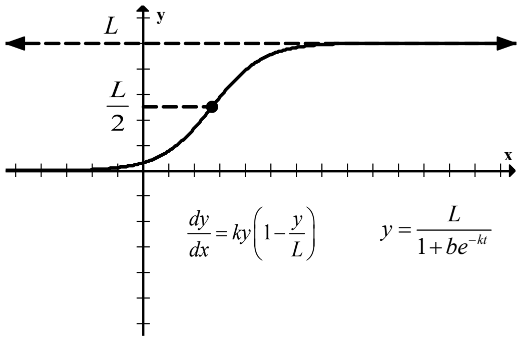


Logistic Differential Equation $\frac{dy}{dx} = ky\left(1-\frac{y}{L}\right)$ is used to model situations like population growth with a carrying capacity. The rate of change is proportional to the value of *y*.

If $f(x)$ is a solution to the logistic differential equation, then

$\lim_{x\to\infty} f(x) = L$

$f(x)$ has an inflection point when $y = \frac{L}{2}$

In the context of population growth, the logistic differential equation is often expressed as $\frac{dP}{dt} = kP\left(1-\frac{P}{L}\right)$.

Note that some factoring may need to be performed to achieve the "1" in the logistic differential equation.

$$\frac{dP}{dt} = 5P(7-P) \leftrightarrow \frac{dP}{dt} = 35P\left(1-\frac{P}{7}\right)$$

A rumor spreads among a population of *N* people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If *p* denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time *t*, where *k* is a positive constant?

(a)	(b)	(c)	(d)	(e)
$\frac{dp}{dt} = kp$	$\frac{dp}{dt} = kp(N-p)$	$\frac{dp}{dt} = kp(p-N)$	$\frac{dp}{dt} = kt(N-t)$	$\frac{dp}{dt} = kt(t-N)$

A population of wolves is modeled by the function *P* and grows according to the logistic differential equation $\frac{dP}{dt} = 5P\left(1-\frac{P}{5000}\right)$, where *t* is the time in years and $P(0) = 1000$. Which of the following statements are true?

I. $\lim_{t\to\infty} P(t) = 5000$ II. $\frac{dP}{dt}$ is positive for $t > 0$ III. $\frac{d^2P}{dt^2}$ is positive for $t > 0$

(a) I only (b) II only (c) I and II only (d) I and III only (e) I, II, and III

Released FRQ focusing on the Logistic Differential Equation.

AP Calculus BC 2004 #5: parts (a) and (b)

A population is modeled by a function *P* that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right)$$

(a) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$? If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

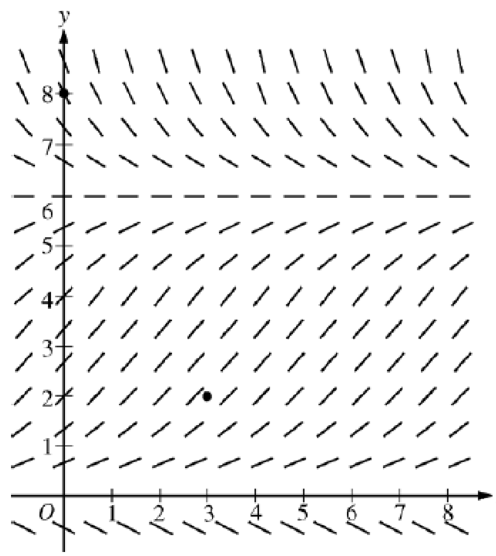
(b) If $P(0) = 3$, for what value of *P* is the population growing the fastest?

AP Calculus BC 2008 #6: parts (a) and (d)

Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

(a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3,2)$ and $(0,8)$

(d) What is the range of *f* for $t \geq 0$? $6 < y \leq 8$



AP Calculus BC 2006 Form B #5: parts (b) and (c)

Let *f* be a function with $f(4) = 1$ such that all points (x, y) on the graph of *f* satisfy the differential equation

$$\frac{dy}{dx} = 2y(3 - x)$$

Let *g* be a function with $g(4) = 1$ such that all points (x, y) on the graph of *g* satisfy the logistic differential equation

$$\frac{dy}{dx} = 2y(3 - y)$$

(b) Given that $g(4) = 1$, find $\lim_{x\to\infty} g(x)$ and $\lim_{x\to\infty} g'(x)$. It is not necessary to solve for $g(x)$ to show how you arrived at your answers.

(c) For what value of *y* does the graph of *g* have a point of inflection? Find the slope of the graph of *g* at the point of inflection. (It is not necessary to solve for $g(x)$).