L'Hopital's Rule:

If $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}=\dfrac{0}{0}$ or $\dfrac{\pm\infty}{\pm\infty}$ **, then** $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to c}\dfrac{f'(x)}{g'(x)}$

Tips:

Indeterminate forms other than $\dfrac{0}{0}$ or $\dfrac{\pm\infty}{\pm\infty}$ that can be handled by L'Hopital's Rule:

$\boxed{0\cdot\infty}$	0^0	1^∞	∞^0
$0\cdot\infty=\dfrac{0}{\left(\frac{1}{\infty}\right)}$ $\downarrow$ $=\dfrac{0}{0}$ *OR* $0\cdot\infty=\dfrac{\infty}{\left(\frac{1}{0}\right)}$ $\downarrow$ $=\dfrac{\infty}{\infty}$	$y=0^0$ $\downarrow$ $\ln(y)=\ln(0^0)$ $\ln(y)=0\cdot\ln(0)$ $\ln(y)=\boxed{0\cdot-\infty}$ $\ln(y)=\dfrac{-\infty}{\frac{1}{0}}$ $\downarrow$ $\ln(y)=\dfrac{-\infty}{\infty}$	$y=1^\infty$ $\downarrow$ $\ln(y)=\ln(1^\infty)$ $\ln(y)=\infty\cdot\ln(1)$ $\ln(y)=\boxed{\infty\cdot 0}$ $\downarrow$ $\ln(y)=\dfrac{0}{\frac{1}{\infty}}$ $\ln(y)=\dfrac{0}{0}$	$y=\infty^0$ $\downarrow$ $\ln(y)=\ln(\infty^0)$ $\ln(y)=0\cdot\ln(\infty)$ $\ln(y)=\boxed{0\cdot\infty}$ $\ln(y)=\dfrac{\infty}{\frac{1}{0}}$ $\downarrow$ $\ln(y)=\dfrac{\infty}{\infty}$

You should recognize the limit definition of the number *e*: $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)^n=e$

Furthermore, if *a* is a constant where $a\neq 0$, then $\lim\limits_{n\to\infty}\left(1+\dfrac{a}{n}\right)^n=e^a$

Don't forget your properties of logarithms!

$$\log_b(xy)=\log_b(x)+\log_b(y)$$
$$\log_b\left(\frac{x}{y}\right)=\log_b(x)-\log_b(y)$$
$$\log_b(x^n)=n\cdot\log_b(x)$$
$$\log_b(b^m)=m$$
$$b^{\log_b(m)}=m$$

$$\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = k$$

$$\ln\left[\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n\right] = \ln(k)$$

$$\lim_{n\to\infty}\ln\left(1+\frac{a}{n}\right)^n = \ln(k)$$

$$\lim_{n\to\infty} n\cdot\ln\left(1+\frac{a}{n}\right) = \ln(k)$$

$$\lim_{n\to\infty}\frac{\ln\left(1+\frac{a}{n}\right)}{\frac{1}{n}} = \ln(k)$$

Note: $\lim_{n\to\infty}\frac{\ln\left(1+\frac{a}{n}\right)}{\frac{1}{n}} \sim \frac{0}{0}$

$$\downarrow$$

$$\lim_{n\to\infty}\frac{\frac{1}{1+\frac{a}{n}}\cdot -an^{-2}}{-n^{-2}} = \ln(k)$$

$$\lim_{n\to\infty}\frac{\frac{1}{1+\frac{a}{n}}\cdot a\left(\cancel{-n^{-2}}\right)}{\cancel{-n^{-2}}} = \ln(k)$$

$$\lim_{n\to\infty}\frac{1}{1+\frac{a}{n}}\cdot a = \ln(k)$$

$$\lim_{n\to\infty}\frac{1}{1+\frac{a}{n}}\cdot\lim_{n\to\infty} a = \ln(k)$$

$$1\cdot a = \ln(k)$$

$$a = \ln(k)$$

$$e^a = k$$

$\lim\limits_{x\to0} x^x \sim 0^0$	$\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^x \sim 1^\infty$	$\lim\limits_{x\to\infty} x^{\frac{1}{x}} \sim \infty^0$
$\lim\limits_{x\to0} x^x = y$ $\ln\left(\lim\limits_{x\to0} x^x\right)=\ln(y)$ $\lim\limits_{x\to0}\ln\left(x^x\right)=\ln(y)$ $\lim\limits_{x\to0}\underbrace{x\cdot\ln(x)}_{0\cdot\infty}=\ln(y)$ $\lim\limits_{x\to0}\frac{\ln(x)}{\left(\frac{1}{x}\right)}=\ln(y)$ $\frac{\infty}{\infty}$ $\downarrow$ $\lim\limits_{x\to0}\frac{\left[\ln(x)\right]'}{\left[x^{-1}\right]'}=\ln(y)$ $\lim\limits_{x\to0}\frac{\frac{1}{x}}{-x^{-2}}=\ln(y)$ $\lim\limits_{x\to0}\frac{\frac{1}{x}}{-\frac{1}{x^2}}=\ln(y)$ $\lim\limits_{x\to0}-x=\ln(y)$ $0=\ln(y)$ $e^0=e^{\ln(y)}$ $1=y$ $\downarrow$ $\lim\limits_{x\to0} x^x=1$	$\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^x=y$ $\ln\left(\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^x\right)=\ln(y)$ $\lim\limits_{x\to\infty}\ln\left[\left(1+\frac{2}{x}\right)^x\right]=\ln(y)$ $\lim\limits_{x\to\infty}\underbrace{x\cdot\ln\left(1+\frac{2}{x}\right)}_{\infty\cdot0}=\ln(y)$ $\lim\limits_{x\to\infty}\underbrace{\frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}}}_{\frac{0}{0}}=\ln(y)$ $\lim\limits_{x\to\infty}\frac{\left[\ln\left(1+\frac{2}{x}\right)\right]'}{\left[\frac{1}{x}\right]'}=\ln(y)$ $\lim\limits_{x\to\infty}\frac{\left(\frac{1}{1+\frac{2}{x}}\right)\cdot-2x^{-2}}{-x^{-2}}=\ln(y)$ $\lim\limits_{x\to\infty}\frac{\left(\frac{1}{1+\frac{2}{x}}\right)\cdot-2\cancel{x^{-2}}}{-\cancel{x^{-2}}}=\ln(y)$ $\lim\limits_{x\to\infty}\left(\frac{1}{1+\frac{2}{x}}\right)\cdot2=\ln(y)$ $2=\ln(y)$ $e^2=e^{\ln(y)}$ $y=e^2$ $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^x=e^2$	$\lim\limits_{x\to\infty} x^{\frac{1}{x}}=y$ $\ln\left(\lim\limits_{x\to\infty} x^{\frac{1}{x}}\right)=\ln(y)$ $\lim\limits_{x\to\infty}\ln\left(x^{\frac{1}{x}}\right)=\ln(y)$ $\lim\limits_{x\to\infty}\frac{1}{x}\cdot\ln(x)=\ln(y)$ $\lim\limits_{x\to\infty}\frac{\ln(x)}{x}=\ln(y)$ $\downarrow$ $\lim\limits_{x\to\infty}\frac{\left[\ln(x)\right]'}{\left[x\right]'}=\ln(y)$ $\lim\limits_{x\to\infty}\frac{\left(\frac{1}{x}\right)}{1}=\ln(y)$ $0=\ln(y)$ $e^0=e^{\ln(y)}$ $1=y$ $\downarrow$ $\lim\limits_{x\to\infty} x^{\frac{1}{x}}=1$

$$\lim_{x\to 0}\frac{5^x-4^x}{3^x-2^x}=\lim_{x\to 0}\frac{5^x-2^{2x}}{3^x-2^x}$$

$$=\lim_{x\to 0}\frac{\frac{5^x}{2^x}-\frac{2^{2x}}{2^x}}{\frac{3^x}{2^x}-1}$$

$$=\lim_{x\to 0}\frac{\left(\frac{5}{2}\right)^x-2^x}{\left(\frac{3}{2}\right)^x-1}$$

$$\downarrow$$

$$=\lim_{x\to 0}\frac{\ln\left(\frac{5}{2}\right)\left(\frac{5}{2}\right)^x-\ln(2)2^x}{\ln\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)^x}$$

$$=\lim_{x\to 0}\frac{\ln\left(\frac{5}{2}\right)}{\ln\left(\frac{3}{2}\right)}\left(\frac{5}{3}\right)^x-\frac{\ln(2)}{\ln\left(\frac{3}{2}\right)}\left(\frac{4}{3}\right)^x$$

$$=\frac{\ln\left(\frac{5}{2}\right)}{\ln\left(\frac{3}{2}\right)}(1)-\frac{\ln(2)}{\ln\left(\frac{3}{2}\right)}(1)$$

$$\approx 0.55033\ldots$$