2002 #2 Calculator Allowed: The rate are which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t)=\frac{15600}{\left(t^2-24t+160\right)}$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t)=\frac{9890}{\left(t^2-38t+370\right)}$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measure in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t=9$, there are no people in the park.

(a) How many people have entered the park by 5:00pm $(t=17)$? Round your answer to the nearest whole number.

$\int_9^{17} E(t)\,dt \approx 6004.270$. 6004 people have entered the park by 5 pm.

$$3:\begin{cases}1:\ \text{limits}\\ 1:\ \text{integrand}\\ 1:\ \text{answer}\end{cases}$$

(b) The price of admission to the park is \$15 until 5:00pm $(t=17)$. After 5:00pm, the price of admission to the park is \$11. How many dollars are collected from admission to the park on the given day? Round your answer to the nearest whole number.

$$\begin{aligned}\text{Revenue} &= 15\cdot\int_9^{17} E(t)\,dt + 11\cdot\int_{17}^{23} E(t)\,dt\\ &\approx 104{,}048.165\end{aligned} \quad \text{or} \quad \begin{aligned}\text{Revenue} &= 15\cdot 6004 + 11\cdot\int_{17}^{23} E(t)\,dt\\ &\approx 104{,}044.110\end{aligned}$$

$$\begin{aligned}\text{Revenue} &= 15\cdot 6004 + 11\cdot 1271\\ &\approx 104{,}041.00\end{aligned}$$

\$104,048 or \$104,044 or \$104,041 have been collected from admission on the given day.

1: setup

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(c) Let $H(t)=\int_9^t \left(E(x)-L(x)\right)dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.

$$H(t)=\int_9^t \left(E(x)-L(x)\right)dx$$

$$\downarrow$$

$$H'(t)=E(t)-L(t)$$

$$H'(17)\approx -380.281$$

$$3:\begin{cases}1:\ \text{value of } H'(17)\\ 2:\ \text{meanings}\\ \quad 1:\ \text{meaning of } H(17)\\ \quad 1:\ \text{meaning of } H'(17)\\ \quad \langle -1\rangle \text{ if no reference to } t=17\end{cases}$$

$H(17)=3725$ means that the number of people in the park at time $t=17$ is 3725.

$H'(17)\approx -380.281$ means that the number of people in the park is decreasing at a rate of 380.281 people per hour.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

$$E(t)-L(t)=0$$
$$E(t)=L(t)$$
$$\downarrow$$
$$t=15.7948...$$

$$H(9)=0$$
$$H(15.7948...)=3950.680$$
$$H(23)=1.013(4)$$

$$2:\begin{cases}1: E(t)-L(t)=0\\ 1:\ \text{answer}\end{cases}$$

The model predicts that the number of people in the park is at a maximum when $t\approx 15.7948...$ hours.

2019 #1 Calculator allowed: Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds, and t is measured in days

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

$$\underset{0\le t\le 30}{AROC} \text{ of } A(t) = \frac{A(30) - A(0)}{30 - 0} \approx -0.1968... \ \frac{\text{pounds}}{\text{day}}$$

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

$$A(t) = 6.687(0.931)^t$$
$$\downarrow$$
$$A'(t) = 6.687 \cdot \ln(0.931) \cdot (0.931)^t$$
$$A'(15) = 6.687 \cdot \ln(0.931) \cdot (0.931)^{15}$$
$$\approx -0.1635...$$

$A'(15)$ represents the rate at which the grass clippings are decomposing, in pounds per day, at $t = 15$ days.

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(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

$$\underset{0 \le t \le 30}{\text{Average Value}} \text{ of } A(t) = \frac{1}{30-0}\int_0^{30} A(t)\,dt$$
$$\approx 2.752(3)$$

$$\frac{1}{30-0}\int_0^{30} A(t)\,dt = A(t)$$
$$\downarrow$$
$$t \approx 12.414(5)$$

The time at which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$ is approximately 12.414(5) days.

(d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

$$A(30) \approx 0.7829...$$
$$A'(30) \approx -0.5597...$$
$$L(t)$$
$$\downarrow$$
$$y - A(30) = A'(30)(t-30)$$
$$y = -0.5597...(t-30) + 0.7829...$$
$$L(t) = -0.5597...(t-30) + 0.7829...$$

$$L(t) = 0.5 \rightarrow t \approx 35.054$$

2005 #2 Calculator allowed: Tide removes sand from Sandy Point Beach at a rate modeled by the function *R*, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$$

A pumping station adds sand to the beach at a rate modeled by the function *S*, given by

$$S(t) = \frac{15t}{1+3t}$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and *t* is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) [3 points] How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

$\int_0^6 R(t)\,dt \approx 31.8159...$ The tide removes 31.8159… cubic yards of sand from the beach during this six hour period.

(b) [3 points] Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time *t*.

$$Y(t) = 2500 + \int_0^t S(x) - R(x)\,dx$$

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(c) [2 points] Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

$$Y'(t) = S(t) - R(t)$$
$$Y'(4) = S(4) - R(4)$$
$$= -1.9087...$$

The rate at which the total amount of sand on the beach is changing at time $t = 4$ is $-1.9087...$ cubic yards per hour.

(d) [4 points] For $0 \le t \le 6$, at what time *t* is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

$Y'(t) = 0$ or $S(t) - R(t) = 0$ when $x = 5.1178...$

$$Y(0) = 2500$$
$$Y(5.1178...) = 2500 + \int_0^{5.1178...} S(t) - R(t)\,dt = 2492.3694...$$
$$Y(6) = 2500 + \int_0^{6} S(t) - R(t)\,dt = 2493.2766...$$

The amount of sand on the beach will be at a minimum when $t = 5.1178...$ hours.