Continuity

A function f is continuous at a point $x = c$ if all three of the following hold:

I. $f(c)$ is defined/exists.

II. $\lim\limits_{x \to c} f(x)$ exists (must be ***a finite number***)

III. $\boxed{\lim\limits_{x \to c} f(x) = f(c)}$

Alternately, one f is continuous at a point $x = c$ if:

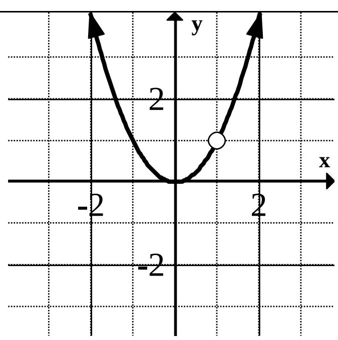
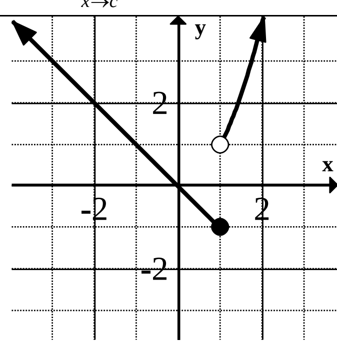
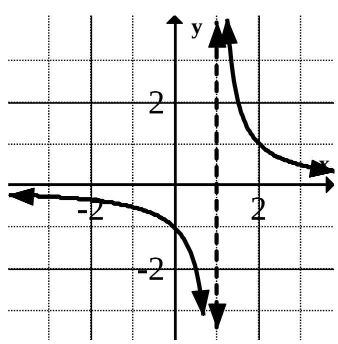
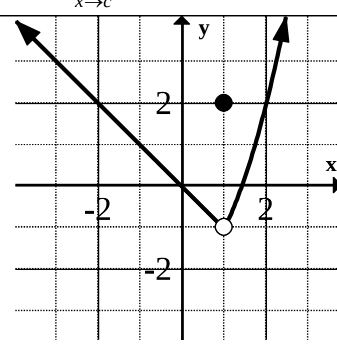
$$\lim_{x \to c^-} f(x) = f(c) = \lim_{x \to c^+} f(x)$$

Or

$$\boxed{\lim_{x \to c} \left[f(x) - f(c) \right] = 0}$$

That is, the left-hand limit, the right-hand limit, and function value all exist, and are all the same.

A function f is not continuous/discontinuous at $x = c$ if any one of the following conditions are met:

f is not defined at $x = c$	$\lim\limits_{x \to c} f(x)$ DNE	$\lim\limits_{x \to c} f(x) \neq f(c)$
	 	

A function f is continuous on an open interval (a,b) if f is continuous for at every point in the interval.
A function f is continuous everywhere if it is continuous for all real numbers.

A function f is continuous on a closed interval $[a,b]$ if

I. f is continuous on (a,b).

II. $\lim_{x\to a^+} f(x) = f(a)$.

III. $\lim_{x\to b^-} f(x) = f(b)$.

The conditions of continuity are relaxed at the endpoints since a two-sided limit analysis is impossible.

Properties of Continuity:

I. Scalar Multiple of a continuous function is continuous
II. Sum/Difference of two continuous functions is continuous
III. Product of two continuous functions is continuous
IV. Quotient of two continuous functions is continuous (provided denominator $\neq 0$)

The following functions are continuous *on their domains*:
Polynomial functions: $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$

Rational functions: $r(x) = \dfrac{p(x)}{q(x)}$ provided $q(x) \neq 0$

Radical Functions: $\sqrt[n]{x}$
Exponential Functions: a^x and e^x
Logarithmic functions: $\log_b(x)$ and $\ln(x)$
Trigonometric Functions: $\sin(x), \cos(x), \tan(x), \csc(x), \sec(x), \cot(x)$ *on their domains*.

Intermediate Value Theorem:

If f is a continuous function on $[a,b]$, and k is a value *between* $f(a)$ and $f(b)$, then there is a c in $[a,b]$ such that $f(c) = k$.