Section 11-7 Homework Hints

1. Direct Comparison to $\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n$

2. Use the Root Test $\sum_{n=1}^{\infty}\left(\frac{2n+1}{n^2}\right)^n$

3. Test $\lim_{n\to\infty}(\text{summand})$

4. Use Alternating Series Test

5. Use Root Test with $\sum_{n=1}^{\infty}\frac{1}{2}\cdot n^2\left(\frac{2}{5}\right)^n$

6. Use Limit Comparison Test or Direct Comparison Test with $\sum_{n=1}^{\infty}\frac{1}{n}$

7. Use Integral Test $\sum_{n=2}^{\infty}\frac{1}{n\sqrt{\ln(n)}} \sim \int_2^{\infty}\frac{1}{x\sqrt{\ln(x)}}dx$ with u-sub of $u=\ln(x)$

8. Use Ratio Test

9. Use Integral Test with Integration by Parts, or the root test (root test is easier).

10. Use Integral Test with u-sub of $u=-n^3$, or the root test.

11. $\sum_{n=1}^{\infty}\frac{1}{n^3}+\frac{1}{3^n}=\sum_{n=1}^{\infty}\frac{1}{n^3}+\sum_{n=1}^{\infty}\frac{1}{3^n}$: p-series plus geometric series

12. Use Limit Comparison with $\sum_{k=1}^{\infty}\frac{1}{k^2}$

13. $\sum_{n=1}^{\infty}\frac{3^n n^2}{n!}$ Use Ratio Test

14. Use Direct Comparison Test $\sum_{n=1}^{\infty}\left|\frac{\sin(2n)}{1+2^n}\right| < \sum_{n=1}^{\infty}\left|\frac{1}{1+2^n}\right| < \sum_{n=1}^{\infty}\left|\frac{1}{2^n}\right| = \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n$

15. $\sum_{k=1}^{\infty}\frac{2^{k-1}3^{k-1}}{k^k}=\sum_{k=1}^{\infty}\frac{1}{2}\cdot\frac{1}{3}\frac{2^k3^k}{k^k}=\sum_{k=1}^{\infty}\frac{1}{6}\cdot\left(\frac{6}{k}\right)^k$ Use the Root Test

16. Use Limit Comparison with $\sum_{n=1}^{\infty}\frac{1}{n}$

17. Use the Ratio Test and keep track of canceling terms

$$\frac{1\cdot3\cdot5\cdot\dots\cdot(2(n+1)-1)}{2\cdot5\cdot8\cdot\dots\cdot(3(n+1)-1)}=\frac{1\cdot3\cdot5\cdot\dots\cdot(2n+1)}{2\cdot5\cdot8\cdot\dots\cdot(3n+2)}$$
$$=\frac{1\cdot3\cdot5\cdot\dots\cdot(2n-1)(2n+1)}{2\cdot5\cdot8\cdot\dots\cdot(3n-1)(3n+2)}$$

18. Use the Alternating Series Test

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20. Use the Limit Comparison Test with $\sum_{k=1}^{\infty} \frac{1}{k^{\frac{7}{6}}}$

21. $\lim_{k\to\infty}(\text{summand}) \neq 0$

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23. Use the Integral Test $\lim_{b\to\infty}\int_1^b \tan\left(\frac{1}{x}\right)dx$

24. $\lim_{n\to\infty}(\text{summand}) \neq 0$ with L'Hopital's Rule

25. Use Ratio Test

26. Use Root Test

27. $\sum_{k=1}^{\infty}\frac{k\ln(k)}{(k+1)^3} < \sum_{k=1}^{\infty}\frac{k\ln(k)}{k^3} < \boxed{\sum_{k=1}^{\infty}\frac{\ln(k)}{k^2}} \leftarrow$ Use Integral Test with Integration by Parts along with the Direct Comparison Test.

28. Use Direct Comparison Test with $\sum_{n=1}^{\infty}\frac{3}{n^2}$

29. Do not attempt, we have not worked with hyperbolic-cosine: $\cosh(x)$

30. Use the alternating series test.

31. $\sum_{k=1}^{\infty}\frac{5^k}{3^k+4^k} = \sum_{k=1}^{\infty}\frac{5^k}{3^k+4^k}\cdot\frac{\left(\frac{1}{4^k}\right)}{\left(\frac{1}{4^k}\right)} = \sum_{k=1}^{\infty}\frac{\left(\frac{5}{4}\right)^k}{\left(\frac{3}{4}\right)^k+1}$, then demonstrate that $\lim_{k\to\infty}(\text{summand}) \neq 0$

32. Use the Root Test

33. Use the root test and $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^{-n} = \lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$

34. $\frac{1}{n+n\cos^2(n)} \geq \frac{1}{2n}$ for all *n*. Direct Comparison with $\sum_{n=1}^{\infty}\frac{1}{2n}$