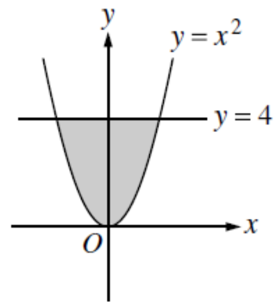
Area and Volume Released Questions

AB–2 / BC–2 1999

2. The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated by revolving R about the x axis.
   (c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.



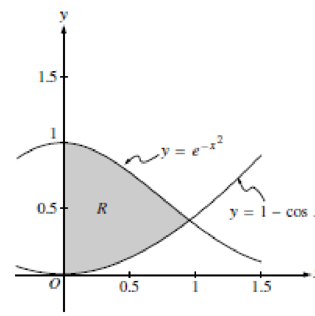
AP Calculus AB–1 / BC–1 2000

Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



2001 SCORING GUIDELINES

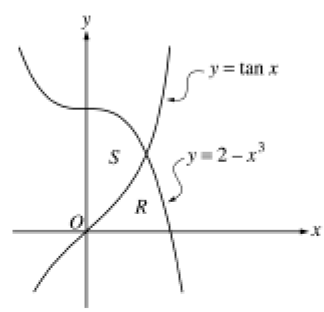
Question 1

Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.



AP® CALCULUS AB 2002 SCORING GUIDELINES

Question 1

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

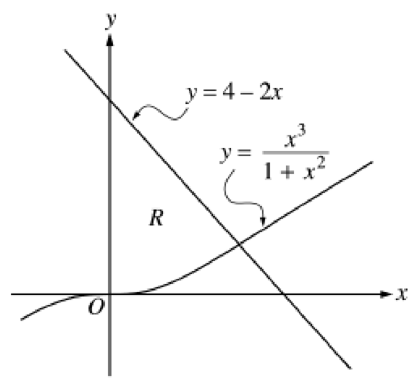
(a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

2002 SCORING GUIDELINES (Form B)

Question 1



Let R be the region bounded by the y-axis and the graphs of $y = \frac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.

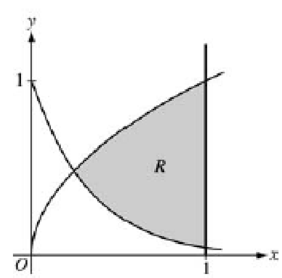
(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

2003 SCORING GUIDELINES

Question 1



Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

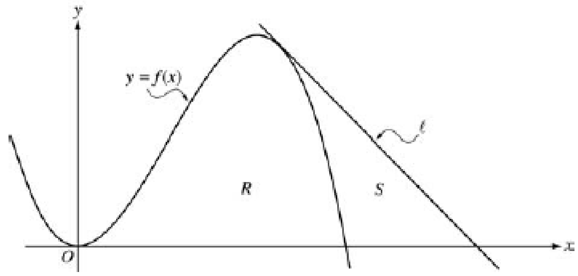
(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

2003 SCORING GUIDELINES (Form B)

Question 1

Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.



(a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

(b) Find the area of S.

(c) Find the volume of the solid generated when R is revolved about the x-axis.

2003 Form B #3 Scoring Guidelines

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

3. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

   (a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

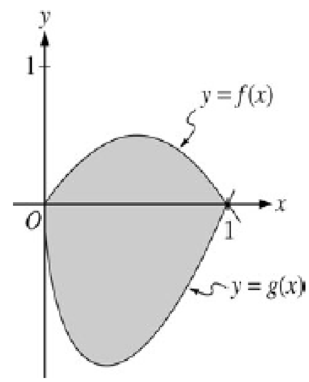
   (b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

   (c) Using correct units, explain the meaning of $\pi \int_{125}^{275} \left(\frac{B(x)}{2} \right)^2 dx$ in terms of the blood vessel.

   (d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

2004 SCORING GUIDELINES

Question 2



Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1 - x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

2004 SCORING GUIDELINES (Form B)

Question 1

Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x-axis.

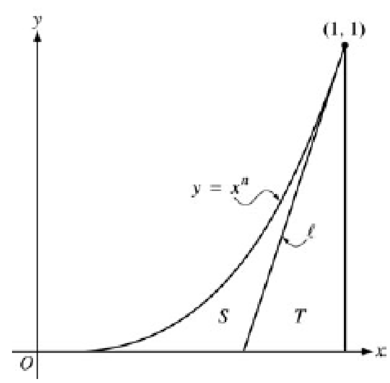
(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.

(c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.

2004 SCORING GUIDELINES (Form B)

Question 6



Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

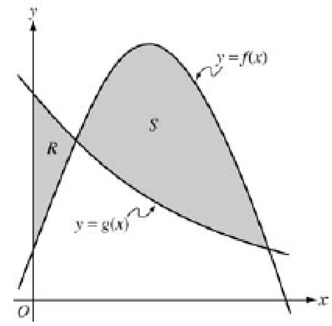
(a) Find $\int_0^1 x^n\,dx$ in terms of n.

(b) Let T be the triangular region bounded by ℓ, the x-axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.

(c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ, and the x-axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S.

2005 SCORING GUIDELINES

Question 1



Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.

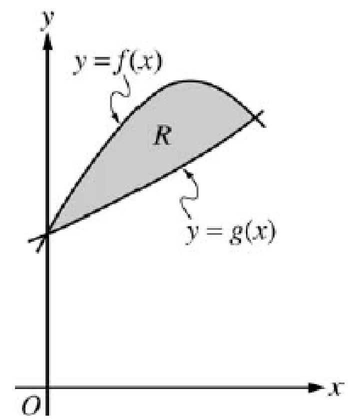
(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

2005 SCORING GUIDELINES (Form B)

Question 1



Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

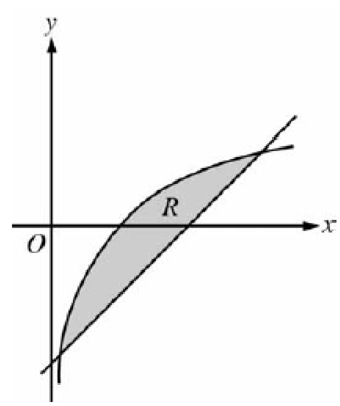
(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

2006 SCORING GUIDELINES

Question 1



Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

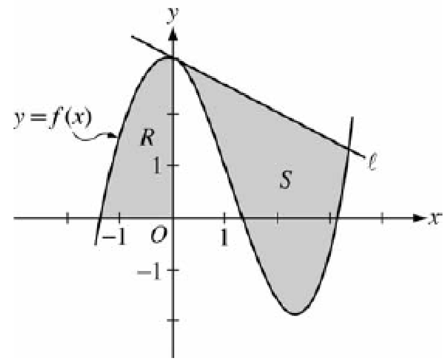
(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

2006 SCORING GUIDELINES (Form B)

Question 1

Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an integral expression that can be used to find the area of S.

2007 SCORING GUIDELINES

Question 1

Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

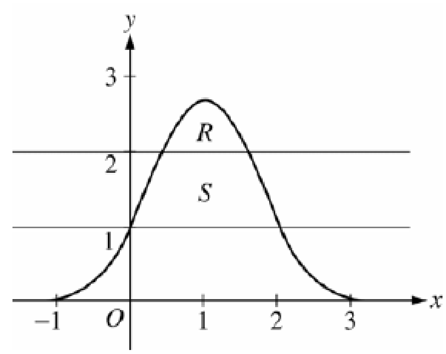
(b) Find the volume of the solid generated when R is rotated about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

2007 SCORING GUIDELINES (Form B)

Question 1

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.



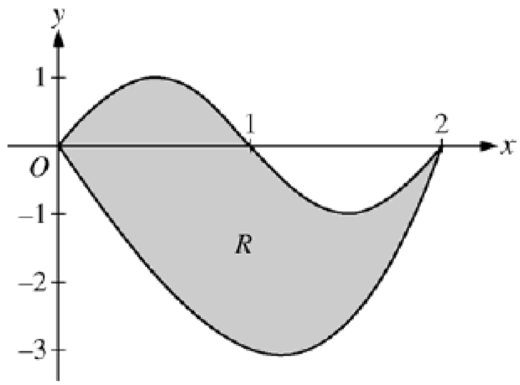
(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

2008 SCORING GUIDELINES

Question 1



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

2008 SCORING GUIDELINES (Form B)

Question 1

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

2008 SCORING GUIDELINES (Form B)

Question 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

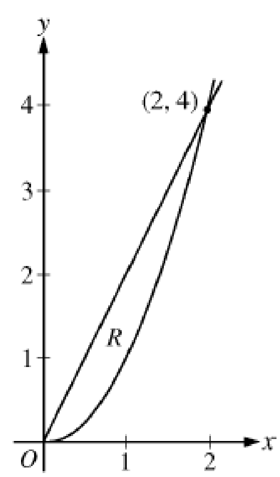
(c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

(d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

2009 SCORING GUIDELINES

Question 4

Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.



(a) Find the area of R.

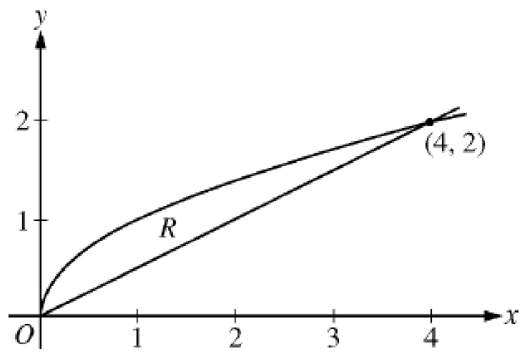
(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

2009 SCORING GUIDELINES (Form B)

Question 4

Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.



(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

AP® CALCULUS AB
2009 SCORING GUIDELINES (Form B)

Question 1

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

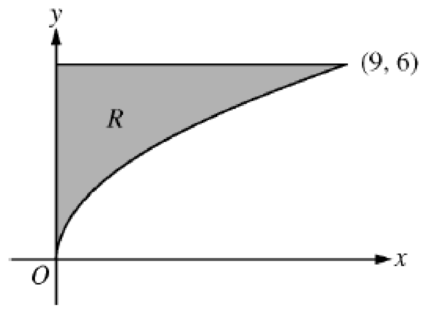
(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\,dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

2010 SCORING GUIDELINES

Question 4



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

2010 SCORING GUIDELINES (Form B)

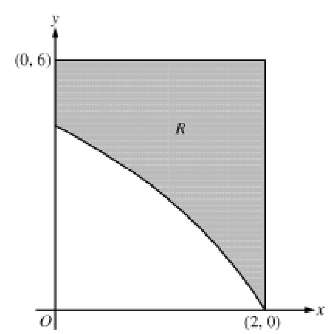
Question 1

In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

(a) Find the area of R.

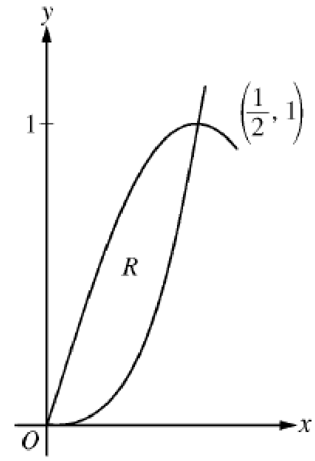
(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.



2011 SCORING GUIDELINES

Question 3



Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

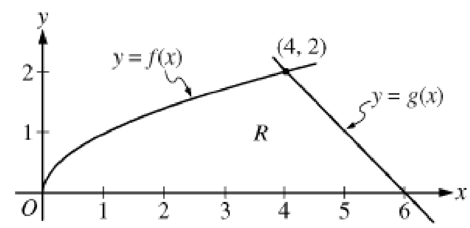
(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

2011 SCORING GUIDELINES (Form B)

Question 3



The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

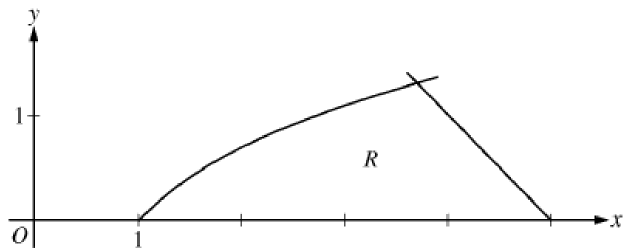
(a) Find the area of R.

(b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

2012 SCORING GUIDELINES

Question 2



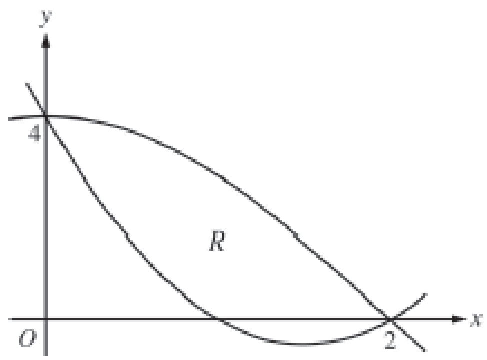
Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

2013 AB



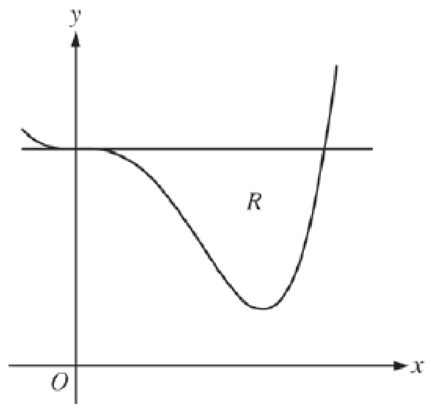
5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

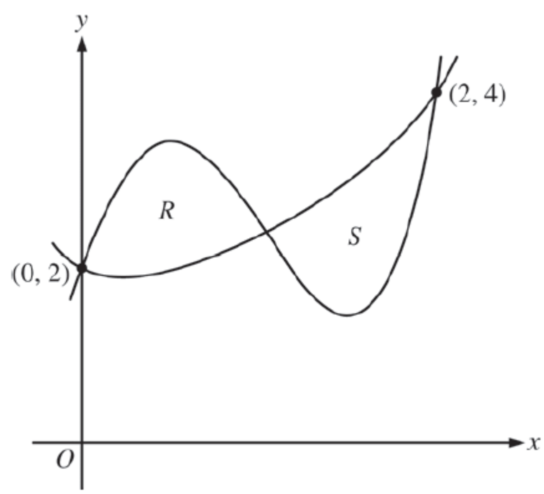
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

2014 AB



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.
   (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
   (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.
   (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

2015 AB #2 Calculator Required

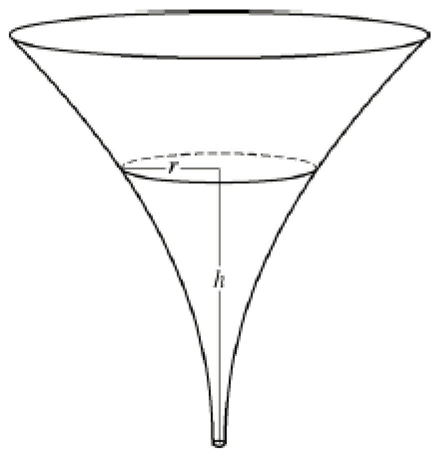


2. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2 - 2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.
   (a) Find the sum of the areas of regions R and S.
   (b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.
   (c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

No Calculator

2016 SCORING GUIDELINES

Question 5



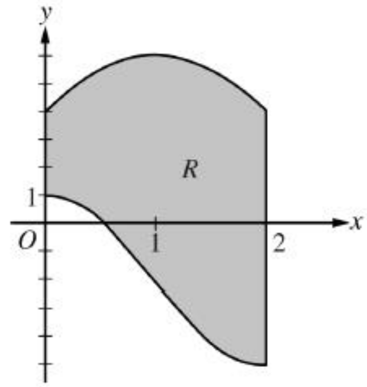
The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

2019 # 5 No Calculator



5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure above.

   (a) Find the area of R.

   (b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \dfrac{1}{x+3}$. Find the volume of the solid.

   (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

BC Released Question 1998

1. Let R be the region in the first quadrant bounded by the graph of $y = 8 - x^{\frac{2}{3}}$, the x-axis, and the y-axis.
   (a) Find the area of the region R.
   (b) Find the volume of the solid generated when R is revolved about the x-axis.
   (c) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

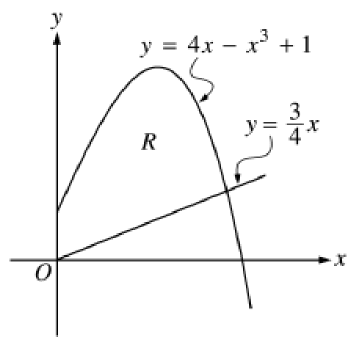
2002 SCORING GUIDELINES (Form B)
BC Released Question
Question 3

Let R be the region in the first quadrant bounded by the y-axis and the graphs of $y = 4x - x^3 + 1$ and $y = \frac{3}{4}x$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Write an expression involving one or more integrals that gives the perimeter of R. Do not evaluate.

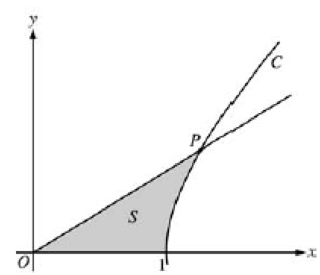


2003 SCORING GUIDELINES
BC Released Question
Question 3

The figure above shows the graphs of the line $x = \frac{5}{3}y$ and the curve C given by $x = \sqrt{1 + y^2}$. Let S be the shaded region bounded by the two graphs and the x-axis. The line and the curve intersect at point P.

(a) Find the coordinates of point P and the value of $\frac{dx}{dy}$ for curve C at point P.

(b) Set up and evaluate an integral expression with respect to y that gives the area of S.



2004 SCORING GUIDELINES (Form B)

BC Released Question

Question 5

Let g be the function given by $g(x) = \frac{1}{\sqrt{x}}$.

(a) Find the average value of g on the closed interval $[1, 4]$.

(b) Let S be the solid generated when the region bounded by the graph of $y = g(x)$, the vertical lines $x = 1$ and $x = 4$, and the x-axis is revolved about the x-axis. Find the volume of S.

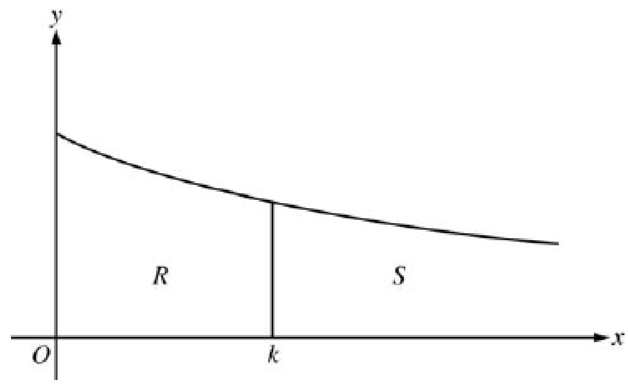
(c) For the solid S, given in part (b), find the average value of the areas of the cross sections perpendicular to the x-axis.

(d) The average value of a function f on the unbounded interval $[a, \infty)$ is defined to be $\lim_{b\to\infty}\left[\frac{\int_a^b f(x)\,dx}{b-a}\right]$. Show that the improper integral $\int_4^{\infty} g(x)\,dx$ is divergent, but the average value of g on the interval $[4, \infty)$ is finite.

2005 SCORING GUIDELINES (Form B)

BC Released Question

Question 6



Consider the graph of the function f given by $f(x) = \frac{1}{x+2}$ for $x \ge 0$, as shown in the figure above. Let R be the region bounded by the graph of f, the x- and y-axes, and the vertical line $x = k$, where $k \ge 0$.

(a) Find the area of R in terms of k.

(b) Find the volume of the solid generated when R is revolved about the x-axis in terms of k.

(c) Let S be the unbounded region in the first quadrant to the right of the vertical line $x = k$ and below the graph of f, as shown in the figure above. Find all values of k such that the volume of the solid generated when S is revolved about the x-axis is equal to the volume of the solid found in part (b).

2008 SCORING GUIDELINES (Form B)

BC Released Question

Question 4

Let f be the function given by $f(x) = kx^2 - x^3$, where k is a positive constant. Let R be the region in the first quadrant bounded by the graph of f and the x-axis.

(a) Find all values of the constant k for which the area of R equals 2.

(b) For $k > 0$, write, but do not evaluate, an integral expression in terms of k for the volume of the solid generated when R is rotated about the x-axis.

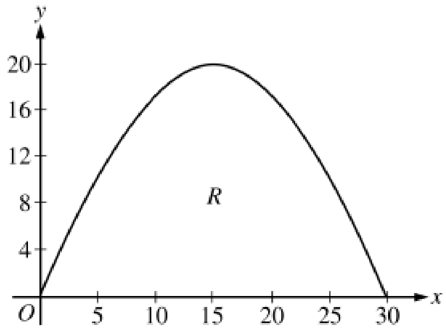
(c) For $k > 0$, write, but do not evaluate, an expression in terms of k, involving one or more integrals, that gives the perimeter of R.

2009 SCORING GUIDELINES (Form B)

BC Released Question

Question 1

A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \le x \le 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure above. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.



(a) The region R is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.

(b) The cake is a solid with base R. Cross sections of the cake perpendicular to the x-axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?

(c) Find the perimeter of the base of the cake.

2010 SCORING GUIDELINES (Form B)

BC Released Question

Question 5

Let f and g be the functions defined by $f(x) = \frac{1}{x}$ and $g(x) = \frac{4x}{1 + 4x^2}$, for all $x > 0$.

(a) Find the absolute maximum value of g on the open interval $(0, \infty)$ if the maximum exists. Find the absolute minimum value of g on the open interval $(0, \infty)$ if the minimum exists. Justify your answers.

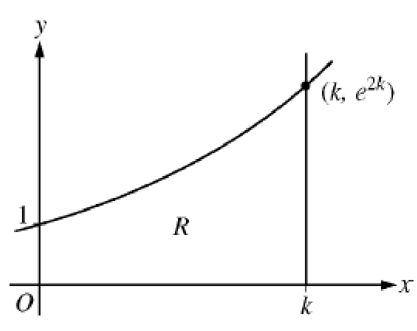
(b) Find the area of the unbounded region in the first quadrant to the right of the vertical line $x = 1$, below the graph of f, and above the graph of g.

2011 SCORING GUIDELINES

BC Released Question

Question 3

Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.



(a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k.

(b) The region R is rotated about the x-axis to form a solid. Find the volume, V, of the solid in terms of k.

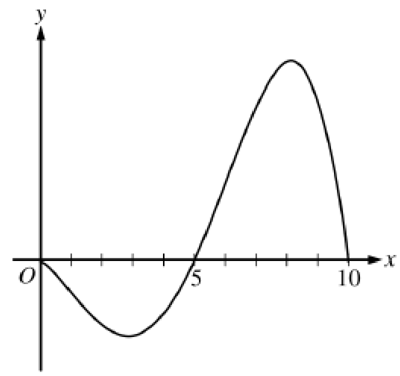
(c) The volume V, found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$.

2011 SCORING GUIDELINES (Form B)

BC Released Question

Question 4

The graph of the differentiable function $y = f(x)$ with domain $0 \le x \le 10$ is shown in the figure above. The area of the region enclosed between the graph of f and the x-axis for $0 \le x \le 5$ is 10, and the area of the region enclosed between the graph of f and the x-axis for $5 \le x \le 10$ is 27. The arc length for the portion of the graph of f between $x = 0$ and $x = 5$ is 11, and the arc length for the portion of the graph of f between $x = 5$ and $x = 10$ is 18. The function f has exactly two critical points that are located at $x = 3$ and $x = 8$.



Graph of f

(a) Find the average value of f on the interval $0 \le x \le 5$.

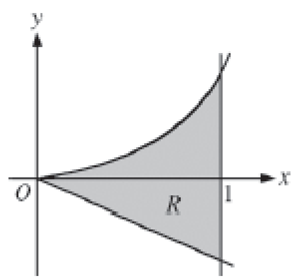
(b) Evaluate $\int_0^{10} (3f(x) + 2)\,dx$. Show the computations that lead to your answer.

(c) Let $g(x) = \int_5^x f(t)\,dt$. On what intervals, if any, is the graph of g both concave up and decreasing? Explain your reasoning.

(d) The function h is defined by $h(x) = 2f\left(\frac{x}{2}\right)$. The derivative of h is $h'(x) = f'\left(\frac{x}{2}\right)$. Find the arc length of the graph of $y = h(x)$ from $x = 0$ to $x = 20$.

2014 BC # 5

BC Released Question



5. Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R.