Simple Harmonic Motion Equations:

$$x = A\cos(\omega t + \varphi) \qquad \omega = 2\pi f = 2\pi/T \qquad \omega = \sqrt{k/m}$$

Damped Harmonic Motion

This gives $\boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0.}$

If b is small, a solution of the form

$$x = Ae^{-\gamma t}\cos\omega' t$$

will work, with

$$\gamma = \frac{b}{2m}$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}.$$

oct. 8, 2012 57

Damped Sinusoidally driven Harmonic Motion:

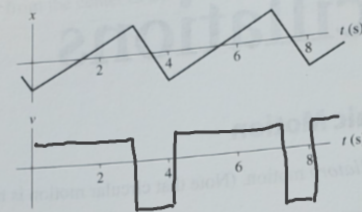
$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0)$$

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Assignment: 250 Oscillations Solutions

Page 2 / 6

3. Consider the particle whose motion is represented by the x-versus-t graph below.



a. Is this periodic motion? yes

b. Is this motion SHM? No (not sinusoidal)

c. What is the period? 4 s

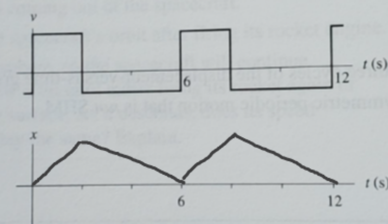
d. What is the frequency? $\frac{1}{4}$ Hz

e. You learned in Chapter 2 to relate velocity graphs to position graphs. Use that knowledge to draw the particle's velocity-versus-time graph on the axes provided.

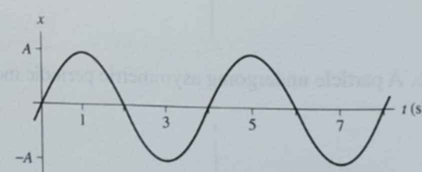
4. Shown below is the velocity-versus-time graph of a particle.

a. What is the period of the motion? 6 s

b. Draw the particle's position-versus-time graph, starting from $x = 0$ at $t = 0$ s.



5. The figure shows the position-versus-time graph of a particle in SHM.



a. At what times is the particle moving to the right at maximum speed?

0 s, 4 s, 8 s

b. At what times is the particle moving to the left at maximum speed?

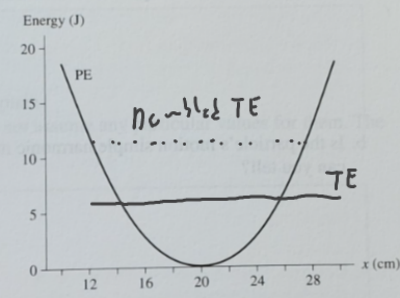
2 s, 6 s

c. At what times is the particle instantaneously at rest?

1 s, 3 s, 5 s, 7 s

Printed from Classkick

10. The figure shows the potential-energy diagram of a particle oscillating on a spring.



a. What is the spring's equilibrium length?

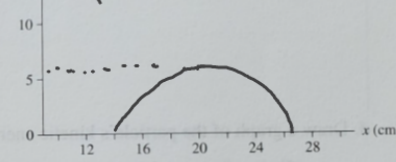
$x = 20$ cm (minimum PE = equilibrium for a spring)

b. The particle's turning points are at 14 cm and 26 cm. Draw the total energy line and label it TE.

c. What is the particle's maximum kinetic energy?

About 5 or 6 J

(K + PE = TE)



d. Draw a graph of the particle's kinetic energy as a function of position.

e. What will be the turning points if the particle's total energy is doubled?

About $x = 12$ cm and 28 cm

11. A block oscillating on a spring has an amplitude of 20 cm. What will be the block's amplitude if its total energy is tripled? Explain.

$TE = PE_{max} = \frac{1}{2}kA^2$.

$\Rightarrow A \propto \sqrt{TE}$. so tripling TE gives A multiplied by $\sqrt{3}$. $A_{new} = 20\sqrt{3}$ cm

12. A block oscillating on a spring has a maximum speed of 20 cm/s. What will be the block's maximum speed if its total energy is tripled? Explain.

$TE = K_{max} = \frac{1}{2}mv_{max}^2$

$\Rightarrow v_{max} \propto \sqrt{TE}$

so tripling TE multiplies v_{max} by $\sqrt{3}$!

$v_{max,\,new} = 20\sqrt{3}\ \frac{cm}{s}$

14.5 Vertical Oscillations

$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$

15. A block oscillating on a spring has period $T = 4$ s.

a. What is the period if the block's mass is halved? Explain.
Note: You do not know values for either m or k. Do *not* assume any particular values for them. The required analysis involves thinking about ratios.

$T \propto \sqrt{m}$, so multiplying m by 0.5 multiplies T by $\sqrt{0.5} = 1/\sqrt{2}$

$T_{new} = \frac{4s}{\sqrt{2}}$

b. What is the period if the value of the spring constant is quadrupled?

$T \propto 1/\sqrt{k}$. multiplying k by 4 multiplies T by $\sqrt{\frac{1}{4}} = \frac{1}{2}$

so $T_{new} = \frac{4s}{2} = 2s$

c. What is the period if the oscillation amplitude is doubled while m and k are unchanged?

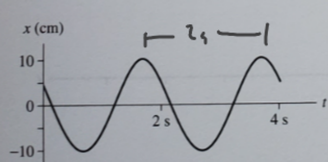
Amplitude doesn't affect period! so still 4s

16. For graphs a and b, determine:
- The angular frequency ω.
- The oscillation amplitude A.
- ~~The phase constant ϕ_0.~~

$\omega = \frac{2\pi}{T}$

Note: Graphs a and b are independent. Graph b is *not* the velocity graph of a.

a.

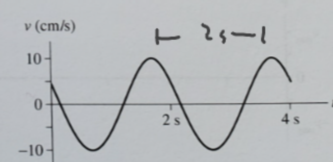


$\omega = \frac{2\pi \text{ rad}}{2s} = \pi$ rad/s

$A =$ 10 cm

~~$\phi_0 =$~~

b.

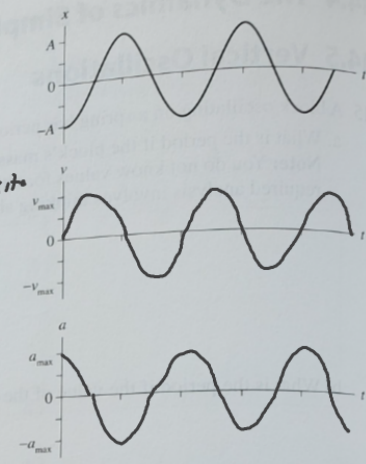


$\omega =$ π rad/s (as in a)

~~$A =$~~

~~$\phi_0 =$~~

Assignment: 250 Oscillations Solutions



17. The graph on the right is the position-versus-time graph for a simple harmonic oscillator.

a. Draw the v-versus-t and a-versus-t graphs.

b. When x is greater than zero, is a ever greater than zero? If so, at which points in the cycle?

Nope. Since $F = ma = -k\cancel{x}x$, $a = -\frac{k}{m}x$ → Signs of a and x are opposite

c. When x is less than zero, is a ever less than zero? If so, at which points in the cycle?

Nope - see above

d. Can you make a general conclusion about the relationship between the sign of x and the sign of a?

Yes - see above

e. When x is greater than zero, is v ever greater than zero? If so, how is the oscillator moving at those times?

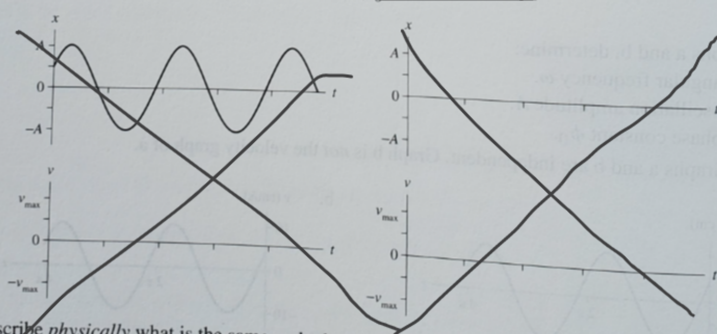
Sure! It is to the right of the origin but moving to the right: → v

~~18. For the oscillation shown on the left below:~~

~~a. What is the phase constant ϕ_0?~~

~~b. Draw the corresponding v-versus-t graph on the axes below the x-versus-t graph.~~

~~c. On the axes on the right, sketch two cycles of the x-versus-t and the v-versus-t graphs if the value of ϕ_0 found in part a is replaced by its negative, $-\phi_0$.~~



~~d. Describe *physically* what is the same and what is different about the initial conditions for two oscillators having "equal but opposite" phase constants ϕ_0 and $-\phi_0$.~~

Assignment: 250 Oscillations Solutions

14.8 Driven Oscillations and Resonance

26. What is the difference between the driving frequency and the natural frequency of an oscillator?

- Natural frequency is the frequency of oscillation if the system is undisturbed.
- Driving frequency is the frequency of some external stimulus to the system.

27. A car drives along a bumpy road on which the bumps are equally spaced. At a speed of 20 mph, the frequency of hitting bumps is equal to the natural frequency of the car bouncing on its springs.

a. Draw a graph of the car's vertical bouncing amplitude as a function of its speed if the car has new shock absorbers (large damping coefficient).

b. Draw a graph of the car's vertical bouncing amplitude as a function of its speed if the car has worn out shock absorbers (small damping coefficient).

Draw both graphs on the same axes, and label them as to which is which.