Maclaurin Series that must be memorized for the AP Calculus BC exam:

Function	**Summation**	**Expansion**	**Interval of Convergence**
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots$	All real numbers
$\sin(x)$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$	$x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}-\cdots$	All real numbers
$\cos(x)$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$	$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$	All real numbers
$\frac{1}{1+x}$	$\sum_{n=0}^{\infty} (-1)^n x^n$	$1-x+x^2-x^3+\cdots+(-1)^n x^n+\cdots$	$-1<x<1$

Use the Taylor Series to find the given expansion:

$f(x)=e^{x^2}$ centered at $x=0$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\downarrow$$

$$f(x)=e^{x^2} = \sum_{n=0}^{\infty} \frac{\left(x^2\right)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$$

$g(x)=\sin(\pi x)$ centered at $x=0$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\downarrow$$

$$\sin(\pi x) = \sum_{n=0}^{\infty} (-1)^n \frac{(\pi x)^{2n+1}}{(2n+1)!}$$

$h(x)=\cos(4x)$ centered at $x=0$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\downarrow$$

$$\cos(4x) = \sum_{n=0}^{\infty} (-1)^n \frac{(4x)^{2n}}{(2n)!}$$

Use the table above to help determine the value of the series:

$$\sum_{n=0}^{\infty} \frac{1}{2^n n!} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)^n}{n!} = e^x\Big|_{x=\frac{1}{2}} = e^{\frac{1}{2}}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^n (2n!)} = \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{2}{3}\right)^n}{(2n!)} = \sum_{n=0}^{\infty} (-1)^n \frac{\left(\sqrt{\frac{2}{3}}\right)^{2n}}{(2n!)} = \cos(x)\Bigg|_{x=\sqrt{\frac{2}{3}}} = \cos\left(\sqrt{\frac{2}{3}}\right)$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{3^{2n+1}(2n+1)!} = \sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{1}{3}\right)^{2n+1}}{(2n+1)!} = \sin(x)\Big|_{x=\frac{1}{3}} = \sin\left(\frac{1}{3}\right)$$