#27

$$\int_1^3 r^3 \ln(r)\,dr$$

$$u = \ln(r) \quad v' = r^3$$

$$u' = \frac{1}{r} \quad v = \frac{1}{4}r^4$$

$$\begin{aligned}
\int r^3 \ln(r)\,dr &= \ln(r)\cdot\frac{1}{4}r^4 - \int \frac{1}{r}\cdot\frac{1}{4}r^4 dr \\
&= \ln(r)\cdot\frac{1}{4}r^4 - \frac{1}{4}\int r^3 dr \\
&= \ln(r)\cdot\frac{1}{4}r^4 - \frac{1}{4}\left[\frac{1}{4}r^4\right] + C \\
&= \ln(r)\cdot\frac{1}{4}r^4 - \frac{1}{16}r^4 + C
\end{aligned}$$

$$\begin{aligned}
\int_1^3 r^3 \ln(r)\,dr &= \left[\ln(r)\cdot\frac{1}{4}r^4 - \frac{1}{16}r^4\right]_1^3 \\
&= \left[\ln(3)\cdot\frac{1}{4}(3)^4 - \frac{1}{16}(3)^4\right] - \left[\ln(1)\cdot\frac{1}{4}(1)^4 - \frac{1}{16}(1)^4\right]
\end{aligned}$$

#13

$$\int t\cdot\sec^2(2t)\,dt$$

$$u=t \quad v'=\sec^2(2t)$$
$$u'=1 \quad v=\frac{1}{2}\tan(2t)$$

$$\int t\cdot\sec^2(2t)\,dt = t\cdot\frac{1}{2}\tan(2t)-\int 1\cdot\tan(2t)\,dt$$
$$= t\cdot\frac{1}{2}\tan(2t)-\int\frac{\sin(2t)}{\cos(2t)}dt$$
$$= t\cdot\frac{1}{2}\tan(2t)-\int\frac{1}{\cos(2t)}\cdot\sin(2t)\,dt$$
$$= t\cdot\frac{1}{2}\tan(2t)-\left(-\frac{1}{2}\right)\int\frac{1}{\cos(2t)}\cdot\sin(2t)\cdot(-2)\,dt$$
$$= t\cdot\frac{1}{2}\tan(2t)+\frac{1}{2}\ln\left|\cos(2t)\right|+C$$

$$w=\cos(2t)$$
$$dw=-\sin(2t)\cdot 2dt$$
$$\left(-\frac{1}{2}\right)\int\frac{1}{\cos(2t)}\cdot\sin(2t)\cdot(-2)\,dt$$
$$\left(-\frac{1}{2}\right)\int\frac{1}{w}dw$$
$$-\frac{1}{2}\ln|w|$$

#7

$$\int\left(x^2+2x\right)\cos(x)\,dx$$

u	v'
x^2+2x	$\cos(x)$
$2x+2$	$\sin(x)$
2	$-\cos(x)$
0	$-\sin(x)$

$$\int\left(x^2+2x\right)\cos(x)\,dx=\left(x^2+2x\right)\sin(x)-(2x+2)\left(-\cos(x)\right)+2\left(-\sin(x)\right)+C$$

#9

$\int \ln\left(\sqrt[3]{x}\right)dx$

$$\int \ln\left(\sqrt[3]{x}\right)dx = \int \ln\left(x^{\frac{1}{3}}\right)dx$$
$$= \int \frac{1}{3}\ln(x)\,dx$$

$$u = \ln(x) \quad v' = \frac{1}{3}$$
$$u' = \frac{1}{x} \qquad v = \frac{1}{3}x$$

$$\int \ln\left(\sqrt[3]{x}\right)dx = \int \ln\left(x^{\frac{1}{3}}\right)dx$$
$$= \int \frac{1}{3}\ln(x)\,dx$$
$$= \ln(x)\cdot\frac{1}{3}x - \int \frac{1}{x}\cdot\frac{1}{3}x\,dx$$
$$= \ln(x)\cdot\frac{1}{3}x - \int \frac{1}{3}dx$$
$$= \ln(x)\cdot\frac{1}{3}x - \frac{1}{3}x + C$$

#10

$\int \arcsin(x)\,dx$

$$\int \arcsin(x)\,dx = \int 1\cdot \arcsin(x)\,dx$$

$$u = \arcsin(x) \quad v' = 1$$

$$u' = \frac{1}{\sqrt{1-x^2}} \qquad v = x$$

$$\begin{aligned}
\int \arcsin(x)\,dx &= \int 1\cdot \arcsin(x)\,dx \\
&= \arcsin(x)\cdot x - \int \frac{1}{\sqrt{1-x^2}}\cdot x\,dx \\
&= \arcsin(x)\cdot x - \left(-\frac{1}{2}\right)\int \frac{1}{\sqrt{1-x^2}}\cdot(-2)\,x\,dx \\
&= \arcsin(x)\cdot x + \frac{1}{2}\cdot 2\cdot\left(1-x^2\right)^{\frac{1}{2}} + C \\
&= \arcsin(x)\cdot x + \left(1-x^2\right)^{\frac{1}{2}} + C
\end{aligned}$$

$$w = 1-x^2$$

$$dw = \boxed{-2}\,x\,dx$$

$$\boxed{-\frac{1}{2}}\int \frac{1}{\sqrt{1-x^2}}\left(\boxed{-2}\,x\right)dx = -\frac{1}{2}\int \frac{1}{\sqrt{w}}\,dw$$

$$= -\frac{1}{2}\left[2w^{\frac{1}{2}}\right]$$

#8

$\int t^2 \sin(\beta t)\,dt$

u	v'
t^2	$\sin(\beta t)$
$2t$	$-\frac{1}{\beta}\cos(\beta t)$
2	$-\frac{1}{\beta^2}\sin(\beta t)$
0	$\frac{1}{\beta^3}\cos(\beta t)$

$$\int t^2 \sin(\beta t)\,dt = t^2 \cdot \left(-\frac{1}{\beta}\cos(\beta t)\right) - 2t\left(-\frac{1}{\beta^2}\sin(\beta t)\right) + 2\left(\frac{1}{\beta^3}\cos(\beta t)\right) + C$$

#11

$\int \arctan(4t)\,dt$

$$\int \arctan(4t)\,dt = \int 1\cdot \arctan(4t)\,dt$$

$$u = \arctan(4t) \qquad v' = 1$$

$$u' = \frac{1}{1+(4t)^2}\cdot 4 \qquad v = t$$

$$\begin{aligned}\int \arctan(4t)\,dt &= \int 1\cdot \arctan(4t)\,dt \\ &= \arctan(4t)\cdot t - \int \frac{1}{1+(4t)^2}\cdot 4\cdot t\,dt \\ &= \arctan(4t)\cdot t - \int \frac{1}{1+16t^2}\cdot 4t\,dt \\ &= \arctan(4t)\cdot t - \frac{1}{8}\int \frac{1}{1+16t^2}\cdot 4\cdot 8t\,dt \\ &= \arctan(4t)\cdot t - \frac{1}{8}\int \frac{1}{1+16t^2}\cdot 32t\,dt \\ &= \arctan(4t) - \frac{1}{8}\ln\left|1+16t^2\right| + C\end{aligned}$$

$$w = 1+16t^2$$

$$dw = 32t\,dt$$

$$\begin{aligned}\int \frac{1}{1+16t^2}\cdot 4t\,dt &= \frac{1}{8}\int \frac{1}{1+16t^2}\cdot 4\cdot 8t\,dt \\ &= \frac{1}{8}\int \frac{1}{1+16t^2}\cdot 32t\,dt \\ &= \frac{1}{8}\int \frac{1}{w}\,dw \\ &= \frac{1}{8}\ln|w| + C \\ &= \frac{1}{8}\ln\left|1+16t^2\right|\end{aligned}$$