A function $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$.

"an" is emphasized since there are *many* functions which are an antiderivative of $f(x)$.

Consider $f(x) = 2x$

$F_1(x) = x^2$
$F_2(x) = x^2 + 3$
$F_2(x) = x^2 - 5$
$\vdots$

are all antiderivatives of $f(x)$.

(all are vertical translations/shifts by C units of the graph of $y = x^2$)

Theorem: If F is an antiderivative of f on an interval I, then G is an antiderivative of f on the interval I if and only if G is of the form $G(x) = F(x) + C$ for all x in I, where C is a constant.

The "*C*" is called the *constant of integration.*

$G(x)$ is called the *general antiderivative of f.*

We say that $G(x)$ is the *general solution* to the differential equation $G'(x) = f(x)$.

A *specific solution* to the differential equation is a solution that passes through a given point (a, b). Making the solution pass through a given point will *uniquely* fix the value of C.

Finding the solution is called *Antidifferentiation* or *Indefinite Integration.*

Variable of Integration

Integral Symbol (elongated "S") $\rightarrow \int f(x)\,dx = F(x) + C$

Integrand

Constant of Integration

Indefinite Integral $\leftrightarrow$ Antiderivative

Antidifferentiation is the inverse operation of differentiation:

$\int f'(x)\,dx = f(x) + C$	$\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x)$

Indefinite Integration Rules:

I. $\int 0\,dx = C$

II. $\int k\,dx = kx + C$

III. $\int k \cdot f(x)\,dx = k \cdot \int f(x)\,dx$

IV. $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Note:

$$\int f(x)g(x)\,dx \neq \int f(x)\,dx \cdot \int g(x)\,dx$$

$$\int x\cos(x)\,dx \neq \frac{1}{2}x^2 \cdot \sin(x) + C$$

Basic Integration Formulas: *a* and *k* are constants.

$$\int k\cdot f(u)\,du = k\int f(u)\,du \qquad \int f(u)\pm g(u)\,du = \int f(u)\,du \pm \int g(u)\,du$$

$$\int k\,du = k\cdot u + C \qquad \int u^n\,du = \frac{1}{n+1}u^{n+1}+C$$

$$\int a^u\,du = \frac{1}{\ln(a)}a^u + C \qquad \int e^u\,du = e^u + C \qquad \int \frac{1}{u}\,du = \ln|u| + C$$

$$\int \cos(u)\,du = \sin(u)+C \qquad \int \sec(u)\tan(u)\,du = \sec(u)+C \qquad \int \sec^2(u)\,du = \tan(u)+C$$

$$\int \sin(u)\,du = -\cos(u)+C \qquad \int \csc(u)\cot(u)\,du = -\csc(u)+C \qquad \int \csc^2(u)\,du = -\cot(u)+C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\left(\frac{u}{a}\right)+C \qquad \int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C \qquad \int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right)+C$$

Solve for $f(x)$: $f''(x) = 2+\cos(x)$ where $f'(0)=0$ $\quad f\left(\frac{\pi}{2}\right)=0$

$f''(x)=2+\cos(x)$ $f'(x)=\int f''(x)\,dx$ $=\int 2+\cos(x)\,dx$ $=\int 2dx+\int\cos(x)\,dx$ $=2x+C_1+\sin(x)+C_2$ $=2x+\sin(x)+[C_1+C_2]$ $=2x+\sin(x)+C$	$f'(0)=0$ $\downarrow$ $0=2(0)+\sin(0)+C$ $0=C$ $\downarrow$ $f'(x)=2x+\sin(x)$	$f(x)=\int f'(x)\,dx$ $=\int 2x+\sin(x)\,dx$ $=x^2-\cos(x)+C$ $f\left(\frac{\pi}{2}\right)=0$ $\downarrow$ $0=\left(\frac{\pi}{2}\right)^2-\cos\left(\frac{\pi}{2}\right)+C$ $C=-\frac{\pi^2}{4}$ $\downarrow$ $f(x)=x^2-\cos(x)-\frac{\pi^2}{4}$