AP Calculus BC
Stewart Chapter 2 Test (Limits)
Fall 2020 Solutions

Name:____________________________________

Date:________________________ Period:_________

Free Response Section: *NO CAS Calculator Permitted.*
You have the remainder of the period to complete this section.
Once you submit your Free Response Section, you will not be allowed to revisit it.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. To be eligible for partial credit, methods, reasoning, and conclusions should be presented clearly.

- Justifications require that you give mathematical (non-calculator) reasons. Students should use complete sentences in responses that include explanations or justifications.

ALL LIMITS MUST BE DETERMINED ANALYTICALLY!
No use of L'Hopital's Rule or Differentiation Rules are permitted.

1. $\displaystyle\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$	**2.** $\displaystyle\lim_{x\to 0}\frac{x\sin(x)}{\lvert x\rvert}$
$\displaystyle\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}=\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}\cdot\frac{\left(\sqrt{x+2}+\sqrt{2}\right)}{\left(\sqrt{x+2}+\sqrt{2}\right)}$ $\displaystyle=\lim_{x\to 0}\frac{(x+2)-(2)}{x\left(\sqrt{x+2}+\sqrt{2}\right)}$ $\displaystyle=\lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}\left(\sqrt{x+2}+\sqrt{2}\right)}$ $\displaystyle=\lim_{x\to 0}\frac{1}{\sqrt{x+2}+\sqrt{2}}$ $\displaystyle=\frac{2}{\sqrt{2}+\sqrt{2}}=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$	$\displaystyle\lim_{x\to 0}\frac{x\sin(x)}{\lvert x\rvert}=0$ $\displaystyle\lim_{x\to 0^-}\frac{x\sin(x)}{\lvert x\rvert}=\lim_{x\to 0^-}\frac{x\sin(x)}{-x}$ $\displaystyle=\lim_{x\to 0^-}\left[-\sin(x)\right]$ $=0$ $\displaystyle\lim_{x\to 0^+}\frac{x\sin(x)}{\lvert x\rvert}=\lim_{x\to 0^+}\frac{x\sin(x)}{x}$ $\displaystyle=\lim_{x\to 0^+}\left[\sin(x)\right]$ $=0$

3. $\lim\limits_{x\to 0}\dfrac{1-\cos^2(x)}{\sin(x)\left(1+\cos(x)\right)}$	**4.** $\lim\limits_{x\to\infty}\dfrac{3x^2+1}{\sqrt{x^3-3x^2+x+1}}$
$\begin{aligned}\lim_{x\to 0}\frac{1-\cos^2(x)}{\sin(x)\left(1+\cos(x)\right)} &= \lim_{x\to 0}\frac{\sin^2(x)}{\sin(x)\left(1+\cos(x)\right)}\\ &= \lim_{x\to 0}\frac{\sin(x)}{\left(1+\cos(x)\right)}\\ &= \frac{\lim\limits_{x\to 0}\left[\sin(x)\right]}{\lim\limits_{x\to 0}\left(1+\cos(x)\right)}\\ &= \frac{0}{2}=0\end{aligned}$	$\begin{aligned}\lim_{x\to\infty}\frac{3x^2+1}{\sqrt{x^3-3x^2+x+1}} &\sim \lim_{x\to\infty}\frac{3x^2}{\sqrt{x^3}}\\ &= \lim_{x\to\infty}\frac{3x^2}{\sqrt{x^3}}\\ &= \lim_{x\to\infty}\frac{3x^2}{x^{\frac{3}{2}}}\\ &= \lim_{x\to\infty}3x^{\frac{1}{2}}\\ &= \lim_{x\to\infty}3\sqrt{x}\end{aligned}$ DNE or $+\infty$

5. $\lim\limits_{h\to3}\dfrac{3(h-2)^2-h}{h-3}$	**6.** $\lim\limits_{h\to0}\dfrac{\frac{1}{3-h}-\frac{1}{3}}{h}$
$\begin{aligned}\lim_{h\to3}\frac{3(h-2)^2-h}{h-3}&=\lim_{h\to3}\frac{3(h^2-4h+4)-h}{h-3}\\&=\lim_{h\to3}\frac{3h^2-12h+12-h}{h-3}\\&=\lim_{h\to3}\frac{3h^2-13h+12}{h-3}\\&=\lim_{h\to3}\frac{\cancel{(h-3)}(3h-4)}{\cancel{h-3}}\\&=\lim_{h\to3}[3h-4]\\&=5\end{aligned}$	$\begin{aligned}\lim_{h\to0}\frac{\frac{1}{3-h}-\frac{1}{3}}{h}&=\lim_{h\to0}\frac{\frac{1}{3-h}\cdot\frac{3}{3}-\frac{1}{3}\cdot\frac{(3-h)}{(3-h)}}{h}\\&=\lim_{h\to0}\frac{\frac{3}{3(3-h)}-\frac{3-h}{3(3-h)}}{h}\\&=\lim_{h\to0}\frac{\left[\frac{h}{3(3-h)}\right]}{h}\\&=\lim_{h\to0}\frac{\cancel{h}}{3\cancel{h}(3-h)}\\&=\lim_{h\to0}\frac{1}{3(3-h)}\\&=\frac{1}{9}\end{aligned}$

7. $\lim\limits_{x\to\frac{3\pi}{4}}\dfrac{\sin(x)-\cos(x)}{\tan(x)-1}$	**8.** $\lim\limits_{x\to0}\dfrac{5^x-5^{2x}}{1-5^x}$
$\begin{aligned}\lim_{x\to\frac{3\pi}{4}}\frac{\sin(x)-\cos(x)}{\tan(x)-1}&=\lim_{x\to\frac{3\pi}{4}}\frac{\sin(x)-\cos(x)}{\tan(x)-1}\\&=\frac{\sin\left(\frac{3\pi}{4}\right)-\cos\left(\frac{3\pi}{4}\right)}{\tan\left(\frac{3\pi}{4}\right)-1}\\&=\frac{\frac{1}{\sqrt{2}}-\left(-\frac{1}{\sqrt{2}}\right)}{(-1)-1}\\&=-\frac{\sqrt{2}}{2}\end{aligned}$	$\begin{aligned}\lim_{x\to0}\frac{5^x-5^{2x}}{1-5^x}&=\lim_{x\to0}\frac{5^x-5^x\cdot5^x}{1-5^x}\\&=\lim_{x\to0}\frac{5^x\cancel{(1-5^x)}}{\cancel{1-5^x}}\\&=\lim_{x\to0}5^x\\&=1\end{aligned}$

9. Explain why the function is not continuous using limits.

$$f(x)=\begin{cases}1-x^2 & x\neq 1\\ 2 & x=1\end{cases}$$

$\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^-}1-x^2$ $=0$	$f(1)=2$	$\lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1^+}1-x^2$ $=0$

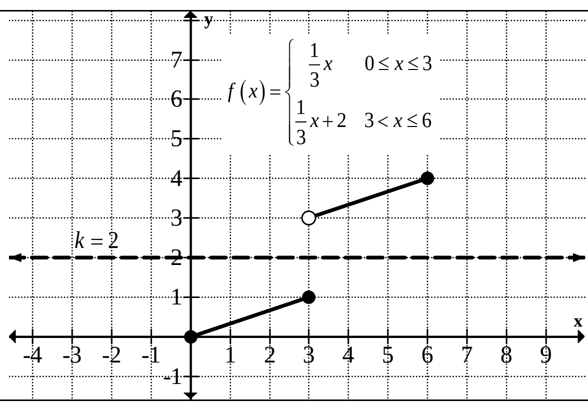
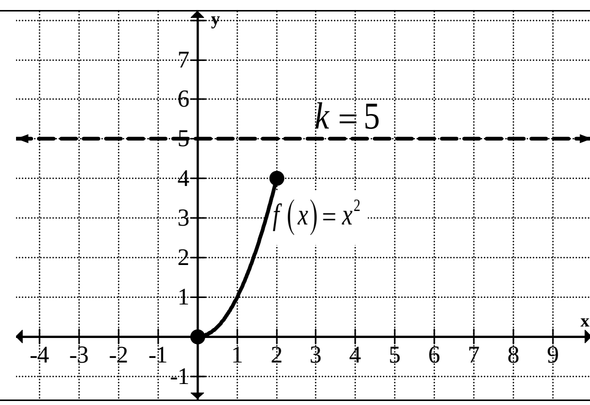
Any of the following statements below are acceptable, so long as each limit value has been demonstrated.

Since $\lim\limits_{x\to 1} f(x)\neq f(1)$ $f(x)$ is not continuous at $x=1$.

Since $\lim\limits_{x\to 1^-} f(x)\neq f(1)$ $f(x)$ is not continuous at $x=1$.

Since $\lim\limits_{x\to 1^+} f(x)\neq f(1)$ $f(x)$ is not continuous at $x=1$.

10. Sketch an example of a function on a closed interval for which the conclusion of the Intermediate Value Theorem is not true. Identify the value of *k* in your example, and explain what condition(s) of the hypothesis of the theorem are not met.

	
Here, the value of *k* is 2 and the interval is the closed interval $[0,6]$. The conclusion of the hypothesis of the Intermediate Value Theorem does not apply since $f(x)$ is not continuous on the closed interval $[0,6]$.	Here, the value of *k* is 5 and the interval is the closed interval $[0,2]$. The conclusion of the hypothesis of the Intermediate Value Theorem does not apply because the value of *k* is not between $f(0)$ and $f(2)$.