MATH-253-YJH-CRN82680 Exam 4

David Yang

TOTAL POINTS

66 / 70

QUESTION 1

1 Line integral over line segment **9 / 10**

✓ **- 1 pts** **Bad arithmetic**

QUESTION 2

2 Line integral over line segment 2 **10 / 10**

✓ **- 0 pts** **Correct**

QUESTION 3

3 Line integral over two line segments **10 / 10**

✓ **- 0 pts** **Correct**

QUESTION 4

Conservative vector field 10 pts

4.1 Find the potential **5 / 5**

✓ **- 0 pts** **Correct**

4.2 Find the line integral **5 / 5**

✓ **- 0 pts** **Correct**

QUESTION 5

5 Arc length **10 / 10**

✓ **- 0 pts** **Correct**

QUESTION 6

6 Volume **10 / 10**

✓ **- 0 pts** **Correct**

QUESTION 7

Green Monster 10 pts

7.1 Acceleration, velocity, position **2 / 5**

✓ **- 3 pts** **Need to fill in all the letters**

7.2 What does the ball do? **5 / 5**

✓ **- 0 pts** **Correct**

Fourth Exam
Math 252
Hasson

Honors pledge:

I will not use notes, books, other paper sources, "cheat sheet," digital sources, or any computer source of information or computation during this exam.

I will not ask or consult with anyone or get any kind of work or help from another person during this exam.

Signed: David Yang

How to do the exam:

- **Show all work. No credit for just answers or incomplete work.**
- If you can print the exam, then write all of your work and answers on the printed exam.
- If you can't print the exam, then write all of your work and answers on notebook paper.
- Either way, when you are done with the exam, then take pictures of the pages using your cell phone camera.
- Then use CamScanner or a similar app to tie the pictures (JPEGs) into a single pdf.
- Go to Canvas and click on Gradescope.
- Follow the directions to submit your exam
- Also use Gradescope to match the exam problems with pages of your exam. Minus two points for not doing this step or for doing a poor job of it.
- Do not send me a bunch of JPEGs. Do not send me a Google Docs link (so don't do all this in Google Docs).

1. C is the line segment traced from $(1, 2)$ to $(5, 8)$. Find $\int_C xy\, ds$.

Line Segment Parameterized: $(1-t)\langle x_0, y_0\rangle + (t)\langle x_1, y_1\rangle$ and $0 \le t \le 1$

Bounds for C: $0 \le t \le 1$, $(1-t)\langle 1, 2\rangle + t\langle 5, 8\rangle = \langle 1+4t, 2+6t\rangle = \vec{r}(t)$

x y

$|\vec{r}\,'(t)| = \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{4^2 + 6^2} = \sqrt{\boxed{24} + 36} = \sqrt{60}$

① continued

$x = 1 + 4t$

$y = 2 + 6t$

$\int_C xy\, ds =$

$\int_0^1 (1+4t)(2+6t) \cdot |\vec{r}\,'| dt$

$\int_0^1 (24t^2 + 14t + 2) \cdot \sqrt{60} \cdot dt$

$\sqrt{60} \int_0^1 (24t^2 + 14t + 2) dt$

$\sqrt{60} \cdot \left(8t^3 + 7t^2 + 2t\right)\Big|_0^1$

$\sqrt{60} \cdot (8 + 7 + 2) =$

$\sqrt{60} \cdot 17 =$

$2\sqrt{15} \cdot 17 =$

$34\sqrt{15}$

1 Line integral over line segment 9 / 10

✓ - 1 pts Bad arithmetic

2. C is the line segment traced from $(1,2)$ to $(5,8)$. Find $\int_C xy\,dx + x\,dy$.

$\vec{F} = \langle xy, x \rangle$ Bounds: $0 \le t \le 1$

From last problem → $\vec{r}(t) = \langle 1+4t, 2+6t \rangle$

$$\int_C xy\,dx + x\,dy = \int_C \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

$$\vec{F}(\vec{r}(t)) = \vec{F}(\langle 1+4t, 2+6t \rangle) = \langle 24t^2 + 14t + 2, 1+4t \rangle$$

$$\vec{r}\,'(t) = \langle (1+4t)\,dt, (2+6t)\,dt \rangle = \langle 4, 6 \rangle$$

$$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = \langle 24t^2 + 14t + 2, 4t+1 \rangle \cdot \langle 4, 6 \rangle$$
$$= (96t^2 + 56t + 8) + (24t + 6) = 96t^2 + 80t + 14$$

$$\int_C \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt = \int_0^1 96t^2 + 80t + 14\,dt = 32t^3 + 40t^2 + 14t \Big]_0^1$$

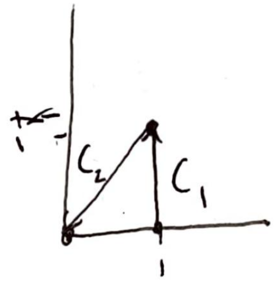
$$= (32 + 40 + 14) - 0 = 86$$

2 Line integral over line segment 2 **10 / 10**

✓ **- 0 pts** **Correct**

3. C is the path from $(1, 0)$ to $(1, 1)$ to $(0, 0)$ by line segments. Find
$$\int_C xy\, dx + x\, dy.$$

$\vec{F} = \langle xy, x \rangle$



$$\int_C \vec{F}\, ds = \int_{C_1} \vec{F}\, ds + \int_{C_2} \vec{F}\, ds$$

$F(r(t)) = F(1, t) = \langle t, 1 \rangle$

$\int_{C_1} \vec{F}\, ds$, Line Segment: $(1-t)\langle 1, 0 \rangle + t\langle 1, 1 \rangle = \langle 1, t \rangle = \vec{r}(t)$

$\vec{r}'(t) = \langle 0, 1 \rangle$, $0 \le t \le 1$

$F(r(t)) = F(1-t, 1-t) = \langle t^2 - 2t + 1, 1-t \rangle$

$\int_{C_2} \vec{F}\, ds$, Line Segment: $(1-t)\langle 1, 1 \rangle + t\langle 0, 0 \rangle = \langle 1-t, 1-t \rangle = \vec{r}(t)$

$\vec{r}'(t) = \langle -1, -1 \rangle$, $0 \le t \le 1$

$$\int_{C_1} \vec{F}\, ds = \int_0^1 \vec{F}(\vec{r}(t)) \cdot r'(t)\, dt = \int_0^1 \langle t, 1 \rangle \cdot \langle 0, 1 \rangle dt = \int_0^1 1\, dt = 1$$

$$\int_{C_2} \vec{F}\, ds = \int_0^1 \vec{F}(\vec{r}(t)) \cdot r'(t)\, dt = \int_0^1 \langle t^2 - 2t + 1, 1-t \rangle \cdot \langle -1, -1 \rangle dt$$

$$= \int_0^1 (-t^2 + 2t - 1) + (t - 1)\, dt = \int_0^1 -t^2 + 3t - 2\, dt = \left(-\frac{t^3}{3} + \frac{3t^2}{2} - 2t\right)\Big|_0^1$$

$$= -\frac{1}{3} + \frac{3}{2} - 2 = -\frac{1}{3} + -\frac{1}{2} = \frac{-5}{6}$$

$$\int_C \vec{F}\, ds = \int_{C_1} \vec{F}\, ds + \int_{C_2} \vec{F}\, ds = 1 - \frac{5}{6}\ 3 = \frac{1}{6}$$

3 Line integral over two line segments **10 / 10**

✓ **- 0 pts** **Correct**

4. $\vec{F}(x,y) =< 2xy + y + 2x, x^2 + x + 2y >$. $\vec{F}$ is conservative.

(a) Find a potential for $\vec{F}$.

$f_x(x,y) = 2xy + 2x + y, \quad f_y(x,y) = x^2 + x + 2y$

$\int f_x\,dx = \int 2xy + 2x + y\,dx = x^2y + x^2 + xy + C$

$\int f_y\,dy = \int x^2 + x + 2y\,dy = x^2y + xy + y^2 + C$

$\boxed{f(x,y) = x^2y + x^2 + y^2 + xy + C}$

(b) Find $\int_C \vec{F} \cdot d\vec{r}$ if C is the curve $y = x^2$ from $(0,0)$ to $(1,1)$.

Let $x = t,\ y = t^2,\ \vec{r}(t) = \langle t, t^2 \rangle,\ \vec{r}\,'(t) = \langle 1, 2t \rangle\ 0 \le t \le 1$

$\int_C \vec{F}\,d\vec{r} = \int_0^1 \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt = \int_0^1 \langle 2t^3 + t^2 + 2t, 2t^2 + t^2 + t \rangle \cdot \langle 1, 2t \rangle\,dt$

$\int_0^1 \langle 2t^3 + t^2 + 2t, 3t^2 + t \rangle \cdot \langle 1, 2t \rangle\,dt = \int_0^1 (2t^3 + t^2 + 2t) + (6t^3 + 2t^2)\,dt$

$\int_0^1 8t^3 + 3t^2 + 2t\,dt = 2t^4 + t^3 + t\Big]_0^1 = (2+1+1) - (0) = 4$

4.1 Find the potential 5 / 5

✓ - 0 pts Correct

4. $\vec{F}(x,y) =< 2xy + y + 2x, x^2 + x + 2y >$. $\vec{F}$ is conservative.

(a) Find a potential for $\vec{F}$.

$f_x(x,y) = 2xy + 2x + y, \quad f_y(x,y) = x^2 + x + 2y$

$\int f_x \, dx = \int 2xy + 2x + y \, dx = x^2y + x^2 + xy + C$

$\int f_y \, dy = \int x^2 + x + 2y \, dy = x^2y + xy + y^2 + C$

$\boxed{f(x,y) = x^2y + x^2 + y^2 + xy + C}$

(b) Find $\int_C \vec{F} \cdot d\vec{r}$ if C is the curve $y = x^2$ from $(0,0)$ to $(1,1)$.

Let $x = t,\ y = t^2,\ \vec{r}(t) = \langle t, t^2 \rangle,\ \vec{r}\,'(t) = \langle 1, 2t \rangle \quad 0 \le t \le 1$

$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt = \int_0^1 \langle 2t^3 + t^2 + 2t, 2t^2 + t^2 + t \rangle \cdot \langle 1, 2t \rangle \, dt$

$\int_0^1 \langle 2t^3 + t^2 + 2t, 3t^2 + t \rangle \cdot \langle 1, 2t \rangle \, dt = \int_0^1 (2t^3 + t^2 + 2t) + (6t^3 + 2t^2)\, dt$

$\int_0^1 8t^3 + 3t^2 + 2t \, dt = 2t^4 + t^3 + t \Big]_0^1 = (2+1+1) - (0) = 4$

4.2 Find the line integral 5 / 5

✓ - 0 pts Correct

5. Curve C is $y = \frac{1}{2}x^2 - \frac{1}{4}\ln x$ for $1 \leq x \leq 2$. Find the arc length.

Let $x = t$, $y = \frac{t^2}{2} - \frac{\ln(t)}{4}$

$\vec{r}(t) = \langle t, \frac{t^2}{2} - \frac{\ln(t)}{4} \rangle$

$\vec{r}'(t) = \cancel{\langle 1, \frac{t^3}{6} - \frac{1}{4t} \rangle}$ $\langle 1, t - \frac{1}{4t} \rangle$

$$\text{Arc Length} = \int_1^2 |\vec{r}'(t)|\,dt = \int_1^2 \left|\langle 1, t - \tfrac{1}{4}t \rangle\right| dt$$

$$= \int_1^2 \sqrt{1^2 + \left(t - \frac{1}{4t}\right)^2}$$

$$= \int_1^2 t + \frac{1}{4t}\,dt =$$

$$= \frac{t^2}{2} + \frac{\ln(t)}{4}\Big]_1^2 = \left(\frac{4}{2} + \frac{\ln(2)}{4}\right) - \left(\frac{1}{2} + \frac{\ln(1)}{4}\right)$$

$$= \boxed{\frac{3}{2} + \frac{\ln(2)}{4}}$$

	t	$-\frac{1}{4t}$
t	t^2	$-\frac{1}{4}$
$-\frac{1}{4t}$	$-\frac{1}{4}$	$\frac{1}{16t^2}$

$\sqrt{1 + t^2 - \frac{1}{2} + \frac{1}{16t^2}}$ $\quad t + \frac{1}{4t}$

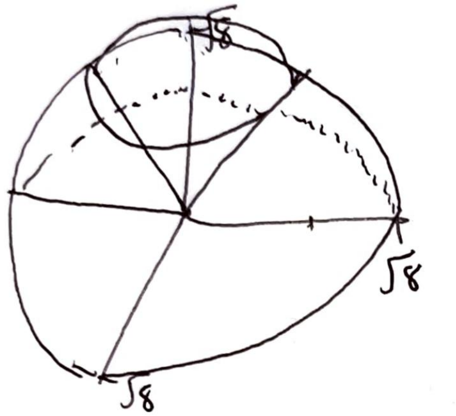
$= \sqrt{t^2 + \frac{1}{2} + \frac{1}{16t^2}} =$

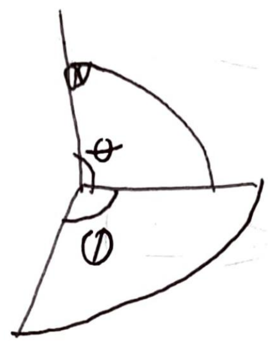
Simplify the square root

5 Arc length 10 / 10

✓ **- 0 pts** **Correct**

6. Find the volume of the region above the xy plane inside the hemisphere $x^2+y^2+z^2=8$ with $z \geq 0$ and outside the cone $z=\sqrt{x^2+y^2}$.





Find $\iiint_E 1 \, dV$

$0 \le \theta \le 2\pi$

$\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

$0 \le \rho \le \sqrt{8}$

$$\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^{\sqrt{8}} 1 \quad \rho^2 \sin\theta \, d\rho\, d\theta\, d\phi$$

$$\frac{\rho^3}{3}\sin\theta \Big]_0^{\sqrt{8}} = \frac{\sqrt{8}^3}{3}\sin\theta$$

$$\int_0^{2\pi}\int_{\pi/4}^{\pi/2} \frac{\sqrt{8}^3}{3}\sin\theta \, d\theta\, d\phi$$

$$-\frac{\sqrt{8}^3}{3}\cos\theta\Big]_{\pi/4}^{\pi/2} = -\frac{\sqrt{8}^3}{3}\left(\cos\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{4}\right)\right) = -\frac{\sqrt{8}^3}{3}\cdot -\frac{\sqrt{2}}{2}$$

$$= 4 \cdot \frac{\sqrt{8}^2}{6}$$

$$= \frac{4 \cdot 8}{6} = \frac{32}{6}$$

$$\int_0^{2\pi} \frac{32}{6}\, d\phi = \frac{32}{6}\cdot 2\pi = \boxed{\frac{32\pi}{3}}$$

6 Volume 10 / 10

✓ **- 0 pts** Correct

7. The Green Monster is a wall 37 feet high. It is 310 feet from the batter. The batter hits the ball at an angle of 60° and a speed of 110 feet per second. On planet Earth acceleration due to gravity is −32 feet per second squared.

(a) Find the acceleration, velocity, and position vector functions for the motion of the ball as a function of time t with the batter at the origin.

$\vec{a}(t) = \langle 0, -32 \rangle$

$\vec{v}(t) = \int \vec{a}\, dt = \langle C, -32t + D \rangle$

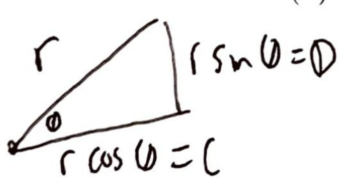
$\vec{x}(t) = \int \vec{v}\, dt = \langle \boxed{Ct} + E, -16t^2 + Dt + F \rangle$

$x(0) = $ origin, $E, F = 0$

$\vec{x}(t) = \langle \boxed{Ct}, -16t^2 + Dt \rangle$

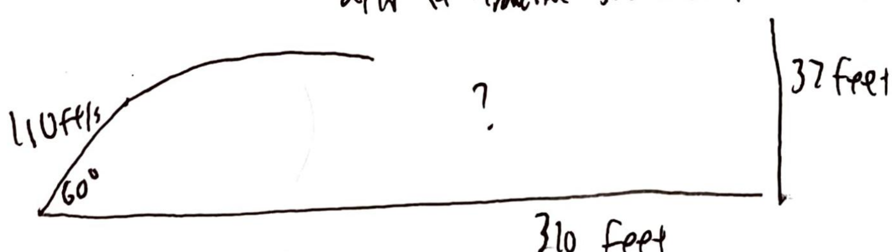
(b) Does the ball go over the Green Monster? Explain. (Decimals okay for this question.)

$\boxed{\text{No}}$, the ball does not go over the GM. The ball will only reach about 28.6385 feet high after it travelled 310 feet horizontally.



$C = 55$

$D = 55\sqrt{3}$



Vertical velocity $= 110 \sin 60 = 110 \cdot \frac{\sqrt{3}}{2} = 55\sqrt{3}$ ft/s

Time until ball hits ground =

$-16t^2 + Dt = 0$, $55\sqrt{3}\, t = 16t^2$, $55\sqrt{3} = 16t$, $t = \frac{55\sqrt{3}}{16} = 5.9539$

Horizontal velocity = 55 ft/s

Horizontal travel $= Ct = 55 \cdot 5.9539 = 327.4645$ feet long

Time for ball to travel 310 feet $\frac{310}{55} = 5.63636$

Vertical displacement $= -16(5.63636)^2 + 55\sqrt{3}(5.63636) = 28.6385$ feet high.

7.1 Acceleration, velocity, position 2 / 5

✓ **- 3 pts Need to fill in all the letters**

7. The Green Monster is a wall 37 feet high. It is 310 feet from the batter. The batter hits the ball at an angle of 60° and a speed of 110 feet per second. On planet Earth acceleration due to gravity is −32 feet per second squared.

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$\vec{a}(t) = \langle 0, -32 \rangle$

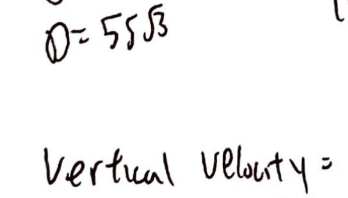
$\vec{v}(t) = \int \vec{a}\, dt = \langle C, -32t + D \rangle$

$\vec{x}(t) = \int \vec{v}\, dt = \langle Ct + E, -16t^2 + Dt + F \rangle$

$x(0) = \text{origin}$, $E, F = 0$

$\vec{x}(t) = \langle Ct, -16t^2 + Dt \rangle$

(b) Does the ball go over the Green Monster? Explain. (Decimals okay for this question.) $\boxed{\text{No}}$, the ball does not go over the GM. The ball will only reach about 28.6385 feet high after it travelled 310 feet horizontally.



$C = 55$

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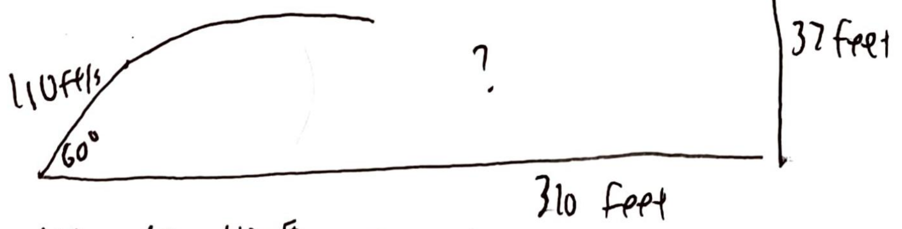

Vertical velocity $= 110 \sin 60 = 110 \cdot \frac{\sqrt{3}}{2} = 55\sqrt{3}$ ft/s

Time until ball hits ground =

$-16t^2 + Dt = 0$, $55\sqrt{3}\,t = 16t^2$, $55\sqrt{3} = 16t$, $t = \frac{55\sqrt{3}}{16} = 5.9539$

Horizontal velocity = 55 ft/s

Horizontal travel $= Ct = 55 \cdot 5.9539 = 327.465$ feet long

Time for ball to travel 310 feet $\frac{310}{55} = 5.63636$

Vertical displacement $= -16(5.63636)^2 + 55\sqrt{3}\,(5.63636) = 28.6385$ feet high

7.2 What does the ball do? **5 / 5**

✓ **- 0 pts** Correct