Computational Photonics

Wed 14:15 – 15:45
Seminarraum 001, 1105
Prof. Dr. Ayhan Demircan

Institut für Quantenoptik (IQ)

Date	Topic	Lecturer
Mi, 13.04.22	Introduction	Demircan
Di, 19.04.22	Phython Basics	Melchert/Babushkin/Demircan
Mi, 22.04.22	Monte Carlo Simulation I	Melchert
Mi, 27.04.22	Monte Carlo Simulation II	Melchert
Di, 03.05.22	Monte Carlo Simulation	Melchert
Mi, 04.05.22	FFT/ Numerical Differentiation	Demircan
Mi, 11.05.22	Nonlinear Optics	Demircan
Di, 17.05.22	FFT	Melchert/Demircan
Mi, 18.05.22	Nonlinear Pulse Propagation/Solitons	Demircan
Mi, 25.05.22	Beam Propagation Method	Melchert
Di, 31.05.22	Nonlinear Pulse Propagation	Melchert/Demircan
Mi, 01.06.22	Numerical Integration/Runge Kutta	Demircan
Mi, 08.06.21	Holiday	
Di, 14.06.22	IAC Meeting	
Mi, 15.06.22	IAC Meeting	
Di, 21.06.22	Numerical Integration/Runge-Kutta	Melchert/Demircan
Mi, 22.06.22	Light in Nanostructures	Babushkin
Mi, 29.06.21	Atoms in Strong Fields	Babushkin
Di, 05.07.22	Atoms in Strong Fields	Babushkin
Mi, 06.07.22	Parallel Computing	Babushkin
Mi, 13.07.22	Stochastic Differential Equations	Melchert
Di, 19.07.22	Parallel Computing	Babushkin
Mi, 20.07.22	Probeklausur	

Wave Equation

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0\left(\frac{\partial \vec{j}}{\partial t} + \frac{\partial^2 \vec{P}}{\partial t^2}\right) + \frac{\partial}{\partial t}\nabla \times \vec{M} + \nabla\left(\nabla \vec{E}\right) \qquad c_0 = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

Non magnetic media with no free charges and currents due to free charges

$\vec{M} = 0\,,\ \vec{j} = 0\,,\ \rho = 0$ with $\vec{D} = \epsilon(\vec{r})\vec{E} = \epsilon_0 \epsilon_r(\vec{r})\vec{E}$

We obtain $\nabla(\epsilon(\vec{r})\vec{E}) = 0$ and for homogeneous media $\nabla \vec{E} = 0$

Wave equation driven by the polarization in the medium

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$$

Light Generated in a Nonlinear Medium

We model non-sinusoidal motion using higher-order (nonlinear) terms in the applied light electric field E:

$$P_i = P_i^0 + \chi_{ij}^{(1)} E_j + \chi_{ijk}^{(2)} E_j E_k + \chi_{ijkl}^{(3)} E_j E_k E_l + \ldots$$

What are the effects of such nonlinear terms?

Frequencies in the polarization become emitted frequencies.

Consider the second-order term:

Since $\mathrm{E}(t) \propto \mathrm{Re}\left\{E_1 \exp(i\omega t)\right\} \propto E_1 \exp(i\omega t) + E_1^* \exp(-i\omega t)$,

$$\mathrm{E}(t)^2 \propto E_1^2 \exp(2i\omega t) + 2\left|E\right|^2 + E_1^{*2} \exp(-2i\omega t)$$

2ω = 2nd harmonic!

Self trapping of light

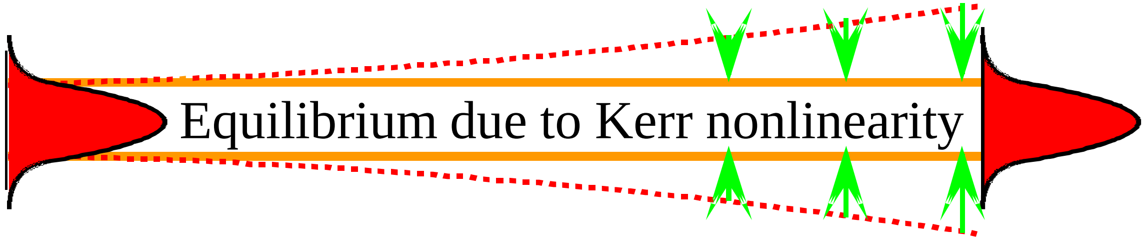
Self-focusing by optical Kerr effect

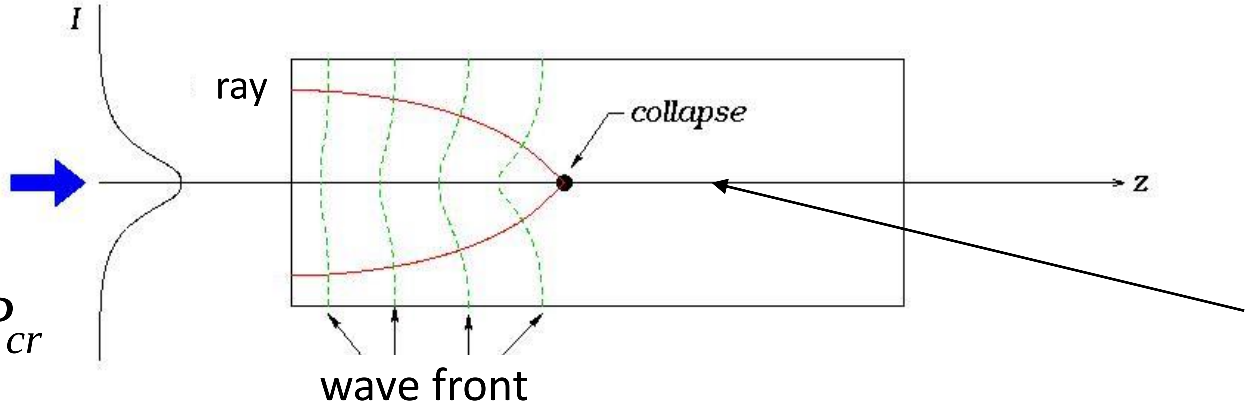
$$n = n_0 + n_2 I(r)$$

$P < P_{cr}$

$$P_{cr} = 3.77 \frac{\lambda^2}{8\pi n_0 n_2}$$

$P = P_{cr}$

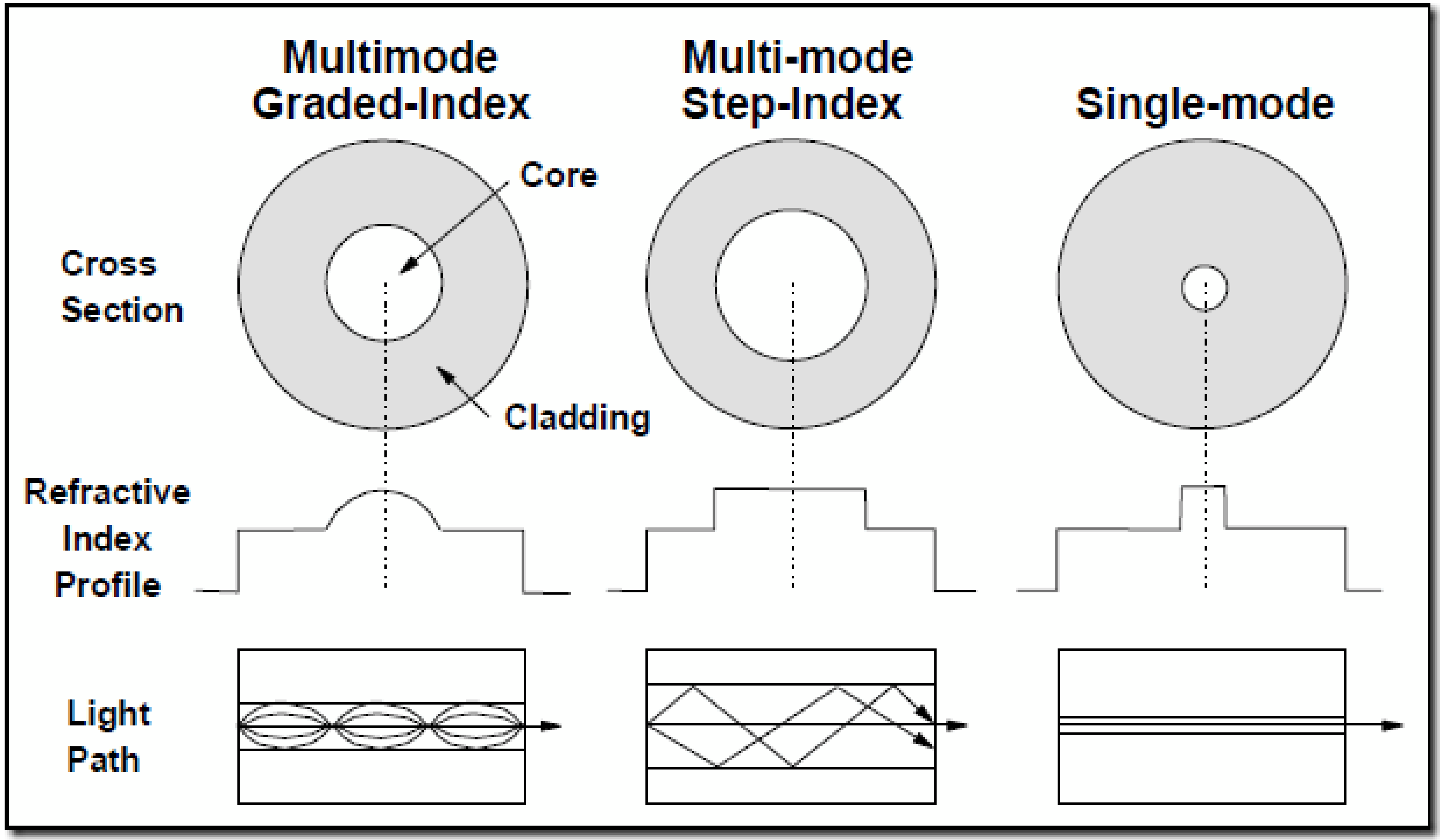




$P > P_{cr}$

$$z_c = \frac{z_R}{\sqrt{P/P_{cr} - 1}}$$

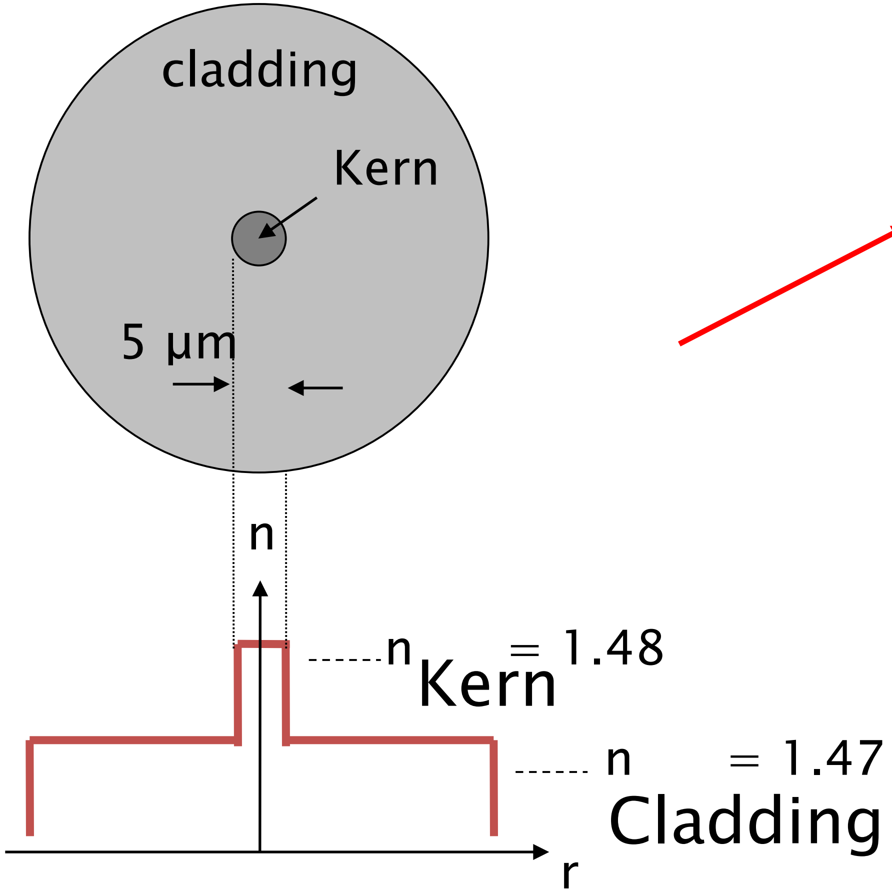
Fiber Types



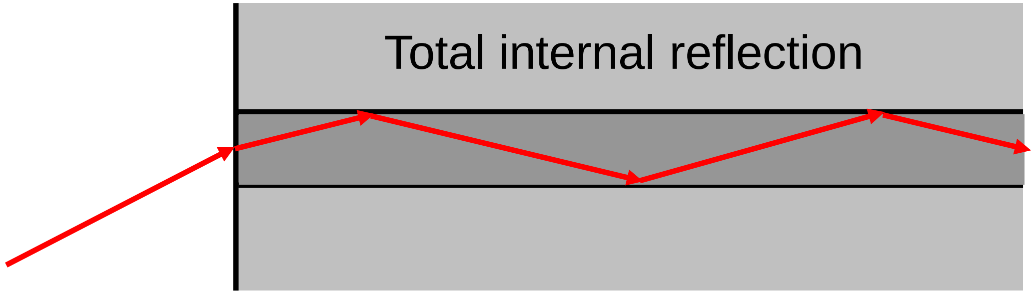
Multimode Graded-Index
Multi-mode Step-Index
Single-mode
Core
Cross Section
Cladding
Refractive Index Profile
Light Path

Standard optical Fiber

Optical Fiber – circular dielectric waveguide



Step index profile

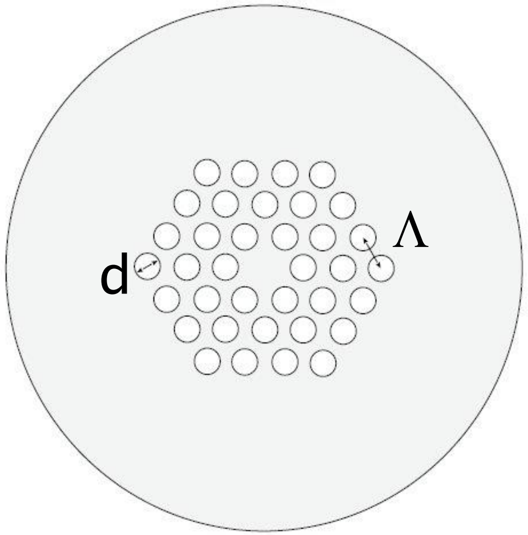


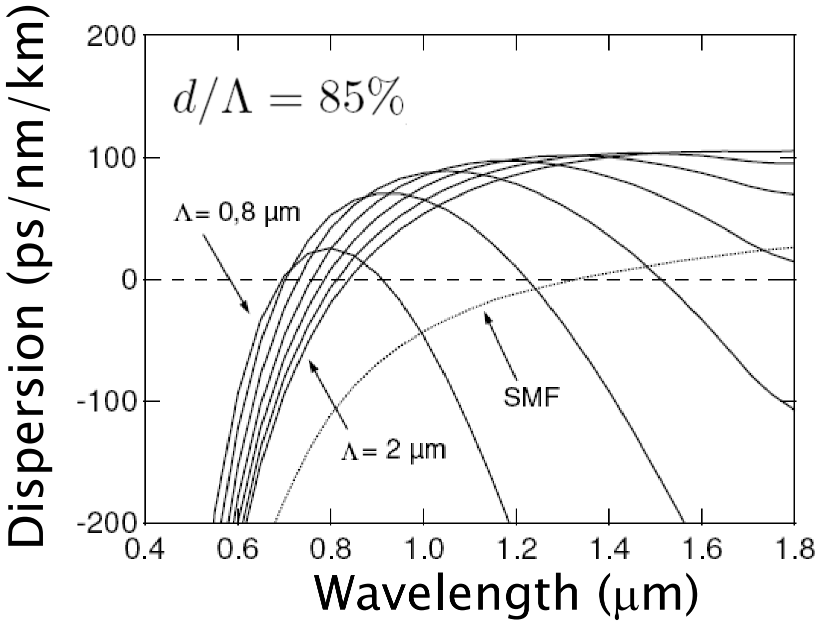
Propagationsconstant – Dispersion

Confinement – Nonlinearity

Properties of highly nonlinear PCF

- Usually based on a single material with a high air-fill fraction





- Dispersion very sensitive to structure

- Attenuation ~ 1 -10 dB/km, elevated nonlinearity $\gamma_{ZDW} \sim 100\text{–}200\ \mathrm{W}^{-1}\ \mathrm{km}^{-1}$

Linear propagation effects

$$k(\omega) = \frac{2\pi}{\lambda} n(\omega) = \frac{2\pi\nu}{c} n(\omega) = \frac{\omega}{c} n(\omega)$$

Taylor expansion

$$k(\omega) = k_0 + \boxed{\left(\frac{dk}{d\omega}\right)_{\omega_0}} (\omega - \omega_0) + \frac{1}{2}\left(\frac{d^2k}{d\omega^2}\right)_{\omega_0} (\omega - \omega_0)^2 + \frac{1}{6}\left(\frac{d^3k}{d\omega^3}\right)_{\omega_0} (\omega - \omega_0)^3 + \ldots$$

$$\left(\frac{dk}{d\omega}\right)_{\omega_0} = \frac{d}{d\omega}\left(\frac{\omega}{c} n\right)_{\omega_0} = \frac{n(\omega_0)}{c} + \left(\frac{\omega}{c}\frac{dn}{d\omega}\right)_{\omega_0} = \frac{1}{c}\left[n(\omega_0) + \omega_0\left(\frac{dn}{d\omega}\right)\right] = \frac{n_{gr}}{c} = \frac{1}{v_{gr}}$$

Hierarchy:

$v_{ph} = \dfrac{c}{n}$	$v_{gr} = \dfrac{c}{n_{gr}}$	$n_{gr} = n(\omega_0) + \omega_0 \left(\dfrac{dn}{d\omega}\right)_{\omega_0}$
Phase velocity	Group velocity	Group index

Optical Kerr-effect

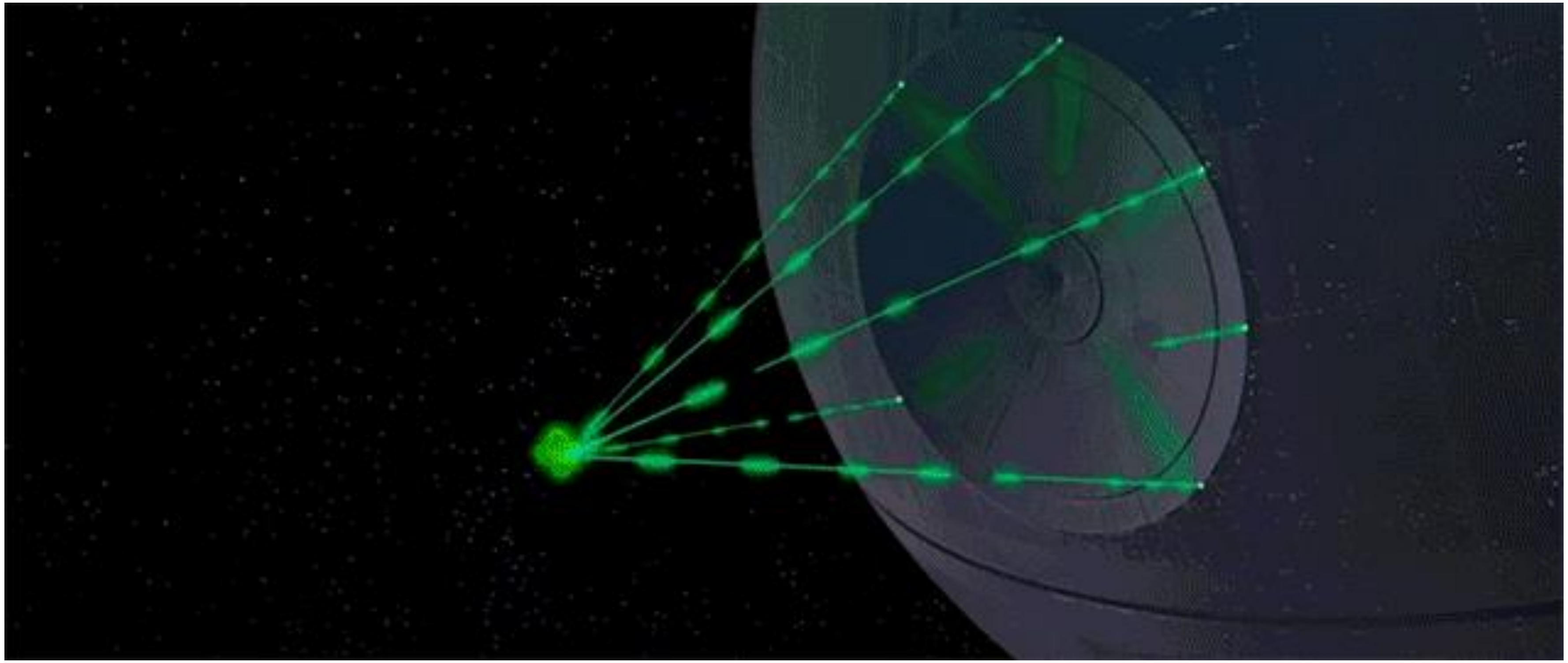
$$\chi^{(3)}_{ijkl}(\omega;\omega,\omega,-\omega)$$

$$\vec{E}(z,t) = \vec{A}(z,t)\exp(-i\omega t) + c.c.$$

$$\vec{P}^{(3)}_{\omega} \quad = \quad \sum_i \vec{e}_i (P^{(3)}_{\omega})_i = \sum_i \vec{e}_i \left[\epsilon_0 \frac{3}{4} \sum_{j,k,l} \chi^{(3)}_{ijkl}(\omega;\omega,\omega,-\omega)(A_\omega)_j (A_\omega)_k (A_{-\omega})_l \right]$$

$$\vec{P}^{(3)}_{\omega} = \epsilon_0 (3/4) \vec{e}_x \chi^{(3)}_{xxxx} |A^x_\omega|^2 A^x_\omega$$

$$\left(\Delta - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \right) \vec{E} = \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$$



Kerr-Nonlinearity

Insertion into the general slowly varying propagation equation

$$i\frac{\partial}{\partial z}A(z,t) - \left(\frac{1}{2k_0}\sum_{n=1}^{\infty}\frac{(-i)^n}{n!}\left.\frac{\partial^n k^2(\omega)}{\partial\omega^n}\right|_{\omega_0}\frac{\partial^n}{\partial t^n}\right)A(z,t) = -\frac{\mu_0\omega_0}{2k_0}\bar{P}^{NL}(z,t)$$

$$v_g^{-1} = \frac{d\omega}{dk}, \quad \beta_n = \left.\frac{d^n k}{d\omega^n}\right|_{\omega_0}$$ und $$k_0 = \frac{\omega_0 n_0}{c}$$ $$\overline{P}_{SPM}(t,z) = \frac{3}{4}\varepsilon_0\chi^{(3)}|A|^2 A$$

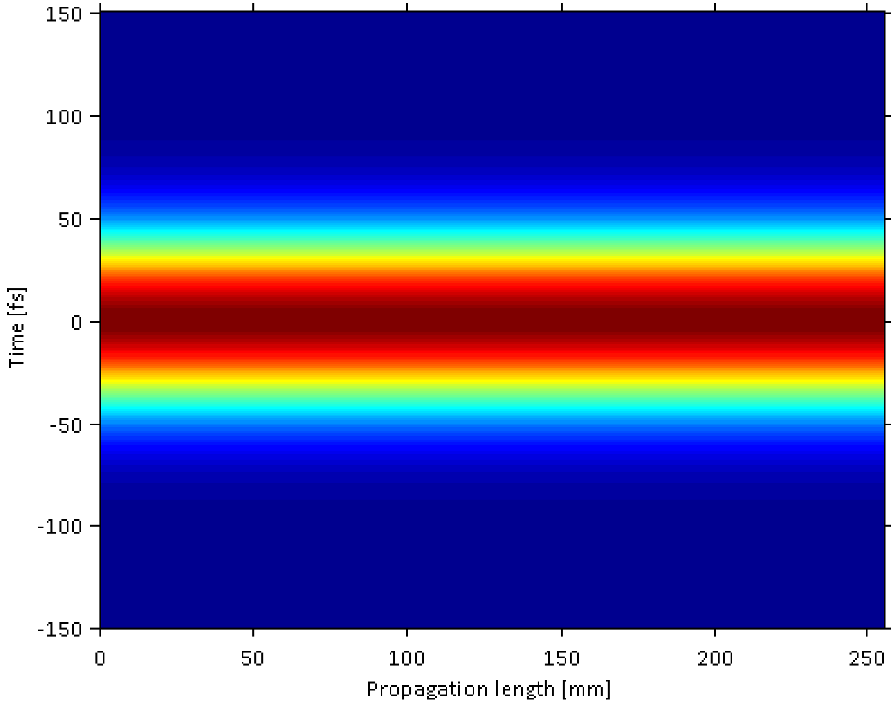
$$\boxed{\left(i\frac{\partial}{\partial z} - i\frac{1}{v_g}\frac{\partial}{\partial t} + \frac{1}{2k_0}\sum_{n=2}^{\infty}\frac{(-i)^n}{n!}\beta_n\frac{\partial^n}{\partial t^n}\right)A(z,t) = -\frac{3\mu_0\omega_0^2}{8cn_0}\chi^{(3)}|A(z,t)|^2 A(z,t)}$$

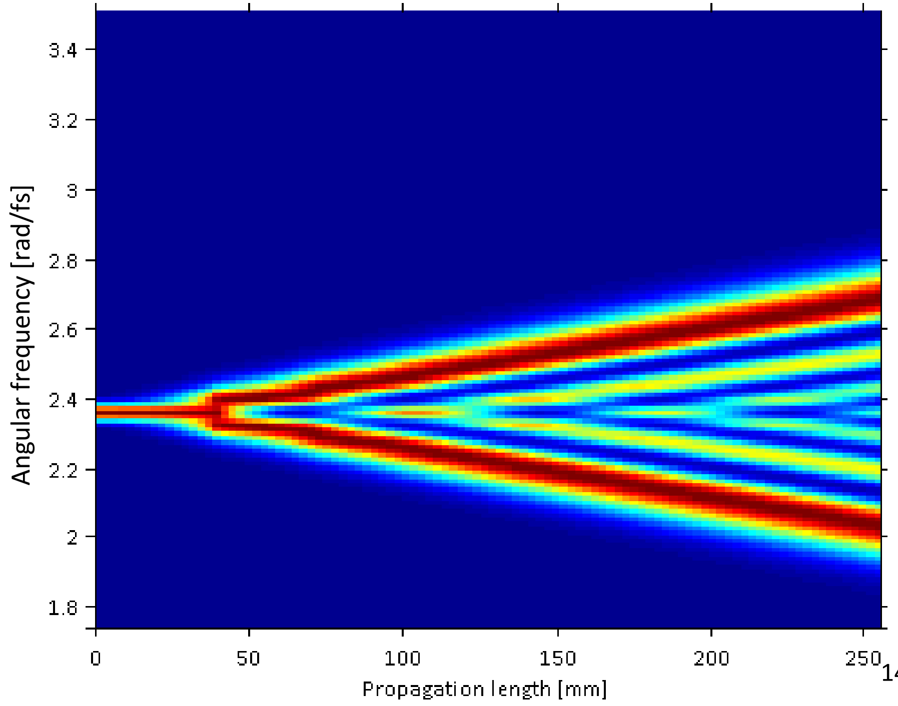
Propagation equation for ultrashort pulses
with dispersion and Kerr nonlinearity

Self-phase modulation (SPM)

We take only the nonlinear term into account

The SPM does not change the temporal pulse shape, but broadens the spectrum considerably.

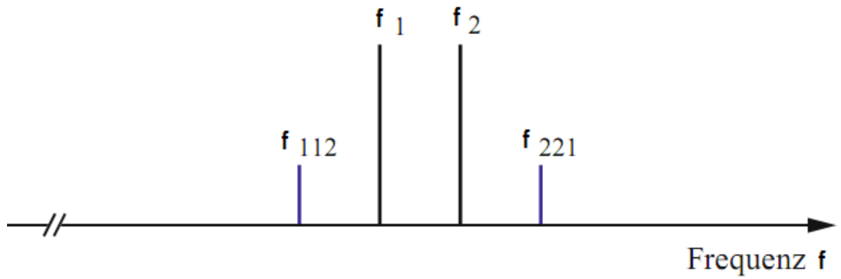




Degenerated four-wave mixing

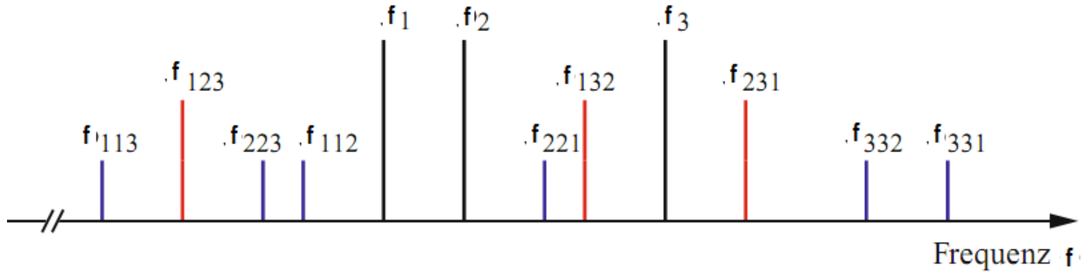
Two input frequencies:

$$f_{112} = 2f_1 - f_2$$
$$f_{221} = 2f_2 - f_1$$

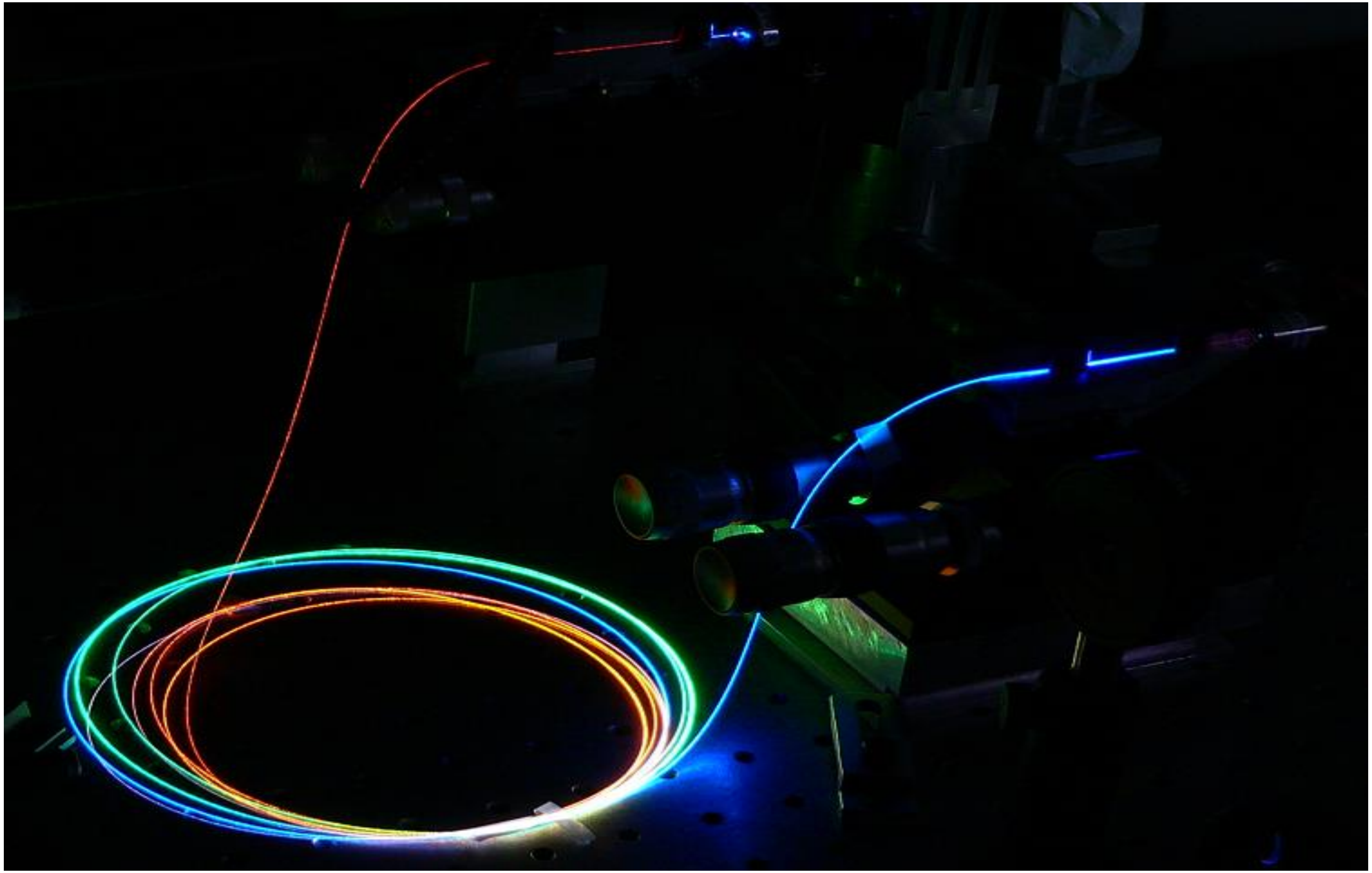


Three input frequencies:

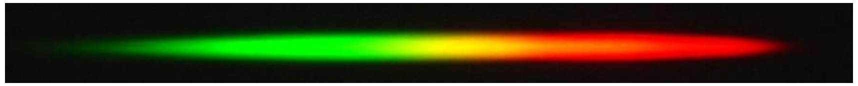
$$f_{123} = f_1 + f_2 - f_3$$
$$f_{132} = f_1 - f_2 + f_3$$
$$-f_{231} = f_1 - f_2 - f_3$$



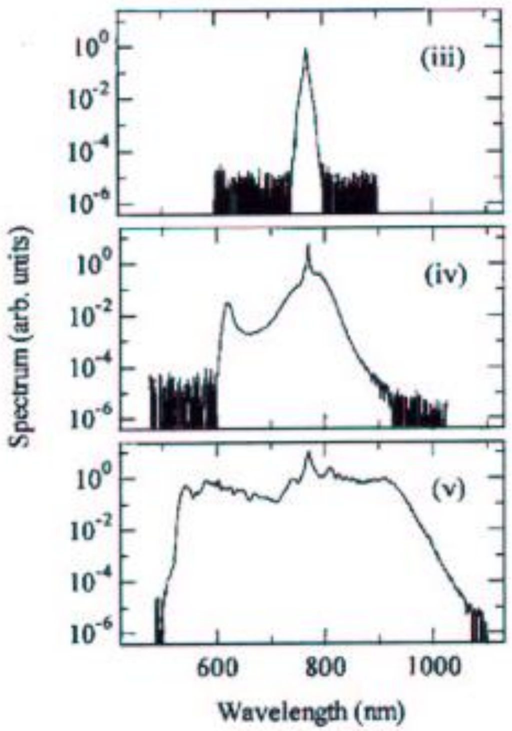
Generation of white laser light

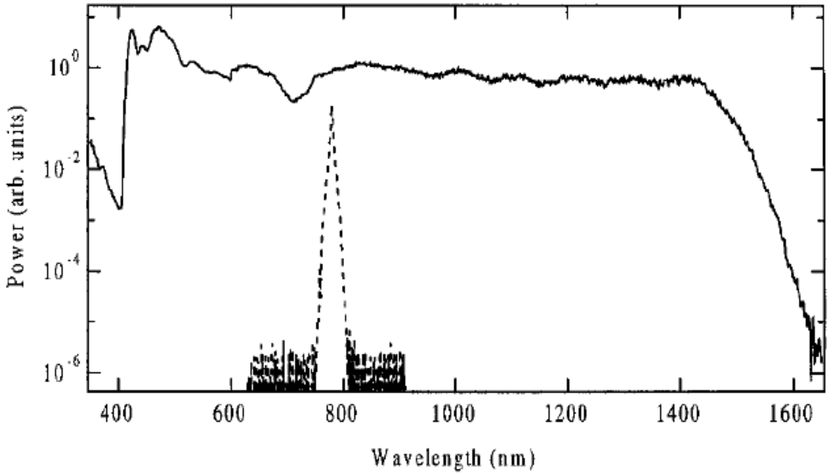


Dramatic spectral broadening



nJ Energie, femtosecond
Ti:Sapphire
Anomalous GVD
Pump wavelength

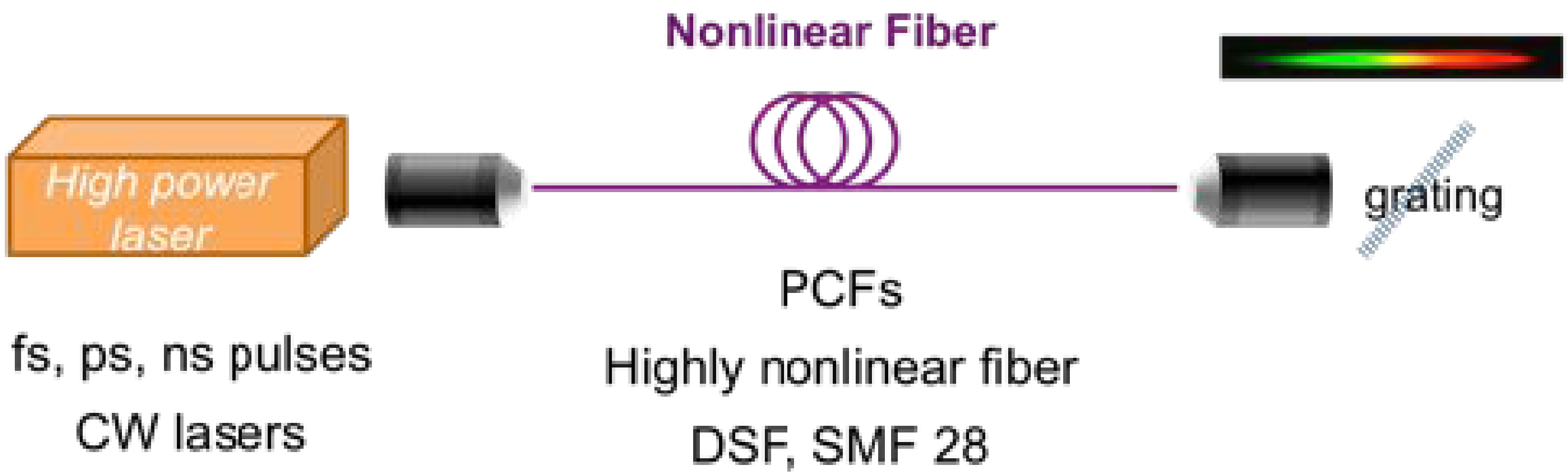




Ranka *et al.* Optics Letters 25, 25 (2000)

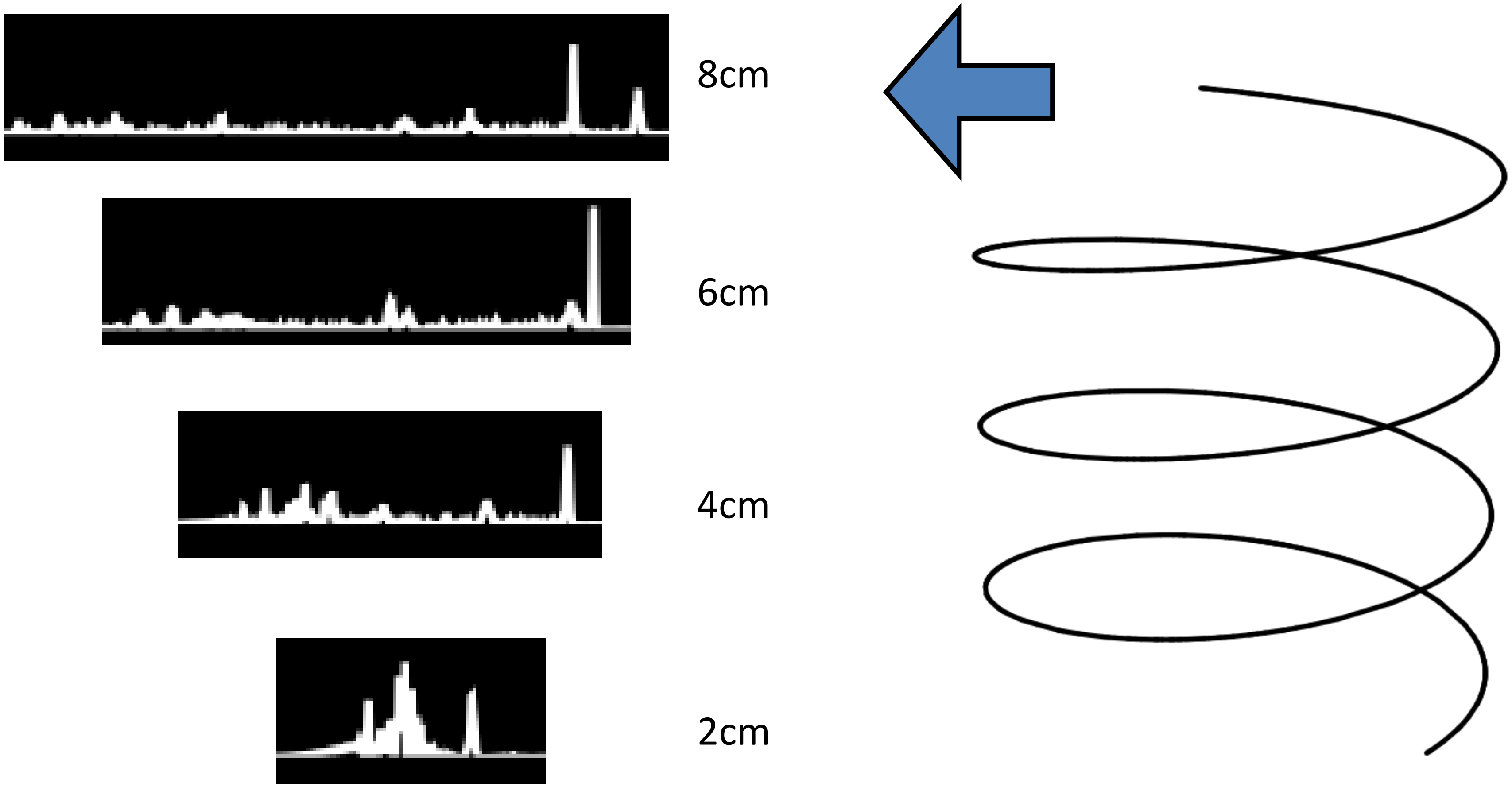
~ 75 cm PCF

Simple experimental realization of supercontinuum generation



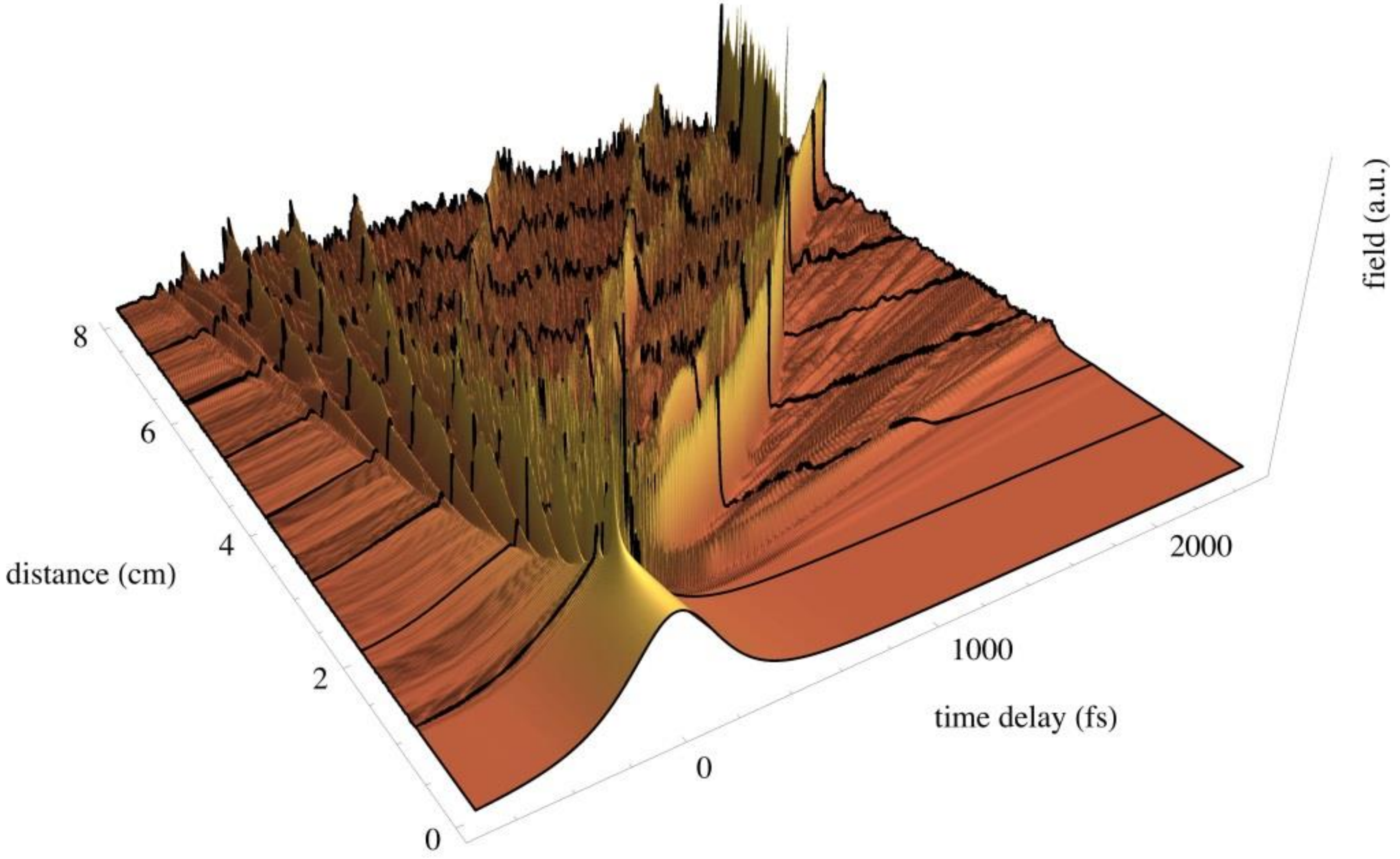
Highly complicated propagation dynamics

Complex propagation dynamics



Higher-order Soliton

Soliton fission (evolution in time)



Modeling ultra-short pulse propagation

Generalized Nonlinear Schrödinger equation

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2} - \sum_{k\geq 2} \frac{i^{k+1}}{k!}\beta_k \frac{\partial^k A}{\partial T^k} = i\gamma \left(1 + \tau_{shock}\frac{\partial}{\partial T}\right)\left(A(z,t)\int_{-\infty}^{\infty} R(T')|A(z,T-T')|^2 dT'\right)$$

Unidirectional forward Maxwell Equation

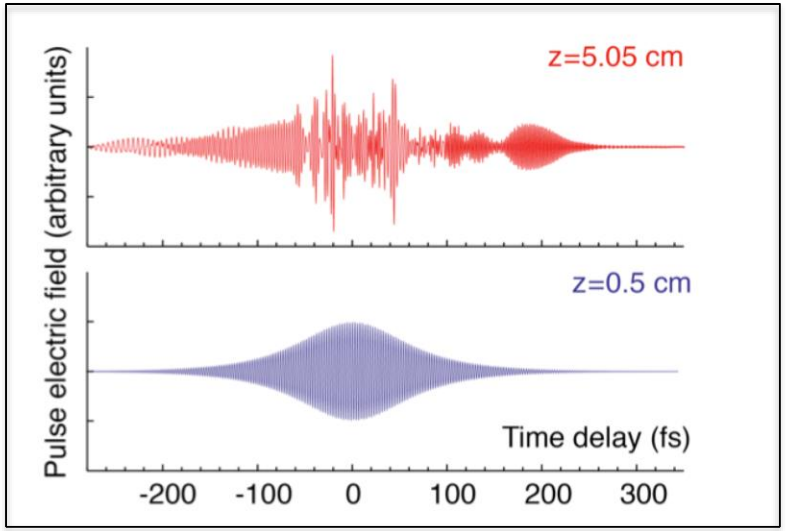
$$i\partial_z E_\omega + \beta(\omega)E_\omega + \frac{\omega^2\chi}{2c^2\beta(\omega)}(E^3)_\omega = 0$$

$$E(z,t) = \sum_\omega E_\omega(z)e^{-i\omega t}$$

Non-envelope unidirectional propagation equation

$$\left[i\partial_z + \beta(\omega)\right]\mathcal{E}_\omega + \frac{3\omega^2\chi^{(3)}}{8c^2\beta(\omega)}\left[(1-f_R)|\mathcal{E}|^2\mathcal{E} + f_R\mathcal{I}_R\mathcal{E}\right]_\omega^+ = 0$$

Raman effect: $$\mathcal{I}_R = \int_0^\infty h(\tau)|\mathcal{E}(t-\tau)|^2 d\tau = \sum_\omega \underbrace{\frac{\nu_1^2+\nu_2^2}{\nu_1^2-(\omega+i\nu_2)^2}}_{h(\omega)}\left(|\mathcal{E}|^2\right)_\omega e^{-i\omega t}$$



Complex wave field:

$$\mathcal{E}(z,t) = 2\sum_{\omega>0} E_\omega(z)e^{-i\omega t}$$

- Few-cycle optical pulse propagation
- Two color pumping

Amiranashvili, Demircan, Physical Review A 82 (1), 013812 (2010)

Propagation Equation

Generalized Nonlinear Schrödinger Equation (GNLSE)

$$\boxed{\begin{aligned}\frac{\partial A}{\partial z} = \; & \underbrace{-\frac{i}{2}\beta_2\frac{\partial^2 A}{\partial \tau^2} + \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial \tau^3} + \frac{i}{24}\beta_4\frac{\partial^4 A}{\partial \tau^4}}_{\text{Dispersion}} \; \underbrace{-\frac{\alpha}{2}A}_{\text{Absorption}} \\ & + \underbrace{i\gamma|A|^2 A}_{\text{SPM}} \underbrace{-\frac{\gamma}{\omega_0}\frac{\partial}{\partial \tau}\left(|A|^2 A\right)}_{\text{Self-steepening}} \underbrace{-i\gamma T_R A\frac{\partial}{\partial \tau}\left(|A|^2\right)}_{\text{Raman Scattering}}\end{aligned}}$$

Propagation of (sub-)picosecond Pulses along $z\,(\tau = t - z/v_g)$

Pseudospectral Method

periodic boundary conditions: $\omega_n = n\,2\pi/T$, n = 0, 1, 2, . . .

$$A(z,\tau) = \sum_{n=-N/2}^{N/2} \tilde{A}_n(z) \mathrm{e}^{i\omega_n \tau}$$

$$\boxed{\begin{aligned}\frac{\partial \tilde{A}_n(z)}{\partial z} = {} & \frac{i}{2}\beta_2\omega_n^2\tilde{A}_n(z) - \frac{\alpha}{2}\tilde{A}_n(z) - \frac{i}{6}\beta_3\omega_n^3\tilde{A}_n(z) + \frac{i}{24}\beta_4\omega_n^4\tilde{A}_n(z) \\ & + i\gamma\tilde{\Psi}_n(z) + a_1\tilde{\Phi}_n(z) - ia_2\tilde{\Xi}_n(z)\end{aligned}}$$

$$\tilde{\Psi}_n(z) = \sum_{l-m+k=n} \tilde{A}_l(z)\tilde{A}_m^*(z)\tilde{A}_k(z)$$ SPM

$$\tilde{\Phi}_n(z) = i\omega_n \sum_{l-m+k=n} \tilde{A}_l(z)\tilde{A}_m^*(z)\tilde{A}_k(z)$$ Self-Steepening

$$\tilde{\Xi}_n(z) = i \sum_{l+m=n} \tilde{A}_l(z)\omega_m \sum_{k-q=m} \tilde{A}_k(z)\tilde{A}_q^*(z)$$ Intrapulse Ramanscattering

Fourier Transform and Convolution

Spatial Domain *(x)*		Frequency Domain *(u)*
$g = f * h$	$\longleftrightarrow$	$G = FH$
$g = fh$	$\longleftrightarrow$	$G = F * H$

So, we can find *g(x)* by Fourier transform

$$\begin{tikzcd}
g & = & f \arrow[d, "\mathbf{FT}"] & * & h \arrow[d, "\mathbf{FT}"] \\
G \arrow[u, "\mathbf{IFT}"] & = & F & \times & H
\end{tikzcd}$$

Pseudospectral method

$$
\begin{tikzcd}
\tilde{A}_n \arrow[r, "\text{IFFT}"] & A \arrow[d, "N(A)"] \\
\tilde{\Psi}_n & {|A|^2 A} \arrow[l, "\text{FFT}"']
\end{tikzcd}
$$

- Equation is solved in the Fourier-domain

- Nonlinear functions are evaluated in the time-domain to avoid the calculation of 'expensive' convolution sums

$$\Psi = |A|^2 A\,,\ \Phi = \frac{\partial}{\partial\tau}\left(|A|^2 A\right)\ ,\ \Xi = A\frac{\partial}{\partial\tau}\left(|A|^2\right)$$

$$\tilde{A}_n,\ i\omega\tilde{A}_n \xrightarrow{\text{IFFT}} A,\ \partial_\tau A \xrightarrow{\times} |A|^2 A = \Psi,\ A\partial_\tau|A|^2 = \Phi \xrightarrow{\text{FFT}} \tilde{\Psi}_n,\ \tilde{\Phi}_n,\ \tilde{\Xi}_n$$

Raman requires repeated FFT

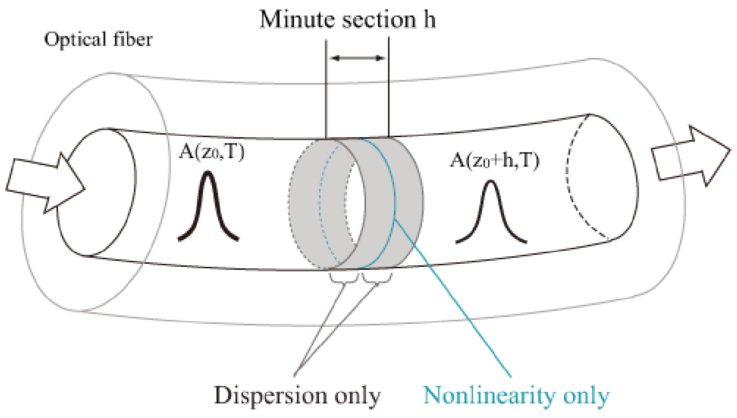
Integration: Split-Step Method

General nonlinear equation

$$\frac{\partial A}{\partial z} = \hat{D}A + \hat{N}A$$

Separation of the linear and nonlinear part

$$\frac{\partial A}{\partial z} = \hat{D}A \quad \text{and} \quad \frac{\partial A}{\partial z} = \hat{N}A$$



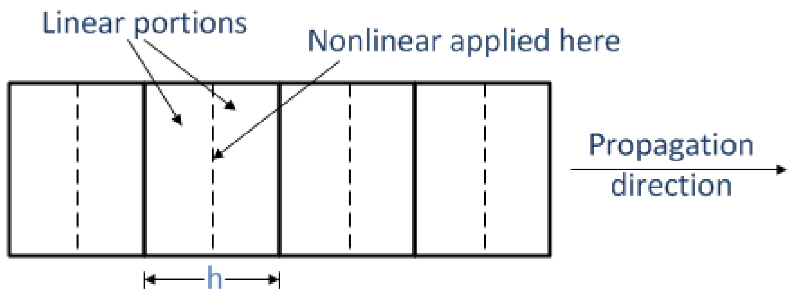
Solving separately and recomposing the solutions

$$\boxed{A(z+h,T) \cong \exp\left(\frac{h}{2}\hat{D}\right)\exp\left(\int_z^{z+h}\hat{N}(z')dz'\right)\exp\left(\frac{h}{2}\hat{D}\right)A(z,T)}$$

F- Fourier transformation $\exp\left(\frac{h}{2}\hat{D}\right)A(z,T) = \left(F^{-1}\exp\left[h\hat{D}(-i\omega)\right]F\right)A(z,T)$

Operated at a center of a tiny space $\int_z^{z+h}\hat{N}(z')dz' \approx \frac{h}{2}\left[\hat{N}(z) + \hat{N}(z+h)\right]$

Split Step Method



$$\hat{D} = -\frac{1}{2}\beta_2\frac{\partial^2}{\partial T^2} + \frac{1}{6}\beta_3\frac{\partial^3}{\partial T^3} - \frac{\alpha}{2}$$

$$\hat{N} = i\gamma\left(|A|^2 + \frac{i}{\omega_0 A}\frac{\partial}{\partial T}\left(|A|^2 A\right) - T_R\frac{\partial |A|^2}{\partial T}\right)$$

Linear and nonlinear Operator do not commute

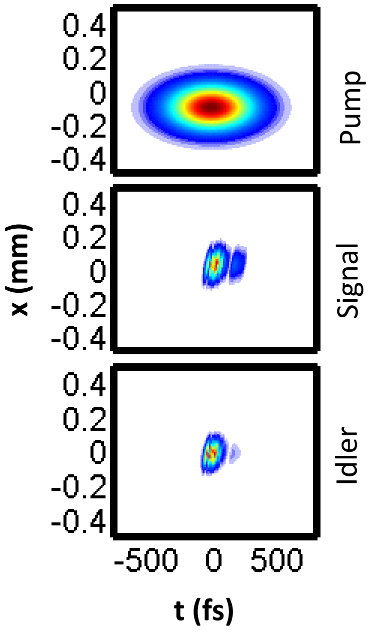
$$\exp(\hat{D})\exp(\hat{N})A = \exp\left(\hat{D} + \hat{N} + \frac{1}{2}[\hat{D}, \hat{N}] + \frac{1}{12}[\hat{D} - \hat{N}, [\hat{D}, \hat{N}]] + \cdots\right)$$

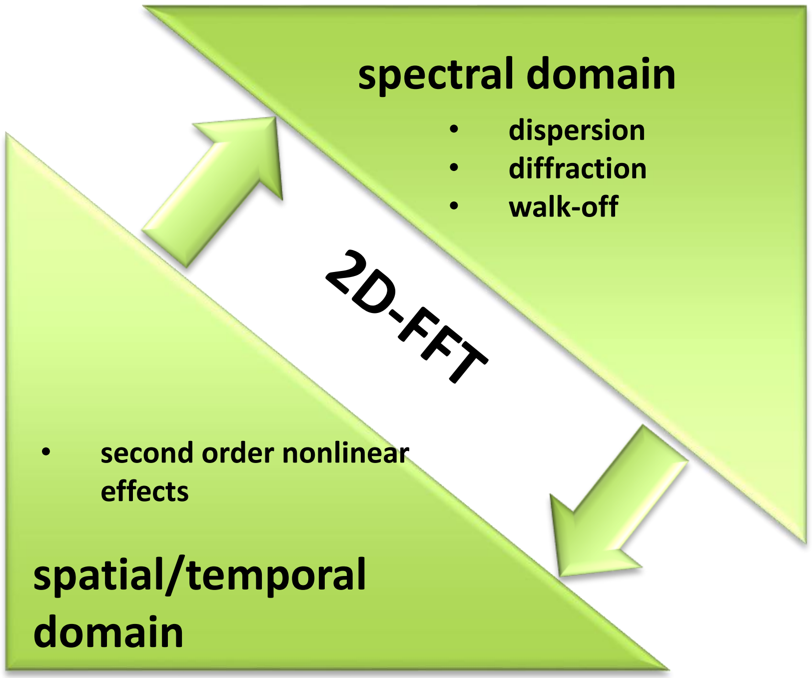
Increasing order of the Split-Step method

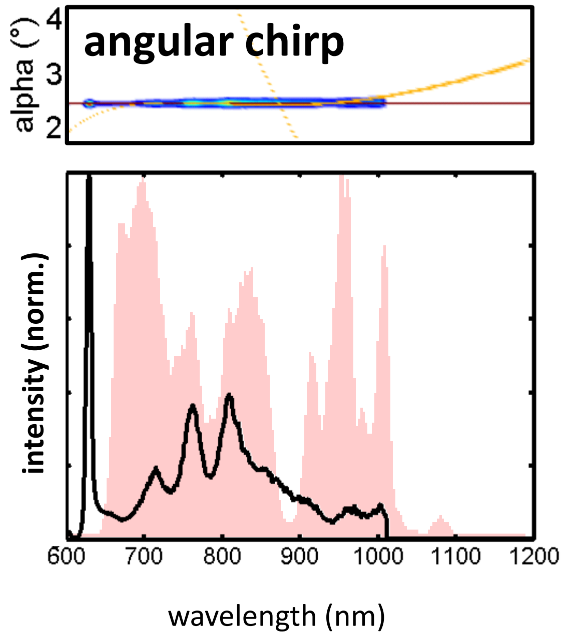
$$A(z+h) \approx \exp(c_1 h\hat{D})\exp(d_1 h\hat{N})\exp(c_2 h\hat{D})\exp(d_2 h\hat{N})\cdots\exp(c_k h\hat{D})\exp(d_k h\hat{N})A(z)$$

Fast simulation model for χ^2 - nonlinear processes

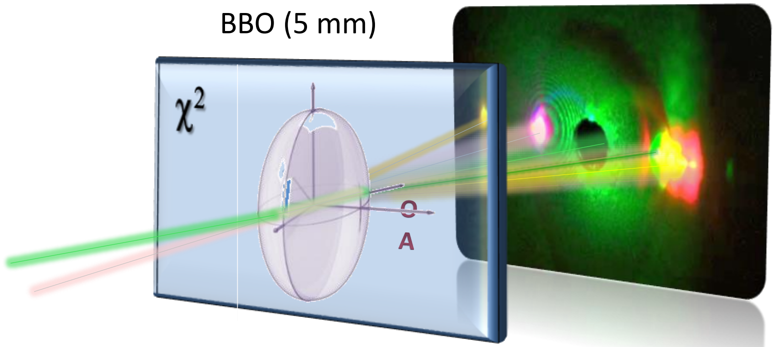
- Results (experiment vs. simulation)







– OPA-System:



- 27 µJ pump pulse energy / 515 nm / 500 fs

- ultra broadband CEP-stabilized seed pulse TL < 5 fs

Heuristic derivation of the NLSE

In media with (cubic) Kerr nonlinearity

$$n = n(\omega, |E|^2) \quad , \quad k = k(\omega, |E|^2)$$

Taylor series expansion:

$$k - k_0 = \left.\frac{\partial k}{\partial \omega}\right|_{\omega_0} (\omega - \omega_0) + \frac{1}{2} \left.\frac{\partial^2 k}{\partial \omega^2}\right|_{\omega_0} (\omega - \omega_0)^2 + \frac{\partial k}{\partial |E|^2} |E|^2 + \dots$$

Electromagnetic field: $$E = E_0 \exp(i\left[(\omega - \omega_0)t - (k - k_0)z\right])$$

Correspondence between the differential operators and the multipliers in the Taylor series:

$$\frac{\partial E}{\partial z} = -i(k - k_0)E \quad => \quad \frac{\partial}{\partial z} \leftrightarrow -i(k - k_0)$$

$$\frac{\partial E}{\partial t} = i(\omega - \omega_0)E \quad => \quad \frac{\partial}{\partial t} \leftrightarrow i(\omega - \omega_0)$$

$$\frac{\partial^2 E}{\partial t^2} = -(\omega - \omega_0)^2 E \quad => \quad \frac{\partial^2}{\partial t^2} \leftrightarrow -(\omega - \omega_0)^2$$

Heuristic derivation of the NLSE

The formal substitution in the Taylor series yields

$$i\frac{\partial E}{\partial z} = -\frac{\partial k}{\partial \omega}\frac{\partial E}{\partial t} - \frac{1}{2}\frac{\partial^2 k}{\partial \omega^2}\frac{\partial^2 E}{\partial t^2} + \frac{\partial k}{\partial |E|^2}|E|^2 E$$

Recalling that

$$\beta_1 = \left(\frac{dk}{d\omega}\right)_{\omega_0} = \frac{1}{v_{gr}} \qquad \beta_2 = \left(\frac{d^2 k}{d\omega^2}\right)_{\omega_0} = \frac{d}{d\omega}\left(\frac{1}{v_{gr}}\right)_{\omega_0} = -\frac{1}{v_{gr}^2}\left(\frac{dv_{gr}}{d\omega}\right)_{\omega_0}$$

=> $$i\frac{\partial E}{\partial z} + \frac{1}{v_{gr}}\frac{\partial E}{\partial t} - \frac{\beta_2}{2}\frac{\partial^2 E}{\partial t^2} + \frac{\partial k}{\partial |E|^2}|E|^2 E = 0$$

Heuristic element – Kerr type nonlinearity $n = n_0 + n_2|E|^2$

$$\boxed{i\frac{\partial E}{\partial z} + \frac{1}{v_{gr}}\frac{\partial E}{\partial t} - \frac{\beta_2}{2}\frac{\partial^2 E}{\partial t^2} + \frac{\omega}{c}n_2|E|^2 E = 0}$$

Nonlinear Schrödinger Equation (NLSE)

$$\left(i\frac{\partial}{\partial z} - i\frac{1}{v_{gr}}\frac{\partial}{\partial t} - \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}\right)A(z,t) = -\frac{3\mu_0\omega_0^2}{8k_0}\epsilon_0\chi^{(3)}|A(z,t)|^2A(z,t)$$

Transformation into moving frame of the pulse and $\gamma = \frac{\mu_0\omega_0^2}{2k_0}\frac{3}{4}\epsilon_0\chi^{(3)}$

NLS

$$\boxed{i\frac{\partial}{\partial z}A(z,T) - \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A(z,T) + \gamma|A(z,T)|^2A(z,T) = 0}$$

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) + \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) - V\psi(x,t) = 0$$

(Comparison to quantum mech. Schrödinger Eq.)

Dimensionless NLSE

Normalization

$$U = \frac{A}{\sqrt{P_0}}$$

Normalized amplitude U and P_0 peakintensity

$$Z = \frac{z}{L_D}$$

Normalized distance

$$T = \frac{\tau}{T_0}$$

Timescale normalized to the input pulse width T_0

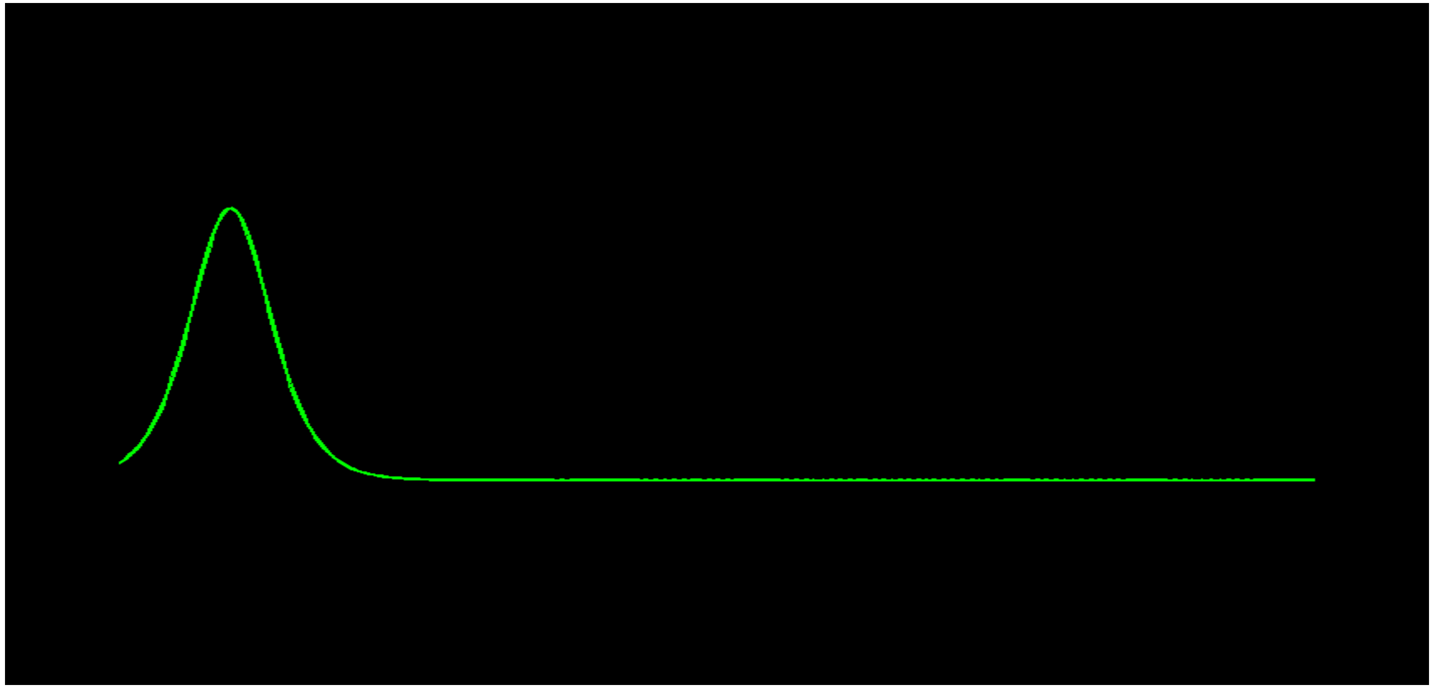
$$i\frac{\partial}{\partial z}U(Z,T) - \mathrm{sgn}(\beta_2)\frac{1}{2}\frac{\partial^2}{\partial T^2}U(z,T) + \frac{L_D}{L_{NL}}|U(Z,T)|^2 U(Z,T) = 0$$

Characteristic lengths depend on the material as well as on the pulse parameters

Dispersionlength $L_D = \frac{T_0^2}{|\beta_2|}$ Nonlinear length $L_{NL} = \frac{1}{\gamma P_0}$ $N = \sqrt{\frac{L_D}{L_{NL}}} = \sqrt{P_0 T_0^2 \frac{\gamma}{|\beta_2|}}$

$u = NU$ => $$\boxed{i\frac{\partial}{\partial Z}u \pm \frac{1}{2}\frac{\partial^2}{\partial T^2}u + |u|^2 u = 0}$$

Soliton (solitary solution)



Solitons

Union Canal, Heriot-Watt University, 1995

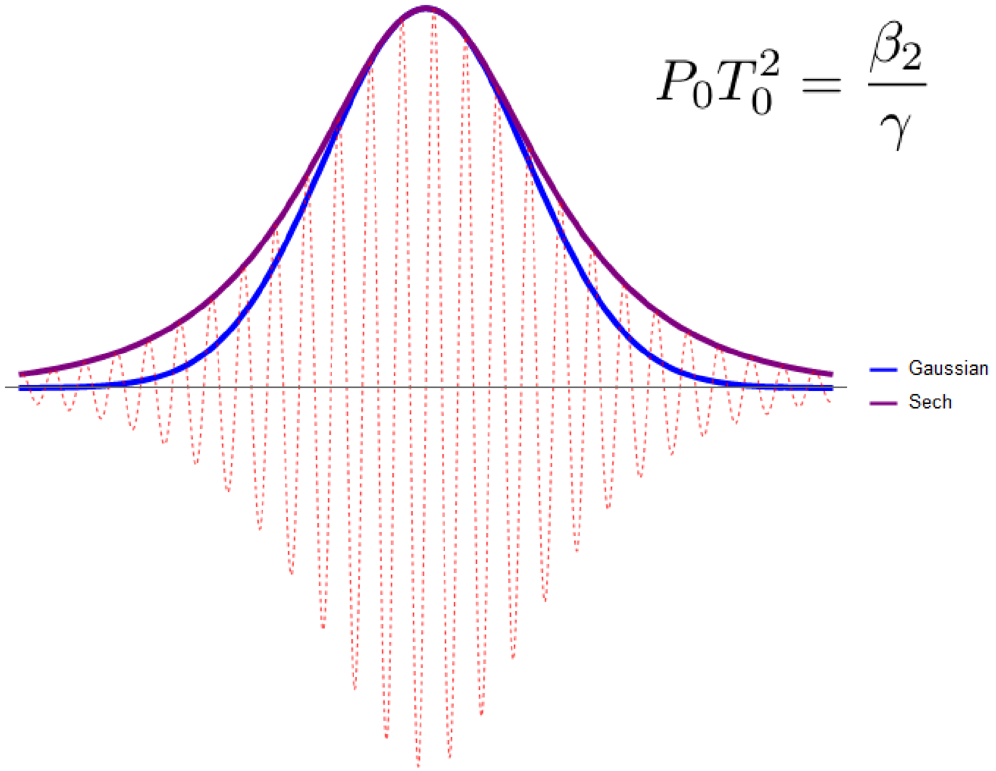
John Scott Russell, "Report on Waves" (Report of the 14th meeting of the British Association for the Advancement of Science, York, September 1844 (London 1845), pp 311-390, Plates XLVII-LVII).

Stable solutions of the NLSE - Solitons

$$\boxed{i\frac{\partial}{\partial Z}u + \frac{1}{2}\frac{\partial^2}{\partial T^2}u + |u|^2 u = 0}$$

$$L_D = L_{NL} \Leftrightarrow N = 1$$

- Balance between SPM and GVD
- Valid for $n_2 > 0$ and $\beta_2 < 0$
- Anomalous dispersion



$$P_0 T_0^2 = \frac{\beta_2}{\gamma}$$

Fundamental Soliton:

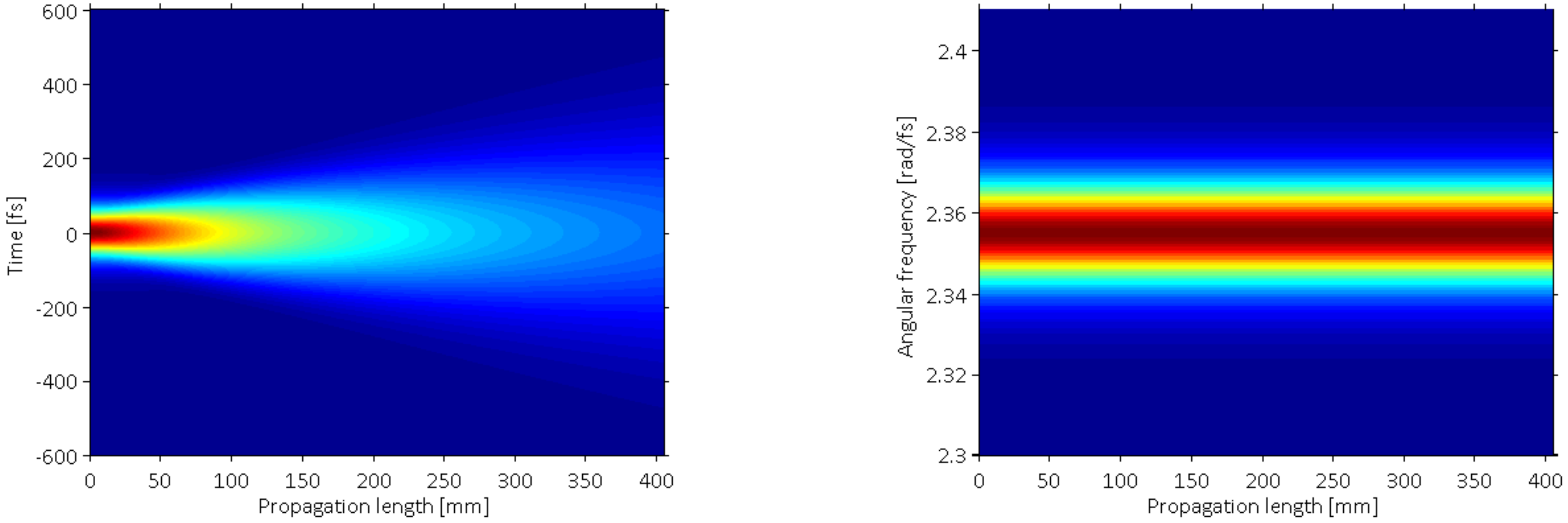
$$\boxed{u = \mathrm{sech}(T) e^{\frac{1}{2} i Z}}$$

Hyperbolic Secant:

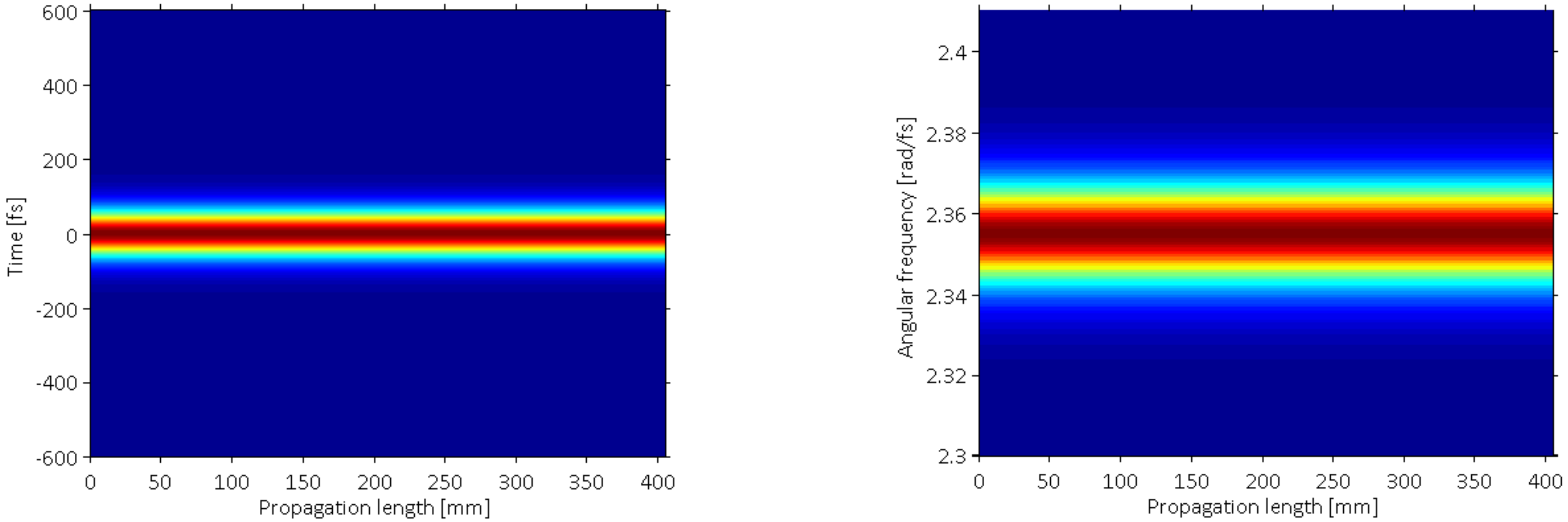
$$\mathrm{sech}(x) = \frac{2}{e^x + e^{-x}} = \cosh^{-1}(x)$$

The fundamental soliton (N=1)

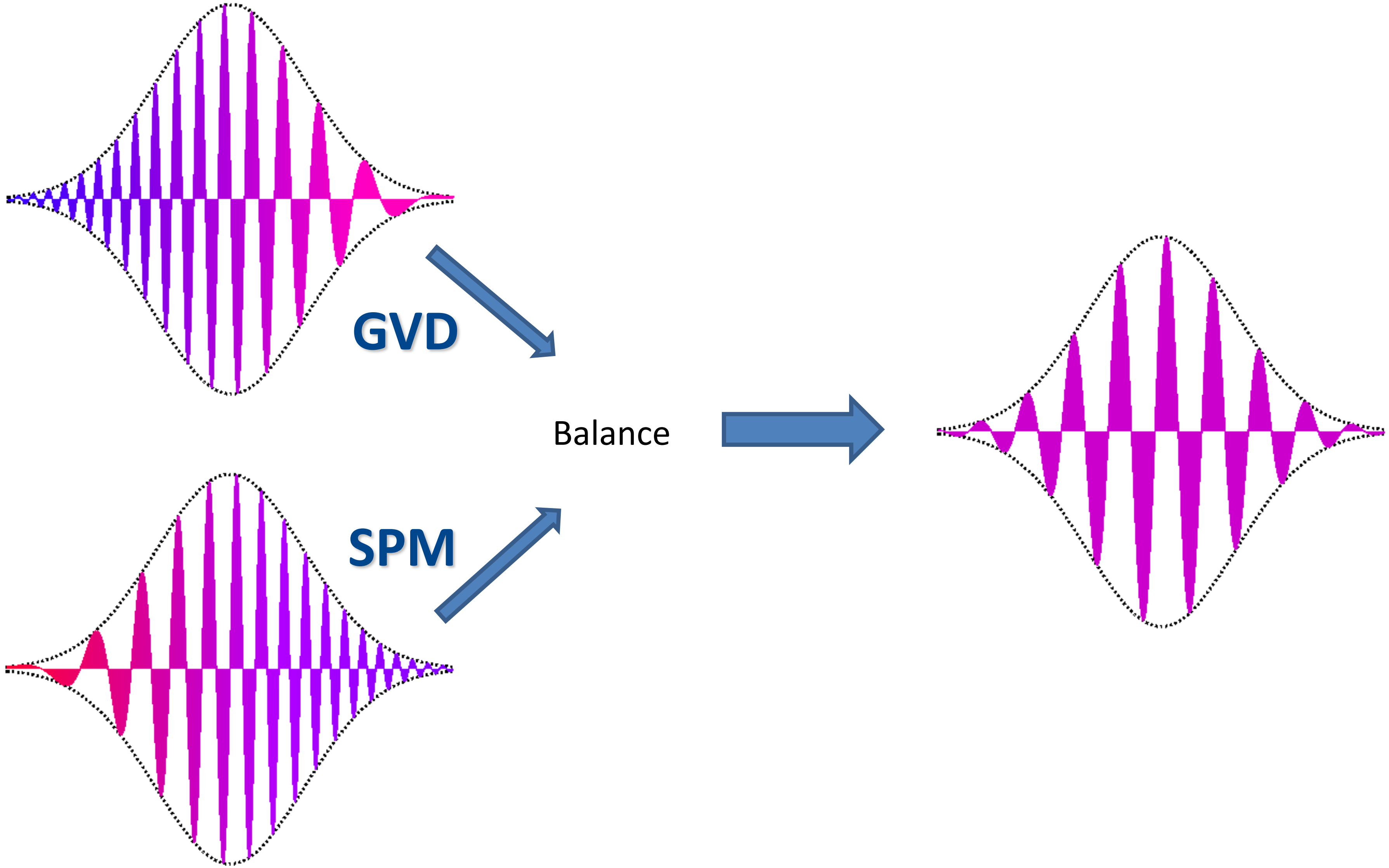
Dispersive propagation



Soliton: The negative GVD cancels the SPM’s positive one.

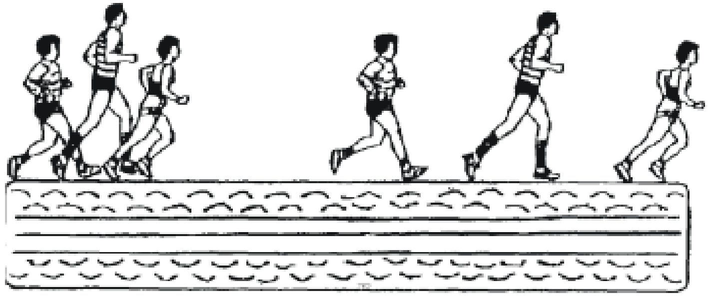


GVD & SPM - Solitons

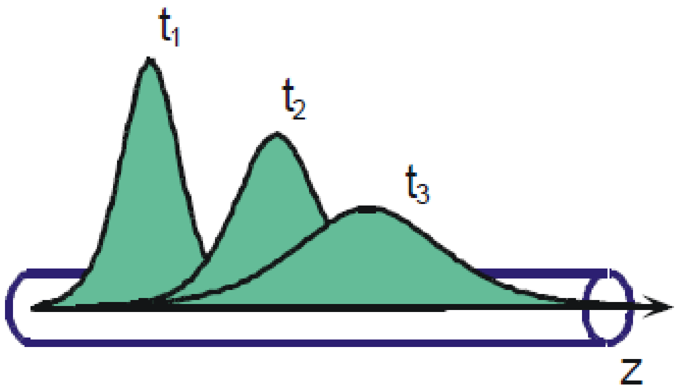


"runners«wave packet" *

ultrashort pulses in optical fibers



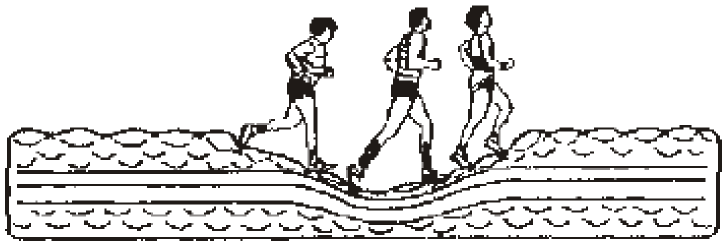
dispersion



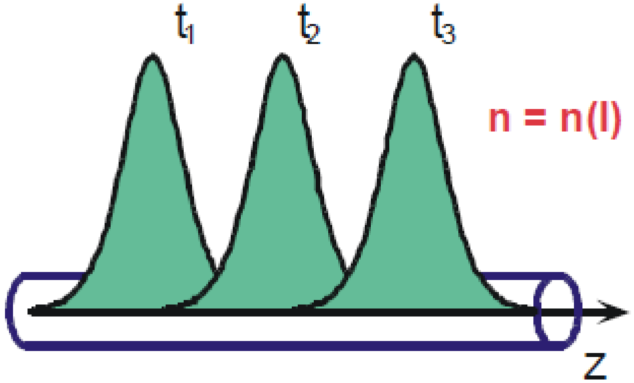
"soft mat"

nonlinearity

Kerr-effekt



soliton



(* Mollenauer)

Nonlinear Schrödinger Equation

Nonlinear Schrödinger Equation (NLSE)

$$i\frac{\partial A(z,T)}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 A(z,T)}{\partial T^2} - \gamma\,|A(z,T)|^2\,A(z,T)$$

Co-moving frame $T = t - z/v_g = t - \beta_1 z$

Kerr nonlinearity $\gamma = n_2\omega_0/cA_{eff}$

Instantaneous power (W) $|A(z,T)|^2$

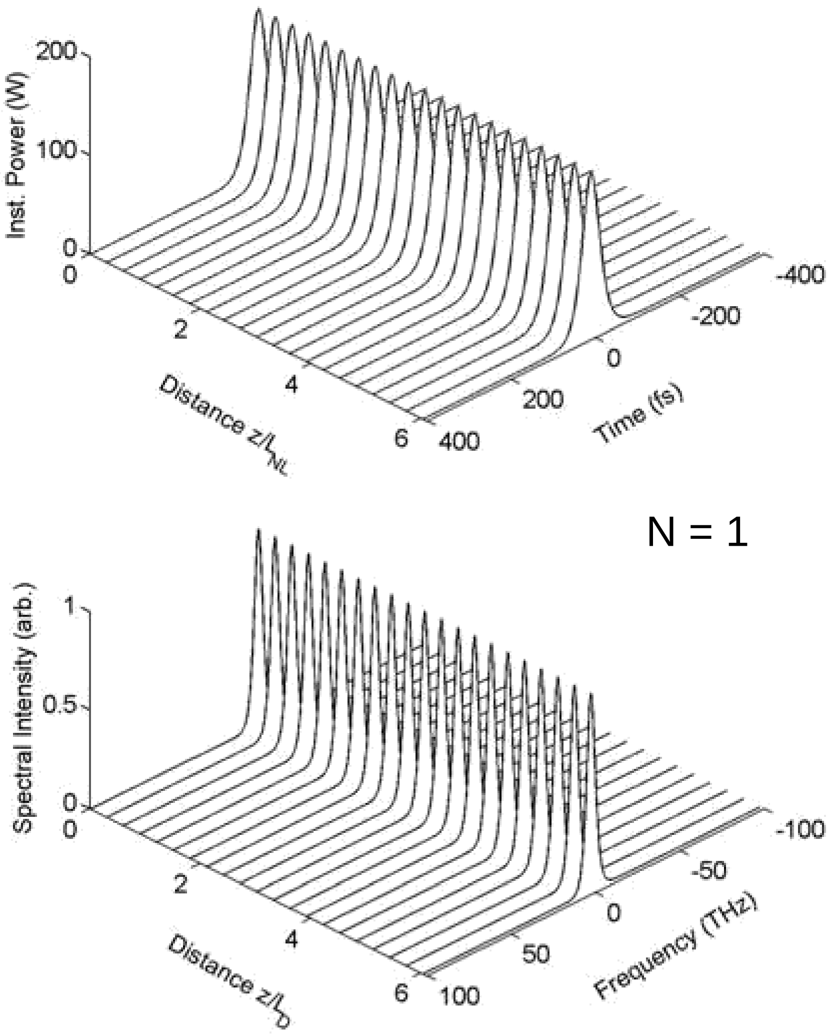
Consider propagation in highly nonlinear PCF: **ZDW at 780 nm**

λ = 850 nm
β_2 = -13 ps^2 km^{-1}
γ = 100 W^{-1}km^{-1}

$$A(z=0,T) = \sqrt{P_0}\,\mathrm{sech}(T/T_0)$$

T_0 = 28 fs (FWHM 50 fs)

Fundamental solitons (N=1)



Initial condition

$$A(z=0,T) = \sqrt{P_0}\,\mathrm{sech}(T/T_0)$$

$$\frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} = 1$$

$$\implies \quad P_0 = \frac{|\beta_2|}{\gamma T_0^2}$$

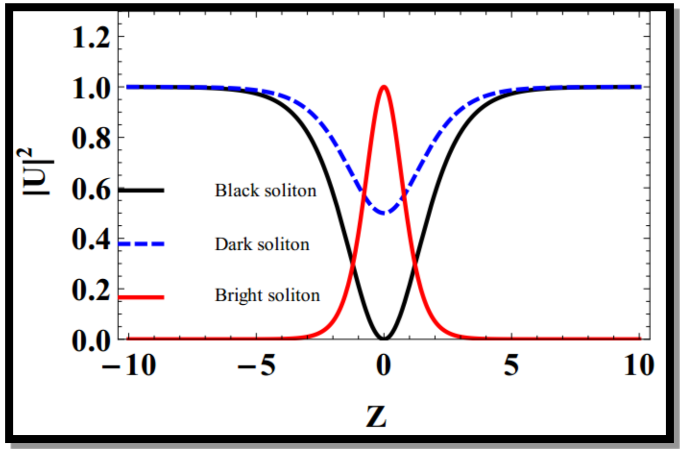
Invariant evolution

$$A(z,T) = \sqrt{P_0}\,\mathrm{sech}(T/T_0)\exp(\mathrm{i}k_{\mathrm{sol}}z)$$

$$k_{sol} = \gamma P_0/2$$

soliton wavenumber

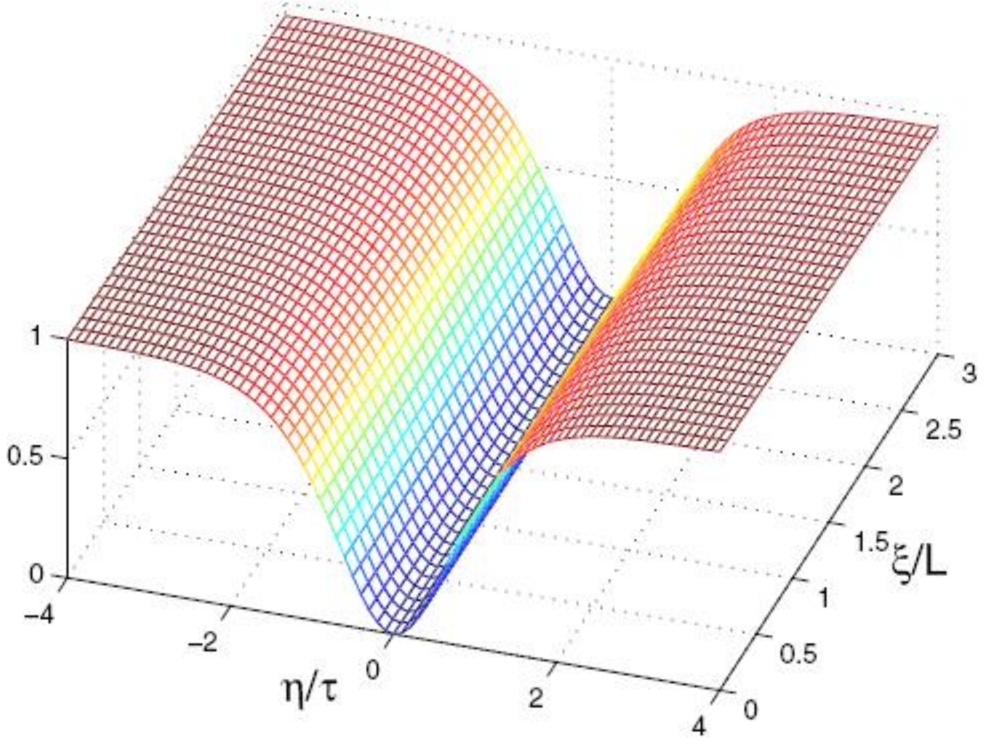
Dark solitons



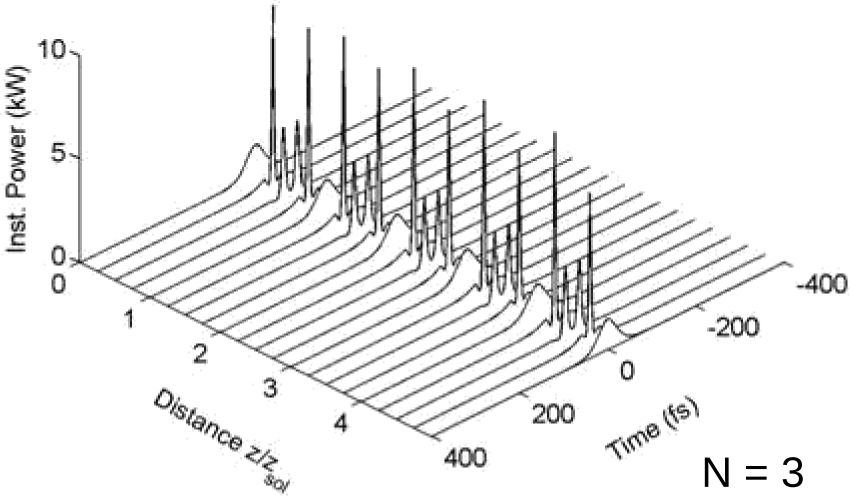
Soliton solution in the normal dispersion regime

$$i\frac{\partial}{\partial Z}u - \frac{1}{2}\frac{\partial^2}{\partial T^2}u + |u|^2 u = 0$$

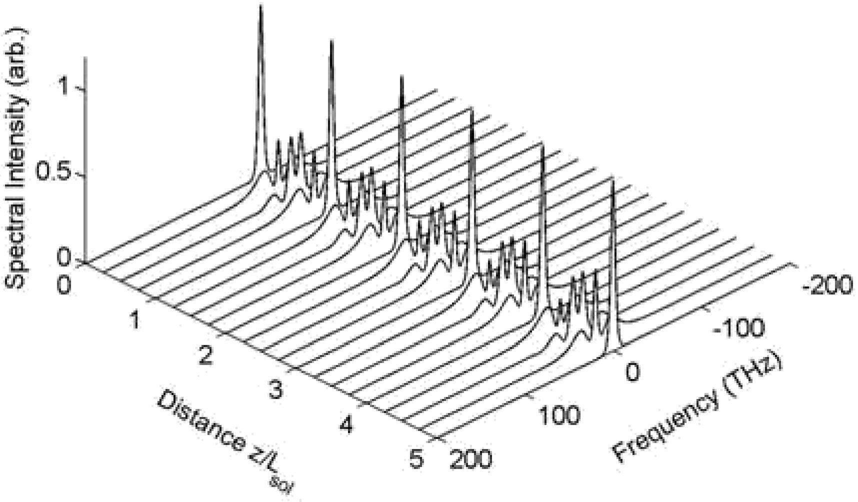
$$u(Z,T) = u_0 \tanh(u_0 T)e^{iu_0^2 Z}$$



Higher-order solitons



N = 3



Initial condition

$$A(z=0,T) = \sqrt{P_0}\,\mathrm{sech}(T/T_0)$$

$$\frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} = N^2$$

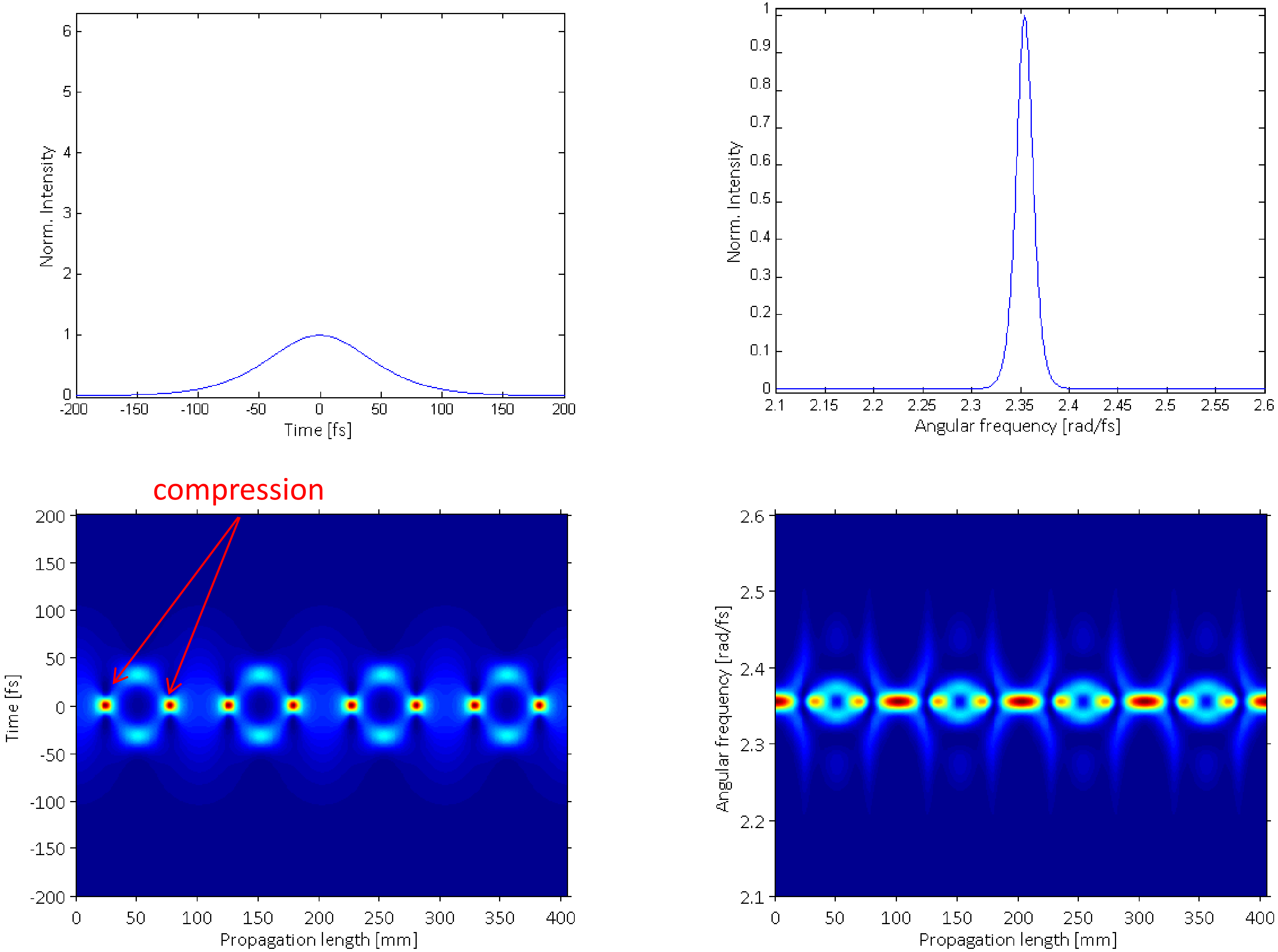
$$N = 2, 3, 4, 5, \ldots$$

$$\implies \quad P_0 = N^2 \frac{|\beta_2|}{\gamma T_0^2}$$

Periodic evolution

$$z_{\mathrm{sol}} = \frac{\pi}{2} L_D = 10 \text{ cm}$$

Higher-order soliton (N=3)

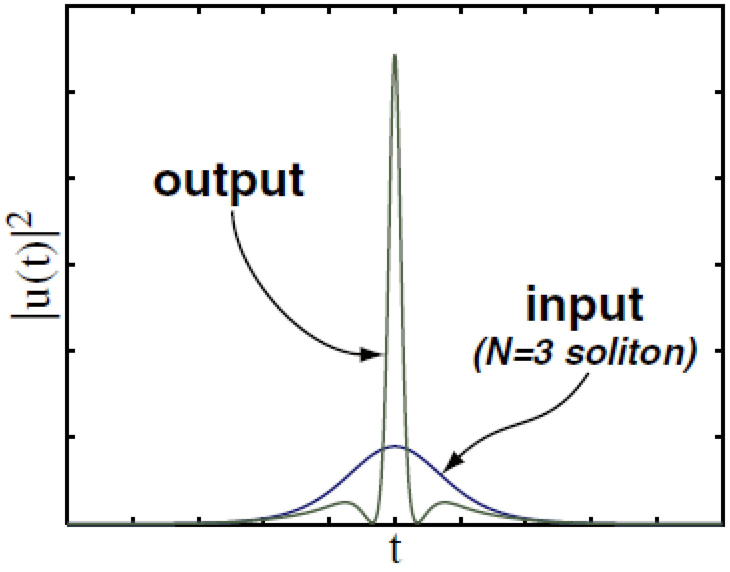


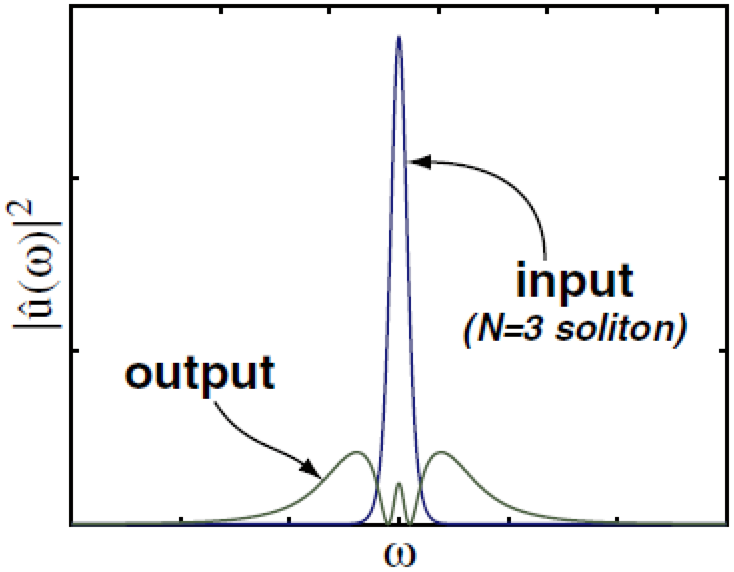
'High-order' soliton compression

- First step of higher-order soliton propagation
- Propagation lengths with optimal compression
- Energy is transferred into the pedestals
- Empirical relation:

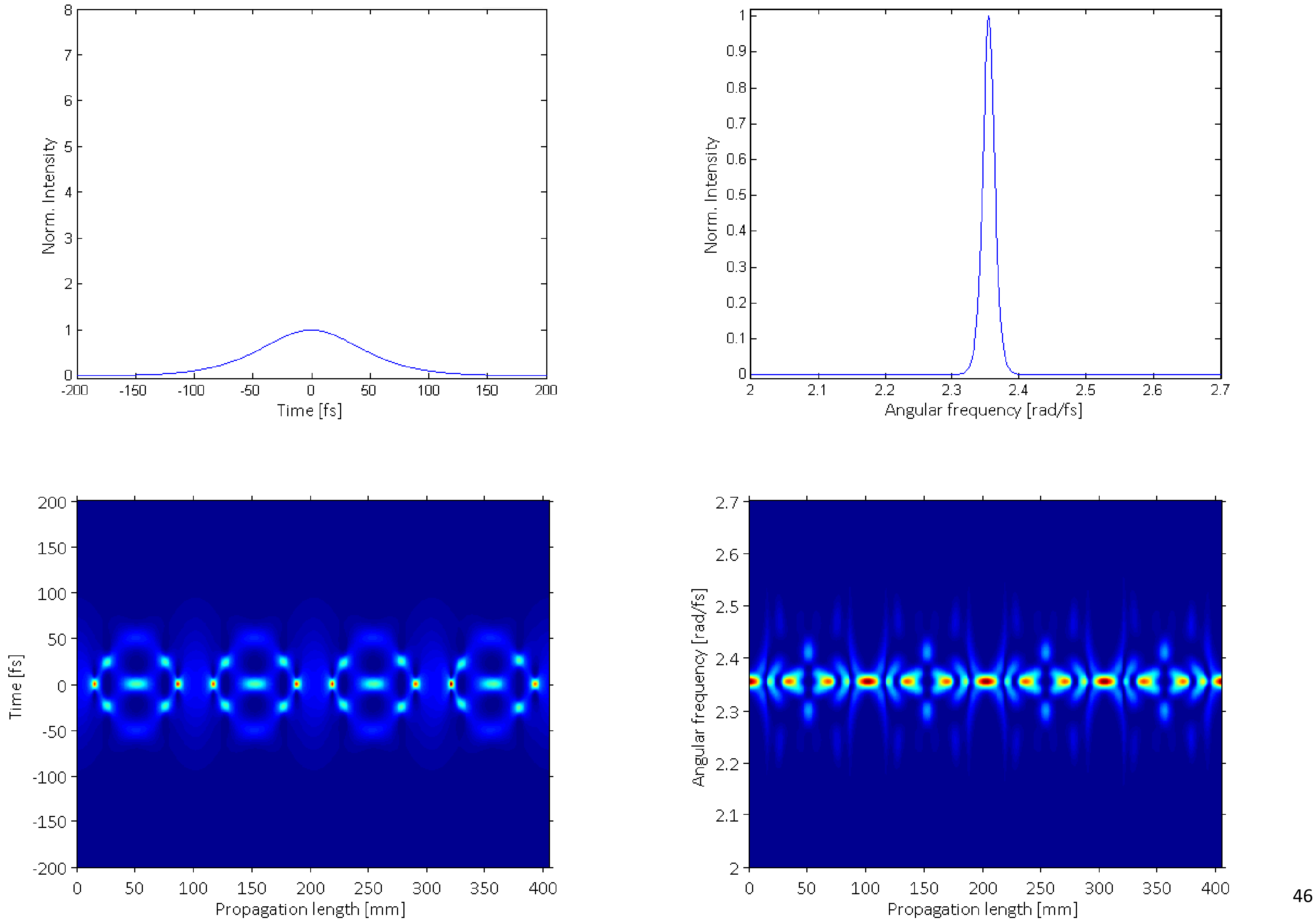
$$\frac{L_{opt}}{z_0} = \frac{0.32}{N} + \frac{1.1}{N^2}$$

Example: N=3 soliton compression (~12X):

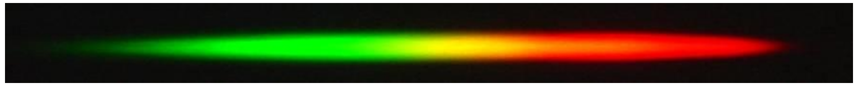




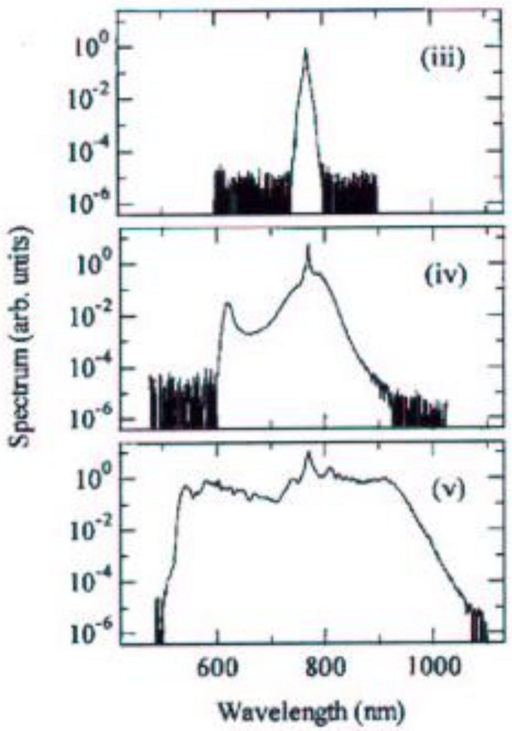
4th-Order Soliton (N=4)

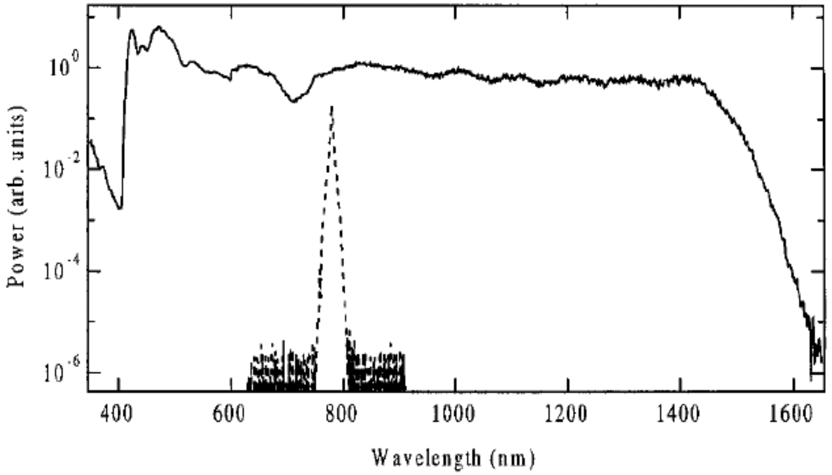


Dramatic spectral broadening



nJ Energie, femtosecond
Ti:Sapphire
Anomalous GVD
Pump wavelength





~ 75 cm PCF

Ranka *et al*. Optics Letters 25, 25 (2000)

Propagation Equation

Generalized Nonlinear Schrödinger Equation (GNLS)

$$\frac{\partial A}{\partial z} = \underbrace{-\frac{i}{2}\beta_2\frac{\partial^2 A}{\partial \tau^2} + \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial \tau^3} + \frac{i}{24}\beta_4\frac{\partial^4 A}{\partial \tau^4}}_{\text{Dispersion}} \underbrace{-\frac{\alpha}{2}A}_{\text{Absorption}}$$

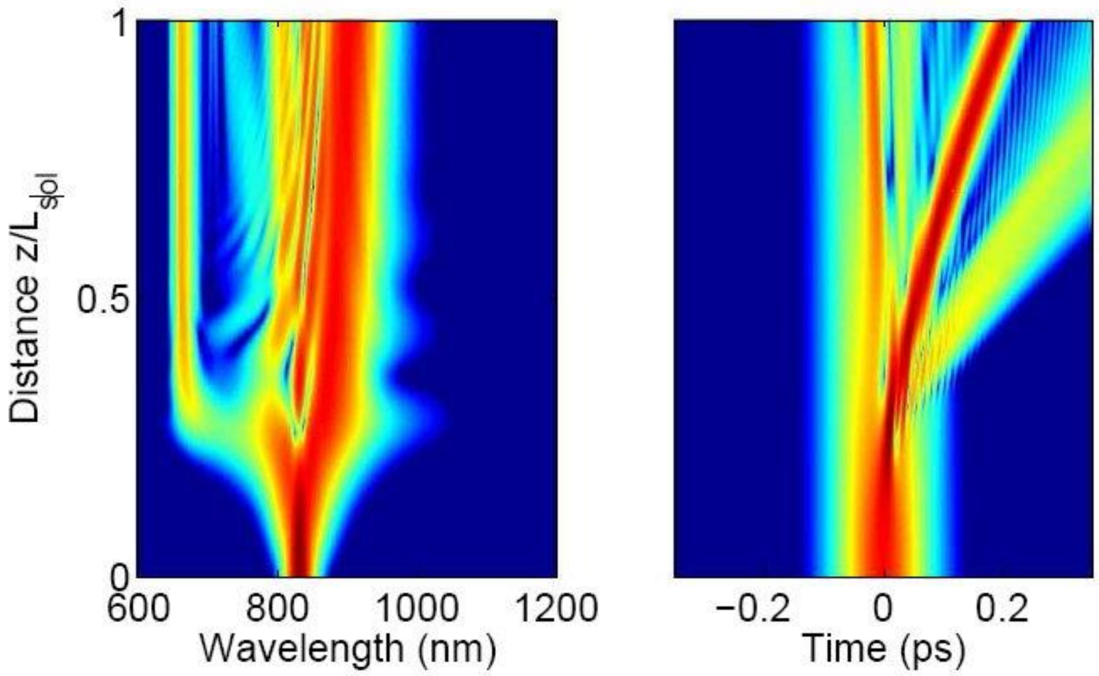
$$+\underbrace{i\gamma|A|^2 A}_{\text{SPM}} \underbrace{-\frac{\gamma}{\omega_0}\frac{\partial}{\partial \tau}\left(|A|^2 A\right)}_{\text{Self-steepening}} \underbrace{-i\gamma T_R A\frac{\partial}{\partial \tau}\left(|A|^2\right)}_{\text{Raman Scattering}}$$

Propagation of (sub–)picosecond Pulses along $z\,(\tau = t - z/v_g)$

Supercontinuum propagation dynamics

- Injection of a higher-order soliton in the anomalous dispersion regime

spectral evolution time evolution



SPM and GVD

ideal periodic evolution

N = 3

$L_{sol} = \pi/2\ T_0^2/|\beta_2|$

Perturbation (Raman, higher-order dispersion…)

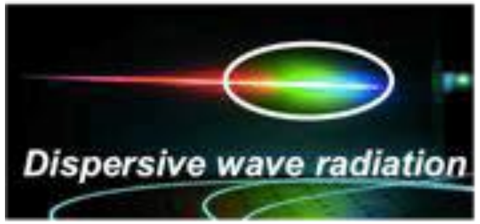
Induced fission into fundamental solitons

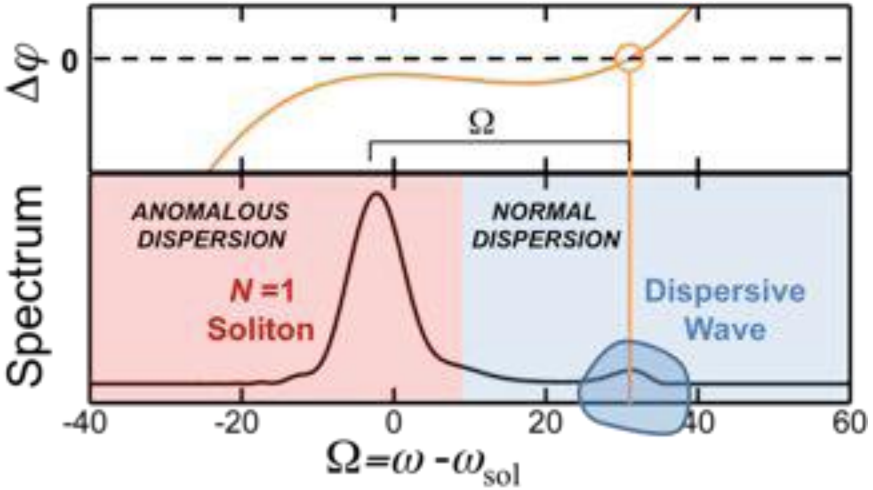
Solitons create blue-shifted radiation

Third order dispersion perturbation on N=1 Soliton

- Higher-order dispersion: fundamental Soliton looses energy to radiation
- Radiation in the normal dispersion regime (dispersive wave)
  - Phase–matching condition: phase$^{(soliton)}$=phase$^{(disp.wave)}$

$$\boxed{\beta(\omega_s) - \omega_s\beta_1(\omega_s) + \gamma P_0/2 = \beta(\omega_r) - \omega_r\beta_1(\omega_r)}$$



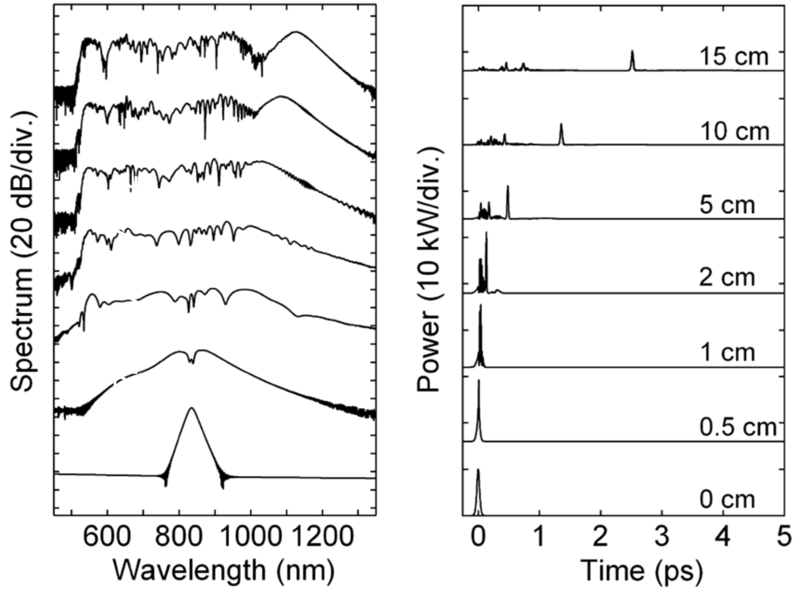


Modeling supercontinuum generation

Generalized NLSE (Dudley, Genty, Coen, *Rev. Mod. Phys.* 78, 1135 (2006))

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2} - \sum_{k\geq 2} \frac{i^{k+1}}{k!}\beta_k \frac{\partial^k A}{\partial T^k} = i\gamma\left(1 + \tau_{shock}\frac{\partial}{\partial T}\right)\left(A(z,t)\int_{-\infty}^{\infty} R(T')|A(z,T-T')|^2 dT'\right)$$

Spectral Evolution   Temporal Evolution



FWHM 50 fs, 835 nm, 0.5 nJ, 15 cm PCF, N ~ 9

Beaud et al. JQE (1987)

Islam et al. JOSAB (1989)

……

Herrmann et al. PRL (2002)

Gaeta OL (2002)

Brown JOSAB (2004)

Demircan et al. APB (2007)

……

Modeling supercontinuum generation

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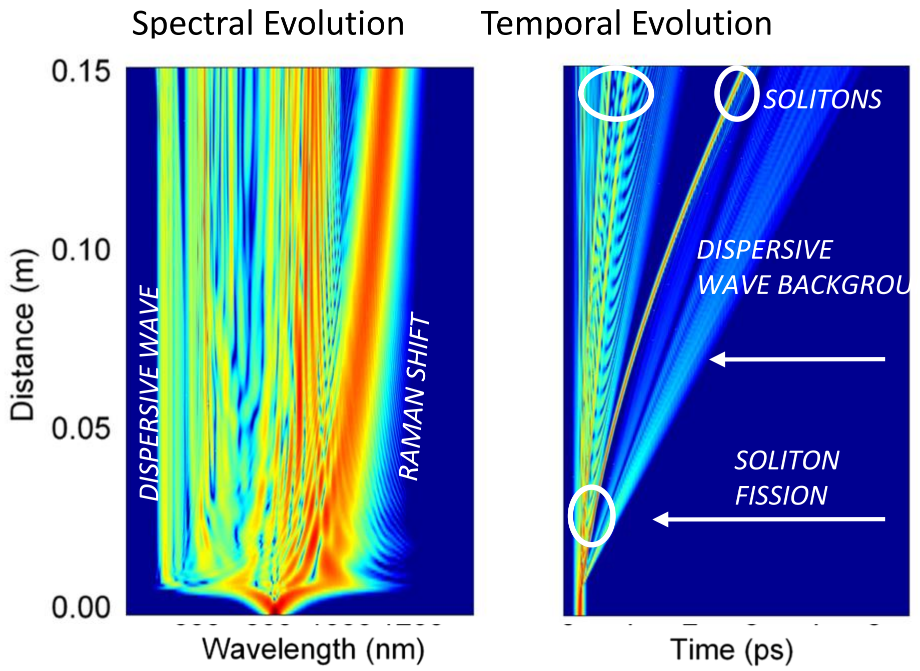

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……

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Gaeta OL (2002)

Brown JOSAB (2004)

Demircan et al. APB (2007)

……

ROGUE WAVES

TIME

Thursday, Mar. 04, 2010

Cruise-Ship Disaster: How Do 'Rogue Waves' Work?

By Bryan Walsh

The New York Times
nytimes.com

July 11, 2006

Rogue Giants at Sea

By WILLIAM J. BROAD

The Economist

Rogue waves

Monsters of the deep

Huge, freak waves may not be as rare as once thought

Sep 17th 2009 | From the print edition

SCIENTIFIC AMERICAN™

Permanent Address: http://www.scientificamerican.com/article/rogue-waves-ocean-energy-forecasting/

The Sciences » News

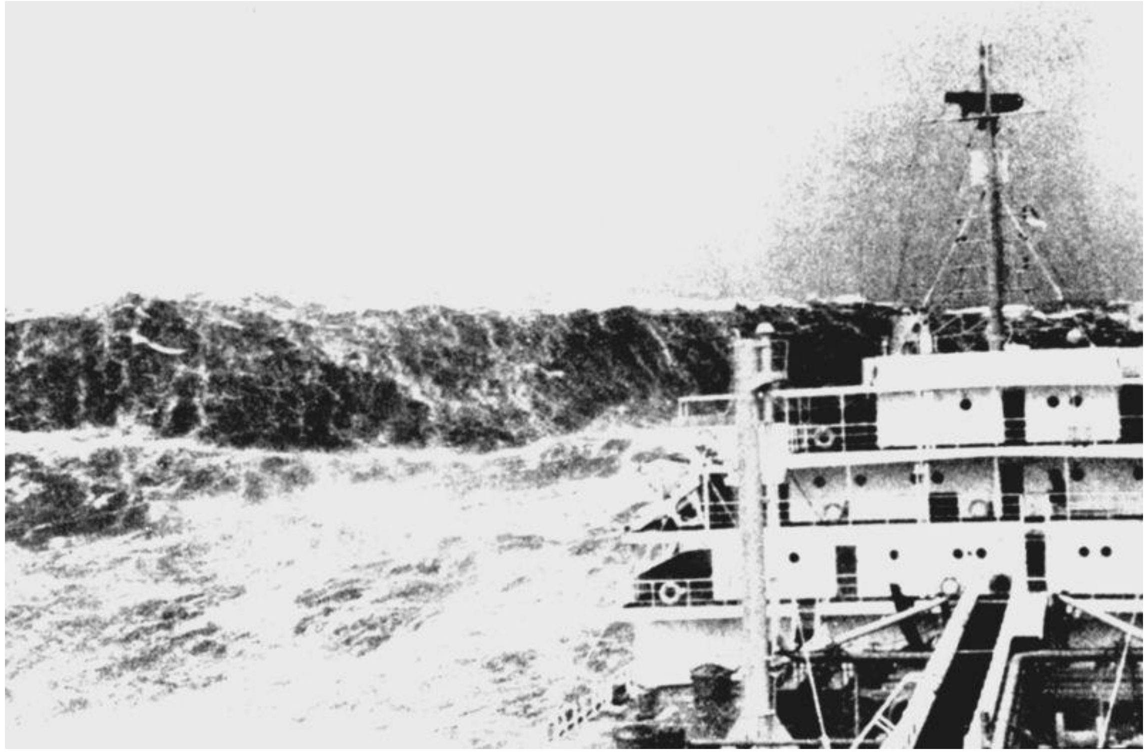
The Real Sea Monsters: On the Hunt for Rogue Waves

Scientists hope a better understanding of when, where and how mammoth oceanic waves form can someday help ships steer clear of danger

By Lynne Peeples | September 2, 2009 | 0

Rogue Waves, Freak Waves, White Wall,Three Sisters

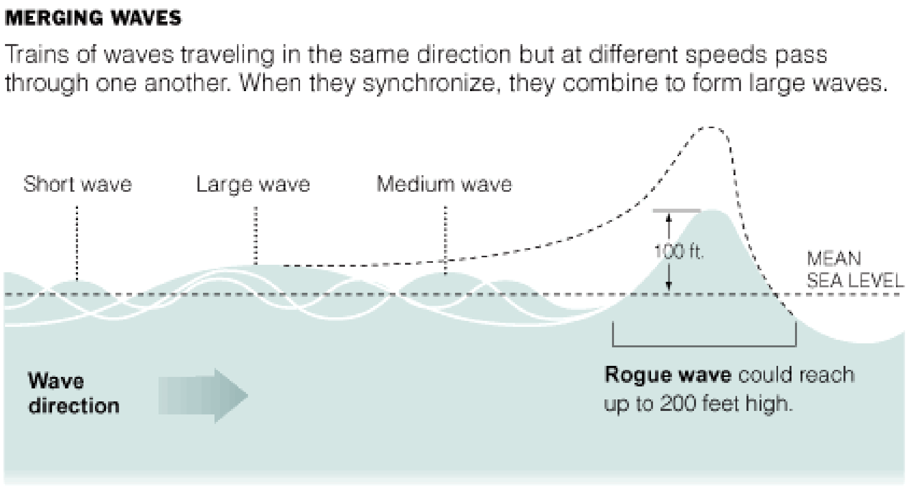
Unexplained Phenomenon in the open ocean

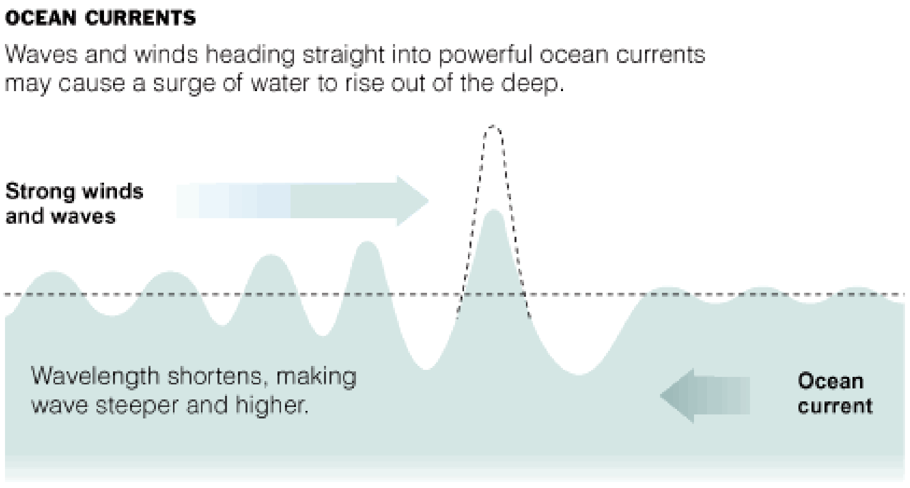


" … just yarn spun by imaginative sailors"


WILSTAR


© Reuters



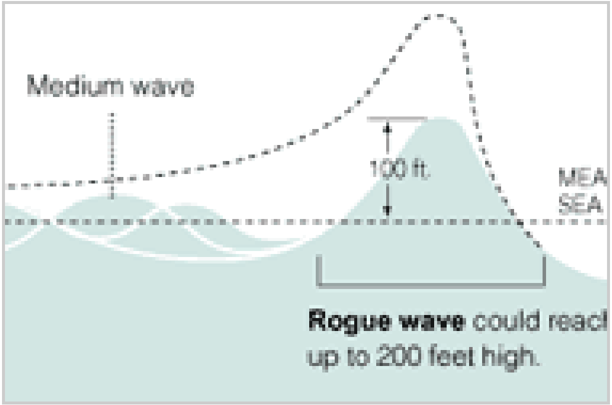


ForschungsZentrum Küste (FZK)
Großer Wellenkanal (GWK)

Leibniz Universität Hannover
Technischen Universität Braunschweig

307 m long
7.00 m deep und 5.00 m wide

Motivation and Overview

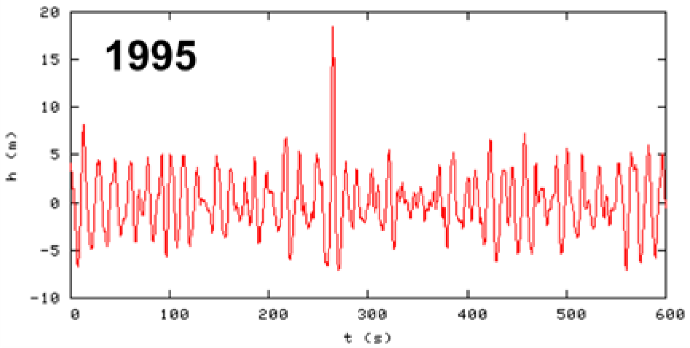


Characteristics:

- High amplitude
- Localized in time and space
- Unpredictability
- Defy Gaussian statistics (observed more often)

Measurements in 1990's have established long-tailed statistics

Wave Height (m)



p (%)

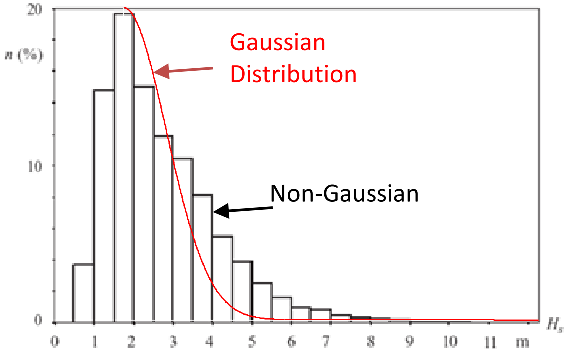


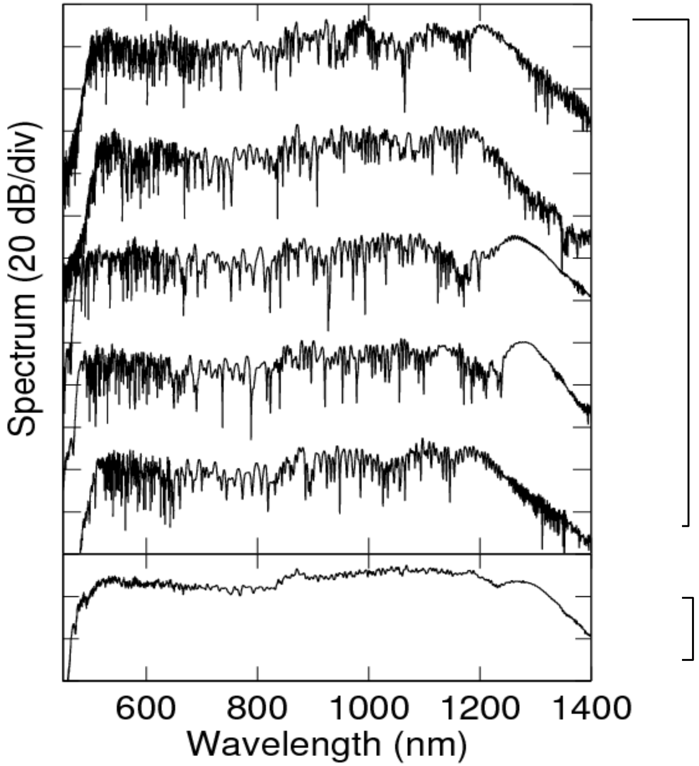
Figure 4.19 The histogram of the significant wave height for the years 1980–2003 for NODC buoy 46005 of Fig. 4.17 (n is the percentage of the total number of occurrences in the interval $\Delta H_s = 0.5$ m).

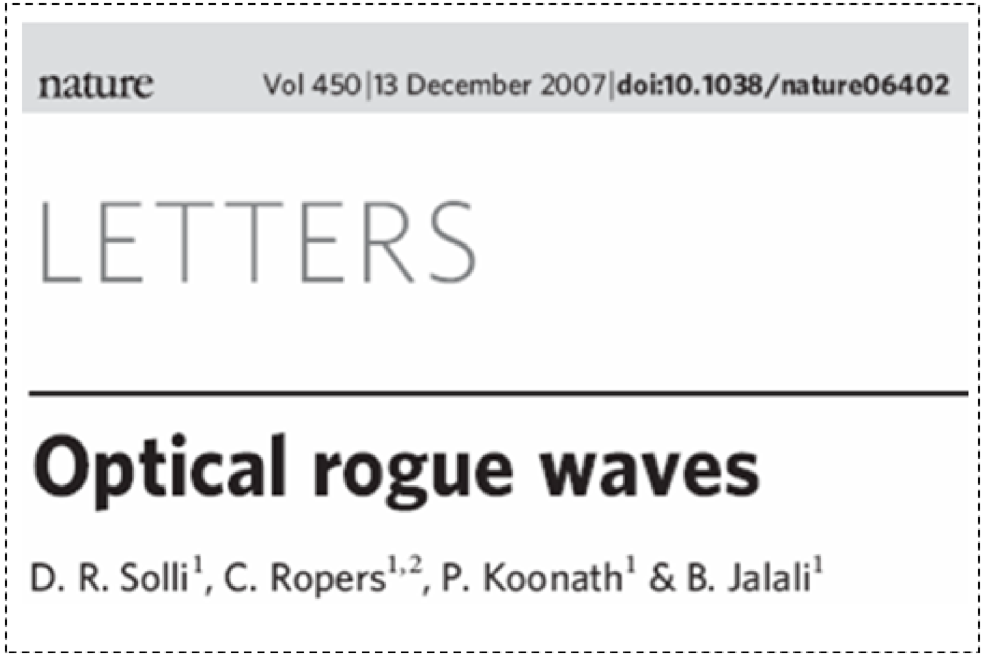
h (m)

Generalized nonlinear Schrödinger equation

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A - \sum_{k\geq 2}\frac{\mathrm{i}^{k+1}}{k!}\beta_k\frac{\partial^k A}{\partial T^k} = i\gamma\left(1+\mathrm{i}\tau_{\text{shock}}\frac{\partial}{\partial T}\right)\left(A(z,t)\int_{-\infty}^{+\infty}R(T')|A(z,T-T')|^2 dT'\right)$$

Stochastic simulations




nature Vol 450|13 December 2007|doi:10.1038/nature06402

LETTERS

Optical rogue waves

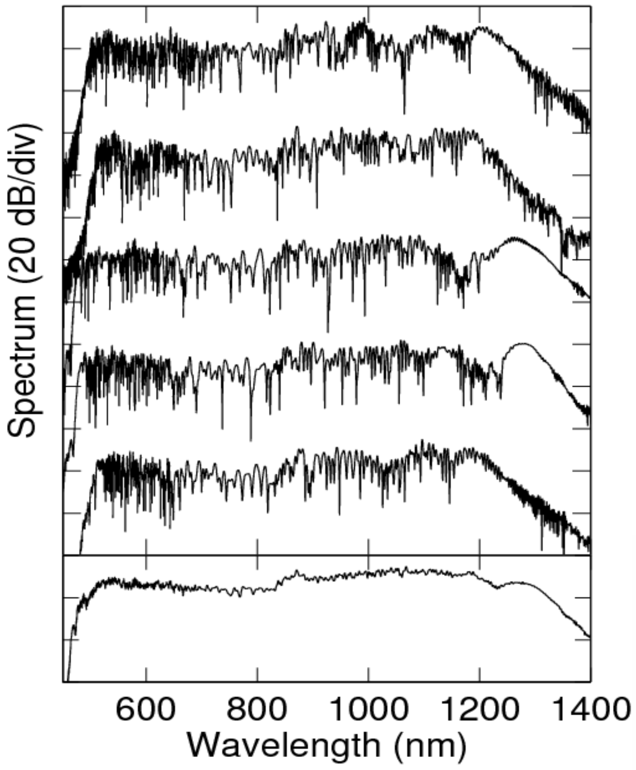
D. R. Solli[1], C. Ropers[1,2], P. Koonath[1] & B. Jalali[1]

835 nm, 150 fs 10 kW, 10 cm

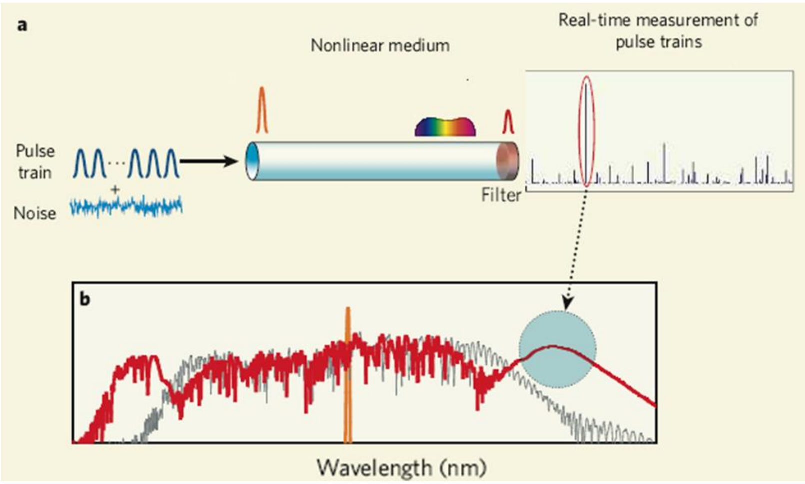
Spectral instabilities

Initial “optical rogue wave” paper detected these spectral fluctuations

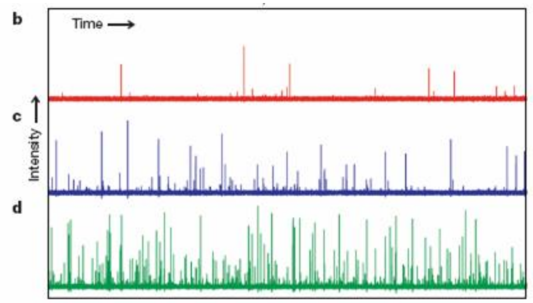
Stochastic simulations



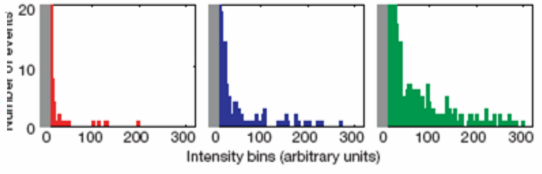
Schematic



Time Series

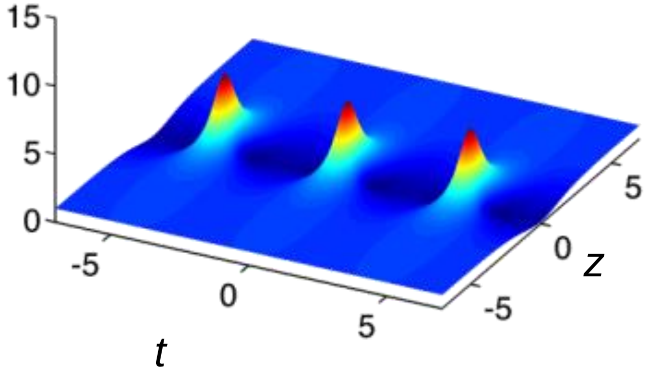


Histograms

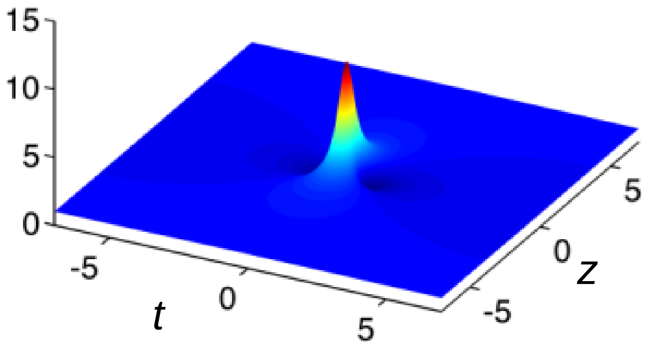


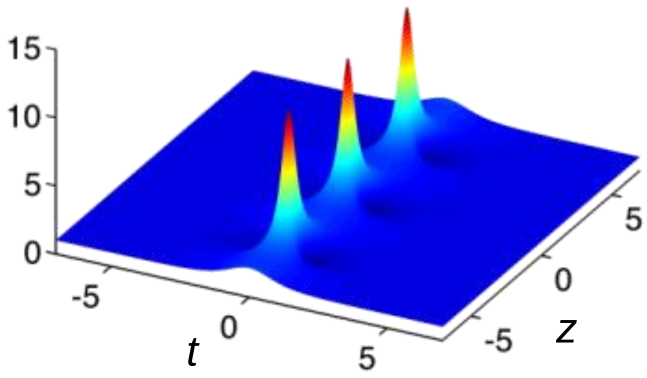
Solitons on Finite Background (SFB)

Akhmediev breather (AB)
Hammani *et al.*,OL (2011)

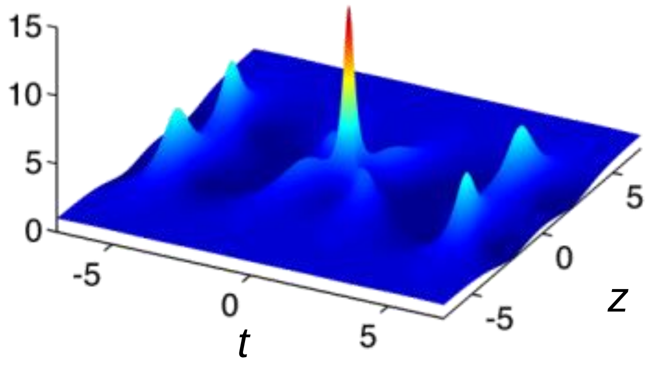


Peregrine soliton (PS)
Kibler *et al.*, Nat. Phys. (2010)





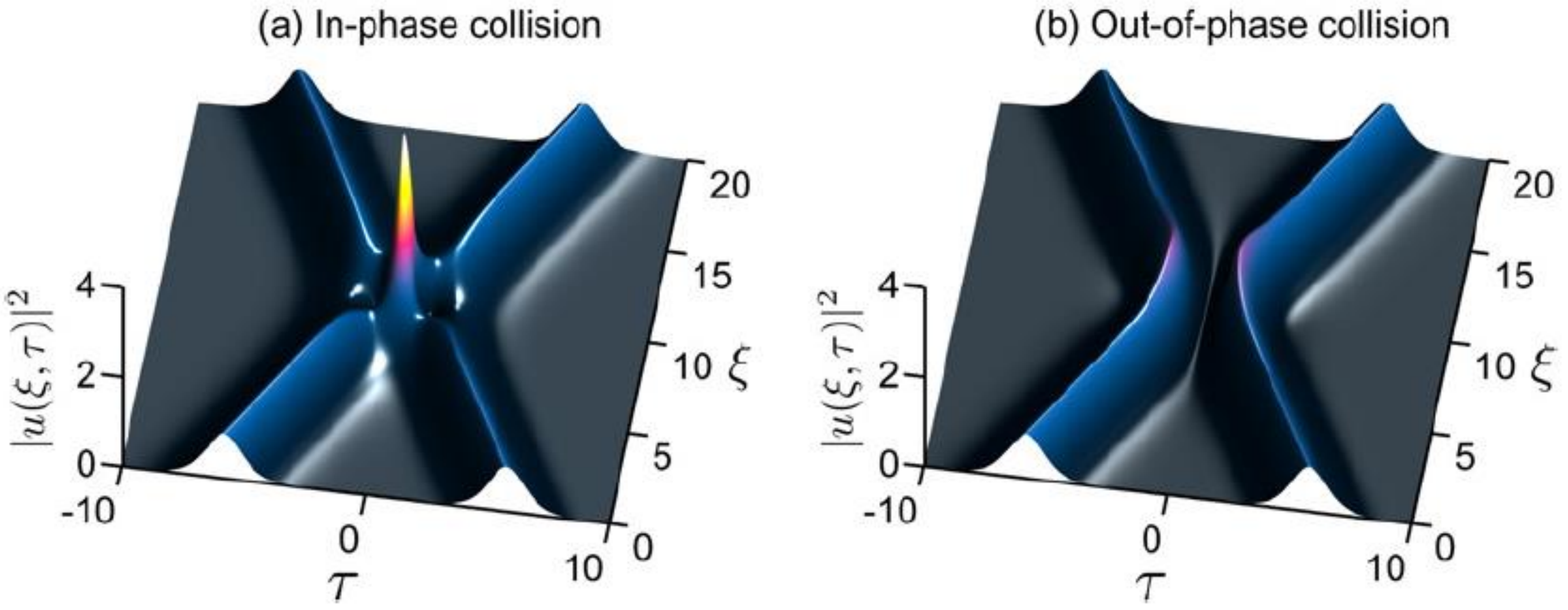
Kuznetsov-Ma soliton (KM)
Kibler *et al.*, Sci. Rep. (2012)



Higher-order AB
Frisquet *et al.*, PRX (2013)

WAVE-BLOCKING

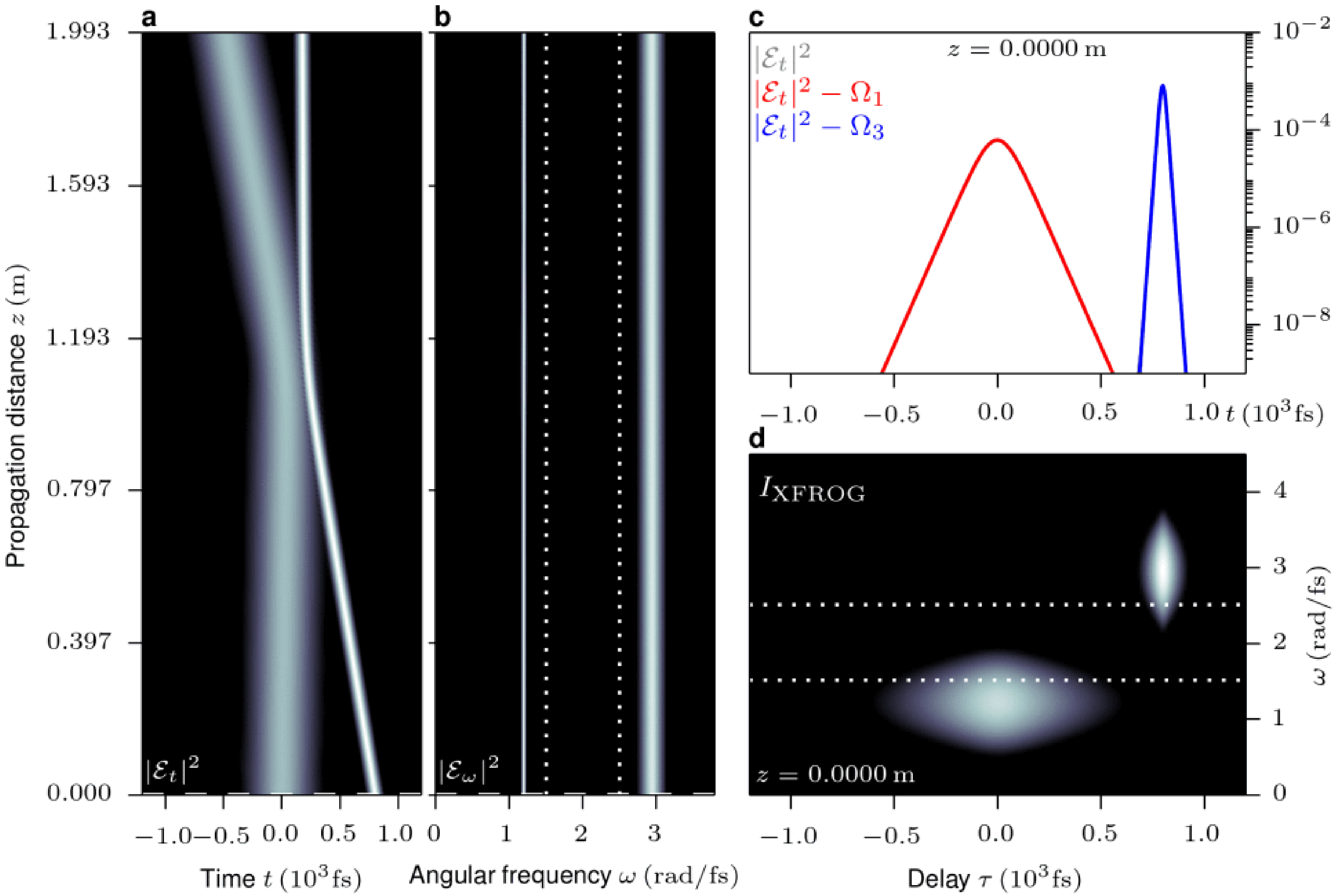
Soliton collision



- Two solitons propagating in the same direction with different group velocities

- Pulse shapes unchanged after the collision

- Particle like behavior (depending on the phase)



a
b
c
d
Propagation distance z (m)
1.993
1.593
1.193
0.797
0.397
0.000
$|\mathcal{E}_t|^2$
$|\mathcal{E}_\omega|^2$
Time t (10^3fs)
Angular frequency ω (rad/fs)
$z = 0.0000$ m
$|\mathcal{E}_t|^2$
$|\mathcal{E}_t|^2 - \Omega_1$
$|\mathcal{E}_t|^2 - \Omega_3$
t (10^3fs)
10^{-2}
10^{-4}
10^{-6}
10^{-8}
I_{XFROG}
$z = 0.0000$ m
ω (rad/fs)
Delay τ (10^3fs)

Two-Pulse Interaction