MATH-253-YJH-CRN82680 Final

David Yang

TOTAL POINTS

114 / 119

QUESTION 1

1 Region E **10 / 10**

✓ **- 0 pts Correct**

QUESTION 2

2 Double int over D **10 / 10**

✓ **- 0 pts Correct**

QUESTION 3

Max/min 9 pts

3.1 Criticals **3 / 3**

✓ **- 0 pts Correct**

3.2 D **3 / 3**

✓ **- 0 pts Correct**

3.3 Classify points **2 / 3**

✓ **- 1 pts Need to explain why max -- it's not just because $$D>0$$.**

QUESTION 4

4 Lagrange **10 / 10**

✓ **- 0 pts Correct**

QUESTION 5

5 Surface area **9 / 10**

✓ **- 1 pts Bad algebra**

QUESTION 6

6 FTLI **10 / 10**

✓ **- 0 pts Correct**

QUESTION 7

Plane 10 pts

7.1 Tan plane **5 / 5**

✓ **- 0 pts Correct**

7.2 Normal to the plane **5 / 5**

✓ **- 0 pts Correct**

QUESTION 8

Reverse integration 10 pts

8.1 First **5 / 5**

✓ **- 0 pts Correct**

8.2 Second **5 / 5**

✓ **- 0 pts Correct**

QUESTION 9

9 Dot product **10 / 10**

✓ **- 0 pts Correct**

QUESTION 10

10 Line integral of scalar **10 / 10**

✓ **- 0 pts Correct**

QUESTION 11

11 Directional derivative **10 / 10**

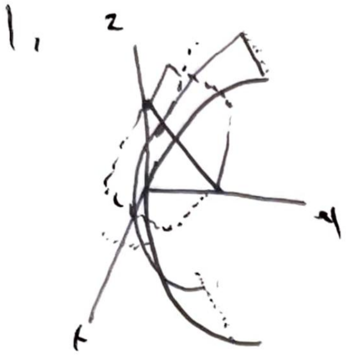
✓ **- 0 pts Correct**

QUESTION 12

12 Stokes **7 / 10**

✓ **- 3 pts Bad $$\nabla \times \vec{F}$$ or curl $$\vec{F}$$**

1.



~~$0 \le z \le 1$~~

~~$0 \le y \le 1$~~

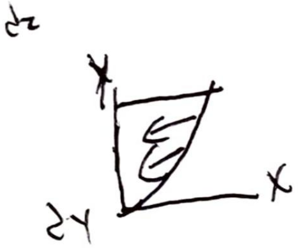
~~$-1 \le x \le 1$~~

not Integral bounds

$0 \le z \le 1-y$

$x^2 \le y \le 1$

$-1 \le x \le 1$



$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} 1 \, dz\, dy\, dx = \int_{-1}^{1}\int_{x^2}^{1} 1-y \, dy\, dx$$

$$= \int_{-1}^{1} \left[y - \frac{y^2}{2}\right]_{x^2}^{1} dx = \int_{-1}^{1} 0.5 - x^2 + \frac{x^4}{2} \, dx$$

$$= 0.5x - \frac{x^3}{3} + \frac{x^5}{10}\Big]_{-1}^{1} = \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{10}\right) - \left(\frac{-1}{2} + \frac{1}{3} - \frac{1}{10}\right)$$

$$= 1 - \frac{2}{3} + \frac{1}{5} = \frac{15}{15} - \frac{10}{15} + \frac{3}{15} = \boxed{\frac{8}{15}}$$

---

2. ~~$f(x,y) = -x^4 + 4xy - 2y^2 + 1$~~

$$\iint_D \frac{xy}{\sqrt{x^2+y^2}}$$ where D is disk $x^2+y^2 \le 4$

Convert to polar

$x = r\cos(\theta)$

$y = r\sin(\theta)$

$x^2+y^2 = r^2$

θ bounds

$0 \le \theta \le 2\pi$

$0 \le r \le 2$

$$\int_0^{2\pi}\int_0^{2} \frac{r^2\cos(\theta)\sin(\theta)}{\sqrt{r^2}}\, r\, dr\, d\theta = \int_0^{2\pi}\int_0^{2} r^2 \cos\theta \sin\theta \, dr\, d\theta$$

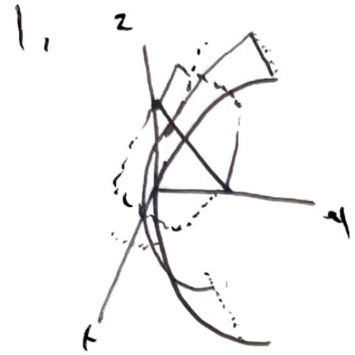
$$\int_0^{2\pi} \frac{r^3}{3}\Big]_0^2 \cos(\theta)\sin(\theta)\, d\theta = \frac{8}{3}\int_0^{2\pi} \cos(\theta)\sin(\theta)\, d\theta$$

$$= \frac{8}{3}\left[\frac{\sin^2(x)}{2}\right]_0^{2\pi} = \frac{8}{3}\left[\frac{0}{2} - \frac{0}{2}\right] = \frac{8}{3}(0) = 0$$

1 Region E 10 / 10

✓ - 0 pts Correct

1.



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not Integral bounds

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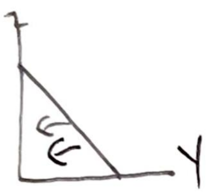
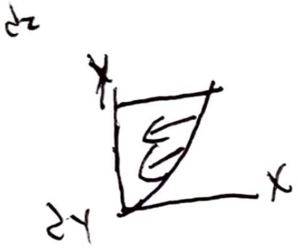
$$x^2 \le y \le 1$$
$$-1 \le x \le 1$$





$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} 1 \, dz\, dy\, dx = \int_{-1}^{1}\int_{x^2}^{1} 1-y \, dy\, dx$$

$$= \int_{-1}^{1} \left[y-\frac{y^2}{2}\right]_{x^2}^{1} dx = \int_{-1}^{1} 0.5 - x^2 + \frac{x^4}{2} \, dx$$

$$= 0.5x - \frac{x^3}{3} + \frac{x^5}{10}\Big]_{-1}^{1} = \left(\frac{1}{2}-\frac{1}{3}+\frac{1}{10}\right) - \left(\frac{-1}{2}+\frac{1}{3}-\frac{1}{10}\right)$$

$$= 1 - \frac{2}{3} + \frac{1}{5} = \frac{15}{15} - \frac{10}{15} + \frac{3}{15} = \boxed{\frac{8}{15}}$$

---

2. ~~$f(x,y) = -x^4 + 4xy - 2y^2 + 1$~~

$$\iint_D \frac{xy}{\sqrt{x^2+y^2}} \quad \text{where } D \text{ is disk } x^2+y^2 \le 4$$

Convert to polar

$x = r\cos(\theta)$
$y = r\sin(\theta)$
$x^2+y^2 = r^2$

θ bounds
$0 \le \theta \le 2\pi$
$0 \le r \le 2$

$$\int_0^{2\pi}\int_0^{2} \frac{r^2\cos(\theta)\sin(\theta)}{\sqrt{r^2}} r \, dr\, d\theta = \int_0^{2\pi}\int_0^{2} r^2 \cos\theta \sin\theta \, dr\, d\theta$$

$$\int_0^{2\pi} \frac{r^3}{3}\Big]_0^2 \cos(\theta)\sin(\theta)\, d\theta = \frac{8}{3}\int_0^{2\pi} \cos(\theta)\sin(\theta)\, d\theta$$

$$= \frac{8}{3}\left[\frac{\sin^2(x)}{2}\right]_0^{2\pi} = \frac{8}{3}\left[\frac{0}{2}-\frac{0}{2}\right] = \frac{8}{3}(0) = 0$$

2 Double int over D 10 / 10

✓ **- 0 pts** Correct

gradescope

3. $f = -x^4 + 4xy - 2y^2 + 1$

$f_x = -4x^3 + 4y = 0$

$f_y = 4x - 4y = 0$

3 a. $(-1,-1)$ $(0,0)$ $(1,1)$	b. $D = 48x^2 - 16$	c. $(-1,-1)$ and $(1,1)$ are relative max; $(0,0)$ is saddle

$4x - 4y = 0$

$4x = 4y$

$x = y$

$-4x^3 + 4y = 0$

$-4x^3 + 4x = 0$

$4x = 4x^3$

$x = x^3$

$(0,0)$ $(1,1)$ $(-1,-1)$

Criticals

$D(-1,-1) = 32$

$D(-1,-1) > 0$:

$f_{yy} = -4 < 0$: rel max

$D(0,0) = -16 < 0 =$ saddle

$D(1,1) = 32$:

$f_{yy} = -4 < 0$: rel max

$D = f_{xx}(x,y) \cdot f_{yy}(x,y) - (f_{xy}(x,y))^2$

$f_{xx} = -12x^2$

$f_{yy} = -4$

$f_{xy} = 4$

$D = (-12x^2 \cdot -4) - (4^2) = 48x^2 - 16$

4.

$f(x,y) = x^2 + y^2$

$g(x,y) = 1 - x - y$

$\nabla f = \langle 2x, 2y \rangle$, $\nabla g = \langle -1, -1 \rangle$

$2x = \lambda \cdot -1$

$2y = \lambda \cdot -1$

$y = 1 - x$

$2(1-x) = -\lambda$

$2 - 2x = 2x$, $4x = 2$, $x = \frac{1}{2}$

$y = 1 - x$, $y = 1 - \frac{1}{2}$, $y = \frac{1}{2}$

$2 \cdot \frac{1}{2} = -\lambda$

$1 = -\lambda$

$\lambda = -1$

$x = y = \frac{1}{2}$, $\lambda = -1$

$f(\frac{1}{2}, \frac{1}{2}) = \frac{1}{2}$

Check bounds

$f(0,1) = 1$

$f(1,0) = 1$

$\frac{1}{2} < 1$ so at $x = \frac{1}{2}$, $y = \frac{1}{2}$ $f(\frac{1}{2}, \frac{1}{2})$ is minimum

~~Minimize $f(x,y)$ subject to $g(x,y)$~~

~~$f(x,y) = x^2 + y^2$, $g(x,y) = 1 - x = y$~~

~~$\nabla f = \langle 2x, 2y \rangle$, $\nabla g = \langle -1, 0 \rangle$~~

~~$2x = \lambda \cdot -1$~~

~~$2y = \lambda \cdot 0$, $y = 0$~~

~~$y = 1 - x$~~

~~$0 = 1 - x$, $x = 1$~~

~~$x = 1$, $y = 0$~~

~~$2x = -\lambda$~~

~~$-2 = \lambda$~~

~~$x = 1$, $y = 0$ $\to \lambda = -2$~~

3.1 Criticals 3 / 3

✓ **- 0 pts** Correct

3. $f = -x^4 + 4xy - 2y^2 + 1$

$f_x = -4x^3 + 4y = 0$

$f_y = 4x - 4y = 0$

3 a.	b.	c.
$(-1,-1)$ $(0,0)$ $(1,1)$	$D = 48x^2 - 16$	$(-1,-1)$ and $(1,1)$ are relative max $(0,0)$ is saddle

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$f_{xx} = -12x^2$

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$D = (-12x^2 \cdot -4) - (4^2) = 48x^2 - 16$

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$2 - 2x = 2x$, $4x = 2$, $x = \frac{1}{2}$

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$x = y = \frac{1}{2}, \lambda = -1$

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~~$2y = \lambda \cdot 0$, $y = 0$~~

~~$y = 1 - x$~~

~~$0 = 1 - x$, $x = 1$~~

~~$x = 1$~~
~~$y = 0$~~

~~$2x = -\lambda$~~

~~$-2 = \lambda$~~

~~$x = 1$~~
~~$y = 0$~~
~~$\rightarrow \lambda = -2$~~

3.2 D **3 / 3**

✓ **- 0 pts** Correct

3. $f = -x^4 + 4xy - 2y^2 + 1$

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~~$x = 1$ $y = 0$ $\rightarrow$~~

~~$2x = -\lambda$~~

~~$-2 = \lambda$~~

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3.3 Classify points 2 / 3

✓ **- 1 pts** **Need to explain why max -- it's not just because $$D>0$$.**

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$f_x = -4x^3 + 4y = 0$

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b. $D = 48x^2 - 16$

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~~$x = 1$, $y = 0$, $\to \lambda = -2$~~

4 Lagrange 10 / 10

✓ - 0 pts Correct

5.

paraboloid is bounded by

$z=0$, $z=1$

Surface $= x^2+y^2=z$

$$\iint_S dS = \iint_R \sqrt{(2x)^2+(2y)^2+1}\, dA = \iint_R \sqrt{4x^2+4y^2+1}\, dA$$

$$\int_0^{2\pi}\int_0^1 \sqrt{4r^2+1}\, r\, dr\, d\theta = \int_0^{2\pi} \left.\frac{(4r^2+1)^{3/2}}{1.5 * 8}\right]_0^1 d\theta = \int_0^{2\pi}\left(\frac{5^{3/2}}{12}-\frac{1^{3/2}}{12}\right)d\theta$$

$$\frac{5^{3/2}-1}{12}\cdot 2\pi = \boxed{\frac{5^{3/2}\cdot\pi-\pi}{6}} = \boxed{\frac{5\sqrt{5}(\pi-1)}{6}}$$

6.

$$\int_{(0,1)}^{(1,3)} 6xy^3\,dx + 9x^2y^2\,dy$$

$\vec{F} = \langle 6xy^3, 9x^2y^2 \rangle$

is $\vec{F}$ conservative?

$\nabla f = F$?

$f = 3x^2y^3$ works

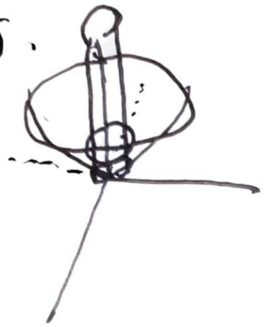
$$\int_C \vec{F}\cdot d\vec{s} = \int_C \nabla f\cdot d\vec{s} = f(\vec{r}(b)) - f(\vec{r}(a)) = f(1,3) - f(0,1)$$

$$= 3\cdot 27 - 0 = \boxed{81}$$

5 Surface area 9 / 10

✓ **- 1 pts** **Bad algebra**

5.



paraboloid is bounded by

$z=0,\ z=1$

Surface $= x^2+y^2=z$

$$\iint_S dS = \iint_R \sqrt{(2x)^2+(2y)^2+1}\; dA = \iint_R \sqrt{4x^2+4y^2+1}\; dA$$

$$\int_0^{2\pi}\int_0^1 \sqrt{4r^2+1}\; r\,dr\,d\theta = \int_0^{2\pi} \frac{(4r^2+1)^{3/2}}{1.5 * 8}\Big]_0^1 d\theta = \int_0^{2\pi}\left(\frac{5^{3/2}}{12}-\frac{1^{3/2}}{12}\right)d\theta$$

$$\frac{5^{3/2}-1}{12}\cdot 2\pi = \boxed{\frac{5^{3/2}\cdot\pi-\pi}{6}} = \boxed{\frac{5\sqrt{5}\,(\pi-1)}{6}}$$

---

6. $$\int_{(0,1)}^{(1,3)} 6xy^3\,dx + 9x^2y^2\,dy$$

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$$\boxed{= 3\cdot 27 - 0} = \boxed{81}$$

6 FTLI 10 / 10

✓ - 0 pts Correct

7. $f(x,y,z) = x^2+y^2+z^2 = 49$

$\nabla f = \langle 2x, 2y, 2z \rangle$ $\quad \nabla f(6,2,3) = \langle 12, 4, 6 \rangle = \vec{N}$

$\vec{N} \cdot \langle x-6, y-2, z-3 \rangle = 0$

$12(x-6) + 4(y-2) + 6(z-3) = 0$

$12x + 4y + 6z = 72 + 8 + 18$

$\boxed{12x + 4y + 6z = 98}$

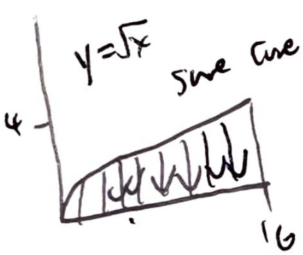
Parametric

$\langle 6,2,3 \rangle + t\langle 12,4,6 \rangle$

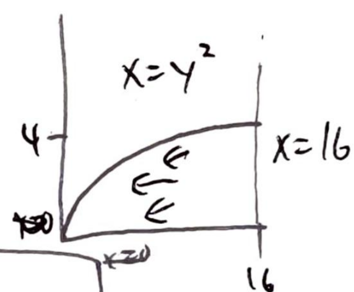
$$\boxed{\begin{aligned} x(t) &= 6 + 12t \\ y(t) &= 2 + 4t \\ z(t) &= 3 + 6t \end{aligned}}$$

8.

a.

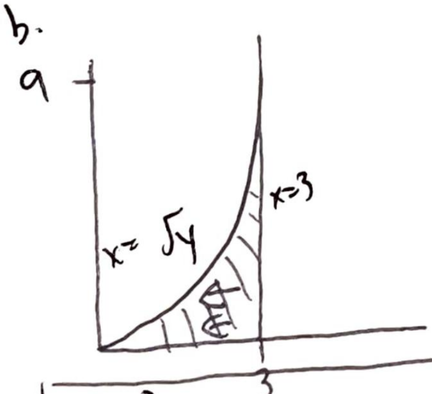


convert to $dx\,dy$

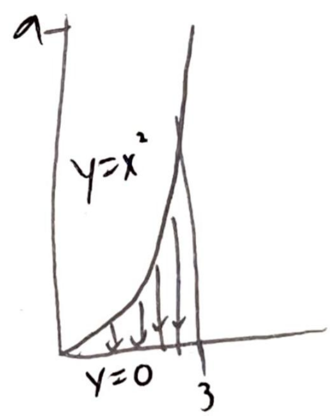


$$\boxed{\int_0^{16}\int_0^{\sqrt{x}} f(x,y)\,dy\,dx = \int_0^{4}\int_{y^2}^{16} f(x,y)\,dx\,dy}$$

b.



convert to $dy\,dx$



$$\boxed{\int_0^{9}\int_{\sqrt{y}}^{3} f(x,y)\,dx\,dy = \int_0^{3}\int_0^{x^2} f(x,y)\,dy\,dx}$$

7.1 Tan plane 5 / 5

✓ - 0 pts Correct

7. $f(x,y,z) = x^2+y^2+z^2 = 49$

$\nabla f = \langle 2x, 2y, 2z \rangle \quad \nabla f(6,2,3) = \langle 12,4,6 \rangle = \vec{N}$

$\vec{N} \cdot \langle x-6, y-2, z-3 \rangle = 0$

$12(x-6) + 4(y-2) + 6(z-3) = 0$

$12x + 4y + 6z = 72 + 8 + 18$

$$\boxed{12x + 4y + 6z = 98}$$

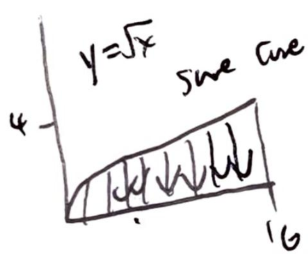
Parametric

$\langle 6,2,3 \rangle + t\langle 12,4,6 \rangle$

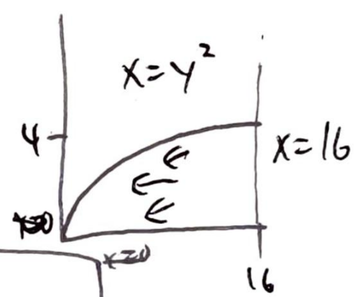
$$\boxed{\begin{aligned} x(t) &= 6 + 12t \\ y(t) &= 2 + 4t \\ z(t) &= 3 + 6t \end{aligned}}$$

8.

a.

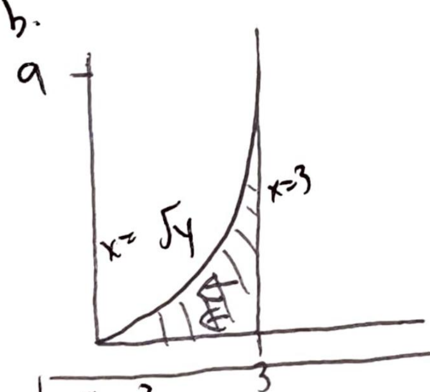


convert to $dx\,dy$

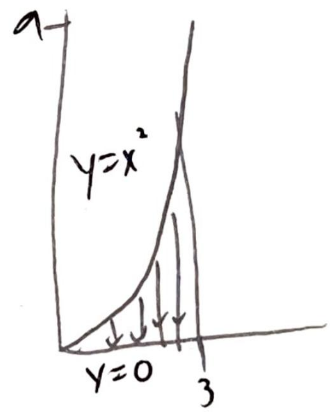


$$\boxed{\int_0^{16}\int_0^{\sqrt{x}} f(x,y)\,dy\,dx = \int_0^{4}\int_{y^2}^{16} f(x,y)\,dx\,dy}$$

b.



Convert to $dy\,dx$



$$\boxed{\int_0^{9}\int_{\sqrt{y}}^{3} f(x,y)\,dx\,dy = \int_0^{3}\int_0^{x^2} f(x,y)\,dy\,dx}$$

7.2 Normal to the plane 5 / 5

✓ **- 0 pts** Correct

7. $f(x,y,z) = x^2+y^2+z^2 = 49$

$\nabla f = \langle 2x, 2y, 2z \rangle$ $\quad \nabla f(6,2,3) = \langle 12, 4, 6 \rangle = \vec{N}$

$\vec{N} \cdot \langle x-6, y-2, z-3 \rangle = 0$

$12(x-6) + 4(y-2) + 6(z-3) = 0$

$12x + 4y + 6z = 72 + 8 + 18$

$\boxed{12x + 4y + 6z = 98}$

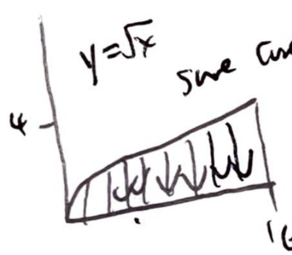
Parametric

$\langle 6,2,3 \rangle + t\langle 12,4,6 \rangle$

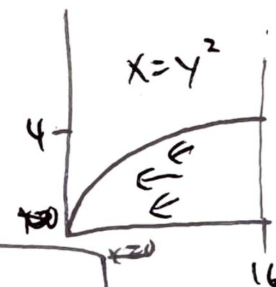
$$\boxed{\begin{aligned} x(t) &= 6 + 12t \\ y(t) &= 2 + 4t \\ z(t) &= 3 + 6t \end{aligned}}$$

8.

a.

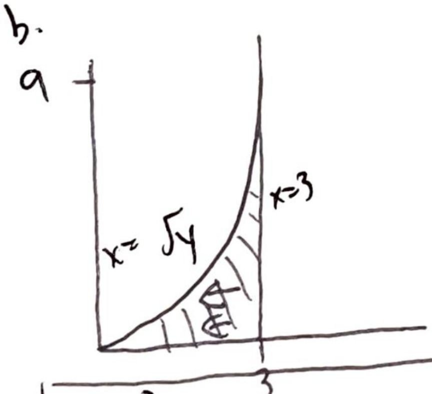


convert to $dx\,dy$

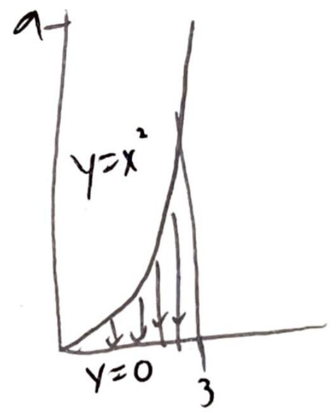


$$\boxed{\int_0^{16}\int_0^{\sqrt{x}} f(x,y)\,dy\,dx = \int_0^{4}\int_{y^2}^{16} f(x,y)\,dx\,dy}$$

b.



Convert to $dy\,dx$



$$\boxed{\int_0^{9}\int_{\sqrt{y}}^{3} f(x,y)\,dx\,dy = \int_0^{3}\int_0^{x^2} f(x,y)\,dy\,dx}$$

8.1 First 5 / 5

✓ **- 0 pts** Correct

7. $f(x,y,z) = x^2+y^2+z^2 = 49$

$\nabla f = \langle 2x, 2y, 2z \rangle$ $\quad \nabla f(6,2,3) = \langle 12, 4, 6 \rangle = \vec{N}$

$\vec{N} \cdot \langle x-6, y-2, z-3 \rangle = 0$

$12(x-6) + 4(y-2) + 6(z-3) = 0$

$12x + 4y + 6z = 72 + 8 + 18$

$\boxed{12x + 4y + 6z = 98}$

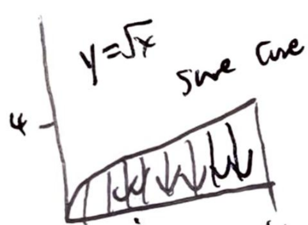
Parametric

$\langle 6,2,3 \rangle + t\langle 12,4,6 \rangle$

$$\boxed{\begin{aligned} x(t) &= 6 + 12t \\ y(t) &= 2 + 4t \\ z(t) &= 3 + 6t \end{aligned}}$$

8.

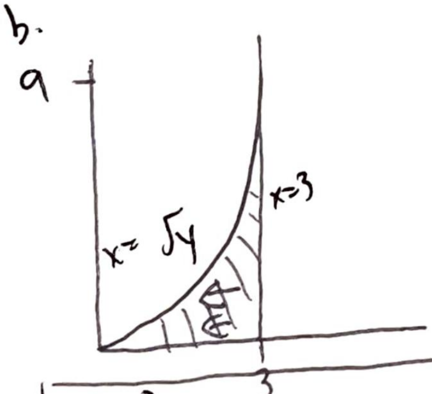
a.



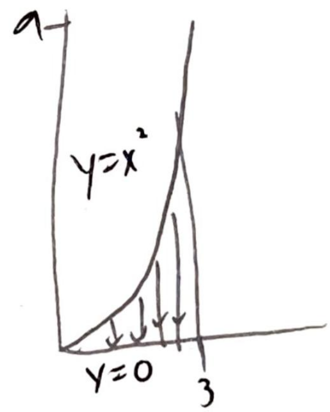
conved to $dx\,dy$

$$\boxed{\int_0^{16}\int_0^{\sqrt{x}} f(x,y)\,dy\,dx = \int_0^4\int_{y^2}^{16} f(x,y)\,dx\,dy}$$

b.



Convert to $dy\,dx$



$$\boxed{\int_0^9\int_{\sqrt{y}}^3 f(x,y)\,dx\,dy = \int_0^3\int_0^{x^2} f(x,y)\,dy\,dx}$$

8.2 Second **5 / 5**

✓ **- 0 pts** **Correct**

9.

$(3\vec{a}-\vec{b})\cdot(2\vec{a}+5\vec{b})$

$\vec{a}\cdot\vec{a} = |a||a|\cos(\theta) = |a|^2$

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

$6\vec{a}\cdot\vec{a} + 15\vec{a}\cdot\vec{b} - 2\vec{b}\cdot\vec{a} - 5\vec{b}\cdot\vec{b}$

$6|a|^2 + 15\vec{a}\cdot\vec{b} - 2\vec{a}\cdot\vec{b} - 5|b|^2$

$6\cdot 2^2 - 5\cdot 3^2 + 13\vec{a}\cdot\vec{b}$

$24 - 45 + 13(|a||b|\cos(30°)) = -21 + 13(2\cdot 3\cdot \frac{\sqrt{3}}{2}) = \boxed{-21+39\sqrt{3}}$

10.

$\int_C f(x,y,z)\,ds = \int_C x^4 z\,ds = \int_C x^4 z\cdot \|r'(t)\|\,dt$

Curve is line segments

$(1-t)\langle 1,2,3\rangle + t\langle 3,3,4\rangle$

$= \langle 1,2,3\rangle + t\langle 2,1,1\rangle$

$0 \le t \le 1$

$x = 1+2t$

$y = 2+t$

$z = 3+t$

$\frac{dx}{dt} = 2$

$\frac{dy}{dt} = 1$

$\frac{dz}{dt} = 1$

$\int_0^1 x^4 z\cdot\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\,dt$

$\int_0^1 (1+2t)^4(3+t)(\sqrt{6})\,dt$

$\sqrt{6}\int_0^1 16t^5+80t^4+120t^3+80t^2+25t+3\,dt$

$\sqrt{6}\left[\frac{16t^6}{6}+16t^5+30t^4+\frac{80}{3}t^3+\frac{25}{2}t^2+3t\right]_0^1$

$\sqrt{6}\left[\frac{16}{6}+16+30+\frac{160}{6}+\frac{75}{6}+3\right]$

$\sqrt{6}\left[\frac{251}{6}+49\right]$

$\boxed{\frac{545\sqrt{6}}{6}}$

	$4x^2$	$4x$	1
$4x^2$	$16x^4$	$16x^3$	$4x^2$
$4x$	$16x^3$	$16x^2$	$4x$
1	$4x^2$	$4x$	1

$16x^4+32x^3+24x^2+8x+1$

	$16x^4$	$32x^3$	$24x^2$	$8x$	1
x	$16x^5$	$32x^4$	$24x^3$	$8x^2$	x
$+3$	$48x^4$	$96x^3$	$72x^2$	$24x$	3

$16x^5+80x^4+120x^3+80x^2+25x+3$

9 Dot product 10 / 10

✓ **- 0 pts** **Correct**

9.

$(3\vec{a}-\vec{b})\cdot(2\vec{a}+5\vec{b})$

$\vec{a}\cdot\vec{a} = |a||a|\cos(0) = |a|^2$

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

$6\vec{a}\cdot\vec{a} + 15\vec{a}\cdot\vec{b} - 2\vec{b}\cdot\vec{a} - 5\vec{b}\cdot\vec{b}$

$6|a|^2 + 15\vec{a}\cdot\vec{b} - 2\vec{a}\cdot\vec{b} - 5|b|^2$

$6\cdot 2^2 - 5\cdot 3^2 + 13\vec{a}\cdot\vec{b}$

$24 - 45 + 13(|a||b|\cos(30^\circ)) = -21 + 13(2\cdot 3\cdot \frac{\sqrt{3}}{2}) = \boxed{-21+39\sqrt{3}}$

10.

Curve is line segments

$\int_C f(x,y,z)\,ds = \int_C x^4 z\,ds = \int_C x^4 z\cdot \|r'(t)\|\,dt$

$(1-t)\langle 1,2,3\rangle + t\langle 3,3,4\rangle$

$= \langle 1,2,3\rangle + t\langle 2,1,1\rangle$

$\int_0^1 x^4 z \cdot \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\,dt$

$0\le t\le 1$

$x = 1+2t$

$y = 2+t$

$z = 3+t$

$\frac{dx}{dt} = 2$

$\frac{dy}{dt} = 1$

$\frac{dz}{dt} = 1$

$\int_0^1 (1+2t)^4(3+t)(\sqrt{6})\,dt$

$\sqrt{6}\int_0^1 16t^5+80t^4+120t^3+80t^2+25t+3\,dt$

$\sqrt{6}\left[\frac{16t^6}{6}+16t^5+30t^4+\frac{80}{3}t^3+\frac{25}{2}t^2+3t\right]_0^1$

$\sqrt{6}\left[\frac{16}{6}+16+30+\frac{160}{6}+\frac{75}{6}+3\right]$

$\sqrt{6}\left[\frac{251}{6}+49\right]$

$\boxed{\frac{545\sqrt{6}}{6}}$

	$16x^4$	$32x^3$	$24x^2$	$8x$	1
x	$16x^5$	$32x^4$	$24x^3$	$8x^2$	x
$+3$	$48x^4$	$96x^3$	$72x^2$	$24x$	3

$16x^5+80x^4+120x^3+80x^2+25x+3$

	$4x^2$	$4x$	1
$4x^2$	$16x^4$	$16x^3$	$4x^2$
$4x$	$16x^3$	$16x^2$	$4x$
1	$4x^2$	$4x$	1

$16x^4+32x^3+24x^2+8x+1$

10 Line integral of scalar **10 / 10**

✓ **- 0 pts** **Correct**

11

$D_{\vec{u}} f(x,y) = \nabla f(x,y) \cdot \vec{u}$

$\frac{1}{5} = \nabla f(1,2) \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle = \frac{3}{5} f_x + \frac{4}{5} f_y$

$1 = 3f_x + 4f_y$

$\frac{4}{\sqrt{5}} = \nabla f(1,2) \cdot \langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \rangle = \frac{2}{\sqrt{5}} f_x + \frac{1}{\sqrt{5}} f_y$

$4 = 2f_x + 1f_y$

$4(1 = 3f_x + 4f_y) = 4 = 12f_x + 16f_y$

$- 4 = 2f_x + 1f_y$

$0 = 10f_x + 15f_y$

$10f_x = -15f_y$

$f_x = -1.5f_y$

oops

$16 = 8f_x + 4f_y$

$1 = 3f_x + 4f_y$

$\cancel{16} = 15 = 5f_x$

$f_x = 3$

$3 = -1.5 f_y$

$f_y = -2$

$f_x = 3, f_y = -2, \boxed{\nabla f(x,y) = \langle 3, -2 \rangle}$

11 Directional derivative 10 / 10

✓ **- 0 pts** Correct

10. Find $\int_C x^4 z \, ds$ if C is the line segment from $(1, 2, 3)$ to $(3, 3, 4)$

11. $f(x, y)$ at (1,2) has directional derivative of $1/5$ in the direction $< 3, 4 >$. It has directional derivative of $4/\sqrt{5}$ in the direction $< 2, 1 >$. Find $\nabla f(1, 2)$.

12. Surface S is the part of the $z = 1 - y$ plane that is inside of the cylinder $x^2 + y^2 = 1$ oriented up. C is the boundary curve of S traced counter clockwise as seen from above. Use Stokes to compute

$$\int_C (x + y) \, dx + (y + z) \, dy + (x + z) \, dz$$

$$F = \langle x+y, y+z, x+z \rangle$$

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot (z_x \times z_y) \, dA$$

$$\text{curl}(\vec{F}) = \nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle$$

$$= \langle -1, \boxed{1,} -1 \rangle$$

$z = 1 - y$

$z_x = 0$

$z_y = -1$

$$z_x \times z_y = \langle 1, 0, 0 \rangle \times \langle 0, 1, -1 \rangle = \langle 0, 1, 1 \rangle$$

$$\iint_S \langle -1, 1, -1 \rangle \cdot \langle 0, 1, 1 \rangle \, dA = \iint_S 0 \, dA = 0$$ or $\boxed{\text{zero}}$

12 Stokes **7 / 10**

- ✓ **- 3 pts** **Bad $$\nabla \times \vec{F}$$ or curl $$\vec{F}$$**