Stewart Section 11-8 Complete Homework Solutions.

#3

$\sum_{n=1}^{\infty}(-1)^n \cdot n \cdot x^n$

$$\begin{aligned}\lim_{n\to\infty}\sqrt[n]{\left|(-1)^n \cdot n \cdot x^n\right|} &= \lim_{n\to\infty}\sqrt[n]{n \cdot |x|^n} \\ &= \lim_{n\to\infty}\left[\sqrt[n]{n} \cdot \sqrt[n]{|x|^n}\right] \\ &= |x|\end{aligned}$$

The series will converge so long as $|x| < 1$

Therefore the series is centered at $x = 0$ and has a radius of convergence $= 1$.

If $x = -1$	If $x = 1$
$\sum_{n=1}^{\infty}(-1)^n \cdot n \cdot (-1)^n = \sum_{n=1}^{\infty}(-1)^{2n} \cdot n$ $= \sum_{n=1}^{\infty}\left[(-1)^2\right]^n \cdot n$ $= \sum_{n=1}^{\infty} n$ $\downarrow$ ∞	$\sum_{n=1}^{\infty}(-1)^n \cdot n \cdot (1)^n = \sum_{n=1}^{\infty}(-1)^n \cdot n$ Since $\lim_{n\to\infty}(-1)^n \cdot n \neq 0$, the limit of the n^{th} term test, $\sum_{n=1}^{\infty}(-1)^n \cdot n \cdot (1)^n$ does not converge.

Therefore the interval of convergence is $-1 < x < 1$.

#4

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot x^n}{\sqrt[3]{n}}$$

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(-1)^n \cdot x^n}{\sqrt[3]{n}}\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{|x|^n}{\sqrt[3]{n}}}$$

$$= \lim_{n\to\infty} \frac{|x|}{1}$$

$$= |x|$$

The series will converge so long as $|x| < 1$

Therefore the series is centered at $x = 0$ and has radius of convergence $= 1$

If $x = -1$	If $x = 1$
$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (-1)^n}{\sqrt[3]{n}} = \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{\sqrt[3]{n}}$ $= \sum_{n=1}^{\infty} \frac{\left[(-1)^2\right]^n}{\sqrt[3]{n}}$ $= \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ $= \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{3}}}$ This is a divergent *p*-series.	$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (1)^n}{\sqrt[3]{n}} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{\sqrt[3]{n}}$ This is an alternating series with $\lim_{n\to\infty} \frac{1}{\sqrt[3]{n}} = 0$. Therefore the series converges by the alternating series test.

Therefore the interval of convergence is $-1 < x \le 1$

#5

$$\sum_{n=1}^{\infty}\frac{x^n}{2n-1}$$

$$\begin{aligned}\lim_{n\to\infty}\sqrt[n]{\left|\frac{x^n}{2n-1}\right|} &= \lim_{n\to\infty}\sqrt[n]{\frac{|x|^n}{2n-1}} \\ &= \lim_{n\to\infty}\frac{\sqrt[n]{|x|^n}}{\sqrt[n]{2n-1}} \\ &= |x|\end{aligned}$$

Therefore the series will converge so long as $|x|<1$

The series is centered at $x=0$ and has radius of convergence $=1$

If $x=-1$	If $x=1$
$\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}=\sum_{n=1}^{\infty}(-1)^n\cdot\frac{1}{2n-1}$ This is an alternating series with $\lim_{n\to\infty}\frac{1}{2n-1}=0$, therefore the series converges by the alternating series test.	$\sum_{n=1}^{\infty}\frac{(1)^n}{2n-1}=\sum_{n=1}^{\infty}\frac{1}{2n-1}$ Since $\frac{1}{2n}<\frac{1}{2n-1}$ $\sum_{n=1}^{\infty}\frac{1}{2n}<\sum_{n=1}^{\infty}\frac{1}{2n-1}$ And $\sum_{n=1}^{\infty}\frac{1}{2n}=\frac{1}{2}\cdot\left[\sum_{n=1}^{\infty}\frac{1}{n}\right]$ Since $\sum_{n=1}^{\infty}\frac{1}{n}$ is the divergent harmonic series, $\sum_{n=1}^{\infty}\frac{(1)^n}{2n-1}$ diverges by the direct comparison test.

Therefore the interval of convergence is $-1\le x<1$

#6

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot x^n}{n^2}$$

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(-1)^n \cdot x^n}{n^2}\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{|x|^n}{n^2}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[n]{|x|^n}}{\sqrt[n]{n^2}}$$

$$= |x|$$

This series will converge so long as $|x| < 1$

Therefore the series is centered at $x = 0$ and has a radius of convergence $= 1$

If $x = -1$	If $x = 1$
$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (-1)^n}{n^2} = \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n^2}$ $= \sum_{n=1}^{\infty} \frac{\left[(-1)^2\right]^n}{n^2}$ $= \sum_{n=1}^{\infty} \frac{1}{n^2}$ This is a convergent *p*-series	$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (1)^n}{n^2} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n^2}$ This is an alternating series with $\lim_{n\to\infty} \frac{1}{n^2} = 0$. Therefore by the alternating series test, $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (1)^n}{n^2}$ converges.

Therefore the interval of convergence is $-1 \le x \le 1$.

#7

$$\sum_{n=1}^{\infty}\frac{x^n}{n!}$$

$$\begin{aligned}
\lim_{n\to\infty}\left|\frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}}\right| &= \lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)!}\cdot\frac{n!}{x^n}\right| \\
&= \lim_{n\to\infty}\left|\frac{x^{n+1}}{x^n}\cdot\frac{n!}{(n+1)!}\right| \\
&= \lim_{n\to\infty}\left|\frac{x^n\cdot x}{x^n}\cdot\frac{n!}{(n+1)\cdot[n!]}\right| \\
&= \lim_{n\to\infty}\left|\frac{x}{n+1}\right| \\
&= 0
\end{aligned}$$

This series will converge for all values of *x*.

This series is centered at $x=0$ and has a radius of convergence of ∞.

#8

$$\sum_{n=1}^{\infty} n^n \cdot x^n$$

$$\begin{aligned}\lim_{n\to\infty} \sqrt[n]{n^n \cdot x^n} &= \lim_{n\to\infty} \sqrt[n]{n^n} \cdot \sqrt[n]{|x|^n} \\ &= n \cdot |x|\end{aligned}$$

This series will converge only if $x = 0$.
This series is centered at $x = 0$ and has a radius of convergence of 0.

#9

$$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n^2 \cdot x^n}{2^n}$$

$$\begin{aligned}\lim_{n\to\infty} \sqrt[n]{\left|(-1)^n \cdot \frac{n^2 \cdot x^n}{2^n}\right|} &= \lim_{n\to\infty} \sqrt[n]{\frac{n^2 \cdot |x|^n}{2^n}} \\ &= \lim_{n\to\infty} \frac{\sqrt[n]{n^2} \cdot \sqrt[n]{|x|^n}}{\sqrt[n]{2^n}} \quad \text{t} \\ &= \frac{|x|}{2}\end{aligned}$$

This series will converge so long as

$$\begin{aligned}\frac{|x|}{2} &< 1 \\ |x| &< 2\end{aligned}$$

Therefore this series is centered at $x = 0$ and has a radius of convergence $= 2$.

If $x = -2$	If $x = 2$
$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n^2 \cdot (-2)^n}{2^n} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{n^2 \cdot (-1)^n (2)^n}{2^n}$ $= \sum_{n=1}^{\infty} (-1)^{2n} \cdot \frac{n^2 (2)^n}{2^n}$ $= \sum_{n=1}^{\infty} \left[(-1)^2\right]^n \cdot \frac{n^2 2^n}{2^n}$ $= \sum_{n=1}^{\infty} n^2$ Since $\lim_{n\to\infty} n^2 \neq 0$, this series will not converge the limit of the n^{th} term test.	$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n^2 \cdot (2)^n}{2^n} = \sum_{n=1}^{\infty} (-1)^n \cdot n^2$ Since $\lim_{n\to\infty} (-1)^n \cdot n^2 \neq 0$, this series will not converge the limit of the n^{th} term test.

Therefore the interval of convergence is $-2 < x < 2$.

#10

$$\sum_{n=1}^{\infty} \frac{10^n \cdot x^n}{n^3}$$

$$\begin{aligned} \lim_{n\to\infty} \sqrt[n]{\left|\frac{10^n \cdot x^n}{n^3}\right|} &= \lim_{n\to\infty} \sqrt[n]{\frac{10^n \cdot |x|^n}{n^3}} \\ &= \lim_{n\to\infty} \frac{\sqrt[n]{10^n} \cdot \sqrt[n]{|x|^n}}{\sqrt[n]{n^3}} \\ &= 10 \cdot |x| \end{aligned}$$

This series will converge so long as

$$\begin{aligned} 10 \cdot |x| &< 1 \\ |x| &< \frac{1}{10} \end{aligned}$$

Therefore this series is centered at $x = 0$ and has radius of convergence $= \frac{1}{10}$

If $x = -\frac{1}{10}$	If $x = \frac{1}{10}$
$\sum_{n=1}^{\infty} \frac{10^n \cdot \left(-\frac{1}{10}\right)^n}{n^3} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n^3}$ This is an alternating series with $\lim_{n\to\infty} \frac{1}{n^3} = 0$, therefore the series converges by the alternating series test.	$\sum_{n=1}^{\infty} \frac{10^n \cdot \left(\frac{1}{10}\right)^n}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n^3}$ This is a convergent *p*-series.

Therefore the interval of convergence is $-\frac{1}{10} \le x \le \frac{1}{10}$.

#11

$$\sum_{n=1}^{\infty}\frac{(-3)^n}{n\sqrt{n}}\cdot x^n$$

$$\begin{aligned}\lim_{n\to\infty}\sqrt[n]{\left|\frac{(-3)^n}{n\sqrt{n}}\cdot x^n\right|} &= \lim_{n\to\infty}\sqrt[n]{\frac{3^n}{n\sqrt{n}}\cdot|x|^n}\\ &= \lim_{n\to\infty}\frac{\sqrt[n]{3^n}}{\sqrt[n]{n}\sqrt[n]{\sqrt{n}}}\cdot\sqrt[n]{|x|^n}\\ &= \frac{3}{1\cdot 1}\cdot|x|\\ &= 3|x|\end{aligned}$$

This series will converge so long as

$$3|x|<1$$

$$|x|<\frac{1}{3}$$

The series is centered at $x=0$ and has radius of convergence $=\frac{1}{3}$

If $x=-\frac{1}{3}$	If $x=\frac{1}{3}$
$\sum_{n=1}^{\infty}\frac{(-3)^n}{n\sqrt{n}}\cdot\left(-\frac{1}{3}\right)^n=\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}}$ $=\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$ This series is a convergent *p*-series.	$\sum_{n=1}^{\infty}\frac{(-3)^n}{n\sqrt{n}}\cdot\left(\frac{1}{3}\right)^n=\sum_{n=1}^{\infty}\frac{(-1)^n}{n\sqrt{n}}$ $=\sum_{n=1}^{\infty}(-1)^n\cdot\frac{1}{n\sqrt{n}}$ This series is alternating with $\lim_{n\to\infty}\frac{1}{n\sqrt{n}}=0$, therefore this series converges by the alternating series test.

The interval of convergence is

$$-\frac{1}{3}\le x\le\frac{1}{3}$$

#12

$$\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 3^n}$$

$$\begin{aligned}\lim_{n\to\infty} \sqrt[n]{\left|\frac{x^n}{n \cdot 3^n}\right|} &= \lim_{n\to\infty} \sqrt[n]{\frac{|x|^n}{n \cdot 3^n}} \\ &= \lim_{n\to\infty} \frac{\sqrt[n]{|x|^n}}{\sqrt[n]{n} \cdot \sqrt[n]{3^n}} \\ &= \frac{|x|}{3}\end{aligned}$$

This series will converge if

$$\begin{aligned}\frac{|x|}{3} &< 1 \\ |x| &< 3\end{aligned}$$

Therefore the series is centered at $x = 0$ and has radius of convergence $= 3$

If $x = -3$	If $x = 3$
$\sum_{n=1}^{\infty} \frac{(-3)^n}{n \cdot 3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 3^n}{n \cdot 3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n}$ This is an alternating series with $\lim_{n\to\infty} \frac{1}{n} = 0$, therefore the series converges by the alternating series test.	$\sum_{n=1}^{\infty} \frac{(3)^n}{n \cdot 3^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ This is the harmonic series, which diverges.

The interval of convergence is $-3 \le x < 1$

#13

$$\sum_{n=1}^{\infty}(-1)^n \cdot \frac{x^n}{4^n \cdot \ln(n)}$$

$$\lim_{n\to\infty}\sqrt[n]{\left|(-1)^n \cdot \frac{x^n}{4^n \cdot \ln(n)}\right|} = \lim_{n\to\infty}\sqrt[n]{\frac{|x|^n}{4^n \cdot \ln(n)}}$$

$$= \lim_{n\to\infty}\frac{\sqrt[n]{|x|^n}}{\sqrt[n]{4^n} \cdot \sqrt[n]{\ln(n)}}$$

$$= \frac{|x|}{4}$$

This series will converge so long as

$$\frac{|x|}{4} < 1$$

$$|x| < 4$$

The series is centered at $x = 0$ and has radius of convergence $= 4$

If $x = -4$	If $x = 4$
$\sum_{n=1}^{\infty}(-1)^n \cdot \frac{(-4)^n}{4^n \cdot \ln(n)} = \sum_{n=1}^{\infty}(-1)^n \cdot \frac{(-1)^n \cdot 4^n}{4^n \cdot \ln(n)}$ $= \sum_{n=1}^{\infty}(-1)^{2n} \cdot \frac{4^n}{4^n \cdot \ln(n)}$ $= \sum_{n=1}^{\infty}\left[(-1)^2\right]^n \cdot \frac{1}{\ln(n)}$ $= \sum_{n=1}^{\infty}\frac{1}{\ln(n)}$ Since $\frac{1}{n} < \frac{1}{\ln(n)}$ $\sum_{n=1}^{\infty}\frac{1}{n} < \sum_{n=1}^{\infty}\frac{1}{\ln(n)}$ And $\sum_{n=1}^{\infty}\frac{1}{n}$ is the divergent harmonic series, $\sum_{n=1}^{\infty}(-1)^n \cdot \frac{(-4)^n}{4^n \cdot \ln(n)}$ diverges by the direct comparison test.	$\sum_{n=1}^{\infty}(-1)^n \cdot \frac{(4)^n}{4^n \cdot \ln(n)} = \sum_{n=1}^{\infty}(-1)^n \cdot \frac{1}{\ln(n)}$ This series is an alternating series with $\lim_{n\to\infty}\frac{1}{\ln(n)} = 0$, therefore by the alternating series test the series converges.

The interval of convergence is $-4 < x \le 4$.

#14

$$\sum_{n=1}^{\infty}(-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!}$$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}\cdot\frac{x^{2(n+1)+1}}{(2(n+1)+1)!}}{(-1)^n\cdot\frac{x^{2n+1}}{(2n+1)!}}\right| = \lim_{n\to\infty}\left|\frac{\frac{x^{2(n+1)+1}}{(2(n+1)+1)!}}{\frac{x^{2n+1}}{(2n+1)!}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x^{2n+3}}{(2n+3)!}\cdot\frac{(2n+1)!}{x^{2n+1}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(2n+1)!}{(2n+3)!}\cdot\frac{x^{2n+3}}{x^{2n+1}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(2n+1)!}{(2n+3)(2n+2)\left[(2n+1)!\right]}\cdot\frac{x^{2n+1}\cdot x^2}{x^{2n+1}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x^2}{(2n+3)(2n+2)}\right|$$

$$= 0$$

This series will converge for all values of *x*.
This series is centered at $x = 0$ and has a radius of convergence of ∞.

#15

$$\sum_{n=1}^{\infty}\frac{(x-2)^n}{n^2+1}$$

$$\begin{aligned}\lim_{n\to\infty}\sqrt[n]{\left|\frac{(x-2)^n}{n^2+1}\right|} &= \lim_{n\to\infty}\sqrt[n]{\frac{|x-2|^n}{n^2+1}}\\ &= \lim_{n\to\infty}\frac{\sqrt[n]{|x-2|^n}}{\sqrt[n]{n^2+1}}\\ &= |x-2|\end{aligned}$$

This series will converge if $|x-2|<1$

The series is centered at $x=2$ and the radius of convergence is 1

$$|x-2|<1$$

$$-1<x-2<1$$

$$1<x<3$$

If $x=1$	If $x=3$
$\sum_{n=1}^{\infty}\frac{(1-2)^n}{n^2+1}=\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+1}$ $=\sum_{n=1}^{\infty}(-1)^n\cdot\frac{1}{n^2+1}$ This is an alternating series with $\lim_{n\to\infty}\frac{1}{n^2+1}=0$, therefore by the alternating series test, this series converges.	$\sum_{n=1}^{\infty}\frac{(3-2)^n}{n^2+1}=\sum_{n=1}^{\infty}\frac{(1)^n}{n^2+1}$ $=\sum_{n=1}^{\infty}\frac{1}{n^2+1}$ Since $\frac{1}{n^2+1}<\frac{1}{n^2}$ $\sum_{n=1}^{\infty}\frac{1}{n^2+1}<\sum_{n=1}^{\infty}\frac{1}{n^2}$ And $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is a convergent *p*-series, $\sum_{n=1}^{\infty}\frac{(3-2)^n}{n^2+1}$ converges by the direct comparison test.

The interval of convergence is $1\le x\le 3$.

#16

$$\sum_{n=1}^{\infty}(-1)^n \cdot \frac{(x-3)^n}{2n+1}$$

$$\lim_{n\to\infty}\sqrt[n]{\left|(-1)^n \cdot \frac{(x-3)^n}{2n+1}\right|} = \lim_{n\to\infty}\sqrt[n]{\frac{|x-3|^n}{2n+1}}$$

$$= \lim_{n\to\infty}\frac{\sqrt[n]{|x-3|^n}}{\sqrt[n]{2n+1}}$$

$$= |x-3|$$

This series will converge if $|x-3|<1$

The series is centered at $x=3$ and the radius of convergence is 1

$$|x-3|<1$$

$$-1<x-3<1$$

$$2<x<4$$

<table>
<tr><th>If $x=2$</th><th>If $x=4$</th></tr>
<tr><td>

$$\sum_{n=1}^{\infty}(-1)^n \cdot \frac{(2-3)^n}{2n+1} = \sum_{n=1}^{\infty}(-1)^n \cdot \frac{(-1)^n}{2n+1}$$

$$= \sum_{n=1}^{\infty}(-1)^{2n} \cdot \frac{1}{2n+1}$$

$$= \sum_{n=1}^{\infty}\left[(-1)^2\right]^n \cdot \frac{1}{2n+1}$$

$$= \sum_{n=1}^{\infty}\frac{1}{2n+1}$$

Since

$$\lim_{n\to\infty}\frac{1}{2n+1} = \lim_{n\to\infty}\frac{1}{2n}$$

and

$$\lim_{n\to\infty}\frac{\left(\frac{1}{2n}\right)}{\left[\frac{1}{2n+1}\right]} = \lim_{n\to\infty}\frac{\left(\frac{1}{2n}\right)}{\left[\frac{1}{2n}\right]} = 1$$

Since $\sum_{n=1}^{\infty}\frac{1}{2n} = \frac{1}{2}\cdot\left[\sum_{n=1}^{\infty}\frac{1}{n}\right]$ is a multiple of the divergent harmonic series, by the limit comparison test $\sum_{n=1}^{\infty}(-1)^n \cdot \frac{(2-3)^n}{2n+1}$ diverges.

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$$\sum_{n=1}^{\infty}(-1)^n \cdot \frac{(4-3)^n}{2n+1} = \sum_{n=1}^{\infty}(-1)^n \cdot \frac{(1)^n}{2n+1}$$

$$= \sum_{n=1}^{\infty}(-1)^n \cdot \frac{1}{2n+1}$$

This is an alternating series with $\lim_{n\to\infty}\frac{1}{2n+1} = 0$, therefore by the alternating series test $\sum_{n=1}^{\infty}(-1)^n \cdot \frac{(4-3)^n}{2n+1}$ converges.

</td></tr>
</table>

The interval of convergence is $2<x\le 4$.

#17

$$\sum_{n=1}^{\infty}\frac{3^n\cdot(x+4)^n}{\sqrt{n}}$$

$$\begin{aligned}
\lim_{n\to\infty}\sqrt[n]{\left|\frac{3^n\cdot(x+4)^n}{\sqrt{n}}\right|} &= \lim_{n\to\infty}\sqrt[n]{\frac{3^n\cdot|x+4|^n}{\sqrt{n}}}\\
&= \lim_{n\to\infty}\frac{\sqrt[n]{3^n}\cdot\sqrt[n]{|x+4|^n}}{\sqrt[n]{\sqrt{n}}}\\
&= \frac{3\cdot|x+4|}{1}\\
&= 3\cdot|x+4|
\end{aligned}$$

This series will converge if

$$3\cdot|x+4|<1$$

$$|x+4|<\frac{1}{3}$$

The series is centered at $x=-4$ and has radius of convergence $=\frac{1}{3}$

$$|x+4|<\frac{1}{3}$$

$$-\frac{1}{3}<x+4<\frac{1}{3}$$

$$-\frac{13}{3}<x<-\frac{11}{3}$$

If $x=-\frac{13}{3}$	If $x=-\frac{11}{3}$
$\sum_{n=1}^{\infty}\frac{3^n\cdot\left(-\frac{13}{3}+4\right)^n}{\sqrt{n}}=\sum_{n=1}^{\infty}\frac{3^n\cdot\left(-\frac{1}{3}\right)^n}{\sqrt{n}}$ $=\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ $=\sum_{n=1}^{\infty}(-1)^n\cdot\frac{1}{\sqrt{n}}$ Since this is an alternating series with $\lim_{n\to\infty}\frac{1}{\sqrt{n}}0$, by the alternating series test, the series converges.	$\sum_{n=1}^{\infty}\frac{3^n\cdot\left(-\frac{11}{3}+4\right)^n}{\sqrt{n}}=\sum_{n=1}^{\infty}\frac{3^n\cdot\left(\frac{1}{3}\right)^n}{\sqrt{n}}$ $=\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ $=\sum_{n=1}^{\infty}\frac{1}{n^{\frac{1}{2}}}$ This is a divergent *p*-series

The interval of convergence is $-\frac{13}{3}\le x<\frac{11}{3}$

#18

$$\sum_{n=1}^{\infty} \frac{n}{4^n} \cdot (x+1)^n$$

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{n}{4^n} \cdot (x+1)^n\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{4^n} \cdot |x+1|^n}$$

$$= \lim_{n\to\infty} \frac{\sqrt[n]{n}}{\sqrt[n]{4^n}} \cdot \sqrt[n]{|x+1|^n}$$

$$= \frac{1}{4} \cdot |x+1|$$

The series will converge when

$$\frac{1}{4} \cdot |x+1| < 1$$

$$|x+1| < 4$$

The series is centered at $x=0$ and the radius of convergence is 4

$$|x+1| < 4$$

$$-4 < x+1 < 4$$

$$-5 < x < 3$$

If $x=-5$	If $x=3$
$\sum_{n=1}^{\infty} \frac{n}{4^n} \cdot (-5+1)^n = \sum_{n=1}^{\infty} \frac{n}{4^n} \cdot (-4)^n$ $= \sum_{n=1}^{\infty} \frac{n}{4^n} \cdot (-1)^n \cdot 4^n$ $= \sum_{n=1}^{\infty} (-1)^n \cdot n$ Since the series is alternating and $\lim_{n\to\infty} n \neq 0$, the series does not converge by the alternating series test.	$\sum_{n=1}^{\infty} \frac{n}{4^n} \cdot (3+1)^n = \sum_{n=1}^{\infty} \frac{n}{4^n} \cdot 4^n$ $= \sum_{n=1}^{\infty} n$ Since $\lim_{n\to\infty} n \neq 0$, the series does not converge by the limit of the n^{th} term test.

The interval of convergence is $-5 < x < 3$

#19

$$\sum_{n=1}^{\infty}\frac{(x-2)^n}{n^n}$$

$$\begin{aligned}\lim_{n\to\infty}\sqrt[n]{\left|\frac{(x-2)^n}{n^n}\right|} &= \lim_{n\to\infty}\sqrt[n]{\frac{|x-2|^n}{n^n}}\\ &= \lim_{n\to\infty}\frac{\sqrt[n]{|x-2|^n}}{\sqrt[n]{n^n}}\\ &= \lim_{n\to\infty}\frac{|x-2|}{n}\\ &= 0\end{aligned}$$

Therefore the series converges for all real numbers

The series is centered at $x=2$ and has radius of convergence ∞

#20

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \cdot \sqrt{n}}$$

$$\begin{aligned}
\lim_{n\to\infty} \sqrt[n]{\left|\frac{(2x-1)^n}{5^n \cdot \sqrt{n}}\right|} &= \lim_{n\to\infty} \sqrt[n]{\frac{|2x-1|^n}{5^n \cdot \sqrt{n}}} \\
&= \lim_{n\to\infty} \frac{\sqrt[n]{|2x-1|^n}}{\sqrt[n]{5^n} \cdot \sqrt[n]{\sqrt{n}}} \\
&= \frac{|2x-1|}{5}
\end{aligned}$$

The series will converge when

$$\frac{|2x-1|}{5} < 1$$

$$|2x-1| < 5$$

The series is centered at $x = \frac{1}{2}$ and has radius of convergence of 5

$$|2x-1| < 5$$

$$-5 < 2x-1 < 5$$

$$-4 < 2x < 6$$

$$-2 < x < 3$$

If $x=-2$	If $x=3$
$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \cdot \sqrt{n}} = \sum_{n=1}^{\infty} \frac{(2(-2)-1)^n}{5^n \cdot \sqrt{n}}$ $= \sum_{n=1}^{\infty} \frac{(-5)^n}{5^n \cdot \sqrt{n}}$ $= \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 5^n}{5^n \cdot \sqrt{n}}$ $= \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{\sqrt{n}}$ This is an alternating series with $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$, by the alternating series test the series converges.	$\sum_{n=1}^{\infty} \frac{(2(3)-1)^n}{5^n \cdot \sqrt{n}} = \sum_{n=1}^{\infty} \frac{5^n}{5^n \cdot \sqrt{n}}$ $= \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ $= \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$ This is a divergent *p*-series

The interval of convergence is $-2 \le x < 3$.