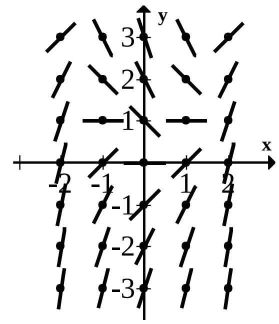
1. Which of the following differential equations matches the slope field at right?



$\frac{dy}{dx} \neq x + y$ or $\frac{dy}{dx} = 0$ along the line $y = -x$

$\frac{dy}{dx} \neq xy$ or $\frac{dy}{dx} = 0$ along the x-axis and along the y-axis

$\frac{dy}{dx} \neq x + y^2$ since $\left.\frac{dy}{dx}\right|_{(1,1)} \neq 2$

$\frac{dy}{dx} \neq x - y^2$ since $\left.\frac{dy}{dx}\right|_{(1,2)} \neq -1$

(a) $\frac{dy}{dx} = x + y^2$
(b) $\frac{dy}{dx} = x - y^2$
(c) $\frac{dy}{dx} = xy$
(d) $\frac{dy}{dx} = x + y$
(e) $\frac{dy}{dx} = x^2 - y$

2. **<u>(Calculator Required):</u>** A cup of coffee is heated to boiling (212°F), and taken out of a microwave and placed in a 72°F room at time $t = 0$ minutes. The coffee cools at the rate of $16e^{-0.112t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the coffee at time $t = 5$ minutes?

$$\begin{aligned} T(5) &= T(0) + \int_0^5 T'(x)\,dx \\ &= 212 + \int_0^5 -16e^{-0.112x}\,dx \\ &\approx 150.7441... \end{aligned}$$

(a) 105°F
(b) 133°F
(c) 166°F
(d) 151°F
(e) 203°F

3. The table below gives values of the differentiable functions f and g at $x=-1$. If $h(x)=\dfrac{f(x)-g(x)}{2f(x)}$, then $h'(-1)=$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	-2	4	e	-3

$$h(x)=\frac{f(x)-g(x)}{2f(x)}$$

$$\downarrow$$

$$h'(x)=\frac{\left[f'(x)-g'(x)\right]\cdot 2f(x)-\left[f(x)-g(x)\right]\cdot 2f'(x)}{\left[2f(x)\right]^2}$$

$$h'(-1)=\frac{\left[f'(-1)-g'(-1)\right]\cdot 2f(-1)-\left[f(-1)-g(-1)\right]\cdot 2f'(-1)}{\left[2f(-1)\right]^2}$$

$$=\frac{\left[e-(-3)\right]\cdot 2(-2)-\left[(-2)-4\right]\cdot 2e}{\left[2(-2)\right]^2}$$

$$=\frac{-4[e+3]-[-6]\cdot 2e}{[-4]^2}$$

$$=\frac{-4e-12+12e}{16}$$

$$=\frac{2e-3}{4}$$

(a) $\dfrac{-e-3}{4}$ (b) $\dfrac{e+3}{2e}$ (c) $\dfrac{e-6}{8}$ (d) $\dfrac{2e-3}{4}$ (e) $\dfrac{-4e-3}{4}$

4. If $f(x)$ is an antiderivative of $\dfrac{\sin^2 x}{x^2+2}$ such that $f(2)=\dfrac{1}{2}$, then $f(0)$ is given by

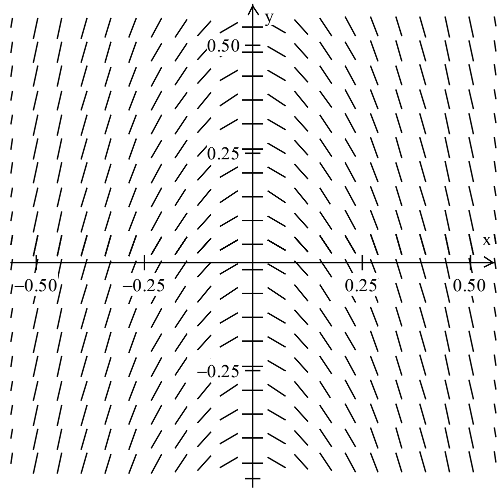
$$f(b)=f(a)+\int_a^b f'(x)\,dx$$

$$f(0)=f(2)+\int_2^0 f'(x)\,dx$$

(a) $\displaystyle\int_0^2 \frac{\sin^2(x)}{x^2+2}dx$ (b) $\displaystyle\int_2^0 \frac{\sin^2(x)}{x^2+2}dx$ (c) $\displaystyle\frac{1}{2}+\int_2^0 \frac{\sin^2(x)}{x^2+2}dx$

(d) $\displaystyle\frac{1}{2}+\int_0^2 \frac{\sin^2(x)}{x^2+2}dx$ (e) $\displaystyle 2+\int_2^0 \frac{\sin^2(x)}{x^2+2}dx$

5. Shown at right is the slope field of a differential equation. Which of the following could be a solution to the differential equation?



The only function whose graph is consistent in the slope field is $y = -5x^2$

(a) $y = e^x$

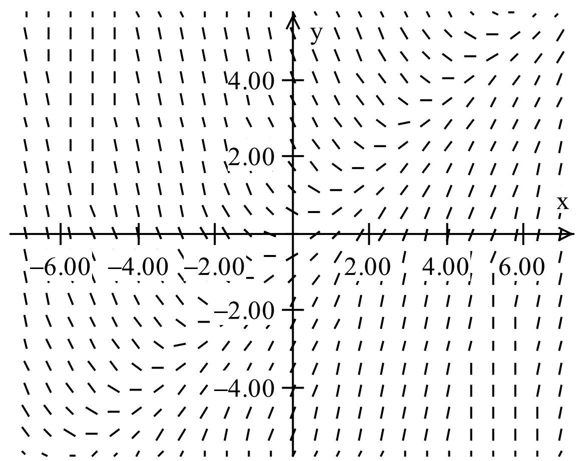
(b) $y = x^3$

(c) $y = -5x^2$

(d) $y = x$

(e) $y = x^2$

6. Which of the following differential equations is represented by the slope field at right?



(a) $\dfrac{dy}{dx} = 1 + y^2$ slopes are not consistent in horizontal rows

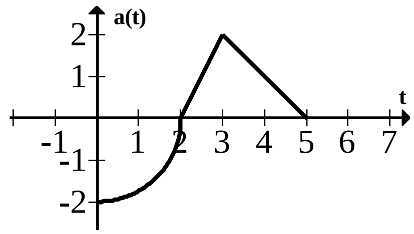
(b) $\dfrac{dy}{dx} = x - y$

(c) $\dfrac{dy}{dx} = 2x^2$ slopes are not consistent in vertical columns

(d) $\dfrac{dy}{dx} = 1 + x^2$ slopes are not consistent in vertical columns

(e) $\dfrac{dy}{dx} = 1 - y^2 + x^2$ $\left.\dfrac{dy}{dx}\right|_{(0,0,)} \neq 1$

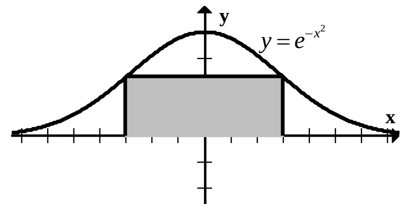
7. The graph at right shows an object's acceleration in $\frac{\text{ft}}{\text{sec}^2}$. It consists of a quarter circle, and two line segments. If the object was at rest at $t = 5$ seconds, what was its initial velocity?

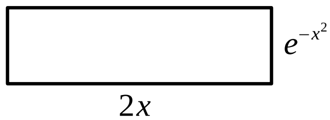


$$\begin{aligned} v(0) &= v(5) + \int_5^0 a(t)\,dt \\ &= 0 + \int_5^0 a(t)\,dt \\ &= -\frac{1}{2}(3)(2) + \frac{1}{4}\pi\left(2^2\right) \\ &= \pi - 3 \end{aligned}$$

(a) $-2\frac{\text{ft}}{\text{sec}}$ (b) $3-\pi\frac{\text{ft}}{\text{sec}}$ (c) $0\frac{\text{ft}}{\text{sec}}$ (d) $\pi-3\frac{\text{ft}}{\text{sec}}$ (e) $\pi+3\frac{\text{ft}}{\text{sec}}$

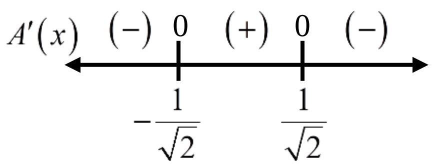
8. The area of the largest rectangle that can be drawn with one side along the x-axis and two vertices on the curve $y = e^{-x^2}$ is





$$A(x) = 2xe^{-x^2}$$
$$\downarrow$$
$$A'(x) = 2e^{-x^2} + 2x\left[-2xe^{-x^2}\right]$$
$$= 2e^{-x^2} + -4x^2e^{-x^2}$$
$$= 2e^{-x^2}\left[1-2x^2\right]$$

$$A'(x) = 0 \text{ or } DNE$$
$$\downarrow$$
$$0 = 2e^{-x^2}\left[1-2x^2\right]$$
$$0 = \underbrace{\frac{2}{e^{x^2}}}_{(+)}\left[1-2x^2\right]$$
$$1-2x^2 = 0$$
$$1 = 2x^2$$
$$\frac{1}{2} = x^2$$
$$x = \pm\frac{1}{\sqrt{2}}$$



$A(x)$ will be at a maximum when $x = \frac{1}{\sqrt{2}}$ because $A'(x)$ changes sign from positive to negative.

$$A\left(\frac{1}{\sqrt{2}}\right) = 2\left(\frac{1}{\sqrt{2}}\right)e^{-\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{2}\cancel{\sqrt{2}} \cdot \frac{1}{\cancel{\sqrt{2}}} e^{-\frac{1}{2}}$$

$$= \sqrt{2} \cdot \left(e^{\frac{1}{2}}\right)^{-1}$$

$$= \sqrt{2} \cdot \left(\sqrt{e}\right)^{-1}$$

$$= \frac{\sqrt{2}}{\sqrt{e}}$$

$$= \sqrt{\frac{2}{e}}$$

(a) $\sqrt{\frac{2}{e}}$ (b) $\sqrt{2e}$ (c) $\frac{2}{e}$ (d) $\frac{1}{\sqrt{2e}}$ (e) $\frac{2}{e^2}$