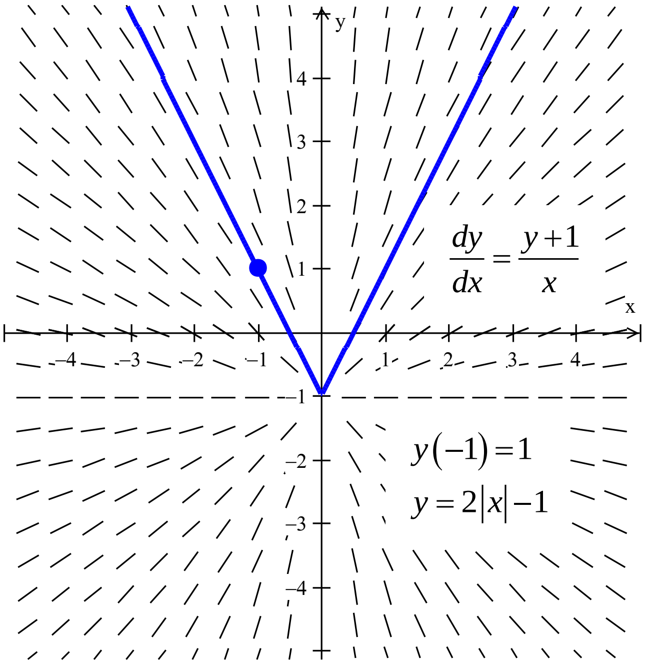



AB 2006 #5

$$\frac{dy}{dx}=\frac{y+1}{x}$$

$$\frac{1}{y+1}dy=\frac{1}{x}dx$$

$$\int\frac{1}{y+1}dy=\int\frac{1}{x}dx$$

$$\ln|y+1|=\ln|x|+C$$

$$e^{\ln|y+1|}=e^{\ln|x|+C}$$

$$|y+1|=e^{\ln|x|}e^{C}$$

$$|y+1|=A|x|$$

$$y+1=\pm A|x|$$

$$y+1=A|x|$$

$$y=A|x|-1$$

$$y(-1)=1$$

$$\downarrow$$

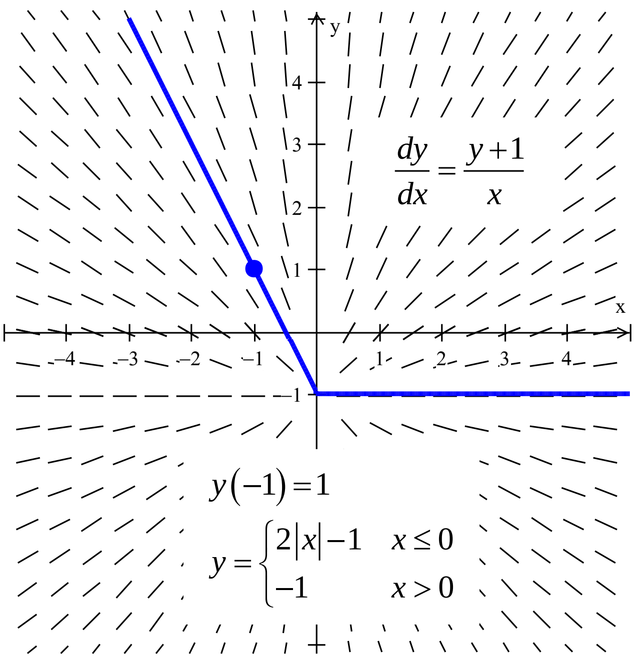
$$1=A|-1|-1$$

$$1=A-1$$

$$A=2$$

$$\downarrow$$

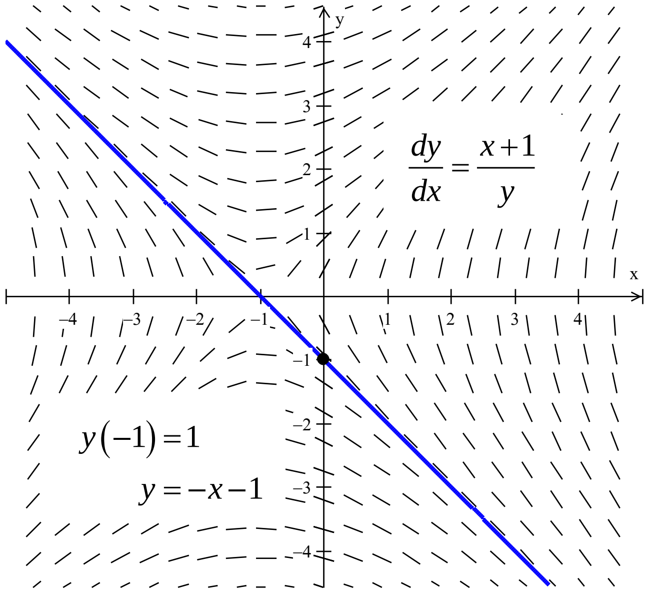
$$y=2|x|-1$$




Another possible solution to the differential equation is $y=\begin{cases}2|x|-1 & x\le 0\\ -1 & x>0\end{cases}$. This function passes through $(-1,1)$ and is consistent in the slope field.

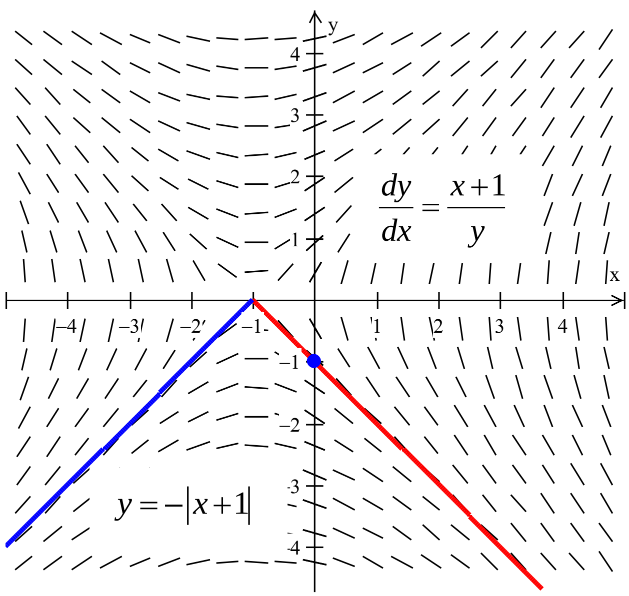
Since there are two possible functions passing through $(-1,1)$ that satisfy the differential equation, we restrict the domain of the solution to be the domain of the graph that is common to both/all solutions passing through $(-1,1)$. Therefore the domain should be restricted to $x<0$.

AB 2010 Form B #5




$$\frac{dy}{dx}=\frac{x+1}{y}$$
$$ydy=(x+1)dx$$
$$\int ydy=\int(x+1)dx$$
$$\frac{1}{2}y^2=\frac{1}{2}x^2+x+C$$
$$y^2=x^2+2x+C$$
$$\sqrt{y^2}=\sqrt{x^2+2x+C}$$
$$|y|=\sqrt{x^2+2x+C}$$
$$y=\pm\sqrt{x^2+2x+C}$$

$$y(0)=-1$$
$$\downarrow$$
$$-1=-\sqrt{(0)^2+2(0)+C}$$
$$1=\sqrt{C}$$
$$C=1$$
$$\downarrow$$
$$y=-\sqrt{x^2+2x+1}$$
$$=-\sqrt{(x+1)^2}$$
$$=-|x+1|$$
$$=\begin{cases}-x-1 & x>-1\\ x+1 & x<-1\end{cases}$$




There is a solution $y=-|x+1|$. However, if one started sketching the solution from the point $(0,-1)$, one would be lead to believe the solution should look like $y=-x-1$, which is not the case. Since $y'$ at $y=0$ is $-1$, which is not consistent with the slope field.

Since $\frac{dy}{dx}$ DNE could signify either a sharp corner or discontinuity, the graph of the solution can change behavior at that location dramatically.

A *relation* that would satisfy this differential equation, be consistent with the slope field , and pass through $(0,-1)$ would be $|y|-1=x$.

Since all solutions to the differential equation consistent with the slope field, that pass through $(0,-1)$ are guaranteed to have the branch of the graph in red, we state that the solution to the differential equation passing through $(0,-1)$ is $y=-x-1$ with the restriction that $x>-1$.