

# Hw 6 Counting

Paul Yang

6.1.

3. a.  $4^{10}$   
b.  $5^{10}$

7.  $26^3$  become  
26 letters  
and 3 slots

21. 7 nums  
a. 56, 63, 70, 77,  
84, 91, 98  
b. <sup>5 nums</sup> 55, 66, 77, 88,  
99.  
c. 1 num 77

28.

Sum rule

let  $a$  = num ways for 3 dig, 3 letters

let  $b$  = num ways for 3 letters, 3 dig

$a = 10 \times 10 \times 10 \times 26 \times 26 \times 26$

$b = 26 \times 26 \times 26 \times 10 \times 10 \times 10$

$a = b$

answer =  $(a + b)$

35.

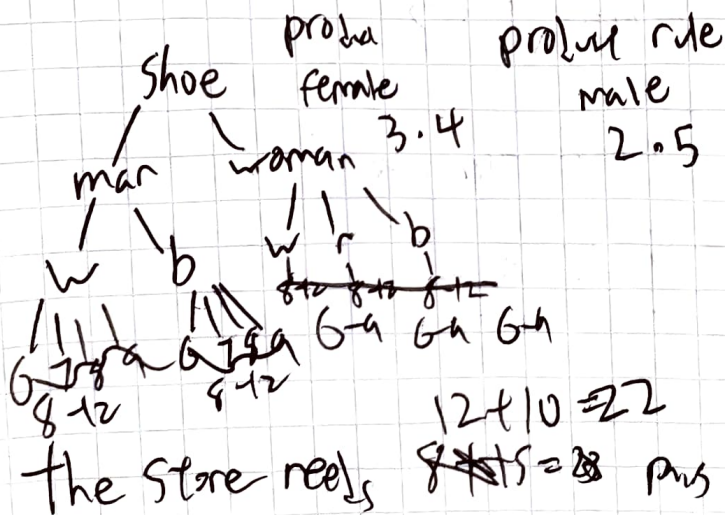
a. 0

b.  $5!$

c.  $6!$

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$   
 $= \frac{7!}{2!}$

69.



Dom: Yrs  
6, 2

3.  
a. 3 socks  
b. 14 socks

8, 12, 16, 20, 24, 28,

18.

a. If a class  
has  $\geq 5$  students  
this is true.

~~If not~~, it must  
have  $< 5$  students.

The ~~smallest~~ <sup>closest</sup> number before 5  
is 4.

If there are 4 males, there must  
be  $9 - 4 = 5$  females.

A B C D E

5. set of mod by 4  
 $\{0, 1, 2, 3\}$

set of 5 numbers

$\{a_1, a_2, a_3, a_4, a_5\}$

Since this function is  
onto, there must exist  
a remainder with  $\geq 2$   
ints.

7.

$a_1, a_2, a_3, \dots, a_{n-1}$

We know that

$a_i \bmod n$   
is distinct for  
the values of  
the indices.

Since there  
are  $N$  values of  
 $\bmod$ , and  
 $N$  numbers in set,  
the function  
is bijective.

23.



Daniel Kang

6.3

3. b.

123	134	145
124	135	
125		
234	245	
235		
345		

a.

123	132	213	312
124	142	214	412
125	152	215	512
134	143	314	413
135	153	315	513
145	154	415	514

5.

a

$$6_03 \cdot 3! = 720/6 = 120$$

$$b. 6_05 \cdot 5! = 720/1 = 720$$

$$c. 8_01 \cdot 1! = 40320/5040 = 8$$

$$6. 5_01 = 5$$

$$b. 5_03 = \frac{4 \cdot 5}{2} = 10$$

$$c. 8_04 = \frac{5 \cdot 6 \cdot 7 \cdot 8}{2 \cdot 3 \cdot 4} = 70$$

NB

Since we can

ignore this constant and

Realize the 2 ways that

this is possible are:

ABAB...

BABAB...

So...

$$2 \cdot (N! \cdot N!)$$

21

$$a. 4! = 120$$

$$b. 3! \cdot 4 = 24$$

$$c. 120$$

$$d. 24$$

$$e. 6$$

$$f. 0$$

30.

$$a. C(16, 5)$$

$$= C(9, 5)$$

$$= 424$$

$$b. 4242$$

$$= C(7, 5)$$

$$= 4221$$

Find Yang

$$\begin{array}{r}
 x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6 \\
 \begin{array}{ccccccc}
 & & & & 1 & & \\
 & & & 1 & & & \\
 & & 1 & & & & \\
 & 1 & & & & & \\
 & & 1 & & & & \\
 & & & 1 & & & \\
 & & & & 1 & & \\
 & & & & & 1 & \\
 & & & & & & 1
 \end{array}
 \end{array}$$

1.

Theorem

$$(x+y)^n = \sum_{i=0}^n \binom{n}{i} x^{n-i} y^i$$

$$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$\binom{4}{0}, \binom{4}{1}, \binom{4}{2}, \dots, \binom{4}{4}$$

7.  $2^{10} \cdot -1 \cdot 92378$

8.  $\binom{17}{8} \cdot 3^8 \cdot 2^n$

13.

$$1 \ 9 \ 36 \ 84 \ 126 \ 126 \ 84 \ 36 \ 9 \ 1$$

16.

Show that if ~~n > 1~~,  $n > 1$ , then

$$\binom{n}{n/2} \geq 2^n/n$$

idk

27

Prove:

$$\sum_{k=0}^r \binom{n+k}{k} = \binom{n+r+1}{r}$$

$n+r+1$

$$\begin{array}{c}
 \swarrow \quad \searrow \\
 \binom{n+r}{r} \quad \binom{n+r}{r-1}
 \end{array}$$

David 1/2

7.1

$$\begin{aligned} AA &= \frac{1}{4} \\ AB &= \frac{1}{4} \\ BA &= \frac{1}{4} \\ BB &= \frac{1}{4} \end{aligned} \quad = \frac{1}{2}$$

7.

$$A \cdot A \cdot A \cdot A \cdot A \cdot A = A^6 = \frac{1}{64}$$

13

$$\begin{aligned} &C(4,1) + \\ &C(4,2) + \\ &C(4,3) + \\ &C(4,4) \end{aligned} \quad \cdot C(48,4) = 1 + 4 + 6 + 4 = 15$$

20,

$$4 \cdot \frac{1}{2598960}$$

$$194580 - 15 = 2918700$$

25a

$$\frac{1}{5006} = \frac{1}{15840700}$$

35.

- a.  $18/38$
- b.  $(18/38)^2$
- c.  $2/38$
- d.  $(36/38)^5$
- e.  $6/38 \cdot 32/38$