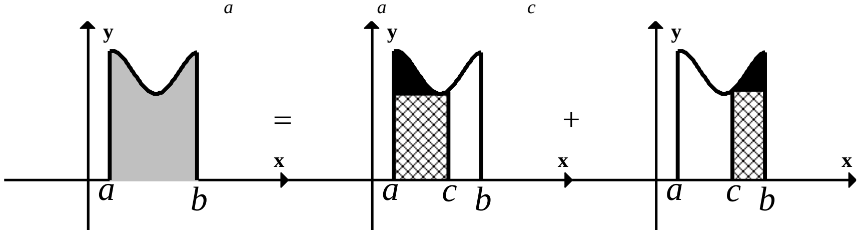
Properties of Definite Integrals

I. $\int_a^a f(x)\,dx = 0$

II. $\int_b^a f(x)\,dx = -\left[\int_a^b f(x)\,dx\right]$ "switch the direction, switch the sign"

III. If $a \le c \le b$, then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

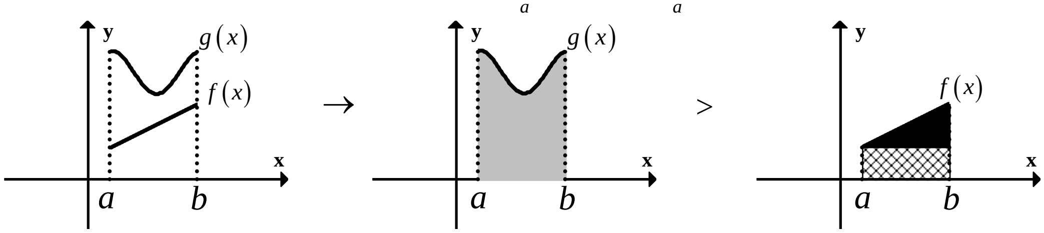


IV. If k is a constant then $\int_a^b k \cdot f(x)\,dx = k \cdot \int_a^b f(x)\,dx$

V. $\int_a^b f(x) \pm g(x)\,dx = \left[\int_a^b f(x)\,dx\right] \pm \left[\int_a^b (x)\,dx\right]$

VI. If f is integrable and non-negative on $[a,b]$, then $0 \le \int_a^b f(x)\,dx$

VII. If $f(x) \le g(x)$ for all x in $[a,b]$, then $\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$



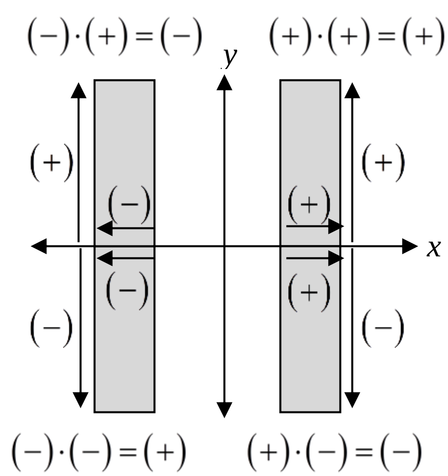
VIII. The definite integral can take on negative, zero, and positive values depending on

a. The direction of integration

b. The sign of $f(x)$

$a \xrightarrow{\Delta x} b \qquad \Delta x > 0$

$a \xleftarrow{\Delta x} b \qquad \Delta x < 0$



Fundamental Theorem of Calculus

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If $f(x)$ is continuous on $[a,b]$, then

$$\int_a^b f(x)\,dx = \left[F(x)\right]_a^b$$

$$= F(b) - F(a)$$

where $F'(x) = f(x)$.

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If $f(t)$ is integrable on $[a,b]$, then

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

Note:

(1) Lower bound must be a constant

(2) Upper bound and integrand must be of different variables.

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I. $$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f\left(g(x)\right)\cdot g'(x)$$

II. $$\frac{d}{dx}\left[\int_{h(x)}^{b} f(t)\,dt\right] = \frac{d}{dx}\left[-\int_b^{h(x)} f(t)\,dt\right] = -\frac{d}{dx}\left[\int_b^{h(x)} f(t)\,dt\right] = -f\left(h(x)\right)\cdot h'(x)$$

III. $$\begin{aligned}\frac{d}{dx}\left[\int_{k(x)}^{m(x)} f(t)\,dt\right] &= \frac{d}{dx}\left[\int_a^{m(x)} f(t)\,dt - \int_a^{k(x)} f(t)\,dt\right] \\ &= \frac{d}{dx}\left[\int_a^{m(x)} f(t)\,dt\right] - \frac{d}{dx}\left[\int_a^{k(x)} f(t)\,dt\right] \\ &= f\left(m(x)\right)\cdot m'(x) - f\left(k(x)\right)\cdot k'(x)\end{aligned}$$

Basic Integration Formulas:

$$\int k\cdot f(u)\,du = k\int f(u)\,du \qquad \int f(u)\pm g(u)\,du = \int f(u)\,du \pm \int g(u)\,du \qquad \int du = u + C$$

$$\int a^u\,du = \frac{1}{\ln(a)}a^u + C \qquad \int e^u\,du = e^u + C \qquad \int \frac{1}{u}\,du = \ln|u| + C$$

$$\int \cos(u)\,du = \sin(u) + C \qquad \int \sin(u)\,du = -\cos(u) + C \qquad \int \sec^2(u)\,du = \tan(u) + C$$

$$\int \csc^2(u)\,du = -\cot(u) + C \qquad \int \sec(u)\tan(u)\,du = \sec(u) + C \qquad \int \csc(u)\cot(u)\,du = -\csc(u) + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\left(\frac{u}{a}\right) + C \qquad \int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C \qquad \int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right) + C$$

Substitution Rule: If $u = g(x)$ is a differentiable function of *x*, then

$$\int f\left(g(x)\right)g'(x)\,dx = \int f(u)\,du \qquad \int_a^b f\left(g(x)\right)g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$