Directions for Larson Section 3-4 #27-40

"In exercises 27-40, find all relative extrema. Use the Second Derivative Test where applicable."

Step 1: Demonstrate $f'(x)$ and determine the critical values $\left(f'(x)=0 \text{ or DNE when } x=c_1,c_2,\ldots c_n\right)$

Step 2: Depends on whether $f'(c_i)=0$ or $f'(c_i)$ DNE

<table>
<tr><th colspan="2">If $f'(c_i)=0$</th><th>If $f'(c_i)$ DNE</th></tr>
<tr><td colspan="2">Demonstrate $f''(x)$ and demonstrate the value of $f''(c_i)$</td><td rowspan="3">MUST make a sign chart for $f'(x)$ and use the First Derivative Test</td></tr>
<tr><td>If $f''(c_i)\neq 0$</td><td>$f''(c_i)=0$</td></tr>
<tr><td>If $f'(c_i)=0$ and $f''(c_i)>0$, then $f(x)$ has a relative minimum at $x=c_i$

If $f'(c_i)=0$ and $f''(c_i)<0$, then $f(x)$ has a relative maximum at $x=c_i$</td><td>MUST make a sign chart for $f'(x)$ and use the First Derivative Test</td></tr>
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"In exercises 27-40, find all relative extrema. ***<u>Use the Second Derivative Test where applicable</u>***."

#33 $f(x)=x^2(x-3)^3$

$f'(x)=x(x-3)^2(5x-6)$	$f''(x)=2(x-3)(10x^2-24x+9)$
$f'(x)=0$ when $x=0,3,\frac{6}{5}$	$f''(3)=0 \leftarrow$ Must use First Derivative Test $f''\left(\frac{6}{5}\right)=\frac{486}{25}=19.44 \leftarrow$ Must use Second Derivative Test $f''(0)=-54 \leftarrow$ Must use Second Derivative Test

$f(x)$ has a relative maximum at $x=0$ because $f'(0)=0$ and $f''(0)=-54<0$.

$f(x)$ has a relative minimum at $x=\frac{6}{5}$ because $f'\left(\frac{6}{5}\right)=0$ and $f''\left(\frac{6}{5}\right)=19.44>0$.

$f'(x)$	$(+)$	0	$(-)$	0	$(+)$	0	$(+)$
		0		$\frac{6}{5}$		3	

$f(x)$ has neither a min nor a max at $x=3$ because $f'(x)$ does not change sign.

Extrema Decision Tree

Find the extrema of a function $f(x)$

Find the Critical Values of $f(x)$

$f'(x) = 0$ or DNE $\rightarrow x = c_1, c_2, \cdots, c_i, \cdots, c_n$

Closed Interval
Absolute Min/Max

USE EVT

Test Endpoints and all critical values in the closed interval:

$$f(a) = y_a$$
$$\vdots$$
$$f(c_i) = y_i$$
$$\vdots$$
$$f(b) = y_b$$

Claim the greatest y_i value as the Absolute Max
Claim the least y_i value as the Absolute Min
Note, the Absolute Min/Max occurs at $x = c_i$
State "$f(x)$ has an absolute max/min at $(c_i, f(c_i))$"

Not a closed interval
Relative Min/Max

You CAN Make a Sign Chart for $f'(x)$

$f'(c_i) = 0$ or DNE

1st Derivative Test:
Make a labeled Sign Chart for $f'(x)$

- $f(x)$ has a relative max at $x = c_i$ because $f'(x)$ goes from $+ \rightarrow -$
- $f(x)$ has a relative min at $x = c_i$ because $f'(x)$ goes from $- \rightarrow +$

$f'(c_i) = 0$

You Cannot Make a Sign Chart for $f'(x)$ [Differential Equations]

2nd Derivative Test:
Demonstrate the values of $f'(c_i)$ and $f''(c_i)$

- $f(x)$ has a relative max at $x = c_i$ because $f'(c_i) = 0$ and $f''(c_i) < 0$
- $f(x)$ has a relative min at $x = c_i$ because $f'(c_i) = 0$ and $f''(c_i) > 0$
- $f''(c_i) = 0 \rightarrow$ *MUST* Use First Derivative Test