Midterm 3 (Short Answer)

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1.

a. procedure $(a_1, a_2, \ldots, a_n$: integers with $n \geq 3)$

for $i := 1$ to 3
  if $(a_{n-2} > a_{n-1})$ then interchange a_{n-2} and a_{n-1}
  if $(a_{n-1} > a_n)$ then interchange a_{n-1} and a_n

$\{a_{n-2}, a_{n-1}, a_n$ is in increasing order $\}$

b.

$i=1$	$i=2$	$i=3$
7 8 3 4 7 9 5	7 8 3 4 7 5 9	7 8 3 4 5 7 9
7 8 3 4 7 9 5	7 8 3 4 5 7 9	7 8 3 4 5 7 9
7 8 3 4 7 5 9	7 8 3 4 5 7 9	7 8 3 4 5 7 9

The first line for each i is the state before any if statements are executed. The next line is the next if statement

2.

a. $\frac{6n^3-n+3}{2n^2-1} \leq C \cdot n$ for $\cancel{k \geq}$? $n \geq k$

We can observe that when taking leading terms, we get $\frac{6n^3}{2n^2} = 3n$.

Keeping into account the $2n^2-1$ term, we can bound $C=4$ and $k>1$.

$f(2) = 7 < 4 \cdot 2$
$f(3) = 9.529\ldots < 4 \cdot 3$
...

$$\lim_{n \to \infty} \frac{6n^3-n+3}{2n^2-1} = \frac{6n^3}{2n^2} = 3n$$

2h.

$$\frac{6n^3-n+3}{2n^2-1} + \log(n) \le C \cdot n \quad \text{for } n \ge k$$

$O(n)$, $C=4$ $K=2$ because the $\log(n)$ ~~factor~~ term doesn't change the leading term, which is $\frac{6n^3-n+3}{2n^2-1}$. Since Big-O is only affected by leading terms, the $(+\log(n))$ term does nothing to change the $O(n)$ status.

4. $O(n^2)$. This is because the for-loop's k-variable is not being modified outside the loop. Therefore, the for loop will go from $k=n$ to $k=n^2$.

5. Let $T(n)$ be that (n^3+3n^2+2n) is divisible by 3.

Basis Step: $T(1) = 6$ = true. ~~$T(2) = 24$ $T(3) = 60$~~

Inductive Step: Assuming $T(n)$ is true, $T(n+1)$ is equal to $(n+1)(n+2)(n+3)$ and true.

We know that $(n+1)(n+2)(n+3) = n^3+6n^2+11n+6$ which is equivalent to $(n^3+3n^2+2n) + 3n^2+9n+6$.

Since $T(n) = n^3+3n^2+2n$ = true, and $3(n^2+3n+2)$ is a multiple of 3, $T(n+1)$ is true.

Conclusion. We know we have finished the inductive and basis steps. For an $n \ge 1$ and $T(n)$ is true, then $T(n+1)$ is true

6. Let $P(n)$ be the statement that postage of n stamps can be formed with 2-cent and 7-cent stamps.

Basis Step: $P(6)$ can be formed with 3 - 2cent stamps
$P(7)$ can be formed with 1 - 7 cent stamp

Inductive Step:
Hypotesis: Assuming $P(d)$ is true for $6 \leq d \leq e$, where e is an integer with $e \geq 7$.

We assume we can form postage of d cents with only 2-cent and 7-cent postage for $6 \leq d \leq e$.

To create postage of $e+1$ cents, we can first assume $P(e-1)$ is true because $7-1 \geq 6$.

Since we know we can create postage of $e-1$ cents, we can make $e+1$ cents by adding a 2-cent stamp.

Thus, if the induction hypothesis is true, $P(k+1)$ is true.

Conclusion:
We have finished the inductive and the basis step. We know $P(n)$ is true for $n \geq 6$.

7.

a.

$f(0) = 0$

$f(n) = 2n + f(n-1)$

I figured this out from $\sum_{1}^{n} n = \frac{(n)(n+1)}{2}$

b.

$f(0) = 0$

$f(1) = 2$

$f(2) = 4 + f(1) = 6$

$f(3) = 6 + f(2) = 10 + f(1) = 12$

$f(4) = 8 + f(3) = 14 + f(2) = 18 + f(1) = 20$

$f(5) = 10 + f(4) = 18 + f(3) = 24 + f(2) = 28 + f(1) = 30$

$f(6) = 12 + f(5) = 22 + f(4) = 30 + f(3) = 36 + f(2) = 40 + f(1) = 42$

$a_6 = 42$