Find det $\begin{pmatrix} 1 & 1 & -5 & -5 \\ -2 & 2 & 1 & -4 \\ -1 & 5 & 2 & 1 \\ 2 & -2 & 3 & 1 \end{pmatrix} = \det \begin{bmatrix} 1 & 1 & -5 & -5 \\ 0 & 4 & -9 & -14 \\ 0 & 6 & -3 & -4 \\ 0 & -4 & 13 & 11 \end{bmatrix}$

$= \det \begin{bmatrix} 4 & -9 & -14 \\ 6 & -3 & -4 \\ -4 & 13 & 11 \end{bmatrix} = 4(-33 - -52) + 9(66 - 16)$
$-14(78-12)$

$= 19 \times 4 + 9 \times 50 - 14 \times 66$

$= -398$

Find derivative of the function

$f(x) = \det \begin{bmatrix} 4 & 7 & -8 & 1 & -6 \\ 8 & 0 & -1 & 2 & 6 \\ -8 & 0 & 0 & 5 & -6 \\ x & -9 & 7 & 9 & 2 \\ 3 & 0 & 0 & 0 & 3 \end{bmatrix} = 4 \det() - 8 \det$
$+ (-8) \det() - x \det()$
$+ 3 \det()$

$f'(x) = -\det \begin{pmatrix} 7 & -8 & 1 & -6 \\ 0 & -1 & 2 & 6 \\ 0 & 0 & 5 & -6 \\ 0 & 0 & 0 & 3 \end{pmatrix} = 7 \cdot -1 \cdot 5 \cdot 3 = -105 \times -1 = 105$

Given matrix $A = \begin{pmatrix} a & 5 & 3 \\ a & -1 & 4 \\ 4 & 2 & a \end{pmatrix}$ find all values of a so $|A| = 0$

$a \cdot (-a-8) - a(5a-6) + 4(20+3) = 0$

$-a^2 - 8a - 5a^2 + 6a + 92 = 0$

$-6a^2 - 2a + 92 = 0$

Basis in Euclidean Space

①

A. A single vector is linearly dependent No.

B. The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$

Yes because only iff a matrix is LI, the matrix is invertible. Thus the matrix is LI and span $\mathbb{R}^n$.

C. A basis is a spanning set as large as possible. No. Duh.

D. Linear dependence can be modified by elementary row operations. No. Duh.

E. If $H = \text{span}\{b_1, b_2, b_3 \ldots b_p\}$ then $\{b_1, b_2, b_3 \ldots b_p\}$ is a basis for H.

H must have p columns, and the basis is already a span, thereby LI. ~~Also~~ However, the problem never assumes that the vectors are LI, or that the span is not redundant so no.

The vectors form a basis for $\mathbb{R}^3$ iff $k \neq$?

$$V_1 = \begin{bmatrix} 6 \\ -4 \\ 0 \end{bmatrix} \quad V_2 = \begin{pmatrix} -4 \\ 6 \\ 5 \end{pmatrix} \quad V_3 = \begin{pmatrix} 0 \\ 10 \\ k \end{pmatrix}$$

If N vectors are LI they span $\mathbb{R}^N$.

Check for Linear Independence

$$\begin{pmatrix} 6 & -4 & 0 & 0 \\ -4 & 6 & 10 & 0 \\ 0 & 5 & k & 0 \end{pmatrix} \rightarrow \begin{bmatrix} -4 & 6 & 10 & 0 \\ 6 & -4 & 0 & 0 \\ 0 & 5 & k & 0 \end{bmatrix}$$

Find null space of A where

$$A = \begin{bmatrix} 4 & -3 & 4 & 1 & 5 \\ 4 & -4 & 1 & -4 & -3 \\ -5 & 2 & -2 & -1 & -1 \end{bmatrix}$$

Null space is where $A\vec{x} = \vec{0}$

$$\begin{bmatrix} 4 & -3 & 4 & 1 & 5 \\ 4 & -4 & 1 & -4 & -3 \\ -5 & 2 & -2 & -1 & -1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ a \\ b \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve as augmented matrix

$$\left[\begin{array}{ccccc|c} 4 & -3 & 4 & 1 & 5 & 0 \\ 4 & -4 & 1 & -4 & -3 & 0 \\ -5 & 2 & -2 & -1 & -1 & 0 \end{array}\right]$$

$$\begin{matrix} 4 & -3 & 4 & 1 & 5 & 0 \\ 0 & -1 & -3 & -5 & -8 & 0 \\ -5 & 2 & -2 & -1 & -1 & 0 \end{matrix}$$

Is $\begin{pmatrix}1\\2\\3\end{pmatrix}$ in Column Space of $\begin{bmatrix}1 & 1 & 3\\1 & 0 & 2\\1 & -1 & 1\end{bmatrix}$?

Show that $\begin{pmatrix}1\\2\\3\end{pmatrix}$ can be written as a linear combination of the cols

solve $x\begin{pmatrix}1\\1\\1\end{pmatrix} + y\begin{pmatrix}1\\0\\-1\end{pmatrix} + z\begin{bmatrix}3\\2\\1\end{bmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$

Subspaces in Euclidean Space

Determine if subset of $S = \{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{ in } \mathbb{R}^3 \mid abc = 0 \}$ is a subspace

No. $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ are in span, but $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is not.

Which of the following are subspaces of $\mathbb{R}^3$?

A $\{(x,y,z) \mid 8x - 3y - 7z = 5\}$ no

B $\{(x,y,z) \mid 4x - 2y - 6z = 0\}$

C $\{x,y,z \mid -4x + 2y = 0, 6x - 5z = 0\}$

D $\{-5x, -8x, 3x \mid x \text{ arbitrary number}\}$

E $\{4, y, z \mid y, z \text{ anything}\}$ no, $\vec{0}$ not exist

F $\{(x,y,z) \mid x < y < z\}$ no, $\vec{0} \notin \mathbb{R}^3$

B. Probably yes, but we need to confirm. Show S can be written as a span.

$4x - 6z = 2y$

$y = 2x - 3z$

so span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix} \right\}$

$4x - 2(2x - 3z) - 6z = 0$

C. Probably Yes. Confirm

$2y = 4x,\ y = 2x$

$6x = 5z,\ z = \frac{6}{5}x$

Span $\left\{ \begin{matrix} 1 \\ 2 \\ \frac{6}{5} \end{matrix} \right\}$

D. Yes, obviously. Span $\left\{ \begin{matrix} -5 \\ -8 \\ 3 \end{matrix} \right\}$

Determine if $\{ \begin{pmatrix} a \\ b \end{pmatrix} \text{ in } \mathbb{R}^2 \mid a + b = 1 \}$ is subspace.

$\vec{0}$ exists? No. Scalar multiplication? No. Vector Addition? No.

Not a subspace at all :-