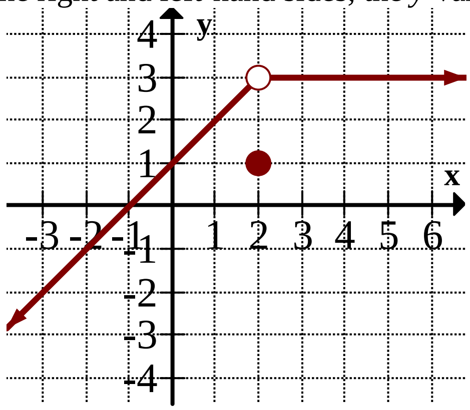
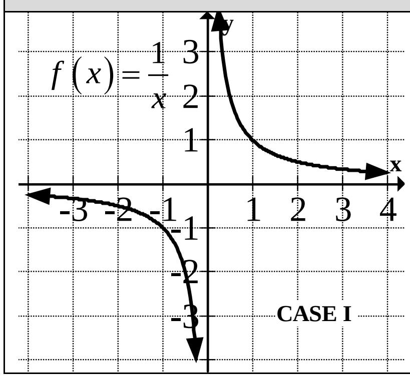
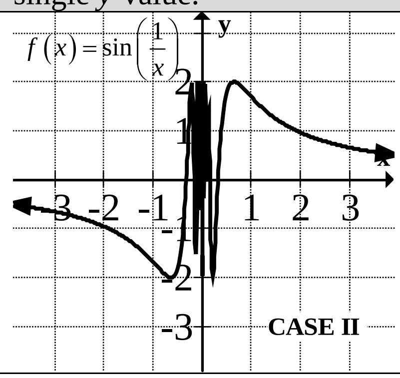
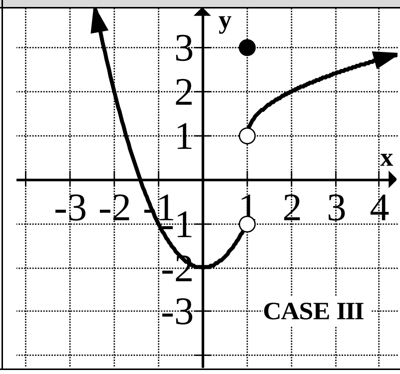
$f\left(x\right)$ Unit Summary for Limits:

1. Describe in words and with a diagram/graph what $\lim_{x\to 2} f\left(x\right)=3$ means.

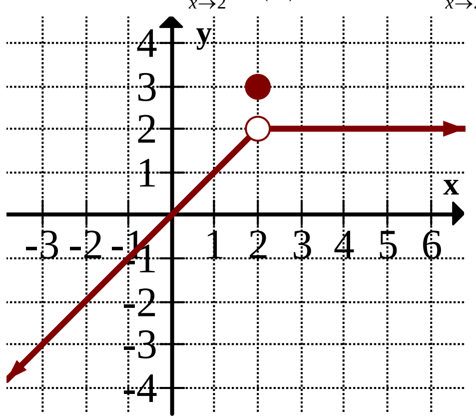
As x approaches 2 from both the right and left-hand sides, the y-value approaches 3.



2. Describe in words, mathematical notation, or diagrams all cases in which $\lim_{x\to c} f\left(x\right)$ DNE.

$\lim_{x\to c} f\left(x\right)$ DNE because the y-value goes to $\pm\infty$	$\lim_{x\to c} f\left(x\right)$ DNE because the y-value does not converge to a single y-value.	$\lim_{x\to c} f\left(x\right)$ DNE because $\lim_{x\to c^-} f\left(x\right)\neq\lim_{x\to c^+} f\left(x\right)$
		

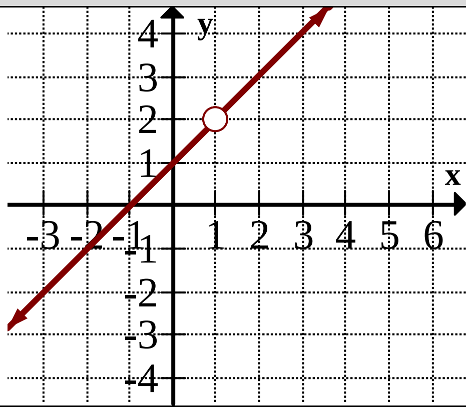
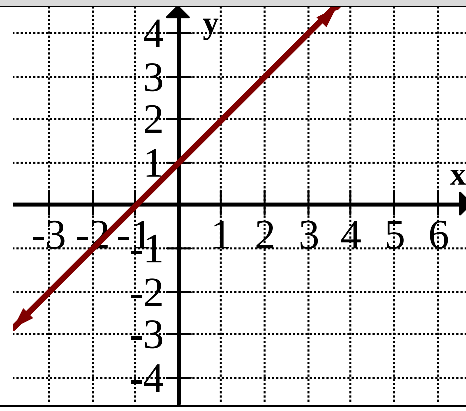
3. Sketch a function that demonstrates that $\lim_{x\to 2} f\left(x\right)$ exists and $\lim_{x\to 2} f\left(x\right)\neq f\left(2\right)$.



4. Explain why $f\left(x\right)=\dfrac{\left(x+1\right)\left(x-1\right)}{x-1}$ is not defined at $x=1$. $g\left(x\right)=x+1$ is not the same as $f\left(x\right)$. Explain why cancelling the factors of $x-1$ changes the graph of $f\left(x\right)$, and therefore the function.

$f\left(x\right)=\dfrac{\left(x+1\right)\left(x-1\right)}{x-1}$ is not defined at $x=1$ because when 1 is substituted for *x*, the resulting expression is of the form $\dfrac{0}{0}$. Division by zero is the reason for DNE.

Canceling the factors of $\left(x-1\right)$ will remove the division by zero. This removal will eliminate the removable discontinuity at $x=1$.

$f\left(x\right)=\dfrac{\left(x+1\right)\left(x-1\right)}{x-1}$	$f\left(x\right)=x+1$
	

5. Given the exercise $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+2x-7}}{-x}$, explain in words why it is acceptable to state $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+2x-7}}{-x}=\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2}}{-x}$.

Since the limit is *x* approaching infinity, as *x* becomes very very large, the polynomial x^2+2x+7 behaves like it's leading term. $\left(\text{very large}\right)^2$ is significantly larger than $2\left(\text{very large}\right)-7$.

6. A function $f(x)$ is continuous at $x = c$ if $\lim_{x \to c} f(x) = f(c)$. An alternate notation for the definition of continuity is $\lim_{x \to c}\left[f(x) - f(c) \right] = 0$. Choose one of the definitions and explain in words how this calculus definition is of continuity is **interpreted visually.**

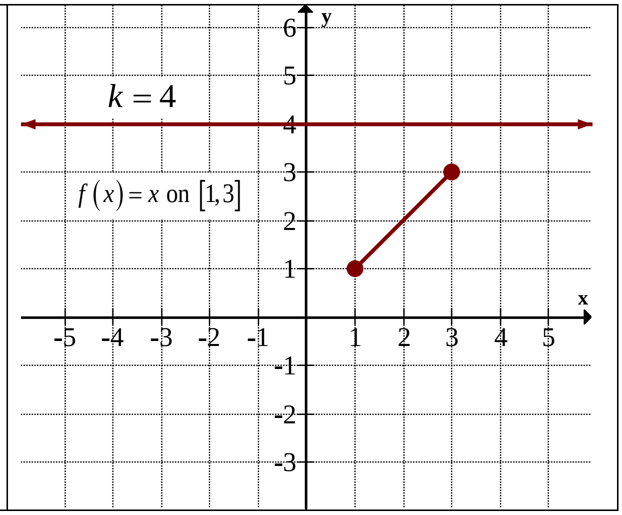
$\lim_{x \to c} f(x) = f(c)$	$\lim_{x \to c}\left[f(x) - f(c) \right] = 0$
As *x* approaches *c* from both the right and lef-hand sides, the corresponding *y*-values converge to $f(c)$.	As *x* approaches *c* from both the right and left-hand sides, the difference between the corresponding *y*-values and $f(c)$ go to zero.

Note: Many students lost points on this exercise because they did not interpret visually - they only mentioned limits and not what those limits mean visually.
Students also lost points because they described continuous functions, but did not link their explanations to the equation.

7. The Intermediate Value Theorem states that if $f(x)$ is continuous on a closed interval $[a,b]$ and k is a value between $f(a)$ and $f(b)$, then there exists a value c where $a \le c \le b$ and $f(c) = k$.

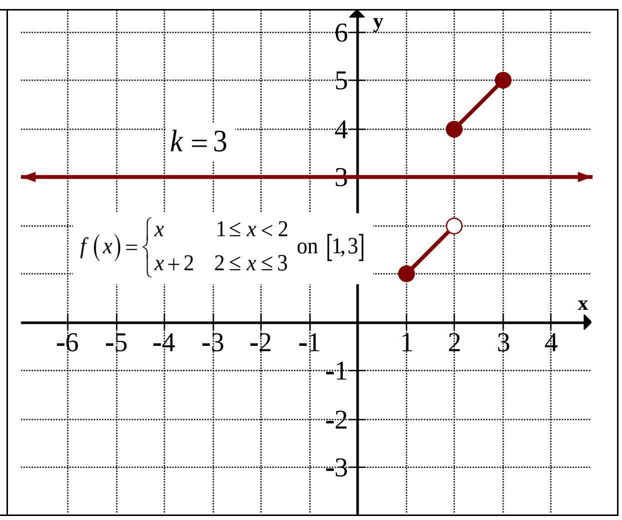
(a) Explain in words and with a diagram why the conclusion of the Intermediate Value Theorem does not hold if k is not between $f(a)$ and $f(b)$.

If the value of k is not between $f(a)$ and $f(b)$, then the function may be continuous on the closed interval $[a,b]$ and never reach k. That is, the range of the function on the closed interval $[a,b]$ may be restricted to the interval $f(a)$ and $f(b)$.



(b) Explain in words and with a diagram why the conclusion of the Intermediate Value Theorem does not hold if $f(x)$ is not continuous on $[a,b]$.

If $f(x)$ is not continuous on $[a,b]$, then $f(x)$ may not have the value of k in the range of $f(x)$ on the closed interval $[a,b]$. That is, the discontinuity could allow the function to "skip" over the line $y = k$.



8. Explain how the concept of infinitely small is applied to limits of the form $\lim_{x \to c} f(x)$ where c is a finite value. Explain how the concept of infinitely large is applied to limits of the form $\lim_{x \to \pm\infty} f(x)$.

Infinitely Small	Infinitely Large
In limits of the form $\lim_{x \to c} f(x)$, x is getting very very close to the value of c. Therefore, the distance between x and c goes to zero. That is the distance between x and c is infinitely small (but never equal to zero).	In limits of the form $\lim_{x \to \pm\infty} f(x)$, the value of x continues to become a larger and larger positive/negative number. Therefore the magnitude of x continues to increase making the value of x infinitely large. The value of x is never equal to infinity, since infinity is not a value on the number line (it is a conceptual value).