

Writing Definite Integrals as Riemann Sums

$$\int_a^b f(x) dx \Leftrightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(a + k\left(\frac{b-a}{n}\right)\right) \cdot \frac{b-a}{n}$$

$$\Leftrightarrow \lim_{n \rightarrow \infty} \sum_{k=1}^n f(a + k(\Delta x)) \cdot \Delta x$$

Integral	$\Delta x = \frac{b-a}{n}$	$x \rightarrow a + k \cdot \Delta x$	Summation Notation
$\int_3^5 \frac{1}{2+x} dx$	$\frac{5-3}{n} = \frac{2}{n}$	$x \rightarrow \begin{cases} 3 + k\left(\frac{2}{n}\right) \\ 3 + \frac{2k}{n} \end{cases}$	$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left[\frac{1}{2 + \left(3 + \frac{2k}{n}\right)} \right] \cdot \frac{2}{n} = \frac{2}{n} \left[\frac{1}{5 + \frac{2}{n}} + \frac{1}{5 + \frac{4}{n}} + \dots + \frac{1}{5 + \frac{2n}{n}} \right]$
$\int_1^2 \sin(3x) dx$			
$\int_0^5 \frac{1}{1+x^2} dx$			
$\int_1^3 \sqrt{x+7} + 2 dx$			

Integral	$\Delta x = \frac{b-a}{n}$	$x \rightarrow a + k \cdot \Delta x$	Summation Notation
$\int_1^2 \sin(3x) dx$	$\Delta x = \frac{2-1}{n} = \frac{1}{n}$	$x \rightarrow \begin{cases} 1 + k\left(\frac{1}{n}\right) \\ 1 + \frac{k}{n} \end{cases}$	$\lim_{n \rightarrow \infty} \sum_{k=1}^n \sin\left(3 + \frac{3k}{n}\right) \cdot \frac{1}{n} = \frac{1}{n} \left[\left(3 + \frac{3}{n}\right) + \left(3 + \frac{6}{n}\right) + \cdots + \left(3 + \frac{3n}{n}\right) \right]$
$\int_0^5 \frac{1}{1+x^2} dx$	$\Delta x = \frac{5-0}{n} = \frac{5}{n}$	$x \rightarrow \begin{cases} 0 + k\left(\frac{5}{n}\right) \\ \frac{5k}{n} \end{cases}$	$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{1 + \left(\frac{5k}{n}\right)^2} \cdot \frac{5}{n} = \lim_{n \rightarrow \infty} \frac{5}{n} \left[\frac{1}{1 + \frac{25}{n^2}} + \frac{1}{1 + \frac{100}{n^2}} + \cdots + \frac{1}{1 + \frac{25n^2}{n^2}} \right]$
$\int_1^3 \sqrt{x+7} + 2 dx$	$\Delta x = \frac{3-1}{n} = \frac{2}{n}$	$x \rightarrow \begin{cases} 1 + k\left(\frac{2}{n}\right) \\ 1 + \frac{2k}{n} \end{cases}$	$\begin{aligned} \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\sqrt{1 + \frac{2k}{n}} + 7 + 2 \right) \cdot \frac{2}{n} &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\sqrt{8 + \frac{2k}{n}} + 2 \right) \cdot \frac{2}{n} \\ &= \lim_{n \rightarrow \infty} \left[\left(\sqrt{8 + \frac{2}{n}} + 2 \right) + \left(\sqrt{8 + \frac{4}{n}} + 2 \right) + \cdots + \left(\sqrt{8 + \frac{n}{n}} + 2 \right) \right] \end{aligned}$