<u>Integral Test Explained</u>

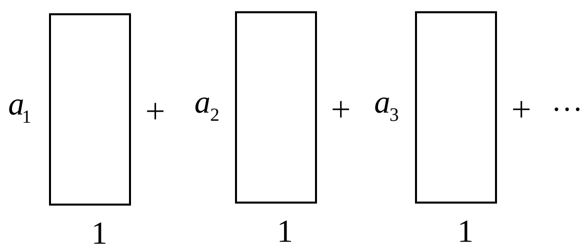
If a sequence a_n is modeled by the function $f(x)$ such that $f(n)=a_n$, then we can use integrals to determine whether or not $\sum_{n=k}^{\infty} a_n$ converges so long as the function $f(x)$ is (1) positive and (2) constantly decreasing for some interval $[k,\infty)$.

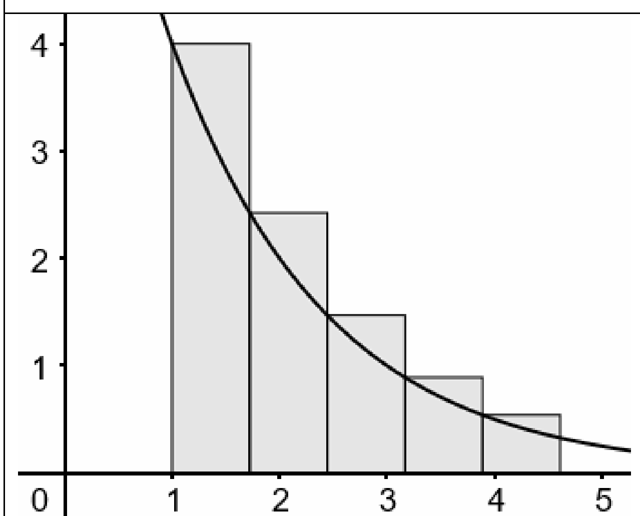
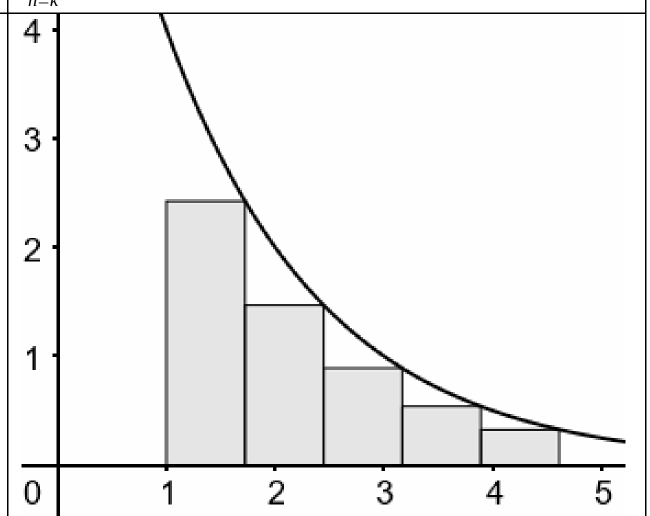
Consider the sum:

$$a_1 + a_2 + a_3 + \cdots$$

This sum cane be expressed as the sum of the areas of rectangles that have width of 1 and height of a_i

$$a_1 + a_2 + a_3 + \cdots$$
$$1 \cdot a_1 + 1 \cdot a_2 + 1 \cdot a_3 + \cdots$$

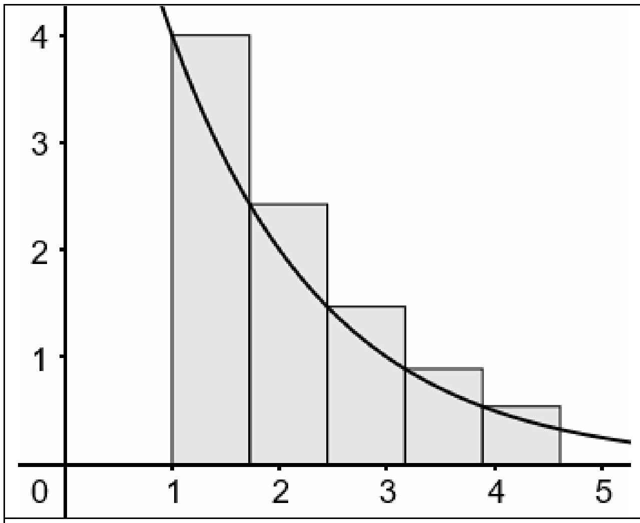
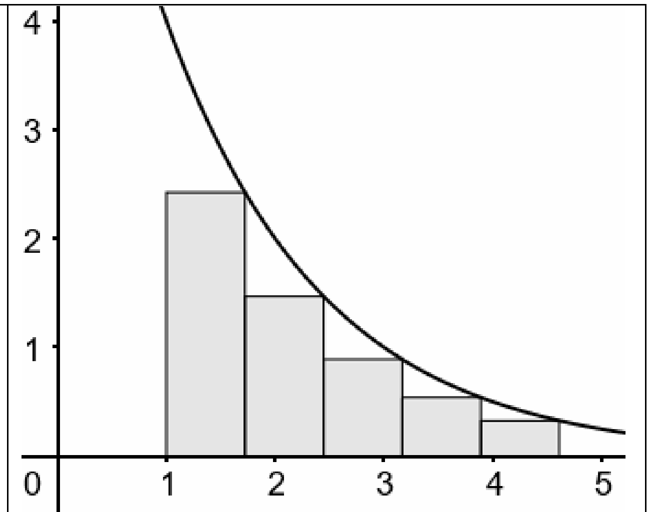


If the sum of the areas of the rectangles is more than the area under the curve, and the area of the curve $\to \infty$, then $\sum_{n=k}^{\infty} a_n$ does not converge.	If the sum of the areas of the rectangles is less than the area under the curve, and the area of the cruve is finite as $x \to \infty$, then $\sum_{n=k}^{\infty} a_n$ converges.
	
$\int_1^{\infty} f(x)dx < \sum_{n=k}^{\infty} a_n$ Since $\int_1^{\infty} f(x)dx \to \infty$, then $\sum_{n=k}^{\infty} a_n \to \infty$	$\sum_{n=k}^{\infty} a_n < \int_1^{\infty} f(x)dx$ Since $\int_1^{\infty} f(x)dx$ converges, so does $\sum_{n=k}^{\infty} a_n$

How do we know if the rectangles are above or below the function?

We don’t know until we determine whether $\int_1^\infty f(x)\,dx$ converges or does not converge.

Notice that the differences between the two graphs is whether we are using a left-sum or right-sum to approximate the area under the curve and whether or not we include the first rectangle of the diagram on the left.

	
The first rectangle is missing from the other diagram. However, including/excluding this rectangle from the sum does not affect whether the sum converges or not because convergence is **tail-end dependent.**	All of these rectangles appear in both diagrams! In the other diagram, these rectangles are over the function and the

If $\int_1^\infty f(x)\,dx \to \infty$,we will choose the perspective of the diagram on the left.

If $\int_1^\infty f(x)\,dx$ converges, we will choose the perspective the diagram on the right.

<u>THERE IS NO NEED TO WORRY ABOUT THESE RECTANGLES!!</u>

If $\int_1^\infty f(x)\,dx$ converges, so does $\sum_{n=1}^\infty a_n$

$\int_1^\infty f(x)\,dx \to \infty$, so does $\sum_{n=1}^\infty a_n$

p-series Test Explained

$$\sum_{n=1}^{\infty}\frac{1}{n^p}$$

If $p<0$ then $\lim_{n\to\infty}\frac{1}{n^p}=\lim_{n\to\infty}\frac{1}{n^{\text{something negative}}}=\lim_{n\to\infty}n^{\text{something positive}}\neq 0$, so $\sum_{n=1}^{\infty}\frac{1}{n^p}$ does not converge.

Since a *p*-series is a series of the form $\sum_{n=1}^{\infty}\frac{1}{n^p}$,we can use the integral test to determine the values of *p* that will make the series converge.

If $p=1$	If $p>0$

$\int_1^{\infty}\frac{1}{x}dx=\lim_{t\to\infty}\int_1^{t}\frac{1}{x}dx$
$=\lim_{t\to\infty}\Big[\ln\lvert x\rvert\Big]_1^t$
$=\lim_{t\to\infty}\left(\ln\lvert t\rvert-\ln\lvert 1\rvert\right)$
$\downarrow$
∞
By the Integral Test, $\sum_{n=1}^{\infty}\frac{1}{n^p}$ does not converge.
	$\int_1^{\infty}\frac{1}{x^p}dx=\lim_{t\to\infty}\int_1^{t}x^{-p}dx$
	$=\lim_{t\to\infty}\left[\frac{1}{-p+1}x^{-p+1}\right]_1^t$
	$=\lim_{t\to\infty}\left(\left[\frac{1}{-p+1}t^{-p+1}\right]-\left[\frac{1}{-p+1}1^{-p+1}\right]\right)$
	$=\lim_{t\to\infty}\frac{1}{-p+1}\cdot\frac{1}{t^{p-1}}+\frac{1}{-p+1}$
	If $p>1$, then
	$\lim_{t\to\infty}\frac{1}{t^{p-1}}=\lim_{t\to\infty}\frac{1}{t^{\text{something positive}}}=0$
	Therefore by the Integral Test $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges
	If $0<p<1$, then
	$\lim_{t\to\infty}\frac{1}{t^{p-1}}=\lim_{t\to\infty}\frac{1}{t^{\text{something negative}}}=\lim_{t\to\infty}t^{\text{something positive}}\to\infty$
	Therefore by the Integral Test $\sum_{n=1}^{\infty}\frac{1}{n^p}$ does not converge.