David Yang

HW 1A: Logic and Proofs

Section 1.1

10. Let p and q be the propositions “The election is decided” and “The votes have been counted,” respectively. Express the following in English.

a. $\neg p\ =\ It\,is\ not\,the\,case\,the\,election\,\hat{}\,is\,decided$
b. $p \vee q\ =\ The\,election\,is\,decided,\ or\,the\,votes\,have\,been\,counted.$
c. $\neg p\ \vee q\ =\ The\,election\,is\,not\,decided,\ or\,the\,votes\,have\,been\,counted$
d. $q\ \rightarrow\ p\ =\ If\,the\,votes\,have\,been\,counted,\ the\,election\,is\,decided.$
e. $\neg q\ \rightarrow\ \neg p\ =\ If\,the\,votes\,have\,not\,been\,counted,\ the\,election\,is\,not\,decided.$
f. $\neg p\ \rightarrow\ \neg q =\ If\,the\,election\,is\,not\,decided,\ the\,votes\,have\,not\,been\,counted.$
g. $p \Leftrightarrow q\ =\ The\,election\,is\,decided\,if\,and\,only\,if\,The\,votes\,have\,been\,counted$
h. $\neg q \vee (\neg p \wedge q)\ =\ The\,votes\,have\,not\,been\,counted\,or,\ The\,election\,is\,not\,decided\,and\,the\,votes\,have\,been\,counted.$

13. Let p and q be the propositions p : You drive over 65 miles per hour. q : You get a speeding ticket. Write these propositions using p and q and logical connectives (including negations).

a. $\neg p$
b. $p\ \wedge \neg q$
c. $p \rightarrow q$
d. $\neg p \rightarrow\ \neg q$
e. $p \rightarrow\ q$
f. $\neg p\ \wedge q$
g. $q\ \rightarrow\ p$

21. For each of these sentences, state what the sentence means if the logical connective or is an inclusive or (that is, a disjunction) versus an exclusive or. Which of these meanings of or do you think is intended?

a. To take discrete mathematics, you must have taken calculus or a course in computer science.
   If the “or” is exclusive, this means that you must satisfy both calculus and computer science. If inclusive, this means you must satisfy one of the requirements.
b. When you buy a new car from Acme Motor Company, you get \$2000 back in cash or a 2% car loan.
   If the “or” is inclusive, this means you will receive either one of the 2000 dollars or a 2% car loan. If the “or” is exclusive, this means you will receive both.
c. Dinner for two includes two items from column A or three items from column B.
   If the “or” is inclusive, this means you will receive either one of the 2 items from column A or 3 items from column B. If the “or” is exclusive, this means you will receive both.
d. School is closed if more than 2 feet of snow falls or if the wind chill is below −100.
   If the “or” is inclusive, then school will be closed if either statement: >2 feet of snow falls, wind chill is below -100, is true. If the “or” is exclusive, then school will be closed only if both statements are true.

29. How many rows appear for each of these compound propositions?

a. $p \rightarrow \neg p$ will have 2 rows. Since there is only 1 variable, 2^1 = 2.
b. $(p \vee \neg r) \wedge (q \vee \neg s)$ will have 16 rows. Since there are 4 variables, 2^4 = 16.
c. $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$ will have 64 rows. Since there are 6 variables, 2^6=64.

d. $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$ will have 16 rows. Since there are 4 variables, 2^4 = 16.

33. Construct truth table.

a. $(p \vee q) \rightarrow (p \oplus q)$

p	q	p or q	p xor q	$(p \vee q) \rightarrow (p \oplus q)$
0	0	0	0	1
0	1	1	1	1
1	0	1	1	1
1	1	1	0	0

b. $(p \oplus q) \rightarrow (p \wedge q)$

p	q	p xor q	p or q	$(p \oplus q) \rightarrow (p \wedge q)$
0	0	0	0	1
0	1	1	1	1
1	0	1	1	1
1	1	0	1	1

c. $(p \vee q) \oplus (p \wedge q)$

p	q	p and q	p or q	$(p \vee q) \oplus (p \wedge q)$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

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5. Use a truth table to verify the distributive law p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r).

q	r	p	qvr	p∧(q ∨ r)	(p∧q)	p ∧ r	(p ∧ q) ∨ (p ∧ r)

0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	0	1	1
1	0	0	1	0	0	0	0
1	0	1	1	1	1	0	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

As you can see, The p∧(q ∨ r) column is identical to (p ∧ q) ∨ (p ∧ r) column.

7.

a. Let p=rich, q=happy. $\neg p \wedge \neg q$
b. Let p=bicycle, q=run. $\neg p \vee \neg q$
c. Let p=walk, q=takes the bus $\neg p \vee \neg q$
d. Let p= smart, q= hard working. $\neg p \wedge \neg q$

22. Show that(p → q) ∧ (p → r) and p → (q ∧ r) are logically equivalent.

p	q	r	p->q	p->r	(p→ q) ∧ (p → r)	q∧ r	p → q∧ r
0	0	0	1	1	1	0	1
0	0	1	1	1	1	0	1
0	1	0	1	1	1	0	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	1	0	0	0
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

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27. Show that p ↔ q and (p → q) ∧ (q → p) are logically equivalent

p	q	p ↔ q	p->q	q->p	(p → q) ∧ (q→ p)
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	1	1	1	1

Section 1.4

5. Let P (x) be the statement “x spends more than five hours every weekday in class,” where the domain for x consists of all students. Express each of these quantifications in English.

a. $\exists x P(x)$ =There exists a student that works more than 5 hours every weekday in class.
b. $\forall x P(x)$= For all students, the student works more than 5 hours every weekday in class.
c. $\exists x \neg P(x)$= There exists a student that does not work more than 5 hours every weekday in class.
d. $\forall x \neg P(x)$= For all students, the student does not work more than 5 hours every weekday in class.

9.Let P (x) be the statement “x can speak Russian” and let Q(x) be the statement “x knows the computer language C++.” Express each of these sentences in terms of P (x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian and who knows C++.

$\exists x(P(x) \wedge Q(x))$

b) There is a student at your school who can speak Russian but who doesn’t know C++.

$\exists x(P(x) \wedge \neg Q(x))$

c) Every student at your school either can speak Russian or knows C++.

$\forall x(P(x) \vee Q(x))$

d) No student at your school can speak Russian or knows C++.
$\forall x \neg (P(x) \vee Q(x))$

14. Determine the truth value of each of these statements if the domain consists of all real numbers.

a) ∃x(x^3 = −1) = true, for x=-1.

b) ∃x(x^4 < x^2) = true, because for x=0.5 this is true.

c) ∀x((−x)^2 = x^2) = true, because (-x)^2 == x^2

d) ∀x(2x > x) = false, because -1 breaks the statement.

21.For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

a) Everyone is studying discrete mathematics.
Domain True = all students in MTH 8. Domain false = whole school.
b) Everyone is older than 21 years.
Domain True = Club/Bar. Domain false = elementary school.
c) Every two people have the same mother.
Domain True = siblings. Domain false = earth
d) No two different people have the same grandmother
Domain True = Barack Obama and Michelle Obama. Domain false = earth

26abc.Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

a) Someone in your school has visited Uzbekistan.
Let the domain of x denote the people in the school, U(x) denote whether one has visited Uzbekistan.
$\exists x U(x)$
Let the domain of x denote the people in the entire world now. S(x) denotes whether one goes to the school. U(x) remains constant.
$\exists x (S(x) \wedge U(x))$
Let U(x,y) denote if person x visited place y.
$\exists x (S(x) \ \wedge U(x, \, Uzbekis\tan))$

b) Everyone in your class has studied calculus and C++.
Let the domain of x denote the people in the class, C(x) denote whether one studied calculus, c(x) denotes whether one studied C++.
$\forall x (C(x) \wedge c(x))$
Let the domain of x denote the people in the entire world now. S(x) denotes whether one goes to the class.

$\forall x(S(x) \wedge C(x) \wedge c(x))$

Let U(x,y) denote if person x studied topic y.

$\forall x(S(x) \wedge U(x,\text{''}\ Calculus) \wedge U(x, C++))$

c) No one in your school owns both a bicycle and a motorcycle.

Let B(x) denote “x owns a bicycle” and M(x) denote “x owns a motorcycle”. The domain of x is the school.

$\neg\exists x(B(x) \wedge M(x))$

Let the domain of x be the whole world. Let S(x) denote “x goes to your school”.

$\neg\exists x(S(x)\ \wedge B(x)\ \wedge M(x))$

Let U(x,y) denote “x owns a y”.

$\neg\exists x(S(x) \wedge U(x,\ bicycle) \wedge U(x, motorcycle))$

29.Express each of these statements using logical operators, predicates, and quantifiers.

a) Some propositions are tautologies.

Let T(x) denote “x is a tautology. The domain of x is all propositions.

$\exists x T(x)$

b) The negation of a contradiction is a tautology.

Let T(x) denote “x is a tautology”. Let C(x) denote “x is a contradiction” The domain of x is all propositions.

$\forall x(\neg C(x) \to T(x))$

c) The disjunction of two contingencies can be a tautology.

Let C(x) denote “x is not a contradiction”. Let T(x) denote “x is not a tautology”. Let c(x) denote C(x)^T(x). The domain of x and y is all propositions.

$\exists x \exists y\ =\ (T(x \vee y) \wedge c(x) \wedge c(y))$

d) The conjunction of two tautologies is a tautology

Let T(x) denote “x is a tautology”.The domain of x and y is all propositions.

$\forall x \forall y((T(x) \wedge T(y)) \to\ T(x \wedge y))$

Section 1.5

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3. Let Q(x, y) be the statement “x has sent an e-mail message to y,” where the domain for both x and y consists of all students in your class. Express each of these quantifications in English.

a) ∃x∃yQ(x, y) = There exists a person x and a person y such that x has sent an email message to y.

b) ∃x∀yQ(x, y)= There exists a person x such that for all people y, x has sent an email message to y.

c) ∀x∃yQ(x, y) = For all persons x, there exists a y such that x has sent an email message to y.

d) ∃y∀xQ(x, y) = There exists a person y such that for all persons x, x has sent an email message to y.

e) ∀y∃xQ(x, y) = For all persons y, there exists a person x such that x has sent an email message to y.

f) ∀x∀yQ(x, y)= For all persons x, there exists all persons y, such that x has sent an email message to y.

8abcd.

Let Q(x, y) be the statement “student x has been a contestant on quiz show y.” Express each of these sentences in terms of Q(x, y), quantifiers, and logical connectives, where the domain for x consists of all students at your school and for y consists of all quiz shows on television.

a) There is a student at your school who has been a contestant on a television quiz show.

$\exists x \exists y Q(x, y)$

b) No student at your school has ever been a contestant on a television quiz show.

$\neg \exists x (\exists y \, Q(x, y))$

c) There is a student at your school who has been a contestant on Jeopardy and on Wheel of Fortune.

$\exists x Q(x,\ Jeopardy)\ \wedge Q(x, Wheel\, of\, Fortune)$

d) Every television quiz show has had a student from your school as a contestant.

$\forall y \exists x \, Q(x, y)$

e) At least two students from your school have been contestants on Jeopardy.

$\exists x \exists y \, (x \neq y)\ \wedge Q(x, Jeopardy) \wedge Q(y, Jeopardy)$

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15.Use quantifiers and predicates with more than one variable to express these statements.

a) Every computer science student needs a course in discrete mathematics.

Let S(x,y) denote “Computer Science Student x needs a course in y”.

$\forall x S(x,\, discrete\, mathematics)$

b) There is a student in this class who owns a personal computer.

Let S(x,y) denote “student x in this class owns a y”

$\exists x S(x,\, personal\, comp\,\mathbf{t}er)$

c) Every student in this class has taken at least one computer science course.

Let S(x,y) denote “student x has taken course y”. Let C(x) denote “course x is a computer science course”.

$\forall x \exists y\, S(x,y) \land C(y)$

d) There is a student in this class who has taken at least one course in computer science.

Let S(x,y) denote “student x has taken course y”. Let C(x) denote “course x is a computer science course”.

$\exists x \exists y\, S(x,y) \land C(y)$

e) Every student in this class has been in every building on campus.

Let S(x,y) denote “student x in this class has been in building y on campus”.

$\forall x \forall y S(x,y)$

f) There is a student in this class who has been in every room of at least one building on campus.

Let S(x,y) denote “student x in this class has been in building y on campus”.

$\exists x \exists y S(x,y)$

g) Every student in this class has been in at least one room of every building on campus

$\forall x \exists y S(x,y)$