#3

$$\sum_{n=1}^{\infty}\frac{n}{2n^3+1}$$

Since $\frac{n}{2n^3+1}<\frac{n}{2n^3}=\frac{1}{2n^2}$

$$\sum_{n=1}^{\infty}\frac{n}{2n^3+1}<\sum_{n=1}^{\infty}\frac{1}{2n^2}$$

Since $\sum_{n=1}^{\infty}\frac{1}{2n^2}=\frac{1}{2}\cdot\sum_{n=1}^{\infty}\frac{1}{n^2}$ is a convergent *p*-series, $\sum_{n=1}^{\infty}\frac{n}{2n^3+1}$ converges by the direct comparison test.

OR

$$\lim_{n\to\infty}\frac{n}{2n^3+1}\sim\lim_{n\to\infty}\frac{n}{2n^3}=\lim_{n\to\infty}\frac{1}{2n^2}$$

$\sum_{n=1}^{\infty}\frac{1}{2n^2}$ is a convergent *p*-series.

$$\lim_{n\to\infty}\frac{\left(\frac{1}{2n^2}\right)}{\left(\frac{n}{2n^3+1}\right)}=\lim_{n\to\infty}\frac{2n^3+1}{2n^3}=1$$

Therefore, by the limit comparison test, $\sum_{n=1}^{\infty}\frac{n}{2n^3+1}$ converges.

#4

$$\sum_{n=2}^{\infty}\frac{n^3}{n^4-1}$$

Since $\frac{n^3}{n^4-1} > \frac{n^3}{n^4} = \frac{1}{n}$

$$\sum_{n=1}^{\infty}\frac{1}{n} < \sum_{n=2}^{\infty}\frac{n^3}{n^4-1}$$

Since $\sum_{n=1}^{\infty}\frac{1}{n}$ is divergent, the Harmonic Series, then by the direct comparison test $\sum_{n=2}^{\infty}\frac{n^3}{n^4-1}$ diverges.

OR

$$\lim_{n\to\infty}\frac{n^3}{n^4-1} \sim \lim_{n\to\infty}\frac{n^3}{n^4} = \lim_{n\to\infty}\frac{1}{n}$$

$\sum_{n=1}^{\infty}\frac{1}{n}$ is the divergent harmonic series.

$$\lim_{n\to\infty}\frac{\left(\frac{1}{n}\right)}{\left(\frac{n^3}{n^4-1}\right)} = \lim_{n\to\infty}\frac{n^4-1}{n^4} = 1$$

By the limit comparison test, $\sum_{n=2}^{\infty}\frac{n^3}{n^4-1}$ diverges.

#5

$$\sum_{n=1}^{\infty}\frac{n+1}{n\sqrt{n}}$$

Since $\frac{n+1}{n\sqrt{n}} > \frac{n}{n\sqrt{n}} = \frac{1}{\sqrt{n}} = \frac{1}{n^{\frac{1}{2}}}$

$$\sum_{n=1}^{\infty}\frac{1}{n^{\frac{1}{2}}} < \sum_{n=1}^{\infty}\frac{n+1}{n\sqrt{n}}$$

Since $\sum_{n=1}^{\infty}\frac{1}{n^{\frac{1}{2}}}$ is a divergent *p*-series, $\sum_{n=1}^{\infty}\frac{n+1}{n\sqrt{n}}$ diverges by the direct comparison test.

OR

$$\lim_{n\to\infty}\frac{n+1}{n\sqrt{n}} \sim \lim_{n\to\infty}\frac{n}{n\sqrt{n}} = \lim_{n\to\infty}\frac{1}{n^{\frac{1}{2}}}$$

$\sum_{n=1}^{\infty}\frac{1}{n^{\frac{1}{2}}}$ is a divergent *p*-series.

$$\lim_{n\to\infty}\frac{\left(\frac{1}{n^{\frac{1}{2}}}\right)}{\left(\frac{n+1}{n\sqrt{n}}\right)} = \lim_{n\to\infty}\frac{n\sqrt{n}}{n^{\frac{3}{2}}+n^{\frac{1}{2}}} \sim \lim_{n\to\infty}\frac{n^{\frac{3}{2}}}{n^{\frac{3}{2}}} = 1$$

By the limit comparison test, $\sum_{n=1}^{\infty}\frac{n+1}{n\sqrt{n}}$ diverges.

#6

$$\sum_{n=1}^{\infty}\frac{n-1}{n^2\sqrt{n}}$$

Since $\frac{n-1}{n^2\sqrt{n}} < \frac{n}{n^2\sqrt{n}} = \frac{1}{n\sqrt{n}} = \frac{1}{n^{\frac{3}{2}}}$

$$\sum_{n=1}^{\infty}\frac{n-1}{n^2\sqrt{n}} < \sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$$

Since $\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$ is a convergent *p*-series, $\sum_{n=1}^{\infty}\frac{n-1}{n^2\sqrt{n}}$ converges by the direct comparison test.

OR

$$\lim_{n\to\infty}\frac{n-1}{n^2\sqrt{n}} \sim \lim_{n\to\infty}\frac{n}{n^2\sqrt{n}} = \lim_{n\to\infty}\frac{n}{n^{\frac{5}{2}}} = \lim_{n\to\infty}\frac{1}{n^{\frac{3}{2}}}$$

$\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$ is a convergent *p*-series

$$\lim_{n\to\infty}\frac{\left(\frac{1}{n^{\frac{3}{2}}}\right)}{\left(\frac{n-1}{n^2\sqrt{n}}\right)} = \lim_{n\to\infty}\frac{n^2\sqrt{n}}{n^{\frac{5}{2}} - n^{\frac{3}{2}}} \sim \lim_{n\to\infty}\frac{n^{\frac{5}{2}}}{n^{\frac{5}{2}}} = 1$$

By the limit comparison test, $\sum_{n=1}^{\infty}\frac{n-1}{n^2\sqrt{n}}$ converges.

#7

$$\sum_{n=1}^{\infty}\frac{9^n}{3+10^n}$$

Since $\frac{9^n}{3+10^n} < \frac{9^n}{10^n} = \left(\frac{9}{10}\right)^n$

$$\sum_{n=1}^{\infty}\frac{9^n}{3+10^n} < \sum_{n=1}^{\infty}\left(\frac{9}{10}\right)^n$$

Since $\sum_{n=1}^{\infty}\left(\frac{9}{10}\right)^n$ is a convergent geometric series, $\sum_{n=1}^{\infty}\frac{9^n}{3+10^n}$ converges by the direct comparison test.

OR

$$\lim_{n\to\infty}\frac{9^n}{3+10^n} \sim \lim_{n\to\infty}\frac{9^n}{10^n}$$

$\sum_{n=1}^{\infty}\left(\frac{9}{10}\right)^n$ is a convergent geometric series.

$$\lim_{n\to\infty}\frac{\left(\frac{9}{10}\right)^n}{\left(\frac{9^n}{3+10^n}\right)} \sim \lim_{n\to\infty}\frac{\left(\frac{9}{10}\right)^n}{\left(\frac{9^n}{10^n}\right)} = \lim_{n\to\infty}\frac{\left(\frac{9}{10}\right)^n}{\left(\frac{9}{10}\right)^n} = 1$$

By the limit comparison test, $\sum_{n=1}^{\infty}\frac{9^n}{3+10^n}$ converges.

#8

$$\sum_{n=1}^{\infty}\frac{6^n}{5^n-1}$$

Since $\frac{6^n}{5^n-1} > \frac{6^n}{5^n} = \left(\frac{6}{5}\right)^n$

$$\sum_{n=1}^{\infty}\left(\frac{6}{5}\right)^n < \sum_{n=1}^{\infty}\frac{6^n}{5^n-1}$$

Since $\sum_{n=1}^{\infty}\left(\frac{6}{5}\right)^n$ is a divergent geometric series, $\sum_{n=1}^{\infty}\frac{6^n}{5^n-1}$ diverges by the direct comparison test.

OR

$$\lim_{n\to\infty}\frac{6^n}{5^n-1} \sim \lim_{n\to\infty}\frac{6^n}{5^n} = \lim_{n\to\infty}\left(\frac{6}{5}\right)^n$$

$\sum_{n=1}^{\infty}\left(\frac{6}{5}\right)^n$ is divergent geometric series.

$$\lim_{n\to\infty}\frac{\left(\frac{6}{5}\right)^n}{\left(\frac{6^n}{5^n-1}\right)} \sim \lim_{n\to\infty}\frac{\left(\frac{6}{5}\right)^n}{\left(\frac{6^n}{5^n}\right)} = \lim_{n\to\infty}\frac{\left(\frac{6}{5}\right)^n}{\left(\frac{6}{5}\right)^n} = 1$$

By the limit comparison test, $\sum_{n=1}^{\infty}\frac{6^n}{5^n-1}$ diverges.

#9

$$\sum_{k=1}^{\infty}\frac{\ln(k)}{k}$$

Since $\frac{1}{k}<\frac{\ln(k)}{k}$

$$\sum_{k=1}^{\infty}\frac{1}{k}<\sum_{k=1}^{\infty}\frac{\ln(k)}{k}$$

Since $\sum_{k=1}^{\infty}\frac{1}{k}$ is the divergent harmonic series, $\sum_{k=1}^{\infty}\frac{\ln(k)}{k}$ diverges by the direct comparison test.

#10

$$\sum_{k=1}^{\infty}\frac{k\cdot\sin^2(k)}{1+k^3}$$

Since $\frac{k\cdot\sin^2(k)}{1+k^3}<\frac{k\cdot[1]}{k^3}<\frac{k}{k^3}<\frac{1}{k^2}$

$$\sum_{k=1}^{\infty}\frac{k\cdot\sin^2(k)}{1+k^3}<\sum_{k=1}^{\infty}\frac{1}{k^2}$$

Since $\sum_{k=1}^{\infty}\frac{1}{k^2}$ is a convergent *p*-series, $\sum_{k=1}^{\infty}\frac{k\cdot\sin^2(k)}{1+k^3}$ converges by the direct comparison test.

#11

$$\sum_{k=1}^{\infty}\frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}}$$

Since $\frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}}<\frac{\sqrt[3]{k}}{\sqrt{k^3}}=\frac{k^{\frac{1}{3}}}{k^{\frac{3}{2}}}=\frac{1}{k^{\frac{7}{6}}}$

$$\sum_{k=1}^{\infty}\frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}}<\sum_{k=1}^{\infty}\frac{1}{k^{\frac{7}{6}}}$$

Since $\sum_{k=1}^{\infty}\frac{1}{k^{\frac{7}{6}}}$ is a convergent *p*-series $\sum_{k=1}^{\infty}\frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}}$ converges by the direct comparison test.

OR

$$\lim_{n\to\infty}\frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}}\sim\lim_{n\to\infty}\frac{\sqrt[3]{k}}{\sqrt{k^3}}=\lim_{n\to\infty}\frac{k^{\frac{1}{3}}}{k^{\frac{3}{2}}}=\lim_{n\to\infty}\frac{1}{k^{\frac{7}{6}}}$$

$$\lim_{k\to\infty}\frac{\left(\frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}}\right)}{\left(\frac{1}{k^{\frac{7}{6}}}\right)}\sim\lim_{k\to\infty}\frac{\left(\frac{\sqrt[3]{k}}{\sqrt{k^3}}\right)}{\left(\frac{1}{k^{\frac{7}{6}}}\right)}=\lim_{k\to\infty}\frac{\left(\frac{1}{k^{\frac{7}{6}}}\right)}{\left(\frac{1}{k^{\frac{7}{6}}}\right)}=1$$

$\sum_{k=1}^{\infty}\frac{1}{k^{\frac{7}{6}}}$ is convergent *p*-series.

By the limit comparison test, $\sum_{k=1}^{\infty}\frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}}$ converges.

#12

$$\sum_{k=1}^{\infty}\frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$$

Since $\frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}<\frac{2k^3}{k^5}=\frac{2}{k^2}$

$$\sum_{k=1}^{\infty}\frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}<\sum_{k=1}^{\infty}\frac{2}{k^2}$$

Since $\sum_{k=1}^{\infty}\frac{2}{k^2}$ is a convergent *p*-series $\sum_{k=1}^{\infty}\frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$ converges by the direct comparison test.

OR

$$\lim_{n\to\infty}\frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}\sim\lim_{n\to\infty}\frac{2k^3}{k^5}=\lim_{n\to\infty}\frac{2}{k^2}$$

$$\lim_{k\to\infty}\frac{\left(\frac{2}{k^2}\right)}{\left(\frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}\right)}\sim\lim_{k\to\infty}\frac{\left(\frac{2}{k^2}\right)}{\left(\frac{2}{k^2}\right)}=1$$

$\sum_{k=1}^{\infty}\frac{2}{k^2}$ is a convergent *p*-series.

By the limit comparison test, $\sum_{k=1}^{\infty}\frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$ converges.

#13

$$\sum_{n=1}^{\infty}\frac{\arctan(n)}{n^{1.2}}$$

Since $\frac{\arctan(n)}{n^{1.2}} < \frac{\left(\frac{\pi}{2}\right)}{n^{1.2}} = \frac{\pi}{2}\cdot\frac{1}{n^{1.2}}$

$$\sum_{n=1}^{\infty}\frac{\arctan(n)}{n^{1.2}} < \sum_{n=1}^{\infty}\frac{\pi}{2}\cdot\frac{1}{n^{1.2}} = \frac{\pi}{2}\cdot\left[\sum_{n=1}^{\infty}\frac{1}{n^{1.2}}\right]$$

Since $\sum_{n=1}^{\infty}\frac{1}{n^{1.2}}$ is a convergent *p*-series, $\sum_{n=1}^{\infty}\frac{\arctan(n)}{n^{1.2}}$ converges by the direct comparison test.

OR

$$\lim_{n\to\infty}\frac{\arctan(n)}{n^{1.2}} \sim \lim_{n\to\infty}\frac{\left(\frac{\pi}{2}\right)}{n^{1.2}} = \frac{\pi}{2}\cdot\lim_{n\to\infty}\left[\frac{1}{n^{1.2}}\right]$$

$\sum_{n=1}^{\infty}\frac{1}{n^{1.2}}$ is a convergent *p*-series.

$$\lim_{n\to\infty}\frac{\left(\frac{\arctan(n)}{n^{1.2}}\right)}{\left(\frac{1}{n^{1.2}}\right)} = \lim_{n\to\infty}\left[\arctan(n)\right] = \frac{\pi}{2}$$

By the limit comparison test, $\sum_{n=1}^{\infty}\frac{\arctan(n)}{n^{1.2}}$ converges.

#14

$$\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n-1}$$

Since $\frac{1}{n^{\frac{1}{2}}}=\frac{\sqrt{n}}{n}<\frac{\sqrt{n}}{n-1}$

$$\sum_{n=1}^{\infty}\frac{1}{n^{\frac{1}{2}}}<\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n-1}$$

Since $\sum_{n=1}^{\infty}\frac{1}{n^{\frac{1}{2}}}$ is a divergent *p*-series, $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n-1}$ diverges by the direct comparison test.

OR

$$\lim_{n\to\infty}\frac{\sqrt{n}}{n-1}\sim\lim_{n\to\infty}\frac{\sqrt{n}}{n}=\lim_{n\to\infty}\frac{1}{n^{\frac{1}{2}}}$$

$$\lim_{n\to\infty}\frac{\left(\frac{1}{n^{\frac{1}{2}}}\right)}{\left(\frac{\sqrt{n}}{n-1}\right)}\sim\lim_{n\to\infty}\frac{\left(\frac{1}{n^{\frac{1}{2}}}\right)}{\left(\frac{\sqrt{n}}{n}\right)}=\lim_{n\to\infty}\frac{\left(\frac{1}{n^{\frac{1}{2}}}\right)}{\left(\frac{1}{n^{\frac{1}{2}}}\right)}=1$$

$\sum_{n=1}^{\infty}\frac{1}{n^{\frac{1}{2}}}$ is a divergent *p*-series.

By the limit comparison test, $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n-1}$ diverges.

#15

$$\sum_{n=1}^{\infty}\frac{4^{n+1}}{3^n-2}$$

Since $4\cdot\left(\frac{4}{3}\right)^n=\frac{4\cdot 4^n}{3^n}=\frac{4^{n+1}}{3^n}<\frac{4^{n+1}}{3^n-2}$

$$4\cdot\sum_{n=1}^{\infty}\left(\frac{4}{3}\right)^n<\sum_{n=1}^{\infty}\frac{4^{n+1}}{3^n-2}$$

Since $4\cdot\sum_{n=1}^{\infty}\left(\frac{4}{3}\right)^n$ is a divergent geometric series, $\sum_{n=1}^{\infty}\frac{4^{n+1}}{3^n-2}$ diverges by the direct comparison test.

OR

$$\lim_{n\to\infty}\frac{4^{n+1}}{3^n-2}\sim\lim_{n\to\infty}\frac{4^{n+1}}{3^n}=\lim_{n\to\infty}\frac{4\cdot 4^n}{3^n}=4\cdot\lim_{n\to\infty}\frac{4^n}{3^n}=4\cdot\lim_{n\to\infty}\left(\frac{4}{3}\right)^n$$

$$\lim_{n\to\infty}\frac{\left(\frac{4}{3}\right)^n}{\left(\frac{4^{n+1}}{3^n-2}\right)}\sim\lim_{n\to\infty}\frac{\left(\frac{4}{3}\right)^n}{\left(\frac{4^{n+1}}{3^n}\right)}=\lim_{n\to\infty}\frac{\left(\frac{4}{3}\right)^n}{4\cdot\left(\frac{4^n}{3^n}\right)}=\frac{1}{4}$$

$\sum_{n=1}^{\infty}\left(\frac{4}{3}\right)^n$ is divergent geometric series.

By the limit comparison test, $\sum_{n=1}^{\infty}\frac{4^{n+1}}{3^n-2}$ diverges.

#16

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{3n^4+1}}$$

Since $\frac{1}{\sqrt[3]{3n^4+1}} < \frac{1}{\sqrt[3]{3n^4}} = \frac{1}{\sqrt[3]{3}}\cdot\frac{1}{n^{\frac{4}{3}}}$

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{3n^4+1}} < \frac{1}{\sqrt[3]{3}}\cdot\left[\sum_{n=1}^{\infty}\frac{1}{n^{\frac{4}{3}}}\right]$$

Since $\sum_{n=1}^{\infty}\frac{1}{n^{\frac{4}{3}}}$ is a convergent *p*-series, $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{3n^4+1}}$ converges by the direct comparison test.

OR

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{3n^4+1}} \sim \lim_{n\to\infty}\frac{1}{\sqrt[3]{3n^4}} = \lim_{n\to\infty}\frac{1}{\sqrt[3]{3}}\cdot\frac{1}{\sqrt[3]{n^4}} = \frac{1}{\sqrt[3]{3}}\cdot\lim_{n\to\infty}\frac{1}{n^{\frac{4}{3}}}$$

$$\lim_{n\to\infty}\frac{\left(\frac{1}{n^{\frac{4}{3}}}\right)}{\left(\frac{1}{\sqrt[3]{3n^4+1}}\right)} \sim \lim_{n\to\infty}\frac{\left(\frac{1}{n^{\frac{4}{3}}}\right)}{\left(\frac{1}{\sqrt[3]{3n^4}}\right)} = \lim_{n\to\infty}\frac{\left(\frac{1}{n^{\frac{4}{3}}}\right)}{\left(\frac{1}{\sqrt[3]{3}\cdot\sqrt[3]{n^4}}\right)} == \lim_{n\to\infty}\frac{\left(\frac{1}{n^{\frac{4}{3}}}\right)}{\left(\frac{1}{\sqrt[3]{3}}\right)\left(\frac{1}{n^{\frac{4}{3}}}\right)} = \sqrt[3]{3}$$

$\sum_{n=1}^{\infty}\frac{1}{n^{\frac{4}{3}}}$ is a convergent geometric series.

By the limit comparison test, $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{3n^4+1}}$ converges.

#17

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$$

Since $\lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{1}{n}$

$$\lim_{n\to\infty} \frac{\left[\frac{1}{\sqrt{n^2+1}}\right]}{\left(\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{\left[\frac{1}{n}\right]}{\left(\frac{1}{n}\right)} = 1$$

Since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series), so does $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ by the limit comparison test.

OR

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}} \sim \lim_{n\to\infty} \frac{1}{\sqrt{n^2}} = \lim_{n\to\infty} \frac{1}{n}$$

$\sum_{n=1}^{\infty} \frac{1}{n}$ is the divergent harmonic series.

$$\lim_{n\to\infty} \frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{\sqrt{n^2+1}}\right)} \sim \lim_{n\to\infty} \frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{\sqrt{n^2}}\right)} = \lim_{n\to\infty} \frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{n}\right)} = 1$$

By the limit comparison test, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ diverges.

#18

$$\sum_{n=1}^{\infty}\frac{1}{2n+3}$$

Since $\lim_{n\to\infty}\frac{1}{2n+3}=\lim_{n\to\infty}\frac{1}{2n}$

and

$$\lim_{n\to\infty}\frac{\left[\frac{1}{2n+3}\right]}{\left(\frac{1}{2n}\right)}=\lim_{n\to\infty}\frac{\left[\frac{1}{2n}\right]}{\left(\frac{1}{2n}\right)}=1$$

Since $\sum_{n=1}^{\infty}\frac{1}{2n}=\frac{1}{2}\cdot\left[\sum_{n=1}^{\infty}\frac{1}{n}\right]$ and $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges (harmonic series), $\sum_{n=1}^{\infty}\frac{1}{2n+3}$ diverges by the limit comparison test.

OR

$$\lim_{n\to\infty}\frac{1}{2n+3}\sim\lim_{n\to\infty}\frac{1}{2n}=\frac{1}{2}\cdot\lim_{n\to\infty}\left[\frac{1}{n}\right]$$

$\sum_{n=1}^{\infty}\frac{1}{n}$ is the divergent harmonic series.

$$\lim_{n\to\infty}\frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{2n+3}\right)}\sim\lim_{n\to\infty}\frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{2n}\right)}=\lim_{n\to\infty}\frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{n}\right)}=2$$

By the limit comparison test, $\sum_{n=1}^{\infty}\frac{1}{2n+3}$ diverges.

#19

$$\sum_{n=1}^{\infty}\frac{1+4^n}{1+3^n}$$

Since $\lim\limits_{n\to\infty}\frac{1+4^n}{1+3^n}=\lim\limits_{n\to\infty}\frac{4^n}{3^n}=\lim\limits_{n\to\infty}\left(\frac{4}{3}\right)^n\to\infty$ , the series diverges by the $n^{\text{th}}$ term test.

OR

$$\lim_{n\to\infty}\frac{1+4^n}{1+3^n}\sim\lim_{n\to\infty}\frac{4^n}{3^n}=\lim_{n\to\infty}\left(\frac{4}{3}\right)^n$$

$\sum\limits_{n=1}^{\infty}\left(\frac{4}{3}\right)^n$ is a divergent geometric series.

$$\lim_{n\to\infty}\frac{\left(\frac{4}{3}\right)^n}{\left(\frac{1+4^n}{1+3^n}\right)}\sim\lim_{n\to\infty}\frac{\left(\frac{4}{3}\right)^n}{\left(\frac{4^n}{3^n}\right)}=\lim_{n\to\infty}\frac{\left(\frac{4}{3}\right)^n}{\left(\frac{4}{3}\right)^n}=1$$

By the limit comparison test, $\sum\limits_{n=1}^{\infty}\frac{1+4^n}{1+3^n}$ diverges.

#20

$$\sum_{n=1}^{\infty}\frac{n+4^n}{n+6^n}$$

Since $\lim_{n\to\infty}\frac{n+4^n}{n+6^n}=\lim_{n\to\infty}\frac{4^n}{6^n}=\lim_{n\to\infty}\left(\frac{2}{3}\right)^n$

and

$$\lim_{n\to\infty}\frac{\left[\frac{n+4^n}{n+6^n}\right]}{\left(\frac{2}{3}\right)^n}=\lim_{n\to\infty}\frac{\left[\frac{2}{3}\right]^n}{\left(\frac{2}{3}\right)^n}=1$$

Since $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ is a convergent geometric series, $\sum_{n=1}^{\infty}\frac{n+4^n}{n+6^n}$ converges by the limit comparison test.

#21

$$\sum_{n=1}^{\infty}\frac{\sqrt{n+2}}{2n^2+n+1}$$

Since $\lim\limits_{n\to\infty}\frac{\sqrt{n+2}}{2n^2+n+1}=\lim\limits_{n\to\infty}\frac{\sqrt{n}}{2n^2}=\lim\limits_{n\to\infty}\frac{1}{2n^{\frac{3}{2}}}$

and

$$\lim_{n\to\infty}\frac{\left[\frac{\sqrt{n+2}}{2n^2+n+1}\right]}{\left(\frac{1}{2n^{\frac{3}{2}}}\right)}=\lim_{n\to\infty}\frac{\left[\frac{1}{2n^{\frac{3}{2}}}\right]}{\left(\frac{1}{2n^{\frac{3}{2}}}\right)}=1$$

Since $\sum_{n=1}^{\infty}\frac{1}{2n^{\frac{3}{2}}}=\frac{1}{2}\cdot\left[\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}\right]$ and $\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$ is a convergent *p*-series, $\sum_{n=1}^{\infty}\frac{\sqrt{n+2}}{2n^2+n+1}$ converges by the limit comparison test.

#22

$$\sum_{n=3}^{\infty}\frac{n+2}{(n+1)^3}$$

Since $\lim\limits_{n\to\infty}\frac{n+2}{(n+1)^3}=\lim\limits_{n\to\infty}\frac{n}{n^3}=\lim\limits_{n\to\infty}\frac{1}{n^2}$

And

$$\lim_{n\to\infty}\frac{\left[\frac{n+2}{(n+1)^3}\right]}{\left(\frac{1}{n^2}\right)}=\lim_{n\to\infty}\frac{\left[\frac{1}{n^2}\right]}{\left(\frac{1}{n^2}\right)}=1$$

Since $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is a convergent *p*-series, $\sum_{n=3}^{\infty}\frac{n+2}{(n+1)^3}$ converges by the limit comparison test.

#23

$$\sum_{n=1}^{\infty}\frac{5+2n}{\left(1+n^2\right)^2}$$

Since $\lim\limits_{n\to\infty}\frac{5+2n}{\left(1+n^2\right)^2}=\lim\limits_{n\to\infty}\frac{2n}{n^4}=\lim\limits_{n\to\infty}\frac{2}{n^3}$

And

$$\lim_{n\to\infty}\frac{\left[\frac{5+2n}{\left(1+n^2\right)^2}\right]}{\left(\frac{2}{n^3}\right)}=\lim_{n\to\infty}\frac{\left[\frac{2}{n^3}\right]}{\left(\frac{2}{n^3}\right)}=1$$

Since $\sum\limits_{n=1}^{\infty}\frac{2}{n^3}=2\cdot\left[\sum\limits_{n=1}^{\infty}\frac{1}{n^3}\right]$ and $\sum\limits_{n=1}^{\infty}\frac{1}{n^3}$ is a convergent *p*-series, $\sum\limits_{n=1}^{\infty}\frac{5+2n}{\left(1+n^2\right)^2}$ converges by the limit comparison test.

#24

$$\sum_{n=1}^{\infty}\frac{n^2-5n}{n^3+n+1}$$

Since $\lim\limits_{n\to\infty}\frac{n^2-5n}{n^3+n+1}=\lim\limits_{n\to\infty}\frac{n^2}{n^3}=\lim\limits_{n\to\infty}\frac{1}{n}$

And

$$\lim_{n\to\infty}\frac{\left[\frac{n^2-5n}{n^3+n+1}\right]}{\left(\frac{1}{n}\right)}=\lim_{n\to\infty}\frac{\left[\frac{1}{n}\right]}{\left(\frac{1}{n}\right)}=1$$

Since $\sum\limits_{n=1}^{\infty}\frac{1}{n}$ is the divergent harmonic series, $\sum\limits_{n=1}^{\infty}\frac{n^2-5n}{n^3+n+1}$ diverges by the limit comparison test.

#25

$$\sum_{n=1}^{\infty}\frac{\sqrt{n^4+1}}{n^3+n^2}$$

Since $\lim\limits_{n\to\infty}\frac{\sqrt{n^4+1}}{n^3+n^2}=\lim\limits_{n\to\infty}\frac{\sqrt{n^4}}{n^3}=\lim\limits_{n\to\infty}\frac{n^2}{n^3}=\lim\limits_{n\to\infty}\frac{1}{n}$

And

$$\lim_{n\to\infty}\frac{\left[\frac{\sqrt{n^4+1}}{n^3+n^2}\right]}{\left(\frac{1}{n}\right)}=\lim_{n\to\infty}\frac{\left[\frac{1}{n}\right]}{\left(\frac{1}{n}\right)}=1$$

Since $\sum\limits_{n=1}^{\infty}\frac{1}{n}$ is the divergent harmonic series, $\sum\limits_{n=1}^{\infty}\frac{\sqrt{n^4+1}}{n^3+n^2}$ diverges by the limit comparison test.

#26

$$\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n^2-1}}$$

Since $\lim\limits_{n\to\infty}\frac{1}{n\sqrt{n^2-1}}=\lim\limits_{n\to\infty}\frac{1}{n\sqrt{n^2}}=\lim\limits_{n\to\infty}\frac{1}{n^2}$

And

$$\lim_{n\to\infty}\frac{\left[\frac{1}{n\sqrt{n^2-1}}\right]}{\left(\frac{1}{n^2}\right)}=\lim_{n\to\infty}\frac{\left[\frac{1}{n^2}\right]}{\left(\frac{1}{n^2}\right)}=1$$

Since $\sum\limits_{n=1}^{\infty}\frac{1}{n^2}$ is a convergent *p*-series, $\sum\limits_{n=1}^{\infty}\frac{1}{n\sqrt{n^2-1}}$ converges by the limit comparison test.

#27

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^2 e^{-n}$$

Since $\left(1+\frac{1}{n}\right)^2 e^{-n}<5e^{-n}=\frac{5}{e^n}$

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^2 e^{-n}<\sum_{n=1}^{\infty}\frac{5}{e^n}=5\cdot\left[\sum_{n=1}^{\infty}\frac{1}{e^n}\right]=5\cdot\left[\sum_{n=1}^{\infty}\left(\frac{1}{e}\right)^n\right]$$

Since $\sum\limits_{n=1}^{\infty}\left(\frac{1}{e}\right)^n$ is a convergent geometric series, $\sum\limits_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^2 e^{-n}$ converges by the direct comparison test.

#28

$$\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n}$$

Since $\frac{1}{n} < \frac{e^{\frac{1}{n}}}{n}$

$$\sum_{n=1}^{\infty} \frac{1}{n} < \sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n}$$

Since $\sum_{n=1}^{\infty} \frac{1}{n}$ is the divergent harmonic series, $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n}$ diverges by the direct comparison test.