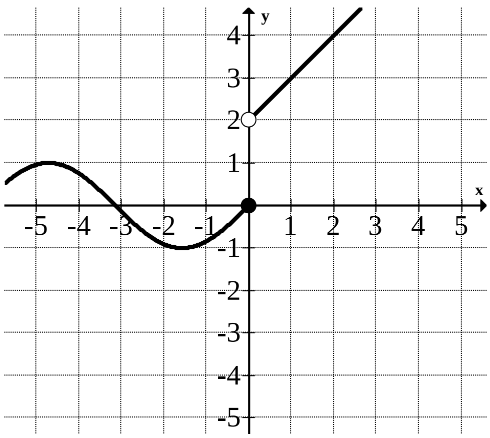
Given a function $f(x)=\begin{cases} g(x) & \text{for } x \le c \\ h(x) & \text{for } x > c \end{cases}$, how to demonstrate that $f(x)$ is continuous at $x=c$, or not continuous at $x=c$.

Not continuous at $x=c$:	**Continuous at $x=c$:**
Must demonstrate that $\lim\limits_{x\to c} f(x) \ne f(c)$. That is $\lim\limits_{x\to c^-} f(x) = f(c) = \lim\limits_{x\to c^+} f(x)$ i.e. $\lim\limits_{x\to c^-} g(x) = g(c) = \lim\limits_{x\to c^+} h(x)$ FAILS	Must demonstrate that $\lim\limits_{x\to c} f(x) = f(c)$. That is $\lim\limits_{x\to c^-} f(x) = f(c) = \lim\limits_{x\to c^+} f(x)$ i.e. $\lim\limits_{x\to c^-} g(x) = g(c) = \lim\limits_{x\to c^+} h(x)$ IS TRUE
To show $\lim\limits_{x\to c} f(x) \ne f(c)$, show ONE of the following: $\lim\limits_{x\to c^-} f(x) \ne f(c)$ *or* $f(c) \ne \lim\limits_{x\to c^+} f(x)$ *or* $\lim\limits_{x\to c^-} f(x) \ne \lim\limits_{x\to c^+} f(x)$ AND *Claim* that $f(x)$ is not continuous at $x=c$ because of the statement you demonstrated.	To show $\lim\limits_{x\to c} f(x) = f(c)$, you must demonstrate ALL of the following ✓ The value of $\lim\limits_{x\to c^-} f(x)$ ✓ The value of $\lim\limits_{x\to c^+} f(x)$ ✓ The value of $f(c)$ ✓ State that all the values are equal ✓ *Claim* that $f(x)$ is continuous at $x=c$ because $\lim\limits_{x\to c} f(x) = f(c)$

Given $f(x)=\begin{cases}\sin(x) & \text{for } x\le 0\\ x+2 & \text{for } x>0\end{cases}$	Given $f(x)=\begin{cases}x+2 & \text{for } x\le 0\\ 2 & \text{for } x>0\end{cases}$



Demonstrate analytically that $f(x)$ *is not* continuous at $x=0$.

Method 1:

$$f(0)=\sin(0)$$
$$=0$$

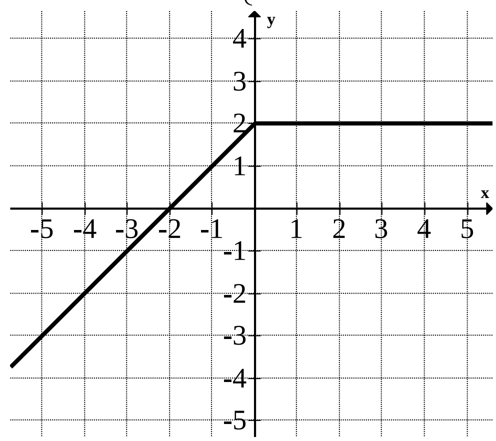
$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+} x+2$$
$$=2$$

$f(x)$ is not continuous at $x=0$ because $\lim_{x\to 0^+} f(x)\ne f(0)$.

Method 2:

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\sin(x) \qquad \lim_{x\to 0^+} f(x)=\lim_{x\to 0^+} x+2$$
$$=0 \qquad\qquad =2$$

$f(x)$ is not continuous at $x=0$ because $\lim_{x\to 0^-} f(x)\ne \lim_{x\to 0^+} f(x)$.



Demonstrate analytically that $f(x)$ *is* continuous at $x=0$.

Only one way to demonstrate continuity:

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-} x+2$$
$$=2$$
$$f(0)=0+2$$
$$=2$$
$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+} 2$$
$$=2$$

$f(x)$ is continuous at $x=0$, because $\lim_{x\to 0} f(x)=f(0)$.