

# ***Scalability in Multi-agent systems***

## Analysis and Control

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Coláiste na hInnealtóireachta agus na hAiltireachta



# Presentation outline

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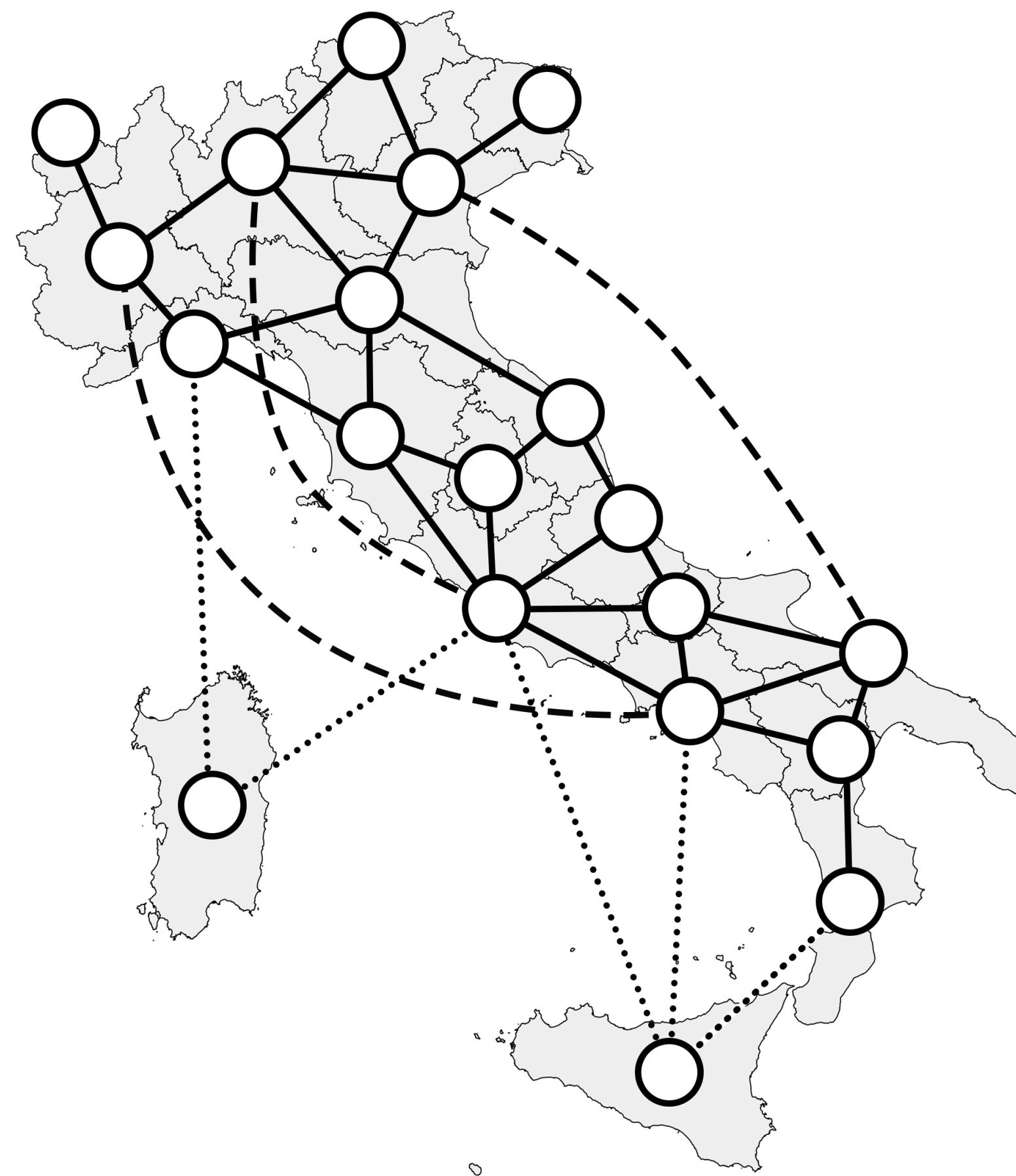
- Motivation and Challenges
- Contributions and Publications
- Setting up the Problem: Scalability
- Rejecting Polynomial Disturbances: Multiplex Architecture
- Scalability for COVID-19 Mitigation
- Conclusion and Future Plan

# Presentation outline

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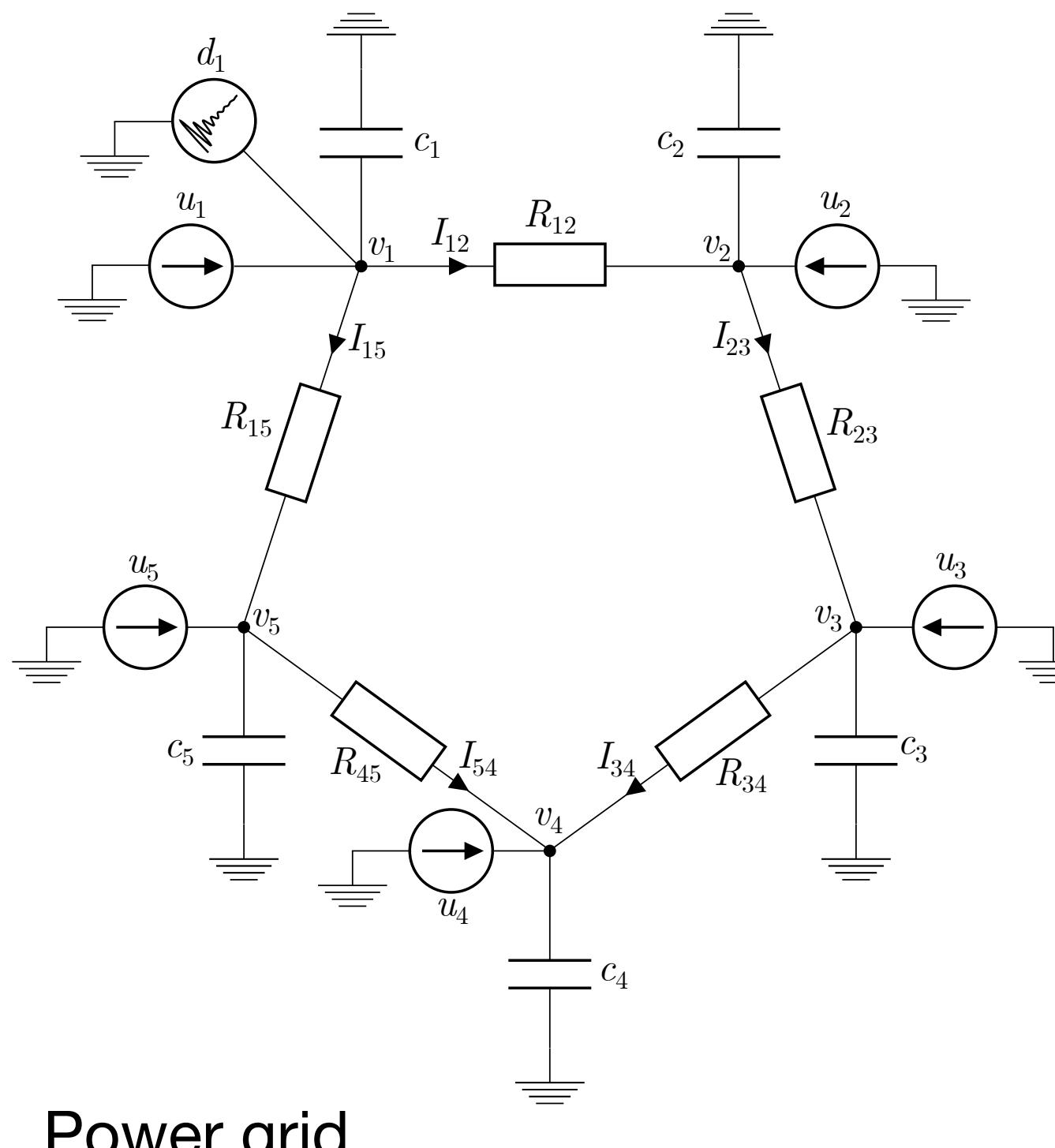
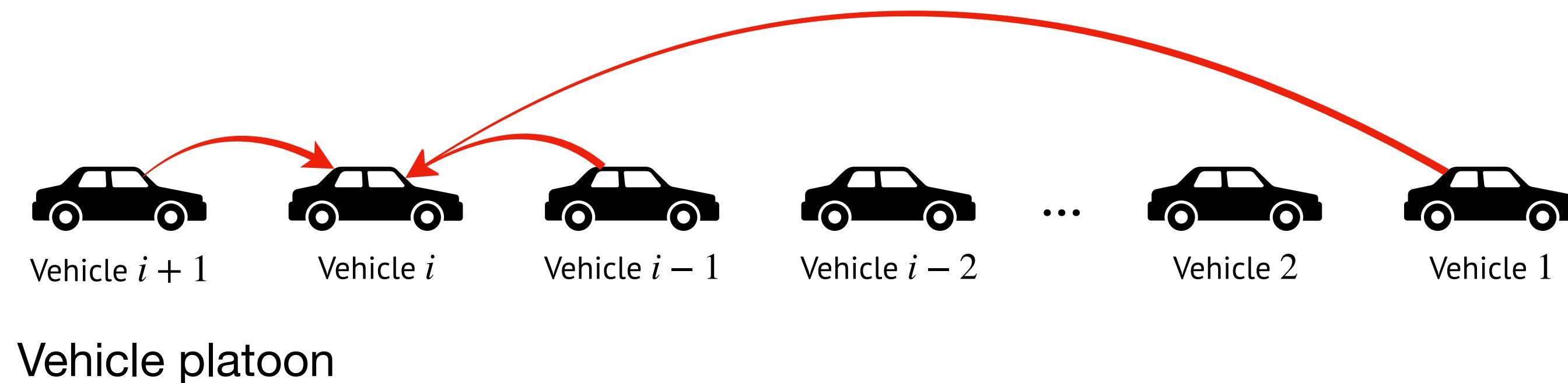
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# Examples of MASs and the challenges



[Della Rossa et al. Nat Commun, 2020]

Regional epidemic model



## Key task:

- Design control protocols to achieve some desired behaviours

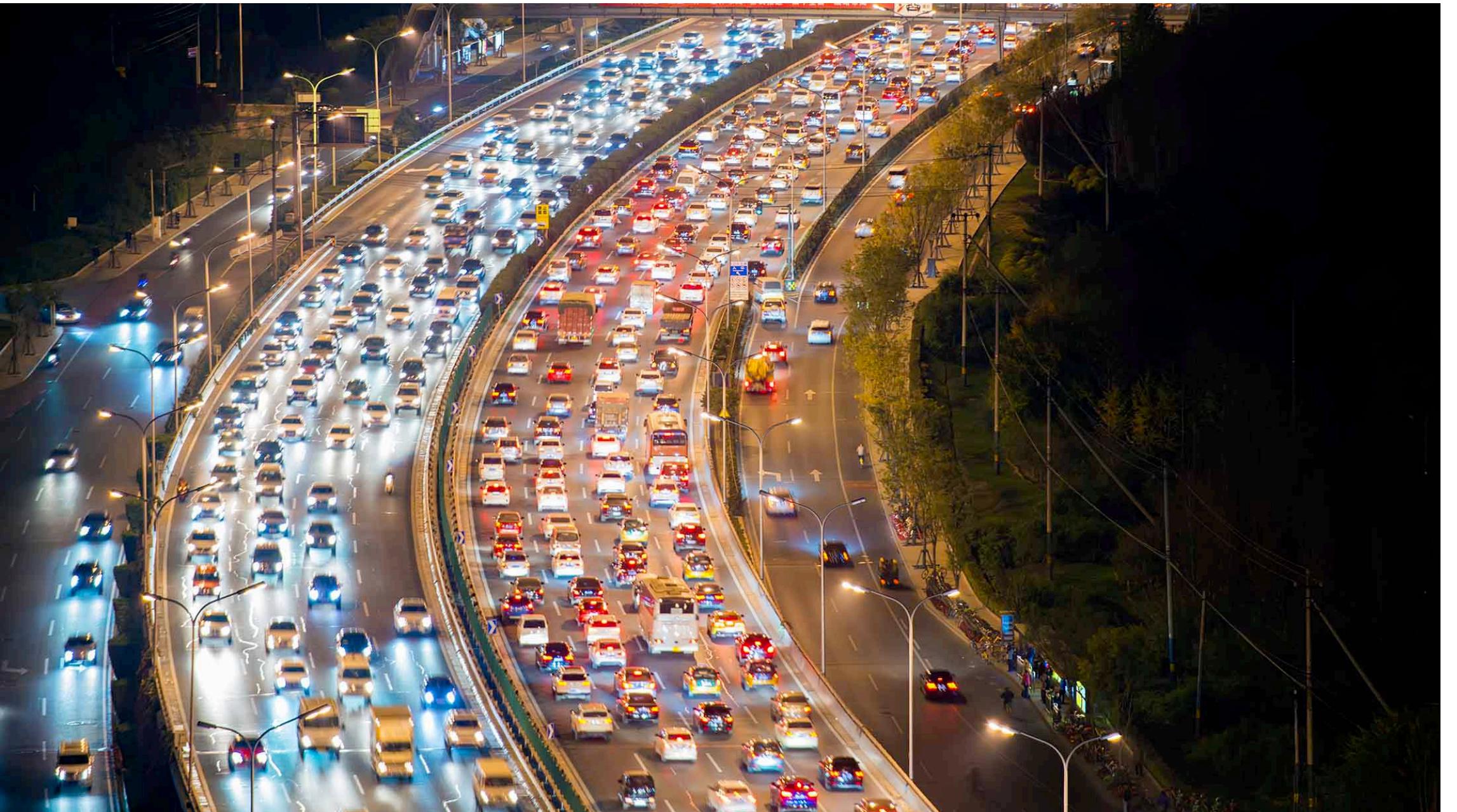
## Challenges:

- Delays occurred during interaction (communicating, sensing...)
- External disturbances

How do disturbances **propagate** within the MAS?

# Amplified disturbances can lead to ...

- Night traffic of Beijing, China, 2017



## Key features:

- ▶ Achieve desired behaviours
- ▶ Guarantee non-amplification of disturbances

→ **Scalability!**



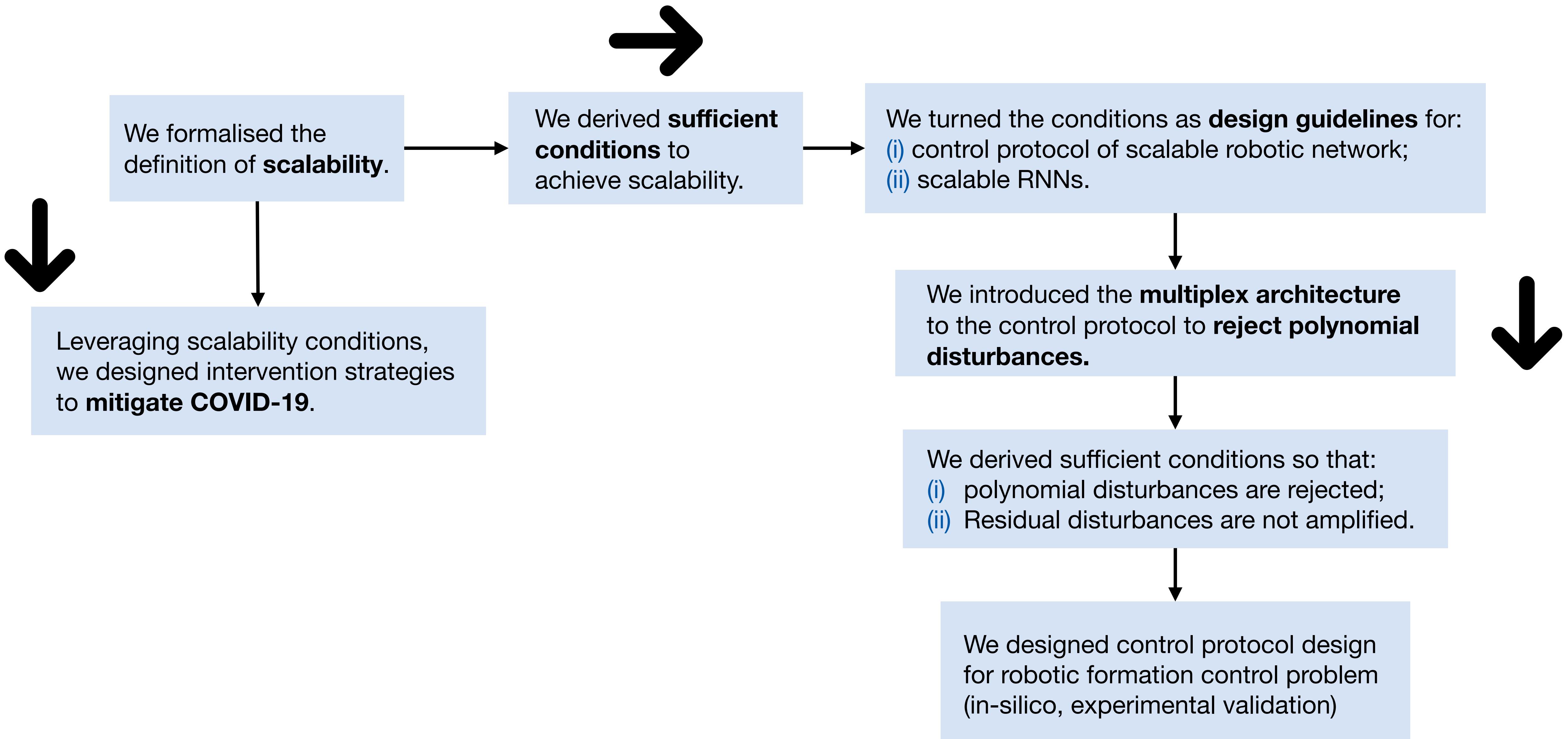
- Drone light show in Xi'an, China, 2018

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# Contributions



# Publications and preprints

- Journal:

- ▶ **Xie, S.**, Russo, G. and Middleton, R.H., 2021. Scalability in nonlinear network systems affected by delays and disturbances. *IEEE Transactions on Control of Network Systems*, 8(3), pp.1128-1138.
- ▶ **Xie, S.** and Russo, G., 2022. On the Design of Integral Multiplex Control Protocols for Nonlinear Network Systems with Delays. [Submitted to *Automatica*, major revision].

- Conference:

- ▶ **Xie, S.** and Russo, G., 2022. On the design of scalable networks rejecting first order disturbances. *IFAC-PapersOnLine*, 55(13), pp.216-221.
- ▶ Coraggio, M., **Xie, S.**, De Lellis, F., Russo, G. and di Bernardo, M., 2021, December. Intermittent non-pharmaceutical strategies to mitigate the COVID-19 epidemic in a network model of Italy via constrained optimization. In 2021 60th IEEE Conference on Decision and Control (CDC) (pp. 3538-3543).
- ▶ **Xie, S.** and Russo, G., 2022. On the design of multiplex control to reject disturbances in nonlinear network systems affected by heterogeneous delays. [Submitted to American Control Conference (ACC) 2023].

**Scalability in Nonlinear Network Systems Affected by Delays and Disturbances**  
Shihao Xie<sup>a</sup>, Giovanni Russo<sup>b</sup>, Senior Member, IEEE, and Richard H. Middleton<sup>a</sup>, Fellow, IEEE  
*Abstract*—This article is concerned with the study of scalability in nonlinear heterogeneous networks affected by communication delays and disturbances. After formalizing the notion of scalability, we give two sufficient conditions noted in, e.g., [8], there is an intrinsic limit (this limit appears to be approximately 14% of the network size) for recurrent networks that precludes them to store an arbitrarily large number in memory. Instead, we propose a new notion of scalability that is based on the ability to store an arbitrary number of states in memory. We show that this new notion of scalability is equivalent to the one proposed in [8] for recurrent networks.  
On the Design of Integral Multiplex Control Protocols for Nonlinear Network Systems with Delays<sup>\*</sup>  
Shihao Xie<sup>a</sup>, Giovanni Russo<sup>b</sup>  
<sup>a</sup>School of Electrical and Electronic Engineering, University College Dublin, Ireland (e-mail: shihao.xie@ucdconnect.ie)  
<sup>b</sup>Department of Information and Electrical Engineering and Applied Mathematics, University of Salerno, Italy (e-mail: giovanni.russo@unisa.it)  
2022 Available online at [www.sciencedirect.com](http://www.sciencedirect.com)  
ScienceDirect  
IFAC PapersOnLine 55-13 (2022) 216-221  
**On the design of scalable networks rejecting first order disturbances**  
Shihao Xie<sup>a</sup>, Giovanni Russo<sup>\*\*</sup>  
<sup>a</sup>School of Electrical and Electronic Engineering, University College Dublin, Ireland (e-mail: shihao.xie@ucdconnect.ie)  
<sup>\*\*</sup>Department of Information and Electrical Engineering and Applied Mathematics, University of Salerno, Italy (e-mail: giovanni.russo@unisa.it)  
*Abstract*: This paper is concerned with the problem of designing distributed control protocols for nonlinear network systems affected by time-varying delays and disturbances. The goal is to reject polynomial disturbances affecting the network. To satisfy these requirements, we propose an integral control design implemented via a multiplex architecture. We give sufficient conditions for the desired disturbance rejection and stability properties by leveraging tools from contraction theory. We illustrate the effectiveness of the results via a numerical example that involves the control of a multi-terminal high-voltage DC grid.  
2021 60th IEEE Conference on Decision and Control (CDC)  
December 13-15, 2021, Austin, Texas  
**Intermittent non-pharmaceutical strategies to mitigate the COVID-19 epidemic in a network model of Italy via constrained optimization**  
Marco Coraggio<sup>\*, 1</sup>, Shihao Xie<sup>\*, 2</sup>, Francesco De Lellis<sup>1</sup>, Giovanni Russo<sup>\*, 3</sup>, Mario di Bernardo<sup>\*, 1</sup>  
*Abstract*—This paper is concerned with the design of intermittent non-pharmaceutical strategies to mitigate the spread of the COVID-19 epidemic in a network epidemiological model. Specifically, by studying a variant extension for the dynamics of the epidemic, we propose, using contraction arguments, a condition that can be used to guarantee that the effective reproduction number is less than unity. This condition, whereas in [8] the benefits were shown of adopting feedback intermittent regional control strategies triggered by occupancy levels of the intensive care units (ICUs) in each region. Furthermore, several epidemic control strategies were recently proposed, based on model predictive control (MPC).  
On the design of multiplex control to reject disturbances in nonlinear network systems affected by heterogeneous delays  
Shihao Xie<sup>1</sup> and Giovanni Russo<sup>2</sup>  
*Abstract*—We consider the problem of designing control protocols for nonlinear network systems affected by heterogeneous, time-varying delays and disturbances. For these networks, the goal is to reject polynomial disturbances affecting the network. To satisfy these requirements, we propose an integral control design implemented via a multiplex architecture. We give sufficient conditions for the desired disturbance rejection and stability properties by leveraging tools from contraction theory. We illustrate the effectiveness of the results via a numerical example that involves the control of a multi-terminal high-voltage DC grid.  
I. INTRODUCTION

# Presentation outline

- Motivation and Challenges
- Contributions and Publications
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IEEE TRANSACTIONS ON CONTROL OF NETWORK SYSTEMS, VOL. 8, NO. 3, SEPTEMBER 2021



## Scalability in Nonlinear Network Systems Affected by Delays and Disturbances

Shihao Xie , Giovanni Russo , Senior Member, IEEE, and Richard H. Middleton , Fellow, IEEE

**Abstract**—This article is concerned with the study of scalability in nonlinear heterogeneous networks affected by communication delays and disturbances. After formalizing the notion of scalability, we give two sufficient conditions to assess this property. Our results can be used to study leader-follower and leaderless networks and allow us to consider the case when the desired configuration of the system changes over time. We show how our conditions can be turned into design guidelines to guarantee scalability and illustrate their effectiveness via numerical examples.

**Index Terms**—Networks of autonomous agents, nonlinear systems, scalability of networks, stability.

I. INTRODUCTION

# Network dynamics

- Consider a **nonlinear control-affine** network system...

Intrinsic dynamics      Control input      Disturbance

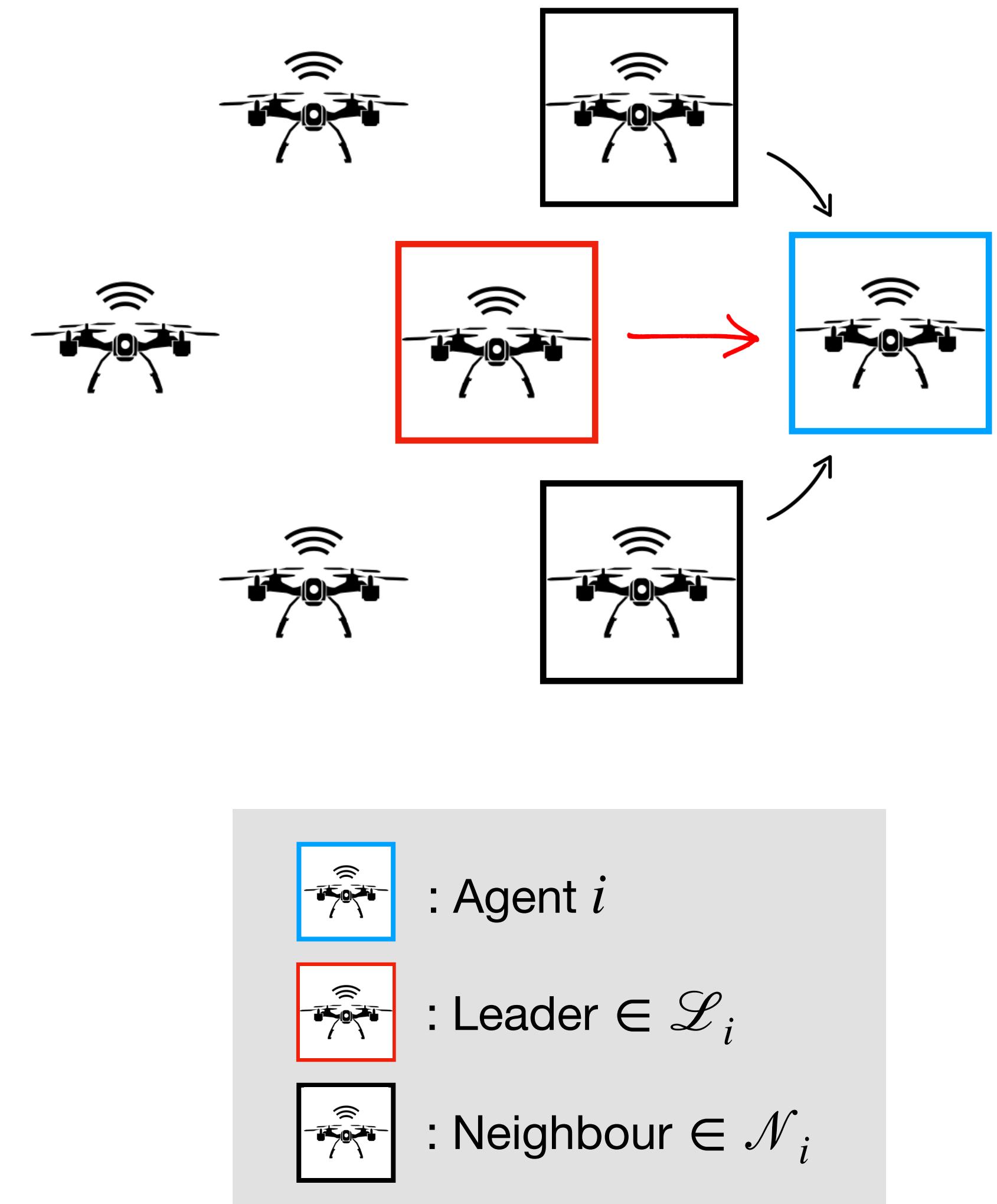
$$\dot{x}_i(t) = \underline{f}_i(x_i, t) + \underline{u}_i(t) + \underline{b}_i(x_i, t)\underline{d}_i(t), \quad t \geq 0 \quad (1)$$

$$y_i(t) = g_i(x_i) \quad \text{Disturbance intensity function}$$

The control satisfies:

	Delayed coupling	Neighbour coupling
$u_i(t) =$	$\sum_{j \in \mathcal{N}_i} h_{ij}^{(\tau)}(x_i(t - \tau(t)), x_j(t - \tau(t)), t) + \sum_{j \in \mathcal{N}_i} h_{ij}(x_i(t), x_j(t), t)$	
$+ \sum_{l \in \mathcal{L}_i} h_{il}^{(\tau)}(x_i(t - \tau(t)), x_l(t - \tau(t)), t) + \sum_{l \in \mathcal{L}_i} h_{il}(x_i(t), x_l(t), t)$		

- ▶ Interactions between agents are affected by a **time-varying delay**  $\tau(t)$
- ▶ Coupling from both **neighbours** and **possible leaders** are included
- ▶ **Leader-follower** and **leaderless** networks



# Definition of scalability

- The desired solution  $x^*$ :

$$\begin{aligned}\dot{x}_i(t) &= f_i(x_i, t) + \cancel{u_i(t)} + b_i(x_i, t)d_i(t), \quad t \geq 0 \\ y_i(t) &= g_i(x_i)\end{aligned} \quad \rightarrow \quad \begin{aligned}\dot{x}_i^*(t) &= f_i(x_i^*, t) \\ y_i^*(t) &= g_i(x_i^*)\end{aligned}$$

**Definition 1.** The network system (1) is  $\mathcal{L}_\infty$ -scalable-Input-to-State Stable ( $\mathcal{L}_\infty$ -sISS) if there exists some class  $\mathcal{KL}$  function,  $\beta$ , and class  $\mathcal{K}$  function,  $\gamma$ , such that, for all  $t \geq 0$ ,

$$\max_i \|x_i(t) - x_i^*(t)\|_2 \leq \beta \left( \max_i \sup_{-\tau_0 \leq s \leq 0} \|x_i(s) - x_i^*(s)\|_2, t \right) + \gamma \left( \max_i \|d_i(\cdot)\|_{\mathcal{L}_\infty} \right), \quad \forall N$$

- Maximum deviation of the state from the desired solution is **independent** on the size of the network
- Sufficient conditions are derived for networks to achieve this property...

# Interpretation of the conditions

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- Condition 1:

$$h_{ij}(x_i^*, x_j^*, t) = h_{ij}^{(\tau)}(x_i^*, x_j^*, t) = \underline{h}_{il}(x_i^*, x_l, t) = \underline{h}_{il}^{(\tau)}(x_i^*, x_l, t) = 0, \forall j, \forall l;$$

- ▶  $u_i(t) = 0$  at the desired solution. It guarantees  $\dot{x}_i^*(t) = f_i(x_i^*, t)$ .

- Condition 2:

$$\mu_2 \left( \partial_1 f_i(x_i, t) + \sum_{l \in \mathcal{L}_i} \partial_1 \underline{h}_{il}(x_i, x_l, t) + \sum_{j \in \mathcal{N}_i} \partial_1 h_{ij}(x_i, x_j, t) \right) + \sum_{j \in \mathcal{N}_i} \left\| \partial_2 h_{ij}(x_i, x_j, t) \right\|_2 \leq -\bar{\sigma},$$

- ▶ C2 contains the Jacobian of the **delay-free** part of the dynamics

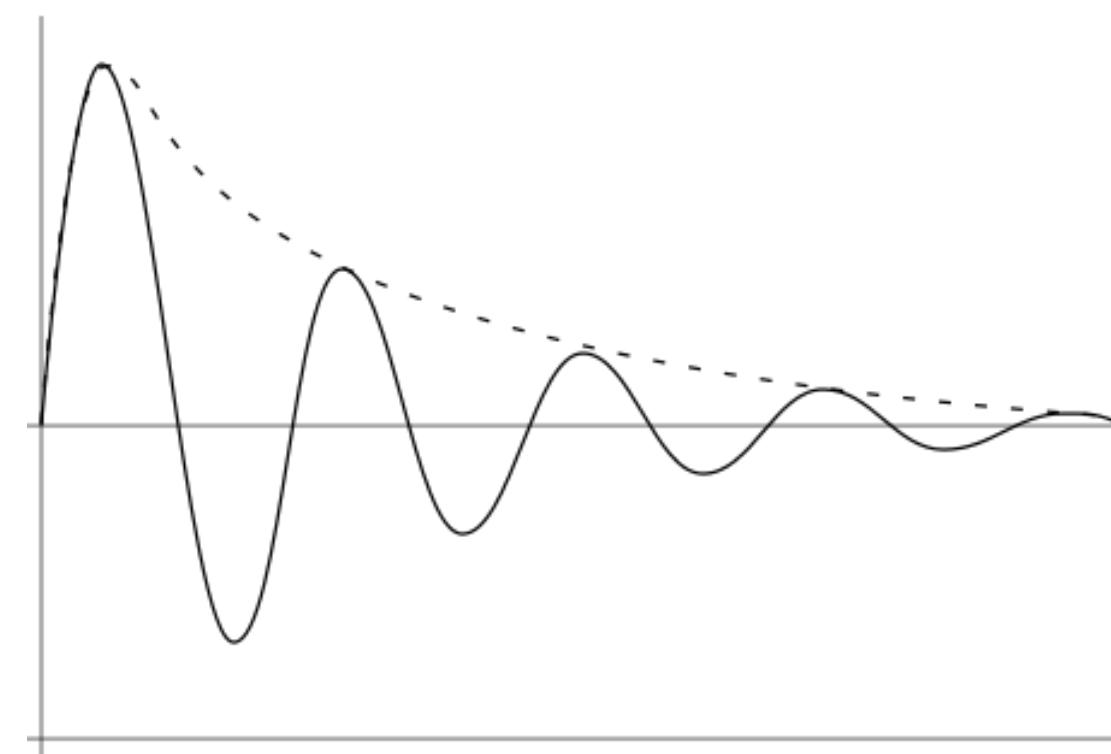
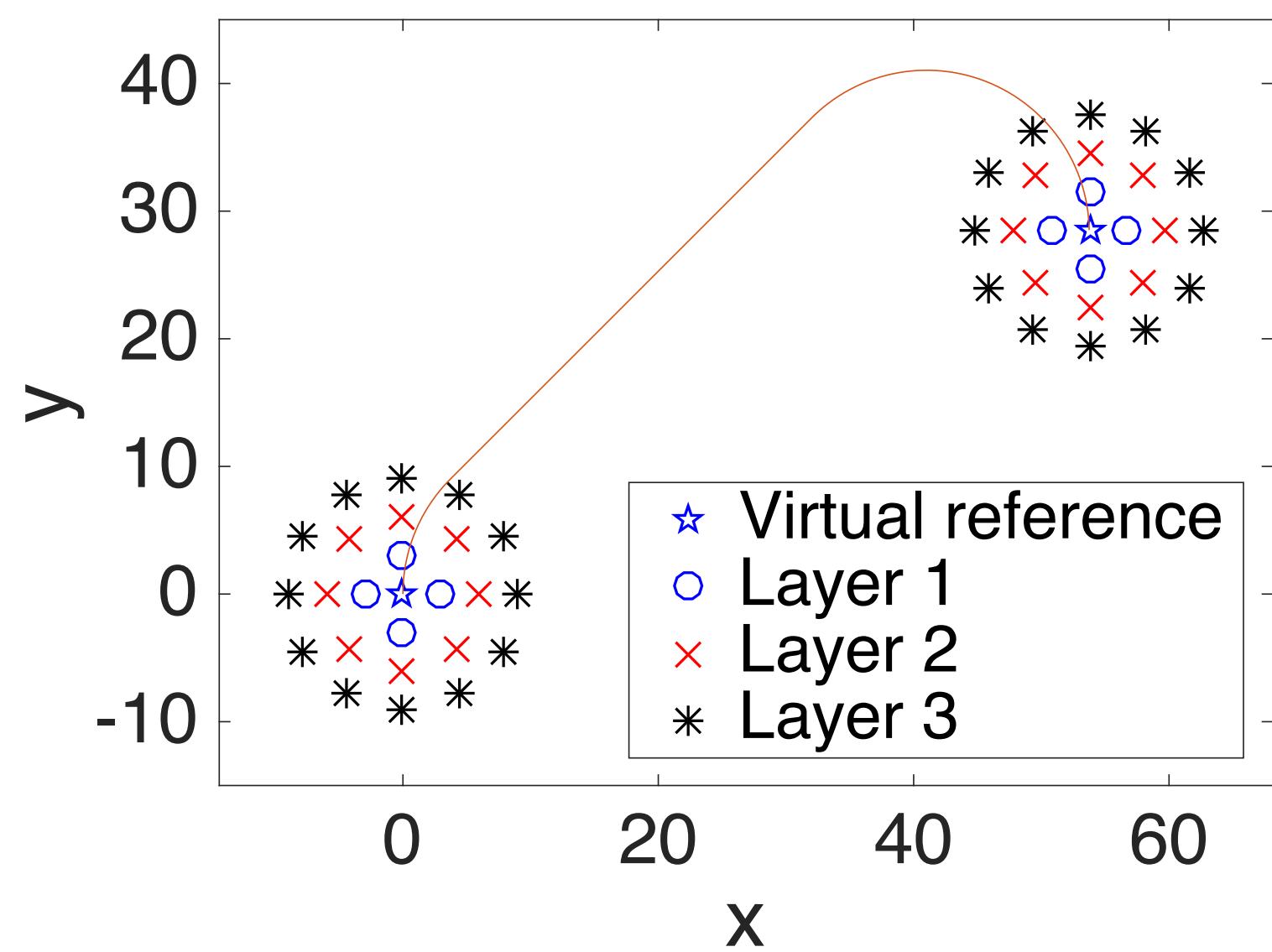
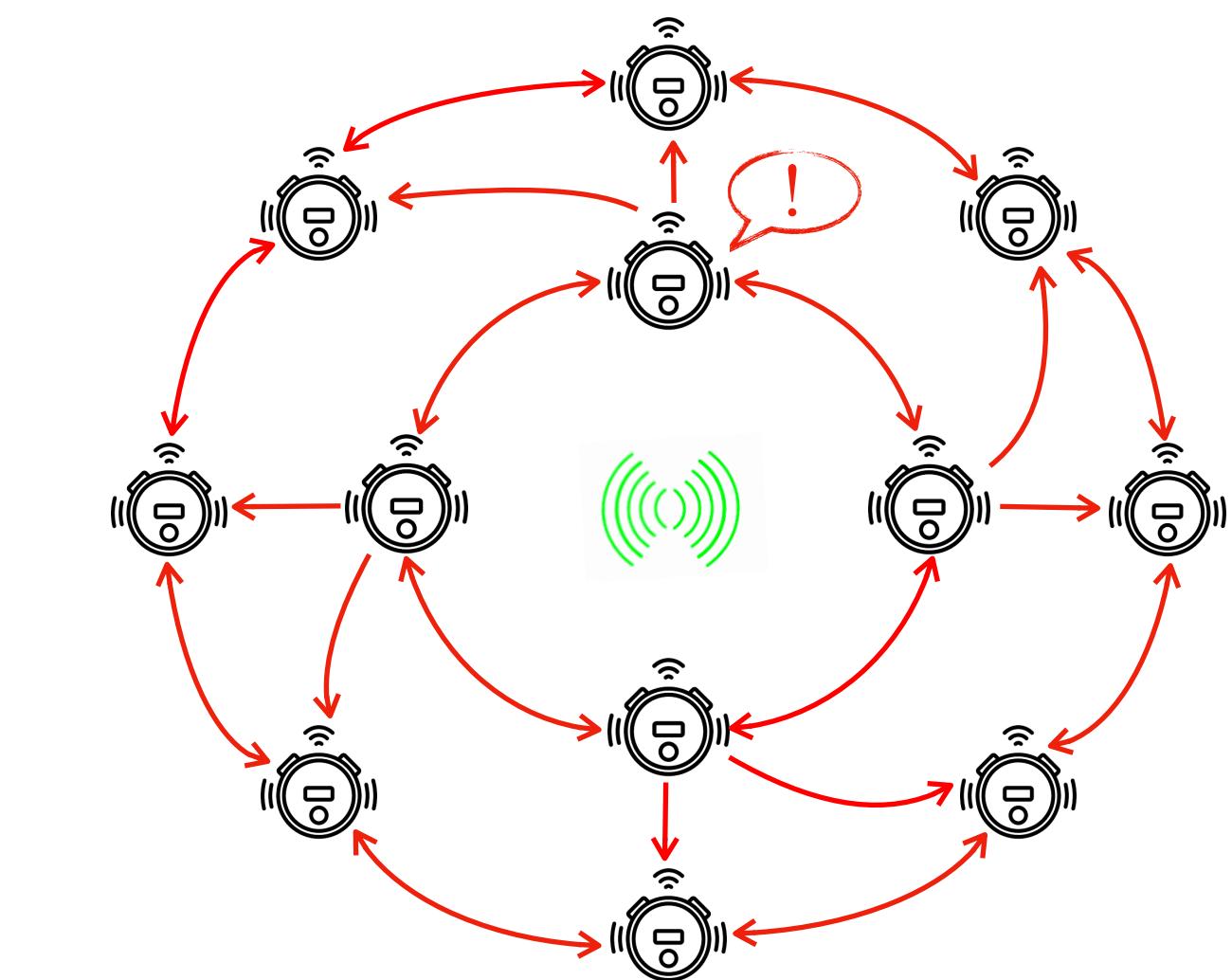
- Condition 3:

$$\left\| \sum_{l \in \mathcal{L}_i} \partial_1 \underline{h}_{il}^{(\tau)}(x_i, x_l, t) + \sum_{j \in \mathcal{N}_i} \partial_1 h_{ij}^{(\tau)}(x_i, x_j, t) \right\|_2 + \sum_{j \in \mathcal{N}_i} \left\| \partial_2 h_{ij}^{(\tau)}(x_i, x_j, t) \right\|_2 \leq \underline{\sigma};$$

- ▶ C3 contains the Jacobian of the **delayed** part of the dynamics.
- ▶ Delay-free dynamics should be able to **compensate** the delayed dynamics;
- ▶ C2 and C3 can be used to **shape the coupling functions** between the agents and to **determine the maximum number of neighbours** for each agent;

# Application: robotic formation control (problem setup)

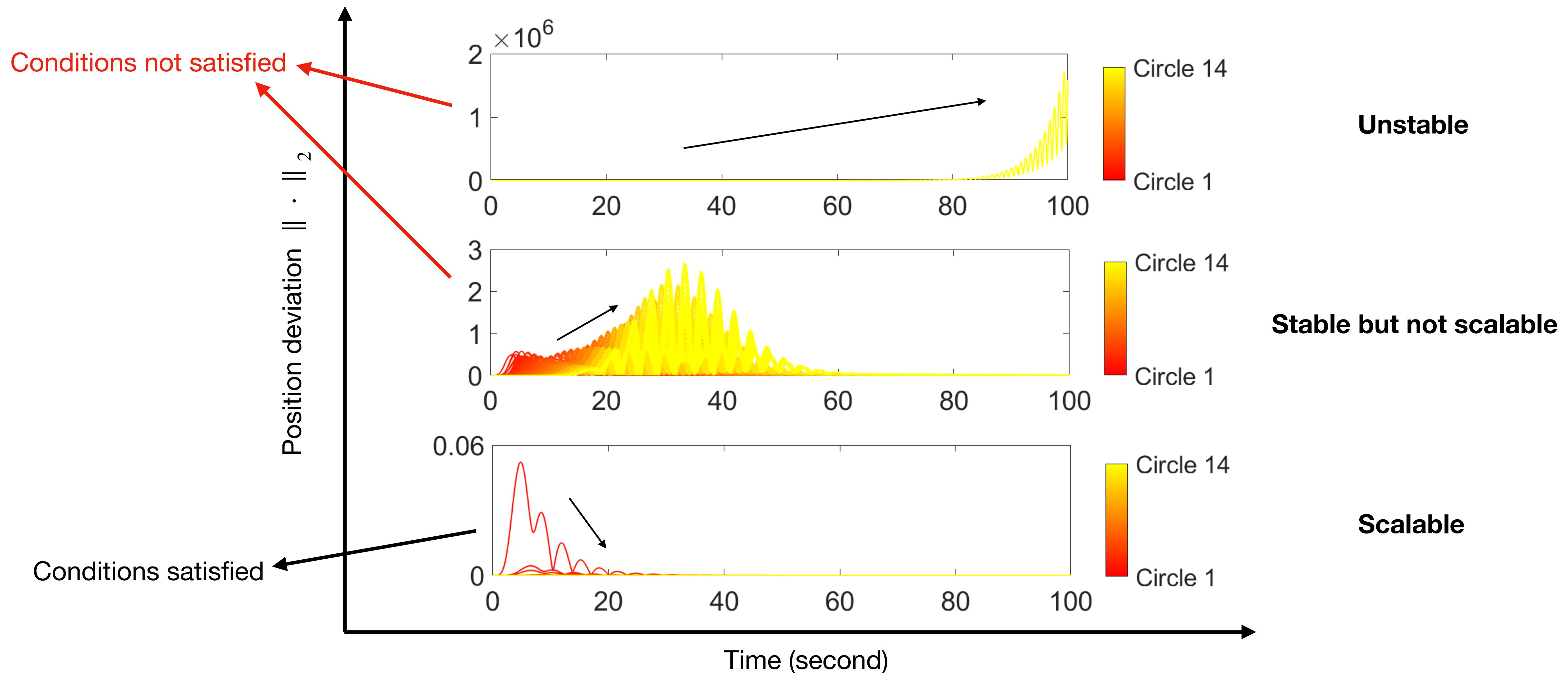
- Formation pattern: 14 concentric circles, circle  $i$  has  $4*i$  unicycle robots.
- Topology: robots are connected to its neighbours on same layer, and closest robot on inner circle (if any).
- Desired solution/control goal:**
  - follow a trajectory provided by reference signal
  - Keep desired offset from each other



- Reference signal  $x_l(t)$ ;  
 - Delayed communication;  
 - External disturbances.

# Application: robotic formation control (simulation results)

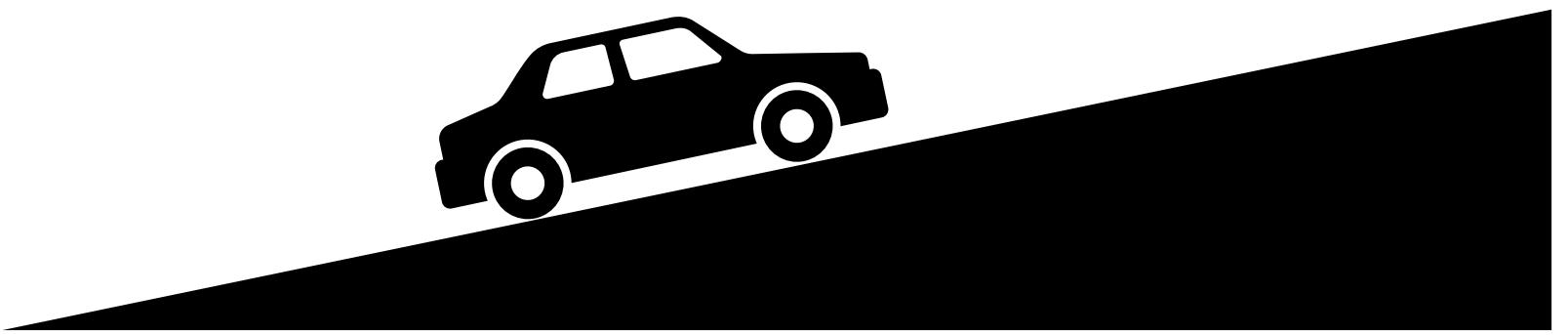
[Xie, S. et al. IEEE Trans. Control. Netw. Syst., 2021]



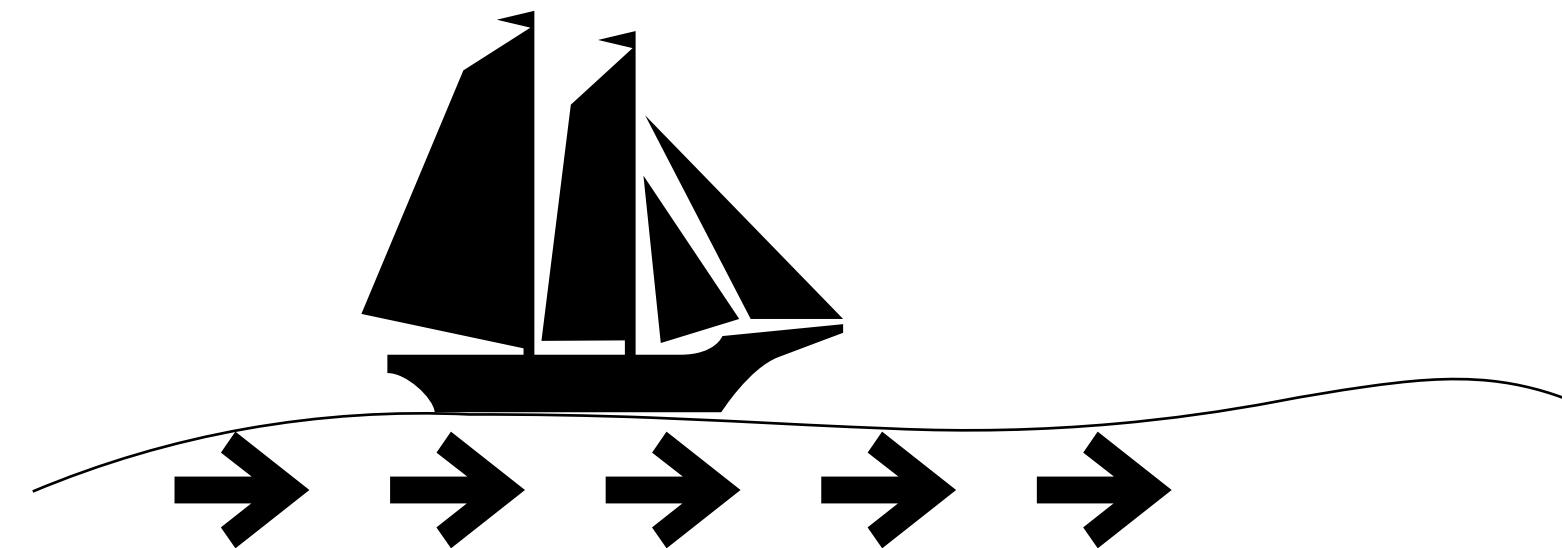
# A challenge

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- **Polynomial disturbances** are commonly seen in real world applications...



Car hitting a slope



Current affecting boats

Can we design control protocols to **reject** such polynomial disturbances?

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## On the Design of Integral Multiplex Control Protocols for Nonlinear Network Systems with Delays\*

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<sup>b</sup>Department of Information and Electrical Engineering and Applied Mathematics, University of Salerno, Italy (e-mail: giovarusso@unisa.it)

[eess.SY] 7 Jun 2022

### Abstract

We consider the problem of designing control protocols for possibly nonlinear networks with delays that not only allow the fulfillment of some desired behaviour, but also simultaneously guarantee the rejection of polynomial disturbances and the non-amplification of other classes of disturbances across the network. To address this problem, we propose the systematic use of multiplex architectures to deliver integral control protocols ensuring the desired disturbance rejection and non-amplification properties. We then present a set of sufficient conditions to assess these properties and hence to design the multiplex architecture for both leaderless and leader-follower networks with time-varying references consisting of possibly heterogeneous nonlinearly coupled agents affected by communication delays. The effectiveness of our conditions, which are also turned into an optimisation problem allowing protocol design, is illustrated via both in-silico and experimental validations with a real hardware set-up.



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IFAC PapersOnLine 55-13 (2022) 216–221



## On the design of scalable networks rejecting first order disturbances

Shihao Xie\* Giovanni Russo\*\*

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\*\* Department of Information and Electrical Engineering and Applied Mathematics, University of Salerno, Italy (e-mail: giovarusso@unisa.it)

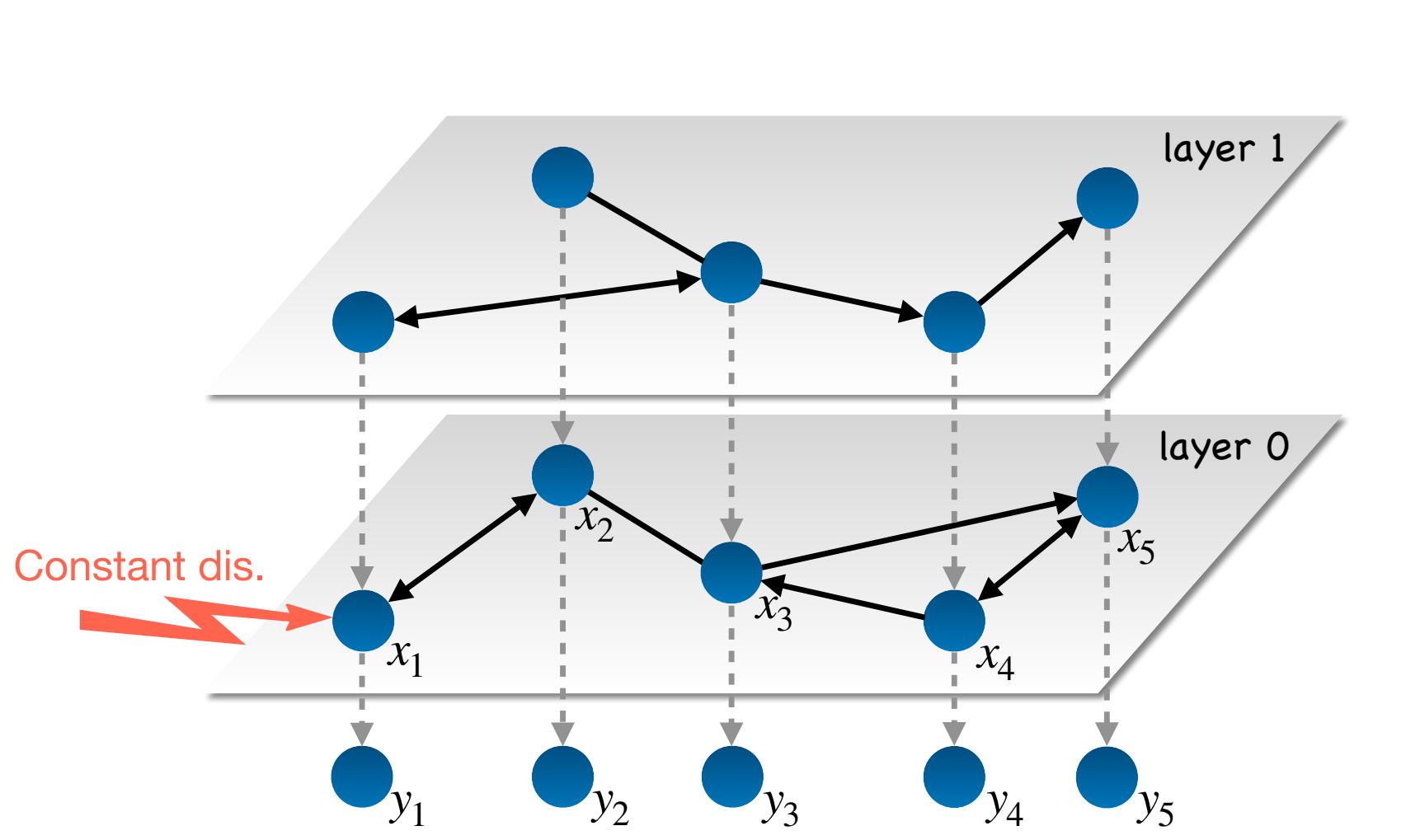
**Abstract:** This paper is concerned with the problem of designing distributed control protocols for network systems affected by delays and disturbances consisting of a first-order polynomial component and a residual signal. Specifically, we propose the use of a multiplex architecture to design distributed control protocols to reject polynomial disturbances up to ramps and guarantee a scalability property that prohibits the amplification of residual disturbances. For this architecture, we give a sufficient condition on the control protocols to guarantee scalability and ramps rejection. The effectiveness of the result, which can be used to study networks of nonlinearly coupled nonlinear agents, is illustrated via a robot formation control problem.

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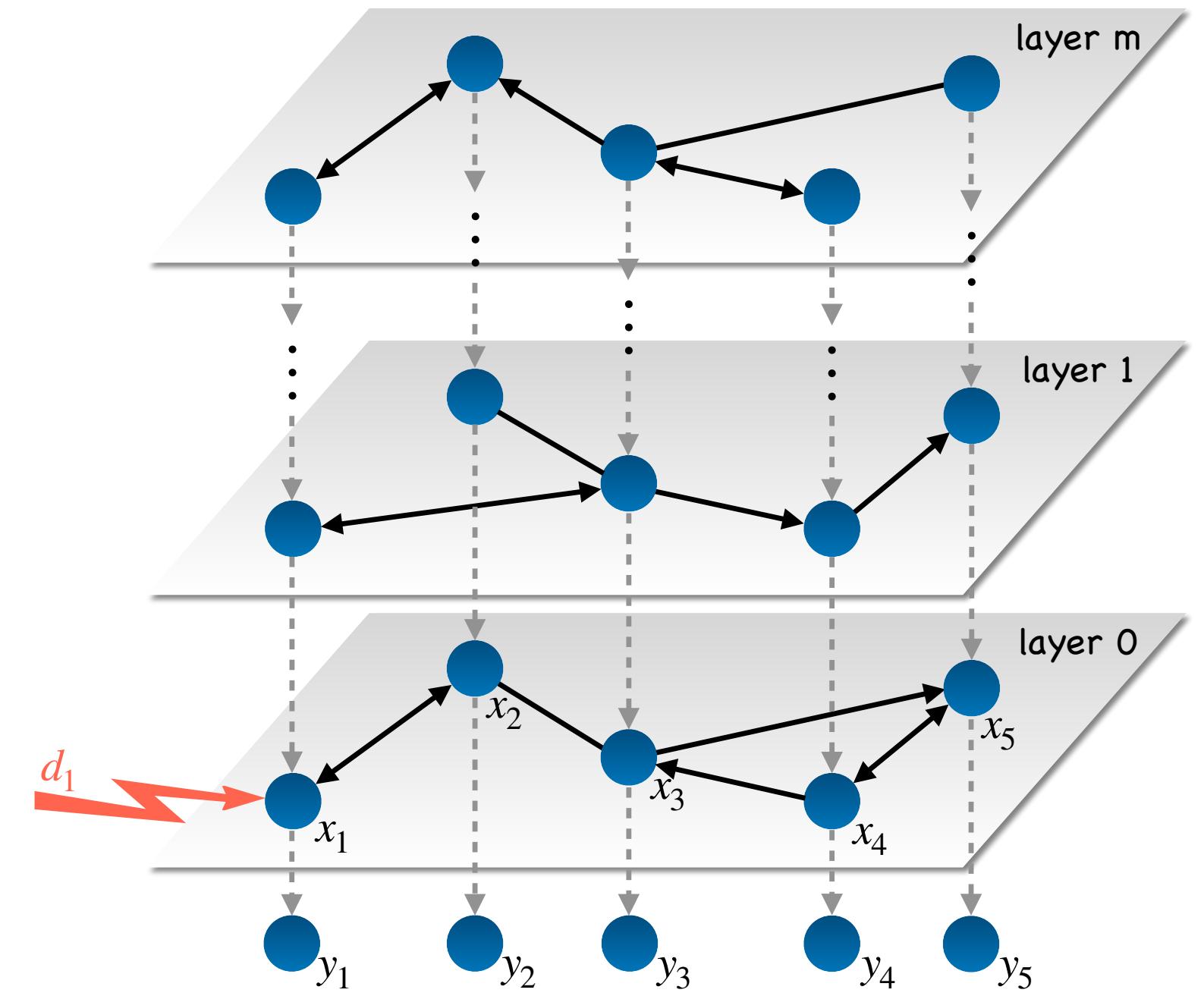
# Rejecting polynomial disturbances

Control goals:

- Reject polynomial disturbances → **HOW?**
- Scalability with respect to the residual disturbance



→ Add more layers?



# Network dynamics

- The system dynamics follows

$$\begin{array}{c} \text{Intrinsic dynamics} \quad \text{Control input} \\ \dot{x}_i(t) = f_i(x_i, t) + u_i(t) + \underline{d}_i(t), \quad t \geq 0 \\ y_i(t) = g_i(x_i) \\ \text{Disturbance} \end{array} \quad (2)$$

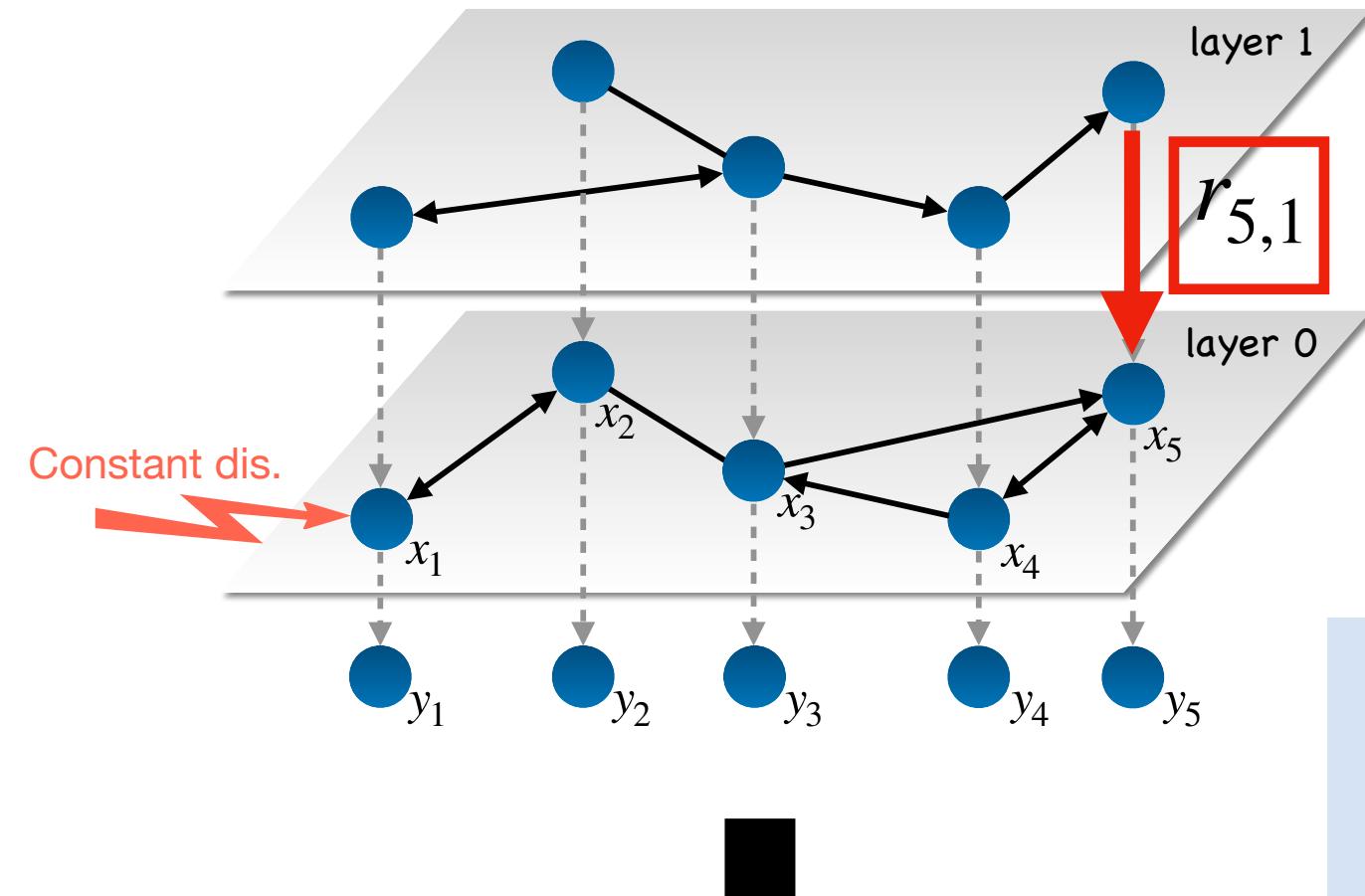
where the disturbance satisfies

$$d_i(t) = \underline{w}_i(t) + \overline{\bar{d}}_i(t) := w_i(t) + \sum_{k=0}^{m-1} \bar{d}_{i,k} \cdot t^k, \quad (3)$$

Polynomial disturbances  
 $\underline{w}_i(t)$        $\overline{\bar{d}}_i(t)$   
Residual disturbance

$\bar{d}_{i,k}$ : constant vector

# Multiplex control protocol



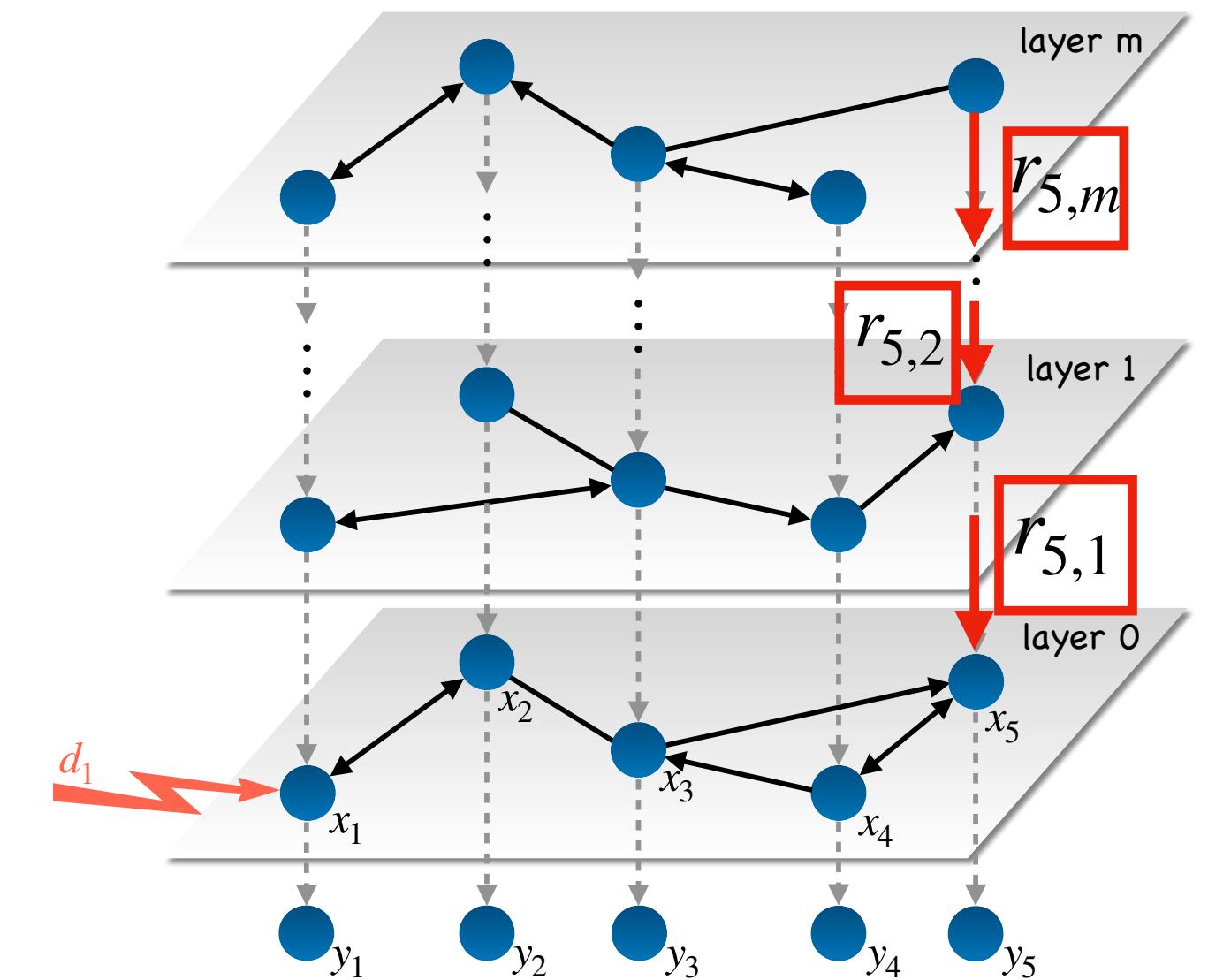
Note:

- Topology on each layer is **independent** on each other

The control is:

$$u_i(t) = h_{i,0}(x(t), x_l(t), t) + h_{i,0}^{(\tau)}(x(t - \tau(t)), x_l(t - \tau(t)), t) + r_{i,1}(t)$$

$$\dot{r}_{i,1}(x, t) = h_{i,1}(x(t), x_l(t), t) + h_{i,1}^{(\tau)}(x(t - \tau(t)), x_l(t - \tau(t)), t)$$



The control is:

$$u_i(t) = h_{i,0}(x(t), x_l(t), t) + h_{i,0}^{(\tau)}(x(t - \tau(t)), x_l(t - \tau(t)), t) + r_{i,1}(t)$$

$$\dot{r}_{i,1}(x, t) = h_{i,1}(x(t), x_l(t), t) + h_{i,1}^{(\tau)}(x(t - \tau(t)), x_l(t - \tau(t)), t) + r_{i,2}(t)$$

⋮

$$\dot{r}_{i,m}(x, t) = h_{i,m}(x(t), x_l(t), t) + h_{i,m}^{(\tau)}(x(t - \tau(t)), x_l(t - \tau(t)), t)$$

## Adaptation of the scalability definition

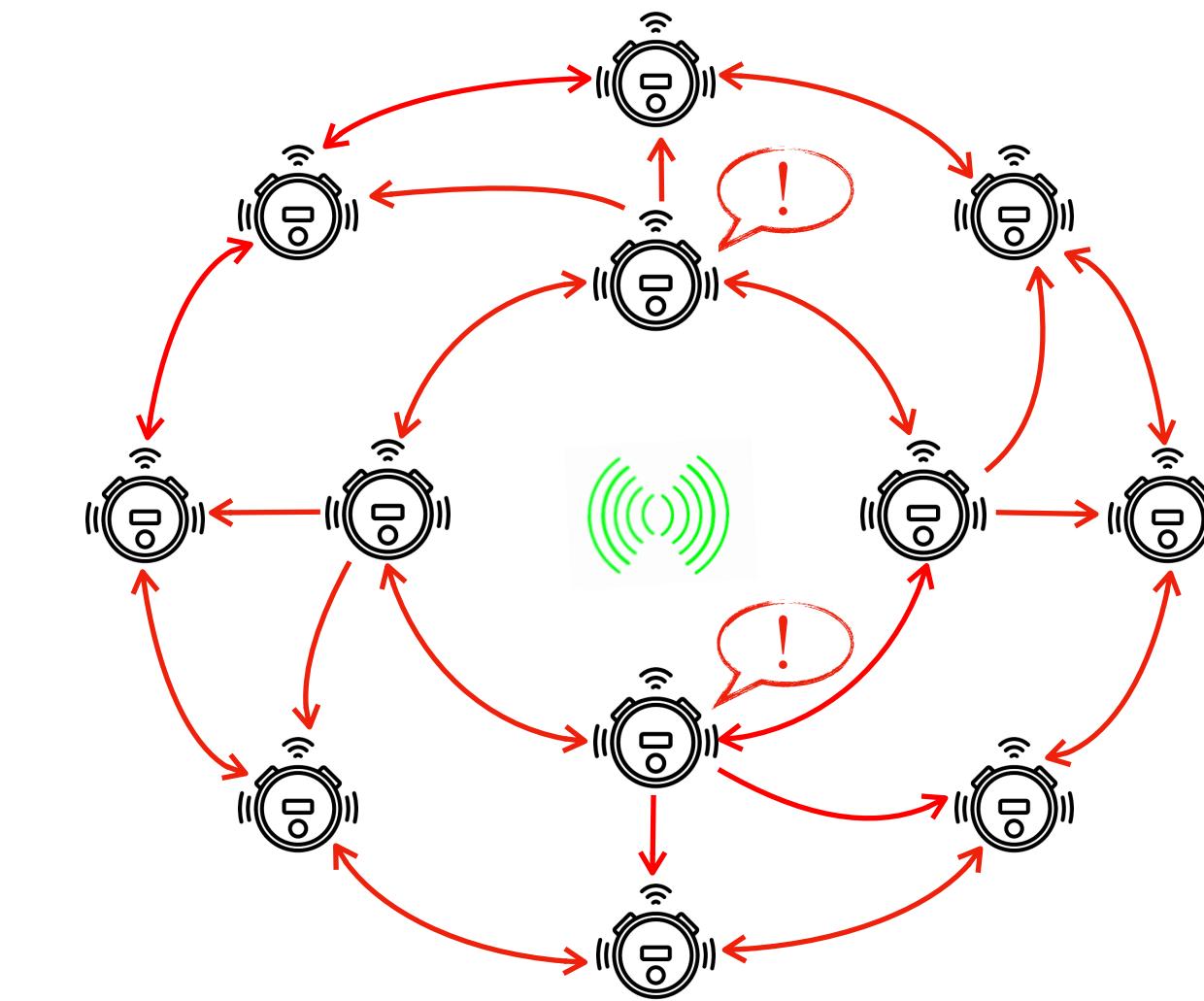
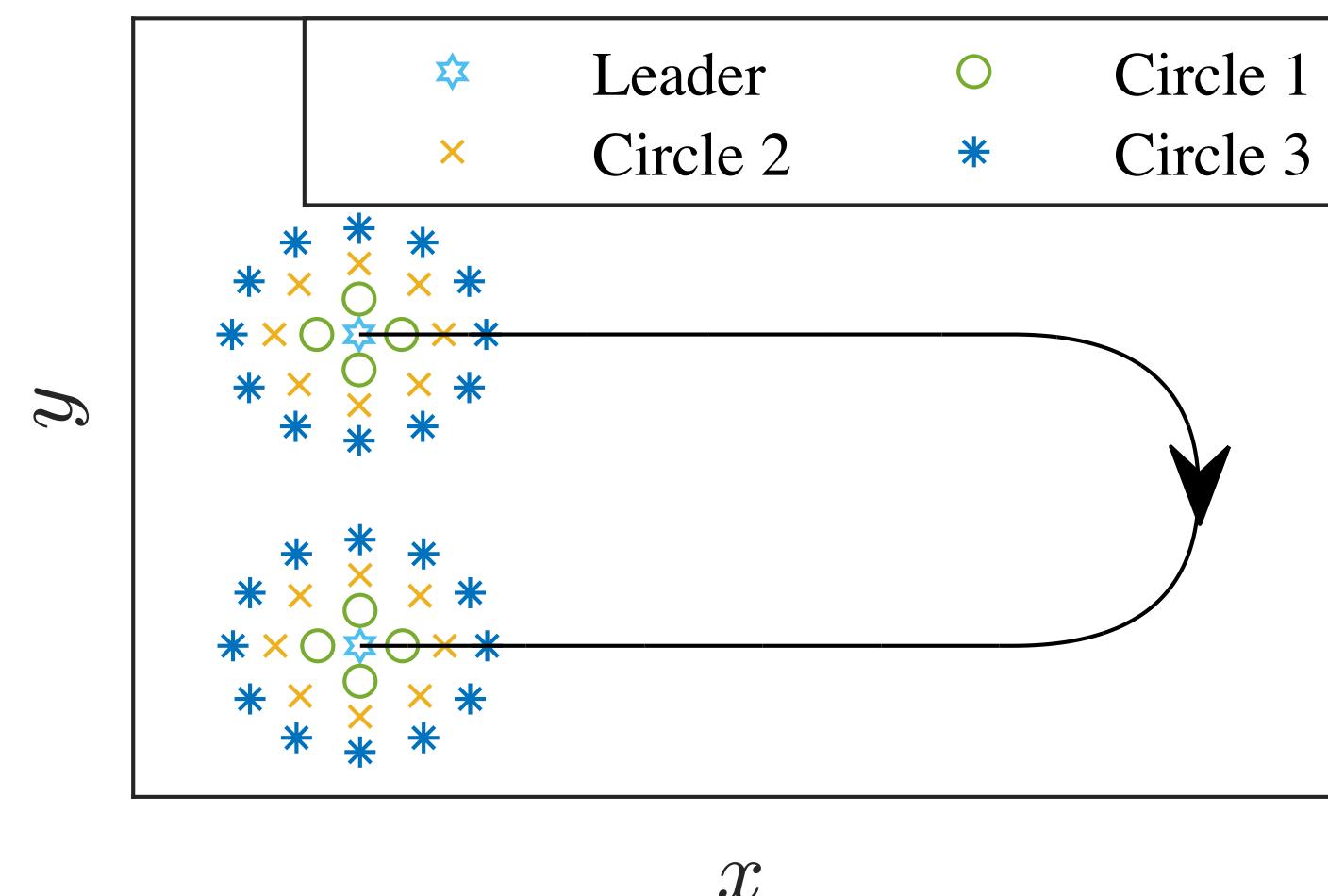
**Definition 2:** Consider network (2) affected by disturbance (3). The system is  $\mathcal{L}_\infty^p$ -**Input-to-State Scalable** if there exists class  $\mathcal{KL}$  function,  $\alpha, \beta$ , a class- $\mathcal{K}$  function,  $\gamma$ , such that, for any initial conditions and  $\forall t \geq t_0$ , we have,

$$\max_i \|x_i(t) - x_i^*(t)\|_p \leq \alpha \left( \max_i \sup_{-\tau_{\max} \leq s \leq t_0} \|x_i(s) - x_i^*(s)\|_p, t - t_0 \right) + \beta \left( \max_i \sup_{-\tau_{\max} \leq s \leq t_0} \sum_{j=0}^m \|r_{i,j}(s) + \bar{d}^{(j)}(t)\|_p, t - t_0 \right) + \gamma \left( \max_i \|w_i(\cdot)\|_{\mathcal{L}_\infty^p} \right), \forall N;$$

- ▶ Def. 2 requires the effect of the polynomial disturbances be attenuating
- ▶ Building on our framework, sufficient conditions to assess this property are derived...

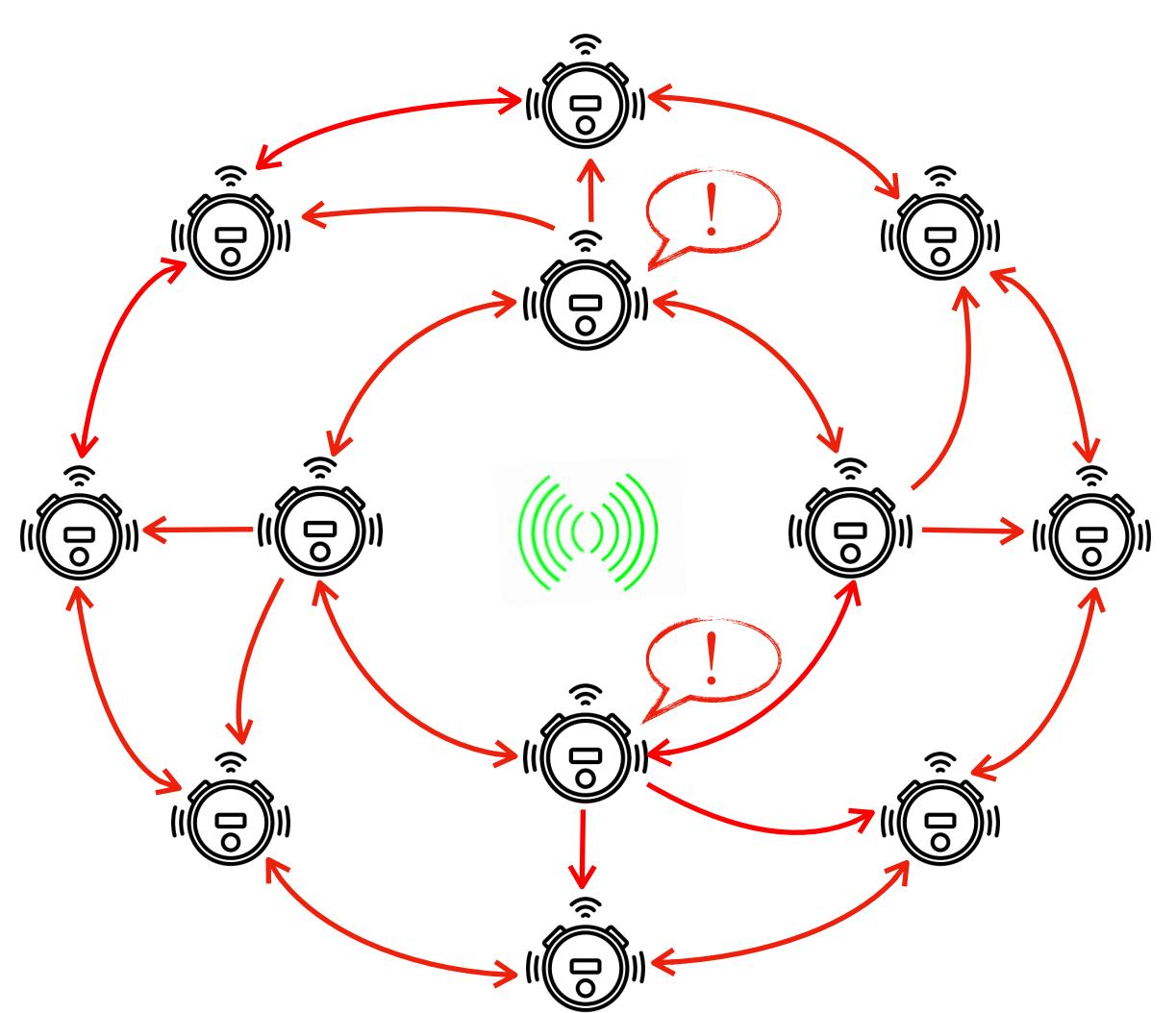
# Application: robotic formation control (problem setup)

- Formation pattern: 14 concentric circles, circle  $i$  has  $4*i$  unicycle robots.
- Topology: robots are connected to its neighbours on same layer, and closest robot on inner circle (if any).
- Desired solution/control goal:**
  - follow a trajectory provided by leader
  - Keep desired offset from each other

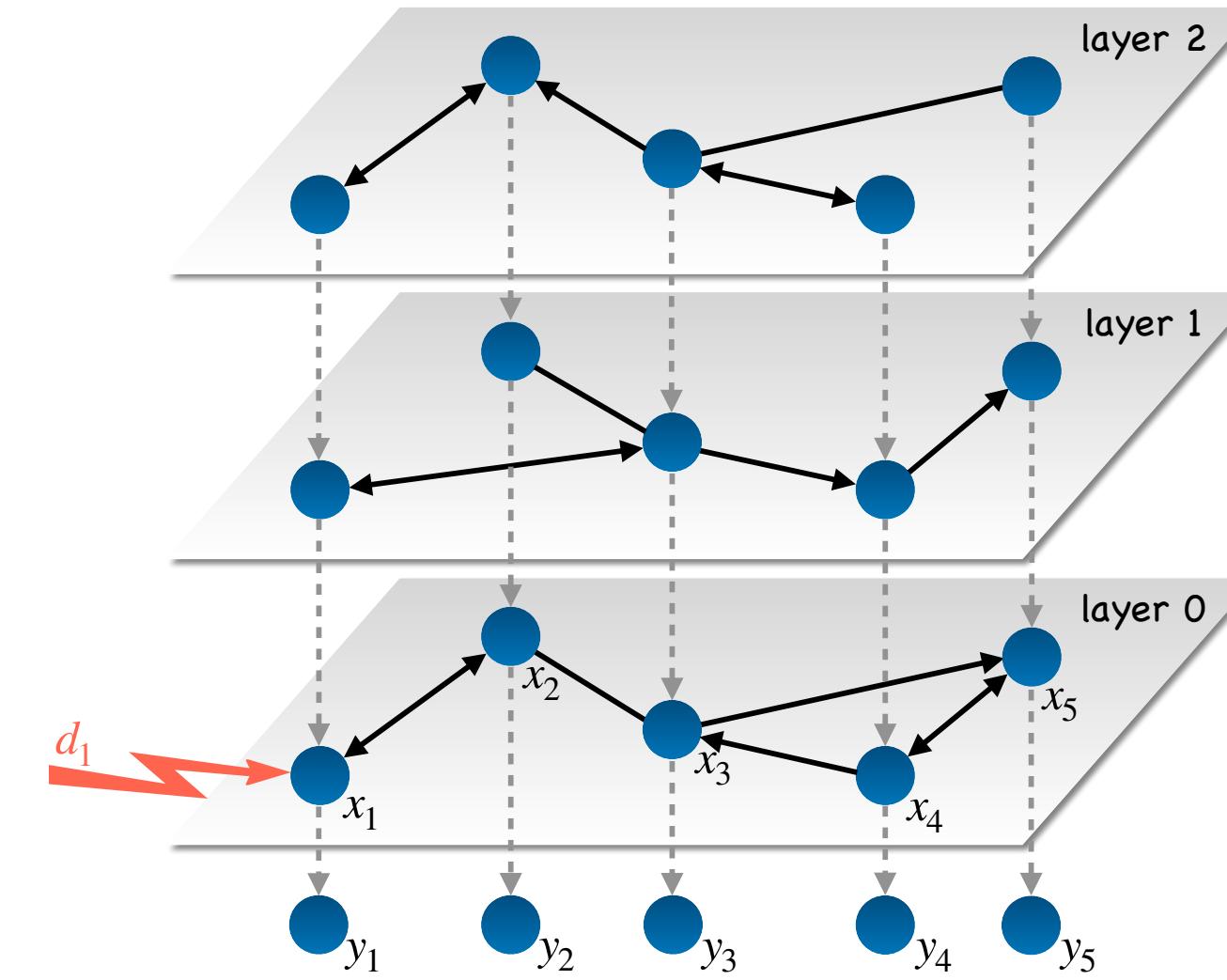
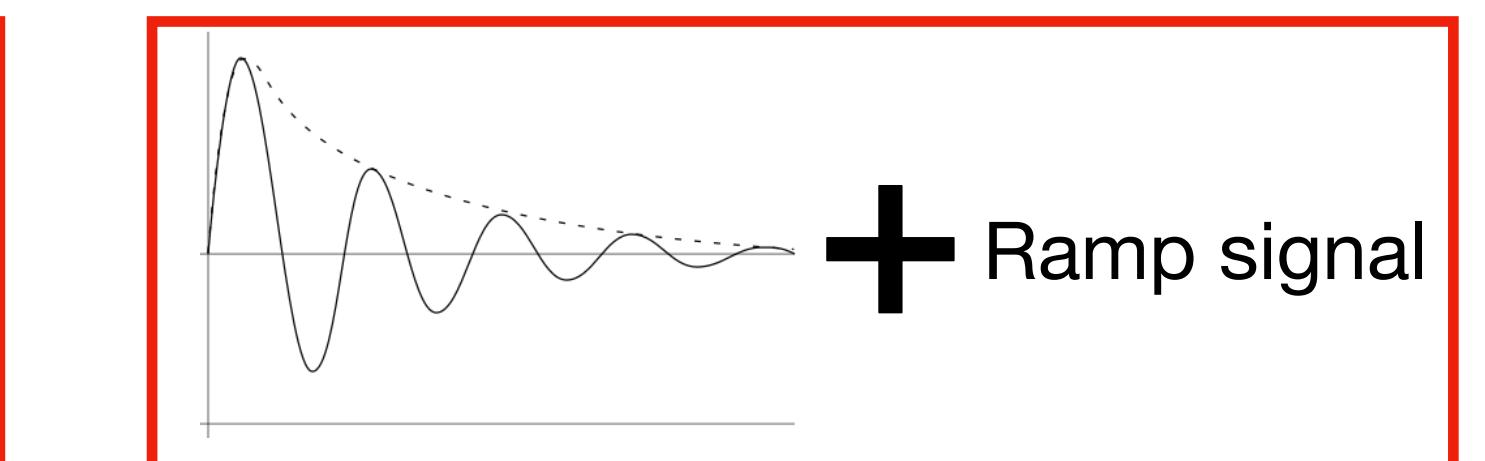
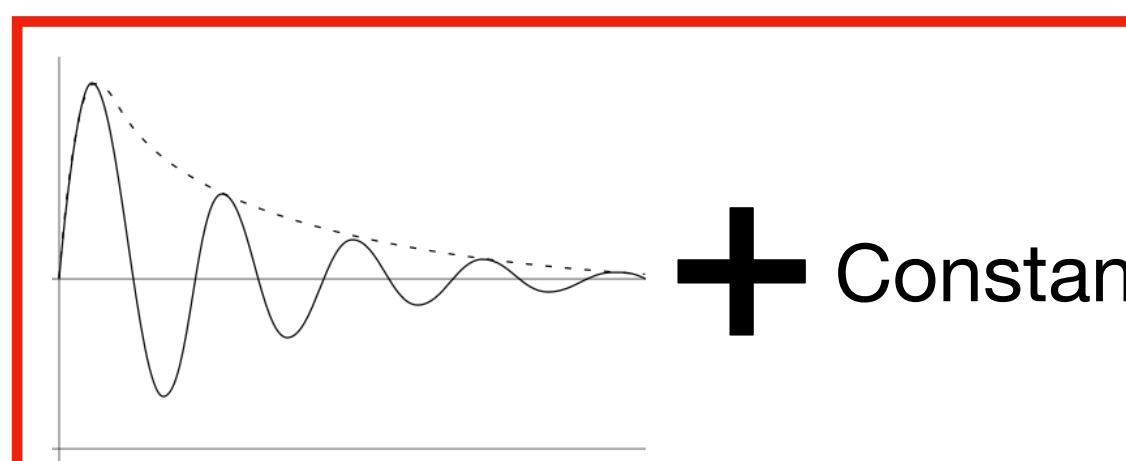


[Xie, S. et al. ArXiv, 2022]

# Application: robotic formation control (control design)



- Reference signal  $x_l(t)$ ;
- Delayed communication;
- External disturbances



The control should be:

$$u_i(t) = h_{i,0}(x(t), x_l(t), t) + h_{i,0}^{(\tau)}(x(t - \tau(t)), x_l(t - \tau(t)), t) + r_{i,1}(t)$$

$$\dot{r}_{i,1}(x, t) = h_{i,1}(x(t), x_l(t), t) + h_{i,1}^{(\tau)}(x(t - \tau(t)), x_l(t - \tau(t)), t) + r_{i,2}(t)$$

$$\dot{r}_{i,2}(x, t) = h_{i,2}(x(t), x_l(t), t) + h_{i,2}^{(\tau)}(x(t - \tau(t)), x_l(t - \tau(t)), t)$$

**How many layers at least  
should we use?**

# Application: robotic formation control (hardware description)

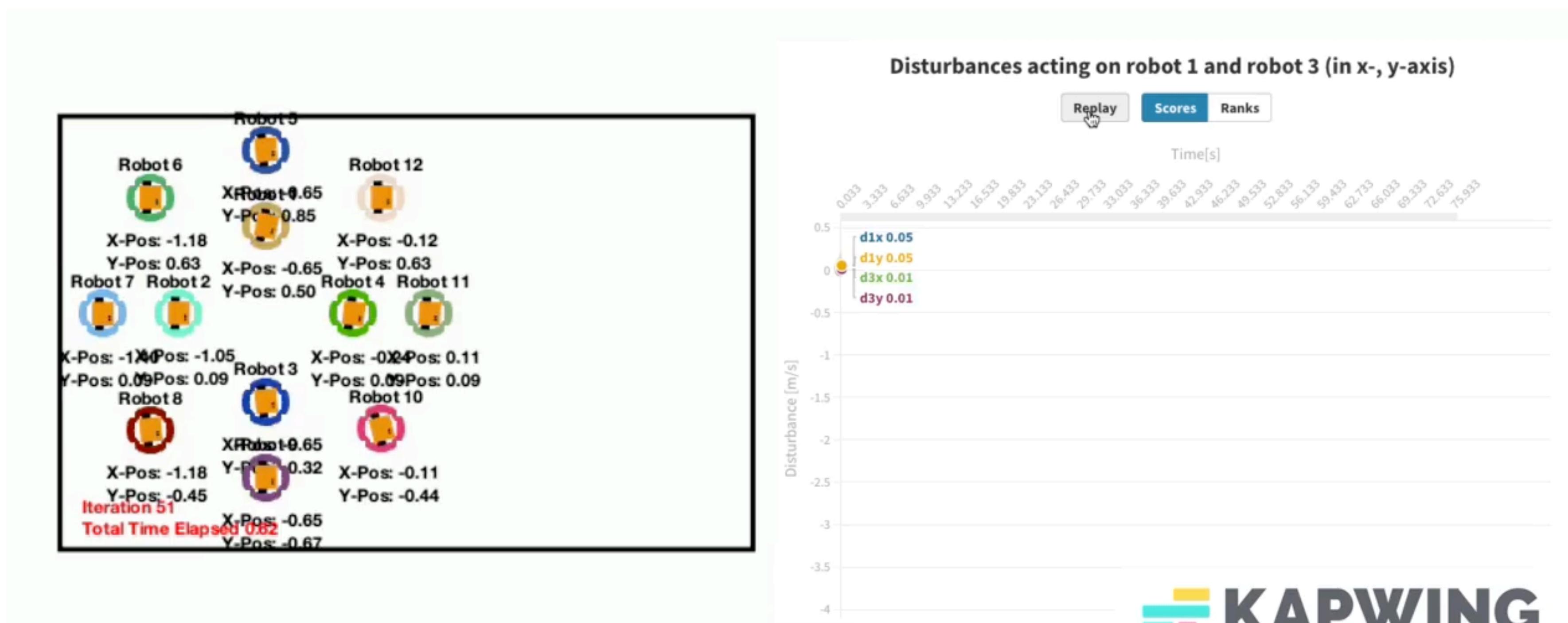


[Sean Wilson, et al., IEEE Control Systems Magazine, 2020.]

## Experimental setup:

- ▶ There is a **central computer** which collects the information and sends the control input to each individual robot. This process takes **approximately 0.033 seconds**.

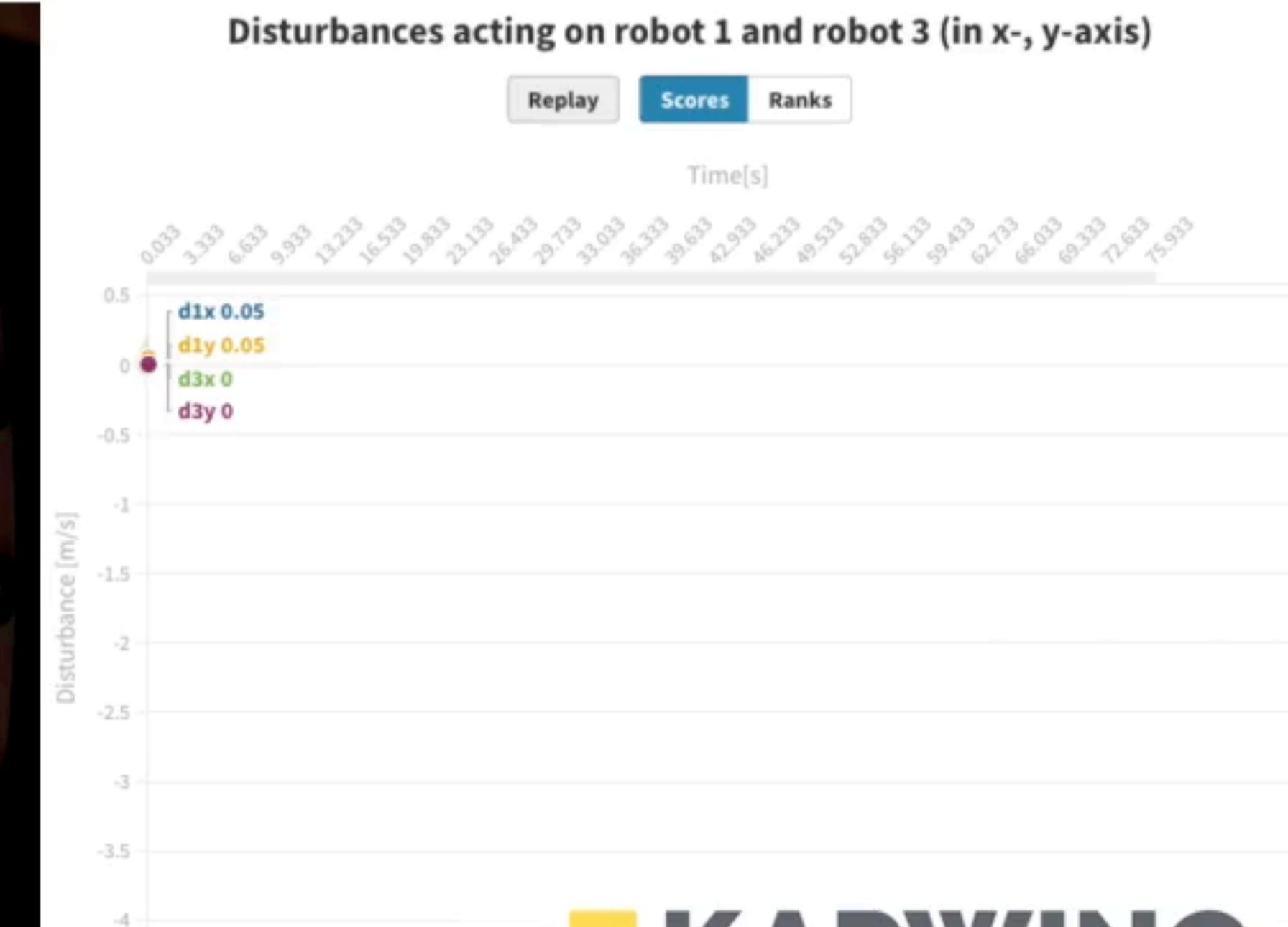
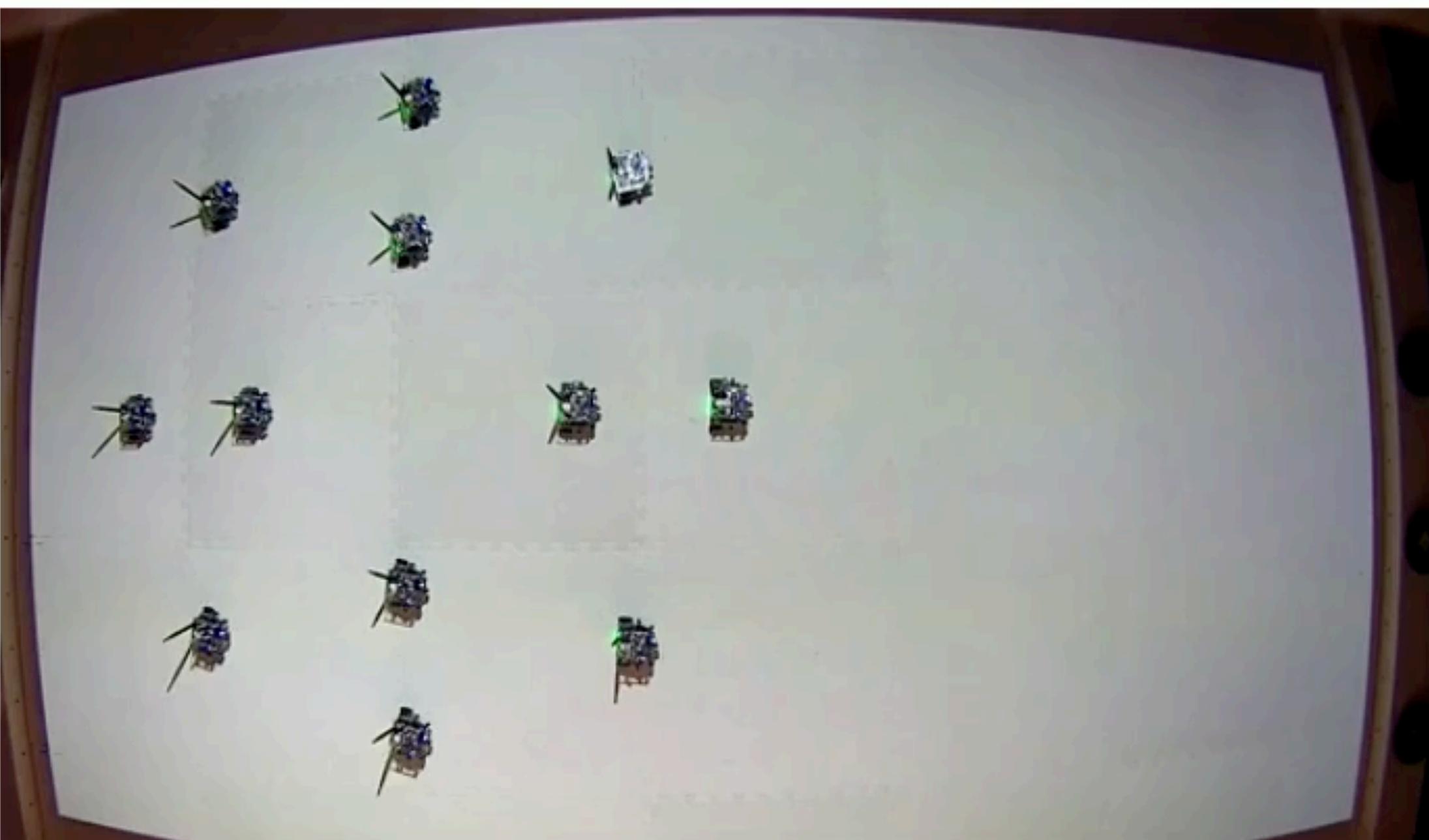
# Application: robotic formation control (high-fidelity simulator)



 KAPWING

# Application: robotic formation control (hardware experiment)

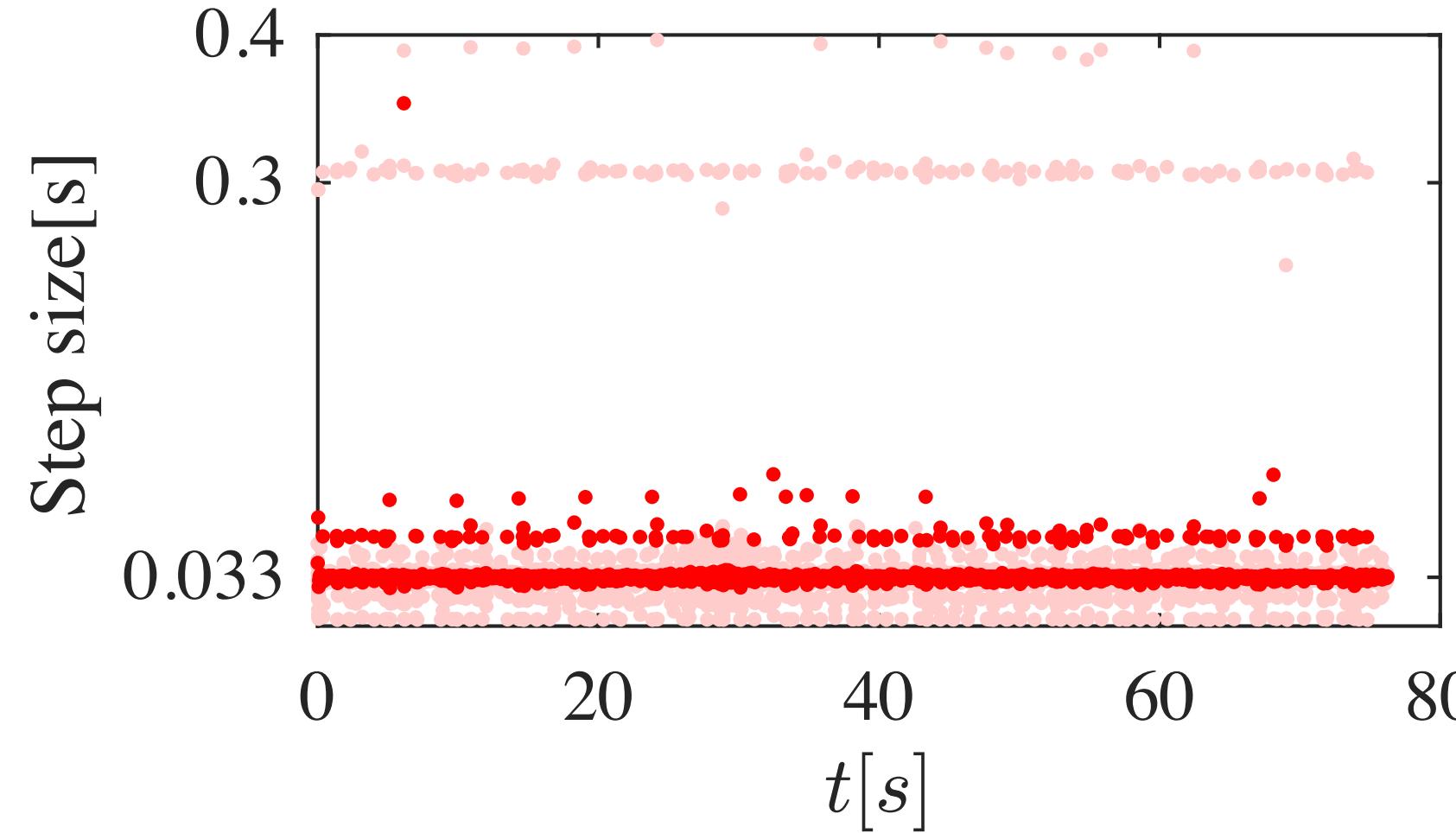
Why are we observing the **oscillations**?



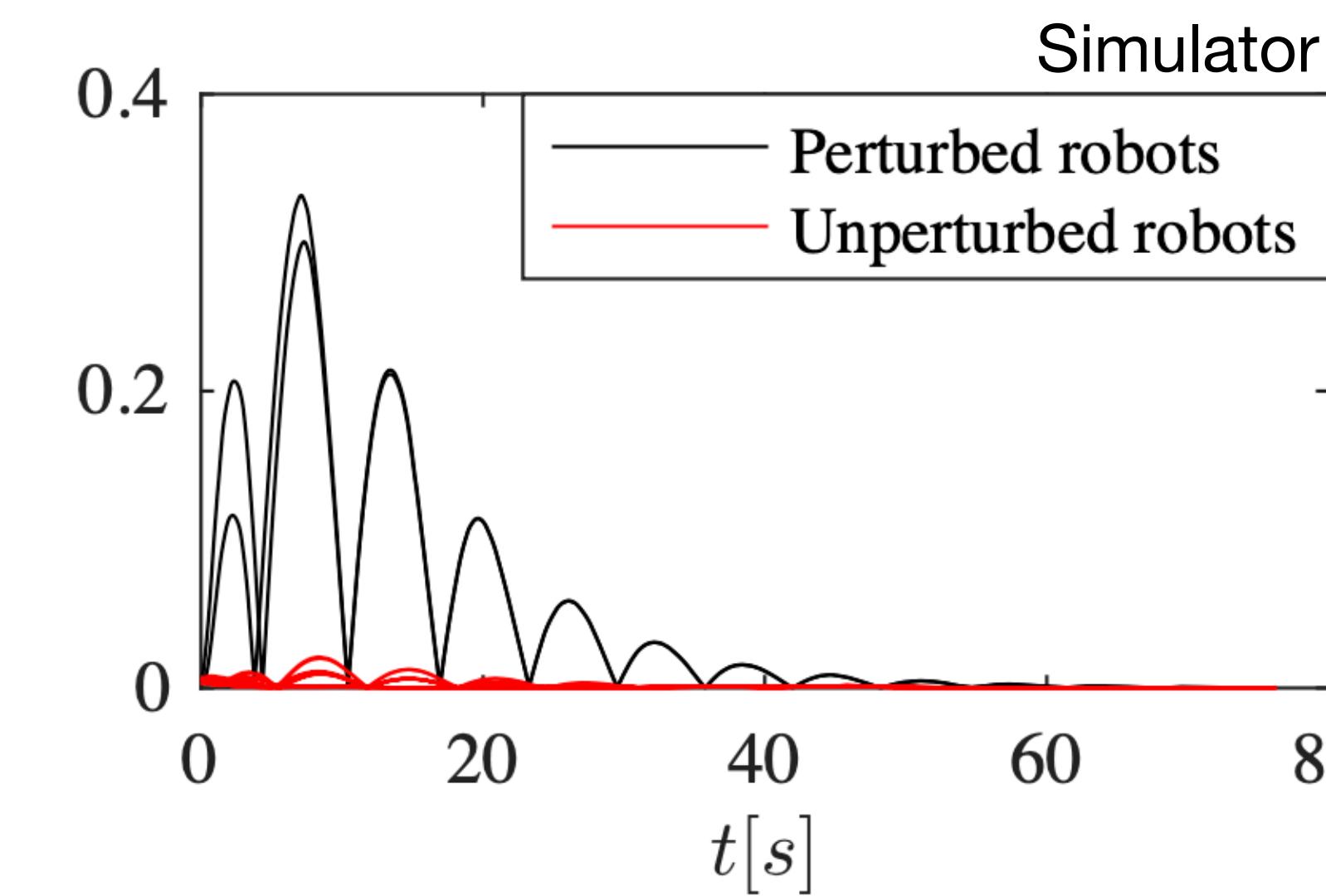
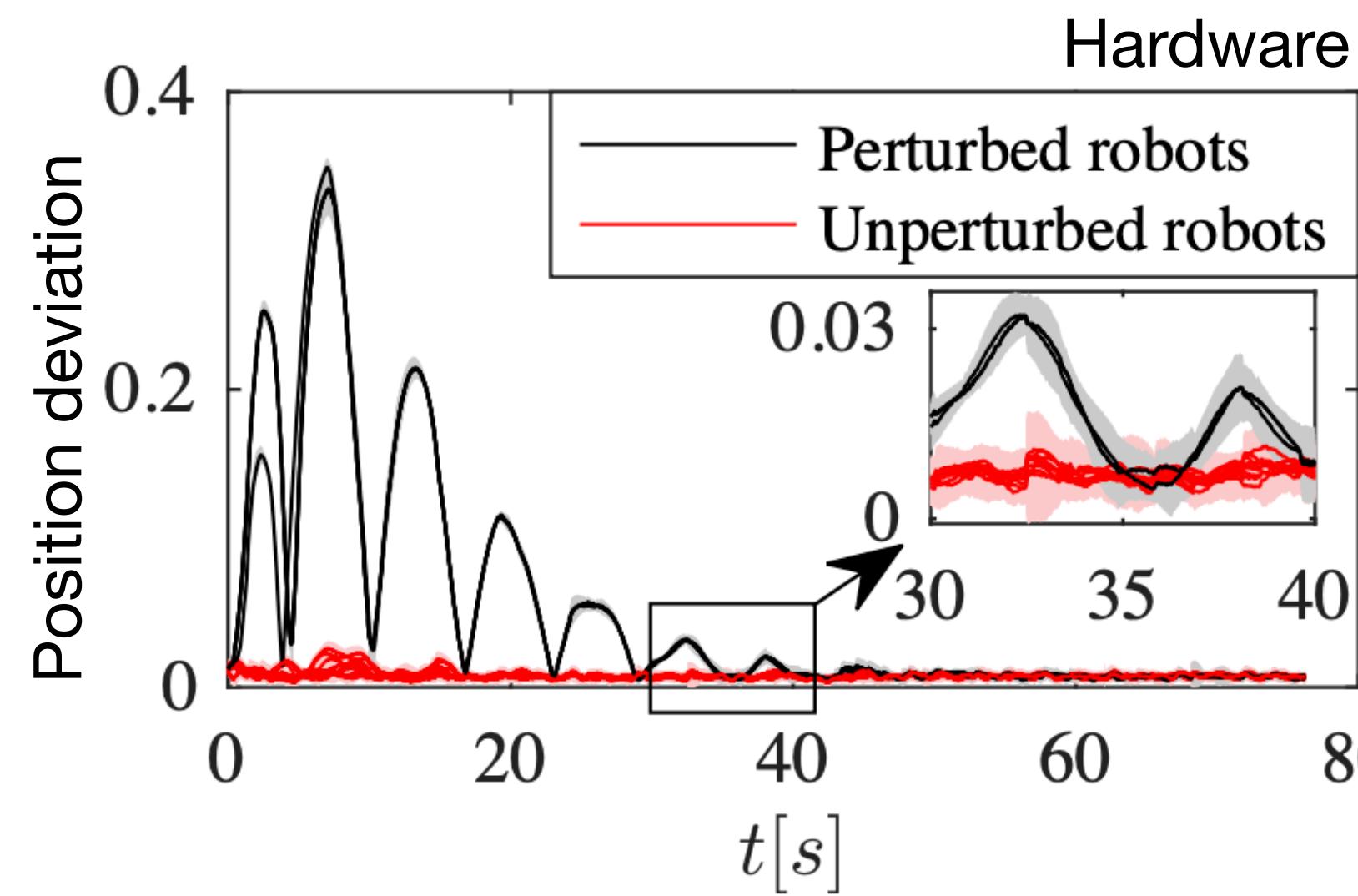
 KAPWING

# Application: robotic formation control (conclusions)

[Xie, S. et al. ArXiv, 2022]



- The **inconsistent step size** during hardware implementation leads to the robot oscillation behaviour.
- The position deviation obtained from the hardware experiments matches the results from the high-fidelity simulator.
- Polynomial disturbance is rejected. Residual disturbances are not amplified.



# Presentation outline

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- Motivation and Challenges
- Contributions and Publications
- Setting up the Problem: Scalability
- Rejecting Polynomial Disturbances: Multiplex Architecture
- Scalability for COVID-19 Mitigation
- Conclusion and Future Plan

2021 60th IEEE Conference on Decision and Control (CDC)  
December 13-15, 2021. Austin, Texas

Intermittent non-pharmaceutical strategies to mitigate the COVID-19 epidemic in a network model of Italy via constrained optimization

Marco Coraggio<sup>\* 1</sup>, Shihao Xie<sup>\* 2</sup>, Francesco De Lellis<sup>1</sup>, Giovanni Russo<sup># 3</sup>, Mario di Bernardo<sup># 1</sup>

*Abstract*—This paper is concerned with the design of intermittent non-pharmaceutical strategies to mitigate the spread of the COVID-19 epidemic exploiting network epidemiological models. Specifically, by studying a variational equation for the dynamics of the infected, we derive, using contractivity arguments, a condition that can be used to guarantee that the effective reproduction number is less than unity. This condition (i) is easily computable, (ii) is interpretable, being directly related to the model parameters, and (iii) can be used to enforce a scalability condition that prohibits the amplification of disturbances within the network system. We then include satisfaction of such a condition as a constraint in a Model Predictive Control problem so as to mitigate (or suppress)

[7], several feedback intervention strategies are presented, whereas in [8] the benefits were shown of adopting feedback intermittent regional control strategies triggered by occupancy levels of the intensive care units (ICUs) in each region. Furthermore, several epidemic control strategies were recently proposed, based on model predictive control (MPC). These include [9], where MPC is used to decide whether to enforce social distancing in a region/country of interest, using a SIRASDC model (*S*: susceptible, *I*: infected, *R*: recovered, *A*: asymptomatic, *S*: symptomatic, *D*: dead, *C*: with control) and including hard constraints on ICUs occupancy and min-

# Collaboration

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Marco Coraggio



Francesco De Lellis



Giovanni Russo

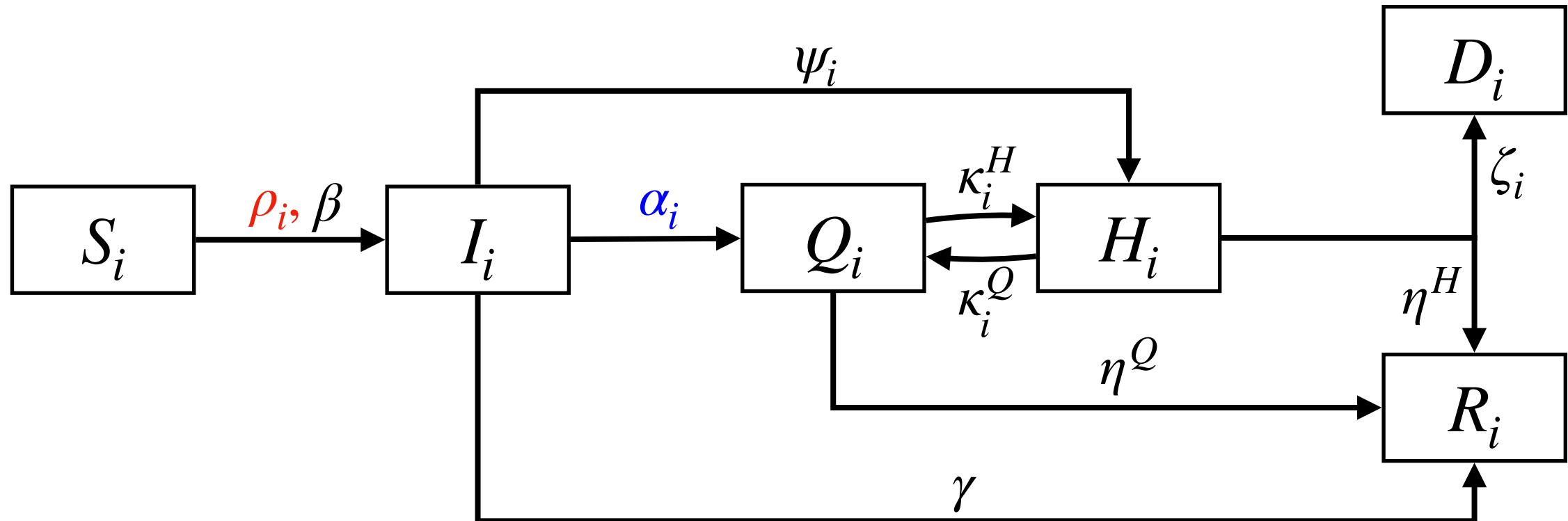


Mario di Bernardo

# Problem setup

## Model (SIQHDR)

S: Suspected; I: Infected, R: Recovered, Q: Quarantined, H: Hospitalised, D: Deceased



$$S_i(t+1) = S_i(t) - \beta S_i(t) \sum_{j=1}^M \frac{\rho_j(t) \phi_{ij}(t)}{N_j^p(t)} \sum_{k=1}^M \phi_{kj}(t) I_k(t),$$

$$I_i(t+1) = I_i(t) + \beta S_i(t) \sum_{j=1}^M \frac{\rho_j(t) \phi_{ij}(t)}{N_j^p(t)} \sum_{k=1}^M \phi_{ki}(t) I_k(t) - (\gamma + \alpha_i(t) + \psi_i) I_i(t),$$

$$Q_i(t+1) = Q_i(t) + \alpha_i(t) I_i(t) - (\kappa_i^H + \eta_i^Q) Q_i(t) + \kappa_i^Q H_i(t),$$

$$H_i(t+1) = H_i(t) + \kappa_i^H Q_i(t) + \psi_i I_i(t) - \left( \eta_i^H + \kappa_i^Q + \zeta(H_i(t)) \right) H_i(t),$$

$$D_i(t+1) = D_i(t) + \zeta(H_i(t)) H_i(t),$$

$$R_i(t+1) = R_i(t) + \gamma I_i(t) + \eta_i^Q Q_i(t) + \eta_i^H H_i(t),$$



### Goal:

- **Mitigation:** to keep the epidemic within a **controllable level**, i.e. health care systems operating within capacity.

**How:** Intermittent control strategies (**social distancing  $\rho$** , **travel restrictions  $\phi_{ij}$** , **testing  $\alpha$** )

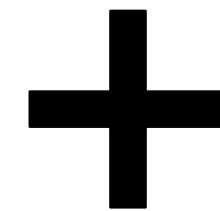
# Setting up the MPC

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- **Goal:** minimise the economic cost caused by the intervention strategies

- **Constraints:**

- ▶ SIQRDH model;
- ▶ The strength of the intervention strategies should be selected from a ‘pool’;
- ▶ The strategies should last for at least a minimum dwell time.



- ▶ **Scalability condition** → the **maximum number of infected** across all regions be bounded and **decrease to 0**

- Triggered only when: (1) **occupancy of ICU**, or (2) **effective reproduction number**, reach some threshold.

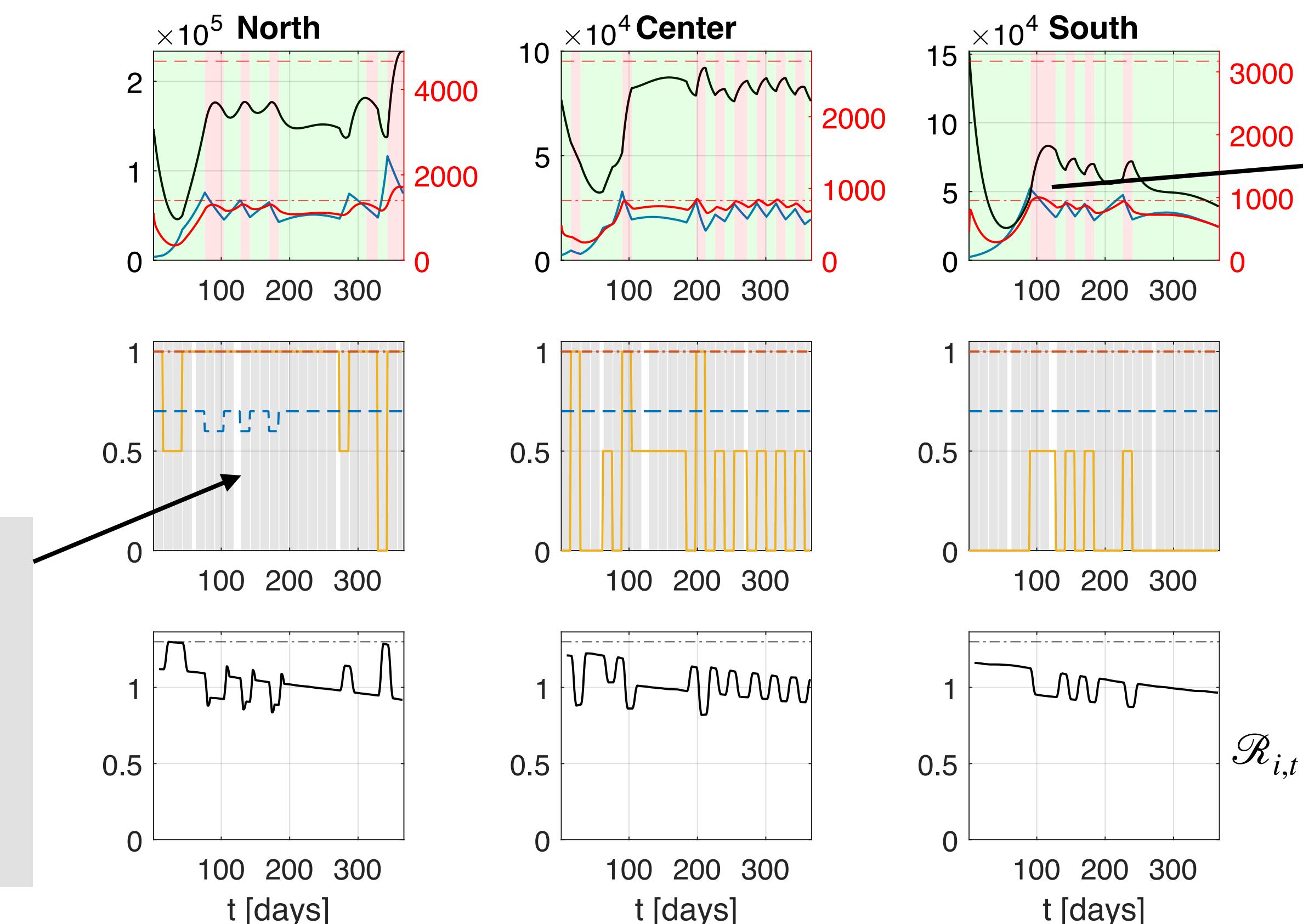
# Simulation results

[Coraggio, M. et al. . IEEE CDC, 2021]

- Economic cost:

$$\text{€ } 262.379 \cdot 10^9$$

- Blue: social distancing  
(1=open, 0=closed)
- Orange: travel restrictions  
(1=open, 0=closed)
- Gold: testing (1=a lot, 0=few)



- Black:  $Q_i$
- Blue:  $I_i$
- Red:  $0.1H_i$

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# Conclusion

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## Theoretical:

- We formalised the notion of scalability
- We designed multiplex integral control protocols to reject polynomial disturbances and ensure scalability w.r.t. residual disturbances
- We validated our results via simulation

## Practical:

- We applied our designed control protocols to hardware robots.
- We used scalability to derive conditions for COVID-19 mitigation

# Future plan

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## Theoretical:

- Investigate the conservativeness of the conditions, e.g. investigating conditions for the loss of system scalability
- Derive conditions for the design of scalable RNNs which is able to deal with sequence of inputs
- Extend the results to time-scale dynamics
- Extend the results to systems with stochasticity

## Practical:

- We can further investigate the design of control protocols in hardwares, taking into consideration the stochasticity
- We can consider the robustness of the intervention strategies to model uncertainties, measurement error etc.



**Thank you for your attention.**