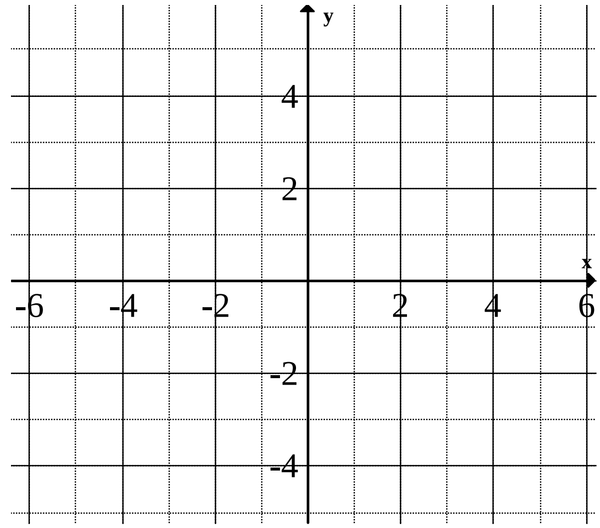
When the First Derivative Test Fails, and the Second Derivative Test Succeeds: Graphing Relations and Implicit Differentiation

Ellipse: $\frac{x^2}{25}+\frac{y^2}{4}=1$



Consider the point $(0,2)$ on the graph of the ellipse at right. Visually we can identify this point as a relative maximum of the ellipse. To justify this claim using calculus, we use the Second Derivative Test.

$$\frac{x^2}{25}+\frac{y^2}{4}=1$$

$$\downarrow$$

$$\frac{2}{25}x+\frac{1}{2}yy'=0$$

$$\frac{1}{2}yy'=-\frac{2}{25}x$$

$$y'=-\frac{4}{25}\cdot\boxed{\frac{x}{y}}$$

Since y' involves both <u>x and y</u>, you cannot make a sign chart – the First Derivative Test is not applicable. Since $y'\big|_{(0,2)}=-\frac{4}{25}\cdot\frac{0}{2}=0$ and $y''\big|_{(0,2)}<0$ we can justify that the graph has a relative maximum at $(0,2)$ by the Second Derivative Test.

$$y'=-\frac{4}{25}\cdot\frac{x}{y}$$

$$=-\frac{4}{25}xy^{-1}$$

$$\downarrow$$

$$y''=-\frac{4}{25}\left(y^{-1}-xy^{-2}y'\right)$$

$$=-\frac{4}{25}\left(y^{-1}-xy^{-2}\left[-\frac{4}{25}\cdot\frac{x}{y}\right]\right)$$

$$y''\big|_{(0,2)}=-\frac{4}{25}\left(2^{-1}-(0)(2)^{-2}\left[-\frac{4}{25}\cdot\frac{0}{2}\right]\right)$$

$$=-\frac{4}{25}\left(\frac{1}{2}+0\right)$$

$$<0$$