AP Calculus BC
Stewart Chapter 3 Test (Derivatives)
Fall 2019

Name:____________________________________

Date:______________________

Period:________

Multiple Choice Section: 7 Questions. *No Calculator Permitted.* Suggested Time: 40 minutes.

Once you submit your Multiple Choice section, you will not be allowed to revisit it.

Free Response Section: 7 Questions. No Calculator Permitted. Suggested Time: 50 ≈ minutes.

Multiple Choice Scoring Procedures: Each exercise is worth 5 points.	
If an response is **CIRCLED**	If no response is circled
▪ 5 points awarded if circled response is correct. ▪ 0 points awarded if circled response is incorrect.	▪ 1 point awarded for each incorrect response eliminated. ▪ 0 points awarded if the correct response is eliminated.

1. Find $f'(x)$ for $f(x)=\sqrt{\sin(4x)}$

$$f(x)=\sqrt{\sin(4x)}$$
$$=\left[\sin(4x)\right]^{\frac{1}{2}}$$
$$\downarrow$$
$$f'(x)=\frac{1}{2}\left[\sin(4x)\right]^{-\frac{1}{2}}\cdot\left(\cos(4x)\right)\cdot 4$$
$$=\frac{2\cos(4x)}{\sqrt{\sin(4x)}}$$

a) $\dfrac{2}{\sqrt{\sin(4x)}}$

b) $\sin(8x)$

c) $\dfrac{\cos(4x)}{2\sqrt{\sin(4x)}}$

d) $\dfrac{2\cos(4x)}{\sqrt{\sin(4x)}}$

e) None of these

2. Find $\frac{d^2y}{dx^2}$ for $y=\left(3x^2-1\right)^3$

$$y=\left(3x^2-1\right)^3$$

$$\downarrow$$

$$y'=3\left(3x^2-1\right)^2\cdot 6x$$

$$=18x\left(3x^2-1\right)^2$$

$$\downarrow$$

$$y''=18\left(3x^2-1\right)^2+(18x)\left[2\left(3x^2-1\right)^1\cdot 6x\right]$$

$$=18\left(3x^2-1\right)^2+216x^2\left(3x^2-1\right)$$

$$=18\left(3x^2-1\right)\left[3x^2-1+12x^2\right]$$

$$=18\left(3x^2-1\right)\left(15x^2-1\right)$$

a) $18\left(3x^2-1\right)\left(15x^2-1\right)$ b) $6\left(3x^2-1\right)$ c) $18x\left(3x^2-1\right)^2$

d) $216x\left(3x^2-1\right)$ e) None of these

3. Differentiate: $y = x^{e^x}$

$$y = x^{e^x}$$
$$\ln(y) = \ln\left(x^{e^x}\right)$$
$$\ln(y) = e^x \cdot \ln(x)$$
$$\downarrow$$
$$\frac{1}{y} \cdot y' = e^x \ln(x) + e^x \frac{1}{x}$$
$$y' = y\left[e^x \ln(x) + e^x \frac{1}{x}\right]$$
$$= x^{e^x}\left[e^x \ln(x) + e^x \frac{1}{x}\right]$$

a) $y' = e^x x^{e^x - 1}$

b) $y' = e^x$

c) $y' = x^{e^x}\left[\frac{e^x}{x} - \ln(x) e^x\right]$

d) $y' = xe^x + e^x$

e) None of these.

4. Find $\frac{dy}{dx}$ for $y = \arcsin\left(\frac{x}{3}\right)$

$$y = \arcsin\left(\frac{x}{3}\right)$$
$$\downarrow$$
$$y' = \frac{1}{\sqrt{1 - \left(\frac{x}{3}\right)^2}} \cdot \frac{1}{3}$$
$$= \frac{\frac{1}{3}}{\sqrt{1 - \frac{x^2}{9}}}$$
$$= \frac{\frac{1}{3}}{\frac{1}{3}\sqrt{9 - x^2}}$$
$$= \frac{1}{\sqrt{9 - x^2}}$$

a) $\frac{1}{\sqrt{9 - x^2}}$

b) $\frac{3}{\sqrt{9 - x^2}}$

c) $\frac{x}{\sqrt{9 - x^2}}$

d) $\frac{3}{\sqrt{x^2 - 9}}$

e) $\frac{3}{\sqrt{9 + x^2}}$

5. Find y' if $x = \tan(x+y)$

$$x = \tan(x+y)$$

$$\downarrow$$

$$1 = \sec^2(x+y)\cdot(1+y')$$

$$1 = \sec^2(x+y) + \sec^2(x+y)\cdot y'$$

$$\frac{1-\sec^2(x+y)}{\sec^2(x+y)} = y'$$

$$\frac{-\tan^2(x+y)}{\sec^2(x+y)} =$$

$$\frac{-\tan^2(x+y)}{\frac{1}{\cos^2(x+y)}} =$$

$$\frac{-\frac{\sin^2(x+y)}{\cos^2(x+y)}}{\frac{1}{\cos^2(x+y)}} =$$

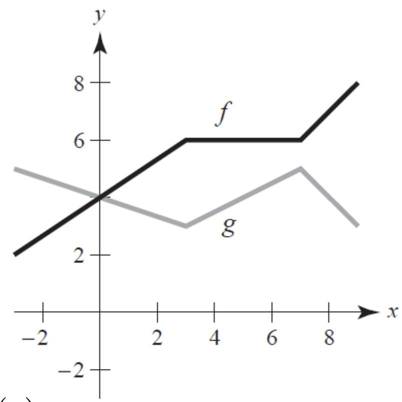
$$-\frac{\sin^2(x+y)}{\cos^2(x+y)}\cdot\frac{\cos^2(x+y)}{1} =$$

$$-\sin^2(x+y) = y'$$

a) $-\sin^2(x+y)$ b) $\sec^2(x+y)$ c) $-\tan^2(x+y)$

d) $\dfrac{1-\sec^2(x)}{\sec^2(y)}$ e) None of these.

6. Let $q(x)=\dfrac{f(x)}{g(x)}$. Use the figure at right to find $q'(5)$.



We need to find the following:

$f(5)=6$

$f'(5)=0$

$g(5)=4$

$g'(5)=\dfrac{5-3}{7-3}=\dfrac{1}{2}$

$$q'(5)=\frac{f'(5)g(5)-f(5)g'(5)}{\left[g(5)\right]^2}$$

$$=\frac{0\cdot 4-6\left(\frac{1}{2}\right)}{4^2}$$

$$=\frac{-3}{16}$$

a) 3 b) $\frac{3}{2}$ c) $-\frac{3}{16}$

d) $-\frac{3}{4}$ e) None of these

7. Assume $f'(c)=-4$. Find $f'(-c)$ if *f* is an odd function. Note that *f* is an odd function if $f(-c)=-f(c)$ for all real numbers *c*.

$$\frac{d}{dc}\left[f(-c)\right]=\frac{d}{dc}\left[-f(c)\right]$$

$$f'(-c)\cdot -1=-f'(c)$$

$$f'(-c)=f'(c)$$

a) 4 b) 0 c) -3

d) -4 e) None of these.

8. Find an equation for the tangent line at the point where $x=2$ on the graph of the function $f(x)=5^{\frac{x}{2}}$.

$$f(x)=5^{\frac{x}{2}} \rightarrow f(2)=5$$

$$\downarrow$$

$$f'(x)=\ln(5)\cdot 5^{\frac{x}{2}}\cdot\frac{1}{2}$$

$$f'(2)=\ln(5)\cdot 5^{\frac{2}{2}}\cdot\frac{1}{2}$$

$$=\frac{5}{2}\ln(5)$$

$$y-5=\frac{5}{2}\ln(5)(x-2)$$

$$y=\frac{5}{2}\ln(5)x-5\ln(5)+5$$

a) $y=\frac{5}{2}x+3$

b) $y=\frac{5}{2}x+5$

c) $y=\frac{5}{2}\left[\ln(5)\cdot x+2\ln(5)\right]$

d) $y=\frac{5}{2}\left[\ln(5)\cdot x-2\ln(5)+2\right]$

e) None of these

9. Find $f^{-1}(x)$ if $f(x)=9\sqrt[7]{8x-7}$

$$f(x)=9\sqrt[7]{8x-7}$$

$$y=9\sqrt[7]{8x-7}$$

$$\downarrow$$

$$x=9\sqrt[7]{8y-7}$$

$$\frac{x}{9}=\sqrt[7]{8y-7}$$

$$\left(\frac{x}{9}\right)^7=8y-7$$

$$\left(\frac{x}{9}\right)^7+7=8y$$

$$\frac{1}{8}\left(\left(\frac{x}{9}\right)^7+7\right)=y=f^{-1}(x)$$

a) $f^{-1}(x)=\frac{1}{9}\left(\left(\frac{x}{9}\right)^7+7\right)$

b) $f^{-1}(x)=\frac{1}{9}(8x-7)^7$

c) $f^{-1}(x)=\frac{1}{8}\left(\left(\frac{x}{9}\right)^7+7\right)$

d) $f^{-1}(x)$ does not exist

e) $f^{-1}(x)=\frac{1}{8}\left(\left(\frac{x}{9}\right)^7-7\right)$

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Free Response Section: *NO Calculator Permitted.*
You have the remainder of the period to complete this section.
Once you submit your Free Response Section, you will not be allowed to revisit it.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. To be eligible for partial credit, methods, reasoning, and conclusions should be presented clearly.

- Justifications require that you give mathematical (non-calculator) reasons. Students should use complete sentences in responses that include explanations or justifications.

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

10. A point moves along the curve $y = 2x^2 - 1$ in such a way that the y value is decreasing at the rate of 2 units per second. At what rate is x changing when $x = -\frac{3}{2}$?

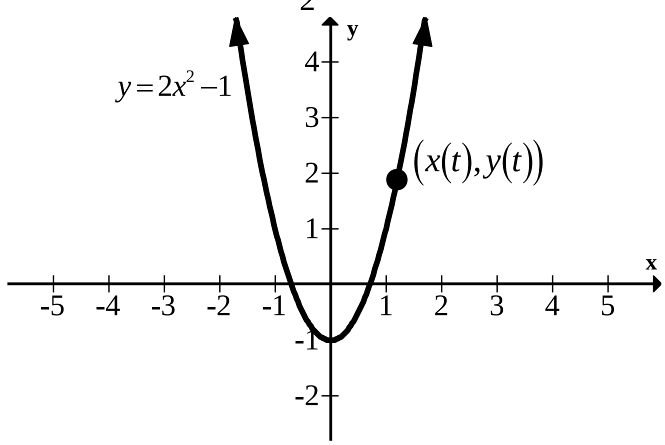
$$y(t) = 2\left[x(t)\right]^2 - 1$$

$$\downarrow$$

$$y'(t) = 4x(t)\cdot x'(t)$$

$$-2 = 4\left[-\frac{3}{2}\right]\cdot x'(t)$$

$$x'(t) = \frac{1}{3}$$



When $x = -\frac{3}{2}$, the rate at which x is changing is $\frac{1}{3}$ units per second.

11. Given that f and g are differentiable functions and

$$f(a)=-4,\ g(a)=c,\ g(c)=10,\ f(c)=15$$

$$f'(a)=8,\ g'(a)=b,\ g'(c)=5,\ f'(c)=6$$

If $h(x)=f(g(x))$, find $h'(a)$.

$$h(x)=f(g(x))$$
$$\downarrow$$
$$\begin{aligned} h'(a) &= f'(g(a))\cdot g'(a) \\ &= f'(c)\cdot b \\ &= 6b \end{aligned}$$

12. Differentiate: $y=3^{2t}(2t)^3$

$$y=3^{2t}(2t)^3$$
$$\downarrow$$
$$\begin{aligned} y' &= \left[\ln(3)\cdot 3^{2t}\cdot 2\right]\cdot(2t)^3+3^{2t}\left[3(2t)^2\cdot 2\right] \\ &= 16\cdot\ln(3)\cdot t^3\cdot 3^{2t}+24t^2 3^{2t} \\ &= 8t^2\cdot 3^{2t}\left[2\ln(3)t+3\right] \end{aligned}$$

13. Differentiate $y=(x-1)^{2x}$

$$y=(x-1)^{2x}$$
$$\ln(y)=\ln\left[(x-1)^{2x}\right]$$
$$\ln(y)=2x\cdot\ln\left[(x-1)\right]$$
$$\downarrow$$
$$\frac{1}{y}\cdot y'=2\ln\left[(x-1)\right]+2x\frac{1}{x-1}$$
$$\begin{aligned} y' &= y\left(2\ln\left[(x-1)\right]+2x\frac{1}{x-1}\right) \\ &= (x-1)^{2x}\left(2\ln\left[(x-1)\right]+2x\frac{1}{x-1}\right) \end{aligned}$$

14. Let f be a function defined by $f(x)=\ln\left(\frac{x}{x+1}\right)$

a. What is the domain of f?
b. Find $f'(x)$
c. Find an equation of for the tangent line to the graph of f at the point $(1, f(1))$
d. Find $g'(-1)$ if $g(x)=f^{-1}(x)$ accurate to three decimal places (rounded or truncated).

a) The domain of the function would be all values for which $\frac{x}{x+1}>0$.

This occurs at the intersection of the following sets:

$$\begin{array}{ccc} x>0 \text{ and } x+1>0 & & x<0 \text{ and } x+1<0 \\ x>-1 & \text{along with} & x<-1 \\ \downarrow & & \downarrow \\ x>0 & & x<-1 \end{array}$$

So the domain of the function is $(-\infty,-1)\cup(0,\infty)$

b)

$$f(x)=\ln\left(\frac{x}{x+1}\right)$$

$$\downarrow$$

$$\begin{aligned} f'(x)&=\frac{1}{\frac{x}{x+1}}\cdot\frac{1(x+1)-x(1)}{(x+1)^2} \\ &=\frac{x+1}{x}\cdot\frac{1}{(x+1)^2} \\ &=\frac{1}{x(x+1)} \end{aligned}$$

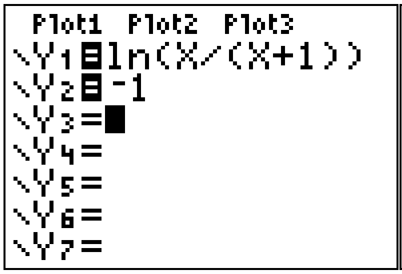
c)

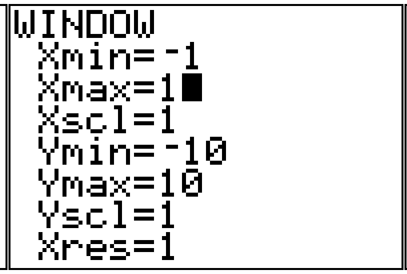
$$\begin{aligned} f(1)&=\ln\left(\frac{1}{1+1}\right) \\ &=\ln\left(\frac{1}{2}\right) \end{aligned} \quad \text{and} \quad \begin{aligned} f'(1)&=\frac{1}{1(1+1)} \\ &=\frac{1}{2} \end{aligned}$$

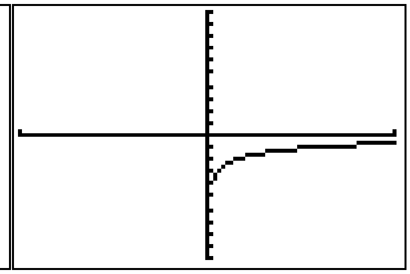
$$y-\ln\left(\frac{1}{2}\right)=\frac{1}{2}(x-1)$$

$$y=\frac{1}{2}x-\frac{1}{2}+\ln\left(\frac{1}{2}\right)$$

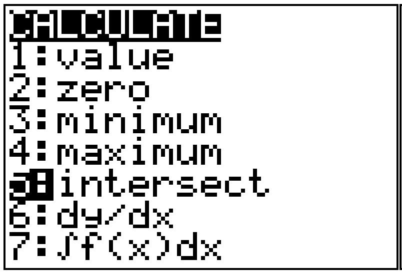
d) Using your graphing calculator, you can determine what x value corresponds to $y = -1$.

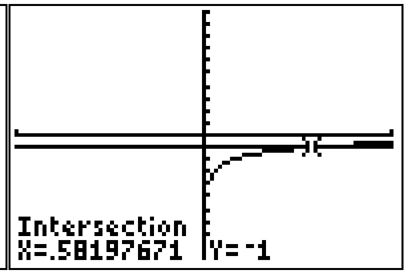




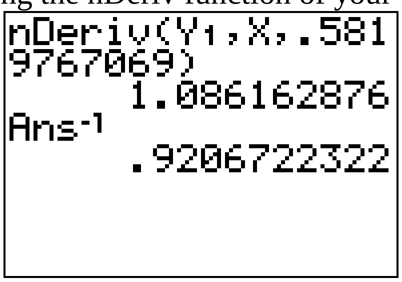


Using the intersect function with a guess of $x = \frac{1}{2}$ you get





Using the nDeriv function of your calculator, you can estimate the derivative to be



Therefore

$$g'(-1) = \frac{1}{f'(\text{the input that give you } -1 \text{ as an output})}.$$
$$\approx 0.920 \text{ or } 0.921$$