Integral as Net Change Released AP Questions

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Integral as Net Change Released AP Questions

AP Calculus AB–4 **2000**

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

2001 SCORING GUIDELINES

Question 2

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

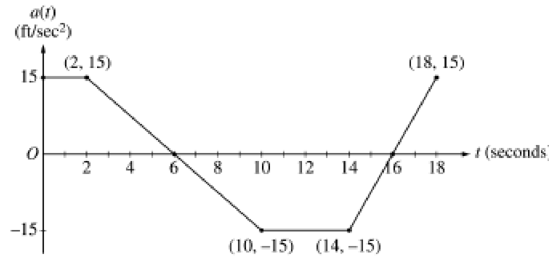
(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

2001 # 3



A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

2002 # 2

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

2002 Form B # 2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

2002 SCORING GUIDELINES (Form B)

Question 3

A particle moves along the x-axis so that its velocity v at any time t, for $0 \le t \le 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

(a) On the axes provided, sketch the graph of $v(t)$ for $0 \le t \le 16$.

(b) During what intervals of time is the particle moving to the left? Give a reason for your answer.

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

(d) Is there any time t, $0 < t \le 16$, at which the particle returns to the origin? Justify your answer.

2003 # 2

A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

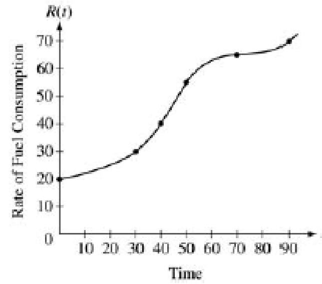
(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

2003 # 3

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

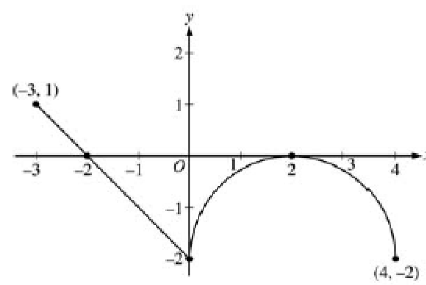
(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

2003 # 4

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.



Graph of f'

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $(0,3)$.

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

2003 Form B # 2

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12\sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

2003 SCORING GUIDELINES (Form B)

Question 3

A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

(a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

(b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

(c) Using correct units, explain the meaning of $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

(d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

2003 SCORING GUIDELINES (Form B)

Question 4

A particle moves along the x-axis with velocity at time $t \ge 0$ given by $v(t) = -1 + e^{1-t}$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(c) Find all values of t at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

2004 SCORING GUIDELINES

Question 1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

2004 SCORING GUIDELINES

Question 3

A particle moves along the y-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \ge 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

2004 SCORING GUIDELINES (Form B)

Question 2

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

2004 SCORING GUIDELINES (Form B)

Question 3

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

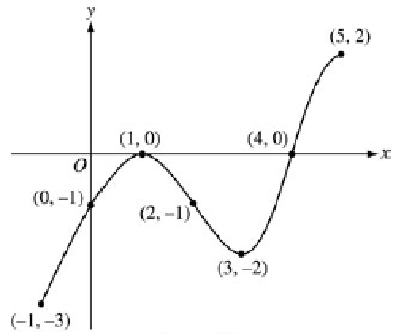
(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicates units of measure.

(d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

2004 SCORING GUIDELINES (Form B)

Question 4



Graph of f'

The figure above shows the graph of f', the derivative of the function f, on the closed interval $-1 \le x \le 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.

(a) Find the x-coordinate of each of the points of inflection of the graph of f. Give a reason for your answer.

(b) At what value of x does f attain its absolute minimum value on the closed interval $-1 \le x \le 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \le x \le 5$? Show the analysis that leads to your answers.

(c) Let g be the function defined by $g(x) = x f(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

2005 SCORING GUIDELINES

Question 2

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

2005 SCORING GUIDELINES

Question 3

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

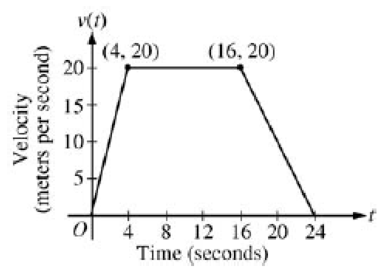
(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

2005 SCORING GUIDELINES

Question 5

A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.



(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

2005 SCORING GUIDELINES (Form B)

Question 2

A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \le t \le 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

(a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?

(b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?

(c) At what time t, for $0 \le t \le 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.

(d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

2005 SCORING GUIDELINES (Form B)

Question 3

A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$. The particle is at position $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

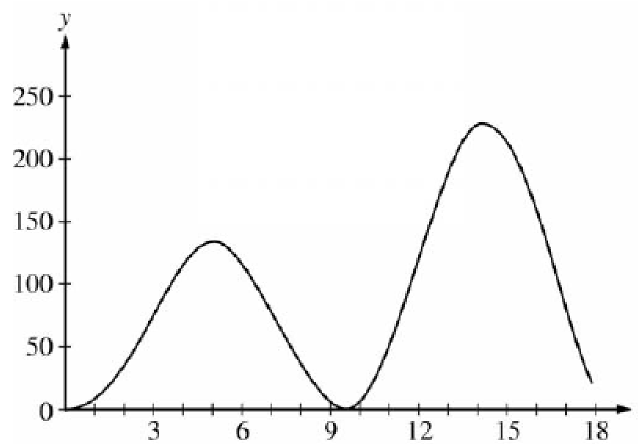
(c) Find the position of the particle at time $t = 2$.

(d) Find the average speed of the particle over the interval $0 \le t \le 2$.

2006 SCORING GUIDELINES

Question 2

At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.



(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

2006 SCORING GUIDELINES

Question 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

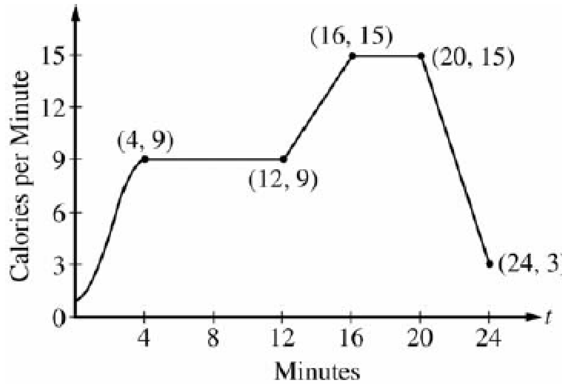
(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

2006 SCORING GUIDELINES (Form B)

Question 4

The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f. In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \le t \le 4$ and f is piecewise linear for $4 \le t \le 24$.



(a) Find $f'(22)$. Indicate units of measure.

(b) For the time interval $0 \le t \le 24$, at what time t is f increasing at its greatest rate? Show the reasoning that supports your answer.

(c) Find the total number of calories burned over the time interval $6 \le t \le 18$ minutes.

(d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find c so that an average of 15 calories per minute is burned during the time interval $6 \le t \le 18$.

2006 SCORING GUIDELINES (Form B)

Question 6

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	-20	-30	-20	-14	-10	0	10
$a(t)$ $\left(\text{ft/sec}^2\right)$	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\, dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\, dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\, dt$.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

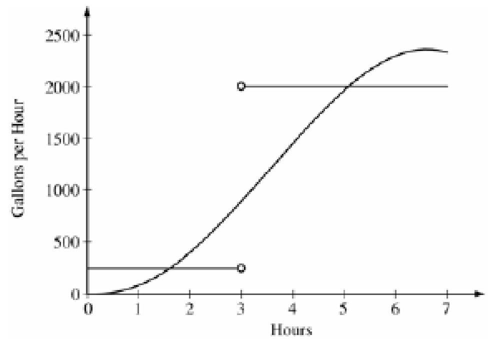
2007 SCORING GUIDELINES

Question 2

The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \le t \le 7$.

(ii) The rate at which water leaves the tank is
$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases}$$ gallons per hour.



The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

2007 SCORING GUIDELINES (Form B)

Question 2

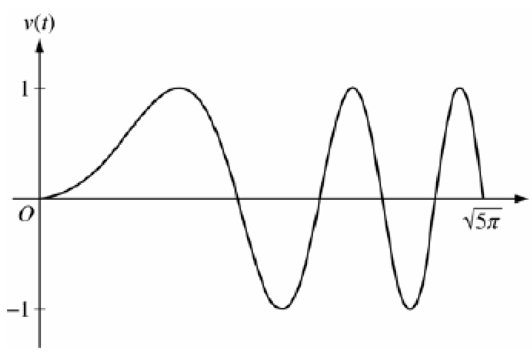
A particle moves along the x-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown above for $0 \le t \le \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \le t \le \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.



2007 SCORING GUIDELINES (Form B)

Question 3

The wind chill is the temperature, in degrees Fahrenheit $(°F)$, a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is $32°F$, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

(a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

(b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant $32°F$. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

2008 SCORING GUIDELINES

Question 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

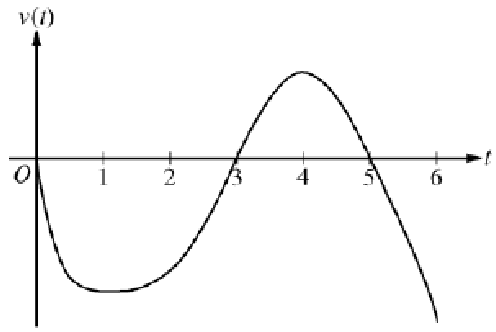
(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

2008 SCORING GUIDELINES

Question 4



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

2008 SCORING GUIDELINES (Form B)

Question 2

For time $t \ge 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

2008 SCORING GUIDELINES (Form B)

Question 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

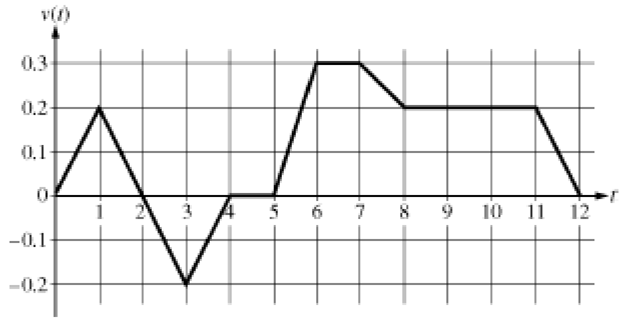
(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

(c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

(d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

2009 SCORING GUIDELINES

Question 1



Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

(d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

2009 SCORING GUIDELINES

Question 2

The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

(a) How many people are in the auditorium when the concert begins?

(b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

(c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t. The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

(d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

2009 SCORING GUIDELINES

Question 3

Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

(a) Find Mighty's profit on the sale of a 25-meter cable.

(b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\,dx$ in the context of this problem.

(c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

(d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

2009 SCORING GUIDELINES

Question 5

x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

(a) Estimate $f'(4)$. Show the work that leads to your answer.

(b) Evaluate $\int_{2}^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

(c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_{2}^{13} f(x)\,dx$. Show the work that leads to your answer.

(d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

2009 SCORING GUIDELINES (Form B)

Question 2

A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \frac{1}{2\sqrt{t}} - \sin t$.

(a) What was the distance between the road and the edge of the water at the end of the storm?

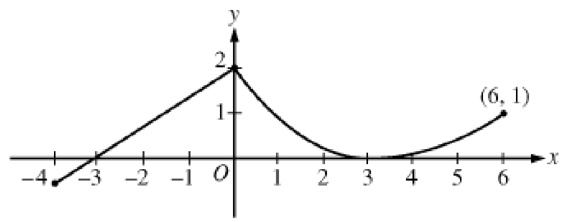
(b) Using correct units, interpret the value $f'(4) = 1.007$ in terms of the distance between the road and the edge of the water.

(c) At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.

(d) After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

2009 SCORING GUIDELINES (Form B)

Question 3



Graph of f

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

(a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

(b) For how many values of a, $-4 \le a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0 ? Give a reason for your answer.

(c) Is there a value of a, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

(d) The function g is defined by $g(x) = \int_0^x f(t)\,dt$ for $-4 \le x \le 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

2009 SCORING GUIDELINES (Form B)

Question 6

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

(c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

2010 SCORING GUIDELINES

Question 1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

2010 SCORING GUIDELINES

Question 2

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

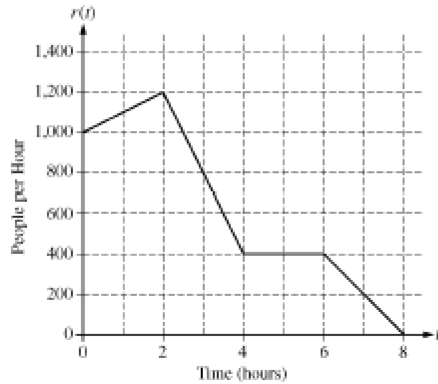
(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

2010 SCORING GUIDELINES

Question 3



There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

(a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

(b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

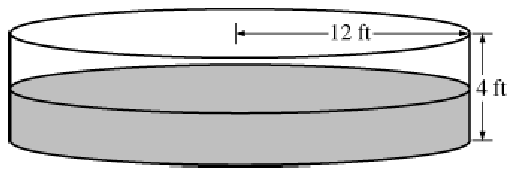
(c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

(d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

2010 SCORING GUIDELINES (Form B)

Question 3

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

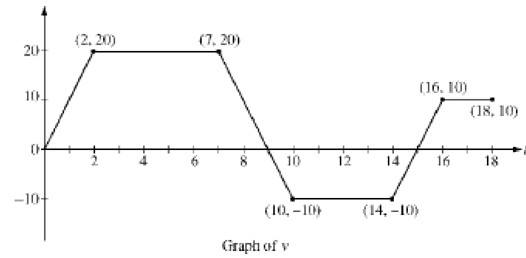
(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

(d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

2010 SCORING GUIDELINES (Form B)

Question 4



A squirrel starts at building A at time $t = 0$ and travels along a straight wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at this time?

(c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

2010 SCORING GUIDELINES (Form B)

Question 6

Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

(a) For $0 \le t \le 6$, find all times t during which particle R is moving to the right.

(b) For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.

(c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

(d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3$.

2011 SCORING GUIDELINES

Question 1

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

2011 SCORING GUIDELINES

Question 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selectec values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

2011 SCORING GUIDELINES

Question 6

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

2011 SCORING GUIDELINES (Form B)

Question 1

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(a) According to the model, what is the height of the water in the can at the end of the 60-day period?

(b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

(c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

2011 SCORING GUIDELINES (Form B)

Question 2

A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

2011 SCORING GUIDELINES (Form B)

Question 5

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\,dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\,dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

(d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

2012 SCORING GUIDELINES

Question 1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

2012 SCORING GUIDELINES

Question 6

For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

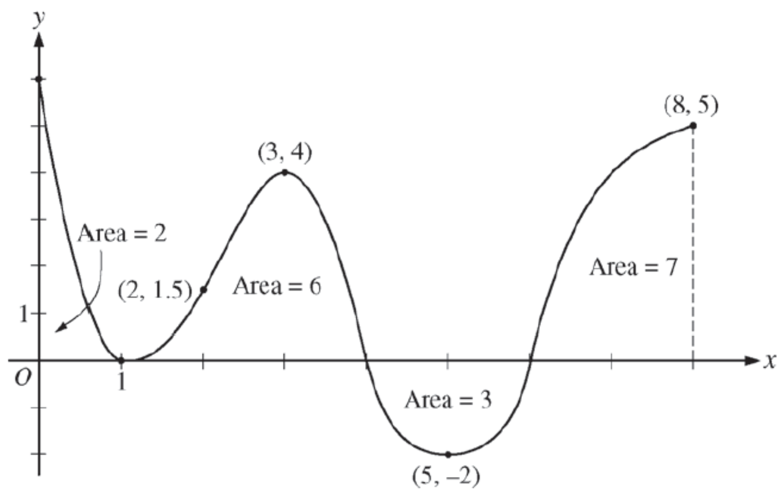
(a) For $0 \le t \le 12$, when is the particle moving to the left?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

(d) Find the position of the particle at time $t = 4$.

2013 AB/BC



Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

2013 AB/BC

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

2013 AB/BC

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

2013 AB

2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

2014 AB #1

1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

   (a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

   (b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

   (c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

   (d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

2014 AB #4

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	–120	–150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

   (a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

   (b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

   (c) At time $t = 2$, train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.

   (d) A second train, train B, travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

2015 #1 Calculator Required

1. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

   (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

   (b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

   (c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

   (d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

2015 #3 No Calculator Permitted

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

3. Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

   (a) Use the data in the table to estimate the value of $v'(16)$.

   (b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem.

   Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

   (c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute.

   Find Bob's acceleration at time $t = 5$.

   (d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

Calculator Permitted

2016 SCORING GUIDELINES

Question 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

Calculuator Permitted

2016 SCORING GUIDELINES

Question 2

For $t \ge 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the position of the particle at time $t = 0$.

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

No Calculator Permitted

2016 SCORING GUIDELINES

Question 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

AP Calculus AB 2017 Calculuator Required

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

   (a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

   (b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

   (c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

   (d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

AP Calculus AB 2017 Calculator Required

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \le 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \le 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

(a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

(b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

(c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

(d) How many pounds of bananas are on the display table at time $t = 8$?

AP Calculus AB 2017 No Calculator Permitted:

5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

(a) For $0 \le t \le 8$, when is particle P moving to the left?

(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

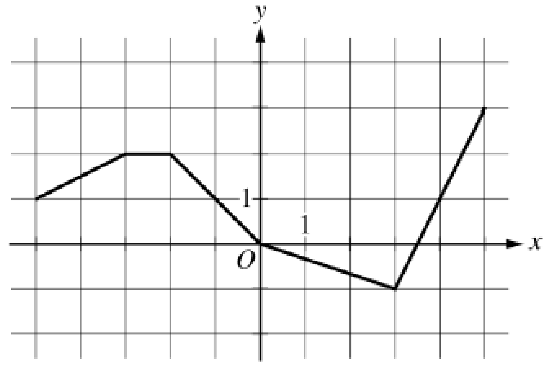
(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

(d) Find the position of particle Q the first time it changes direction.

AP Calculus AB 2017 No Calculator Permitted

2017

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

2018 Calculator Required

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

2018 Calculator Required

2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

2018 No Calculator Allowed

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

   (a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

   (b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

   (c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

   (d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \dfrac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

2018 No Calculator Allowed

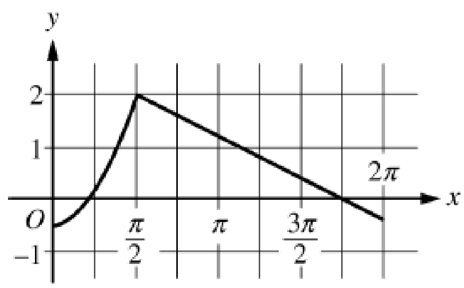
5. Let f be the function defined by $f(x) = e^x \cos x$.

   (a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

   (b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

   (c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

   (d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'