Conservation of Energy Problems Methods

The Model

For any system and any process where non-mechanical energy is not converted to mechanical energy

$$\boldsymbol{E_f = E_i + W_{other}}$$

Where mechanical energy is defined as

$$\boldsymbol{E = U + K}$$

And $\boldsymbol{W_{other}}$ is the work done by forces other than conservative forces on that system[1].

In this class we have two types of potential energy:

$$\boldsymbol{U_g = mgh} \qquad \text{and} \qquad \boldsymbol{U_{el} = \frac{1}{2}kx^2}$$

And of course we have kinetic energy:

$$\boldsymbol{K = \frac{1}{2}mv^2}$$

Problem Solving Steps

1) Organize and Plan
    a. Draw a clear diagram of the motion, labeling the key moments in time.
    b. Establish coordinate systems if using potential energies.
    c. Make and FBD for the object at a representative location in the middle of the motion.
    d. Determine which forces are conservative and which are non-conservative.
2) Solve
    a. Calculate the work done by each force, except for conservative forces. Usually, using these shortcuts is the way to go:
        i. If a force and the path are always perpendicular to each other, the work done by that force is zero.
        ii. If a force and the path are always in the same direction, and the magnitude of the force is constant, then the work is $F * d$.
        iii. If a force and the path are always in the opposite direction, and the magnitude of the force is constant, then the work is $-F * d$.
        iv. If a force is of constant magnitude and the angle between the force and the path is always θ, the work is $F * d * \cos(\theta)$.
        v. If a force is of varying magnitude, but the angle between the force and the path is always θ, then the work is $\cos(\theta) \int_A^B F(s) * ds$.
    b. Add up all of the "other" works from above to get the total "other" work.

c. Break down each term for “E” in the energy conservation equation into kinetic and potential energies. (In some problems you may have both gravitational and elastic potential energy).
d. Evaluate the initial and final energies using what you know, and add in the “other” work.
e. Solve for what you don’t know!

3) Reflect
   a. Does the answer make sense?
   b. Are the units correct?
   c. Did you use any new techniques?
   d. Any other insights?

1. Technical note: $\boldsymbol{W_{other}}$ really represents work done by two types of forces: forces of any type that are external to the system, and forces that are internal to the system but are dissipative. For our purposes, we will always consider conservative forces to be internal to our systems, with systems implicitly defined to include the bodies involved in creating the conservative forces. This means that we can treat $\boldsymbol{W_{other}}$ as representing the work done by non-conservative forces.