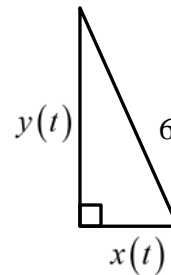$$x'(t) = \pm 0.5 \ \frac{m}{sec}$$
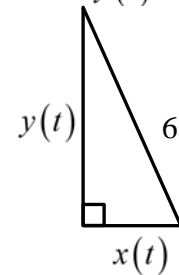
$x'(t) > 0$ if the ladder is moving **away** from the wall

$x'(t) < 0$ if the ladder is moving **towards** from the wall

$y(t) = 5\ m$

Slide Ladder
Show dy/dt
y = 5
y = 3
y = 1

6m

y = 5.6m y(t)

y(t)

x(t)

x = 2.3m

At any time t

$y(t)$ 6

$x(t)$

When $y(t) = 5$

$y(t)$ 6

$x(t)$

Relation equation:	At the moment that $y(t) = 5$ we can use this value to find
$\left[x(t) \right]^2 + \left[y(t) \right]^2 = 6^2$	$x(t)$ at the same moment.

Relation equation:

$$\left[x(t) \right]^2 + \left[y(t) \right]^2 = 6^2$$

$$\left[x(t) \right]^2 + \left[y(t) \right]^2 = 6^2$$

$\downarrow$

$$2x(t)\,x'(t) + 2y(t)\,y'(t) = 0$$
$$x(t)\,x'(t) + y(t)\,y'(t) = 0$$

At the moment that $y(t) = 5$ we can use this value to find $x(t)$ at the same moment.

$$5^2 + \left[x(t) \right]^2 = 6^2$$
$$25 + \left[x(t) \right]^2 = 36$$
$$\left[x(t) \right]^2 = 11$$
$$x(t) = \sqrt{11}$$

$$\underbrace{x(t)}_{\sqrt{11}\ m}\,\underbrace{x'(t)}_{0.5\ \frac{m}{sec}} + \underbrace{y(t)}_{5m}\,\underbrace{y'(t)}_{unknown} = 0$$
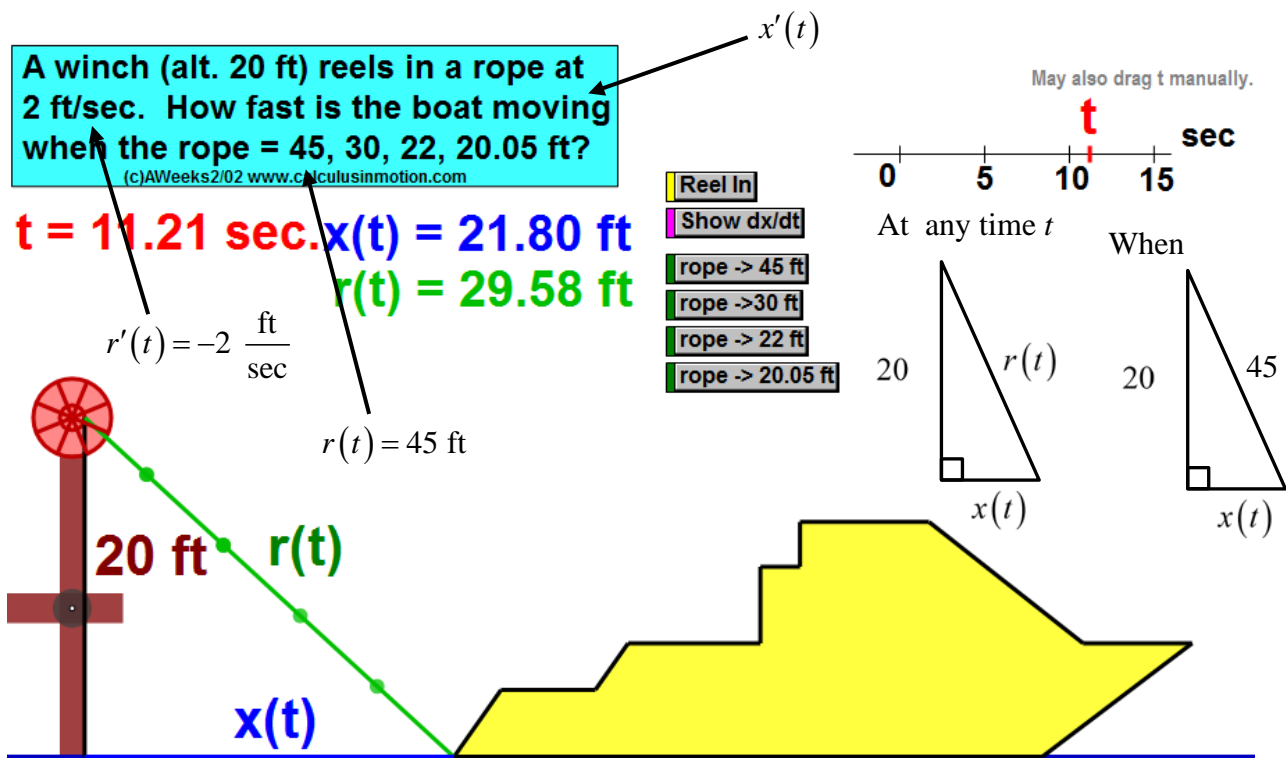
$$\sqrt{11}\left(\frac{1}{2} \right) + 5 \cdot y'(t) = 0$$

$$5 \cdot y'(t) = -\frac{\sqrt{11}}{2}$$

$$y'(t) = -\frac{\sqrt{11}}{10} = -0.3316... \ \frac{m}{sec}$$

The top of the ladder is moving down at a rate of $\dfrac{\sqrt{11}}{10} = 0.3316... \ \dfrac{m}{sec}$ when the bottom of the ladder is being pushed out at a rate of 0.5 $\dfrac{m}{sec}$ and $y(t) = 5$ meters.

The top of the ladder is moving at a rate of $-\dfrac{\sqrt{11}}{10} = -0.3316... \ \dfrac{m}{sec}$ when the bottom of the ladder is being pushed out at a rate of 0.5 $\dfrac{m}{sec}$ and $y(t) = 5$ meters..

Related Rates Exercises Solutions

$x'(t)$

May also drag t manually.

t sec

0   5   10   15

At any time t

When

t = 11.21 sec. **x(t) = 21.80 ft**
r(t) = 29.58 ft

Reel In
Show dx/dt
rope -> 45 ft
rope ->30 ft
rope -> 22 ft
rope -> 20.05 ft

$r'(t) = -2 \dfrac{ft}{sec}$

$r(t) = 45$ ft

20

$r(t)$

$x(t)$

20

45

$x(t)$

20 ft **r(t)**

x(t)

Relation equation: $20^2 + \left[x(t)\right]^2 = \left[r(t)\right]^2$

$20^2 + \left[x(t)\right]^2 = \left[r(t)\right]^2$ $\downarrow$ $2 \cdot x(t) \cdot x'(t) = 2 \cdot r(t) \cdot r'(t)$	At the moment that $r(t) = 45$: $$20^2 + \left[x(t)\right]^2 = 45^2$$ $$400 + \left[x(t)\right]^2 = 2025$$ $$\left[x(t)\right]^2 = 1625$$ $$x(t) = 5\sqrt{65}$$
$2 \cdot \underbrace{x(t)}_{5\sqrt{65}} \cdot \underbrace{x'(t)}_{unknown} = 2 \cdot \underbrace{r(t)}_{45} \cdot \underbrace{r'(t)}_{-2}$ $\downarrow$ $2\left[5\sqrt{65}\right] \cdot x'(t) = 2(45)(-2)$ $x'(t) = \dfrac{2(45)(-2)}{2\left[5\sqrt{65}\right]} \approx -2.2326... \dfrac{ft}{sec}$	The boat is moving towards the dock at a rate of $2.2326\ldots \dfrac{ft}{sec}$ when the length of the rope is 45 ft and the rope is being reeled in at $2 \dfrac{ft}{sec}$. The velocity of the boat is $-2.2326... \dfrac{ft}{sec}$ when the length of the rope is 45 ft and the rope is being reeled in at $2 \dfrac{ft}{sec}$.
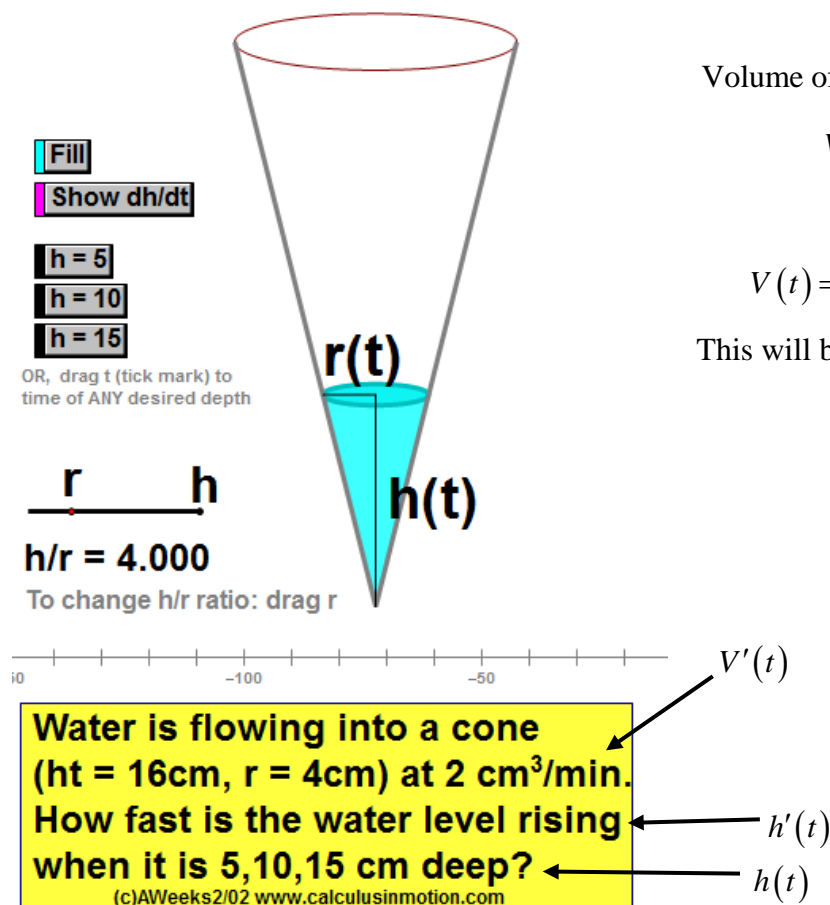
Volume of a right circular cone:

$$V = \frac{1}{3}\pi r^2 \cdot h$$

$$\downarrow$$

$$V(t) = \frac{1}{3}\pi \left[r(t) \right]^2 \cdot h(t)$$

This will be given on the AP Exam

Fill

Show dh/dt

h = 5
h = 10
h = 15

OR, drag t (tick mark) to time of ANY desired depth

r(t)

h(t)

r   h

h/r = 4.000

To change h/r ratio: drag r

i0   −100   −50

$V'(t)$

Water is flowing into a cone (ht = 16cm, r = 4cm) at 2 cm³/min. How fast is the water level rising when it is 5,10,15 cm deep?

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$h'(t)$

$h(t)$

$$V(t) = \frac{1}{3}\pi \left[r(t) \right]^2 \cdot h(t)$$

$$\downarrow$$

$$\underbrace{V'(t)}_{2} = \frac{1}{3}\pi \left[2\, \underbrace{r(t)}_{\text{unkown}}\, \underbrace{r'(t)}_{\text{unkown}} \cdot \underbrace{h(t)}_{\text{known}} + \underbrace{\left[r(t) \right]}_{\text{unkown}}^{2} \underbrace{h'(t)}_{\text{unknown}} \right]$$

The problem here is that after differentiating the relation equation, we have too many unknowns. If we had only one unknown, and it was the rate we are looking for, then we would be ok.

If you ever run into a situation like this, you will need to use substitute a secondary relationship into the equation you differentiated to reduce the number of functions that you are working with. In my experience, the two most common secondary relationships that arise in these types of exercises are (1) similar triangles, and (2) Pythagorean Theorem.

A solution is presented on the next page.

r(t)

r(t)

h(t)

r
h

h/r = 4.000

To change h/r ratio: drag r

i0   −100   −50

Water is flowing into a cone (ht = 16cm, r = 4cm) at 2 cm³/min. How fast is the water level rising when it is 5,10,15 cm deep?
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$r(t)$

$h(t)$

4

16

Using Similar Triangles

$$\frac{r(t)}{4} = \frac{h(t)}{16}$$

$$\downarrow$$

$$r(t) = \frac{1}{4}h(t)$$

$$V(t) = \frac{1}{3}\pi\left[r(t)\right]^2 \cdot h(t)$$

$$V(t) = \frac{1}{3}\pi\left[\frac{1}{4}h(t)\right]^2 \cdot h(t)$$

$$V(t) = \frac{\pi}{48}\left[h(t)\right]^3$$

$$\downarrow$$

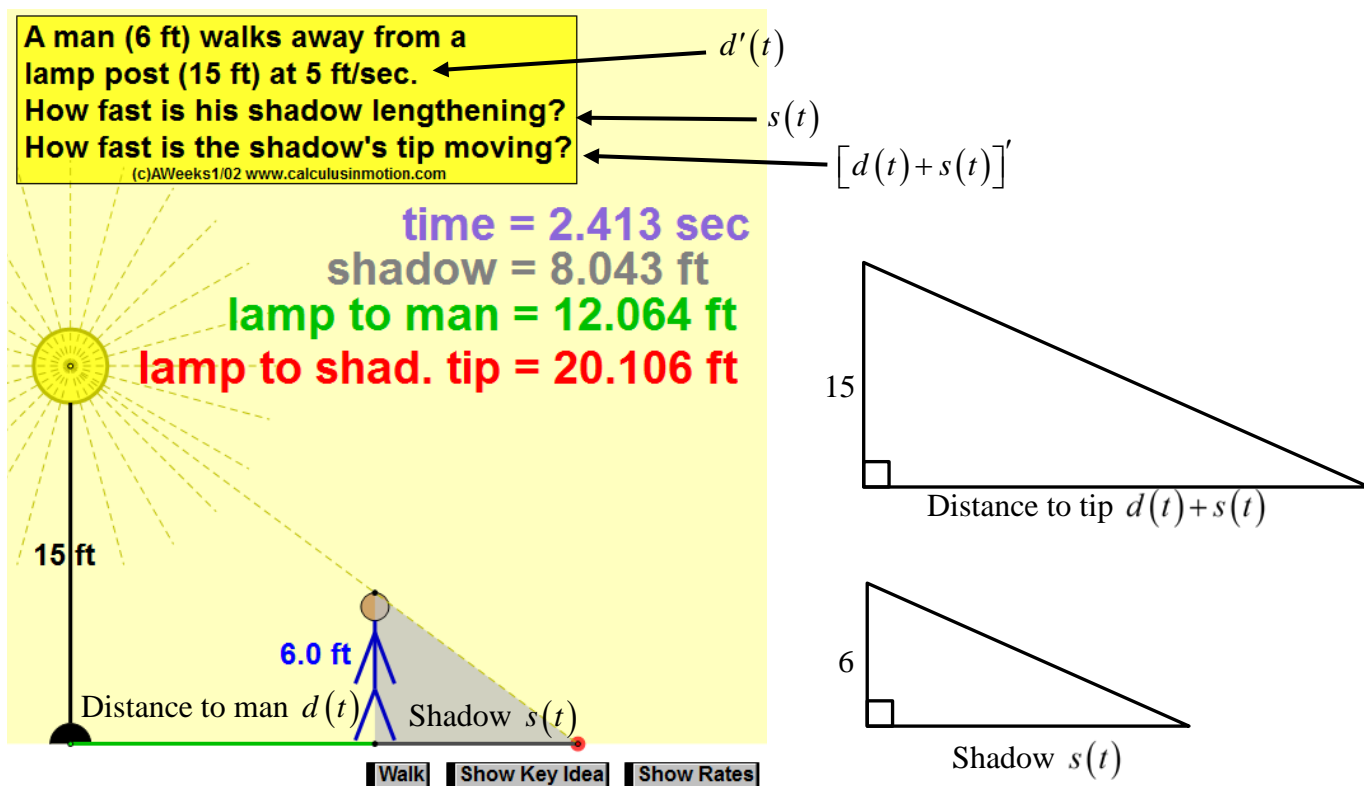$$\underbrace{V'(t)}_{2} = \frac{\pi}{16}\underbrace{\left[h(t)\right]^2}_{5} \cdot \underbrace{h'(t)}_{unknown}$$

$$2 = \frac{25\pi}{16}h'(t)$$

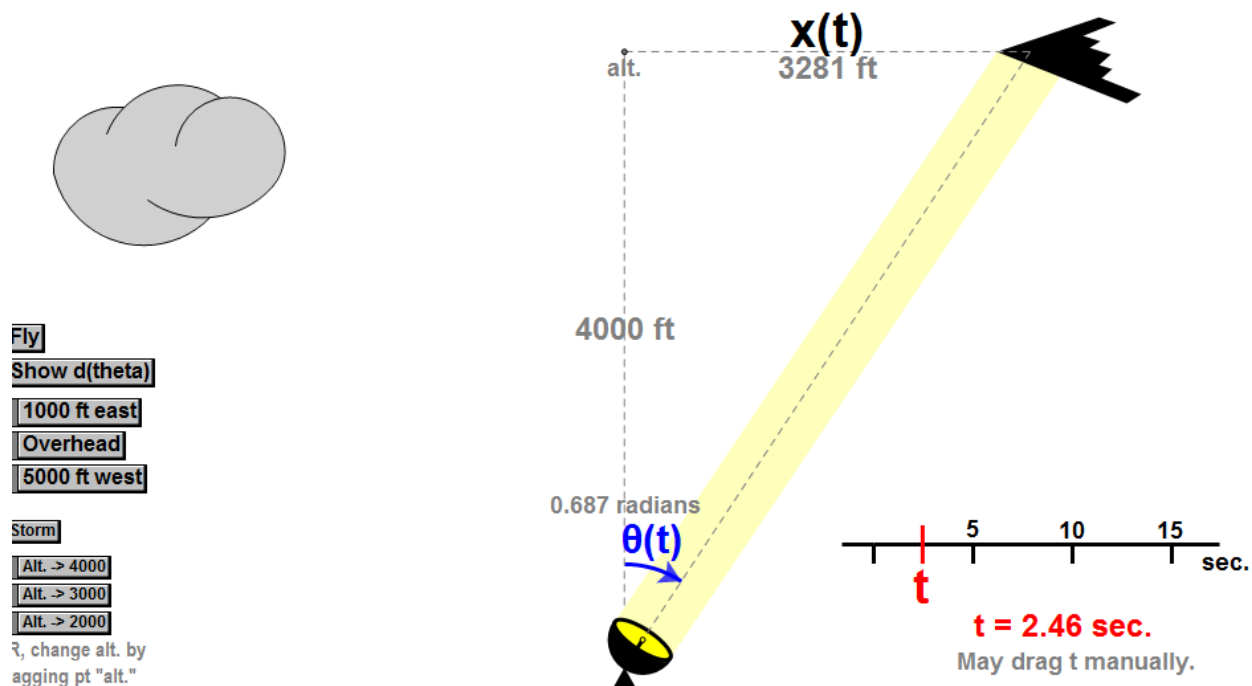$$h'(t) = \frac{32}{25\pi}$$

$$h'(t) = 0.4074... \frac{cm}{min}$$

The height of the water is increasing at a rate of 0.4074… $\frac{cm}{min}$

when $h(t) = 5$ cm and $V'(t) = 2 \frac{cm^3}{min}$

A man (6 ft) walks away from a lamp post (15 ft) at 5 ft/sec.
How fast is his shadow lengthening?
How fast is the shadow's tip moving?

$d'(t)$

$s(t)$

$\left[d(t)+s(t)\right]'$

time = 2.413 sec
shadow = 8.043 ft
lamp to man = 12.064 ft
lamp to shad. tip = 20.106 ft

15 ft

6.0 ft

Distance to man $d(t)$ Shadow $s(t)$

Walk   Show Key Idea   Show Rates

15

Distance to tip $d(t)+s(t)$

6

Shadow $s(t)$

$\dfrac{6}{15}=\dfrac{s(t)}{s(t)+d(t)}$ $6\left[s(t)+d(t)\right]=15s(t)$ $6s(t)+6d(t)=15s(t)$ $6d(t)=9s(t)$ $s(t)=\dfrac{2}{3}d(t)$ $\downarrow$ $s'(t)=\dfrac{2}{3}d(t)$	$s'(t)=\dfrac{2}{3}\underbrace{d(t)}_{5}$ $s'(t)=\dfrac{10}{3}\ \dfrac{\text{ft}}{\text{sec}}$ The length of the shadow is increasing at rate of $2\ \dfrac{\text{ft}}{\text{sec}}$ when the man is walking away at $5\ \dfrac{\text{ft}}{\text{sec}}$.	Tip of the shadow moving is given by: $\left[d(t)+s(t)\right]'$ $\downarrow$ $d'(t)+s'(t)$ $5+\dfrac{10}{3}$ $\dfrac{25}{3}\ \dfrac{\text{ft}}{\text{sec}}$

The tip of the shadow is moving at a rate of $\dfrac{25}{3}\ \dfrac{\text{ft}}{\text{sec}}$ when the man is walking away at $5\ \dfrac{\text{ft}}{\text{sec}}$.

Related Rates Exercises Solutions

x(t)
alt.   **3281 ft**

4000 ft

0.687 radians

θ(t)

| | 5 | 10 | 15 | sec. |

t

t = 2.46 sec.

May drag t manually.

Fly
Show d(theta)
1000 ft east
Overhead
5000 ft west

Storm
Alt. -> 4000
Alt. -> 3000
Alt. -> 2000
R, change alt. by
agging pt "alt."

A plane (alt. 4000 ft) is flying west at 700 ft/s. A searchlight, under its path, tracks it.
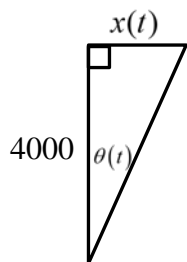How fast is the light pivoting when the plane is 1000 ft east, overhead, 5000 ft west?
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$\theta'(t)$

$x'(t) = -700$ $\left[\text{negative since } x(t) \text{ is decreasing}\right]$

Option 1	Option 2
$\tan\left(\theta(t)\right) = \dfrac{x(t)}{4000}$	$\theta(t) = \arctan\left[\dfrac{x(t)}{4000}\right]$
↓	↓
$\sec^2\left(\theta(t)\right)\cdot\theta'(t) = \dfrac{1}{4000}x'(t)$	$\theta'(t) = \dfrac{1}{1+\left[\dfrac{x(t)}{4000}\right]^2}\dfrac{1}{4000}x'(t)$

At any time t

$x(t)$

4000 $\theta(t)$

$\tan\left(\theta(t)\right) = \dfrac{x(t)}{4000}$

$\theta(t) = \arctan\left(x(t)\right)$

When $x(t) = 1000$

1000

4000 $\theta(t)$

$\tan\left(\theta(t)\right) = \dfrac{1000}{4000}$

$\theta(t) = \arctan\left(\dfrac{1}{4}\right)$

Related Rates Exercises Solutions
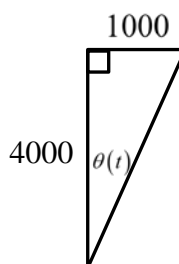
When $x(t) = 1000$

$$\tan(\theta(t)) = \frac{1000}{4000}$$

$$\theta(t) = \arctan\left(\frac{1}{4}\right)$$

Option 1
$$\sec^2(\theta(t)) \cdot \theta'(t) = \frac{1}{4000} x'(t)$$ $$\downarrow$$ $$\sec^2\left(\arctan\left(\frac{1}{4}\right)\right) \cdot \theta'(t) = \frac{1}{4000}(-700)$$ $$\theta'(t) = \frac{1}{4000}(-700) \cdot \frac{1}{\sec^2\left(\arctan\left(\frac{1}{4}\right)\right)}$$ $$= -0.1647...$$
Option 2
$$\theta'(t) = \frac{1}{1 + \left[\frac{1000}{4000}\right]^2} \frac{1}{4000}(-700) = -0.1647...$$

The spotlight is rotating at a rate of $-0.1647...\dfrac{\text{rad}}{\text{sec}}$ when the plane is travelling east at $700 \ \dfrac{\text{ft}}{\text{sec}}$ and is 100ft East.

The angle θ is decreasing at a rate of $0.1647...\dfrac{\text{rad}}{\text{sec}}$ when the plane is travelling east at $700 \ \dfrac{\text{ft}}{\text{sec}}$ and is 100ft East.