1. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{2^n}{n+1}$ II. $\sum_{n=1}^{\infty} \frac{3}{n}$ III. $\sum_{n=1}^{\infty} \frac{\cos(2\pi n)}{n^2}$

$\lim_{n\to\infty} \frac{2^n}{n+1} \neq 0$

$\sum_{n=1}^{\infty} \frac{3}{n} = 3\cdot\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic

$\sum_{n=1}^{\infty} \frac{\cos(2\pi n)}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$
Convergent p-series; $p = 2$

(a) I only (b) II only (c) III only
(d) I and II only (e) I and III only

2. If $\sum_{n=0}^{\infty} a_n (x-c)^n$ is a Taylor series that converges to $f(x)$ for all real numbers x, then $f''(x) =$

$$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \cdots$$

$$f'(x) = \sum_{n=0}^{\infty} n\cdot a_n (x-c)^{n-1} = a_1 + 2\cdot a_2 (x-c)^1 + 3\cdot a_3 (x-c)^2 + \cdots$$

$$f'(x) = \sum_{n=0}^{\infty} n\cdot(n-1) a_n (x-c)^{n-2} = 2\cdot a_2 + 3\cdot 2\cdot a_3 (x-c)^1 + \cdots$$

(a) 0

(b) $(n)(n-1)a_n$

(c) $\sum_{n=0}^{\infty} n\cdot a_n (x-c)^{n-1}$

(d) $\sum_{n=0}^{\infty} a_n$

(e) $\sum_{n=0}^{\infty} n(n-1)a_n (x-c)^{n-2}$

3. What are all the values for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n\sqrt{n}\cdot 3^n}$ converges?

$\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} = \lim_{n\to\infty} \sqrt[n]{\left\lvert \frac{(x+2)^n}{n\sqrt{n}\cdot 3^n} \right\rvert}$ $= \lim_{n\to\infty} \frac{\sqrt[n]{\lvert (x+2) \rvert^n}}{\sqrt[n]{n^{1.5}}\cdot \sqrt[n]{3^n}}$ $= \frac{\lvert x+2 \rvert}{3}$	$\frac{\lvert x+2 \rvert}{3} < 1$ $\lvert x+2 \rvert < 3$ $-3 < x+2 < 3$ $-5 < x < 1$
$\sum_{n=1}^{\infty} \frac{(-5+2)^n}{n\sqrt{n}\cdot 3^n} = \sum_{n=1}^{\infty} \frac{(-3)^n}{n\sqrt{n}\cdot 3^n}$ $= \sum_{n=1}^{\infty} \frac{(-1)^n (3)^n}{n\sqrt{n}\cdot 3^n}$ $= \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1.5}}$ Since $\lim_{n\to\infty} \frac{1}{n^{1.5}} = 0$, the series converges by the Alternating Series Test	$\sum_{n=1}^{\infty} \frac{(1+2)^n}{n\sqrt{n}\cdot 3^n} = \sum_{n=1}^{\infty} \frac{3^n}{n\sqrt{n}\cdot 3^n}$ $= \sum_{n=1}^{\infty} \frac{1}{n^{1.5}}$ Convergent. *p*-series; $p = 1.5$

(a) $-3 < x < 3$
(b) $-3 \le x \le 3$
(c) $-5 < x < 1$
(d) $-5 < x \le 1$
(e) $-5 \le x \le 1$

4. Calculator required: The sum of the infinite geometric series $\frac{4}{5} + \frac{8}{35} + \frac{16}{245} + \frac{32}{1715} + \cdots$ is

$$r = \frac{\left(\frac{8}{35}\right)}{\left(\frac{4}{5}\right)} = \frac{8}{35}\cdot\frac{5}{4} = \frac{2}{7}$$

$$\frac{4}{5} + \frac{8}{35} + \frac{16}{245} + \frac{32}{1715} + \cdots = \sum_{n=0}^{\infty} \frac{4}{5}\left(\frac{2}{7}\right)^n = \frac{\left(\frac{4}{5}\right)}{1-\frac{2}{7}}$$

(a) 0.622 (b) 0.893 (c) 1.120
(d) 1.429 (e) 2.800

5. For what integer $k>1$ will both $\sum_{n=1}^{\infty}\frac{(-1)^{kn}}{n^2}$ and $\sum_{n=1}^{\infty}\left(\frac{k}{3}\right)^n$ converge:

$\sum_{n=1}^{\infty}\left(\frac{k}{3}\right)^n$ will not converge for $k\ge 3$. Therefore the anwers is (a)

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

6. What are all the values of *x* for which the series $\sum_{n=1}^{\infty}\frac{(2x+3)^n}{\sqrt{n}}$ converges?

$\lim_{n\to\infty}\sqrt[n]{\left\lvert\frac{(2x+3)^n}{\sqrt{n}}\right\rvert}=\lim_{n\to\infty}\frac{\sqrt[n]{\lvert(2x+3)^n\rvert}}{\sqrt[n]{n^{0.5}}}$ $=\frac{\lvert 2x+3\rvert}{1}$ $=\lvert 2x+3\rvert$	$\lvert 2x+3\rvert<1$ $-1<2x+3<1$ $-4<2x<-2$ $-2<x<-1$
$\sum_{n=1}^{\infty}\frac{(2(-2)+3)^n}{\sqrt{n}}=\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ By the Alternating Series Test, since $\lim_{n\to\infty}\frac{1}{\sqrt{n}}=0$, $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ converges.	$\sum_{n=1}^{\infty}\frac{(2(-1)+3)^n}{\sqrt{n}}=\sum_{n=1}^{\infty}\frac{1^n}{\sqrt{n}}$ $=\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ Diverges. *p*-series; $p<1$

(a) $-2<x<-1$
(b) $-2\le x<-1$
(c) $-2<x\le -1$
(d) $-2\le x\le -1$
(e) $-2\le x<1$

7. The Taylor polynomial of degree 3 centered at $x=0$ for $f(x)=\sqrt{1+x}$ is

$$\begin{aligned} f(x)&=\sqrt{1+x} & f(0)&=\sqrt{1+0}=1 \\ f'(x)&=\frac{1}{2}(1+x)^{-\frac{1}{2}} & f'(0)&=\frac{1}{2}(1+0)^{-\frac{1}{2}}=\frac{1}{2} \\ f''(x)&=-\frac{1}{4}(1+x)^{-\frac{3}{2}} \rightarrow & f''(0)&=-\frac{1}{4}(1+0)^{-\frac{3}{2}}=-\frac{1}{4} \\ f'''(x)&=\frac{3}{8}(1+x)^{-\frac{5}{2}} & f'''(0)&=\frac{3}{8}(1+0)^{-\frac{5}{2}}=\frac{3}{8} \end{aligned}$$

$$\begin{aligned} T_3(x)&=f(0)+f'(0)(x-0)+\frac{f''(0)(x-0)^2}{2!}+\frac{f'''(0)(x-0)^3}{3!} \\ &=1+\frac{1}{2}x+\frac{\left(-\frac{1}{4}\right)}{2!}x^2+\frac{\left(\frac{3}{8}\right)}{3!}x^3 \\ &=1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3 \end{aligned}$$

(a) $1+\frac{1}{2}x-\frac{1}{4}x^2+\frac{3}{8}x^3$

(b) $1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3$

(c) $1-\frac{1}{2}x-\frac{1}{8}x^2-\frac{1}{16}x^3$

(d) $1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{8}x^3$

(e) $1-\frac{1}{2}x+\frac{1}{4}x^2-\frac{3}{8}x^3$

8. Which of the following series is divergent?

$$\lim_{n\to\infty}\frac{n}{\sqrt{4n^2-1}}\sim\lim_{n\to\infty}\frac{n}{\sqrt{4n^2}}=\lim_{n\to\infty}\frac{n}{2|n|}=\frac{1}{2}\neq 0$$

(a) $\sum_{n=1}^{\infty}\frac{1}{n^2}$ (b) $\sum_{n=1}^{\infty}\frac{1}{n^2+n}$ (c) $\sum_{n=1}^{\infty}\frac{n}{n^3+1}$

(d) $\sum_{n=1}^{\infty}\frac{n}{\sqrt{4n^2-1}}$ (e) None of these

9. Which one of the following series is convergent?

$$\sum_{n=1}^{\infty}\frac{2}{n^2-5}\sim\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$\lim_{n\to\infty}\frac{\left(\frac{1}{n^2}\right)}{\left(\frac{2}{n^2-5}\right)}=\lim_{n\to\infty}\frac{1}{n^2}\cdot\frac{n^2-5}{2}=\frac{1}{2}$. Since $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is a convergent *p*-series, $p=2$, by the Limit Comparison Test, $\sum_{n=1}^{\infty}\frac{2}{n^2-5}$ also converges.

(a) $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$ (b) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ (c) $\sum_{n=1}^{\infty}\frac{1}{n}$

(d) $\sum_{n=1}^{\infty}\frac{1}{10n-1}$ (e) $\sum_{n=1}^{\infty}\frac{2}{n^2-5}$

10. Which of the following statements are false?

(a) $\sum_{n=1}^{\infty}a_n=\sum_{n=k}^{\infty}a_n$ where k is any positive integer.

(b) If $\sum_{n=1}^{\infty}a_n$ converges, then so does $\sum_{n=1}^{\infty}c\cdot a_n$, where $c\neq 0$.

(c) $\sum_{n=1}^{\infty}a_n$ and $\sum_{n=1}^{\infty}b_n$ converge, so does $\sum_{n=1}^{\infty}(c\cdot a_n+b_n)$ where $c\neq 0$

(d) If 1000 terms are added to a convergent series, the new series also converges.

(e) Rearranging the terms of a positive convergent series will not affect its convergence or sum.

11. The series $(x-2)+\frac{(x-2)^2}{4}+\frac{(x-2)^3}{9}+\frac{(x-2)^4}{16}+\cdots$ converges for

$$(x-2)+\frac{(x-2)^2}{4}+\frac{(x-2)^3}{9}+\frac{(x-2)^4}{16}+\cdots=\sum_{n=1}^{\infty}\frac{(x-2)^n}{n^2}$$

$\lim_{n\to\infty}\left\|\frac{a_{n+1}}{a_n}\right\|=\lim_{n\to\infty}\left\|\frac{\frac{(x-2)^{n+1}}{(n+1)^2}}{\frac{(x-2)^n}{n^2}}\right\|$ $=\lim_{n\to\infty}\left\|\frac{(x-2)^{n+1}}{(n+1)^2}\cdot\frac{n^2}{(x-2)^n}\right\|$ $=\lim_{n\to\infty}\left\|\frac{n^2}{(n+1)^2}\cdot\frac{(x-2)^{n+1}}{(x-2)^n}\right\|$ $=\lim_{n\to\infty}\left\|\frac{n^2}{(n+1)^2}\right\|\cdot\lim_{n\to\infty}\left\|\frac{(x-2)^n(x-2)}{(x-2)^n}\right\|$ $=\lim_{n\to\infty}\left\|\frac{(x-2)^n(x-2)}{(x-2)^n}\right\|$ $=\|x-2\|$	$\|x-2\|<1$ $-1<x-2<1$ $1<x<3$
$\sum_{n=1}^{\infty}\frac{(1-2)^n}{n^2}=\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ Since $\lim_{n\to\infty}\frac{1}{n^2}=0$, by the Alternating Series Test $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ converges.	$\sum_{n=1}^{\infty}\frac{(3-2)^n}{n^2}=\sum_{n=1}^{\infty}\frac{(1)^n}{n^2}=\sum_{n=1}^{\infty}\frac{1}{n^2}$ This is a convergent *p*-series, $p=2$.

(a) $1\le x\le 3$ (b) $1\le x<3$ (c) $1<x\le 3$
(d) $0\le x\le 4$ (e) None of these

12. The radius of convergence of the series $\frac{x}{4}+\frac{x^2}{4^2}+\frac{x^3}{4^3}+\cdots+\frac{x^n}{4^n}+\cdots$ is

$$\frac{x}{4}+\frac{x^2}{4^2}+\frac{x^3}{4^3}+\cdots+\frac{x^n}{4^n}+\cdots=\sum_{n=1}^{\infty}\frac{x^n}{4^n}$$

$$\lim_{n\to\infty}\sqrt[n]{\left|\frac{x^n}{4^n}\right|}=\left|\frac{x}{4}\right|$$

$$\left|\frac{x}{4}\right|<1$$

$$\frac{|x|}{4}<1$$

$$|x|<4$$

(a) 0 (b) 1 (c) 2
(d) 4 (e) All real numbers

13. Which of the following series are conditionally convergent?

I. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n+1}$

II. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{\cos(n)}{3^n}$

III. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$

$\displaystyle\sum_{n=1}^{\infty}\left\lvert(-1)^{n+1}\frac{1}{2n+1}\right\rvert=\sum_{n=1}^{\infty}\frac{1}{2n+1}\sim\sum_{n=1}^{\infty}\frac{1}{n}$ $\displaystyle\lim_{n\to\infty}\frac{\left(\frac{1}{n}\right)}{\left(\frac{1}{2n+1}\right)}=\lim_{n\to\infty}\frac{1}{n}\cdot\frac{2n+1}{1}=1$ Since $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$ diverges (Harmonic Series), by the Limit Comparison Test, so does $\displaystyle\sum_{n=1}^{\infty}\left\lvert(-1)^{n+1}\frac{1}{2n+1}\right\rvert$	$\displaystyle\sum_{n=1}^{\infty}\left\lvert(-1)^{n+1}\frac{1}{\sqrt{n}}\right\rvert=\sum_{n=1}^{\infty}\frac{1}{n^{0.5}}$ This is a divergent p-series $p=0.5$
Since $\displaystyle\lim_{n\to\infty}\frac{1}{2n+1}=0$, by the Alternating Series Test, $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n+1}$ converges.	Since $\displaystyle\lim_{n\to\infty}\frac{1}{\sqrt{n}}=0$, by the Alternating Series Test, $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$ converges.
Therefore $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n+1}$ is conditionally convergent	Therefore $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$ is conditionally convergent.
$\displaystyle\sum_{n=1}^{\infty}\left\lvert(-1)^{n}\frac{\cos(n)}{3^n}\right\rvert=\sum_{n=1}^{\infty}\frac{\lvert\cos(n)\rvert}{3^n}\le\sum_{n=1}^{\infty}\frac{1}{3^n}$. Since $\displaystyle\sum_{n=1}^{\infty}\frac{1}{3^n}$ is a convergent Geometric series, $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{\cos(n)}{3^n}$ is absolutely convergent.	

(a) I only (b) II only (c) I, II, and III
(d) I and III only (e) I and II only

14. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor Series about $x=0$ for which of the following functions?

$$\underset{\substack{\text{centered} \\ x=0}}{e^x} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\underset{\substack{\text{centered} \\ x=0}}{e^{-x}} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$$

(a) $\sin(x)$ (b) $\cos(x)$ (c) e^x

(d) e^{-x} (e) $\ln(1+x)$

15. $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{2n} =$

$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{2n} = \sum_{n=1}^{\infty} \left(\frac{1}{9}\right)^{n}$ is a Geometric Series

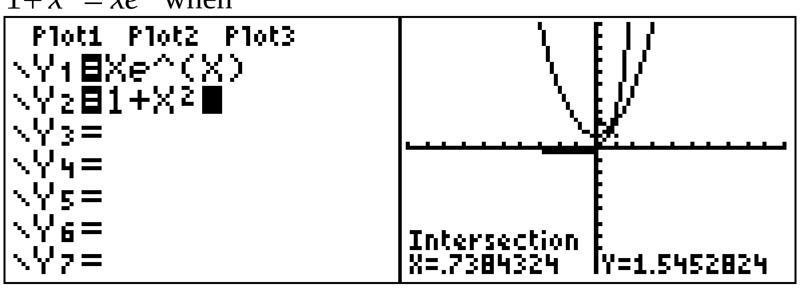
$$\sum_{n=1}^{\infty} \left(\frac{1}{9}\right)^{n} = \frac{\text{first term}}{1-\text{common ratio}} = \frac{\frac{1}{9}}{1-\frac{1}{9}} = \frac{\frac{1}{9}}{\frac{8}{9}} = \frac{1}{8}$$

(a) $\frac{1}{8}$ (b) $\frac{1}{3}$ (c) 1 (d) $\frac{9}{8}$ (e) ∞

16. Calculator required: The graph of the function represented by the Maclaurin series $x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$ intersects the graph of $y = 1 + x^2$ at the point where $x =$

$$x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!} = \sum_{n=0}^{\infty} \frac{x^n \cdot x}{n!} = x \cdot \sum_{n=0}^{\infty} \frac{x^n}{n!} = xe^x$$

$1 + x^2 = xe^x$ when



(a) 0.718 (b) 0.738 (c) 0.758
(d) 0.778 (e) 0.798