Stewart Chapter 11 Review Homework Hints

Determine whether the series is convergent or divergent

11. Use the Limit Comparison Test
$$\sum_{n=1}^{\infty} \frac{n}{n^3 + 1} \sim \sum_{n=1}^{\infty} \frac{1}{n^2}$$

12. Use the Limit Comparison Test with
$$\sum_{n=1}^{\infty} \frac{1}{n}$$

- 13. Use the Ratio Test or Root Test
- 14. Use Alternating Series Test

15. Use Integral Test
$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln(n)}} \leftrightarrow \int_{1}^{\infty} \frac{1}{x\sqrt{\ln(x)}} dx$$

16. Use Telescoping Series
$$\sum_{n=1}^{\infty} \ln \left(\frac{n}{3n+1} \right) = \sum_{n=1}^{\infty} \ln (n) - \ln (3n+1)$$

17. Use Direct Comparison Test
$$\sum_{n=1}^{\infty} \frac{\cos(3n)}{1+(1.2^n)} \le \sum_{n=1}^{\infty} \frac{1}{(1.2)^n} = \sum_{n=1}^{\infty} \left(\frac{1}{1.2}\right)^n$$

18. Use the Root Test
$$\sum_{n=1}^{\infty} \frac{n^{2n}}{(1+2n^2)^n} = \sum_{n=1}^{\infty} \left(\frac{n^2}{(1+2n^2)}\right)^n$$

- 19. Use the Ratio Test
- **20.** Use the Root Test $\sum_{n=1}^{\infty} \frac{\left(-5\right)^{2n}}{n^2 9^n} = \sum_{n=1}^{\infty} \frac{25^n}{n^2 9^n}$
- 21. Use the Alternating Series Test

22.
$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n} = \sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n} \cdot \frac{\left(\sqrt{n+1} + \sqrt{n-1}\right)}{\left(\sqrt{n+1} + \sqrt{n-1}\right)} = \sum_{n=1}^{\infty} \frac{n}{n\left(\sqrt{n+1} + \sqrt{n-1}\right)}$$
 Then use

the Limit comparison Test with $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}}$

Find the sum of the series

27.
$$\sum_{n=1}^{\infty} \frac{\left(-3\right)^{n-1}}{2^{3n}} = \sum_{n=1}^{\infty} \frac{\left(-3\right)^{n-1}}{2^{3n}} \cdot \frac{\left(-3\right)}{\left(-3\right)} = \sum_{n=1}^{\infty} -\frac{1}{3} \cdot \left(\frac{-3}{8}\right)^n$$
 Use Geometric Series Formula

28.
$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)} = \sum_{n=1}^{\infty} \frac{1}{3n} - \frac{1}{3(n+3)}$$
 Use Telescoping Series

29. Use Telescoping Series

30.
$$\sum_{n=0}^{\infty} \frac{\left(-1\right)^{n} \pi^{n}}{3^{2n} (2n)!} = \sum_{n=0}^{\infty} \frac{\left(-1\right)^{n} \left(\sqrt{\pi}\right)^{2n}}{3^{2n} (2n)!} = \sum_{n=0}^{\infty} \left(-1\right)^{n} \frac{\left(\frac{\sqrt{\pi}}{3}\right)^{2n}}{(2n)!}$$

31.
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \rightarrow e^{(-e)} = \sum_{n=0}^{\infty} \frac{(-e)^n}{n!} = 1 + (-e) + \frac{(-e)^2}{2!} + \frac{(-e)^3}{3!} + \cdots$$