Practice Differential Equations Multiple Choice Questions

1. No Calculator: If $\dfrac{dy}{dx}=\dfrac{3x^2+2}{y}$, and $y=4$ when $x=2$, then when $x=3$, $y=$

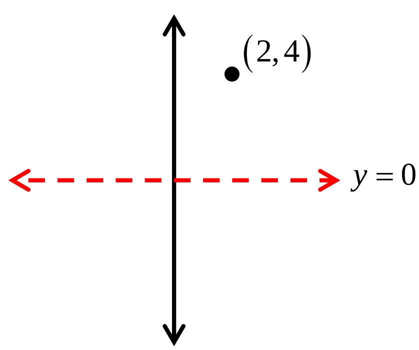
$$\frac{dy}{dx}=\frac{3x^2+2}{y}$$
$$ydy=\left(3x^2+2\right)dx$$
$$\int ydy=\int 3x^2+2dx$$
$$\frac{1}{2}y^2=x^3+2x+C$$
$$y^2=2x^3+4x+C$$

$$y^2=2x^3+4x+C$$
$$\downarrow$$
$$4^2=2(2)^3+4(2)+C$$
$$16=16+8+C$$
$$-8=C$$

$$y^2=2x^3+4x-8$$
$$y^2=2(3)^3+4(3)-8$$
$$y^2=54+12-8$$
$$y^2=58$$
$$y=\pm\sqrt{58}$$

Since the slope field is undefined for $y=0$ (i.e. the x-axis) and the solution passes through the coordinate $(2,4)$, we must use the positive square root value as the solution.

Solutions to the differential equation must lie either entirely above the line $y=0$ or entirely below the line $y=0$



(a) 18 (b) $\sqrt{66}$ (c) 58 (d) $\sqrt{74}$ (e) $\sqrt{58}$

2. No Calculator: The function f is continuous on the closed interval $[0,4]$, and twice differentiable on the open interval $(0,4)$. If $f'(2)=-5$ and $f''(x)>0$ over the interval $(0,4)$, which of the following could be a table of values for f?

Check to see that each of entries in the table is consistent with a decreasing function since $f'(2)<0$. All are consistent with decreasing functions.

Check to see that each table reflects a function that are concave up since $f''(x)>0$ for $x\in(0,4)$. That is, check to see that *<u>successive slope values are</u>* ***<u>increasing.</u>*** The only table for which this is true is (c).

A

x	y	
0	10	–2.5
1	7.5	–1.5
2	6	–1.5
3	4.5	–2.5
4	2	

B

x	y	
0	10	–2.5
1	7.5	–1
2	6.5	–3
3	3.5	–1.5
4	2	

C

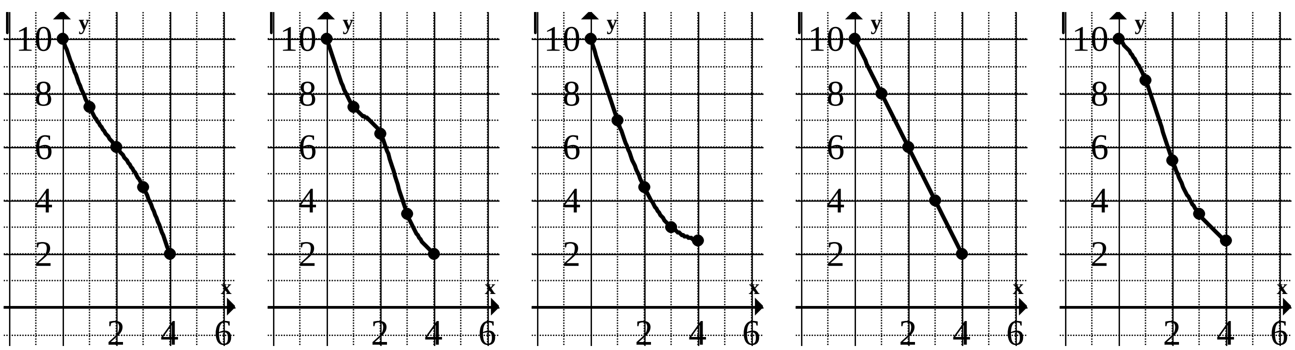
x	y	
0	10	–3
1	7	–2.5
2	4.5	–1.5
3	3	–0.5
4	2.5	

D

x	y	
0	10	–2
1	8	–2
2	6	–2
3	4	–2
4	2	

E

x	y	
0	10	–1.5
1	8.5	–3
2	5.5	–2
3	3.5	–1
4	2.5	



3. No Calculator: If $f'(x)=12x^2-6x+3$ and $f(1)=15$. What is $f(x)$?

$$f'(x)=12x^2-6x+3$$
$$f(x)=\int 12x^2-6x+3dx$$
$$=4x^3-3x^2+3x+C$$

$$f(x)=4x^3-3x^2+3x+C$$
$$\downarrow$$
$$15=4(1)^3-3(1)^2+3(1)+C$$
$$15=4+C$$
$$C=11$$

(a) $4x^3-3x^2+3x+1$

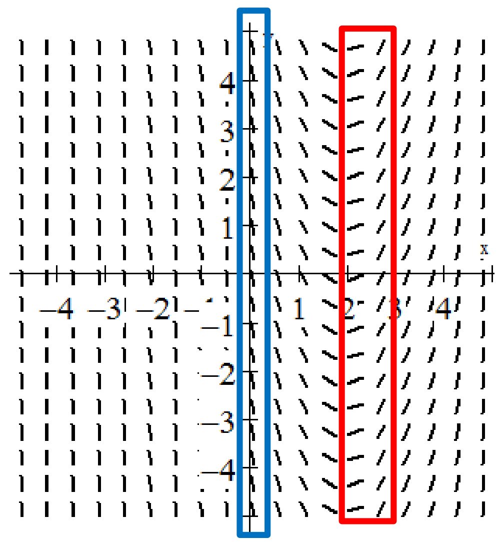
(b) $4x^3-3x^2+3x+11$

(c) $4x^3-6x^2+3x+1$

(d) $12x^3-6x-12$

(e) $4x^3-3x^2+3x-11$

4. No Calculator: Shown at right is the slope field for which of the following differential equations?



Vertical bands have the same slope. This indicates that the equation of *the differential equation is a function of x only*. This eliminates (d) and (e).

Looking at $x=0$, the slopes must be negative. This eliminates answer choices (a) and (c).

By process of elimination, the correct answer choice is (b).

(a) $\frac{dy}{dx}=2x$

(b) $\frac{dy}{dx}=2x-4$

(c) $\frac{dy}{dx}=4-2x$

(d) $\frac{dy}{dx}=y$

(e) $\frac{dy}{dx}=x+y$

5. No Calculator: Water is being pumped continuously from a water pool at a rate proportional to the amount of water left in the pool; that is $\frac{dy}{dt} = ky$, where y is the amount of water left in the pool at any time t, and k is a constant. Initially, there were 500,000 gallons of water in the pool, and 10 days later, there were 100,000 gallons of water in the pool. What is the equation for y, the amount of water remaining in the pool at any time t ?

$$\frac{dy}{dt} = ky$$
$$\frac{1}{y}dt = kdt$$
$$\int \frac{1}{y}dt = \int kdt$$
$$\ln|y| = kt + C$$
$$e^{\ln|y|} = e^{kt+C}$$
$$|y| = e^{kt} \cdot e^{C} \quad \text{let } A = e^{C}$$
$$|y| = Ae^{kt}$$
$$y = \pm Ae^{kt}$$
$$y = Ae^{kt}$$

$$y = Ae^{kt}$$
$$500,000 = Ae^{k(0)}$$
$$A = 500,000$$

$$y = 500,000e^{kt}$$
$$100,000 = 500,000e^{k(10)}$$
$$\frac{1}{5} = e^{10k}$$
$$\ln\left(\frac{1}{5}\right) = \ln\left(e^{10k}\right)$$
$$\ln\left(\frac{1}{5}\right) = 10k$$
$$k = \frac{1}{10}\ln\left(\frac{1}{5}\right)$$

$$y = 500,000e^{\frac{1}{10}\ln\left(\frac{1}{5}\right)t}$$
$$= 500,000\left(e^{\ln\left(\frac{1}{5}\right)}\right)^{\frac{t}{10}}$$
$$= 500,000\left(\frac{1}{5}\right)^{\frac{t}{10}}$$

(a) $y(t) = 500,000\left(\frac{1}{2}\right)^{\frac{t}{10}}$

(b) $y(t) = 500,000e^{\frac{1}{5}t}$

(c) $y(t) = 500,000\left(\frac{1}{10}\right)^{\frac{t}{10}}$

(d) $y(t) = 500,000\left(\frac{1}{5}\right)^{\frac{t}{10}}$

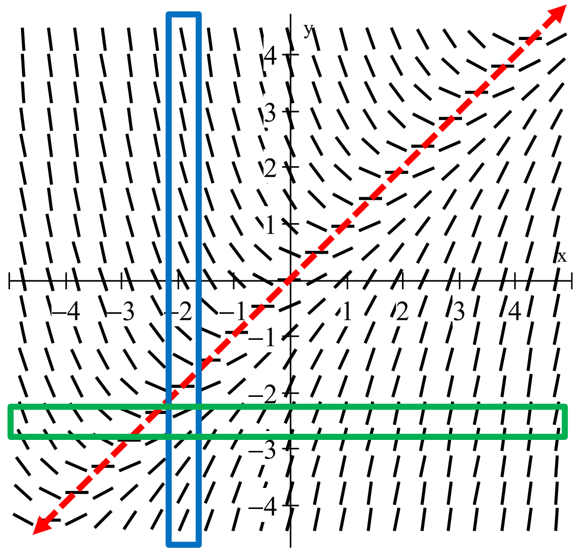
(e) $y(t) = 500,000e^{10t}$

6. No Calculator: Let f be the function whose derivative is $\frac{dy}{dx}=\frac{1+e^x}{x^2}$ and whose graph passes through the point $(3,6)$. What is the approximate value of $f(3.1)$ if $\int_3^{3.1} f'(x)\,dx \approx 0.2377$?

$$f(3.1)=f(3)+\int_3^{3.1} f'(x)\,dx$$
$$\approx 6+0.2377$$
$$\approx 6.2377$$

(a) 6.238 (b) 2.414 (c) 6.1 (d) –5.762 (e) –2.414

7. No Calculator: Shown at right is the slope field for which of the following differential equations?



Notice that all the slopes along the line $y=x$ are equal to zero. The only answer choice for which $\frac{dy}{dx}=0$ when $y=x$ is (a).
Also notice that the slopes are not all the same in vertical and horizontal bands. This indicates that $\frac{dy}{dx}$ is a function of both x *and* y. This eliminates (d) and (e).
(c) can be eliminated because if it were the anwer, when $x=0$ or $y=0$, then $\frac{dy}{dx}=0$ - which is not the case in this slope field.

(a) $\frac{dy}{dx}=x-y$

(b) $\frac{dy}{dx}=x+y$

(c) $\frac{dy}{dx}=xy$

(d) $\frac{dy}{dx}=e^x$

(e) $\frac{dy}{dx}=x^2$

8. No Calculator: Let $\frac{dy}{dx} = e^{x-y}$, which of the following is a solution to the differential equation such that $y(0) = 1$?

$$\frac{dy}{dx} = e^{x-y} \qquad y = \ln\left(e^x + C\right)$$
$$\frac{dy}{dx} = \frac{e^x}{e^y} \qquad \downarrow$$
$$e^y dy = e^x dx \qquad 1 = \ln\left(e^{(0)} + C\right)$$
$$\int e^y dy = \int e^x dx \qquad 1 = \ln(1 + C) \qquad y = \ln\left(e^x + e - 1\right)$$
$$e^y = e^x + C \qquad e^1 = e^{\ln(1+C)}$$
$$\ln\left(e^y\right) = \ln\left(e^x + C\right) \qquad e = 1 + C$$
$$C = e - 1$$
$$y = \ln\left(e^x + C\right)$$

(a) $y = \ln(x)$ (b) $y = \ln\left(e^x + e\right)$ (c) $y = x$ (d) $y = e^x$ (e) $y = \ln\left(e^x + e - 1\right)$

9. No Calculator: The slope field for a certain differential equation is shown at right. Which of the following could be a specific solution to that differential equation.

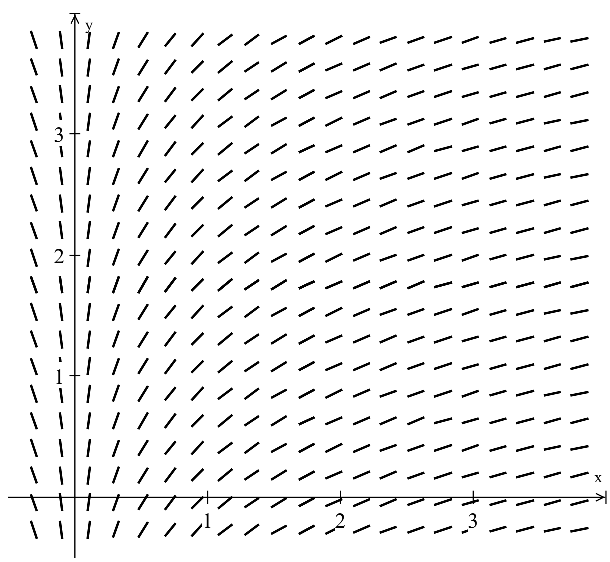
(a) $y = x^3$

(b) $y = e^x$

(c) $y = e^{-x}$

(d) $y = \cos(x)$

(e) $y = \ln(x)$



The only function that "fits" and "goes with the flow" of the slope field is $y = \ln(x)$.

Notice that $\frac{dy}{dx}$ DNE when $x = 0$ (i.e. you'd get vertical slopes). Therefore $\frac{dy}{dx}$ is undefined at $x = 0$. The only function for which $y' \leftrightarrow \frac{dy}{dx}$ is undefined at $x = 0$ is $y = \ln(x)$.

10. No Calculator: Shown at right is a slope field for which of the following differential equations?

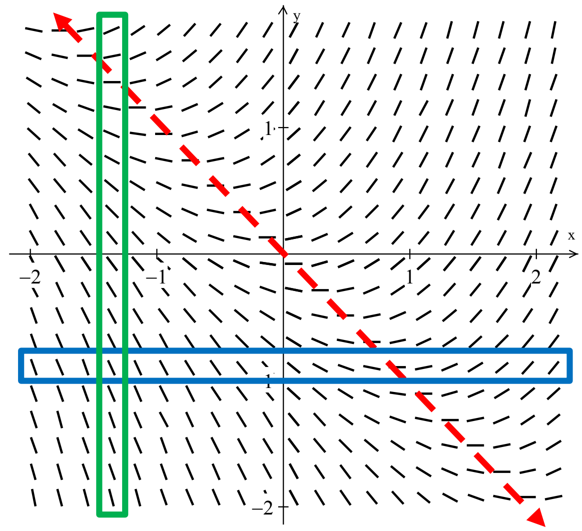
(a) $\frac{dy}{dx} = 1 + x$

(b) $\frac{dy}{dx} = x^2$

(c) $\frac{dy}{dx} = x + y$

(d) $\frac{dy}{dx} = \frac{x}{y}$

(e) $\frac{dy}{dx} = \ln(x)$



Note that the slopes along the line $y = -x$ are all equal to zero. Therefore $\frac{dy}{dx} = 0$ when $y = -x$. The only differential equation for which this is true is (c).

Note that the slopes are not the same in horizontal and vertical bands. Therefore this indicates that the differential equation is a function of both x and y – which eliminates (a), (b), and (e).

If the answer choice were (d), then $\frac{dy}{dx}$ would be undefined when $y = 0$, which is not the case in this slope field – eliminating (d).

By process of elimination, the answer choice must be (c).

11. No calculator: Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5?

$$f(1) = 2$$

$$\begin{aligned} f(1.5) &\approx f(1) + \left[\left. \frac{dy}{dx} \right|_{(1,2)} \right] \cdot (0.5) \\ &\approx 2 + [1+2] \cdot (0.5) \\ &\approx 2 + 1.5 \\ &\approx 3.5 \end{aligned}$$

$$\begin{aligned} f(2) &\approx f(1.5) + \left[\left. \frac{dy}{dx} \right|_{(1.5,3.5)} \right] \cdot (0.5) \\ &\approx 3.5 + [1.5 + 3.5](0.5) \\ &\approx 3.5 + 2.5 \\ &\approx 6 \end{aligned}$$

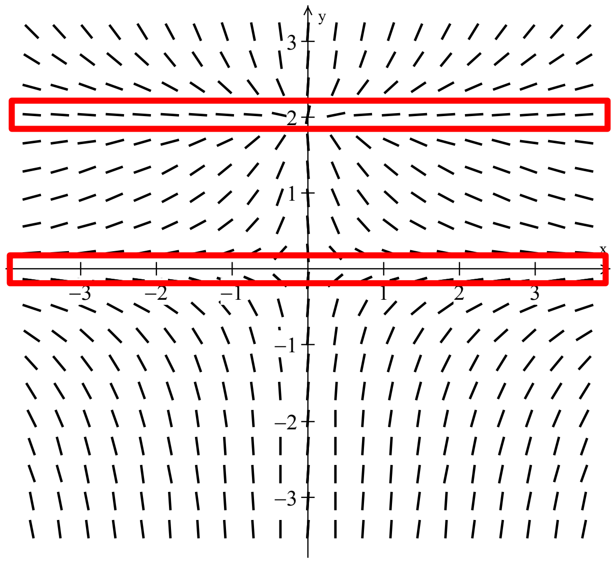
(a) 3 (b) 5 (c) 6 (d) 10 (e) 12

12. Calculator required: A pizza heated to a temperature of 475°F is taken out of an oven and placed in a 105°F room at $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-256e^{-0.7t}$ °F/minute. To the nearest degree, what is the temperature of the pizza at $t = 9$ minutes?

$$\begin{aligned} T(9) &= T(0) + \int_0^9 T'(x)\,dx \\ &= 475 + \int_0^9 -256e^{-0.7t}\,dt \\ &\approx 109.957 \end{aligned}$$

(a) 100 (b) 80 (c) 110 (d) 115 (e) 120

13. No Calculator: Shown at right is the slope field of which differential equation?



$\frac{dy}{dx} = 0$ when $y = 2$. This eliminates (c), (b), (d).

$\frac{dy}{dx} = 0$ when $y = 0$. This eliminates (e).

Therefore the correct answer is (a).

(a) $\frac{dy}{dx} = \frac{y^2 - 2y}{x}$

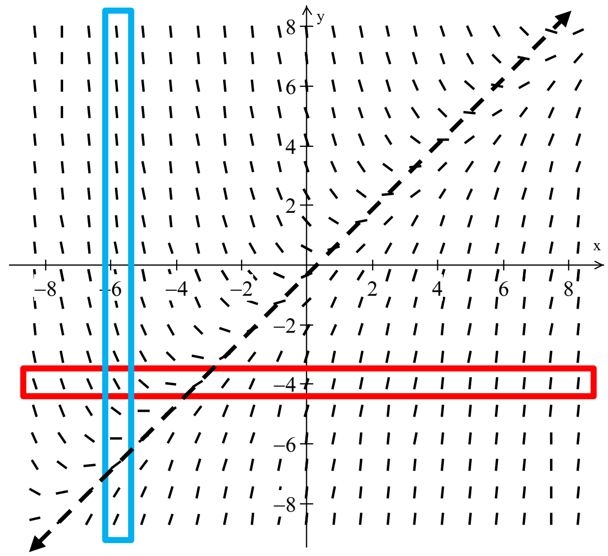
(b) $\frac{dy}{dx} = \frac{3x^2 - 4}{x}$

(c) $\frac{dy}{dx} = \frac{3y^2 - 4}{4}$

(d) $\frac{dy}{dx} = \frac{y - 3}{x}$

(e) $\frac{dy}{dx} = \frac{y - 2}{2x}$

14. No Calculator: Shown at right is the slope field for which differential equation?



$\frac{dy}{dx}$ is not the same in horizontal rows, therefore this differential equation is not solely a function of *y*.

$\frac{dy}{dx}$ is not the same in vertical columns, therefore this differential equation is not solely a function of *x*.

This eliminates (a), (c), and (d).

$\frac{dy}{dx} = 0$ along the line $y = x$. This is the case for (b) only.

(a) $\frac{dy}{dx} = 1 + y^2$

(b) $\frac{dy}{dx} = x - y$

(c) $\frac{dy}{dx} = 2x^2$

(d) $\frac{dy}{dx} = 1 + x^2$

(e) $\frac{dy}{dx} = 1 - y^2 + x^2$