1.

$y'' - y' - 30y = 0$ $\quad y = e^{rx}$

$y' = re^{rx}$

$y'' = r^2 e^{rx}$

$r^2 e^{rx} - re^{rx} - 30e^{rx} = 0$

$r^2 - r - 30 = 0$

$(r+5)(r-6)$ so $r = 6, -5$

General solution is $y = Ce^{6x} + De^{-5x}$

2.

$y'' + 12y' + 36y = 0$

$r^2 + 12r + 36 = 0$

$(r+6)^2$, $r = -6$ → now reduce order :)

$y_1 = e^{-6x}$

$y_2 = uy_1$, ue^{-6x}

$y' = u'e^{-6x} - 6ue^{-6x}$

$y'' = u''e^{-6x} - 6u'e^{-6x} - 6u'e^{-6x} + 36ue^{-6x}$

$u''e^{-6x} - 12u'e^{-6x} + 36ue^{-6x} + 12u'e^{-6x} - 72ue^{-6x} + 36ue^{-6x} = 0$

$u''e^{-6x} = 0$

$u'' = 0$

u can be like... anything! but we need Lin Ind sols

so we choose $u = x$, so $y = Ce^{-6x} + Dxe^{-6x}$:)

3. $y'' - 8y' + 17y = 0$

$r^2 - 8r + 17 = 0$, $r = 4+i, 4-i$

For 2 distinct complex roots where $r_1 = a + iB$, $r_2 = a - iB$

The gen sol is $y = e^{at}(C\cos(Bx) + D\sin(Bx))$

$y = e^{4x}(C\cos(x) + D\sin(x))$

6. $y''' + 3y'' + 3y' + y = 0$

$r^3 + 3r^2 + 3r + 1 = 0$

$(r+1)^3 = 0$, $r = -1$