Simple Harmonic Motion Equations:

$$x = A\cos(\omega t + \varphi) \qquad \omega = 2\pi f = 2\pi/T \qquad \omega = \sqrt{k/m}$$

Damped Harmonic Motion

This gives $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0.$

If b is small, a solution of the form

$$x = Ae^{-\gamma t}\cos\omega' t$$

will work, with

$$\gamma = \frac{b}{2m}$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}.$$

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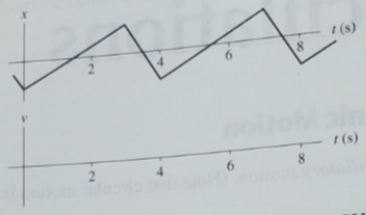
Damped Sinusoidally driven Harmonic Motion:

$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0)$$

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

3. Consider the particle whose motion is represented by the x-versus-t graph below.



a. Is this periodic motion? _____ b. Is this motion SHM? _____

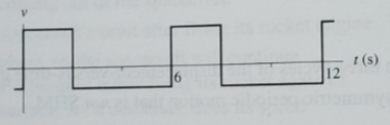
c. What is the period? _____ d. What is the frequency? _____

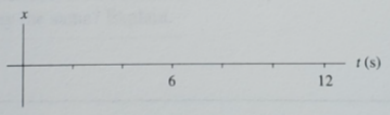
e. You learned in Chapter 2 to relate velocity graphs to position graphs. Use that knowledge to draw the particle's velocity-versus-time graph on the axes provided.

4. Shown below is the velocity-versus-time graph of a particle.

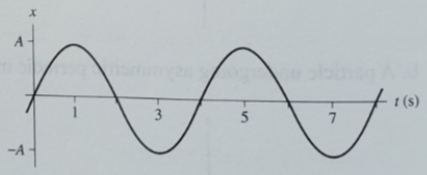
a. What is the period of the motion? _____

b. Draw the particle's position-versus-time graph, starting from $x = 0$ at $t = 0$ s.





5. The figure shows the position-versus-time graph of a particle in SHM.

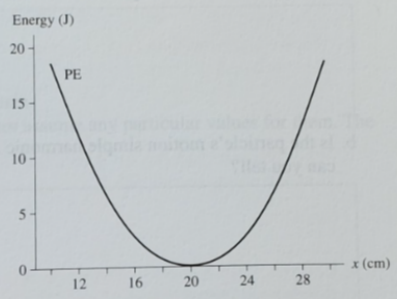


a. At what times is the particle moving to the right at maximum speed?

b. At what times is the particle moving to the left at maximum speed?

c. At what times is the particle instantaneously at rest?

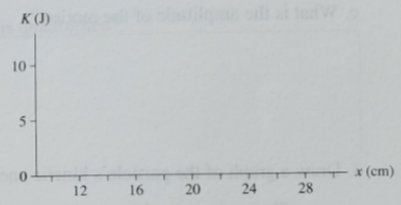
10. The figure shows the potential-energy diagram of a particle oscillating on a spring.



a. What is the spring's equilibrium length?

b. The particle's turning points are at 14 cm and 26 cm. Draw the total energy line and label it TE.

c. What is the particle's maximum kinetic energy?



d. Draw a graph of the particle's kinetic energy as a function of position.

e. What will be the turning points if the particle's total energy is doubled?

11. A block oscillating on a spring has an amplitude of 20 cm. What will be the block's amplitude if its total energy is tripled? Explain.

12. A block oscillating on a spring has a maximum speed of 20 cm/s. What will be the block's maximum speed if its total energy is tripled? Explain.

14.5 Vertical Oscillations

15. A block oscillating on a spring has period $T = 4$ s.

a. What is the period if the block's mass is halved? Explain.
Note: You do not know values for either m or k. Do *not* assume any particular values for them. The required analysis involves thinking about ratios.

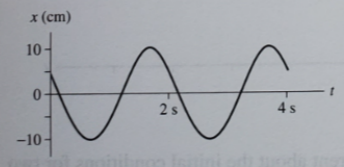
b. What is the period if the value of the spring constant is quadrupled?

c. What is the period if the oscillation amplitude is doubled while m and k are unchanged?

16. For graphs a and b, determine:
- The angular frequency ω.
- The oscillation amplitude A.
- The phase constant ϕ_0.

Note: Graphs a and b are independent. Graph b is *not* the velocity graph of a.

a.

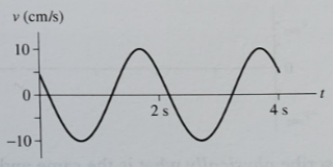


$\omega =$ _____

$A =$ _____

$\phi_0 =$ _____

b.

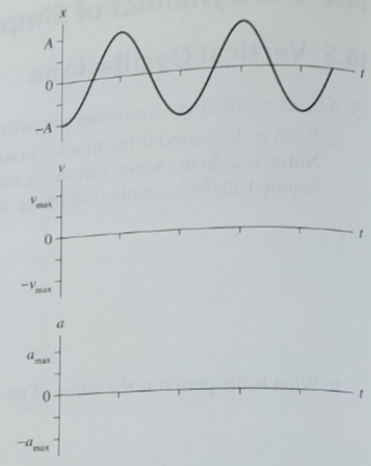


$\omega =$ _____

$A =$ _____

$\phi_0 =$ _____

17. The graph on the right is the position-versus-time graph for a simple harmonic oscillator.



a. Draw the v-versus-t and a-versus-t graphs.

b. When x is greater than zero, is a ever greater than zero? If so, at which points in the cycle?

c. When x is less than zero, is a ever less than zero? If so, at which points in the cycle?

d. Can you make a general conclusion about the relationship between the sign of x and the sign of a?

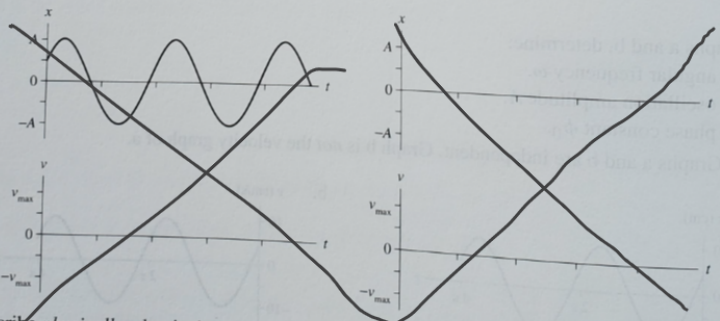
e. When x is greater than zero, is v ever greater than zero? If so, how is the oscillator moving at those times?

18. For the oscillation shown on the left below:

a. What is the phase constant ϕ_0?

b. Draw the corresponding v-versus-t graph on the axes below the x-versus-t graph.

c. On the axes on the right, sketch two cycles of the x-versus-t and the v-versus-t graphs if the value of ϕ_0 found in part a is replaced by its negative, $-\phi_0$.



d. Describe *physically* what is the same and what is different about the initial conditions for two oscillators having "equal but opposite" phase constants ϕ_0 and $-\phi_0$.

14.8 Driven Oscillations and Resonance

26. What is the difference between the driving frequency and the natural frequency of an oscillator?

27. A car drives along a bumpy road on which the bumps are equally spaced. At a speed of 20 mph, the frequency of hitting bumps is equal to the natural frequency of the car bouncing on its springs.

a. Draw a graph of the car's vertical bouncing amplitude as a function of its speed if the car has new shock absorbers (large damping coefficient).

b. Draw a graph of the car's vertical bouncing amplitude as a function of its speed if the car has worn out shock absorbers (small damping coefficient).

Draw both graphs on the same axes, and label them as to which is which.