

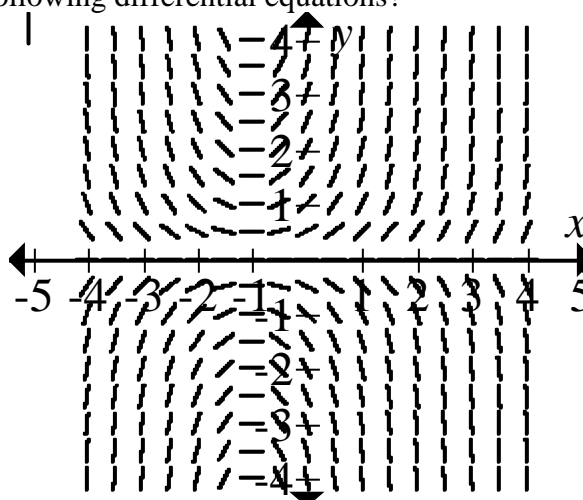
No Calculator Permitted

1. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t , where k is a positive constant?

- (a) $\frac{dp}{dt} = kp$ (b) $\frac{dp}{dt} = kp(N - p)$ (c) $\frac{dp}{dt} = kp(p - N)$ (d) $\frac{dp}{dt} = kt(N - t)$ (e) $\frac{dp}{dt} = kt(t - N)$

2. Shown at right is a slope field for which of the following differential equations?

- (a) $\frac{dy}{dx} = xy$
 (b) $\frac{dy}{dx} = xy - y$
 (c) $\frac{dy}{dx} = xy + y$
 (d) $\frac{dy}{dx} = xy + x$
 (e) $\frac{dy}{dx} = (x+1)^3$



3. Given that $y(1) = -3$ and $\frac{dy}{dx} = 2x + y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5 starting at $x = 1$?

- (a) -5 (b) -4.25 (c) -4 (d) -3.75 (e) -3.5

4. Which differential equation does the function $y = e^{-3t}$ satisfy?

- (a) $y'' + y' + 12y = 0$
 (b) $y'' + y' - 12y = 0$
 (c) $y'' - y' - 12y = 0$
 (d) $y'' - y' + 12y = 0$
 (e) $y'' - 3y' + 12y = 0$

5. The acceleration a of a body moving in a straight line is given in terms of time t by $a(t) = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t , what is $s(4) - s(2)$?

- (a) 20 (b) 24 (c) 28 (d) 32 (e) 42

6. $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h} =$

- (a) 0 (b) 1 (c) $\sin(x)$ (d) $\cos(x)$ (e) DNE

7. The average value of $y = \frac{1}{x}$ on the interval closed interval $[1, 3]$ is

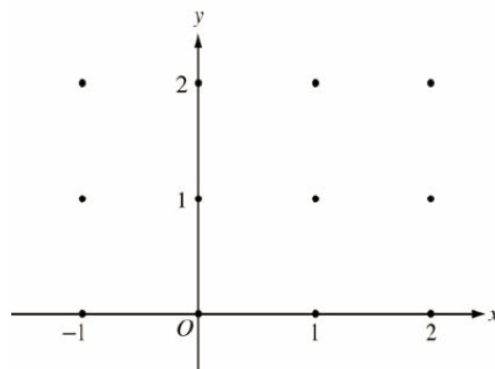
- (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{\ln(2)}{2}$ (d) $\frac{\ln(3)}{2}$ (e) $\ln(3)$

Calculator Required

8. Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let

$y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

- On the axis provided, sketch a slope field for the given differential equation at the twelve points indicated.
- Write an equation for the line tangent to the graph of f at $x = -1$.



Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

- c. Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

9. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x , where $f(3) = 25$.

a. Find $f''(3)$

b. Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x , where $f(3) = 25$.

- c. Use Euler's Method starting at $x = 3$ with two steps of equal size to approximate the value of $f(5)$. Show the work that leads to your answer.