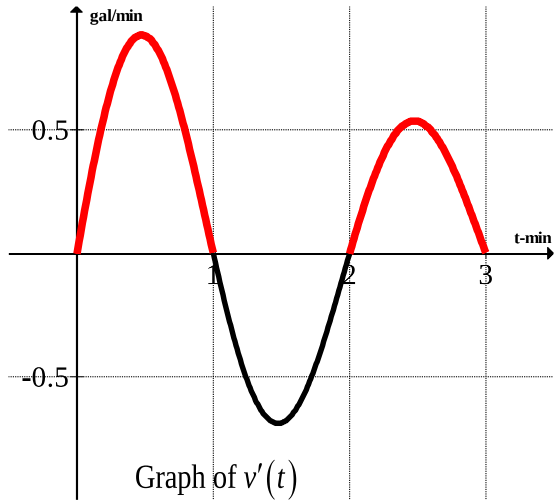
The Definite Integral of the Rate of Change is NET Change

A water tank has 3 gallons of water in it at time $t = 0$ minutes. The graph shows $v'(t)$, measured in gallons per minute, of the rate of change of volume of water in the tank for $0 \le t \le 3$.



1. When is the amount of water in the tank increasing? Justify your answer.

The amount of water in the tank is increasing when $v'(t) > 0$, i.e. is above the t-axis. This occurs on the intervals $(0,1)$ and $(2,3)$.

2. Write, but do not evaluate an expression involving an integral for each of the following

a. The amount of water in the tank at $t = 1$: $3 + \int_0^1 v'(t)\,dt$

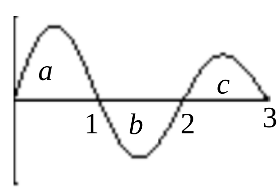
b. The amount of water in the tank at $t = 2$: $3 + \int_0^2 v'(t)\,dt$

c. The amount of water in the tank at $t = 3$: $3 + \int_0^3 v'(t)\,dt$

d. The amount of water in the tank at $t = x$: $3 + \int_0^x v'(t)\,dt$

(assume $0 \le x \le 3$)

As before, a water tank is initially empty. The graph shows $f(t)$, measured in gallons per minute, of the rate of change of volume of water in the tank for $0 \le t \le 3$ minutes. The horizontal intercepts are at 0, 1, 2, and 3.



In this graph *a*, *b*, and *c* are the areas of the three regions determined by $f(t)$ and the *t*-axis. These areas are all positive, and $0 < c < b < a$.

3. In terms of *a*, *b* and *c*, write expressions for

a. The amount of water in the tank at $t = 1$: $3 + a$

b. The amount of water in the tank at $t = 2$: $3 + a - b$

c. The amount of water in the tank at $t = 3$: $3 + a - b + c$

4. Interpret the statement "$b = 0.4382$" in the context of the problem. Use appropriate units.

The net change in volume of water in the tank is -0.4832 gallons from $t = 1$ to $t = 2$.

The volume of water in the tank dropped by 0.4832 gallons from $t = 1$ to $t = 2$.

5. Given that $v'(t) = e^{-0.25t}\sin(\pi t)$ gal/minute at time *t* minutes, use your method in Problem 2 to find

a. The amount of water in the tank at $t = 1$: $3 + \int_0^1 v'(t)\,dt = 3 + \int_0^1 e^{-0.25t}\sin(\pi t)\,dt \approx 3.562(3)$ gallons

The amount of water in the tank at $t = 1$ is 3.562(3) gallons.

b. The amount of water in the tank at $t = 2$: $3 + \int_0^2 v'(t)\,dt = 3 + \int_0^2 e^{-0.25t}\sin(\pi t)\,dt \approx 3.124$ gallons

The amount of water in the tank at $t = 2$ is 3.124 gallons.

c. The amount of water in the tank at $t = 3$: $3 + \int_0^3 f(t)\,dt = 3 + \int_0^3 e^{-0.25t}\sin(\pi t)\,dt \approx 3.465(6)$ gallons

The amount of water in the tank at $t = 3$ is 3.465(6) gallons.

6. A tank initially has 13 gallons in it. From $t = 0$ to $t = 3$ min the rate of change of water in the tank is $r(t) = \left(t^3 - 4.7t^2 + 5.1t\right)e^{-0.25t}$ gal/min.

a. Find the amount of water in the tank when $t = 3$ min.

$$13 + \int_0^3 r(t)\,dt = 13 + \int_0^3 \left(t^3 - 4.7t^2 + 5.1t\right)e^{-0.25t}\,dt \approx 13.978(9) \text{ gallons}$$

There are $13.978(9)$ gallons of water in the tank at time $t = 3$.

b. Write an integral expression for the amount of water in the tank at time *t*, where $0 \le t \le 3$.

$$13 + \int_0^x r(t)\,dt = 13 + \int_0^x \left(t^3 - 4.7t^2 + 5.1t\right)e^{-0.25t}\,dt$$

c. Find the maximum amount of water in the tank on the closed interval $[0,3]$. You must demonstrate an analysis using EVT.

$$\left(t^3 - 4.7t^2 + 5.1t\right)e^{-0.25t} = 0 \text{ or } DNE$$

$$\downarrow$$

$$t = 1.7$$

$$13 + \int_0^0 \left(t^3 - 4.7t^2 + 5.1t\right)e^{-0.25t}\,dt = 13$$

$$13 + \int_0^{1.7} \left(t^3 - 4.7t^2 + 5.1t\right)e^{-0.25t}\,dt \approx 14.453(4) \leftarrow \text{Absolute Maximum}$$

$$13 + \int_0^3 \left(t^3 - 4.7t^2 + 5.1t\right)e^{-0.25t}\,dt \approx 13.978(9)$$