1. Explain what a critical value of f (x) is.

A critical value of f(x) is a value c that lies in the domain of f, such that f'(c)=0 or DNE

2. Explain what a vertical tangent and horizontal tangent are, and why they are of interest in Calculus.

A vertical tangent is when the graph of f has a tangent line that is vertical and the slope of the tangent DNE. Vertical tangents are of interest because they may be locations of an inflection point.

A horizontal tangent is a tangent line that has a slope of 0, and it is horizontal. Horizontal Tangents are of interest because they may be the location of a relative min or max.

3. The derivative of a function f (x), denoted f'(x), can tell you about the visual qualities of the graph of f(x). What visual qualities of f(x) can be determined from solely from the graph of f'(x) ? List all possible qualities of the graph of f(x) that can be determined from the graph of f'(x).

Using only the graph of f'(x), you can see when f(x) is increasing, decreasing, or is not changing. If f'(x) <0, f(x) must be decreasing. If f'(x)> 0, f(x) must be increasing. If f'(x)=0, f(x) is not changing, and has a horizontal tangent at x. At f'(x)=0 or DNE, x is a critical point. Therefore, if f'(x) switches from positive to negative, f(x) is a local maximum, and if f'(x) switches from negative to positive, f(x) is a local minimum.

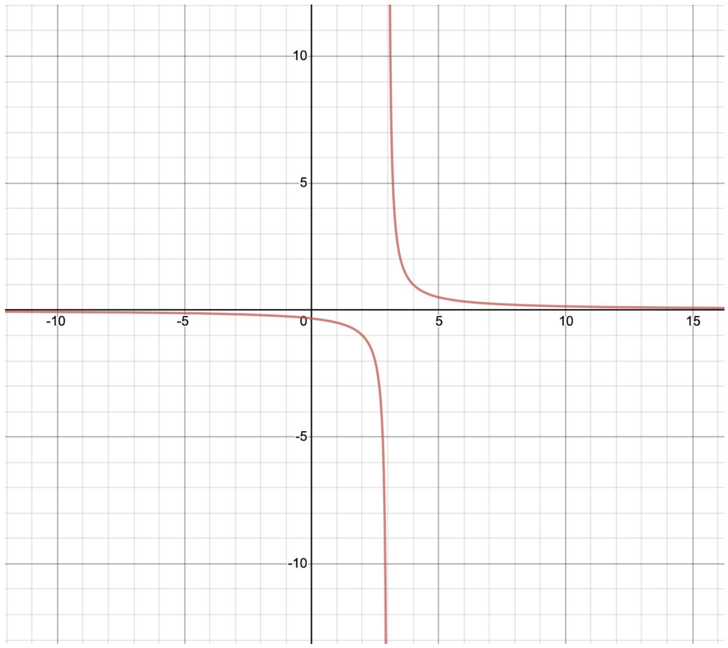
If f’(x) is increasing, f(x) is concave up. If f’(x) is decreasing, f(x) is concave down. At a point x=c, you can determine a point of inflection from f’(c) if f’(c) is increasing and then decreasing, or vice versa.

4. (a) Explain what a point of inflection on the graph of f (x) is visually.
(b) The location of a point of inflection of the graph of f(x) can be identified by using the graph of the first derivative or the graph of the second derivative. Explain how to identify the location of a point of inflection using the graph of f’(x) and the graph of f’’(x) .

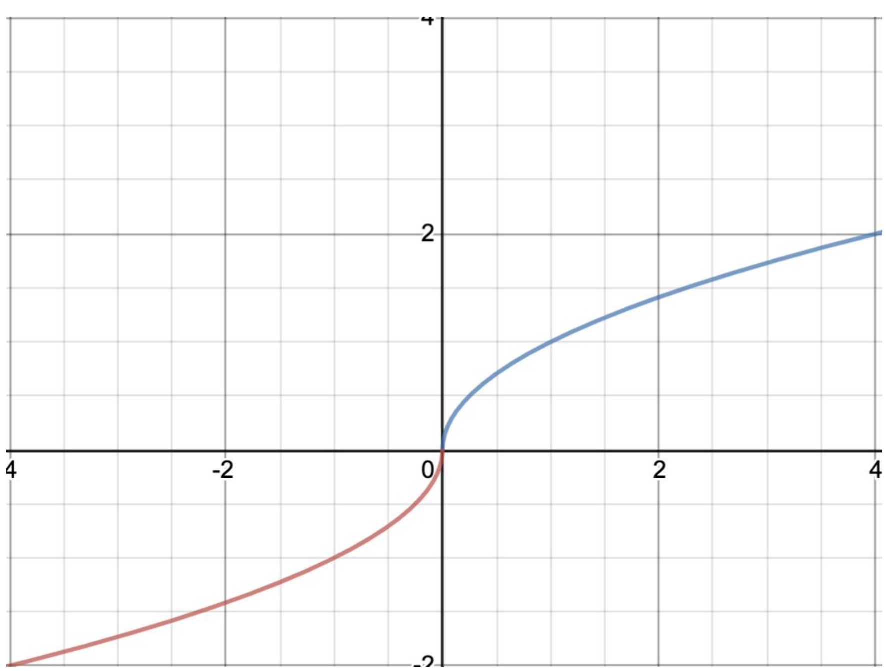
(A): A point of inflection on the graph of f(x) is a point where the graph changes from concave downward to concave upward, or vice versa.
(B):
Using the graph of f’(x), you can find the point of inflection of the graph f(x) when f’(x) goes from increasing to decreasing, or vice versa.
Using the graph of f’’(x), you can find the points of inflection of the graph f(x) by identifying points when f’’(x) crosses the x axis/ changes signs(positive to negative, vv). Candidates for inflection points can be found when f’’(x)=0, but f’’(x) must cross the x axis in order for (x,f(x)) to be an inflection point.

5. The following statement is not true: “If f’(x) does not exist at x=c, then the graph of f(x) has a sharp corner at x=c .” Provide two counterexamples as to why this is not necessarily true.
This statement is not necessarily true because:

1. f(x) could be undefined. For a function to be differentiable at x=c, the function must be continuous at x=c. The below graph shows the function f not defined at x=3.



2. f(x) may have a vertical tangent, where the slope is undefined. The below graph shows a vertical tangent at x=0.



6. To find a relative minimum or relative maximum, the following methods can be used:

• The Extreme Value Theorem

• The First Derivative Test

• The Second Derivative Test

(a) State the conditions necessary for each method to be applied.

(b) When solving an optimization exercise, when must EVT be used? When must First/Second Derivative Test be used?

• The Extreme Value Theorem requires that a function must be continuous on a closed interval.

• The First Derivative Test requires that a function *f* must be differentiable at a point x=c, that x is in the domain of f, and f'(c)=0 or DNE.

• The Second Derivative Test requires that a function *f* must be twice differentiable at a point x=c, that x is in the domain of f, (f'(c)=0) and (f''(c) must not be 0) .

When solving an optimization exercise, EVT must be used when the feasible domain is bounded. The First/Second Derivative Test must be used if the feasible domain is unbounded.

7. A line tangent to the graph of f (x) is constructed at (c, f(c)). The tangent line is used to estimate the value of f(x) at x=d. Explain how the second derivative can be used to determine if the tangent line approximation for f(d) is and underestimate or an overestimate for the true/exact value of f(d).

Denote t(d) as the tangent line approximation for f(x) at x=d.

t(d) is an underapproximation of f(d) if f''(x)>0 for all x between x=c and x=d. Concave Up)

t(d) is an overapproximation of f(d) if f''(x)<0 for all x between x=c and x=d. (Concave Down)