#3

$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$:

$$\begin{aligned}
\int_1^{\infty} \frac{1}{\sqrt[5]{x}}\,dx &= \lim_{t\to\infty}\int_1^{t} \frac{1}{\sqrt[5]{x}}\,dx \\
&= \lim_{t\to\infty}\int_1^{t} x^{-\frac{1}{5}}dx \\
&= \lim_{t\to\infty}\left[\frac{5}{4}x^{\frac{4}{5}}\right]_1^t \\
&= \lim_{t\to\infty}\left(\left[\frac{5}{4}(t)^{\frac{4}{5}}\right]-\left[\frac{5}{4}(1)^{\frac{4}{5}}\right]\right) \\
&= \lim_{t\to\infty}\left(\left[\frac{5}{4}\cdot\sqrt[5]{t^4}\right]-\left[\frac{5}{4}(1)^{\frac{4}{5}}\right]\right)
\end{aligned}$$

$$\downarrow$$

$$\infty$$

Therefore, the series diverges by the Integral test.

#4

$\sum_{n=1}^{\infty} \frac{1}{n^5}$

$$\begin{aligned}
\int_1^{\infty} \frac{1}{x^5}\,dx &= \lim_{t\to\infty}\int_1^{t} \frac{1}{x^5}\,dx \\
&= \lim_{t\to\infty}\int_1^{t} x^{-5}dx \\
&= \lim_{t\to\infty}\left[-\frac{1}{4}x^{-4}\right]_1^t \\
&= \lim_{t\to\infty}\left(\left[-\frac{1}{4}(t)^{-4}\right]-\left[-\frac{1}{4}(1)^{-4}\right]\right) \\
&= \lim_{t\to\infty}\left(\left[-\frac{1}{4}\cdot\frac{1}{t^4}\right]-\left[-\frac{1}{4}(1)^{-4}\right]\right) \\
&= \frac{1}{4}
\end{aligned}$$

Therefore, series converges by the Integral Test.

#5

$$\sum_{n=1}^{\infty}\frac{1}{(2n+1)^3}\,dx$$

$$\begin{aligned}
\int_1^{\infty}\frac{1}{(2x+1)^3}\,dx &= \lim_{t\to\infty}\int_1^{t}\frac{1}{(2x+1)^3}\,dx \\
&= \lim_{t\to\infty}\int_1^{t}(2x+1)^{-3}\,dx \\
&= \lim_{t\to\infty}\left[-\frac{1}{4}(2x+1)^{-2}\right]_1^t \\
&= \lim_{t\to\infty}\left(\left[-\frac{1}{4}(2(t)+1)^{-2}\right]-\left[-\frac{1}{4}(2(1)+1)^{-2}\right]\right) \\
&= \lim_{t\to\infty}\left(\left[-\frac{1}{4}(2(t)+1)^{-2}\right]-\left[-\frac{1}{4}(2(1)+1)^{-2}\right]\right) \\
&= \lim_{t\to\infty}\left(\left[-\frac{1}{4}\cdot\frac{1}{(2(t)+1)^2}\right]-\left[-\frac{1}{4}(2(1)+1)^{-2}\right]\right) \\
&= -\frac{1}{4}(2(1)+1)^{-2}
\end{aligned}$$

Therefore, series converges by the Integral Test.

#6

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+4}}$$

$$\begin{aligned}
\int_1^{\infty}\frac{1}{\sqrt{x+4}}\,dx &= \lim_{t\to\infty}\int_1^{t}\frac{1}{\sqrt{x+4}}\,dx \\
&= \lim_{t\to\infty}\int_1^{t}(x+4)^{-\frac{1}{2}}\,dx \\
&= \lim_{t\to\infty}\left[2(x+4)^{\frac{1}{2}}\right]_1^t \\
&= \lim_{t\to\infty}\left(\left[2(t+4)^{\frac{1}{2}}\right]-\left[2(1+4)^{\frac{1}{2}}\right]\right) \\
&= \lim_{t\to\infty}\left(\left[2\sqrt{t+4}\right]-\left[2(1+4)^{\frac{1}{2}}\right]\right) \\
&\downarrow \\
&\infty
\end{aligned}$$

Therefore, the series diverges by the Integral test.

#7

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

$$\begin{aligned}
\int_1^{\infty} \frac{x}{x^2+1}\,dx &= \lim_{t\to\infty}\int_1^t \frac{x}{x^2+1}\,dx \\
&= \lim_{t\to\infty}\frac{1}{2}\int_1^t \frac{1}{x^2+1}\cdot 2x\,dx \\
&= \frac{1}{2}\cdot\lim_{t\to\infty}\int_1^t \frac{1}{x^2+1}\cdot 2x\,dx \\
&= \frac{1}{2}\cdot\lim_{t\to\infty}\Big[\ln\left|x^2+1\right|\Big]_1^t \\
&= \frac{1}{2}\cdot\lim_{t\to\infty}\Big[\ln\left|(t)^2+1\right| - \ln\left|(1)^2+1\right|\Big] \\
&\downarrow \\
&\ \infty
\end{aligned}$$

Therefore, the series diverges by the Integral test.

#8

$$\sum_{n=1}^{\infty} n^2 e^{-n^3}$$

$$\begin{aligned}
\int_1^{\infty} x^2 e^{-x^3}\,dx &= \lim_{t\to\infty}\int_1^t x^2 e^{-x^3}\,dx \\
&= -\frac{1}{3}\lim_{t\to\infty}\int_1^t e^{-x^3}\cdot\left(-3x^2\right)dx \\
&= -\frac{1}{3}\lim_{t\to\infty}\Big[e^{-x^3}\Big]_1^t \\
&= -\frac{1}{3}\left(\lim_{t\to\infty}\Big[e^{-(t)^3} - e^{-(1)^3}\Big]\right) \\
&= -\frac{1}{3}\left(\lim_{t\to\infty}\left[\frac{1}{e^{t^3}} - e^{-(1)^3}\right]\right) \\
&= -\frac{1}{3}\Big[-e^{-(1)^3}\Big]
\end{aligned}$$

Therefore, series converges by the Integral Test.

#9

$$\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}}$$

$\int_1^{\infty} \frac{1}{x^{\sqrt{2}}}\,dx = \lim_{t\to\infty} \int_1^{t} \frac{1}{x^{\sqrt{2}}}\,dx$ $= \lim_{t\to\infty} \int_1^{t} x^{-\sqrt{2}}\,dx$ $= \lim_{t\to\infty} \left[\frac{1}{-\sqrt{2}+1} x^{-\sqrt{2}+1} \right]_1^t$ $= \lim_{t\to\infty} \left(\left[\frac{1}{-\sqrt{2}+1} (t)^{-\sqrt{2}+1} \right] - \left[\frac{1}{-\sqrt{2}+1} (1)^{-\sqrt{2}+1} \right] \right)$ $= -\left[\frac{1}{-\sqrt{2}+1} (1)^{-\sqrt{2}+1} \right]$ Therefore, series converges by the Integral Test.	$\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}}$ Is a *p*-series with $p > 1$ Therefore the series converges.

#10

$$\sum_{n=1}^{\infty} n^{-0.9999}$$

$\int_1^{\infty} x^{-0.9999}\,dx = \lim_{t\to\infty} \int_1^{t} x^{-0.9999}\,dx$ $= \lim_{t\to\infty} \left[\frac{1}{0.0001} x^{0.0001} \right]_1^t$ $= \lim_{t\to\infty} \left(\left[\frac{1}{0.0001} (t)^{0.0001} \right] - \left[\frac{1}{0.0001} (1)^{0.0001} \right] \right)$ $= \lim_{t\to\infty} \left(\left[\frac{1}{0.0001} \sqrt[10,000]{t} \right] - \left[\frac{1}{0.0001} (1)^{0.0001} \right] \right)$ $\downarrow$ ∞ Therefore, series diverges by the Integral Test.	$\sum_{n=1}^{\infty} n^{-0.9999} = \sum_{n=1}^{\infty} \frac{1}{n^{0.9999}}$ Is a *p*-series with $p < 1$ Therefore the series diverges.

#11

$$1+\frac{1}{8}+\frac{1}{27}+\frac{1}{64}+\frac{1}{125}+\cdots=\sum_{n=1}^{\infty}\frac{1}{n^3}$$

$\int_1^{\infty}\frac{1}{x^3}dx=\lim_{t\to\infty}\int_1^{t}\frac{1}{x^3}dx$ $=\lim_{t\to\infty}\int_1^{t}x^{-3}dx$ $=\lim_{t\to\infty}\left[-\frac{1}{2}x^{-2}\right]_1^t$ $=\lim_{t\to\infty}\left(\left[-\frac{1}{2}(t)^{-2}\right]-\left[-\frac{1}{2}(1)^{-2}\right]\right)$ $=\lim_{t\to\infty}\left(\left[-\frac{1}{2}\cdot\frac{1}{t^2}\right]-\left[-\frac{1}{2}(1)^{-2}\right]\right)$ $=-\left[-\frac{1}{2}(1)^{-2}\right]$ Therefore, series converges by the Integral Test.	$\sum_{n=1}^{\infty}\frac{1}{n^3}$ Is a p-series with $p>1$ Therefore the series converges.

#12

$$1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\frac{1}{5\sqrt{5}}+\cdots=\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}}$$

$\int_1^{\infty}\frac{1}{x\sqrt{x}}dx=\lim_{t\to\infty}\int_1^{t}\frac{1}{x\sqrt{x}}dx$ $=\lim_{t\to\infty}\int_1^{t}x^{-\frac{3}{2}}dx$ $=\lim_{t\to\infty}\left[-2x^{-\frac{1}{2}}\right]_1^t$ $=\lim_{t\to\infty}\left(\left[-2(t)^{-\frac{1}{2}}\right]-\left[-2(1)^{-\frac{1}{2}}\right]\right)$ $=\lim_{t\to\infty}\left(\left[-2\cdot\frac{1}{\sqrt{t}}\right]-\left[-2(1)^{-\frac{1}{2}}\right]\right)$ $=-\left[-2(1)^{-\frac{1}{2}}\right]$ Therefore, series converges by the Integral Test.	$\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}}=\sum_{n=1}^{\infty}\frac{1}{n^{1.5}}$ Is a p-series with $p>1$ Therefore the series converges.

#13

$$1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\cdots=\sum_{n=0}^{\infty}\frac{1}{2n+1}$$

$$\begin{aligned}\int_0^{\infty}\frac{1}{2x+1}\,dx &= \lim_{t\to\infty}\int_0^{t}\frac{1}{2x+1}\,dx\\ &= \lim_{t\to\infty}\left[\frac{1}{2}\ln|2x+1|\right]_0^t\\ &= \lim_{t\to\infty}\left(\left[\frac{1}{2}\ln|2(t)+1|\right]-=\lim_{t\to\infty}\left[\frac{1}{2}\ln|2(0)+1|\right]\right)\end{aligned}$$

$$\downarrow$$

$$\infty$$

Therefore, the series diverges by the Integral test.

#14

$$\frac{1}{5}+\frac{1}{8}+\frac{1}{11}+\frac{1}{14}+\frac{1}{17}+\cdots=\sum_{n=1}^{\infty}\frac{1}{3n+2}$$

$$\begin{aligned}\int_1^{\infty}\frac{1}{3x+2}\,dx &= \lim_{t\to\infty}\int_1^{t}\frac{1}{3x+2}\,dx\\ &= \lim_{t\to\infty}\left[\frac{1}{3}\ln|3x+2|\right]_1^t\\ &= \lim_{t\to\infty}\left(\left[\frac{1}{3}\ln|3(t)+2|\right]-=\lim_{t\to\infty}\left[\frac{1}{3}\ln|3(1)+2|\right]\right)\end{aligned}$$

$$\downarrow$$

$$\infty$$

Therefore, the series diverges by the Integral test.

#15

$$\sum_{n=1}^{\infty}\frac{\sqrt{n}+4}{n^2}$$

$\int_1^{\infty}\frac{\sqrt{x}+4}{x^2}dx=\lim_{t\to\infty}\int_1^t\frac{\sqrt{x}+4}{x^2}dx$ $=\lim_{t\to\infty}\int_1^t\frac{\sqrt{x}}{x^2}+\frac{4}{x^2}dx$ $=\lim_{t\to\infty}\int_1^t x^{-\frac{3}{2}}+4x^{-2}dx$ $=\lim_{t\to\infty}\left[-2x^{-\frac{1}{2}}-4x^{-1}\right]_1^t$ $=\lim_{t\to\infty}\left(\left[-2(t)^{-\frac{1}{2}}-4(t)^{-1}\right]-\left[-2(1)^{-\frac{1}{2}}-4(1)^{-1}\right]\right)$ $=\lim_{t\to\infty}\left(\left[-2\cdot\frac{1}{\sqrt{t}}-4\cdot\frac{1}{t}\right]-\left[-2(1)^{-\frac{1}{2}}-4(1)^{-1}\right]\right)$ $=-\left[-2(1)^{-\frac{1}{2}}-4(1)^{-1}\right]$ Therefore, series converges by the Integral Test.	$\sum_{n=1}^{\infty}\frac{\sqrt{n}+4}{n^2}=\sum_{n=1}^{\infty}\left[\frac{\sqrt{n}}{n^2}+\frac{4}{n^2}\right]$ $=\left[\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2}\right]+\left[\sum_{n=1}^{\infty}\frac{4}{n^2}\right]$ $=\left[\sum_{n=1}^{\infty}\frac{1}{n^{1.5}}\right]+4\cdot\left[\sum_{n=1}^{\infty}\frac{1}{n^2}\right]$ Both are *p*-series with $p>1$ Therefore the series converges.

#16

$$\sum_{n=1}^{\infty}\frac{n^2}{n^3+1}$$

$$\begin{aligned}
\int_1^{\infty}\frac{x^2}{x^3+1}dx &= \lim_{t\to\infty}\int_1^t\frac{x^2}{x^3+1}dx \\
&= \frac{1}{3}\lim_{t\to\infty}\int_1^t\frac{1}{x^3+1}\cdot 3x^2dx \\
&= \frac{1}{3}\lim_{t\to\infty}\left[\ln\left|x^3+1\right|\right]_0^t \\
&= \frac{1}{3}\lim_{t\to\infty}\left[\ln\left|(t)^3+1\right|-\ln\left|(1)^3+1\right|\right] \\
&\downarrow \\
&\infty
\end{aligned}$$

Therefore, the series diverges by the Integral test.

#17

$$\sum_{n=1}^{\infty}\frac{1}{n^2+4}$$

$$\begin{aligned}\int_1^{\infty}\frac{1}{x^2+4}dx &= \lim_{t\to\infty}\int_1^t\frac{1}{x^2+4}dx\\ &= \lim_{t\to\infty}\int_1^t\frac{1}{x^2+(2)^2}dx\\ &= \lim_{t\to\infty}\left[\frac{1}{2}\arctan\left(\frac{x}{2}\right)\right]_1^t\\ &= \lim_{t\to\infty}\left(\left[\frac{1}{2}\arctan\left(\frac{t}{2}\right)\right]-\left[\frac{1}{2}\arctan\left(\frac{1}{2}\right)\right]\right)\\ &= \left[\frac{1}{2}\cdot\frac{\pi}{2}\right]-\left[\frac{1}{2}\arctan\left(\frac{1}{2}\right)\right]\end{aligned}$$

Therefore, series converges by the Integral Test.

#18

$$\sum_{n=3}^{\infty}\frac{3n-4}{n^2-2n}$$

$$\begin{aligned}\int_3^{\infty}\frac{3x-4}{x^2-2x}dx &= \lim_{t\to\infty}\int_3^t\frac{3x-4}{x^2-2x}dx\\ &= \lim_{t\to\infty}\int_3^t\frac{1}{x-2}+\frac{2}{x}dx \quad \text{used partial fractions :(}\\ &= \lim_{t\to\infty}\left[\ln|x-2|+2\ln|x|\right]_1^t\\ &= \lim_{t\to\infty}\left(\left[\ln|t-2|+2\ln|t|\right]-\lim_{t\to\infty}\left[\ln|3-2|+2\ln|3|\right]\right)\end{aligned}$$

$$\downarrow$$

$$\infty$$

Therefore, the series diverges by the Integral test.

#19

$$\sum_{n=1}^{\infty}\frac{\ln(n)}{n^3}$$

$$\begin{aligned}
\int_1^{\infty}\frac{\ln(x)}{x^3}dx &= \lim_{t\to\infty}\int_1^{t}\frac{\ln(x)}{x^3}dx \\
&= \lim_{t\to\infty}\int_1^{t}x^{-3}\ln(x)dx \\
&= \lim_{t\to\infty}\left[-\frac{1}{2}x^{-2}\ln(x)-\frac{1}{4}x^{-2}\right]_1^t \\
&= \lim_{t\to\infty}\left(\left[-\frac{1}{2}(t)^{-2}\ln(t)-\frac{1}{4}(t)^{-2}\right]-\left[-\frac{1}{2}(1)^{-2}\ln(1)-\frac{1}{4}(1)^{-2}\right]\right) \\
&= \lim_{t\to\infty}\left(\left[-\frac{1}{2}\cdot\frac{\ln(t)}{t^2}-\frac{1}{4}\cdot\frac{1}{t^2}\right]-\left[-\frac{1}{2}(1)^{-2}\ln(1)-\frac{1}{4}(1)^{-2}\right]\right) \\
&= -\left[-\frac{1}{2}(1)^{-2}\ln(1)-\frac{1}{4}(1)^{-2}\right]
\end{aligned}$$

$$u=\ln(x) \quad v'=x^{-3}$$

$$u'=\frac{1}{x} \qquad v=-\frac{1}{2}x^{-2}$$

$$\begin{aligned}
\int x^{-3}\ln(x)dx &= -\frac{1}{2}x^{-2}\ln(x)-\int\frac{1}{x}\left(-\frac{1}{2}x^{-2}\right)dx \\
&= -\frac{1}{2}x^{-2}\ln(x)+\frac{1}{2}\int x^{-3}dx \\
&= -\frac{1}{2}x^{-2}\ln(x)+\frac{1}{2}\left(-\frac{1}{2}x^{-2}\right)+C \\
&= -\frac{1}{2}x^{-2}\ln(x)-\frac{1}{4}x^{-2}+C
\end{aligned}$$

Therefore, series converges by the Integral Test.

#20

$$\sum_{n=1}^{\infty}\frac{1}{n^2+6n+13}$$

$$\begin{aligned}\int_1^{\infty}\frac{1}{x^2+6x+13}dx &= \lim_{t\to\infty}\int_1^t\frac{1}{x^2+6x+13}dx\\ &= \lim_{t\to\infty}\int_1^t\frac{1}{\left(x^2+6x+9\right)+4}dx\\ &= \lim_{t\to\infty}\int_1^t\frac{1}{\left(x+3\right)^2+2^2}dx\\ &= \lim_{t\to\infty}\left[\frac{1}{2}\arctan\left(\frac{x+3}{2}\right)\right]_1^t\\ &= \lim_{t\to\infty}\left(\left[\frac{1}{2}\arctan\left(\frac{t+3}{2}\right)\right]-\left[\frac{1}{2}\arctan\left(\frac{(1)+3}{2}\right)\right]\right)\\ &= \left[\frac{1}{2}\cdot\frac{\pi}{2}\right]-\left[\frac{1}{2}\arctan\left(\frac{(1)+3}{2}\right)\right]\end{aligned}$$

Therefore, series converges by the Integral Test.

#21

$$\sum_{n=2}^{\infty}\frac{1}{n\ln(n)}$$

$$\begin{aligned}\int_2^{\infty}\frac{1}{x\ln(x)}dx &= \lim_{t\to\infty}\int_2^t\frac{1}{x\ln(x)}dx\\ &= \lim_{t\to\infty}\left[\ln\left(\ln(x)\right)\right]_2^{\infty}\\ &= \lim_{t\to\infty}\left(\left[\ln\left(\ln(t)\right)\right]-\lim_{t\to\infty}\left[\ln\left(\ln(2)\right)\right]\right)\\ &\downarrow\\ &\infty\end{aligned}$$

Therefore, the series diverges by the Integral test.

Do a u-sub with $u=\ln(x)$ and $du=\frac{1}{x}dx$

#22

$$\sum_{n=2}^{\infty}\frac{1}{n\left(\ln\left(n\right)\right)^2}$$

$$\begin{aligned}
\int_2^{\infty}\frac{1}{x\left(\ln\left(x\right)\right)^2}\,dx &= \lim_{t\to\infty}\int_2^{t}\frac{1}{x\left(\ln\left(x\right)\right)^2}\,dx \\
&= \lim_{t\to\infty}\left[-\left(\ln\left(x\right)\right)^{-1}\right]_2^t \\
&= \lim_{t\to\infty}\left(\left[-\left(\ln\left(t\right)\right)^{-1}\right]-\left[-\left(\ln\left(2\right)\right)^{-1}\right]\right) \\
&= \lim_{t\to\infty}\left(\left[-\frac{1}{\ln\left(t\right)}\right]-\left[-\left(\ln\left(2\right)\right)^{-1}\right]\right) \\
&= -\left[-\left(\ln\left(2\right)\right)^{-1}\right]
\end{aligned}$$

Therefore, series converges by the Integral Test.

Do a u-sub with $u=\ln\left(x\right)$ and $du=\frac{1}{x}dx$

#23

$$\sum_{n=1}^{\infty}\frac{e^{\frac{1}{n}}}{n^2}$$

$$\begin{aligned}
\int_1^{\infty}\frac{e^{\frac{1}{x}}}{x^2}\,dx &= \lim_{t\to\infty}\int_1^{t}\frac{e^{\frac{1}{x}}}{x^2}\,dx \\
&= \lim_{t\to\infty}\int_1^{t}e^{x^{-1}}\cdot\left(x^{-2}\right)dx \\
&= -\lim_{t\to\infty}\int_1^{t}e^{x^{-1}}\cdot\left(-x^{-2}\right)dx \\
&= -\lim_{t\to\infty}\left[e^{x^{-1}}\right]_2^{\infty} \\
&= -\lim_{t\to\infty}\left(\left[e^{(t)^{-1}}\right]-\left[e^{(2)^{-1}}\right]\right) \\
&= -\lim_{t\to\infty}\left(e^{\frac{1}{t}}-\left[e^{(2)^{-1}}\right]\right) \\
&= -\left(1-\left[e^{(2)^{-1}}\right]\right)
\end{aligned}$$

Therefore, series converges by the Integral Test.

#24

$$\sum_{n=3}^{\infty}\frac{n^2}{e^n}$$

$$\begin{aligned}
\int_3^{\infty}\frac{x^2}{e^x}dx &= \lim_{t\to\infty}\int_3^{t}\frac{x^2}{e^x}dx \\
&= \lim_{t\to\infty}\int_3^{t}x^2e^{-x}dx \\
&= \lim_{t\to\infty}\left[-x^2e^{-x}-2xe^{-x}-2e^{-x}\right]_3^t \\
&= \lim_{t\to\infty}\left(\left[-(t)^2e^{-(t)}-2(t)e^{-(t)}-2e^{-(t)}\right]-\left[-(3)^2e^{-(3)}-2(3)e^{-(3)}-2e^{-(3)}\right]\right) \\
&= \lim_{t\to\infty}\left(\left[-(t)^2\cdot\frac{1}{e^t}-2(t)\cdot\frac{1}{e^t}-2\cdot\frac{1}{e^t}\right]-\left[-(3)^2e^{-(3)}-2(3)e^{-(3)}-2e^{-(3)}\right]\right) \\
&= -\left[-(3)^2e^{-(3)}-2(3)e^{-(3)}-2e^{-(3)}\right]
\end{aligned}$$

u	v'
x^2	e^{-x}
$2x$	$-e^{-x}$
2	e^{-x}
0	$-e^{-x}$

Therefore, series converges by the Integral Test.

#25

$$\sum_{n=1}^{\infty}\frac{1}{n^2+n^3}$$

$$\begin{aligned}
\int_1^{\infty}\frac{1}{x^2+x^3}dx &= \lim_{t\to\infty}\int_1^{t}\frac{1}{x^2+x^3}dx \\
&= \lim_{t\to\infty}\int_1^{t}\frac{1}{x+1}-\frac{1}{x}+\frac{1}{x^2}dx \\
&= \lim_{t\to\infty}\left[\ln|x+1|-\ln|x|+x^{-1}\right]_1^t \\
&= \lim_{t\to\infty}\left(\left[\ln|t+1|-\ln|t|+t^{-1}\right]-\left[\ln|1+1|-\ln|1|+(1)^{-1}\right]\right) \\
&= \lim_{t\to\infty}\left(\left[\ln\left|\frac{t+1}{t}\right|+t^{-1}\right]-\left[\ln|1+1|-\ln|1|+(1)^{-1}\right]\right) \\
&= \left(\left[\ln\left|\lim_{t\to\infty}\frac{t+1}{t}\right|+\lim_{t\to\infty}t^{-1}\right]-\left[\ln|1+1|-\ln|1|+(1)^{-1}\right]\right) \\
&= \left(\left[\ln|1|+0\right]-\left[\ln|1+1|-\ln|1|+(1)^{-1}\right]\right)
\end{aligned}$$

Therefore, series converges by the Integral Test.

#26

$$\sum_{n=1}^{\infty}\frac{n}{n^4+1}$$

$$\begin{aligned}
\int_1^{\infty}\frac{x}{x^4+1}dx &= \lim_{t\to\infty}\int_1^{t}\frac{x}{x^4+1}dx \\
&= \lim_{t\to\infty}\int_1^{t}\frac{x}{\left(x^2\right)^2+1^2}dx \\
&= \frac{1}{2}\lim_{t\to\infty}\int_1^{t}\frac{1}{\left(x^2\right)^2+1^2}\cdot 2xdx \\
&= \frac{1}{2}\lim_{t\to\infty}\Big[\arctan\left(x^2\right)\Big]_1^t \\
&= \frac{1}{2}\lim_{t\to\infty}\left[\arctan\left(t^2\right)-\arctan\left(1^2\right)\right] \\
&= \frac{1}{2}\left[\frac{\pi}{2}-\arctan\left(1^2\right)\right]
\end{aligned}$$