Section 3-5 Homework Help

#15

$$e^{\frac{x}{y}} = x - y$$

$$e^{\left(xy^{-1}\right)} = x - y$$

$$\downarrow$$

$$e^{\left(xy^{-1}\right)} \cdot \left(1 \cdot y^{-1} + x \cdot \left[-1y^{-2} \cdot y'\right]\right) = 1 - y'$$

$$e^{\left(xy^{-1}\right)} \cdot y^{-1} + e^{\left(xy^{-1}\right)} \cdot x \cdot \left(-1y^{-2} \cdot y'\right) = 1 - y'$$

$$e^{\left(xy^{-1}\right)} \cdot y^{-1} - e^{\left(xy^{-1}\right)} \cdot x \cdot y^{-2} \cdot y' = 1 - y'$$

$$-e^{\left(xy^{-1}\right)} \cdot x \cdot y^{-2} \cdot y' + y' = 1 - e^{\left(xy^{-1}\right)} \cdot y^{-1}$$

$$y'\left[-e^{\left(xy^{-1}\right)} \cdot x \cdot y^{-2} + 1\right] = 1 - e^{\left(xy^{-1}\right)} \cdot y^{-1}$$

$$y' = \frac{1 - e^{\left(xy^{-1}\right)} \cdot y^{-1}}{-e^{\left(xy^{-1}\right)} \cdot x \cdot y^{-2} + 1}$$

#26

$$\sin\left(x + y\right) = 2x - 2y$$

$$\downarrow$$

$$\cos\left(x + y\right) \cdot \left(1 + y'\right) = 2 - 2y'$$

$$y'\big|_{(\pi,\pi)}$$

$$\downarrow$$

$$\cos\left(\pi + \pi\right) \cdot \left(1 + y'\right) = 2 - 2y'$$

$$\cos\left(2\pi\right) \cdot \left(1 + y'\right) = 2 - 2y'$$

$$1 + y' = 2 - 2y'$$

$$3y' = 1$$

$$y' = \frac{1}{3}$$

$$\downarrow$$

$$y - y_1 = m\left(x - x_1\right)$$

$$y - \pi = \frac{1}{3} \cdot \left(x - \pi\right)$$

#17

$$\tan^{-1}\left(x^2y\right)=x+xy^2$$

$$\arctan\left(x^2y\right)=x+xy^2$$

$$\frac{d}{dx}\left[\arctan\left(x^2y\right)\right]=\frac{d}{dx}\left[x+xy^2\right]$$

$$\frac{1}{1+\left(x^2y\right)^2}\cdot\left(2xy+x^2y'\right)=1+y^2+x\cdot 2y\cdot y'$$

$$\frac{1}{1+\left(x^2y\right)^2}\cdot 2xy+\frac{1}{1+\left(x^2y\right)^2}\cdot x^2y'=1+y^2+x\cdot 2y\cdot y'$$

$$\frac{1}{1+\left(x^2y\right)^2}\cdot x^2y'-x\cdot 2y\cdot y'=1+y^2-\frac{1}{1+\left(x^2y\right)^2}\cdot 2xy$$

$$y'\left[\frac{1}{1+\left(x^2y\right)^2}\cdot x^2-x\cdot 2y\right]=1+y^2-\frac{1}{1+\left(x^2y\right)^2}\cdot 2xy$$

$$y'=\frac{\left[1+y^2-\frac{1}{1+\left(x^2y\right)^2}\cdot 2xy\right]}{\left[\left[\frac{1}{1+\left(x^2y\right)^2}\cdot x^2-x\cdot 2y\right]\right]}$$

#19

$$e^y\cos\left(x\right)=1+\sin\left(xy\right)$$

$$\frac{d}{dx}\left[e^y\cos\left(x\right)\right]=\frac{d}{dx}\left[1+\sin\left(xy\right)\right]$$

$$e^y\cdot y'\cdot\cos\left(x\right)+e^y\cdot\left(-\sin\left(x\right)\right)=\cos\left(xy\right)\cdot\left(1\cdot y+x\cdot y'\right)$$

$$e^y\cdot y'\cdot\cos\left(x\right)+e^y\cdot\left(-\sin\left(x\right)\right)=\cos\left(xy\right)\cdot 1\cdot y+\cos\left(xy\right)\cdot x\cdot y'$$

$$e^y\cdot y'\cdot\cos\left(x\right)-\cos\left(xy\right)\cdot x\cdot y'=\cos\left(xy\right)\cdot 1\cdot y+e^y\cdot\sin\left(x\right)$$

$$y'\cdot\left[e^y\cdot\cos\left(x\right)-\cos\left(xy\right)\cdot x\right]=\cos\left(xy\right)\cdot 1\cdot y+e^y\cdot\sin\left(x\right)$$

$$y'=\frac{\cos\left(xy\right)\cdot 1\cdot y+e^y\cdot\sin\left(x\right)}{e^y\cdot\cos\left(x\right)-\cos\left(xy\right)\cdot x}$$

#28

$$x^2 + (2x)\cdot y - y^2 + x = 2$$

$$\downarrow$$

$$2x + 2y + 2xy' - 2y\cdot y' + 1 = 0$$

$$2xy' - 2y\cdot y' = -1 - 2x - 2y$$

$$y'\cdot[2x - 2y] = -1 - 2x - 2y$$

$$y' = \frac{-1 - 2x - 2y}{2x - 2y}$$

$$y'\big|_{(1,2)} = \frac{-1 - 2(1) - 2(2)}{2(1) - 2(2)}$$

Let $\boxed{m = \frac{-1 - 2(1) - 2(2)}{2(1) - 2(2)}}$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \boxed{m}(x - 1)$$

$$x^2 + (2x)\cdot y - y^2 + x = 2$$

$$\downarrow$$

$$2x + 2y + 2xy' - 2y\cdot y' + 1 = 0$$

$$y'\big|_{(1,2)}$$

$$\downarrow$$

$$2(1) + 2(2) + 2(1)y' - 2(2)\cdot y' + 1 = 0$$

$$7 - 2y' = 0$$

$$y' = \frac{7}{2}$$

#37

$$x^3 + y^3 = 1$$

$$\downarrow$$

$$3x^2 + 3y^2 \cdot y' = 0$$

$$y' = -\frac{x^2}{y^2}$$

$$y' = -x^2 y^{-2}$$

$$\downarrow$$

$$y'' = -2xy^{-2} + \left(-x^2\right)\cdot\left(-2y^{-3} \cdot y'\right)$$

$$y'' = -2xy^{-2} + \left(-x^2\right)\cdot\left(-2y^{-3} \cdot \left[-x^2 y^{-2}\right]\right)$$

#20

$$\tan\left(x-y\right) = \frac{y}{1+x^2}$$

$$= y\left(1+x^2\right)^{-1}$$

$$\downarrow$$

$$\sec^2\left(x-y\right)\cdot\left(1-y'\right) = y'\left(1+x^2\right)^{-1} + y\cdot\left[-\left(1+x^2\right)^{-2}\cdot 2x\right]$$

$$\sec^2\left(x-y\right)\cdot 1 - \sec^2\left(x-y\right)\cdot y' = y'\left(1+x^2\right)^{-1} + y\cdot\left[-\left(1+x^2\right)^{-2}\cdot 2x\right]$$

$$-\sec^2\left(x-y\right)\cdot y' - y'\left(1+x^2\right)^{-1} = y\cdot\left[-\left(1+x^2\right)^{-2}\cdot 2x\right] - \sec^2\left(x-y\right)\cdot 1$$

$$y'\left[-\sec^2\left(x-y\right) - \left(1+x^2\right)^{-1}\right] = y\cdot\left[-\left(1+x^2\right)^{-2}\cdot 2x\right] - \sec^2\left(x-y\right)\cdot 1$$

$$y' = \frac{y\cdot\left[-\left(1+x^2\right)^{-2}\cdot 2x\right] - \sec^2\left(x-y\right)\cdot 1}{\left[-\sec^2\left(x-y\right) - \left(1+x^2\right)^{-1}\right]}$$

Example of logarithmic differentiation

$$
\begin{aligned}
y &= x^{\tan(x)} \\
\ln(y) &= \ln\left(x^{\tan(x)}\right) \\
\ln(y) &= \tan(x)\cdot\ln(x) \\
&\downarrow \\
\frac{1}{y}\cdot y' &= \sec^2(x)\cdot\ln(x)+\tan(x)\cdot\frac{1}{x} \\
y' &= y\cdot\left[\sec^2(x)\cdot\ln(x)+\tan(x)\cdot\frac{1}{x}\right] \\
y' &= x^{\tan(x)}\cdot\left[\sec^2(x)\cdot\ln(x)+\tan(x)\cdot\frac{1}{x}\right]
\end{aligned}
$$

$$
\begin{aligned}
y &= \left[\ln(x)\right]^x \\
\ln(y) &= \ln\left(\left[\ln(x)\right]^x\right) \\
\ln(y) &= \ln\left(x\cdot\ln(x)\right) \\
\ln(y) &= \ln(x)+\ln\left(\ln(x)\right) \\
&\downarrow \\
\frac{1}{y}\cdot y' &= \frac{1}{x}+\frac{1}{\ln(x)}\cdot\frac{1}{x} \\
y' &= y\cdot\left[\frac{1}{x}+\frac{1}{\ln(x)}\cdot\frac{1}{x}\right] \\
y' &= \left[\ln(x)\right]^x\cdot\left[\frac{1}{x}+\frac{1}{\ln(x)}\cdot\frac{1}{x}\right]
\end{aligned}
$$