Calculus Homework Guidelines Regarding Limits:

Ex: Find the limit analytically:

$$\begin{aligned}
\lim_{x\to 0}\frac{\sin(x)\left(1-\cos(x)\right)}{-2x^2} &= -\frac{1}{2}\lim_{x\to 0}\frac{\sin(x)}{x}\cdot\frac{1-\cos(x)}{x}\\
&= -\frac{1}{2}\left[\lim_{x\to 0}\frac{\sin(x)}{x}\right]\cdot\left[\lim_{x\to 0}\frac{1-\cos(x)}{x}\right]\\
&= -\frac{1}{2}[1][0]\\
&= 0
\end{aligned}$$

$$\begin{aligned}
\lim_{x\to -9}\frac{x^2+6x-27}{x+9} &= \lim_{x\to -9}\frac{(x+9)(x-3)}{x+9}\\
&= \lim_{x\to -9} x-3\\
&= -9-3\\
&= -12
\end{aligned}$$

- ✓ All work must be done vertically, aligned at the equals sign.
- ✓ The limit must be written at each step until you can use direct substitution or a special limit property. You must show the substitution so that the second to last step is an expression that no longer has a limit and has no variables.
- ✓ Intermediate steps must be shown, within reason. No "giant leaps" to the correct answer. The use of the properties of limits should be demonstrated within reason.
- ✓ If you want to reason that a limit is $\pm\infty$ because of division by small numbers close to zero, your solution must demonstrate a limit such as :

  $$\lim_{x\to k^{\pm}}\frac{c}{x-k}=\frac{c}{0^{\pm}}\text{ , provided } c\in\mathbb{R}\ c\neq 0.$$
- ✓ You may not plug in numbers close to the limit and use your calculator to guess at the correct answer. Analytic ↔ Algebraic.
- ✓ The method you use to achieve your final answer must be correct/valid. Work that so happens to lead to the final result, but does not make sense, will be penalized.

$$\lim_{x\to-\infty}\left(3x+\sqrt{9x^2-x}\right)=\lim_{x\to-\infty}\frac{3x+\sqrt{9x^2-x}}{1}\cdot\frac{3x-\sqrt{9x^2-x}}{3x-\sqrt{9x^2-x}}$$

$$=\lim_{x\to-\infty}\frac{9x^2-\left(9x^2-x\right)}{3x-\sqrt{9x^2-x}}$$

$$=\lim_{x\to-\infty}\frac{x}{3x-\sqrt{9x^2-x}}$$

$$\downarrow$$

$$=\lim_{x\to-\infty}\frac{x}{3x-\sqrt{9x^2}}$$

$$=\lim_{x\to-\infty}\frac{x}{3x-3|x|}$$

$$=\lim_{x\to-\infty}\frac{x}{3x-3(-x)}$$

$$=\lim_{x\to-\infty}\frac{x}{6x}$$

$$=\lim_{x\to-\infty}\frac{1}{6}$$

$$=\frac{1}{6}$$

$$|x|=\begin{cases}-x & \text{if } x<0\\ x & \text{if } x\geq 0\end{cases}$$

$$\lim_{s\to0}\frac{\frac{1}{\sqrt{s+1}}-1}{s}=\lim_{s\to0}\frac{\frac{1}{\sqrt{s+1}}-\frac{\sqrt{s+1}}{\sqrt{s+1}}}{s}$$

$$=\lim_{s\to0}\frac{\frac{1-\sqrt{s+1}}{\sqrt{s+1}}}{s}$$

$$=\lim_{s\to0}\frac{1-\sqrt{s+1}}{s\sqrt{s+1}}$$

$$=\lim_{s\to0}\frac{1-\sqrt{s+1}}{s\sqrt{s+1}}\cdot\frac{1+\sqrt{s+1}}{1+\sqrt{s+1}}$$

$$=\lim_{s\to0}\frac{1-(s+1)}{s\sqrt{s+1}\left(1+\sqrt{s+1}\right)}$$

$$=\lim_{s\to0}\frac{-s}{s\sqrt{s+1}\left(1+\sqrt{s+1}\right)}$$

$$=\lim_{s\to0}\frac{-1}{\sqrt{s+1}\left(1+\sqrt{s+1}\right)}$$

$$=-\frac{1}{2}$$

$$\lim_{\theta\to0}\frac{\cos(\theta)\tan(\theta)}{\theta}=\lim_{\theta\to0}\frac{\cos(\theta)\frac{\sin(\theta)}{\cos(\theta)}}{\theta}$$

$$=\lim_{\theta\to0}\frac{\sin(\theta)}{\theta}$$

$$=1$$

Use the following information to evaluate the limits:

$$\lim_{x\to c}f(x)=2\quad \lim_{x\to c}g(x)=\frac{3}{4}$$

$$\lim_{x\to c}\left[f(x)g(x)\right]=\left[\lim_{x\to c}f(x)\right]\cdot\left[\lim_{x\to c}g(x)\right]$$

$$=(2)\left(\frac{3}{4}\right)$$

$$=\frac{3}{2}$$

$$\lim_{x\to c}\left[4f(x)+3\left[g(x)\right]^2\right]=\lim_{x\to c}\left[4f(x)\right]+\lim_{x\to c}\left[3\left[g(x)\right]^2\right]$$

$$=4\cdot\lim_{x\to c}f(x)+3\cdot\lim_{x\to c}\left[g(x)\right]^2$$

$$=4\cdot\lim_{x\to c}f(x)+3\cdot\left[\lim_{x\to c}g(x)\right]^2$$

$$=4\cdot(2)+3\cdot\left(\frac{3}{4}\right)^2$$

$$=8+\frac{9}{16}$$

$$=\frac{137}{16}$$