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Reflection and Capture of a Quasi-Monochromatic Pulse in the Interaction with Co-Propagating Extremely Short Pulses

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Abstract—We study the modes of reflection, capture, and tunneling of a quasi-monochromatic pulse in the nonlinear interaction with an intense extremely short pulse. Conditions and parameters of the pulses and the environment are formulated.

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INTRODUCTION

The ability to control the propagation of light by means of light is of considerable interest in nonlinear optics and photonics. The number of studies on the effect of light reflecting from a moving inhomogeneity induced by an intense laser pulse [1–3], or from a response or defects in the medium itself [4, 5], has grown recently.

The essence of the above effects of reflection is that during the parametric interaction of simultaneously propagating pulses in a nonlinear dispersive medium, the signs of both the group velocity mismatch and the pulse frequency change.

Extremely short pulses (ESPs) of laser radiation with durations of several oscillations are becoming increasingly important in optical technologies, so it is interesting to consider the interaction of objects with quasi-monochromatic pulses (QMPs).

Note that dispersion spreading can hinder the manifestation of the reflection effect [3]. This is particularly important for ESPs, which have a very wide spectrum that distorts the profile of an inhomogeneity. An exception is the propagation of pulses in the soliton mode, and we consider such cases below in investigating various modes of controlling the speed and frequency of quasi-monochromatic pulses in their parametric interaction with extremely short high-intensity pulses.

MODEL EQUATIONS

Let us consider the propagation of a linearly polarized laser pulse in a nonlinear medium. Suppose there is interaction between two pulses whose characteristic frequencies ω and Ω differ considerably: $\omega \ll \Omega$. To be definite, we assume that the frequency of ω lies in the optical range, while that of Ω lies in the far-infrared or terahertz frequency range.

Assuming that the optical pulse has a well-defined carrier frequency ω, wave number k, and envelope ψ, let us write the field E_o of the optical pulse in the form

$$E_o = \psi \exp\left[i\left(\omega t - kz\right)\right] + c.c. \qquad (1)$$

At the same time, we must write the equation for the low-frequency component directly for its field E_Ω.

We consider the case of a quadratically nonlinear medium in which the spectrum of ESPs lies in the terahertz range, below the characteristic frequency of optical phonons. The corresponding self-consistent system of equations is [6, 7]

$$i\left(\frac{\partial\psi}{\partial z} + \frac{1}{\upsilon_g}\frac{\partial\psi}{\partial t}\right) + \frac{k_2}{2}\frac{\partial^2\psi}{\partial t^2} - a\omega E_\Omega \psi + k_2 g|\psi|^2\psi = 0, \qquad (2)$$

$$\frac{\partial E_\Omega}{\partial z} + \frac{n_\Omega}{c}\frac{\partial E_\Omega}{\partial t} - \sigma\frac{\partial^3 E_\Omega}{\partial t^3} + \frac{\partial}{\partial t}\left(\eta E_\Omega^2 + b|\psi|^2\right) = 0, \qquad (3)$$

where n_Ω and σ are a massless index of refraction and a dispersion coefficient in the terahertz frequency range, and $\nu_g = (\partial k/\partial\omega)^{-1}$ and $k_2 = \partial^2 k/\partial\omega^2$ are the group velocity and the group velocity dispersion coefficient of the optical pulse. Coefficients a, b, and η in the nonlinear terms are expressed in terms of the corresponding frequency components of the nonlinear susceptibility tensor χ_2. Here we also consider the contribution from third-order nonlinearity ($g \neq 0$) in the equation for the optical pulse.

When there is no low-frequency component, instead of (2) we have a nonlinear Schrödinger equation (NSE), which admits solutions such as bright solitons at $g > 0$. Below, we assume the sign of the coefficient to be like this.

In the absence of an optical component. Eq. (3) becomes the Korteweg–de Vries (KdV) equation, which has single-soliton solutions like a single spike of the field

$$E_\Omega = -\frac{6\sigma}{\eta\tau_p^2}\operatorname{sech}^2\left(\frac{t - z/\nu_s}{\tau_p}\right). \tag{4}$$

Here, $1/\nu_s = n_\Omega/c - 4\sigma/\tau_p^2$ is the velocity and τ_p is the duration of the ESP.

An associated mode of pulse propagation in the soliton mode for $g = \sigma = \eta = 0$, $\nu_g = c/n_\Omega$ is also possible. In this case, we have a reduced version of the Zakharov equation (the Yajima–Oikawa system), which is integrable like the two cases described above [8].

REFLECTION OR CAPTURE OF A PULSE IN AN INHOMOGENEITY

Let us consider the effect of the interaction of a weak probe optical pulse with strong inhomogeneity created by an extremely short pulse. In this case, $|\psi| \ll E_\Omega$, and we can ignore the effect of the optical pulse on the low frequency pulse, omitting the corresponding term in Eq. (3). We also assume that the amplitude of QMPs is sufficient to form a single soliton of the NSE.

The equation for the trajectory of the probe pulse can be obtained in the approximation of geometrical optics. Let $\psi = A\exp(iS)$. Solving the equation for the eikonal S in the form of

$$\frac{\partial S}{\partial z} + \delta\frac{\partial S}{\partial \tau} + \frac{k_2}{2}\left(\frac{\partial S}{\partial \tau}\right)^2 = -U, \tag{5}$$

we then obtain [1]

$$S = -F(\theta)z - \delta\tau/k_2 \pm \int_{-\infty}^{\tau}\sqrt{\delta^2/k_2^2 - (U(\tau') - U(\theta))}\,d\tau', \tag{6}$$

where $\tau = t - z/\upsilon$, υ is the group velocity of ESPs, $\delta = 1/\nu_g - 1/\upsilon$, θ is the time delay between the signal optical pulse and a reference ESP, and $U(\tau) = 2k_2^{-1}a\omega E_\Omega(\tau)$ has the meaning of inhomogeneity in the refractive index. The equation for the trajectory [1] follows from (6)

$$\frac{d\tau}{dz} = \pm\sqrt{\delta^2 - k_2^2(U(\tau) - U(\theta))}. \tag{7}$$

The instantaneous change in the frequency of the light pulse $\Delta\omega = \partial S/\partial\tau$ can be calculated using expression (6).

Below, we consider the interaction of the probe pulse with ESPs of the form of (4) so that we can explicitly obtain the analytical results to illustrate the resulting effects.

Note that the problem of the evolution of the pulse of Eq. (2) can be formally identified in the linearized case via the well-known quantum mechanical problem in [9] on the motion of a particle with energy $E = \delta^2/k_2^2 + U_0\operatorname{sech}^2(\theta/\tau_p)$ in the potential $U(\tau) = U_0\operatorname{sech}^2(\tau/\tau_p)$, where $U_0 = -12a\sigma\omega/(k_2\eta\tau_p^2)$. By this analogy, expressions (5)–(7) correspond to the classical trajectory obtained in the VKB approximation.

In the geometric optics approximation for $E > |U_0|$ the pulse passes through the uniformity, and at $E < U_0$ ($U_0 > 0$) it is reflected. For the same signs of quadratic nonlinearities in (2) and (3) in the case of a positive sign of the coefficient of group dispersion, $k_2 > 0$, we thus have a potential well; in the opposite case, $k_2 < 0$, we have a potential barrier.

The trajectory of the pulse in the limit of $\theta/\tau_p \gg 1$ in the case of propagation takes the form

$$\sinh(\tau/\tau_p) = \sqrt{1 \pm \frac{\delta_c^2}{\delta^2}}\sinh(\delta z/\tau_p), \tag{8}$$

where $\delta_c^2 = k_2^2|U_0|$, and the plus and minus signs correspond to the cases $U_0 < 0$ and $U_0 > 0$. If the group-velocity mismatch is no greater than the critical $\delta^2 < \delta_c^2$, we obtain the trajectory in the case of reflection:

$$\sinh(\tau/\tau_p) = \sqrt{\frac{\delta_c^2}{\delta^2} - 1}\cosh(\delta z/\tau_p). \tag{9}$$

Note that even if the group-velocity mismatch is below the critical value, a part of the pulse can penetrate through the inhomogeneity, as is in the case of quantum tunneling. The propagation coefficient in general form is given by the following expression [9]

$$D = \left(1 + \cosh^2(\pi\delta\tau_p/k_2)/\sinh^2\left(\pi\sqrt{U_0\tau_p^2 - 1/4}\right)\right)^{-1}. \tag{10}$$

Note that formula (10) is valid for any sign of U_0. With a strong inhomogeneity expressed by the condition $\tau_p^2 U_0 = 12a\sigma\omega/(|k_2|\eta) \gg 1$, and near-critical group-velocity mismatch $\delta \approx \delta_c$, we have $D \approx \left(1 + \exp\left[-\pi\tau_p(\delta^2 - \delta_c^2)/(\delta_c|k_2|)\right]\right)^{-1}$.

The above analysis is confirmed by numerical calculations.

We now consider the case where $U_0 < 0$ and $E < 0$, i.e., $\delta^2 < \delta_c^2\operatorname{sech}^2(\theta/\tau_p)$. It makes sense to dwell on the capture of the pulse by the inhomogeneity. In the geometric optics approximation, the trajectory of the pulse has the form

$$\sinh(\tau/\tau_p) = \sqrt{\delta_c^2\left(1 - \operatorname{sech}^2(\theta/\tau_p)\right) + \delta^2} \times \frac{\sin\left(\sqrt{\delta_c^2\operatorname{sech}^2(\theta/\tau_p) - \delta^2}\, z/\tau_p\right)}{\sqrt{\delta_c^2\operatorname{sech}^2(\theta/\tau_p) - \delta^2}}. \tag{11}$$

The motion thus occurs with the inhomogeneity.

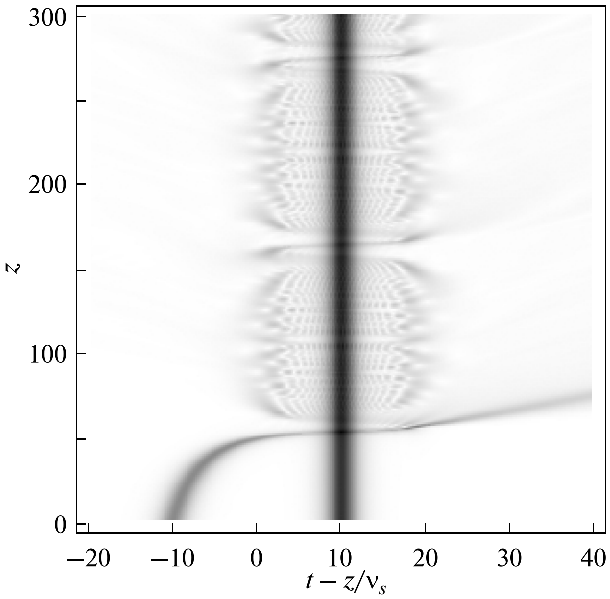

Fig. 1. Effect of the capture of a weak optical pulse by a low-frequency ESP. A bound state with a high number of m is excited. The grey scale shows the intensity of the pulses.

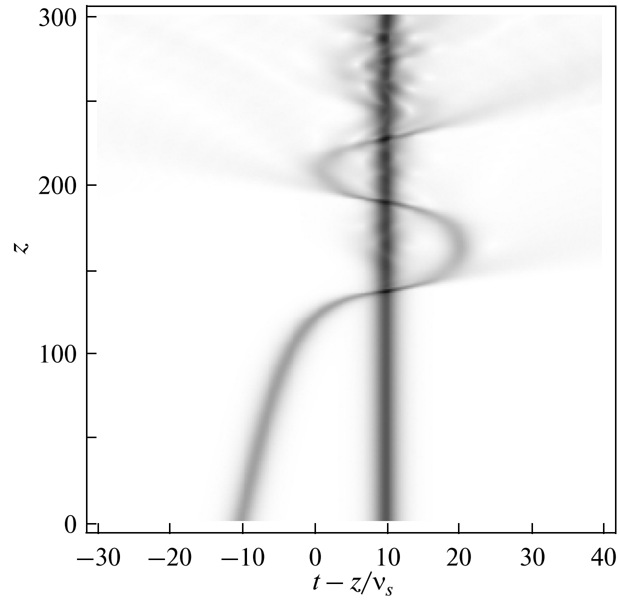

Fig. 2. Effect of the capture of a weak optical pulse by a low-frequency ESP. The bound state with a low number of m is excited. The grey scale shows the intensity of the pulses.

In the case of strong inhomogeneity, the field of the pulse in its profile is defined by a discrete set of frequencies:

$$\omega_m = \omega - \delta_m / k_2, \tag{12}$$

where

$$\delta_m^2 = \delta_c^2 \operatorname{sech}^2(\theta/\tau_p) - [\rho - m]^2 k_2^2/\tau_p^2,$$
$$0 \le \rho - \delta_c^2 k_2^2 \tau_p^2 \operatorname{sech}^2(\theta/\tau_p) \le m \le \rho, \quad m = 0, 1, 2, \ldots,$$
$$\rho = \sqrt{|U_0|\tau_p^2 + 1/4} - 1/2.$$

Solutions are expressed [9] in terms of hypergeometric functions:

$$\begin{aligned}\psi_m(\tau, z) &= \cosh^{\rho+1}(\tau/\tau_p)[A_m \sinh(\tau/\tau_p)_2 \\ &\times F_1(\alpha + 1/2, \beta + 1/2, \ 3/2, \ \sinh^2(\tau/\tau_p)) \\ &+ B_{m2} F_1(\alpha, \beta, 1/2, \ -\sinh^2(\tau/\tau_p))] \\ &\times \exp[-i(\delta_m \tau + \delta_c^2 \operatorname{sech}(\theta/\tau_p) z/2)/k_2].\end{aligned} \tag{13}$$

Here, the coefficient A_m corresponds to the solution with an odd-numbered m, and B_m corresponds to the solution with an even-numbered m; $\alpha = \rho + 1/2 - \tau_p\sqrt{\delta_c^2 \operatorname{sech}(\theta/\tau_p) - \delta^2/(2|k_2|)}$, $\beta = \rho + 1/2 + \tau_p\sqrt{\delta_c^2 \operatorname{sech}(\theta/\tau_p) - \delta^2/(2|k_2|)}$.

A general solution for trapped pulse formation was studied by numerical simulation. Note that for the excitation of a bound state there must be set a very small group-velocity mismatch between pulses, so in practice we used group-velocity mismatch near the critical one, which also allowed us to investigate the effect of over-barrier reflection.

Figs. 1 and 2 show the results from calculating the interaction of a probe pulse with an inhomogeneity like the potential well, when bound states with different internal modes are excited. In Fig. 1, we have a captured state with a high value of m, while Fig. 2 shows the case of excitation of the state with a low value of m.

Figure 3 shows the results from calculations in which there is interaction between a probe pulse and an almost impenetrable barrier, and the bound state only on one side of the center of the inhomogeneity is excited.

CONCLUSIONS

We have considered three possible scenarios of the interaction of light pulses with an effective inhomogeneity of the refractive index created by an extremely short pulse.

The modes of interaction of the pulse with the inhomogeneity were considered on the basis of a formal analogy with the quantum mechanical problem of the motion of a particle through a potential well or barrier. When the inhomogeneity profile is equivalent to the potential well, a part of the pulse can be captured by the inhomogeneity. In the opposite case, we observe the propagation or reflection of the pulse. If there is a sufficiently strong inhomogeneity and the group-velocity mismatch does not exceed the critical value, the tunneling of the pulse through the inhomogeneity can be ignored. Note the analog of total internal reflection in [1–3].

An important condition for the possible ocurrence of this effect is the stability of the profile of the

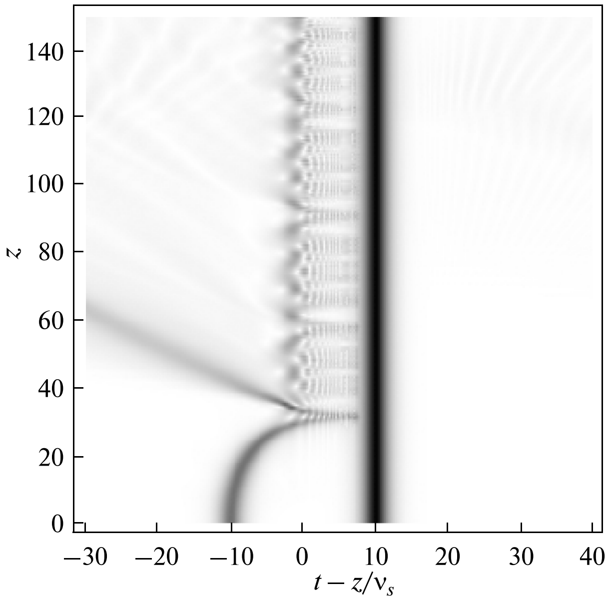

Fig. 3. Effect of the capture of a weak optical pulse by a low-frequency ESP. The light pulse does not penetrate through the inhomogeneity. The bound state with a high number of *m* is excited on one side of the inhomogeneity. The grey scale shows the intensity of the pulses.

extremely short pulse at the required distance of propagation. It is therefore necessary to use soliton modes of ESP propagation.

This phenomenon could be used in optical information processing systems that require all-optical control of the speed and frequency of pulses of different duration and spectral composition.

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