AP Calculus BC
Stewart Chapter 4 Test
Fall 2020 Solutions

Name:________________________________

Date:____________________ Period:________

Multiple Choice Section: 8 Questions. *No Calculator Permitted.* Suggested Time: 35 minutes.

Once you submit your Multiple Choice section, you will not be allowed to revisit it.

Free Response Section: 3 Questions. Calculator Permitted. Suggested Time: 55≈ minutes.

Multiple Choice Scoring Procedures: Each exercise is worth 5 points.	
If an response is **CIRCLED**	If no response is circled
▪ 5 points awarded if circled response is correct. ▪ 0 points awarded if circled response is incorrect.	▪ 1 point awarded for each incorrect response eliminated. ▪ 0 points awarded if the correct response is eliminated.

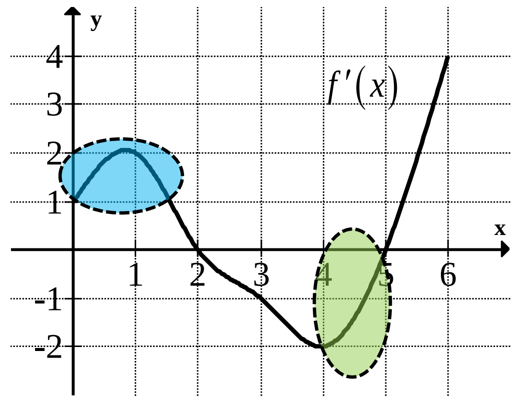
1. Evaluate the limit: $\lim_{x\to\infty}\frac{x^5}{e^{4x}}$

$$\begin{aligned}\lim_{x\to\infty}\frac{x^5}{e^{4x}} &= \lim_{x\to\infty}\frac{5x^4}{4e^{4x}}\\ &= \lim_{x\to\infty}\frac{5\cdot 4x^3}{4^2e^{4x}}\\ &= \lim_{x\to\infty}\frac{5\cdot 4\cdot 3x^2}{4^3e^{4x}}\\ &= \lim_{x\to\infty}\frac{5\cdot 4\cdot 3\cdot 2x^1}{4^3e^{4x}}\\ &= \lim_{x\to\infty}\frac{5!}{4^3e^{4x}}\\ &= 0\end{aligned}$$

(a) $\frac{5}{4}$ (b) $\frac{15}{128}$ (c) 1 (d) 0 (e) DNE

Use the graph of $f'(x)$ for exercises 2 and 3.

2. Where does $f(x)$ have a local maximum on the interval $[0,6]$?



(a) $x=1$ (b) $x=2$ (c) $x=4$ (d) $x=5$ (e) nowhere

3. Which of the following statements are true about the graph of $f''(x)$?

I. f'' is decreasing on $(0,1)$ $\left[f'(x)\right]'' < 0$ because f' is concave down

II. f'' is positive on $(4,5)$ because $f'(x)$ is increasing

III. f'' has a local minimum when $x=5$

(a) I only (b) II only (c) I and II only (d) II and III only (e) I, II, and III

4. On what interval is the function $\dfrac{e^x}{x+1}$ decreasing?

$$f(x)=\frac{e^x}{x+1}$$

$$f'(x)=\frac{e^x(x+1)-e^x(1)}{(x+1)^2}$$

$$=\frac{xe^x}{(x+1)^2}$$

$$\frac{x\cdot\underbrace{e^x}_{\text{always }(+)}}{\underbrace{(x+1)^2}_{\text{always }(+)}}$$

$$\frac{xe^x}{(x+1)}<0$$

$$\downarrow$$

$$x<0$$

(a) $(0,\infty)$ (b) $(-1,0)$ (c) $(0,1)$ (d) $(-\infty,-1)$ (e) $(-\infty,0)$

5. In Newton's Method, the $(n+1)^{th}$ approximation of the zero is given by:

$$y - f(x_n) = f'(x_n)(x - x_n)$$
$$0 - f(x_n) = f'(x_n)(x - x_n)$$
$$-\frac{f(x_n)}{f'(x_n)} = x - x_n$$
$$x_n - \frac{f(x_n)}{f'(x_n)} = x$$

(a) $x_{n+1} = x_n - \dfrac{f'(x_n)}{f(x_n)}$

(b) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

(c) $x_n = \dfrac{f(x_n)}{f'(x_n)} - 1$

(d) $x_{n+1} = f'(x_n) - f(x_n)$

(e) None of these

6. The graph of $y = \dfrac{x-1}{3+x}$ is concave down on which of the following intervals?

$$y = \frac{x-1}{3+x}$$
$$y' = \frac{(1)(3+x)-(x-1)(1)}{(3+x)^2}$$
$$= \frac{2}{(3+x)^2}$$
$$= 2(3+x)^{-2}$$
$$y'' = -4(3+x)^{-3}$$
$$= \frac{-4}{(3+x)^3}$$

$$-\frac{4}{(3+x)^3} < 0$$
$$\frac{4}{(3+x)^3} > 0$$
$$\downarrow$$
$$(3+x)^3 > 0$$
$$3 + x > 0$$
$$x > -3$$

(a) $(-3,\infty)$ only (b) $(-\infty,-3)$ only (c) $(-\infty,-3)\cup(-3,\infty)$ (d) All real numbers (e) Never

7. How many critical values does the function $f(x)=(x-2)^5(x+3)^4$ have?

$$
\begin{aligned}
f(x) &= (x-2)^5(x+3)^4 \\
f'(x) &= 5(x-2)^4(x+3)^4+4(x+3)^3(x-2)^5 \\
&= (x-2)^4(x+3)^3\left[5(x+3)+4(x-2)\right] \\
&= (x-2)^4(x+3)^3\left[9x+7\right]
\end{aligned}
$$

(a) 1 (b) 2 (c) 3 (d) 5 (e) 9

8. How many inflection points does the function $f(x)=3x^4-5x^3-9x+2$ have?

$$
\begin{aligned}
f(x) &= 3x^4-5x^3-9x+2 \\
f'(x) &= 12x^3-15x^2-9 \\
f''(x) &= 36x^2-30x \\
&= 2x(18x-15)
\end{aligned}
$$

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

AP Calculus BC
Stewart Chapter 4 Test
Fall 2020

Name:___________________________________

Date:______________________ Period:________

Free Response Section: ***<u>NO Calculator Permitted.</u>***
You have the remainder of the period to complete this section.
Once you submit your Free Response Section, you will not be allowed to revisit it.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. To be eligible for partial credit, methods, reasoning, and conclusions should be presented clearly.

- Justifications require that you give mathematical (non-calculator) reasons. Students should use complete sentences in responses that include explanations or justifications.

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

  A graphing calculator appropriate for use on the exam is expected to have the built-in capability to:
  (1) plot the graph of a function within an arbitrary viewing window,
  (2) find the zeros of functions (solve equations numerically),
  (3) numerically calculate the derivative of a function, and
  (4) numerically calculate the value of a definite integral.

- For results obtained using one of the four required calculator capabilities, students are required to write the mathematical setup that leads to the solution along with the result produced by the calculator. These setups include the equation being solved, the derivative being evaluated, or the definite integral being evaluated. In general, in a calculator-active problem that requires the value of a definite integral, students may use a calculator to determine the value; they do not need to compute an antiderivative as an intermediate step. Similarly, if a calculator-active problem requires the value of a derivative of a given function at a specific point, students may use a calculator to determine the value; they do not need to state the symbolic derivative expression. For solutions obtained using a calculator capability other than one of the four required, students must show the mathematical steps necessary to produce their results; a calculator result alone is not sufficient. For example, if students are asked to find a relative minimum value of a function, they are expected to use calculus and show the mathematical steps that lead to the answer. It is not sufficient to graph the function or use a calculator application that finds minimum values.

- Students may bring to the exam one or two (but no more than two) graphing calculators from the approved list. Calculator memories will not be cleared. Students are allowed to bring calculators containing whatever programs they want. They are expected to bring calculators that are set to radian mode.

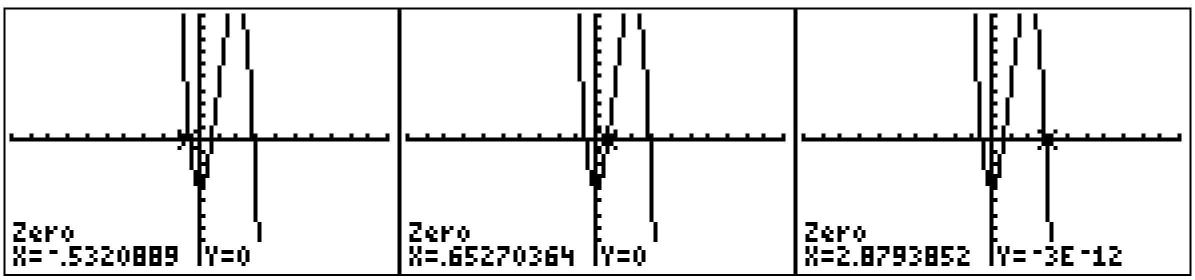
9. Given $f(x) = -x^4 + 4x^3 - 4x + 1$ determine the following:

- ✓ The intervals on which *f* is increasing.
- ✓ The intervals on which *f* is decreasing.
- ✓ The location(s) at which *f* has a relative minimum.
- ✓ The location(s) at which *f* has a relative minimum.
- ✓ The location(s) at which *f* has a horizontal tangent.
- ✓ The location(s) at which *f* has a horizontal tangent(s), if any.
- ✓ The intervals on which *f* is concave up.
- ✓ The intervals on which *f* is concave down.
- ✓ The locations at which *f* has a point(s) of inflection, if any.

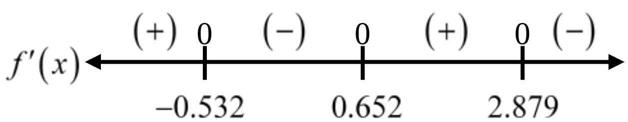
<u>Justify all your responses.</u>

$f(x) = -x^4 + 4x^3 - 4x + 1$

$f'(x) = -4x^3 + 12x^2 - 4$



$f''(x) = -12x^2 + 24x$ $f''(x) = 0$ when $x = 0, 2$.



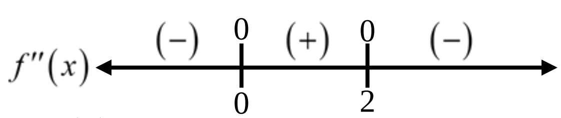
$f(x)$ is decreasing on $(-0.532, 0.652) \cup (2{,}879, \infty)$ because $f'(x) < 0$.

$f(x)$ is increasing on $(-\infty, -0.532) \cup (0.652, 2.879)$ because $f'(x) > 0$.

$f(x)$ has a maximum at $x \approx -0.532$ and $x \approx 2.879$ because $f'(x)$ changes sign from positive to negative.

$f(x)$ has a relative minimum at $x \approx 0.652$ because $f'(x)$ changes sign from negative to positive.

$f(x)$ has horizontal tangents at $x = -0.5320...$, $0.6527...$, and $2.8793...$ because $f'(x) = 0$.



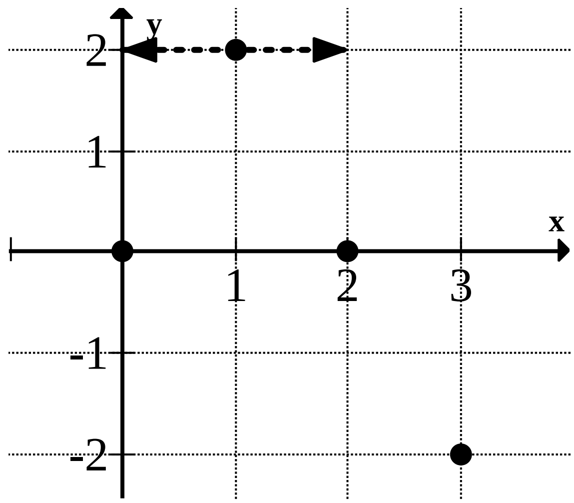
$f(x)$ is concave down on $(-\infty, 0) \cup (2, \infty)$ because $f''(x) < 0$.

$f(x)$ is concave up on $(0, 2)$ because $f''(x) < 0$.

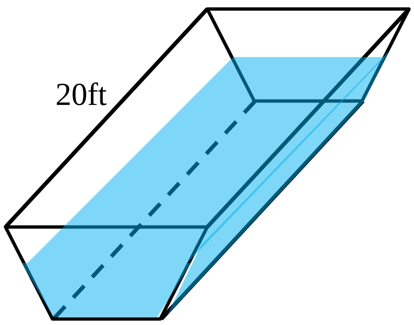
$f(x)$ has inflection points at $x = 0$ and $x = 2$ because $f''(x)$ changes sign.

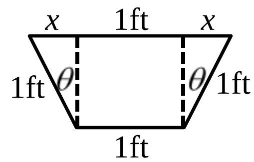
10. Sketch the graph of a continuous function *f* on the interval $[0,3]$ that satisfies the following criteria:

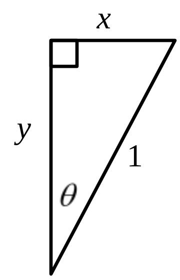
x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3
$f(x)$	0	+	2	+	0	–	–2
$f'(x)$	+	+	0	–	DNE	–	–
$f''(x)$	0	–	–	–	DNE	–	0



11. The trough in the figure below is to be made with the dimensions shown. Only the angle of θ can be varied. What value of θ will maximize the trough's volume. Justify your answer using calculus.







$$\sin(\theta) = \frac{x}{1} \qquad \cos(\theta) = \frac{y}{1}$$

$$x = \sin(\theta) \qquad y = \cos(\theta)$$

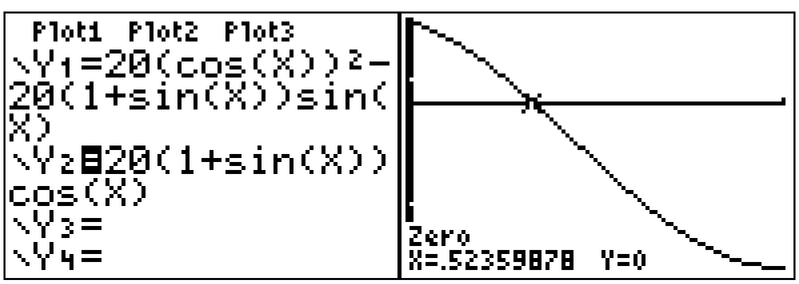
Feasible Domain: $0 \le \theta \le \frac{\pi}{2}$

$$\begin{aligned} A &= \frac{1}{2}(b_1 + b_2) \cdot h \\ &= \frac{1}{2}(1 + 2\sin(\theta) + 1) \cdot \cos(\theta) \\ &= \frac{1}{2}(2 + 2\sin(\theta)) \cdot \cos(\theta) \\ &= (1 + \sin(\theta)) \cdot \cos(\theta) \end{aligned}$$

$$\begin{aligned} V &= (\text{Base area}) \cdot \text{height} \\ &= 20(1 + \sin(\theta))\cos(\theta) \end{aligned}$$

$$\downarrow$$

$$V' = 20\cos^2(\theta) - 20(1 + \sin(\theta))\sin(\theta)$$



$V'(\theta) = 0$ when $\theta \approx 0.52359878...$

$$V(0) = 20$$

$$V(0.5235...) = 25.9807...$$

$$V\left(\frac{\pi}{2}\right) = 0$$

The volume of the trough will be at a maximum when $\theta \approx 0.523$ or 0.524 radians.