Pointers for Definite Integrals

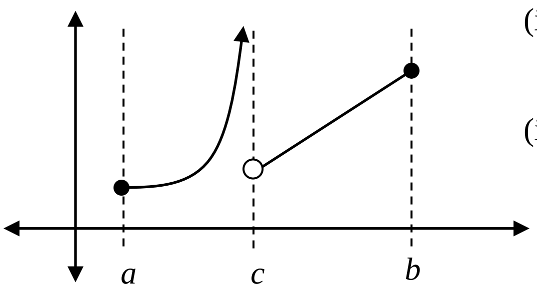
#1 First and foremost, the function you are integrating must be *continuous on the interval and have an antiderivative* for the interval you are integrating over *in order to use the Fundamental Theorem of Calculus*. That is, if you are evaluating:

$$\int_a^b f(x)\,dx$$

then $f(x)$, must be continuous on the closed interval $[a,b]$ and have an antiderivative on the interval to use $\int_a^b f(x)\,dx = F(b) - F(a)$.

#2 If your function does have a discontinuity within the interval, there are two possibilities:

a) If there is an infinite discontinuity within the interval then

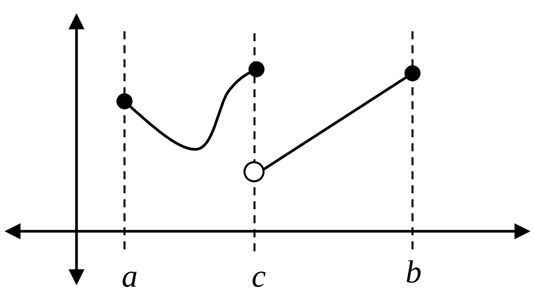


(i) in AP Calculus AB $\int_a^b f(x)\,dx$ Does not exist

(ii) In AP Calculus BC

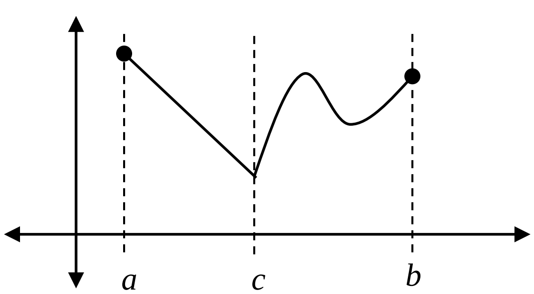
$$\int_a^b f(x)\,dx = \lim_{k\to c^-}\int_a^k f(x)\,dx + \lim_{w\to c^+}\int_w^b f(x)\,dx$$

b) If the only discontinuities are jump-discontinuities then you can handle the discontinuities by splitting the interval into subintervals. See the example below

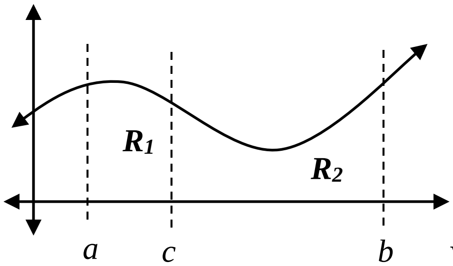


$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

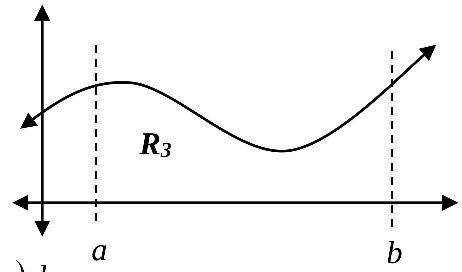
#3 If your function is not differentiable at a given value in the integral you are integrating, you must break up the integral using that value as one of the endpoints.



$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



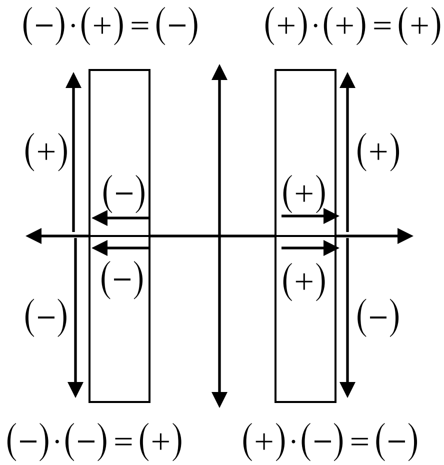
Basic Integration Rules



$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

When $a \le c \le b$ then $\underbrace{\int_a^b f(x)\,dx}_{R_3} = \underbrace{\int_a^c f(x)\,dx}_{R_1} + \underbrace{\int_c^b f(x)\,dx}_{R_2}$



$$\int_a^b k\cdot f(x)\,dx = k\cdot\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

If $f(x)\ge 0$ on $[a,b]$, then $\int_a^b f(x)\,dx \ge 0$

If $f(x)\le 0$ on $[a,b]$, then $\int_a^b f(x)\,dx \le 0$ nsh

If $f(x)\ge g(x)$ on $[a,b]$, then $\int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

Fundamental Theorem of Calculus

Part I: If $f(x)$ is a continuous function on $[a,b]$, and $F(x)$ is an antiderivative of $f(x)$ on $[a,b]$, then $\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)$.

Part II:

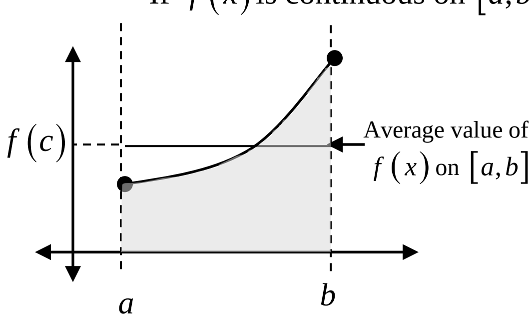
$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f(g(x))\cdot g'(x)$$

$$\frac{d}{dx}\left[\int_{m(x)}^{n(x)} f(t)\,dt\right] = \frac{d}{dx}\left[\int_a^{n(x)} f(t)\,dt - \int_a^{m(x)} f(t)\,dt\right] = f(n(x))\cdot n'(x) - f(m(x))\cdot m'(x)$$

Mean Value Theorem for Integrals

If $f(x)$ is continuous on $[a,b]$, then there exists a number c in $[a,b]$ such that



$$\underbrace{\int_a^b f(x)\,dx}_{\text{area under curve}} = \underbrace{f(c)\cdot(b-a)}_{\text{area of rectangle}}$$

$$\updownarrow$$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$