#2

$$\frac{2}{3}-\frac{2}{5}+\frac{2}{7}-\frac{2}{9}+\frac{2}{11}-\cdots=\sum_{n=1}^{\infty}(-1)^n\cdot\frac{2}{2n+1}$$

Since the series is alternating and $\lim_{n\to\infty}\left(\frac{2}{2n+1}\right)=0$, the series converges by the alternating series test.

#3

$$-\frac{2}{5}+\frac{4}{6}-\frac{6}{7}+\frac{8}{8}-\frac{10}{9}+\cdots=\sum_{n=1}^{\infty}(-1)^n\frac{2n}{n+4}$$

Since the series is alternating and $\lim_{n\to\infty}\left[\frac{2n}{n+4}\right]\sim\lim_{n\to\infty}\left[\frac{2n}{n}\right]=2$, the series does not converge by the alternating series test.

#4

$$\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{6}}-\cdots=\sum_{n=2}^{\infty}(-1)^n\cdot\frac{1}{\sqrt{n}}$$

Since the series is alternating and $\lim_{n\to\infty}\left[\frac{1}{\sqrt{n}}\right]=0$, the series converges by the alternating series test.

#5

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n+1}$$

Since the series is alternating and $\lim_{n\to\infty}\left[\frac{1}{2n+1}\right]=0$, the series converges by the alternating series test.

#6

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\ln(n+4)}$$

Since the series is alternating and $\lim_{n\to\infty}\left[\frac{1}{\ln(n+4)}\right]=0$, the series converges by the alternating series test.

#7

$$\sum_{n=1}^{\infty}(-1)^n\cdot\frac{3n-1}{2n+1}$$

Since the series is alternating and $\lim_{n\to\infty}\left[\frac{3n-1}{2n+1}\right]=\frac{3}{2}$, the series does not converge by the alternating series test.

#8 $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{n}{\sqrt{n^3+2}}$

Since the series is alternating and $\lim_{n\to\infty}\left[\frac{n}{\sqrt{n^3+2}}\right]\sim\lim_{n\to\infty}\left[\frac{n}{\sqrt{n^3}}\right]=\lim_{n\to\infty}\left[\frac{n}{n^{\frac{3}{2}}}\right]=\lim_{n\to\infty}\left[\frac{1}{n^{\frac{1}{2}}}\right]=0$, the series converges by the alternating series test.

#9

$$\sum_{n=1}^{\infty}(-1)^n\cdot\left(e^{-n}\right)$$

Since the series is alternating and $\lim_{n\to\infty}\left[e^{-n}\right]=\lim_{n\to\infty}\left[\frac{1}{e^n}\right]=0$, the series converges by the alternating series test.

#10

$$\sum_{n=1}^{\infty}(-1)^n\cdot\frac{\sqrt{n}}{2n+3}$$

Since the series is alternating and $\lim_{n\to\infty}\left[\frac{\sqrt{n}}{2n+3}\right]\sim\lim_{n\to\infty}\left[\frac{\sqrt{n}}{2n}\right]=\lim_{n\to\infty}\left[\frac{1}{2n^{\frac{1}{2}}}\right]=0$, the series converges by the alternating series test.

#11

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2}{n^3+4}$$

Since the series is alternating and $\lim_{n\to\infty}\left[\frac{n^2}{n^3+4}\right]\sim\lim_{n\to\infty}\left[\frac{n^2}{n^3}\right]=\lim_{n\to\infty}\left[\frac{1}{n}\right]=0$, the series converges by the alternating series test.

#12 $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot ne^{-n}$

Since the series is alternating and $\lim_{n\to\infty}\left[ne^{-n}\right]=\lim_{n\to\infty}\left[\frac{n}{e^n}\right]=0$, the series converges by the alternating series test.

#13

$\sum_{n=1}^{\infty}(-1)^{n-1}\cdot e^{\frac{2}{n}}$

Since the series is alternating and $\lim_{n\to\infty}\left[e^{\frac{2}{n}}\right]=\lim_{n\to\infty}\left[\left(\sqrt[n]{e}\right)^2\right]=1$, the series does not converge by the alternating series test.

#14

$\sum_{n=1}^{\infty}(-1)^{n-1}\arctan(n)$

Since the series is alternating and $\lim_{n\to\infty}\left[\arctan(n)\right]=\frac{\pi}{2}$, the series does not converge by the alternating series test.

#15

$$\sum_{n=1}^{\infty}\frac{\sin\left[\left(n+\frac{1}{2}\right)\pi\right]}{1+\sqrt{n}}=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{1+\sqrt{n}}$$

Since the series is alternating and $\lim_{n\to\infty}\left[\frac{1}{1+\sqrt{n}}\right]=0$, the series converges by the alternating series test.

#16

$$\sum_{n=1}^{\infty}\frac{n\cdot\cos(n\pi)}{2^n}=\sum_{n=1}^{\infty}\frac{n\cdot(-1)^n}{2^n}=\sum_{n=1}^{\infty}(-1)^n\cdot\frac{n}{2^n}$$

Since the series is alternating and $\lim_{n\to\infty}\left[\frac{n}{2^n}\right]=0$, the series converges by the alternating series test.

#17

$$\sum_{n=1}^{\infty}(-1)^n \sin\left(\frac{\pi}{n}\right)$$

Since the series is alternating and $\lim_{n\to\infty}\left[\sin\left(\frac{\pi}{n}\right)\right]=\sin(0)=0$, the series converges by the alternating series test.

#18

$$\sum_{n=1}^{\infty}(-1)^n \cos\left(\frac{\pi}{n}\right)$$

Since the series is alternating and $\lim_{n\to\infty}\left[\cos\left(\frac{\pi}{n}\right)\right]=\cos(0)=1$, the series does not converge by the alternating series test.