

## Mid-Term Exams

Fayoum University

1<sup>st</sup> Civil Eng.

Differential Equations

Faculty of Engineering

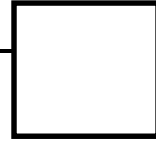
Mid -Term exam

Apr., 29, 2018

Examiner: Assoc. Prof. Karem Mahmoud

Time: 40 Min.

**Choose the correct answer as this Figure** ●



1. Find the degree and order of D.E: $\sin x + y^2 + y' = 3$	(A) 4, 3	(B) 3, 2	(C) 2, 1	(D) 1, 1
2. Find the independent variable in the code <code>&gt;&gt; y=dsolve('Dy=y^3*exp(-2*x)')</code>	(A) y	(B) x	(C) t	(D) D
3. Solve the D.E: $y' = \sin^2(x - y + 1), y(0) = -1,$	(A) $x - 1 - \tan^{-1}(x)$	(B) $x - \tan^{-1}(x + c)$	(C) $x - \tan^{-1}(x)$	(D) $x - \tan(x)$
4. $(2xy + \tan x) y' = -(y^2 + y / \cos^2 x)$ is .... D.E:	(A) linear	(B) Bernoulli	(C) exact	(D) separable
5. $xy' + y = y^2 x^2$	(A) $1/(-x^2 - cx)$	(B) $1/(x^2 + cx)$	(C) $1/(x^2 - cx)$	(D) $(1+c)/x^2$
6. Find the resistance force in the mechanical system: $mg - kv^2 = m dv/dt$	(A) $-kv^2$	(B) $mg$	(C) $kv^2$	(D) $-mg - kv^2$
7. The problem: $y''' + 2xy'' + y' - y = x, y(0)=1, y'(0)=-1, y''(0)=0.$ is ....	(A) non-linear	(B) BVP	(C) IVP	(D) particular
8. Find the linearly dependent functions	(A) $e^{-x}, e^x$	(B) $x, e^x$	(C) $x, 2x$	(D) $\sin x, \sin(2x)$
9. Solve the D.E: $4y'' - 12y' + 5y = 0, y(0)=0, y'(0)=1.$	(A) $e^{x/2}(c - 2ce^{2x})$	(B) $e^{x/2}(1 - e^{2x})$	(C) $ce^{x/2}(1 - e^{2x})$	(D) $ce^{x/2}(1 + e^{2x})$
10. Find output of the code <code>&gt;&gt; dsolve('D2y+y=2*t','y(0)=0','Dy(0)=1','t')</code>	(A) $\sin(t) + 2*t$	(B) $2*\sin(t) + t$	(C) $-\sin(t) + 2*t$	(D) $\sin(x) + 2*x$
11. If the characteristic equation has repeated root ( $m=1, r=3$ ), then the bases are	(A) $e^x, e^x, e^x$	(B) $xe^x, x^2 e^x, x^3 e^x$	(C) $e^x, xe^x, x^2 e^x$	(D) $e^{-x}, xe^{-x}, x^2 e^{-x}$

<p>12. Consider the D.E. <math>y'''' - 4y''' + 4y'' = 0</math>. Find root of its characteristic equation</p> <p>(A) 0, 0, 4, 4      (B) 0, 0, -2, -2      (C) 0, 0, 2, 2      (D) 0, 2</p>
<p>13. the D.E. <math>y'' + ay' + by =</math> cannot be solved using undetermined coefficients method if</p> <p>(A) 1      (B) <math>(1 + e^{2x})/e^x</math>      (C) <math>e^x/(1 + e^{2x})</math>      (D) <math>\sin x + \cos x</math></p>
<p>14. Find <math>Y_p</math> in the D.E. <math>y''' + 2y'' = 2</math></p> <p>(A) 2      (B) <math>2x^2</math>      (C) <math>x^2</math>      (D) 0, 0, -2</p>
<p>15. Find a solution of <math>y''' - y'' + y' - y = 0</math></p> <p>(A) <math>e^{-jx}</math>      (B) <math>e^{2jx}</math>      (C) <math>e^{jx}</math>      (D) <math>e^{jx/2}</math></p>
<p>16. Find <math>(w_1 - c_1)</math> or <math>(w_2 - c_2)</math> in the solution of <math>y'' + y = \sin^2 x</math> by variation of parameters method.</p> <p>(A) <math>(\cos^3 x)/3</math>      (B) <math>-(\cos^3 x)/3</math>      (C) <math>(\sin^3 x)/3</math>      (D) <math>(\cos^3 x)/3 - \cos x</math></p>
<p>17. Find number of dependent variables in the system: <math>y' = y + z</math>, <math>z' = y + z + x</math></p> <p>(A) 0      (B) 1      (C) 2      (D) 3</p>
<p>18. Find <math>z</math> in the system: <math>y' = z</math>, <math>z' = y + x</math>.</p> <p>(A) <math>ae^x + be^{-x} - x</math>      (B) <math>ae^x + be^{-x} + 1</math>      (C) <math>ae^x + be^{-x} - 1</math>      (D) <math>ae^x + be^{-x} + x</math></p>
<p>19. The D.E. governs the shown circuit is ....</p> <div data-bbox="304 1339 616 1464" data-label="Diagram"> </div> <p>(A) Non-linear      (B) homogeneous      (C) Linear      (D) separable</p>
<p>20. Find the D.E. governs the deflection of shown beam.</p> <div data-bbox="1066 1541 1385 1675" data-label="Diagram"> </div> <p>(A) <math>EI \frac{d^4 y}{dx^4} + w(x) = 0</math>      (B) <math>EI \frac{d^2 y}{dx^2} + w(x) = 0</math>      (C) <math>EI \frac{d^4 y}{dx^4} + w_0 = 0</math>      (D) <math>\frac{d^4 y}{dx^4} + w(x) = 0</math></p>

Fayoum University

1<sup>st</sup> Civil Eng.

Differential Equations

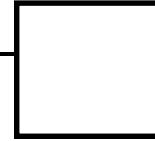
Faculty of Engineering

Makeup -Term exam

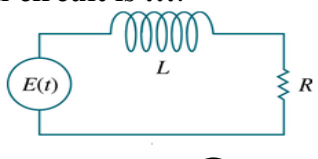
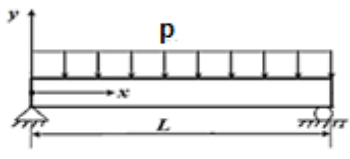
May, 13, 2018

Examiner: Assoc. Prof. Karem Mahmoud

Time: 35 Min.

**Choose the correct answer as this Figure ● ..**

2. Find the degree and order of D.E: $\sin x + y^3 + (y')^2 = 0$ .	(A) 2, 1	(B) 2, 1	(C) 3, 1	(D) 3, 2
21. Find the independent variable in the code <code>&gt;&gt; y=dsolve('Dy=y^3*exp(-2*y)', 'x')</code>	(A) y	(B) x	(C) t	(D) D
22. Solve the D.E: $y' = \sin^2(x - y + 1), y(0) = -1$	(A) $x - 1 - \tan^{-1}(x)$	(B) $x - \tan^{-1}(x)$	(C) $x - \tan^{-1}(x + c)$	(D) $x - \tan(x)$
23. $(2x + \tan x) y' = -(y^2 + y / \cos y)$ is .... D.E:	(A) linear	(B) Bernoulli	(C) exact	(D) separable
24. Solve the D.E: $xy' + y = y^2 x^2$	(A) $1/(-x^2 - cx)$	(B) $1/(x^2 + cx)$	(C) $(1+c)/x^2$	(D) $1/(x^2 - cx)$
25. Find the resistance force in the mechanical system: $mg + kv^2 - \lambda v = m dv/dt$	(A) $kv^2 - \lambda v$	(B) $mg$	(C) $-mg - kv^2$	(D) $\lambda v$
26. The problem: $y''' + 2xy'' + y' - y = x, y(0)=1, y'(0)=-1, y''(1)=0$ . is ....	(A) non-linear	(B) IVP	(C) BVP	(D) particular
27. Find the linearly dependent functions	(A) $e^{-x}, e^x$	(B) $x, e^x$	(C) $\exp(x), \exp(2+x)$	(D) $\sin x, \sin(2x)$
28. Solve the D.E: $4y'' - 12y' + 5y = 0, y(0)=0, y'(0)=1$ .	(A) $e^{x/2}(c - 2ce^{2x})$	(B) $e^{x/2}(1 - e^{2x})$	(C) $ce^{x/2}(1 + e^{2x})$	(D) $ce^{x/2}(1 - e^{2x})$
29. Find output of the code <code>&gt;&gt; dsolve('D2y+y=2*t', 'y(0)=0', 'Dy(0)=1', 't')</code>	(A) $-\sin(t) + 2*t$	(B) $\sin(t) + 2*t$	(C) $2*\sin(t) + t$	(D) $\sin(x) + 2*x$
30. If the characteristic equation has repeated root ( $m=1, r=3$ ), then the bases are				

(A) $e^x, xe^x, x^2 e^x$	(B) $e^x, 2e^x, 3e^x$	(C) $xe^x, x^2 e^x, x^3 e^x$	(D) $e^{-x}, xe^{-x}, x^2 e^{-x}$
31. Consider the D.E. $y'''' - 6y''' + 9y'' = 0$ . Find root of its characteristic equation			
(A) 0, 0, 3, 3	(B) 0, 0, 6, 6	(C) 0, 0, -3, -3	(D) 3, 3
32. The D.E. $y'' + ay' + by = F$ cannot be solved using undetermined coefficients method if $F = \dots$			
(A) $\sinh x \cdot \cos x$	(B) 1	(C) $(1 + e^{2x})/e^x$	(D) $\sin x + \cos x$
33. Find $Y_p$ in the D.E. $y''' - y'' = 2$			
(A) $-x^2$	(B) $x^2$	(C) $2x^2$	(D) 0, 0, 2
34. Find a solution of $y''' - y'' + y' - y = 0$			
(A) $e^{-jx}$	(B) $e^{2jx}$	(C) $e^{jx}$	(D) $e^{jx/2}$
35. Find $(w_1 - c_1)$ or $(w_2 - c_2)$ in the solution of $y'' + y = \sin^2 x$ by variation of parameters method.			
(A) $(\cos^3 x)/3$	(B) $-(\cos^3 x)/3$	(C) $(\sin^3 x)/3$	(D) $(\cos^3 x)/3 - \cos x$
36. Find number of independent variables in the system: $\dot{y} = y + z$ , $\dot{z} = y + z + x$ .			
(A) 2	(B) 1	(C) 0	(D) 3
37. Find $Z$ in the system: $\dot{y} = z$ , $\dot{z} = y + x$ .			
(A) $ae^x + be^{-x} + 1$	(B) $ae^x + be^{-x} - 1$	(C) $ae^x + be^{-x} - x$	(D) $ae^x + be^{-x} + x$
38. The D.E. governs the shown circuit is ...			
			
(A) exact	(B) Non-homogeneous	(C) Non-linear	(D) separable
39. Find the D.E. governs the deflection of shown beam.			
			
(A) $EI \frac{d^2 y}{dx^2} + p(x) = 0$	(B) $EI \frac{d^4 y}{dx^4} + p = 0$	(C) $EI \frac{d^4 y}{dx^4} + w(x) = 0$	(D) $\frac{d^4 y}{dx^4} + w(x) = 0$

Fayoum University

1<sup>st</sup> Civil Eng.

Differential Equations

Faculty of Engineering

Mid -Term exam

March, 31, 2019

Examiner: Assoc. Prof. Karem Mahmoud

Time: 45 Min.

Choose the correct answer as this Figure .

1. Find the degree and order of D.E: $\sin x + y^2 + y''' = 3$ .	(A) 3, 1	(B) 1, 1	(C) 2, 1	(D) 1, 3
2. The D.E: $y y'' + y''' = 3x$ is .....	(A) third-order	(B) Linear	(C) Non-linear	(D) homogenous
3. Solve the D.E: $y' + 1 = 1/(x+y)$ .	(A) $\sqrt{x+c}$	(B) $\pm \sqrt{2x+c} - x$	(C) $\sqrt{x-c}$	(D) $\pm \sqrt{2c-2x}$
4. The D.E: $(2xy + \tan x) y' = -(y^2 + y/\cos^2 x)$ is .....	(A) exact	(B) Bernoulli	(C) linear	(D) separable
5. The D.E: $xy' + y = y^2 x^2$ is .....	(A) linear	(B) Bernoulli	(C) exact	(D) separable
6. Solve the D.E: $y' = y/x + x/y$ , $y(1) = 0$ .	(A) $y^2 = c x^2 \ln x^2$	(B) $y = c x \ln x^2$	(C) $y^2 = x^2 \ln x^2$	(D) $y = 3x \ln x^2$
7. The problem: $y''' + 2xy'' + y' - y = x$ , $y(0) = 1$ , $y'(0) = -1$ , $y''(1) = 0$ . is .....	(A) non-linear	(B) IVP	(C) particular	(D) BVP
8. Find the linearly dependent bases.	(A) $e^{-x}, e^x$	(B) $x, e^x$	(C) $\sin x, \sin(2x)$	(D) $\ln x, \ln(x^2)$
9. Solve the D.E: $y'' + y = 0$ , $y(0) = 1, y'(0) = 0$ .	(A) $e^x$	(B) $\sin x$	(C) $\cos x$	(D) $x e^{-x}$
10. The D.E $y''' + 2xy'' + y' - y = x$ has ....coefficients.	(A) constant	(B) variable	(C) homogenous	(D) degree
11. If the characteristic equation has repeated root ( $m=1, r=3$ ), then the bases are:	(A) $e^x, x e^x, x^2 e^x$	(B) $e^x, e^x, e^x$	(C) $x e^x, x^2 e^x, x^3 e^x$	(D) $e^{-x}, x e^{-x}, x^2 e^{-x}$
12. Consider the D.E. $y^{(4)} - 4y^{(3)} + 13y^{(2)} - 36y^{(1)} + 36y = 0$ . Find roots of its characteristic equation.				

Associate Prof. Dr. / Karem Mahmoud Ewis

(A) 3j, 2	(B) 3j, -3j, 2, 2	(C) 3j, -3j, 2	(D) 2j, -2j, 3, -3
13. The D.E $y'' + ay' + by = F$ cannot be solved using undetermined coefficients method if $F=.....$			
(A) 1	(B) $\ln 2x / \ln x$	(C) $1/(1+e^{2x})$	(D) $\sin x + \cos x$
14. Find $F(x)$ in the D.E. $y''' + 2y'' + x^2 = 1$ .			
(A) 1	(B) $y'''$	(C) $x^2$	(D) $1 - x^2$
15. Find a solution of $y''' - y'' + y' - y = 0$ .			
(A) $e^{-2jx}$	(B) $e^{2jx}$	(C) $e^{jx}$	(D) $e^{2x}$
16. Find $(w_1 - c_1)$ or $(w_2 - c_2)$ in the solution of $y'' + y = \sin^2 x$ by variation of parameters method.			
(A) $(\cos^3 x)/3$	(B) $-(\cos^3 x)/3$	(C) $(\sin^3 x)/3$	(D) $(\cos^3 x)/3 - \cos x$
17. Find the dependent variable in the D.E.: $\frac{dz}{dx} = z + 2x$ .			
(A) y	(B) x	(C) z	(D) x, z
18. Find $Y_p$ in the system:, $y'' = y + x$ .			
(A) -x	(B) x	(C) -2x	(D) -x/2
19. The characteristic equation $m^5 - 2m^4 + m^3 = 0$ has .....roots.			
(A) 5	(B) 4	(C) 3	(D) 2
20. Find the non-homogenous D.E.			
(A) $\frac{d^2 y}{dx^2} + \sin x = 0$	(B) $\frac{d^4 y}{dx^4} + A \frac{d^2 y}{dx^2} = 0$	(C) $\frac{d^4 y}{dx^4} + Ay = 0$	(D) $\frac{d^4 y}{dx^4} + xy = 0$

Fayoum University  
Faculty of Engineering

1<sup>st</sup> Civil Eng.  
Make-up exam

Differential Equations  
May., 5, 2019

Examiner: Assoc. Prof. Karem Mahmoud

Time: 40 Min.

**Choose the correct answer as this Figure** ●

1. Find the degree and order of D.E: $\sin x + y^2 + y' = 3$	(A) 4, 3	(B) 3, 2	(C) 1, 1	(D) 2, 1
2. Find roots of its characteristic equation. In D.E. $y^{(4)} - 4y^{(3)} + 13y^{(2)} - 36y^{(1)} + 36y = 0$ .	(A) 3j, 2	(B) 3j, -3j, 2	(C) 3j, -3j, 2, 2	(D) 2j, -2j, 3, -3
3. Solve the D.E: $y' = \sin^2(x - y + 1), y(0) = -1$ ,	(A) $x - 1 - \tan^{-1}(x)$	(B) $x - \tan^{-1}(x + c)$	(C) $x - \tan^{-1}(x)$	(D) $x - \tan(x)$
4. $(2xy + \tan x) y' = -(y^2 + y / \cos^2 x)$ is .... D.E:	(A) linear	(B) Bernoulli	(C) separable	(D) exact
5. $xy' + y = y^2 x^2$	(A) $1/(-x^2 - cx)$	(B) $1/(x^2 + cx)$	(C) $(1+c)/x^2$	(D) $1/(x^2 - cx)$
6. Solve the D.E: $y' + 1 = 1/(x + y)$ .	(A) $\sqrt{x + c}$	(B) $\sqrt{x - c}$	(C) $\pm \sqrt{2c - 2x}$	(D) $\pm \sqrt{2x + c} - x$
7. The problem: $y''' + 2xy'' + y' - y = x, y(0) = 1, y'(0) = -1, y''(0) = 0$ . is ....	(A) non-linear	(B) BVP	(C) IVP	(D) particular
8. Find the linearly dependent functions	(A) $e^{-x}, e^x$	(B) $x, e^x$	(C) $x, 2x$	(D) $\sin x, \sin(2x)$
9. Solve the D.E: $4y'' - 12y' + 5y = 0, y(0) = 0, y'(0) = 1$ .	(A) $e^{x/2}(c - 2ce^{2x})$	(B) $e^{x/2}(1 - e^{2x})$	(C) $ce^{x/2}(1 - e^{2x})$	(D) $ce^{x/2}(1 + e^{2x})$
10. Find $Y_p$ in the system:, $y'' = y + x$ .	(A) $-x$	(B) $2*\sin(t) + t$	(C) $-\sin(t) + 2*t$	(D) $\sin(x) + 2*x$
11. If the characteristic equation has repeated root ( $m=1, r=3$ ), then the bases are	(A) $e^x, xe^x, x^2e^x$	(B) $xe^x, x^2e^x, x^3e^x$	(C) $e^x, e^x, e^x$	(D) $e^{-x}, xe^{-x}, x^2e^{-x}$

12. Consider the D.E. $y'''' - 4y''' + 4y'' = 0$ . Find root of its characteristic equation			
<input checked="" type="radio"/> (A) 0, 0, 2, 2	<input type="radio"/> (B) 0, 0, -2, -2	<input type="radio"/> (C) 0, 0, 4, 4	<input type="radio"/> (D) 0, 2
13. $y'' + ay' + by = F$ is solved by undetermined coefficients method if $F=...$			
<input type="radio"/> (A) $e^x/(1+e^{2x})$	<input type="radio"/> (B) $(1+e^{2x})/(2+e^x)$	<input checked="" type="radio"/> (C) $\sin x \cdot \cos x$	<input type="radio"/> (D) $\sin x / \cos x$
14. Find $Y_p$ in the D.E. $y''' + 2y'' = 2$			
<input type="radio"/> (A) 2	<input type="radio"/> (B) $2x^2$	<input checked="" type="radio"/> (C) $x^2$	<input type="radio"/> (D) 0, 0, -2
15. Find a solution of $y''' - y'' + y' - y = 0$			
<input type="radio"/> (A) $e^{-jx}$	<input type="radio"/> (B) $e^{2jx}$	<input checked="" type="radio"/> (C) $e^{jx}$	<input type="radio"/> (D) $e^{jx/2}$
16. Find $(w_1 - c_1)$ or $(w_2 - c_2)$ in the solution of $y'' + y = \sin^2 x$ by variation of parameters method.			
<input type="radio"/> (A) $(\cos^3 x)/3$	<input type="radio"/> (B) $-(\cos^3 x)/3$	<input checked="" type="radio"/> (C) $(\sin^3 x)/3$	<input type="radio"/> (D) $(\cos^3 x)/3 - \cos x$
17. Find number of dependent variables in the system: $y' = y + z$ , $z' = y + z + x$			
<input type="radio"/> (A) 0	<input type="radio"/> (B) 1	<input checked="" type="radio"/> (C) 2	<input type="radio"/> (D) 3
18. Find $z$ in the system: $y' = z$ , $z' = y + x$ .			
<input type="radio"/> (A) $ae^x + be^{-x} - x$	<input type="radio"/> (B) $ae^x + be^{-x} + 1$	<input checked="" type="radio"/> (C) $ae^x + be^{-x} - 1$	<input type="radio"/> (D) $ae^x + be^{-x} + x$
19. The characteristic equation $m^5 - 2m^4 + m^3 = 0$ has .....roots.			
<input checked="" type="radio"/> (A) 5	<input type="radio"/> (B) 3	<input type="radio"/> (C) Linear	<input type="radio"/> (D) no
20. Find the non-homogenous D.E.			
<input checked="" type="radio"/> (A) $\frac{d^2 y}{dx^2} + \sin x = 0$	<input type="radio"/> (B) $EI \frac{d^2 y}{dx^2} + w(x) = 0$	<input type="radio"/> (C) $EI \frac{d^4 y}{dx^4} + w_0 = 0$	<input type="radio"/> (D) $\frac{d^4 y}{dx^4} + w(x) = 0$



## Final-Term Exams



Fayoum University Final Term Exam. Differential Equations 1<sup>st</sup> Year Civil Eng. Dept.  
Faculty of Engineering June, 2, 2018 Max. Points: 60 Time: 3 Hrs.

Examiner: Assoc. Prof. Karem Mahmoud Ewis

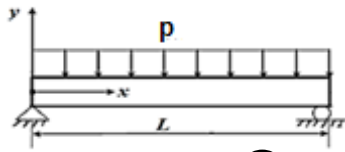
مسموح ب ٨ صفحات قوانين - يجيب الطالب في نفس أوراق الأسئلة و في ظهورها الفارغة  
درجات السؤال الأول موزعة بانتظام على جزئياته و تظليل الاختيار يكون كما هو موضح في رأس السؤال

### Exam instructions

- 1- Only one sheet answer is allowed for every student.
- 2- It is not allowed to borrow the tools (pen, pencils, drawing tools, calculator ...etc.).
- 3- It is not allowed to use the cell phone or any of its application during the time of exam.

Question No 1: Choose the correct answer as this Figure  (40 M)

1. Find $a_0$ in the Fourier series of the periodic function $f(x) = \pi^2 - x^2$ , where, $-\pi < x < \pi$	(A) 6.58	(B) 13.16	(C) 0	(D) 4.39
2. Find $F_s(n)$ of $f(x) = \pi$ , where, $p = \pi$ and $n=6$ .	(A) $\pi(1 - (-1)^n)/n$	(B) 0	(C) 1.05	(D) $2\pi/(2m-1)$
3. Solve the D.E: $y' = \sin^2(x-y+1), y(0)=1$ .	(A) $x-1-\tan^{-1}(x)$	(B) $x-\tan^{-1}(x)+1$	(C) $x-\tan^{-1}(x)+1$	(D) $x-\tan(x)+1$
4. Find inverse Laplace transform of $s/[(s-2)(s-1)]$	(A) $(e^t - e^{2t})/2$	(B) $e^t - e^{2t}$	(C) $e^{2t} + e^t$	(D) $2e^{2t} - e^t$
5. Solve the D.E: $xy' + y = y^2x^2$	(A) $1/(-x^2 - cx)$	(B) $1/(x^2 + cx)$	(C) $(1+c)/x^2$	(D) $1/(x^2 - cx)$
6. Find Laplace transform of $u_1(t) \sin(t-1)$ , where, $s=2$ .	(A) $\frac{e^{-s}}{s^2+1}$	(B) 0.027	(C) 0.135	(D) 0.068
7. The problem: $y''' + 2xy'' + y' - y = x, y(0)=1, y'(0)=-1, y''(1)=0$ . is .....	(A) non-linear	(B) IVP	(C) BVP	(D) particular
8. Find Laplace transform of $t^{100}$ .	(A) $101!/s^{101}$	(B) $100!/s^{100}$	(C) $100!/s^{101}$	(D) $100/s^{101}$
9. Solve the D.E: $4y'' - 12y' + 5y = 0, y(0)=0, y'(0)=1$ .	(A) $e^{x/2}(c - 2ce^{2x})$	(B) $e^{x/2}(1 - e^{2x})$	(C) $ce^{x/2}(1 + e^{2x})$	(D) $ce^{x/2}(1 - e^{2x})$
10. Find output of the code <code>&gt;&gt; dsolve('D2y+y=2*t','y(0)=0','Dy(0)=1','t')</code>				

(A) $\sin(t)+2*t$	(B) $-\sin(t)+2*t$	(C) $2*\sin(t)+t$	(D) $\sin(x)+2*x$
11. If the characteristic equation has repeated root ( $m=1, r=3$ ), then the bases are			
(A) $e^x, xe^x, x^2e^x$	(B) $e^x, 2e^x, 3e^x$	(C) $xe^x, x^2e^x, x^3e^x$	(D) $e^{-x}, xe^{-x}, x^2e^{-x}$
12. Consider the D.E. $y'''' - 6y''' + 9y'' = 0$ . Find root of its characteristic equation			
(A) 0, 0, 3, 3	(B) 0, 0, 6, 6	(C) 0, 0, -3, -3	(D) 3, 3
13. The D.E. $y'' + ay' + by = F$ cannot be solved using undetermined coefficients method if $F=.....$			
(A) $\sinh x * \cos x$	(B) 1	(C) $(1 + e^{2x})De^x$	(D) $\sin x + \cos x$
14. Find $Y_p$ in the D.E. $y''' - y'' = 2$			
(A) $-x^2$	(B) $x^2$	(C) $2x^2$	(D) 0, 0, 2
15. Find a solution of $y''' - y'' + y' - y = 0$			
(A) $e^{-jx^2}$	(B) $e^{2jx}$	(C) $e^{jx}$	(D) $e^{jx/2}$
16. Find ( $w_1 - c_1$ ) or ( $w_2 - c_2$ ) in the solution of $y'' + y = \sin^2 x$ by variation of parameters method.			
(A) $(\cos^3 x)/3$	(B) $-(\cos^3 x)/3$	(C) $(\sin^3 x)/6$	(D) $\left(\frac{\cos^3 x}{3}\right) - \cos x$
17. Find number of independent variables in the system: $y' = y + z, z' = y + z + x$ .			
(A) 2	(B) 1	(C) 0	(D) 3
18. Find $Z$ in the system: $y' = z, z' = y + x$ .			
(A) $ae^x + be^{-x} + 1$	(B) $ae^x + be^{-x}$	(C) $ae^x + be^{-x} - 1$	(D) $ae^x + be^{-x} + x$
19. The series: $\sum_{k=1}^{\infty} \frac{x^k}{k * 2^{2k}}$ convergences around origin in the interval .....			
(A) $[-4, 4]$	(B) $(-4, 4)$	(C) $(-2, 2)$	(D) $[-2, 2]$
20. Find the D.E. governs the deflection of shown beam.			
			
(A) $EI \frac{d^2 y}{dx^2} + p(x) = 0$	(B) $EI \frac{d^4 y}{dx^4} + p = 0$	(C) $EI \frac{d^4 y}{dx^4} + w(x) = 0$	(D) $\frac{d^4 y}{dx^4} + w(x) = 0$

**Question No 2 (10 Marks)**

Find a power series solution of the D.E.  $y'' + (5/2)y' + y = 2x$  up to  $x^4$ .

**Solution**

Consider the solution:  $y(x) = \sum_{k=0}^{\infty} c_k x^k$ , then,

$$y'(x) = \sum_{k=1}^{\infty} k c_k x^{k-1} \quad \text{and} \quad y''(x) = \sum_{k=2}^{\infty} k(k-1) c_k x^{k-2}.$$

We can substitute from table (1) the present D.E. thus, we find that

$$y'' + 2.5y' + y = 2x \rightarrow$$

$$\sum_{k=2}^{\infty} k(k-1) c_k x^{k-2} + \sum_{k=1}^{\infty} 2.5k c_k x^{k-1} + \sum_{k=0}^{\infty} c_k x^k = 2x$$

$$\sum_{i=2}^{\infty} i(i-1) c_i x^{i-2} + \sum_{j=1}^{\infty} 2.5j c_j x^{j-1} + \sum_{k=0}^{\infty} c_k x^k = 2x$$

To find coefficients of  $x^k$  in R.H.S, we may use the following substitution.

$$i-2 = k \quad j-1 = k.$$

$$\sum_{k=0}^{\infty} (k+2)(k+1) c_{k+2} x^k + \sum_{k=0}^{\infty} 2.5(k+1) c_{k+1} x^k + \sum_{k=0}^{\infty} c_k x^k = 2x$$

$$\sum_{k=0}^{\infty} \{ (k+2)(k+1) c_{k+2} + 2.5(k+1) c_{k+1} + c_k \} x^k = 2x$$

This equation shows that coefficients of  $x^k$  in L.H.S equal to zero except coefficients of  $x$  equal to 2. Thus a recurrence relation between coefficients are written as

$$c_{k+2} = \begin{cases} -\frac{2.5(k+1)c_{k+1} + c_k}{(k+2)(k+1)}, & k \neq 1 \\ \frac{2 - 2.5(k+1)c_{k+1} - c_k}{(k+2)(k+1)}, & k = 1 \end{cases}$$

We can obtain the following coefficients by successive substitutions in the above recurrence relation.

**Table coefficients in example**

<b>k</b>	<b>0</b>	<b>1</b>	<b>2</b>
<b><math>c_{k+2} =</math></b>	<b><math>c_2 =</math></b>	<b><math>c_3 =</math></b>	<b><math>c_4 =</math></b>
<b>Expression</b>	$-\frac{c_0}{2} - \frac{5c_1}{4}$	$\frac{5c_0}{12} + \frac{7c_1}{8} + \frac{1}{3}$	$-\frac{7c_0}{32} - \frac{85c_1}{192} - \frac{1}{24}$

The solution may be obtained by substituting in the power series expansion of  $\sum_{k=0}^{\infty} c_k x^k$  as

$$y(x) = (c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + c_6 x^6 + \dots)$$

$$y(x) = c_0 \left(1 - \frac{x^2}{2} + \frac{5x^3}{12} - \frac{7x^4}{32} + \dots\right) + c_1 \left(x - \frac{5x^2}{4} + \frac{7x^3}{8} - \frac{85x^4}{192} + \dots\right) + \frac{x^3}{3} - \frac{5x^4}{24} + \dots$$

$$= c_0 (1 - 0.5x^2 + 0.117x^3 - 0.219x^4 + \dots) + c_1 (x - 1.25x^2 + 0.875x^3 - 0.443x^4 + \dots) + 0.333x^3 - 0.208x^4 + \dots$$

### Question No 3 (10 Marks)

a) Use the Fourier transform to solve the following problem.

$$y'' - y = f(x), \text{ where, } f(x) = \begin{cases} 1 & 0 < x < 0.5 \\ 0 & 0.5 \leq x < 1 \end{cases}, \quad y'(0) = -1, \quad y'(1) = 0.$$

b) Use the Laplace transform to solve the following problem.

$$\ddot{y} + 5\dot{y} + 4y = 0, \quad y(0) = 1, \quad \dot{y}(0) = 1.$$

### Solution

a) Taking the Fourier cosine transform of the given D.E, we find that

$$F_c \left( \frac{d^2 y}{dx^2} - y \right) = F_c(f(x))$$

$$-\left(\frac{n\pi}{p}\right)^2 Y_c(n) + (-1)^n y'(p) - y'(0) - Y_c(n) = F_c(f(x))$$

$$((n\pi)^2 - 1) Y_c(n) + 1 = \int_0^{0.5} \cos(2n\pi x) dx + 0 = \frac{1}{n\pi} \sin(n\pi x)_0^{0.5} = \frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

$$(n^2 \pi^2 - 1) Y_c(n) + 1 = \begin{cases} 0.5, & (n = 0) \\ 1 & (n = 1, 5, 9, \dots) \\ -1 & (n = 3, 7, 11, \dots) \\ 0 & (n = 2, 4, 6, \dots) \end{cases},$$

$$Y_c(n) = \begin{cases} \frac{-0.5}{(n^2 \pi^2 - 1)}, & (n = 0) \\ 0 & (n = 1, 5, 9, \dots) \\ \frac{-2}{(n^2 \pi^2 - 1)} & (n = 3, 7, 11, \dots) \\ \frac{-1}{(n^2 \pi^2 - 1)} & (n = 2, 4, 6, \dots) \end{cases}$$

Taking the inverse Fourier cosine transform of last result, we find that

$$y(x) = \frac{F_c(0)}{p} + \frac{2}{p} \sum_{n=1}^{\infty} F_c(n) \cos \frac{n\pi x}{p} \rightarrow$$

$$y(x) = \frac{-0.5}{(n^2 \pi^2 - 1)} + 2 \sum_{n=1}^{\infty} F_c(n) \cos(n\pi x)$$

**b)  $\ddot{y} + 5\dot{y} + 4y = 0$ ,  $y(0) = 1$ ,  $\dot{y}(0) = 1$ .**

**$\ddot{y} - \dot{y} - 4y = 0$ ,  $y(0) = 2$ ,  $\dot{y}(0) = 3$**

**Taking Laplace transform (L.T) of the D.E,**

$$\mathcal{L}(\ddot{y} + 5\dot{y} + 4y) = 0 \rightarrow s^2 Y(s) - sy(0) - \dot{y}(0) + 5sY(s) - 5y(0) + 4Y(s) = 0$$

$$(s^2 + 5s + 4)Y(s) = 2s + 13 \rightarrow Y(s) = \frac{2s + 13}{s^2 + 5s + 4} \text{ (quadratic expression)}$$

$$Y(s) = \frac{2s + 13}{(s + 5/2)^2 - 25/4 + 4} = 2 \frac{s + 2.5 + 4}{(s + 5/2)^2 - (3/2)^2} = 2 \frac{s + 5/2}{(s + 5/2)^2 - (3/2)^2} + \frac{16}{3} \frac{3/2}{(s + 5/2)^2 - (3/2)^2}$$

**Taking inverse Laplace transform (I.L.T), we find the final solution as**

$$y(t) = 2e^{-5t/2} [\cosh(3t/2) + (8/3)\sinh(3t/2)]$$

**Another inverse (using partial fraction)**

$$\frac{2s + 13}{s^2 + 5s + 4} = \frac{2s + 13}{(s + 1)(s + 4)} = \frac{A}{(s + 1)} + \frac{B}{(s + 4)} = \frac{11}{3(s + 1)} - \frac{5}{3(s + 4)}$$

$$y(t) = 11e^{-t}/3 - 5e^{-4t}/3$$

## امتحان التخلف

Fayoum University Final Term Exam. Differential Equations 1<sup>st</sup> Year Civil Eng. Dept.  
Faculty of Engineering May, 7, 2019 Max. Points: 60 Time: 3 Hrs.



Examiner: Assoc. Prof. Karem Mahmoud Ewis

مسموح ب ٨ صفحات قوانين - يجيب الطالب في نفس أوراق الأسئلة و في ظهورها الفارغة  
درجات السؤال الأول موزعة بانتظام على جزئياته و تظليل الاختيار يكون كما هو موضح في رأس  
السؤال

**Exam instructions**

- 4- Only one sheet answer is allowed for every student.
- 5- It is not allowed to borrow the tools (pen, pencils, drawing tools, calculator ...etc.).
- 6- It is not allowed to use the cell phone or any of its application during the time of exam.

**Question No 1: Choose the correct answer as this Figure (60 M)**

1. Find $a_0$ in the Fourier series of the periodic function $f(x) = x^3 \sin x$ , where, $-\pi < x < \pi$ .	(A) 6.58	(B) 0	(C) 0	(D) 4.39
2. Find $F_s(n)$ of $f(x) = \pi$ , where, $p = \pi$ and $n=2$ .	(A) $\pi(1 - (-1)^n)/n$	(B) $\pi$	(C) 1.05	(D) $2\pi/(2m-1)$
3. Solve the D.E: $y' = y^3 e^{-2x}$ .	(A) $1/\sqrt{e^{-2x} - 2c}$	(B) $\pm 1/\sqrt{e^{-2x} - 2c}$	(C) $\pm 1/\sqrt{e^{2x} - 2c}$	(D) $\pm \sqrt{e^{-2x} - 2c}$
4. Find inverse Laplace transform of $s/[(s-2)(s-1)]$	(A) $(e^t - e^{2t})/2$	(B) $e^t - e^{2t}$	(C) $e^{2t} + e^t$	(D) $2e^{2t} - e^t$
5. Solve the D.E: $x^2 y' y' + y^2 x = x^3$	(A) $\pm x \sqrt{\frac{1 - cx^{-4}}{2}}$	(B) $x \sqrt{\frac{1 - cx^{-4}}{2}}$	(C) $\pm \sqrt{\frac{1 - cx^{-4}}{2}}$	(D) $\pm x \sqrt{\frac{1 + cx^{-4}}{2}}$
6. Find Laplace transform of $u_1(t) \sin(t-1)$ , where, $s=2$ .	(A) $\frac{e^{-s}}{s^2 + 1}$	(B) 0.027	(C) 0.135	(D) 0.068
7. The problem: $y''' + 5xy' - y = x$ , $y(0)=1$ , $y'(0)=5$ , $y''(1)=0$ . is .....	(A) non-linear	(B) IVP	(C) BVP	(D) particular
8. Find Laplace transform of $t^{10}$ .	(A) $11!/s^{11}$	(B) $10!/s^{10}$	(C) $10!/s^{11}$	(D) $10/s^{11}$
9. Solve the D.E: $y'' - 8y' + 16y = 0$ .	(A) $(c_1 + c_2 x)e^{-2x}$	(B) $(c_1 + c_2 x)e^{-4x}$	(C) $(c_1 + c_2 x)e^{2x}$	(D) $(c_1 + c_2 x)e^{4x}$

<p>10. Find output of the code <code>&gt;&gt; dsolve('D2y+2*y=2*t','y(0)=0','Dy(0)=1','t')</code></p> <p>(A) <math>\sin(t)+2*t</math>      (B) <math>t</math>      (C) <math>2*\sin(t)+t</math>      (D) <math>\sin(x)+2*x</math></p>
<p>11. If the characteristic equation has repeated root (<math>m=1, r=2</math>), then the bases are</p> <p>(A) <math>e^x, xe^x</math>      (B) <math>e^x, 2e^x</math>      (C) <math>xe^x, x^2e^x</math>      (D) <math>e^{-x}, xe^{-x}</math></p>
<p>12. Consider the D.E. <math>y'''' - 12y''' + 36y'' = 0</math>. Find root of its characteristic equation</p> <p>(A) 0, 0, 6, 6      (B) 0, 0, 3, 3      (C) 0, 0, -3, -3      (D) 3, 3</p>
<p>13. The D.E <math>y'' + ay' + by = F</math> cannot be solved using undetermined coefficients method if <math>F=.....</math></p> <p>(A) <math>e^x/(1+e^{2x})</math>      (B) <math>x</math>      (C) <math>e^{2x}</math>      (D) <math>\sin x + \cos x</math></p>
<p>14. Find <math>Y_p</math> in the D.E. <math>y''' + y'' = 10</math>.</p> <p>(A) <math>5x^2</math>      (B) <math>-5x^2</math>      (C) <math>10x^2</math>      (D) 0, 0, 1</p>
<p>15. Find a solution of <math>y''' + y' = 0</math>.</p> <p>(A) <math>e^x</math>      (B) <math>e^{2jx}</math>      (C) <math>e^{jx}</math>      (D) <math>e^{jx/2}</math></p>
<p>16. Find (<math>w_1 - c_1</math>) or (<math>w_2 - c_2</math>) in the solution of <math>y'' + y = \sin^2 x</math> by variation of parameters method.</p> <p>(A) <math>(\cos^3 x)/3</math>      (B) <math>-(\cos^3 x)/3</math>      (C) <math>(\sin^3 x)/3</math>      (D) <math>\left(\frac{\cos^3 x}{3}\right) - \cos x</math></p>
<p>17. Find number of dependent variables in the system: <math>xy' = y - 2z</math>, <math>z' = y' + z + x</math>.</p> <p>(A) 1      (B) 2      (C) 0      (D) 3</p>
<p>18. <math>F_s\left(\frac{dy}{dx}\right) = .....</math></p> <p>(A) <math>\frac{n\pi}{p} Y_c(n)</math>      (B) <math>-\frac{n\pi}{p} Y_c(n)</math>      (C) <math>-\frac{n\pi}{p} Y_s(n)</math>      (D) <math>\frac{n\pi}{p} Y_s(n)</math></p>
<p>19. The series: <math>\sum_{k=1}^{\infty} \frac{x^k}{k * 3^{2k}}</math> convergences around origin in the interval .....</p> <p>(A) [-9, 9]      (B) (-9, 9)      (C) (-3, 3)      (D) [-3, 3]</p>
<p>20. <math>L(\dot{y}(t)) = .....</math></p> <p>(A) <math>sY(s)</math>      (B) <math>sY(s) - y(0)</math>      (C) <math>Y(s)</math>      (D) <math>sY(s) - y'(0)</math></p>

**Question No 2 (20 Marks)**

Find a power series solution of the D.E.  $y'' + x y' + y = 1$  up to  $x^5$ .

**Solution**

Consider the solution:  $y(x) = \sum_{k=0}^{\infty} c_k x^k$ , then,

$$y'(x) = \sum_{k=1}^{\infty} k c_k x^{k-1} \quad \text{and} \quad y''(x) = \sum_{k=2}^{\infty} k(k-1) c_k x^{k-2}.$$

We can substitute from in D.E. thus, we find that

$$y'' + x y' + y = 1 \rightarrow$$

$$\sum_{k=2}^{\infty} k(k-1) c_k x^{k-2} + \sum_{k=1}^{\infty} k c_k x^k + \sum_{k=0}^{\infty} c_k x^k = 2x$$

To find coefficients of  $x^k$  in R.H.S, we may use the following replacement.

$$\sum_{k=0}^{\infty} (k+2)(k+1) c_{k+2} x^k + \sum_{k=0}^{\infty} k c_k x^k + \sum_{k=0}^{\infty} c_k x^k = 2x$$

This equation shows that coefficients of  $x^k$  in L.H.S equal to zero except coefficients of  $x^0$  equal to 2. Thus a recurrence relation between coefficients are written as

$$c_{k+2} = \begin{cases} \frac{1-(k+1)c_k}{(k+2)(k+1)}, & k=0 \\ \frac{-c_k}{(k+2)}, & k \neq 0 \end{cases}$$

We can obtain the following coefficients by successive substitutions in the above recurrence relation.

**Table coefficients in example**

<b>k</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
<b><math>c_{k+2}</math></b>	<b><math>c_2 =</math></b>	<b><math>c_3 =</math></b>	<b><math>c_4 =</math></b>	<b><math>c_5 =</math></b>
<b>Expression</b>	$\frac{1}{2} - \frac{c_0}{2}$	$-\frac{c_1}{3}$	$\frac{c_0}{8} - \frac{1}{8}$	$\frac{c_1}{15}$

The solution may be obtained by substituting in the power series expansion of  $\sum_{k=0}^{\infty} c_k x^k$  as

$$y(x) = (c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + \dots)$$

$$\begin{aligned} y &= c_0 \left(1 - \frac{x^2}{2} + \frac{x^4}{8} + \dots\right) + c_1 \left(x - \frac{x^3}{3} - \frac{x^5}{15} + \dots\right) + \frac{x^2}{2} - \frac{x^4}{8} + \dots \\ &= c_0 (1 - 0.5x^2 + 0.125x^4 + \dots) + c_1 (x - 0.333x^3 - 0.0667x^5 + \dots) + 0.5x^2 - 0.125x^4 + \dots \end{aligned}$$

**Question No 3 (20 Marks)**



a) Use the finite Fourier transforms to solve the following problem.

$$\frac{d^2 y}{dx^2} + y = x, \text{ with the boundary conditions: } y(0) = 1 \text{ and } y(1) = 0$$

b) Use the Laplace transform to solve the following problem.

$$\ddot{y} - \dot{y} - 4y = 0, y(0) = 2, \dot{y}(0) = 3.$$

### Solution

a) Taking the Fourier sine transform of the given D.E, we find that

$$F_s \left( \frac{d^2 y}{dx^2} + y \right) = F_s(x)$$

$$-(n\pi)^2 Y_s(n) + n\pi (y(0) + (-1)^{n+1} y(1)) + Y_s(n) = \underbrace{\int_0^1 x \sin(n\pi x) dx}_{\text{by parts}}$$

$$(1 - (n\pi)^2) Y_s(n) + n\pi = \frac{(-1)^{n+1}}{n\pi} \rightarrow Y_s(n) = \frac{(-1)^n}{n\pi(1 - (n\pi)^2)}$$

Taking the inverse Fourier sine transform of last result, we find that

$$y_s(x) = \frac{2}{p} \sum_{n=1}^{\infty} F_s(n) \sin \frac{n\pi x}{p} = y_s(x) = 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n\pi(1 - (n\pi)^2)} \sin n\pi x$$

$$y(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n(1 - (n\pi)^2)} \sin n\pi x$$

b)  $\ddot{y} - \dot{y} - 4y = 0, y(0) = 2, \dot{y}(0) = 3$

Taking Laplace transform (L.T) of the D.E,

$$\mathcal{L}(\ddot{y} - \dot{y} - 4y) = 0 \rightarrow s^2 Y(s) - s y(0) - \dot{y}(0) - s Y(s) + y(0) - 4Y(s) = 0$$

$$(s^2 - s - 4)Y(s) = 2s + 1 \rightarrow Y(s) = \frac{2s + 1}{s^2 - s - 4} \text{ (quadratic expression)}$$

$$Y(s) = \frac{2s + 1}{(s - 1/2)^2 - 17/4} = 2 \frac{s - 1/2 + 1/2}{(s - 1/2)^2 - 17/4} = 2 \frac{s - 1/2}{(s - 1/2)^2 - 17/4} + \frac{1}{(s - 1/2)^2 - 17/4}$$

Taking inverse Laplace transform (I.L.T), we find the final solution as

$$y(t) = 2e^{t/2} \left( \cosh(\sqrt{17}t/2) + (2/\sqrt{17}) \sinh(\sqrt{17}t/2) \right)$$

**Examiner: Ass. Prof Dr. Karem Mahmoud Ewis**
**Exam instruction**

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 8- It is not allowed to borrow the tools (pen, pencils, drawing tools, calculator ...etc.).  
 9- It is not allowed to use the cell phone or any of its application during the time of exam.

**Question No 1 (24 Marks) Choose the correct answer as this Figure ●**

1. Find the degree and order of D.E: $\sin x + y^2 + y' = 3$	(A) 4, 3	(B) 1, 1	(C) 2, 1	(D) 3, 2
2. Find the independent variable in the code <code>&gt;&gt; y=dsolve('Dy=y^3*exp(-2*y))</code>	(A) y	(B) t	(C) x	(D) D
3. Solve the D.E: $y' = \sin^2(x - y + 1), y(0) = -1$	(A) $x - 1 - \tan^{-1}(x)$	(B) $x - \tan^{-1}(x)$	(C) $x - \tan^{-1}(x + e)$	(D) $x - \tan(x)$
4. $(2xy + \tan x) y' = -(y^2 + y / \cos^2 x)$ is .... D.E:	(A) exact	(B) Bernoulli	(C) separable	(D) linear
5. $xy' + y = y^2 x^2$	(A) $1/(x^2 - cx)$	(B) $1/(x^2 + cx)$	(C) $(1+c)/x^2$	(D) $1/(-x^2 - cx)$
6.	(A) $kv^2$	(B) $mg$	(C) $-mg - kv^2$	(D) $-kv^2$
7. The problem: $y''' + 2xy'' + y' - y = x, y(0)=1, y'(0)=-1, y''(0)=0$ . is ....	(A) non-linear	(B) BVP	(C) particular	(D) IVP
8. Find the linearly dependent functions	(A) $e^{-x}, e^x$	(B) $x, e^x$	(C) $\sin x, \sin(2x)$	(D) $x, 2x$
9. Solve the D.E: $4y'' - 12y' + 5y = 0, y(0)=0, y'(0)=1$ .	(A) $e^{x/2}(c - 2ce^{2x})$	(B) $e^{x/2}(1 - e^{2x})$	(C) $ce^{x/2}(1 - e^{2x})$	(D) $ce^{x/2}(1 + e^{2x})$
10. Find output of the code <code>&gt;&gt; dsolve('D2y+y=2*t','y(0)=0','Dy(0)=1','t')</code>	(A) $-\sin(t) + 2*t$	(B) $\sin(t) + 2*t$	(C) $2*\sin(t) + t$	(D) $\sin(x) + 2*x$
11. If the characteristic equation has repeated root ( $m=1, r=3$ ), then the bases are	(A) $xe^x, x^2 e^x, x^3 e^x$	(B) $e^x, 2e^x, 3e^x$	(C) $xe^x, e^x, x^2 e^x$	(D) $e^{-x}, xe^{-x}, x^2 e^{-x}$
12. Consider the D.E. $y'''' - 4y''' + 4y'' = 0$ . Find root of its characteristic equation				

(A)	-2	0, 0, -2,	(B)	0, 0, 4, 4	(C)	0, 0, 2, 2	(D)	0, 2
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**Question No 2 (10 Marks)**

a) Find a power series solution of the D.E.  $y'' + 5y' + y = 2x$  up to  $x^4$ .

b) Find the interval of convergence of the series:  $\sum_{k=1}^{\infty} \frac{x^{2k+1}}{k^2 2^{2k}}$ .

**Question No 3 (10 Marks)**

a) Find the Fourier series of the periodic function  $f(x) = \pi^2 - x^2$  on the interval:  $-\pi < x < \pi$ .

b) Use the Fourier transform to solve the following problem.

$$y'' - y = f(x), \text{ where, } f(x) = \begin{cases} 1 & 0 < x < 0.5 \\ 0 & 0.5 \leq x < 1 \end{cases}, \quad y'(0) = -1, \quad y'(1) = 0.$$

**Question No 4 (10 Marks)**

a) Find Laplace transform (L.T) of the following functions:

$$f(t) = e^{6t} + \sin(5t) + e^{at} \cosh(bt) \text{ and } \frac{d^3 y}{dt^3}.$$

b) Find inverse Laplace transform (I.L.T) of the following functions:

$$H(s) = \frac{3}{s} - \frac{5}{s^2 + 4} - \frac{30}{s^7} \text{ and } F(s) = \left(\frac{1}{s}\right) \left(\frac{1}{s^2 - 16}\right)$$

**Question No 5 (6 Marks)**

Use the Laplace transform to solve the following problem.

$$\ddot{y} + 5\dot{y} + 4y = 0, \quad y(0) = 1, \quad \dot{y}(0) = 1.$$

### **References**

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