<u>Guidelines for Related Rates:</u>

* Note: *<u>All angle measures must be converted to radians!!</u>*

*Remember all diagrams represent a snapshot of an instant in time. All lengths, areas, volumes, etc. that are changing are a function of time, and should be labeled as such (i.e. $h(t), V(t)$, etc.)

<u>Step 1:</u> Identify all **given quantities**, and **quantities to be determined.** Identify rates in terms of $h'(t), V'(t)$, etc. .

<u>Step 2:</u> ***<u>Draw two pictures</u>*** to model the situation - one with values for the specific instant in time, and one for any time *t* in general. Label your drawings with the known information.

<u>Step 3:</u> Identify any formulas that can relate the given information together. Think of what dimensions/volumes/etc. can be differentiated to get the desired rate. Try to reduce the general formula you will differentiate to involve as few functions as possible (mainly the desired ones). This oftentimes involves using similar triangles in some way .

Ways to Relate Quantities Together		To Reduce # Quantities
Pythagorean Theorem Volume Formula ⋮	SOH CAH TOA Area Formula ⋮	Use Similar Triangles 99% of the time

<u>SUBSTITUTE FUNCTIONS BEFORE YOU DIFFERENTIATE!!!</u>

<u>Step 4:</u> Use the chain rule & implicit differentiation to differentiate both sides of an equation with respect to *t*.

<u>Step 5:</u> Substitute any known information into the resulting equation, and solve for the desired rate of change.

<u>SUBSTITUTE VALUES AFTER YOU DIFFERENTIATE!!!</u>

<u>Step 6:</u> Make sure you identify the units of the desired rate of change.

At a sand and gravel plant, sand is falling off a conveyor belt onto a conical pile at a rate of 5 cubic feet per minute. The diameter of the base of the pile is twice the height of the pile. At what rate is the height of the pile changing when the pile is 20 feet high?

STEP 1

$$V'(t) = 5 \text{ and } h(t) = 20$$

$$2h(t) = 2r(t)$$

$$h(t) = r(t)$$

Looking for $h'(t)$

STEP 2

$$V(t) = \frac{1}{3}\pi\left[r(t)\right]^2 \cdot h(t) \quad \text{and} \quad \begin{aligned} 2h(t) &= 2r(t) \\ h(t) &= r(t) \end{aligned}$$

STEP 3: "Related" part of Related Rates

$$\begin{aligned} V(t) &= \frac{1}{3}\pi\left[r(t)\right]^2 \cdot h(t) \\ &= \frac{1}{3}\pi\left[h(t)\right]^2 \cdot h(t) \\ &= \frac{1}{3}\pi\left[h(t)\right]^3 \end{aligned}$$

$$\downarrow$$

STEP 4

$$V'(t) = \pi\left[h(t)\right]^2 \cdot h'(t)$$

$$5 = \pi\left[20\right]^2 \cdot h'(t)$$

STEP 5

$$h'(t) = \frac{1}{80\pi} \text{ ft/min}$$

$$\approx 0.003(4) \text{ ft/min}$$

STEP 6

The height of the sand pile is changing at a rate of 0.003(4) ft/min **when $\boldsymbol{h(t) = 20}$ and $\boldsymbol{V'(t) = 5}$**.