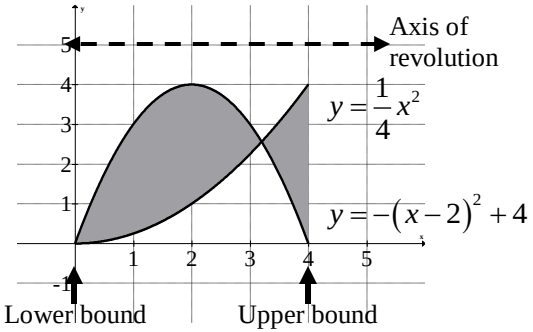


Horizontal Axis of Revolution

↓

Integrate with respect to $x \rightarrow \int \underline{\underline{dx}}$

↓

Determine the greatest and least x-coordinates of your region

Greatest→ Upper Bound

Least →Lower Bound

↓

Determine each subinterval on the x-axis for which the greater and lesser functions change.

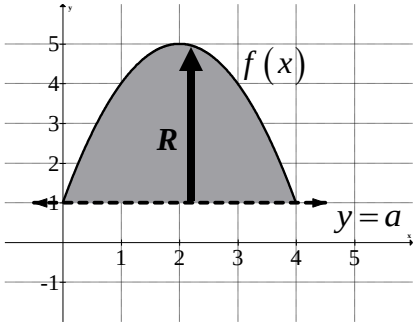
FOR EACH SUBINTERVAL

↓

Determine whether your region lies above or below the axis of revolution

Above the axis of revolution

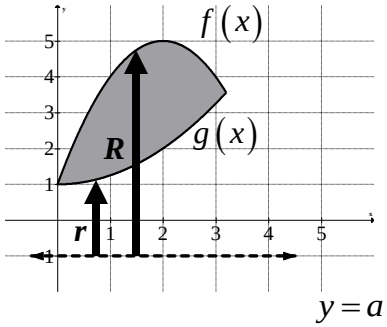
Adjacent to axis of revolution



$$\pi\int_c^d \left[R(x)\right]^2 dx$$

$$R(x)=f(x)-a$$

Not adjacent to axis of revolution



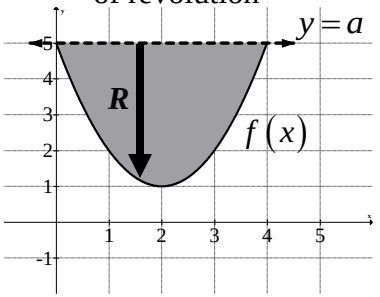
$$\pi\int_c^d \left[R(x)\right]^2-\left[r(x)\right]^2 dx$$

$$R(x)=f(x)-a$$

$$r(x)=g(x)-a$$

Below the axis of revolution

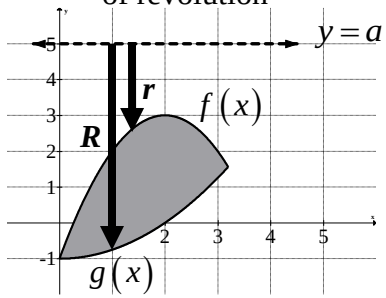
Adjacent to axis of revolution



$$\pi\int_c^d \left[R(x)\right]^2 dx$$

$$R(x)=a-f(x)$$

Not adjacent to axis of revolution



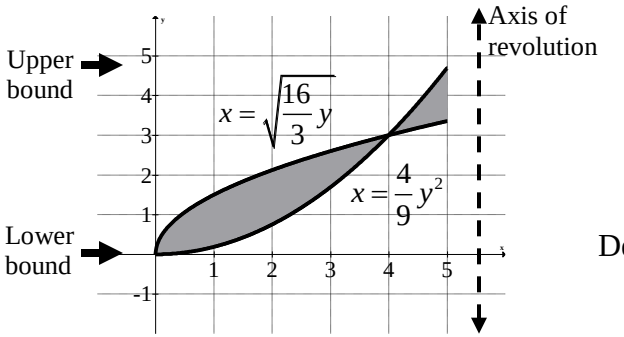
$$\pi\int_c^d \left[R(x)\right]^2-\left[r(x)\right]^2 dx$$

$$R(x)=a-g(x)$$

$$r(x)=a-f(x)$$

$R(x)$ is always measured from the axis of revolution to the function farthest from axis of revolution (i,e. outside edge of the region.)

$r(x)$ is always measured from the axis of revolution to the function closest to the axis of revolution (i.e. inside edge of the region.)



Vertical Axis of Revolution

↓

Integrate with respect to $y \rightarrow \int dy$

Rewrite your functions as functions of y: $x = f(y)$

↓

Determine the greatest and least y-coordinates of your region

Greatest→ Upper Bound

Least →Lower Bound

↓

Determine each subinterval on the y-axis for which the greater and lesser functions change.

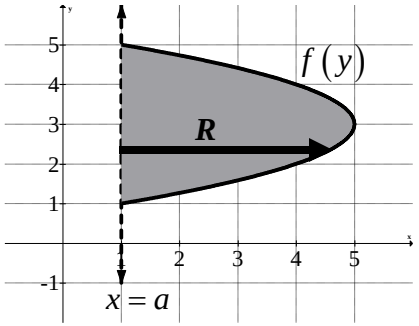
FOR EACH SUBINTERVAL

↓

Determine whether your region lies to the right or to the left of the axis of revolution

To the right of the axis of revolution

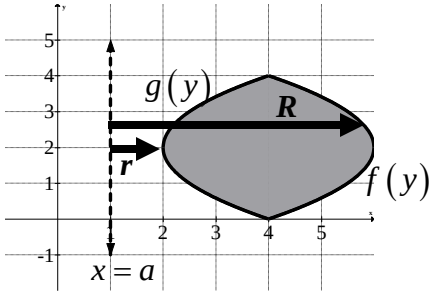
Adjacent to axis of revolution



$$\pi\int_c^d \left[R(y)\right]^2 dy$$

$$R(y) = f(y) - a$$

Not adjacent to axis of revolution



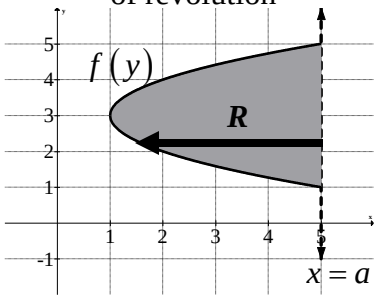
$$\pi\int_c^d \left[R(y)\right]^2 - \left[r(y)\right]^2 dy$$

$$R(y) = f(y) - a$$

$$r(y) = g(y) - a$$

To left of the axis of revolution

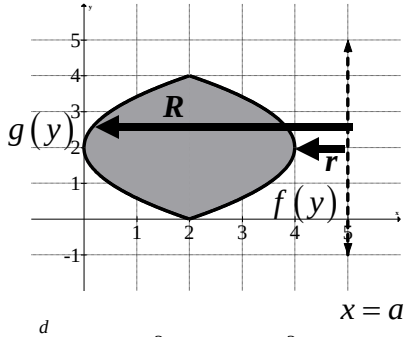
Adjacent to axis of revolution



$$\pi\int_c^d \left[R(y)\right]^2 dy$$

$$R(y) = a - f(y)$$

Not adjacent to axis of revolution



$$\pi\int_c^d \left[R(y)\right]^2 - \left[r(y)\right]^2 dy$$

$$R(y) = a - g(y)$$

$$r(y) = a - f(y)$$

$R(x)$ is always measured from the axis of revolution to the function farthest from axis of revolution (i,e. outside edge of the region.)

$r(x)$ is always measured from the axis of revolution to the function closest to the axis of revolution (i.e. inside edge of the region.)