<u>Integral Test Explained</u>

If a sequence a_n is modeled by the function $f(x)$ such that $f(n)=a_n$, then we can use integrals to determine whether or not $\sum_{n=k}^{\infty} a_n$ converges so long as the function $f(x)$ is (1) positive and (2) constantly decreasing for some interval $[k,\infty)$.

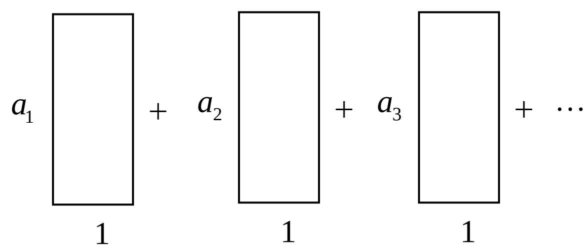
Consider the sum:

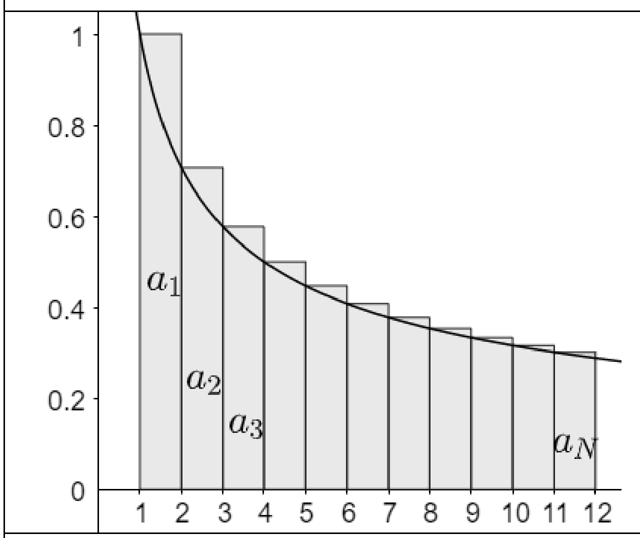
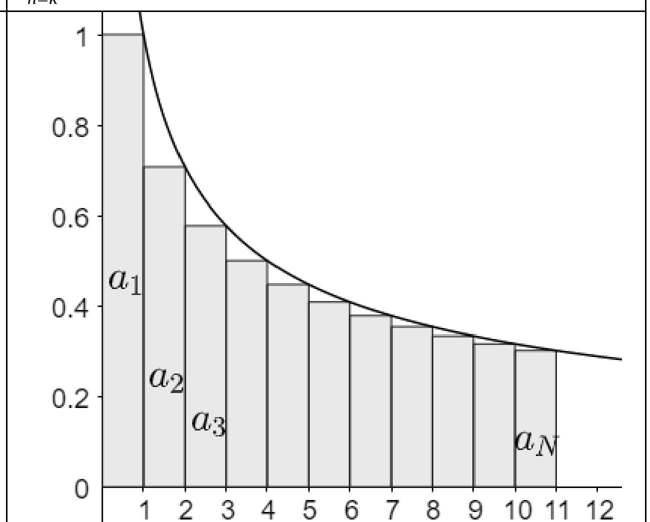
$$a_1+a_2+a_3+\cdots$$

This sum cane be expressed as the sum of the areas of rectangles that have width of 1 and height of a_i

$$a_1+a_2+a_3+\cdots$$

$$1\cdot a_1+1\cdot a_2+1\cdot a_3+\cdots$$



If the sum of the areas of the rectangles is more than the area under the curve, and the area of the curve $\to\infty$, then $\sum_{n=k}^{\infty} a_n$ does not converge.	If the sum of the areas of the rectangles is less than the area under the curve, and the area of the cruve is finite as $x\to\infty$, then $\sum_{n=k}^{\infty} a_n$ converges.
	
$\int_1^{\infty} f(x)\,dx < \sum_{n=k}^{\infty} a_n$ Since $\int_1^{\infty} f(x)\,dx \to\infty$, then $\sum_{n=k}^{\infty} a_n \to\infty$	$\sum_{n=k}^{\infty} a_n < \int_1^{\infty} f(x)\,dx$ Since $\int_1^{\infty} f(x)\,dx$ converges, so does $\sum_{n=k}^{\infty} a_n$

How do we know if the rectangles are above or below the function?

We don't know until we determine whether $\int_1^{\infty} f(x)\,dx$ converges or does not converge.

Notice that the differences between the two graphs is whether we are using a left-sum or right-sum to approximate the area under the curve.

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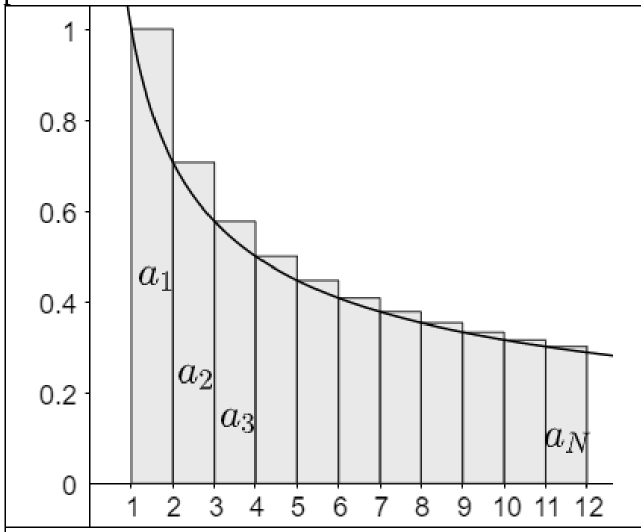
<td>
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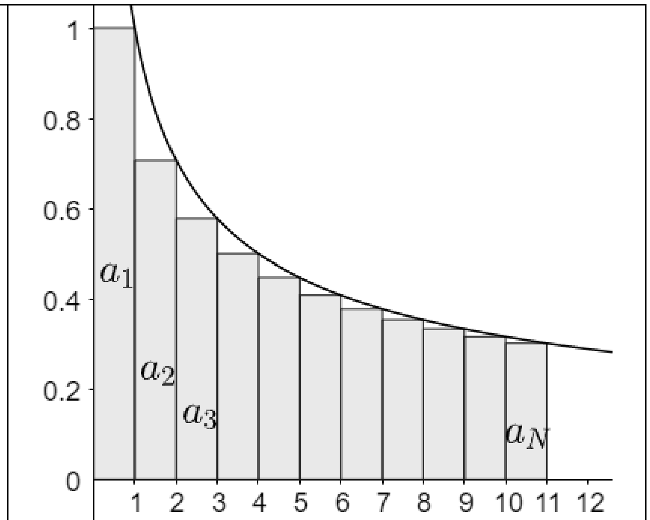
<td>
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<td>Each rectangle is over the function in the case that the area under the curve goes to infinity.</td>
<td>Each rectangle is under the function in the case that the area under the curve converges to a finite value.</td>
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If $\int_1^{\infty} f(x)\,dx \to \infty$,we will choose the perspective of the diagram on the left.

If $\int_1^{\infty} f(x)\,dx$ converges, we will choose the perspective the diagram on the right.

<u>THERE IS NO NEED TO WORRY ABOUT THESE RECTANGLES!!</u>

If $\int_1^{\infty} f(x)\,dx$ converges, so does $\sum_{n=1}^{\infty} a_n$

$\int_1^{\infty} f(x)\,dx \to \infty$, so does $\sum_{n=1}^{\infty} a_n$

***p*-series Test Explained**

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

If $p<0$ then $\lim\limits_{n\to\infty}\frac{1}{n^p}=\lim\limits_{n\to\infty}\frac{1}{n^{\text{something negative}}}=\lim\limits_{n\to\infty} n^{\text{something positive}} \neq 0$, so $\sum\limits_{n=1}^{\infty}\frac{1}{n^p}$ does not converge.

Since a *p*-series is a series of the form $\sum\limits_{n=1}^{\infty}\frac{1}{n^p}$,we can use the integral test to determine the values of *p* that will make the series converge.

If $p=1$	If $p>0$

$\int_1^{\infty}\frac{1}{x}dx=\lim\limits_{t\to\infty}\int_1^{t}\frac{1}{x}dx$
$=\lim\limits_{t\to\infty}\Big[\ln\lvert x\rvert\Big]_1^t$
$=\lim\limits_{t\to\infty}\big(\ln\lvert t\rvert-\ln\lvert 1\rvert\big)$
$\downarrow$
∞
By the Integral Test, $\sum\limits_{n=1}^{\infty}\frac{1}{n^p}$ does not converge.
	$\int_1^{\infty}\frac{1}{x^p}dx=\lim\limits_{t\to\infty}\int_1^{t}x^{-p}dx$
	$=\lim\limits_{t\to\infty}\left[\frac{1}{-p+1}x^{-p+1}\right]_1^t$
	$=\lim\limits_{t\to\infty}\left(\left[\frac{1}{-p+1}t^{-p+1}\right]-\left[\frac{1}{-p+1}1^{-p+1}\right]\right)$
	$=\lim\limits_{t\to\infty}\frac{1}{-p+1}\cdot\frac{1}{t^{p-1}}+\frac{1}{-p+1}$
	If $p>1$, then
	$\lim\limits_{t\to\infty}\frac{1}{t^{p-1}}=\lim\limits_{t\to\infty}\frac{1}{t^{\text{something positive}}}=0$
	Therefore by the Integral Test $\sum\limits_{n=1}^{\infty}\frac{1}{n^p}$ converges
	If $0<p<1$, then
	$\lim\limits_{t\to\infty}\frac{1}{t^{p-1}}=\lim\limits_{t\to\infty}\frac{1}{t^{\text{something negative}}}=\lim\limits_{t\to\infty}t^{\text{something positive}}\to\infty$
	Therefore by the Integral Test $\sum\limits_{n=1}^{\infty}\frac{1}{n^p}$ does not converge.