

1. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{2^n}{n+1}$ II. $\sum_{n=1}^{\infty} \frac{3}{n}$ III. $\sum_{n=1}^{\infty} \frac{\cos(2\pi n)}{n^2}$

- (a) I only (b) II only (c) III only
(d) I and II only (e) I and III only

2. If $\sum_{n=0}^{\infty} a_n (x-c)^n$ is a Taylor series that converges to $f(x)$ for all real numbers x , then $f''(x) =$

- (a) 0
(b) $(n)(n-1)a_n$
(c) $\sum_{n=0}^{\infty} n \cdot a_n (x-c)^{n-1}$
(d) $\sum_{n=0}^{\infty} a_n$
(e) $\sum_{n=0}^{\infty} n(n-1)a_n (x-c)^{n-2}$

3. What are all the values for which the

series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n\sqrt{n} \cdot 3^n}$ converges?

- (a) $-3 < x < 3$
(b) $-3 \leq x \leq 3$
(c) $-5 < x < 1$
(d) $-5 < x \leq 1$
(e) $-5 \leq x \leq 1$

4. Calculator required: The sum of the infinite geometric series

$$\frac{4}{5} + \frac{8}{35} + \frac{16}{245} + \frac{32}{1715} + \dots$$

- (a) 0.622 (b) 0.893 (c) 1.120
(d) 1.429 (e) 2.800

5. For what integer $k > 1$ will both

$$\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n^2} \text{ and } \sum_{n=1}^{\infty} \left(\frac{k}{3}\right)^n$$
 converge:

- (a) 2 (b) 3 (c) 4 (d) 5 (e) 6

6. What are all the values of x for which

the series $\sum_{n=1}^{\infty} \frac{(2x+3)^n}{\sqrt{n}}$ converges?

- (a) $-2 < x < -1$
(b) $-2 \leq x < -1$
(c) $-2 < x \leq -1$
(d) $-2 \leq x \leq -1$
(e) $-2 \leq x < 1$

7. The Taylor polynomial of degree 3 centered at $x=0$ for $f(x) = \sqrt{1+x}$ is

- (a) $1 + \frac{1}{2}x - \frac{1}{4}x^2 + \frac{3}{8}x^3$
(b) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$
(c) $1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3$
(d) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{8}x^3$
(e) $1 - \frac{1}{2}x + \frac{1}{4}x^2 - \frac{3}{8}x^3$

8. Which of the following series is divergent?

- (a) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ (c) $\sum_{n=1}^{\infty} \frac{n}{n^3 + 1}$
(d) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{4n^2 - 1}}$ (e) None of these

9. Which one of the following series is convergent?

- (a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ (b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ (c) $\sum_{n=1}^{\infty} \frac{1}{n}$
 (d) $\sum_{n=1}^{\infty} \frac{1}{10n-1}$ (e) $\sum_{n=1}^{\infty} \frac{2}{n^2-5}$

10. Which of the following statements are false?

(a) $\sum_{n=1}^{\infty} a_n = \sum_{n=k}^{\infty} a_n$ where k is any positive integer.

(b) If $\sum_{n=1}^{\infty} a_n$ converges, then so does

$$\sum_{n=1}^{\infty} c \cdot a_n, \text{ where } c \neq 0.$$

(c) $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge, so does

$$\sum_{n=1}^{\infty} (c \cdot a_n + b_n) \text{ where } c \neq 0$$

(d) If 1000 terms are added to a convergent series, the new series also converges.

(e) Rearranging the terms of a positive convergent series will not affect its convergence or sum.

11. The series

$$(x-2) + \frac{(x-2)^2}{4} + \frac{(x-2)^3}{9} + \frac{(x-2)^4}{16} + \dots$$

converges for

- (a) $1 \leq x \leq 3$ (b) $1 \leq x < 3$ (c) $1 < x \leq 3$
 (d) $0 \leq x \leq 4$ (e) None of these

12. The radius of convergence of the series

$$\frac{x}{4} + \frac{x^2}{4^2} + \frac{x^3}{4^3} + \dots + \frac{x^n}{4^n} + \dots$$

- (a) 0 (b) 1 (c) 2
 (d) 4 (e) All real numbers

13. Which of the following series are conditionally convergent?

I. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$

II. $\sum_{n=1}^{\infty} (-1)^n \frac{\cos(n)}{3^n}$

III. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$

- (a) I only (b) II only (c) I, II, and III
 (d) I and III only (e) I and II only

14. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor Series about $x=0$ for which of the following functions?

- (a) $\sin(x)$ (b) $\cos(x)$ (c) e^x
 (d) e^{-x} (e) $\ln(1+x)$

15. $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{2n} =$

- (a) $\frac{1}{8}$ (b) $\frac{1}{3}$ (c) 1 (d) $\frac{9}{8}$ (e) ∞

16. Calculator required: The graph of the function represented by the Maclaurin series

$$x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \dots = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$$

intersects the graph of $y = 1 + x^2$ at the point where $x =$

- (a) 0.718 (b) 0.738 (c) 0.758
 (d) 0.778 (e) 0.798