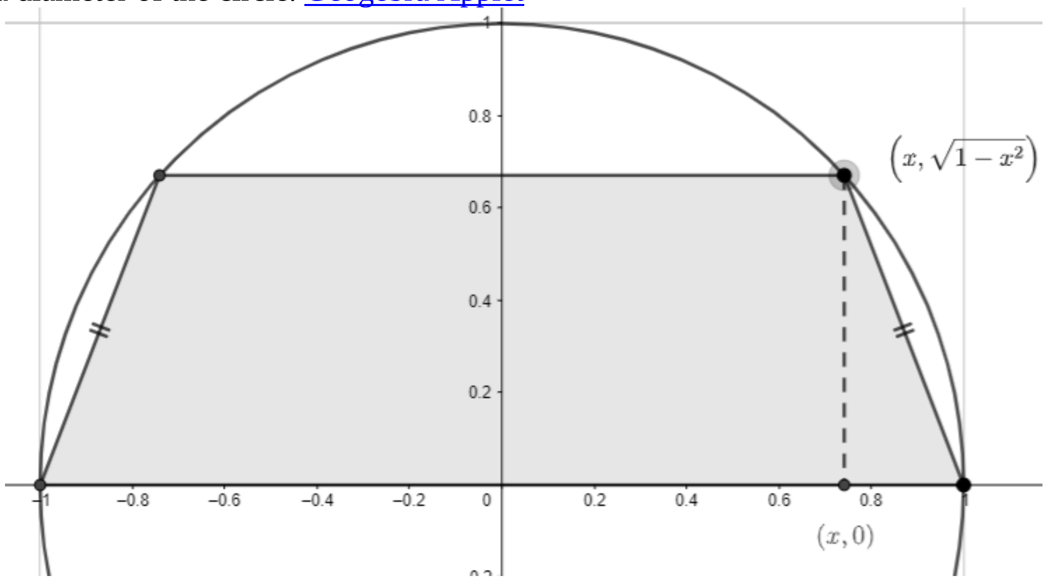
Stewart Section 4-7 Optimization Exercises:

#26 Find the area of the largest trapezoid that can be inscribed in a circle of radius 1 and whose base is a diameter of the circle. Geogebra Applet



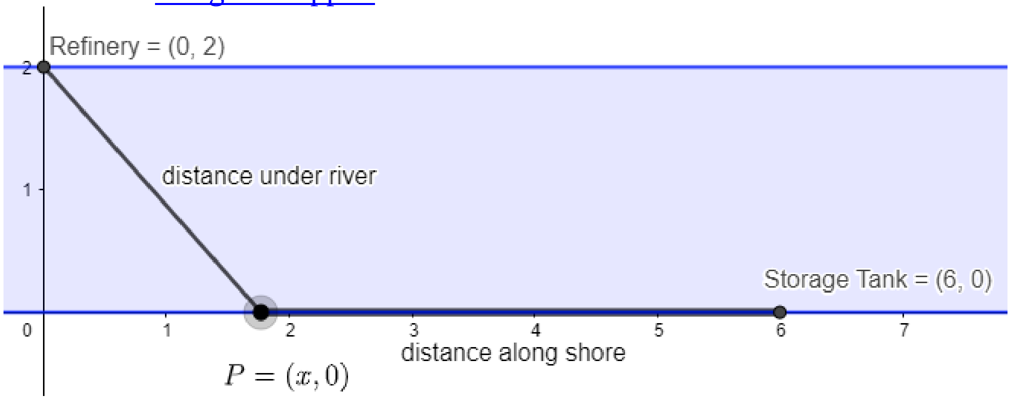
Feasible domain for x is $(0,1)$. Since the interval is bounded, we will use EVT with the closed interval $[0,1]$.

Area of the trapezoid is given by

$A=\frac{1}{2}(b_1+b_2)\cdot h$ $A(x)=\frac{1}{2}(2+2x)\cdot\sqrt{1-x^2}$ $=\frac{1}{2}(2+2x)(1-x^2)^{\frac{1}{2}}$	$A'(x)=\frac{1}{2}\left[2(1-x^2)^{\frac{1}{2}}+(2+2x)\cdot\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot 2x\right]$ $A'(x)=0$ or DNE when $x=\frac{1}{2}$
$A(0)=0$ $A\left(\frac{1}{2}\right)=\frac{3\sqrt{3}}{4}\approx 1.2990...$ $A(1)=0$	

The area of the largest trapezoid that can be inscribed in a circle of radius 1 and whose base is a diameter of the circle is $\frac{3\sqrt{3}}{4}$ units.

#49 An oil refinery is located on the north bank of a straight river that is 2 km wide. A pipeline is to be constructed from the refinery to storage tanks located on the south bank of the river 6 km east of the refinery. The cost of laying pipe is $400,000/km over land to a point *P* on the south bank and $800,000 under the river to the storage tanks. To minimize the cost of the pipeline, where should be *P* located? [Geogebra Applet]



The feasible domain for x is $[0,6]$ so EVT must be used.

Cost of the pipeline is given by

$$\text{Cost} = (\text{km under water})\left(\frac{\$}{\text{km}}\text{ underwater}\right) + (\text{km on land})\left(\frac{\$}{\text{km}}\text{ on land}\right)$$

$$\begin{aligned} C(x) &= \left(\sqrt{x^2+2^2}\right)(800,000)+(6-x)(400,000) \\ &= 400,000\left[2\sqrt{4+x^2}+(6-x)\right] \\ &= 400,000\left[2\left(4+x^2\right)^{\frac{1}{2}}+(6-x)\right] \end{aligned}$$

$$\downarrow$$

$$C'(x) = 400,000\left[\left(4+x^2\right)^{-\frac{1}{2}}\cdot 2x-1\right]$$

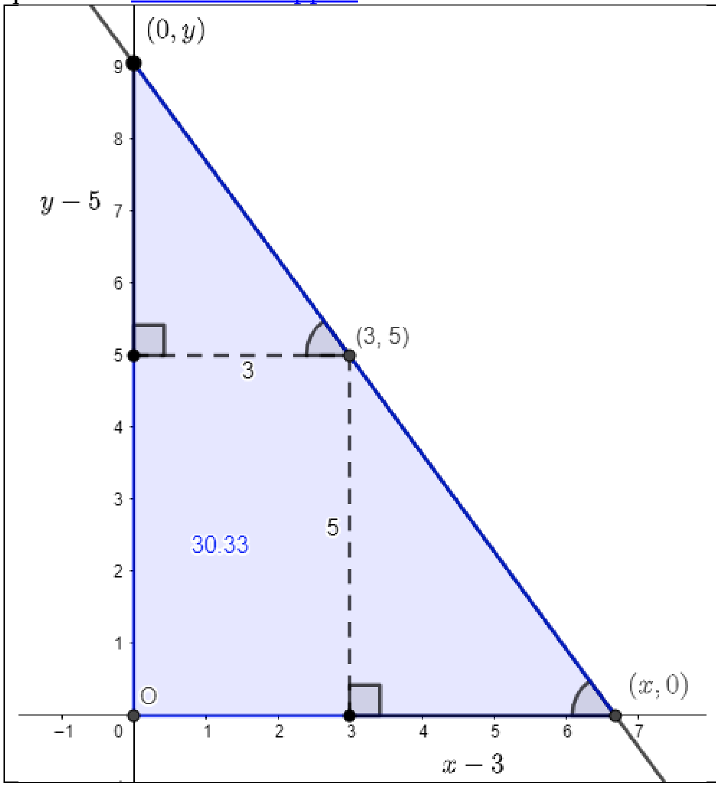
$C'(x)=0$ when $x=\dfrac{2\sqrt{3}}{3}\approx 1.1547...$

$$C(0)=4,000,000$$

$$C\left(\frac{2\sqrt{3}}{3}\right)=3,785,640.6460...$$

$$C(6)=5.059,644.2562...$$

#52 Find an equation of the line through the point $(3,5)$ that cuts off the least area from the first quadrant. [GeoGebra Applet]



Using Similar Triangles we have the following proportion.

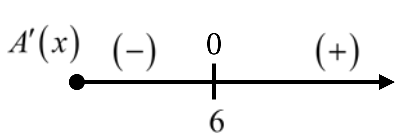
$$\frac{3}{x-3}=\frac{y-5}{5}$$

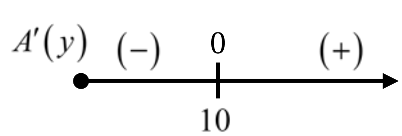
$$15=(y-5)(x-3)$$

$$\downarrow$$

$$x=\frac{15}{y-5}+3 \qquad y=\frac{15}{x-3}+5$$

$A=\frac{1}{2}xy$	
$A(x)=\frac{1}{2}x\left(\frac{15}{x-3}+5\right)$ $\downarrow$ $A'(x)=\frac{1}{2}\left(\frac{15}{x-3}+5\right)+\frac{1}{2}x\left(-15(x-3)^{-2}\right)$ $A'(x)=0$ when $x=6$	$A(y)=\frac{1}{2}y\left(\frac{15}{y-5}+3\right)$ $\downarrow$ $A'(y)=\frac{1}{2}\left(\frac{15}{y-5}+3\right)+\frac{1}{2}y\left(-15(y-5)^{-2}\right)$ $A'(y)=0$ when $y=10$





The area of the triangle is minimized when $x=6\,/\,y=10$ because $A'(x)\,/\,A'(y)$ changes sign from negative to positive.

An equation of the line that will minimize the area in the first quadrant is $y-5=-\frac{5}{3}(x-3)$.

#60 During the summer months Terry makes and sells necklaces on the beach. Last summer he sold the necklaces for $10 each and his sales averaged 20 per day. When he increased the price by $1, he found that the average decreased by two sales per day.
(a) Find the demand function, assuming that it is linear.

Let P be the price of the necklace. Let n be the number of necklaces sold at price P. Then

$$n = 20 - 2(P - 10) \text{ where } 10 \le P \le 20$$

(b) If the material for each necklace costs Terry $6, what should the selling price be to maximize his profit?

$$\begin{aligned}
\text{Profit} &= \text{Revenue} - \text{Cost} \\
&= (\text{number of necklaces sold})(\text{price per necklace}) - (\text{number of necklaces sold})(\text{cost per necklace}) \\
&= \left[20 - 2(P - 10)\right](P) - 6\left[20 - 2(P - 10)\right] \\
&= -2P^2 + 52P - 240 \\
&\downarrow
\end{aligned}$$

$$[\text{Profit}]' = 52 - 4x$$

$$[\text{Profit}]' = 0 \text{ or } DNE \text{ when } P = 13$$

$$\begin{aligned}
\text{Profit}(10) &= 80 \\
\text{Profit}(13) &= 98 \\
\text{Profit}(20) &= 0
\end{aligned}$$

Terry should sell necklaces at a cost of $13 per necklace to maximize profit.