1. At $x=0$, which of the following is true of the function $f(x)=\sin(x)+e^{-x}$?

$f(x)=\sin(x)+e^{-x}$

$f'(x)=\cos(x)-e^{-x}$ $\quad f'(0)=\cos(0)-e^{-(0)}=0$

$f''(x)=-\sin(x)+e^{-x}$ $\quad f''(0)=-\sin(0)+e^{-(0)}=1$

$f(x)$ is continuous at zero because $f(x)$ is the sum of two continuous functions.

(a) f is increasing
(b) f is decreasing
(c) f is discontinuous
(d) The graph of *f* is concave up.
(e) The graph of *f* is concave down.

2. Which of the following are true about the function $f(x)$ if its derivative is defined by

$$f'(x)=(x-1)^2(4-x)?$$

I. f is decreasing for all $x<4$ FALSE: $f'(x)>0$ for all $x<4$.

II. f has a local maximum at $x=1$. FALSE: $f'(x)$ does not change sign at $x=1$

III. The graph of *f* is concave up for all $1<x<3$.

$$\begin{aligned} f'(x)&=(x-1)^2(4-x) \\ f''(x)&=2(x-1)^1(4-x)+(x-1)^2(-1) \\ &=(x-1)\left[2(4-x)-(x-1)\right] \\ &=(x-1)(9-3x) \\ &=3(x-1)(3-x) \end{aligned}$$

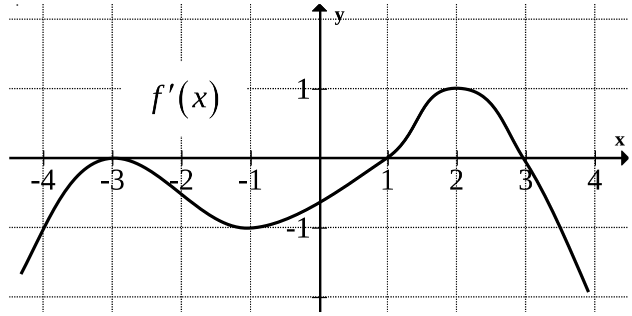
(a) I only (b) II only (c) III only (d) II and III only (e) I, II, and III

3. The figure at right shows the graph of $f'(x)$, the derivative of the function $f(x)$. The domain of $f(x)$ is $-4 \le x \le 4$. Which of the following must be true about the graph of f ?

I. At the points where $x = -3$ and $x = 2$, the graph of f has horizontal tangents.

II. At the point where $x = 1$ the graph of f has a relative minimum.

III. At the point where $x = -3$, the graph of f has a point of inflection.



(a) Nonne (b) II only (c) III only (d) II and III only (e) I, II, and III

4. A particle moves on the x-axis in such a way that its position at time t, $t > 0$, is given by $x(t) = [\ln(t)]^2$. At what value of t does the velocity of the particle attain its maximum?

$$x(t) = [\ln(t)]^2$$

$$x'(t) = 2[\ln(t)] \cdot \frac{1}{t} = \frac{2\ln(t)}{t}$$

$$x''(t) = \frac{2\left(\frac{1}{t}\right)t - 2\ln(t)(1)}{t^2} = \frac{2 - 2\ln(t)}{t^2}$$

$x''(t) = 0$ or *DNE* when $x = e$

$x''(t)$ $(+)$ 0 $(-)$

e

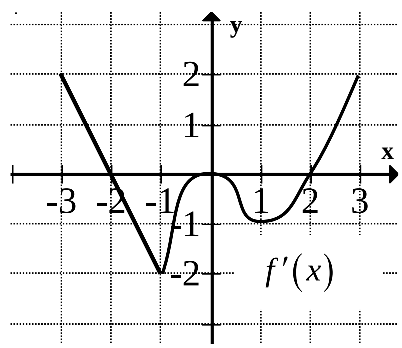
(a) 1 (b) $e^{\frac{1}{2}}$ (c) e (d) $e^{\frac{3}{2}}$ (e) e^2

5. At right is the graph of $f'(x)$, the derivative of $f(x)$. The domain of f is $-3 \le x \le 3$. Which of the following must be true about the graph of f ?

I. f is increasing on $-3 < x < -2$.

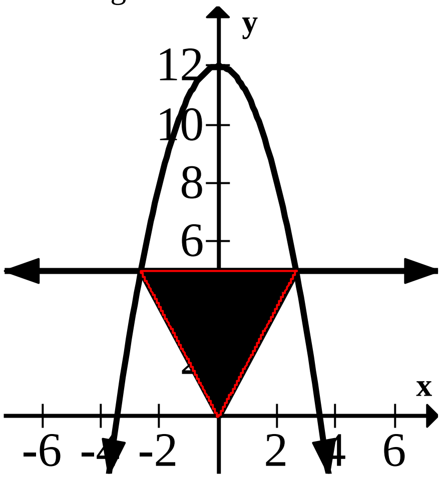
II. The graph of f is concave down on $-3 < x < -1$.

~~III. The graph of f has two relative minimums.~~



(a) I only (b) IIIonly (c) I and II only (d) II and III only (e) None

6. An isosceles triangle has one vertex at the origin and the other two points where a line parallel to and above the x-axis intersects the curve $y = 12 - x^2$. The maximum area of the triangle is



$0 \le x \le 2\sqrt{3}$

$A(x) = \frac{1}{2}(2x)(12 - x^2)$

$= 12x - x^3$

$A'(x) = 12 - 3x^2$

$A'(x) = 0$ or DNE

when $x = \pm 2$

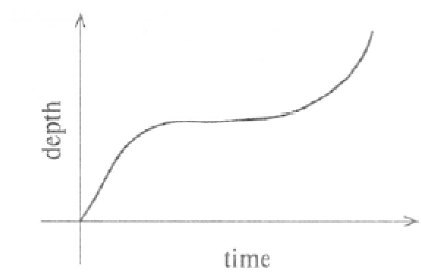
$A(0) = 0$

$A(2) = 16$

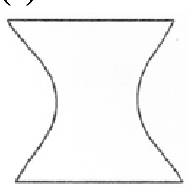
$A(2\sqrt{3}) = 0$

(a) 40 (b) 32 (c) 24 (d) 16 (e) 8

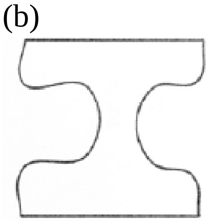
7. Every cross section perpendicular to the axis of a container is a circle. Water is flowing into a the container at a constant rate. A graph of the depth of the water as a function of time is shown at right. Which of the following best describes the profile of the container?



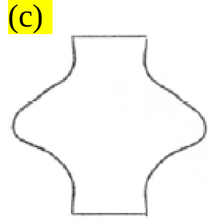
(a)



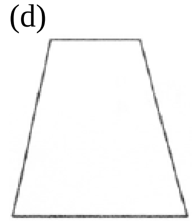
(b)



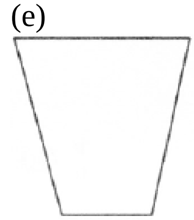
(c)



(d)

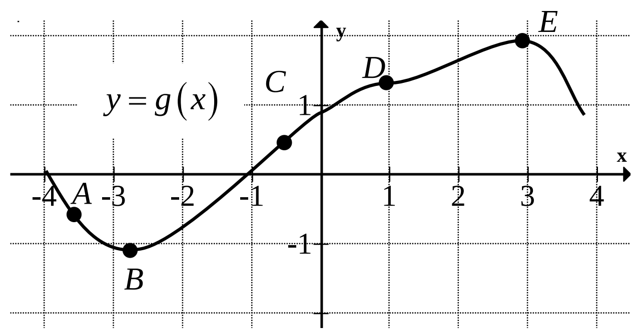


(e)

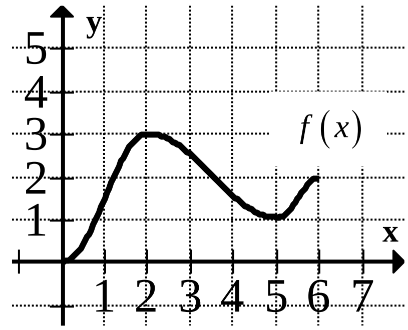


8. At which point on the graph of $y = g(x)$ at right is $g'(x) = 0$ and $g''(x) < 0$?

(a) A (b) B (c) C (d) D (e) E



9. A graph of the function $f(x)$ is shown at right. Which of the following statements are true?



I. $\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h}=f'(5)$

II. $\dfrac{f(5)-f(2)}{5-2}=-\dfrac{2}{3}$

III. $f''(1.5)\le f''(5)$

(a) I and II only (b) I and III only (c) II and III only (d) I, II, and III (e) None