

Table FRQ

Table FRQ #s:

| Exam | # | Exam | # |
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| 2001 | 2 | 2011 | 2 |
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2001 SCORING GUIDELINES

Question 2

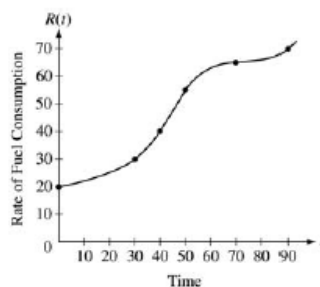
The temperature, in degrees Celsius ($^{\circ}\text{C}$), of the water in a pond is a differentiable function W of time t . The table above shows the water temperature as recorded every 3 days over a 15-day period.

| t (days) | $W(t)$ ($^{\circ}\text{C}$) |
|---------------|----------------------------------|
| 0 | 20 |
| 3 | 31 |
| 6 | 28 |
| 9 | 24 |
| 12 | 22 |
| 15 | 21 |

- Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.
- Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.
- A student proposes the function P , given by $P(t) = 20 + 10te^{(-t/8)}$, as a model for the temperature of the water in the pond at time t , where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.
- Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.

2003 # 3

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t . The graph of R and a table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes, are shown above.



| t (minutes) | $R(t)$ (gallons per minute) |
|------------------|--------------------------------|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

- Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.
- The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.
- Approximate the value of $\int_0^{90} R(t) dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t) dt$? Explain your reasoning.
- For $0 < b \leq 90$ minutes, explain the meaning of $\int_0^b R(t) dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b} \int_0^b R(t) dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

2003 SCORING GUIDELINES (Form B)

Question 3

A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points

| | | | | | | | |
|-------------------------|----|----|-----|-----|-----|-----|-----|
| Distance x (mm) | 0 | 60 | 120 | 180 | 240 | 300 | 360 |
| Diameter $B(x)$ (mm) | 24 | 30 | 28 | 30 | 26 | 24 | 26 |

along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

- Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.
- Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.
- Using correct units, explain the meaning of $\pi \int_{125}^{275} \left(\frac{B(x)}{2} \right)^2 dx$ in terms of the blood vessel.
- Explain why there must be at least one value x , for $0 < x < 360$, such that $B''(x) = 0$.

2004 SCORING GUIDELINES (Form B)

Question 3

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t . Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table above.

| | | | | | | | | | |
|--------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| t (min) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
| $v(t)$ (mpm) | 7.0 | 9.2 | 9.5 | 7.0 | 4.5 | 2.4 | 2.4 | 4.3 | 7.3 |

- Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t) dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t) dt$ in terms of the plane's flight.
- Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.
- The function f , defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.
- According to the model f , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \leq t \leq 40$?

2005 SCORING GUIDELINES

Question 3

| | | | | | |
|--|-----|----|----|----|----|
| Distance x (cm) | 0 | 1 | 5 | 6 | 8 |
| Temperature $T(x)$ ($^{\circ}\text{C}$) | 100 | 93 | 70 | 62 | 55 |

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius ($^{\circ}\text{C}$), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

- Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.
- Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- Find $\int_0^8 T'(x) dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x) dx$ in terms of the temperature of the wire.
- Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

2006 SCORING GUIDELINES

Question 4

| | | | | | | | | | |
|-----------------------------|---|----|----|----|----|----|----|----|----|
| t (seconds) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| $v(t)$ (feet per second) | 5 | 14 | 22 | 29 | 35 | 40 | 44 | 47 | 49 |

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table above.

- Find the average acceleration of rocket A over the time interval $0 \leq t \leq 80$ seconds. Indicate units of measure.
- Using correct units, explain the meaning of $\int_{10}^{70} v(t) dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t) dt$.
- Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

2006 SCORING GUIDELINES (Form B)

Question 6

| | | | | | | | |
|----------------------------------|-----|-----|-----|-----|-----|----|----|
| t (sec) | 0 | 15 | 25 | 30 | 35 | 50 | 60 |
| $v(t)$ (ft/sec) | -20 | -30 | -20 | -14 | -10 | 0 | 10 |
| $a(t)$ (ft/sec ²) | 1 | 5 | 2 | 1 | 2 | 4 | 2 |

A car travels on a straight track. During the time interval $0 \leq t \leq 60$ seconds, the car's velocity v , measured in feet per second, and acceleration a , measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

- (a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)| dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)| dt$ using a trapezoidal approximation with the three subintervals determined by the table.
- (b) Using appropriate units, explain the meaning of $\int_0^{30} a(t) dt$ in terms of the car's motion. Find the exact value of $\int_0^{30} a(t) dt$.
- (c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.
- (d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

2008 SCORING GUIDELINES

Question 2

| | | | | | | | |
|-----------------|-----|-----|-----|-----|-----|----|---|
| t (hours) | 0 | 1 | 3 | 4 | 7 | 8 | 9 |
| $L(t)$ (people) | 120 | 156 | 176 | 126 | 150 | 80 | 0 |

Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \leq t \leq 9$. Values of $L(t)$ at various times t are shown in the table above.

- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For $0 \leq t \leq 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for $0 \leq t \leq 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number?

2008 SCORING GUIDELINES (Form B)

Question 3

| | | | | | |
|---------------------------------------|---|---|----|----|----|
| Distance from the river's edge (feet) | 0 | 8 | 14 | 22 | 24 |
| Depth of the water (feet) | 0 | 7 | 8 | 2 | 0 |

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin(\sqrt{t+10})$ for $0 \leq t \leq 120$ minutes.

- Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.
- The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.
- The scientist proposes the function f , given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.
- Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \leq t \leq 60$ minutes. Does this value indicate that the water must be diverted?

2009 SCORING GUIDELINES

Question 5

| | | | | | |
|--------|---|---|----|---|----|
| x | 2 | 3 | 5 | 8 | 13 |
| $f(x)$ | 1 | 4 | -2 | 3 | 6 |

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \leq x \leq 13$.

- Estimate $f'(4)$. Show the work that leads to your answer.
- Evaluate $\int_2^{13} (3 - 5f'(x)) dx$. Show the work that leads to your answer.
- Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x) dx$. Show the work that leads to your answer.
- Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \leq x \leq 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \leq 4$. Use the secant line for the graph of f on $5 \leq x \leq 8$ to show that $f(7) \geq \frac{4}{3}$.

2009 SCORING GUIDELINES (Form B)

Question 6

| | | | | | | |
|-------------------------------|---|---|-----|----|----|----|
| t (seconds) | 0 | 8 | 20 | 25 | 32 | 40 |
| $v(t)$ (meters per second) | 3 | 5 | -10 | -8 | -4 | 7 |

The velocity of a particle moving along the x -axis is modeled by a differentiable function v , where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

- (a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.
- (b) Using correct units, explain the meaning of $\int_{20}^{40} v(t) dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t) dt$.
- (c) For $0 \leq t \leq 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.
- (d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

2010 SCORING GUIDELINES

Question 2

| | | | | | |
|---------------------------------|---|---|----|----|----|
| t (hours) | 0 | 2 | 5 | 7 | 8 |
| $E(t)$ (hundreds of entries) | 0 | 4 | 13 | 21 | 23 |

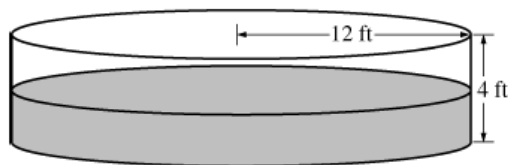
A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t = 0$) and 8 P.M. ($t = 8$). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \leq t \leq 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

- (a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.
- (b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8} \int_0^8 E(t) dt$. Using correct units, explain the meaning of $\frac{1}{8} \int_0^8 E(t) dt$ in terms of the number of entries.
- (c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P , where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \leq t \leq 12$. According to the model, how many entries had not yet been processed by midnight ($t = 12$)?
- (d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

2010 SCORING GUIDELINES (Form B)

Question 3

| | | | | | | | |
|--------|---|----|----|----|----|----|----|
| t | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
| $P(t)$ | 0 | 46 | 53 | 57 | 60 | 62 | 63 |



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t . During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

- Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \leq t \leq 12$ hours. Show the computations that lead to your answer.
- Calculate the total amount of water that leaked out of the pool during the time interval $0 \leq t \leq 12$ hours.
- Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.
- Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

2011 SCORING GUIDELINES

Question 2

| | | | | | |
|-----------------------------|----|----|----|----|----|
| t (minutes) | 0 | 2 | 5 | 9 | 10 |
| $H(t)$ (degrees Celsius) | 66 | 60 | 52 | 44 | 43 |

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \leq t \leq 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

- Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.
- Using correct units, explain the meaning of $\frac{1}{10} \int_0^{10} H(t) dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10} \int_0^{10} H(t) dt$.
- Evaluate $\int_0^{10} H'(t) dt$. Using correct units, explain the meaning of the expression in the context of this problem.
- At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

2011 SCORING GUIDELINES (Form B)

Question 5

| | | | | |
|-------------------------------|-----|-----|-----|-----|
| t (seconds) | 0 | 10 | 40 | 60 |
| $B(t)$ (meters) | 100 | 136 | 9 | 49 |
| $v(t)$ (meters per second) | 2.0 | 2.3 | 2.5 | 4.6 |

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t , measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t .

- (a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.
- (b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)| dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)| dt$ using a left Riemann sum with the subintervals indicated by the data in the table.
- (c) For $40 \leq t \leq 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.
- (d) A light is directly above the western end of the track. Ben rides so that at time t , the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

2012 SCORING GUIDELINES

Question 1

| | | | | | |
|-----------------------------|------|------|------|------|------|
| t (minutes) | 0 | 4 | 9 | 15 | 20 |
| $W(t)$ (degrees Fahrenheit) | 55.0 | 57.1 | 61.8 | 67.9 | 71.0 |

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W , where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F . The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

- (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate $\int_0^{20} W'(t) dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t) dt$ in the context of this problem.
- (c) For $0 \leq t \leq 20$, the average temperature of the water in the tub is $\frac{1}{20} \int_0^{20} W(t) dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_0^{20} W(t) dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For $20 \leq t \leq 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t} \cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

Table FRQ

2013 AB/BC

| | | | | | | | |
|--------------------|---|-----|-----|------|------|------|------|
| t (minutes) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $C(t)$ (ounces) | 0 | 5.3 | 8.8 | 11.2 | 12.8 | 13.8 | 14.5 |

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.
- Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.
 - Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.
 - Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6} \int_0^6 C(t) dt$. Using correct units, explain the meaning of $\frac{1}{6} \int_0^6 C(t) dt$ in the context of the problem.
 - The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

2014 AB #4

| | | | | | |
|-----------------------------|---|-----|----|------|------|
| t (minutes) | 0 | 2 | 5 | 8 | 12 |
| $v_A(t)$ (meters/minute) | 0 | 100 | 40 | -120 | -150 |

4. Train A runs back and forth on an east-west section of railroad track. Train A 's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.
- Find the average acceleration of train A over the interval $2 \leq t \leq 8$.
 - Do the data in the table support the conclusion that train A 's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.
 - At time $t = 2$, train A 's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A , in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.
 - A second train, train B , travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

Table FRQ

2015 #3 No Calculator Permitted

| | | | | | |
|-------------------------------|---|-----|-----|------|-----|
| t (minutes) | 0 | 12 | 20 | 24 | 40 |
| $v(t)$ (meters per minute) | 0 | 200 | 240 | -220 | 150 |

3. Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v . Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.
- (a) Use the data in the table to estimate the value of $v'(16)$.
- (b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)| dt$ in the context of the problem.
- Approximate the value of $\int_0^{40} |v(t)| dt$ using a right Riemann sum with the four subintervals indicated in the table.
- (c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.
- (d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \leq t \leq 10$.

Calculator Permitted

2016 SCORING GUIDELINES

Question 1

| | | | | | |
|---------------------------|------|------|-----|-----|-----|
| t (hours) | 0 | 1 | 3 | 6 | 8 |
| $R(t)$ (liters / hour) | 1340 | 1190 | 950 | 740 | 700 |

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

- (a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.
- (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
- (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
- (d) For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

Calculator Permitted

No Calculator Permitted

2016 SCORING GUIDELINES

Question 6

| x | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| 1 | -6 | 3 | 2 | 8 |
| 2 | 2 | -2 | -3 | 0 |
| 3 | 8 | 7 | 6 | 2 |
| 6 | 4 | 5 | 3 | -1 |

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x .

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x) dx$.

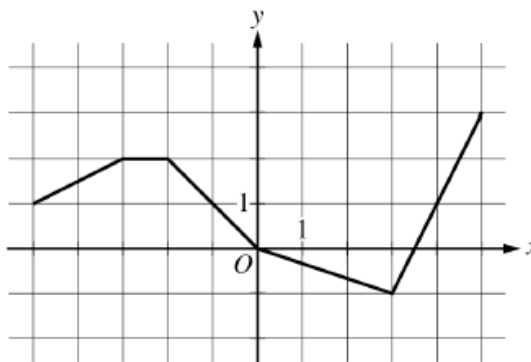
AP Calculus AB 2017 Calculator Required

| h (feet) | 0 | 2 | 5 | 10 |
|-------------------------|------|------|-----|-----|
| $A(h)$ (square feet) | 50.3 | 14.4 | 6.5 | 2.9 |

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A , where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

- (a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.
- (b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.
- (c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \frac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.
- (d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

| x | $g(x)$ | $g'(x)$ |
|-----|--------|---------|
| -5 | 10 | -3 |
| -4 | 5 | -1 |
| -3 | 2 | 4 |
| -2 | 3 | 1 |
| -1 | 1 | -2 |
| 0 | 0 | -3 |

Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x .

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

- Find the slope of the line tangent to the graph of f at $x = \pi$.
- Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.
- Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.
- Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

Table FRQ

2018 No Calculator Allowed

| | | | | | |
|--------------------|-----|---|---|----|----|
| t (years) | 2 | 3 | 5 | 7 | 10 |
| $H(t)$ (meters) | 1.5 | 2 | 6 | 11 | 15 |

4. The height of a tree at time t is given by a twice-differentiable function H , where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.
- Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.
 - Explain why there must be at least one time t , for $2 < t < 10$, such that $H'(t) = 2$.
 - Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \leq t \leq 10$.
 - The height of the tree, in meters, can also be modeled by the function G , given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?