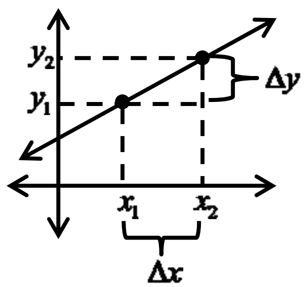
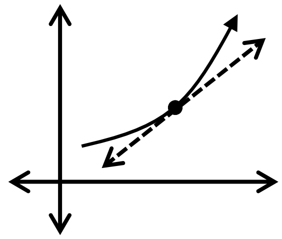
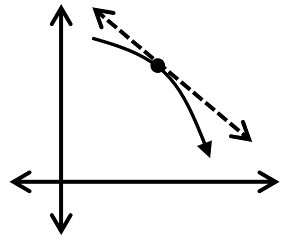
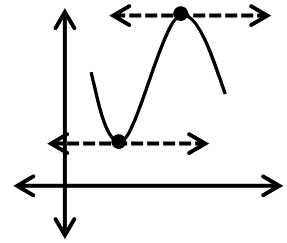



Slope is positive


Slope is negative


Slope is zero

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

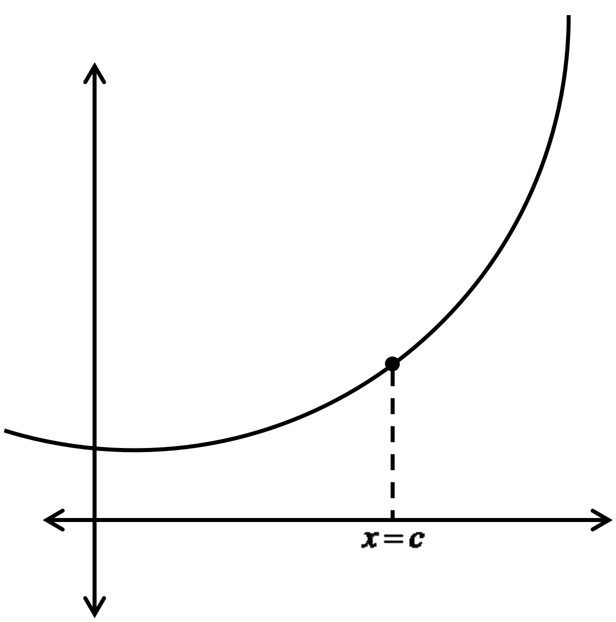
We can determine the slope of a linear function at *any* given *x* value by using the slope formula. When it comes to the slope of a *smooth* curve, the best we can do is identify whether the slope at a given point is positive, negative, or zero without some formal calculus.

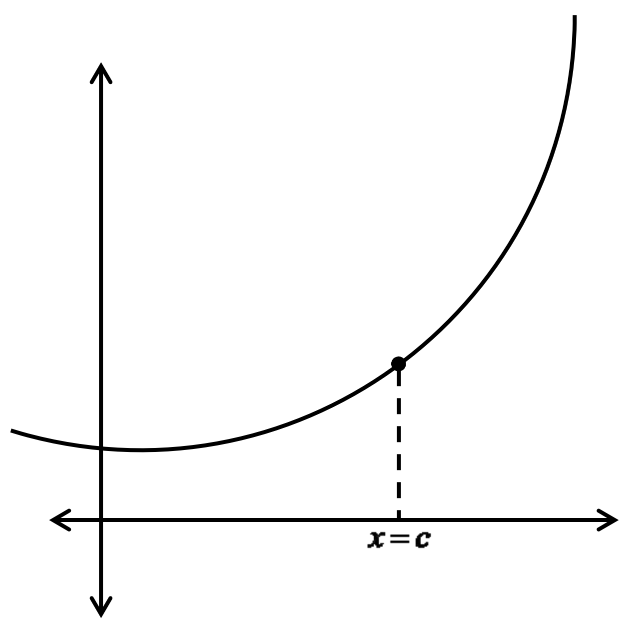
If a smooth function is increasing at $x = c$, the slope should be positive.
If a smooth function is decreasing at $x = c$, then the slope should be negative.
If a smooth function has a horizontal tangent at $x = c$, then the slope should be zero.

Therefore, the slope of a smooth curve at $x = c$ is directly related to the slope of the line tangent to the curve at $x = c$.

To get a numerical value for the slope of the tangent line, we must start with an estimate of the slope, using two points on the curve under the following conditions.

I. One point on the curve will be the point of tangency. This point must remain fixed.
II. A second point must be on the curve, and must be close to the point of tangency.
    a. To improve the estimate of the slope of the tangent line, we move this point closer and closer along the curve towards $(c, f(c))$.





We estimate the slope the tangent at $(c, f(c))$ by using the slope of the secant line between the points $(c, f(c))$ and $(c+h, f(c+h))$:

$$\begin{aligned}\text{slope of tangent} &\approx \frac{y_2 - y_1}{x_2 - x_1} \\ &\approx \frac{f(c+h) - f(c)}{(c+h)-(c)} \\ &\approx \frac{f(c+h) - f(c)}{h}\end{aligned}$$

To get a better estimate for the slope of the tangent line, we use points closer and closer to the point of tangency. Moving the point $(c+h, f(c+h))$ closer to the point of tangency means that $h \to 0$. Now we can take the limit of the expression, and ***<u>if the limit exists</u>***, we shall claim that the slope of the tangent line is the value of this limit. That is

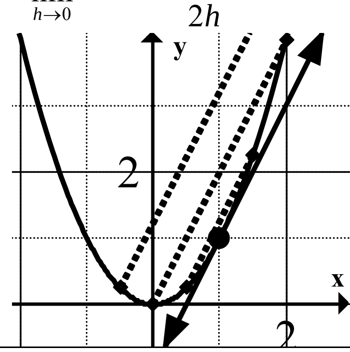
$$\boxed{\underset{(\text{at } x=c)}{\text{slope of tangent line}} = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}}$$

<u>This is a two-sided limit.</u>

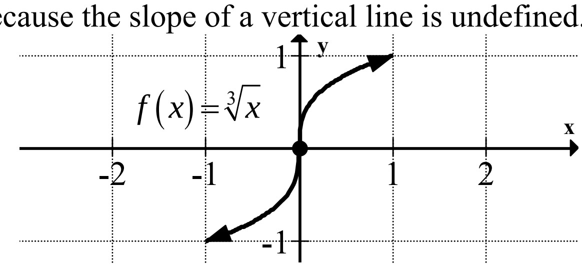
$$\underbrace{\lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h}}_{\text{slopes using points to the left}} = \underbrace{\lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h}}_{\text{slopes using points to the right}}$$

Alternate forms for the slope of the line tangent to the graph of $f(x)$ at $x = c$

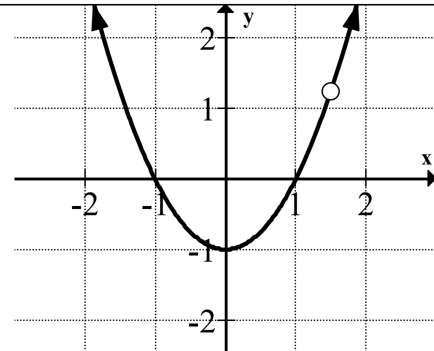
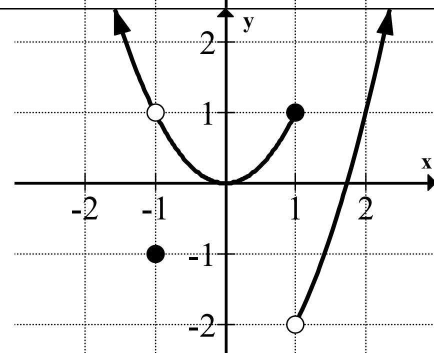
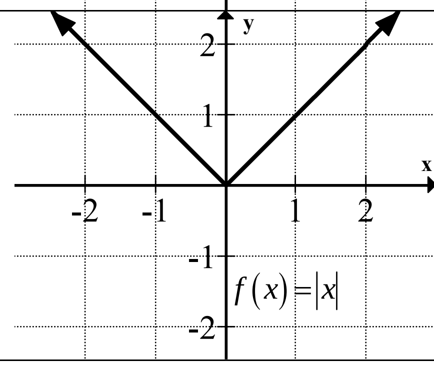
Using Δx instead of h	Symmetric Difference Quotient	Alternate form
$\lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$	$\lim_{h \to 0} \frac{f(c+h) - f(c-h)}{2h}$	$\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$



If the two-sided limit turns out to be $\pm\infty$, then we say that the function has a vertical "tangent" line at $x = c$. An example of such a function is $f(x) = \sqrt[3]{x}$ at $x = 0$. The slope of the tangent does not exist at $x = 0$ because the slope of a vertical line is undefined.



The two sided limit can fail under some other circumstances as well:

$f(x)$ is not defined at $x=c$	
$f(x)$ is discontinuous at $x=c$	
slopes from the left $\neq$ slopes from the right $\lim\limits_{h\to 0^-}\dfrac{f(c+h)-f(c)}{h}\neq\lim\limits_{h\to 0^+}\dfrac{f(c+h)-f(c)}{h}$	

If $\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}=k$, then we say that the derivative of $f(x)$ at $x=c$ is *k*. We denote this as $f'(c)=k$, read "*f*-prime of *c* equals *k*."

The verb – **<u>differentiate.</u>** The process of taking the derivative is called **<u>differentiation.</u>**

A function is **<u>differentiable</u>** at a point $x=c$ if the derivative exists at $x=c$.
A function is **<u>differentiable on an open interval</u>** (a,b) if it is differentiable at all *x* in (a,b).

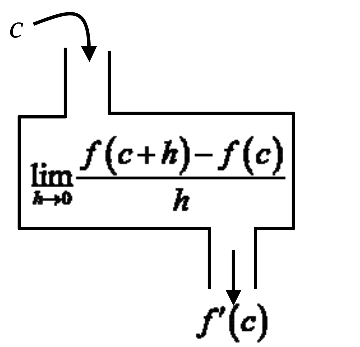
Differentiation is always done <u>*with respect* to a given variable.</u> In this course, we will always differentiate with respect to *x*.

Different notation to express the derivative:

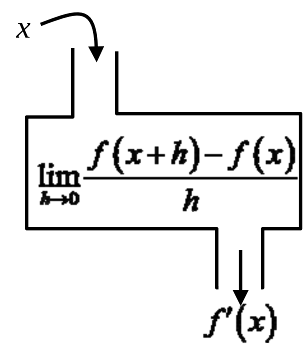
$f'(x)$	$\frac{dy}{dx}$	$\frac{d}{dx}\left[f(x)\right]$	y'

$\frac{d}{dx}[\cdots]$ or $[\cdots]'$ indicates to <u>take the derivative of</u> $[\cdots]$

Finding the derivative at each value of *c* is a tedious process, and it would be helpful if there were faster means to do so. It would be nice if there was a function that would tell you the derivative of a given function *at any value of x*.



The result is a **<u>value.</u>** Information only useful at $x=c$



The result is a **<u>function.</u>** Information useful at *any* value of *x*.

Higher order derivatives:

First derivative	$f'(x)$	$\frac{dy}{dx}$	$\frac{d}{dx}\left[f(x)\right]$	y'
Second derivative	$f''(x)$	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}\left[f(x)\right]$	y''
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n^{th} derivative	$f^{(n)}(x)$	$\frac{d^ny}{dx^n}$	$\frac{d^n}{dx^n}\left[f(x)\right]$	$y^{(n)}$