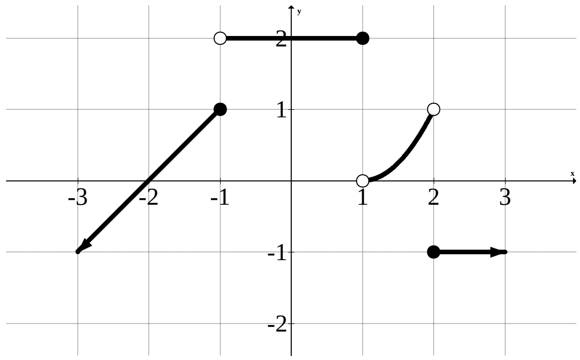
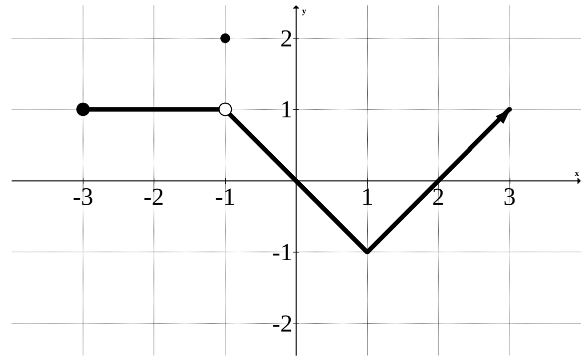
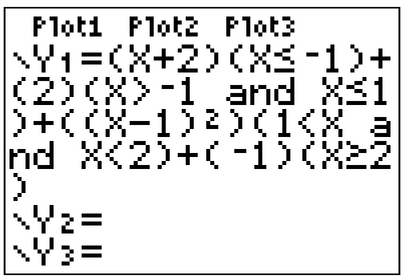
The graphs of f and g are given below.

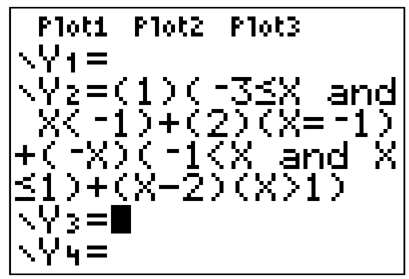


$f(x)$



$g(x)$





1. Determine whether the following limits exist. If they do, determine the value of the limit.

(a) $\lim\limits_{x\to -1} f(x)$ DNE

$\lim\limits_{x\to -1^-} f(x)=1$ and $\lim\limits_{x\to -1^+} f(x)=2$

(b) $\lim\limits_{x\to 1} f(x)$ DNE

$\lim\limits_{x\to 1^-} f(x)=2$ and $\lim\limits_{x\to 1^+} f(x)=0$

(c) $\lim\limits_{x\to -1} g(x)=1$

$\lim\limits_{x\to -1^-} g(x)=1$ and $\lim\limits_{x\to -1^+} g(x)=1$

(d) $\lim\limits_{x\to 1} g(x)=-1$

$\lim\limits_{x\to 1^-} g(x)=-1$ and $\lim\limits_{x\to 1^+} g(x)=-1$

(e) $\lim\limits_{x\to -1}\left[f(x)+g(x)\right]$ DNE

$$\begin{aligned}\lim_{x\to -1^-}\left[f(x)+g(x)\right]&=\left[\lim_{x\to -1^-} f(x)\right]+\left[\lim_{x\to -1^-} g(x)\right]\\&=1+1\\&=2\end{aligned}$$

$$\begin{aligned}\lim_{x\to -1^+}\left[f(x)+g(x)\right]&=\left[\lim_{x\to -1^+} f(x)\right]+\left[\lim_{x\to -1^+} g(x)\right]\\&=2+1\\&=3\end{aligned}$$

(f) $\lim\limits_{x\to 0}\left[2f(x)+3g(x)\right]=4$

$$\begin{aligned}\lim_{x\to 0^-}\left[2f(x)+3g(x)\right]&=\left[\lim_{x\to 0^-} 2\cdot f(x)\right]+\left[\lim_{x\to 0^-} 3\cdot g(x)\right]\\&=2\left[\lim_{x\to 0^-} f(x)\right]+3\left[\lim_{x\to 0^-} g(x)\right]\\&=2(2)+3(0)\\&=4\end{aligned}$$

$$\begin{aligned}\lim_{x\to 0^+}\left[2f(x)+3g(x)\right]&=\left[\lim_{x\to 0^+} 2f(x)\right]+\left[\lim_{x\to 0^+} 3g(x)\right]\\&=2\left[\lim_{x\to 0^+} f(x)\right]+3\left[\lim_{x\to 0^+} g(x)\right]\\&=2(2)+3(0)\\&=4\end{aligned}$$

(g) $\lim_{x\to -1^-}\left[f(x)g(x)\right]$ DNE

$$\lim_{x\to -1^-}\left[f(x)g(x)\right]=\left[\lim_{x\to -1^-} f(x)\right]\left[\lim_{x\to -1^-} g(x)\right]=[1]\cdot[1]=1$$

$$\lim_{x\to -1^+}\left[f(x)g(x)\right]=\left[\lim_{x\to -1^+} f(x)\right]\left[\lim_{x\to -1^+} g(x)\right]=[2]\cdot[1]=2$$

(h) $\lim_{x\to 2}\left[f(x)g(x)\right]=0$

$$\lim_{x\to 2^-}\left[f(x)g(x)\right]=\left[\lim_{x\to 2^-} f(x)\right]\left[\lim_{x\to 2^-} g(x)\right]=[1]\cdot[0]=0$$

$$\lim_{x\to 2^+}\left[f(x)g(x)\right]=\left[\lim_{x\to 2^+} f(x)\right]\left[\lim_{x\to 2^+} g(x)\right]=[-1]\cdot[0]=0$$

(i) $\lim_{x\to 0}\dfrac{f(x)}{g(x)}$ DNE

$$\lim_{x\to 0^-}\frac{f(x)}{g(x)}=\frac{\lim_{x\to 0^-} f(x)}{\lim_{x\to 0^-} g(x)}=\frac{2}{0^+}\to DNE/\infty$$

$$\lim_{x\to 0^+}\frac{f(x)}{g(x)}=\frac{\lim_{x\to 0^+} f(x)}{\lim_{x\to 0^+} g(x)}=\frac{2}{0^-}\to DNE/-\infty$$

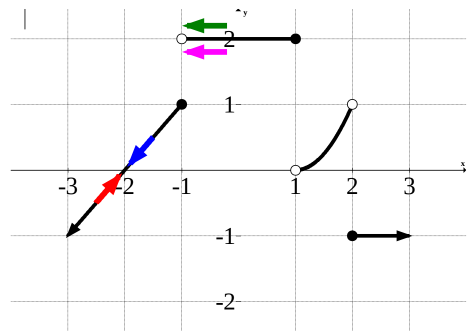
(j) $\lim_{x\to 0}\dfrac{g(x)}{f(x)}=0$

$$\lim_{x\to 0^+}\frac{g(x)}{f(x)}=\frac{\lim_{x\to 0^+} g(x)}{\lim_{x\to 0^+} f(x)}=\frac{0}{2}=0$$

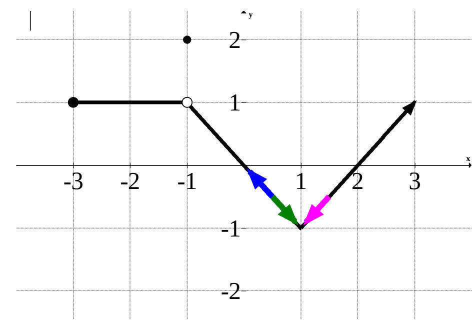
$$\lim_{x\to 0^-}\frac{g(x)}{f(x)}=\frac{\lim_{x\to 0^-} g(x)}{\lim_{x\to 0^-} f(x)}=\frac{0}{2}=0$$

2. Determine what should be written in $\boxed{?}$. Include the value and $^{\pm}$.

$\lim_{x\to 0^-} f(x+2)$ $\downarrow$ $\lim_{x\to 2^-} f(x)=1$	$f(-0.1+2)=f(1.9)$ $f(-0.01+2)=f(1.99)$ $f(-0.001+2)=f(1.999)$	$\lim_{x\to -1^-} f(x^2)$ $\downarrow$ $\lim_{x\to 1^+} f(x)=0$	$f\left([-1.1]^2\right)=f(1.21)$ $f\left([-1.01]^2\right)=f(1.0201)$ $f\left([-1.001]^2\right)=f(1.002001)$



$f(x)$



$g(x)$

$$\lim_{x\to -2} g(f(x))$$

$$\lim_{x\to 1} f(g(x))$$

$$\lim_{x\to -2^-} g(f(x))$$

$$\lim_{x\to -2^-} f(x) = 0^-$$

$$\lim_{x\to 0^-} g(x) = 0^+$$

$$\downarrow$$

$$\lim_{x\to 2^-} g(f(x)) = 0$$

$$\lim_{x\to -2^+} g(f(x))$$

$$\lim_{x\to -2^+} f(x) = 0^+$$

$$\lim_{x\to 0^+} g(x) = 0^-$$

$$\downarrow$$

$$\lim_{x\to -2^+} g(f(x)) = 0$$

$$\lim_{x\to 1^-} f(g(x))$$

$$\lim_{x\to 1^-} g(x) = -1^+$$

$$\lim_{x\to -1^+} f(x) = 2$$

$$\downarrow$$

$$\lim_{x\to 1^-} f(g(x)) = 2$$

$$\lim_{x\to 1^+} f(g(x))$$

$$\lim_{x\to 1^+} g(x) = -1^+$$

$$\lim_{x\to -1^+} f(x) = 2$$

$$\downarrow$$

$$\lim_{x\to 1^+} f(g(x)) = 2$$