Section 11-1 Homework Solutions:

#3	#4	#5
$a_n = \frac{2n}{n^2+1}$ $a_1 = \frac{2(1)}{(1)^2+1} = 1$ $a_2 = \frac{2(2)}{(2)^2+1} = \frac{1}{4}$ $a_3 = \frac{2(3)}{(3)^2+1} = \frac{3}{5}$ $a_4 = \frac{2(4)}{(4)^2+1} = \frac{8}{17}$ $a_5 = \frac{2(5)}{(5)^2+1} = \frac{5}{13}$	$a_n = \frac{3^n}{1+2^n}$ $a_1 = \frac{3^{(1)}}{1+2^{(1)}} = 1$ $a_2 = \frac{3^{(2)}}{1+2^{(2)}} = \frac{9}{5}$ $a_3 = \frac{3^{(3)}}{1+2^{(3)}} = 3$ $a_4 = \frac{3^{(4)}}{1+2^{(4)}} = \frac{81}{17}$ $a_5 = \frac{3^{(5)}}{1+2^{(5)}} = \frac{81}{11}$	$a_n = \frac{(-1)^{n-1}}{5^n}$ $a_1 = \frac{(-1)^{(1)-1}}{5^{(1)}} = -\frac{1}{5}$ $a_2 = \frac{(-1)^{(2)-1}}{5^{(2)}} = \frac{1}{25}$ $a_3 = \frac{(-1)^{(3)-1}}{5^{(3)}} = -\frac{1}{125}$ $a_4 = \frac{(-1)^{(4)-1}}{5^{(4)}} = \frac{1}{625}$ $a_5 = \frac{(-1)^{(5)-1}}{5^{(5)}} = -\frac{1}{3125}$

#6	#7	#8
$a_n = \cos\left(\frac{n\cdot\pi}{2}\right)$ $a_1 = \cos\left(\frac{(1)\cdot\pi}{2}\right) = 0$ $a_2 = \cos\left(\frac{(2)\cdot\pi}{2}\right) = -1$ $a_3 = \cos\left(\frac{(3)\cdot\pi}{2}\right) = 0$ $a_4 = \cos\left(\frac{(4)\cdot\pi}{2}\right) = 1$ $a_5 = \cos\left(\frac{(5)\cdot\pi}{2}\right) = 0$	$a_n = \frac{1}{(n+1)!}$ $a_1 = \frac{1}{((1)+1)!} = \frac{1}{2}$ $a_2 = \frac{1}{((2)+1)!} = \frac{1}{3\cdot2\cdot1} = \frac{1}{6}$ $a_3 = \frac{1}{((3)+1)!} = \frac{1}{4\cdot3\cdot2\cdot1} = \frac{1}{24}$ $a_4 = \frac{1}{((4)+1)!} = \frac{1}{5\cdot4\cdot3\cdot2\cdot1} = \frac{1}{120}$ $a_5 = \frac{1}{((5)+1)!} = \frac{1}{6\cdot5\cdot4\cdot3\cdot2\cdot1} = \frac{1}{720}$	$a_n = \frac{(-1)^n\cdot n}{n!+1}$ $a_1 = \frac{(-1)^{(1)}\cdot(1)}{(1)!+1} = -\frac{1}{2}$ $a_1 = \frac{(-1)^{(2)}\cdot(2)}{(2)!+1} = \frac{2}{3}$ $a_3 = \frac{(-1)^{(3)}\cdot(\#)}{(3)!+1} = -\frac{3}{7}$ $a_4 = \frac{(-1)^{(4)}\cdot(4)}{(4)!+1} = \frac{4}{25}$ $a_5 = \frac{(-1)^{(5)}\cdot(5)}{(5)!+1} = -\frac{5}{121}$

#13 $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \cdots\right\} \to \frac{1}{2n+1}$ starting with $n=0$

#14 $\left\{1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, \cdots\right\} \to (-1)^n \cdot \left(\frac{1}{3}\right)^n = \frac{(-1)^n}{3^n} = \left(-\frac{1}{3}\right)^n$ starting with $n=0$

#15 $\left\{-3, 2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \cdots\right\} \to 3 \cdot \left(\frac{2}{3}\right)^n$ starting with $n=0$

#16 $\{5, 8, 11, 14, 17, \cdots\} \to 5+3n$ starting with $n=0$

#17 $\left\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \cdots\right\} \to \frac{(-1)^{n+1} \cdot n^2}{n+1}$ starting with $n=1$

#18 $\{1, 0, -1, 0, 1, 0, -1, 0, \cdots\} \to \sin\left(n \cdot \frac{\pi}{2}\right)$ starting with $n=1$

$\to \cos\left(n \cdot \frac{\pi}{2}\right)$ starting with $n=0$

23-33

#23 $\lim_{n\to\infty}\left[1-(0.2)^n\right] = \lim_{n\to\infty}\left[1-\left(\frac{1}{5}\right)^n\right] = 1$

#24 $\lim_{n\to\infty}\left[\frac{n^3}{n^3+1}\right] \sim \lim_{n\to\infty}\left[\frac{n^3}{n^3}\right] = 1$

#25 $\lim_{n\to\infty}\left[\frac{3+5n^2}{n+n^2}\right] \sim \lim_{n\to\infty}\left[\frac{5n^2}{n^2}\right] = 5$

#26 $\lim_{n\to\infty}\frac{n^3}{n+1} \sim \lim_{n\to\infty}\frac{n^3}{n} = \lim_{n\to\infty}\left[n^2\right] \to \infty$

#27 $\lim_{n\to\infty} e^{\frac{1}{n}} = e^{\lim_{n\to\infty}\frac{1}{n}} = e^0 = 1$

#28 $\lim_{n\to\infty}\left[\frac{3^{n+2}}{5^n}\right] = \lim_{n\to\infty}\left[\frac{3^2 \cdot 3^n}{5^n}\right] = \lim_{n\to\infty}\left[9 \cdot \frac{3^n}{5^n}\right] = \lim_{n\to\infty}\left[9 \cdot \left(\frac{3}{5}\right)^n\right] = 0$

#29 $\lim_{n\to\infty}\left[\tan\left(\frac{2\pi n}{1+8n}\right)\right] = \tan\left(\lim_{n\to\infty}\frac{2\pi n}{1+8n}\right) \sim \tan\left(\lim_{n\to\infty}\frac{2\pi n}{8n}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

#30 $\lim_{n\to\infty}\sqrt{\frac{n+1}{9n+1}} = \sqrt{\lim_{n\to\infty}\left[\frac{n+1}{9n+1}\right]} \sim \sqrt{\lim_{n\to\infty}\left[\frac{n}{9n}\right]} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

#31 $\lim_{n\to\infty}\frac{n^2}{\sqrt{n^3+4n}} \sim \lim_{n\to\infty}\left[\frac{n^2}{\sqrt{n^3}}\right] = \lim_{n\to\infty}\left[\frac{n^2}{n^{\frac{3}{2}}}\right] = \lim_{n\to\infty} n^{\frac{1}{2}} \to \infty$

#32 $\lim_{n\to\infty} e^{\frac{2n}{n+2}} = e^{\lim_{n\to\infty}\left[\frac{2n}{n+2}\right]} \sim e^{\lim_{n\to\infty}\left[\frac{2n}{n}\right]} = e^2$

#33 $\lim_{n\to\infty}\left[\frac{(-1)^n}{2\sqrt{n}}\right] = 0$

#44

$$\begin{aligned}
\lim_{n\to\infty}\left[\sqrt[n]{2^{1+3n}}\right] &= \lim_{n\to\infty}\left(2^{1+3n}\right)^{\frac{1}{n}} \\
&= \lim_{n\to\infty}\left(2^{\frac{1+3n}{n}}\right) \\
&= 2^{\lim_{n\to\infty}\left[\frac{1+3n}{n}\right]} \\
&\sim 2^{\lim_{n\to\infty}\left[\frac{3n}{n}\right]} \\
&= 2^3
\end{aligned}$$

#47

$$
\begin{aligned}
y &= \lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n \\
\ln(y) &= \ln\left[\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n\right] \\
\ln(y) &= \lim_{n\to\infty}\ln\left[\left(1+\frac{2}{n}\right)^n\right] \\
\ln(y) &= \lim_{n\to\infty}\left[n\cdot\ln\left(1+\frac{2}{n}\right)\right] \\
\ln(y) &= \lim_{n\to\infty}\left[\frac{\ln\left(1+\frac{2}{n}\right)}{\frac{1}{n}}\right] \text{ use L'Hopital's Rule} \\
\ln(y) &= \lim_{n\to\infty}\left[\frac{\left(\frac{1}{1+\frac{2}{n}}\right)\cdot\left(-2n^{-2}\right)}{\left(-n^{-2}\right)}\right] \\
\ln(y) &= 2 \\
y &= e^2
\end{aligned}
$$