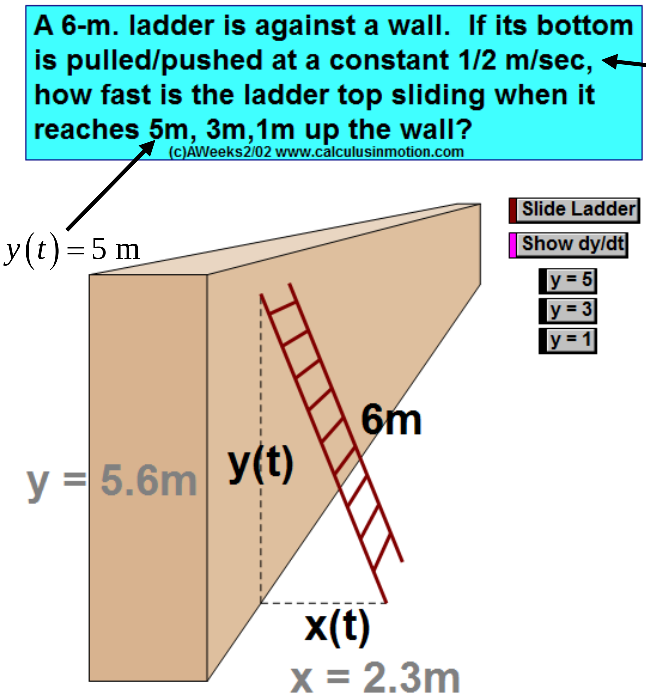


$$x'(t)=\pm 0.5\ \frac{\text{m}}{\text{sec}}$$

$x'(t)>0$ if the ladder is moving **away** from the wall

$x'(t)<0$ if the ladder is moving **towards** from the wall

Relation equation:

$$[x(t)]^2+[y(t)]^2=6^2$$

$$[x(t)]^2+[y(t)]^2=6^2$$
$$\downarrow$$
$$2x(t)x'(t)+2y(t)y'(t)=0$$
$$x(t)x'(t)+y(t)y'(t)=0$$

At the moment that $y(t)=5$ we can use this value to find $x(t)$ at the same moment.

$$5^2+[x(t)]^2=6^2$$
$$25+[x(t)]^2=36$$
$$[x(t)]^2=11$$
$$x(t)=\sqrt{11}$$

$$\underbrace{x(t)}_{\sqrt{11}\text{ m}}\underbrace{x'(t)}_{0.5\ \frac{\text{m}}{\text{sec}}}+\underbrace{y(t)}_{5\text{m}}\underbrace{y'(t)}_{\text{unknown}}=0$$

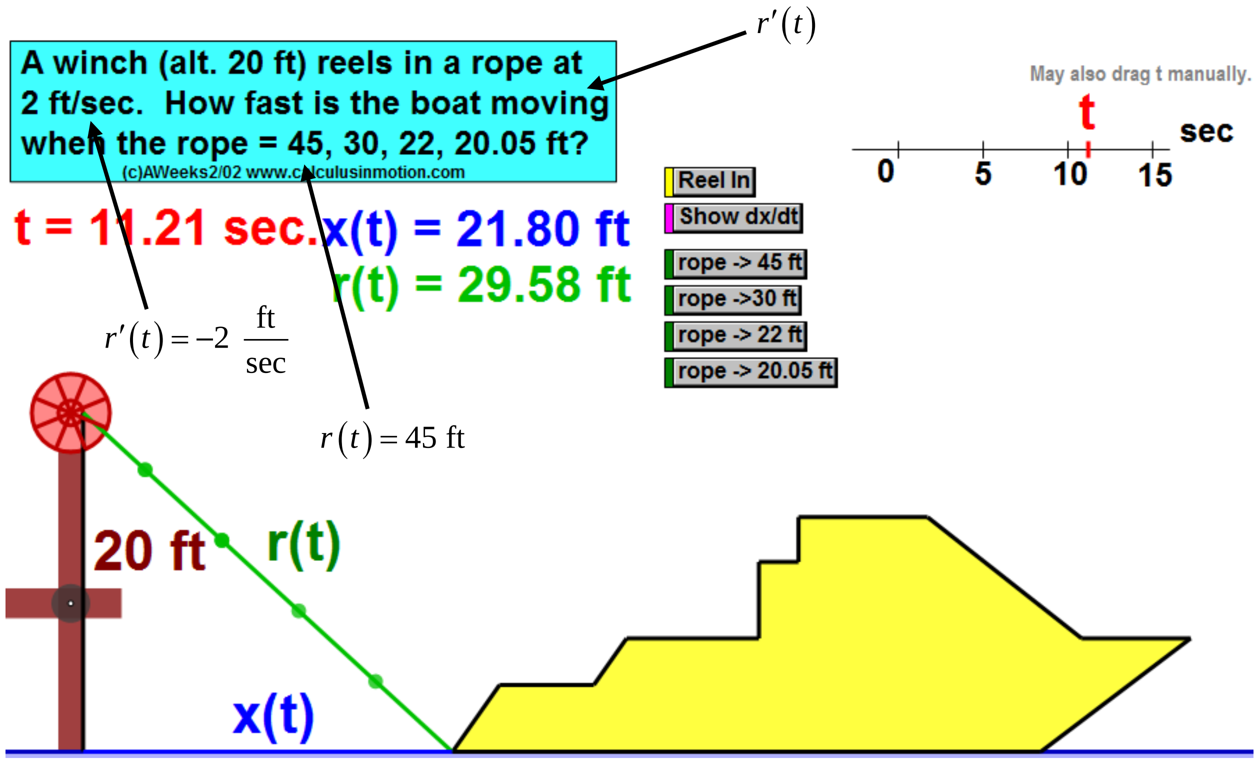
$$\sqrt{11}\left(\frac{1}{2}\right)+5\cdot y'(t)=0$$

$$5\cdot y'(t)=-\frac{\sqrt{11}}{2}$$

$$y'(t)=-\frac{\sqrt{11}}{10}=-0.3316\ldots\ \frac{\text{m}}{\text{sec}}$$

The top of the ladder is moving down at a rate of $\frac{\sqrt{11}}{10}=0.3316\ldots\ \frac{\text{m}}{\text{sec}}$ when the bottom of the ladder is being pushed out at a rate of 0.5 $\frac{\text{m}}{\text{sec}}$.

The top of the ladder is moving at a rate of $-\frac{\sqrt{11}}{10}=-0.3316\ldots\ \frac{\text{m}}{\text{sec}}$ when the bottom of the ladder is being pushed out at a rate of 0.5 $\frac{\text{m}}{\text{sec}}$.



Relation equation: $20^2+\left[x(t)\right]^2=\left[r(t)\right]^2$

$20^2+\left[x(t)\right]^2=\left[r(t)\right]^2$ $\downarrow$ $2\cdot x(t)\cdot x'(t)=2\cdot r(t)\cdot r'(t)$	At the moment that $r(t)=45$: $20^2+\left[x(t)\right]^2=45^2$ $400+\left[x(t)\right]^2=2025$ $\left[x(t)\right]^2=1625$ $x(t)=5\sqrt{65}$
$2\cdot\underbrace{x(t)}_{5\sqrt{65}}\cdot\underbrace{x'(t)}_{\text{unknown}}=2\cdot\underbrace{r(t)}_{45}\cdot\underbrace{r'(t)}_{-0.2}$ $\downarrow$ $2\left[5\sqrt{65}\right]\cdot x'(t)=2(45)(-0.2)$ $x'(t)=\dfrac{2(45)(-0.2)}{2\left[5\sqrt{65}\right]}\approx-2.2326\ldots\ \dfrac{\text{ft}}{\text{sec}}$	The boat is moving towards the dock at a rate of 2.2326… $\dfrac{\text{ft}}{\text{sec}}$ when the length of the rope is 45 ft and the rope is being reeled in at 2 $\dfrac{\text{ft}}{\text{sec}}$. The velocity of the boat is $-2.2326\ldots\ \dfrac{\text{ft}}{\text{sec}}$ when the length of the rope is 45 ft and the rope is being reeled in at 2 $\dfrac{\text{ft}}{\text{sec}}$.

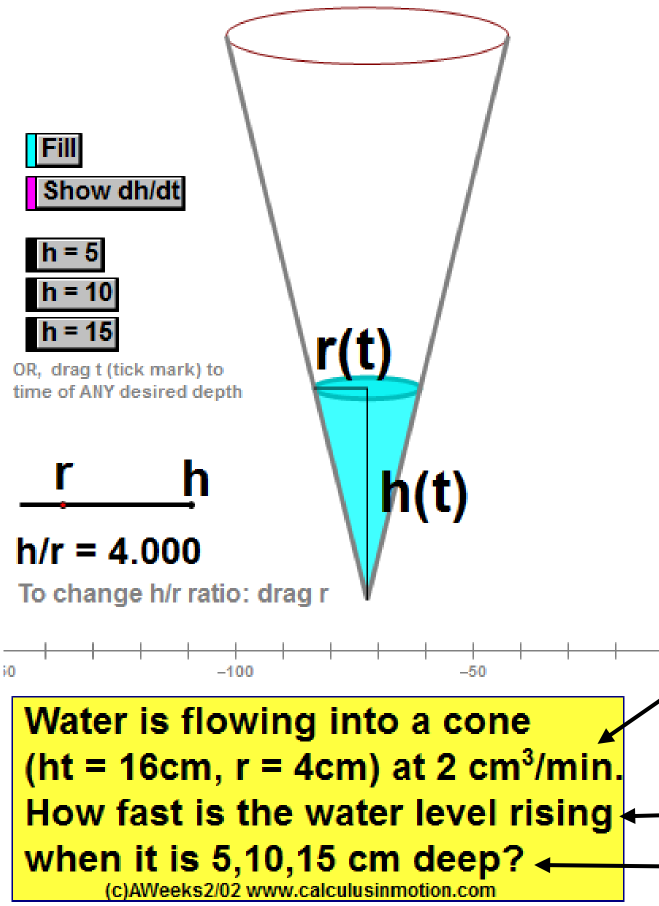
Volume of a right circular cone:

$$V = \frac{1}{3}\pi r^2 \cdot h$$

$$\downarrow$$

$$V(t) = \frac{1}{3}\pi \left[r(t)\right]^2 \cdot h(t)$$

This will be given on the AP Exam



$V'(t)$

$h'(t)$

$h(t)$

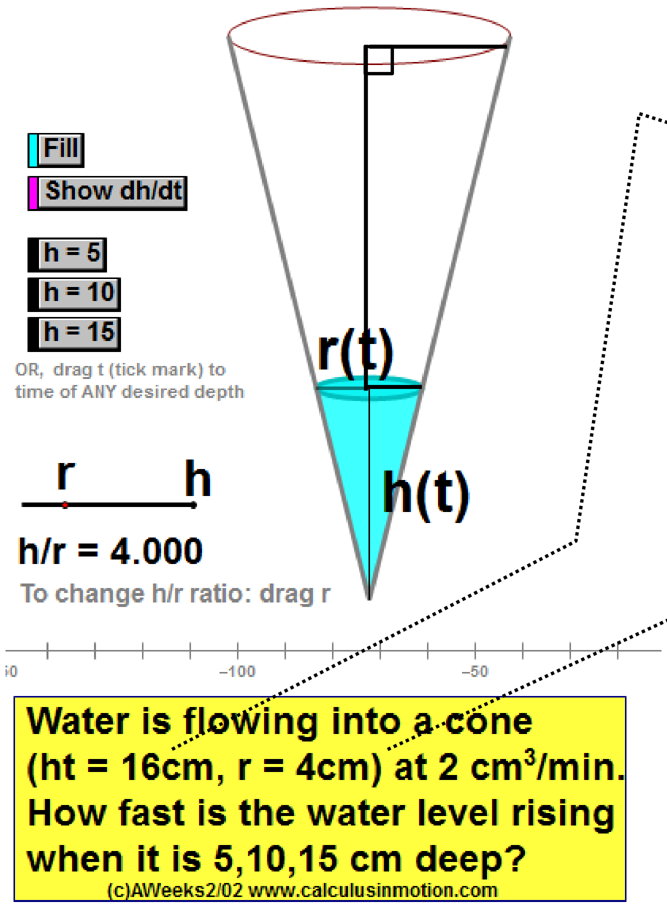
$$V(t) = \frac{1}{3}\pi \left[r(t)\right]^2 \cdot h(t)$$

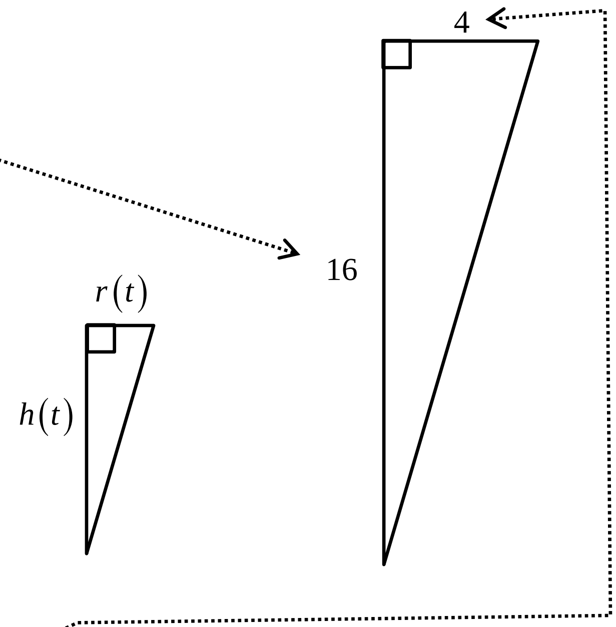
$$\downarrow$$

$$\underbrace{V'(t)}_{2} = \frac{1}{3}\pi \left[2\underbrace{r(t)}_{\text{unkown}}\underbrace{r'(t)}_{\text{unkown}} \cdot \underbrace{h(t)}_{\text{known}} + \left[\underbrace{r(t)}_{\text{unkown}}\right]^2 \underbrace{h'(t)}_{\text{unknown}} \right]$$

The problem here is that after differentiating the relation equation, we have too many unknowns. If we had only one unknown, and it was the rate we are looking for, then we would be ok.
If you ever run into a situation like this, you will need to use substitute a secondary relationship into the equation you differentiated to reduce the number of functions that you are working with. In my experience, the two most common secondary relationships that arise in these types of exercises are (1) similar triangles, and (2) Pythagorean Theorem.

A solution is presented on the next page.





Using Similar Triangles

$$\frac{r(t)}{4} = \frac{h(t)}{16}$$

$$\downarrow$$

$$r(t) = \frac{1}{4}h(t)$$

$$V(t) = \frac{1}{3}\pi\left[r(t)\right]^2 \cdot h(t)$$

$$V(t) = \frac{1}{3}\pi\left[\frac{1}{4}h(t)\right]^2 \cdot h(t)$$

$$V(t) = \frac{\pi}{48}\left[h(t)\right]^3$$

$$\downarrow$$

$$\underbrace{V'(t)}_{2} = \frac{\pi}{16}\left[\underbrace{h(t)}_{5}\right]^2 \cdot \underbrace{h'(t)}_{\text{unknown}}$$

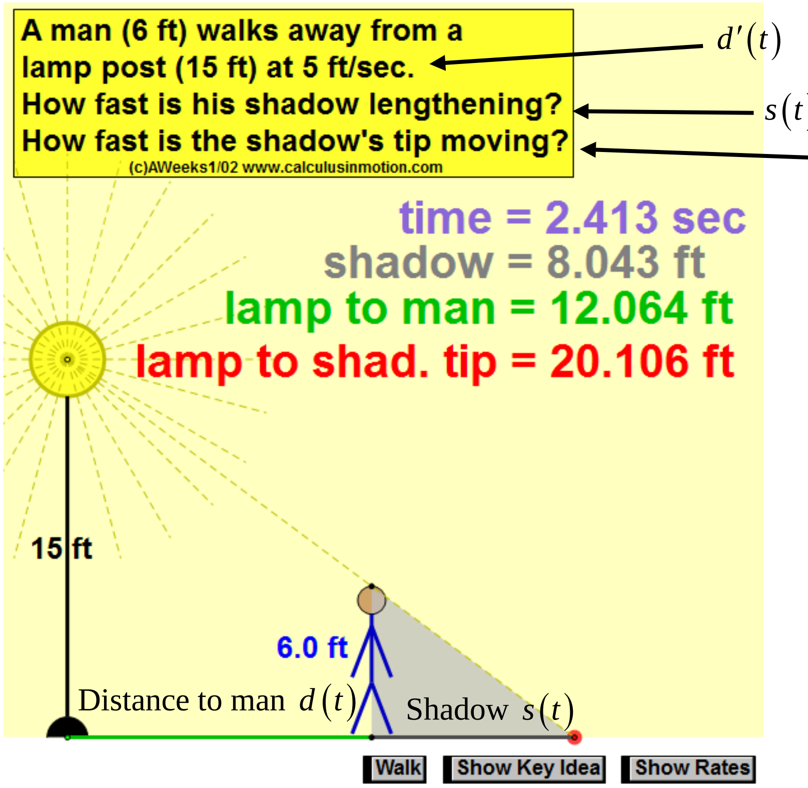
$$2 = \frac{25\pi}{16}h'(t)$$

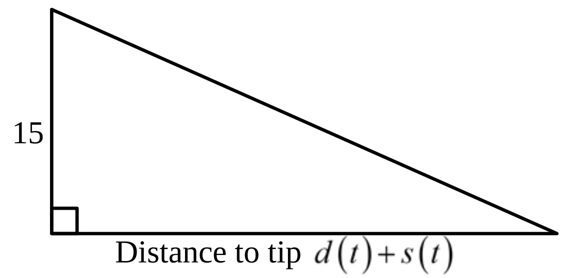
$$h'(t) = \frac{32}{25\pi}$$

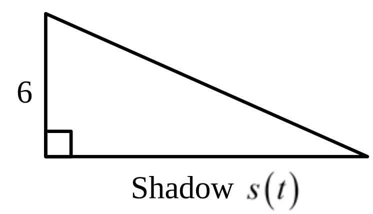
$$h'(t) = 0.4074\ldots\frac{\text{cm}}{\text{min}}$$

The height of the water is increasing at a rate of $0.4074\ldots\ \frac{\text{cm}}{\text{min}}$

when $h(t) = 5$ cm and $V'(t) = 2\ \frac{\text{cm}^3}{\text{min}}$

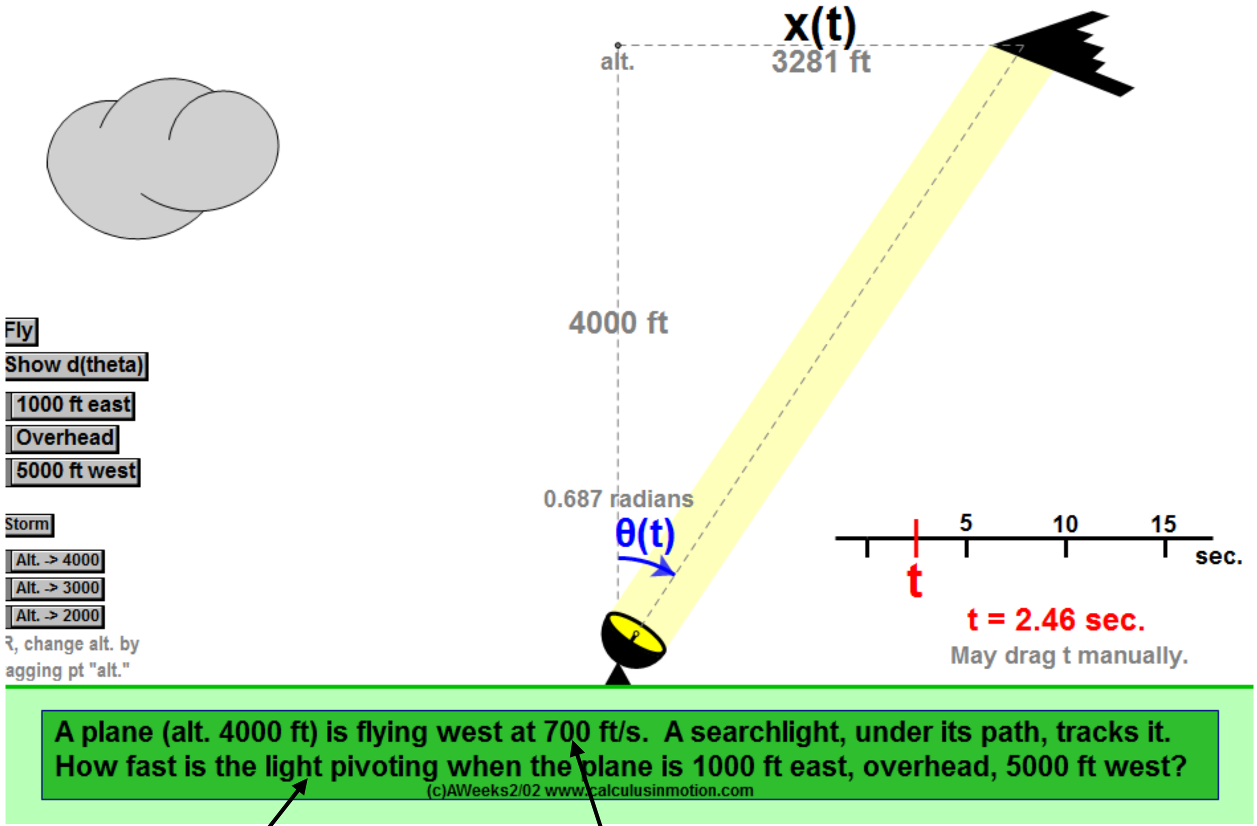






$\frac{6}{15}=\frac{s(t)}{s(t)+d(t)}$ $6[s(t)+d(t)]=15s(t)$ $6s(t)+6d(t)=15s(t)$ $6d(t)=9s(t)$ $s(t)=\frac{2}{3}d(t)$ $\downarrow$ $s'(t)=\frac{2}{3}d(t)$	$s'(t)=\frac{2}{3}\underbrace{d(t)}_{5}$ $s'(t)=\frac{10}{3}\ \frac{\text{ft}}{\text{sec}}$ The length of the shadow is increasing at rate of 2 $\frac{\text{ft}}{\text{sec}}$ when the man is walking away at 5 $\frac{\text{ft}}{\text{sec}}$.	Tip of the shadow moving is given by: $[d(t)+s(t)]'$ $\downarrow$ $d'(t)+s'(t)$ $5+\frac{10}{3}$ $\frac{25}{3}\ \frac{\text{ft}}{\text{sec}}$

The tip of the shadow is moving at a rate of $\frac{25}{3}\ \frac{\text{ft}}{\text{sec}}$ when the man is walking away at 5 $\frac{\text{ft}}{\text{sec}}$.



$\theta'(t)$

$x'(t) = -700$ [negative since $x(t)$ is decreasing]

Option 1	Option 2
$\tan(\theta(t)) = \dfrac{x(t)}{4000}$ $\downarrow$ $\sec^2(\theta(t)) \cdot \theta'(t) = \dfrac{1}{4000} x'(t)$	$\theta(t) = \arctan\left[\dfrac{x(t)}{4000}\right]$ $\downarrow$ $\theta'(t) = \dfrac{1}{1+\left[\dfrac{x(t)}{4000}\right]^2} \dfrac{1}{4000} x'(t)$

When $x(t)=1000$

$$\tan(\theta(t))=\frac{1000}{4000}$$
$$\theta(t)=\arctan\left(\frac{1}{4}\right)$$

Option 1
$\sec^2(\theta(t))\cdot\theta'(t)=\frac{1}{4000}x'(t)$ $\downarrow$ $\sec^2\left(\arctan\left(\frac{1}{4}\right)\right)\cdot\theta'(t)=\frac{1}{4000}(-700)$ $\theta'(t)=\frac{1}{4000}(-700)\cdot\frac{1}{\sec^2\left(\arctan\left(\frac{1}{4}\right)\right)}$ $=-0.1647...$
Option 2
$\theta'(t)=\frac{1}{1+\left[\frac{1000}{4000}\right]^2}\frac{1}{4000}(-700)=-0.1647...$

The spotlight is rotating at a rate of $-0.1647...\frac{\text{rad}}{\text{sec}}$ when the plane is travelling east at 700 $\frac{\text{ft}}{\text{sec}}$ and is 100ft East.

The angle θ is decreasing at a rate of $0.1647...\frac{\text{rad}}{\text{sec}}$ when the plane is travelling east at 700 $\frac{\text{ft}}{\text{sec}}$ and is 100ft East.