3. You don't need to bother looking at the ditch! simply use cons. of energy, noting that we have both gravitational and elastic P.E.

$$E_f = E_i + \cancel{W_{other}}^{0}$$

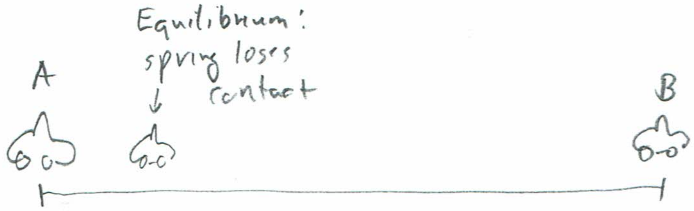
$$U_{gf} + \cancel{U_{ef}}^{0} + \cancel{K_f}^{0} = U_{gi} + U_{ei} + \cancel{K_i}^{0}$$

(disconnected from the spring) (at rest) (at rest at end)

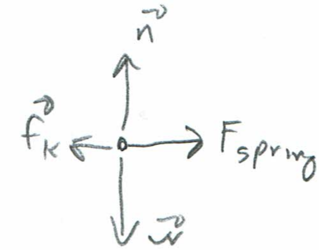
~~mgh_f~~

$$\Rightarrow \quad mgh_f = mgh_i + \frac{1}{2}kx_i^2$$

$$\Rightarrow \quad h_f = \frac{mgh_i + \frac{1}{2}kx^2}{mg} = 21.48 \text{ m}$$ above the "~~[illegible]~~" bottom of the ditch



FBD:



$\vec{n}$ and $\vec{w}$ are perpendicular to motion and do no work

$\vec{F}_{spring}$ is conservative: use potential energy

$W_{f_K} = -f_K d$ (f_K is constant and opposite motion)

$$E_B = E_A + W_{other}$$

only friction provides "other" work

at rest

$$\cancel{K_B}^{0} + \cancel{U_{eB}}^{0} = \cancel{K_A}^{0} + U_{eA} + W_{fK}$$

at rest

spring disconnected

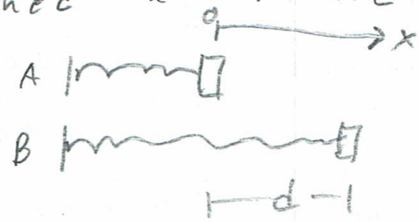
so $\quad 0 = U_{eA} + W_{fK}$

$\Rightarrow \quad 0 = \frac{1}{2} k x_A^2 - f_K d$

$$\Rightarrow \quad k = \frac{2 f_K d}{x_A^2} = \frac{2(0.1N)(6m)}{(0.1m)^2} = 120\,N/m$$

4) a) b has units of $\frac{N}{m^3}$ so that bx^3 has units of meters

b) $\Delta U = -W_{cons} = -W_{spring}$ in this case. Imagine the spring is stretched a distance d from its equilibrium position:



$$W_{spring} = \int_A^B \vec{F}_s \cdot \vec{ds} \quad \text{Here } \vec{ds} = dx\,\hat{\imath}$$

$$\text{so } W_s = \int_A^B (-bx^3\hat{\imath}) \cdot (dx\,\hat{\imath})$$

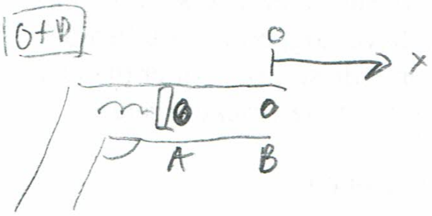
$$W_s = \int_0^d -bx^3\,dx = \frac{-bd^4}{4}$$

$$\text{so } \Delta U_{s\,AB} = U_B - \cancel{U_A}^{0} = -\left(\frac{-bd^4}{4}\right)$$

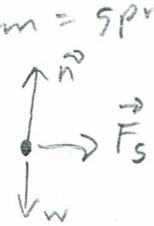
$$\Rightarrow U_{sB} = \frac{bd^4}{4}$$

in general, if stretched to position x, the spring has $U_s = \frac{bx^4}{4}$ potential energy.

c) $\boxed{\text{O+P}}$



System = spring + ball



$\vec{n}$ and $\vec{w}$ do no work.

$\boxed{\text{Solve}}$

$$E_B = E_A + \cancel{W_{diss}}^{0\text{ (none)}} + \cancel{W_{ext,\,non\text{-}diss}}^{0\text{ (none)}}$$

$$\cancel{U_B}^{0} + K_B = U_A + \cancel{K_A}^{0} \quad \text{(starts at rest)}$$

(equilib. position)

$$\frac{1}{2}mv_B^2 = \frac{bx_A^4}{4}$$

$$\Rightarrow v_B = \sqrt{\frac{bx_A^4}{2m}} = \sqrt{\frac{(40\,000\ \text{N/m}^3)(0.10\text{m})^4}{2 \cdot 0.02\text{kg}}}$$

$$= \cancel{300}\ \text{m/s}$$
$$10$$

$\boxed{\text{Ref}}$

We had to use $\Delta U = -W_{cons}$ to define a new pot. energy