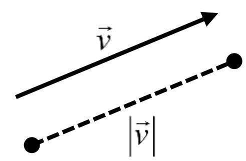
Position, Velocity, and Acceleration.

From an earlier time in your life, you should be familiar with the following relationship:

$$\text{Distance} = \text{Rate} \times \text{Time}$$
$$D = R \cdot T$$

Rate in this case can be thought of as ***velocity***. That is, velocity is a *vector*. It has both (1) magnitude and (2) direction. Vectors in their simplest form are arrows – where the direction of the arrow indicates the direction of the vector, and the length of the arrow is the magnitude of the vector.



In AP Calculus, you must know some key points and differences between velocity and speed:

Velocity	Speed
Is a vector $\vec{v}$ Has direction and magnitude Direction can be conveyed by sign $\pm$, the words left/right, north/south, etc. Can take on negative, zero, and positive values.	Is a scalar (i.e. real number) Must be a non-negative real number. Is defined as the magnitude of velocity. $\text{speed} = \lvert\vec{v}\rvert$

You must also be familiar with the difference between position and distance.

Position	Distance
Is a location Can take on negative, zero, and positive values Is relative to a fixed point (i.e. origin)	Measurement of the space between locations Must be a non-negative real number $\lvert\Delta\text{position}\rvert$

With these ideas in mind, and using $D = R \cdot T$, and letting $s(t)$ represent position at time *t*, we can define Average Velocity:

$$\text{Average Velocity on } [t_1, t_2] = \frac{\Delta\text{Position}}{\Delta\text{Time}} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Note that *average velocity is taken over an interval of time.*

To get *instantaneous velocity*, we need to take the limit of average velocity as $\Delta t \to 0$.

Instantaneous Velocity at $t = c$:

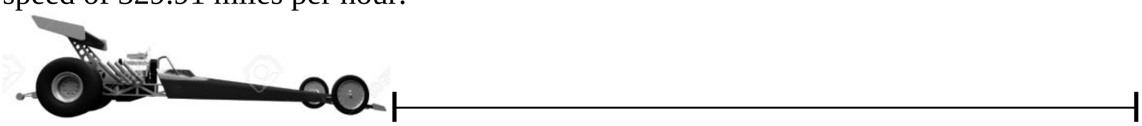
$$v(c) = \lim_{\Delta t \to 0} \frac{s(c + \Delta t) - s(c)}{\Delta t} \quad \text{or} \quad v(c) = \lim_{h \to 0} \frac{s(c+h) - s(c)}{h}$$

Just as we outlined the differences between the similar/related concepts above, we must do the same with the average velocity and instantaneous velocity:

Average Velocity	Instantaneous Velocity
Taken over a <u>time interval</u> $[t_1, t_2]$ $\dfrac{s(t_2)-s(t_1)}{t_2-t_1}$ Is the <u>average rate of change of position</u> on $[t_1, t_2]$	Taken at a <u>moment in time</u> t $\lim\limits_{h\to 0}\dfrac{s(c+h)-s(c)}{h}$ Is the <u>limit of the average velocity</u>

Top Fuel Dragsters and Top Speed: https://www.youtube.com/watch?v=f84aBtmTDd8

Top fuel dragsters are cars that race down a track that is 0.25 miles long. When cars race, the elapsed time from start to crossing the finish line is displayed, along with top speed. The 2012 world record holder for NHRA top fuel dragster had an elapsed time of 3.728 seconds and top speed of 329.91 miles per hour.



What is the average velocity of the top fuel dragster over the course of the track?

Why is the average velocity of the top fuel dragster not 329.91 miles per hour?

How is the measurement of the top speed of the top fuel dragster calculated?