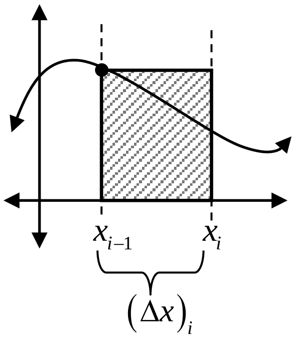
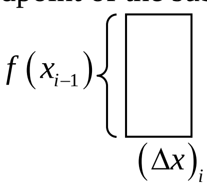


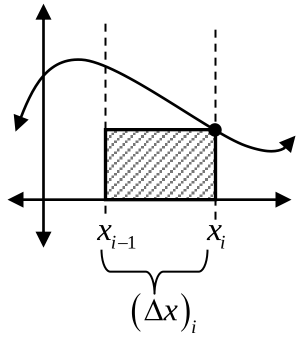
Left Sum

$f\left(x_{i-1}\right)\cdot(\Delta x)_i$

Create rectangles with width Δx, and height using the function value at the left endpoint of the subinterval



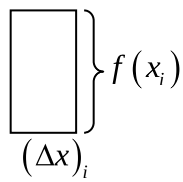
$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f\left(x_{i-1}\right)(\Delta x)_i$$



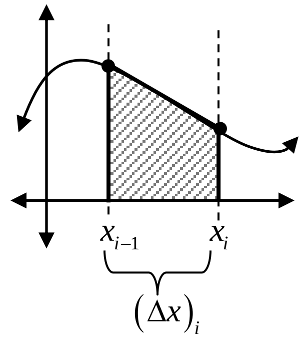
Right Sum

$f\left(x_i\right)\cdot(\Delta x)_i$

Create rectangles with width Δx, and height using the function value at the right endpoint of the subinterval



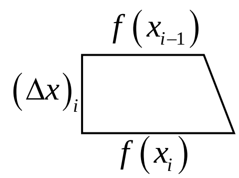
$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f\left(x_i\right)(\Delta x)_i$$



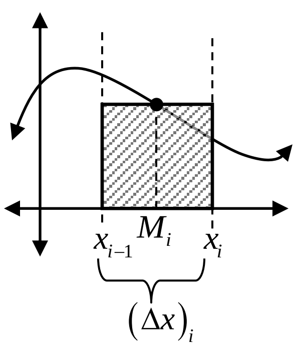
Trapezoidal Sum

$\frac{1}{2}\left(f\left(x_{i-1}\right)+f\left(x_i\right)\right)\cdot(\Delta x)_i$

Create trapezoids with height Δx, and bases using the function values of at the endpoints of the subinterval



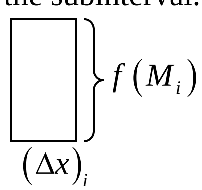
$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} \frac{1}{2}\left[f\left(x_{i-1}\right)+f\left(x_i\right)\right](\Delta x)_i$$



Midpoint Sum:

$f\left(M_i\right)\cdot(\Delta x)_i$

Create rectangles with width Δx, and height using the function value at the midpoint of the subinterval.



$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f\left(M_i\right)(\Delta x)_i$$

For the following tables, write and evaluate the following sums with the intervals indicated by the table:

Example:Left Sum, Right Sum, Trapezoidal Sum. Explain why it is impossible to do a Midpoint Sum with the given information.

t	0	1	3	4	7	8	9
$L(t)$	120	156	176	126	150	80	0

Left Sum: $(120)(1-0)+(156)(3-1)+(176)(4-3)+(126)(7-4)+(150)(8-7)+(80)(9-8)$

Right Sum: $(156)(1-0)+(176)(3-1)+(126)(4-3)+(150)(7-4)+(80)(8-7)+(0)(9-8)$

Trapezoidal Sum:

$\frac{1}{2}(120+156)(1-0)+\frac{1}{2}(156+176)(3-1)+\frac{1}{2}(176+126)(4-3)+\frac{1}{2}(126+150)(7-4)+\frac{1}{2}(150+180)(8-7)+\frac{1}{2}(80+0)(9-8)$

You cannot do any Midpoint Sum with the given table because you cannot partition the interval $[0,9]$ into subintervals in such a way that the midpoint value of every subinterval is given in the table.

Table # 1

Approximate $\int_0^{80} v(t)\,dt$ using the Left Sum, Right Sum, Trapezoidal Sum, and Midpoint Sum with 4 subintervals of equal length.

t	0	10	20	30	40	50	60	70	80
$v(t)$	5	14	22	29	35	40	44	47	49

Table # 2

Approximate $\int_0^{12} r'(t)\,dt$ using the Left Sum, Right Sum, and Trapezoidal Sum with the subintervals indicated by the table. State why it is impossible to do a Midpoint Sum to approximate $\int_0^{12} r'(t)\,dt$ with the given information.

t	0	2	5	7	11	12
$r'(t)$	5.7	4.0	2.0	1.2	0.6	0.5

Reimann Sums with subintervals of equal length:

Write out an expression for the Left, Right, Midpoint, and Trapezoidal Sums of $f(x)=x^2-2x$ on the interval $[0,3]$ with 6 subintervals of equal length.

Approximate $\int_0^4 x^2-3x\,dx$ using Left, Right, Midpoint, and Trapezoidal Sums with 8 subintervals of equal length.

For the following tables, write and evaluate the following sums with the intervals indicated by the table:

Example:Left Sum, Right Sum, Trapezoidal Sum. Explain why it is impossible to do a Midpoint Sum with the given information.

t	0	1	3	4	7	8	9
$L(t)$	120	156	176	126	150	80	0

Left Sum: $(120)(1-0)+(156)(3-1)+(176)(4-3)+(126)(7-4)+(150)(8-7)+(80)(9-8)$

Right Sum: $(156)(1-0)+(176)(3-1)+(116)(4-3)+(150)(7-4)+(80)(8-7)+(0)(9-8)$

Trapezoidal Sum:

$\frac{1}{2}(120+156)(1-0)+\frac{1}{2}(156+176)(3-1)+\frac{1}{2}(176+126)(4-3)+\frac{1}{2}(126+150)(7-4)+\frac{1}{2}(150+180)(8-7)+\frac{1}{2}(80+0)(9-8)$

You cannot do any Midpoint Sum with the given table because you cannot partition the interval $[0,9]$ into subintervals in such a way that the midpoint value of every subinterval is given in the table.

Table # 1

Approximate $\int_0^{80} v(t)\,dt$ using the Left Sum, Right Sum, Trapezoidal Sum, and Midpoint Sum with 4 subintervals of equal length.

t	0	10	20	30	40	50	60	70	80
$v(t)$	5	14	22	29	35	40	44	47	49

Left Sum:
$$(5)(20-0)+(22)(40-20)+(35)(60-40)+(44)(80-60)$$
$$5\cdot 20+22\cdot 20+35\cdot 20+44\cdot 20$$
$$[5+22+35+44]\cdot 20$$

Right Sum:
$$(22)(20-0)+(35)(40-20)+(44)(60-40)+(49)(80-60)$$
$$22\cdot 20+35\cdot 20+44\cdot 20+49\cdot 20$$
$$[22+35+44+49]\cdot 20$$

Trapezoidal Sum:
$$\frac{1}{2}[5+22]\cdot(20-0)+\frac{1}{2}[22+35]\cdot(40-20)+\frac{1}{2}[35+44]\cdot(60-40)+\frac{1}{2}[44+49]\cdot(80-60)$$
$$\frac{1}{2}[5+22]\cdot 20+\frac{1}{2}[22+35]\cdot 20+\frac{1}{2}[35+44]\cdot 20+\frac{1}{2}[44+49]\cdot 20$$
$$\frac{1}{2}[5+2\cdot 22+2\cdot 35+2\cdot 44+49]\cdot 20$$

Midpoint Sum:
$$14\cdot(20-0)+29\cdot(40-20)+40\cdot(60-40)+47\cdot(80-60)$$
$$14\cdot 20+29\cdot 20+40\cdot 20+47\cdot 20$$

Table # 2

Approximate $\int_{0}^{12} r'(t)\,dt$ using the Left Sum, Right Sum, and Trapezoidal Sum with the subintervals indicated by the table. State why it is impossible to do a Midpoint Sum to approximate $\int_{0}^{12} r'(t)\,dt$ with the given information.

t	0	2	5	7	11	12
$r'(t)$	5.7	4.0	2.0	1.2	0.6	0.5

Left Sum: $\begin{matrix}(5.7)(2-0)+(4.0)(5-2)+(2.0)(7-5)+(1.2)(11-7)+(0.6)(12-11)\\(5.7)(2)+(4.0)(3)+(2.0)(2)+(1.2)(4)+(0.6)(1)\end{matrix}$

Right Sum: $\begin{matrix}(4.0)(2-0)+(2.0)(5-2)+(1.2)(7-5)+(0.6)(11-7)+(0.5)(12-11)\\(4.0)(2)+(2.0)(3)+(1.2)(2)+(0.6)(4)+(0.5)(1)\end{matrix}$

Trapezoidal Sum:

$$\frac{1}{2}[5.7+4.0](2-0)+\frac{1}{2}[4.0+2.0](5-2)+\frac{1}{2}[2.0+1.2](7-5)+\frac{1}{2}[1.2+0.6](11-7)+\frac{1}{2}[0.6+0.5](12-11)$$

$$\frac{1}{2}[9.7](2)+\frac{1}{2}[6.0](3)+\frac{1}{2}[3.2](2)+\frac{1}{2}[1.8](4)+\frac{1}{2}[1.1](1)$$

Midpoint Sum: Does not work because $x=2$ is not the midpoint of the subinterval $[0,5]$.

Reimann Sums with subintervals of equal length:

Write out an expression for the Left, Right, Midpoint, and Trapezoidal Sums of $f(x)=x^2-2x$ on the interval $[0,3]$ with 6 subintervals of equal length.

$\Delta x=\frac{3-0}{6}=\frac{1}{2}=0.5\rightarrow\Delta=\{3,\ 3.5,\ 4,\ 4.5,\ 5,\ 5.5,\ 6\}$ and midpoints $M_i=\{3.25,\ 3.75,\ 4.25,\ 4.75,\ 5.25,\ 5.75\}$

Left Sum:

$$\left[f(3)+f(3.5)+f(4)+f(4.5)+f(5)+f(5.5)\right]\cdot(0.5)$$

$$\left[\left[(3)^2-2(3)\right]+\left[(3.5)^2-2(3.5)\right]+\left[(4)^2-2(4)\right]+\left[(4.5)^2-2(4.5)\right]+\left[(5)^2-2(5)\right]+\left[(5.5)^2-2(5.5)\right]\right]\cdot(0.5)$$

Right Sum:

$$\left[f(3.5)+f(4)+f(4.5)+f(5)+f(5.5)+f(6)\right]\cdot(0.5)$$

$$\left[\left[(3.5)^2-2(3.5)\right]+\left[(4)^2-2(4)\right]+\left[(4.5)^2-2(4.5)\right]+\left[(5)^2-2(5)\right]+\left[(5.5)^2-2(5.5)\right]+\left[(6)^2-2(6)\right]\right]\cdot(0.5)$$

Trapezoidal Sum:

$$\frac{1}{2}\left[f(3)+2\cdot f(3.5)+2\cdot f(4)+2\cdot f(4.5)+2\cdot f(5)+2\cdot f(5.5)+f(6)\right]\cdot(0.5)$$

$$\frac{1}{2}\cdot\begin{bmatrix}\left[(3.5)^2-2(3.5)\right]+2\cdot\left[(3.5)^2-2(3.5)\right]+2\cdot\left[(4)^2-2(4)\right]+2\cdot\left[(4.5)^2-2(4.5)\right]\\+2\cdot\left[(5)^2-2(5)\right]+2\cdot\left[(5.5)^2-2(5.5)\right]+\left[(6)^2-2(6)\right]\end{bmatrix}\cdot(0.5)$$

Midpoints Sum:

$$\left[f(3.25)+f(3.75)+f(4.25)+f(4.75)+f(5.25)+f(5.75)\right]\cdot(0.5)$$

$$\begin{bmatrix}\left[(3.25)^2-2(3.25)\right]+\left[(3.75)^2-2(3.75)\right]+\left[(4.25)^2-2(4.25)\right]\\+\left[(4.75)^2-2(4.75)\right]+\left[(5.25)^2-2(5.25)\right]+\left[(5.75)^2-2(5.75)\right]\end{bmatrix}\cdot(0.5)$$

Approximate $\int_0^4 x^2 - 3x dx$ using Left, Right, Midpoint, and Trapezoidal Sums with 8 subintervals of equal length.

$$\Delta x = \frac{4-0}{8} = \frac{1}{2} = 0.5 \to \Delta = \{0,\ 0.5,\ 1,\ 1.5,\ 2,\ 2.5,\ 3,\ 3.5,\ 4\}$$

$$M_i = \{0.25,\ 0.75,\ 1.25,\ 1.75,\ 2.25,\ 2.75,\ 3.25,\ 3.75\}$$

$$\left[f(0) + f(0.5) + f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5)\right]\cdot(0.5)$$

Left Sum: $$\begin{bmatrix} \left[(0)^2 - 3(0)\right] + \left[(0.5)^2 - 3(0.5)\right] + \left[(1)^2 - 3(1)\right] + \left[(1.5)^2 - 3(1.5)\right] + \left[(2)^2 - 3(2)\right] \\ + \left[(2.5)^2 - 3(2.5)\right] + \left[(3)^2 - 3(3)\right] + \left[(3.5)^2 - 3(3.5)\right] \end{bmatrix}\cdot(0.5)$$

$$\left[f(0.5) + f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5) + f(4)\right]\cdot(0.5)$$

Right Sum: $$\begin{bmatrix} \left[(0.5)^2 - 3(0.5)\right] + \left[(1)^2 - 3(1)\right] + \left[(1.5)^2 - 3(1.5)\right] + \left[(2)^2 - 3(2)\right] \\ + \left[(2.5)^2 - 3(2.5)\right] + \left[(3)^2 - 3(3)\right] + \left[(3.5)^2 - 3(3.5)\right] + \left[(4)^2 - 3(4)\right] \end{bmatrix}\cdot(0.5)$$

$$\frac{1}{2}\cdot\left[f(0) + 2f(0.5) + 2f(1) + 2f(1.5) + 2f(2) + 2f(2.5) + 2f(3) + 2f(3.5) + f(4)\right]\cdot(0.5)$$

Trapezoidal Sum: $$\begin{bmatrix} \left[(0)^2 - 3(0)\right] + 2\cdot\left[(0.5)^2 - 3(0.5)\right] + 2\cdot\left[(1)^2 - 3(1)\right] + 2\cdot\left[(1.5)^2 - 3(1.5)\right] + 2\cdot\left[(2)^2 - 3(2)\right] \\ +2\cdot\left[(2.5)^2 - 3(2.5)\right] + 2\cdot\left[(3)^2 - 3(3)\right] + 2\cdot\left[(3.5)^2 - 3(3.5)\right] + \left[(4)^2 - 3(4)\right] \end{bmatrix}\cdot(0.5)$$

Midpoint Sum:

$$\left[f(0.25) + f(0.75) + f(1.25) + f(1.75) + f(2.25) + f(2.75) + f(3.25) + f(3.75)\right]\cdot(0.5)$$

$$\begin{bmatrix} \left[(0.25)^2 - 3(0.25)\right] + \left[(0.75)^2 - 3(0.75)\right] + \left[(1.25)^2 - 3(1.25)\right] + \left[(1.75)^2 - 3(1.75)\right] + \left[(2.25)^2 - 3(2.25)\right] \\ + \left[(2.75)^2 - 3(2.75)\right] + \left[(3.25)^2 - 3(3.25)\right] + \left[(3.75)^2 - 3(3.75)\right] \end{bmatrix}\cdot(0.5)$$