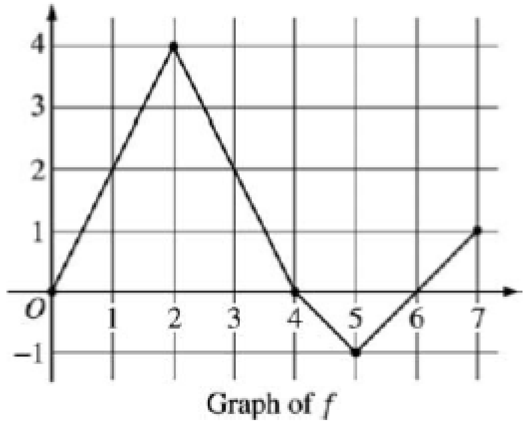
2003 Form B #5 No Calculator:



Let f be a function defined on the closed interval $[0,7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x)=\int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

$$g(x)=\int_2^x f(t)\,dt \qquad g(3)=\int_2^3 f(t)\,dt = 2+\frac{1}{2}(1)(2)=3$$

$$g'(x)=f(x) \qquad \text{so} \quad g'(3)=f(3)=2$$

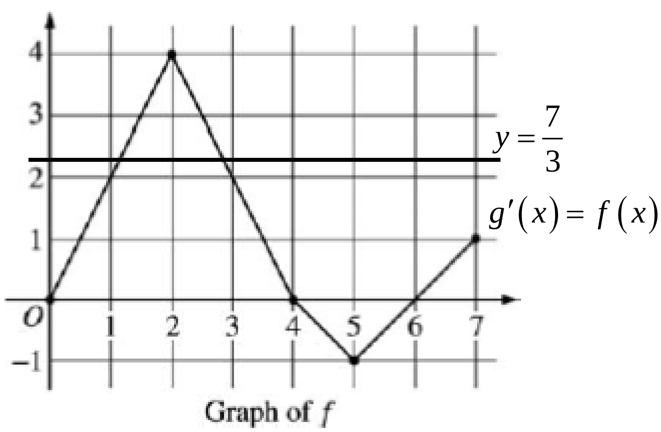
$$g''(x)=f'(x) \qquad g''(3)=f'(3)=-2$$

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

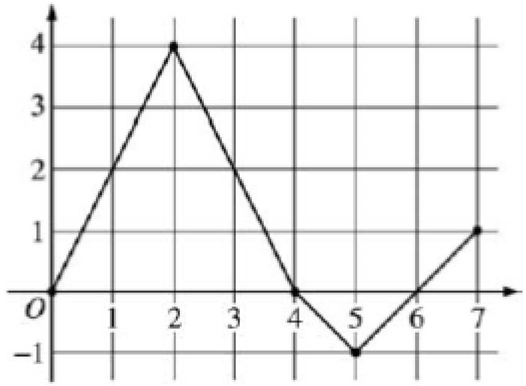
$$AROC=\frac{g(3)-g(0)}{3-0}=\frac{3-\int_2^0 f(t)\,dt}{3}=\frac{3-\left[-\frac{1}{2}(2)(4)\right]}{3}=\frac{7}{3}$$

(c) For how many values c, where $0<c<3$, is $g'(c)$ equal to the average rate of change found in part (b)? Explain your reasoning.

$g'(x)=\frac{7}{3}$ twice on the interval $0<x<3$



2003 Form B #5 No Calculator:



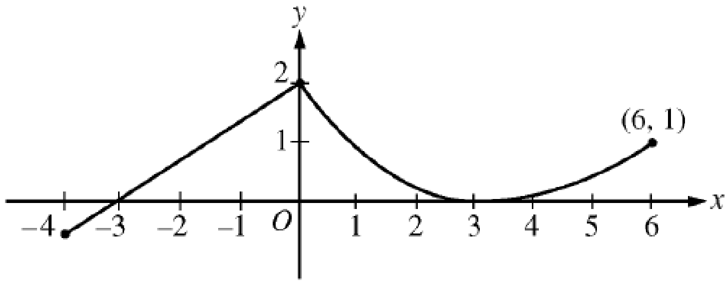
Graph of f

Let f be a function defined on the closed interval $[0,7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x)=\int_2^x f(t)\,dt$.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0<x<7$. Justify your answer.

$g(x)$ has points of inflection when $g''(x)=f'(x)$ changes sign. This occurs at $x=2$ and $x=5$.

2009 Form B #3 Calculator Allowed:



Graph of f

A continuous function *f* is defined on the closed interval $-4 \le x \le 6$. The graph of *f* consists of a line segment and a curve that is tangent to the *x*-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function *f* is twice differentiable, with $f''(x) > 0$.

(a) Is *f* differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

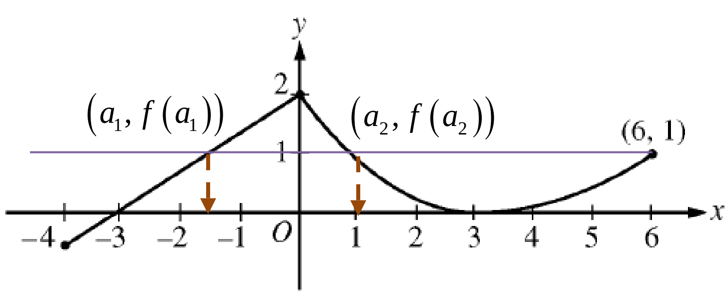
$$\lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h} = \frac{3}{4} \text{ [slope of the line to the left of zero]}$$

$$\lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h} < 0 \text{ [visually]}$$

Since $\lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h} \ne \lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h}$, $f(x)$ is not differentiable at $x = 0$.

(b) For how many values of *a*, $-4 \le a < 6$ is the average rate of change of *f* on the interval $[a, 6]$ equal to 0? Give a reason for your answer.

The average rate of change of *f* on $[a, 6]$ is given by $\frac{f(6) - f(a)}{6 - a}$, which represents that slope between the two points $(a, f(a))$ and $(6, f(6))$. The average rate of change of *f* on the interval $[a, 6]$ equal to 0 when the slope of the line between $(a, f(a))$ and $(6, f(6))$ is zero. This will occur twice for $-4 \le a < 6$.

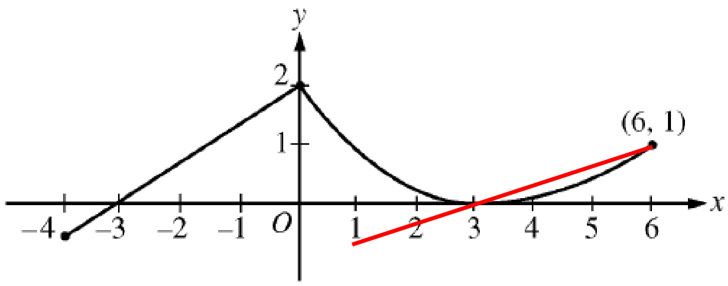


Graph of f

(c) Is there a value of a, $-4 \le a < 6$ for which the Mean Value Theorem, applied to the interval $[a,6]$ guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$. Justify your answer.

We need to see if there is a point $(a, f(a))$ for $-4 \le a < 6$ where $\frac{f(6) - f(a)}{6-a} = \frac{1}{3}$.

There is such a point, and the value of a is 3.



Graph of f

$$\frac{f(6) - f(3)}{6-3} = \frac{1-0}{3} = \frac{1}{3}$$

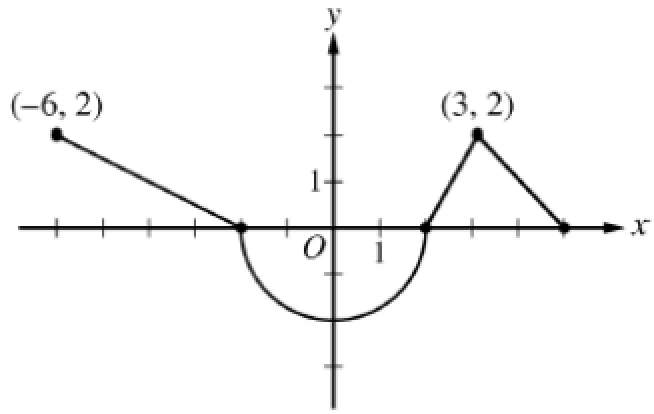
(d) The function g is defined by $g(x) = \int_0^x f(t)\,dt$ for $-4 \le x < 6$. On what intervals contained in $[-4,6]$ is the graph of g concave up? Explain your reasoning.

$g(x)$ is concave up when $g''(x) > 0$

$$g(x) = \int_0^x f(t)\,dt$$
$$g'(x) = f(x)$$
$$g''(x) = f'(x)$$

$g(x)$ is concave up when $-4 < x < 0$ and $3 < x < 6$.

2017 #3 No Calculator:



Graph of f'

The function *f* is differentiable on the closed interval $[-6,5]$ and satisfies $f(-2)=7$. The graph of f', the derivative of *f*, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

$$f(-6)=f(-2)+\int_{-2}^{-6} f'(x)\,dx=7-\frac{1}{2}(4)(2)=3$$

$$f(5)=f(-2)+\int_{-2}^{5} f'(x)\,dx=7-\frac{1}{2}\pi(2)^2+\frac{1}{2}(3)(2)=10-2\pi\approx 3.7168...$$

(b) On what intervals is *f* increasing? Justify your answer.

$f(x)$ is increasing on $-6<x<-2$ and $2<x<5$ because $f'(x)>0$.

(c) Find the absolute minimum value of *f* on the closed interval $[-6,5]$. Justify your answer.

$f'(x)$ changes sign from negative to positive at $x=2$.

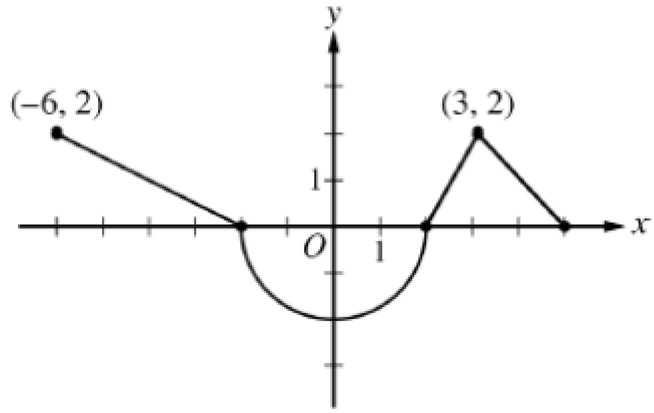
$$f(-6)=3$$

$$f(2)=f(-2)+\int_{-2}^{2} f'(x)\,dx=7-\frac{1}{4}\pi(2)^2=7-\pi\approx 3.8584...$$

$$f(5)=10-\pi\approx 6.8584...$$

The absolute minimum of *f* on the closed interval $[-6,5]$ is 3

2017 #3 No Calculator:



Graph of f'

The function *f* is differentiable on the closed interval $[-6,5]$ and satisfies $f(-2)=7$. The graph of f', the derivative of *f*, consists of a semicircle and three line segments, as shown in the figure above.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

$f''(-5)=-\frac{1}{2}$. $f''(3)$ does not exist because

$$\lim_{h\to 0^-}\frac{f(3+h)-f(3)}{h}\neq\lim_{h\to 0^+}\frac{f(3+h)-f(3)}{h}$$
$$2\neq -1$$