Unit Summary for Applications of the Derivative

1. Explain what a critical value of $f(x)$ is.

2. Explain what a vertical tangent and horizontal tangent are, and why they are of interest in Calculus.

3. The derivative of a function $f(x)$, denoted $f'(x)$, can tell you about the visual qualities of the graph of $f(x)$. What visual qualities of $f(x)$ can be determined from solely from **the graph** of $f'(x)$? List all possible qualities of the graph of $f(x)$ that can be determined from **the graph** of $f'(x)$.

4.

(a) Explain what a point of inflection on the graph of $f(x)$ is **visually.**

(b) The location of a point of inflection of the graph of $f(x)$ can be identified by using the graph of the first derivative or the graph of the second derivative. Explain how to identify the location of a point of inflection using the graph of $f'(x)$ and the graph of $f''(x)$.

5. The following statement is not true: "If $f'(x)$ does not exist at $x=c$, then the graph of $f(x)$ has a sharp corner at $x=c$." Provide two counterexamples as to why this is not necessarily true.

6. To find a relative minimum or relative maximum, the following methods can be used:

- The Extreme Value Theorem
- The First Derivative Test
- The Second Derivative Test

(a) State the conditions necessary for each method to be applied.

(b) When solving an optimization exercise, when must EVT be used? When must First/Second Derivative Test be used?

7. A line tangent to the graph of $f(x)$ is constructed at $(c, f(c))$. The tangent line is used to estimate the value of $f(x)$ at $x=d$. Explain how the second derivative can be used to determine if the tangent line approximation for $f(d)$ is and underestimate or an overestimate for the true/exact value of $f(d)$.