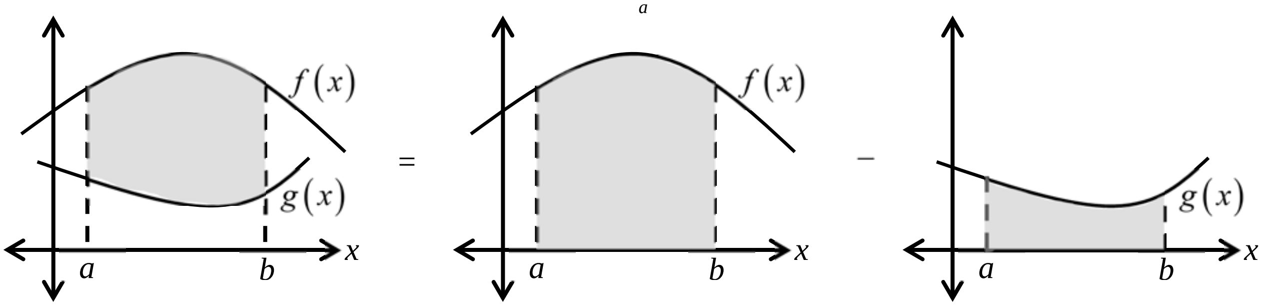
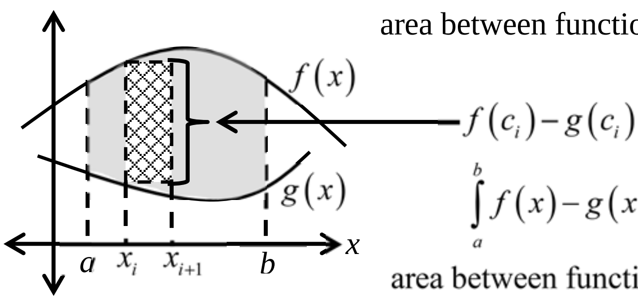
If $f$ and $g$ are continuous on $\left[a,b\right]$ and $g\left(x\right)\le f\left(x\right)$ for all $x$ in $\left[a,b\right]$, then the region bounded by graph of $f\left(x\right)$, $g\left(x\right)$, $x=a$, and $x=b$ is given by:

$$A=\int_a^b f\left(x\right)-g\left(x\right)dx$$



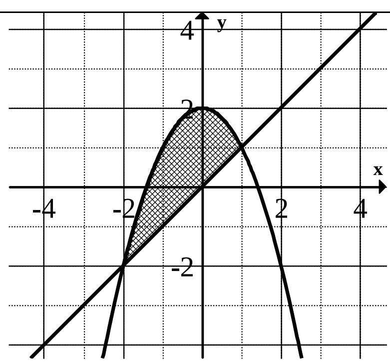
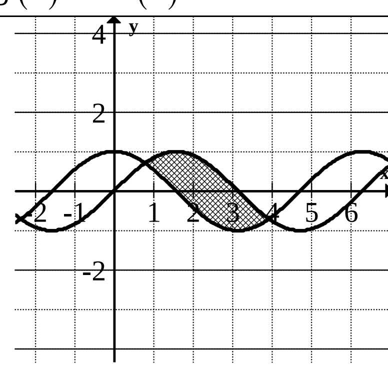

$$\int_a^b f\left(x\right)-g\left(x\right)dx=\int_a^b f\left(x\right)dx-\int_a^b g\left(x\right)dx$$

area between functions = difference of the areas




$$\int_a^b f\left(x\right)-g\left(x\right)dx=\lim_{\|\Delta\|\to 0}\sum_{i=1}^{n}\left(f\left(c_i\right)-g\left(c_i\right)\right)\left(\Delta x\right)_i$$

area between functions = sum of the areas of the rectangles between the functions

| $f\left(x\right)=2-x^2$ $x=-2$ to $x=1$<br>$g\left(x\right)=x$ | $f\left(x\right)=\sin\left(x\right)$ $x=\frac{\pi}{4}$ to $x=\frac{5\pi}{4}$<br>$g\left(x\right)=\cos\left(x\right)$ |
|---|---|
| <br>$\int_{-2}^{1}\left(2-x^2\right)-xdx$<br>or<br>$\int_{-2}^{1}\left(2-x^2\right)dx-\int_{-2}^{1}xdx$ | <br>$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\sin\left(x\right)-\cos\left(x\right)dx$<br>or<br>$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\sin\left(x\right)dx-\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\cos\left(x\right)dx$ |

Area bounded by the functions

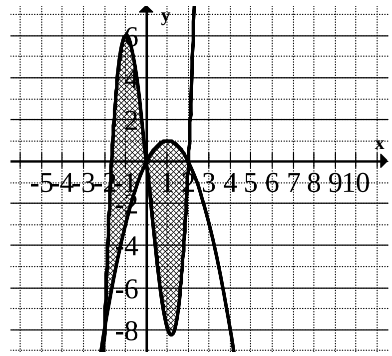
$f(x) = 3x^3 - x^2 - 10x$

$g(x) = -x^2 + 2x$

$$\text{Area} = \int_{-2}^{0} f(x) - g(x)\,dx + \int_{0}^{2} g(x) - f(x)\,dx$$

$$= \int_{-2}^{2} \left| f(x) - g(x) \right| dx$$

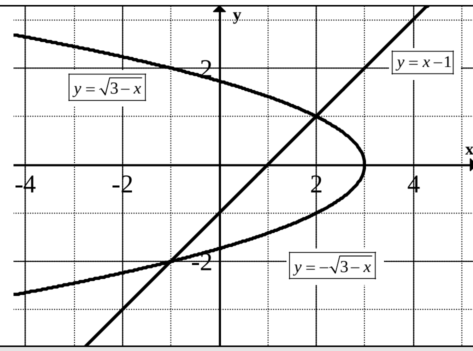
$$= \int_{-2}^{2} \left| g(x) - f(x) \right| dx$$



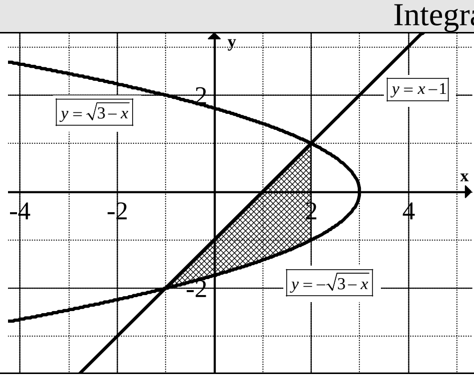
Consider $\begin{matrix} x = 3 - y^2 \\ x = y + 1 \end{matrix}$ The curves intersect at the points $(2,1)$ and $(-1,-2)$.

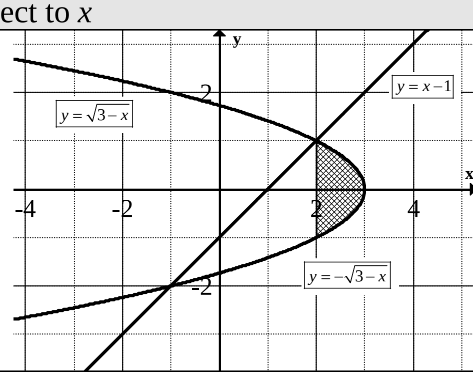
$$x = 3 - y^2 \quad \leftrightarrow \quad y = \pm\sqrt{3-x}$$

$$x = y + 1 \quad \leftrightarrow \quad y = x - 1$$




Integrate with respect to *x*




$$\int_{-1}^{2} (x-1) - \left(-\sqrt{3-x}\right) dx$$

$$\int_{2}^{3} \left(\sqrt{3-x}\right) - \left(-\sqrt{3-x}\right) dx$$

Integrate with Respect to *y*

$$\int_{-2}^{1} \left(3 - y^2\right) - (y+1)\,dy$$