Note: These solutions were originally done using a different way to express the conservation of energy as an equation. The markups change the solutions to the notation we are using.

Standard Problems 8. Conservation of Mechanical Energy

1)~~Introductory problem.~~

W_other

$$E_f = E_i + \cancel{W_{diss} + W_{ext,non-diss}}$$

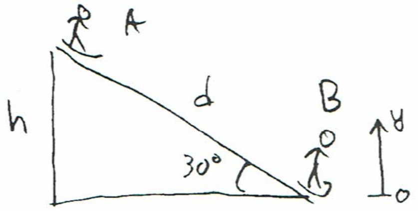
$$W = \int \overrightarrow{F_{net}} \cdot \overrightarrow{ds} = \int_{q_A}^{q_B} \overrightarrow{F_{net}} \cdot (x'(q)\hat{\imath} + y'(q)\hat{\jmath})dq$$

mgh

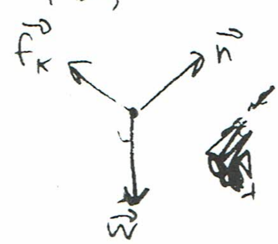
$$K = \frac{1}{2}mv^2 \qquad U_g = \cancel{[illegible]h} \qquad U_s = \frac{k}{2}x^2$$

A 30 kg child sits on a sled of negligible mass at the top of a straight snowy slope 10 m in elevation. The slope has a steepness of 30 degrees above the horizontal, so the length of the slope itself is 20 m. The child slides down the slope on the sled, and as she does so, a constant 50 N kinetic frictional force is exerted on her. What is her speed when she reaches the bottom of the slope?

Use conservation of energy to solve this problem. Take you system to be the child and Earth.



FBD, Child



$\vec{n}$ does no work (⊥ to path)

"other"

$\vec{f_K}$ does ~~dissipative~~ work

$\vec{w}$ does (internal) conservative work, so we'll treat that as Pot. energy.

$E_f = U_f + K_f = \frac{1}{2}mv_f^2 + mg\cancel{h_f}^0$

$E_i = U_i + K_i = mgh_i + \frac{1}{2}m\cancel{v_i}^0$

$W_{\cancel{diss}} = W_{f_K} = -f_K d$ since f_K is constant and opposite the displacement.

other

~~Using a temporary tilted coord system~~

~~$W_{ext, non-diss} = 0$ (none of these forces do work)~~ so

Note that $\sin 30 = \frac{h}{d} = 0.5$

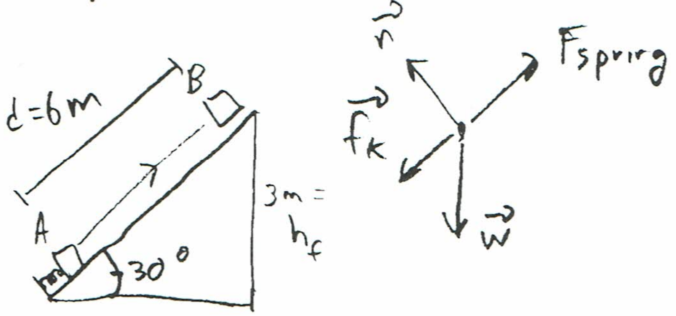
so $d = 2h$

$\frac{1}{2}mv_f^2 = mgh_i + (-f_K d)$

$\Rightarrow v_f = \sqrt{\frac{2mgh_i - 2f_K d}{m}} = \sqrt{2gh_i - \frac{4f_K h}{m}} = 11.4$ m/s

this is > 20 mph, so hopefully the kid is wearing a helmet!

2) (System = block + spring + Earth)

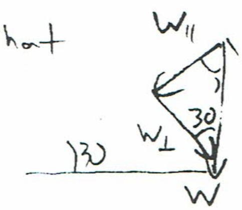


$\vec{n}$ does no work

$\vec{F}_{spring}$ and $\vec{w}$ are conservative: treat with potential energy ~~internal and non-diss so they don't matter~~

$\vec{f}_K$ is dissipative

To find f_K, note that



$W_\perp = w\cos 30$

and $n = W_\perp$ b/c the block stays on the ramp.

so $f_K = \mu_K mg\cos 30$

So $W_{\cancel{\text{diss}}\ \text{other}} = W_{fk}$

$= -(\mu_K mg\cos 30)(d)$

$E_f = \frac{1}{2}mv_f^2 + mgh_f + \cancel{U_{s,f}}^{0}$ (spring @ equilibrium)

$E_i = \frac{1}{2}\cancel{mv_i^2}^{0} + \cancel{mgh_i}^{0} + U_{spring,i}$

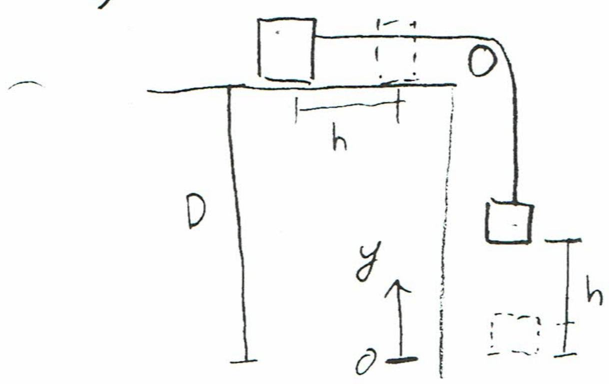
so $\frac{1}{2}mv_f^2 + mgh_f = U_{spring,i} - \mu_K mgd\cos 30$

$\Rightarrow U_{spring,i} = \frac{1}{2}mv_f^2 + mgh_f + \mu_K mg\,d\cos 30$

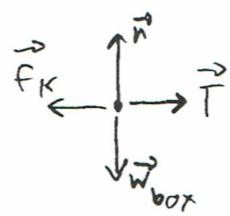
$= 118\,J$

3)

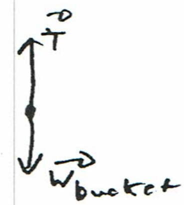
(System = both box and bucket, rope, and Earth.)



Box :



Bucket



$\vec{n}$ and $\vec{w}_{box}$ do no work ($\perp$ to motion of box)

~~The Tensions are internal and non-dissipative so while they do work, that work does not factor into our energy cons equation~~

The tensions will do work, but however much work one does, the other will do negative the same amount. So overall they do no net work.

$\vec{f}_K$ is ~~dissipative~~ "other"

$\vec{w}_{bucket}$ is ~~also internal and non-dissipative~~ conservative

$$E_f = K_f + U_f = \frac{1}{2}(m_{box} + m_{bucket}) v_f^2 + U_{g\,box} + U_{g\,bucket}$$

$$= \frac{1}{2}(m_{box} + m_{bucket}) v_f^2 + m_{box}\, g D + m_{bucket}\, g (0\,m)$$

$$E_i = K_i + U_i = \underset{\text{Not moving}}{0} + U_{g\,box} + U_{g\,bucket} = m_{box} g D + m_{bucket} g h$$

$$W_{diss} = W_{f_K} = -f_K h = -\mu_K m_{box} g h$$

$W_{ext,\,non\text{-}diss} = 0$ so

$$\frac{1}{2}(m_{box} + m_{bucket}) v_f^2 + m_{box} g D = m_{box} g D + m_{bucket} g h - \mu_K m_{box} g h$$

$$\Rightarrow v_f = \sqrt{\frac{2(m_{bucket} g h - \mu_K m_{box} g h)}{m_{box} + m_{bucket}}} = 2.99\ m/s$$

Note that, had we chosen our system as just the box or just the bucket, we would have had to know the tension force. By including both in our system, we can avoid looking at internal non-dissipative forces