E+03**
	Std	4.29E+04	1.54E+04	3.63E+03	1.35E+03	3.47E+03	5.26E+04	2.13E+03	5.58E+03	5.67E+03	6.45E+03	1.10E+04	1.38E+03
	Rank(W)	11(+)	9(+)	3(+)	2(=)	5(+)	12(+)	4(+)	7(+)	6(+)	10(+)	8(+)	1
F_{16}	*Mean*	2.49E+03	2.31E+03	2.25E+03	2.52E+03	2.36E+03	2.50E+03	3.19E+03	2.07E+03	2.22E+03	**1.98E+03**	2.59E+03	2.01E+03
	Std	2.23E+02	2.15E+02	2.97E+02	2.88E+02	2.74E+02	3.33E+02	1.50E+02	3.58E+02	2.54E+02	3.84E+02	2.53E+02	2.80E+02
	Rank(W)	8(+)	6(+)	5(+)	10(+)	7(+)	9(+)	12(+)	3(=)	4(+)	1(=)	11(+)	2
F_{17}	*Mean*	2.01E+03	1.92E+03	1.90E+03	2.09E+03	1.98E+03	2.01E+03	2.29E+03	1.84E+03	1.89E+03	**1.74E+03**	2.19E+03	1.76E+03
	Std	2.09E+02	1.27E+02	1.30E+02	1.32E+02	1.64E+02	1.88E+02	3.36E+02	8.37E+01	1.46E+02	1.04E+01	2.63E+02	5.23E+01
	Rank(W)	9(+)	6(+)	5(+)	10(+)	7(+)	8(+)	12(+)	3(+)	4(+)	1(=)	11(+)	2
F_{18}	*Mean*	4.80E+05	1.13E+05	2.36E+05	3.41E+04	4.04E+04	1.97E+05	1.99E+04	3.36E+05	1.60E+05	3.37E+05	**1.95E+04**	6.81E+04
	Std	6.19E+05	5.20E+04	1.51E+05	3.20E+04	4.06E+04	5.98E+04	7.44E+03	3.56E+05	1.91E+05	1.16E+05	1.78E+04	2.61E+04
	Rank(W)	12(+)	6(+)	9(+)	3(-)	4(-)	8(+)	2(-)	10(+)	7(=)	11(+)	1(-)	5

Table 5
(Continued of Table 5).

Func	Criteria	GPSO	TVACPSO	SLPSO	MPSO	TAPSO	AWPSO	APDPSO	AMSEPSO	PSOsono	CDLPSO	EAPSO	LDPSO
F_{19}	*Mean*	3.51E+05	2.44E+04	7.35E+03	5.69E+03	7.09E+03	7.22E+04	1.36E+04	7.35E+03	5.45E+03	3.19E+04	1.40E+04	**4.42E+03**
	Std	1.06E+06	1.85E+04	3.72E+03	3.58E+03	7.39E+03	7.63E+04	2.46E+03	6.78E+03	7.00E+03	2.02E+04	1.46E+04	2.72E+03
	Rank(W)	12(+)	9(+)	6(+)	3(+)	4(=)	11(+)	7(+)	5(+)	2(-)	10(+)	8(+)	1
F_{20}	*Mean*	2.25E+03	2.17E+03	2.25E+03	2.35E+03	2.28E+03	2.25E+03	2.76E+03	2.10E+03	2.34E+03	**2.05E+03**	2.42E+03	2.15E+03
	Std	1.50E+02	8.72E+01	1.82E+02	1.36E+02	1.43E+02	3.86E+01	3.26E+02	8.16E+01	1.21E+02	1.25E+01	2.24E+02	2.84E+01
	Rank(W)	6(+)	4(=)	5(+)	10(+)	8(+)	7(+)	12(+)	2(-)	9(+)	1(-)	11(+)	3
F_{21}	*Mean*	2.40E+03	2.37E+03	2.34E+03	2.36E+03	2.36E+03	2.40E+03	2.40E+03	2.35E+03	2.34E+03	2.43E+03	2.39E+03	**2.33E+03**
	Std	1.92E+01	1.39E+01	9.24E+00	2.36E+01	1.37E+01	1.01E+01	3.34E+01	3.82E+01	1.69E+01	2.74E+01	3.37E+01	4.00E+01
	Rank(W)	9(+)	7(+)	2(+)	5(+)	6(+)	10(+)	11(+)	4(=)	3(+)	12(+)	8(+)	1
F_{22}	*Mean*	4.36E+03	3.41E+03	2.79E+03	2.70E+03	2.94E+03	4.57E+03	2.86E+03	2.53E+03	2.49E+03	2.36E+03	4.92E+03	**2.30E+03**
	Std	1.39E+03	1.34E+03	1.01E+03	1.17E+03	1.19E+03	1.02E+03	4.29E+02	8.79E+02	7.32E+02	1.07E+02	1.55E+03	0.00E+00
	Rank(W)	10(+)	9(+)	6(+)	5(+)	8(+)	11(+)	7(+)	4(+)	3(+)	2(+)	12(+)	1
F_{23}	*Mean*	2.82E+03	2.78E+03	2.70E+03	2.72E+03	2.73E+03	2.87E+03	2.83E+03	2.69E+03	2.70E+03	2.73E+03	2.72E+03	**2.67E+03**
	Std	6.77E+01	4.01E+01	1.09E+01	2.19E+01	2.00E+01	1.16E+01	3.56E+01	3.69E+01	1.64E+01	3.19E+01	2.45E+01	6.94E+00
	Rank(W)	10(+)	9(+)	4(+)	5(+)	8(+)	12(+)	11(+)	2(+)	3(+)	7(+)	6(+)	1
F_{24}	*Mean*	3.04E+03	2.98E+03	2.87E+03	2.90E+03	2.91E+03	3.01E+03	2.94E+03	2.89E+03	2.86E+03	3.01E+03	2.90E+03	**2.85E+03**
	Std	5.83E+01	3.66E+01	1.52E+01	2.17E+01	1.70E+01	1.85E+01	2.56E+01	4.28E+01	1.46E+01	1.99E+01	2.45E+01	4.25E+01
	Rank(W)	12(+)	9(+)	3(+)	5(+)	7(+)	11(+)	8(+)	4(+)	2(+)	10(+)	6(+)	1
F_{25}	*Mean*	2.93E+03	2.90E+03	2.89E+03	2.93E+03	2.89E+03	2.92E+03	2.95E+03	2.90E+03	2.91E+03	2.89E+03	**2.89E+03**	2.89E+03
	Std	3.85E+01	1.64E+01	9.19E+00	3.35E+01	9.90E+00	2.90E+01	1.70E+01	8.08E+00	1.46E+01	6.82E-02	1.60E+00	2.60E-01
	Rank(W)	10(+)	6(+)	5(+)	11(+)	4(+)	9(+)	12(+)	7(+)	8(+)	2(-)	1(=)	3
F_{26}	*Mean*	4.70E+03	4.90E+03	4.05E+03	3.73E+03	4.42E+03	5.29E+03	5.26E+03	3.92E+03	4.10E+03	4.32E+03	4.40E+03	**3.16E+03**
	Std	4.18E+02	4.55E+02	1.51E+02	1.09E+03	6.92E+02	2.84E+02	5.08E+02	3.57E+02	2.86E+02	6.36E+01	5.18E+02	4.38E+02
	Rank(W)	9(+)	10(+)	4(+)	2(+)	8(+)	12(+)	11(+)	3(+)	5(+)	6(+)	7(+)	1
F_{27}	*Mean*	3.29E+03	3.24E+03	3.22E+03	3.24E+03	3.21E+03	3.34E+03	3.30E+03	3.23E+03	3.22E+03	3.22E+03	3.23E+03	**3.21E+03**
	Std	4.66E+01	2.35E+01	1.14E+01	1.97E+01	1.10E+01	1.23E+01	5.11E+01	9.85E+00	2.81E+01	2.34E+01	1.56E+01	6.91E+00
	Rank(W)	10(+)	9(+)	4(+)	8(+)	2(=)	12(+)	11(+)	6(+)	5(=)	3(=)	7(+)	1
F_{28}	*Mean*	3.41E+03	3.32E+03	3.24E+03	3.32E+03	3.11E+03	3.34E+03	3.35E+03	3.24E+03	3.17E+03	3.30E+03	3.14E+03	**3.11E+03**
	Std	1.61E+02	2.50E+02	2.82E+01	4.58E+01	3.92E+01	7.30E+01	6.83E+01	5.13E+01	6.24E+01	1.06E+02	5.76E+01	3.48E+01
	Rank(W)	12(+)	9(+)	5(+)	8(+)	2(+)	10(+)	11(+)	6(+)	4(+)	7(+)	3(+)	1
F_{29}	*Mean*	3.66E+03	3.56E+03	3.55E+03	3.61E+03	3.64E+03	3.56E+03	4.62E+03	3.45E+03	3.77E+03	3.36E+03	3.77E+03	**3.33E+03**
	Std	1.83E+02	1.33E+02	1.27E+02	1.82E+02	1.73E+02	1.10E+02	3.24E+02	1.33E+02	2.17E+02	3.83E+01	1.99E+02	3.89E+01
	Rank(W)	9(+)	6(+)	4(+)	7(+)	8(+)	5(+)	12(+)	3(+)	10(+)	2(+)	11(+)	1
F_{30}	*Mean*	1.25E+06	5.91E+04	8.59E+03	6.75E+03	7.32E+03	5.49E+05	1.48E+05	7.17E+03	4.13E+04	1.56E+04	9.47E+03	**6.09E+03**
	Std	2.60E+06	1.05E+05	2.14E+03	1.79E+03	2.16E+03	6.11E+05	2.15E+05	1.29E+03	8.37E+04	8.59E+02	2.66E+03	1.01E+03
	Rank(W)	12(+)	9(+)	5(+)	2(+)	4(+)	11(+)	10(+)	3(+)	8(+)	7(+)	6(+)	1
	#Best	0	0	3	0	2	0	1	1	1	4	3	14
	#Defeated	29	26	26	26	20	29	27	21	23	21	20	
	#Equal	0	2	3	1	5	0	0	6	4	6	6	
	#Win	0	1	0	2	4	0	2	2	2	2	3	

Table 6
Comparisons between LDPSO and PSO variants under 50-D.

Func	Criteria	GPSO	TVACPSO	SLPSO	MPSO	TAPSO	AWPSO	APDPSO	AMSEPSO	PSOsono	CDLPSO	EAPSO	LDPSO
F_1	*Mean*	8.16E+09	2.24E+09	6.75E+03	1.42E+09	3.76E+03	1.20E+10	8.80E+09	3.18E+03	2.72E+03	2.11E+08	8.49E+03	**2.70E+03**
	Std	4.12E+09	2.59E+09	7.38E+03	1.05E+09	6.25E+03	5.49E+09	2.89E+09	4.81E+03	2.79E+03	1.16E+09	8.95E+03	3.23E+03
	Rank(W)	10(+)	9(+)	5(+)	8(+)	4(=)	12(+)	11(+)	3(=)	2(=)	7(+)	6(+)	1
F_3	*Mean*	3.50E+04	9.32E+02	7.28E+03	6.94E+02	6.32E+02	5.23E+04	1.18E+05	1.38E+04	4.10E+02	7.96E+03	4.04E+02	**3.01E+02**
	Std	3.83E+04	1.65E+03	5.32E+03	1.13E+03	5.70E+02	2.78E+04	2.35E+04	7.13E+03	1.13E+02	1.90E+03	3.59E+02	2.49E+00
	Rank(W)	10(+)	6(+)	7(+)	5(+)	4(+)	11(+)	12(+)	9(+)	3(+)	8(+)	2(-)	1
F_4	*Mean*	1.69E+03	7.21E+02	5.50E+02	7.60E+02	4.43E+02	1.39E+03	1.20E+03	6.55E+02	6.37E+02	5.95E+02	**4.27E+02**	4.63E+02
	Std	9.89E+02	1.36E+02	5.57E+01	1.80E+02	4.78E+01	3.60E+02	2.24E+02	2.14E+01	8.44E+01	3.53E+01	3.86E+01	3.29E+01
	Rank(W)	12(+)	8(+)	4(+)	9(+)	2(-)	11(+)	10(+)	7(+)	6(+)	5(+)	1(-)	3
F_5	*Mean*	6.85E+02	6.52E+02	5.79E+02	6.47E+02	6.26E+02	6.83E+02	7.56E+02	5.95E+02	5.89E+02	7.24E+02	6.69E+02	**5.36E+02**
	Std	3.41E+01	3.86E+01	2.07E+01	3.24E+01	3.24E+01	4.31E+01	4.29E+00	8.76E+01	2.38E+01	1.01E+02	4.67E+01	6.35E+00
	Rank(W)	10(+)	7(+)	2(+)	6(+)	5(+)	9(+)	12(+)	4(=)	3(+)	11(+)	8(+)	1
F_6	*Mean*	6.10E+02	6.06E+02	6.00E+02	6.02E+02	6.00E+02	6.07E+02	6.33E+02	6.00E+02	6.04E+02	6.00E+02	6.07E+02	**6.00E+02**
	Std	4.02E+00	2.63E+00	1.47E-01	2.16E+00	1.92E-02	2.86E+00	3.61E+00	2.42E-02	3.38E+00	1.81E-01	7.42E+00	1.75E-08
	Rank(W)	11(+)	8(+)	5(+)	6(+)	3(+)	9(+)	12(+)	2(+)	7(+)	4(+)	10(+)	1
F_7	*Mean*	9.38E+02	9.24E+02	8.22E+02	9.22E+02	8.72E+02	1.00E+03	1.17E+03	9.06E+02	8.66E+02	1.08E+03	9.53E+02	**8.21E+02**
	Std	6.35E+01	4.18E+01	1.31E+01	4.97E+01	2.94E+01	4.99E+01	8.08E+01	1.17E+02	5.23E+01	1.36E+01	6.27E+01	9.85E+01
	Rank(W)	8(+)	7(+)	2(+)	6(+)	4(+)	10(+)	12(+)	5(+)	3(+)	11(+)	9(+)	1
F_8	*Mean*	1.00E+03	9.43E+02	8.70E+02	9.65E+02	9.31E+02	9.69E+02	1.08E+03	8.74E+02	9.03E+02	1.09E+03	9.72E+02	**8.39E+02**
	Std	4.05E+01	3.48E+01	1.51E+01	3.29E+01	2.50E+01	2.81E+01	7.73E+01	7.57E+01	2.34E+01	7.78E+01	4.21E+01	8.23E+00
	Rank(W)	10(+)	6(+)	2(+)	7(+)	5(+)	8(+)	11(+)	3(=)	4(+)	12(+)	9(+)	1
F_9	*Mean*	3.78E+03	2.08E+03	9.80E+02	1.84E+03	1.82E+03	4.29E+03	7.53E+03	9.08E+02	1.12E+03	9.09E+02	6.81E+03	**9.02E+02**
	Std	1.66E+03	3.63E+02	9.32E+01	7.51E+02	5.59E+02	1.74E+03	3.80E+03	9.66E+00	2.39E+02	2.85E+01	3.82E+03	1.42E+00
	Rank(W)	9(+)	8(+)	4(+)	7(+)	6(+)	10(+)	12(+)	2(+)	5(+)	3(-)	11(+)	1
F_{10}	*Mean*	6.91E+03	7.27E+03	5.51E+03	6.75E+03	5.68E+03	8.29E+03	1.14E+04	7.44E+03	6.94E+03	1.16E+04	7.05E+03	**4.29E+03**
	Std	7.23E+02	9.52E+02	6.57E+02	1.14E+03	8.70E+02	2.84E+02	2.20E+03	7.34E+02	8.14E+02	1.17E+03	8.67E+02	7.52E+02
	Rank(W)	5(+)	8(+)	2(+)	4(+)	3(+)	10(+)	11(+)	9(+)	6(+)	12(+)	7(+)	1
F_{11}	*Mean*	1.82E+03	1.42E+03	1.33E+03	1.33E+03	1.23E+03	1.38E+03	3.34E+03	1.24E+03	1.34E+03	**1.14E+03**	1.24E+03	1.14E+03
	Std	6.77E+02	1.24E+02	5.53E+01	1.22E+02	2.98E+01	8.16E+01	1.29E+03	3.86E+01	6.12E+01	1.08E+01	5.68E+01	7.27E+00
	Rank(W)	11(+)	10(+)	6(+)	7(+)	3(+)	9(+)	12(+)	4(+)	8(+)	1(-)	5(+)	2
F_{12}	*Mean*	2.33E+09	5.27E+08	1.74E+06	6.32E+07	1.14E+05	5.83E+09	1.76E+09	2.04E+06	1.18E+06	4.54E+08	**5.81E+04**	8.59E+04
	Std	1.48E+09	6.52E+08	9.02E+05	1.01E+08	2.41E+05	2.38E+09	1.04E+09	1.62E+06	1.93E+06	8.87E+08	2.83E+04	3.66E+04
	Rank(W)	11(+)	9(+)	5(+)	7(+)	3(=)	12(+)	10(+)	6(+)	4(+)	8(+)	1(-)	2
F_{13}	*Mean*	6.97E+08	7.52E+07	1.13E+04	1.01E+05	8.67E+03	1.67E+08	2.17E+08	4.61E+03	7.38E+03	4.30E+08	1.01E+04	**2.58E+03**
	Std	1.24E+09	1.06E+08	9.25E+03	2.02E+05	6.44E+03	1.06E+08	6.19E+08	4.26E+03	4.38E+03	9.25E+08	7.96E+03	1.06E+03
	Rank(W)	12(+)	8(+)	6(+)	7(+)	4(+)	9(+)	10(+)	2(=)	3(+)	11(+)	5(+)	1
F_{14}	*Mean*	6.46E+05	5.27E+04	8.77E+04	1.20E+04	1.72E+04	5.03E+05	1.48E+05	6.90E+04	8.99E+03	1.92E+05	**5.13E+03**	1.54E+04
	Std	1.57E+06	6.36E+04	6.36E+04	1.98E+04	2.89E+04	5.61E+05	3.65E+05	7.47E+04	7.22E+03	4.59E+05	2.25E+03	8.22E+03
	Rank(W)	12(+)	6(=)	8(+)	3(-)	5(-)	11(+)	9(-)	7(+)	2(-)	10(+)	1(-)	4
F_{15}	*Mean*	1.64E+07	2.41E+06	9.50E+03	1.20E+04	9.30E+03	8.17E+04	6.54E+03	5.21E+03	**4.68E+03**	8.47E+07	9.43E+03	8.25E+03
	Std	7.70E+07	1.30E+07	5.22E+03	6.04E+03	8.22E+03	4.95E+04	2.06E+03	4.27E+03	3.24E+03	2.03E+08	7.02E+03	4.63E+03
	Rank(W)	11(+)	10(+)	7(=)	8(+)	5(=)	9(+)	3(-)	2(-)	1(-)	12(+)	6(=)	4
F_{16}	*Mean*	3.53E+03	3.25E+03	2.56E+03	2.98E+03	2.73E+03	3.00E+03	3.70E+03	2.83E+03	3.15E+03	**1.94E+03**	3.22E+03	2.03E+03
	Std	3.24E+02	5.03E+02	3.72E+02	3.71E+02	4.32E+02	4.94E+02	6.91E+02	5.38E+02	2.91E+02	1.33E+02	4.50E+02	1.64E+02
	Rank(W)	11(+)	10(+)	3(+)	6(+)	4(+)	7(+)	12(+)	5(+)	8(+)	1(-)	9(+)	2
F_{17}	*Mean*	2.95E+03	2.95E+03	2.60E+03	2.94E+03	2.85E+03	3.46E+03	3.46E+03	2.62E+03	2.91E+03	**1.97E+03**	3.03E+03	2.28E+03
	Std	2.12E+02	3.69E+02	2.16E+02	3.46E+02	3.24E+02	3.39E+02	2.90E+02	3.88E+02	2.55E+02	1.67E+02	3.58E+02	2.01E+02
	Rank(W)	8(+)	9(+)	3(+)	7(+)	5(+)	11(+)	12(+)	4(+)	6(+)	1(-)	10(+)	2
F_{18}	*Mean*	1.94E+06	2.38E+05	7.10E+05	1.10E+05	4.49E+04	4.65E+05	9.54E+04	1.22E+06	1.23E+06	1.36E+06	**1.59E+04**	5.15E+04
	Std	5.04E+06	2.06E+05	7.59E+05	6.27E+04	2.65E+04	6.39E+05	6.73E+04	1.02E+06	3.60E+06	9.14E+05	7.57E+03	1.87E+04
	Rank(W)	12(+)	6(+)	8(+)	5(+)	2(=)	7(+)	4(+)	9(+)	10(+)	11(+)	1(-)	3

Table 6
Continued of Table 6.

Func	Criteria	GPSO	TVACPSO	SLPSO	MPSO	TAPSO	AWPSO	APDPSO	AMSEPSO	PSOsono	CDLPSO	EAPSO	LDPSO
F_{19}	*Mean*	1.47E+06	2.17E+05	2.06E+04	1.76E+04	1.68E+04	2.80E+06	7.58E+04	**1.09E+04**	1.68E+04	1.71E+05	1.95E+04	1.92E+04
	Std	2.82E+06	3.63E+05	8.66E+03	4.75E+03	9.83E+03	3.27E+06	8.05E+04	6.01E+03	6.02E+03	8.76E+05	1.37E+04	4.96E+03
	Rank(W)	11(+)	10(=)	7(=)	4(=)	2(=)	12(+)	8(+)	1(-)	3(=)	9(-)	6(=)	5
F_{20}	*Mean*	2.78E+03	2.78E+03	2.71E+03	2.85E+03	2.72E+03	3.08E+03	4.04E+03	2.64E+03	2.61E+03	2.18E+03	2.97E+03	**2.16E+03**
	Std	2.02E+02	2.36E+02	2.56E+02	2.93E+02	1.99E+02	1.48E+02	2.15E+02	2.59E+02	2.78E+02	2.68E+02	3.22E+02	2.39E+02
	Rank(W)	7(+)	8(+)	5(+)	9(+)	6(+)	11(+)	12(+)	4(+)	3(+)	2(=)	10(+)	1
F_{21}	*Mean*	2.50E+03	2.46E+03	2.39E+03	2.44E+03	2.45E+03	2.54E+03	2.58E+03	2.39E+03	2.39E+03	2.53E+03	2.45E+03	**2.35E+03**
	Std	3.32E+01	2.69E+01	1.70E+01	3.22E+01	3.32E+01	5.20E+01	4.21E+01	9.00E+01	1.62E+01	1.02E+02	4.16E+01	6.58E+01
	Rank(W)	9(+)	8(+)	2(+)	5(+)	6(+)	11(+)	12(+)	4(=)	3(+)	10(+)	7(+)	1
F_{22}	*Mean*	8.99E+03	8.31E+03	6.59E+03	7.01E+03	6.91E+03	8.60E+03	1.25E+04	8.62E+03	7.79E+03	7.74E+03	8.67E+03	**4.80E+03**
	Std	1.67E+03	1.70E+03	1.05E+03	2.95E+03	2.06E+03	7.47E+02	1.48E+03	2.65E+03	2.25E+03	4.22E+03	9.90E+02	1.67E+03
	Rank(W)	11(+)	7(+)	2(+)	4(+)	3(+)	8(+)	12(+)	9(+)	6(+)	5(=)	10(+)	1
F_{23}	*Mean*	3.27E+03	3.05E+03	2.83E+03	2.93E+03	2.91E+03	3.21E+03	3.18E+03	2.80E+03	2.86E+03	2.92E+03	2.89E+03	**2.76E+03**
	Std	1.33E+02	8.88E+01	2.37E+01	6.95E+01	4.84E+01	8.04E+01	2.31E+01	7.05E+01	4.40E+01	5.23E+01	4.59E+01	1.30E+01
	Rank(W)	12(+)	9(+)	3(+)	8(+)	6(+)	11(+)	10(+)	2(+)	4(+)	7(+)	5(+)	1
F_{24}	*Mean*	3.44E+03	3.25E+03	3.00E+03	3.05E+03	3.12E+03	3.47E+03	3.32E+03	3.01E+03	2.99E+03	3.15E+03	3.06E+03	**2.96E+03**
	Std	1.39E+02	1.29E+02	2.61E+01	4.70E+01	3.66E+01	4.74E+01	5.69E+01	8.64E+01	1.58E+01	7.14E+01	4.68E+01	8.93E+01
	Rank(W)	11(+)	9(+)	3(+)	5(+)	7(+)	12(+)	10(+)	4(+)	2(+)	8(+)	6(+)	1
F_{25}	*Mean*	3.40E+03	3.10E+03	3.06E+03	3.30E+03	3.03E+03	3.17E+03	3.45E+03	3.09E+03	3.17E+03	**2.98E+03**	3.03E+03	3.04E+03
	Std	4.10E+02	5.83E+01	2.84E+01	1.33E+02	3.91E+01	1.98E+01	2.82E+01	1.66E+01	3.37E+01	4.62E-02	3.60E+01	2.85E+01
	Rank(W)	11(+)	7(+)	5(+)	10(+)	2(=)	9(+)	12(+)	6(+)	8(+)	1(-)	3(=)	4
F_{26}	*Mean*	7.46E+03	6.83E+03	4.68E+03	5.68E+03	5.33E+03	7.59E+03	8.22E+03	4.22E+03	4.83E+03	5.07E+03	5.63E+03	**3.59E+03**
	Std	8.78E+02	7.13E+02	1.76E+02	1.68E+03	1.31E+03	9.93E+02	2.93E+02	3.97E+02	1.01E+03	4.65E+02	4.66E+02	5.38E+02
	Rank(W)	10(+)	9(+)	3(+)	8(+)	6(+)	11(+)	12(+)	2(+)	4(+)	5(+)	7(+)	1
F_{27}	*Mean*	3.72E+03	3.61E+03	3.43E+03	3.49E+03	3.33E+03	3.81E+03	3.86E+03	3.33E+03	3.61E+03	3.61E+03	3.41E+03	**3.25E+03**
	Std	2.67E+02	1.52E+02	6.18E+01	1.23E+02	5.25E+01	2.05E+02	1.39E+02	5.10E+01	8.38E+01	1.68E+02	7.07E+01	1.61E+01
	Rank(W)	10(+)	8(+)	5(+)	6(+)	2(+)	11(+)	12(+)	3(+)	7(+)	9(+)	4(+)	1
F_{28}	*Mean*	4.78E+03	3.61E+03	3.34E+03	3.83E+03	3.28E+03	4.74E+03	4.24E+03	3.34E+03	3.46E+03	4.32E+03	3.30E+03	**3.28E+03**
	Std	1.06E+03	3.67E+02	3.03E+01	2.64E+02	2.18E+01	7.90E+02	2.17E+02	2.32E+01	6.32E+01	1.13E+03	3.33E+01	2.37E+01
	Rank(W)	12(+)	7(+)	5(+)	8(+)	2(=)	11(+)	9(+)	4(+)	6(+)	10(+)	3(+)	1
F_{29}	*Mean*	4.66E+03	4.15E+03	3.92E+03	4.17E+03	3.86E+03	4.41E+03	6.03E+03	3.56E+03	4.15E+03	3.63E+03	4.17E+03	**3.36E+03**
	Std	5.74E+02	2.57E+02	3.03E+02	4.18E+02	2.57E+02	4.74E+02	4.36E+02	3.15E+02	3.77E+02	5.78E+02	3.73E+02	1.03E+02
	Rank(W)	11(+)	6(+)	5(+)	8(+)	4(+)	10(+)	12(+)	2(+)	7(+)	3(=)	9(+)	1
F_{30}	*Mean*	3.02E+07	4.78E+06	1.15E+06	1.42E+06	**7.62E+05**	2.19E+07	3.50E+07	1.14E+06	4.93E+06	9.01E+05	8.34E+05	8.44E+05
	Std	8.48E+07	3.65E+06	2.41E+05	6.19E+05	7.41E+04	5.06E+06	1.49E+07	2.80E+05	4.04E+06	4.94E+05	1.46E+05	4.37E+04
	Rank(W)	11(+)	8(+)	6(+)	7(+)	1(-)	10(+)	12(+)	5(+)	9(+)	4(-)	2(=)	3
	#Best	0	0	0	0	1	0	0	1	1	4	4	18
	#Defeated	27	28	24	29	26	26	25	22	26	29	27	
	#Equal	1	1	5	0	2	1	2	7	3	0	2	
	#Win	1	0	0	0	1	2	2	0	0	0	0	

Table 7
Comparisons between LDPSO and PSO variants under 100-D.

Func	Criteria	GPSO	TVACPSO	SLPSO	MPSO	TAPSO	AWPSO	APDPSO	AMSEPSO	PSOsono	CDLPSO	EAPSO	LDPSO
F_1	*Mean*	4.54E+10	1.50E+10	8.69E+03	1.94E+10	1.15E+04	6.40E+10	8.95E+10	1.37E+05	1.67E+04	4.67E+08	1.43E+04	**5.16E+03**
	Std	1.69E+10	7.98E+09	1.19E+04	7.87E+09	1.07E+04	1.85E+10	1.44E+10	6.65E+05	4.97E+04	2.37E+09	1.53E+04	6.86E+03
	Rank(W)	10(+)	8(+)	2(+)	9(+)	3(+)	11(+)	12(+)	6(+)	5(+)	7(=)	4(+)	1
F_3	*Mean*	1.71E+05	4.13E+04	1.84E+05	1.65E+04	3.75E+04	1.13E+05	3.15E+05	1.29E+05	8.43E+03	9.68E+04	6.63E+04	**5.21E+03**
	Std	4.92E+04	1.64E+04	5.95E+04	7.48E+03	1.13E+04	5.63E+04	1.94E+04	3.25E+04	1.28E+04	1.09E+04	2.73E+04	1.36E+03
	Rank(W)	10(+)	5(+)	11(+)	3(+)	4(+)	8(+)	12(+)	9(+)	2(=)	7(+)	6(+)	1
F_4	*Mean*	5.01E+03	1.73E+03	6.58E+02	3.02E+03	5.95E+02	5.06E+03	1.22E+04	8.40E+02	1.01E+03	7.54E+02	**5.15E+02**	5.89E+02
	Std	1.74E+03	4.58E+02	3.22E+01	1.24E+03	5.12E+01	1.39E+03	1.59E+03	4.82E+01	1.81E+02	6.49E+01	6.74E+01	3.42E+01
	Rank(W)	10(+)	8(+)	4(+)	9(+)	3(=)	11(+)	12(+)	6(+)	7(+)	5(+)	1(-)	2
F_5	*Mean*	1.02E+03	9.02E+02	6.74E+02	9.82E+02	8.97E+02	1.09E+03	1.52E+03	6.17E+02	7.79E+02	1.11E+03	9.42E+02	**5.94E+02**
	Std	6.15E+01	4.83E+01	2.23E+01	5.75E+01	5.23E+01	6.60E+01	2.34E+02	1.16E+02	3.27E+01	1.12E+02	7.37E+01	1.26E+01
	Rank(W)	9(+)	6(+)	3(+)	8(+)	5(+)	10(+)	12(+)	2(=)	4(+)	11(+)	7(+)	1
F_6	*Mean*	6.24E+02	6.19E+02	6.00E+02	6.15E+02	6.00E+02	6.28E+02	6.62E+02	6.00E+02	6.10E+02	6.00E+02	6.25E+02	**6.00E+02**
	Std	4.77E+00	3.16E+00	1.42E-01	4.25E+00	2.95E-02	4.30E+00	5.71E+00	2.54E-01	3.69E+00	2.36E-02	7.66E+00	2.01E-02
	Rank(W)	9(+)	8(+)	4(+)	7(+)	3(+)	11(+)	12(+)	5(+)	6(+)	2(+)	10(+)	1
F_7	*Mean*	1.58E+03	1.50E+03	9.70E+02	1.53E+03	1.30E+03	1.54E+03	2.34E+03	1.20E+03	1.18E+03	1.59E+03	1.57E+03	**8.86E+02**
	Std	1.39E+02	1.69E+02	3.37E+01	1.56E+02	9.40E+01	1.13E+02	2.27E+02	2.86E+02	7.49E+01	1.31E+01	1.18E+02	1.24E+01
	Rank(W)	10(+)	6(+)	2(+)	7(+)	5(+)	8(+)	12(+)	4(+)	3(+)	11(+)	9(+)	1
F_8	*Mean*	1.35E+03	1.23E+03	9.73E+02	1.28E+03	1.21E+03	1.34E+03	1.66E+03	9.47E+02	1.11E+03	1.55E+03	1.27E+03	**9.10E+02**
	Std	7.33E+01	5.66E+01	3.76E+01	6.42E+01	6.27E+01	6.60E+01	1.54E+01	1.49E+02	6.91E+01	1.07E+01	9.30E+01	1.27E+02
	Rank(W)	10(+)	6(+)	3(+)	8(+)	5(+)	9(+)	12(+)	2(=)	4(+)	11(+)	7(+)	1
F_9	*Mean*	2.50E+04	5.68E+03	1.13E+03	9.39E+03	1.01E+04	1.17E+04	4.88E+04	1.03E+03	3.84E+03	1.08E+03	1.66E+04	**9.73E+02**
	Std	1.49E+04	1.47E+03	2.77E+02	2.93E+03	2.83E+03	5.52E+03	7.24E+03	1.06E+02	1.49E+03	8.79E+01	6.43E+03	2.29E+01
	Rank(W)	11(+)	6(+)	4(+)	7(+)	8(+)	9(+)	12(+)	2(+)	5(+)	3(+)	10(+)	1
F_{10}	*Mean*	1.51E+04	1.48E+04	**9.62E+03**	1.47E+04	1.35E+04	1.64E+04	3.00E+04	1.61E+04	1.47E+04	2.35E+04	1.43E+04	1.26E+04
	Std	1.54E+03	1.16E+03	1.24E+03	1.06E+03	1.24E+03	6.35E+02	4.33E+03	1.23E+03	1.07E+03	3.80E+03	1.67E+03	5.69E+03
	Rank(W)	8(+)	7(+)	1(-)	5(+)	3(+)	10(+)	12(+)	9(+)	6(+)	11(+)	4(+)	2
F_{11}	*Mean*	1.16E+04	2.72E+03	3.08E+03	2.95E+03	1.45E+03	1.32E+04	8.23E+04	1.99E+03	5.09E+03	1.42E+03	1.75E+03	**1.23E+03**
	Std	1.36E+04	4.31E+02	1.39E+03	6.95E+02	8.40E+01	1.26E+04	2.72E+04	2.26E+02	1.96E+03	3.44E+01	1.54E+02	4.02E+01
	Rank(W)	10(+)	6(+)	8(+)	7(+)	3(+)	11(+)	12(+)	5(+)	9(+)	2(+)	4(+)	1
F_{12}	*Mean*	1.52E+10	4.38E+09	3.03E+06	2.62E+09	5.88E+05	1.75E+10	1.59E+10	1.83E+07	5.44E+07	1.72E+09	**1.99E+05**	2.56E+05
	Std	9.41E+09	4.27E+09	2.54E+06	2.35E+09	3.30E+05	5.94E+09	6.63E+09	9.79E+06	6.65E+07	4.10E+08	9.78E+04	7.84E+04
	Rank(W)	10(+)	9(+)	4(+)	8(+)	3(+)	12(+)	11(+)	5(+)	6(+)	7(+)	1(-)	2
F_{13}	*Mean*	2.06E+09	3.76E+08	7.07E+03	1.39E+07	6.88E+03	1.31E+09	1.01E+09	4.04E+03	9.60E+03	7.48E+08	8.44E+03	**3.58E+03**
	Std	1.57E+09	5.23E+08	4.62E+03	4.26E+07	5.67E+03	7.96E+08	7.45E+08	2.26E+03	6.64E+03	3.34E+08	8.58E+03	2.76E+03
	Rank(W)	12(+)	8(+)	4(+)	7(+)	3(+)	11(+)	10(+)	2(=)	6(+)	9(+)	5(+)	1
F_{14}	*Mean*	3.78E+06	5.03E+05	4.71E+05	8.57E+04	2.49E+04	1.21E+06	5.78E+05	1.21E+06	3.39E+05	2.67E+05	**2.15E+04**	3.26E+04
	Std	6.37E+06	4.63E+05	3.17E+05	6.92E+04	1.59E+04	9.81E+05	3.87E+05	7.90E+05	9.14E+05	6.28E+04	1.64E+04	9.73E+03
	Rank(W)	12(+)	8(+)	7(+)	4(+)	2(-)	11(+)	9(+)	10(+)	6(+)	5(+)	1(-)	3
F_{15}	*Mean*	8.33E+08	1.34E+08	4.18E+03	5.95E+04	3.35E+03	4.95E+08	3.51E+08	**3.12E+03**	1.90E+04	3.24E+05	7.33E+03	3.69E+03
	Std	9.30E+08	3.55E+08	2.24E+03	2.32E+05	3.55E+03	5.44E+08	4.71E+08	3.43E+03	7.86E+04	1.63E+06	7.19E+03	2.38E+03
	Rank(W)	12(+)	9(+)	4(=)	7(+)	2(=)	11(+)	10(+)	1(=)	6(+)	8(+)	5(+)	3
F_{16}	*Mean*	6.81E+03	5.64E+03	3.83E+03	5.90E+03	5.27E+03	5.43E+03	7.46E+03	5.38E+03	5.62E+03	**2.29E+03**	5.19E+03	2.88E+03
	Std	1.15E+03	7.24E+02	5.86E+02	7.58E+02	6.12E+02	5.17E+02	7.14E+02	1.18E+03	7.75E+02	4.55E+02	6.23E+02	2.81E+02
	Rank(W)	11(+)	9(+)	3(+)	10(+)	5(+)	7(+)	12(+)	6(+)	8(+)	1(-)	4(+)	2
F_{17}	*Mean*	7.02E+03	5.38E+03	3.72E+03	4.70E+03	4.35E+03	6.99E+03	6.91E+03	5.05E+03	4.61E+03	3.66E+03	4.58E+03	**2.94E+03**
	Std	1.93E+03	6.61E+02	5.49E+02	5.78E+02	3.82E+02	7.98E+02	1.64E+03	8.74E+02	4.25E+02	1.46E+03	6.34E+02	2.98E+02
	Rank(W)	12(+)	9(+)	3(+)	7(+)	4(+)	11(+)	10(+)	8(+)	6(+)	2(=)	5(+)	1
F_{18}	*Mean*	5.07E+06	1.10E+06	5.26E+05	2.41E+05	**9.18E+04**	4.00E+06	8.92E+05	2.43E+06	9.86E+05	8.70E+05	9.94E+04	1.06E+05
	Std	5.30E+06	1.60E+06	3.37E+05	1.11E+05	4.27E+04	3.21E+06	5.63E+05	1.35E+06	1.88E+06	3.81E+05	5.70E+04	2.37E+04
	Rank(W)	12(+)	9(+)	5(+)	4(+)	1(-)	11(+)	7(+)	10(+)	8(+)	6(+)	2(=)	3

Table 7
Continued of Table 7.

Func	Criteria	GPSO	TVACPSO	SLPSO	MPSO	TAPSO	AWPSO	APDPSO	AMSEPSO	PSOsono	CDLPSO	EAPSO	LDPSO
F_{19}	*Mean*	5.96E+08	1.85E+08	5.54E+03	2.94E+04	6.25E+03	5.10E+08	1.38E+08	**3.31E+03**	5.59E+03	2.08E+08	6.45E+03	3.33E+03
	Std	7.82E+08	2.57E+08	3.68E+03	7.54E+04	4.76E+03	3.36E+08	1.20E+08	1.71E+03	3.80E+03	2.57E+08	5.02E+03	2.04E+03
	Rank(W)	12(+)	9(+)	3(+)	7(+)	5(+)	11(+)	8(+)	1(=)	4(+)	10(+)	6(+)	2
F_{20}	*Mean*	4.90E+03	4.86E+03	3.74E+03	4.40E+03	4.40E+03	5.00E+03	6.39E+03	5.07E+03	4.38E+03	**2.66E+03**	5.13E+03	3.00E+03
	Std	5.82E+02	4.69E+02	4.32E+02	5.39E+02	4.39E+02	4.20E+02	1.15E+03	5.18E+02	3.02E+02	5.88E+02	4.94E+02	4.89E+02
	Rank(W)	8(+)	7(+)	3(+)	5(+)	6(+)	9(+)	12(+)	10(+)	4(+)	1(-)	11(+)	2
F_{21}	*Mean*	3.09E+03	2.90E+03	2.53E+03	2.83E+03	2.76E+03	3.17E+03	3.32E+03	**2.43E+03**	2.63E+03	2.69E+03	2.78E+03	2.44E+03
	Std	8.64E+01	7.75E+01	3.17E+01	9.84E+01	5.96E+01	1.11E+02	5.05E+01	2.04E+01	8.16E+01	2.40E+02	7.26E+01	1.25E+02
	Rank(W)	10(+)	9(+)	3(+)	8(+)	6(+)	11(+)	12(+)	1(=)	4(+)	5(+)	7(+)	2
F_{22}	*Mean*	1.77E+04	1.65E+04	1.21E+04	1.60E+04	1.57E+04	1.86E+04	2.70E+04	1.84E+04	1.56E+04	1.19E+04	1.72E+04	**1.17E+04**
	Std	1.42E+03	2.10E+03	1.04E+03	4.50E+03	1.37E+03	1.27E+03	1.66E+03	1.17E+03	1.83E+03	5.60E+03	1.65E+03	5.28E+03
	Rank(W)	9(+)	7(+)	3(=)	6(+)	5(+)	11(+)	12(+)	10(+)	4(+)	2(=)	8(+)	1
F_{23}	*Mean*	4.48E+03	3.88E+03	3.06E+03	3.41E+03	3.15E+03	4.39E+03	4.49E+03	2.95E+03	3.22E+03	3.51E+03	3.25E+03	**2.91E+03**
	Std	2.66E+02	1.75E+02	3.41E+01	1.77E+02	4.46E+01	2.56E+02	8.06E+01	1.98E+01	8.56E+01	8.69E+01	7.77E+01	1.50E+01
	Rank(W)	11(+)	9(+)	3(+)	7(+)	4(+)	10(+)	12(+)	2(+)	5(+)	8(+)	6(+)	1
F_{24}	*Mean*	5.61E+03	5.11E+03	3.51E+03	3.95E+03	3.82E+03	5.77E+03	5.89E+03	3.43E+03	3.69E+03	4.79E+03	3.75E+03	**3.35E+03**
	Std	4.06E+02	4.08E+02	5.63E+01	2.22E+02	7.16E+01	4.23E+02	9.05E+01	2.62E+01	6.69E+01	5.43E+02	9.36E+01	1.38E+01
	Rank(W)	10(+)	9(+)	3(+)	7(+)	6(+)	11(+)	12(+)	2(+)	4(+)	8(+)	5(+)	1
F_{25}	*Mean*	5.10E+03	4.15E+03	3.33E+03	5.07E+03	**3.25E+03**	5.33E+03	7.77E+03	3.60E+03	3.83E+03	3.28E+03	3.25E+03	3.27E+03
	Std	7.52E+02	6.44E+02	5.02E+01	6.42E+02	7.30E+01	9.66E+02	7.36E+02	4.78E+01	1.19E+02	1.46E+01	7.17E+01	4.41E+01
	Rank(W)	10(+)	8(+)	5(+)	9(+)	1(=)	11(+)	12(+)	6(+)	7(+)	4(+)	2(=)	3
F_{26}	*Mean*	2.11E+04	2.06E+04	8.03E+03	1.80E+04	1.03E+04	2.19E+04	2.64E+04	7.06E+03	9.28E+03	1.92E+04	1.11E+04	**6.39E+03**
	Std	3.22E+03	3.59E+03	4.22E+02	4.68E+03	2.57E+03	3.76E+03	2.85E+03	3.76E+02	8.16E+02	1.64E+03	9.81E+02	1.67E+02
	Rank(W)	10(+)	9(+)	3(+)	7(+)	5(+)	11(+)	12(+)	2(+)	4(+)	8(+)	6(+)	1
F_{27}	*Mean*	4.51E+03	4.05E+03	3.50E+03	3.74E+03	3.54E+03	4.67E+03	4.20E+03	3.47E+03	4.02E+03	3.60E+03	3.56E+03	**3.42E+03**
	Std	3.71E+02	3.00E+02	4.64E+01	1.16E+02	6.77E+01	4.02E+02	1.76E+02	2.97E+01	1.87E+02	5.73E+01	8.54E+01	2.70E+01
	Rank(W)	11(+)	9(+)	3(+)	7(+)	4(+)	12(+)	10(+)	2(+)	8(+)	6(+)	5(+)	1
F_{28}	*Mean*	9.14E+03	6.78E+03	3.45E+03	7.05E+03	3.35E+03	1.03E+04	1.27E+04	3.60E+03	4.20E+03	7.26E+03	**3.32E+03**	3.38E+03
	Std	2.41E+03	2.21E+03	2.95E+01	1.04E+03	3.04E+01	9.07E+02	1.25E+03	3.50E+01	3.72E+02	1.83E+03	4.67E+01	4.30E+01
	Rank(W)	10(+)	7(+)	4(+)	8(+)	2(-)	11(+)	12(+)	5(+)	6(+)	9(+)	1(-)	3
F_{29}	*Mean*	7.32E+03	6.84E+03	5.42E+03	6.82E+03	6.16E+03	7.24E+03	1.13E+04	5.99E+03	6.85E+03	**4.21E+03**	6.31E+03	4.47E+03
	Std	6.42E+02	6.07E+02	4.80E+02	5.61E+02	4.86E+02	5.21E+02	1.23E+03	1.04E+03	7.56E+02	1.86E+02	3.78E+02	3.52E+02
	Rank(W)	11(+)	8(+)	3(+)	7(+)	5(+)	10(+)	12(+)	4(+)	9(+)	1(-)	6(+)	2
F_{30}	*Mean*	2.27E+09	2.61E+08	1.47E+04	4.03E+07	9.36E+03	2.10E+09	2.25E+09	1.79E+04	8.85E+05	2.44E+05	9.62E+03	**9.26E+03**
	Std	1.61E+09	4.43E+08	7.23E+03	7.55E+07	3.29E+03	1.40E+09	9.73E+08	6.02E+03	1.77E+06	1.85E+05	3.38E+03	2.30E+03
	Rank(W)	12(+)	9(+)	4(+)	8(+)	2(=)	10(+)	11(+)	5(+)	7(+)	6(+)	3(=)	1
	#Best	0	0	1	0	2	0	0	3	0	3	4	16
	#Defeated	27	29	27	29	27	28	28	23	27	29	29	
	#Equal	0	0	2	0	1	1	0	6	2	0	0	
	#Win	2	0	0	0	1	0	1	0	0	0	0	

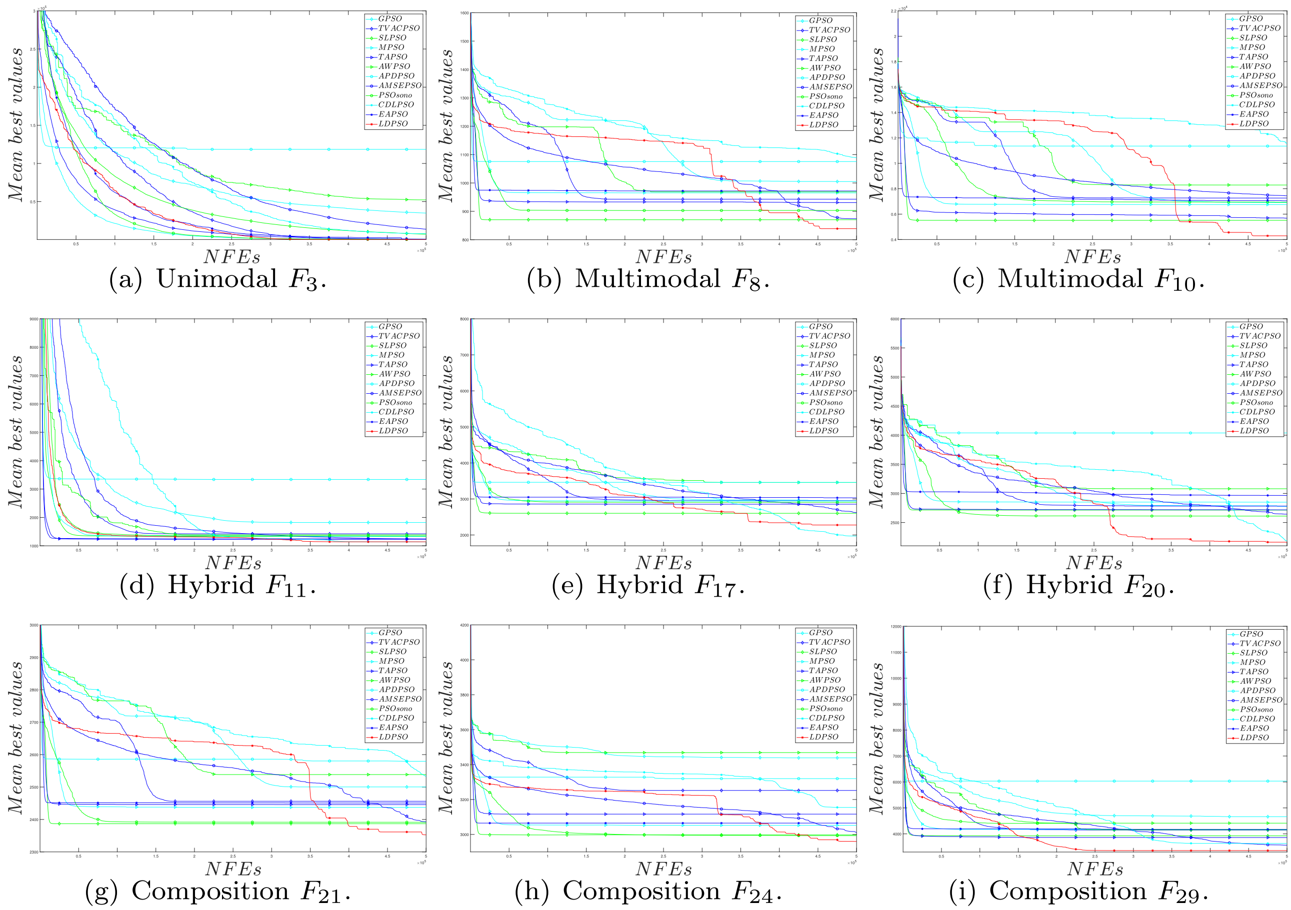


Fig. 9. Convergence performance on some representative functions under 50-D. a. Unimodal F_3. b. Multimodal F_8. c. Multimodal F_{10}. d. Hybrid F_{11}. e. Hybrid F_{17}. f. Hybrid F_{20}. g. Composition F_{21}. h. Composition F_{24}. i. Composition F_{29}.

Table 8
The Friedman rank-and-sum results for the comparison with PSO variants.

Algorithm	30-D		50-D		100-D	
	rank	p-value	rank	p-value	rank	p-value
GPSO	9.85		10.18		10.28	
TVACPSO	7.38		7.92		7.78	
SLPSO	4.65		4.55		3.95	
MPSO	6.02		6.55		6.98	
TAPSO	4.95		4.02		3.98	
AWPSO	9.75	1.54E-26	9.95	2.50E-35	10.22	2.03E-40
APDPSO	9.75		10.48		11.08	
AMSEPSO	4.78		4.52		4.95	
PSOsono	5.45		4.98		5.65	
CDLPSO	6.62		6.92		6.08	
EAPSO	6.72		5.98		5.25	
LDPSO	**2.08**		**1.95**		**1.78**	

lutions compared to both GPSO and CDLPSO. Therefore, the increased time consumption in LDPSO is acceptable.

4.6. Comparison between LDPSO and state-of-the-art non-PSO methods

To further evaluate the competitiveness of LDPSO, we compare it with state-of-the-art non-PSO methods, including coati optimization algorithm (COA) (Dehghani, Montazeri, Trojovská, & Trojovskỳ, 2023), kepler optimization algorithm (KOA) (Abdel-Basset, Mohamed, Azeem, Jameel, & Abouhawwash, 2023), optical microscope algorithm (OMA) (Cheng & Sholeh, 2023), rime-ice based optimizer (RIME) (Su et al., 2023), snow ablation optimizer (SAO) (Deng & Liu, 2023), electric eel foraging optimization algorithm (EEFO) (Zhao et al., 2024b), human evolutionary optimization algorithm (HEOA) (Lian & Hui, 2024), walrus optimizer (WO) (Han et al., 2024), chaotic evolution optimization algorithm (CEO) (Dong, Zhang, Zhang, Zhou, & Jiang, 2025), dream optimization algorithm (DOA) (Lang & Gao, 2025), and weighted average algorithm (WAA) (Cheng & De Waele, 2024). The parameter settings for these methods are consistent with their respective references. Each method is executed independently 30 times, and the computational results are reported in Tables 9–11.

The results presented in Table 9 show that the LDPSO yields the best results on 22 out of 29 functions and ranks the top three across all tested problems. DOA performs the best on two functions (F_{10} and F_{25}), while KOA is good with hybrid problem F_{19}. Also, OMA demonstrates superior performance on hybrid function F_{14}. RIME, SAO, EEFO yield the best results on F_{20}, F_4, and F_{18}, respectively. According to the results of the Wilcoxon test, LDPSO outperforms the non-PSO methods on the majority of functions. It “wins over”, “equals to”, and “under-performs” the compared methods on 295, 17, and 7 out of 319 (29×11) cases, respectively.

As shown in Table 10, the LDPSO yields the best results on 25 functions and ranks in the top three on 28 out of 29 problems, which confirms its competitiveness. The RIME benefits from various search mechanisms and obtains the best results on 2 hybrid functions (F_{15} and F_{19}). For composition functions, LDPSO ranks second on F_{26} and F_{30}, slightly underperforms WO and KOA, respectively. The Wilcoxon test results listed in Table 10 indicate that LDPSO outperforms the other methods

Table 9
Comparisons between LDPSO and non-PSO methods under 30-D.

Func	Criteria	COA	KOA	OMA	RIME	SAO	EEFO	HEOA	WO	CEO	DOA	WAA	LDPSO
F_1	*Mean*	5.00E+10	7.23E+04	2.51E+03	7.39E+03	3.31E+03	2.29E+03	1.59E+08	6.51E+03	1.18E+11	1.89E+05	1.25E+06	**1.57E+03**
	Std	5.29E+09	2.93E+04	2.93E+03	3.30E+03	2.96E+03	2.45E+03	3.40E+07	5.61E+03	1.30E+10	3.49E+04	6.26E+04	1.60E+03
	Rank	11(+)	7(+)	3(=)	6(+)	4(+)	2(=)	10(+)	5(+)	12(+)	8(+)	9(+)	1
F_3	*Mean*	8.36E+04	1.07E+04	5.42E+03	3.03E+02	4.46E+04	3.89E+02	1.19E+04	1.55E+03	3.78E+05	9.95E+03	3.10E+02	**3.00E+02**
	Std	1.63E+03	3.09E+03	3.80E+03	1.08E+00	1.91E+04	7.24E+01	3.21E+03	9.32E+02	5.93E+04	3.61E+03	2.02E+00	3.66E-10
	Rank	11(+)	8(+)	6(+)	2(+)	10(+)	4(+)	9(+)	5(+)	12(+)	7(+)	3(+)	1
F_4	*Mean*	1.28E+04	5.11E+02	5.02E+02	4.98E+02	**4.46E+02**	4.85E+02	5.41E+02	4.97E+02	2.31E+04	4.90E+02	5.14E+02	4.57E+02
	Std	2.15E+03	8.92E+00	2.71E+01	1.36E+01	2.53E+01	2.23E+01	1.57E+01	1.46E+01	2.10E+03	1.94E+01	4.32E-01	1.53E+01
	Rank	11(+)	8(+)	7(+)	6(+)	1(-)	3(+)	10(+)	5(+)	12(+)	4(+)	9(+)	2
F_5	*Mean*	9.27E+02	5.56E+02	6.15E+02	5.66E+02	5.52E+02	5.93E+02	7.47E+02	5.89E+02	1.18E+03	5.46E+02	8.09E+02	**5.20E+02**
	Std	1.57E+01	1.77E+01	3.00E+01	4.98E+00	1.48E+01	1.96E+01	4.56E+01	5.23E+01	2.18E+01	7.38E+00	7.02E+00	2.62E+01
	Rank	11(+)	4(+)	8(+)	5(+)	3(+)	7(+)	9(+)	6(+)	12(+)	2(+)	10(+)	1
F_6	*Mean*	6.93E+02	6.00E+02	6.10E+02	6.00E+02	6.00E+02	6.03E+02	6.59E+02	6.04E+02	7.40E+02	6.00E+02	6.71E+02	**6.00E+02**
	Std	4.59E+00	6.01E-02	4.71E+00	1.04E-01	1.86E-01	1.91E+00	9.36E+00	4.22E+00	9.41E-01	1.27E-02	7.65E+00	0.00E+00
	Rank	11(+)	5(+)	8(+)	4(+)	2(+)	6(+)	9(+)	7(+)	12(+)	3(+)	10(+)	1
F_7	*Mean*	1.46E+03	8.20E+02	1.02E+03	8.07E+02	7.72E+02	8.65E+02	1.14E+03	8.23E+02	3.52E+03	8.20E+02	1.36E+03	**7.62E+02**
	Std	3.80E+01	1.06E+01	6.50E+01	1.01E+01	1.19E+01	3.72E+01	7.78E+01	4.16E+01	1.16E+02	4.68E+00	3.15E+01	4.45E+01
	Rank	11(+)	5(+)	8(+)	3(+)	2(+)	7(+)	9(+)	6(+)	12(+)	4(+)	10(+)	1
F_8	*Mean*	1.14E+03	8.67E+02	8.92E+02	8.74E+02	8.54E+02	8.90E+02	1.01E+03	8.90E+02	1.35E+03	8.65E+02	1.00E+03	**8.46E+02**
	Std	1.73E+01	7.80E+00	2.22E+01	6.53E+00	1.37E+01	2.00E+01	3.00E+01	5.12E+01	2.96E+01	4.02E+00	2.18E+01	5.85E+01
	Rank	11(+)	4(+)	8(+)	5(+)	2(+)	7(+)	10(+)	6(+)	12(+)	3(+)	9(+)	1
F_9	*Mean*	1.00E+04	1.16E+03	1.57E+03	9.36E+02	9.05E+02	1.45E+03	9.22E+03	1.55E+03	3.23E+04	9.52E+02	6.53E+03	**9.00E+02**
	Std	7.78E+02	5.99E+01	4.86E+02	1.55E+01	9.49E+00	2.91E+02	2.26E+03	8.07E+02	1.19E+02	2.64E+01	8.29E+02	1.39E-01
	Rank	11(+)	5(+)	8(+)	3(+)	2(+)	6(+)	10(+)	7(+)	12(+)	4(+)	9(+)	1
F_{10}	*Mean*	8.80E+03	4.25E+03	7.42E+03	3.27E+03	3.83E+03	3.99E+03	6.33E+03	6.55E+03	9.27E+03	**3.00E+03**	5.27E+03	3.13E+03
	Std	2.91E+02	8.77E+02	9.54E+02	4.22E+02	6.77E+02	3.64E+02	5.32E+02	2.12E+03	1.85E-12	3.56E+02	7.82E+02	1.31E+03
	Rank	11(+)	6(+)	10(+)	3(+)	4(+)	5(+)	8(+)	9(+)	12(+)	1(=)	7(+)	2
F_{11}	*Mean*	8.24E+03	1.20E+03	1.20E+03	1.19E+03	1.18E+03	1.18E+03	1.27E+03	1.23E+03	5.47E+04	1.20E+03	1.16E+03	**1.13E+03**
	Std	7.75E+02	3.11E+01	3.42E+01	2.25E+01	5.08E+01	3.41E+01	2.90E+01	4.99E+01	1.86E+01	5.04E+01	2.11E+00	2.70E+01
	Rank	11(+)	8(+)	6(+)	5(+)	4(+)	3(+)	10(+)	9(+)	12(+)	7(+)	2(+)	1
F_{12}	*Mean*	1.38E+10	5.51E+05	1.42E+05	4.62E+06	4.69E+04	7.02E+04	2.59E+07	4.20E+06	2.95E+10	1.65E+06	3.44E+06	**1.88E+04**
	Std	2.15E+09	2.59E+05	1.63E+05	1.30E+06	3.03E+04	7.62E+04	1.09E+07	3.27E+06	3.84E+09	1.47E+06	1.21E+06	6.38E+03
	Rank	11(+)	5(+)	4(+)	9(+)	2(+)	3(+)	10(+)	8(+)	12(+)	6(+)	7(+)	1
F_{13}	*Mean*	6.00E+09	3.97E+04	1.77E+04	9.99E+03	1.88E+04	1.14E+04	2.73E+06	2.93E+04	3.01E+10	4.16E+04	1.42E+05	**9.85E+03**
	Std	4.29E+09	1.25E+04	1.58E+04	5.57E+03	1.84E+04	1.37E+04	1.46E+06	2.10E+04	2.52E+09	2.44E+04	8.50E+04	6.29E+03
	Rank	11(+)	7(+)	4(=)	2(=)	5(=)	3(=)	10(+)	6(+)	12(+)	8(+)	9(+)	1
F_{14}	*Mean*	5.60E+06	1.02E+04	**3.30E+03**	1.27E+04	9.32E+03	6.38E+03	5.83E+04	4.24E+04	1.04E+07	2.38E+05	3.47E+04	3.30E+03
	Std	3.38E+06	8.17E+03	3.06E+03	1.90E+04	4.34E+03	5.90E+03	5.89E+04	3.85E+04	1.95E+06	1.64E+05	4.40E+03	1.26E+03
	Rank	11(+)	5(+)	1(-)	6(=)	4(+)	3(=)	9(+)	8(+)	12(+)	10(+)	7(+)	2
F_{15}	*Mean*	8.86E+08	6.39E+03	1.04E+04	1.07E+04	4.21E+03	7.55E+03	6.03E+04	7.77E+03	6.70E+09	8.51E+03	1.94E+04	**2.52E+03**
	Std	3.26E+08	9.34E+03	7.38E+03	4.18E+03	3.54E+03	7.98E+03	3.89E+04	6.07E+03	2.36E+09	1.72E+03	3.06E+03	1.38E+03
	Rank	11(+)	3(=)	7(+)	8(+)	2(+)	4(+)	10(+)	5(+)	12(+)	6(+)	9(+)	1
F_{16}	*Mean*	5.66E+03	2.56E+03	2.83E+03	2.25E+03	2.33E+03	2.41E+03	2.97E+03	2.45E+03	6.85E+03	2.56E+03	3.53E+03	**2.01E+03**
	Std	1.01E+03	2.26E+02	2.03E+02	8.55E+01	2.51E+02	2.92E+02	3.07E+02	3.40E+02	3.95E+02	1.38E+02	1.19E+02	2.80E+02
	Rank	11(+)	7(+)	8(+)	2(+)	3(+)	4(+)	9(+)	5(+)	12(+)	6(+)	10(+)	1
F_{17}	*Mean*	3.78E+03	1.77E+03	2.05E+03	2.12E+03	2.06E+03	2.04E+03	2.17E+03	2.08E+03	2.91E+04	1.92E+03	2.76E+03	**1.76E+03**
	Std	9.06E+02	4.34E+00	8.77E+01	6.03E+01	2.02E+02	1.84E+02	2.15E+02	2.93E+02	7.07E+03	4.43E+01	3.34E+02	5.23E+01
	Rank	11(+)	2(+)	5(+)	8(+)	6(+)	4(+)	9(+)	7(+)	12(+)	3(+)	10(+)	1
F_{18}	*Mean*	5.87E+07	7.03E+04	1.07E+05	9.95E+04	1.05E+05	**4.61E+04**	1.06E+06	4.50E+05	2.22E+08	2.56E+05	1.47E+05	6.81E+04
	Std	4.48E+07	4.72E+04	6.63E+04	6.57E+04	7.80E+04	4.08E+04	5.00E+05	2.96E+05	5.87E+01	2.03E+05	8.26E+04	2.61E+04
	Rank	11(+)	3(=)	6(+)	4(=)	5(=)	1(-)	10(+)	9(+)	12(+)	8(+)	7(+)	2

Table 9
Continued of Table 9.

Func	Criteria	COA	KOA	OMA	RIME	SAO	EEFO	HEOA	WO	CEO	DOA	WAA	LDPSO
F_{19}	*Mean*	4.44E+08	**4.34E+03**	1.19E+04	8.53E+03	8.34E+03	6.23E+03	3.90E+05	4.38E+03	1.05E+10	1.28E+04	7.00E+04	4.42E+03
	Std	1.59E+08	4.59E+03	1.17E+04	6.08E+03	9.38E+03	5.35E+03	2.67E+05	2.46E+03	3.37E+09	5.60E+03	3.47E+04	2.72E+03
	Rank	11(+)	1(-)	7(+)	6(+)	5(=)	4(=)	10(+)	2(=)	12(+)	8(+)	9(+)	3
F_{20}	*Mean*	2.99E+03	2.31E+03	2.34E+03	**2.14E+03**	2.29E+03	2.31E+03	2.64E+03	2.37E+03	3.60E+03	2.36E+03	2.78E+03	2.15E+03
	Std	1.37E+02	1.33E+02	6.74E+01	1.26E+02	1.52E+02	1.44E+02	2.16E+02	2.46E+02	7.11E+01	1.21E+02	1.21E+02	2.84E+01
	Rank	11(+)	5(+)	6(+)	1(-)	3(+)	4(+)	9(+)	8(+)	12(+)	7(+)	10(+)	2
F_{21}	*Mean*	2.71E+03	2.35E+03	2.39E+03	2.35E+03	2.36E+03	2.38E+03	2.52E+03	2.37E+03	2.96E+03	2.36E+03	2.66E+03	**2.33E+03**
	Std	4.32E+01	1.29E+01	2.38E+01	3.07E+00	1.45E+01	1.87E+01	6.73E+01	5.31E+01	4.63E-13	6.13E+00	6.48E+01	4.00E+01
	Rank	11(+)	2(+)	8(+)	3(+)	4(+)	7(+)	9(+)	6(+)	12(+)	5(+)	10(+)	1
F_{22}	*Mean*	9.03E+03	2.31E+03	2.30E+03	2.81E+03	2.49E+03	2.30E+03	7.39E+03	2.30E+03	1.03E+04	3.08E+03	7.73E+03	**2.30E+03**
	Std	8.35E+02	4.72E+00	3.36E+00	1.17E+03	7.19E+02	1.76E+00	1.55E+03	1.41E+00	1.26E+02	1.00E+03	4.45E+02	0.00E+00
	Rank	11(+)	5(+)	4(+)	7(+)	6(+)	3(+)	9(+)	2(+)	12(+)	8(+)	10(+)	1
F_{23}	*Mean*	3.50E+03	2.71E+03	2.80E+03	2.76E+03	2.71E+03	2.77E+03	3.01E+03	2.74E+03	3.46E+03	2.71E+03	3.53E+03	**2.67E+03**
	Std	1.31E+02	8.55E+00	2.85E+01	1.32E+01	1.44E+01	3.08E+01	7.81E+01	4.02E+01	7.83E+01	3.86E+00	1.35E+02	6.94E+00
	Rank	11(+)	3(+)	8(+)	6(+)	2(+)	7(+)	9(+)	5(+)	10(+)	4(+)	12(+)	1
F_{24}	*Mean*	3.80E+03	2.92E+03	2.99E+03	2.93E+03	2.88E+03	2.94E+03	3.23E+03	2.91E+03	3.76E+03	2.92E+03	3.81E+03	**2.85E+03**
	Std	1.48E+02	1.15E+01	3.80E+01	1.49E+01	1.55E+01	3.34E+01	7.03E+01	2.07E+01	6.74E+01	8.04E+00	2.43E+02	4.25E+01
	Rank	11(+)	5(+)	8(+)	6(+)	2(+)	7(+)	9(+)	3(+)	10(+)	4(+)	12(+)	1
F_{25}	*Mean*	5.17E+03	2.89E+03	2.91E+03	2.89E+03	2.89E+03	2.90E+03	2.92E+03	2.89E+03	1.87E+04	**2.89E+03**	2.89E+03	2.89E+03
	Std	2.12E+02	2.06E+01	2.40E+01	3.92E-01	6.97E+00	2.10E+01	1.35E+01	1.28E+01	2.77E+03	2.01E+00	1.27E+00	2.60E-01
	Rank	11(+)	7(-)	9(+)	3(+)	4(-)	8(+)	10(+)	6(+)	12(+)	1(=)	5(+)	2
F_{26}	*Mean*	1.09E+04	4.71E+03	5.47E+03	4.49E+03	4.04E+03	4.51E+03	4.67E+03	4.17E+03	1.18E+04	3.82E+03	9.55E+03	**3.16E+03**
	Std	3.50E+02	1.72E+02	4.14E+02	3.06E+02	4.35E+02	9.68E+02	1.83E+03	7.71E+02	1.01E+03	6.72E+02	4.75E+02	4.38E+02
	Rank	11(+)	8(+)	9(+)	5(+)	3(+)	6(+)	7(+)	4(+)	12(+)	2(+)	10(+)	1
F_{27}	*Mean*	4.30E+03	3.23E+03	3.27E+03	3.22E+03	3.22E+03	3.25E+03	3.31E+03	3.22E+03	3.56E+03	3.22E+03	3.77E+03	**3.21E+03**
	Std	2.06E+02	5.68E+00	3.15E+01	5.53E+00	1.31E+01	1.58E+01	3.54E+01	1.06E+01	1.59E+02	5.91E+00	2.30E+02	6.91E+00
	Rank	12(+)	6(+)	8(+)	3(+)	5(+)	7(+)	9(+)	4(+)	10(+)	2(+)	11(+)	1
F_{28}	*Mean*	6.64E+03	3.21E+03	3.25E+03	3.22E+03	3.16E+03	3.20E+03	3.31E+03	3.27E+03	6.94E+03	3.21E+03	3.17E+03	**3.11E+03**
	Std	3.82E+02	1.24E+01	2.52E+01	2.27E+01	6.50E+01	3.33E+01	2.17E+01	4.72E+01	7.76E+01	2.15E+00	4.65E+01	3.48E+01
	Rank	11(+)	5(+)	8(+)	7(+)	2(+)	4(+)	10(+)	9(+)	12(+)	6(+)	3(+)	1
F_{29}	*Mean*	6.76E+03	3.58E+03	3.84E+03	3.61E+03	3.69E+03	3.74E+03	4.18E+03	3.73E+03	5.68E+03	3.45E+03	5.01E+03	**3.33E+03**
	Std	5.48E+02	1.27E+02	1.74E+02	1.21E+02	1.66E+02	1.49E+02	2.42E+02	2.67E+02	1.97E+02	9.18E+01	4.78E+02	3.89E+01
	Rank	12(+)	3(+)	8(+)	4(+)	5(+)	7(+)	9(+)	6(+)	11(+)	2(+)	10(+)	1
F_{30}	*Mean*	1.43E+09	1.08E+04	8.37E+03	1.39E+04	8.40E+03	8.10E+03	4.00E+06	3.25E+04	2.86E+09	2.38E+04	2.86E+05	**6.09E+03**
	Std	4.87E+08	1.01E+03	2.06E+03	5.96E+03	2.89E+03	2.22E+03	1.66E+06	3.24E+04	3.50E+08	6.06E+03	1.39E+05	1.01E+03
	Rank	11(+)	5(+)	3(+)	6(+)	4(+)	2(+)	10(+)	8(+)	12(+)	7(+)	9(+)	1
	#Best	0	1	1	1	1	1	0	0	0	2	0	22
	#Defeated	29	25	26	25	24	24	29	28	29	27	29	
	#Equal	0	2	2	3	3	4	0	1	0	2	0	
	#Win	0	2	1	1	2	1	0	0	0	0	0	

Table 10
Comparisons between LDPSO and non-PSO methods under 50-D.

Func	Criteria	COA	KOA	OMA	RIME	SAO	EEFO	HEOA	WO	CEO	DOA	WAA	LDPSO
F_1	*Mean*	1.05E+11	1.98E+05	2.87E+06	3.14E+04	4.66E+03	3.41E+03	3.53E+08	6.28E+03	2.44E+11	3.04E+06	4.75E+06	**2.70E+03**
	Std	6.86E+09	5.27E+04	1.17E+07	2.64E+04	4.57E+03	4.60E+03	7.21E+07	5.94E+03	1.52E+10	7.87E+05	4.11E+05	3.23E+03
	Rank	11(+)	6(+)	7(+)	5(+)	3(=)	2(=)	10(+)	4(+)	12(+)	8(+)	9(+)	1
F_3	*Mean*	1.86E+05	5.03E+04	3.77E+04	3.58E+02	1.58E+05	1.50E+03	3.69E+04	1.41E+04	7.30E+05	2.20E+04	4.09E+02	**3.01E+02**
	Std	7.59E+03	8.30E+03	6.84E+03	2.10E+01	4.67E+04	7.56E+02	4.79E+03	3.44E+03	3.99E+04	3.28E+03	2.72E+01	2.49E+00
	Rank	11(+)	9(+)	8(+)	2(+)	10(+)	4(+)	7(+)	5(+)	12(+)	6(+)	3(+)	1
F_4	*Mean*	3.73E+04	5.48E+02	5.91E+02	5.25E+02	4.65E+02	5.37E+02	6.55E+02	5.69E+02	6.28E+04	5.65E+02	5.36E+02	**4.63E+02**
	Std	5.17E+03	1.32E+01	6.62E+01	1.62E+01	4.15E+01	5.59E+01	5.94E+01	5.11E+01	1.41E+03	3.65E+01	4.81E-01	3.29E+01
	Rank	11(+)	6(+)	9(+)	3(+)	2(=)	5(+)	10(+)	8(+)	12(+)	7(+)	4(+)	1
F_5	*Mean*	1.19E+03	6.21E+02	7.67E+02	6.84E+02	6.18E+02	7.64E+02	9.28E+02	6.71E+02	1.69E+03	6.44E+02	9.02E+02	**5.36E+02**
	Std	3.33E+01	1.29E+01	4.18E+01	3.31E+01	2.63E+01	5.26E+01	5.79E+01	3.97E+01	2.30E+01	6.31E+00	3.05E+00	6.35E+00
	Rank	11(+)	3(+)	8(+)	6(+)	2(+)	7(+)	10(+)	5(+)	12(+)	4(+)	9(+)	1
F_6	*Mean*	6.98E+02	6.00E+02	6.30E+02	6.01E+02	6.00E+02	6.07E+02	6.70E+02	6.17E+02	7.40E+02	6.00E+02	6.70E+02	**6.00E+02**
	Std	7.12E+00	1.16E-01	9.28E+00	5.41E-01	1.69E-01	2.50E+00	6.85E+00	1.62E+01	4.63E-13	6.32E-02	4.47E-01	1.75E-08
	Rank	11(+)	3(+)	8(+)	5(+)	2(+)	6(+)	10(+)	7(+)	12(+)	4(+)	9(+)	1
F_7	*Mean*	2.04E+03	9.02E+02	1.36E+03	8.91E+02	8.70E+02	1.11E+03	1.57E+03	9.95E+02	5.88E+03	9.50E+02	1.82E+03	**8.21E+02**
	Std	3.91E+01	3.83E+01	1.53E+02	1.53E+01	9.54E+01	7.66E+01	1.02E+02	7.12E+01	3.39E+02	2.65E+01	1.99E+01	9.85E+01
	Rank	11(+)	4(+)	8(+)	3(+)	2(+)	7(+)	9(+)	6(+)	12(+)	5(+)	10(+)	1
F_8	*Mean*	1.49E+03	9.18E+02	1.04E+03	9.26E+02	9.15E+02	1.04E+03	1.26E+03	9.71E+02	2.02E+03	9.34E+02	1.18E+03	**8.39E+02**
	Std	2.39E+01	1.81E+01	4.99E+01	1.62E+01	3.10E+01	3.82E+01	4.58E+01	3.18E+01	5.25E+01	1.53E+01	2.85E+01	8.23E+00
	Rank	11(+)	3(+)	7(+)	4(+)	2(+)	8(+)	10(+)	6(+)	12(+)	5(+)	9(+)	1
F_9	*Mean*	3.54E+04	2.46E+03	9.82E+03	4.59E+03	9.71E+02	5.03E+03	2.83E+04	6.38E+03	8.13E+04	1.14E+03	1.85E+04	**9.02E+02**
	Std	2.12E+03	2.15E+02	3.21E+03	2.34E+03	8.63E+01	1.35E+03	4.01E+03	7.75E+03	1.14E+04	1.70E+02	7.96E+02	1.42E+00
	Rank	11(+)	4(+)	8(+)	5(+)	2(+)	6(+)	10(+)	7(+)	12(+)	3(+)	9(+)	1
F_{10}	*Mean*	1.52E+04	5.67E+03	1.37E+04	6.16E+03	5.98E+03	6.81E+03	1.01E+04	1.08E+04	1.32E+04	5.02E+03	8.50E+03	**4.29E+03**
	Std	2.90E+02	3.56E+02	4.11E+02	8.77E+02	7.32E+02	6.63E+02	1.00E+03	4.10E+03	3.84E+02	3.66E+02	5.16E+02	7.52E+02
	Rank	12(+)	3(+)	11(+)	5(+)	4(+)	6(+)	8(+)	9(+)	10(+)	2(+)	7(+)	1
F_{11}	*Mean*	2.47E+04	1.32E+03	1.25E+03	1.35E+03	1.25E+03	1.25E+03	1.48E+03	1.34E+03	1.01E+05	1.46E+03	1.39E+03	**1.14E+03**
	Std	3.09E+03	3.79E+01	4.27E+01	4.91E+01	6.82E+01	4.05E+01	7.26E+01	5.84E+01	3.25E+03	3.35E+01	2.11E+01	7.27E+00
	Rank	11(+)	5(+)	3(+)	7(+)	4(+)	2(+)	10(+)	6(+)	12(+)	9(+)	8(+)	1
F_{12}	*Mean*	8.01E+10	2.19E+06	1.51E+06	1.22E+07	5.16E+05	1.08E+06	1.24E+08	1.87E+07	1.21E+11	8.88E+06	1.15E+07	**8.59E+04**
	Std	7.77E+09	1.13E+06	9.01E+05	4.97E+06	2.97E+05	6.18E+05	3.83E+07	8.66E+06	7.78E+09	4.85E+06	6.88E+06	3.66E+04
	Rank	11(+)	5(+)	4(+)	8(+)	2(+)	3(+)	10(+)	9(+)	12(+)	6(+)	7(+)	1
F_{13}	*Mean*	3.71E+10	6.02E+03	4.87E+03	3.34E+04	7.55E+03	5.24E+03	1.11E+07	4.85E+04	6.90E+10	6.99E+04	5.39E+05	**2.58E+03**
	Std	1.55E+10	3.59E+03	3.14E+03	5.88E+03	7.16E+03	4.32E+03	6.20E+06	1.47E+04	6.90E+09	3.57E+04	2.25E+05	1.06E+03
	Rank	11(+)	4(+)	2(+)	6(+)	5(+)	3(+)	10(+)	7(+)	12(+)	8(+)	9(+)	1
F_{14}	*Mean*	4.51E+07	2.81E+04	2.65E+04	5.10E+04	2.94E+04	3.97E+04	4.31E+05	3.46E+05	2.12E+08	1.75E+06	6.71E+04	**1.54E+04**
	Std	3.28E+07	4.11E+03	2.44E+04	2.35E+04	2.36E+04	2.81E+04	2.41E+05	2.42E+05	6.35E+07	1.25E+06	6.33E+04	8.22E+03
	Rank	11(+)	3(+)	2(=)	6(+)	4(+)	5(+)	9(+)	8(+)	12(+)	10(+)	7(+)	1
F_{15}	*Mean*	8.09E+09	1.15E+04	6.15E+03	**4.88E+03**	1.12E+04	9.89E+03	6.08E+05	2.13E+04	3.56E+10	1.70E+04	5.82E+04	8.25E+03
	Std	4.00E+09	3.89E+03	4.75E+03	6.09E+03	6.46E+03	5.22E+03	2.31E+05	5.81E+03	1.56E+10	5.34E+03	3.17E+04	4.63E+03
	Rank	11(+)	6(+)	2(-)	1(-)	5(=)	4(=)	10(+)	8(+)	12(+)	7(+)	9(+)	3
F_{16}	*Mean*	8.91E+03	2.83E+03	3.56E+03	2.96E+03	2.93E+03	3.05E+03	3.89E+03	3.20E+03	1.32E+04	2.92E+03	4.11E+03	**2.03E+03**
	Std	8.16E+02	2.92E+02	3.66E+02	6.23E+01	4.90E+02	4.72E+02	4.12E+02	5.49E+02	1.63E+02	2.08E+02	4.92E+02	1.64E+02
	Rank	11(+)	2(+)	8(+)	5(+)	4(+)	6(+)	9(+)	7(+)	12(+)	3(+)	10(+)	1
F_{17}	*Mean*	9.68E+03	3.02E+03	3.18E+03	2.43E+03	2.77E+03	2.82E+03	3.39E+03	2.93E+03	1.90E+06	2.61E+03	3.86E+03	**2.28E+03**
	Std	4.12E+03	3.69E+02	2.62E+02	2.01E+02	3.55E+02	3.11E+02	2.97E+02	5.37E+02	8.61E+05	8.60E+01	1.04E+02	2.01E+02
	Rank	11(+)	7(+)	8(+)	2(=)	4(+)	5(+)	9(+)	6(+)	12(+)	3(+)	10(+)	1
F_{18}	*Mean*	1.45E+08	1.04E+06	2.19E+05	4.13E+05	1.72E+05	1.31E+05	2.57E+06	1.55E+06	4.29E+08	3.29E+06	4.86E+05	**5.15E+04**
	Std	6.42E+07	4.41E+05	1.19E+05	3.59E+05	1.36E+05	1.20E+05	1.85E+06	8.47E+05	2.46E+07	7.08E+05	5.16E+04	1.87E+04
	Rank	11(+)	7(+)	4(+)	5(+)	3(+)	2(+)	9(+)	8(+)	12(+)	10(+)	6(+)	1

Table 10
Continued of Table 10.

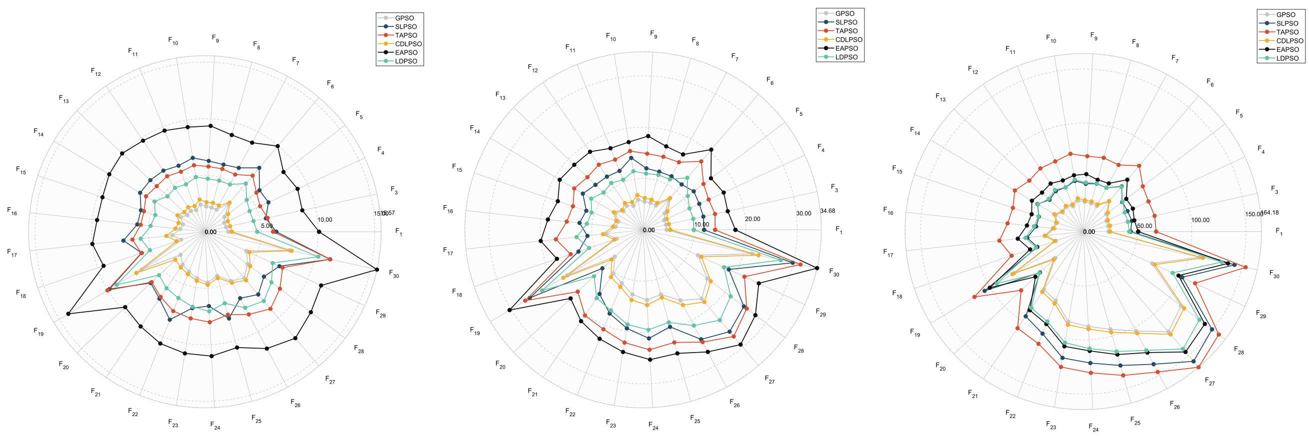
Func	Criteria	COA	KOA	OMA	RIME	SAO	EEFO	HEOA	WO	CEO	DOA	WAA	LDPSO
F_{19}	*Mean*	2.40E+09	2.22E+04	1.27E+04	**2.36E+03**	1.96E+04	1.86E+04	5.17E+05	2.51E+04	1.29E+10	1.43E+04	2.46E+05	1.92E+04
	Std	7.03E+08	3.65E+03	9.16E+03	6.03E+02	1.31E+04	8.59E+03	1.79E+05	1.44E+04	1.82E+09	4.93E+03	1.68E+05	4.96E+03
	Rank	11(+)	7(=)	2(-)	1(-)	6(=)	4(=)	10(+)	8(=)	12(+)	3(-)	9(+)	5
F_{20}	*Mean*	4.09E+03	2.99E+03	3.31E+03	3.17E+03	2.73E+03	2.74E+03	3.48E+03	3.14E+03	4.94E+03	2.61E+03	3.63E+03	**2.16E+03**
	Std	4.93E+01	1.47E+02	2.58E+02	1.33E+02	2.49E+02	2.59E+02	2.89E+02	6.23E+02	1.85E+01	2.89E+02	5.88E+02	2.39E+02
	Rank	11(+)	5(+)	8(+)	7(+)	3(+)	4(+)	9(+)	6(+)	12(+)	2(+)	10(+)	1
F_{21}	*Mean*	3.22E+03	2.43E+03	2.50E+03	2.45E+03	2.41E+03	2.49E+03	2.73E+03	2.46E+03	3.40E+03	2.44E+03	3.12E+03	**2.35E+03**
	Std	8.04E+01	1.35E+01	3.96E+01	4.93E+00	1.90E+01	3.35E+01	4.79E+01	5.69E+01	6.81E+00	8.07E+00	1.12E+02	6.58E+01
	Rank	11(+)	3(+)	8(+)	5(+)	2(+)	7(+)	9(+)	6(+)	12(+)	4(+)	10(+)	1
F_{22}	*Mean*	1.69E+04	7.75E+03	1.08E+04	8.27E+03	7.00E+03	7.80E+03	1.12E+04	1.10E+04	1.54E+04	8.17E+03	1.04E+04	**4.80E+03**
	Std	5.02E+02	5.07E+02	6.12E+03	3.48E+02	2.06E+03	2.29E+03	1.20E+03	5.41E+03	3.62E+02	2.92E+02	1.37E+03	1.67E+03
	Rank	12(+)	3(+)	8(+)	6(+)	2(+)	4(+)	10(+)	9(+)	11(+)	5(+)	7(+)	1
F_{23}	*Mean*	4.66E+03	2.87E+03	3.19E+03	2.93E+03	2.85E+03	3.00E+03	3.40E+03	2.90E+03	4.53E+03	2.88E+03	4.33E+03	**2.76E+03**
	Std	1.15E+02	4.32E+00	1.19E+02	1.23E+01	2.88E+01	7.02E+01	1.13E+02	4.54E+01	2.22E+02	1.16E+01	2.93E+02	1.30E+01
	Rank	12(+)	3(+)	8(+)	6(+)	2(+)	7(+)	9(+)	5(+)	11(+)	4(+)	10(+)	1
F_{24}	*Mean*	4.71E+03	3.01E+03	3.34E+03	3.14E+03	3.01E+03	3.20E+03	3.75E+03	3.06E+03	4.43E+03	3.23E+03	4.16E+03	**2.96E+03**
	Std	7.15E+01	9.62E+00	9.08E+01	4.36E+01	2.39E+01	6.33E+01	1.42E+02	3.63E+01	1.22E+02	2.00E+01	1.29E+02	8.93E+01
	Rank	12(+)	2(+)	8(+)	5(+)	3(+)	6(+)	9(+)	4(+)	11(+)	7(+)	10(+)	1
F_{25}	*Mean*	1.55E+04	3.04E+03	3.13E+03	3.06E+03	3.05E+03	3.07E+03	3.20E+03	3.06E+03	5.55E+04	3.08E+03	3.07E+03	**3.04E+03**
	Std	1.07E+03	1.51E+01	3.22E+01	1.11E+01	3.26E+01	3.24E+01	3.55E+01	2.93E+01	4.10E+03	9.92E+00	4.15E+01	2.85E+01
	Rank	11(+)	2(=)	9(+)	4(=)	3(=)	6(+)	10(+)	5(+)	12(+)	8(+)	7(+)	1
F_{26}	*Mean*	1.75E+04	5.29E+03	8.62E+03	5.53E+03	4.98E+03	5.74E+03	7.37E+03	**3.12E+03**	2.27E+04	5.09E+03	1.29E+04	3.59E+03
	Std	8.82E+02	3.41E+02	1.11E+03	1.71E+02	3.22E+02	2.95E+03	2.95E+03	7.00E+02	1.81E+03	2.46E+02	1.71E+02	5.38E+02
	Rank	11(+)	5(+)	9(+)	6(+)	3(+)	7(+)	8(+)	1(=)	12(+)	4(+)	10(+)	2
F_{27}	*Mean*	7.22E+03	3.56E+03	3.72E+03	3.32E+03	3.36E+03	3.59E+03	3.99E+03	3.45E+03	4.50E+03	3.30E+03	5.51E+03	**3.25E+03**
	Std	7.52E+02	1.41E+02	1.18E+02	1.83E+01	7.91E+01	1.02E+02	2.05E+02	8.92E+01	1.66E+01	2.21E+01	4.80E+02	1.61E+01
	Rank	12(+)	6(+)	8(+)	3(+)	4(+)	7(+)	9(+)	5(+)	10(+)	2(+)	11(+)	1
F_{28}	*Mean*	1.15E+04	3.30E+03	3.43E+03	3.31E+03	3.29E+03	3.32E+03	3.49E+03	3.31E+03	1.07E+04	3.32E+03	3.31E+03	**3.28E+03**
	Std	1.60E+03	1.47E+01	4.09E+01	5.50E+00	2.73E+01	3.17E+01	6.37E+01	2.71E+01	3.22E+02	2.17E+01	3.53E+00	2.37E+01
	Rank	12(+)	3(+)	9(+)	4(+)	2(=)	8(+)	10(+)	6(+)	11(+)	7(+)	5(+)	1
F_{29}	*Mean*	6.61E+04	3.67E+03	5.06E+03	4.10E+03	3.96E+03	4.18E+03	4.94E+03	4.10E+03	1.73E+04	3.74E+03	6.41E+03	**3.36E+03**
	Std	5.79E+04	2.26E+02	2.79E+02	6.55E+01	3.32E+02	2.90E+02	3.57E+02	3.25E+02	2.33E+02	6.30E+01	2.50E+02	1.03E+02
	Rank	12(+)	2(+)	9(+)	6(+)	4(+)	7(+)	8(+)	5(+)	11(+)	3(+)	10(+)	1
F_{30}	*Mean*	5.21E+09	**7.56E+05**	9.89E+05	1.82E+06	9.70E+05	9.73E+05	2.53E+07	1.78E+06	1.82E+10	9.60E+05	1.33E+07	8.44E+05
	Std	5.12E+08	2.58E+04	1.93E+05	2.17E+05	2.10E+05	2.28E+05	4.61E+06	7.58E+05	4.25E+09	6.09E+04	5.93E+05	4.37E+04
	Rank	11(+)	1(-)	6(+)	8(+)	4(=)	5(+)	10(+)	7(+)	12(+)	3(+)	9(+)	2
	#Best	0	1	0	2	0	0	0	1	0	0	0	25
	#Defeated	29	26	26	25	22	26	29	27	29	28	29	
	#Equal	0	2	1	2	7	3	0	2	0	0	0	
	#Win	0	1	2	2	0	0	0	0	0	1	0	

Table 11
Comparisons between LDPSO and non-PSO methods under 100-D.

Func	Criteria	COA	KOA	OMA	RIME	SAO	EEFO	HEOA	WO	CEO	DOA	WAA	LDPSO
F_1	*Mean*	2.67E+11	7.36E+05	6.07E+08	7.26E+04	1.01E+04	5.17E+03	1.42E+09	2.73E+04	6.49E+11	2.81E+07	2.78E+07	**5.16E+03**
	Std	4.30E+09	1.54E+05	1.05E+09	1.58E+04	1.31E+04	4.64E+03	3.23E+08	9.20E+04	3.98E+10	1.06E+07	9.31E+05	6.86E+03
	Rank	11(+)	6(+)	9(+)	5(+)	3(=)	2(=)	10(+)	4(+)	12(+)	8(+)	7(+)	1
F_3	*Mean*	3.43E+05	1.53E+05	1.86E+05	8.30E+03	5.70E+05	3.39E+04	1.43E+05	8.59E+04	1.49E+06	1.13E+05	1.30E+04	**5.21E+03**
	Std	1.07E+04	2.10E+04	2.07E+04	7.86E+02	1.42E+05	7.33E+03	1.54E+04	1.14E+04	1.07E+05	1.47E+04	1.16E+03	1.36E+03
	Rank	10(+)	8(+)	9(+)	2(+)	11(+)	4(+)	7(+)	5(+)	12(+)	6(+)	3(+)	1
F_4	*Mean*	1.13E+05	7.41E+02	1.02E+03	6.89E+02	6.18E+02	6.84E+02	1.20E+03	6.88E+02	1.88E+05	7.76E+02	6.48E+02	**5.89E+02**
	Std	1.11E+04	2.95E+01	1.82E+02	3.11E+01	2.68E+01	6.35E+01	1.17E+02	4.32E+01	8.00E+03	5.99E+01	1.68E+01	3.42E+01
	Rank	11(+)	7(+)	9(+)	6(+)	2(+)	4(+)	10(+)	5(+)	12(+)	8(+)	3(+)	1
F_5	*Mean*	2.14E+03	8.31E+02	1.20E+03	9.70E+02	8.30E+02	1.22E+03	1.52E+03	9.80E+02	3.13E+03	1.03E+03	1.42E+03	**5.94E+02**
	Std	1.98E+01	3.90E+01	7.00E+01	8.50E+01	5.76E+01	8.39E+01	6.30E+01	6.98E+01	7.26E+01	3.87E+01	1.07E+01	1.26E+01
	Rank	11(+)	3(+)	7(+)	4(+)	2(+)	8(+)	10(+)	5(+)	12(+)	6(+)	9(+)	1
F_6	*Mean*	7.11E+02	6.01E+02	6.49E+02	6.04E+02	6.01E+02	6.16E+02	6.78E+02	6.37E+02	7.64E+02	6.01E+02	6.68E+02	**6.00E+02**
	Std	7.56E-01	1.05E-01	8.18E+00	2.14E+00	8.37E-01	4.51E+00	5.73E+00	2.75E+01	8.17E+00	1.51E-01	1.66E+00	2.01E-02
	Rank	11(+)	2(+)	8(+)	5(+)	3(+)	6(+)	10(+)	7(+)	12(+)	4(+)	9(+)	1
F_7	*Mean*	4.03E+03	1.44E+03	2.85E+03	1.17E+03	1.68E+03	2.11E+03	3.13E+03	1.59E+03	1.37E+04	1.51E+03	3.45E+03	**8.86E+02**
	Std	4.19E+01	9.31E+01	4.14E+02	4.66E+01	2.67E+02	1.79E+02	1.96E+02	1.34E+02	2.30E+02	8.61E+01	2.33E+01	1.24E+01
	Rank	11(+)	3(+)	8(+)	2(+)	6(+)	7(+)	9(+)	5(+)	12(+)	4(+)	10(+)	1
F_8	*Mean*	2.58E+03	1.28E+03	1.55E+03	1.18E+03	1.12E+03	1.55E+03	1.99E+03	1.25E+03	3.71E+03	1.34E+03	1.91E+03	**9.10E+02**
	Std	4.46E+01	3.22E+01	8.27E+01	4.70E+01	5.22E+01	1.07E+02	7.51E+01	5.59E+01	2.31E-12	4.69E+01	6.11E+00	1.27E+02
	Rank	11(+)	5(+)	8(+)	3(+)	2(+)	7(+)	10(+)	4(+)	12(+)	6(+)	9(+)	1
F_9	*Mean*	7.54E+04	1.19E+04	5.39E+04	6.49E+03	4.13E+03	1.95E+04	5.54E+04	5.56E+04	2.24E+05	9.11E+03	4.55E+04	**9.73E+02**
	Std	3.45E+03	1.30E+03	7.22E+03	2.58E+03	3.00E+03	2.14E+03	6.96E+03	2.48E+04	6.34E+03	3.13E+03	1.76E+03	2.29E+01
	Rank	11(+)	5(+)	8(+)	3(+)	2(+)	6(+)	9(+)	10(+)	12(+)	4(+)	7(+)	1
F_{10}	*Mean*	3.25E+04	1.41E+04	2.98E+04	1.41E+04	1.44E+04	1.47E+04	2.09E+04	2.44E+04	3.36E+04	1.76E+04	1.73E+04	**1.26E+04**
	Std	3.45E+02	6.51E+02	1.77E+03	9.54E+02	1.50E+03	1.04E+03	1.29E+03	8.58E+03	1.15E+03	7.15E+02	6.06E+02	5.69E+03
	Rank	11(+)	2(+)	10(+)	3(+)	4(+)	5(+)	8(+)	9(+)	12(+)	7(+)	6(+)	1
F_{11}	*Mean*	2.18E+05	3.26E+03	2.45E+03	2.78E+03	6.85E+04	1.93E+03	5.79E+03	2.31E+03	7.06E+05	3.77E+03	3.03E+03	**1.23E+03**
	Std	2.99E+04	1.00E+02	6.74E+02	1.79E+02	2.43E+04	1.61E+02	1.20E+03	2.23E+02	8.15E+04	6.59E+02	2.09E+02	4.02E+01
	Rank	11(+)	7(+)	4(+)	5(+)	10(+)	2(+)	9(+)	3(+)	12(+)	8(+)	6(+)	1
F_{12}	*Mean*	2.09E+11	1.48E+07	2.09E+07	1.15E+08	1.46E+06	5.33E+06	4.75E+08	5.50E+07	3.06E+11	5.23E+07	4.07E+07	**2.56E+05**
	Std	3.80E+09	2.78E+06	1.32E+07	1.45E+07	8.99E+05	2.64E+06	1.26E+08	1.93E+07	4.16E+08	1.86E+07	4.12E+06	7.84E+04
	Rank	11(+)	4(+)	5(+)	9(+)	2(+)	3(+)	10(+)	8(+)	12(+)	7(+)	6(+)	1
F_{13}	*Mean*	4.84E+10	4.96E+03	9.01E+03	8.93E+04	6.57E+03	7.12E+03	1.11E+07	5.02E+04	8.72E+10	2.88E+04	5.74E+05	**3.58E+03**
	Std	2.31E+09	6.91E+02	5.10E+03	2.26E+04	5.16E+03	4.53E+03	4.64E+06	2.84E+04	0.00E+00	1.99E+04	3.18E+04	2.76E+03
	Rank	11(+)	2(+)	5(+)	8(+)	3(+)	4(+)	10(+)	7(+)	12(+)	6(+)	9(+)	1
F_{14}	*Mean*	9.15E+07	3.70E+05	1.87E+05	5.08E+05	4.50E+04	1.64E+05	2.10E+06	5.49E+05	3.91E+08	3.85E+06	2.26E+05	**3.26E+04**
	Std	1.91E+07	2.35E+05	5.53E+04	3.42E+05	1.98E+04	6.44E+04	5.08E+05	2.61E+05	4.37E+07	1.08E+06	1.11E+04	9.73E+03
	Rank	11(+)	6(+)	4(+)	7(+)	2(+)	3(+)	9(+)	8(+)	12(+)	10(+)	5(+)	1
F_{15}	*Mean*	2.49E+10	**2.10E+03**	3.26E+03	9.20E+03	3.65E+03	4.11E+03	2.28E+06	2.11E+04	5.10E+10	1.34E+04	1.88E+05	3.69E+03
	Std	3.83E+09	3.46E+02	1.54E+03	1.75E+03	2.03E+03	2.24E+03	8.35E+05	1.22E+04	3.93E+09	6.68E+03	3.17E+04	2.38E+03
	Rank	11(+)	1(=)	2(=)	6(+)	3(=)	5(=)	10(+)	8(+)	12(+)	7(+)	9(+)	4
F_{16}	*Mean*	2.46E+04	5.77E+03	5.95E+03	6.06E+03	4.90E+03	5.58E+03	7.35E+03	5.56E+03	2.48E+04	5.41E+03	6.80E+03	**2.88E+03**
	Std	1.75E+03	4.72E+02	6.32E+02	4.54E+02	6.47E+02	6.95E+02	6.84E+02	1.30E+03	2.55E+03	4.80E+02	5.04E+02	2.81E+02
	Rank	11(+)	6(+)	7(+)	8(+)	2(+)	5(+)	10(+)	4(+)	12(+)	3(+)	9(+)	1
F_{17}	*Mean*	4.36E+06	4.60E+03	5.30E+03	4.31E+03	4.49E+03	4.82E+03	5.59E+03	4.95E+03	4.82E+07	4.45E+03	5.39E+03	**2.94E+03**
	Std	1.71E+06	2.40E+02	4.55E+02	3.89E+02	4.75E+02	4.80E+02	6.11E+02	5.60E+02	6.89E+06	5.44E+02	6.75E+02	2.98E+02
	Rank	11(+)	5(+)	8(+)	2(+)	4(+)	6(+)	10(+)	7(+)	12(+)	3(+)	9(+)	1
F_{18}	*Mean*	1.87E+08	1.79E+06	4.69E+05	2.02E+06	5.46E+05	2.72E+05	2.74E+06	1.25E+06	7.61E+08	5.41E+06	9.22E+05	**1.06E+05**
	Std	3.62E+07	4.82E+05	1.86E+05	3.29E+05	2.87E+05	9.94E+04	4.70E+05	5.90E+05	8.25E+07	2.26E+06	2.85E+05	2.37E+04
	Rank	11(+)	7(+)	3(+)	8(+)	4(+)	2(+)	9(+)	6(+)	12(+)	10(+)	5(+)	1

Table 11
Continued of Table 11.

Func	Criteria	COA	KOA	OMA	RIME	SAO	EEFO	HEOA	WO	CEO	DOA	WAA	LDPSO
F_{19}	*Mean*	2.98E+10	5.06E+03	4.96E+03	9.20E+03	5.47E+03	4.54E+03	3.56E+06	1.67E+04	4.64E+10	1.54E+04	2.11E+06	**3.33E+03**
	Std	3.34E+09	1.28E+03	2.99E+03	1.44E+03	5.39E+03	2.54E+03	9.49E+05	1.15E+04	6.90E+09	8.86E+03	3.21E+04	2.04E+03
	Rank	11(+)	4(+)	3(+)	6(+)	5(+)	2(+)	10(+)	8(+)	12(+)	7(+)	9(+)	1
F_{20}	*Mean*	7.89E+03	4.66E+03	5.95E+03	5.38E+03	4.53E+03	4.61E+03	5.60E+03	5.75E+03	8.60E+03	4.57E+03	6.18E+03	**3.00E+03**
	Std	2.46E+02	2.10E+02	5.81E+02	5.91E+01	4.90E+02	3.76E+02	4.21E+02	1.41E+03	9.52E+01	3.38E+02	9.03E+01	4.89E+02
	Rank	11(+)	5(+)	9(+)	6(+)	2(+)	4(+)	7(+)	8(+)	12(+)	3(+)	10(+)	1
F_{21}	*Mean*	4.96E+03	2.73E+03	2.95E+03	2.72E+03	2.68E+03	2.92E+03	3.48E+03	2.79E+03	5.09E+03	2.86E+03	4.40E+03	**2.44E+03**
	Std	2.02E+02	4.91E+01	8.77E+01	4.49E+01	7.83E+01	8.71E+01	1.25E+02	8.64E+01	1.58E+01	4.56E+01	1.24E+01	1.25E+02
	Rank	11(+)	4(+)	8(+)	3(+)	2(+)	7(+)	9(+)	5(+)	12(+)	6(+)	10(+)	1
F_{22}	*Mean*	3.48E+04	1.77E+04	3.20E+04	1.82E+04	1.63E+04	1.77E+04	2.42E+04	2.57E+04	3.26E+04	1.93E+04	2.03E+04	**1.17E+04**
	Std	3.88E+02	3.33E+02	5.81E+02	7.23E+02	1.44E+03	3.27E+03	1.43E+03	1.10E+04	1.28E+03	1.20E+03	1.08E+03	5.28E+03
	Rank	12(+)	3(+)	10(+)	5(+)	2(+)	4(+)	8(+)	9(+)	11(+)	6(+)	7(+)	1
F_{23}	*Mean*	6.75E+03	3.10E+03	3.65E+03	3.13E+03	3.16E+03	3.45E+03	4.06E+03	3.18E+03	6.71E+03	3.09E+03	5.20E+03	**2.91E+03**
	Std	2.98E+02	1.99E+01	9.91E+01	4.52E+01	5.29E+01	1.06E+02	1.41E+02	5.48E+01	8.27E+01	3.05E+01	1.56E+02	1.50E+01
	Rank	12(+)	3(+)	8(+)	4(+)	5(+)	7(+)	9(+)	6(+)	11(+)	2(+)	10(+)	1
F_{24}	*Mean*	1.03E+04	3.74E+03	5.15E+03	3.66E+03	3.59E+03	4.28E+03	4.93E+03	3.76E+03	9.68E+03	3.74E+03	6.46E+03	**3.35E+03**
	Std	2.58E+02	6.70E+01	2.82E+02	4.38E+01	5.70E+01	1.34E+02	1.30E+02	6.75E+01	6.09E+01	3.84E+01	1.84E+02	1.38E+01
	Rank	12(+)	5(+)	9(+)	3(+)	2(+)	7(+)	8(+)	6(+)	11(+)	4(+)	10(+)	1
F_{25}	*Mean*	2.97E+04	**3.24E+03**	3.66E+03	3.25E+03	3.24E+03	3.32E+03	3.79E+03	3.34E+03	1.43E+05	3.39E+03	3.32E+03	3.27E+03
	Std	1.30E+03	6.66E+01	1.04E+02	1.81E+01	4.78E+01	5.88E+01	6.77E+01	4.83E+01	2.00E+04	6.47E+01	9.03E+00	4.41E+01
	Rank	11(+)	1(-)	9(+)	3(-)	2(=)	5(+)	10(+)	7(+)	12(+)	8(+)	6(+)	4
F_{26}	*Mean*	5.23E+04	1.08E+04	2.68E+04	1.01E+04	9.34E+03	1.91E+04	2.15E+04	1.06E+04	6.49E+04	1.05E+04	2.66E+04	**6.39E+03**
	Std	1.48E+03	3.64E+02	3.19E+03	3.25E+02	5.70E+02	2.23E+03	7.06E+03	2.39E+03	2.70E+03	1.21E+03	8.92E+02	1.67E+02
	Rank	11(+)	6(+)	10(+)	3(+)	2(+)	7(+)	8(+)	5(+)	12(+)	4(+)	9(+)	1
F_{27}	*Mean*	1.45E+04	3.57E+03	4.58E+03	3.48E+03	3.44E+03	3.97E+03	4.38E+03	3.54E+03	7.37E+03	3.48E+03	6.08E+03	**3.42E+03**
	Std	1.79E+03	4.97E+01	2.91E+02	7.21E+00	5.12E+01	1.41E+02	2.17E+02	8.77E+01	7.82E+02	2.73E+01	1.48E+03	2.70E+01
	Rank	12(+)	6(+)	9(+)	4(+)	2(=)	7(+)	8(+)	5(+)	11(+)	3(+)	10(+)	1
F_{28}	*Mean*	2.90E+04	3.47E+03	3.93E+03	3.47E+03	**3.36E+03**	3.42E+03	3.81E+03	3.45E+03	2.66E+04	3.48E+03	3.37E+03	3.38E+03
	Std	9.90E+02	2.31E+01	1.85E+02	5.35E+01	2.84E+01	4.01E+01	5.86E+01	4.26E+01	1.85E-11	3.88E+01	1.87E+01	4.30E+01
	Rank	12(+)	6(+)	10(+)	7(+)	1(=)	4(+)	9(+)	5(+)	11(+)	8(+)	2(=)	3
F_{29}	*Mean*	6.07E+05	6.03E+03	8.28E+03	6.99E+03	5.99E+03	6.82E+03	8.42E+03	6.64E+03	9.79E+06	6.01E+03	9.72E+03	**4.47E+03**
	Std	3.04E+05	3.54E+02	6.20E+02	3.10E+02	4.05E+02	5.54E+02	5.62E+02	5.24E+02	6.23E+06	3.46E+02	8.79E+01	3.52E+02
	Rank	11(+)	4(+)	8(+)	7(+)	2(+)	6(+)	9(+)	5(+)	12(+)	3(+)	10(+)	1
F_{30}	*Mean*	4.57E+10	1.03E+04	4.30E+06	2.11E+06	1.04E+04	1.80E+04	4.61E+07	2.06E+05	5.42E+10	7.62E+04	6.49E+06	**9.26E+03**
	Std	4.19E+09	9.36E+02	2.26E+07	8.49E+05	3.60E+03	7.67E+03	1.81E+07	1.03E+05	5.93E+09	3.04E+04	2.41E+06	2.30E+03
	Rank	11(+)	2(+)	8(+)	7(+)	3(=)	4(+)	10(+)	6(+)	12(+)	5(+)	9(+)	1
	#Best	0	2	0	0	1	0	0	0	0	0	0	26
	#Defeated	29	27	28	28	23	27	29	29	29	29	28	
	#Equal	0	1	1	0	6	2	0	0	0	0	1	
	#Win	0	1	0	1	0	0	0	0	0	0	0	



(a) 30-D (b) 50-D (c) 100-D

Fig. 10. Execution time of different algorithms on benchmark functions (seconds).

on most functions. It “wins over”, “equals to”, and “under-performs” the compared methods in 296, 17, and 6 out of 319 cases, respectively.

Table 11 presents the comparison results under 100-dimension. It can be observed that the LDPSO has outstanding behavior. It acquires the best results on 26 functions and ranks in the top three on 27 functions. KOA performs best on F_{15} and F_{25}, while the SAO acquires the best results on F_{28}. Additionally, the Wilcoxon statistical analysis confirms that, for the majority of functions, the results produced by LDPSO are statistically superior to those generated by non-PSO methods.

Furthermore, in Table 12, the Friedman rank-and-sum statistical results indicate that LDPSO consistently ranks first across all tested dimensions, with p-values below 0.05, highlighting its competitiveness.

Table 12
The Friedman rank-and-sum results for the comparison with non-PSO variants.

Algorithm	30-D		50-D		100-D	
	rank	p-value	rank	p-value	rank	p-value
COA	10.92		11.08		10.98	
KOA	5.12		4.28		4.48	
OMA	6.65		6.85		7.38	
RIME	4.82		4.85		5.02	
SAO	3.75		3.48		3.38	
EEFO	4.82	9.08E-45	5.32	4.47E-46	4.98	9.49E-45
HEOA	9.25		9.25		9.05	
WO	6.08		6.32		6.22	
CEO	11.58		11.52		11.65	
DOA	5.08		5.28		5.75	
WAA	8.48		8.32		7.65	
LDPSO	**1.45**		**1.45**		**1.45**	

4.7. Diversity analysis

The experimental results indicate that LDPSO demonstrates promising search ability while maintaining good population diversity. To investigate this property, this section focuses on the diversity comparison between LDPSO and TVACPSO, TAPSO, CDLPSO, and EAPSO. These competitors are selected since TVACPSO is considered a canonical variant of PSO, and the other methods have shown strong performance. Nine functions are selected to examine the population diversity, as presented in Fig. 11.

It can be observed that, the population diversity of CDLPSO and EAPSO[3]. have been maintained at a higher level. LDPSO exhibits reasonable population diversity in the early stage, which then steadily decreases as the search progresses. Although its population diversity is not as higher as that of its competitors (e.g., EAPSO), it still successfully avoids entrapment and rapidly converges to the global optimum. More specifically, LDPSO demonstrates the characteristic of extensively exploring the search space in the early stage, trying to avoid the attraction of local optima in the middle stage, and achieving precise local exploitation in the later stage.

For unimodal problems, since only one optimum exists, the duration of higher diversity is very short, and it stably converges towards the global optima. This suggests that when addressing unimodal problems, the leaders exhibit stronger guidance abilities, while the followers quickly cluster together, thereby exhibiting effective local search performance.

For complex problems (multimodal function, hybrid function, and composition function), properly maintaining population diversity is crucial. As illustrated in Fig. 11b–i, the population diversity of LDPSO decreases slowly during the search process. In the early stage, particles can explore a broader search space to find as many potential regions as possible. One reason is that the bidirectional search strategy provides more diversified guidance for leaders, enhancing their exploration capabilities. Once promising regions are identified, the unidirectional search strategy gradually directs followers toward these areas, resulting in a slow decrease in population diversity. Furthermore, the jump-out strategy transfers high-quality and diverse information to stagnant particles, equipping them with the ability to escape from local optima.

5. LDPSO for solving flexible job shop scheduling problem

Flexible job shop scheduling problem (FJSP) is a widely used production model in the manufacturing industry (Brucker & Schlie, 1990; Gao et al., 2019). This problem involves a set of jobs, each comprising multiple operations, where each operation must be processed on one of its eligible machines (Zhao & Zhou, 2025). The primary objective of FJSP is to determine an optimal sequence and production routes (processing machine) for these operations to minimize total completion time or meet other performance criteria. Consequently, decisions must be made not only regarding the sequence of operations (i.e., deciding which operation should be processed next) but also concerning the machine assignment for each operation.

5.1. The mathematical model for FJSP

In the FJSP, there is a set of jobs ($I = \{1, 2, \cdots, n\}$) that needs to be processed on a set of machines ($K = \{1, 2, \cdots, m\}$). Each job i ($1 \le i \le n$) consists of a sequence of operations ($J_i = \{O_{i,1}, O_{i,2}, \cdots, O_{i,n_i}\}$), where

[3] The source code for EAPSO is download from: https://github.com/jsuzyy/EAPSO

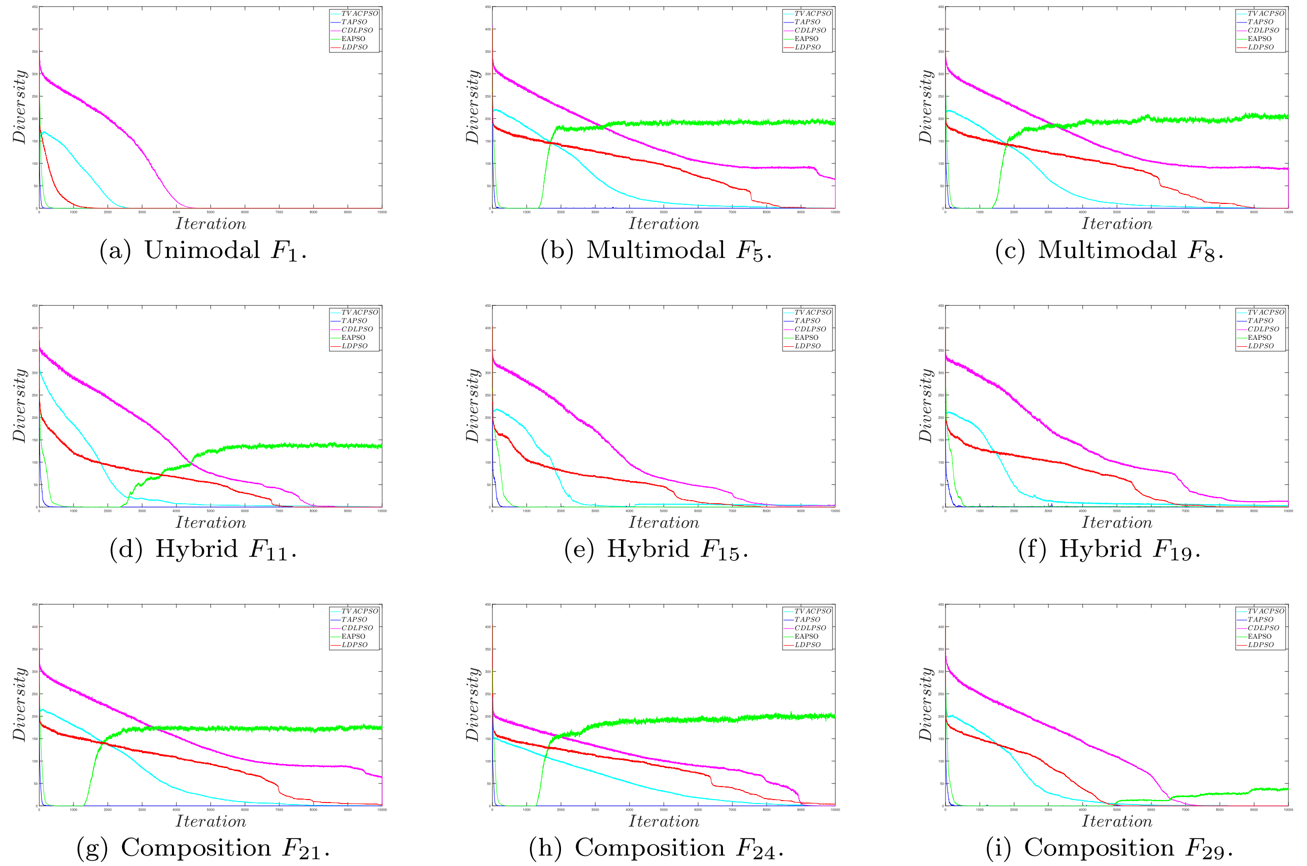


Fig. 11. Population diversity comparison on representative functions under 50-D. a. Unimodal F_1. b. Multimodal F_5. c. Multimodal F_8. d. Hybrid F_{11}. e. Hybrid F_{15}. f. Hybrid F_{19}. g. Composition F_{21}. h. Composition F_{24}. i. Composition F_{29}.

each operation $O_{i,j}$ can be processed on one of the qualified machines $K_{i,j}$ ($k \in K_{i,j} \subseteq M$). The goal is to find an optimal scheduling plan that assigns each operation to a qualified machine, determines the start time, and minimizes the completion time. During the scheduling process, the processing times and qualified machines for each operation are known in advance. All machines are available at the beginning, and each machine can handle only one operation at a time. Additionally, the scheduling is non-preemptive, meaning that once an operation begins, it cannot be interrupted or paused. Furthermore, the transportation and setup times are not considered.

The notations and decision variables in this Section are presented in Table 13.

To accurately describe the FJSP, a mixed integer linear programming model (MILP) is presented in this subsection (Zhao, Zhou, Zhao, & Wang, 2024a), the objective is to minimize the completion time ($C_{\max}$).

$$min\ f = C_{max} \tag{10}$$

$$\sum_{k \in K_{i,j}} \sum_{t \in P_k} X_{i,j,k,t} = 1, \forall i \in I, j \in J_i \tag{11}$$

$$\sum_{i \in I} \sum_{j \in J_i} X_{i,j,k,t} \leq 1, \forall k \in K, t \in P_k \tag{12}$$

$$\sum_{i \in I} \sum_{j \in J_i} X_{i,j,k,t} \geq \sum_{i' \in I} \sum_{j' \in J_i} X_{i',j',k,t+1}, \forall k \in K, t \in P'_k \tag{13}$$

$$S_{k,t+1} \geq S_{k,t} + \sum_{i \in I} \sum_{j \in J_i} (p_{i,j,k} \cdot X_{i,j,k,t}), \forall k \in K, t \in P'_k \tag{14}$$

Table 13
Notations and variables used in this section.

Symbol	Description
Indices:	
i, i'	indices for jobs
j, j'	indices for operations of jobs
k, k'	indices for machines
t	index for position on machine
l	index for time window
o	index for position on time window
n	total number of jobs
m	total number of machines
I	job set and $I = \{1, 2, \cdots, n\}$
n_i	operation number of job i
J_i	operation set of job i and $J_i = \{1, 2, \cdots, n_i\}$
$O_{i,j}$	the $j-th$ operation of job i
$K_{i,j}$	set of machines that can process $O_{i,j}$
$m_{i,j}$	number of machines that can process $O_{i,j}$
K	set of machines and $K = \{1, 2, \cdots, m\}$
$x_{i,j,k}$	binary constraints and it takes 1 if $O_{i,j}$ can be processed by machine k, otherwise, it takes 0
$p_{i,j,k}$	processing time for $O_{i,j}$ on machine k
G	a large positive number
p_k	number of positions on machine k and $p_k = \sum_{i \in I} \sum_{j \in J_i} x_{i,j,k}$
P_k	set of positions on machine k and $P_k = \{1, 2, \cdots, p_k\}$
P'_k	set of top $p_k - 1$ positions on machine k and $P_k = \{1, 2, \cdots, p_k - 1\}$
Decision variables:	
$X_{i,j,k,t}$	it takes 1 if the operation $O_{i,j}$ is assigned to the position t on machine k; otherwise, it takes 0
$C_{i,j}$	complete time of operation $O_{i,j}$ (continuous variable)
$B_{i,j}$	starting time of operation $O_{i,j}$ (continuous variable)
$S_{k,t}$	starting time of $t-th$ position on machine k (continuous variable)
$F_{k,t}$	complete time of $t-th$ position on machine k (continuous variable)

$$B_{i,j+1} \geq B_{i,j} + \sum_{k \in K_{i,j}} \sum_{t \in P_k} (p_{i,j,k} \cdot X_{i,j,k,t}), \forall i \in I, j \in \{1, 2, \cdots, n_i - 1\} \quad (15)$$

$$C_{i,j} \geq B_{i,j} + \sum_{k \in K_{i,n_i}} \sum_{t \in P_k} (p_{i,j,k} \cdot X_{i,j,k,t}), \forall i \in I, j \in J_i \quad (16)$$

$$C_{max} \geq B_{i,n_i} + \sum_{k \in K_{i,n_i}} \sum_{t \in P_k} (p_{i,n_i,k} \cdot X_{i,n_i,k,t}), \forall i \in I \quad (17)$$

$$S_{k,t} \geq B_{i,j} - G \cdot (1 - X_{i,j,k,t}), \forall i \in I, j \in J_i, k \in K_{i,j}, t \in P_k \quad (18)$$

$$S_{k,t} \leq B_{i,j} + G \cdot (1 - X_{i,j,k,t}), \forall i \in I, j \in J_i, k \in K_{i,j}, t \in P_k \quad (19)$$

$$S_{k,t} \geq 0, \forall k \in K, t \in P'_k \quad (20)$$

$$B_{i,j} \geq 0, \forall i \in I, j \in J_i \quad (21)$$

where Eq. 10 is the optimization objective, referred to completion time. Eq. 11 enforces that each operation can only be assigned to one of the positions in the qualified machine. Eq. 12 states that each operation can be processed only once. Subsequently, Eq. 13 ensures that an operation can be assigned to a later position only if all preceding positions are already occupied by other operations. Eq. 14 states that on the same machine, an operation can only begin after the previous operation is completed. Similarly, Eq. 15 ensures that for a given job, the next operation cannot start until the preceding operation is finished. Then, Eq. 16 presents the relationship between $C_{i,j}$ and $B_{i,j}$. Eq. 17 specifies that the completion time must be greater than or equal to the finish time of the final operation of all jobs. Eqs. 18 and 19 define the relationship between $S_{k,t}$ and $B_{i,j}$. Finally, Eqs. 20 and 21 state that the decision variables must be non-negative.

5.2. Encoding and decoding

As mentioned above, in FJSP, two decisions must be made: operation sequencing (OS) and machine assignment (MA) (Dauzère-Pérès, Ding, Shen, & Tamssaouet, 2024). The OS determines the processing order for all operations, while the MA specifies the corresponding machine. Consequently, each particle is represented by a two-dimensional vector (Zhang, Shao, Li, & Gao, 2009). To clearly illustrate the encoding and decoding mechanisms, an example involving four jobs is illustrated in Table 14. A feasible solution based on continuous numbers is presented in Fig. 12.

Table 14
Data for the example.

Job	Operation	M_1	M_2	M_3	M_4
J_1	$O_{1,1}$	8	5	-	-
J_1	$O_{1,2}$	6	5	2	7
J_1	$O_{1,3}$	2	-	4	-
J_2	$O_{2,1}$	-	2	5	3
J_2	$O_{2,2}$	-	7	-	4
J_3	$O_{3,1}$	8	-	-	-
J_3	$O_{3,2}$	-	7	-	2
J_3	$O_{3,3}$	-	-	8	5
J_4	$O_{4,1}$	5	7	-	-
J_4	$O_{4,2}$	-	2	4	-
J_4	$O_{4,3}$	2	3	7	5

As shown in Fig. 12, the lengths of OS and MA are equal, both of which are $\sum_{i=1}^{n} n_i$. To convert the solution into a feasible schedule, the following conversion strategies are required.

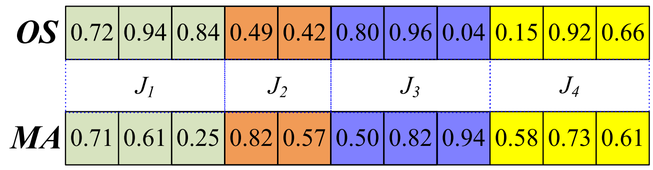


Fig. 12. A feasible encoding representation for Table 14.

For the OS vector:

1. As shown in Fig. 13, sort the numerical values in the OS in ascending order and provide their corresponding rankings;
2. Based on the ranking, the sequence of operations is determined. For example, the lowest value (0.04) belongs the job 3. Thus, the first operation is $O_{3,1}$, followed by $O_{4,1}$, $O_{2,1}$, $\cdots$, $O_{3,3}$.

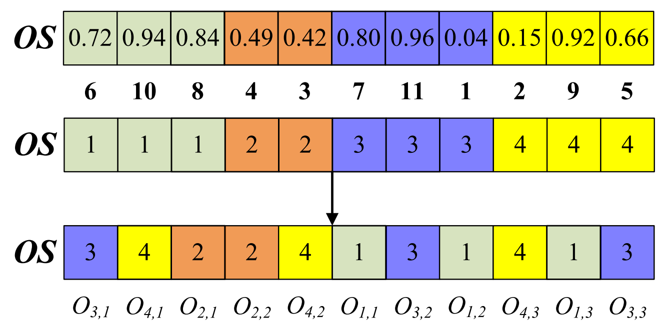


Fig. 13. Decoding process for OS vector.

For the MA vector:

1. Determine the number of machines that can process the operation. For example, two machines can process $O_{3,2}$.
2. The interval [0,1] is mapped to the machine index. That is, interval [0,0.5] corresponds to the first machine (M_2), and interval [0.5,1] belongs to the second machine (M_4).
3. Based on the value of MA, determine the machine assignment. For example, the value for $O_{3,2}$ is 0.82 and belongs to the second interval ([0.5,1]). As shown in Fig. 14, M_4 is prepared for it.

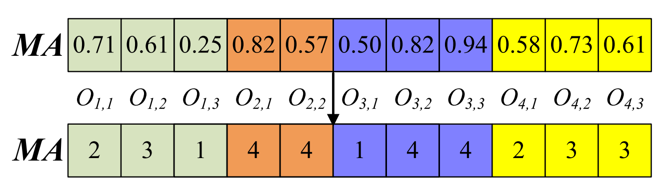


Fig. 14. Decoding process for MA vector.

Based on the above strategies, the final Gantt chart is presented in Fig. 15.

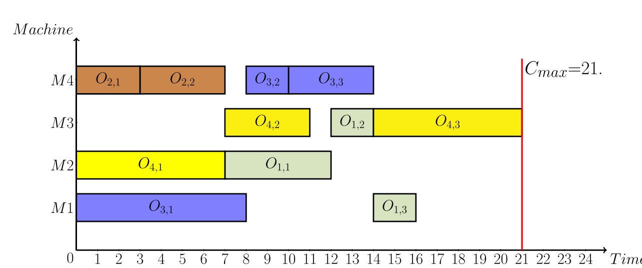
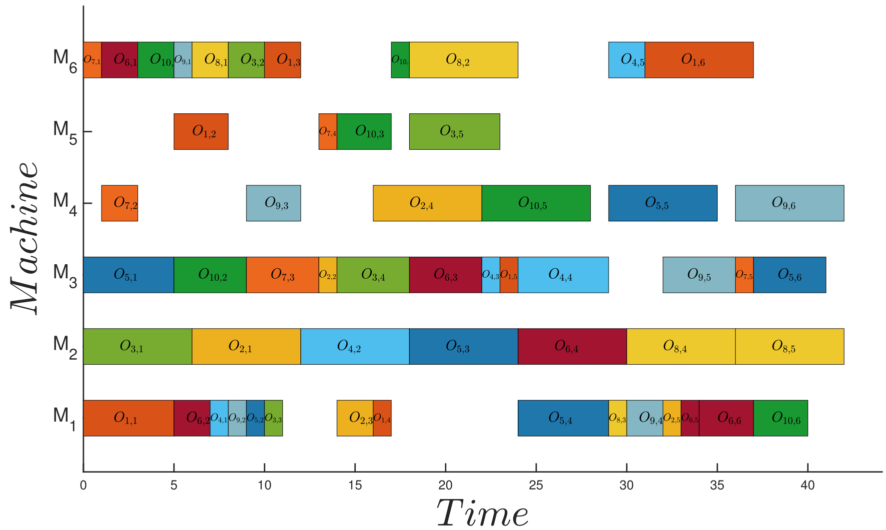
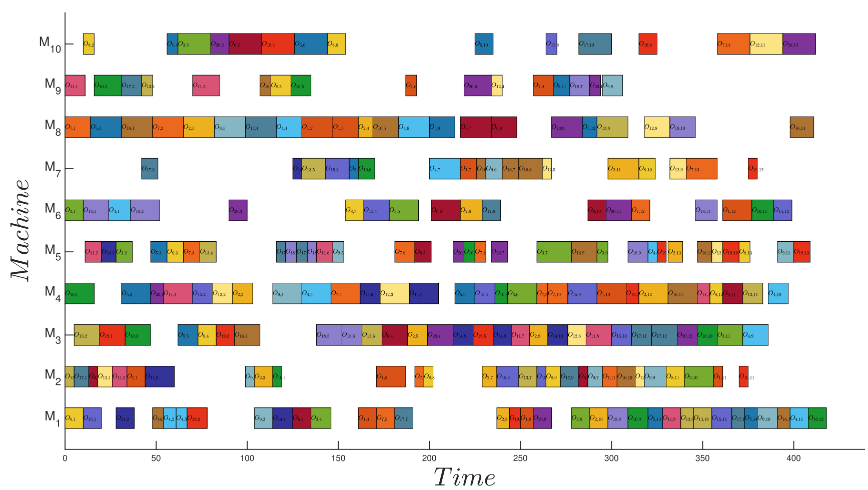


Fig. 15. Corresponding Gantt chart of the solution presented in Figure 12.



(a) Gantt chart for MK01.



(b) Gantt chart for MK09.

Fig. 16. Gantt chart for the best schedules searched by LDPSO. a. Gantt chart for MK01. b. Gantt chart for MK09.

5.3. Compare with other algorithms

In this subsection, LDPSO is compared with two algorithms to demonstrate its effectiveness. The first algorithm is GPSO, which represents the classic PSO algorithm. The second algorithm is CDLPSO, which is the most competitive PSO variant compared in the last Section.

Ten standard flexible job shop scheduling problems from Brandimarte (MK01∼MK10) are utilized to validate the performance of LDPSO. For these problems, the maximum number of fitness evaluations is set to $MaxNFEs = 200 \cdot \sum_{i=1}^{n} n_i$. The best results and mean values are presented in Table 15.

It can be observed that LDPSO is the most efficient algorithm, achieving the best results on 8 problems, and only slightly underperforming GPSO on MK05 and MK10. Additionally, in terms of mean values, it ranks first on 6 problems. GPSO achieves the best results on 4 problems and shares the top position with LDPSO on two problems (MK01 and MK02). In terms of mean values, GPSO performs best on 2 problems (MK02 and MK06), slightly surpassing LDPSO. Moreover, the results of the Friedman rank-and-sum test are presented in Table 16. It can be seen that, LDPSO is the best algorithm for the FJSP. Furthermore, the optimal schedules for MK01 and MK09 are presented in Fig. 16. It is evident that LDPSO is highly effective in solving the FJSP problem, demonstrating its broad potential for application.

Table 15
The comparison results for MK01∼MK10.

	GPSO		CDLPSO		LDPSO	
	Best	Mean	Best	Mean	Best	Mean
MK01	**42**	49.43	43	46.17	**42**	**45.27**
MK02	**38**	**43.60**	44	50.70	**38**	47.70
MK03	250	273.57	248	270.87	**217**	**262.13**
MK04	78	84.33	75	80.00	**73**	**79.40**
MK05	**190**	204.07	193	**200.27**	196	200.60
MK06	117	**130.10**	109	139.47	**98**	132.90
MK07	184	209.77	182	195.43	**169**	**190.63**
MK08	565	585.93	559	577.30	**558**	**575.07**
MK09	437	468.27	439	**462.30**	**418**	462.60
MK10	**363**	398.83	386	404.53	368	**394.77**

Table 16
The Friedman rank-and-sum results for the comparison.

Algorithm	rank	p-value
GPSO	2.50	
CDLPSO	2.00	0.08
LDPSO	1.50	

6. Conclusion

In this paper, a leaders-driven particle swarm optimizer (LDPSO) was proposed. Based on the performance of the particles, the population was divided into leaders and followers. The leaders, characterized by superior performance, are responsible for identifying more promising search areas and guiding the search direction, while the followers concentrate on enhancing local search capabilities. A bidirectional search strategy was introduced for the leaders, allowing them to explore the search space as broadly as possible while maintaining search quality. Additionally, a unidirectional search strategy was designed for the followers, which enables the underperforming group to fully utilize the promising information provided by the leaders. Moreover, to help stagnating particles in breaking free from local optima, a leader-follower collaborative jump-out strategy has been designed. Furthermore, the number of leaders gradually decreases throughout the search process, and high-quality global diversification gives way to enhanced local centralization. Finally, the strategies were organically coupled, effectively balancing search efficiency and population diversity.

The comprehensive contribution analyses revealed that each essential component of LDPSO (i.e., bidirectional search strategy, unidirectional search strategy, leader-follower collaborative jump-out strategy, and leader number adjustment strategy) offers a positive impact and collectively contributes the most. Subsequently, the optimization performance of LDPSO was evaluated against widely used and recently developed PSO variants, as well as non-PSO methods. The comparison confirmed that LDPSO performs well in both search efficiency and population diversity. Moreover, LDPSO demonstrated superior search capability and robustness, more stable convergence properties, and higher solution accuracy. Diversity analysis demonstrated that LDPSO achieves rapid convergence on unimodal problems while maintaining reasonable population diversity on multimodal functions. This balance ensures that population diversity is neither too high to disrupt the exploitation, nor too low, which could lead to premature convergence. Finally, LDPSO was applied to solve flexible job shop scheduling problems. The numerical simulations demonstrated that LDPSO generally outperforms other methods in most instances, exhibiting strong search capabilities in identifying optimal solutions for complex scheduling challenges.

Although LDPSO has demonstrated superior performance, there remains potential for further enhancement. Its effectiveness on hybrid functions ($F_{11}\sim F_{20}$) is not as robust as on composition functions ($F_{21}\sim F_{30}$), which could potentially be enhanced by introducing an adaptive parameter adjustment mechanism. In the future, we intend to extend the co-evolutionary mechanism of leaders and followers to multi-objective evolutionary algorithms. Additionally, integrating LDPSO with other optimization algorithms may enhance its exploration capabilities. For instance, the crossover and mutation operations of genetic algorithms can cover a broader search space, while the particle movement of LDPSO can refine the search for optimal solutions. Furthermore, reinforcement learning techniques have demonstrated encouraging performance in dynamic decision-making scenarios. The use of reinforcement learning to dynamically adjust relevant parameters, select learning exemplars, or choose evolutionary mechanisms appears to be a promising direction.

CRediT authorship contribution statement

Shicun Zhao: Conceptualization, Methodology, Software, Formal analysis, Writing – original draft, Investigation; **Hong Zhou:** Conceptualization, Supervision, Funding acquisition, Writing – review & editing; **Han Zhou:** Investigation, Formal analysis.

Data availability

Data will be made available on request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

Abdel-Basset, M., Mohamed, R., Azeem, S. A. A., Jameel, M., & Abouhawwash, M. (2023). Kepler optimization algorithm: A new metaheuristic algorithm inspired by kepler's laws of planetary motion. *Knowledge-Based Systems*, *268*, 110454.

Brucker, P., & Schlie, R. (1990). Job-shop scheduling with multipurpose machines. *Computing*, *45*(4), 369–375.

Chen, H.-t., Wang, W.-c., Chen, X.-n., & Qiu, L. (2020). Multi-objective reservoir operation using particle swarm optimization with adaptive random inertia weights. *Water Science and Engineering*, *13*(2), 136–144.

Chen, K., Zhou, F., Yin, L., Wang, S., Wang, Y., & Wan, F. (2018). A hybrid particle swarm optimizer with sine cosine acceleration coefficients. *Information Sciences*, *422*, 218–241.

Cheng, J., & De Waele, W. (2024). Weighted average algorithm: A novel meta-heuristic optimization algorithm based on the weighted average position concept. *Knowledge-Based Systems*, *305*, 112564.

Cheng, M.-Y., & Sholeh, M. N. (2023). Optical microscope algorithm: A new metaheuristic inspired by microscope magnification for solving engineering optimization problems. *Knowledge-Based Systems*, *279*, 110939.

Cheng, R., & Jin, Y. (2015). A social learning particle swarm optimization algorithm for scalable optimization. *Information Sciences*, *291*, 43–60.

Dauzère-Pérès, S., Ding, J., Shen, L., & Tamssaouet, K. (2024). The flexible job shop scheduling problem: A review. *European Journal of Operational Research*, *314*(2), 409–432.

Dehghani, M., Montazeri, Z., Trojovská, E., & Trojovský, P. (2023). Coati optimization algorithm: A new bio-inspired metaheuristic algorithm for solving optimization problems. *Knowledge-Based systems*, *259*, 110011.

Deng, L., & Liu, S. (2023). Snow ablation optimizer: A novel metaheuristic technique for numerical optimization and engineering design. *Expert Systems with Applications*, *225*, 120069.

Dong, Y., Zhang, S., Zhang, H., Zhou, X., & Jiang, J. (2025). Chaotic evolution optimization: A novel metaheuristic algorithm inspired by chaotic dynamics. *Chaos, Solitons & Fractals*, *192*, 116049.

Gao, K., Cao, Z., Zhang, L., Chen, Z., Han, Y., & Pan, Q. (2019). A review on swarm intelligence and evolutionary algorithms for solving flexible job shop scheduling problems. *IEEE/CAA Journal of Automatica Sinica*, *6*(4), 904–916.

Gong, Y.-j., & Zhang, J. (2013). Small-world particle swarm optimization with topology adaptation. In *Proceedings of the 15th annual conference on genetic and evolutionary computation* (pp. 25–32).

Guo, W., Li, L., Chen, M., Ni, W., Wang, L., & Li, D. (2025). Balancing convergence and diversity preservation in dual search space for large scale particle swarm optimization. *Applied Soft Computing*, *169*, 112617.

Han, M., Du, Z., Yuen, K. F., Zhu, H., Li, Y., & Yuan, Q. (2024). Walrus optimizer: A novel nature-inspired metaheuristic algorithm. *Expert Systems with Applications*, *239*, 122413.

Jiyue, E., Liu, J., & Wan, Z. (2023). A novel adaptive algorithm of particle swarm optimization based on the human social learning intelligence. *Swarm and Evolutionary Computation*, *80*, 101336.

Karaboga, D. et al. (2005). An idea based on honey bee swarm for numerical optimization. Technical Report Technical report-tr06, Erciyes university, engineering faculty, computer engineering department.

Kennedy, J., & Eberhart, R. (1995). Particle swarm optimization. In *Proceedings of ICNN'95-international conference on neural networks* (pp. 1942–1948). IEEE (*vol. 4*).

Lang, Y., & Gao, Y. (2025). Dream optimization algorithm (DOA): A novel metaheuristic optimization algorithm inspired by human dreams and its applications to real-world engineering problems. *Computer Methods in Applied Mechanics and Engineering*, *436*, 117718.

Li, W., Jing, J., Chen, Y., & Chen, Y. (2023). A cooperative particle swarm optimization with difference learning. *Information Sciences*, *643*, 119238.

Lian, J., & Hui, G. (2024). Human evolutionary optimization algorithm. *Expert Systems with Applications*, *241*, 122638.

Liang, J. J., Qin, A. K., Suganthan, P. N., & Baskar, S. (2006). Comprehensive learning particle swarm optimizer for global optimization of multimodal functions. *IEEE Transactions on Evolutionary Computation*, *10*(3), 281–295.

Lin, A., Sun, W., Yu, H., Wu, G., & Tang, H. (2019). Global genetic learning particle swarm optimization with diversity enhancement by ring topology. *Swarm and Evolutionary Computation*, *44*, 571–583.

Liu, H., Zhang, X.-W., & Tu, L.-P. (2020). A modified particle swarm optimization using adaptive strategy. *Expert Systems with Applications*, *152*, 113353.

Liu, Q., Li, J., Ren, H., & Pang, W. (2022). All particles driving particle swarm optimization: superior particles pulling plus inferior particles pushing. *Knowledge-Based Systems*, *249*, 108849.

Liu, W., Wang, Z., Yuan, Y., Zeng, N., Hone, K., & Liu, X. (2019). A novel sigmoid-function-based adaptive weighted particle swarm optimizer. *IEEE Transactions on Cybernetics*, *51*(2), 1085–1093.

Liu, W., Wang, Z., Zeng, N., Yuan, Y., Alsaadi, F. E., & Liu, X. (2021). A novel randomised particle swarm optimizer. *International Journal of Machine Learning and Cybernetics*, *12*, 529–540.

Lynn, N., & Suganthan, P. N. (2017). Ensemble particle swarm optimizer. *Applied Soft Computing*, *55*, 533–548.

Mendes, R., Kennedy, J., & Neves, J. (2004). The fully informed particle swarm: Simpler, maybe better. *IEEE Transactions on Evolutionary Computation*, *8*(3), 204–210.

Meng, X., & Li, H. (2024). Heterogeneous pbest-guided comprehensive learning particle swarm optimization. *Applied Soft Computing*, (p. 111874).

Meng, Z., Zhong, Y., Mao, G., & Liang, Y. (2022). Pso-sono: A novel pso variant for single-objective numerical optimization. *Information Sciences*, *586*, 176–191.

Peram, T., Veeramachaneni, K., & Mohan, C. K. (2003). Fitness-distance-ratio based particle swarm optimization. In *Proceedings of the 2003 IEEE swarm intelligence symposium. SIS'03 (cat. no. 03EX706)* (pp. 174–181). IEEE.

Qu, B.-Y., Suganthan, P. N., & Das, S. (2012). A distance-based locally informed particle swarm model for multimodal optimization. *IEEE Transactions on Evolutionary Computation*, *17*(3), 387–402.

Ratnaweera, A., Halgamuge, S. K., & Watson, H. C. (2004). Self-organizing hierarchical particle swarm optimizer with time-varying acceleration coefficients. *IEEE Transactions on Evolutionary Computation*, *8*(3), 240–255.

Ren, C., Xu, Q., Meng, Z., & Pan, J.-S. (2024). Surrogate-assisted fully-informed particle swarm optimization for high-dimensional expensive optimization. *Applied Soft Computing*, *167*, 112464.

Shi, Y., & Eberhart, R. (1998). A modified particle swarm optimizer. In *1998 IEEE international conference on evolutionary computation proceedings. IEEE world congress on computational intelligence (cat. no. 98TH8360)* (pp. 69–73). IEEE.

Su, H., Zhao, D., Heidari, A. A., Liu, L., Zhang, X., Mafarja, M., & Chen, H. (2023). Rime: A physics-based optimization. *Neurocomputing*, *532*, 183–214.

Suganthan, P. N. (1999). Particle swarm optimiser with neighbourhood operator. In *Proceedings of the 1999 congress on evolutionary computation-CEC99 (cat. no. 99TH8406)* (pp. 1958–1962). IEEE (*vol. 3*).

Sun, B., Li, G., Wang, S., & Xie, C. (2025). Two-dimensional bin-packing problem with conflicts and load balancing: A hybrid chaotic and evolutionary particle swarm optimization algorithm. *Computers & Industrial Engineering*, *200*, 110851.

Tang, Y., Huang, K., Tan, Z., Fang, M., & Huang, H. (2024). Multi-subswarm cooperative particle swarm optimization algorithm and its application. *Information Sciences*, (p. 120887).

Tian, D., Xu, Q., Yao, X., Zhang, G., Li, Y., & Xu, C. (2024). Diversity-guided particle swarm optimization with multi-level learning strategy. *Swarm and Evolutionary Computation*, *86*, 101533.

Wang, D., Zhao, S., Chen, K., & Liu, S. (2021). Parameter estimation of the classical fractal map based on a given julia set's shape. *Fractals*, *29*(08), 2150247.

Wang, X., Snášel, V., Mirjalili, S., Pan, J.-S., Kong, L., & Shehadeh, H. A. (2024). Artificial protozoa optimizer (APO): A novel bio-inspired metaheuristic algorithm for engineering optimization. *Knowledge-Based Systems*, *295*, 111737.

Wu, G., Mallipeddi, R., & Suganthan, P. N. (2017). Problem definitions and evaluation criteria for the CEC 2017 competition on constrained real-parameter optimization. *National university of defense technology, Changsha, Hunan, PR China and Kyungpook national university, Daegu, South Korea and Nanyang technological university, Singapore, Technical Report.*

Xia, X., Gui, L., Yu, F., Wu, H., Wei, B., Zhang, Y.-L., & Zhan, Z.-H. (2019). Triple archives particle swarm optimization. *IEEE Transactions on Cybernetics*, *50*(12), 4862–4875.

Xu, G., Cui, Q., Shi, X., Ge, H., Zhan, Z.-H., Lee, H. P., Liang, Y., Tai, R., & Wu, C. (2019). Particle swarm optimization based on dimensional learning strategy. *Swarm and Evolutionary Computation*, *45*, 33–51.

Xu, Y., Wang, D., Zhang, M., Yang, M., & Liang, C. (2025). Quantum particle swarm optimization with chaotic encoding schemes for flexible job-shop scheduling problem. *Swarm and Evolutionary Computation*, *93*, 101836.

Yang, J., Yu, J., & Huang, C. (2022). Adaptive multistrategy ensemble particle swarm optimization with signal-to-noise ratio distance metric. *Information Sciences*, *612*, 1066–1094.

Zhan, Z.-H., Zhang, J., Li, Y., & Chung, H. S.-H. (2009a). Adaptive particle swarm optimization. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, *39*(6), 1362–1381.

Zhan, Z.-H., Zhang, J., & Liu, O. (2009b). Orthogonal learning particle swarm optimization. In *Proceedings of the 11th annual conference on genetic and evolutionary computation* (pp. 1763–1764).

Zhan, Z.-h., Zhang, J., & Shi, Y.-h. (2010). Experimental study on PSO diversity. In *Third international workshop on advanced computational intelligence* (pp. 310–317). IEEE.

Zhang, G., Shao, X., Li, P., & Gao, L. (2009). An effective hybrid particle swarm optimization algorithm for multi-objective flexible job-shop scheduling problem. *Computers & Industrial Engineering*, *56*(4), 1309–1318.

Zhang, X., & Lin, Q. (2022). Three-learning strategy particle swarm algorithm for global optimization problems. *Information Sciences*, *593*, 289–313.

Zhang, Y. (2023). Elite archives-driven particle swarm optimization for large scale numerical optimization and its engineering applications. *Swarm and Evolutionary Computation*, *76*, 101212.

Zhao, S., & Wang, D. (2022). Elite-ordinary synergistic particle swarm optimization. *Information Sciences*, *609*, 1567–1587.

Zhao, S., & Zhou, H. (2025). Learning-driven memetic algorithm for solving integrated distributed production and transportation scheduling problem. *Swarm and Evolutionary Computation*, *96*, 101945.

Zhao, S., Zhou, H., Zhao, Y., & Wang, D. (2024a). Dql-assisted competitive evolutionary algorithm for energy-aware robust flexible job shop scheduling under unexpected disruptions. *Swarm and Evolutionary Computation*, *91*, 101750.

Zhao, W., Wang, L., Zhang, Z., Fan, H., Zhang, J., Mirjalili, S., Khodadadi, N., & Cao, Q. (2024b). Electric eel foraging optimization: A new bio-inspired optimizer for engineering applications. *Expert Systems with Applications*, *238*, 122200.

Zhao, Y., Wu, F., Pang, J., & Zhong, W. (2025). New heterogeneous comprehensive learning particle swarm optimizer enhanced with low-discrepancy sequences and conjugate gradient method. *Swarm and Evolutionary Computation*, *93*, 101848.

Zheng, K., Yuan, X., Xu, Q., Dong, L., Yan, B., & Chen, K. (2022). Hybrid particle swarm optimizer with fitness-distance balance and individual self-exploitation strategies for numerical optimization problems. *Information Sciences*, *608*, 424–452.

Zhu, D., Shen, J., Zhang, Y., Li, W., Zhu, X., Zhou, C., Cheng, S., & Yao, Y. (2024). Multi-strategy particle swarm optimization with adaptive forgetting for base station layout. *Swarm and Evolutionary Computation*, *91*, 101737.