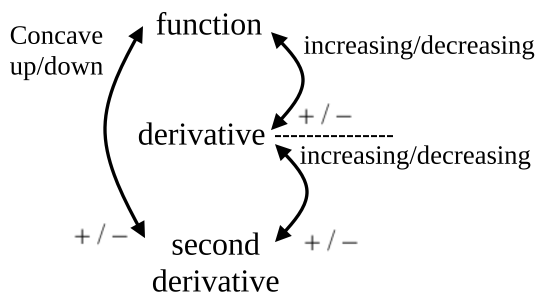
Connections between $f(x)$, $f'(x)$, and $f''(x)$.



$f(x)$ is increasing if an only if $f'(x)$ is positive.
$f(x)$ is decreasing if and only if $f'(x)$ is negative.

$f(x)$ has a relative minimum if and only if $f'(x)$ changes sign from negative to positive.
$f(x)$ has a relative maximum if and only if $f'(x)$ changes sign from positive to negative.

$f(x)$ has a relative minimum at $x = c$ because $f'(c) = 0$ and $f''(c)$ is positive.
$f(x)$ has a relative maximum at $x = c$ because $f'(c) = 0$ and $f''(c)$ is negative.

$f(x)$ is concave up if and only if $f''(x)$ is positive.
$f(x)$ is concave down if and only if $f''(x)$ is negative.

$f(x)$ is concave up if and only if $f'(x)$ is increasing.
$f(x)$ is concave down if and only if $f'(x)$ is decreasing.

$f(x)$ has an inflection point at $x = c$ if and only if $f''(x)$ changes sign.
$f(x)$ has an inflection point at $x = c$ if $f'(x)$ changes from [decreasing to increasing] or [increasing to decreasing].

<u>Extrema Decision Tree</u>

Find the critical values of f

$$\boxed{f'(x)=0 \text{ or DNE } \rightarrow x=c_1,c_2,\cdots,c_i,\cdots,c_n}$$

Closed Interval
<u>Absolute Min/Max</u>

<u>USE EVT</u>

Test Endpoints and all critical values in the closed interval:

$$f(a)=y_a$$
$$\vdots$$
$$f(c_i)=y_i$$
$$\vdots$$
$$f(b)=y_b$$

Claim the greatest y_i value as the Absolute Max
Claim the least y_i value as the Absolute Min
Note, the Absolute Min/Max occurs at $x=c_i$
State "$f(x)$ has an absolute max/min at $(c_i, f(c_i))$"

Not Closed Interval
Relative Min/Max

You <u>CAN</u> Make a Sign Chart for $f'(x)$

You <u>Cannot</u> Make a Sign Chart for $f'(x)$ [Differential Equations]

$f'(c_i)=0$ or DNE

1st Derivative Test:
Make a <u>labeled</u>
Sign Chart for $f'(x)$

- $f(x)$ has a relative max at $x=c_i$ because $f'(x)$ goes from $+ \rightarrow -$
- $f(x)$ has a relative min at $x=c_i$ because $f'(x)$ goes from $- \rightarrow +$

$f'(c_i)=0$

2nd Derivative Test:
Demonstrate the values of $f'(c_i)$ and $f''(c_i)$

- $f(x)$ has a relative max at $x=c_i$ because $f'(c_i)=0$ and $f''(c_i)<0$
- $f(x)$ has a relative min at $x=c_i$ because $f'(c_i)=0$ and $f''(c_i)>0$
- $f''(c)=0 \rightarrow$ *MUST* Use First Derivative Test