

Directions for Larson Section 3-4 #27-40

“In exercises 27-40, find all relative extrema. Use the Second Derivative Test where applicable.”

Step 1: Demonstrate $f'(x)$ and determine the critical values ($f'(x) = 0$ or DNE when $x = c_1, c_2, \dots, c_n$)

Step 2: Depends on whether $f'(c_i) = 0$ or $f'(c_i)$ DNE

If $f'(c_i) = 0$		If $f'(c_i)$ DNE
Demonstrate $f''(x)$ and demonstrate the value of $f''(c_i)$		MUST make a sign chart for $f'(x)$ and use the First Derivative Test
If $f''(c_i) \neq 0$	$f''(c_i) = 0$	
<p>If $f'(c_i) = 0$ and $f''(c_i) > 0$, then $f(x)$ has a relative minimum at $x = c_i$</p> <p>If $f'(c_i) = 0$ and $f''(c_i) < 0$, then $f(x)$ has a relative maximum at $x = c_i$</p>	MUST make a sign chart for $f'(x)$ and use the First Derivative Test	

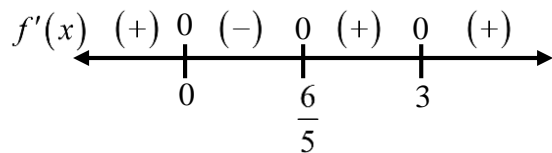
“In exercises 27-40, find all relative extrema. Use the Second Derivative Test where applicable.”

#33 $f(x) = x^2(x-3)^3$

$f'(x) = x(x-3)^2(5x-6)$	$f''(x) = 2(x-3)(10x^2 - 24x + 9)$
$f'(x) = 0$ when $x = 0, 3, \frac{6}{5}$	<p>$f''(3) = 0 \leftarrow$ Must use First Derivative Test</p> <p>$f''\left(\frac{6}{5}\right) = \frac{486}{25} = 19.44 \leftarrow$ Must use Second Derivative Test</p> <p>$f''(0) = -54 \leftarrow$ Must use Second Derivative Test</p>

$f(x)$ has a relative maximum at $x = 0$ because $f'(0) = 0$ and $f''(0) = -54 < 0$.

$f(x)$ has a relative minimum at $x = \frac{6}{5}$ because $f'\left(\frac{6}{5}\right) = 0$ and $f''\left(\frac{6}{5}\right) = 19.44 > 0$.



$f(x)$ has neither a min nor a max at $x = 3$ because $f'(x)$ does not change sign.

Extrema Decision Tree

Find the extrema of a function $f(x)$



Find the Critical Values of $f(x)$

$$f'(x) = 0 \text{ or DNE} \rightarrow x = c_1, c_2, \dots, c_i, \dots, c_n$$

Closed Interval
Absolute Min/Max



USE EVT

Test Endpoints and all critical values in the closed interval:

$$\begin{aligned} f(a) &= y_a \\ &\vdots \\ f(c_i) &= y_i \\ &\vdots \\ f(b) &= y_b \end{aligned}$$

Claim the greatest y_i value as the Absolute Max

Claim the least y_i value as the Absolute Min

*Note, the Absolute Min/Max occurs at $x = c_i$ *

State “ $f(x)$ has an absolute max/min at $(c_i, f(c_i))$ ”

Not a closed interval
Relative Min/Max

You CAN Make a Sign Chart for $f'(x)$

$$f'(c_i) = 0 \text{ or DNE}$$

$$f'(c_i) = 0$$

1st Derivative Test:
Make a labeled
Sign Chart for $f'(x)$

2nd Derivative Test:
Demonstrate the values of $f'(c_i)$ and $f''(c_i)$

- $f(x)$ has a relative max at $x = c_i$ because $f'(c_i) = 0$ and $f''(c_i) < 0$
- $f(x)$ has a relative min at $x = c_i$ because $f'(c_i) = 0$ and $f''(c_i) > 0$
- $f''(c_i) = 0 \rightarrow$ *MUST* Use First Derivative Test

→ $f(x)$ has a relative max at $x = c_i$ because $f'(x)$ goes from $+$ → $-$

→ $f(x)$ has a relative min at $x = c_i$ because $f'(x)$ goes from $-$ → $+$