Repulsion and total reflection with mismatched three-wave interaction of noncollinear optical beams in quadratic media

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The phenomenon of the total reflection of a weak signal beam from a high-power reference beam due to noncollinear mismatched parametric interaction in a quadratic medium is demonstrated. In a planar geometry the conditions of the signal beam reflection from the optical inhomogeneity induced by the reference beam are found. The analytical expression for the critical value of the signal beam tilt is obtained, and it is shown that total reflection occurs if the initial tilt is less than the critical value. The influence of the walk-off and the reference beam focusing on interaction dynamics are discussed. It is shown that in a bulk medium the beam reflection turns into scattering by an induced inhomogeneity. If reflection conditions are fulfilled, the cylindrical reference beam acts as a convex mirror and lunate distortions of the reflected beam profile occur.

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I. INTRODUCTION

Light-by-light control in nonlinear media is of particular interest in nonlinear optics and photonics nowadays [1–7]. In this paper we investigate the effect of repulsion and total reflection of optical beams in a mismatched quadratic nonlinear medium. Such a phenomenon is as follows: A phase mismatch transforms the three-wave interaction into a cascading mechanism of parametric interaction [8–10] when a high-power reference beam creates the effective inhomogeneity of the refractive index for the weak signal wave. The tilted signal beam propagating through an induced inhomogeneity undergoes refraction, so the signal beam trajectory becomes curved and total reflection of signal beam from the reference beam can occur. The elaborated theory of beam-beam interaction shows that the nonlinearly induced reflection emerges if the cascading cubiclike nonlinearity is defocusing and the signal beam tilt is less than a critical value. At the parametric reflection, the pump beam also experiences repulsion and is slightly deflected in the opposite direction. This phenomenon is the spatial analog to nonlinear reflection of pulses colliding in nonlinear dispersive media [11,12].

It is important to note that if total reflection conditions are satisfied the reference beam becomes opaque for the signal wave and acts as a mirror. Therefore after reflection from the cylindrical reference beam a narrow signal beam diverges; additionally, a wide beam surrounds the reference beam and forms typical diffraction patterns.

Beam reflection also can be realized in the solitonic regime. Parametric solitons exist due to three-wave interactions in quadratic media [13–15]. They can repulse and attract each other depending on the phase difference [16–20], but such a method requires beam trapping into solitons and a careful control of phase matching that is not always convenient and achievable.

Also, in this case a reflected wave keeps its frequency, in contrast to the cases described in [21,22] where reflection is accompanied by the efficient frequency conversion.

It should be mentioned that we have previously studied similar effects in media with a defocusing photorefractive and thermal nonlinearity [23,24]. However, quadratic nonlinear media have the advantages of ultrafast nonlinear response and of the absence of reference beam defocusing.

In this work, we found the basic laws of repulsion and total reflection of the beams in quadratic mismatched media by numerical simulation and by the ray optics approach.

II. THEORY OF THE NONCOLLINEAR MISMATCHED THREE-WAVE PARAMETRIC INTERACTION

Consider the dynamics of a three-wave interaction of a high-power reference beam at frequency ω_1, a weak signal beam at frequency ω_2, and a sum wave at frequency $\omega_3 = \omega_2 + \omega_1$. The interaction of wave beams with small divergence and propagating at small angles to the selected axis is well described by introducing the slowly varying amplitudes A_j as $E_j(z,t) = 1/2\{A_j(z,t)\exp[i(\omega_j t - \vec{k}_j \vec{r})] + \text{c.c.}\}$. The slowness of change in amplitude means that the amplitude or envelope of the wave beam varies little at a distance of a wavelength. Using a standard approach [1–3] one can obtain from Maxwell's equations with the quadratic nonlinearity the following system of equations for the complex amplitudes:

$$\begin{aligned} \frac{\partial A_1}{\partial z} + iD_1\Delta_\perp A_1 &= -i\gamma_1 A_3 A_2^* \exp(i\hat{k}z), \\ \frac{\partial A_2}{\partial z} + iD_2\Delta_\perp A_2 &= -i\gamma_2 A_3 A_1^* \exp(i\hat{k}z), \\ \frac{\partial A_3}{\partial z} + iD_3\Delta_\perp A_3 &= -i\gamma_3 A_1 A_2 \exp(-i\hat{k}z). \end{aligned} \tag{1}$$

Here $D_j = (2k_j)^{-1}$ is the diffraction coefficient ($j = 1, 2, 3$), $k_j = |\vec{k}_j| = n_{j0}\omega_j/c$ is the corresponding wave number, n_{j0} is the linear refractive index at the frequency ω_j, γ_j is the quadratic nonlinearity coefficient, $\hat{k} = k_{1z} + k_{2z} - k_{3z}$ is the wave-vector mismatch, and k_{jz} is the z component of the wave vector $\vec{k}_j$. The equations (1) contain oscillating factors arising from the wave mismatch. This factor can be removed by including it in the amplitude of any beam. We

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normalize the sum-frequency beam: $A_3 = \bar{A}_3 \exp(i\hat{k}z)$, and rewrite Eq. (1) in the form

$$\begin{aligned} \frac{\partial A_1}{\partial z} + iD_1\Delta_\perp A_1 &= -i\gamma_1 \bar{A}_3 A_2^*, \\ \frac{\partial A_2}{\partial z} + iD_2\Delta_\perp A_2 &= -i\gamma_2 \bar{A}_3 A_1^*, \\ \frac{\partial \bar{A}_3}{\partial z} + iD_3\Delta_\perp \bar{A}_3 &= i\hat{k}\bar{A}_3 - i\gamma_3 A_1 A_2. \end{aligned} \tag{1a}$$

Let the high-power reference beam propagate along the z axis and have the input envelope $A_1(z=0) = E_1 \exp(-x^2/a_1^2)$; the Gaussian signal beam is tilted at a small angle $\theta_2 \ll 1$, $A_2(z=0) = E_2 \exp(-(x-x_0)^2/a_2^2 + ik_2\theta_2 x)$, and has the smaller amplitude $E_2 \ll E_1$. The sum-frequency wave is absent at the entrance, and therefore its initial amplitude is zero: $\bar{A}_3(z=0) = 0$.

Due to the parametric generation [see the third equation in Eq. (1a)], the sum beam copies the transverse momentum of the signal beam: $k_2 \sin(\theta_2) = k_3 \sin(\theta_3)$ or, taking into account that $\theta_2 \ll 1$, $k_2\theta_2 = k_3\theta_3$.

Thus we obtain the following expression for the wave-vector mismatch:

$$\begin{aligned} \hat{k} &= k_1 + k_2\cos\theta_2 - k_3\cos\theta_3 \approx (k_1 + k_2 - k_3) \\ &\quad -(k_3 - k_2)k_2\theta_2^2/(2k_3) = \hat{k}_m - (k_1 - \hat{k}_m)k_2\theta_2^2/(2k_3), \end{aligned} \tag{2}$$

where $\hat{k}_m = k_1 + k_2 - k_3$ is the wave-vector mismatch in collinear geometry due to dispersion. Note that only the first term of the mismatch is included in the Eqs. (1) directly, while the second part arises from the inclination of the signal beam at the entrance to the medium. Let us assume that $\hat{k}_m = 0$; this condition may be satisfied with the interaction of ordinary and extraordinary waves [25,26] or in periodically polled media [27–29]. Using this condition, one gets $\hat{k} \approx -k_1k_2\theta_2^2/(2k_3)$.

The theory is specified further to cascading interaction at significant values of the effective mismatch [8–10]. In that case the efficiency of sum-frequency generation becomes low. If the widths of the beams are significant and the considered distances are smaller than the diffractive length ($z < k_1a_1^2/2$, $z < k_2a_2^2/2$), the diffractive effects are weak also. So, the equation for the sum frequency contains the large parameter $\hat{k}$ in comparison with the terms in the left-hand side of the equation, and one can simplify it using the asymptotic method [30]. Expanding the amplitude $\bar{A}_3$ into a power series of $1/\hat{k}$ and substituting this expansion to the third equation of Eqs. (1a) one can obtain the simple algebraical equation in the form $\hat{k}\bar{A}_3 = \gamma_3 A_1 A_2$. Here we use only the first term of the expansion. Thus, the obtained expression shows that the sum-frequency wave is generated locally with small amplitude $\bar{A}_3 \approx (\gamma_3/\hat{k})A_1A_2$. Such an approach is widely used for analysis of mismatched parametric processes [9,10]. Further, its validity will be confirmed by the results of our numerical simulations of Eq. (1).

So, substituting this expression to the equation for the signal wave, we obtain the following equation:

$$\frac{\partial A_2}{\partial z} + iD_2\Delta_\perp A_2 = -ik_2 n_{nl} A_2, \tag{3}$$

$$n_{nl} = (\gamma_2\gamma_3/(k_2\hat{k}))|A_1(x,y,z)|^2. \tag{4}$$

Equation (3) describes beam propagation in the medium with parametrically induced inhomogeneity. The sign of the induced inhomogeneity is defined by the sign of the effective wave-vector mismatch, and its profile repeats the reference beam intensity distribution.

Thus in the cascade model of three-wave interaction the behavior of a signal wave is described by only one equation, Eq. (3), which greatly facilitates the analysis of beam interplay.

III. RAY THEORY OF SIGNAL BEAM REFLECTION

The paraxial Eq. (3) describes the propagation of a light beam in the weakly inhomogeneous medium. In such media the beam is deflected from its original direction. The curved trajectory of the signal beam can be found by solving the wave Eq. (3). Along with numerical simulation it is very useful to apply the analytical methods of constructing the trajectory, such as geometrical optics of inhomogeneous media [31].

The flow of energy long the path is directed normal to the wave surface. Therefore, the complex amplitude of the signal beam can be represented by the amplitude profile of B_2 and the phase front S_2 in the form $A_2 = B_2 \exp[-ik_2S_2(x,y,z)]$. Let us substitute this expression into Eq. (3), and according to the method of geometrical optics let $\lambda \to 0$ or $k \to \infty$. This means that a transverse size of the induced inhomogeneity equal to the pump beam width is much larger than the signal wavelength. As a result, we obtain the eikonal equation:

$$\frac{\partial S_2}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial S_2}{\partial x}\right)^2 + \left(\frac{\partial S_2}{\partial y}\right)^2\right] = n_{nl}(x,y,z). \tag{5}$$

Applying to Eq. (5) the method of characteristics [32], we find the equations for the trajectory of the signal beam in the following form:

$$\frac{d^2x}{dz^2} = \frac{\partial n_{nl}(x,y,z)}{\partial x}, \quad \frac{d^2y}{dz^2} = \frac{\partial n_{nl}(x,y,z)}{\partial y}. \tag{6}$$

Next, we consider specific examples of the interaction between the signal and pump beams by numerical simulation of quasioptical equations (1) for slowly varying envelopes and the analytical solution of ray equations (6).

IV. BEAM REFLECTION IN PLANAR GEOMETRY

In one-dimensional geometry the trajectory equations (6) can be rewritten as

$$\frac{d^2x}{dz^2} = \frac{\partial n_{nl}(x,z)}{\partial x}. \tag{7}$$

Equation (7) is rather difficult for theoretical analysis, since $A_1 = E_1(x,z)$, $\hat{k} = \hat{k}(z)$ and, consequently, $n_{nl} = n_{nl}(x,z)$. If the reference beam width is large enough, then the diffractive spreading of the reference beam can be neglected; in addition, reference beam intensity is much larger than signal beam intensity, thus the depletion of the reference beam can be also neglected. Thereby the reference beam remains practically undistorted during the interaction and one can set $A_1 = E_1(x)$. Also, to simplify theoretical analysis one can assume that $\hat{k} = \hat{k}(z=0) = \text{const}$ [see Eq. (2)]. The applicability of such simplifications was confirmed by the results of the numerical simulations of Eq. (1). Thus, under such approximation we can consider $n_{nl} = n_{nl}(x)$, and the solution of Eq. (4) can be written as

$$z = z_0 \pm \sqrt{2}\int_{x_0}^{x} \frac{d\tau}{\sqrt{n_{nl}(\tau) - n_{nl}(x_0) + \theta_2^2/2}}. \quad (8)$$

The plus and minus signs correspond to the different parts of the trajectory: before and after the interaction. The trajectory is parallel to the z axis at turning point z_r where $dx/dz = 0$ so that $-[n_{nl}(x) - n_{nl}(x_0)] = \theta_2^2/2$. Using this expression and assuming that the initial distance between beams is rather large so that $n_{nl}(x_0) = 0$, the conditions of total internal reflection are found to be the following:

$$n_{nl} < 0, \quad \theta_2 < \theta_{\rm cr}. \quad (9)$$

The first condition requires that induced inhomogeneity must be negative (defocusing cascaded nonlinearity) and, consequently, $\hat{k} < 0$. If in a collinear geometry the process is phase matched ($\hat{k}_m = 0$) this condition is satisfied automatically [see Eq. (2)]. The second condition shows that total reflection takes place if the initial tilt angle is less than a critical value. Using Eqs. (2) and (4) one can obtain the expression for the critical tilt angle:

$$\theta_{\rm cr} = [2\max(n_{nl})]^{1/2} = \left(4\gamma_2\gamma_3 k_3 k_1^{-1} k_2^{-2}\right)^{1/4} E_{1\,\max}^{1/2}. \quad (10)$$

Here $E_{1\,\max}$ is the reference beam peak amplitude.

Thus, the elaborated ray theory predicts the effect of beam reflection that is not obvious from Eq. (1). Note that formula (10) works well if the beams are not distorted due to the diffraction. This means that the intersection of beams should occur prior to the diffraction length: $x_0/\theta_2 < k_1 a_1^2/2$. The latter condition limits the minimal value of the signal beam tilt: $\theta_2 > 2x_0/(k_1 a_1^2)$.

V. NUMERICAL SIMULATION OF THREE-BEAM INTERACTION

The theoretical results presented here are confirmed by the data from the numerical simulation of Eq. (1) for slowly varying envelopes (see Fig. 1). For numerical analysis the split-step method was used.

We performed numerical simulations and found that the effect of the signal beam reflection from the reference beam that was predicted by the theory based on cascading and ray optics approximations occurs if the signal beam tilt is less than the critical value. We compared theoretical and numerical dependencies of the critical angle on peak amplitude. It was found that the dependencies obtained numerically and analytically by means of Eq. (10) are virtually identical to each other after the introduction of a numerical factor of the order of 1:

$$\theta_{\rm cr,num} = C\,\theta_{\rm cr}. \quad (11)$$

This correction factor appears because of the approximations that were used to obtain Eq. (10). This implicitly takes into account the influence of weak diffraction effects, the change in energy and the orientation of the wave vectors of the weak beams and other adverse factors.

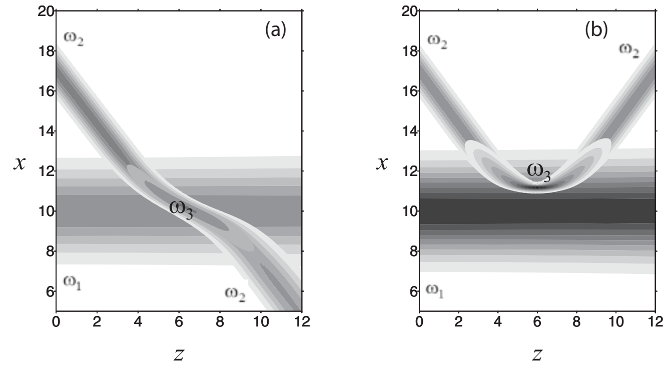

FIG. 1. Intensity distribution on the plane (x,z) for the three-wave interaction in the case $k_3 : k_2 : k_1 = 3:2:1$ (results of numerical simulation of wave equations): (a) Signal beam propagation through the reference beam when the tilt is larger than the critical value, $\theta_2 > \theta_{\rm cr}$; (b) total reflection of the signal beam from the reference beam when the initial tilt is smaller, $\theta_2 < \theta_{\rm cr}$. All quantities are plotted in arbitrary units.

Good correlation between theoretical and numerical results confirms adequacy of our model [Eqs. (3) and (4)] and the developed ray theory. Note that applied approximations allow predicting of the effect of beam reflection and accurately estimating the value of the critical angle despite the fact that the approximation $\hat{k} = $ const and also the cascading approximation $\bar{A}_3 \approx (\gamma_3/\hat{k})A_1A_2$ are not valid in the vicinity of the turning point $z = z_r$, where the signal beam becomes parallel to the z axis. In this point the process becomes phase matched, $\hat{k} = 0$, and efficient sum-frequency generation occurs [see Fig. 2(a)]. However, numerical results demonstrate that the generated sum-frequency wave is located in the small area near the turn point $z = z_r$ [see Figs. 1(b) and 2(a)], and the reflected signal conserves its initial power. In Fig. 2(a), beam power is defined as $P_j = \int_{-\infty}^{\infty} |A_j(x)|^2 dx$.

Analysis of Eq. (1) reveals that the interacting beams obey the law of conservation of the transverse momentum:

$$I_2 = {\rm Im}\int \sum_{n=1}^{3} \frac{1}{\gamma_n} A_n^* \frac{\partial A_n}{\partial x} dx = \sum_{n=1}^{3} I_{2n} = I_{20}. \quad (12)$$

This relation can be rewritten for oblique beams $A_n = |A_n|\exp(ik_n\theta_{nz}x)$ in the form $I_2 = (k_1/\gamma_1)P_1\theta_{1z} + (k_2/\gamma_2)P_2\theta_{2z} + (k_3/\gamma_3)P_3\theta_{3z}$, $I_{20} = (k_2/\gamma_2)P_2(z=0)\theta_2$ as at the entrance $\theta_1 = 0$ and $P_3(z=0) = 0$. As it was confirmed numerically [see Figs. 1(b), 2(a), and 2(b)], after the collision one has $P_3 = 0$, $\theta_{2z} = -\theta_2$, $P_1 = P_1(z=0)$, $P_2 = P_2(z=0)$, and, consequently, $I_2 = (k_1/\gamma_1)P_1\theta_{1z} - (k_2/\gamma_2)P_2\theta_2$. Thus, the reference beam slants after the interaction, and taking into account that $k_1/\gamma_1 \approx k_2/\gamma_2$, one has $\theta_{1z} = 2(P_2/P_1)\theta_2$. Since the signal beam power is much smaller than the reference beam power (in our calculations $P_2/P_1 \sim 10^{-4}$) the reference beam is deflected by a very small angle.

So, varying the signal beam tilt angle or the reference beam peak intensity only, one can control the signal beam trajectory to switch from propagation through the reference beam to the reflection from the reference beam and vice versa (see Fig. 3).

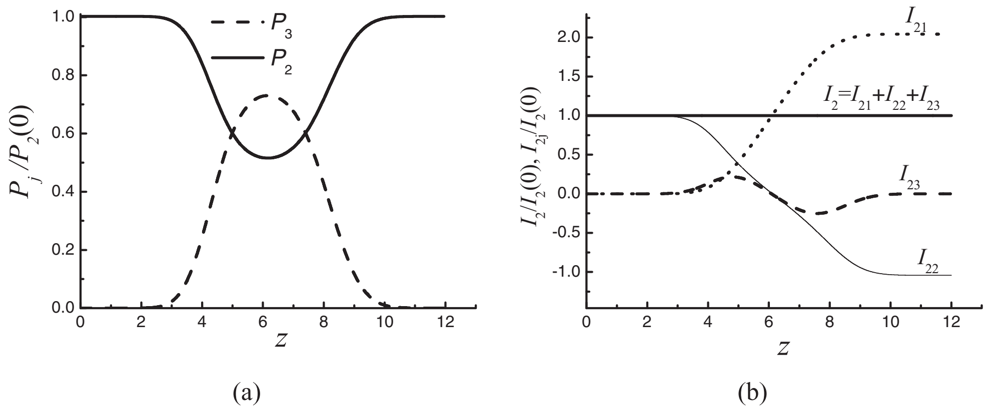

FIG. 2. The distance dependencies of (a) signal (solid line) and sum-frequency (dashed line) beam powers and (b) total transverse impulse and its components in parametric reflection.

VI. SIGNAL BEAM REFLECTION FROM THE FOCUSED REFERENCE BEAM

It is obvious that the value of the critical tilt can be increased by increasing the reference beam peak value. This can be done by appropriate focusing of the reference beam. The intersection region of the beams is located at the distance $l_{\text{int}} \approx x_0/\theta_2$, while the reference beam waist is at a distance of $l_w = R_1/(1 + R_1^2/l_{\text{dif}}^2)$, where $l_{\text{dif}} = k_1 a_1^2/2$ is a diffraction length, and R_1 is the initial radius of curvature of the reference beam wave front. By equating these two lengths we obtain the expression for the optimal curvature radius:

$$R_1 = l_{\text{dif}}(m + \sqrt{m^2 - 1}); \quad m = \theta_2 l_{\text{dif}}/2x_0. \tag{13}$$

In a planar geometry the maximum amplitude in the waist increases by a factor of $\sqrt[4]{1 + (l_{\text{dif}}/R_1)^2}$ which allows an increase in the range of admissible tilt angles.

Let us consider some features of the process. To obtain efficient reflection in the case of strong focusing it is necessary to decrease the signal beam width. This is explained by the fact that the stronger the beam focusing, the smaller the waist length and the reflection region length. During the reflection of a broad signal beam, a part of the beam falls to the region of the reference wave with small amplitude and is not reflected. Indeed the signal beam interacts with the reference beam at the length $\Delta z_1 \approx 2a_2/\theta_2$ and the waist length at the level $N = E_1/E_{1\,\max}$ is $\Delta z_2 \approx 2l_{\text{dif}}\sqrt{N^{-4} - 1}/[1 + (l_{\text{dif}}/R_1)^2]$. Then, the condition $\Delta z_2 > \Delta z_1$ should be satisfied for effective reflection. Another feature of the signal beam reflection from the focused reference beam is the distortion of the reflected beam profile [compare Figs. 4 and Fig. 1(b)]. If the reference beam is not focused and its cross section does not change, the reflection resembles a reflection from a plane surface parallel to the reference beam axis. Upon focusing, the beam waist is formed and the longitudinal curvature of a parametric mirror appears which changes the profile of the reflected signal wave.

VII. SIGNAL BEAM REFLECTION IN THE MEDIUM WITH BIREFRINGENCE

In the more general case of an anisotropic medium, while analyzing the signal beam trajectories birefringence effects should be taken into account [31] and the initial tilt angle θ_2 in Eq. (8) should be replaced by the effective tilt $\theta_2 + \beta_2$, where

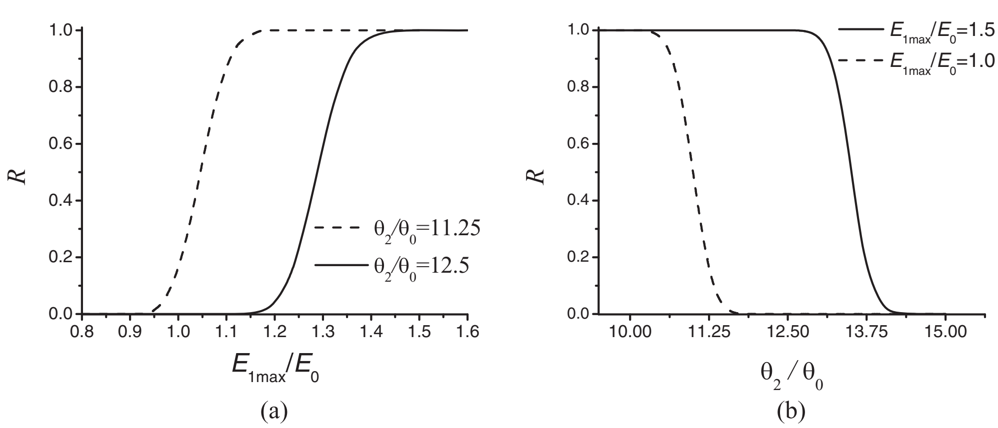

FIG. 3. Reflection coefficient $R$of the signal beam vs (a) the normalized peak amplitude of the reference beam; (b) the normalized signal beam tilt.

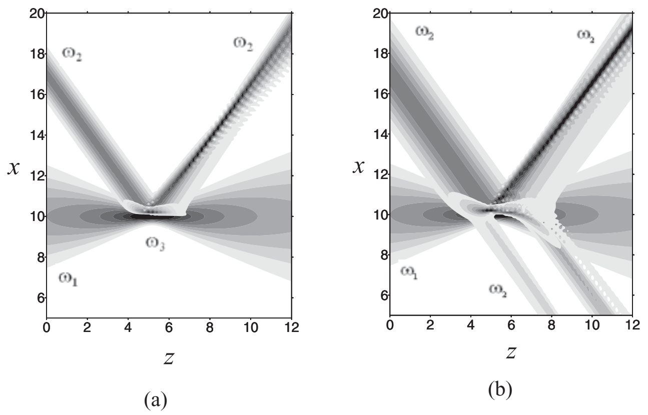

FIG. 4. Intensity distribution on the plane (x, z) for the parametric interaction of the signal beam and focused reference beam (results of numerical simulation of wave equations): (a) Total reflection of the narrow signal beam; (b) partial reflection of the wide signal beam. All quantities are plotted in arbitrary units.

β_2 is the walk-off angle at the signal frequency. So, the signal beam trajectory Eq. (8) should be rewritten as

$$z = z_0 \pm \sqrt{2} \int_{x_0}^{x} \frac{d\tau}{\sqrt{n_{nl}(x) - n_{nl}(x_0) + (\theta_2 + \beta_2)^2/2}}. \quad (14)$$

It is obvious from the analysis of Eq. (14) that reflection occurs if

$$0 < (\theta_2 + \beta_2)^2 < \max\{-2[n_{nl}(x) - n_{nl}(x_0)]\}. \quad (15)$$

To satisfy this condition the induced inhomogeneity should be negative and, consequently, $\hat{k} < 0$. Also, the signal beam must propagate towards the reference beam and, consequently, the effective tilt must be positive, $\theta_2 + \beta_2 > 0$.

It should be noted that, in an anisotropic medium, an effective wave-vector mismatch depends on the signal beam propagation direction. If a spatial walk-off is absent for the reference beam, the expression for the effective mismatch [Eq. (2)] becomes

$$\hat{k} = \hat{k}_m - k_2(k_1 - \hat{k}_m)\theta_2^2/2k_3 + (\beta_3 - \beta_2)k_2\theta_2, \quad (16)$$

where β_3 is the walk-off angle at the sum frequency. In contrast to an isotropic medium ($\beta_2 = \beta_3 = 0$) when an effective mismatch is always negative if $\hat{k}_m = 0$, here at some tilt angles $\hat{k}$ can be positive even if $\hat{k}_m = 0$, and at these angles reflection is impossible for any values of the reference beam peak intensity.

So, using Eqs. (15) and (16) we can present the reflection conditions taking into account a spatial walk-off in the form

$$0 < F(\theta_2) < 1, \; \theta_2 + \beta_2 > 0, \quad (17)$$

where $F(\theta_2) = k_2(\theta_2 + \beta_2)^2[-\hat{k}_m + k_2k_1\theta_2^2/2k_3 + k_2(\beta_2 - \beta_3)\theta_2]/(2\gamma_2\gamma_3E_{1\max}^2)$ is a characteristic function. The inequality $F(\theta_2) > 0$ corresponds to the condition $n_{nl} < 0$, and the inequality $F(\theta_2) < 1$ defines the range of tilt angles which are less than a critical value. So, analysis of characteristic function $F(\theta_2)$ allows determination of tilt angles at which reflection is possible [see Fig. 5(a)].

VIII. BEAM REFLECTION IN THREE-DIMENSIONAL SPACE

In a bulk medium the beam interaction dynamics is more complicated. If the reference beam and the induced inhomogeneity have a cylindrical shape, $A_1 = E_1(r)$, $n_{nl} = n_{nl}(r)$, trajectory Eqs. (6) can be rewritten using cylindrical coordinates:

$$\frac{d^2r}{dz^2} = \frac{a^2\theta_2^2}{r^3} + \frac{\partial n_{nl}(r,z)}{\partial r}, \; \frac{d\varphi}{dz} = a\theta_2/r^2, \quad (18)$$

where r and φ are the polar coordinates, centered on the axis of the reference beam. $a = r_0 \sin(\varphi_2)$ is the aiming parameter, φ_2 is the angle at the initial point $r = r_0$ between the initial propagation direction and the direction to the inhomogeneity maximum—the reference beam center.

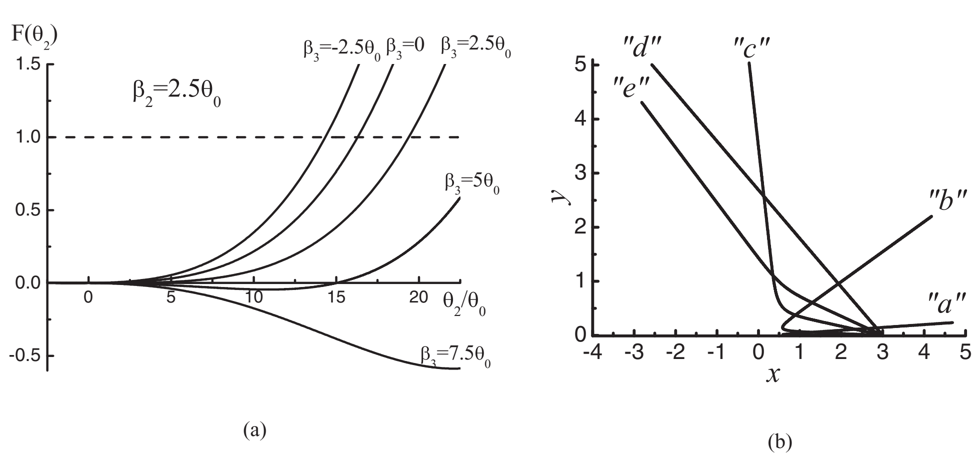

FIG. 5. The description of beam-beam interaction in cascaded quadratic media. (a) Characteristic function F vs signal wave tilt θ_2 at different values of walk-off angles in the case $k_3 : k_2 : k_1 = 3 : 2 : 1$. Range of the tilt angles where reflection can occur is defined by the conditions $0 < F(\theta_2) < 1$ and $\theta_2 + \beta_2 > 0$. (b) Signal beam trajectories in the transverse cross section for different values of aiming parameter a, that is normalized to the reference beam width. Line "a" corresponds to $a = 0.01$, "b" to $a = 0.1$, "c" to $a = 0.5$, "d" to $a = 2.0$, "e" to $a = 1.0$.

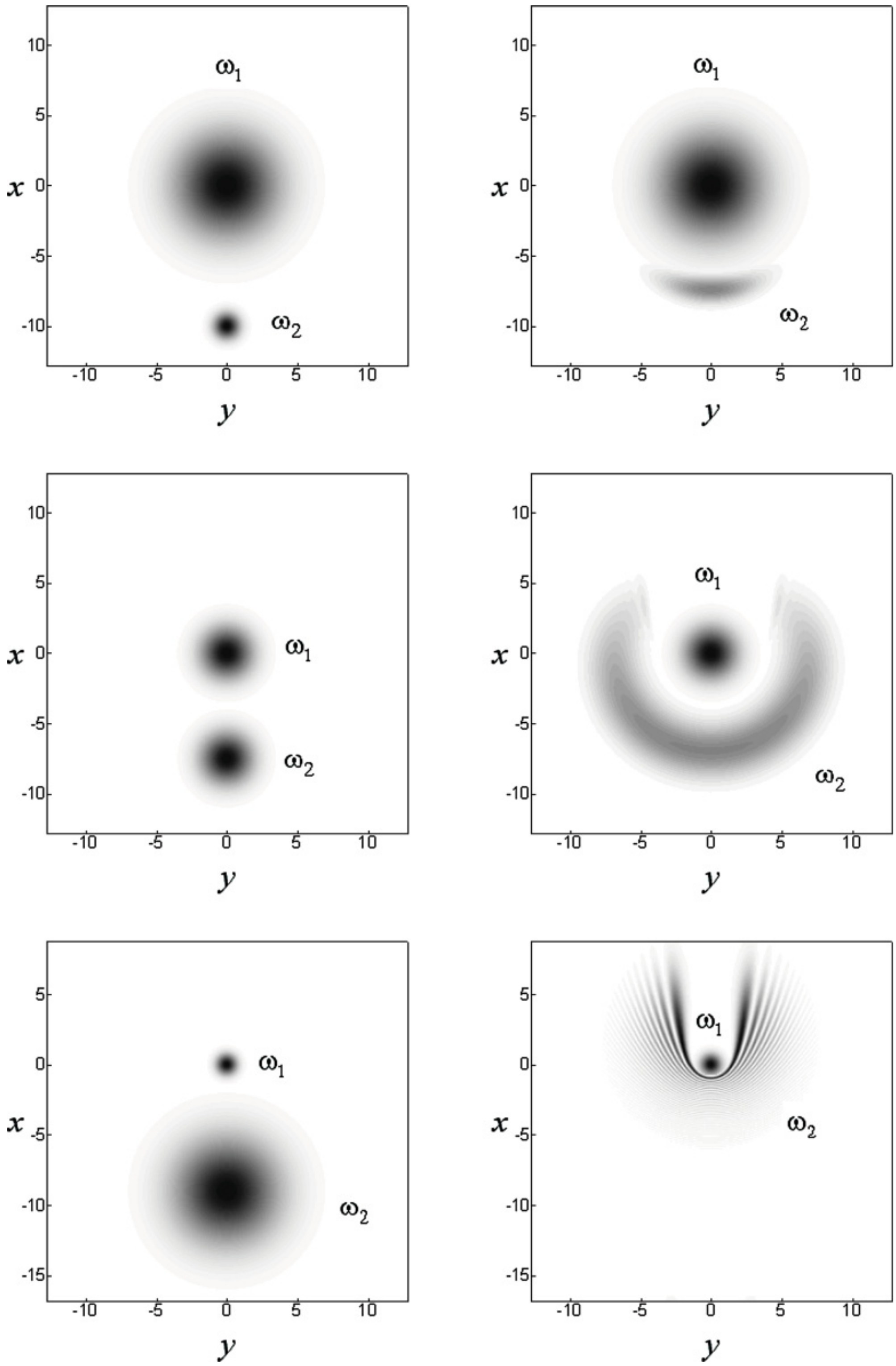

FIG. 6. Reference and signal beam intensity distribution before reflection (left column) and after reflection (right column) for different ratios of beam widths: (top row) $a_1/a_2 = 4$; (middle row) $a_1/a_2 = 1$; (bottom row) $a_1/a_2 = 0.2$. All quantities are plotted in arbitrary units.

Thus, signal beam trajectory depends on both the initial tilt θ_2 and the aiming parameter a. The propagation dynamics of beam cross sections resembles the potential scattering of particles. If the aiming parameter is equal to zero, $a = 0$, interaction can be called "central" [see Eq. (18)]. Trajectory equations for such interactions are the same as for planar geometry. In such a case, parametric reflection occurs for tilt angles less than the critical value, $\theta_2 < \theta_{\mathrm{cr}}$. If an initial signal wave vector is not exactly directed to the reference axis "noncentral" interaction takes place. In such case the result depends on the magnitude of the aiming parameter a. If this parameter is small enough, the signal beam is reflected back. If the deviation increases the reflection angle decreases. In this case, the aiming parameter is great, the interaction is weak, and the signal propagation direction coincides with the initial direction [see Fig. 5(b)].

One more feature of the considered process is presented: If reflection conditions are satisfied the reference beam becomes nontransparent for the signal wave and acts as a mirror. In the case of a two-dimensional cylindrical reference beam, since the inhomogeneity profile repeats the reference beam intensity distribution [see Eq. (4)] the induced mirror possesses a considerable curvature. Thus, the interaction process looks like a reflection from the convex mirror. If the signal beam width is comparable with the reference beam width, the reflected signal beam becomes divergent. To minimize such divergence one can use focused signal beams.

If the signal beam is much wider than the reference beam, the signal beam rounds the induced inhomogeneity and the interaction process looks like the diffraction of the signal wave at a reflective wire located in the medium [33] (see Fig. 6).

IX. CONCLUSION

In summary, the effect of the total reflection of the weak signal beam from the high-power reference beam at noncollinear mismatched parametric interaction is reported. By means of ray optics approximation the signal beam trajectory equation is obtained. In planar geometry the conditions of the signal beam reflection from the inhomogeneity induced by the pump beam are found. It is shown that reflection occurs if the initial tilt is less than the critical value, for which the analytical expression was obtained. The influence of the birefringence and the initial reference beam focusing on interaction dynamics are analyzed. It is shown that the beam reflection from a cylindrical beam turns into scattering on induced inhomogeneity. If reflection conditions are satisfied, the cylindrical reference beam acts as a convex mirror.

ACKNOWLEDGMENTS

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