

USA Tutor

Farmer John Solves 3Sum

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Statement Summary

We have $N \leq 5000$ numbers and $Q \leq 1e5$ queries

A query consists of 2 indices as the bound

Answer this query:

Find how many triples of indices in the range (a,b) have a sum 0.

Brute Force

We can create a new subarray for each query and do this for 0 credit

Or we could answer each query in N time



Bad 3 Pointers

2 0 -1 1 -2 3 3
↑ ↑ ↑
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Why was this wrong

We want to fix a range (a,b)

Then we sweep in between



Query 1

2 0
↑ ↑

-1 1 -2

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Query 1

2 0
↑ ↑

-1 1 -2
↑
= 1

Query 1

2 0
↑ ↑

-1 1 -2
↑
= 3

Query 1

2 0
↑

-1 1 -2
= 2

Query 1

$$\begin{array}{ccccccc} 2 & 0 & -1 & 1 & -2 \\ \uparrow & \uparrow & & & \uparrow \\ & & = 0 & \text{😊} & \end{array}$$

Query 1

$$\begin{array}{ccccccc} 2 & 0 & -1 & 1 & -2 \\ \uparrow & & \downarrow & & \uparrow \\ & & = & -1 & \end{array}$$

Query 1

Handwritten mathematical expression for Query 1:

$$2 \quad 0 \quad -1 \quad 1 \quad -2$$

Arrows point to the terms 2, 1, and -2. Below the expression, there is an equals sign followed by a vertical line:

$$= |$$

Query 1

2 0
↑

-1 1 -2
↑

Query 1

$$\begin{array}{ccccccc} 2 & 0 & -1 & 1 & -2 \\ & \uparrow & & \uparrow & \\ & & \uparrow & \uparrow & \\ = 0 & & & & \end{array}$$

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Query 1

2 0
↑

-1 1 -2
↑ ↑ ↑

= -3


Query 1

$$\begin{array}{ccccccc} 2 & 0 & -1 & 1 & -2 \\ & \uparrow & & \uparrow & \uparrow \\ & & = & - & | \end{array}$$

Query 1

2 0

-1 1 -2




A diagram showing two arrows pointing upwards. The first arrow is blue and points to the number -1. The second arrow is green and points to the number 1. The arrows are positioned below the numbers -1 and 1 respectively.

Query 1

2 0

-1 1 -2



A diagram consisting of three vertical lines. Below the first line is a blue arrow pointing upwards. Below the second line is a green arrow pointing upwards. Below the third line is a red arrow pointing upwards.

= -2



A diagram consisting of a green equals sign followed by a green minus sign and a green number 2.

How can we save time?

Previous Solution works in $Q * N^3$ time

That is a lot

The range (1,5) also includes (2,5) and (3,5) and (4,5)

The range (1,5) also includes (1,4) and (1,3) and (1,2)

Prefix summing for (1,4) to (1,5)

You can just say `prefix[i][j] += prefix[i][j-1]`

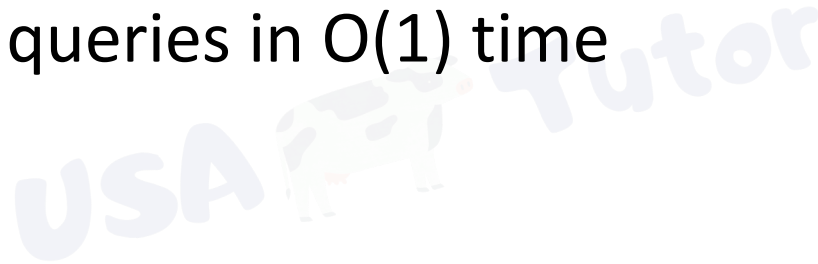
Very simple



Kicking out the Q factor

Let $\text{answer}[a][b]$ be the answer to your query, a b

Then you can answer queries in $O(1)$ time



Kicking out the Last N^3 to N^2

For sweeping in between, you can store a map of numbers

Then, you can query if a number exists in your map, then add the amount of times that it showed up before



Kicking out the Last N^3 to N^2

If you are at $i=0$ and $j=5$
you came from $i=0$ and $j=4$

This means you can store all of the numbers that you saw from
 $i=0$ $j=4$, and add in $j=5$ into your map

Map stuff

We reset our map at every new i (starting point)

We do not add in the starting point into our map, or the ending point

And you query for $-(arr[i] + arr[j])$ in your map