$f(x)$ Unit Summary for Limits:

1. Describe in words and with a diagram/graph what $\lim_{x \to 2} f(x) = 3$ means.
2. Describe in words, mathematical notation, or diagrams all cases in which $\lim_{x \to c} f(x)$ DNE.
3. Sketch a function that demonstrates that $\lim_{x \to 2} f(x)$ exists and is $\lim_{x \to 2} f(x) \neq f(2)$.
4. Explain why $f(x) = \frac{(x+1)(x-1)}{x-1}$ is not defined at $x = 1$. $g(x) = x + 1$ is not the same as $f(x)$. Explain why cancelling the factors of $x - 1$ changes the graph of $f(x)$, and therefore the function.
5. Given the exercise $\lim_{x \to \infty} \frac{\sqrt{x^2 + 2x - 7}}{-x}$, explain in words why it is acceptable to state $\lim_{x \to \infty} \frac{\sqrt{x^2 + 2x - 7}}{-x} = \lim_{x \to \infty} \frac{\sqrt{x^2}}{-x}$.
6. A function $f(x)$ is continuous at $x = c$ if $\lim_{x \to c} f(x) = f(c)$. An alternate notation for the definition of continuity is $\lim_{x \to c} \left[f(x) - f(c) \right] = 0$. Choose one of the definitions and explain in words how this calculus definition is of continuity is interpreted visually.
7. The Intermediate Value Theorem states that if $f(x)$ is continuous on a closed interval $[a,b]$ and k is a value between $f(a)$ and $f(b)$, then there exists a value c where $a \leq c \leq b$ and $f(c) = k$. Explain in words and with a diagram why the conclusion of the Intermediate Value Theorem does not hold if k is not between $f(a)$ and $f(b)$. Explain in words and with a diagram why the conclusion of the Intermediate Value Theorem does not hold if $f(x)$ is not continuous on $[a,b]$.
8. Explain how the concept of infinitely small is applied to limits of the form $\lim_{x \to c} f(x)$ where c is a finite value. Explain how the concept of infinitely large is applied to limits of the form $\lim_{x \to \pm\infty} f(x)$.