MATH-253-YJH-CRN82680 Exam 1

David Yang

TOTAL POINTS

66.5 / 70

QUESTION 1

1 Equation of a plane **10 / 10**

✓ - 0 pts Correct

QUESTION 2

2 Line of intersection of planes **10 / 10**

✓ - 0 pts Correct

QUESTION 3

Partial derivatives 10 pts

3.1 f_x **2 / 2**

✓ - 0 pts Correct

3.2 f_y **2 / 2**

✓ - 0 pts Correct

3.3 f_xx **2 / 2**

✓ - 0 pts Correct

3.4 f_yy **2 / 2**

✓ - 0 pts Correct

3.5 f_xy **2 / 2**

✓ - 0 pts Correct

QUESTION 4

4 Equations of a line **10 / 10**

✓ - 0 pts Correct

QUESTION 5

5 Left turn problem **10 / 10**

✓ - 0 pts Correct

QUESTION 6

Dot and cross products 10 pts

6.1 Dot product **3.5 / 5**

✓ - 1.5 pts Bad algebra

6.2 Cross product magnitude **5 / 5**

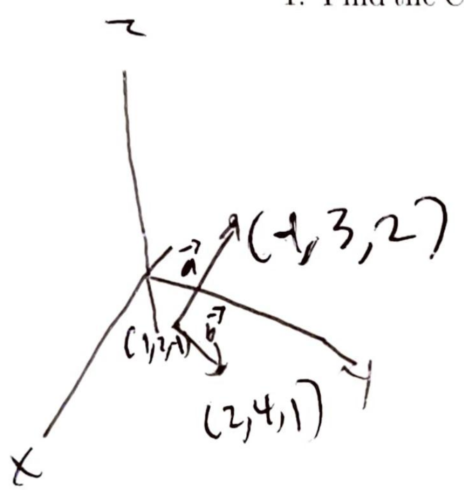
✓ - 0 pts Correct

QUESTION 7

7 Implicit partial derivative **8 / 10**

✓ - 2 pts Derivative error

1. Find the Cartesian equation of the plane through the points $(1, 2, -1)$, $(2, 4, 1)$, and $(-1, 3, 2)$



$$\vec{a} = \langle -1-1,\ 3-2,\ 2--1\rangle = \langle -2, 1, 3\rangle$$

$$\vec{b} = \langle 2-1,\ 4-2,\ 1--1\rangle = \langle 1, 2, 2\rangle$$

$$\vec{N} = \vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ -2 & 1 & 3 \\ 1 & 2 & 2 \end{vmatrix} = (2-6)i - (-4-3)j + (-4-1)k$$

$$= -4i + 7j - 5k = \langle -4, 7, -5\rangle$$

$$\text{Plane} = \vec{N} \cdot \langle x-1,\ y-2,\ z--1\rangle = 0$$

$$= \langle -4, 7, -5\rangle \cdot \langle x-1,\ y-2,\ z+1\rangle = 0$$

$$= -4(x-1) + 7(y-2) - 5(z+1) = 0$$

$$-4x + 4 + 7y - 14 - 5z - 5 = 0$$

$$\boxed{-4x + 7y - 5z = 15}$$

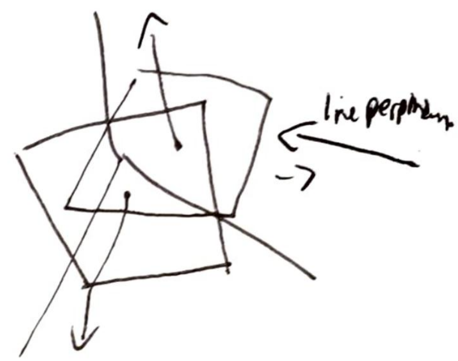
1 Equation of a plane **10 / 10**

✓ **- 0 pts** Correct

2. Two planes have equations $x - 2y - z = 6$ and $2x + y + 2z = 8$ Find parametric equations of the line of intersection.

Solve

~~Reverse~~ Engineer



Need a line perpendicular to the two planes.

By diagram, the $\perp$ line is also $\perp$ to both normal vectors.

Therefore the cross product gives us the direction/magnitude of line.

Then need initial coordinates.

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① $\vec{a} = \vec{N_1} = \langle 1, -2, -1 \rangle$, $\vec{b} = \vec{N_2} = \langle 2, 1, 2 \rangle$

$$\vec{x} = \vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & -2 & -1 \\ 2 & 1 & 2 \end{vmatrix} = (-4-(-1))i - (2-(-2))j + (1-(-4))k$$

$$= -3i - 4j + 5k = \langle -3, -4, 5 \rangle$$

② $x - 2y - z = 6$, $x - 2y - 0 = 6$, $x - 2y = 6$, ③ $2x - 4y = 12$

$2x + y + 2z = 8$, $2x + y + 0 = 8$, $2x + y = 8$, $\underline{-2x + y = 8}$

$-5y = 4$, $y = \frac{-4}{5}$

$x - 2(\frac{-4}{5}) = 6$

$x + \frac{8}{5} = 6$

$x = \frac{30}{5} - \frac{8}{5} = \frac{22}{5}$

$z = 0$.

④ Cartesian $=$

$\langle \frac{22}{5}, \frac{-4}{5}, 0 \rangle + t\langle -3, -4, 5 \rangle$

Parametric

$$\boxed{\begin{aligned} x &= \tfrac{22}{5} - 3t \\ y &= -\tfrac{4}{5} - 4t \\ z &= 5t \end{aligned}}$$

2 Line of intersection of planes 10 / 10

✓ **- 0 pts** Correct

3. $f(x, y) = x\cos(xy)$. Find (simplified):

(a) f_x

$$\frac{\partial f}{\partial x} = \frac{d}{dx}[x]\cdot\cos(xy) + \frac{d}{dx}[\cos(xy)]\cdot x$$

$$= 1\cdot\cos(xy) - y\,\cancel{\cos}\sin(xy)\cdot x$$

$$= \cancel{\cos(xy) - xy\cos} = \boxed{\cos(xy) - xy\sin(xy)}$$

(b) f_y

$$\frac{\partial f}{\partial y} = \frac{d}{dy}[x\cos(xy)] = x\frac{d}{dy}[\cos(xy)]$$

$$= x(-\sin(xy)\cdot x)$$

$$= \boxed{-x^2\sin(xy)}$$

(c) f_{xx}

Let $g(x,y) = f_x$

$$\frac{\partial g}{\partial x} = \frac{d}{dx}[\cos(xy)] - \frac{d}{dx}[xy\sin(xy)]$$

$$= -y\sin(xy) - y\frac{d}{dx}[x\sin(xy)]$$

$$= -y\sin(xy) - y(\sin(xy) + xy\cos(xy)) = \boxed{-2y\sin(xy) - xy^2\cos(xy)}$$

(d) f_{yy}

Let $h(x,y) = f_y$

$$\frac{\partial h}{\partial y} = \frac{d}{dy}[-x^2\sin(xy)] = -x^2\frac{d}{dy}[\sin(xy)]$$

$$= -x^2(x\cos(xy))$$

$$= \boxed{-x^3\cos(xy)}$$

(e) f_{xy}

Let $g(x,y) = f_x$

$$\frac{\partial g}{\partial y} = \frac{d}{dy}[\cos(xy) - xy\sin(xy)] = \frac{d}{dy}[\cos(xy)] - \frac{d}{dy}[xy\sin(xy)]$$

$$= -x\sin(xy) - x\left\{\frac{d}{dy}[y\sin(xy)]\right\} = -x\sin(xy) - x(\sin(xy) + y\cos(xy)\cdot x)$$

$$= \boxed{-2x\sin(xy) - x^2y\cos(xy)}$$

3.1 f_x **2 / 2**

✓ **- 0 pts** Correct

3. $f(x, y) = x\cos(xy)$. Find (simplified):

(a) f_x

$$\frac{\partial f}{\partial x} = \frac{d}{dx}[x]\cdot\cos(xy) + \frac{d}{dx}[\cos(xy)]\cdot x$$

$$= 1\cdot\cos(xy) - y\,\sin(xy)\cdot x$$

$$= \boxed{\cos(xy) - xy\sin(xy)}$$

(b) f_y

$$\frac{\partial f}{\partial y} = \frac{d}{dy}[x\cos(xy)] = x\frac{d}{dy}[\cos(xy)]$$

$$= x(-\sin(xy)\cdot x)$$

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$$\frac{\partial h}{\partial y} = \frac{d}{dy}[-x^2\sin(xy)] = -x^2\frac{d}{dy}[\sin(xy)]$$

$$= -x^2(x\cos(xy))$$

$$= \boxed{-x^3\cos(xy)}$$

(e) f_{xy}

Let $g(x,y) = f_x$

$$\frac{\partial g}{\partial y} = \frac{d}{dy}[\cos(xy) - xy\sin(xy)] = \frac{d}{dy}[\cos(xy)] - \frac{d}{dy}[xy\sin(xy)]$$

$$= -x\sin(xy) - x\frac{d}{dy}[y\sin(xy)] = -x\sin(xy) - x(\sin(xy) + y\cos(xy)\cdot x)$$

$$= \boxed{-2x\sin(xy) - x^2y\cos(xy)}$$

3.2 f_y **2 / 2**

✓ **- 0 pts** Correct

3. $f(x,y) = x\cos(xy)$. Find (simplified):

(a) f_x

$$\frac{\partial f}{\partial x} = \frac{d}{dx}[x]\cdot\cos(xy) + \frac{d}{dx}[\cos(xy)]\cdot x$$

$$= 1\cdot\cos(xy) - y\,\overset{\sin}{\cancel{\cos}}(xy)\cdot x$$

$$= \cancel{\cos(xy)\ xy\cos} = \boxed{\cos(xy) - xy\sin(xy)}$$

(b) f_y

$$\frac{\partial f}{\partial y} = \frac{d}{dy}[x\cos(xy)] = x\frac{d}{dy}[\cos(xy)]$$

$$= x(-\sin(xy)\cdot x)$$

$$= \boxed{-x^2\sin(xy)}$$

(c) f_{xx}

Let $g(x,y) = f_x$

$$\frac{\partial g}{\partial x} = \frac{d}{dx}[\cos(xy)] - \frac{d}{dx}[xy\sin(xy)]$$

$$= -y\sin(xy) - y\frac{d}{dx}[x\sin(xy)]$$

$$= -y\sin(xy) - y(\sin(xy) + xy\cos(xy)) = \boxed{-2y\sin(xy) - xy^2\cos(xy)}$$

(d) f_{yy}

Let $h(x,y) = f_y$

$$\frac{\partial h}{\partial y} = \frac{d}{dy}[-x^2\sin(xy)] = -x^2\frac{d}{dy}[\sin(xy)]$$

$$= -x^2(x\cos(xy))$$

$$= \boxed{-x^3\cos(xy)}$$

(e) f_{xy}

Let $g(x,y) = f_x$

$$\frac{\partial g}{\partial y} = \frac{d}{dy}[\cos(xy) - xy\sin(xy)] = \frac{d}{dy}[\cos(xy)] - \frac{d}{dy}[xy\sin(xy)]$$

$$= -x\sin(xy) - x\left\{\frac{d}{dy}[y\sin(xy)]\right\} = -x\sin(xy) - x(\sin(xy) + y\cos(xy)\cdot x)$$

$$= \boxed{-2x\sin(xy) - x^2y\cos(xy)}$$

3.3 f_xx **2 / 2**

✓ **- 0 pts** Correct

3. $f(x,y) = x\cos(xy)$. Find (simplified):

(a) f_x

$$\frac{\partial f}{\partial x} = \frac{d}{dx}[x]\cdot\cos(xy) + \frac{d}{dx}[\cos(xy)]\cdot x$$

$$= 1\cdot\cos(xy) - y\,\cancel{\cos}\,\sin(xy)\cdot x$$

$$= \cancel{\cos(xy) - xy\cos} = \boxed{\cos(xy) - xy\sin(xy)}$$

(b) f_y

$$\frac{\partial f}{\partial y} = \frac{d}{dy}[x\cos(xy)] = x\frac{d}{dy}[\cos(xy)]$$

$$= x(-\sin(xy)\cdot x)$$

$$= \boxed{-x^2\sin(xy)}$$

(c) f_{xx}

Let $g(x,y) = f_x$

$$\frac{\partial g}{\partial x} = \frac{d}{dx}[\cos(xy)] - \frac{d}{dx}[xy\sin(xy)]$$

$$= -y\sin(xy) - y\frac{d}{dx}[x\sin(xy)]$$

$$= -y\sin(xy) - y(\sin(xy) + xy\cos(xy)) = \boxed{-2y\sin(xy) - xy^2\cos(xy)}$$

(d) f_{yy}

Let $h(x,y) = f_y$

$$\frac{\partial h}{\partial y} = \frac{d}{dy}[-x^2\sin(xy)] = -x^2\frac{d}{dy}[\sin(xy)]$$

$$= -x^2(x\cos(xy))$$

$$= \boxed{-x^3\cos(xy)}$$

(e) f_{xy}

Let $g(x,y) = f_x$

$$\frac{\partial g}{\partial y} = \frac{d}{dy}[\cos(xy) - xy\sin(xy)] = \frac{d}{dy}[\cos(xy)] - \frac{d}{dy}[xy\sin(xy)]$$

$$= -x\sin(xy) - x\frac{d}{dy}[y\sin(xy)] = -x\sin(xy) - x(\sin(xy) + y\cos(xy)\cdot x)$$

$$= \boxed{-2x\sin(xy) - x^2y\cos(xy)}$$

3.4 f_yy 2 / 2

✓ - 0 pts Correct

3. $f(x, y) = x\cos(xy)$. Find (simplified):

(a) f_x

$$\frac{\partial f}{\partial x} = \frac{d}{dx}[x] \cdot \cos(xy) + \frac{d}{dx}[\cos(xy)] \cdot x$$

$$= 1 \cdot \cos(xy) - y\,\cancel{\cos}\,\sin(xy) \cdot x$$

$$= \cancel{\cos(xy) \; xy\cos} = \boxed{\cos(xy) - xy\sin(xy)}$$

(b) f_y

$$\frac{\partial f}{\partial y} = \frac{d}{dy}[x\cos(xy)] = x\frac{d}{dy}[\cos(xy)]$$

$$= x(-\sin(xy) \cdot x)$$

$$= \boxed{-x^2 \sin(xy)}$$

(c) f_{xx}

Let $g(x,y) = f_x$

$$\frac{\partial g}{\partial x} = \frac{d}{dx}[\cos(xy)] - \frac{d}{dx}[xy\sin(xy)]$$

$$= -y\sin(xy) - y\frac{d}{dx}[x\sin(xy)]$$

$$= -y\sin(xy) - y(\sin(xy) + xy\cos(xy)) = \boxed{-2y\sin(xy) - xy^2\cos(xy)}$$

(d) f_{yy}

Let $h(x,y) = f_y$

$$\frac{\partial h}{\partial y} = \frac{d}{dy}[-x^2\sin(xy)] = -x^2\frac{d}{dy}[\sin(xy)]$$

$$= -x^2(x\cos(xy))$$

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(e) f_{xy}

Let $g(x,y) = f_x$

$$\frac{\partial g}{\partial y} = \frac{d}{dy}[\cos(xy) - xy\sin(xy)] = \frac{d}{dy}[\cos(xy)] - \frac{d}{dy}[xy\sin(xy)]$$

$$= -x\sin(xy) - x\frac{d}{dy}[y\sin(xy)] = -x\sin(xy) - x(\sin(xy) + y\cos(xy) \cdot x)$$

$$= \boxed{-2x\sin(xy) - x^2y\cos(xy)}$$

3.5 f_xy 2 / 2

✓ - 0 pts Correct

4. Find parametric equations of the line through $(1, 5, 2)$ that is perpendicular to the plane with equation $x - y + 2z = 5$.

$\vec{N} = \langle 1, -1, 2 \rangle$

$\vec{N}$ is perpendicular to the plane...

$\langle 1, 5, 2 \rangle$ is given as a point on the plane

Cartesian line $= \langle 1, 5, 2 \rangle + t\langle 1, -1, 2 \rangle$

Parametric

$$\boxed{\begin{aligned} x &= 1 + 1t \\ y &= 5 - 1t \\ z &= 2 + 2t \end{aligned}}$$

Uh this was a little bit too simple?

Proof that $\vec{N} = \langle 1, -1, 2 \rangle$. Recall equation of plane $0 = \vec{N} \cdot \langle x - x_0, y - y_0, z - z_0 \rangle$

$0 = N_1(x - x_0) + N_2(y - y_0) + N_3(z - z_0)$, $0 = N_1 x + N_2 y + N_3 z - N_1 x_0 - N_2 y_0 - N_3 z_0$

$N_1 x_0 + N_2 y_0 + N_3 z_0 = N_1 x + N_2 y + N_3 z$

$N_1 x + N_2 y + N_3 z = \text{Constant}$

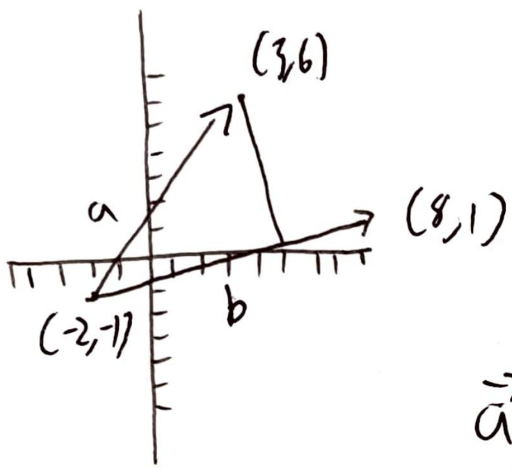
For the plane

$1x - 1y + 2z = 5$, $N_1 = 1$, $N_2 = -1$, $N_3 = 2$, $\vec{N} = \langle 1, -1, 2 \rangle$

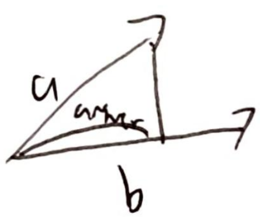
4 Equations of a line 10 / 10

✓ - 0 pts Correct

5. You start at $(-2,-1)$ and you walk in the direction of the point $(8,1)$. When the point $(3,6)$ is exactly on your left, then you make a right angle left turn and walk to $(3,6)$. Where did you turn?



$$\text{Proj}_{\vec b}\vec a = \underbrace{\frac{\vec a\cdot\vec b}{|\vec b|}}_{\text{Scalar}} \times \underbrace{\frac{\vec b}{|\vec b|}}_{\text{vector}} = \frac{\vec a\cdot\vec b}{|\vec b|^2}\vec b$$



$$\vec a = \langle 3--2, 6--1\rangle = \langle 5,7\rangle$$

$$\vec b = \langle 8--2, 1--1\rangle = \langle 10, 2\rangle$$

$$\vec a\cdot\vec b = 50+14 = 64$$

$$|\vec b| = \sqrt{10^2+2^2} = \sqrt{104}$$

$$|\vec b|^2 = 104$$

$$\text{Proj}_{\vec b}\vec a = \frac{\vec a\cdot\vec b}{|\vec b|^2}\vec b = \frac{64}{104}\vec b = \frac{32}{52}\vec b = \frac{16}{26}\vec b = \frac{8}{13}\vec b = \frac{8}{13}\langle 10,2\rangle$$

Solve for coordinates

$$\underbrace{\left\langle \frac{80}{13}, \frac{16}{13}\right\rangle}_{\text{Scalar Projection}\,\cdot\,\text{vector}} + \underbrace{\langle -2,-1\rangle}_{\text{Original point}} = \left\langle \frac{80}{13}-\frac{26}{13}, \frac{16}{13}-\frac{13}{13}\right\rangle$$

$$\boxed{=\left\langle \frac{54}{13}, \frac{3}{13}\right\rangle}$$

= ~~Direction~~ new vector with right magnitude & with same direction

5 Left turn problem 10 / 10

✓ **- 0 pts** Correct

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

$\vec{a}\cdot\vec{b} = |a||b|\cos\theta$

$\vec{a}\cdot\vec{a} = |a|^2\cos\theta = |a|^2$

6. $|\vec{a}| = 2$, $|\vec{b}| = 4$, and the angle between $\vec{a}$ and $\vec{b}$ is $45°$. Find

(a) $(2\vec{a} - \vec{b}) \cdot (3\vec{a} + 4\vec{b})$

$= \boxed{2}\vec{a}\cdot\vec{a} + 8\vec{a}\cdot\vec{b} - 3\vec{b}\cdot\vec{a} - 4\vec{b}\cdot\vec{b}$

$= 2|a|^2 + 8\vec{a}\cdot\vec{b} - 3\vec{a}\cdot\vec{b} - 4|b|^2$

$= 2\cdot 2^2 - 4\cdot 4^2 + 5\vec{a}\cdot\vec{b}$

$= 8 - 64 + 5\cdot 2\cdot 4\cdot\cos(45°)$

$= \frac{40\sqrt{2}}{2} - 56$

$= \boxed{20\sqrt{2} - \boxed{56}}$

$\vec{a}\times\vec{b} = |a||b|\sin\theta$

$\vec{a}\times\vec{a} = |a|^2\cdot\sin 0 = 0$

$\vec{a}\times\vec{b} = -\vec{b}\times$

(b) $|(2\vec{a} - \vec{b}) \times (3\vec{a} + 4\vec{b})|$

$= 6\vec{a}\times\vec{a} + 8\vec{a}\times\vec{b} - 3\vec{b}\times\vec{a} - 4\vec{b}\times\vec{b}$

$= 0 + 8\vec{a}\times\vec{b} + 3\vec{a}\times\vec{b} + 0$

$= 11(\vec{a}\times\vec{b})$

$= 11(2\cdot 4\cdot\sin(45°))$

$= 11(8\frac{\sqrt{2}}{2})$

$= \boxed{44\sqrt{2}}$

6.1 Dot product **3.5 / 5**

✓ **- 1.5 pts** Bad algebra

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

$\vec{a}\cdot\vec{b} = |a||b|\cos\theta$

$\vec{a}\cdot\vec{a} = |a|^2\cos\theta = |a|^2$

6. $|\vec{a}| = 2$, $|\vec{b}| = 4$, and the angle between $\vec{a}$ and $\vec{b}$ is 45°. Find

(a) $(2\vec{a} - \vec{b}) \cdot (3\vec{a} + 4\vec{b})$

$= 2\vec{a}\cdot\vec{a} + 8\vec{a}\cdot\vec{b} - 3\vec{b}\cdot\vec{a} - 4\vec{b}\cdot\vec{b}$

$= 2|a|^2 + 8\vec{a}\cdot\vec{b} - 3\vec{a}\cdot\vec{b} - 4|b|^2$

$= 2\cdot 2^2 - 4\cdot 4^2 + 5\vec{a}\cdot\vec{b}$

$= 8 - 64 + 5\cdot 2\cdot 4\cdot\cos(45^\circ)$

$= \frac{40\sqrt{2}}{2} - 56$

$= \boxed{20\sqrt{2} - 56}$

$\vec{a}\times\vec{b} = |a||b|\sin\theta$

$\vec{a}\times\vec{a} = |a|^2\cdot\sin 0 = 0$

$\vec{a}\times\vec{b} = -\vec{b}\times$

(b) $|(2\vec{a} - \vec{b}) \times (3\vec{a} + 4\vec{b})|$

$= 6\vec{a}\times\vec{a} + 8\vec{a}\times\vec{b} - 3\vec{b}\times\vec{a} - 4\vec{b}\times\vec{b}$

$= 0 + 8\vec{a}\times\vec{b} + 3\vec{a}\times\vec{b} + 0$

$= 11(\vec{a}\times\vec{b})$

$= 11(2\cdot 4\cdot\sin(45^\circ))$

$= 11(8\frac{\sqrt{2}}{2})$

$= \boxed{44\sqrt{2}}$

6.2 Cross product magnitude 5 / 5

✓ - 0 pts Correct

7. The equation is: $x^2yz + xyz^2 = xy^2$. Find (simplified) $\dfrac{\partial z}{\partial x}$.

$\cancel{f(x,y,z) = x^2yz + xyz^2 - xy^2}$

$$\frac{\partial}{\partial x}\left[x^2yz + xyz^2\right] = \frac{\partial}{\partial x}\left[xy^2\right]$$

$$= y\left(2xz + x^2\frac{\partial z}{\partial x}\right) + y\left(z^2 + 2xz\frac{\partial z}{\partial x}\right) = y^2(1)$$

$$= 2xyz + \boxed{yz^2}\,\frac{\partial z}{\partial x} + yz^2 + 2xyz\frac{\partial z}{\partial x} = y^2$$

$$y^2 - 2xyz - yz^2 = \left(yz^2 + 2xyz\right)\frac{\partial z}{\partial x}$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{y^2 - 2xyz - yz^2}{2xyz + \boxed{yz^2}}}$$

7 Implicit partial derivative **8 / 10**

✓ **- 2 pts** **Derivative error**