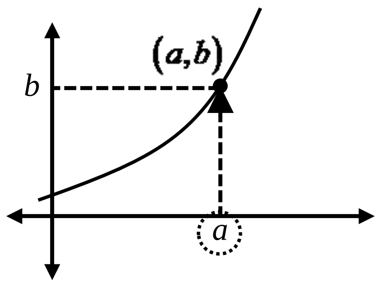
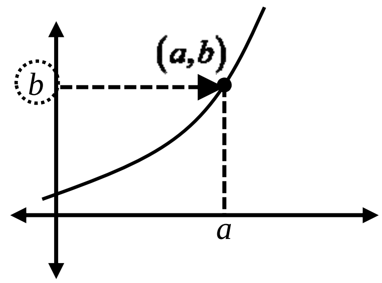
Derivative of the Inverse Function

Let (a,b) be a coordinate on the graph of $y = f(x)$.
$f(x)$ takes *a* in as an input, does something with that value, and produces the output *b*.
f^{-1} takes in *b* as an input, does something with that value, and produces the output *a*.

$f(a) = b$



$f^{-1}(b) = a$



To figure out the value of $\left(f^{-1}\right)'$, we differentiate the following equation that we know from the properties of inverse functions:

$$f^{-1}\left(f(x)\right) = x$$

$$\left[f^{-1}\left(f(x)\right)\right]' = [x]'$$

$$\left(f^{-1}\right)'\left(f(x)\right) \cdot f'(x) = 1$$

$$\left(f^{-1}\right)'\left(f(x)\right) = \frac{1}{f'(x)}$$

What good does this do us? Well, if $(a,b) \sim (x,y)$ we can substitute to get the following:

$$\left(f^{-1}\right)'(b) = \frac{1}{f'(a)}$$

I like to think of it in less formal terms:

$$\boxed{\left(f^{-1}\right)'(b) = \frac{1}{f'\begin{pmatrix}\text{whatever makes} \\ f(x) = b\end{pmatrix}}}$$

1. The following table gives the values of a differentiable function *f*, and its derivative f' at given values of x.

x	f	f'
1	2	$\frac{1}{2}$
2	3	1
3	4	2
4	6	4

If $g(x)$ is the inverse function of $f(x)$, then what is the value of $g'(4)$?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) 2

2. If $f(x)=x^3-3x^2+8x+5$ and $g(x)=f^{-1}(x)$, then $g'(5)=$

(a) 8 (b) $\frac{1}{8}$ (c) 1 (d) $\frac{1}{53}$ (e) 5

1. The following table gives the values of a differentiable function *f*, and its derivative f' at given values of *x*.

x	f	f'
1	2	$\frac{1}{2}$
2	3	1
3	4	2
4	6	4

If $g(x)$ is the inverse function of $f(x)$, then what is the value of $g'(4)$?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) 2

2. If $f(x)=x^3-3x^2+8x+5$ and $g(x)=f^{-1}(x)$, then $g'(5)=$

(a) 8 (b) $\frac{1}{8}$ (c) 1 (d) $\frac{1}{53}$ (e) 5

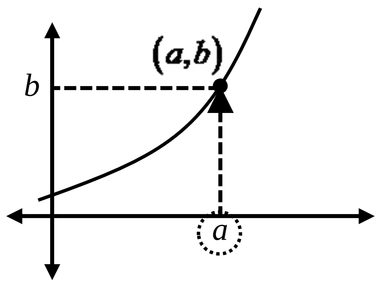
Derivative of the Inverse Function

Let (a,b) be a coordinate on the graph of $y = f(x)$.

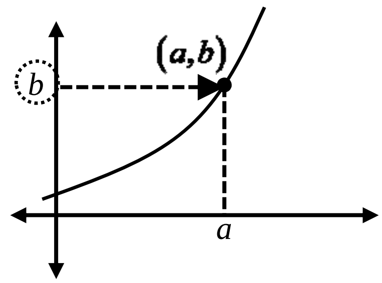
$f(x)$ takes *a* in as an input, does something with that value, and produces the output *b*.

f^{-1} takes in *b* as an input, does something with that value, and produces the output *a*.

$f(a)=b$



$f^{-1}(b)=a$



To figure out the value of $\left(f^{-1}\right)'$, we differentiate the following equation that we know from the properties of inverse functions:

$$f^{-1}\left(f(x)\right)=x$$

$$\left[f^{-1}\left(f(x)\right)\right]'=[x]'$$

$$\left(f^{-1}\right)'\left(f(x)\right)\cdot f'(x)=1$$

$$\left(f^{-1}\right)'\left(f(x)\right)=\frac{1}{f'(x)}$$

What good does this do us? Well, if $(a,b)\sim(x,y)$ we can substitute to get the following:

$$\left(f^{-1}\right)'(b)=\frac{1}{f'(a)}$$

I like to think of it in less formal terms:

$$\boxed{\left(f^{-1}\right)'(b)=\frac{1}{f'\begin{pmatrix}\text{whatever makes}\\ f(x)=b\end{pmatrix}}}$$