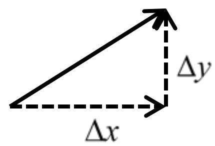
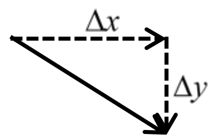
Increasing/Decreasing Functions and the First Derivative Test

A function <u>*f* is increasing on an interval *I*,</u> if for any two numbers x_1 and x_2 in *I* where $x_1 < x_2$ then $f(x_1) < f(x_2)$.



$$\text{slope} = \frac{\Delta y}{\Delta x} \sim \frac{(+)}{(+)} \to (+)$$

A function <u>*f* is decreasing on an interval *I*</u>, if for any two numbers x_1 and x_2 in *I* where $x_1 < x_2$ then $f(x_1) > f(x_2)$.



$$\text{slope} = \frac{\Delta y}{\Delta x} \sim \frac{(-)}{(+)} \to (-)$$

Since slope is directly related to whether a function is increasing or decreasing, we can conclude the following:

Let *f* be a continuous function on $[a,b]$ and differentiable on (a,b), then

I. $f'(x) > 0$ for all x in $(a,b) \to f$ is increasing on $[a,b]$.

II. $f'(x) < 0$ for all x in $(a,b) \to f$ is decreasing on $[a,b]$.

III. $f'(x) = 0$ for all x in $(a,b) \to f$ is constant on $[a,b]$.

If *f* is differentiable at $x = c$, then

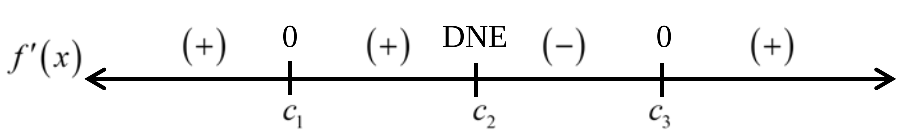
I. $f'(c) > 0 \to f(x)$ is increasing at $x = c$.

II. $f'(c) < 0 \to f(x)$ is decreasing at $x = c$.

III. $f'(c) = 0 \to f(x)$ <u>has a horizontal tangent</u> at $x = c$.

To determine the intervals on which a function $f(x)$ is increasing or decreasing:

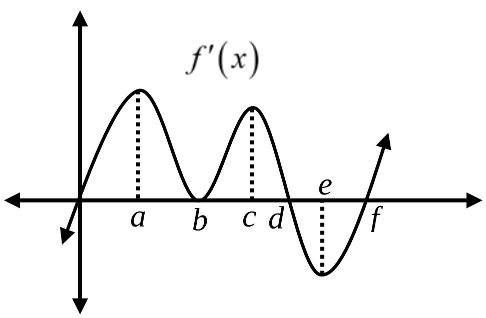
1. Determine $f'(x)$
2. Find the critical values of f : $f'(x) = 0$ or DNE at $x = c_1, c_2, \ldots, c_n$
3. Make a labeled sign chart for $f'(x)$
   a. Test values in between critical values to determine the sign of $f'(x)$ on those intervals.



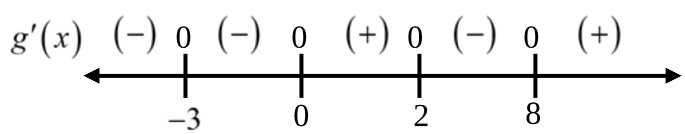
Having a labeled sign chart can help you identify whether $f(x)$ has a relative minimum, relative maximum, or neither at any critical value.

<u>First Derivative Test:</u>
Let c be a critical value of a function f that is continuous on an open interval around c. If f is differentiable on an open interval around c (except possibly at c itself), then $f(c)$ can be classified as follows:

I. If $f'(x)$ changes sign from positive to negative at $x = c$, then $f(x)$ has a relative maximum at $(c, f(c))$.
II. If $f'(x)$ changes sign from negative to positive at $x = c$, then $f(x)$ has a relative minimum at $(c, f(c))$.
III. If $f'(x)$ changes sign from positive to positive, or negative to negative at $x = c$, then $f(x)$ has neither a relative maximum nor relative minimum at $(c, f(c))$.



$$g'(x) = \underbrace{x}_{\text{cross}} \cdot \underbrace{(x+3)^2}_{\text{bounce}} \underbrace{(x-2)^3}_{\text{cross}} \underbrace{(x-8)}_{\text{cross}}$$



$f(x)$ is increasing for $0 < x < b$, $b < x < d$ and $x > f$ because $f'(x) > 0$.
$f(x)$ is deceasing for $x < 0$ and $d < x < f$ because $f'(x) < 0$.
$f(x)$ has a relative maximum at $x = d$ because $f'(x)$ changes sign from positive to negative.
$f(x)$ has a relative minimum at $x = 0$ and $x = f$ because $f'(x)$ changes from $(-)$ to $(+)$.
$f(x)$ has horizontal tangents at $x = 0, a, b, d,$ and f because $f'(x) = 0$.

$g(x)$ is increasing for $0 < x < 2$ and $x > 8$ because $f'(x) > 0$.
$g(x)$ is decreasing for $x < -3$, $-3 < x < 0$, and $2 < x < 8$ because $f'(x) < 0$.
$g(x)$ has a relative maximum at $x = 0$ because $g'(x)$ changes sign from $(+)$ to $(-)$.
$g(x)$ has a relative minimum at $x = 2$ because $g'(x)$ changes sign from $(-)$ to $(+)$.
$g(x)$ has horizontal tangents at $x = -3, 0, 2,$ and 8 because $g'(x) = 0$.