Why/How to use MVT & IVT

Exercise: Let $f(x)$ be a function that is twice differentiable on the interval $[3,8]$. Selected values of $f(x)$ are given in the table below. Prove that $f'(x)=0$ at least once in the interval $(3,8)$.

x	3	5	8
$f(x)$	2	4	0

Solution:

Whenever an exercise states that a function $f(x)$ is **<u>TWICE DIFFERENTIABLE</u>** you should state that this makes $\boxed{f(x)}$ **<u>and</u>** $\boxed{f'(x)}$ **<u>are both continuous and differentiable functions</u>**. This is true because if a function $f(x)$ is differentiable, then $f(x)$ is continuous.

Since $f(x)$ is twice differentiable, that means that $f(x)$ is differentiable, and $f'(x)$ is differentiable. Therefore $f(x)$ *and* $f'(x)$ are continuous functions.

Since $f(x)$ is continuous on $[3,8]$ *and* differentiable on $(3,8)$, we can use MVT.

By MVT there exists a c_1 where $3<c_1<5$ and $f'(c_1)=\dfrac{4-2}{5-3}=1$.

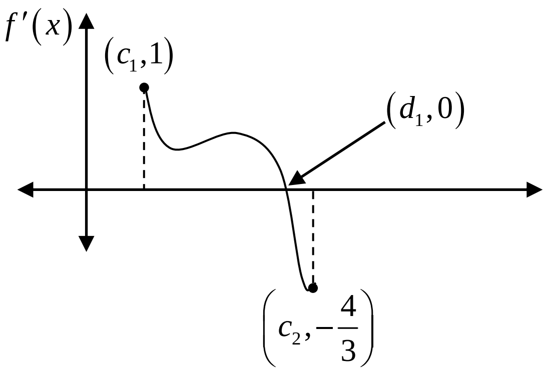
By MVT there exists a c_2 where $5<c_2<8$ and $f'(c_2)=\dfrac{0-4}{8-5}=-\dfrac{4}{3}$

$\boxed{c_1}$ $\boxed{c_2}$

x	3	5	8
$f(x)$	2	4	0

MVT gives us these values of $f'(x)$ $\boxed{1}$ $\boxed{-\frac{4}{3}}$

Now since $f'(x)$ is continuous on $(3,8)$, and therefore $[c_1, c_2]$ we can use IVT to demonstrate that $f'(x)=0$ at least once in the interval $[c_1, c_2]$. Since $f'(x)$ is continuous and $f(c_1)=1$ and $f(c_2)=-\frac{4}{3}$, by IVT there exists a d_1 such that $c_1 < d_1 < c_2$ and $f'(d_1)=0$.



Exercise: 2009 Practice AB Exam

t (minutes)	0	4	8	12	16
$H(t)$ $(°C)$	65	68	73	80	90

The temperature, in degrees Celsius $(°C)$, of an oven being heated is modeled by an increasing differentiable function H of time t, where t is measured in minutes. The table above gives the temperature recorded every 4minutes over a 16-minute period.
Are the data in the table consistent, or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

Solution: $H(t)$ is continuous because $H(t)$ is differentiable. Since $H(t)$ is continuous on $[0,16]$ and differentiable on $(0,16)$, by MVT…

There exists a c_1, such that $0 < c_1 < 4$ and $H'(c_1) = \frac{68-65}{4-0} = \frac{3}{4}$

There exists a c_2, such that $4 < c_2 < 8$ and $H'(c_2) = \frac{73-68}{8-4} = \frac{5}{4}$

There exists a c_3, such that $8 < c_3 < 12$ and $H'(c_3) = \frac{80-73}{12-8} = \frac{7}{4}$

There exists a c_4, such that $12 < c_4 < 16$ and $H'(c_4) = \frac{90-80}{16-12} = \frac{10}{4}$

If the temperature of the oven is increasing at an increasing rate, as the value of t increases so will the value of $H'(t)$.

t (minutes)	0	4	8	12	16
$H(t)$ $(°C)$	65	68	73	80	90

c_1 c_2 c_3 c_4

MVT gives us these values of $H'(t)$ $\frac{3}{4}$ $\frac{5}{4}$ $\frac{7}{4}$ $\frac{10}{4}$

Since the value of $H'(t)$ increases as t increases, we can conclude that the data in the table is consistent with the claim that the temperature of the oven is increasing at an increasing rate.

2003 AB Form B #3 Part (d)

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

(d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

Solution:

Since $B(x)$ is ***twice differentiable***, we know that $B(x)$ AND $B'(x)$ are both continuous and differentiable.

Option #1:

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

c_1 $\qquad$ c_2

MVT gives used with $B(x)$ gives these values of $B'(x)$ ---> $\frac{1}{10}$ $\quad \boxed{-\frac{1}{30}}$ $\quad \frac{1}{30}$ $\quad -\frac{1}{15}$ $\quad \boxed{-\frac{1}{30}}$ $\quad \frac{1}{30}$ --->

d

MVT gives used with $B'(x)$ gives this value of $B''(x)$ ----------> 0

<u>Formally:</u> Since $B(x)$ is continuous on $[0,360]$ and differentiable on $(0,360)$, by MVT...

There exists a c_1, such that $60 < c_1 < 120$ and $B'(c_1) = \frac{28-30}{120-60} = -\frac{1}{30}$

There exists a c_2, such that $240 < c_2 < 300$ and $B'(c_2) = \frac{24-26}{300-240} = -\frac{1}{30}$

Since $B'(x)$ is continuous on $[0,360]$ and differentiable on $(0,360)$ and $B'(c_1) = -\frac{1}{30}$ and $B'(c_2) = -\frac{1}{30}$, by Rolle's theorem, there exists a d such that $c_1 < d < c_2$ and $B''(d) = 0$.

Option #2:

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

c_1 $\qquad$ c_2

MVT used with $B(x)$ gives these values of $B'(x)$ $\dashrightarrow$ $\frac{1}{10}$ $\quad -\frac{1}{30}$ $\quad \boxed{\frac{1}{30}}$ $\quad -\frac{1}{15}$ $\quad -\frac{1}{30}$ $\quad \boxed{\frac{1}{30}}$ $\dashrightarrow$

d

MVT gives used with $B'(x)$ gives this value of $B''(x)$ $\dashrightarrow$ 0

<u>Formally:</u> Since $B(x)$ is continuous on $[0,360]$ and differentiable on $(0,360)$, by MVT...

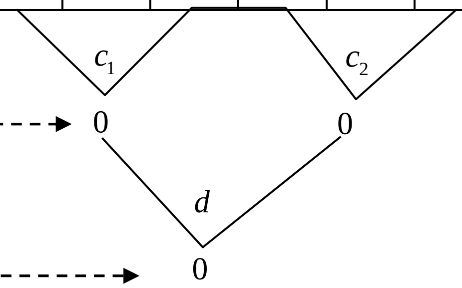
There exists a c_1, such that $120 < c_1 < 180$ and $B'(c_1) = \dfrac{30-28}{180-120} = \dfrac{1}{30}$

There exists a c_2, such that $300 < c_2 < 360$ and $B'(c_2) = \dfrac{26-24}{360-300} = \dfrac{1}{30}$

Since $B'(x)$ is continuous on $[0,360]$ and differentiable on $(0,360)$ and $B'(c_1) = \dfrac{1}{30}$ and $B'(c_2) = \dfrac{1}{30}$, by Rolle's theorem, there exists a d such that $c_1 < d < c_2$ and $B''(d) = 0$.

Option #3:

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26



Rolle's Theorem used with $B(x)$ gives these values of $B'(x)$

Rolle's Theorem used with $B'(x)$ gives this value of $B''(x)$

Formally: Since $B(x)$ is continuous on $[0,360]$ and differentiable on $(0,360)$, by MVT...

There exists a c_1, such that $60 < c_1 < 120$ and $B'(c_1) = \dfrac{30-30}{180-60} = 0$

There exists a c_2, such that $240 < c_2 < 300$ and $B'(c_2) = \dfrac{26-26}{360-240} = 0$

Since $B'(x)$ is continuous on $[0,360]$ and differentiable on $(0,360)$ and $B'(c_1)=0$ and $B'(c_2)=0$, by Rolle's theorem, there exists a d such that $c_1 < d < c_2$ and $B''(d)=0$.

2005 AB #3

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

Option #1:

Since $T(x)$ is twice differentiable, we know that...

$T(x)$ is continuous on $[0,8]$ and differentiable on $(0,8)$

$T'(x)$ is continuous on $[0,8]$ and differentiable on $(0,8)$

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

c_1 $\quad$ c_2 $\quad$ c_3 $\quad$ c_4

MVT used with $T(x)$ gives these values of $T'(x)$ ⟶ $-7 \quad -\frac{23}{4} \quad -8 \quad -7$ ⟶

d_1 $\quad$ d_2 $\quad$ d_3

MVT used with $T'(x)$ gives these values of $T''(x)$

$$\frac{\left(-\frac{23}{4}\right)-(-7)}{c_2-c_1} \qquad \frac{-8-\left(-\frac{23}{4}\right)}{c_3-c_2} \qquad \frac{-7-(-8)}{c_4-c_3}$$

$$\frac{(+)}{(+)} \qquad \frac{(-)}{(+)} \qquad \frac{(+)}{(+)}$$

Formally: Since $T(x)$ is continuous on $[0,8]$ and differentiable on $(0,8)$, by MVT...

There exists a c_2, such that $1 < c_2 < 5$ and $T'(c_2) = -\frac{23}{4}$

There exists a c_3, such that $5 < c_3 < 6$ and $T'(c_3) = -8$

Since $T'(x)$ is continuous on $[0,8]$ and differentiable on $(0,8)$, by MVT...

There exists a d_2, such that $c_2 < d_2 < c_3$ and $T''(d_2) = \dfrac{-8-\left(-\frac{23}{4}\right)}{c_3-c_2} < 0$.

Hence the data in the table is not consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$.

Option #2:

Since $T(x)$ is decreasing, if $T(x)$ is also concave up for $0 \le x \le 8$, then the graph of $T(x)$ must be decreasing and concave up. That is, the slopes of $T(x)$ should be negative, and getting less negative as x increases.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

c_1 c_2 c_3 c_4

-7 $\quad -\frac{23}{4}$ $\quad -8$ $\quad -7$

Since $T(x)$ is continuous and differentiable on $[0,8]$, by MVT there exists a c_2 in $(1,5)$ such that $T'(c_2) = \frac{70-93}{5-1} = -\frac{23}{4} = -5.75$, and there is a c_3 in $(5,6)$ such that $T'(c_3) = \frac{62-70}{6-5} = -8$. If $T''(x) > 0$ for $0 < x < 8$, then $T'(c_3) > T'(c_2)$, and this is not the case. Therefore the data in the table is not consistent with the assertion that $T''(x) > 0$ for all $0 < x < 8$.

SOLUTIONS TO THE EXERCISES:

1999 AB #3

t (hours)	0	3	6	9	12	15	18	21	24
$R(t)$ (gallons per hour)	**9.6**	10.4	10.8	11.2	11.4	11.3	10.7	10.2	**9.6**

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24 hour period.

(b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

Since $R(t)$ is differentiable and $R(0) = 9.6$ and $R(24) = 9.6$ by Rolle's Theorem/MVT there exists a $c \in (0,24)$ such that $R'(t) = \frac{R(24) - R(0)}{24-0} = \frac{0}{24} = 0$.

2002 AB #6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let *f* be a function that is differentiable for all real numbers. The table above gives values of *f* and the derivative f' for selected points *x* in the closed interval $-1.5 \le x \le 1.5$. The second derivative of *f* has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(c) Find a positive number *r* having the property that there must exist a value *c*, with $0 < c < 0.5$, and $f''(c) = r$. Give a reason for your answer.

Since $f'(x)$ is differentiable on $0 < x < 0.5$, $f'(x)$ is continuous on $0 < x < 0.5$. By Mean Value Theorem, there exists a *c*, $0 < c < 0.5$ such that $f''(c) = \dfrac{f'(0.5) - f'(0)}{0.5 - 0} = \dfrac{3-0}{0.5} = 6$

2004 AB Form B #3

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time *t* minutes, where *v* is a differentiable function of *t*. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

$a(t) = v'(t)$.

Since $v(t)$ is differentiable on $[0,15]$ and $v(0) = 7$ and $v(15) = 7$, by Rolles Theorem or MVT there exists a c_1 where $0 < c_1 < 7$ such that $v'(c_1) = 0$.

Since $v(t)$ is differentiable on $[25,30]$ and $v(25) = 2.4$ and $v(30) = 2.4$, by Rolles Theorem or MVT there exists a c_2 where $25 < c_2 < 30$ such that $v'(c_2) = 0$.

Therefore the minimum number of times that $a(t) = 0$ on the open interval $0 < t < 40$ is TWO times.

2007 AB #3

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

The functions *f* and *g* are differentiable for all real numbers, and *g* is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of *x*. The function *h* is given by $h(x) = f(g(x)) - 6$.

$h(x)$ is differentiable because the composition of two differentiable functions is differentiable.

$h(x)$ is continuous because it is differentiable.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

$$\begin{aligned} h(1) &= f(g(1)) - 6 \\ &= f(2) - 6 \\ &= 9 - 6 \\ &= 3 \end{aligned} \qquad \begin{aligned} h(3) &= f(g(3)) - 6 \\ &= f(4) - 6 \\ &= -1 - 6 \\ &= -7 \end{aligned}$$

Since $h(x)$ is continuous and $h(1) = 3$ and $h(3) = -7$, by IVT there exists a c such that $1 < c < 3$ and $h(c) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

Since $h(x)$ is differentiable and $h(1) = 3$ and $h(3) = -7$, by MVT there exists a d in $(1,3)$ such that

$$\begin{aligned} h'(d) &= \frac{h(3) - h(1)}{3 - 1} \\ &= \frac{-7 - 3}{3 - 1} \\ &= -5 \end{aligned}$$

2007 AB Form B #6

Let f be a twice-differentiable function such that $f(2)=5$ and $f(5)=2$. Let g be the function given by $g(x)=f(f(x))$.

(a) Explain why there must be a value c for $2<c<5$ such that $f'(c)=-1$.

Since $f(x)$ is continuous on $2\le x\le 5$ and differentiable on $2<x<5$, by Mean Value Theorem, there exists a c, $2<c<5$, such that $f'(c)=\dfrac{f(5)-f(2)}{5-2}=\dfrac{2-5}{3}=-1$.

(b) Show that $g'(2)=g'(5)$. Use this result to explain why there must be a value of k for $2<k<5$ such that $g''(k)=0$.

$$g(x)=f(f(x))$$
$$\downarrow$$
$$g'(x)=f'(f(x))\cdot f'(x)$$

$$\begin{aligned} g'(2) &= f'(f(2))\cdot f'(2) \\ &= f'(5)\cdot f'(2) \\ g'(5) &= f'(f(5))\cdot f'(5) \\ &= f'(2)\cdot f'(5) \end{aligned}$$

Since $f'(x)$ is continuous on $2\le x\le 5$ and differentiable on $2<x<5$, by Rolle’s Theorem/ Mean Value Theorem, there exists a k, $2<k<5$, such that $f''(k)=0$

2008 AB #2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t=0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(c) For $0 < t < 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

Since $L(t)$ is twice differentiable, $L(t)$ is continuous, and $L'(t)$ are continuous.

Since $L(t)$ is differentiable and $L(3)=176$ and $L(1)=156$, by MVT there exists a c_1 in $(1,3)$ such that $L'(c_1)=\dfrac{L(3)-L(1)}{3-1}=\dfrac{176-156}{2}=10$.

Since $L(t)$ is differentiable and $L(3)=176$ and $L(4)=126$, by MVT there exists a c_2 in $(3,4)$ such that $L'(c_2)=\dfrac{L(4)-L(3)}{4-3}=\dfrac{126-176}{2}=-25$.

Since $L(t)$ is differentiable and $L(4)=126$ and $L(7)=150$, by MVT there exists a c_3 in $(4,7)$ such that $L'(c_3)=\dfrac{L(7)-L(4)}{7-4}=\dfrac{150-126}{3}=8$.

Since $L(t)$ is differentiable and $L(7)=150$ and $L(8)=80$, by MVT there exists a c_4 in $(7,8)$ such that $L'(c_4)=\dfrac{L(8)-L(7)}{8-7}=\dfrac{150-180}{1}=-70$.

Since $L'(t)$ is continuous and $L'(c_1)=10$ and $L'(c_2)=-25$, by IVT there exists a d_1 in (c_1,c_2) such that $L'(d_1)=0$.

Since $L'(t)$ is continuous and $L'(c_2)=-25$ and $L'(c_3)=8$, by IVT there exists a d_2 in (c_2,c_3) such that $L'(d_2)=0$.

Since $L'(t)$ is continuous and $L'(c_3)=8$ and $L'(c_4)=-70$, by IVT there exists a d_2 in (c_3,c_4) such that $L'(d_3)=0$.

Therefore the minimum number of times that $L'(t)=0$ in $(0,9)$ is THREE times.

2009 AB Form B #6

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	–10	–8	–4	7

The velocity of a particle moving along the *x*-axis is modeled by a differentiable function *v*, where the position *x* is measured in meters, and time *t* is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x=7$ meters when $t=0$ seconds.

(c) For $0<t<40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

Since $v(t)$ is differentiable, it is continuous.

Since $v(8)>0$ and $v(20)<0$ and $v(t)$ is continuous, the particle must change direction in the interval $(8,20)$

Since $v(32)<0$ and $v(40)>0$ and $v(t)$ is continuous, the particle must change direction in the interval $(32,40)$

2011 AB Form B # 1

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of the water in the can is modeled by the function *S*, where $S(t)$ is measured in millimeters and *t* is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 2.5$.

(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsson Mountain is modeled by the function *M*, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and *t* is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function *D* on the interval $0 \le t \le 60$ to justify that there exists a time *t*, $0 < t < 60$, at which the heights of the water in the two cans are changing at the same rate.

$$D(0) = M'(0) - S'(0) \qquad D(60) = M'(60) - S'(60)$$
$$= -2.5 \qquad \approx 1395.0523...$$

Since $M(t)$ and $S(t)$ are continuous, is continuous. By IVT since $D(0) > 0$ and $D(60) > 0$, by IVT, there exists a *c*, where $0 < c < 60$, such that

$$D(c) = 0$$
$$M(c) - S(c) = 0$$
$$M(c) = S(c)$$

Therefore, at time $t = c$, the heights of the water in the two cans are changing at the same rate.

2011 AB Form B # 5

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

Since $B(t)$ is differentiable and $B(40) = 9$ and $B(60) = 49$ by MVT there exists a $c \in (40, 60)$ such that $B'(c) = v(c) = \dfrac{B(60) - B(40)}{60 - 40} = \dfrac{49 - 9}{20} = 2\dfrac{\text{m}}{\text{sec}}$.

2013 AB # 3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time $t, 0 \le t \le 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

Since $C(t)$ is differentiable and $C(4) = 12.8$ and $C(2) = 8.8$, by MVT there exists a $c \in (2,4)$ such that $C'(c) = \dfrac{C(4) - C(2)}{4-2} = \dfrac{12.8 - 8.8}{2} = 2$.

2014 AB # 4

Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table below.

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters / minute)	0	100	40	–120	–150

(b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

Since $v_A(t)$ is differentiable, $v_A(t)$ is continuous. Since $v_A(5) = 40$ and $v_A(8) = -120$, by IVT there exists a c, such that $5 < c < 8$ such that $v_A(c) = 100$.

Yes, the data in the table supports the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$.

2014 AB # 5

The twice differentiable function f and g are defined for all real numbers x. Value of f, f', g, and g' for various values of x are given in the table below.

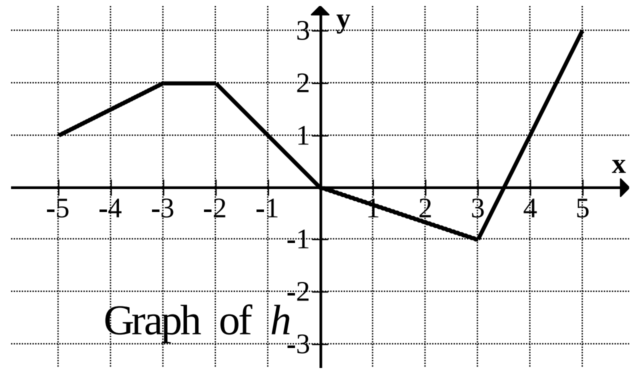
x	-2	$-2<x<1$	-1	$-1<x<1$	1	$1<x<3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

(b) Explain why there must be a value c, for $-1<c<1$, such that $f''(c)=0$.

Since f is twice differentiable, f' is differentiable. Since $f'(-1)=f'(1)=0$ by MVT/Rolle's Theorem, there must exist a value c, for $-1<c<1$, such that $f''(c)=0$.

2017 AB # 6

x	$g(x)$	$g'(x)$
-5	10	-3
-4	5	-1
-3	2	4
-2	3	1
-1	1	-2
0	0	-3



Let f be the function defined by $f(x) = \cos(2x) + e^{\sin(x)}$. Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x. Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(d) Is there a number c in the closed interval $[-5,3]$ such that $g'(c) = -4$? Justify your answer.

Since $g(x)$ is continuous and differentiable, by MVT there exists a c, where $-5 < c < -3$, such that $g'(c) = \dfrac{g(-5) - g(-3)}{-5-(-3)} = -4$

For 2003 AB Form B #3 part (d), there are three other ways to demonstrate that $B''(x)=0$ on the interval $0<x<360$. Write up a solution similar to Option #1 as described earlier, for each of these ways.

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

I. Use MVT with $[0,120]$ and $[120,180]$. Then use MVT again with the those results.

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

$$\frac{28-24}{120-0} \qquad \frac{30-28}{180-120}$$

$$\frac{1}{30} \qquad \frac{1}{30}$$

$$0$$

Since $B(x)$ is continuous and differentiable on $[0,120]$, by MVT there exists a c_1 in $(0,120)$ such that $B'(c_1)=\frac{28-24}{120-0}=\frac{1}{30}$.

Since $B(x)$ is continuous and differentiable on $[120,180]$, by MVT there exists a c_2 in $(120,180)$ such that $B'(c_2)=\frac{30-28}{180-120}=\frac{1}{30}$.

Since $B'(x)$ is continuous and differentiable on $[c_1,c_2]$, by MVT there exists a d in (c_1,c_2) such that $B''(d)=\frac{\frac{1}{30}-\frac{1}{30}}{c_2-c_1}=\frac{0}{(+)}=0$.

Alternately, since $B'(x)$ is continuous and differentiable on $[c_1,c_2]$ and $B'(c_1)=B'(c_2)$, by Rolle's Theorem, there exists a d in (c_1,c_2) such that $B''(d)=0$.

II. Use MVT with $[0,120]$ and $[300,360]$. Then use MVT again with the those results.

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

$$\frac{28-24}{120-0} \qquad\qquad \frac{26-24}{360-300}$$

$$\frac{1}{30} \qquad\qquad \frac{1}{30}$$

$$0$$

Since $B(x)$ is continuous and differentiable on $[0,120]$, by MVT there exists a c_1 in $(0,120)$ such that $B'(c_1)=\frac{28-24}{120-0}=\frac{1}{30}$.

Since $B(x)$ is continuous and differentiable on $[300,360]$, by MVT there exists a c_2 in $(300,360)$ such that $B'(c_2)=\frac{26-24}{360-300}=\frac{1}{30}$.

Since $B'(x)$ is continuous and differentiable on $[c_1,c_2]$, by MVT there exists a *d* in (c_1,c_2) such that $B''(d)=\frac{\frac{1}{30}-\frac{1}{30}}{c_2-c_1}=\frac{0}{(+)}=0$.

Alternately, since $B'(x)$ is continuous on $[c_1,c_2]$, differentiable on (c_1,c_2), and $B'(c_1)=B'(c_2)$, by Rolle's Theorem there exists a *d* in (c_1,c_2) such that $B''(d)=0$.

III. Use MVT with $[0,180]$ and $[300,360]$. Then use MVT again with those results.

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

$$\frac{30-24}{180-0} \qquad\qquad \frac{26-24}{360-300}$$

$$\frac{1}{30} \qquad\qquad \frac{1}{30}$$

$$0$$

Since $B(x)$ is continuous and differentiable on $[0,180]$, by MVT there exists a c_1 in $(0,180)$ such that $B'(c_1)=\frac{30-24}{180-0}=\frac{1}{30}$.

Since $B(x)$ is continuous and differentiable on $[360,300]$, by MVT there exists a c_2 in $(360,300)$ such that $B'(c_2)=\frac{26-24}{360-300}=\frac{1}{30}$.

Since $B'(x)$ is continuous and differentiable on $[c_1,c_2]$, by MVT there exists a d in (c_1,c_2), such that $B''(d)=\frac{\frac{1}{30}-\frac{1}{30}}{c_2-c_1}=\frac{0}{(+)}=0$.

Alternatively, since $B'(x)$ is continuous and differentiable on $[c_1,c_2]$, and $B'(c_1)=B'(c_2)$, by Rolle's Theorem there exists a d in (c_1,c_2) such that $B'(d)=0$.

For 2005 AB #3 Part (d), there are two other ways to demonstrate that $T''(x) < 0$. Write up a solution similar to solution presented earlier for these alternate ways.

Distance *x* (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

I. Use MVT with $[0,1]$ and $[5,6]$. Then use MVT again with those results.

Distance *x* (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

$$\frac{93-100}{1-0} \qquad \frac{62-70}{6-5}$$

$$-7 \qquad -8$$

$$\frac{-8-(-7)}{(+)} = \frac{(-)}{(+)} = (-)$$

Since $T(x)$ is continuous and differentiable on $[0,1]$, by MVT there exists a c_1 in $(0,1)$ such that $T'(c_1) = \frac{93-100}{1-0} = -8$.

Since $T(x)$ is continuous and differentiable on $[5,6]$, by MVT there exists a c_2 in $(5,6)$ such that $T'(c_2) = \frac{62-70}{6-5} = -7$.

Since $T'(x)$ is continuous and differentiable on $[c_1, c_2]$, by MVT there exists a *d* in (c_1, c_2) such that $T'(d) = \frac{-8-(-7)}{(+)} = \frac{(-)}{(+)} = (-)$.

Therefore the data in the table is not consistent with the assertion that $T''(x) > 0$.

II. Use MVT with $[0,5]$ and $[5,6]$. Then use MVT again with those results.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

$$\frac{70-100}{5-0} \qquad \frac{62-70}{6-5}$$

$$-6 \qquad -8$$

$$\frac{-8-(-6)}{(+)}=\frac{-2}{(+)}=(-)$$

Since $T(x)$ is continuous and differentiable on $[0,5]$, by MVT there exists a c_1 in $(0,5)$ such that $T'(c_1)=\frac{70-100}{5-0}=-6$.

Since $T(x)$ is continuous and differentiable on $[5,6]$, by MVT there exists a c_2 in $(5,6)$ such that $T'(c_2)=\frac{62-70}{6-5}=-8$.

Since $T'(x)$ is continuous and differentiable on (c_1,c_2), by MVT there exists a d in (c_1,c_2) such that $T''(d)=\frac{-8-(-6)}{(+)}=\frac{-2}{(+)}=(-)$.

Therefore the data in the table is not consistent with the assertion that $T''(x)>0$.

1. The function f is continuous for $-2 \le x \le 1$, and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

(a) There exists a c, where $-2 < c < 1$, such that $f(c) = 0$ Guaranteed by IVT
(b) There exists a c, where $-2 < c < 1$, such that $f'(c) = 0$ FALSE!!!!
(c) There exists a c, where $-2 < c < 1$, such that $f(c) = 3$ Guaranteed by IVT
(d) There exists a c, where $-2 < c < 1$, such that $f'(c) = 3$ Guaranteed by MVT
(e) There exists a c, where $-2 \le c \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$. Guaranteed by EVT

2. For all x in the closed interval $[2,5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

~~(a)~~

x	$f(x)$
2	7
3	9
4	12
5	16

(b)

x	$f(x)$
2	7
3	11
4	14
5	16

~~(c)~~

x	$f(x)$
2	16
3	12
4	9
5	7

~~(d)~~

x	$f(x)$
2	16
3	14
4	11
5	7

~~(e)~~

x	$f(x)$
2	16
3	13
4	10
5	7

f has a positive first derivative $\rightarrow$ $f(x)$ increases as x increases. This eliminates (c), (d), and (e)

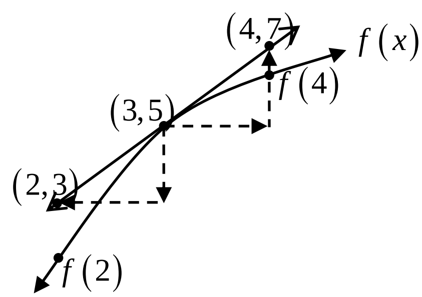
f has a negative second derivative $\rightarrow$ $f(x)$ increases by a smaller amount as x increases. This eliminates (a).

3. The function f is continuous on the closed interval $[2,4]$ and twice differentiable on the open interval $(2,4)$. If $f'(3)=2$ and $f''(x)<0$ on the open interval $(2,4)$, which of the following could be a table of values for f?

(a)		(b)		(c)		(d)		(e)	
x	$f(x)$	x	$f(x)$	x	$f(x)$	x	$f(x)$	x	$f(x)$
2	2.5	2	2.5	2	3	2	3	2	3.5
3	5	3	5	3	5	3	5	3	5
4	6.5	4	7	4	6.5	4	7	4	7.5

Since $f''(x)<0$ on the open interval $(2,4)$, $f(x)$ is concave down on $(2,4)$. Therefore the tangent line to $f(x)$ at $x=3$ will produce overestimates of $f(x)$ around $x=3$. The equation of the tangent line to $f(x)$ at $x=3$ is given by

$$y-5=2(x-3)$$
$$y=2x-1$$



$f(4)\approx 2(4)-1=7$

$f(2)\approx 2(2)-1=3$

Since these approximations are overestimates of $f(x)$ on $(2,4)$, $f(4)<7$ and $f(2)<3$. Therefore the only answer where $f(4)<7$ and $f(2)<3$ is (a).

4. The polynomial function f has selected values of its second derivative f'' given in the table below. Which of the following statements must be true?

x	0	1	2	3
$f''(x)$	5	0	-7	4

If $f(x)$ is a polynomial function, then all orders of its derivatives are continuous and differentiable. That is $f, f', f'', f''', \cdots$ are all continuous and differentiable. If this is the case, the data in the table would indicate that since f'' changes sign somewhere in the interval, then f changes concavity somewhere in the interval.

(a) f is increasing on the interval $(0,2)$
(b) f is decreasing on the interval $(0,2)$
(c) f has a local maximum at $x=1$
(d) The graph of f has a point of inflection at $x=1$
(e) The graph of f changes concavity in the interval $(0,2)$

5. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following must be true?

If $f'(x)$ exists for all x in $-2 \le x \le 2$, then Rolle's Theorem would apply since this is the only other part of the condition of the hypothesis which is not met. If Rolle's Theorem does not apply, then the only wat for this not to occur is if there is a point in the interval $-2 \le x \le 2$ where $f'(x)$ DNE.

(a) For $-2 < k < 2$, $f'(k) > 0$
(b) For $-2 < k < 2$, $f'(k) < 0$
(c) For $-2 < k < 2$, $f'(k)$ exists
(d) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous
(e) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

6. The function f is continuous and differentiable on the closed interval $[0,4]$. The table below gives selected values of f on this interval. Which of the following statements must be true?

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

Since f is continuous and differentiable on the closed interval $[0,4]$ and $f(0) = f(4)$, by Rolle's Theorem there exists a c in $(0,4)$ such that $f'(c) = 0$.
Since f is continuous and differentiable on the closed interval $[1,3]$ and $f(1) = f(3)$, by Rolle's Theorem there exists a c in $(1,3)$ such that $f'(c) = 0$.

(a) The minimum value of f on $[0,4]$ is 2.
(b) The maximum value of f on $[0,4]$ is 4.
(c) $f(x) > 0$ for $0 < x < 4$
(d) $f'(x) < 0$ for $2 < x < 4$
(e) There exists a c, with $0 < c < 4$, for which $f'(c) = 0$