AP Calculus AB 2010 #2 Calculator Allowed

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t=0$) and 8 P.M. ($t=8$). The number of entries in the box t hours after noon is modeled by a differentiable function $E(t)$ for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table at right.

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of Entries)	0	4	13	21	23

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t=6$. Show the computations that lead to your answer.

$E'(6) \approx \dfrac{E(7)-E(5)}{7-5} = \dfrac{21-13}{2} = 4$ hundred entries per hour.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$ .Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

$$\begin{aligned}\frac{1}{8}\int_0^8 E(t)\,dt &\approx \frac{1}{8}\left[\frac{1}{2}(4+0)(2)+\frac{1}{2}(4+13)(3)+\frac{1}{2}(13+21)(2)+\frac{1}{2}(23+21)(1)\right]\\ &\approx \frac{1}{8}[85.5]\\ &\approx 10.6875\end{aligned}$$

$\frac{1}{8}\int_0^8 E(t)\,dt$ represents the average number of hundreds of entries in the box from time $t=0$ to time $t=8$ (i.e. Noon to 8pm).

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t (hours)	0	2	5	7	8
$E(t)$ (hundreds of Entries)	0	4	13	21	23

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function $P(t)$, where $P(t)=t^3-30t^2+298t-976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight ($t=12$)?

At 8pm there was a total of 2,300 entries in the box. The amount of entries that had not been processed by midnight is given by

$$23-\int_8^{12} P(t)\,dt = 7$$

There were 7 hundred entries that had not been processed by midnight, $t=12$.

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

The entries are being processed the most quickly when $p(t)$ is at a maximum on $[8,12]$. Therefore we must use EVT to determine the maximum value of $p(t)$ on $[8,12]$

$$P'(t)=3t^2-60t+298$$
$$0=3t^2-60t+298$$

$P'(t)=0$ when $t \approx 9.1835...$ or $t \approx 10.8164...$

$$P(8)=0$$
$$P(9.1835...) \approx 5.0886...$$
$$P(10.8164...) \approx 2.9113...$$
$$P(12)=8$$

Entries are being processed the quickest at midnight, at a rate of 8 hundred entries per hour.

t (minutes)	0	4	9	15	20
$W(t)$ (Degrees Farenheit	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t=0$, the temperature of the water is $55\,°\mathrm{F}$. The water is heated for 30 minutes, beginning at time $t=0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) [4 points] Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$\begin{aligned} W'(12) &\approx \frac{W(15)-W(9)}{15-9} \\ &\approx \frac{67.9-61.8}{15-9} \\ &\approx \frac{61}{60} \\ &\approx 1.0166... \end{aligned}$$

The temperature of the water at time $t=12$ is increasing by approximately $1.016\ °\mathrm{F/min}$.

(b) [4 points] Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\begin{aligned} \int_0^{20} W'(t)\,dt &= \big[W(t)\big]_0^{20} \\ &= W(20)-W(0) \\ &= 16°\mathrm{F} \end{aligned}$$

$\int_0^{20} W'(t)\,dt$ represents the net change in temperature from time $t=0$ to time $t=20$.

$\int_0^{20} W'(t)\,dt$ means the temperature of the water in the rub rose by 16°F from time $t=0$ to time $t=20$.

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t=0$, the temperature of the water is $55\,^{\circ}\mathrm{F}$. The water is heated for 30 minutes, beginning at time $t=0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

t (minutes)	0	4	9	15	20
$W(t)$ (Degrees Farenheit	55.0	57.1	61.8	67.9	71.0

(c) [4points] For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$

Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$$\begin{aligned}\frac{1}{20}\int_0^{20} W(t)\,dt &\approx \frac{1}{20}\left[(55)(4-0)+(57.1)(9-4)+(61.8)(15-9)+(67.9)(20-15)\right]\\ &\approx \frac{1}{20}(1215.8)\\ &\approx 60.79\end{aligned}$$

The average temperature of the water from time $t=0$ to $t=20$ is $60.79°\mathrm{F}$.
The approximation is an underestimate because $W(t)$ is strictly increasing on the interval $[0,20]$ and the sum is a Left Riemann Sum.

(d) [4points] For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$ Based on the model, what is the temperature of the water at time $t=25$?

$$\begin{aligned}W(25) &= W(20) + \int_{20}^{25} W'(t)dt\\ &= 71 + \int_{20}^{25} 0.4\sqrt{t}\cos(0.06t)\,dt\\ &\approx 73.043\end{aligned}$$

The temperature of the water at time $t=25$ is 73.043°F.

AP Calculus AB 2005 #3 Calculator Allowed

A metal wire of length 8 centimeters (cm) is heated at one end. The table below gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ $°C$	100	93	70	62	55

(a) [5 points] Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

$$\begin{aligned} T'(7) &\approx \frac{T(8)-T(6)}{8-6} \\ &\approx \frac{55-62}{8-6} \\ &\approx -3.5 \ \frac{°C}{\text{cm}} \end{aligned}$$

(b) [6 points] Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

$$\begin{aligned} \frac{1}{8-0}\int_0^8 T(x)\,dx &\approx \frac{1}{8}\left[\frac{1}{2}(93+100)(1)+\frac{1}{2}(93+70)(4)+\frac{1}{2}(70+62)(1)+\frac{1}{2}(62+55)(2)\right] \\ &\approx \frac{1}{8}[605.5] \\ &\approx 75.6875...°C \end{aligned}$$

A metal wire of length 8 centimeters (cm) is heated at one end. The table below gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ °C	100	93	70	62	55

(c) [4 points] Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

$$\begin{aligned}\int_0^8 T'(x)\,dx &= T(8) - T(0) \\ &= 55 - 100 \\ &= -45°C\end{aligned}$$

$\int_0^8 T'(x)\,dx$ represents the net change in temperature in degrees Celsius from the distance of 0 cm to the distance of 8 cm.