

The Sum of the First n -terms of a Geometric Series (with index starting at $i = 0$) is given by

$$\begin{aligned}\sum_{i=0}^{n-1} ar^i &= a + ar + ar^2 + \cdots + ar^{n-1} \\ &= \frac{a(1-r^n)}{1-r}\end{aligned}$$

* Note: If the index starts at zero, then the first n -terms ends with the exponent of $n-1$.

** Note: The exponent in the fraction is always *one more* than the exponent of the final term.

The Sum of an infinite Geometric Series with initial term ar^k where $k > 0$

$$\begin{aligned}\sum_{i=k}^{\infty} ar^i &= ar^k + ar^{k+1} + ar^{k+2} + \cdots \\ &= ar^k (1 + r + r^2 + r^3 + \cdots) \\ &= ar^k \sum_{i=0}^{\infty} r^i \\ &= ar^k \left(\frac{1}{1-r} \right) \\ &= \frac{ar^k}{1-r} \\ &= \frac{\text{first term}}{1 - \text{common ratio}}\end{aligned}$$