1. The difference between $\int f(x)dx$ and $\int_a^b f(x)\,dx$ is that $\int f(x)dx$ is an indefinite integral and produces a function. $\int_a^b f(x)\,dx$ is a definite integral and finds a number, the area under the curve of f(x).

2. When f(x) does not change sign on the interval[a,b], then the area under the curve of f(x) within the interval [a,b] will always be either positive or negative. $\int_a^b f(x)\,dx$ integrates from left to right, which means the area under the curve is accumulating forwards. So then if f(x) is positive on the interval [a,b], then $\int_a^b f(x)\,dx$ will be positive. If f(x) is negative on the interval [a,b], then $\int_a^b f(x)\,dx$ will be negative.
$\int_b^a f(x)\,dx$ integrates from right to left, which means the area under the curve is accumulating backwards. So then if f(x) is positive on the interval [a,b], then $\int_b^a f(x)\,dx$ will be negative. If f(x) is negative on the interval [a,b], then $\int_b^a f(x)\,dx$ will be positive.

3. When antidifferentiating a function f(x), you must add +C to the end result because when constants are differentiated they are 0, which means the antiderivative of f(x) could have any constant.
4. To find f(b) given f(a) and f'(x), integrate f'(x) from a to b. Then add that definite integral to f(a). $f(a) + \int_a^b f'(x)\,dx$
5.

Part 1

a. $\int_0^8 T'(x)\,dx$ is the net change of the T(x) from x=0 to x=8.

$\int_0^8 T'(x)\,dx$ = 55 - 100 = -45. The heated end of the metal wire is 45 degrees celsius hotter than the opposite end.

Analysis: $\int_0^8 T'(x)\,dx = \triangle cm * \int_0^8 (\triangle^\circ C/\triangle cm) = 1 * \int_0^8 \triangle^\circ C$ = $\sum \triangle^\circ C$

b. $\int_{30}^{60} |v(t)|\,dt$ is the distance traveled from time t=30 to t=60. Using a trapezoidal approximation, $\int_{30}^{60} |v(t)|dt$ is

$\frac{1}{2}\left(5 * (14+10) + 15 * (10+0) + 10 * (0+10)\right)$ = 185 ft

Analysis:

$\int_{30}^{60} |v(t)|dt = \triangle time * \int_{30}^{60} |(\triangle position/\triangle time)| = 1 * \int_{30}^{60} |\triangle dist| = \sum |\triangle dist|$

$\int_0^{30} a(t)\,dt$ is the change in velocity from time 0 to time 30.

$\int_0^{30} a(t)\,dt$ = $\int_0^{30} v'(t)\,dt = v(30) - v(0) = 6ft/s$

$\int_0^{30} a(t)\,dt = \triangle time * \int_0^{30} \triangle velocity/\triangle time = 1 * \int_0^{30} \triangle velocity = \sum \triangle velocity$

Part 2

a. The diagram shown below shows a graph with the domain of [0,10]. To find $\int_0^{10} f(t)\,dt$, you can break this integral into multiple "pieces" using the linearity rule. In this case, $\int_0^5 f(t)\,dt$ is a mirror of $\int_5^{10} f(t)\,dt$, so we only need to calculate $2\int_0^5 f(t)\,dt$. The interval [0,5] has a negative area from [0,1], a positive area from [1,4], and a negative area from [4,5].
   $\int_0^1 f(t)\,dt = -1,\ \int_1^4 f(t)\,dt = 4,\ \int_4^5 f(t)\,dt = -1,\ so\, 2\int_0^5 f(t)\,dt = 2*(-1+4+-1) = 4.$

b. AP Question
   A. g(4) = $\int_0^4 f(t)\,dt = 3$, $g'(4) = f(4) = 0$, $g''(4) = f'(4) = -4/2$
   B. g(1) has a relative minimum at x=1 because g'(x) = f(x) goes from negative to positive at x=1.
   C. $g(10) = 4$
   Slope of tangent = g'(108) = f(108) = f(108 mod 5) = f(3) = 2.
   Point at x=108 = $\int_0^{105} f(t)\,dt + \int_{105}^{108} f(t)\,dt$ = 42 + 2 = 44.
   2*108 + b = 44. 216 + b =44. 216-44= -b. b= -172.
   Therefore the tangent line of g when x= 108 is y = 2x -172.