Differentiability of Piecewise Functions:

1. For what value of c is the function f continuous on $(-\infty,\infty)$? Justify your answer.

$$f(x)=\begin{cases} cx+7 & x\le 2 \\ cx^2-5 & x>2 \end{cases}$$

Is the function differentiable with this value of c? Justify your answer.

2. Let $f(x)=\begin{cases} x^2+1 & x<1 \\ 2x+1 & x\ge 1 \end{cases}$. Is f differentiable at $x=1$? Justify your answer.

3. Let $f(x)=\begin{cases} 3x^2+x & x\le 0 \\ \sin(x) & x>0 \end{cases}$. Is $f(x)$ differentiable at $x=0$? Justify your answer.

Derivative of Inverse Functions

The following table gives the values of a differentiable function f, and its derivative f' at given values of x.

x	f	f'
1	2	$\frac{1}{2}$
2	3	1
3	4	2
4	6	4

1. If $g(x)$ is the inverse function of $f(x)$, then what is the value of $g'(4)$?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) 2

2. If $f(x)=x^3-3x^2+8x+5$ and $g(x)=f^{-1}(x)$, then $g'(5)=$

(a) 8 (b) $\frac{1}{8}$ (c) 1 (d) $\frac{1}{53}$ (e) 5

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