Unit Summary for Derivatives

1. The slope of a function at a given coordinate is the slope of the tangent to the function at the given coordinate.

(a) Explain in words how the slope of a function $f(x)$ at $x=c$ is **estimated**. You may include diagrams as well.

(b) Explain the process of how these estimates are improved.

2. To formally estimate the slope of a function $f(x)$ at $x=c$, the following expressions may used:

$$\lim_{h\to 0}\frac{f(c+h)-f(c)}{h} \qquad \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

Choose one expression above and explain how the notation in the chosen expression is connected to the way the slope of the tangent to $f(x)$ at $x=c$ is estimated in your answer to 1.

3. Explain in words what each expression represents visually:

$$\lim_{h\to 0^+}\frac{f(c+h)-f(c)}{h} \qquad \lim_{h\to 0^-}\frac{f(c+h)-f(c)}{h}$$

4. The derivative of $f(x)$ at $x=c$, denoted $f'(c)$, is defined as $f'(c)=\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$.

State the similarities and differences between the results/meaning of the following two expressions:

$$\lim_{h\to 0}\frac{f(c+h)-f(c)}{h} \qquad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

5. There are three different types of reasons why $f'(c)$ will not exist. Sketch three different functions for which $f'(4)$ does not exist where the reason for $f'(4)$ not existing is different from the other two graphs.

6. Give an equation of a continuous function that is not differentiable at $x=3$ and sketch its graph.

7.

(a) Explain in words why the line tangent to the function $f(x)$ at $x=c$ is a good linear approximation to $f(x)$ for *x*-values close to *c*, and not a good approximation for *x*-values far away from *c*. Include a sketched diagram to visualize your explanation.

(b) Explain how to estimate the value of $f(2)$ using the tangent to the graph of a function $f(x)$ at $x=1$.

8. Explain how to determine the units of the derivative of a function $f(x)$ given the units on the *x*-axis and the units on the *y*-axis. Sketch a made-up function $f(x)$ on a set of axes. Label the units on each axis with whatever units you like, and demonstrate what the units of $f'(x)$ will be.