AP Calculus AB/BC Writing Prompts

1. $\underbrace{\_\_\_\_\_}_{\text{function}}$ is continuous at $\underset{\text{variable}}{\_\_\_\_} = \underbrace{\_\_\_\_\_}_{\text{value}}$ because $\lim\limits_{\underset{\text{variable}}{\_\_} \to \underset{\text{value}}{\_\_}} \underbrace{\_\_\_\_\_\_}_{\text{function}} = \underset{\text{function}}{\_\_\_\_}\left(\underset{\text{value}}{\_\_\_\_}\right)$

2. $\underbrace{\_\_\_\_\_}_{\text{function}}$ is not continuous at $\underset{\text{variable}}{\_\_\_\_} = \underbrace{\_\_\_\_\_}_{\text{value}}$ because $\left\{\begin{array}{l} \lim\limits_{\underset{\text{variable}}{\_\_} \to \boxed{\underset{\text{value}}{\_\_}}^{+}} \underbrace{\_\_\_\_\_\_}_{\text{function}} \neq \underset{\text{function}}{\_\_\_\_}\left(\underset{\text{value}}{\_\_\_\_}\right) \\ \lim\limits_{\underset{\text{variable}}{\_\_} \to \boxed{\underset{\text{value}}{\_\_}}^{-}} \underbrace{\_\_\_\_\_\_}_{\text{function}} \neq \underset{\text{function}}{\_\_\_\_}\left(\underset{\text{value}}{\_\_\_\_}\right) \\ \lim\limits_{\underset{\text{variable}}{\_\_} \to \boxed{\underset{\text{value}}{\_\_}}^{-}} \underbrace{\_\_\_\_\_\_}_{\text{function}} \neq \lim\limits_{\underset{\text{variable}}{\_\_} \to \boxed{\underset{\text{value}}{\_\_}}^{+}} \underbrace{\_\_\_\_\_\_}_{\text{function}} \end{array}\right\}$.

3. $\underset{\text{function}}{\_\_\_\_}$ is continuous on $\underbrace{\_\_\_\_\_}_{\text{interval}}$ because $\underset{\text{function}}{\_\_\_\_}$ is differentiable on $\underbrace{\_\_\_\_\_}_{\text{interval}}$.

4. $\underset{\text{function}}{\_\_\_\_}$ is $\underbrace{\_\_\_\_\_\_\_\_\_\_}_{\text{increasing / decreasing}}$ on $\underbrace{\_\_\_\_\_}_{\text{interval}}$ because $\underset{\text{derivative}}{\_\_\_\_}$ $\underbrace{\_\_\_\_\_\_\_\_\_\_}_{\substack{\text{is positive / is negative} \\ >0 \text{ or } <0}}$ $\left[\text{on } \underbrace{\_\_\_\_\_}_{\text{interval}}\right]$.

5. $\underset{\text{function}}{\_\_\_\_}$ has a relative $\underbrace{\_\_\_\_}_{\text{min / max}}$ at $\underset{\text{variable}}{\_\_\_\_} = \underbrace{\_\_\_\_\_}_{\text{value}}$ because $\underset{\text{derivative}}{\_\_\_\_}$ changes sign from $\underbrace{\_\_\_\_\_\_\_\_}_{\substack{\text{negative to positive} \\ (-)\to(+) \\ \text{positive to negative} \\ (+)\to(-)}}$ $\left[\text{at } \underset{\text{variable}}{\_\_\_\_} = \underbrace{\_\_\_\_\_}_{\text{value}}\right]$.

6. $\underset{\text{function}}{\_\_\_\_}$ is concave $\underset{\text{up /down}}{\_\_\_\_}$ on $\underbrace{\_\_\_\_\_}_{\text{interval}}$ because $\underbrace{\_\_\_\_\_\_\_}_{\substack{\text{second derivative} \\ \text{OR} \\ \text{first derivative}}}$ $\underbrace{\_\_\_\_\_\_\_\_\_}_{\substack{\text{is positive / is negative} \\ >0 \text{ or } <0 \\ \text{OR} \\ \text{is increasing/ is decreasing}}}$ $\left[\text{on } \underbrace{\_\_\_\_\_}_{\text{interval}}\right]$.

7. (a) $\underset{\text{function}}{\_\_\_\_}$ has an inflection point at $\underset{\text{variable}}{\_\_\_\_} = \underbrace{\_\_\_\_\_}_{\text{value}}$ because $\underbrace{\_\_\_\_\_\_\_}_{\text{second derivative}}$ changes sign $\left[\text{at } \underset{\text{variable}}{\_\_\_\_} = \underbrace{\_\_\_\_\_}_{\text{value}}\right]$.

(b) $\underset{\text{function}}{\_\_\_\_}$ has an inflection point at $\underset{\text{variable}}{\_\_\_\_} = \underbrace{\_\_\_\_\_}_{\text{value}}$ because $\underbrace{\_\_\_\_\_\_\_}_{\text{first derivative}}$ changes from $\underbrace{\_\_\_\_\_\_\_\_\_\_}_{\substack{\text{decreasing to increasing} \\ \text{OR} \\ \text{increasing to decreasing}}}$ $\left[\text{at } \underset{\text{variable}}{\_\_\_\_} = \underbrace{\_\_\_\_\_}_{\text{value}}\right]$.

8. The speed of the object is $\underbrace{\_\_\_\_\_\_\_\_\_\_}_{\text{increasing / decreasing}}$ on $\underbrace{\_\_\_\_\_}_{\text{interval}}$ because $\underbrace{\_\_\_\_\_\_\_}_{\text{velocity function}}$ and $\underbrace{\_\_\_\_\_\_\_\_}_{\text{acceleration function}}$ have $\underbrace{\_\_\_\_\_\_\_\_\_\_\_}_{\text{the same /opposite}}$ sign(s).

9. The speed of the object is $\underbrace{\qquad\qquad}_{\text{increasing / decreasing}}$ at $\underset{\text{variable}}{\underline{\qquad}} = \underbrace{\qquad}_{\text{value}}$ because $\underbrace{\qquad\qquad}_{\substack{\text{velocity funtion}\\ \text{at value}}}$ $\underbrace{\qquad\qquad}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$ and $\underbrace{\qquad\qquad}_{\substack{\text{acceleration funtion}\\ \text{at value}}}$ $\left[\text{at } \underset{\text{variable}}{\underline{\qquad}} = \underbrace{\qquad}_{\text{value}}\right]$.

10. Since $\underbrace{\qquad}_{\text{function}}$ is continuous on $\underbrace{\qquad\quad}_{\text{closed interval}}$, $\underbrace{\qquad\qquad}_{\text{function(left endpoint)}} = \underbrace{\qquad}_{\text{value}}$, and $\underbrace{\qquad\qquad}_{\text{function(right endpoint)}} = \underbrace{\qquad}_{\text{value}}$, **by IVT** there exists a c in $\underbrace{\qquad\quad}_{\text{closed interval}}$ such that $\underset{\text{function}}{\underline{\qquad}}(c) = \underbrace{\qquad\qquad\qquad}_{\substack{\text{value between function(left endpoint)}\\ \text{and function(right endpoint)}}}$.

11. Since $\underbrace{\qquad}_{\text{function}}$ is continuous on $\underbrace{\qquad\quad}_{\text{closed interval}}$, differentiable on $\underbrace{\qquad\quad}_{\text{open interval}}$, $\underbrace{\qquad\qquad}_{\text{function(left endpoint)}} = \underbrace{\qquad\qquad}_{\text{function(right endpoint)}}$, **by Rolle's Theorem**, there exists a c in $\underbrace{\qquad\quad}_{\text{open interval}}$, such that $\underset{\text{function}'}{\underline{\qquad}}(c) = 0$.

12. Since $\underbrace{\qquad}_{\text{function}}$ is continuous on $\underbrace{\qquad\quad}_{\text{closed interval}}$, and differentiable on $\underbrace{\qquad\quad}_{\text{open interval}}$, **by MVT**, there exists a c in $\underbrace{\qquad\quad}_{\text{open interval}}$, such that $\underset{\text{function}'}{\underline{\qquad}}(c) = \underbrace{\qquad\qquad\qquad\qquad\qquad}_{\frac{\text{function(right endpoint)}-\text{function(right endpoint)}}{\text{right endpoint} - \text{left endpoint}}}$.

13. Since $\underset{\text{derivative}}{\underline{\qquad}}$ $\underbrace{\qquad\qquad}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$ and $\underbrace{\qquad\quad}_{\text{second derivative}}$ $\underbrace{\qquad\qquad}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$ on $\underbrace{\qquad\quad}_{\text{interval}}$, the $\underbrace{\qquad\qquad}_{\text{left sum / right sum}}$ is an $\underbrace{\qquad\qquad\qquad}_{\text{underestimate / overestimate}}$ of $\underbrace{\qquad\qquad\qquad\qquad\qquad}_{\substack{\text{definite integral of fucntion}\\ \text{from left endpoint of interval to right endpoint of interval}}}$.

14. $\underbrace{\qquad}_{\substack{\text{definite}\\ \text{integral}}}$ represents the net change in $\underbrace{\qquad}_{\text{function}}$, measured in $\underbrace{\qquad}_{\text{units}}$, from $\underset{\text{variable}}{\underline{\qquad}} = \underbrace{\qquad}_{\substack{\text{lower bound}\\ \text{of integral}}}$ to $\underset{\text{variable}}{\underline{\qquad}} = \underbrace{\qquad}_{\substack{\text{upper bound}\\ \text{of integral}}}$.

<u>**AP Calculus BC Prompts:**</u>

1. The particle is moving $\underbrace{\qquad\qquad}_{\text{towards/ away from}}$ the origin because $r(t)$ and $\dfrac{dr}{dt}$ have $\underbrace{\qquad\qquad\qquad}_{\text{the same /opposite}}$ sign(s).

2. The particle is moving $\underbrace{\qquad\qquad}_{\text{towards/ away from}}$ the origin because $r(t)\underbrace{\qquad\qquad}_{\substack{\text{is positive /is negative}\\ >0 \text{ or } <0}}$ and $\dfrac{dr}{dt}\underbrace{\qquad\qquad}_{\substack{\text{is positive /is negative}\\ >0 \text{ or } <0}}$.

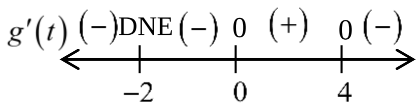
3. The particle is moving $\underbrace{\qquad\qquad}_{\text{towards/ away from}}$ the x-axis because $x(t)\underbrace{\qquad\qquad}_{\substack{\text{is positive /is negative}\\ >0 \text{ or } <0}}$ and $\dfrac{dx}{dt}\underbrace{\qquad\qquad}_{\substack{\text{is positive /is negative}\\ >0 \text{ or } <0}}$.

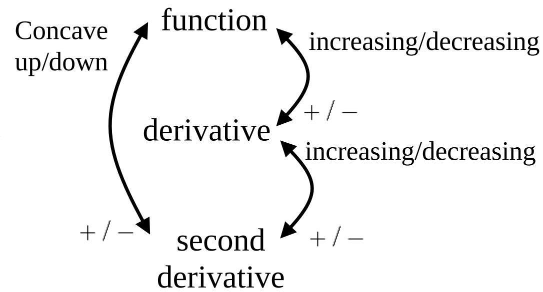
4. The particle is moving $\underbrace{\qquad\qquad}_{\text{towards/ away from}}$ the x-axis because $x(t)$ and $\dfrac{dx}{dt}$ have $\underbrace{\qquad\qquad\qquad}_{\text{the same /opposite}}$ sign(s).

5. The particle is moving $\underbrace{\qquad\qquad}_{\text{towards/ away from}}$ the y-axis because $y(t)\underbrace{\qquad\qquad}_{\substack{\text{is positive /is negative}\\ >0 \text{ or } <0}}$ and $\dfrac{dy}{dt}\underbrace{\qquad\qquad}_{\substack{\text{is positive /is negative}\\ >0 \text{ or } <0}}$.

6. The particle is moving $\underbrace{\qquad\qquad}_{\text{towards/ away from}}$ the y-axis because $y(t)$ and $\dfrac{dy}{dt}$ have $\underbrace{\qquad\qquad\qquad}_{\text{the same /opposite}}$ sign(s).

1. What can you conclude about $g(t)$ from the given sign chart?





$\underset{\text{function}}{g(t)}$ is continuous on $\underbrace{\qquad}_{\text{interval}}$ and $\underbrace{\qquad}_{\text{interval}}$ because $\underset{\text{function}}{\underline{\qquad}}$ is differentiable on $\underbrace{\qquad}_{\text{interval}}$.

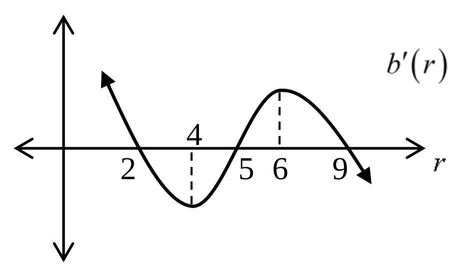
$\underset{\text{function}}{g(t)}$ is $\underbrace{\text{increasing}}_{\text{increasing / decreasing}}$ on $\underbrace{\qquad}_{\text{interval}}$ because $\underset{\text{derivative}}{\underline{\qquad}}$ $\underbrace{\qquad\qquad}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$.

$\underset{\text{function}}{g(t)}$ is $\underbrace{\text{decreasing}}_{\text{increasing / decreasing}}$ on $\underbrace{\qquad}_{\text{interval}}$, $\underbrace{\qquad}_{\text{interval}}$, and $\underbrace{\qquad}_{\text{interval}}$ because $\underset{\text{derivative}}{\underline{\qquad}}$ $\underbrace{\qquad\qquad}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$.

$\underset{\text{function}}{g(t)}$ has a relative $\underset{\text{min / max}}{\underline{\text{min}}}$ at $\underset{\text{variable}}{\underline{\quad}} = \underbrace{\qquad}_{\text{value}}$ because $\underset{\text{derivative}}{\underline{\qquad}}$ changes sign from $\underbrace{\qquad\qquad}_{\substack{\text{negative to positive}\\ (-)\to(+)\\ \text{positive to negative}\\ (+)\to(-)}}$.

$\underset{\text{function}}{g(t)}$ has a relative $\underset{\text{min / max}}{\underline{\text{max}}}$ at $\underset{\text{variable}}{\underline{\quad}} = \underbrace{\qquad}_{\text{value}}$ because $\underset{\text{derivative}}{\underline{\qquad}}$ changes sign from $\underbrace{\qquad\qquad}_{\substack{\text{negative to positive}\\ (-)\to(+)\\ \text{positive to negative}\\ (+)\to(-)}}$.

2. What can you conclude about $b(r)$ given the graph of $b'(r)$ below?



$\underset{\text{function}}{b(r)}$ is continuous on $\underset{\text{interval}}{\mathbb{R}}$ because $\underset{\text{function}}{b(r)}$ is differentiable on $\underset{\text{interval}}{\mathbb{R}}$.

$\underset{\text{function}}{b(r)}$ is $\underbrace{\text{decreasing}}_{\text{increasing / decreasing}}$ on $\underbrace{\qquad}_{\text{interval}}$ because $\underset{\text{derivative}}{\underline{\qquad}}$ $\underbrace{\qquad\qquad}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$.

$\underset{\text{function}}{b(r)}$ is $\underbrace{\text{increasing}}_{\text{increasing / decreasing}}$ on $\underbrace{\qquad}_{\text{interval}}$ and $\underbrace{\qquad}_{\text{interval}}$ because $\underset{\text{derivative}}{\underline{\qquad}}$ $\underbrace{\qquad\qquad}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$.

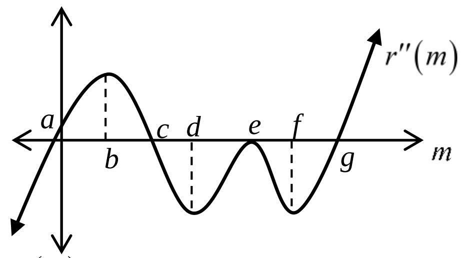
$\underset{\text{function}}{b(r)}$ has a relative $\underset{\text{min / max}}{\underline{\text{min}}}$ at $\underset{\text{variable}}{\underline{\quad}} = \underbrace{\qquad}_{\text{value}}$ because $\underset{\text{derivative}}{\underline{\qquad}}$ changes sign from $\underbrace{\qquad\qquad}_{\substack{\text{negative to positive}\\ (-)\to(+)\\ \text{positive to negative}\\ (+)\to(-)}}$.

$\underset{\text{function}}{b(r)}$ has a relative $\underset{\text{min / max}}{\underline{\text{max}}}$ at $\underset{\text{variable}}{\underline{\quad}} = \underset{\text{value}}{\underline{\qquad}}$ and $\underset{\text{value}}{\underline{\qquad}}$ because $\underset{\text{derivative}}{\underline{\qquad}}$ changes sign from $\underbrace{\qquad\qquad}_{\substack{\text{negative to positive}\\ (-)\to(+)\\ \text{positive to negative}\\ (+)\to(-)}}$.

$\underset{\text{function}}{b(r)}$ is concave $\underset{\text{up /down}}{\underline{\text{up}}}$ on $\underbrace{\qquad}_{\text{interval}}$ because $\underbrace{\qquad\qquad}_{\substack{\text{second derivative}\\ \text{or}\\ \text{derivative}}}$ $\underbrace{\qquad\qquad}_{\substack{\text{is positive / is negative}\\ \text{or}\\ \text{is increasing/decreasing}}}$.

$\underset{\text{function}}{b(r)}$ is concave $\underset{\text{up /down}}{\underline{\text{down}}}$ on $\underbrace{\qquad}_{\text{interval}}$ because $\underbrace{\qquad\qquad}_{\substack{\text{second derivative}\\ \text{or}\\ \text{derivative}}}$ $\underbrace{\qquad\qquad}_{\substack{\text{is positive / is negative}\\ \text{or}\\ \text{is increasing/decreasing}}}$.

3. What can you conclude about $r(m)$ or $r'(m)$ from the graph of $r''(m)$ below?



$\underset{\text{function}}{\underline{r'(m)}}$ is continuous on $\underset{\text{interval}}{\underline{\mathbb{R}}}$ because $\underset{\text{function}}{\underline{r'(m)}}$ is differentiable on $\underset{\text{interval}}{\underline{\mathbb{R}}}$.

$\underset{\text{function}}{\underline{r(m)}}$ is continuous on $\underset{\text{interval}}{\underline{\mathbb{R}}}$ because $\underset{\text{function}}{\underline{r(m)}}$ is differentiable on $\underset{\text{interval}}{\underline{\mathbb{R}}}$.

$\underset{\text{function}}{\underline{r'(m)}}$ is $\underbrace{\text{increasing}}_{\text{increasing / decreasing}}$ on $\underbrace{\_\_\_\_\_\_}_{\text{interval}}$ and $\underbrace{\_\_\_\_\_\_}_{\text{interval}}$ because $\underset{\text{derivative}}{\_\_\_\_\_}$ $\underbrace{\_\_\_\_\_\_\_\_\_\_\_\_}_{\substack{\text{is positive / is negative} \\ >0 \text{ or } <0}}$.

$\underset{\text{function}}{\underline{r'(m)}}$ is $\underbrace{\text{decreasing}}_{\text{increasing / decreasing}}$ on $\underbrace{\_\_\_\_\_\_}_{\text{interval}}$, $\underbrace{\_\_\_\_\_\_}_{\text{interval}}$, and $\underbrace{\_\_\_\_\_\_}_{\text{interval}}$ because $\underset{\text{derivative}}{\_\_\_\_\_}$ $\underbrace{\_\_\_\_\_\_\_\_\_\_\_\_}_{\substack{\text{is positive / is negative} \\ >0 \text{ or } <0}}$.

$\underset{\text{function}}{\underline{r'(m)}}$ has a relative $\underset{\text{min / max}}{\underline{\text{min}}}$ at $\underset{\text{variable}}{\_\_\_\_} = \underset{\text{value}}{\_\_\_\_\_}$ and $\underset{\text{value}}{\_\_\_\_\_}$ because $\underset{\text{derivative}}{\_\_\_\_\_}$ changes sign from $\underbrace{\_\_\_\_\_\_\_\_\_\_}_{\substack{\text{negative to positive} \\ (-)\to(+) \\ \text{positive to negative} \\ (+)\to(-)}}$.

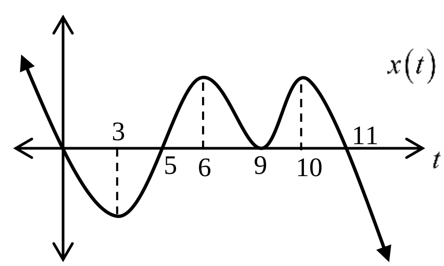
$\underset{\text{function}}{\underline{r'(m)}}$ has a relative $\underset{\text{min / max}}{\underline{\text{max}}}$ at $\underset{\text{variable}}{\_\_\_\_} = \underbrace{\_\_\_\_\_\_}_{\text{value}}$ because $\underset{\text{derivative}}{\_\_\_\_\_}$ changes sign from $\underbrace{\_\_\_\_\_\_\_\_\_\_}_{\substack{\text{negative to positive} \\ (-)\to(+) \\ \text{positive to negative} \\ (+)\to(-)}}$.

$\underset{\text{function}}{\underline{r(m)}}$ is concave $\underset{\text{up /down}}{\underline{\text{up}}}$ on $\underbrace{\_\_\_\_\_\_}_{\text{interval}}$ and $\underbrace{\_\_\_\_\_\_}_{\text{interval}}$ because $\underbrace{\_\_\_\_\_\_\_\_}_{\text{second derivative}}$ $\underbrace{\_\_\_\_\_\_\_\_\_\_\_\_}_{\substack{\text{is positive / is negative} \\ >0 \text{ or } <0}}$.

$\underset{\text{function}}{\underline{r(m)}}$ is concave $\underset{\text{up /down}}{\underline{\text{down}}}$ on $\underbrace{\_\_\_\_\_\_}_{\text{interval}}$, $\underbrace{\_\_\_\_\_\_}_{\text{interval}}$, and $\underbrace{\_\_\_\_\_\_}_{\text{interval}}$ because $\underbrace{\_\_\_\_\_\_\_\_}_{\text{second derivative}}$ $\underbrace{\_\_\_\_\_\_\_\_\_\_\_\_}_{\substack{\text{is positive / is negative} \\ >0 \text{ or } <0}}$.

$\underset{\text{function}}{\underline{r(m)}}$ has an inflection point at $\underset{\text{variable}}{\_\_\_\_} = \underset{\text{value}}{\_\_\_\_\_}$, $\underset{\text{value}}{\_\_\_\_\_}$, and $\underset{\text{value}}{\_\_\_\_\_}$ because $\underbrace{\_\_\_\_\_\_\_\_}_{\text{second derivative}}$ changes sign .

4. A particle is moving along the *x*-axis so that its position at time-*t* seconds is given by the following graph. The graph of $x(t)$ has points of inflection at $t = 5,\ t = 7,$ and $t = 9.5$.



(a) For what interval(s) of *t* is the particle moving to the right? Justify your answer.
(b) For what interval(s) of *t* is the particle moving to the left? Justify your answer.
(c) At what time(s), if any, does the particle change direction? Justify your answer.
(d) Identify the interval(s) of *t* for which the particle's speed increasing. Justify your answer.
(e) Identify the interval(s) of *t* for which particle's speed decreasing? Justify your answer.

1. What can you conclude about $g(t)$ from the given sign chart?

$$g'(t)\quad (-)\ \text{DNE}\ (-)\ \ 0\ \ (+)\ \ 0\ \ (-)$$

$$\underset{(+)}{|}\qquad \underset{0}{|}\qquad \underset{(-)}{|}$$

$\underset{\text{function}}{\underline{g(t)}}$ is continuous on $\underbrace{\begin{Bmatrix}(-\infty,-2)\cup(-2,\infty)\\ t<-2 \text{ and } t>-2\end{Bmatrix}}_{\text{interval}}$ because $\underset{\text{function}}{\underline{g(t)}}$ is differentiable on $\underbrace{\begin{Bmatrix}(-\infty,-2)\cup(-2,\infty)\\ t<-2 \text{ and } t>-2\end{Bmatrix}}_{\text{interval}}$.

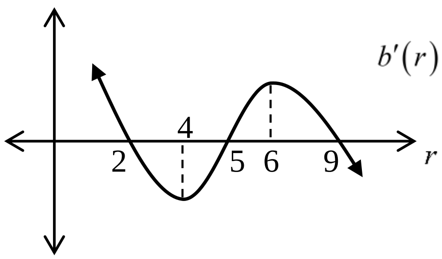
$\underset{\text{function}}{\underline{g(t)}}$ is $\underbrace{\underline{\text{increasing}}}_{\text{increasing / decreasing}}$ on $\underbrace{\begin{Bmatrix}(0,4)\\ 0<t<4\end{Bmatrix}}_{\text{interval}}$ because $\underset{\text{derivative}}{\underline{g'(t)}}$ $\underbrace{\begin{Bmatrix}\text{is positive}\\ >0\end{Bmatrix}}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$.

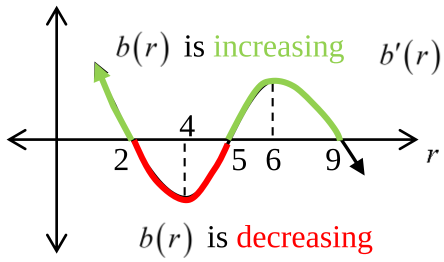
$\underset{\text{function}}{\underline{g(t)}}$ is $\underbrace{\underline{\text{decreasing}}}_{\text{increasing / decreasing}}$ on $\underbrace{\begin{Bmatrix}(-\infty,-2)\cup(-2,0)\cup(4,\infty)\\ t<-2,\ \ -2<t<0, \text{ and } t>4\end{Bmatrix}}_{\text{interval}}$ because $\underset{\text{derivative}}{\underline{g'(t)}}$ $\underbrace{\begin{Bmatrix}\text{is negative}\\ <0\end{Bmatrix}}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$.

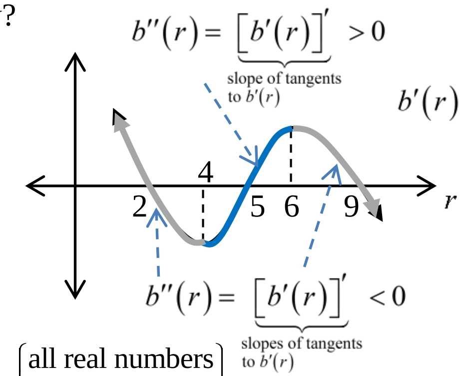
$\underset{\text{function}}{\underline{g(t)}}$ has a relative $\underset{\text{min / max}}{\underline{\text{min}}}$ at $\underset{\text{variable}}{\underline{\ t\ }} = \underset{\text{value}}{\underline{\ 0\ }}$ because $\underset{\text{derivative}}{\underline{g'(t)}}$ changes sign from $\underbrace{\begin{Bmatrix}\text{negative to positive}\\ (-)\to(+)\end{Bmatrix}}_{\substack{\text{negative to positive}\\ (-)\to(+)\\ \text{positive to negative}\\ (+)\to(-)}}$.

$\underset{\text{function}}{\underline{g(t)}}$ has a relative $\underset{\text{min / max}}{\underline{\text{max}}}$ at $\underset{\text{variable}}{\underline{\ t\ }} = \underset{\text{value}}{\underline{\ 4\ }}$ because $\underset{\text{derivative}}{\underline{g'(t)}}$ changes sign from $\underbrace{\begin{Bmatrix}\text{positive to negative}\\ (+)\to(-)\end{Bmatrix}}_{\substack{\text{negative to positive}\\ (-)\to(+)\\ \text{positive to negative}\\ (+)\to(-)}}$.

2. What can you conclude about $b(r)$ given the graph of $b'(r)$ below?







$\underbrace{b(r)}_{\text{function}}$ is continuous on $\underbrace{\left\{\begin{matrix}\text{all real numbers}\\ (-\infty,\infty)\end{matrix}\right\}}_{\text{interval}}$ because $\underbrace{b(r)}_{\text{function}}$ is differentiable on $\underbrace{\left\{\begin{matrix}\text{all real numbers}\\ (-\infty,\infty)\end{matrix}\right\}}_{\text{interval}}$.

$\underbrace{b(r)}_{\text{function}}$ is $\underbrace{\text{increasing}}_{\text{increasing / decreasing}}$ on $\underbrace{\left\{\begin{matrix}(-\infty,2)\cup(5,9)\\ r<2 \text{ and } 5<r<9\end{matrix}\right\}}_{\text{interval}}$ because $\underbrace{b'(r)}_{\text{derivative}}$ $\underbrace{\left\{\begin{matrix}\text{is positive}\\ >0\end{matrix}\right\}}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$.

$\underbrace{b(r)}_{\text{function}}$ is $\underbrace{\text{decreasing}}_{\text{increasing / decreasing}}$ on $\underbrace{\left\{\begin{matrix}(2,5)\cup(9,\infty)\\ 2<r<5 \text{ and } r>9\end{matrix}\right\}}_{\text{interval}}$ because $\underbrace{b'(r)}_{\text{derivative}}$ $\underbrace{\left\{\begin{matrix}\text{is negative}\\ <0\end{matrix}\right\}}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$.

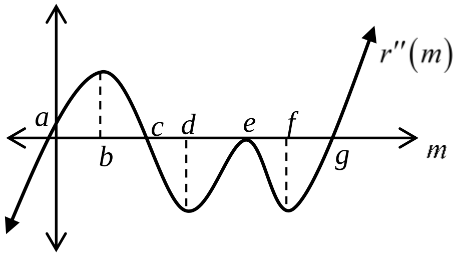
$\underbrace{b(r)}_{\text{function}}$ has a relative $\underbrace{\text{min}}_{\text{min / max}}$ at $\underbrace{r}_{\text{variable}} = \underbrace{5}_{\text{value}}$ because $\underbrace{b'(r)}_{\text{derivative}}$ changes sign from $\underbrace{\left\{\begin{matrix}\text{negative to positive}\\ (-)\to(+)\end{matrix}\right\}}_{\substack{\text{negative to positive}\\ (-)\to(+)\\ \text{positive to negative}\\ (+)\to(-)}}$.

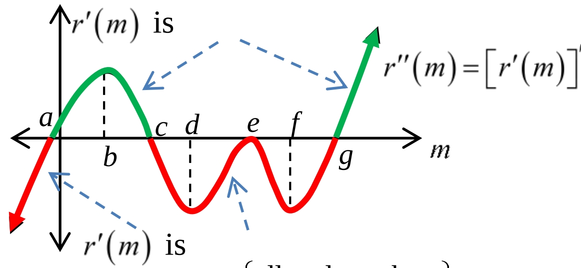
$\underbrace{b(r)}_{\text{function}}$ has a relative $\underbrace{\text{max}}_{\text{min / max}}$ at $\underbrace{r}_{\text{variable}} = \underbrace{2 \text{ and } 9}_{\text{value}}$ because $\underbrace{b'(r)}_{\text{derivative}}$ changes sign from $\underbrace{\left\{\begin{matrix}\text{positive to negative}\\ (+)\to(-)\end{matrix}\right\}}_{\substack{\text{negative to positive}\\ (-)\to(+)\\ \text{positive to negative}\\ (+)\to(-)}}$.

$\underbrace{b(r)}_{\text{function}}$ is concave $\underbrace{\text{up}}_{\text{up /down}}$ on $\underbrace{\left\{\begin{matrix}(4,6)\\ 4<r<6\end{matrix}\right\}}_{\text{interval}}$ because $\underbrace{b''(r)}_{\text{second derivative}}$ $\underbrace{\left\{\begin{matrix}\text{is positive}\\ >0\end{matrix}\right\}}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$.

$\underbrace{b(r)}_{\text{function}}$ is concave $\underbrace{\text{down}}_{\text{up /down}}$ on $\underbrace{\left\{\begin{matrix}(-\infty,4)\cup(6,\infty)\\ r<4 \text{ and } r>6\end{matrix}\right\}}_{\text{interval}}$ because $\underbrace{b''(r)}_{\text{second derivative}}$ $\underbrace{\left\{\begin{matrix}\text{is negative}\\ <0\end{matrix}\right\}}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$.

3. What can you conclude about $r(m)$ or $r'(m)$ from the graph of $r''(m)$ below?





$\underbrace{r'(m)}_{\text{function}}$ is continuous on $\underbrace{\left\{\begin{matrix}\text{all real numbers}\\(-\infty,\infty)\end{matrix}\right\}}_{\text{interval}}$ because $\underbrace{r'(m)}_{\text{function}}$ is differentiable on $\underbrace{\left\{\begin{matrix}\text{all real numbers}\\(-\infty,\infty)\end{matrix}\right\}}_{\text{interval}}$.

$\underbrace{r(m)}_{\text{function}}$ is continuous on $\underbrace{\left\{\begin{matrix}\text{all real numbers}\\(-\infty,\infty)\end{matrix}\right\}}_{\text{interval}}$ because $\underbrace{r(m)}_{\text{function}}$ is differentiable on $\underbrace{\left\{\begin{matrix}\text{all real numbers}\\(-\infty,\infty)\end{matrix}\right\}}_{\text{interval}}$.

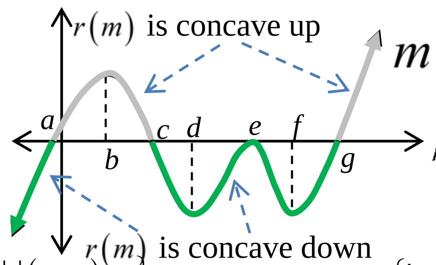
$\underbrace{r'(m)}_{\text{function}}$ is $\underbrace{\text{increasing}}_{\text{increasing / decreasing}}$ on $\underbrace{\left\{\begin{matrix}(a,c)\cup(g,\infty)\\a<m<c\text{ and }m>g\end{matrix}\right\}}_{\text{interval}}$ because $\underbrace{r''(m)}_{\text{derivative}}$ $\underbrace{\left\{\begin{matrix}\text{is positive}\\>0\end{matrix}\right\}}_{\substack{\text{is positive / is negative}\\>0\text{ or }<0}}$.

$\underbrace{r'(m)}_{\text{function}}$ is $\underbrace{\text{decreasing}}_{\text{increasing / decreasing}}$ on $\underbrace{\left\{\begin{matrix}(-\infty,a)\cup(c,e)\cup(e,g)\text{ or }(c,g)\\m<a,\ c<m<e,\text{ and }e<m<g\\\text{or }c<m<g\end{matrix}\right\}}_{\text{interval}}$ because $\underbrace{r''(m)}_{\text{derivative}}$ $\underbrace{\left\{\begin{matrix}\text{is negative}\\<0\end{matrix}\right\}}_{\substack{\text{is positive / is negative}\\>0\text{ or }<0}}$.

$\underbrace{r'(m)}_{\text{function}}$ has a relative $\underbrace{\text{min}}_{\text{min / max}}$ at $\underbrace{m}_{\text{variable}} = \underbrace{a\text{ and }g}_{\text{value}}$ because $\underbrace{r''(m)}_{\text{derivative}}$ changes sign from $\underbrace{\left\{\begin{matrix}\text{negative to positive}\\(-)\to(+)\end{matrix}\right\}}_{\substack{\text{negative to positive}\\(-)\to(+)\\\text{positive to negative}\\(+)\to(-)}}$.

$\underbrace{r'(m)}_{\text{function}}$ has a relative $\underbrace{\text{max}}_{\text{min / max}}$ at $\underbrace{m}_{\text{variable}} = \underbrace{c}_{\text{value}}$ because $\underbrace{r''(m)}_{\text{derivative}}$ changes sign from $\underbrace{\left\{\begin{matrix}\text{positive to negative}\\(+)\to(-)\end{matrix}\right\}}_{\substack{\text{negative to positive}\\(-)\to(+)\\\text{positive to negative}\\(+)\to(-)}}$.

What can you conclude about the graph of $r(m)$ or $r'(m)$ from the graph of $r''(m)$ below?

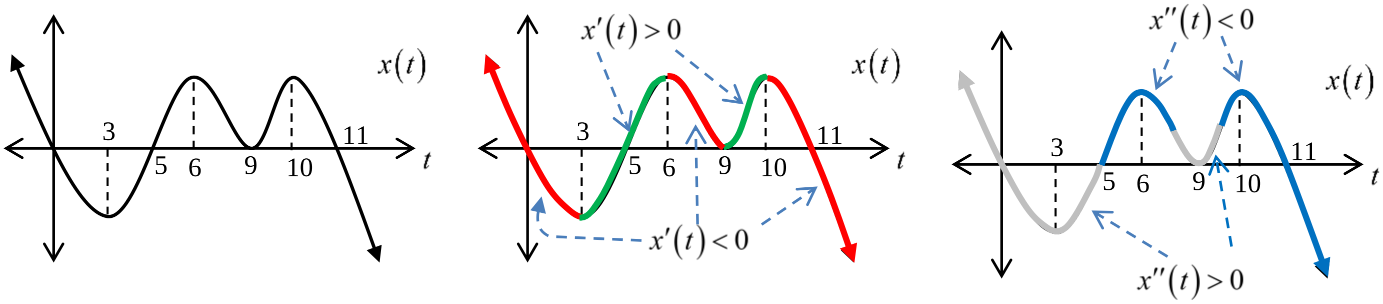


$\underset{\text{function}}{\underline{r(m)}}$ is concave $\underset{\text{up /down}}{\underline{\text{ up }}}$ on $\underbrace{\left\{\begin{matrix}(a,c)\cup(g,\infty)\\ a<m<c \text{ and } m>g\end{matrix}\right\}}_{\text{interval}}$ because $\underbrace{r''(m)}_{\text{second derivative}}$ $\underbrace{\left\{\begin{matrix}\text{is positive}\\ >0\end{matrix}\right\}}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$.

$\underset{\text{function}}{\underline{r(m)}}$ is concave $\underset{\text{up /down}}{\underline{\text{down}}}$ on $\underbrace{\left\{\begin{matrix}(-\infty,a)\cup(c,e)\cup(e,g) \text{ or } (c,g)\\ m<a,\ c<m<e \text{ and } e<m<g\\ \text{or } c<m<g\end{matrix}\right\}}_{\text{interval}}$ because $\underbrace{r''(m)}_{\text{second derivative}}$ $\underbrace{\left\{\begin{matrix}\text{is negative}\\ <0\end{matrix}\right\}}_{\substack{\text{is positive / is negative}\\ >0 \text{ or } <0}}$.

$\underset{\text{function}}{\underline{r(m)}}$ has an inflection point at $\underset{\text{variable}}{\underline{\ m\ }} = \underbrace{a,\ c,\ \text{and } g}_{\text{value}}$ because $\underbrace{r''(m)}_{\text{second derivative}}$ changes sign .

4. A particle is moving along the x-axis so that its position at time-t seconds is given by the following graph. The graph of $x(t)$ has points of inflection at $t = 5,\ t = 7,$ and $t = 9.5$.



(a) For what interval(s) of t is the particle moving to the right? Justify your answer.

The particle is moving to the right on $\left\{\begin{matrix} (3,6)\cup(9,10) \\ 3 < t < 6 \text{ and } 9 < t < 10 \end{matrix}\right\}$ because $x'(t)$ $\left\{\begin{matrix} \text{is positive} \\ > 0 \end{matrix}\right\}$.

(b) For what interval(s) of t is the particle moving to the left? Justify your answer.

The particle is moving to the left on $\left\{\begin{matrix} (-\infty,3)\cup(6,9)\cup(10,\infty) \\ t < 3,\ 6 < t < 9, \text{ and } t > 10 \end{matrix}\right\}$ because $x'(t)$ $\left\{\begin{matrix} \text{is negative} \\ < 0 \end{matrix}\right\}$.

(c) At what time(s), if any, does the particle change direction? Justify your answer.

The particle changes direction at $t = 3, 6, 9,$ and 10 because $x'(t)$ changes sign.

(d) Identify the interval(s) of t for which the particle's speed increasing. Justify your answer.

The particle's speed is increasing on $\left\{\begin{matrix} (3,5)\cup(6,7)\cup(9,9.5)\cup(10,\infty) \\ 3 < t < 5,\ 6 < t < 7,\ 9 < t < 9.5 \text{and } t > 10 \end{matrix}\right\}$ because $x'(t)$ and $x''(t)$ have the same sign.

(e) Identify the interval(s) of t for which particle's speed decreasing? Justify your answer.

The particle's speed is decreasing on $\left\{\begin{matrix} (-\infty,3)\cup(5,6)\cup(7,9)\cup(9.5,10) \\ t < 3,\ 5 < t < 6,\ 7 < t < 9, \text{ and } 9.5 < t < 10 \end{matrix}\right\}$ because $x'(t)$ and $x''(t)$ have opposite signs.