Use the Integral Test to determine whether the series is convergent or divergent.

3. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}} \leftrightarrow \int_{1}^{\infty} \frac{1}{\sqrt[5]{x}}\,dx$

4. $\sum_{n=1}^{\infty} \frac{1}{n^5} \leftrightarrow \int_{1}^{\infty} \frac{1}{x^5}\,dx$

5. $\sum_{n=1}^{\infty} \frac{1}{(2n+1)^3} \leftrightarrow \int_{1}^{\infty} \frac{1}{(2x+1)^3}\,dx$

6. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}} \leftrightarrow \int_{1}^{\infty} \frac{1}{\sqrt{x+4}}\,dx$

7. $\sum_{n=1}^{\infty} \frac{n}{n^2+1} \leftrightarrow \int_{1}^{\infty} \frac{x}{x^2+1}\,dx$

8. $\sum_{n=1}^{\infty} n^2 e^{-n^2} \leftrightarrow \int_{1}^{\infty} x^2 e^{-x^2}\,dx$

Determine whether the series is convergent or divergent

9. $\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}}$ is a *p*-series

10. $\sum_{n=3}^{\infty} n^{-0.9999}$ is a *p*-series

11. $1+\frac{1}{8}+\frac{1}{27}+\frac{1}{64}+\frac{1}{125}+\cdots=\sum_{n=1}^{\infty}\frac{1}{n^3}$

12. $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\frac{1}{5\sqrt{5}}+\cdots=\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}}=\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$

13. $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\cdots=\sum_{n=0}^{\infty}\frac{1}{2n+1} \leftrightarrow \int_{0}^{\infty}\frac{1}{2x+1}\,dx$

14. $\frac{1}{5}+\frac{1}{8}+\frac{1}{11}+\frac{1}{14}+\frac{1}{17}+\cdots=\sum_{n=1}^{\infty}\frac{1}{3n+2} \leftrightarrow \int_{n=1}^{\infty}\frac{1}{3x+2}\,dx$

15. $\sum_{n=1}^{\infty}\frac{\sqrt{n}+4}{n^2}=\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2}+\frac{4}{n^2}=\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2}+\sum_{n=1}^{\infty}\frac{4}{n^2}=\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}+\sum_{n=1}^{\infty}\frac{4}{n^2}$

16. $\sum_{n=1}^{\infty}\frac{n^2}{n^3+1} \leftrightarrow \int_{1}^{\infty}\frac{x^2}{x^3+1}$

17. $\sum_{n=1}^{\infty}\frac{1}{n^2+4} \leftrightarrow \int_{1}^{\infty}\frac{1}{x^2+4}$

18. $\sum_{n=1}^{\infty}\frac{3n^2-4}{n^2-2n} \rightarrow \lim_{n\to\infty}\frac{3n^2-4}{n^2-2n} \neq 0 \rightarrow \text{diverges}$

19. $\sum_{n=1}^{\infty}\frac{\ln(n)}{n^3} \leftrightarrow \int_{1}^{\infty}\frac{\ln(x)}{x^3}\,dx$

20. $\sum_{n=1}^{\infty}\frac{1}{n^2+6n+13}=\sum_{n=1}^{\infty}\frac{1}{\left(n^2+6n+9\right)+13-9}=\sum_{n=1}^{\infty}\frac{1}{(n+3)^2+2^2}$

$\sum_{n=1}^{\infty}\frac{1}{(n+3)^2+2^2}\leftrightarrow\int_1^{\infty}\frac{1}{(n+3)^2+2^2}\,dx$ and use arctan

21. $\sum_{n=2}^{\infty}\frac{1}{n\cdot\ln(n)}\leftrightarrow\int_2^{\infty}\frac{1}{x\ln(x)}\,dx$

22. $\sum_{n=2}^{\infty}\frac{1}{n(\ln(n))^2}\leftrightarrow\int_1^{\infty}\frac{1}{x(\ln(x))^2}$

23. $\sum_{n=1}^{\infty}\frac{e^{\frac{1}{n}}}{n^2}\leftrightarrow\int_1^{\infty}\frac{e^{\frac{1}{x}}}{x^2}\,dx$

24. $\sum_{n=3}^{\infty}\frac{n^2}{e^n}\leftrightarrow\int_1^{\infty}x^2e^{-x}\,dx$

25. $\sum_{n=1}^{\infty}\frac{1}{n^2+n^3}$: $\frac{1}{n^2+n^3}<\frac{1}{n^2}$ use direct comparison test.

$\sum_{n=1}^{\infty}\frac{1}{n^2+n^3}=\sum_{n=1}^{\infty}\frac{1}{n^2(1+n)}\rightarrow\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n}+\frac{1}{n^2}\right)=\underbrace{\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n}\right)}_{(\text{telescoping series})}+\underbrace{\sum_{n=1}^{\infty}\frac{1}{n^2}}_{(p-\text{series})}$

26. $\sum_{n=1}^{\infty}\frac{n}{n^4+1}\leftrightarrow\int_1^{\infty}\frac{x}{\left(x^2\right)^2+1}\,dx$