USA  Tutor

Race

Analysis by David Yang

Statement Summary

Running length K meters K<=1e9

3 choices in any 1 move, then move it:

+1 speed

-1 speed

no change

No more than X<=1e5 meters when you end

What is min time it takes to finish race

Sample with X=1

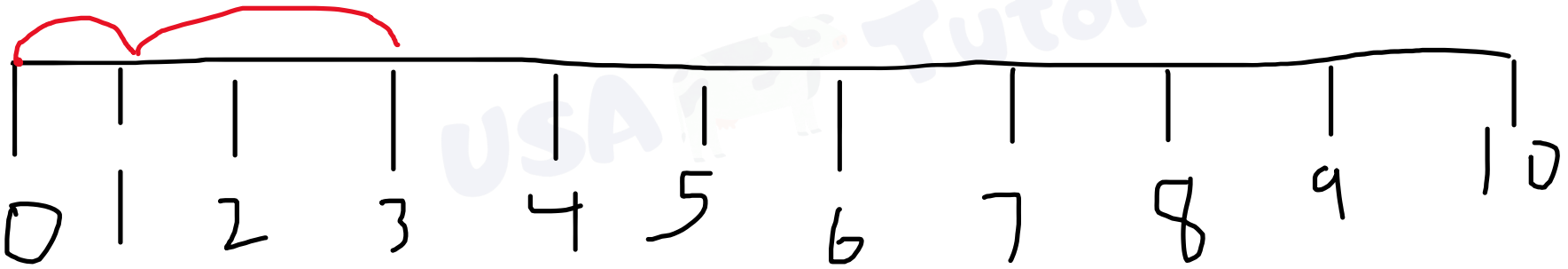
S = 0 → 1

P = 0 → 1

Sample with X=1

S = 1 → 7

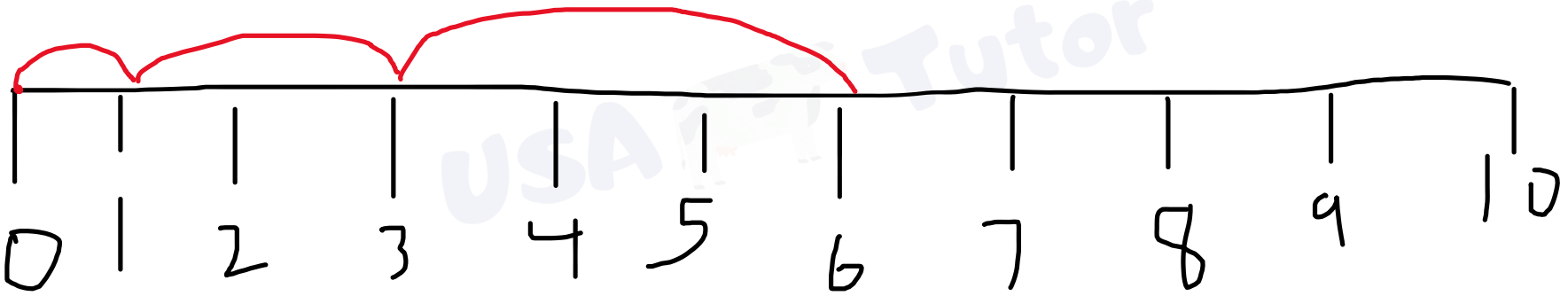
P = 1 → 3



Sample with X=1

S = 2 → 3

P = 3 → 6



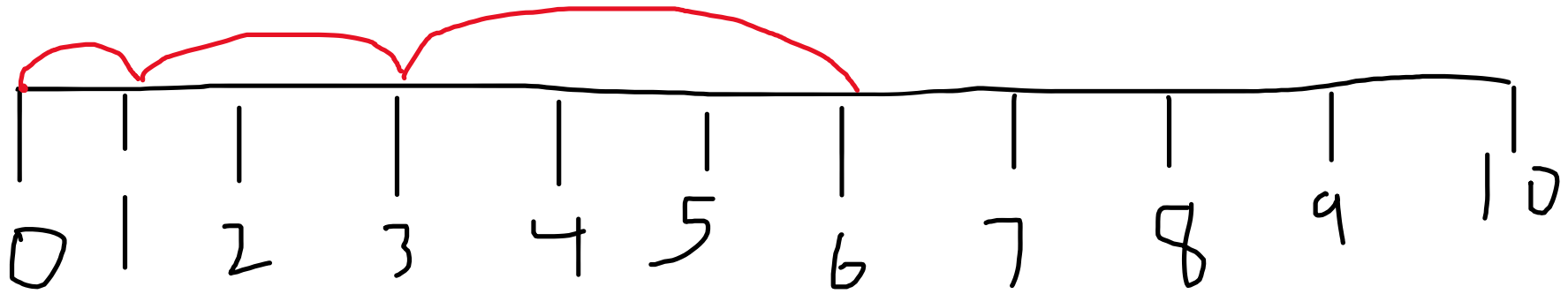
WARNING

Look at this number line. You're at speed 3, position 6.

If you speed up, you will be at speed 4, position 10. NO

If you stay, you will be at speed 3, position 9. You can see for yourself, this will not work either.

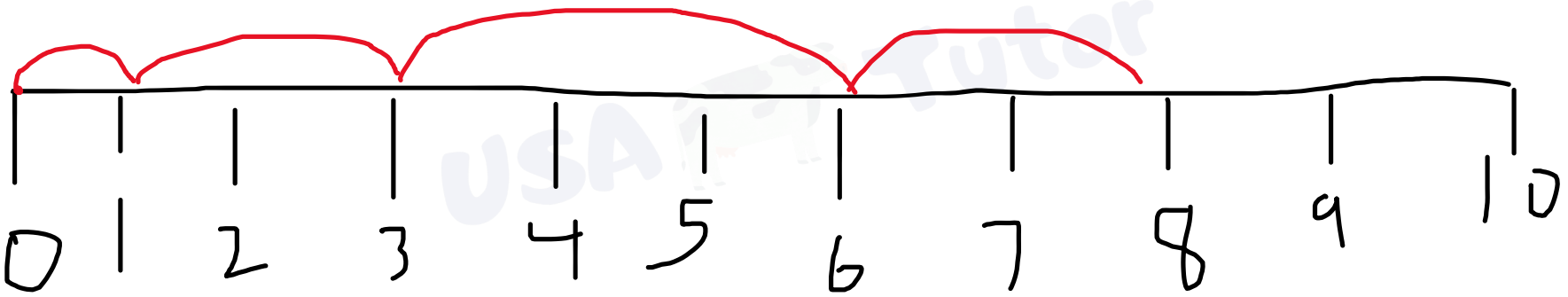
You MUST slow down



Sample with X=1

S = 3 → 7

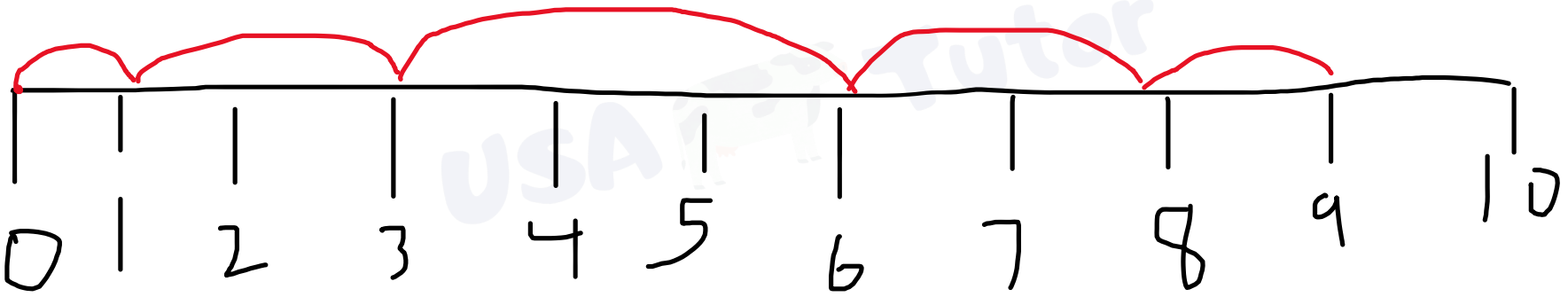
P = 6 → 8



Sample with X=1

S = 2 → 1

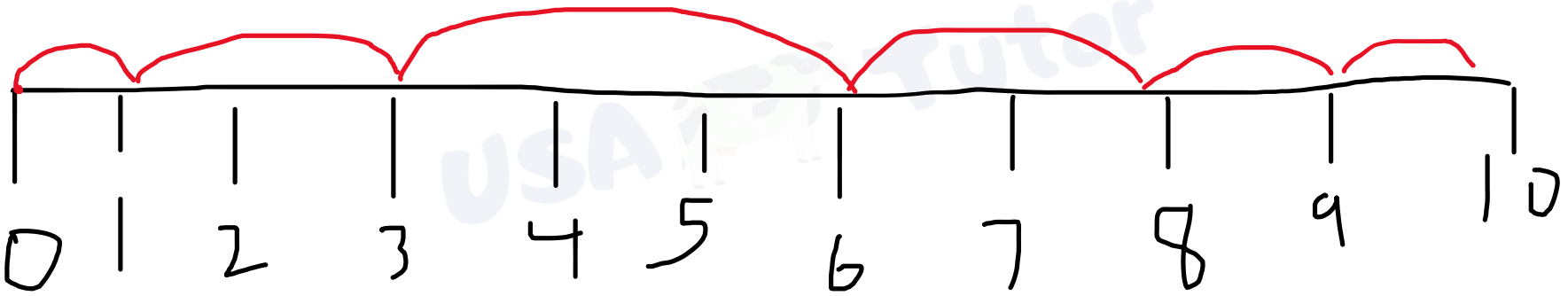
P = 8 → 9



Sample with X=1

S = 1 → 1

P = 9 → 10



Thoughts on X=1

Why did we want to slow down?

We could not have went up or stayed at the same

Priorities:

Go up

Stay

Go down

Greedy Algorithm

Thoughts on X=1

When do we want to speed up?

When we're allowed to

Given S=S+1, We can speed down all the way to X before passing the finish line

When do we want to stay at same speed?

Given S=S, We can speed down all the way to X before passing the finish line

Else Slow down

How do you calculate “speed down dist”

We need the sum(1…X) and sum(1..S)

If you speed up, the distance will just be sum(1..S+1)- sum(1..X)

If you stay, the distance will just be sum(1..S) – sum(1..X)

If you slow down, the distance will just be (1..S-1) - sum(1..X)

Calculating sum(a,b)

We can use a prefix sum array to do this for us

A prefix sum array stores a[i] = a[i-1]+ i

A[0]=0

A[1]=1

A[2]=3

A[3]=5

A[4]=9

A[5]=14

Calculating sum(a,b)

Sum(1..X) = prefix[X] – prefix[1-1]

Sum(1..L) = prefix[L] – prefix[1-1]

Sum(a,b) = prefix[b] – prefix[a-1]

Sum(a,b) = (b)(b+1)/2 – (a-1)(a)/2

Sum(1…N) – sum(1..N-3) = N-2+N-1+N

Complete

You are done with the idea, coding should be easy

Any questions?