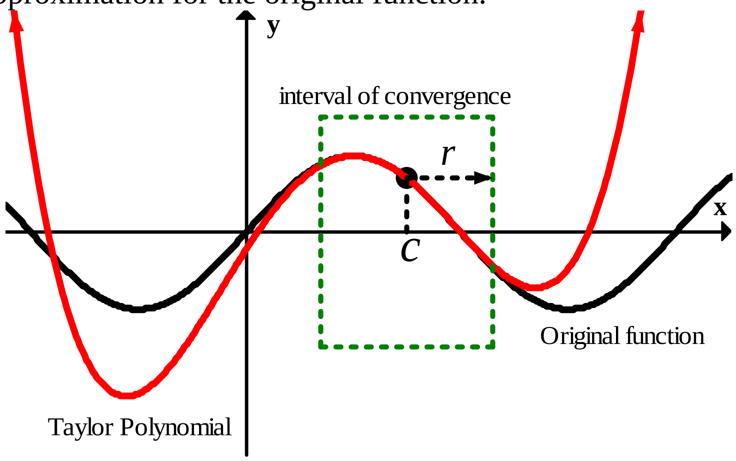
<u>What is the interval of convergence?</u>

As the degree of the Taylor Series increases, the Taylor Series Polynomial starts to match the given function more closely for a larger and larger interval around $x = c$.

If the degree of the Taylor Series Polynomial goes to ∞, the Taylor Series Polynomial will be indistinguishable from the given function, <u>for *x* in the interval of convergence</u>. Outside of the interval of convergence, the Taylor Polynomial is not a good approximation for the original function.



The interval of convergence is found by using either the Ratio Test or Root Test, and testing the endpoint values of the open interval $c - r < x < c + r$ defined by the value of the Radius of Convergence *r*.

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)(x-c)^n}{n!} = f(x) \text{ for all } x \text{ in interval of convergence.}$$

where the interval of convergence could be any of following intervals depending on whether the endpoints are included or not

$$c - r < x < c + r$$

$$c - r < x \le c + r$$

$$c - r \le x < c + r$$

$$c - r \le x \le c + r$$

Whenever a Taylor Polynomial is truncated to estimate the value of $f(x)$ at some value x other than c, there will be some error.

$$f(x) = \underbrace{f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \cdots + \frac{f^{(n)}(c)(x-c)^n}{n!}}_{P_n(x)} + \underbrace{\frac{f^{(n+1)}(c)(x-c)^{n+1}}{(n+1)!} + \cdots}_{R_n}$$

$$f(x) = P_n(x) + R_n$$

$$f(x) - P_n(x) = R_n$$

$$|f(x) - P_n(x)| = \underbrace{|R_n|}_{\text{Error}}$$

$$|f(x) - P_n(x)| = \text{Error}$$

<u>If the series is an Alternating Series</u>, then the Alternating Series Remainder Theorem should be used instead of the Lagrange Error Bound.

If the series is an Alternating Series, then

$$\text{Error} \le |\text{next term}|$$

If the series is not alternating, then the Lagrange Error Bound must be used to bound the error.

The Lagrange Error Bound states that the error in using the degree n Taylor Polynomial to approximate $f(x)$ at some x other than c is bounded by

$$\text{Error} \le \frac{\left[\max\limits_{z \text{ between } x \text{ and } c} f^{(n+1)}(z)\right] \cdot |x-c|^{n+1}}{(n+1)!}$$

<u>What is this</u> $\max\limits_{z \text{ between } x \text{ and } c} \left|f^{(n+1)}(z)\right|$**?** It is a number that is used in finding an upper bound for the error.

To find $\left[\max\limits_{z \text{ between } x \text{ and } c} f^{(n+1)}(z)\right]$, one will have to use one of the methods outlined in the Types of Taylor Series Error Bound exercises handout.

To understand what is meant by "…where z is between x and c." see the graphs provided below.

x* is to the left of *c	***x* is to the right of *c***