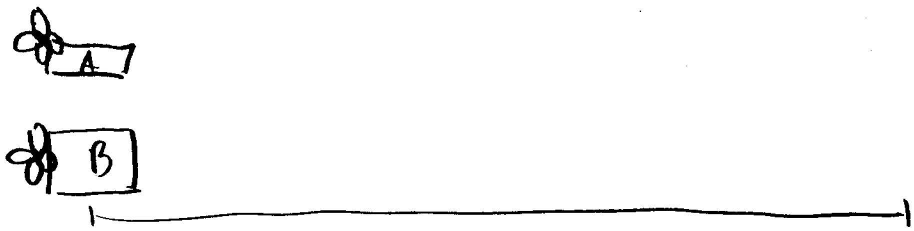
Sample test 3 Solutions.

1.



5m

a) Since the forces acting on the carts are the same, and they act for the same distance in the same direction, both carts ~~expen~~ have equal amounts of work done on them. $W = \Delta K$ for a system with no change in internal energy, so both will end up with the same kinetic energy.

b) $a = \frac{F_{net}}{m}$, and the only horizontal force is the fan, so the acceleration of cart B is smaller. Since both start from rest, this means A will reach the end more quickly

c) The ~~so~~ impulse acting on cart B will be greater since the force acts for longer ($\cancel{p}\ \vec{J} = \vec{F}\Delta t$). Since $\vec{J} = \Delta\vec{p}$, cart B will gain more momentum.

<u>Note</u>: For parts a and c, making arguments about tradeoffs between m and v is <u>not</u> a valid approach.

2. Define our system to be the cart and the Earth.

$$E_f = E_i + \cancel{W_{ext,\,non\text{-}diss} + W_{Diss}}$$

W_other

W_friction

We're told $\cancel{W_{diss}} = 0$

W_{other} ($\cancel{W_{ext,\,non\text{-}diss}}$) is provided by the fan. The fan is always pushing in the direction of motion so at all points along the path $\vec{F}_{fan} \cdot d\vec{s} = |\vec{F}_{fan}||d\vec{s}|$

so $\int |\vec{F}_{fan}||d\vec{s}| = |\vec{F}_{fan}| \int |d\vec{s}| = |\vec{F}_{fan}| D$. where D is the length of the path, 6.0 m.

(This is one of our "short cuts" for finding W)

So the cons. of energy equation becomes.

$$\cancel{U_f}^{0} + K_f = U_i + \cancel{K_i}^{0} + F_{fan} D$$

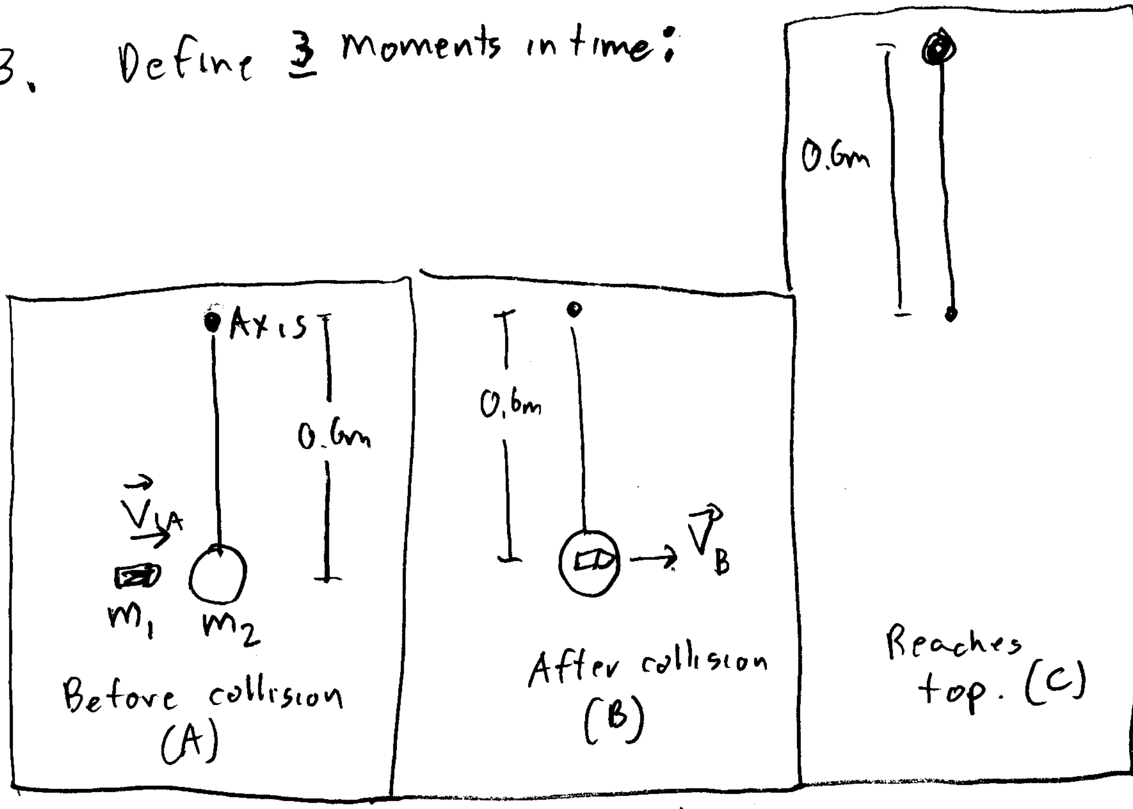
Define h = 0 at bottom

at rest initially.

so $\frac{1}{2} m v_f^2 = m g h_i + F_{fan} D$

$$\Rightarrow v_f = \sqrt{2 g h_i + \frac{2 F_{fan} D}{m}} = 7.56 \text{ m/s}$$

3. Define 3 moments in time:



From A to B, we have a collision, so momentum will be conserved. From B to C, (If we include the Earth in our system) energy will be conserved. If the bob barely gets to the top, then $V_c = 0$. Define $h = 0$ at the lowest position of the bob.

For A to B: $\vec P_f = \vec P_i$ (cons momentum)

$$(m_1 + m_2)\vec V_B = m_1 \vec V_{1A} + m_2 \cancel{\vec V_{2A}}^{\,0}$$

$$\Rightarrow \vec V_B = \frac{m_1}{m_1+m_2}\vec V_{1A} \quad \left(\text{so } V_B = \frac{m_1}{m_1+m_2} V_{1A}\right)$$

(gravity is conservative) W_other is zero

(← ~~internal~~)

(assume negligible)

For B to C: $E_C = E_B + \cancel{W_{ext,\,non-diss}}^{\,0} + \cancel{W_{diss}}^{\,0}$

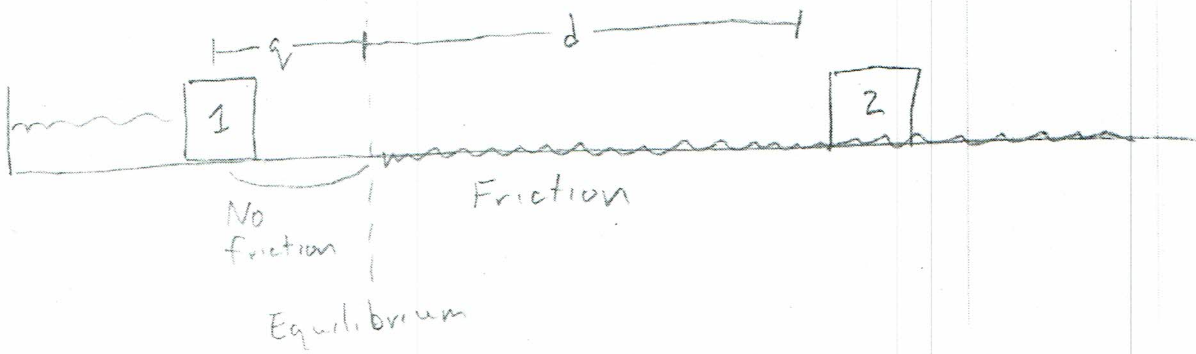
$$\Rightarrow U_C + \cancel{K_C}^{\,0} = \cancel{U_B}^{\,0} + K_B \quad \text{(inserting eqn from A→B)}$$

$$(m_1+m_2) g h_c = \frac{1}{2}(m_1+m_2)V_B^2 = \frac{1}{2}(m_1+m_2)\left(\frac{m_1}{m_1+m_2}\right)^2 V_{1A}^2$$

$$\Rightarrow g h_c = \frac{1}{2}\left(\frac{m_1^2}{m_1+m_2}\right) V_{1A}^2$$

$$\Rightarrow V_{1A} = \sqrt{\frac{2gh_c(m_1+m_2)^2}{m_1^2}} = \sqrt{2gh_c}\left(\frac{m_1+m_2}{m_1}\right) = 732 \text{ m/s}$$

this is ~ 2x the speed of sound: a reasonable speed for a bullet.



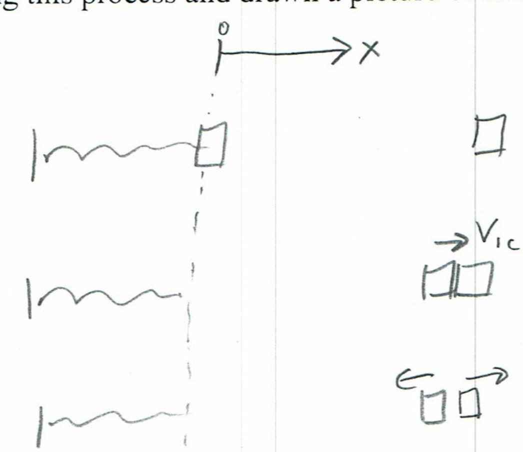
2. A block, mass $m_1 = 2.0\ kg$, is pressed up against a spring with $k = 300\ N/m$ compressed $q = 0.40\ m$ from its equilibrium position. The block slides frictionlessly forwards until it reaches the equilibrium point of the spring. The block then loses contact with the spring and slides over a surface with $\mu_k = 0.20$ for a distance of $d = 2.0\ m$. It then collides inelastically (but NOT perfectly inelastically) with a second block which has mass $m_2 = 6\ kg$. Immediately after the collision, the first block is moving to the left with a speed of $0.5\ m/s$.
    a. List all of the key moments in time during this process and drawn a picture of each moment in time.

A) Initial (see above)

B) 1 reaches equilibrium

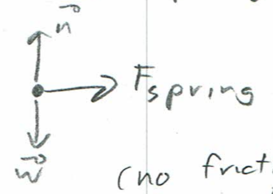
C) 1 starts to collide with 2

D) The end of the collision



b. Find the velocity of the second block immediately after the collision. Show all your work and explain your reasoning. Use the subscripts corresponding to the moments in time you labeled in part a. Do NOT use "initial" and "final" since there are more than two moments in time here.

For AB Use energy conservation. System: m_1 and spring



$E_B = E_A + \cancel{W_{diss}}^{0} + \cancel{W_{ext,NC}}^{0}$ W_other

(no friction from A to B)

The external forces $\vec{n}$ and $\vec{w}$ do no work ($\perp$ to motion). No dissipative forces.

(equilib. position) (starts at rest)

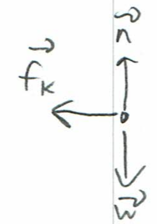
$\cancel{U_B}^{0} + K_B = U_A + \cancel{K_A}^{0}$

(Note $x_A = -q$)

$K_B = \frac{1}{2} m_1 v_B^2 = \frac{1}{2} k x_A^2$

$K_B = \frac{1}{2} k(-q)^2 = \frac{1}{2} k q^2$ $(= 24\,J)$

For BC use W-E theorem. System: box 1



$\vec{n}$ and $\vec{w}$ do no work ($\perp$ to motion).

$W_f = -F_K d = -\mu_K m g d$ $(= -7.84\,J)$

(since $a_y = 0$, $n = w = mg$ and $f_s = \mu_K mg$).

so $\Delta K = K_C - K_B = -\mu_K m g d$

so $K_C = K_B - \mu_K m g d = \frac{1}{2} k q^2 - \mu_K m g d$ $(= 16.16\,J)$

$\frac{1}{2} m v_{1C}^2 = \frac{1}{2} k q^2 - \mu_K m g d$ $\Rightarrow$ $v_{1C} = 4.20$ m/s

Use cons. momentum for CD. (collision)

$P_C = P_D$

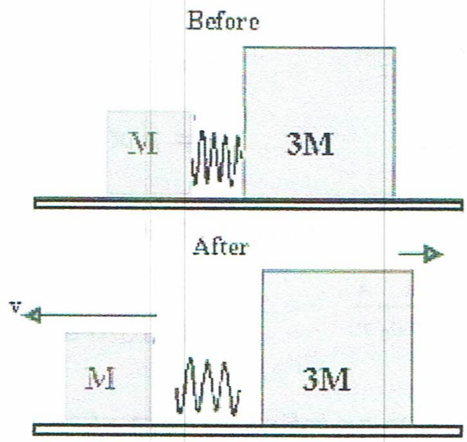
$m_1 v_{1C} + m_2 \cancel{v_{2C}}^{0} = m_1 v_{1D} + m_2 v_{2D}$

$\Rightarrow v_{2D} = \frac{m_1 v_{1C} - m_1 v_{1D}}{m_2} = \frac{(2\,kg)(4.20\,m/s) - (2\,kg)(-0.5\,m/s)}{6\,kg}$

$v_{2D} = 1.51$ m/s.

PHYS250, Test 3 Part 2. Name____________________________________

1. Two boxes are placed on a frictionless surface and a spring of negligible mass is placed between them. They are then pressed together. Finally, they are released, and the spring pressed them apart. As they separate, the spring will exert equal magnitude forces on both of them for equal amounts of time. One box has mass M, one box has mass 3M.



   a. Which box has more momentum after they separate? Explain.

The same forces act on both boxes for the same amount of time. Thus both boxes receive the same amount of impulse and get the same amount of momentum.

   b. Which box will have more energy after they separate? Explain.

Since the left-hand box has smaller mass it will accelerate more quickly and travel a larger distance before losing contact with the spring. The forces are the same, so more work will be done on the left hand block and it will get more energy.

2. Which consumes more energy, a 1.2 kW hair drier used for 10 minutes or a 10 W night light used for 24 hours? Show your work.

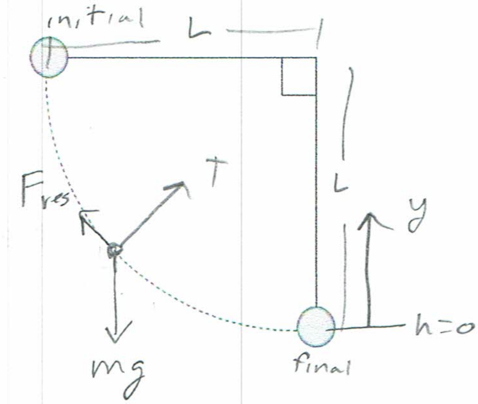
$10\,\text{min} \cdot \frac{60\,\text{s}}{1\,\text{min}} = 600\,\text{s}$ $\quad 24\,\text{hr} \cdot \frac{60\,\text{min}}{1\,\text{hr}} \cdot \frac{60\,\text{s}}{1\,\text{min}} = 86400\,\text{s}$

$W = P\Delta t$ so $W_{drier} = 1200\,\frac{J}{s} \cdot 600\,\text{s} = 720000\,\text{J}$

$W_{light} = 10\,\text{J/s} \cdot 86400 = 864000\,\text{J}$

so the light uses more energy.

3. A pendulum of mass 0.20 kg and length 0.50 m is brought to an initial position of 90 degrees from vertical and then released from rest. As it swings, a constant 0.5 N force of air resistance opposes the motion. (This is unrealistic, but let's ignore that for now). How fast is the pendulum moving when it reaches the bottom of its swing? Show all your work.



Apply conservation of energy to the pendulum + earth.

$$E_f = E_i + W_{diss} + W_{ext,\,non\text{-}diss.}$$

Tension is dissipative.

mg is internal

T does no work ($\perp$ to motion)

so $W_{ext,\,non\,diss} = 0$

$W_{diss} = W_{res} = -F_{res}d$ since the force always opposes the motion and is constant.

The distance travelled is $\frac{1}{4}$ of the circumference: $\frac{2\pi r}{4} = \frac{2\pi L}{4} = \frac{\pi L}{2}$

so $W_{diss} = -F_{res}\left(\frac{\pi L}{2}\right)$

$U_f = 0$ ($h=0$ there)

$K_i = 0$ (starts at rest)

$$\cancel{U_f}^{0} + K_f = U_i + \cancel{K_i}^{0} - F_{res}\left(\frac{\pi L}{2}\right)$$

$$\frac{1}{2}mv_f^2 = mgh_i - F_{res}\left(\frac{\pi L}{2}\right)$$

$$\Rightarrow\ v_f = \sqrt{\frac{2mgh_i - \pi L F_{res}}{m}} = \sqrt{\frac{2(0.2\,kg)(9.8\,m/s^2)(0.5\,m) - \pi(0.5\,m)(0.5\,N)}{0.2\,kg}}$$

$$v_f = 2.43\ m/s$$