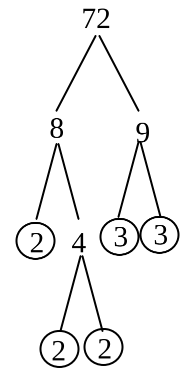
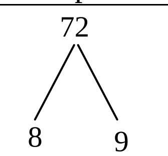
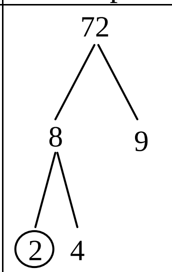
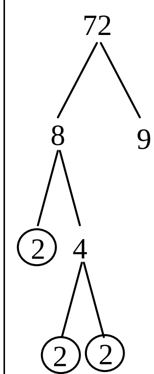
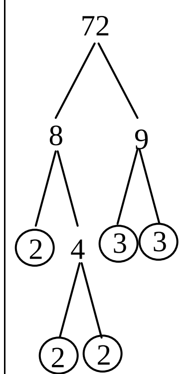
<u>Differentiation is like a Factor Tree</u>



Remember factoring integers like 72 using a factor tree like the one on the right? You thought of two numbers (i.e. factors) that multiplied to 72 and wrote them down as the first two branches of your factor tree. If one of the factors was prime, you “ended” that branch. If not, you created two more branches at that factor and continued the same process for each factor that wasn’t a prime number. Once all of your branches ended in primes you had your prime factorization of 72. That is you figured out that $72 = 2\cdot 2\cdot 2\cdot 3\cdot 3$ by putting together all the ends of your factor tree together.

Note that you didn’t come up with the factor tree all at once. You worked to reach the ends of certain branches, then went back to finish the others un-finished branches. Your factor tree might have evolved as illustrated in the table below.

Step 1	Step 2	Step 3	Step 4
			

What’s interesting is that differentiation is a lot like a factor tree. Oftentimes students believe that there is one rule that will take care of differentiating a function in one fell swoop. That is <u>not</u> the case for complicated functions. Complicated functions need to be broken down in to pieces that need further differentiation, which need further differentiation,... until no further differentiation is needed. This is similar to the way that integers can be factored as a product of two numbers using a factor tree, where each number can be factored further, until you can’t factor any more.

The only problem is that differentiation is a little bit more complicated than factoring integers. At each juncture in your “differentiation tree”, you might have to decide which rule to use (Product Rule, Quotient Rule, Chain Rule, or the Sum & Difference Rules) to get your next smaller piece to differentiate. Each split in the tree would look like one if the following in the table below:

Product Rule	Quotient Rule	Chain Rule	Sum/Difference Rule
$\left[f(x)\cdot g(x)\right]'$ $\downarrow$ $f'(x)g(x)+f(x)g'(x)$	$\left[\dfrac{f(x)}{g(x)}\right]'$ $\downarrow$ $\dfrac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$	$f(g(x))$ $\downarrow$ $f'(g(x))\cdot g'(x)$	$f(x)\pm g(x)$ $\downarrow$ $f'(x)\pm g'(x)$

Once you break down your function to pieces that require only the basic differentiation rules you will not be able to differentiate any further - just like reaching a prime number in a factor tree! Once you have your tree complete, you can piece together what the derivative will be.

The best way to understand the analogy of the process of differentiation to prime factorization is to look at the worked out example on the next page of this handout.

Sum/Difference Rule

$$h(x)=\overbrace{\left(\frac{x^2+3}{x-1}\right)^2+\underbrace{(2x+1)}\underbrace{\left(3x^2+5x+1\right)^8}}$$

Power Rule
(+ Chain Rule)

$$\underbrace{\boxed{2\left(\frac{x^2+3}{x-1}\right)^1}}_{\text{Power Rule}}\cdot\underbrace{\left[\left(\frac{x^2+3}{x-1}\right)\right]'}_{\text{Chain Rule}}$$

Quotient Rule

$$\frac{\underbrace{\left(x^2+3\right)'}_{\boxed{2x}}\cdot\boxed{(x-1)}-\boxed{\left(x^2+3\right)}\underbrace{(x-1)'}_{\boxed{1}}}{\boxed{(x-1)^2}}$$

Product Rule

$$\underbrace{\left[(2x+1)\right]'}_{\boxed{2}}\cdot\boxed{\left(3x^2+5x+1\right)^8}+\boxed{(2x+1)}\left[\left(3x^2+5x+1\right)^8\right]'$$

Power Rule
(+Chain Rule)

$$\underbrace{\boxed{8\left(3x^2+5x+1\right)^7}}_{\text{Power Rule}}\underbrace{\left[3x^2+5x+1\right]'}_{\substack{\text{Chain Rule}\\ \downarrow \\ \boxed{6x+5}}}$$

Result: $h'(x)=2\left(\frac{x^2+3}{x-1}\right)^1\cdot\left(\frac{(2x)\cdot(x-1)-\left(x^2+3\right)}{(x-1)^2}\right)+(2)\cdot\left(3x^2+5x+1\right)^8+(2x+1)\cdot 8\left(3x^2+5x+1\right)^7(6x+5)$