$f''(x)$

+   −   +

$-\infty$ $-.887$ 4.887 ∞

1. [14 points] Given $f(x)=\frac{1}{4}x^4-2x^3-\frac{13}{2}x^2+42x$ determine the following:

- ✓ The intervals on which f is increasing.
- ✓ The intervals on which f is decreasing.
- ✓ The location(s) at which f has a relative minimum or maximum, if any.
- ✓ The intervals on which f is concave up.
- ✓ The intervals on which f is concave down.
- ✓ The locations at which f has a point(s) of inflection, if any.

Justify all your responses using Calculus. Your evidence must include sign charts.

$f'(x)= x^3-6x^2-13x+42$

$f'(x)=0$ ~~and~~ at $x=-3,2,7$ $f''(x)=3x^2-12x-13$

$f'(x)$ −   +   −   +

$-\infty$ -3 2 7 ∞

~~f is decreasing on the intervals (-∞,-3) and (2,7)~~

~~f is increasing on the intervals (-3,2) and (7,∞)~~

~~f has relative minimums at x=-3,7 and relative maximum at x=2~~

~~f is increasing on ...~~

f has an inflection point at $x=-.887$ and $x=4.887$ because f'' changes sign at $x=-.887$ and $x=4.887$.

f is increasing on $(-3,2)$ because f' is positive on $(-3,2)$.

f is increasing on $(7,\infty)$ because f' is positive on $(-3,2)$.

f is decreasing on $(-\infty,3)$ because f' is negative on $(-\infty,3)$

f is decreasing on $(2,7)$ because f' is negative on $(2,7)$.

f has a relative min at $x=-3$ and 7 because f' changes sign from negative to positive at $x=-3$ and 7.

f has a relative max at $x=2$ because f' changes sign from positive to negative at $x=2$.

f is concave up on $(-\infty,-.887)$ and $(4.887,\infty)$ because f'' is positive on $(-\infty,-.887)$ and $(4.887,\infty)$.

f is concave down on $(-.887,4.887)$ because f'' is negative on $(-.887,4.887)$

2. [3 points] Given $f(x)=\cos(x)$ use the line tangent to $f(x)$ at $x=\frac{7\pi}{6}$ to estimate the value of $f\left(\frac{4\pi}{3}\right)$.

$f'(x) = -\sin x$ $\quad y - y_1 = m(x - x_1)$

$f'\left(\frac{7\pi}{6}\right) = 0.5$

$y - \left(-\frac{\sqrt{3}}{2}\right) = 0.5\left(x - \frac{7\pi}{6}\right)$

$y = \frac{x}{2} - \frac{7\pi}{12} - \frac{\sqrt{3}}{2}$

$f(x) = y$

$f\left(\frac{4\pi}{3}\right) = \frac{8\pi}{12} - \frac{7\pi}{12} - \frac{\sqrt{3}}{2}$

$f\left(\frac{4\pi}{3}\right) = \boxed{\frac{\pi}{12} - \frac{\sqrt{3}}{2}}$

3. [4 points] Explain how the second derivative of f on the interval $\left(\frac{7\pi}{6}, \frac{4\pi}{3}\right)$ can be used to determine if the value determined in question 2 is an underestimate of $f\left(\frac{4\pi}{3}\right)$ or an overestimate of $f\left(\frac{4\pi}{3}\right)$.

The second derivative of f on the interval $\left(\frac{7\pi}{6}, \frac{4\pi}{3}\right)$ can be used to determine concavity on intervals. If f(x) is concave up in the interval $\left(\frac{7\pi}{6}, \frac{4\pi}{3}\right)$ then the linear approximation is an underestimate. Since $f''(x) > 0$ on the interval $\left(\frac{7\pi}{6}, \frac{4\pi}{3}\right)$, it is concave up. Thus, the estimated value is an underestimate.

4. Evaluate the limit using L''Hôpital's Rule. Show all your steps. $\lim_{x\to 0}(x+1)^{\frac{\ln(2)}{x}}$

$y = \lim_{x\to 0}(x+1)^{\frac{\ln 2}{x}}$

$\lim_{x\to 0}(x+1)^{\frac{\ln 2}{x}} = 2$

$\ln(y) = \lim_{x\to 0}\left(\frac{\ln 2}{x}\cdot \ln(x+1)\right)$

$\ln(y) = \ln(2)\cdot \lim_{x\to 0}\frac{1}{x}(\ln(x+1))$

$\ln(y) = \ln(2)\cdot \lim_{x\to 0}\frac{\frac{1}{x+1}}{1}$

$\ln(y) = \ln(2)\cdot \lim_{x\to 0}\frac{1}{x+1}$

$\ln(y) = \ln(2)\cdot 1$

$y = 2$

5. [8 points] The total cost, including manufacturing, packaging, and distribution of an electronic calculator is \$21. If the machine sells at x dollars each, then the total number of machines sold, n, is given by $n = \dfrac{200}{x-21} + 10(50-x)$. What selling price x will maximize profit?

Hint: Profit $=$ (Total revenue for n machines) $-$ (Total cost for n machines)

~~domain~~ $\cancel{x = (21, 50)}$

$\cancel{P(x) = \frac{200}{x-21} + 10(50-x)}$

$\cancel{P'(x) = -\frac{200}{(x-21)^2} - 10}$

$P = \left(\frac{200x}{x-21} + 10x(50-x) - 21\right.$

$P'(x) = \frac{-200x}{(x-21)^2} - 10x + 10(50-x) + \frac{200}{x-21} - 21$

domain $x = (0, 21)$ $P'(x) = 0$ or DNE at $15.894, 21$

~~This is not in domain so we cannot~~

Using EVT

$P(0) = 0$

$\cancel{P(15.894) = 4464.4719}$

~~P(21) = undefined~~

P(x) + −

0 15.894 ∞

Thus there is a max at 15.894, as P goes from positive to

x = 15.894 for best profit

6. [7 points] Sketch a graph of the continuous function f on the interval $0 < x < 8$ that satisfies the following criteria:

x	$x<2$	$x=2$	$2<x<4$	$x=4$	$4<x<6$	$x=6$	$x>6$
$f(x)$		1		4		7	
$f'(x)$	$(-)$		$(+)$		$(+)$		$(-)$
$f''(x)$	$(+)$		$(+)$		0		$(+)$

If the graph has a horizontal tangent, label the coordinate with "HT" and do your best to make your graph appear as if it has a horizontal tangent.
If the graph has a vertical tangent, label the coordinate with "VT" and do your best to make your graph appear as if it has a vertical tangent.
If the graph has a sharp corner, label the coordinate with "SC" and do your best to make your graph appear as if it has a sharp corner.
Your work must be done on the graph provided below. If you do not print this exam, this exercise must be done on graph paper, with axes labeled and scaled.