Calculus BC Name:________________________________

Fall 2021 Practice Final Exam Date:_____________________ Period:______

Multiple Choice Scoring Procedures Each exercise is worth 5 points.	
If an response is **CIRCLED**	If no response is circled
▪ 5 points awarded if circled response is correct. ▪ 0 points awarded if circled response is incorrect.	▪ 1 point awarded for each incorrect response eliminated. ▪ 0 points awarded if the correct response is eliminated.

1. A manufacturer has determined that the total cost C of operating a factory is

$$C(x) = 1.5x^2 + 45x + 15000$$

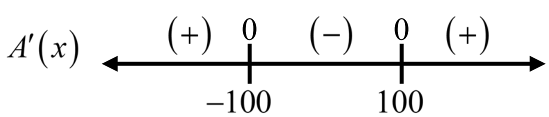
where x is the number of units produced. Which of the following statements is true regarding the **average cost**?

Average cost is given by

$$\begin{aligned} A(x) &= \frac{C(x)}{x} \\ &= \frac{1.5x^2 + 45x + 15000}{x} \\ &= 1.5x + 45 + 15000x^{-1} \end{aligned}$$

$$A'(x) = 1.5 - 15000x^{-2}$$
$$\downarrow$$
$$A'(x) = 0 \text{ when } x = \pm 100$$

$$\begin{aligned} A(100) &= \frac{C(100)}{100} \\ &= 345 \end{aligned}$$



(a) The minimum average cost is 195.
(b) The maximum average cost is 195.
(c) The minimum average cost is 345.
(d) The maximum average cost is 345.
(e) The minimum average cost is 300.

2. $f(x)$ is a polynomial and

$$f'(2) = 0 \qquad\qquad f'(5) = 0$$

$$f''(3) = 0 \qquad f''(x) < 0 \text{ on } (-\infty, 3) \qquad f''(x) > 0 \text{ on } (3, \infty)$$

Which of the following statements are true?

I. $(2, f(2))$ is an inflection point of $f(x)$.

II. $(3, f(3))$ is an inflection point of $f(x)$. $f''(x)$ changes sign at $x = 3$

III. $f(x)$ has a relative maximum at $x = 2$. $f'(2) = 0$ and $f''(2) < 0 \rightarrow$ max

IV. $f(x)$ has a relative minimum at $x = 5$. $f'(5) = 0$ and $f''(5) < 0 \rightarrow$ min

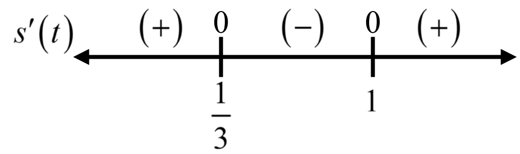
(a) I and III only
(b) I and IV only
(c) II and III only
(d) I, II, and IV only
(e) II, III, and IV only

3. The position function

$$s(t) = t^3 - 2t^2 + t$$

describes the motion of a particle along a line for $t \geq 0$. Choose the correct statement below:

$$s(t) = t^3 - 2t^2 + t$$
$$\downarrow$$
$$s'(t) = 3t^2 - 4t + 1$$



(a) The particle is always moving in a positive direction.
(b) The particle is always moving in a negative direction.
(c) The particle changes direction from a negative direction to a positive direction at $t = \frac{1}{3}$.
(d) The particle changes direction from a negative direction to a positive direction at $t = 1$.
(e) The particle changes direction from a negative direction to a positive direction at $t = 3$.

4. If $h(t) = \sin(3t) + \cos(3t)$, find $h^{(3)}(t)$.

$$h(t) = \sin(3t) + \cos(3t)$$
$$h'(t) = 3\cos(3t) - 3\sin(3t)$$
$$h''(t) = -9\sin(3t) - 9\cos(3t)$$
$$h'''(t) = -27\cos(3t) + 27\sin(3t)$$

(a) $\sin(3t) - \cos(3t)$
(b) $\sin(3t) + \cos(3t)$
(c) $27\sin(3t) - 27\cos(3t)$
(d) $27\sin(3t) + 27\cos(3t)$
(e) $-27\sin(3t) + 27\cos(3t)$

5. Given $f(x)=\dfrac{2(3-x^2)}{\sqrt{3x^2+1}}$, find $f'(1)$

$$f(x)=\frac{2(3-x^2)}{\sqrt{3x^2+1}}$$

$$\downarrow$$

$$f'(x)=\frac{(-4x)(3x^2+1)^{\frac{1}{2}}-2(3-x^2)\cdot\frac{1}{2}(3x^2+1)^{-\frac{1}{2}}6x}{3x^2+1}$$

$$f'(1)=\frac{(-4[1])(3[1]^2+1)^{\frac{1}{2}}-2(3-[1]^2)\cdot\frac{1}{2}(3[1]^2+1)^{-\frac{1}{2}}6[1]}{3[1]^2+1}$$

$$=-\frac{7}{2}$$

(a) $-\frac{7}{2}$ (b) $-\frac{9}{4}$ (c) $-\frac{1}{2}$ (d) $-\frac{13}{6}$ (e) $-\frac{3}{4}$

6. $\lim\limits_{h\to 0}\dfrac{\sin(\pi+h)-\sin(\pi)}{h}=$

$$\lim_{h\to 0}\frac{\sin(\pi+h)-\sin(\pi)}{h}\to\frac{d}{dx}[\sin(x)]\Big|_{x=\pi}$$

$$=\cos(\pi)$$

$$=-1$$

(a) 0 (b) $\cos(x)$ (c) -1 (d) π (e) 1

7. The table below gives values of the differentiable functions *f* and *g* at $x=-1$. If $h(x)=\dfrac{f(x)-g(x)}{2f(x)}$, then $h'(-1)=$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	-2	4	e	-3

$$h(x)=\frac{f(x)-g(x)}{2f(x)}$$

$$\downarrow$$

$$h'(x)=\frac{\left[f'(x)-g'(x)\right]\cdot 2f(x)-\left[f(x)-g(x)\right]\cdot 2f'(x)}{\left[2f(x)\right]^2}$$

$$h'(-1)=\frac{\left[f'(-1)-g'(-1)\right]\cdot 2f(-1)-\left[f(-1)-g(-1)\right]\cdot 2f'(-1)}{\left[2f(-1)\right]^2}$$

$$=\frac{\left[e-(-3)\right]\cdot 2(-2)-\left[(-2)-4\right]\cdot 2e}{\left[2(-2)\right]^2}$$

$$=\frac{[e+3]\cdot(-4)-[-6]\cdot 2e}{16}$$

$$=\frac{-4e-12+12e}{16}$$

$$=\frac{-12+8e}{16}$$

$$=\frac{-3+2e}{4}$$

(a) $\dfrac{-e-3}{4}$ (b) $\dfrac{e+4}{2e}$ (c) $\dfrac{e-6}{8}$

(d) $\dfrac{2e-3}{4}$ (e) $\dfrac{-4e-3}{4}$

8. If $f(x)=2^{x^2-x-1}$, find an equation of the line tangent to the graph of *f* at $x=-1$ is

$$f(x)=2^{x^2-x-1}$$
$$\downarrow$$

$$f(-1)=2^{(-1)^2-(-1)-1} \\ =2$$

$$f'(x)=\ln(2)\cdot 2^{x^2-x-1}\cdot(2x-1)$$
$$f'(-1)=\ln(2)\cdot 2^{[-1]^2-[-1]-1}\cdot(2[-1]-1)$$
$$=-6\ln(2)$$

(a) $y=2x+4$
(b) $y=-6x-4$
(c) $y=2-2\ln(2)(x+1)$
(d) $y=2-6\ln(2)(x+1)$
(e) $y=2-3\ln(2)(x+1)$

9. If $\sin(\pi x)\cos(\pi y)=y$, the value of $\frac{dy}{dx}$ at $(1,0)$ is

$$\sin(\pi x)\cos(\pi y)=y$$
$$\downarrow$$
$$\cos(\pi[1])\cdot\pi\cos(\pi[0])+\sin(\pi[1])\left[-\sin(\pi[0])\cdot\pi y'\right]=y'$$
$$\cos(\pi)\cdot\pi\cos(0)=y'$$
$$-\pi=y'$$

(a) $1+\pi$ (b) $1-\pi$ (c) (d) -1 (e) $-\pi$ (f) π

10. Let f be the function defined below, where c and d are constants. If f is differentiable at $x=-1$, what is the value of $c-d$?

$$f(x)=\begin{cases} x^2+(2c+1)x-d & x\ge -1 \\ e^{2x+2}+cx+3d & x<-1 \end{cases}$$

Continuous	Differentiable
$(-1)^2+(2c+1)(-1)-d=e^{2(-1)+2}+c(-1)+3d$ $1-2c-1-d=1-c+3d$ $-1=c+4d$	$2(-1)+2c+1=e^{2(-1)+2}\cdot 2+c$ $2c-1=2+c$ $c=3$
$-1=c+4d$ $-1=3+4d$ $-4=4d$ $d=-1$	

(a) -2 (b) 0 (c) 2 (d) 3 (e) 4

11. $\dfrac{d}{dx}\left[\displaystyle\int_2^{\sqrt{x}} e^t\,dt\right]=$

$$\frac{d}{dx}\left[\int_2^{\sqrt{x}} e^t\,dt\right]=\frac{d}{dx}\left[\int_2^{x^{\frac{1}{2}}} e^t\,dt\right]$$

$$=e^{x^{\frac{1}{2}}}\cdot\frac{1}{2}x^{-\frac{1}{2}}$$

$$=e^{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}$$

(a) e^t (b) $e^{\sqrt{x}}$ (c) $\frac{1}{2}e^{\sqrt{x}}$ (d) $\frac{1}{2\sqrt{x}}e^{\sqrt{x}}$ (e) e^{2t}

12. The function g is differentiable for all real numbers. The table below gives values of the function and its first derivatives at selected values of x. If g^{-1} is the inverse function of g, what is an equation of the line tangent to the graph of $y = g^{-1}(x)$ at $x = 4$?

x	$g(x)$	$g'(x)$
-2	4	2
4	-3	5

$$\left(g^{-1}\right)'(4) = \frac{1}{g'\left(\begin{matrix}\text{whatever makes}\\ g(x)=4\end{matrix}\right)}$$

$$g^{-1}(4) = -2 \qquad = \frac{1}{g'(-2)}$$

$$= \frac{1}{2}$$

(a) $y + 3 = \frac{1}{5}(x-4)$

(b) $y + 2 = \frac{1}{5}(x-4)$

(c) $y + 2 = 2(x-4)$

(d) $y + 3 = \frac{1}{2}(x-4)$

(e) $y + 2 = \frac{1}{2}(x-4)$

13. The function f is continuous and non-linear for $-3 \le x \le 7$, and $f(-3) = -5$ and $f(7) = 5$. If there is no value of c, where $-3 < c < 7$, for which $f'(c) = 1$, which of the following statements must be true?

(a) For all k, where $-3 < k < 7$, $f'(k) < 1$

(b) For all k, where $-3 < k < 7$, $f'(k) > 1$

(c) For some k, where $-3 < k < 7$, $f'(k) = 0$

(d) For $-3 < k < 7$, $f'(k)$ exists

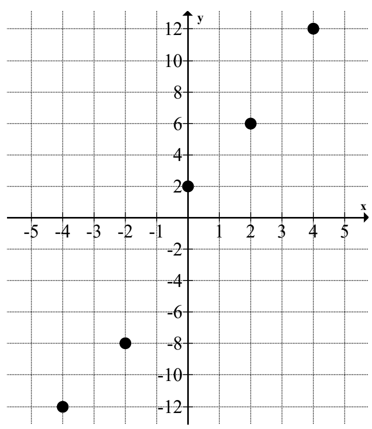
(e) For some k, where $-3 < k < 7$, $f'(k)$ does not exist

14. If the graph of $f(x)=2x^2+\frac{k}{x}$ has a point of inflection at $x=-1$, then the value of *k* is

$$f(x)=2x^2+\frac{k}{x}$$
$$=2x^2+kx^{-1}$$
$$\downarrow$$
$$f'(x)=4x-kx^{-2}$$
$$\downarrow$$
$$f''(x)=4+2kx^{-3}$$
$$0=4+2k(-1)^{-3}$$
$$\downarrow$$
$$k=2$$

(a) 1 (b) -1 (c) 2 (d) -2 (e) 0

15. A continuous and differentiable function *f* is defined on the closed interval $-4\le x\le 4$. Points on the graph of the function, are shown in the figure below. There is a value *a*, $-4\le a<4$, for which the Mean Value Theorem, applied to the interval $[a,4]$ guarantees a value of *c* such that $a\le c<4$ at which $f'(c)=3$. What are the possible values of *a*?



Need $\frac{f(4)-f(a)}{4-a}=3$, which only occurs when

(a) $a=-4$

$$\frac{12-(-12)}{4-(-4)}=\frac{24}{8}=3$$

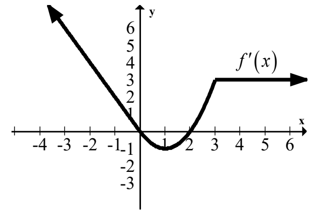
(b) $a=2$

$$\frac{12-6}{4-2}=\frac{6}{2}=3$$

I. -4
II. 0
III. 2

(a) I only
(b) I and III only
(c) III only
(d) II and III only
(e) I, II, and III

16. The graph of f', the derivative of *f*, is shown at right. Which of the following statements is not true?



(a) f is increasing on $2 \le x \le 3$
(b) f has a local minimum at $x = 1$
(c) f has a local maximum at $x = 0$
(d) f is differentiable at $x = 3$
(e) f is concave down on $-2 \le x \le 1$

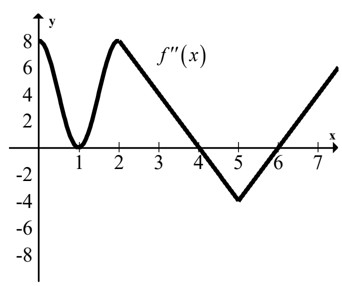
17. Let *g* be a strictly decreasing function such that $g(x) < 0$ for all real numbers *x*. If $f(x) = (x-1)g(x)$, which of the following is true?

$$f(x) = (x-1)g(x)$$
$$\downarrow$$
$$f'(x) = \underline{\underline{g(x)}} + (x-1) \cdot g'(x)$$

We cannot determine the sign of $f'(x)$ without knowledge of the value/graph of $g(x)$.

(a) f has a relative minimum at $x = 1$
(b) f has a relative maximum at $x = 1$
(c) f will be a strictly decreasing function
(d) f will be a strictly increasing function
(e) It cannot be determined if *f* has a relative extrema

18. The graph of f'', the second derivative of *f*, is shown at right. On which of the following intervals is $f'(x)$ decreasing?



$f'(x)$ is decreasing when $f''(x) < 0$.

(a) $[0,1]$
(b) $[0,1]$ and $[2,4]$
(c) $[0,1]$ and $[2,5]$
(d) $[4,5]$
(e) $[4,6]$

19. Let $R(t)$ represent the rate at which water is leaking out of a tank, where t is measured in hours. Which of the following expressions represents the total amount of water that leaks out of the tank in the first three hours?

(a) $R(3)-R(0)$ (b) $\frac{R(3)-R(0)}{3-0}$ (c) $\int_0^3 R(t)\,dt$ (d) $\int_0^3 R'(t)\,dt$ (e) $\frac{1}{3}\int_0^3 R(t)\,dt$

20. If the definite integral $\int_a^b f(x)\,dx$ represents the area of the region bounded by the graph of $y=f(x)$, the x-axis, and the lines $x=a$ and $x=b$, which of the following must be true?

(a) $a>b$ and $f(x)>0$

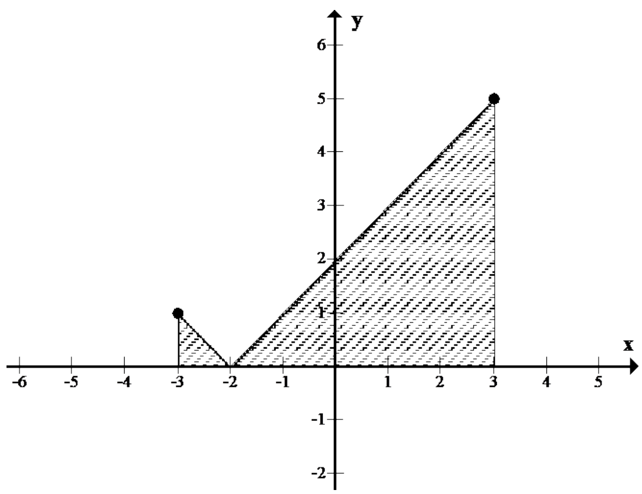
(b) $a>b$ and $f(x)<0$

(c) $a<b$ and $f(x)>0$

(d) $a<b$ and $f(x)<0$

(e) none of the above Since neither (b) nor (c) can be determined with certainty.

21. $\int_{-3}^{3} |x+2|\,dx =$

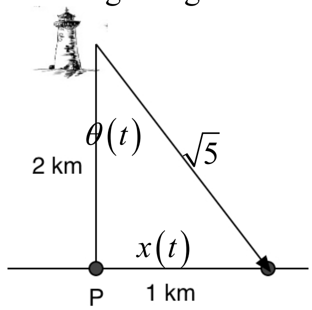


$$\int_{-3}^{3} |x+2|\,dx = \frac{1}{2}(5)(5)+\frac{1}{2}(1)(1)$$

$$=13$$

(a) 0 (b) 8 (c) 13 (d) 17 (e) 21

22. A lighthouse is 2km from a point P along a straight shoreline, and its light makes 1 revolution per minute. How fast, in km/min, is the beam of light moving along the shoreline when it is 1km from point P?



$$\tan(\theta(t)) = \frac{x(t)}{2}$$
$$\downarrow$$
$$\sec^2(\theta(t))\cdot\theta'(t) = \frac{1}{2}x'(t)$$
$$\left(\frac{\sqrt{5}}{2}\right)^2 \cdot \frac{2\pi}{1} = \frac{1}{2}x'(t)$$
$$5\pi = x'(t)$$

(a) 2.5 (b) 10π (c) 20π (d) 5π (e) 10

23. If $f(x) = \sqrt[3]{\cos^2(3x)}$, then $f'(x) =$

$$f(x) = \sqrt[3]{\cos^2(3x)}$$
$$= (\cos(3x))^{\frac{2}{3}}$$
$$\downarrow$$
$$f'(x) = \frac{2}{3}(\cos(3x))^{-\frac{1}{3}} \cdot -\sin(3x)\cdot 3$$
$$= -\frac{2\sin(3x)}{\sqrt[3]{\cos(3x)}}$$

(a) $-\frac{2}{3\sqrt[3]{\sin(3x)}}$ (b) $-\frac{2}{\sqrt[3]{\sin(3x)}}$ (c) $-\frac{2\sin(3x)}{\sqrt[3]{\cos(3x)}}$ (d) $\frac{2}{\sqrt[3]{\cos(3x)}}$ (e) $-\frac{2\sin(3x)}{3\sqrt[3]{\cos(3x)}}$

24. If $k(x) = 5^{g(x)}\cos(-x)$, Then $k'(x) =$

$$k(x) = 5^{g(x)}\cos(-x)$$
$$\downarrow$$
$$k'(x) = \ln(5)\cdot 5^{g(x)}\cdot g'(x)\cdot\cos(-x) + 5^{g(x)}\cdot\left[-\sin(-x)\cdot(-1)\right]$$
$$= \ln(5)5^{g(x)}g'(x)\cos(-x) + 5^{g(x)}\sin(-x)$$

(a) $5^{g'(x)}\sin(-x)$

(b) $5^{g(x)}\cos(-x) + 5^{g(x)}\sin(-x)$

(c) $\ln(5)5^{g(x)}\cos(-x) + 5^{g(x)}\sin(-x)$

(d) $\ln(5)5^{g(x)}\cdot g'(x)\cos(-x) - 5^{g(x)}\sin(-x)$

(e) $\ln(5)5^{g(x)}g'(x)\cos(-x) + 5^{g(x)}\sin(-x)$

25. If $y=\left(1+x^2\right)^x$ then $\dfrac{dy}{dx}=$

$$
\begin{aligned}
y&=\left(1+x^2\right)^x\\
\ln(y)&=\ln\left[\left(1+x^2\right)^x\right]\\
\ln(y)&=x\ln\left(1+x^2\right)\\
&\downarrow\\
\frac{1}{y}\cdot y'&=1\cdot\ln\left(1+x^2\right)+x\cdot\frac{1}{1+x^2}\cdot 2x\\
\frac{1}{y}\cdot y'&=\ln\left(1+x^2\right)+\frac{2x^2}{1+x^2}\\
y'&=y\cdot\left[\ln\left(1+x^2\right)+\frac{2x^2}{1+x^2}\right]\\
&=\left(1+x^2\right)^x\left[\ln\left(1+x^2\right)+\frac{2x^2}{1+x^2}\right]
\end{aligned}
$$

(a) $\left(1+x^2\right)^x\left[\dfrac{2x^2}{1+x^2}+\ln\left(1+x^2\right)\right]$

(b) $2x\left(1+x^2\right)^{x-1}$

(c) $\dfrac{2x}{1+x^2}+\ln\left(1+x^2\right)$

(d) $\left(1+x^2\right)^x\left(\dfrac{2x}{1+x^2}\right)$

(e) $\left(1+x^2\right)^x\left[\dfrac{1}{1+x^2}+\ln\left(1+x^2\right)\right]$

26. If $y=\arcsin(x)-\sqrt{1-x^2}$, then $y'=$

$$
\begin{aligned}
y&=\arcsin(x)-\sqrt{1-x^2}\\
&\downarrow\\
y&=\frac{1}{\sqrt{1-x^2}}-\frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}\cdot -2x\\
&=\frac{1}{\sqrt{1-x^2}}+\frac{x}{\sqrt{1-x^2}}\\
&=\frac{1+x}{\sqrt{1-x^2}}
\end{aligned}
$$

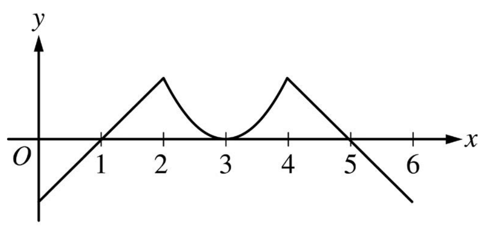
(a) $\dfrac{1}{2\sqrt{1-x^2}}$ (b) $\dfrac{1}{\sqrt{1-x^2}}$ (c) $\dfrac{1+x}{\sqrt{1-x^2}}$ (d) $\dfrac{x^2}{\sqrt{1-x^2}}$ (e) $\dfrac{1}{\sqrt{1+x^2}}$

27. $\int x\sqrt{x^2+1}dx =$

Let $u = x^2+1$ and therefore $du = 2xdx$

$$\int x\sqrt{x^2+1}dx \to \int \frac{1}{2}\sqrt{u}du$$
$$= \frac{1}{2}\int \sqrt{u}du$$
$$= \frac{1}{3}u^{\frac{3}{2}} + C$$
$$\downarrow$$
$$= \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}} + C$$

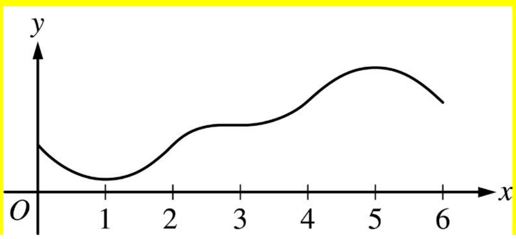
(a) $\dfrac{x}{\sqrt{x^2+1}} + C$ (b) $\dfrac{3}{4}\left(x^2+1\right)^{\frac{3}{2}} + C$ (c) $\dfrac{1}{3}\left(x^2+1\right)^{\frac{3}{2}} + C$ (d) $\dfrac{2}{3}\left(x^2+1\right)^{\frac{3}{2}} + C$ (e) $\dfrac{1}{3}x^2\left(x^2+1\right)^{\frac{3}{2}} + C$



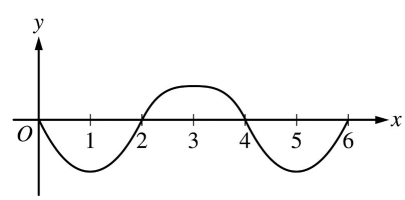
Graph of f'

28. The graph of f', the derivative of the function *f*, is shown above. Which of the following could be the graph of *f*?

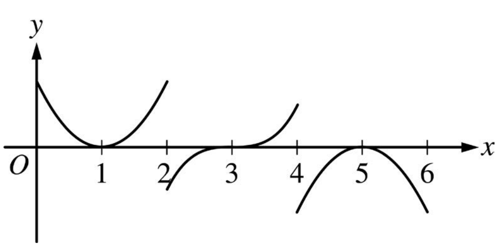
(a)



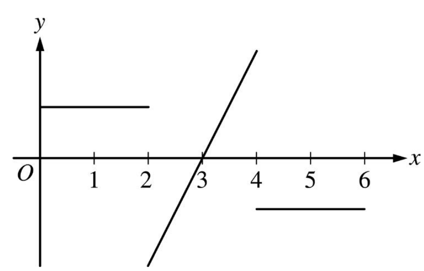
(b)



(c)

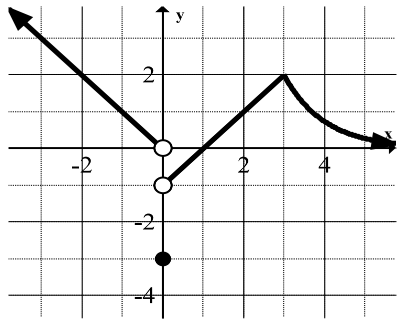


(d)



(e) None of these.

29. For the function $f(x)$ shown below, which of the following statements is false?



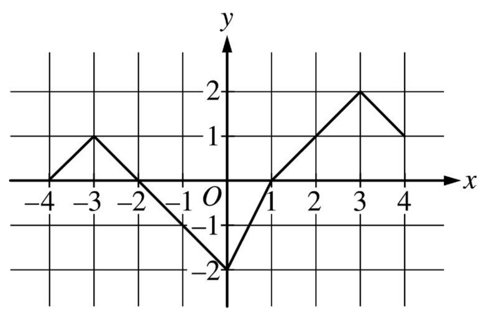
(a) $\lim_{x\to\infty} f(x) = 0$

(b) $\lim_{x\to-\infty} f(x)$ DNE

(c) $\lim_{x\to 0} f(x)$ DNE

(d) $\lim_{x\to 3} f(x)$ DNE

(e) $\lim_{x\to c} f(x) = f(c)$ for $c > 0$



Graph of f

30. The function f is continuous for $-4 \le x \le 4$. The graph of f shown above consists of five line segments. What is the average value of f on the interval $-4 \le x \le 4$?

The average value of f on the interval $-4 \le x \le 4$ is given by

$$\frac{1}{4-(-4)}\int_{-4}^{4} f(x)\,dx = \frac{1}{8}\int_{-4}^{4} f(x)\,dx$$

$$= \frac{1}{8}\left[\int_{-4}^{-2} f(x)\,dx + \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx + \int_{3}^{4} f(x)\,dx\right]$$

$$= \frac{1}{8}\left[1 - 3 + 2 + \frac{3}{2}\right]$$

$$= \frac{1}{8}\left[\frac{3}{2}\right]$$

$$= \frac{3}{16}$$

(a) $\frac{1}{8}$ (b) $\frac{3}{16}$ (c) $\frac{3}{2}$ (d) $\frac{15}{16}$ (e) None of these.

31. Let a and b be real numbers and consider the integral $\int (ax^2+b)\cos(x)\,dx$. Using integration by parts leads to which of the following expressions:

u	v'
ax^2+b	$\cos(x)$
$2ax$	$\sin(x)$

$$\int (ax^2+b)\cos(x)\,dx = (ax^2+b)\sin(x) - \int 2ax\sin(x)\,dx$$

(a) $(ax^2+b)\cos(x) - 2a\int x\cos(x)\,dx$

(b) $(ax^2+b)\cos(x) - 2a\int x\sin(x)\,dx$

(c) $(ax^2+b)\sin(x) - 2a\int x\sin(x)\,dx$

(d) $2ax\sin(x) - 2\int (ax^2+b)\sin(x)\,dx$

(e) $2ax\cos(x) - 2\int (ax^2+b)\sin(x)\,dx$

32. Let $a>3$ be a fixed real number. Evaluate the improper integral $\int_a^\infty \frac{1}{(x-3)^2}\,dx$

$$\begin{aligned}\int_a^\infty \frac{1}{(x-3)^2}\,dx &= \lim_{k\to\infty}\int_a^k \frac{1}{(x-3)^2}\,dx \\ &= \lim_{k\to\infty}\left[-(x-3)^{-1}\right]_a^k \\ &= \lim_{k\to\infty}\left[-(k-3)^{-1}\right]-\left[-(a-3)^{-1}\right] \\ &= \lim_{k\to\infty}\left[-\frac{1}{k-3}\right]-\left[-\frac{1}{a-3}\right] \\ &= \frac{1}{a-3}\end{aligned}$$

(a) $\frac{1}{a-3}$ (b) 0 (c) $\frac{1}{(a+3)^2}$ (d) $\frac{1}{(a-3)^3}$ (e) The integral does not converge.

33. If Newton's Method is used to approximate the root of the equation $x^3-2x^2-1=0$ starting with $x_0=2$, then after one iteration of the method, $x_1=$

$$y-y_1=m(x-x_1)$$
$$\downarrow$$
$$y-f(x_1)=f'(x_1)(x-x_1)$$
$$0-f(x_1)=f'(x_1)(x-x_1)$$
$$\downarrow$$
$$x=x_1-\frac{f(x_1)}{f'(x_1)}$$

$$\begin{aligned}x_1 &= x_0-\frac{f(x_0)}{f'(x_0)} \\ &= 2-\frac{2^3-2(2)^2-1}{3(2)^2-4(2)} \\ &= 2.25\end{aligned}$$

(a) 6 (b) 2.25 (c) 0 (d) 2 (e) None of the above

34. $\lim_{x \to 0} \frac{e^x - 1}{\tan(x)} =$

$$\lim_{x \to 0} \frac{e^x - 1}{\tan(x)} = \lim_{x \to 0} \frac{e^x}{\sec^2(x)}$$
$$= \frac{1}{1}$$
$$= 1$$

(a) -1 (b) 0 (c) 1 (d) e (e) The limit does not exist

35. $\displaystyle\int \frac{1}{(x-1)(x+3)}\,dx =$

$$\int \frac{1}{(x-1)(x+3)}\,dx = \int \frac{A}{x-1} + \frac{B}{x+3}\,dx$$

$$\frac{1}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$\frac{1}{(x-1)(x+3)} = \frac{A}{x-1}\cdot\frac{(x+3)}{(x+3)} + \frac{B}{x+3}\cdot\frac{(x-1)}{(x-1)}$$

$$\frac{1}{(x-1)(x+3)} = \frac{A(x+3)}{(x-1)(x+3)} + \frac{B(x-1)}{(x-1)(x+3)}$$

$$\downarrow$$

$$1 = Ax + 3A + Bx - B$$

$$0x + 1 = (A+B)x + (3A-B)$$

$$A + B = 0$$

$$3A - B = 1$$

$$\downarrow$$

$$A = \frac{1}{4} \quad B = -\frac{1}{4}$$

$$\downarrow$$

$$\int \frac{1}{(x-1)(x+3)}\,dx = \int \frac{1}{4(x-1)} - \frac{1}{4(x+3)}\,dx$$

$$= \frac{1}{4}\ln|x-1| - \frac{1}{4}|x+3| + C$$

$$= \frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$$

(a) $\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$

(b) $\frac{1}{4}\ln\left|\frac{x+3}{x-1}\right| + C$

(c) $\frac{1}{2}\ln\left|(x-1)(x+3)\right| + C$

(d) $\frac{1}{2}\ln\left|\frac{2x+2}{(x-1)(x+3)}\right| + C$

(e) $\ln\left|(x-1)(x+3)\right| + C$