Calculus AB
Section I, Part A
Time-55 Minutes
Number of questions-28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form for the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the answer sheet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

The inverse of a trigonometric function *f* may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}(x) = \arcsin(x)$).

1. If $y = \sin^3(x)$, then $\frac{dy}{dx} =$

$$y = \sin^3(x)$$
$$= [\sin(x)]^3$$
$$\downarrow$$
$$y' = 3\sin^2(x) \cdot \cos(x)$$

(a) $\cos^3(x)$ (b) $3\cos^2(x)$ (c) $3\sin^2(x)$ (d) $-3\sin^2(x)\cos(x)$ (e) $3\sin^2(x)\cos(x)$

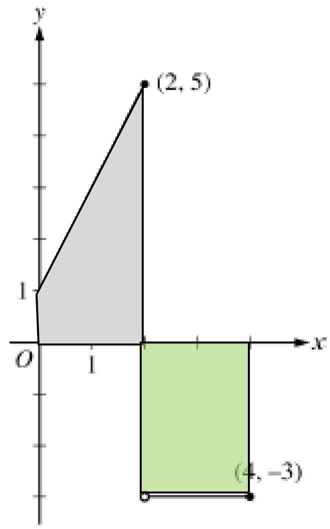
2. The position of a particle moving in the *xy*-plane is given by the parametric equations $x(t) = t^3 - 3t^2$ and $y(t) = 12t - 3t^2$. At which of the following points (x, y) is the particle at rest?

The particle is at rest when $x'(t) = 0$ and $y'(t) = 0$ simultaneously.

$$x(t) = t^3 - 3t^2 \qquad y(t) = 12t - 3t^2$$
$$\downarrow \qquad \downarrow$$
$$x'(t) = 3t^2 - 6t \qquad y'(t) = 12 - 6t$$
$$0 = 3t(t-2) \qquad 0 = 12 - 6t$$
$$\downarrow \qquad \downarrow$$
$$t = 0 \text{ and } t = 2 \qquad t = 2$$
$$x(2) = -4$$
$$y(2) = 12$$

(a) $(-4, 12)$ (b) $(-3, 6)$ (c) $(-2, 9)$ (d) $(0, 0)$ (e) $(3, 4)$

3. The graph of a function f is shown below for $0 \le x \le 4$. What is the value of $\int_0^4 f(x)\,dx$?



Graph of f

$$\int_0^4 f(x)\,dx = \frac{1}{2}(1+5)(2)-(2)(3)$$
$$= 0$$

(a) -1 (b) 0 (c) 2 (d) 6 (e) 12

4. Which of the following integrals gives the length of the curve $y = \ln(x)$ from $x = 1$ t o $x = 2$?

$$\text{length} = \int_1^2 \sqrt{1+\left[f'(x)\right]^2}\,dx$$
$$= \int_1^2 \sqrt{1+\left[\frac{1}{x}\right]^2}\,dx$$
$$= \int_1^2 \sqrt{1+\frac{1}{x^2}}\,dx$$

(a) $\int_1^2 \sqrt{1+\frac{1}{x^2}}\,dx$

(b) $\int_1^2 \left(1+\frac{1}{x^2}\right)dx$

(c) $\int_1^2 \sqrt{1+e^{2x}}\,dx$

(d) $\int_1^2 \sqrt{1+(\ln(x))^2}\,dx$

(e) $\int_1^2 \left(1+(\ln(x))\right)^2 dx$

5. The Maclaurin series for the function f is given by $f(x) = \sum_{n=0}^{\infty}\left(-\frac{x}{4}\right)^n$. What is the value of $f(3)$?

$$f(x) = \sum_{n=0}^{\infty}\left(-\frac{x}{4}\right)^n$$
$$f(3) = \sum_{n=0}^{\infty}\left(-\frac{3}{4}\right)^n \leftarrow \text{Geometric Series } r = -\frac{3}{4}$$
$$= \frac{1}{1-\left(-\frac{3}{4}\right)}$$
$$= \frac{1}{\left(\frac{7}{4}\right)}$$
$$= \frac{4}{7}$$

(a) -3 (b) $-\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{13}{16}$ (e) 4

6. Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x\left(x^2-3\right)^5 dx$ is equal to which of the following?

$$u = x^2 - 3 \qquad u(-1) = -2$$
$$du = 2x\,dx \qquad u(4) = 13$$
$$\int_{-1}^{4} x\left(x^2-3\right)^5 dx = \frac{1}{2}\int_{-1}^{4}\left(x^2-3\right)^5 2x\,dx$$
$$= \frac{1}{2}\int_{-2}^{13} u^5\,du$$

(a) $2\int_{-2}^{13} u^5\,du$

(b) $\int_{-2}^{13} u^5\,du$

(c) $\frac{1}{2}\int_{-2}^{13} u^5\,du$

(d) $\int_{-1}^{4} u^5\,du$

(e) $\frac{1}{2}\int_{-1}^{4} u^5\,du$

7. If $\arcsin(x) = \ln(y)$, then $\frac{dy}{dx} =$

$$\arcsin(x) = \ln(y)$$
$$\downarrow$$
$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{y}y'$$
$$y' = \frac{y}{\sqrt{1-x^2}}$$

(a) $\frac{y}{\sqrt{1-x^2}}$

(b) $\frac{xy}{\sqrt{1-x^2}}$

(c) $\frac{y}{1-x^2}$

(d) $e^{\arcsin(x)}$

(e) $\frac{e^{\arcsin(x)}}{1+x^2}$

8. A tank contains 50 liters of oil at time $t=4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are shown in the table below. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t=15$ hours?

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

$$\begin{aligned} V(15) &= V(0)+\int_0^{15} R(t)\,dt \\ &\approx 50+3(6.2)+5(5.9)+3(5.6) \\ &\approx 50+18.6+29.5+16.8 \\ &\approx 114.9 \end{aligned}$$

(a) 64.9 (b) 68.2 (c) 114.9 (d) 116.6 (e) 118.2

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{8^n}{n!}$ — $n!$ grows faster than 8^n

II. $\sum_{n=1}^{\infty}\frac{n!}{n^{100}}$ — $n!$ grows faster than n^{100}

III. $\sum_{n=1}^{\infty}\frac{n+1}{n(n+2)(n+3)}$ — Similar to $\sum\frac{1}{n^2}$

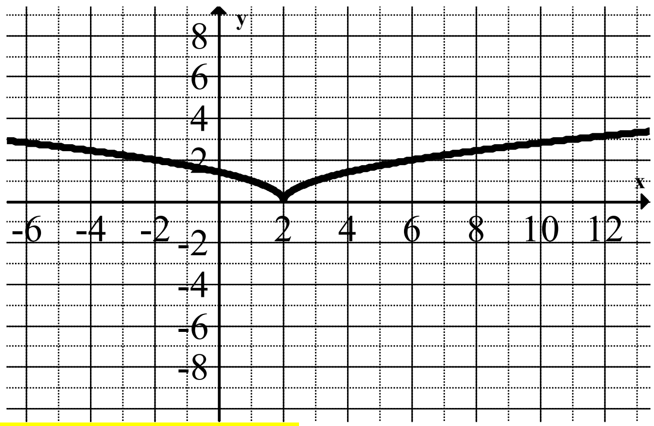
(a) I only (b) II only (c) III only (d) I and III only (e) I, II, and III

10. $\int_1^4 t^{-\frac{3}{2}}dt =$

$$\begin{aligned} \int_1^4 t^{-\frac{3}{2}}dt &= \left[-2t^{-\frac{1}{2}}\right]_1^4 \\ &= \left[-2(4)^{-\frac{1}{2}}\right]-\left[-2(1)^{-\frac{1}{2}}\right] \\ &= -1+2 \\ &= 1 \end{aligned}$$

(a) -1 (b) $-\frac{7}{8}$ (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$ (e) 1

11. Let *f* be the function defined by $f(x)=\sqrt{|x-2|}$ for all *x*. Which of the following statements are true?



(a) f is continuous but not differentiable at $x=2$

(b) f is differentiable at $x=2$

(c) f is not continuous at $x=2$

(d) $\lim_{x\to 2} f(x)\neq 0$

(e) $x=2$ is a vertical asymptote of the graph of f

12. The points $(-1,-1)$ and $(1,-5)$ are on the graph of a function $y=f(x)$ that satisfies the differential equation $\frac{dy}{dx}=x^2+y$. Which of the following must be true?

$$\frac{dy}{dx}=x^2+y$$

$$\frac{d^2y}{dx^2}=2x+y'$$

$$=2x+x^2+y$$

$$\left.\frac{dy}{dx}\right|_{(1,-5)}\neq 0$$

$$\left.\frac{dy}{dx}\right|_{(-1,-1)}=0$$

$$\left.\frac{d^2y}{dx^2}\right|_{(-1,-1)}=2(-1)+(-1)^2+(-1)<0$$

By the second derivative test, since $\left.\frac{dy}{dx}\right|_{(-1,-1)}=0$, and $\left.\frac{d^2y}{dx^2}\right|_{(-1,-1)}<0$, $(-1,-1)$ is a local maximum of f.

(a) $(1,-5)$ is a local maximum of f. $(1,-5)$ is not a critical point of *f*.

(b) $(1,-5)$ is a point of inflection of the graph of f. Cannot make claims about point of inflection without a sign chart for y''

(c) $(-1,-1)$ is a local maximum of f.

(d) $(-1,-1)$ is a local minimum of f.

(e) $(-1,-1)$ is a point of inflection of the graph of f. Cannot make claims about point of inflection without a sign chart for y''

13. What is the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{3^n}$?

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(x-4)^{2(n+1)}}{3^{n+1}}}{\frac{(x-4)^{2n}}{3^n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{3^n(x-4)^{2(n+1)}}{3^{n+1}(x-4)^{2n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(x-4)^2}{3}\right|$$

$$\downarrow$$

$$\left|\frac{(x-4)^2}{3}\right| < 1$$

$$\left|(x-4)^2\right| < 3$$

$$|x-4| < \sqrt{3}$$

(a) $2\sqrt{3}$ (b) 3 (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$ (e) 0

14. Let k be a positive constant. Which of the following is a logistic differential equation?

(a) $\frac{dy}{dt} = kt$

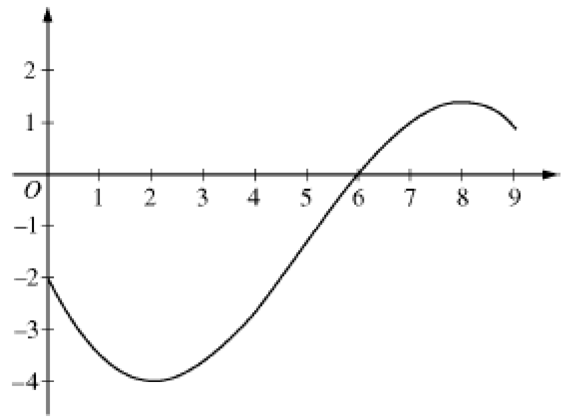
(b) $\frac{dy}{dt} = ky$

(c) $\frac{dy}{dt} = kt(1-t)$

(d) $\frac{dy}{dt} = ky(1-t)$

(e) $\frac{dy}{dt} = ky(1-y)$

15. The graph of a differentiable function f is shown below. If $h(x)=\int_0^x f(t)\,dt$, which of the following is true?



Graph of f

$$h(x)=\int_0^x f(t)\,dt \qquad \rightarrow h(6)<0$$
$$h'(x)=f(x) \qquad \rightarrow h'(6)=0$$
$$h''(x)=f'(x) \qquad \rightarrow h''(6)>0$$

(a) $h(6)<h'(6)<h''(6)$
(b) $h(6)<h''(6)<h'(6)$
(c) $h'(6)<h(6)<h''(6)$
(d) $h''(6)<h(6)<h'(6)$
(e) $h''(6)<h'(6)<h(6)$

16. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - y$ with initial condition $f(1) = 3$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal size starting at $x = 1$?

$$
\begin{aligned}
f(1) &= 3 \\
f(1.5) &= f(1) + \left(\left.\frac{dy}{dx}\right|_{(1,3)} \right)(0.5) \\
&= 3 + (-2)(0.5) \\
&= 2 \\
f(2) &= f(1.5) + \left(\left.\frac{dy}{dx}\right|_{(1.5,2)} \right)(0.5) \\
&= 2 + (-0.25) \\
&= \frac{7}{4}
\end{aligned}
$$

(a) $-\frac{5}{4}$ (b) 1 (c) $\frac{7}{4}$ (d) 4 (e) $\frac{21}{4}$

17. For $x > 0$, the power series $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + (-1)^n \frac{x^{2n}}{(2n+1)!} + \cdots$ converges to which of the following?

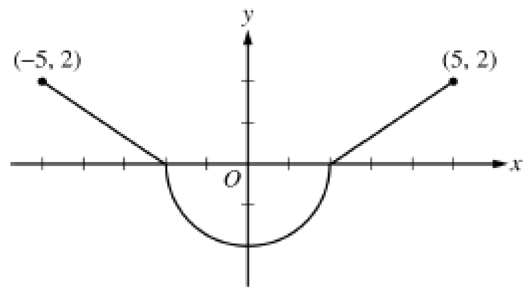
$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\downarrow$$

$$
\begin{aligned}
\frac{\sin(x)}{x} &= \frac{1}{x} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \\
&= \frac{1}{x}\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \right) \\
&= 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots
\end{aligned}
$$

(a) $\cos(x)$ (b) $\sin(x)$ (c) $\frac{\sin(x)}{x}$ (d) $e^x - e^{x^2}$ (e) $1 + e^x - e^{x^2}$

18. The graph of f', the derivative of a fucntion f, consists of two lines segments and a semicircle, as shown in the figure below. If $f(2)=1$, then $f(-5)=$



Graph of f'

$$f(-5)=f(2)+\int_{2}^{-5} f(x)\,dx$$

$$=f(2)-\int_{-5}^{2} f(x)\,dx$$

$$=f(2)-\left[\int_{-5}^{-2} f(x)\,dx+\int_{-2}^{2} f(x)\,dx\right]$$

$$=1-\left[\frac{1}{2}(2)(3)-\frac{1}{2}\pi\cdot 2^2\right]$$

$$=-2+2\pi$$

(a) $2\pi-2$
(b) $2\pi-3$
(c) $2\pi-5$
(d) $6-2\pi$
(e) $4-2\pi$

19. The function f is defined by $f(x)=\dfrac{x}{x+2}$. What points (x,y) on the graph of f have the property that the line tangent to f at (x,y) has slope $\dfrac{1}{2}$?

$$f(x)=\frac{x}{x+2}$$
$$\downarrow$$
$$f'(x)=\frac{(1)(x+2)-(x)(1)}{(x+2)^2}$$
$$=\frac{2}{(x+2)^2}$$
$$\downarrow$$
$$\frac{1}{2}=\frac{2}{(x+2)^2}$$
$$(x+2)^2=4$$
$$|x+2|=2$$
$$\downarrow$$
$$x=0 \text{ or } x=-4$$

(a) $(0,0)$ only

(b) $\left(\dfrac{1}{2},\dfrac{1}{5}\right)$ only

(c) $(0,0)$ and $(-4,2)$

(d) $(0,0)$ and $\left(4,\dfrac{2}{3}\right)$

(e) There are no such points.

20. $\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx =$

$$\int \frac{5x+8}{x^2+3x+2}\,dx = \int \frac{5x+8}{(x+1)(x+2)}\,dx$$

$$\downarrow$$

$$\frac{5x+8}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$\frac{5x+8}{(x+1)(x+2)} = \frac{A(x+2)}{(x+1)(x+2)} + \frac{B(x+1)}{(x+2)(x+1)}$$

$$5x+8 = Ax+2A+Bx+B$$

$$5x+8 = (A+B)x+(2A+B)$$

$$A+B=5$$
$$2A+B=8$$
$$\downarrow$$
$$A=3$$
$$B=2$$

$$\int_0^1 \frac{5x+8}{(x+1)(x+2)}\,dx = \int_0^1 \frac{3}{x+1}+\frac{2}{x+2}\,dx$$
$$= \left[3\ln|x+1|+2\ln|x+2|\right]_0^1$$
$$= \left[3\ln|1+1|+2\ln|1+2|\right]-\left[3\ln|0+1|+2\ln|0+2|\right]$$
$$= 3\ln|2|+2\ln|3|-3\ln|1|-2\ln|2|$$
$$= \ln|2|+2\ln|3|$$
$$= \ln(2)+\ln(3^2)$$
$$= \ln(18)$$

(a) $\ln(8)$ (b) $\ln\left(\frac{27}{2}\right)$ (c) $\ln(18)$ (d) $\ln(288)$ (e) Divergent

21. The line $y=5$ is a horizontal asymptote to the graph of which of the following functions?

Horizontal Asymptote $\leftrightarrow \lim_{x\to\pm\infty} f(x)$

(a) $y=\frac{\sin(5x)}{x}$ $\lim_{x\to\pm\infty}\frac{\sin(5x)}{x}=0$

(b) $y=5x$ $\lim_{x\to\pm\infty} 5x=\pm\infty$

(c) $y=\frac{1}{x-5}$ $\lim_{x\to\pm\infty}\frac{1}{x-5}=0$

(d) $y=\frac{5x}{1-x}$ $\lim_{x\to\pm\infty}\frac{5x}{1-x}=\pm\infty$

(e) $y=\frac{20x^2-x}{1+4x^2}$

22. The power series $\sum_{n=0}^{\infty} a_n (x-3)^n$ converges at $x=5$. Which of the following must be true?

The series is centered at $x=3$. Since the series converges at $x=5$, the series will definitely converge for $1<x<5$. The series may not necessarily converge at $x=1$ because it may be on the "edge" of the radius of convergence, and need to be tested.

(a) The series diverges at $x=0$
(b) The series diverges at $x=1$
(c) The series converges at $x=1$
(d) The series converges at $x=2$
(e) The series converges at $x=6$

23. If $P(t)$ is the size of a population at time *t*, which of the following differential equations describes linear growth in the size of the population?

If the population is growing linearly, then the rate of change of the population is constant.

(a) $\frac{dP}{dt}=200$
(b) $\frac{dP}{dt}=200t$
(c) $\frac{dP}{dt}=100t^2$
(d) $\frac{dP}{dt}=200P$
(e) $\frac{dP}{dt}=100P^2$

24. Let *f* be a differentiable function such that $\int f(x)\sin(x)\,dx = -f(x)\cos(x)+\int 4x^3\cos(x)\,dx$. Which of the following could be $f(x)$?

$$\int uv' = uv - \int u'v$$

$$\int \underbrace{f(x)}_{u}\underbrace{\sin(x)}_{v'}dx = -f(x)\cos(x)+\int 4x^3\cos(x)\,dx$$

$$= f(x)\underbrace{\left[-\cos(x)\right]}_{v} - \int \underbrace{\left[4x^3\right]}_{u'}\underbrace{\left[-\cos(x)\right]}_{v}dx$$

$$\downarrow$$

$$u = f(x)$$
$$u' = 4x^3$$
$$u = x^4 + C = f(x)$$

(a) $\cos(x)$ (b) $\sin(x)$ (c) $4x^3$ (d) $-x^4$ (e) x^4

25. $\int_1^\infty xe^{-x^2}\,dx$ is

$$\begin{aligned}\int_1^\infty xe^{-x^2}\,dx &= \lim_{b\to\infty}\int_1^b xe^{-x^2}\,dx\\ &= \lim_{b\to\infty}\left[-\frac{1}{2}e^{-x^2}\right]_1^b\\ &= \lim_{b\to\infty}\left(\left[-\frac{1}{2}e^{-b^2}\right]-\left[-\frac{1}{2}e^{-1^2}\right]\right)\\ &= [0]+\frac{1}{2e}\end{aligned}$$

(a) $-\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) $\frac{1}{e}$ (d) $\frac{2}{e}$ (e) Divergent

26. What is the slope of the line tangent to the polar curve $r = 1+2\sin(\theta)$ at $\theta = 0$?

$$x = r(\theta)\cos(\theta)$$
$$y = r(\theta)\sin(\theta)$$
$$\downarrow$$
$$\begin{aligned}\frac{dy}{dx} &= \frac{[r(\theta)\sin(\theta)]'}{[r(\theta)\cos(\theta)]'}\\ &= \frac{r'(\theta)\sin(\theta)+r(\theta)\cos(\theta)}{r'(\theta)\cos(\theta)-r(\theta)\sin(\theta)}\end{aligned}$$
$$\downarrow$$
$$= \frac{[2\cos(\theta)]\sin(\theta)+[1+2\sin(\theta)]\cos(\theta)}{[2\cos(\theta)]\cos(\theta)-[1+2\sin(\theta)]\sin(\theta)}$$
$$\downarrow$$

$$r = 1+2\sin(\theta)$$
$$\downarrow$$
$$r'(\theta) = 2\cos(\theta)$$

$$\begin{aligned}\left.\frac{dy}{dx}\right|_{\theta=0} &= \frac{[2\cos(0)]\sin(0)+[1+2\sin(0)]\cos(0)}{[2\cos(0)]\cos(0)-[1+2\sin(0)]\sin(0)}\\ &= \frac{[2](0)+[1+2(0)](1)}{[2](1)-[1+2(0)](0)}\\ &= \frac{1}{2}\end{aligned}$$

(a) 2 (b) $\frac{1}{2}$ (c) 0 (d) $-\frac{1}{2}$ (e) -2

27. For what values of p will both series $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ and $\sum_{n=1}^{\infty} \left(\frac{p}{2}\right)^n$ converge?

$\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ Will converge if $2p > 1$ $\downarrow$ $p > \frac{1}{2}$	$\sum_{n=1}^{\infty} \left(\frac{p}{2}\right)^n$ Will converge if $\left\lvert\frac{p}{2}\right\rvert < 1$ $-1 < \frac{p}{2} < 1$ $-2 < p < 2$

(a) $-2 < p < 2$ only

(b) $-\frac{1}{2} < p < \frac{1}{2}$ only

(c) $\frac{1}{2} < p < 2$ only

(d) $p < \frac{1}{2}$ and $p > 2$

(e) There are no such values of p.

28. Let g be a continually differentiable function with $g(1) = 6$ and $g'(1) = 3$. What is $\lim_{x \to 1} \frac{\int_1^x g(t)\,dt}{g(x) - 6}$?

$$\lim_{x \to 1} \frac{\int_1^x g(t)\,dt}{g(x) - 6} = \frac{0}{0}$$

$$\downarrow$$

$$\begin{aligned} \lim_{x \to 1} \frac{\int_1^x g(t)\,dt}{g(x) - 6} &= \lim_{x \to 1} \frac{\left[\int_1^x g(t)\,dt\right]'}{\left[g(x) - 6\right]'} \\ &= \lim_{x \to 1} \frac{g(x)}{g'(x)} \\ &= \frac{6}{3} \\ &= 2 \end{aligned}$$

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2 (e) DNE

Calculus AB
Section 1, Part B
Time – 50 minutes
Number of questions – 17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

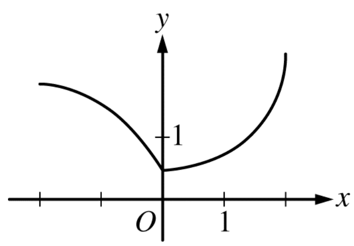
Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the student answer sheet. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1}, or with the prefix "arc" (e.g., $\sin^{-1}(x) = \arcsin(x)$).

76. The function f, whose graph is shown below, is defined on the interval $-2 \le x \le 2$. Which of the following statements about f is false?



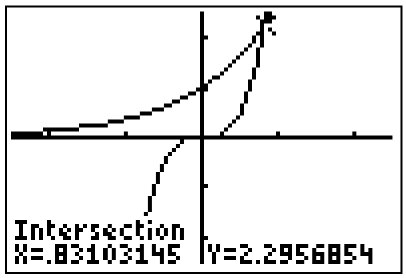
Graph of f

(a) f is continuous at $x = 0$.
(b) f is differentiable at $x = 0$.
(c) f has a critical point at $x = 0$.
(d) f has an absolute minimum at $x = 0$.
(e) The concavity of the graph of f changes at $x = 0$.

77. Let *f* and *g* be the functions given by $f(x)=e^x$ and $g(x)=x^4$. On which what intervals is the rate of change of $f(x)$ greater than the rate of change of $g(x)$?

To find out when $f'(x)>g'(x)$, graph both functions and look for when the graph of $f'(x)$ lies above the graph of $g'(x)$.

$$f(x)=e^x \qquad g(x)=x^4$$
$$\downarrow \qquad\qquad \downarrow$$
$$f'(x)=e^x \qquad g'(x)=4x^3$$



Sinec the answer must include the interval $(-\infty,-0.831)$, the answer must be (b)

(a) $(0.831, 7.384)$ only

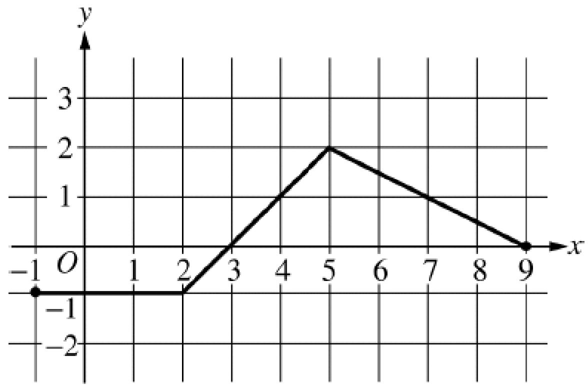
(b) $(-\infty, 0.831)$ and $(7.384, \infty)$

(c) $(-\infty, -0.816)$ and $(1.430, 8.613)$

(d) $(-0.816, 1.430)$ and $(8.613, \infty)$

(e) $(-\infty, \infty)$

78. The graph of the piecewise linear function f is shown below. What is the value of $\int_{-1}^{9} 3f(x)+2dx$?



Graph of f

$$\begin{aligned}\int_{-1}^{9} 3f(x)+2dx &= \int_{-1}^{9} 3f(x)dx + \int_{-1}^{9} 2dx \\ &= 3\int_{-1}^{9} f(x)dx + \int_{-1}^{9} 2dx \\ &= 3\left(-3.5+\frac{1}{2}(6)(2)\right)+20 \\ &= 3(2.5)+20 \\ &= 27.5\end{aligned}$$

(a) 7.5 (b) 9.5 (c) 27.5 (d) 47 (e) 48.5

79. Let *f* be a function having derivatives of all orders for $x > 0$ such that $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for *f* about $x = 3$?

$$T(x) = f(3) + f'(3)(x-3) + \frac{f''(3)(x-3)^2}{2!} + \frac{f'''(3)(x-3)^3}{3!} + \cdots$$

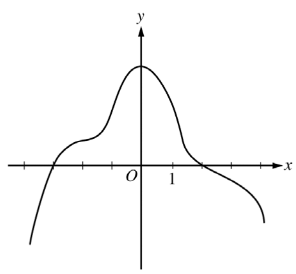
$$\downarrow$$

$$= 2 + (-1)(x-3) + \frac{6(x-3)^2}{2!} + \frac{12(x-3)^3}{3!}$$

$$= 2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$$

(a) $2 - x + 6x^2 + 12x^3$ Not centered at $x = 3$
(b) $2 - x + 3x^2 + 2x^3$ Not centered at $x = 3$
(c) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$
(d) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$
(e) $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

80. The graph of f', the derivative of the function *f*, is shown below. Which of the following statements must be true?



Graph of f'

I. f has a relative minimum at $x = -3$
~~II. The graph of *f* has a point of inflection at $x = -2$.~~
III. The graph of *f* is concave down for $0 < x < 4$

(a) I only (b) II only (c) III only (d) I and II only (e) I and III only

81. Let f be a function that is twice differentiable on $-2 < x < 2$ and satisfies the conditions in the table below. If $f(x) = f(-x)$, what are the x-coordinates of the points of inflection of the graph of f on $-2 < x < 2$?

	$0 < x < 1$	$1 < x < 2$
$f(x)$	Positive	Negative
$f'(x)$	Negative	Negative
$f''(x)$	Negative	Positive

$f(x) = -f(x)$ means that

	$-2 < x < -1$	$-1 < x < 0$	$0 < x < 1$	$1 < x < 2$
$f(x)$	Negative	Positive	Positive	Negative
$f'(x)$	Negative	Negative	Negative	Negative
$f''(x)$	Positive	Negative	Negative	Positive

$f(x)$ will have a point of inflection when $f''(x)$ changes sign. According to the table, this will occur when $x = -1$ and $x = 1$

(a) $x = 0$ only
(b) $x = 1$ only
(c) $x = 0$ and $x = 1$
(d) $x = -1$ and $x = 1$
(e) There are no points of inflection on $-2 < x < 2$.

82. What is the average value of $y = \sqrt{\cos(x)}$ on the interval $0 \le x \le \frac{\pi}{2}$?

$$\text{Average Value} = \frac{1}{\frac{\pi}{2} - 0} \int_0^{\frac{\pi}{2}} \sqrt{\cos(x)}\,dx$$
$$\approx 0.762(3)$$

(a) -0.637 (b) 0.500 (c) 0.763 (d) 1.198 (e) 1.882

83. If the function f is continuous at $x = 3$, which of the following must be true?

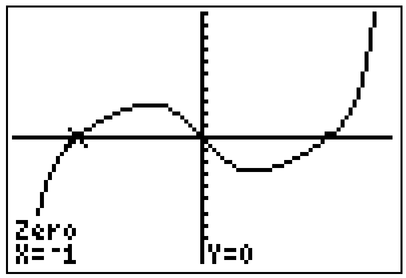
(a) $f(3) < \lim_{x \to 3} f(x)$
(b) $\lim_{x \to 3^-} f(x) \ne \lim_{x \to 3^+} f(x)$
(c) $f(3) = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$
(d) The derivative of f at $x = 3$ exists.
(e) The derivative of f is positive for $x < 3$ and negative for $x > 3$.

84. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{(x^4 - 2x^2 + 1)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

$$f'(x) = e^{(x^4 - 2x^2 + 1)} - 2$$

$$\downarrow$$

$$f''(x) = e^{(x^4 - 2x^2 + 1)} \cdot (4x^3 - 4x)$$



(a) $(-0.418, 0.418)$ only

(b) $(-1, 1)$

(c) $(-1.354, -0.409)$ and $(0.409, 1.354)$

(d) $(-1.5, -1)$ and $(0, 1)$

(e) $(-1.5, -1.354)$, and $(-0.409, 0)$, and $(1.354, 1.5)$

85. The fuel consumption for a car, in miles per gallon (mpg), is modeled by $F(s)=6e^{\left(\frac{s}{20}-\frac{s^2}{2400}\right)}$, where *s* is the speed of the car, in miles per hour. If the car is travelling 50 miles per hour and its speed is changing at the rate of 20 miles/hour^2, which is the rate at which its fuel consumption is changing?

$$F(s)=6e^{\left(\frac{s}{20}-\frac{s^2}{2400}\right)}$$

$$F(s)=6e^{\left(\frac{s(t)}{20}-\frac{[s(t)]^2}{2400}\right)}$$

$$\downarrow$$

$$F'(s)=6e^{\left(\frac{s(t)}{20}-\frac{[s(t)]^2}{2400}\right)}\cdot\left(\frac{1}{20}-\frac{2s}{2400}\right)\cdot s'(t)$$

$$F'(50)=6e^{\left(\frac{50}{20}-\frac{[50]^2}{2400}\right)}\cdot\left(\frac{1}{20}-\frac{2(50)}{2400}\right)\cdot(20)$$

$$\approx 4.299$$

(a) 0.215 mpg per hour
(b) 4.299. mpg per hour
(c) 19.793 mpg per hour
(d) 25.793 mpg per hour
(e) 515.855 mpg per hour

86. If $f'(x)>0$ for all real numbers *x* and $\int_4^7 f(t)\,dt=0$, which of the following could be a table of values for the function *f*?

(a)

x	$f(x)$
4	−4
5	−3
7	0

$\int_4^7 f(t)\,dt<0$

(b)

x	$f(x)$
4	−4
5	−2
7	5

(c)

x	$f(x)$
4	−4
5	6
7	3

(d)

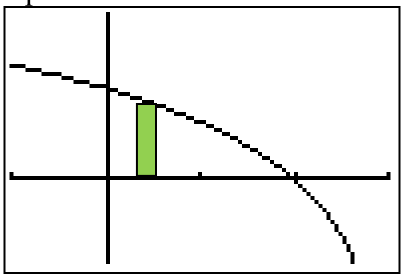
x	$f(x)$
4	0
5	0
7	0

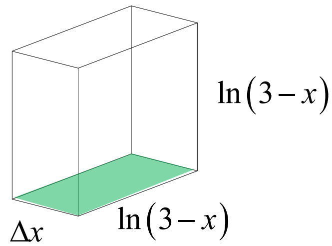
(e)

x	$f(x)$
4	0
5	4
7	6

$\int_4^7 f(t)\,dt>0$

87. Let R be the region in the first quadrant bounded above by the graph of $y = \ln(3-x)$ for $0 \le x \le 2$. R is the base of a solid for which each cross section perpendicular to the x-axis is a square. What is the volume of the solid?





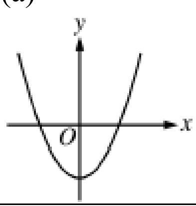
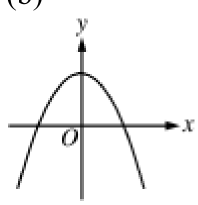
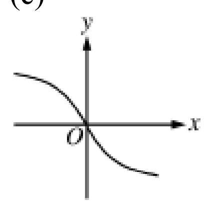
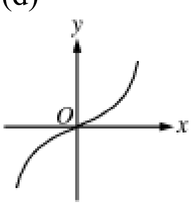
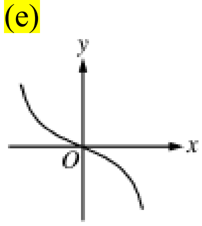
$$\int_0^2 \left(\ln(3-x)\right)^2 dx \approx 1.029$$

(a) 0.442 (b) 1.029 (c) 1.296 (d) 3.233 (e) 4.071

88. The dertivative of a function f is increasing for $x < 0$ and decreasing for $x > 0$. Which of the following could be the graph of f?

$f'(x)$ is increasing $\rightarrow f''(x) > 0 \rightarrow f(x)$ is concave up for $x < 0$

$f'(x)$ is decreasing $\rightarrow f''(x) < 0 \rightarrow f(x)$ is concave down for $x > 0$

(a) 	(b) 	(c) 
(d) 	(e) 	

89. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \dfrac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

$$\begin{aligned} v(3) &= v(0) + \int_0^3 a(t)\,dt \\ &= 5 + \int_0^3 \frac{t+3}{\sqrt{t^3+1}}\,dt \\ &\approx 11.710 \end{aligned}$$

(a) 0.713 (b) 1.134 (c) 6.134 (d) 6.710 (e) 11.710

90. If the series $\sum_{n=1}^{\infty} a_n$ converges and $a_n > 0$ for all *n*, which of the following must be true?

$\sum_{n=1}^{\infty} \dfrac{a_n}{n} \leq \sum_{n=1}^{\infty} a_n$ for all n. By the Direct Comparison Test $\sum_{n=1}^{\infty} \dfrac{a_n}{n}$ converges.

(a) $\lim_{n\to\infty} \left| \dfrac{a_{n+1}}{a_n} \right| = 0$

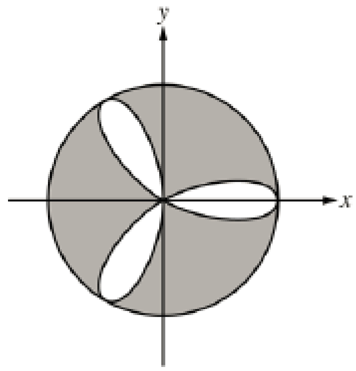
(b) $|a_n| < 1$ for all *n*.

(c) $\sum_{n=1}^{\infty} a_n = 0$

(d) $\sum_{n=1}^{\infty} n a_n$ diverges

(e) $\sum_{n=1}^{\infty} \dfrac{a_n}{n}$ converges

91. The figure below shows the graph of the polar curves $r = 2\cos(3\theta)$ and $r = 2$. What is the sum of the areas of the shaded regions?



$$2\cos(3\theta) = 0$$
$$\cos(3\theta) = 0$$
$$\downarrow$$
$$3\theta = \frac{\pi}{2} + 2\pi k \qquad 3\theta = -\frac{\pi}{2} + 2\pi k$$
$$\theta = \frac{\pi}{6} + \frac{2}{3}\pi k \qquad \theta = -\frac{\pi}{6} + \frac{2}{3}\pi k$$
$$A = \pi \cdot 2^2 - \frac{3}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \left[r_1(\theta)\right]^2 d\theta$$
$$= 4\pi - \frac{3}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \left[2\cos(3\theta)\right]^2 d\theta$$
$$\approx 9.424(5)$$

(a) 0.858 (b) 3.142 (c) 8.566 (d) 9.425 (e) 15.708

92. The function h is differentiable, and for all values of x, $h(x)=h(2-x)$. Which of the following statements must be true?

$$h(x)=h(2-x)$$
$$\downarrow$$
$$h'(x)=h'(2-x)\cdot(-1)$$
$$h'(1)=h'(2-1)\cdot(-1)$$
$$h'(1)=-h'(2-1)$$
$$h'(1)+h'(2-1)=0$$
$$h'(1)+h'(1)=0$$
$$2h'(1)=0$$
$$h'(1)=0$$

I. $\int_0^2 h(x)\,dx>0$

II. $h'(1)=0$

III. $h'(0)=h'(2)=1$

(a) I only
(b) II only
(c) III only
(d) II and III only
(e) I, II, and III

Calculus BC
Section II, Part A
Time – 30 Minutes
Number of Problems – 2
A graphing calculator is required for these problems

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

a. Use the data in the table to estimate $W'(12)$. Show the work that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

$$\begin{aligned} W'(12) &\approx \frac{W(15)-W(9)}{15-9} \\ &\approx \frac{67.9-61.8}{15-9} \\ &\approx \frac{61}{60} \\ &\approx 1.0166... \end{aligned}$$

The rate of change of the temperature of the water at time $t = 12$ is increasing by approximately 1.0166 °F/min.

b. Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

$$\begin{aligned} \int_0^{20} W'(t)\,dt &= \left[W(t)\right]_0^{20} \\ &= W(20) - W(0) \\ &= 16°\text{F} \end{aligned}$$

$\int_0^{20} W'(t)\,dt$ represents the net change in temperature from time $t = 0$ to time $t = 20$.

c. For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$$\begin{aligned}\frac{1}{20}\int_0^{20} W(t)\,dt &\approx \frac{1}{20}\left[(55)(4-0)+(57.1)(9-4)+(61.8)(15-9)+(67.9)(20-15)\right]\\ &\approx \frac{1}{20}(1215.8)\\ &\approx 60.79\end{aligned}$$

The average temperature of the water from time $t=0$ to $t=20$ is $60.79°\text{F}$.
The approximation is an underestimate because $W(t)$ is strictly increasing on the interval $[0,20]$

d. For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

$$\begin{aligned}W(25) &= W(20) + \int_{20}^{25} W'(t)dt\\ &= 71 + \int_{20}^{25} 0.4\sqrt{t}\cos(0.06t)\,dt\\ &\approx 73.043\end{aligned}$$

The temperature of the water at time $t = 25$ is 73.043°F.

2. For $t \ge 0$, a particle is moving along a curve so that its position at time *t* is $\left(x(t), y(t)\right)$. At time $t = 2$ the particle is at position $(1,5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2(t)$.

a. Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

$\left.\frac{dx}{dt}\right|_{t=2} = \frac{\sqrt{2+2}}{e^2} > 0$. Therefore the particle is moving to the right at time $t = 2$ because velocity is positive.

$$\left.\frac{dy}{dx}\right|_{t=2} = \frac{\left.\frac{dy}{dt}\right|_{t=2}}{\left.\frac{dx}{dt}\right|_{t=2}} = \frac{\sin^2(2)}{\left(\frac{\sqrt{2+2}}{e^2}\right)} = 3.0547...$$

b. Find the *x*-coordinate of the particle's position at time $t = 4$?

$$\begin{aligned} x(4) &= x(2) + \int_2^4 x'(t)\,dt \\ &= 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt \\ &\approx 1.2529... \end{aligned}$$

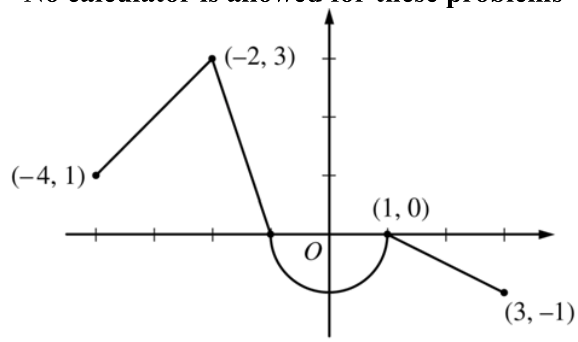
c. Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

$$\begin{aligned} \left|\bar{v}(4)\right| &= \sqrt{\left(\left.\frac{dx}{dt}\right|_{t=4}\right)^2 + \left(\left.\frac{dy}{dt}\right|_{t=4}\right)^2} \\ &= 0.5745... \end{aligned} \qquad \begin{aligned} \bar{a}(4) &= \left\langle \left.\frac{d^2x}{dt^2}\right|_{t=4}, \left.\frac{d^2y}{dt^2}\right|_{t=4} \right\rangle \\ &= \left\langle -0.0411..., 0.9893... \right\rangle \end{aligned}$$

d. Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

$$\begin{aligned} D &= \int_2^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt \\ &\approx 0.6509... \end{aligned}$$

Calculus BC
Section II, Part B
Time – 60 Minutes
Number of Problems – 4
No calculator is allowed for these problems



Graph of f

$$g'(x) = f(x)$$
$$g''(x) = f'(x)$$

3. Let f be the continuous function defined on $[-4,3]$ whose graph consists of three line segments and a semicircle at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

a. Find the value of $g(2)$ and $g(-2)$

$$g(2) = \int_1^2 f(t)\,dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$$

$$g(-2) = \int_1^{-2} f(t)\,dt = \frac{1}{2}\pi(1)^2 - \frac{1}{2}(1)(3)$$

b. For each of $g'(3)$ and $g''(-3)$, find the value or state that it does not exist.

$$g'(-3) = f(-3) = 2$$
$$g''(-3) = f'(-3) = 1$$

c. Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

$g(x)$ has horizontal tangent lines when $g'(x) = f(x) = 0$. This occurs when $x = \pm 1$.

$g(-1)$ is a relative maximum since $g'(x) = f(x)$ changes from positive to negative at $x = -1$.

$g(1)$ is neither a min or max since $g'(x) = f(x)$ does not change sign around $x = 1$.

d. For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

$g(x)$ has an inflection point when $g''(x) = f'(x)$ changes sign. This occurs when $x = -2, 0, 1$

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

4. The function *f* is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of *f*, are given for selected values of *x* in the table above.

a. Write an equation for the line tangent to the graph of *f* at $x = 1$. Use this line to approximate $f(1.4)$

Tangent $y - 15 = 8(x-1)$. $f(1.4) \approx 8(1.4-1)+15$

b. Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

$$\int_1^{1.4} f'(x)\,dx \approx (10)(1.2-1)+(13)(1.4-1.2) = 4.6$$

$$\begin{aligned} f(1.4) &\approx f(1) + \int_1^{1.4} f'(x)\,dx \\ &\approx 15 + 4.6 \\ &\approx 19.6 \end{aligned}$$

c. Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

$$f(1.2) = f(1) + f'(1)(0.2) = 15 + 8(0.2) = 16.6$$
$$f(1.4) = f(1.2) + f'(1.2)(0.2) = 16.6 + 12(0.2) = 19$$

d. Write the second-degree Taylor polynomial for *f* about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

$$\begin{aligned} P_2(x) &= f(1) + f'(1)(x-1) + \frac{f''(1)(x-1)^2}{2!} \\ &= 15 + 8(x-1) + \frac{20(x-1)^2}{2!} \end{aligned}$$
$$P_2(1.4) \approx 15 + 8(1.4-1) + \frac{20(1.4-1)^2}{2!}$$

5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, it is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time *t* days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B)$$

Let $y = B(t)$ be the solution to the differential equation above with the initial condition $B(0) = 20$

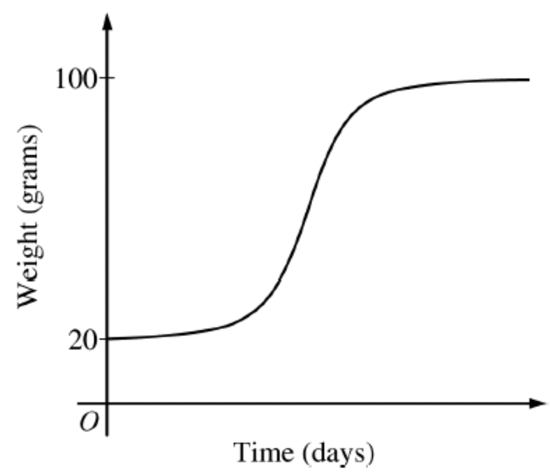
a. Is the bird gaining weight faster when it weighs 40 grams, or when it weighs 70 grams? Explain your reasoning.

$$\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(100 - 40)$$

$$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(100 - 70)$$

The bird is gaining weight faster when it weighs 40 grams because $\left.\frac{dB}{dt}\right|_{B=40} > \left.\frac{dB}{dt}\right|_{B=70}$

b. Find $\frac{d^2B}{dt^2}$ in terms of *B*. Use $\frac{d^2B}{dt^2}$ to explain why the graph of *b* cannot be the following graph:



$$\frac{dB}{dt} = \frac{1}{5}(100 - B) = 20 - \frac{1}{5}B$$

$$\frac{d^2B}{dt^2} = -\frac{1}{5}B' = -\frac{1}{5}\left[\frac{1}{5}(100 - B)\right]$$

The above graph cannot be a solution to the differential equation because

$\frac{d^2B}{dt^2} < 0$ for $B < 100$, which

c. Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

$$\frac{dB}{dt} = \frac{1}{5}(100 - B)$$

$$\frac{1}{(100 - B)} dB = \frac{1}{5} dt$$

$$\int \frac{1}{(100 - B)} dB = \int \frac{1}{5} dt$$

$$-\ln|100 - B| = \frac{1}{5}t + C \text{ where } C \text{ is a constant}$$

$$\ln|100 - B| = -\frac{1}{5}t + C$$

$$e^{\ln|100-B|} = e^{-\frac{1}{5}t+C}$$

$$|100 - B| = Ae^{-\frac{1}{5}t} \text{ where } A = e^C$$

$$100 - B = Ae^{-\frac{1}{5}t}$$

$$B = 100 - Ae^{-\frac{1}{5}t}$$

Given that $B(0) = 20$

$$B = 100 - Ae^{-\frac{1}{5}t}$$

$$20 = 100 - Ae^{-\frac{1}{5}(0)}$$

$$-80 = -A$$

$$A = 80$$

Particular solution is $B = 100 - 80e^{-\frac{1}{5}t}$

6. The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+3}=\frac{x}{3}-\frac{x^3}{5}+\frac{x^7}{7}-\cdots$$

a. Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

$$\lim_{n\to\infty}\left|\frac{(-1)^{(n+1)}\dfrac{x^{2(n+1)+1}}{2(n+1)+3}}{(-1)^n\dfrac{x^{2n+1}}{2n+3}}\right|=\lim_{n\to\infty}\left|\frac{\dfrac{x^{2(n+1)+1}}{2(n+1)+3}}{\dfrac{x^{2n+1}}{2n+3}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{x^{2(n+1)+1}}{2(n+1)+3}\cdot\frac{2n+3}{x^{2n+1}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{2n+3}{2n+5}\cdot\frac{x^{2n+3}}{x^{2n+1}}\right|$$

$$=\lim_{n\to\infty}\left|x^2\right|<1$$

$$\downarrow$$

$$|x|<1$$

When $x=1$

$$\sum_{n=0}^{\infty}(-1)^n\frac{1^{2n+1}}{2n+3}=\sum_{n=0}^{\infty}(-1)^n\frac{1}{2n+3}$$ converges by the Alternating Series Test

When $x=-1$

$$\sum_{n=0}^{\infty}(-1)^n\frac{(-1)^{2n+1}}{2n+3}=\sum_{n=0}^{\infty}\frac{(-1)^{3n+1}}{2n+3}$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^{3n}(-1)}{2n+3}$$

$$=\sum_{n=0}^{\infty}\frac{\left((-1)^3\right)^n(-1)}{2n+3}$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n(-1)}{2n+3}$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{2n+3}$$

Which converges by the Alternating Series Test.

Therefore the interval of convergence is $-1\le x\le 1$

b. The Maclaurin series for g evaluated at $x=\frac{1}{2}$, is an alternating series whose terms decrease in value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ be less than $\frac{1}{200}$.

$$g\left(\frac{1}{2}\right) \approx \frac{\left(\frac{1}{2}\right)}{3} - \frac{\left(\frac{1}{2}\right)^3}{5} \Bigg] + \frac{\left(\frac{1}{2}\right)^5}{7} - \cdots$$

By the Remainder Theorem for Alternating Series, the error is bounded by $\left|\frac{\left(\frac{1}{2}\right)^5}{7}\right| = \frac{1}{2^5 \cdot 7} < \frac{1}{200}$

c. Write the first three nonzero terms and the general term for the Maclaurin series for $g'(x)$.

$$g(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+3}$$

$$\downarrow$$

$$g'(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(2n+1)x^{2n}}{2n+3} = \frac{1}{3} - \frac{3x^2}{5} + \frac{5x^4}{7} - \cdots + (-1)^n \frac{(2n+1)x^{2n}}{2n+3}$$