2000 #3

The Taylor series about $x=5$ for a certain function *f* converges to $f(x)$ for all *x* in the interval of convergence.The $n^{\text{th}}$ derivative of *f* at $z=5$ is given by $f^{(n)}(5)=\dfrac{(-1)^n \cdot n!}{2^n \cdot (n+2)}$ and $f(5)=\dfrac{1}{2}$.

(a) Write the third-degree Taylor polynomial for *f* about $x=5$.
(b) Find the radius of convergence of the Taylor series for *f* about $x=5$.
(c) Show that the sixth-degree Taylor polynomial for *f* about $x=5$ approximates $f(6)$ with error less than $\dfrac{1}{1000}$.

2001 #6
A function *f* is defined by

$$f(x)=\frac{1}{3}+\frac{2}{3^2}\cdot x+\frac{3}{3^3}\cdot x^2+\cdots+\frac{n+1}{3^{n+1}}\cdot x^n+\cdots$$

For all *x* in the interval of convergence of the given power series.

(a) Find the interval of convergence for this power series. Show the work that leads to your answer.
(b) Find $\lim\limits_{x\to 0}\dfrac{f(x)-\frac{1}{3}}{x}$
(c) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)\,dx$.
(d) Find the sum of the series determined in part (c).

2002 Form B #6

(a) The Maclaurin series for $\ln\left(\dfrac{1}{1-x}\right)$ is $\sum\limits_{n=1}^{\infty}\dfrac{x^n}{n}$ with interval of convergence $-1\le x<1$.
(b) Find the Maclaurin series for $\ln\left(\dfrac{1}{1+3x}\right)$ and determine the interval of convergence.
(c) Find the value of $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n}{n}$
(d) Give a value of *p* such that $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n}{n^p}$ converges, but $\sum\limits_{n=1}^{\infty}\dfrac{1}{n^{2p}}$ diverges. Give reasons why your value of *p* is correct.
(e) Give a value of *p* such that $\sum\limits_{n=1}^{\infty}\dfrac{1}{n^p}$ diverges, but $\sum\limits_{n=1}^{\infty}\dfrac{1}{n^{2p}}$ converges. Give reasons why your value of *p* is correct.

2005 #6

Let *f* be a function with derivatives of all orders and for which $f(2)=7$. When *n* is odd, the $n^{\text{th}}$ derivative of *f* at $x=2$ is 0. When *n* is even and $n\geq 2$, the $n^{\text{th}}$ derivative of *f* at $x=2$ is given by $f^{(n)}(2)=\dfrac{(n-1)!}{3^n}$.

(a) Write the sixth-degree Taylor polynomial for *f* about $x=2$.

(b) In the Taylor series for *f* about $x=2$, what is the coefficient of $(x-2)^{2n}$ for $n\geq 1$.

(c) Find the interval of convergence of the Taylor series for *f* about $x=2$. Show the work that leads to your answer.

2007 Form B #6

Let *f* be the function given by $f(x)=6e^{-\frac{x}{3}}$ for all *x*.

(a) Find the first four nonzero terms and the general term for the Taylor series for *f* about $x=0$ .

(b) Let *g* be the function given by $g(x)=\int_0^x f(t)\,dt$. Find the first four nonzero terms and the general term for the Taylor series for *g* about $x=0$.

(c) The function *h* satisfies $h(x)=k\cdot f'(ax)$ for all *x*, where *a* and *k* are constants. The Taylor series for *h* about $x=0$ is given by

$$h(x)=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots$$

Find the values of *a* and *k*.

2009 #6

The Maclaurin series for $e^x=1+x+\dfrac{x^2}{2}+\dfrac{x^3}{6}+\cdots+\dfrac{x^n}{n!}+\cdots$ The continuous function *f* is defined by $f(x)=\dfrac{e^{(x-1)^2}-1}{(x-1)^2}$ for $x\neq 1$ and $f(1)=1$. The function *f* has derivatives of all orders at $x=1$.

(a) Write the first four nonzero terms and the general term of the Taylor series for $e^{(x-1)^2}$ about $x=1$.

(b) Use the Taylor series found in part (a) to write the first four nonzero terms and the general term of the Taylor series for f about $x=1$ .

(c) Use the ratio test to find the interval of convergence for the Taylor series found in part (b)

(d) Use the Taylor series for *f* about $x=1$ to determine whether the graph of *f* has any points of inflection.

2010 Form B #6

The Maclaurin series for the function *f* is given by $f(x)=\sum_{n=2}^{\infty}\frac{(-1)^n\cdot(2x)^n}{n-1}$ on its interval of convergence.

(a) Find the interval of convergence for the Maclaurin series of *f*. Justify your answer.

(b) Show that $y=f(x)$ is a solution to the differential eqution $xy'-y=\frac{4x^2}{1+2x}$ for $|x|<R$, where *R* is the radius of convergence from part (a).

2013 #6

A function *f* has derivatives of all orders at $x=0$. Let $P_n(x)$ denote the $n^{\text{th}}$ degree Taylor polynomial for *f* about $x=0$.

(a) It is known that $f(0)=-4$ and that $P_1\left(\frac{1}{2}\right)=-3$. Show that $f'(0)=2$.

(b) It is known that $f''(0)=-\frac{2}{3}$ and $f'''(0)=\frac{1}{3}$. Find $P_3(x)$.

(c) The function *h* has first derivative given by $h'(x)=f(2x)$. It is known that $h(0)=7$. Find the third-degree Taylor polynomial for *h* about $x=0$.

2016 #6

The function *f* has a Taylor Series about $x=1$ that converges to $f(x)$ for all *x* in the interval of convergence. It is known that $f(1)=1$, $f'(1)=-\frac{1}{2}$, and the $n^{\text{th}}$ derivative of *f* at $x=1$ is given by $f^{(n)}(1)=(-1)^n\cdot\frac{(n-1)!}{2^n}$ for $n\geq 2$.

(a) Write the first four nonzero terms and the general term of the Taylor series for *f* about $x=1$.

(b) The Taylor series for *f* about $x=1$ has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.

(c) The Taylor series for *f* about $x=1$ can be used to represent $f(1.2)$ as an alternating series. Use the first three nonzero terms of the alternating series to approximate $f(1.2)$.

(d) Show that the approximation found in part (c) is within 0.001 of the exact value of $f(1.2)$.

2017 #6

$$f(0) = 0$$
$$f'(0) = 1$$
$$f^{(n+1)}(0) = (-n) \cdot f^{(n)}(0) \text{ for all } n \geq 1$$

A function *f* has derivatives of all order for $-1 < x < 1$. The derivatives of *f* satisfy the conditions above. The Maclaurin series for *f* converges to $f(x)$ for $|x| < 1$.

(a) Show that the first four nonzero terms of the Maclaurin series for *f* are $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$, and write the general term of the Maclaurin series for *f*.

(b) Determine whether the Maclaurin series described in part (a) converges absolutely, converges conditionally, or diverges at $x = 1$. Explain your reasoning.

(c) Write the first four nonzero terms and the general term of the Maclaurin series for $g(x) = \int_0^x f(t)\,dt$.

(d) Let $P_n\left(\frac{1}{2}\right)$ represent the $n^{\text{th}}$ degree Taylor polynomial for *g* about $x = 0$ evaluated at $x = \frac{1}{2}$, where *g* is the function defined in part (c). Use the alternating series error bound to show that $\left|P_4\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right)\right| < \frac{1}{500}$.