Section 11-4 Homework Hints:

3. Limit Comparison to $\sum_{n=1}^{\infty} \frac{1}{n^2}$

4. Limit Comparison to $\sum_{n=1}^{\infty} \frac{1}{n}$

5. Limit Comparison to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

6. Limit Comparison to $\sum_{n=1}^{\infty} \frac{1}{n^{1.5}}$

7. Limit or Direct Comparison to $\sum_{n=1}^{\infty} \left(\frac{9}{10}\right)^n$

8. Limit or Direct Comparison to $\sum_{n=1}^{\infty} \left(\frac{6}{5}\right)^n$

9. Direct Comparison to $\sum_{k=1}^{\infty} \frac{1}{k}$

10. $\sum_{k=1}^{\infty} \frac{k \sin^2(k)}{1+k^3} \le \sum_{k=1}^{\infty} \frac{k}{1+k^3}$. Use Limit Comparison with $\sum_{k=1}^{\infty} \frac{k}{1+k^3}$ and $\sum_{k=1}^{\infty} \frac{1}{k^2}$, and Direct Comparison with the original series.

11. Limit Comparison to $\sum_{k=1}^{\infty} \frac{1}{k^{\frac{7}{6}}}$

12. Limit Comparison $\sum_{k=1}^{\infty} \frac{1}{k^2}$

13. Limit or Direct Comparison to $\sum_{n=1}^{\infty} \frac{\frac{\pi}{2}}{n^{1.2}}$

14. Limit Comparison to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

15. Limit Comparison to $\sum_{n=1}^{\infty} 4\left(\frac{4}{3}\right)^n$

16. Limit or Direct Comparison to $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{3n^4}}$

17. Limit or Direct Comparison Test $\frac{1}{\sqrt{n^2}+1} \sim \frac{1}{n}$

18. Limit Comparison to $\sum_{n=1}^{\infty} \frac{1}{n}$

19. Limit Comparison to $\sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n$

20. Limit Compare to $\sum_{n=1}^{\infty}\left(\frac{4}{6}\right)^n$

21. Limit Compare to $\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$

22. Limit Compare to $\sum_{n=1}^{\infty}\frac{1}{n^2}$

23. Limit Compare to $\sum_{n=1}^{\infty}\frac{1}{n^3}$

24. Limit Compare to $\sum_{n=1}^{\infty}\frac{1}{n}$

25. Limit Compare to $\sum_{n=1}^{\infty}\frac{1}{n^{\frac{11}{12}}}$

26. Limit Compare to $\sum_{n=1}^{\infty}\frac{1}{n^2}$

27. Direct Compare to $\sum_{n=1}^{\infty}\left(\frac{1}{e}\right)^n$

28. Direct Compare to $\sum_{n=1}^{\infty}\frac{1}{n}$

29. Direct Compare to $\sum_{n=1}^{\infty}\frac{1}{n^2}$

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30. Use the following:

$\dfrac{n!}{n^n}$

$\dfrac{1}{1}$

$\dfrac{2}{2}\cdot\dfrac{1}{2}\le\dfrac{2}{2^2}$

$\dfrac{3}{3}\cdot\dfrac{2}{3}\cdot\dfrac{1}{3}=\left(\dfrac{2}{3}\cdot\dfrac{1}{3}\right)\le\dfrac{2}{3^2}$

$\dfrac{4}{4}\cdot\dfrac{3}{4}\cdot\dfrac{2}{4}\cdot\dfrac{1}{4}=\left(\dfrac{2}{4}\cdot\dfrac{1}{4}\right)\left[\dfrac{3}{4}\right]\le\dfrac{2}{4^2}$

$\dfrac{5}{5}\cdot\dfrac{4}{5}\cdot\dfrac{3}{5}\cdot\dfrac{2}{5}\cdot\dfrac{1}{5}=\left(\dfrac{2}{5}\cdot\dfrac{1}{5}\right)\left[\dfrac{4}{5}\cdot\dfrac{3}{5}\right]\le\dfrac{2}{5^2}$

$\dfrac{6}{6}\cdot\dfrac{5}{6}\cdot\dfrac{4}{6}\cdot\dfrac{3}{6}\cdot\dfrac{2}{6}\cdot\dfrac{1}{6}=\left(\dfrac{2}{6}\cdot\dfrac{1}{6}\right)\left[\dfrac{3}{6}\cdot\dfrac{5}{6}\cdot\dfrac{4}{6}\right]\le\dfrac{2}{6^2}$

$\vdots$

$\dfrac{n!}{n^n}\le\dfrac{2}{n^2}$ for $n\ge 2$

$\displaystyle\sum_{n=2}^{\infty}\frac{n!}{n^n}\le\sum_{n=2}^{\infty}\frac{2}{n^2}$. Since $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n^2}$ is a convergent *p*-series [$p=2$], $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{n^n}$ converges by the Direct Comparison Test.

31. Comparison Test $\displaystyle\sum_{n=1}^{\infty}\sin\left(\frac{1}{n}\right)\sim\sum_{n=1}^{\infty}\frac{1}{n}$

$\displaystyle\lim_{n\to\infty}\frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}$ Let $u=\dfrac{1}{n}$, $n\to\infty\Rightarrow u\to 0$ $\displaystyle\lim_{n\to\infty}\frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}=\lim_{u\to 0}\frac{\sin(u)}{u}=1$

32. Comparison Test $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{1+\frac{1}{n}}}=\sum_{n=1}^{\infty}\frac{1}{n\sqrt[n]{n}}\sim\sum_{n=1}^{\infty}\frac{1}{n}$