AP Calculus BC
Stewart Chapter 2 Test (Limits)
Fall 2020

Name: David Yang

Date: 9/11/2020 Period: 4

Free Response Section: *NO CAS Calculator Permitted.*
You have the remainder of the period to complete this section.
Once you submit your Free Response Section, you will not be allowed to revisit it.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. To be eligible for partial credit, methods, reasoning, and conclusions should be presented clearly.

- Justifications require that you give mathematical (non-calculator) reasons. Students should use complete sentences in responses that include explanations or justifications.

ALL LIMITS MUST BE DETERMINED ANALYTICALLY!
No use of L'Hopital's Rule or Differentiation Rules are permitted.

1. $\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}\cdot\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$

$$=\lim_{x\to 0}\frac{4+x-4}{x(\sqrt{4+x}+2)}$$

$$=\lim_{x\to 0}\frac{x}{x(\sqrt{4+x}+2)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{4+x}+2}$$

$$=\frac{1}{\sqrt{4+0}+2}$$

$$=\frac{1}{2+2}$$

$$=\boxed{\frac{1}{4}}$$

2. $\lim_{x\to 0}\frac{x\cos(x)}{|x|}$

$$\lim_{x\to 0}\frac{x\cos x}{|x|}=\lim_{x\to 0^-}\frac{x\cos x}{|x|}=\lim_{x\to 0^+}\frac{x\cos x}{|x|}$$

$$\lim_{x\to 0^+}\frac{x\cos x}{|x|}=\left(\lim_{x\to 0^+}\frac{x}{|x|}\right)\left(\lim_{x\to 0^+}\cos x\right)$$

$$=1\cdot 1=1$$

$$\lim_{x\to 0^-}\frac{x\cos x}{|x|}=\left(\lim_{x\to 0^-}\frac{x}{|x|}\right)\left(\lim_{x\to 0^-}\cos x\right)$$

$$=-1\cdot 1=-1$$

$$\lim_{x\to 0^-}\neq\lim_{x\to 0^+}$$

$$\downarrow$$

$$\lim_{x\to 0}=\text{DNE}$$

3. $\lim\limits_{x\to\frac{\pi}{4}} \frac{1-\tan(x)}{1-\cos(x)}$

$$\lim_{x\to\frac{\pi}{4}} \frac{1-\tan(x)}{1-\cos(x)}$$

$$= \frac{1-\tan\left(\frac{\pi}{4}\right)}{1-\cos\left(\frac{\pi}{4}\right)}$$

$$= \frac{1-1}{1-\cos\left(\frac{\pi}{4}\right)}$$

$$= \frac{0}{1-\frac{\sqrt{2}}{2}} = 0$$

4. $\lim\limits_{x\to 0} \frac{3x(x+1)}{\sqrt{x^4+2x^2+2}}$

$$\lim_{x\to 0} \frac{(3x)(x+1)}{\sqrt{x^4+2x^2+2}}$$

$$= \frac{(3\cdot 0)(0+1)}{\sqrt{0^4+2\cdot 0^2+2}}$$

$$= \frac{0\cdot 1}{\sqrt{2}}$$

$$= \frac{0}{\sqrt{2}}$$

$$= 0$$

5. $\lim_{h\to 2}\frac{2(h-3)^2-h}{h-2}$

$$\lim_{h\to 2}\frac{2(h^2-6h+9)-h}{h-2}$$

$$=\lim_{h\to 2}\frac{2h^2-13h+18}{h-2}$$

$$=\lim_{h\to 2}\frac{(2h-9)(h-2)}{h-2}=\lim_{h\to 2}2h-9=4-9=-5$$

6. $\lim_{h\to 0}\frac{\frac{1}{h-4}+\frac{1}{4}}{h}$

$$\lim_{h\to 0}\frac{\frac{4}{4(h-4)}+\frac{h-4}{4(h-4)}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{h}{4(h-4)}}{h}$$

$$=\lim_{h\to 0}\frac{1}{4(h-4)}=\frac{1}{4(-4)}=\frac{1}{-16}$$

7. $\lim_{x\to 0}\frac{\sin(x)-\cos(x)}{\cot(x)-1}$

$$\frac{\sin(0)-\cos(0)}{\cot(0)-1}\quad\frac{-1}{\cot(0)-1}=\frac{-1}{\infty}=0$$

8. $\lim_{x\to 0}\frac{7(3^x)-3^{2x}}{1-3^x}$

$$\left(\lim_{x\to 0}\frac{1}{1-3^x}\right)\left(\lim_{x\to 0}7\cdot 3^x-3^{2x}\right)$$

$$=\frac{1}{0}\cdot 6$$

$$=\text{DNE}$$

9. Explain why the function is not continuous using limits.

$$f(x)=\begin{cases}3-x & x<2\\ \frac{x}{2}+1 & x\ge 2\end{cases}$$

The function's pieces are at $x<2$ an $x\geq 2$. This means that at $x=2$, if $\lim_{x\to 2}f(x)=f(2)$, then it is continuous.

$\lim_{x\to 2}f(x)$ exists if $\lim_{x\to 2^-}f(x)=\lim_{x\to 2^+}f(x)$.

$$\lim_{x\to 2^-}f(x)=3-x=1\qquad \lim_{x\to 2^+}f(x)=\frac{x}{2}+1=2$$

Since $\lim_{x\to 2^-}f(x)\neq\lim_{x\to 2^+}f(x)$, $\lim_{x\to 2}f(x)=\text{DNE}$, and $f(2)=2$. $2\neq\text{DNE}$ so it is not continuous

10. Use the Intermediate Value Theorem to prove that the function $g(x) = x - 4\cos(x)$ has a zero in the interval $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$. Your response must demonstrate that the conditions of the hypothesis are met, state "...by the Intermediate Value Theorem...", and have the conclusion of the theorem tailored to the conditions of the exercise.

$g(x) = x - 4\cos x$ is a continuous function. In the interval $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right)$, $g\left(\frac{\pi}{3}\right) \approx -.95$ and $g\left(\frac{2\pi}{3}\right) \approx 4.09$.

By the Intermediate Value Theorem, this means that for all y in the range of $\left[g\left(\frac{\pi}{3}\right), g\left(\frac{2\pi}{3}\right)\right]$, we must have some x between $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right)$ such that $g(x) = y$.

Therefore, we must have an x that satisfies $g(x) = 0$ between $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right)$.