$$\frac{N(x)}{D(x)}=\frac{N(x)}{(p_1x+q_1)^{n_1}\cdot(p_2x+q_2)^{n_2}\cdots\cdots(p_kx+q_k)^{n_k}\cdot(a_1x^2+b_1x+c_1)^{m}\cdot(a_2x^2+b_2x+c_2)^{m_2}\cdots\cdots(a_jx^2+b_jx+c_j)^{m_j}}$$

Each Linear Factor $(p_kx+q_k)^{n_k}$ **Contributes**

$$\frac{A_1}{(p_kx+q_k)}+\frac{A_2}{(p_kx+q_k)^2}+\cdots+\frac{A_k}{(p_kx+q_k)^{n_k}}$$

Each Irreducible Quadratic Factor Contributes

$$\frac{B_1x+C_1}{(a_jx^2+b_jx+c_j)}+\frac{B_2x+C_2}{(a_jx^2+b_jx+c_j)^2}+\cdots+\frac{B_jx+C_j}{(a_jx^2+b_jx+c_j)^{m_j}}$$

Total PFD:

$$\frac{A_{1,1}}{(p_kx+q_k)}+\frac{A_{2,1}}{(p_kx+q_k)^2}+\cdots+\frac{A_{n_1,1}}{(p_kx+q_k)^{n_1}}$$

$$+\frac{A_{1,2}}{(p_kx+q_k)}+\frac{A_{2,2}}{(p_kx+q_k)^2}+\cdots+\frac{A_{n_2,2}}{(p_kx+q_k)^{n_2}}$$

$$+$$

$$\vdots$$

$$+\frac{A_{1,k}}{(p_kx+q_k)}+\frac{A_{2,k}}{(p_kx+q_k)^2}+\cdots+\frac{A_{n_k,k}}{(p_kx+q_k)^{n_k}}$$

$$+\frac{B_{1,1}x+C_{1,1}}{(a_jx^2+b_jx+c_j)}+\frac{B_{2,1}x+C_{2,1}}{(a_jx^2+b_jx+c_j)^2}+\cdots+\frac{B_{m_1,1}x+C_{m_1,1}}{(a_jx^2+b_jx+c_j)^{m}}$$

$$+\frac{B_{1,2}x+C_{1,2}}{(a_jx^2+b_jx+c_j)}+\frac{B_{2,2}x+C_{2,2}}{(a_jx^2+b_jx+c_j)^2}+\cdots+\frac{B_{m_2,2}x+C_{m_2,2}}{(a_jx^2+b_jx+c_j)^{m_2}}$$

$$+$$

$$\vdots$$

$$+\frac{B_{1,j}x+C_{1,j}}{(a_jx^2+b_jx+c_j)}+\frac{B_{2,j}x+C_{2,j}}{(a_jx^2+b_jx+c_j)^2}+\cdots+\frac{B_{m_j,j}x+C_{m_j,j}}{(a_jx^2+b_jx+c_j)^{m_j}}$$

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx \qquad\qquad x^3+2x^2+x = x\left(x^2+2x+1\right) = x\left(x+1\right)^2$$

$$\frac{5x^2+20x+6}{x\left(x+1\right)^2} = \frac{A}{x} + \frac{B}{\left(x+1\right)} + \frac{C}{\left(x+1\right)^2}$$

$$\frac{5x^2+20x+6}{x\left(x+1\right)^2} = \frac{A}{x}\cdot\frac{\left(x+1\right)^2}{\left(x+1\right)^2} + \frac{B}{\left(x+1\right)}\cdot\frac{x\left(x+1\right)}{x\left(x+1\right)} + \frac{C}{\left(x+1\right)^2}\cdot\frac{x}{x}$$

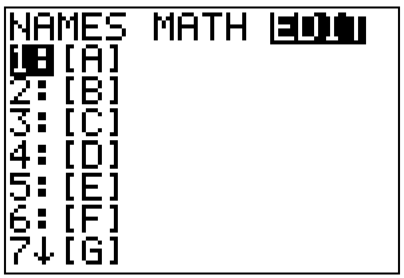
$$\frac{5x^2+20x+6}{x\left(x+1\right)^2} = \frac{Ax^2+2Ax+A}{x\left(x+1\right)^2} + \frac{Bx^2+Bx}{x\left(x+1\right)^2} + \frac{Cx}{x\left(x+1\right)^2}$$

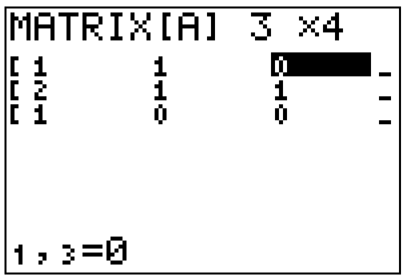
$$5x^2+20x+6 = Ax^2+2Ax+A+Bx^2+Bx+Cx$$

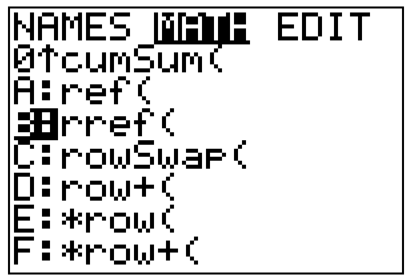
$$5x^2+20x+6 = Ax^2+Bx^2+2Ax+Bx+Cx+A$$

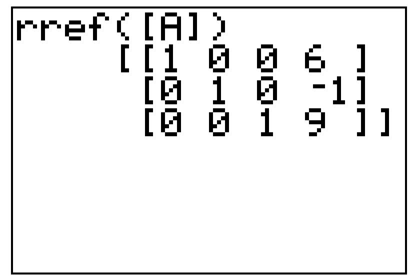
$$\underline{\underline{5}}x^2+\boxed{20}x+\underline{6} = \underline{\underline{\left(A+B\right)}}x^2+\boxed{\left(2A+B+C\right)}x+\underline{A}$$

$$\begin{aligned} 5 &= A+B \\ 20 &= 2A+B+C \\ 6 &= A \end{aligned} \rightarrow \begin{array}{ccc|c} A & B & C & \# \\ \hline 1 & 1 & 0 & 5 \\ 2 & 1 & 1 & 20 \\ 1 & 0 & 0 & 6 \end{array}$$









$A=6\,;B=-1\ ;C=9$

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx = \int \frac{6}{x} - \frac{1}{\left(x+1\right)} + \frac{9}{\left(x+1\right)^2}\,dx$$

$$= 6\ln\left|x\right| - \ln\left|x+1\right| - 9\left(x+1\right)^{-1} + C$$

$$\int \frac{7x}{\left(2x-3\right)\left(x+2\right)}\,dx =$$

(a)	$\frac{3}{2}\ln\left\|2x-3\right\|+2\ln\left\|x+2\right\|+C$
(b)	$3\ln\left\|2x-3\right\|+2\ln\left\|x+2\right\|+C$
(c)	$3\ln\left\|2x-3\right\|-2\ln\left\|x+2\right\|+C$
(d)	$-\frac{6}{\left(2x-3\right)^2}-\frac{2}{\left(x+2\right)^2}+C$
(e)	$-\frac{3}{\left(2x-3\right)^2}-\frac{2}{\left(x+2\right)^2}+C$

FRQ: AP Calc BC 2015 #5