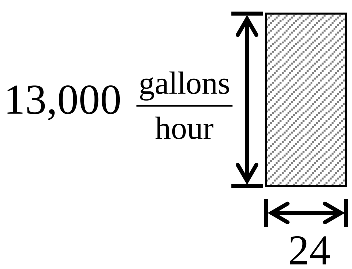
The Definite Integral as Net Change

Key idea: The net change is the definite integral of the rate of change.

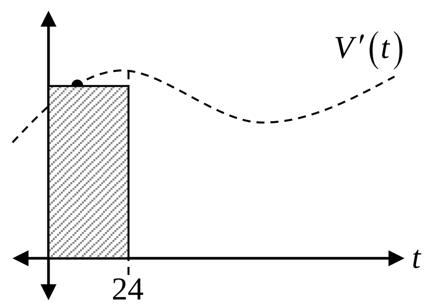
Integral County needs to know about how much water is flowing into its reservoirs. One of Norm of Delta's many jobs is to check the gauging station on the River Calc once each day. The station tells the current rate of flow in thousands of cubic feet per hour. His weekly report shows the following.

	Mon	**Tues**	**Wed**	**Thurs**	**Fri**	**Sat**	**Sun**
Time t_i	9:25 am	1:30 pm	8:00 am	11:30 am	8:30 am	10:20 am	11:30 am
Rate $V'(t_i)$ **1000 ft^3 / hr**	13	17	12	16	10	14	18

What is a reasonable estimate for the amount of water that flowed into the reservoir on Monday? Demonstrate that this value can be visualized as the area of a rectangle under the graph of $V'(t)$. Label the dimensions of the rectangle with proper units, and show the calculations along with unit analysis that leads to your answer.



$$A = \left(13{,}000 \frac{\text{gallons}}{\cancel{\text{hour}}}\right) \cdot \left(24 \ \cancel{\text{hours}}\right)$$
$$= 312{,}000 \text{ gallons}$$



What is a reasonable estimate for the volume of water that flowed into the reservoir for the entire week? Write out, but do not evaluate, a Riemann Sum to determine this value. Label your value with proper units.

$$(13{,}000)(24)+(17{,}000)(24)+(12{,}000)(24)+(16{,}000)(24)+(10{,}000)(24)+(14{,}000)(24)+(18{,}000)(24)$$

$$[13{,}000+17{,}000+12{,}000+16{,}000+10{,}000+14{,}000+18{,}000](24) \text{ gallons}$$

$$[13+17+12+16+10+14+18](24)(1{,}000) \text{ gallons}$$

Write an expression involving an integral that will give the exact value of the volume of water that flowed into the reservoir for the entire week.

$$\text{Water that flowed into reservoir during the week} = \int_0^{7\cdot 24} V'(t)\,dt$$
$$= \int_0^{168} V'(t)\,dt$$

Is it possible to determine the total volume of water in the reservoir at the end of the week? Why or why not?

No, it is not possible to determine the total volume of water in the reservoir at the end of the week. One needs to know the volume of water in the reservoir at the beginning of the week.

In order to determine the total volume of water in the reservoir at the end of the week, Norm of Delta needs to know the volume of water in the reservoir at the beginning of the week.

amount of water at the end of the week		amount of water at the beginning of the week		net change of water from beginning of week to end of week
↓		↓		↓
$V(168)$	$=$	$V(0)$	$+$	$\int_0^{168} V'(t)\,dt$
$V(168)-V(0)$	$=$	$\int_0^{168} V'(t)\,dt$		

<u>FUNDAMENTAL THEOREM OF CALCULUS:</u>

$$\boxed{F(b)-F(a)=\int_a^b f(x)\,dx}$$

or

$$f(b)-f(a)=\int_a^b f'(x)\,dx$$

net change in $f(x)$ from $x=a$ to $x=b$

integral of the rate of change of $f(x)$ from $x=a$ to $x=b$

To find the value of a function $f(x)$ at $x=b$ given:

(1) A point of the graph of f: $(a, f(a))$

(2) The rate of change of f: $f'(x)$

Use the following formula:

$$f(b) = f(a) + \int_a^b f'(x)\,dx$$

final value = starting value + net change

Write an expression that gives the volume of water in the reservoir at time $t=k$ hours, where $0 \le k \le 168$:

$$V(k)=V(0)+\int_0^k V'(t)\,dt$$

Note: The above expression makes <u>V a function of k.</u> To find the value V, one must evaluate a definite integral to do so. This type of function is referred to as an <u>***accumulator function.***</u>