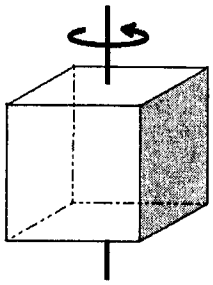
Quiz 7, (Standard Problems 9)

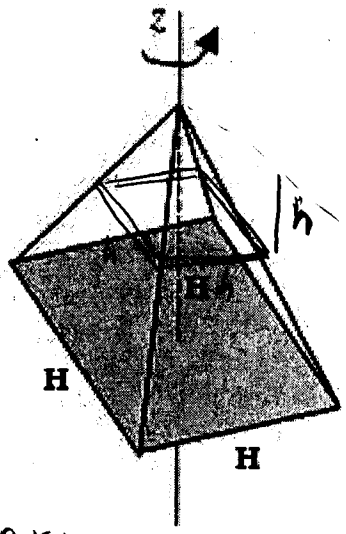
Name Solution Lab section (circle one): 9am 11am

The moment of inertia of a uniform cube of mass M and sides with length L rotating about an axis that passes through the center of two opposing sides, as shown to the right, is given by the equation

$$I = \frac{1}{6}ML^2$$



Use this information to find an expression for the moment of inertia of square-based pyramid with sides of height H and sides also of length H, rotating about an axis that passes through the center of its base and the tip of the pyramid, as shown to the right. All steps must be presented clearly and justified.



First, note that the vertical dimension of the cube doesn't affect the moment of inertia, so $I = \frac{1}{6}ML^2$ applies to a square-based prism as well as a cube.

Chop up the pyramid into square-based prisms as shown. Let h = the ⊥ distance from the point to the prism. The width is dh. The prism will have volume

$dV = h^2\,dh$ (because the side of each prism will be equal to h, since the base of the pyramid also has the same size as the height).

so $dm = \rho\,dV = \rho h^2\,dh$

so $dI = \frac{1}{6}\,dm\,h^2 = \frac{1}{6}\rho h^4\,dh$

And $I = \int_0^H \frac{1}{6}\rho h^4\,dh = \frac{\rho H^5}{30}$