Final Exam

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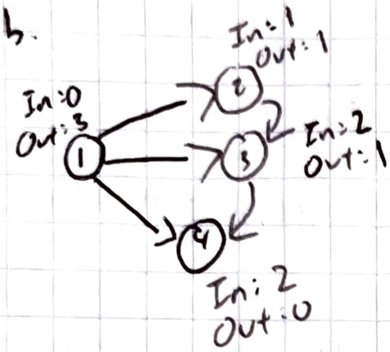
5/26/21

1.

a. Not possible because of the Handshaking Theorem which states $2m = \sum_{v \in V} \deg(v)$. This means the sum of degrees in any graph is an even number.

b.



c. Not possible because a simple clique [Complete graph] has at most $\frac{(n)(n-1)}{2}$ edges.

$\frac{6 \cdot 5}{2} = 15$. $16 > 15$ so it is impossible.

d.

2. $\frac{1}{10} \cdot \frac{1}{10}$

a. $\frac{1}{100} = \boxed{0.01}$

b. There are 100 possible ways to choose numbers by picking ~~this way~~ a random integer from [1, 10]. 10 for the friend and 10 for me $= 10 \times 10 = 100$.

The illegal pairs of possibilities are (9,10), (10,9), (10,10).

~~By Inclusion Exclusion~~

Subtracting unwanted cases by total cases we get $100 - 3 = 97$ ways. Therefore the probability is $\frac{97}{100}$ or $\boxed{0.97}$

c.

	1	2	3	4	5	6	7	8	9	10
1	0	-	-	-	-	-	-	-	-	-
2	+	0	-	-	-	-	-	-	-	-
3	+	+	0	-	-	-	-	-	-	-
4	+	+	+	0	-	-	-	-	-	-
5	+	+	+	+	0	-	-	-	-	-
6	+	+	+	+	+	0	-	-	-	-
7	+	+	+	+	+	+	0	-	-	-
8	+	+	+	+	+	+	+	0	-	-
9	+	+	+	+	+	+	+	+	0	-
10	+	+	+	+	+	+	+	+	+	0

There are $1+2+3+4+5+6+7+8+9$ favorable outcomes and 100 total outcomes.

Therefore $45/100 = \boxed{0.45}$

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Final Exam

3. Let $f(n)$ be the proposition that n^3+3n is divisible by 2.

Basis Step: $f(1)$ is true because $1^3+3\cdot1=4$. 4 is an even number and is therefore divisible by 2.

Inductive Hypothesis: Assume $f(k)$ is valid for an arbitrary value k such that $k \geq 1$.

Inductive Step. ~~For convenience, split the expression n^3+3n into two terms; n^3, $3n$.~~

When we go from $k \to k+1$, we get $(k+1)^3 + (3(k+1))$. Expanded, we get

$$(k^3+3k^2+3k+1)+(3k+3)$$

We know k^3+3k is ~~true~~ valid already.

We need to test if $(3k^2+3k+1)+(3)$ is even.

Firstly, adding an even number to another number x does not change the parity of x.

Our expression simplifies from $3k^2+3k+4$ to $3k^2+3k$.

We can distribute $3k$ out too.

$3k(k+1)$ is the final expression

Multiplying anything by an even number is an even number. Since k or $k+1$ must be even by pigeonhole principle, this expression must be even as well.

(continues on next page)

3 continued Since $3K(K+1)$ is even, we are adding an even number to $f(k)$.

Final Exam

David Yang 5/26/21

This means $f(K+1)$ is also even.

Since we have proved $f(k+1)$ is valid as well, we have completed the inductive step.

We have completed the basis step and inductive step, so by mathematical induction we know $f(n)$ is true for all $n \geq 1$.

4.

a. procedure average min max ($a_1, \ldots, a_n$: integers):

max := a_1
min := a_1

for i := 1 to n
  if a_i > max then max := a_i
  if a_i < min then min := a_i

avg := (min + max) / 2

return avg {avg is average of smallest and largest integers in list}

b.

Step 1:
min = 9
max = 9

Step 2:
4 < min (9) so min = 4
min = 4
max = 9

Step 3:
nothing happens
min = 4
max = 9

Step 4:
12 > max (9) so max = 12
min = 4
max = 12

Step 5
3 < min (4) so min = 3
min = 3
max = 12

Step 6.
avg = (3 + 12) / 2 = 7.5
return 7.5

David Yang
5/26/21

Final Exam

5. To prove a function is a bijection it must be both injective and surjective. ~~Start~~ Start with proving injectiveness.

To prove this, we will use a proof by contradiction.
Suppose $8a-7 = 8b-7$. We simplify this to $8a = 8b$ or $a=b$.
Since $a=b$, the only way $g(x) = g(y)$ is if ~~g(x)~~ $x=y$.
This also means if $x \neq y$, then $g(x) \neq g(y)$.

To prove $g(x)$ is surjective (onto), the range of $g(x)$ must be $= R$, or for every $y \in R$ there exists an $x \in R$ such that $f(x) = y$. We can consider an arbitrary y, 24.8. There exists an x that satisfies $8x-7 = 24.8$. $x = 3.975$

Since $g(x)$ is injective and surjective, it is a bijection.

The inverse of $8x-7=y$ is

$8y-7=x$

$8y = x+7$

$$\boxed{y = \frac{x+7}{8}}$$

~~Question 6:~~

~~Let $x = p \to \neg q$~~

~~p~~	~~q~~	~~$p \to \neg q$~~

Final Exam

David Yang
5/26/21

6.

p	q	¬q	p→¬q	(p→¬q)∧q	((p→¬q)∧q)→¬p
0	0	1	1	0	1
0	1	0	1	1	1
1	0	1	1	0	1
1	1	0	0	0	1

The proposition $((p \to \neg q) \wedge q) \to \neg p$ is a tautology becuse according to the truth table, it always evaluates to true for every combina of p and q.

B. we can simplify this statement of "All flowers are pretty" to let $P(x)$ = flower is pretty. $\forall x\, P(x)$.

a. "All flowers are not pretty" is ambigious becuse in the correct negation, as long as just 1 flower is not pretty, it would be true. In this statement, all flowers must not be pretty in order to be true.

$\neg \forall x\, P(x) = \exists x\, \neg P(x)$

b. "No flowers are pretty" is also wrong becuse the negation of the original statement would still be true if there were pretty flowers as long as there was ≥ 1 not pretty flower. This statement would be false.

c. "Some flowers are pretty" is also wrong because if I had all pretty flowers, the correct negaton would be false but this is true.

d. There exists a flower that is not pretty.