Unit Summary for Integration:

1. What is a difference between the following expressions? How are they connected?

$$\int f(x)\,dx \text{ and } \int_a^b f(x)\,dx \text{ where } a \text{ and } b \text{ are constants.}$$

2. Suppose that $f(x)$ does not change sign on an interval $[a,b]$. Explain how to determine the sign of $\int_a^b f(x)\,dx$ and $\int_b^a f(x)\,dx$ using the direction of integration and sign of $f(x)$.

3. Why is it necessary to add a "$+C$" to an antiderivative when antidifferentiating a function $f(x)$?

4. Write an equation to determine the value of $f(b)$ given $f(a)$ and $f'(x)$.

<u>The unit summary is continued on the next page.</u>

5. The two conceptual interpretations of $\int_a^b f'(x)\,dx$ are the following

- The net change in $f(x)$ from $x=a$ to $x=b$
- The area under the curve of $f'(x)$ from $x=a$ to $x=b$

(a) The idea of the net change in $f(x)$ from $x=a$ to $x=b$ is connected to the notation of the definite integral $\int_a^b f'(x)\,dx$ by rewriting $f'(x)$ as $\frac{\Delta y}{\Delta x}$ and dx as Δx and doing unit analysis (i.e. doing unit analysis on the area of the rectangles of the Reimann sum used to estimate the integral).

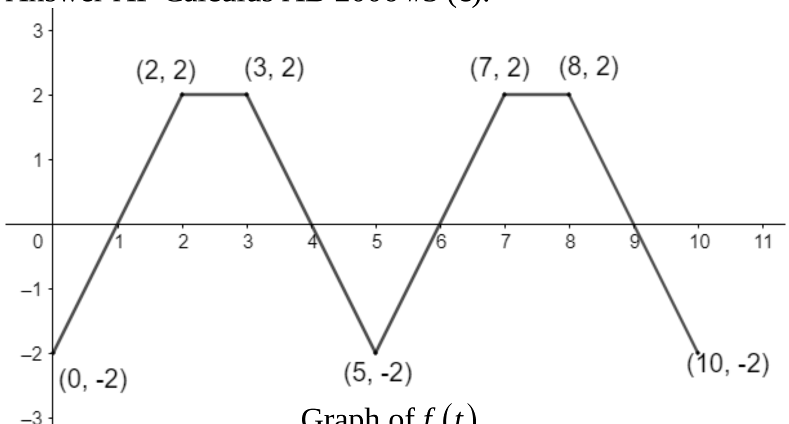
(i) Answer AP Calculus AB 2005 #3 part (c), and use unit analysis to demonstrate how to determine the correct units for $\int_0^8 T'(x)\,dx$.

(ii) Answer AP Calculus AB 2006 Form B #6 parts (a) and (b), and use unit analysis to demonstrate how to determine the correct units for each definite integral given.

(b) The area under the curve of $f(x)$ from $x=a$ to $x=b$ is connected to the notation of the definite integral $\int_a^b f(x)\,dx$ by connecting $f(x)$ and dx to the dimensions of rectangles used to estimate the area under the curve. The value of the definite integral can be negative, zero, or positive - unlike area which is always a non-negative number.

(i) Explain how to determine the value of $\int_0^{10} f(t)\,dt$ by referencing areas in the graph below:

(ii) Answer AP Calculus AB 2006 #3 (c).



Graph of $f(t)$