AP Calculus BC
Practice Taylor Series Test
Spring 2017

Name:________________________________

Date:___________________ Period:_______

No Calculator Permitted

1. Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(a) $\lim_{n\to\infty} \frac{e}{n!} < 1$

(b) $\lim_{n\to\infty} \frac{n!}{e} < 1$

(c) $\lim_{n\to\infty} \frac{n+1}{e} < 1$

(d) $\lim_{n\to\infty} \frac{e}{n+1} < 1$

(e) $\lim_{n\to\infty} \frac{e}{(n+1)!} < 1$

2. Which of the following series converges for all real numbers x?

(a) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(b) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(c) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

(d) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$

(e) $\sum_{n=1}^{\infty} \frac{n! x^n}{e^n}$

3. What are all values of x for which the series $\sum_{n=1}^{\infty} \left(\frac{2}{x^2+1}\right)^n$ converges?

(a) $-1 < x < 1$

(b) $x > 1$ only

(c) $x \geq 1$ only

(d) $x < -1$ and $x > 1$ only

(e) $x \leq -1$ and $x \geq 1$

4. What is the sum of the series $1 + \ln(2) + \frac{(\ln(2))^2}{2!} + \cdots + \frac{(\ln(2))^n}{n!} + \cdots$

(a) $\ln(2)$

(b) $\ln(1+\ln(2))$

(c) 2

(d) e^2

(e) The series diverges

5. $\sum_{n=1}^{\infty} a_n$ diverges and $0 \leq a_n \leq b_n$ for all n, which of the following statements must be true?

(a) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges

(b) $\sum_{n=1}^{\infty} (-1)^n b_n$ converges

(c) $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges

(d) $\sum_{n=1}^{\infty} b_n$ converges

(e) $\sum_{n=1}^{\infty} b_n$ diverges

AP Calculus BC
Chapter 9 Test
Spring 2012

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6. Let $f(x) = \ln\left(1 + x^3\right)$

(a) [5 points] The Maclaurin series for $\ln(1+x)$ is $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + \frac{(-1)^{n+1} x^n}{n} + \cdots$. Use the series to write the first four nonzero terms and the general term of the Maclaurin series for *f*.

(b) [4 points] The radius of convergence of the Maclaurin series for *f* is 1. Determine the interval of convergence. Show the work that leads to your answer.

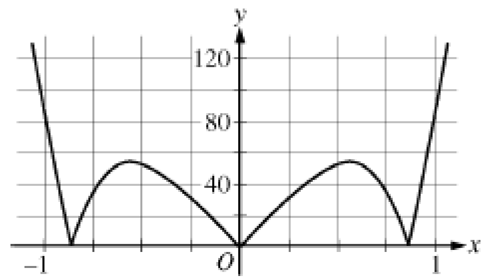
6 Continued

Let $f(x)=\ln\left(1+x^3\right)$ and $\ln(1+x)=x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-\dfrac{x^4}{4}+\cdots+\dfrac{(-1)^{n+1}x^n}{n}+\cdots$

(c) [6 points] Write the first four nonzero terms of the Maclaurin series for $f'\left(t^2\right)$. If $g(x)=\int_0^x f'\left(t^2\right)dt$ use the first two nonzero terms of the Maclaurin series for g to approximate $g(1)$.

(d) [3 points] The Maclaurin series for g, evaluated at $x=1$, is a convergent alternating series with individual terms that decrease in absolute value to 0. Show that your approximation in part (c) must differ from $g(1)$ by less than $\dfrac{1}{5}$.

7. Let $f(x)=\sin\left(x^2\right)+\cos(x)$. The graph of $y=\left|f^{(5)}(x)\right|$ is shown at right.

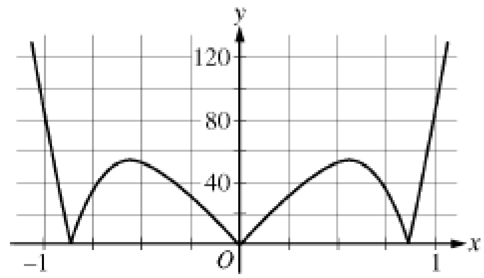


Graph of $y=\left|f^{(5)}(x)\right|$

(a) [6 points] Write the first four nonzero terms of the Taylor series for $\sin(x)$ about $x=0$, and write the first four nonzero terms of the Taylor series for $\sin\left(x^2\right)$ about $x=0$.

(b) [6 points] Write the first four nonzero terms of the Taylor series for $\cos(x)$ about $x=0$. Use this series and the series for $\sin\left(x^2\right)$, found in part (a), to write the first four nonzero terms of the Taylor series for *f* about $x=0$.

7 Continued



Graph of $y = \left|f^{(5)}(x)\right|$

(c) [3 points] Find the value of $f^{(6)}(0)$.

(d) [3 points] Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Using information from the graph of $y = f^{(5)}(x)$ shown at right, show that $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| \le \frac{1}{3000}$.