Extrema Decision Tree

Find the extrema of a function $f(x)$

Find the critical values of f

$$f'(x)=0 \text{ or DNE} \rightarrow x = c_1, c_2, \cdots, c_i, \cdots, c_n$$

Closed Interval
Absolute Min/Max

USE EVT

Test Endpoints and all critical values in the closed interval:

$$f(a) = y_a$$
$$\vdots$$
$$f(c_i) = y_i$$
$$\vdots$$
$$f(b) = y_b$$

Claim the greatest y_i value as the Absolute Max

Claim the least y_i value as the Absolute Min

Note, the Absolute Min/Max occurs at $x = c_i$

State "$f(x)$ has an absolute max/min at $(c_i, f(c_i))$"

Not Closed Interval
Relative Min/Max

You <u>CAN</u> Make a Sign Chart for $f'(x)$

$f'(c_i) = 0$ or DNE

$f'(c_i) = 0$

You <u>Cannot</u> Make a Sign Chart for $f'(x)$ [Differential Equations]

1st Derivative Test:
Make a <u>labeled</u> Sign Chart for $f'(x)$

2nd Derivative Test:
Demonstrate the values of $f'(c_i)$ and $f''(c_i)$

→ $f(x)$ has a relative max at $x = c_i$ because $f'(c_i) = 0$ and $f''(c_i) < 0$

→ $f(x)$ has a relative min at $x = c_i$ because $f'(c_i) = 0$ and $f''(c_i) > 0$

→ $f''(c_i) = 0 \rightarrow$ *MUST* Use First Derivative Test

→ $f(x)$ has a relative max at $x = c_i$ because $f'(x)$ goes from $+ \rightarrow -$

→ $f(x)$ has a relative min at $x = c_i$ because $f'(x)$ goes from $- \rightarrow +$

Extrema Decision Tree

Find the extrema of a function $f(x)$

Find the critical values of f

$f'(x) = 0$ or DNE $\rightarrow x = c_1, c_2, \cdots, c_i, \cdots, c_n$

Closed Interval
Absolute Min/Max

Not Closed Interval
Relative Min/Max

USE EVT

Test Endpoints and all critical values in the closed interval:

$$f(a) = y_a$$
$$\vdots$$
$$f(c_i) = y_i$$
$$\vdots$$
$$f(b) = y_b$$

Claim the greatest y_i value as the Absolute Max
Claim the least y_i value as the Absolute Min
Note, the Absolute Min/Max occurs at $x = c_i$
State "$f(x)$ has an absolute max/min
at $(c_i, f(c_i))$"

You <u>CAN</u> Make a Sign Chart for $f'(x)$

$f'(c_i) = 0$ or DNE

$f'(c_i) = 0$

You <u>Cannot</u> Make a Sign Chart for $f'(x)$ [Differential Equations]

1st Derivative Test:
Make a <u>labeled</u>
Sign Chart for $f'(x)$

2nd Derivative Test:
Demonstrate the values of $f'(c_i)$ and $f''(c_i)$

$\rightarrow f(x)$ has a relative max at $x = c_i$ because $f'(c_i) = 0$ and $f''(c_i) < 0$

$\rightarrow f(x)$ has a relative min at $x = c_i$ because $f'(c_i) = 0$ and $f''(c_i) > 0$

$\rightarrow f''(c) = 0 \rightarrow$ *MUST* Use First Derivative Test

$\rightarrow f(x)$ has a relative max at $x = c_i$ because $f'(x)$ goes from $+ \rightarrow -$

$\rightarrow f(x)$ has a relative min at $x = c_i$ because $f'(x)$ goes from $- \rightarrow +$