Let *f* be defined on an interval *I* containing *c*.

(1) $f(c)$ is the **absolute minimum** of *f* on *I* if $f(c) \le f(x)$ for all *x* in *I*.

(2) $f(c)$ is the **absolute maximum** of *f* on *I* if $f(c) \ge f(x)$ for all *x* in *I*.

Minimum and maximum values of a function are referred to as **extreme values**. A single value is called an **extremum**, and multiple values are called **extrema**.

*Note: $f(c)$ is the absolute min/max value. The min/max value occurs at $x = c$. It is advised to state that *f* has an absolute min/max at $(c, f(c))$. The coordinate communicated both the absolute min/max value and the *x*-value at which the min/max occurs.

Definition of Relative Extrema:

(1)If there exists an open interval *I* containing *c* on which $f(c) \le f(x)$ for all *x* in *I*, then we say that $f(c)$ is called a **relative minimum** of *f*. That is *f* has a relative minimum at $(c, f(c))$.

(2) If there exists an open interval *I* containing *c* on which $f(c) \ge f(x)$ for all *x* in *I*, then we say that $f(c)$ is called a **relative maximum** of *f*. That is *f* has a relative maximum at $(c, f(c))$.

<u>Definition of a Critical Number:</u>

Let $f(x)$ be defined at $x = c$. If $f'(c) = 0$ or if $f'(c)$ DNE, then *c* is a **critical number of *f***.

Theorem: If *f* has a relative minimum or relative maximum at $x = c$, then *c* is a critical number of *f*.

The *x*-values where $f' = 0$ or f' DNE are locations of <u>*candidates*</u> for relative maximums and relative minimums. Further investigation [Frist Derivative Test or Second Derivative Test] is needed to confirm whether that location is a relative minimum, relative maximum, or neither.

<u>Extreme Value Theorem:</u>

If *f* is continuous on a closed interval $[a,b]$, then *f* has both an absolute maximum and an absolute minimum on $[a,b]$.

<u>Guidelines for finding the absolute minimum and absolute maximum of *f* on a closed interval:</u>

To find the absolute minimum or absolute maximum values of a continuous function *f* on a closed interval $[a,b]$:

(1) Demonstrate the derivative of f, f'.

(2) Solve for the values of *x* for which $f'=0$ or f' DNE in the given closed interval

   a. You must write " $f'(x)=0$ or DNE when ..." , and you may use your calculator to determine these values – only the results need to be shown.

$f'=0$ or f' DNE when $x=c_1,c_2,...,c_n$.

(3) Evaluate *f* at each critical value determined in step #2 ***<u>and each endpoint.</u>***

$$f(a)=$$
$$f(c_1)=$$
$$\vdots$$
$$f(c_n)=$$
$$f(b)=$$

(4) <u>The least of the values is the absolute minimum</u>, and <u>the greatest of these values is the absolute maximum.</u>

   a. There is only one absolute minimum value, and there is only one absolute maximum value. However, the absolute min/max can occur at multiple locations.

Try this: Find the absolute minimum value and the absolute maximum value of $f(x)=2\sin(x)-\cos(2x)$ on $[0,2\pi]$.

$f'(x)=2\cos(x)+2\sin(2x)$

$f'(x)=0$ or DNE when $x=$

$\frac{\pi}{2}$	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
1.5707...	3.6651...	4.7123...	5.7595...

$f(0)=-1$

$f\left(\frac{\pi}{2}\right)=3$ ← Absolute Max

$f\left(\frac{7\pi}{6}\right)=-1.5$

$f\left(\frac{3\pi}{2}\right)=-1$

$f\left(\frac{11\pi}{6}\right)=-1.5$ ← Absolute Min

$f(2\pi)=-1$

Note: You can indicate the Absolute Minimum using $f(2\pi)=-1.5$ as well. There is only one absolute minimum value, which is -1. This absolute minimum value occurs at two locations, $x=\frac{\pi}{2}$ and $x=2\pi$.

The absolute minimum of $f(x)$ on $[0,2\pi]$ is -1.5.

The absolute maximum of $f(x)$ on $[0,2\pi]$ is 3.