Standard Problems 3. Projectile Motion

Introductory problem.

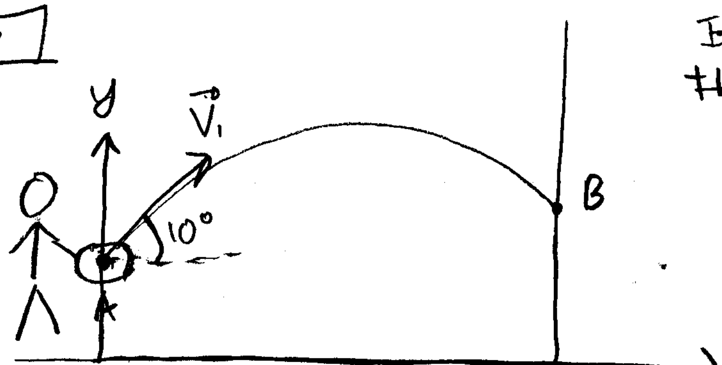
The equations below are valid for X and Y motion:

$$v_B = v_A + a_{AB}\Delta t_{AB} \qquad x_B = x_A + v_A\Delta t_{AB} + \frac{a_{AB}}{2}\Delta t_{AB}{}^2$$

$$v_B^2 = v_A^2 + 2a_{AB}\Delta x_{AB} \qquad x_B = x_A + \left(\frac{v_A+v_B}{2}\right)\Delta t_{AB}$$

A tennis player hits a ball at a height of 1.0 m above the ground. The ball's initial velocity is 50 m/s at an angle of 10 degrees above the horizontal. The ball travels to a vertical wall 10 m away. How high is the ball above the ground when it hits the wall?

$\boxed{\text{O and P}}$



~~I put the origin at the initial~~

$y_A = 1.0\text{ m}$
$y_B = ?$
$x_A = 0\text{ m}$
$x_B = 10\text{ m}$

$V_{xA} = V_A \cos 10 = 49.2\text{ m/s}$
$V_{yA} = V_A \sin 10 = 8.68\text{ m/s}$

$a_{xAB} = 0 \quad a_{yAB} = -g$

I'll use the x-motion to find the time it takes for the ball to hit the wall, then plug that into the y motion.

$\boxed{\text{Solve}}$ x-motion: $X_B = X_A + V_{xA}\Delta t_{AB}$ (const velocity in x direction)

$$\Rightarrow \Delta t_{AB} = \frac{X_B - X_A}{V_{xA}} = \frac{10\text{ m} - 0\text{ m}}{49.2\text{ m/s}} = 0.21\text{ s}.$$

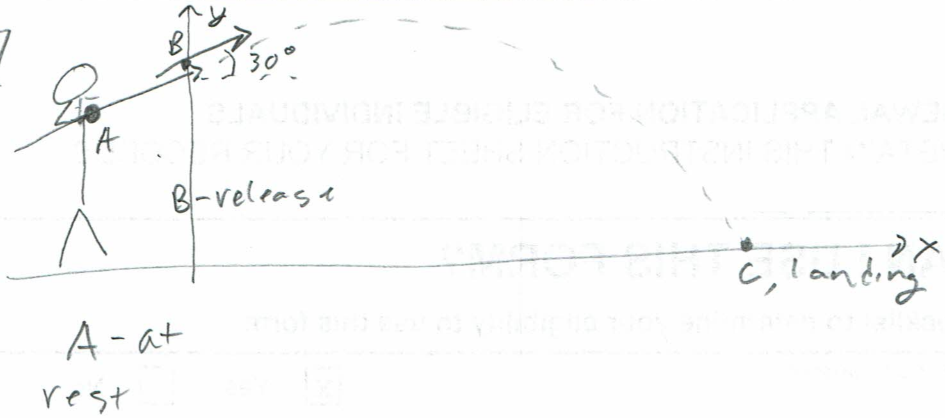
y-motion: $y_B = y_A + V_{yA}\Delta t_{AB} + \frac{1}{2}a_{yAB}\Delta t_{AB}^2$

$$= 1\text{ m} + (8.68\tfrac{\text{m}}{\text{s}})(0.21\text{s}) + \tfrac{1}{2}(-9.8\tfrac{\text{m}}{\text{s}^2})(0.21\text{s})^2$$

$$= 2.56\text{ m}$$ a reasonable height

$\boxed{\text{Reflect}}$ This problem is a bit unusual in that the end point is determined by the x motion instead of the y motion.

O+P



$V_A = 0 \qquad V_{Bx} = V_B \cos 30°$

$V_B = ? \qquad V_{By} = V_B \sin 30°$

$x_B = 0\,m \qquad y_B = 2.0\,m$

$x_c = 63\,m \qquad y_c = 0\,m$

$a_{BC,x} = 0 \qquad a_{BCy} = -g$

We'll use the projectile motion BC to find V_B, then use const. accel 1D motion to analyze AB to find a_{AB}.

Solve For BC

$$x_c = \cancel{x_B}^{0} + V_{Bx}\,\Delta t_{BC} \quad \Rightarrow \quad \Delta t_{BC} = \frac{x_c}{V_{Bx}} = \frac{x_c}{V_B \cos 30°} \quad (1)$$

$$\cancel{y_c}^{0} = y_B + V_{By}\,\Delta t_{BC} + \frac{1}{2} a_{BCy} (\Delta t_{BC})^2 \quad (2)$$

sub in (1) to (2): $$0 = y_B + V_{By}\left(\frac{x_c}{V_B \cos 30°}\right) + \frac{1}{2}(-g)\left(\frac{x_c}{V_B \cos 30°}\right)^2$$

$$\Rightarrow \quad 0 = y_B + \frac{V_B \sin 30°}{V_B \cos 30°} x_c - \frac{g}{2} \frac{x_c^2}{V_B^2 \cos^2 30°}$$

$$\Rightarrow \quad \frac{g}{2} \frac{x_c^2}{V_B^2 \cos^2 30} = y_B + (\tan 30°) x_c$$

$$\Rightarrow \quad 2 V_B^2 \cos^2 30 = \frac{g\, x_c^2}{y_B + (\tan 30)\, x_c}$$

$$\Rightarrow \quad V_B = \sqrt{\frac{g\, x_c^2}{2 \cos^2 30 \left(y_B + \tan(30°)\, x_c\right)}} = \sqrt{\frac{9.8\,m/s^2\,(63\,m)^2}{2 \cdot \left(\frac{3}{4}\right)\left(2\,m + \frac{1}{\sqrt{3}} \cdot 63\,m\right)}}$$

$$= 26.0\,m/s$$

Now along the line AB: (a path we'll define as the "x" direction

$$V_B^2 = \cancel{V_A^2}^{0} + 2a_{AB}\Delta x$$

$$\Rightarrow a_{AB} = \frac{V_B^2}{2\Delta x} = 482.7 \, m/s^2$$

Crazy fast! But makes sense given it's a small distance over which accel occurs.