Standard Problems 5. Newton's Second Law with Vector Components

Introductory problem.

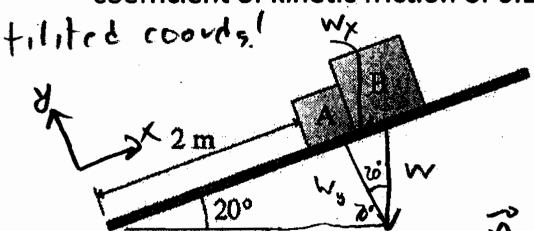
$F_{net} = ma \qquad f_s \le \mu_s n \qquad f_k = \mu_k n \qquad f_r = \mu_r n \qquad w = mg$

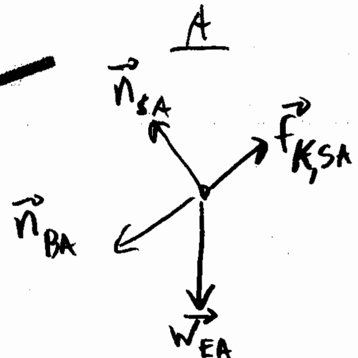
$$v_B = v_A + a_{AB}\Delta t_{AB} \qquad\qquad x_B = x_A + v_A \Delta t_{AB} + \frac{a_{AB}}{2}\Delta t_{AB}{}^2$$

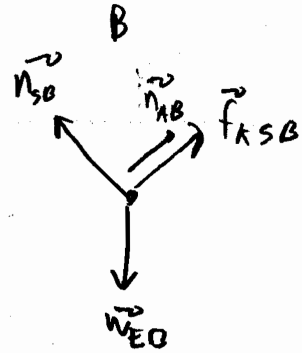
$$v_B^2 = v_A^2 + 2a_{AB}\Delta x_{AB} \qquad\qquad x_B = x_A + \left(\frac{v_A+v_B}{2}\right)\Delta t_{AB}$$

Two packages at UPS start sliding down the 20 degree ramp shown in the figure. Package A has a mass of 4.00 and a coefficient of kinetic friction of 0.200. Package B has a mass of 9.00 and a coefficient of kinetic friction of 0.150. How long does it take package A to reach the bottom?

tilted coords!





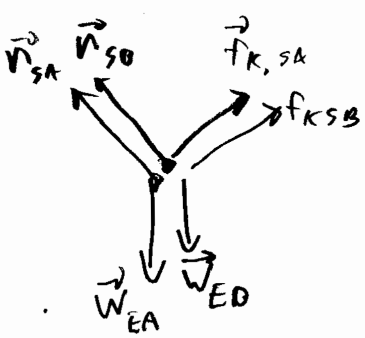


Take components of $\vec{W}_{EA}$ and $\vec{W}_{EB}$: From the diagram:

$W_x = W\sin 20$ for both weights.

$w_y = W\cos 20$

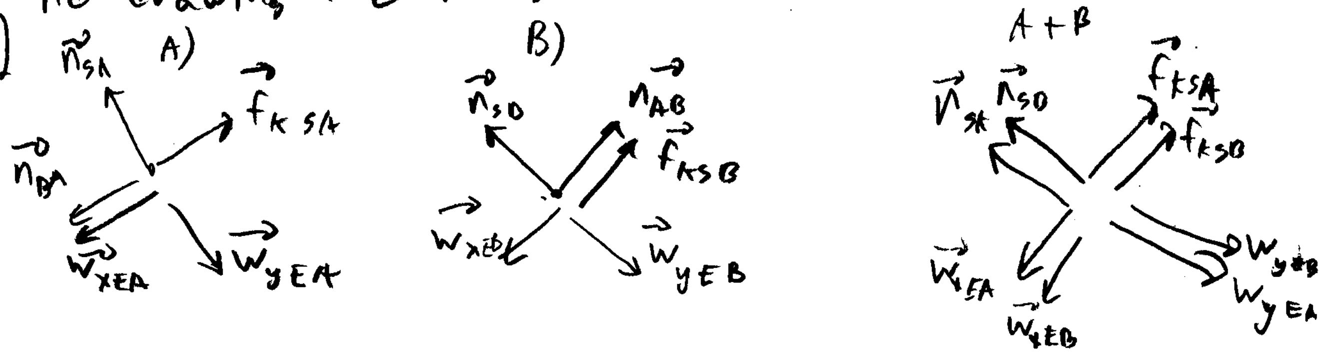
Both objects will move together, because the coefficient of friction of A is > that of B. So we can treat them as a single system:



We know the weights, but we must use the individual blocks to find the frictional forces

Re-drawing the FBDs for everything gives

D+P



We just need kinetic frictions from the analysis of A and B separately, For those all we need are normal forces, since $f_K = \mu_K n$ so analyze the y-directions.

Solve

$$F_{net,y} = m_A \cancel{a_{yA}}^{0}, \text{ because the block stays on the ramp}$$

$$n_{SA} - W_{yEA} = 0 \Rightarrow n_{SA} = W_{yEA} = W_{EA} \cos 20$$
$$= m_A g \cos 20$$

$$F_{net,y} = m_B \cancel{a_{yB}}^{0}$$
$$\Rightarrow n_{SB} = m_B g \cos 20.$$

Now analyze the system (A+B) in the x direction:

$$F_{net,x} = f_{k,SA} + f_{k,SB} - W_{xEA} - W_{xEB} = m_{A+B}\, a_x$$

$$= \mu_{K,A}(n_{SA}) + \mu_{K,B}(n_{SB}) - W_{xEA} - W_{xEB} = (m_A + m_B)\, a_x$$

$$\Rightarrow a_x = \frac{\mu_{KA}(m_A g \cos 20) + \mu_{KB}(m_B g \cos 20) - m_A g \sin 20 - m_B g \,\overset{\sin}{\cancel{\cos}}\, 20}{m_A + m_B}$$

$$= -1.83 \text{ m/s}^2 \quad \text{(negative b/c it slides in the negative direction).}$$

Now looking at the motion, $X_f = 0\text{ m}$, $X_i = 2\text{m}$, $V_i = 0\text{ m/s}$, $a_x = -1.83\text{ m/s}^2$

so

$$\cancel{X_f}^{0} = X_i + \cancel{V_i \Delta t}^{0} + \tfrac{1}{2} a \Delta t^2$$

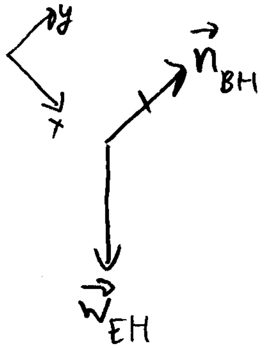
$$\Rightarrow \Delta t = \sqrt{\frac{-2X_i}{a}} = \sqrt{\frac{-2(2\text{m})}{-1.83\text{ m/s}^2}} = 1.48\text{s}.$$

Reflect

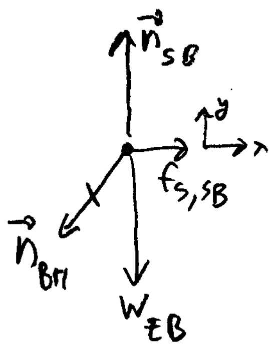
Some conceptual reasoning was key to recognize $a_A = a_B$

1) Hamster:

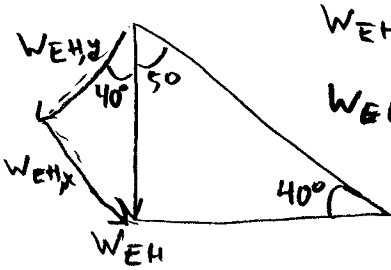
O+P



Block



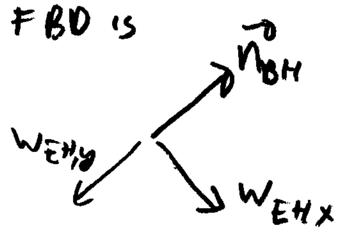
We'll use slanted axes to analyze the Hamster so we can use the fact that it won't come off the block: i.e. $a_y = 0$ in a tilted co-ord system.



$$W_{EH,y} = W_{EH}\cos 40 = m_H g\cos 40$$

$$W_{EH,x} = W_{EH}\sin 40 = m_H g\sin 40$$

The new Hamster FBD is



Solve

Now N's second; to find the normal force:

$$F_{net,y} = m_H a_y = 0 \quad \text{(Hamster stays on block)}$$

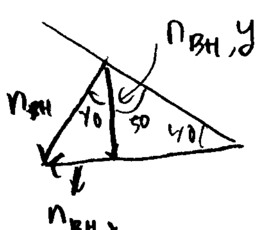
$$n_{BH} - W_{EH,y} = 0 \Rightarrow n_{BH} = W_{EH,y} = m_H g\cos 40$$

~~Now Use N's 2nd on the block:~~

$$\cancel{F_{net,y} = m_B a_y = 0 \quad \text{(Block stationary)}}$$

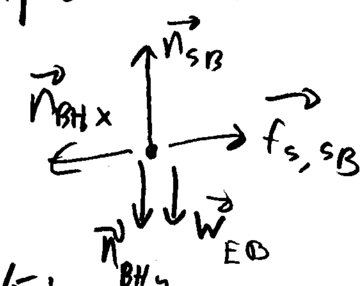
$\cancel{n_{SB}}$

I forgot to take components of n_{BH}:



$$n_{BH,y} = n_{BH}\cos 40$$

$$n_{BH,x} = n_{BH}\sin 40$$



Now N's 2nd on the block:

$$F_{net,y} = m_B a_y = 0 \quad \text{(block stationary)}$$

$$n_{SB} - n_{BH,y} - W_{EB} = 0$$

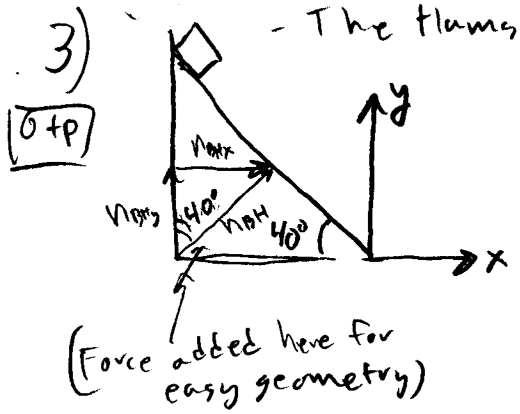
$$\Rightarrow n_{SB} = n_{BH,y} + W_{EB} = n_{BH}\cos 40 + m_B g$$

$$= (m_H g\cos 40)\cos 40 + m_B g = 8.99\,\text{N}$$

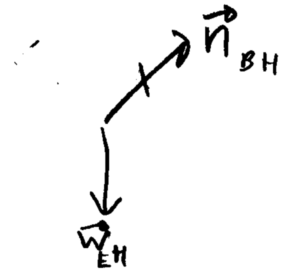
Reflect: we have to carefully apply N's 2nd. shortcuts get us in trouble here.

3) $\boxed{0\text{ tp}}$

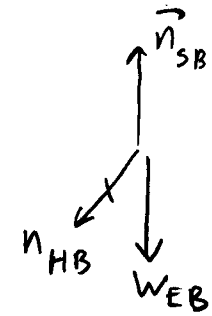
- The Hamster turned into a block.



Hamster:



Block

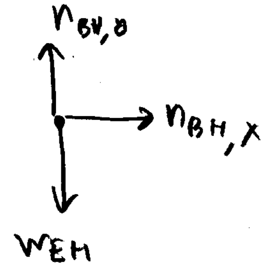


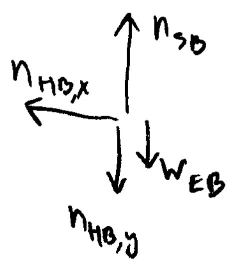
The normal forces n_{BH} and n_{HB} will <u>not</u> be equal to $W_{EH,y}$, because the block will accelerate left, meaning $a_{Hy} \neq 0$ even if the axes are tilted.

Instead, we will consider both block and Hamster with regular axes: $n_{BH,x} = n_{BH} \sin 40$ $\quad n_{BH,y} = n_{BH} \cos 40$ $\Big\}$ see diagram

$= \quad\quad\quad\quad =$

$n_{HB,x} = n_{HB} \sin 40 \quad n_{HB,y} = n_{HB} \cos 40$





N's second gives

① $F_{net,x} = n_{BH,x} = m_H a_{Hx}$

② $F_{net,y} = n_{BH,y} - W_{EH} = m_H a_{Hy}$

③ $F_{net,x} = -n_{HB,x} = -n_{BH,x} = m_B a_{Bx}$

④ $F_{net,y} = n_{SB} - n_{HB,y} - W_{EM} = m_B \cancel{a_{By}}^{0}$

$= n_{SB} - n_{BH,y} - W_{EM} = 0$

Here we have 4 equations and 5 unknowns (a_{Hx}, a_{Hy}, a_{Bx}, n_{BH}, and n_{SB}). So we need more info. The other fact we have is that the hamster will stay on the block. We need to express this idea mathematically.

When the block's lower right is at position X_B, the surface of the block makes a line with slope $-\tan(40°)$, and y-intercept $X_B(\tan 40)$.

3, cont

So the equation of this line is

$$y = -(\tan 40)x + X_B \tan 40.$$

The hamster's position must always be on this line, so $y_H = -(\tan 40) X_H + (\tan 40) X_B = (X_B - X_H)\tan 40$

we care about accelerations, so take the second derivitive w.r.t. time:

$$\frac{d^2 y_H}{dt^2} = \left(\frac{d^2 X_B}{dt^2} - \frac{d^2 X_H}{dt^2}\right)\tan 40$$

$\Rightarrow$ (5) $a_{Hy} = (a_{Bx} - a_{Hx})\tan 40$ — this is the constraint we wanted.

Now we have 5 equations and 5 unknowns.

Solve (1), (2), and (3) for the n_{BH}

(1): $n_{BH,x} = n_{BH}\sin 40 = m_H a_{Hx} \Rightarrow a_{Hx} = \dfrac{n_{BH}\sin 40}{m_H}$

(2): $n_{BH,y} - W_{EH} = n_{BH}\cos 40 - m_H g = m_H a_{Hy}$

$$\Rightarrow a_{Hy} = \frac{n_{BH}\cos 40 - m_H g}{m_H}$$

(3) $-n_{BH,x} = -n_{BH}\sin 40 = m_B a_{Bx} \Rightarrow a_{Bx} = \dfrac{-n_{BH}\sin 40}{m_B}$

sub these all into (5):

$$\frac{n_{BH}\cos 40 - m_H g}{m_H} = \left(\frac{-n_{BH}\sin 40}{m_B} - \frac{n_{BH}\sin 40}{m_H}\right)\tan 40$$

rearrange...

$$n_{BH}\left(\frac{\cos 40}{m_H} + \frac{\sin 40 \tan 40}{m_B} + \frac{\sin 40 \tan 40}{m_H}\right) = g$$

$\Rightarrow n_{BH} = 1.37\ N$

3, cont

Now plug back into the acceleration equations:

$$a_{Hx} = \frac{n_{BH} \sin 40}{m_H} = 4,37\ m/s^2$$

$$a_{Hy} = \frac{n_{BH} \cos 40 - m_H g}{m_H} = -4,59\ m/s^2$$

$$a_{Bx} = \frac{-n_{BH} \sin 40}{m_B} = -1,09\ m/s^2$$

and finally, from 4

$$n_{SB} = n_{BH,y} + W_{Bm} = n_{BH} \cos 40 + m_B g = 8,88\ N$$

Reflect: ~~There~~ In these multi-object problems when one pushes or pulls on another, there is a constraint on their accelerations: they must be related in some way. In this case, the relationship is a bit complex, but once we found it, we were good to go!