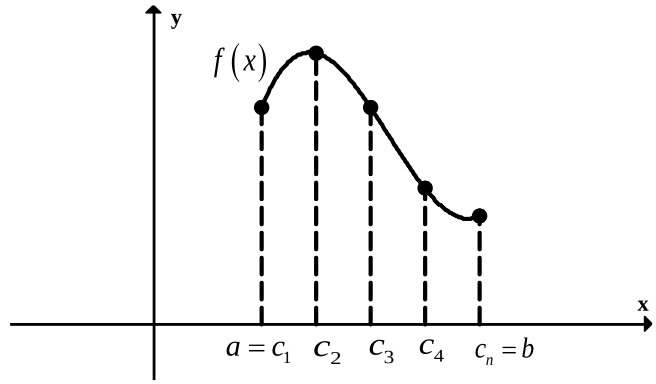
To find the average value of a function, we could start with an estimate using a finite number of sample points to calculate the estimate:

$$\text{Average value } \approx \frac{f\left(c_1\right)+f\left(c_2\right)+\cdots+f\left(c_5\right)}{5}$$

$$=\frac{1}{5}\sum_{i=1}^{5} f\left(c_i\right)$$



To improve the estimate, we would need to use more and more points. As we let $n\to\infty$, we would get

$$\text{Average value } =\lim_{n\to\infty}\frac{1}{n}\cdot\sum_{i=1}^{n} f\left(c_i\right).$$

It may seem as if there is no way to calculate the value of this infinite sum. However, if we partition the interval $[a,b]$ into n subintervals of equal length, then each subinterval will be of length $\Delta x=\dfrac{b-a}{n}$. The distance between each c_i will be uniform, and equal to Δx.

$$\text{Average value } =\frac{1}{n}\left[f\left(x_1\right)+f\left(x_2\right)+\cdots+f\left(x_n\right)\right]=\lim_{n\to\infty}\frac{1}{n}\cdot\sum_{i=1}^{n} f\left(c_i\right)$$

$$=\left[f\left(x_1\right)\cdot\frac{1}{n}+f\left(x_2\right)\cdot\frac{1}{n}+\cdots+f\left(x_n\right)\cdot\frac{1}{n}\right]=\lim_{n\to\infty}\sum_{i=1}^{n} f\left(c_i\right)\cdot\frac{1}{n}$$

$$=\frac{b-a}{b-a}\left[f\left(x_1\right)\cdot\frac{1}{n}+f\left(x_2\right)\cdot\frac{1}{n}+\cdots+f\left(x_n\right)\cdot\frac{1}{n}\right]=\lim_{n\to\infty}\frac{b-a}{b-a}\cdot\sum_{i=1}^{n} f\left(c_i\right)\cdot\frac{1}{n}$$

$$=\frac{1}{b-a}\left[f\left(x_1\right)\cdot\frac{b-a}{n}+f\left(x_2\right)\cdot\frac{b-a}{n}+\cdots+f\left(x_n\right)\cdot\frac{b-a}{n}\right]=\lim_{n\to\infty}\frac{1}{b-a}\cdot\sum_{i=1}^{n} f\left(c_i\right)\cdot\frac{b-a}{n}$$

$$=\frac{1}{b-a}\cdot\lim_{n\to\infty}\sum_{i=1}^{n} f\left(c_i\right)\cdot\frac{b-a}{n}$$

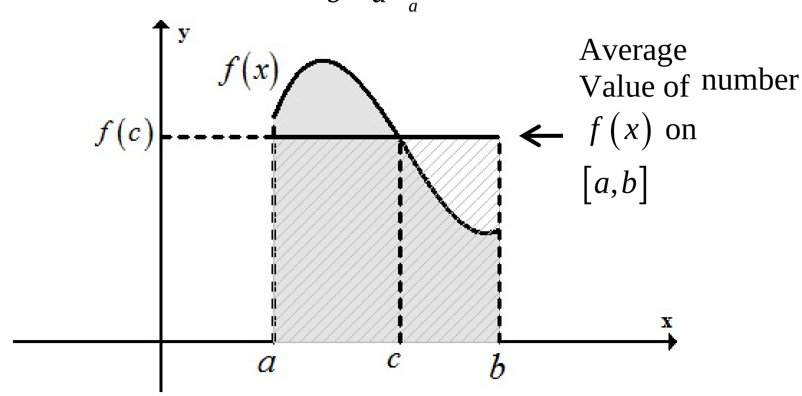
$$=\frac{1}{b-a}\cdot\left[\lim_{n\to\infty}\sum_{i=1}^{n} f\left(c_i\right)\cdot\Delta x\right]$$

$$=\frac{1}{b-a}\cdot\int_a^b f\left(x\right)dx$$

<u>Mean Value Theorem for Integrals:</u> If f is continuous on $[a,b]$, then there exists a c such that $f\left(c\right)\cdot\left(b-a\right)=\int_a^b f\left(x\right)dx$.

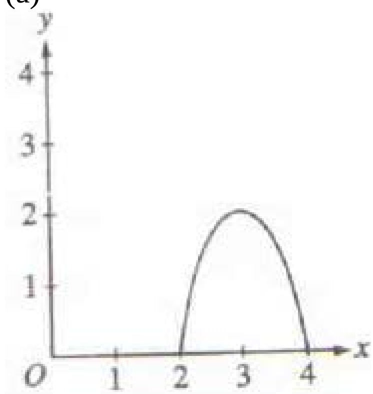
$$\underbrace{f\left(c\right)\cdot\left(b-a\right)}_{\text{area of rectangle}}=\underbrace{\int_a^b f\left(x\right)dx}_{\text{area under curve}}$$

$$f\left(c\right)=\frac{1}{b-a}\cdot\int_a^b f\left(x\right)dx$$

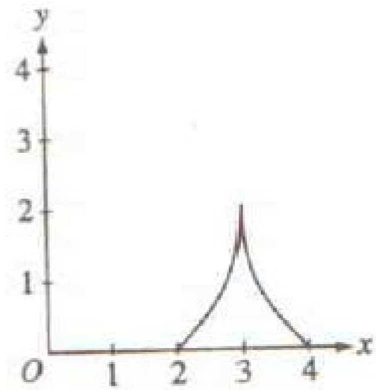


On the closed interval $[2,4]$, which of the following could be a graph of the function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?

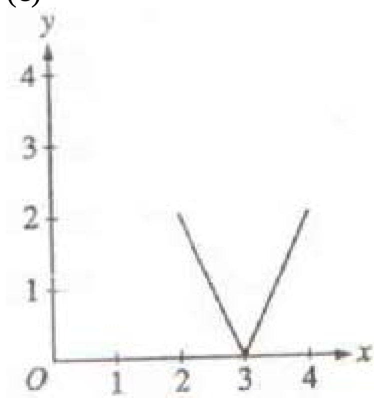
(a)



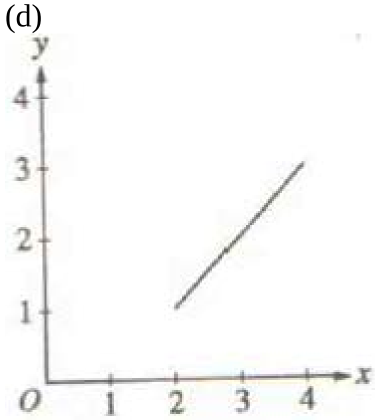
(b)



(c)



(d)



(e)