Things to know/remember for Series:

Sum/Difference Rule

$$\sum_{n=1}^{\infty}(a_n \pm b_n) = \left(\sum_{n=1}^{\infty} a_n\right) \pm \left(\sum_{n=1}^{\infty} b_n\right)$$

Explanation:

$$\begin{aligned}\sum_{n=1}^{\infty}(a_n \pm b_n) &= (a_1 \pm b_1) + (a_2 \pm b_2) + (a_3 \pm b_3) + \cdots \\ &= (a_1 + a_2 + a_3 + \cdots) + (b_1 + b_2 + b_3 + \cdots) \\ &= \left(\sum_{n=1}^{\infty} a_n\right) \pm \left(\sum_{n=1}^{\infty} b_n\right)\end{aligned}$$

Constant Multiple Rule:

$$\sum_{n=1}^{\infty} k \cdot a_n = k \cdot \left(\sum_{n=1}^{\infty} a_n\right)$$

Explanation:

$$\begin{aligned}\sum_{n=1}^{\infty} k \cdot a_n &= k \cdot a_1 + k \cdot a_2 + k \cdot a_3 + \cdots \\ &= k \cdot (a_1 + a_2 + a_3 + \cdots) \\ &= k \cdot \left(\sum_{n=1}^{\infty} a_n\right)\end{aligned}$$

Factorial!

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot (3)(2)(1)$$

Examples:

$$4! = 4 \cdot 3 \cdot 2 \cdot 1$$
$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

Common trick with factorial you will see in your future:

$5! = 4 \cdot 4!$ $102! = 102 \cdot 101 \cdot 100!$	$(n+2)! = (n+2)(n+1) \cdot n!$ $(n+1)! = (n+1) \cdot n!$	$\frac{(n+2)!}{n!} = \frac{(n+2)(n+1) \cdot n!}{n!} = (n+2)(n+1)$

Common Tricks with Exponents and roots:

$$3^{n+2} = 3^n \cdot 3^2 \qquad \frac{3^{n+2}}{3^n} = \frac{3^n \cdot 3^2}{3^2} = 3^2 \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n \qquad \frac{2^{n+2}}{3^n} = \frac{2^2 \cdot 2^n}{3^n} = 4 \cdot \frac{2^n}{3^n} = 4\left(\frac{2}{3}\right)^n \qquad \sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{3} = 3^{\frac{1}{n}}$$

Know L'Hopital's rule!

In particular, remember $\lim_{n\to\infty}\left(1+\frac{k}{n}\right)^n = e^k$. This can be stated without having to demonstrate all of L'Hopital's Rule used to prove it.

You may also want to remember these limits that may need L'Hopital's Rule to be determined:

$$\lim_{n\to\infty}\sqrt[n]{c}=1\ \left(c>0\right)\ ;\ \lim_{n\to\infty}\left(1+\frac{c}{n}\right)^n=e^c\ ;\lim_{n\to\infty}\sqrt[n]{n}=1\ ;\ \lim_{n\to\infty}\sqrt[n]{n!}=\infty\,;\ \lim_{n\to\infty}\sqrt[n]{n^c}=1;$$

Trick with limits of rational functions:

$$\lim_{x\to\infty}\left[\frac{a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0}{b_mx^m+b_{m-1}x^{m-1}+\cdots+b_1x+b_0}\right]\sim\lim_{x\to\infty}\left[\frac{a_nx^n}{b_mx^m}\right]$$

Example: $\lim_{n\to\infty}\frac{n^2+1}{n^7-n^3+7}\sim\lim_{n\to\infty}\frac{n^2}{n^7}=\lim_{n\to\infty}\frac{1}{n^5}$

Know Growth Order!

Slowest growing: $c<\ln\left(\ln\left(n\right)\right)<\ln\left(n\right)<\left[\ln\left(n\right)\right]^c<n^c<c^n<n!<n^n$ Fastest growing

$$\lim_{n\to\infty}\left[\frac{\text{slower growing}}{\text{faster growing}}\right]=0$$

$$\lim_{n\to\infty}\left[\frac{\text{faster growing}}{\text{slower growing}}\right]\to\infty$$

Expressing Even and Odd number sequences:

Even Numbers	$2n$
Odd Numbers	$\left(2n+1\right)$

Property of Absolute Value:

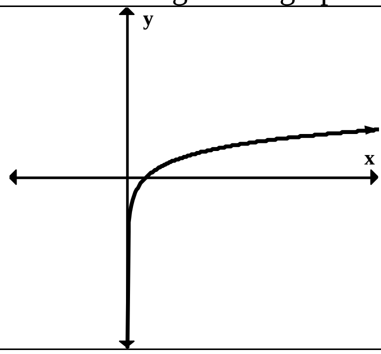
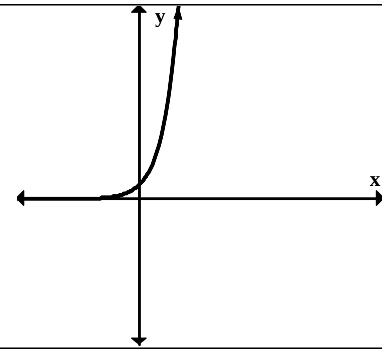
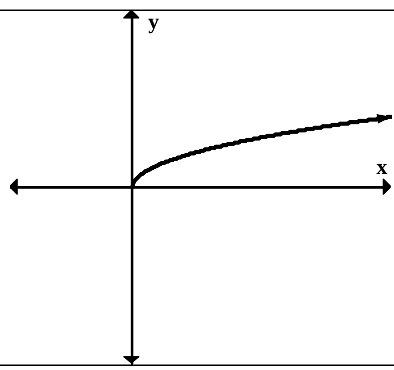
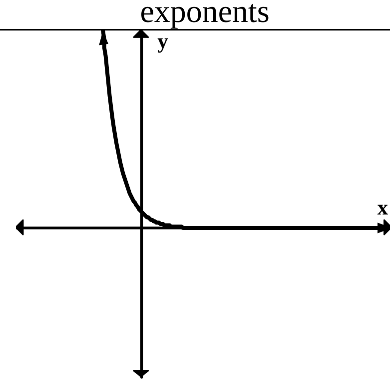
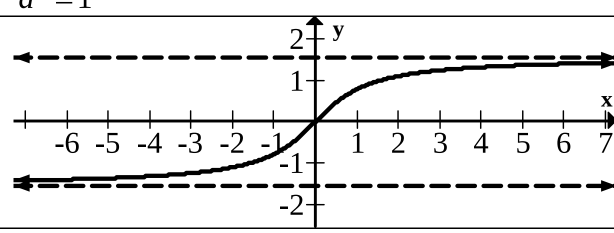
$$\left|a\cdot b\right|=\left|a\right|\cdot\left|b\right| \text{ and } \left|\frac{a}{b}\right|=\frac{\left|a\right|}{\left|b\right|}$$

Example: $\left|-5x\right|=\left|-5\right|\cdot\left|x\right|$

Solve an absolute value inequality:

$$\begin{array}{c}\left|x\right|<2\\ \downarrow\\ -2<x<2\end{array}\quad\text{and}\quad\begin{array}{c}\left|x+2\right|<3\\ \downarrow\\ -3<x+2<3\\ -1<x<5\end{array}\quad\text{and}\quad\begin{array}{c}\left|\frac{x-1}{3}\right|<2\\ \frac{\left|x-1\right|}{3}<2\\ \left|x-1\right|<6\\ \downarrow\\ -6<x-1<6\\ -5<x<7\end{array}$$

Know the basic shape of the following graphs and important features/behavior:

$y=\log_b(x)$ with $b>1$. All basic logarithm graphs	$y=a^x$ with $a>1$ All basic exponential graphs
	
Domain: $x>0$ Range: all real numbers $\lim\limits_{x\to 0^+}\left[\log_b(x)\right]\to -\infty$ $\lim\limits_{x\to\infty}\left[\log_b(x)\right]\to\infty$ $\log_b(1)=0$	Domain: all real numbers Range: $y>0$ $\lim\limits_{x\to -\infty}\left[a^x\right]=0^+$ $\lim\limits_{x\to\infty}\left[a^x\right]\to\infty$ $a^0=1$
$y=\sqrt[k]{x}$ All basic root graphs	$y=a^{-x}$ with $a>1$ or $y=a^x$ with $0<a<1$ All basic exponential graphs with negative exponents
	
Domain: $x>0$ Range: $y>0$ $\lim\limits_{x\to 0^+}\left[\sqrt[k]{x}\right]=0^+$ $\lim\limits_{x\to\infty}\left[\sqrt[k]{x}\right]\to\infty$	Domain: all real numbers Range: $y>0$ $\lim\limits_{x\to\infty}\left[a^x\right]=0^+$ $\lim\limits_{x\to -\infty}\left[a^x\right]\to\infty$ $a^0=1$
Also remember that $\lim\limits_{x\to\infty}\left(\arctan(x)\right)=\frac{\pi}{2}$	

Dividing a fraction by a fraction:

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{x}{y}\right)} = \frac{a}{b} \div \frac{x}{y} = \frac{a}{b} \times \frac{y}{x} = \frac{ay}{bx}$$

$$\frac{\left(\frac{x^2+1}{2x-1}\right)}{\left(\frac{3x-2}{5x^2-1}\right)} = \frac{x^2+1}{2x-1} \cdot \frac{5x^2-1}{3x-2}$$

Know the following fraction comparison techniques:

$\frac{\text{numerator}}{\text{denominator}} < \frac{\text{greater numerator}}{\text{same denominator}}$ $\frac{3}{4} < \frac{5}{4}$	$\frac{\text{same numerator}}{\text{greater denominator}} < \frac{\text{numerator}}{\text{denominator}}$ $\frac{3}{7} < \frac{3}{4}$
$\frac{\text{numerator}}{\text{denominator}} < \frac{\text{greater numerator}}{\text{smaller denominator}}$ $\frac{3}{5} < \frac{5}{2}$	$\frac{\text{smaller numerator}}{\text{greater denominator}} < \frac{\text{numerator}}{\text{denominator}}$ $\frac{2}{7} < \frac{3}{4}$

$$-1 \le \cos(x) \le 1 \text{ and } -1 \le \sin(x) \le 1$$