A function F(x) is an <u>antiderivative</u> of f(x) if F'(x) = f(x).

"an" is emphasized since there are *many* functions which are an antiderivative of f(x).

Consider
$$f(x) = 2x$$
 $F_1(x) = x^2$ are all antiderivatives of $f(x)$.

$$F_2(x) = x^2 + 3$$

$$F_2(x) = x^2 - 5$$

$$\vdots$$
(all are vertical translations/shifts by C units of the graph of $y = x^2$)

Theorem: If F is an antiderivative of f on an interval I, then G is an antiderivative of f on the interval I if and only if G is of the form G(x) = F(x) + C for all x in I, where C is a constant.

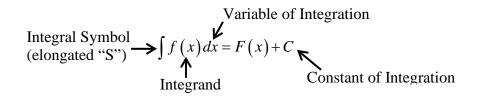
The "C" is called the *constant of integration*.

G(x) is called the *general antiderivative of f.*

We say that G(x) is the <u>general solution</u> to the differential equation G'(x) = f(x).

A <u>specific solution</u> to the differential equation is a solution that passes through a given point (a,b). Making the solution pass through a given point will <u>uniquely</u> fix the value of C.

Finding the solution is called <u>Antidifferentiation</u> or <u>Indefinite Integration</u>.



Indefinite Integral \leftrightarrow Antiderivative

Antidifferentiation is the inverse operation of differentiation:

$$\int f'(x)dx = f(x) + C$$

$$\frac{d}{dx} \Big[\int f(x)dx \Big] = f(x)$$

Indefinite Integration Rules:

I.
$$\int 0 dx = C$$
II.
$$\int k dx = kx + C$$
III.
$$\int k \cdot f(x) dx = k \cdot \int f(x) dx$$
IV.
$$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int f(x) dx$$
Note:
$$\int f(x) g(x) dx \neq \int f(x) dx \cdot \int g(x) dx$$

$$\int x \cos(x) dx \neq \frac{1}{2} x^{2} \cdot \sin(x) + C$$

Basic Integration Formulas: a and k are constants.

$$\int k \cdot f(u) du = k \int f(u) du \qquad \qquad \int f(u) \pm g(u) du = \int f(u) du \pm \int g(u) du$$

$$\int k du = k \cdot u + C \qquad \qquad \int u^n du = \frac{1}{n+1} u^{n+1} + C$$

$$\int a^u du = \frac{1}{\ln(a)} a^u + C \qquad \int e^u du = e^u + C \qquad \qquad \int \frac{1}{u} du = \ln|u| + C$$

$$\int \cos(u) du = \sin(u) + C \qquad \int \sec(u) \tan(u) du = \sec(u) + C \qquad \int \sec^2(u) du = \tan(u) + C$$

$$\int \sin(u) du = -\cos(u) + C \qquad \int \csc(u) \cot(u) du = -\csc(u) + C \qquad \int \csc^2(u) du = -\cot(u) + C$$

$$\int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin\left(\frac{u}{a}\right) + C \qquad \int \frac{1}{a^2 + u^2} du = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C \qquad \int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \operatorname{arcsec}\left(\frac{|u|}{a}\right) + C$$

Solve for f(x): $f''(x) = 2 + \cos(x)$ where f'(0) = 0 $f\left(\frac{\pi}{2}\right) = 0$

$$f''(x) = 2 + \cos(x)$$

$$f'(x) = \int f''(x) dx$$

$$= \int 2 + \cos(x) dx$$

$$= \int 2 dx + \int \cos(x) dx$$

$$= 2x + C_1 + \sin(x) + C_2$$

$$= 2x + \sin(x) + [C_1 + C_2]$$

$$= 2x + \sin(x) + C$$

$$f'(x) = 2x + \sin(x)$$

$$f'(0) = 0$$

$$0 = 2(0) + \sin(0) + C$$

$$0 = C$$

$$0 = C$$

$$0 = (\frac{\pi}{2}) = 0$$

$$0 = (\frac{\pi}{2})^2 - \cos(\frac{\pi}{2}) + C$$

$$C = -\frac{\pi^2}{4}$$

$$0 = (x) = x^2 - \cos(x) - \frac{\pi^2}{4}$$