Limits to know for Series:

$\lim_{n\to\infty} \sqrt[n]{c} = 1$	$\lim_{n\to\infty} \sqrt[n]{n^c} = 1$	$\lim_{n\to\infty} \sqrt[n]{n} = 1$	$\lim_{n\to\infty} \left(1+\frac{c}{n}\right)^n = e^c$	$\lim_{n\to\infty} \sqrt[n]{n!} = \infty$

c is a positive constant, and n is a positive integer.

<u>You do not need to know how to prove these limits!</u>

The proofs for each one of these limits are provided below for entertainment purposes only.

$\lim_{n\to\infty} \sqrt[n]{c} = 1$

$$y = \lim_{n\to\infty} \sqrt[n]{c}$$

$$y = \lim_{n\to\infty} c^{\frac{1}{n}}$$

$$\ln(y) = \ln\left[\lim_{n\to\infty} c^{\frac{1}{n}}\right]$$

$$\ln(y) = \lim_{n\to\infty}\left[\ln\left(c^{\frac{1}{n}}\right)\right]$$

$$\ln(y) = \lim_{n\to\infty}\left[\frac{1}{n}\cdot\ln(c)\right]$$

$$\ln(y) = 0$$

$$y = e^0$$

$$y = 1$$

$$\lim_{n\to\infty}\left(1+\frac{c}{n}\right)^n = e^c$$

$$
\begin{aligned}
y &= \lim_{n\to\infty}\left(1+\frac{c}{n}\right)^n \\
\ln[y] &= \ln\left[\lim_{n\to\infty}\left(1+\frac{c}{n}\right)^n\right] \\
\ln[y] &= \lim_{n\to\infty}\left[\ln\left(1+\frac{c}{n}\right)^n\right] \\
\ln[y] &= \lim_{n\to\infty}\left[n\cdot\ln\left(1+\frac{c}{n}\right)\right] \\
\ln[y] &= \lim_{n\to\infty}\left[\frac{\ln\left(1+\frac{c}{n}\right)}{\frac{1}{n}}\right] \\
\ln[y] &= \lim_{n\to\infty}\left[\frac{\frac{1}{\left(1+\frac{c}{n}\right)}\cdot\left(-cn^{-2}\right)}{-n^{-2}}\right] \\
\ln[y] &= \lim_{n\to\infty}\left[\frac{1}{\left(1+\frac{c}{n}\right)}\cdot(c)\right] \\
\ln[y] &= c \\
y &= e^c
\end{aligned}
$$

Note:

Sometimes you will encounter a limit like $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n$

If so, use the following algebra to rewrite the expression:

$$\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^{-1}\right]^n = \lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{-1}\right]^n = \lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{-1} = \left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\right]^{-1} = e^{-1}$$

$\lim_{n\to\infty} \sqrt[n]{n^c} = 1$

$$
\begin{aligned}
y &= \lim_{n\to\infty} \sqrt[n]{n^c} \\
y &= \lim_{n\to\infty} \left(n^c\right)^{\frac{1}{n}} \\
y &= \lim_{n\to\infty} \left(n^{\frac{c}{n}} \right) \\
\ln(y) &= \ln\left[\lim_{n\to\infty} \left(n^{\frac{c}{n}} \right) \right] \\
\ln(y) &= \lim_{n\to\infty} \left[\ln\left(n^{\frac{c}{n}} \right) \right] \\
\ln(y) &= \lim_{n\to\infty} \left[\frac{c}{n} \cdot \ln(n) \right] \\
\ln(y) &= \lim_{n\to\infty} \left[c \cdot \frac{\ln(n)}{n} \right] \\
\ln(y) &= c \cdot \lim_{n\to\infty} \left[\frac{\ln(n)}{n} \right] \\
\ln(y) &= 0 \\
y &= e^0 \\
y &= 1
\end{aligned}
$$

$\lim_{n\to\infty} \sqrt[n]{n} = 1$

$$
\begin{aligned}
y &= \lim_{n\to\infty} \sqrt[n]{n} \\
y &= \lim_{n\to\infty} \left[n^{\frac{1}{n}} \right] \\
\ln(y) &= \ln\left(\lim_{n\to\infty} \left[n^{\frac{1}{n}} \right] \right) \\
\ln(y) &= \lim_{n\to\infty} \left(\ln\left[n^{\frac{1}{n}} \right] \right) \\
\ln(y) &= \lim_{n\to\infty} \left(\frac{1}{n} \cdot \ln[n] \right) \\
\ln(y) &= \lim_{n\to\infty} \left(\frac{\ln[n]}{n} \right) \\
\ln(y) &= 0 \\
y &= e^{0} \\
y &= 1
\end{aligned}
$$

$\lim\limits_{n\to\infty} \sqrt[n]{n!} = \infty$

$$y = \lim_{n\to\infty} \sqrt[n]{n!}$$

$$y = \lim_{n\to\infty} (n!)^{\frac{1}{n}}$$

$$\ln(y) = \ln\left(\lim_{n\to\infty} (n!)^{\frac{1}{n}}\right)$$

$$\ln(y) = \lim_{n\to\infty} \ln\left((n!)^{\frac{1}{n}}\right)$$

$$\ln(y) = \lim_{n\to\infty} \frac{1}{n} \cdot \ln(n!)$$

$$\ln(y) = \lim_{n\to\infty} \frac{\ln(n!)}{n}$$

$$\ln(y) = \lim_{n\to\infty} \left(\frac{\ln(1\cdot 2\cdot 3\cdot\cdots\cdot n)}{n}\right)$$

$$\ln(y) = \lim_{n\to\infty} \frac{\ln(1)+\ln(2)+\cdots+\ln(n)}{n}$$

$$\ln(y) = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{\ln(k)}{n}$$

Then for $k \geq 2$ we have that $\dfrac{\ln(k)}{n} \geq \dfrac{1}{n}$. Which means that

$$\ln(y) \to \infty$$

Therefore

$$y = \infty$$