1. The position of a particle in the *xy*-plane is given by $x(t)=4t^2$ and $y(t)=\sqrt{t}$. At $t=4$, the acceleration vector is

$$\begin{aligned}\vec{a} &= \langle x''(t), y''(t)\rangle \\ &= \left\langle 8, -\frac{1}{4}t^{-\frac{3}{2}}\right\rangle \\ \vec{a}(4) &= \langle x''(4), y''(4)\rangle \\ &= \left\langle 8, -\frac{1}{32}\right\rangle\end{aligned}$$

(a) $\left\langle 8, -\frac{1}{64}\right\rangle$ (b) $\left\langle 8, -\frac{1}{32}\right\rangle$ (c) $\left\langle 8, \frac{1}{32}\right\rangle$ (d) $\left\langle 32, -\frac{1}{32}\right\rangle$ (e) $\left\langle 32, \frac{1}{4}\right\rangle$

2. The velocity of an object is given by $v(t)=\langle 3\sqrt{t}, 4\rangle$. If the object is at the origin when $t=1$, where was it at $t=0$?

x-coordinate	*y*-coordinate
$x(0)=x(1)+\int_1^0 x'(t)\,dt$ $=0+\int_1^0 3\sqrt{t}\,dt$ $=\int_1^0 3t^{\frac{1}{2}}\,dt$ $=\left[2t^{\frac{3}{2}}\right]_1^0$ $=0-2$ $=-2$	$y(0)=y(1)+\int_1^0 y'(t)\,dt$ $=0+\int_1^0 4\,dt$ $=[4t]_1^0$ $=0-4$ $=-4$

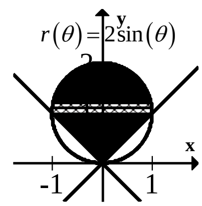
(a) $(-3,-4)$ (b) $(-2,-4)$ (c) $(2,4)$ (d) $\left(\frac{3}{2},0\right)$ (e) $\left(-\frac{3}{2},0\right)$

3. A curve in the xy-plane is defined by the parametric equations $x(t)=t^3+2$ and $y(t)=t^2-5t$. What is the slope of the line tangent to the curve at the point where $x=10$?

$$\begin{array}{c} x(t)=10 \\ \downarrow \\ t=2 \end{array} \qquad \begin{aligned} \frac{dy}{dx} &= \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{2t-5}{3t^2} \\ \left.\frac{dy}{dx}\right|_{t=2} &= \frac{2(2)-5}{3(2)^2} \\ &= -\frac{1}{12} \end{aligned}$$

(a) -12 (b) $-\frac{3}{5}$ (c) $-\frac{1}{8}$ (d) $-\frac{1}{12}$ (e) None of these.

4. The area inside the circle with polar equation $r(\theta)=2\sin(\theta)$ and above the lines with equations $y=x$ and $y=-x$ is given by



$$\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\left[2\sin(\theta)\right]^2 d\theta = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 4\sin^2(\theta)\,d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 2\sin^2(\theta)\,d\theta$$

(a) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 2\sin^2(\theta)\,d\theta$ (b) $\int_{-1}^{1} 2\sin(\theta)\,d\theta$ (c) $\int_{-1}^{1} 2\sin^2(\theta)-1\,d\theta$

(d) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin(\theta)\,d\theta$ (e) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 2\sin^2(\theta)\,d\theta$

5. Find the points on the parametric curve defined by $x(t)=t^3-3t+1$ and $y(t)=t^3-3t^2+1$ where the line tangent to the curve is horizontal

$$\text{horizontal tangent} \rightarrow \frac{dy}{dx}=0 \rightarrow \frac{dy}{dt}=0$$

$y(t)=t^3-3t^2+1$ $y'(t)=3t^2-6t=3t(t-2)$ $y'(t)=0 \rightarrow t=0,2$	$x(0)=1 \quad y(0)=1$ $x(2)=3 \quad y(2)=-3$

(a) $(1,1),(3,-3)$ (b) $(-3,3)$ only (c) $(-1,1),(3,-3)$ (d) $(0,0),(3,-3)$ (e) None of these

6. Find the length of the parametric curve defined by $x(t)=3t^2$ and $y(t)=2t^3$ for $0 \le t \le 1$.

$$\begin{aligned}
\text{length} &= \int_{t_1}^{t_2} \sqrt{\left[x'(t)\right]^2+\left[y'(t)\right]^2}\,dt \\
&= \int_0^1 \sqrt{[6t]^2+\left[6t^2\right]^2}\,dt \\
&= \int_0^1 \sqrt{36t^2+36t^4}\,dt \\
&= \int_0^1 6t\sqrt{1+t^2}\,dt \\
&= \int_0^1 6t\left(1+t^2\right)^{\frac{1}{2}}\,dt \\
&= \left[2\left(1+t^2\right)^{\frac{3}{2}}\right]_0^1 \\
&= \left[2\left(1+(1)^2\right)^{\frac{3}{2}}\right]-\left[2\left(1+(0)^2\right)^{\frac{3}{2}}\right] \\
&= 4\sqrt{2}-2
\end{aligned}$$

(a) $4\sqrt{2}-2$ (b) $2\sqrt{2}-2$ (c) $4\sqrt{2}$ (d) $4\sqrt{2}-1$ (e) None of these

7. Find the length of the polar curve $r(\theta)=7\cos(\theta)$ for $0\le\theta\le\frac{3\pi}{4}$

$$\int_{\theta_1}^{\theta_2}\sqrt{[r'(\theta)]^2+[r(\theta)]^2}\,d\theta=\int_0^{\frac{3\pi}{4}}\sqrt{[-7\sin(\theta)]^2+[7\cos(\theta)]^2}\,d\theta$$

$$=\int_0^{\frac{3\pi}{4}}\sqrt{[-7\sin(\theta)]^2+[7\cos(\theta)]^2}\,d\theta$$

$$=\int_0^{\frac{3\pi}{4}}\sqrt{49\sin^2(\theta)+49\cos^2(\theta)}\,d\theta$$

$$=\int_0^{\frac{3\pi}{4}}\sqrt{49[\sin^2(\theta)+\cos^2(\theta)]}\,d\theta$$

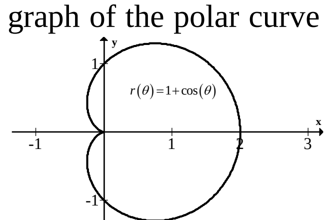
$$=\int_0^{\frac{3\pi}{4}}7\,d\theta$$

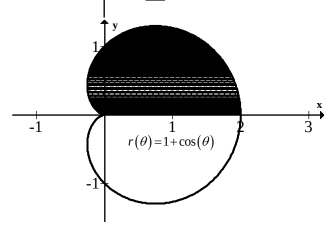
$$=[7\theta]_0^{\frac{3\pi}{4}}$$

$$=\frac{21\pi}{4}$$

(a) $\frac{21}{4}$ (b) $\frac{21\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{21\pi}{11}$ (e) None of these

8. Which of the following gives the area of the region enclosed by the graph of the polar curve $r(\theta)=1+\cos(\theta)$





$$\text{Area}=\frac{1}{2}\int_{\theta_1}^{\theta_2}[r(\theta)]^2\,d\theta$$

$$=2\left[\frac{1}{2}\int_0^{\pi}[1+\cos(\theta)]^2\,d\theta\right]$$

$$=\int_0^{\pi}[1+\cos(\theta)]^2\,d\theta$$

(a) $\int_0^{\pi}1+\cos^2(\theta)\,d\theta$ (b) $\int_0^{\pi}[1+\cos(\theta)]^2\,d\theta$ (c) $\int_0^{2\pi}1+\cos(\theta)\,d\theta$

(d) $\int_0^{2\pi}(1+\cos(\theta))^2\,d\theta$ (e) $\frac{1}{2}\int_0^{2\pi}1+\cos^2(\theta)\,d\theta$

9. Find the points of intersection of the curves $r(\theta)=2$ and $r(\theta)=4\cos(\theta)$

$$2=4\cos(\theta) \qquad r\left(\frac{\pi}{6}\right)=4\cos\left(\frac{\pi}{6}\right)=2$$
$$\frac{1}{2}=\cos(\theta) \qquad r\left(-\frac{\pi}{6}\right)=4\cos\left(-\frac{\pi}{6}\right)=2$$
$$\theta=\frac{\pi}{3},-\frac{\pi}{3}$$

(a) $\left(2,\frac{\pi}{3}\right),\left(2,-\frac{\pi}{3}\right)$ (b) $\left(2,\frac{\pi}{3}\right)$ only (c) $\left(2,\frac{\pi}{4}\right),\left(2,-\frac{\pi}{4}\right)$

(d) $\left(2,\frac{\pi}{6}\right),\left(2,-\frac{\pi}{6}\right)$ (e) $\left(2,\frac{\pi}{6}\right)$ only

10. A particle moves in the *xy*-plane so that at any time *t*, $t>0$, its coordinates are $x(t)=e^t\sin(t)$ and $y(t)=e^t\cos(t)$. The particle's velocity vector at $t=\pi$ is given by

$$\vec{v}=\langle x'(t),y'(t)\rangle$$
$$=\langle e^t\cos(t)-e^t\sin(t),e^t\sin(t)+e^t\cos(t)\rangle$$
$$\vec{v}(\pi)=\langle -e^\pi,-e^\pi\rangle$$

(a) $\langle e^\pi,-e^\pi\rangle$ (b) $\langle 0.-e^\pi\rangle$ (c) $\langle -e^\pi,e^\pi\rangle$ (d) $\langle -e^\pi,-e^\pi\rangle$ (e) $\langle e^\pi,e^\pi\rangle$

11. $\int_1^\infty x^{-\frac{5}{4}}dx$ is

$$\int_1^\infty x^{-\frac{5}{4}}dx=\lim_{b\to\infty}\int_1^b x^{-\frac{5}{4}}dx$$
$$=\lim_{b\to\infty}\left[-4x^{-\frac{1}{4}}\right]_0^b$$
$$=\lim_{b\to\infty}\left[-4(b)^{-\frac{1}{4}}\right]-\left[-4(1)^{-\frac{1}{4}}\right]$$
$$=4$$

(a) $\frac{5}{4}$ (b) $\frac{1}{4}$ (c) 4 (d) -4 (e) Does not exist

12. $\sum_{k=0}^{\infty}\left(-\frac{\pi}{3}\right)^k =$

The common ratio for the Geometric Series is $-\frac{\pi}{3}$, since $\left|-\frac{\pi}{3}\right|>1$, the series diverges.

(a) $\frac{1}{1-\frac{\pi}{3}}$ (b) $\frac{\frac{\pi}{3}}{1-\frac{\pi}{3}}$ (c) $\frac{3}{3+\pi}$ (d) $\frac{\pi}{3+\pi}$ (e) The series does not converge

13. $\lim_{h\to 0}\frac{1}{h}\int_0^h \frac{\sin^2(t)}{t^2}\,dt =$

$$\lim_{h\to 0}\frac{1}{h}\int_0^h \frac{\sin^2(t)}{t^2}\,dt = \lim_{h\to 0}\frac{\left[\int_0^h \frac{\sin^2(t)}{t^2}dt\right]}{h} \to \frac{0}{0}$$

$$\downarrow$$

$$= \lim_{h\to 0}\frac{\frac{d}{dh}\left[\int_0^h \frac{\sin^2(t)}{t^2}\,dt\right]}{\frac{d}{dh}[h]}$$

$$= \lim_{h\to 0}\frac{\left[\frac{\sin^2(h)}{h^2}\right]}{1}$$

$$= \lim_{h\to 0}\left[\frac{\sin(h)}{h}\right]^2$$

$$= \left[\lim_{h\to 0}\frac{\sin(h)}{h}\right]^2$$

$$= 1$$

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2 (e) Does not exist

14. If the substitution $u = 25 - x^2$ is made, the integral $\int_0^3 x\sqrt{25-x^2}\,dx$ is

$u = 25 - x^2 \quad u(3) = 16$ $du = -2x\,dx \quad u(0) = 25$ Since the lower bound must be 25, the answer is (b)	$\int_0^3 x\sqrt{25-x^2}\,dx = -\frac{1}{2}\int_0^3 -2x\sqrt{25-x^2}\,dx$ $= -\frac{1}{2}\int_{u(0)}^{u(3)} \sqrt{u}\,du$ $= -\frac{1}{2}\int_{25}^{16} \sqrt{u}\,du$ $= \frac{1}{2}\int_{16}^{25} \sqrt{u}\,du$

(a) $\frac{1}{2}\int_0^3 \sqrt{u}\,du$ (b) $\frac{1}{2}\int_{25}^{16} \sqrt{u}\,du$ (c) $-\frac{1}{2}\int_0^3 \sqrt{u}\,du$ (d) $\frac{1}{2}\int_{16}^{25} \sqrt{u}\,du$ (e) $2\int_{16}^{25} \sqrt{u}\,du$