The radius of convergence of $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-2)^n}{n\cdot 3^n}$ is

$$\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+2}(x-2)^{n+1}}{(n+1)\cdot 3^{n+1}}}{\frac{(-1)^{n+1}(x-2)^n}{n\cdot 3^n}}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+2}(x-2)^{n+1}n\cdot 3^n}{(n+1)\cdot 3^{n+1}(-1)^{n+1}(x-2)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(x-2)^{n+1}n\cdot 3^n}{(n+1)\cdot 3^{n+1}(x-2)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(x-2)n\cdot 3^n}{(n+1)\cdot 3^{n+1}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(x-2)n}{(n+1)\cdot 3}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(x-2)}{3}\right|$$

$$\downarrow$$

$$\left|\frac{(x-2)}{3}\right| < 1$$

$$|x-2| < 3$$

Whenever you use the ratio or root test and get to a point that looks like $|x-c| < r$, you know that the center of the interval is at $x = c$ and the radius is *r*.

The radius of convergence is 3. The Taylor Series is centered at $x = 2$. Therefore we know that the interval of convergence will be one of the following:

$(2-3, 2+3)$ $\quad$ $(-1,5]$ $\quad$ $[-1,5)$ $\quad$ $[-1,5]$

$(-1,5)$

The only way to test whether or not the endpoint is to be included is to substitute the value of x into the original series and see if that resulting series converges:

At $x=-1$

$$\begin{aligned}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1-2)^n}{n\cdot 3^n} &= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-3)^n}{n\cdot 3^n}\\ &= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1)^n(3)^n}{n\cdot 3^n}\\ &= \sum_{n=1}^{\infty}\frac{(-1)^{2n+1}\cancel{3^n}}{n\cdot \cancel{3^n}}\\ &= \sum_{n=1}^{\infty}\frac{(-1)^{2n}(-1)}{n}\\ &= \sum_{n=1}^{\infty}\frac{\left[(-1)^2\right]^n(-1)}{n}\\ &= \sum_{n=1}^{\infty}-\frac{1}{n}\end{aligned}$$

Since the resulting series is the opposite of the Harmonic series, $x=-1$ is not included in the interval of convergence.

At $x=5$

$$\begin{aligned}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-1-2)^n}{n\cdot 3^n} &= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(3)^n}{n\cdot 3^n}\\ &= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cdot 3^n}{n\cdot 3^n}\\ &= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cancel{3^n}}{n\cdot \cancel{3^n}}\\ &= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\end{aligned}$$

This series *does* converge because it is passes the Alternating Series Test.

Therefore the interval of convergence of $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-2)^n}{n\cdot 3^n}$ is $(-1,5]$.