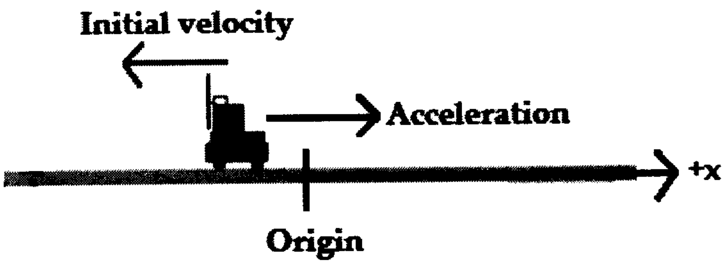
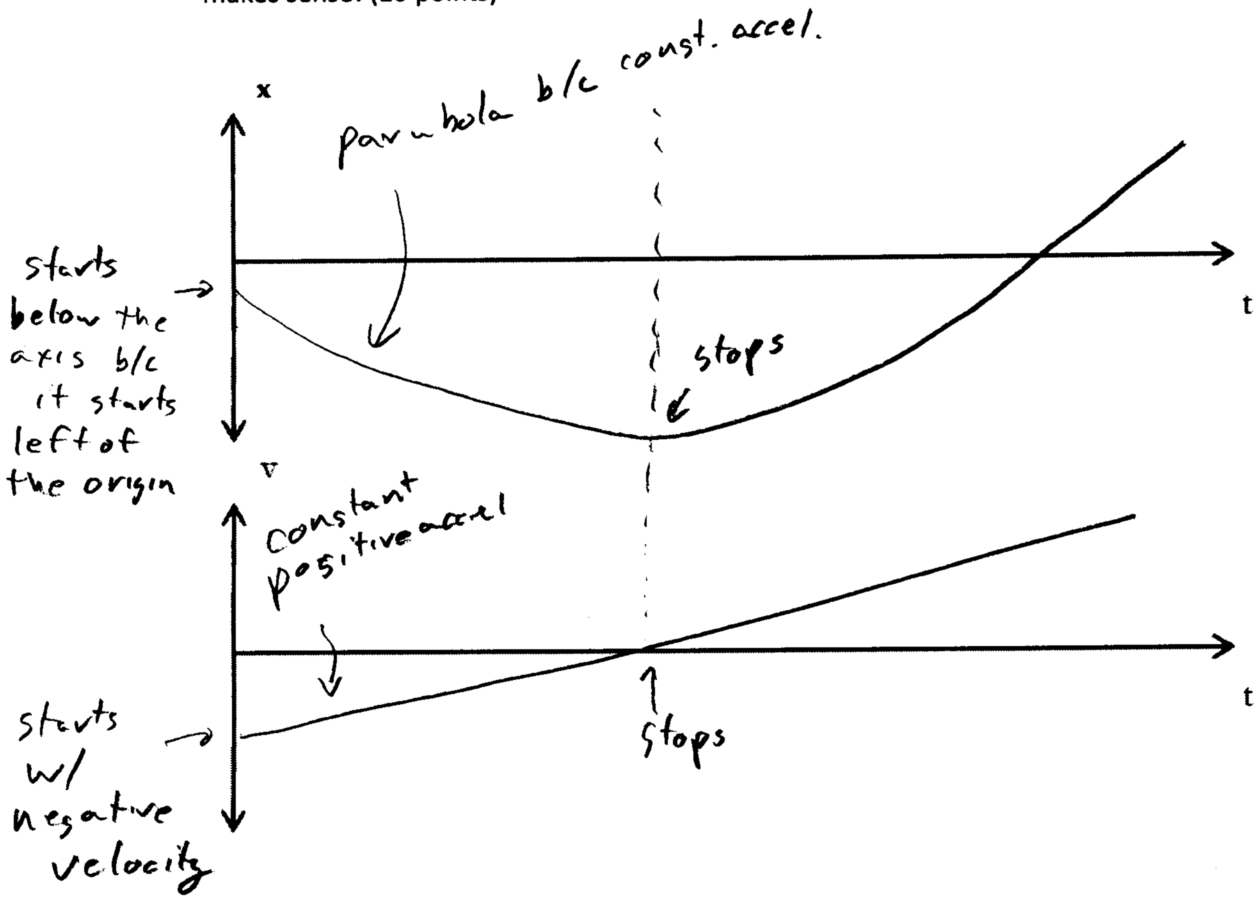
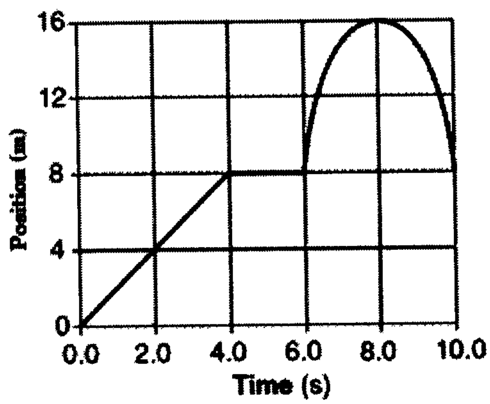
PHYS 250, Sample Test 1

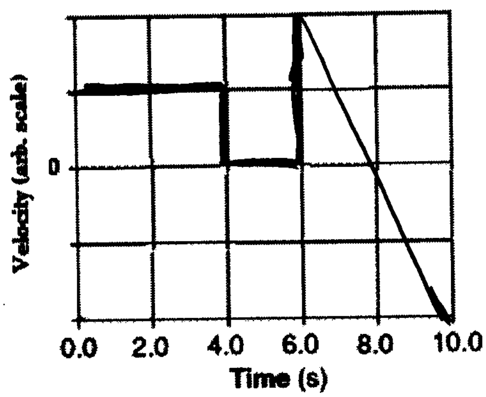
For all problems, show your work! Credit will not be given for the answer only. For maximum partial credit, explain your work in words as well as equations, and be sure to use diagrams! Good luck!



1. The figure above depicts a cart with a fan attached to it moving on a flat track with negligible friction. The fan pushes the cart, causing an acceleration to the right. At time t=0, the cart is moving to the left, and is located to the left of the origin. Draw position vs. time and velocity vs. time graphs that depict the motion of the cart after time t=0. Be sure to use the same time scale on both graphs. REALLY THINK THESE THROUGH! Check to make sure each part of your graph makes sense! (10 points)







2. The graph to the left above shows the position vs. time graph describing the motion of an object. The arc-shaped portion is parabolic.
   a. In the graph to the right, sketch the corresponding velocity vs. time graph. The vertical scale is arbitrary, so you don't need to worry about the values, just the shape. Do note, however, that the line indicating 0 velocity is labeled. (3 points)
   b. What is the instantaneous velocity of the object at time t=2.0 seconds? Show and explain your work. (2 points)

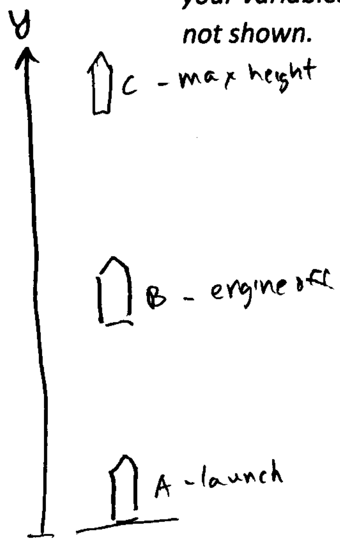
At $t=2$, the inst. velocity is the same as the avg velocity from $t=0$ to $t=4$, since the velocity is constant over that interval. So $v(2) = v_{av(0-4)} = \frac{V(4) - V(0)}{4_s - 0_s} = \frac{8_m - 0_m}{4\ s} = 2\ m/s$

   c. What is the average velocity of the object between times 6.0 s and 8.0 s? Show and explain your work. (2 points)

$V_{av} = \frac{\Delta x}{\Delta t} = \frac{16m - 8m}{8s - 6s} = 4\ m/s$ these are the positions at 6 and 8 seconds

3. A rocket leaves the launch pad and travels straight up with constant acceleration of 1 m/s² to a height of 450 m. The engine then shuts off, and the rocket enters free fall (the effects of air resistance are negligible). What is the maximum height the rocket reaches? (10 points)

Show all of your work. Use proper problem solving steps, including drawing a diagram and listing your variables. Partial credit will be given for showing this work, and you will lose credit if it is not shown.



$y_A = 0\,m$, $y_B = 450\,m$

$V_A = 0\,m/s$

$V_C = 0\,m/s$

$a_{AB} = 1\,m/s^2$

$a_{BC} = -g$

Use AB to find the velocity at B, then use BC to find the height.

For AB: $V_B^2 = \cancel{V_A^2}^{\,0} + 2\,a_{AB}\,\Delta y_{AB}$

$\Rightarrow \quad V_B = \sqrt{2(1\,\tfrac{m}{s^2})(450\,m)} = 30\,m/s$

Now for BC:

$\cancel{V_C^2}^{\,0} = V_B^2 + 2\,a_{BC}\,\Delta y_{BC}$

$\Rightarrow \quad \Delta y_{BC} = \dfrac{-V_B^2}{2a_{BC}} = \dfrac{-(30\,m/s)^2}{2(-9.8\,m/s^2)} = 45.9\,m$

so $y_C = y_B + \Delta y_{BC} = 450\,m + 45.9\,m$
$= 495.9\,m$

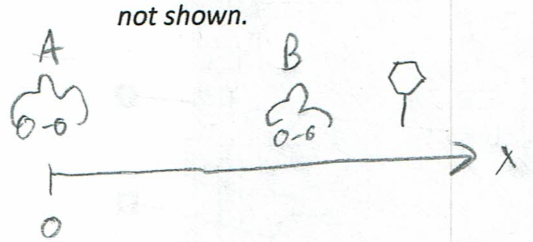
This is kinda wimpy: less than $\frac{1}{2}$ km. Not in space!

4. At time t=0, a car is located ~~50~~ 15 m to the west of a stop sign. Take east to be the +x direction. The car is initially moving with a speed of +~~15~~ 10 m/s. The car's acceleration from t=0 until it stops is given by the equation

$$a(t) = -5m/s^2 + (1\,m/s^3)t$$

Does the car stop before it reaches the stop sign?

Show all of your work. Use proper problem solving steps, including drawing a diagram and listing your variables. Partial credit will be given for showing this work, and you will lose credit if it is not shown.



$x_A = 0$ $t_A = 0$ $V_A = 10\ m/s^2$

$x_B = ?$ $t_B = ?$ $x_{sign} = \cancel{50m}\ 15m$

Integrate a to get v and then v to get x.

Find when $v = 0$ using $v(t)$.

$$v(t) = \int a\,dt = \left(-5\frac{m}{s^2}\right)t + \left(\frac{1}{2}\frac{m}{s^3}\right)t^2 + C_1$$

since $v(0) = V_A = 10\ m/s = C_1$, $C_1 = 10\ m/s$

$$v(t) = \left(-5\frac{m}{s^2}\right)t + \left(\frac{1}{2}\frac{m}{s^3}\right)t^2 + 10\ m/s$$

set equal to zero, use the quad. eqn to find t_B:

$$t_B = \frac{-\left(-5\frac{m}{s^2}\right) \pm \left(\sqrt{\left(5\frac{m}{s}\right)^2 - 4\left(\frac{1}{2}\frac{m}{s^3}\right)\left(10\frac{m}{s}\right)}\right)}{2\left(\frac{1}{2}\frac{m}{s^3}\right)} = 5 \pm \sqrt{5}\ \text{seconds.}$$

the lower root is the first place it stops; $t_B = 5 - \sqrt{5}\ s = 2.76s$.

Now integrate $v(t)$ to find $x(t)$:

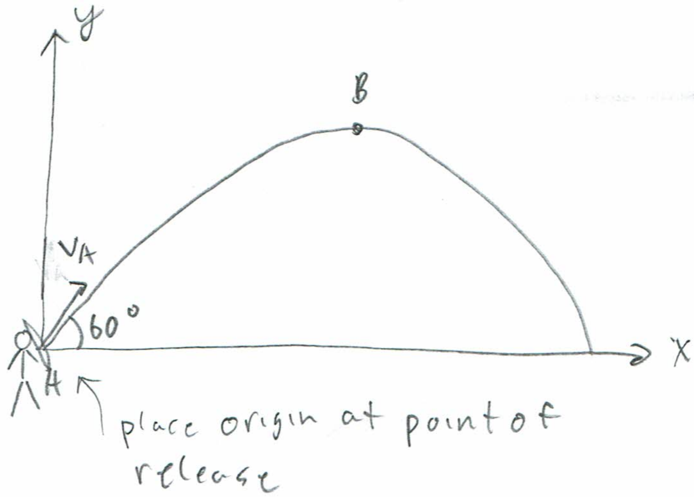
$$x(t) = \int v\,dt = \frac{1}{6}\frac{m}{s^3}t^3 - \frac{5}{2}\frac{m}{s^2}t^2 + 10\frac{m}{s}t + C_2$$

since $x(0) = 0$, $C_2 = 0$.

$$\text{so } x(t_B) = \frac{1}{6}\frac{m}{s^3}(2.76s)^3 - \frac{5}{2}\frac{m}{s^2}(2.76s)^2 + 10\frac{m}{s}(2.76s)$$

$= 12.1\ m$. ~~the car~~ this is $< 15m$ so the car stops before the sign

1)



Knowns: $X_A = 0$ $\quad y_A = 0$

$V_{By} = 0\ m/s$

$V_{Ax} = V_A \cos 60° = \frac{1}{2} V_A$

$V_{Ay} = V_A \sin 60° = \frac{\sqrt{3}}{2} V_A$

$a_x = 0 \quad a_y = -g$

The highest point occurs when $V_B = 0\ m/s$.

a) Analyzing the y-motion we have

$$\cancel{V_{By}^2}^{\,0} = V_{Ay}^2 + 2a_y \Delta y \qquad \text{(const accel in y-direction)}$$

$$0 = \left(V_A \frac{\sqrt{3}}{2}\right)^2 + 2(-g)\Delta y$$

$$\Rightarrow \Delta y = \left(\frac{3V_A^2}{4}\right) / 2g = 860\ m$$

(this is maybe absurdly high: air resistance plays a role in making this lower in reality)

b) To find the position at B, first find Δt_{AB} using the vertical motion:

$$\cancel{V_{By}}^{\,0} = V_{Ay} + a_y \Delta t_{AB} \qquad \text{(const accel in y-direction)}$$

$$\Rightarrow \Delta t_{AB} = \frac{-V_{Ay}}{a_y} = \frac{-V_A \frac{\sqrt{3}}{2}}{-9.8\ m/s^2} = 13.26\ s$$

Now analyze x-motion:

$$X_B = \cancel{X_A}^{\,0} + V_{Ax} \Delta t_{AB} \qquad \text{(const. vel. in x-direction)}$$

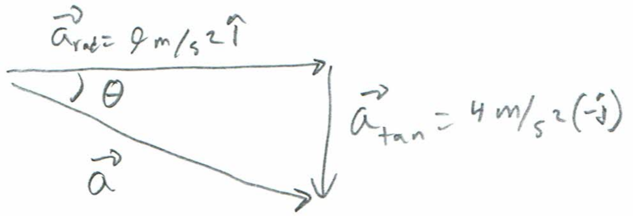
$$= \left(\frac{V_A}{2}\right)(13.26\ s) = 994\ m$$

2) a) If the car stays on the track then $V = V_{tan}$. And $a_{rad} = \frac{V_{tan}^2}{r}$. we know a_{rad} is the x-component of the acceleration, which is $9 \frac{m}{s^2}$

so $V_{tan} = \sqrt{r\, a_{rad}} = \sqrt{100\,m \cdot 9\,m/s^2} = 30\,m/s$

b) The tangential acceleration is downwards - opposite the velocity. That means the car is slowing down.

c) The accel is a vector:



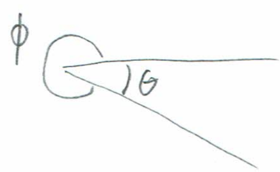
the magnitude is

$$\sqrt{a_{rad}^2 + a_{tan}^2}$$

$$= \sqrt{9^2 + 4^2}\ m/s^2$$

$$= 9.85\ m/s^2$$

the direction can be found using $\theta = \tan^{-1}(4/9) = 24.0°$



the angle that we want is ϕ:

$\phi = 360° - 24.0° = 336.0°$

so $\vec{a} = 9.85 \frac{m}{s^2}$ 336.0° ccw of the x-axis