Definite Integrals with u-substitution:

If $u = g(x)$, then $\int_a^b f\left(\underbrace{g(x)}_{u}\right)\cdot \underbrace{g'(x)\,dx}_{du} \to \int_{u(a)}^{u(b)} f(u)\,du$.

Example: Using the substitution $u = \frac{2x}{3}$, $\int_0^{\frac{\pi}{2}} \cos\left(\frac{2x}{3}\right)dx =$

$u = \frac{2x}{3}$ $\quad u(0) = 0$ $du = \frac{2}{3}dx$ $\quad u\left(\frac{\pi}{2}\right) = \frac{\pi}{3}$	$\int_0^{\frac{\pi}{2}} \cos\left(\frac{2x}{3}\right)dx$ $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos\left(\frac{2x}{3}\right)\cdot\frac{2}{3}dx$ $\downarrow$ $\frac{3}{2}\int_{u(0)}^{u\left(\frac{\pi}{2}\right)} \cos(u)\,du$ $\frac{3}{2}\int_0^{\frac{\pi}{3}} \cos(u)\,du$	in terms of x $\quad$ Must be in terms of u $\int_0^{\frac{\pi}{2}} \cos\left(\frac{2x}{3}\right)dx = \frac{3}{2}\int_{\square}^{\square} \cos(u)\,du$

1. Using the substitution $u = x^2 + 1$, $\int_{-1}^{1} x\left(x^2+1\right)^3 dx =$

2. Using the substitution $u = x^3 + 8$, $\int_{-2}^{4} x^2\left(x^3+8\right)^2 dx =$

3. Using the substitution $u = x^3 + 1$, $\int_{1}^{2} 2x^2\sqrt{x^3+1}\,dx =$

4. Using the substitution $u = 1 - x^2$, $\int_{0}^{1} x\sqrt{1-x^2}\,dx =$

5. Using the substitution $u = 2x + 1$, $\int_{0}^{4} \frac{1}{\sqrt{2x+1}}\,dx =$

6. Using the substitution $u = 1 + 2x^2$, $\int_{0}^{2} \frac{x}{\sqrt{1+2x^2}}\,dx =$

7. Using the substitution $u = 1 + \sqrt{x}$, $\int_{1}^{9} \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}\,dx =$

8. Using the substitution $u = 4 + x^2$, $\int_{0}^{2} x\sqrt[3]{4+x^2}\,dx =$

$\int_{-1}^{1} x\left(x^2+1\right)^3 dx =$ $u = x^2+1 \quad u(-1)=2$ $du = 2xdx \quad u(1)=2$ $\int_{-1}^{1} x\left(x^2+1\right)^3 dx$ $\frac{1}{2}\int_{-1}^{1}\left(x^2+1\right)^3 \cdot 2xdx$ $\downarrow$ $\frac{1}{2}\int_{u(-1)}^{u(1)} u^3 du$ $\frac{1}{2}\int_{2}^{2} u^3 du$	$\int_{-2}^{4} x^2\left(x^3+8\right)^2 dx =$ $u = x^3+8 \quad u(-2)=0$ $du = 3x^2dx \quad u(4)=72$ $\int_{-2}^{4} x^2\left(x^3+8\right)^2 dx$ $\frac{1}{3}\int_{-2}^{4}\left(x^3+8\right)^2 3x^2dx$ $\frac{1}{3}\int_{u(-2)}^{u(4)} u^2 du$ $\frac{1}{3}\int_{0}^{72} u^2 du$	$\int_{1}^{2} 2x^2\sqrt{x^3+1}dx =$ $u = x^3+1 \quad u(1)=2$ $du = 3x^2dx \quad u(2)=9$ $\int_{1}^{2} 2x^2\sqrt{x^3+1}dx$ $\frac{2}{3}\int_{1}^{2}\sqrt{x^3+1}\cdot 3x^2dx$ $\downarrow$ $\frac{2}{3}\int_{u(1)}^{u(2)}\sqrt{u}du$ $\frac{2}{3}\int_{2}^{9}\sqrt{u}du$	$\int_{0}^{1} x\sqrt{1-x^2}dx =$ $u = 1-x^2 \quad u(0)=1$ $du = -2xdx \quad u(1)=0$ $\int_{0}^{1} x\sqrt{1-x^2}dx$ $-\frac{1}{2}\int_{0}^{1}\sqrt{1-x^2}\left(-2x\right)dx$ $\downarrow$ $-\frac{1}{2}\int_{u(0)}^{u(1)}\sqrt{u}du$ $-\frac{1}{2}\int_{1}^{0}\sqrt{u}du$
$\int_{0}^{4}\frac{1}{\sqrt{2x+1}}dx =$ $u = 2x+1 \quad u(0)=1$ $du = 2dx \quad u(4)=9$ $\int_{0}^{4}\frac{1}{\sqrt{2x+1}}dx$ $\frac{1}{2}\int_{0}^{4}\frac{1}{\sqrt{2x+1}}\cdot 2dx$ $\downarrow$ $\frac{1}{2}\int_{u(0)}^{u(4)}\frac{1}{\sqrt{u}}du$ $\frac{1}{2}\int_{1}^{9}\frac{1}{\sqrt{u}}du$	$\int_{0}^{2}\frac{x}{\sqrt{1+2x^2}}dx =$ $u = 1+2x^2 \quad u(0)=1$ $du = 4xdx \quad u(2)=9$ $\int_{0}^{2}\frac{x}{\sqrt{1+2x^2}}dx$ $\frac{1}{4}\int_{0}^{2}\frac{1}{\sqrt{1+2x^2}}4xdx$ $\downarrow$ $\frac{1}{4}\int_{u(0)}^{u(2)}\frac{1}{\sqrt{u}}du$ $\frac{1}{4}\int_{1}^{9}\frac{1}{\sqrt{u}}du$	$\int_{1}^{9}\frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}dx =$ $u = 1+\sqrt{x} \quad u(1)=2$ $du = \frac{1}{2\sqrt{x}}dx \quad u(9)=4$ $\int_{1}^{9}\frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}dx$ $2\int_{1}^{9}\frac{1}{\left(1+\sqrt{x}\right)^2}\cdot\frac{1}{2\sqrt{x}}dx$ $2\int_{u(1)}^{u(9)}\frac{1}{u^2}du$ $2\int_{2}^{4}\frac{1}{u^2}du$	$\int_{0}^{2} x\sqrt[3]{4+x^2}dx =$ $u = 4+x^2 \quad u(0)=4$ $du = 2xdx \quad u(2)=8$ $\int_{0}^{2} x\sqrt[3]{4+x^2}dx$ $\frac{1}{2}\int_{0}^{2}\sqrt[3]{4+x^2}\cdot 2xdx$ $\downarrow$ $\frac{1}{2}\int_{u(0)}^{u(2)}\sqrt[3]{u}du$ $\frac{1}{2}\int_{4}^{8}\sqrt[3]{u}du$