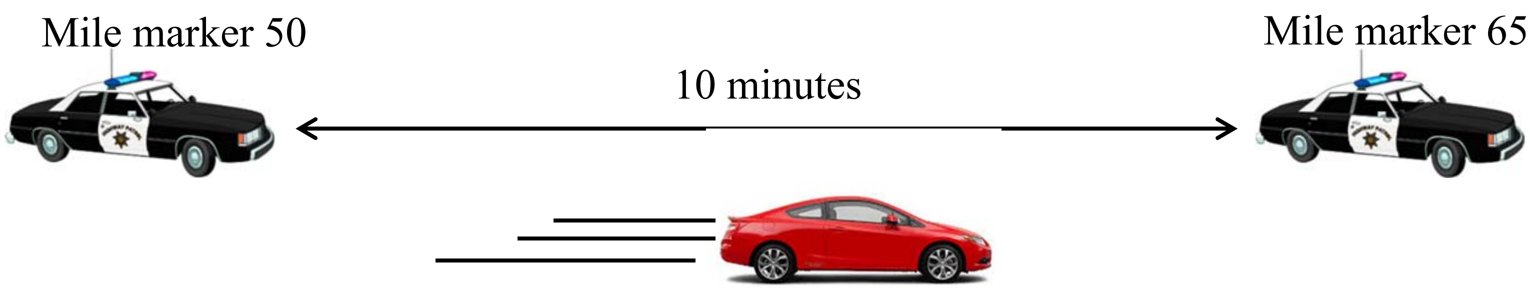


Mr. Rainaldi is driving down the freeway. At mile marker 50, he passes a California Highway Patrol car. The officer radios ahead to keep an eye out for this car. 10 minutes later, Mr. Rainaldi passes another California Highway Patrol car at mile 65. The second officer proceeds to pull him over for speeding. Mr. Rainaldi claims he wasn't speeding. However, the Highway Patrol Officer can prove that Mr. Rainaldi was speeding using Mean Value Theorem. Use Mean Value Theorem to prove that Mr. Rainaldi had to have been speeding at some point during the given 10 minutes.

Let $s(t)$ be the mile marker that Mr. Rainaldi is at time *t*-minutes after passing the first CHP patrol car.

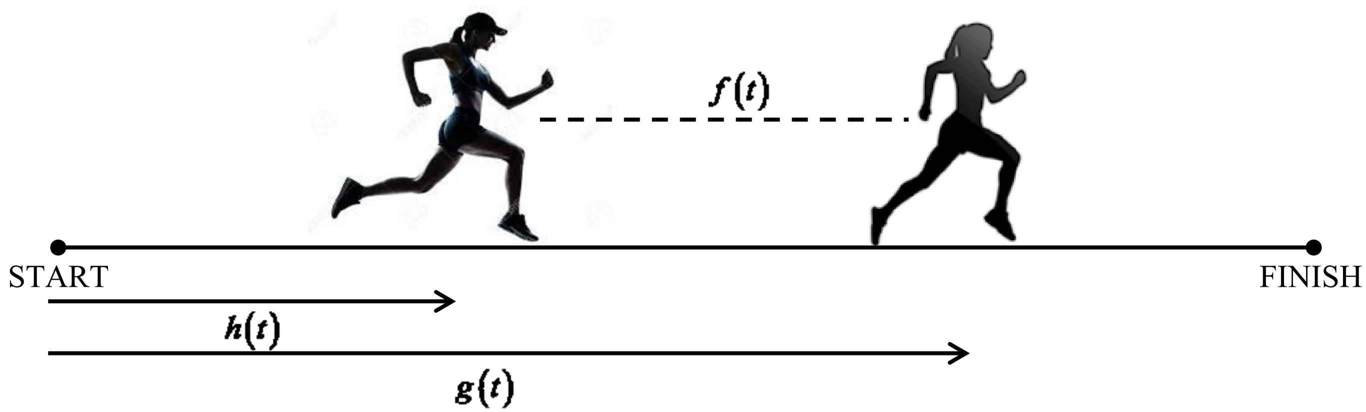
$s(t)$ is continuous on $[0,10]$ and differentiable on $(0,10)$.

By MVT, there exists a *c* in $(0,10)$ such that $s'(c)=\dfrac{s(10)-s(0)}{10-0}=\dfrac{65-50}{10}=\dfrac{15}{10}=\dfrac{3}{2}\dfrac{\text{mi}}{\text{min}}$

$$\frac{3}{2}\frac{\text{mi}}{\text{min}}\times\frac{60\text{min}}{1\text{hour}}=90\frac{\text{mi}}{\text{hr}}$$

Therefore, by MVT, there exists a time *c* in $(0,10)$ such that $s'(t)=90\dfrac{\text{mi}}{\text{hr}}$. Hence Mr. Rainaldi was speeding during the given 10 minutes.

Two runners, Ms. Ofrecio and Ms. DeRosa start a race at the same time, and finish at the same time. Prove that at some time during the race they have the same speed. [Hint: Consider $f(t)=g(t)-h(t)$ where $g(t)$ and $h(t)$ are the position functions for Ms. Ofrecio and Ms. DeRosa respectively.]



Let $g(t)$ be the distance travelled by Ms. Ofrecio *t*-minutes after the race started.

Let $h(t)$ be the distance travelled by Ms. DeRosa *t*-minutes after the race started.

Consider $f(t)=g(t)-h(t)$.

Since Ms. DeRosa and Ms. Ofrecio start the race in the same location at time $t=0$, we know that $f(0)=0$.

Since Ms. DeRosa and Ms. Ofrecio finish the race in the same location at the same time t_F, we know that $f(t_F)=0$.

$f(t)$ is continuous on $[0,t_F]$ since it is the difference of two continuous functions.

$f(t)$ is differentiable on $(0,t_F)$ since it is the difference of two differentiable functions.

By Rolle's Theorem/MVT, there exists a *c* in $(0,t_F)$ such that $f'(c)=0$.

$$f'(c)=g'(c)-h'(c)$$
$$0=g'(c)-h'(c)$$
$$g'(c)=h'(c)$$

Therefore, there exists a time *c* during the race at which Ms. DeRosa and Ms. Ofrecio are moving at the same speed.