Quadratic Solitons: Recent Developments

Lluís Torner, *Senior Member, IEEE,* and Alain Barthélémy

Invited Paper

***Abstract*—Multicolor optical solitons mediated by quadratic nonlinearities have been observed experimentally during the last few years in a variety of materials and geometries in a number of laboratories around the world. Today, many of the basic properties of the corresponding soliton families are theoretically well established. Nevertheless, experiments continuously reveal new surprises, and the topic is entering new theoretical and experimental territories. In this paper, we briefly outline the key features of the quadratic solitons and overview the latest developments.**

***Index Terms*—Frequency conversion, nonlinear optics, optical propagation in nonlinear media, optical solitons.**

I. Introduction

The experimental demonstration in 1995 of spatial soliton formation mediated by parametric wave interactions in crystals with quadratic nonlinearities [1], [2], which were theoretically predicted for the first time in the 1970s [3], opened a whole new area of possibilities in soliton science and its applications. For many years, second-order or quadratic nonlinearities were typically associated only with optical phenomena and devices aimed at the frequency conversion of laser light, with applications in optical disks, biophotonics, optical radar, remote sensing, or spectroscopy. A crucial indication of the existence of much richer opportunities was the proper appreciation of the importance of the concept of *cascading* [4], where cross-induced energy and phase shifts acquired by multiple light waves that parametrically interact in a material with a quadratic nonlinearity are exploited to perform all-optical operations on the signals. Therefore, such energy and phase shifts can also be used to dynamically counteract the spreading caused by diffraction and by group-velocity dispersion, thus forming a self-sustained, localized, nonspreading light packet; i.e., a soliton, or more properly, a solitary wave.

The importance of quadratic solitons is twofold. First, by their very nature, they come as clean, robust, stabilized, particle-like light pulses and beam spots. Thus, they offer potentially unique opportunities in a variety of passive and active, single-pass or cavity, multifrequency photonic systems which can benefit from such features. Second, they constitute a truly unique laboratory for soliton phenomena. Quadratic solitons exist in the spatial and the temporal domains; they exist and are stable and robust in guided and bulk geometries in any physical dimensions, including settings adequate for the formation of three-dimensional (3-D) light bullets. They exist in continuous, as well as in discrete, physical settings, and they are the realization of a universal phenomenon in nature—namely, the nonlinear parametric mixing of coherent waves.

Since their first experimental observation, quadratic solitons have been demonstrated in a variety of materials and geometries in a number of laboratories around the world. Spatial, temporal, and spatio–temporal solitons have been observed experimentally in second-harmonic generation (SHG) and in parametric generation and amplification schemes. Whole families of solitons existing in waveguides and in bulk geometries are known, including those existing in settings with a small Poynting vector walk-off and/or group velocity mismatch. Under conditions where modulational instabilities cannot grow, and with the exception of narrow regions near the cutoff conditions for soliton existence, such families of stationary solitons have been shown to be stable under propagation and robust against different perturbations.

Therefore, today many of the basic properties of the quadratic solitons are well established, and they have been reviewed in detail by several authors [5]–[15]. A comprehensive review of the existence and mathematical properties of different possible types of stable and unstable solitary wave solutions (bright, dark, vortex, gap; continuous, discrete; etc.) and a critical review of the early experimental observations has been reported recently by Buryak *et al.* [15]. Nevertheless, there are plenty of theoretical predictions that are being continuously confirmed experimentally, new experiments reveal new surprises, and the topic is entering new theoretical and experimental territories. In this paper, we briefly overview the key features of the quadratic solitons and focus our main attention on some of the latest developments. We refer to the several reviews mentioned above for detailed mathematical models and to [14] for additional recent experimental progress not included here.

II. Basic Properties

Quadratic solitons form by the mutual trapping and locking of multiple-frequency waves. The simplest case corresponds to the process of SHG or optical parametric generation (OPG), where a fundamental frequency wave and its second-harmonic generate

Manuscript received July 25, 2002; revised September 9, 2002. The work of L. Torner was supported in part by the Generalitat de Catalunya, by the Spanish Government under Grant TIC2000-1010, and by the U.S.–Spain Joint Commission for Scientific and Technological Cooperation.

L. Torner is with the Institute of Photonic Sciences, Universitat Politècnica de Catalunya, Nexus II, E-08034 Barcelona, Spain (e-mail: torner@tsc.upc.es).

A. Barthélémy is with the Institut de Recherche en Communications Optiques et Microondes, Université de Limoges/CNRS, F-87060 Limoges, France (e-mail: abarth@ircom.unilim.fr).

Digital Object Identifier 10.1109/JQE.2002.806189

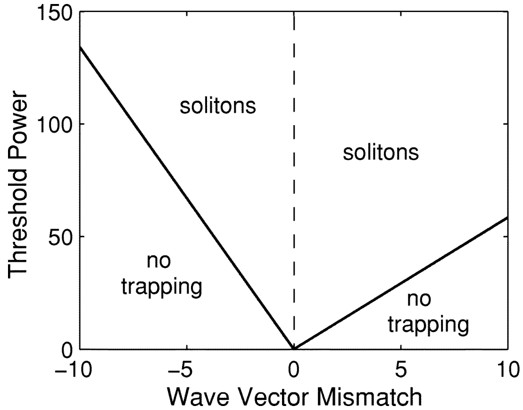


Fig. 1. Calculated threshold power for CW 2-D bright single-spot spatial solitons to exist as a function of wave vector mismatch in the case of type-I SHG/OPG, in conveniently normalized units. Adapted from [18].

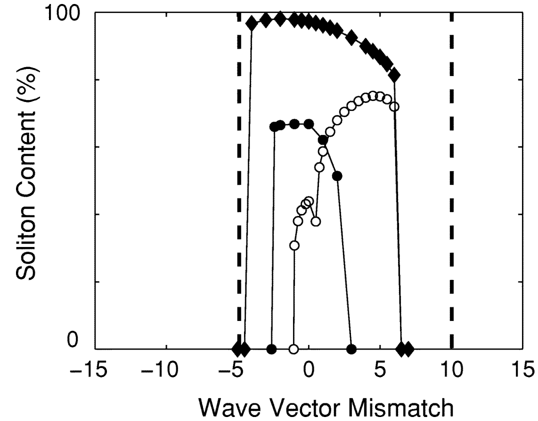


Fig. 2. Calculated soliton content of CW beams with a Gaussian shape versus the wave vector mismatch in a type-I SHG/OPG configuration with a typical input power, in conveniently normalized units. Empty circles: up-conversion, with only a fundamental frequency input beam. Filled circles: down-conversion, with a strong pump second-harmonic beam and a negligible fundamental frequency seed inputs. Diamonds: both fundamental and second-harmonic beams, coherent, in-phase, and with equal powers are assumed to be input into the crystal. The vertical dashed lines show the domain of soliton existence for the total input power considered. The power is set to the same value in all cases. In real units, e.g., in PPLN, it corresponds to a few kilowatts. (Adapted from [20] and [21].)

each other. The resulting soliton contains both the fundamental and harmonic fields, which in the simplest case exhibit a classical bell-shape. Therefore, in contrast to their cubic, Kerr-type, and photorefractive cousins, quadratic solitons are intrinsically *multicolored*.

Solitons form when the material and light propagation conditions inside the quadratic nonlinear crystal are set so that the diffraction and dispersion lengths that measure the spreading of the beams and pulses, and the nonlinear length that measures the strength of the frequency conversion process, are comparable. In real units, the energy flow required for quadratic spatial soliton formation scales as $\lambda^4/\chi^2\eta^2$, where λ is the wavelength, χ is the effective second-order material nonlinear coefficient, and η is the beam width. An analogous scaling holds for temporal solitons.

Ideally, once a soliton has been generated, the energy exchange between the waves that parametrically interact in the crystal ceases, and the envelopes of all waves are phase-locked, regardless the existing material wave vector mismatch. Thus, two key parameters that dictate the formation of quadratic solitons are the light intensity and the existing material wave vector-mismatch between the multiple frequency waves. Whole families of solitons exist above a threshold light intensity for all values of the mismatch. A sketch of how the existence threshold depends on the mismatch is shown in Fig. 1 for the case of single-spot bright spatial solitons propagating in bulk media under conditions of type-I SHG/OPG [16]–[18]. The families of three-wave solitons that exist in type-II phase-matching geometries and in sum and frequency mixing processes are richer than their type-I counterparts [19], but the main features hold in both cases. Almost all members of such soliton families have been proven to be stable under propagation, and many of them have also been found to be robust against perturbations.

However, soliton existence does not guarantee soliton generation. Because quadratic solitons are multiple-wave entities, an obvious but crucial point which is not always properly appreciated is that arbitrary input signals are not necessarily close to the stationary solutions of the governing equations. On the contrary, in the majority of cases, solitons are excited with inputs signals which fall far from those solutions indeed. For example, such is the situation when only one of the participating wavelengths is input into the crystal. Therefore, suitable conditions must be employed in each particular setting for the members of the existing soliton families to be actually, and efficiently, generated. The ultimate goal is the determination of the *soliton content*, or *soliton generation efficiency*, namely the fraction of input energy that is carried by the generated solitons, of arbitrary input light conditions [20], [21]. Fig. 2 shows typical theoretical predictions, awaiting experimental confirmation, of the trends expected with Gaussian-shaped input beams in up-conversion and down-conversion processes, as a function of the material wave vector-mismatch.

One of the key properties of quadratic solitons is the energy sharing amongst the mutually trapped multicolor signals, depending on the total energy that they carry and on the existing material mismatch [22]. Fig. 3 illustrates the point. Such energy partition greatly impacts the overall soliton properties at positive and negative wave vector-mismatch, and in particular, it impacts the outcome of the experimental excitation of solitons with single-frequency pump light. For example, drastically different pump energy thresholds for soliton formation and for soliton generation efficiencies are encountered in up and down-conversion processes, as visible in Fig. 2 (see [23] and [24] for related experimental observations).

Quadratic solitons are able to counteract the spatial and temporal linear walk-off existing between the multiple signals needed to form the solitons. In that case the beams or pulses lock together and propagate stuck to each other in a single light packet, termed a walking soliton, which in general features a curved wave front or a chirp phase and that offers many potential opportunities for soliton steering and gating [25]–[28].

An important point that must be stressed is that even though under appropriate—and often interesting—conditions, some quadratic solitons can be regarded as perturbed cubic-type solitons, this is not the general case. In particular, quadratic

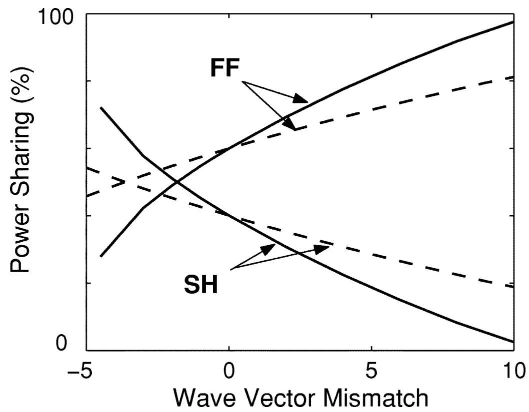


Fig. 3. Power sharing between the fundamental frequency (FF) and the second-harmonic (SH) forming CW 2-D spatial solitons as a function of the wave vector mismatch in a type-I SHG/OPG configuration, for two different soliton powers. The dashed lines correspond to a higher power than the solid lines. (Adapted from [21] and [22].)

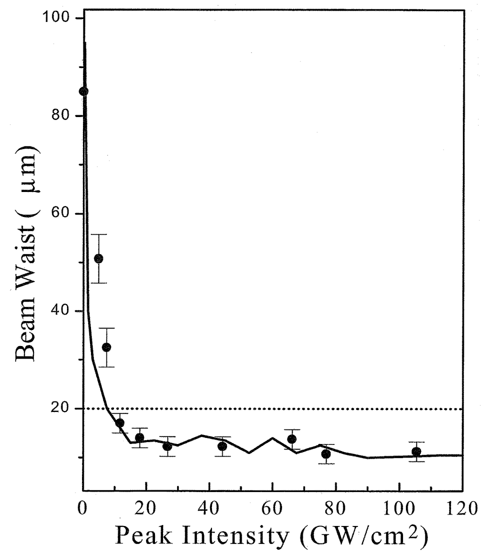
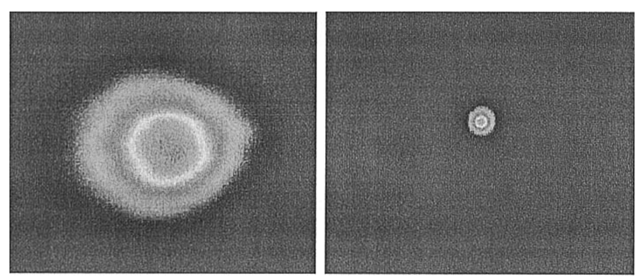


Fig. 4. Data acquired in what is believed to be the first recorded observation of soliton formation in a bulk quadratic crystal. The plot displays the variation of the width of the beam at 1064-nm output of a KTP crystal, as a function of the input peak power, showing the spreading caused by diffraction at low powers and the soliton regime. (Adapted from [1].)

nonlinearities are often viewed as simply perturbed self-focusing, or self-defocusing, Kerr-like nonlinearities at either side of phase matching, respectively. Such intuitive picture holds under appropriate conditions (e.g., SHG with negligible depletion of the fundamental frequency wave and thus a weak second harmonic), but must be used with caution otherwise. Actually, most of the quadratic solitons of highest interest occur near phase matching or under conditions where all the multicolor waves required to form a soliton participate on a similar footing in the parametric wave interaction. Such solitons do not comply with the simple self-focusing/self-defocusing picture mentioned above, and rather they exhibit genuinely quadratic features.



(a) (b)

Fig. 5. Quadratic soliton in a bulk piece of PPKTP. (a) Output of a low-power input. (b) The soliton spot. The experiment was conducted in SHG at 1064 nm in a 1-cm-long PPKTP crystal, with input beams of waist 19 μm and a peak intensity of about 5 GW/cm^2. (Adapted from [29].)

III. Early Experimental Demonstrations

The ingredients employed in routine SHG of focused laser light suffice to form the simplest types of quadratic solitons. The recipe requires the use of high peak-power pulsed laser sources and a material with a reasonable nonlinear coefficient that makes it possible to reach the soliton regime before arriving at the damage threshold. These conditions were satisfied in the schemes employed by Torruellas, Schiek, Stegeman, and coworkers to observe multicolor solitons for the first time in 1995 [1], [2]. The observations where conducted by pumping a few centimeter-long pieces of potassium titanyl phosphate (KTP) and of lithium niobate (LN), respectively, cut for phase matching at the wavelength (1064 nm in [1] and 1320 nm in [2]) delivered by a mode-locked Nd : YAG laser emitting pulses a few tens of picosecond long (35 ps in [1] and 90 ps in [2]). With low peak powers, the infrared beams generate second-harmonic light and both beams diffract. However, when the input peak power reaches a given threshold, the beams no longer diffract and a bright soliton is excited. Such soliton signature is clearly visible in Fig. 4, which displays data acquired in the original experiments by Torruellas *et al.* The clear contrast between a soliton and nonsoliton output is also readily appreciated in Fig. 5, which corresponds to the recent observation by Kim *et al.* of solitons in a bulk piece of periodically poled KTP [29].

Spatial and temporal quadratic solitons are described by similar equations, but their experimental formation faces different challenges. The main difficulties encountered to form temporal quadratic solitons are the small group-velocity dispersion exhibited by the known suitable transparent materials with large quadratic nonlinearities, together with the large group-velocity mismatch experienced by the signals needed to form a soliton. Such difficulty was overcame by Di Trapani *et al.* by using achromatic phase-matching, or tilted-pulse techniques, to observe self-narrowing of sub-picosecond pulses in β–barium borate (BBO), consistent with temporal soliton excitation [30]. Subsequently, Wise *et al.* used the same techniques to observe the formation of two-dimensional (2-D) spatio–temporal soliton formation [31] over several characteristic lengths in BBO and in lithium iodate, and to show the potential of the corresponding light-packets in ultrafast digital logic schemes [32]. Fig. 6 shows the spatio–temporal narrowing of the pulses-beams that was observed. All the above mentioned experiments were conducted under conditions of large phase-mismatch.

On physical grounds, tilted pulse techniques translate the spatial dynamics of a highly elliptical beam into effective group-ve-

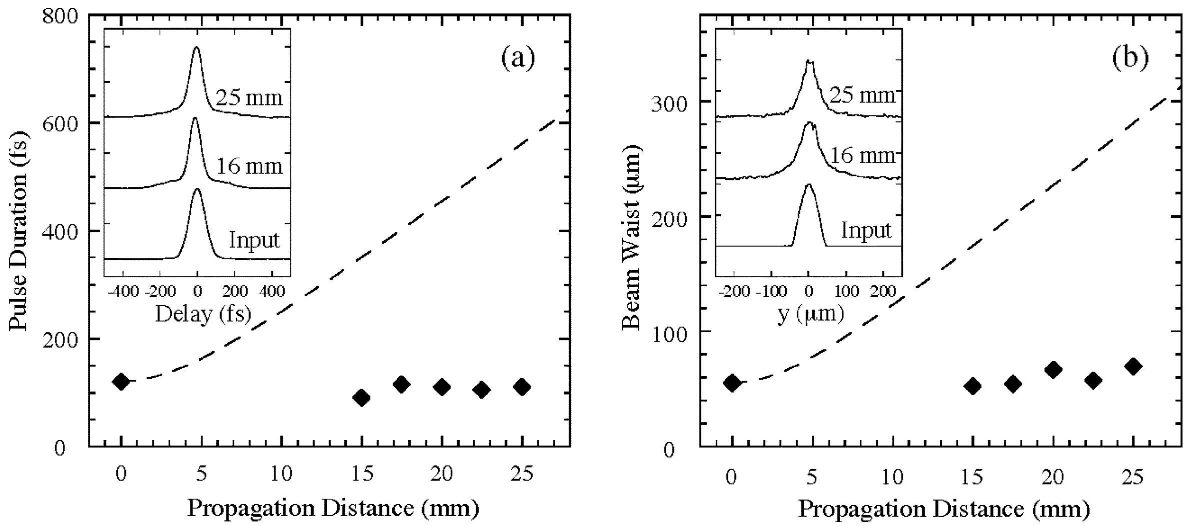


Fig. 6. Spatio–temporal narrowing of beams-pulses in SHG in β-barium borate. The experiment was performed with tilted pump pulses for different effective crystal lengths. Dashed lines: linear spreading. Diamonds: measured pulse duration and beam width. (Adapted from [31] and [32].)

locity dispersion and group-velocity delay of a temporal pulse, i.e., they translate dynamics in one spatial dimension to dynamics in the temporal dimension. Thus, in their present form they cannot be employed to form fully 3-D light bullets. Therefore, new concepts, such as the so-called X-pulses, are currently under investigation to generate spatio–temporal trapping of light [33], [34].

IV. Recent Developments

Quadratic solitons of different dimensionality and nature have been observed in a variety of materials in several laboratories around the world. Scientists are thus now able to form quadratic solitons in many physical configurations, opening the door to the detailed experimental exploration of their properties and features.

For example, when quadratic solitons are excited with only one of the participating wavelengths input into the crystal, the inputs adjust themselves by shedding radiation to form a soliton. For a given input power, there is a finite range of phase mismatch for soliton formation. As shown in Fig. 2, the allowed band is asymmetric and notably different for up-conversion and down-conversion processes; the higher the input power, the larger the mismatch range accessible. However, such is not necessarily the case for the energetic efficiency of soliton formation. In recent experiments by Lopez-Lago *et al.*, such efficiency has been measured in detail as a function of the input light power [35]. The experiments were conducted under conditions of SHG in a 2-cm-long bulk KTP crystal cut for type-II phase matching at 1064 nm. To measure the soliton content of input Gaussian beams, a combination of a knife-edge method and specific data processing techniques were employed. One of the central results that was obtained is shown in Fig. 7. Above an optimum input power, the fraction of input power captured by the excited soliton saturates. At phase matching, the saturation level amounts to about 50%. The other half is shed away in the form of radiation. The dependence of the soliton trapping efficiency with the mismatch is currently under experimental

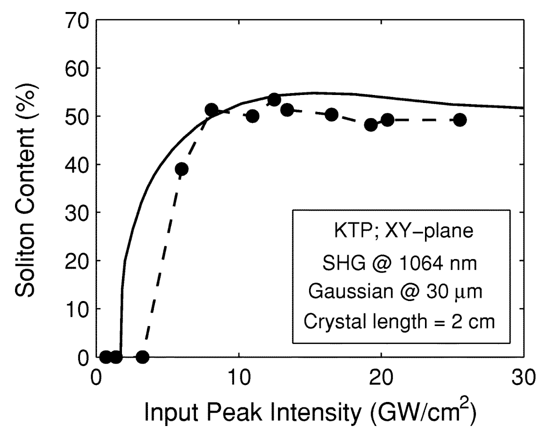


Fig. 7. Efficiency of quadratic soliton generation. Fraction of the input energy carried by a Gaussian beam at the pump wavelength that is captured by the solitons generated in a bulk piece of KTP under conditions of phase-matched SHG. (Adapted from [35].)

investigation in bulk samples made of periodically poled KTP. Several methods aimed at the enhancement of the mismatch bandwidth for soliton excitation, such as the use of properly tailored quasiphase-matched synthetic gratings [36], have been theoretically explored during the last few years, and remain to be experimentally tested.

Crystals with large quadratic coefficients might also exhibit significant cubic nonlinearities, which under appropriate conditions can make relevant contributions to the spatial dynamics of optical beams. The impact of competing quadratic and cubic nonlinearities on solitons has been extensively studied theoretically (see [37]–[39] and references therein). Nevertheless, in the typical conditions suitable for soliton formation under conditions of near-phase-matched SHG, the quadratic nonlinearity has been dominant making effects due to the cubic contributions difficult to detect experimentally. In contrast, in recent down-conversion experiments conducted by Carrasco *et al.*, self-narrowing of light beams which must be attributed to Kerr nonlinearities and two-photon-absorption at green wavelengths in KTP was observed [40]. The experiments were conducted in a special cut of KTP for type-II SHG in the YZ plane (instead of the customary XY plane), where the walk-off angle between the orthogonally-polarized fundamental frequency waves amounts to a large $\rho = 1.8°$, and the quadratic nonlinear coefficient involved is relatively small. The large walk-off and the small quadratic nonlinearity make soliton generation impractical, even of the walking type, and render the strength of the quadratic effects small. Thus, the cubic terms, which included significant two-photon-absorption, dominated the light evolution. However, notice that the observations do not correspond to the excitation of solitons supported by competing nonlinearities, but only to the transient self-narrowing of the input beam.

In this context, a potentially important method to generate engineerable competing quadratic and cubic nonlinearities in settings where frequency conversion is accompanied by optical rectification has been recently put forward theoretically by Torres *et al.* [41]. The scheme is based on the full exploitation of the geometrical conditions that determine the magnitude of the rectified fields, and translates different orientations of an elliptical pump beam into tunable competing nonlinearities. The scheme which may pave the way to the formation of solitons supported by competing nonlinearities is awaiting experimental confirmation.

The potentially detrimental effects introduced by, e.g., the presence of a moderate spatial or temporal walk-off or spatial soliton generation with short pulses, have been also theoretically addressed. The question of spatial soliton formation with pulsed light deserves special attention [42]. Rigorously speaking, spatial solitons are self-trapped beams formed with continuous wave light signals. However, because of the high peak-powers, at the 10s·kW scale, required to form quadratic solitons with feasible crystal lengths in the existing suitable materials, in practice spatial solitons are generated using pulsed laser light. To avoid crystal damage and thermal effects, mode-locked laser sources delivering picosecond, and even subpicosecond pulses are required, depending on the material and laser repetition rate employed. When long enough pulses (typically, a few tens of a picosecond long) are employed, 3-D spatio–temporal simulations show that the space-only, continuous-wave predictions predict qualitatively well the actual light evolution. However, the numerical investigations reveal that with too short pulses group-velocity mismatch may produce large departures from the ideal soliton features, and even prevent soliton formation near phase matching. For example, in the case of SHG in LN pumped at wavelengths around 1550 nm, while with 30-ps pulses the reduced space-only model accurately predicts the actual evolution, with pump pulses a few picoseconds long, Carrasco *et al.* showed that spatial trapping occurs only for large wave vector mismatches [42]. This prediction has been confirmed experimentally recently by Pioger *et al.* in SHG pumped with 4-ps pulses at 1548 nm, launched in a 6-cm-long planar waveguide fabricated by Ti : indiffusion in a Z-cut periodically poled LN crystal [43]. The signature of the spatial beam narrowing at the peak power regime suitable for soliton generation is shown in Fig. 8, which corresponds to $\Delta kL = 18\pi$. No trapping was observed near phase matching, in agreement with the theoretical predictions by Carrasco *et al.*

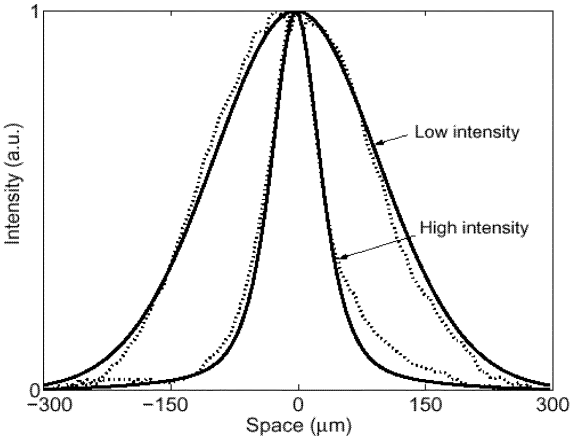


Fig. 8. Spatial trapping of short pulses at large wave vector-mismatch. Spatial profile of the fundamental frequency output in low intensity and in the self-trapped regime (peak intensity 66 MW/cm^2); dotted lines refer to experimental data, solid lines to numerical simulations. Here, $\Delta kL = 18\pi$. The experiment was performed with 4–ps pulses in a 6-cm-long planar waveguide of periodically poled LN. (Adapted from [43].)

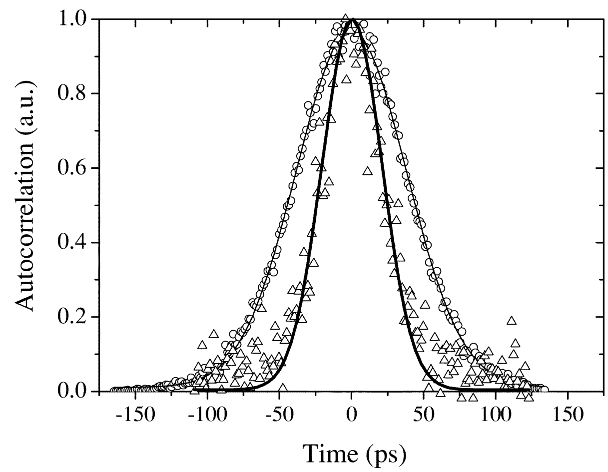


Fig. 9. Reshaping of picosecond pulses by spatial quadratic soliton generation. Circles: autocorrelation traces of the laser pulse. Triangles: autocorrelation trace of the pulses measured after nonlinear propagation and filtering of the soliton part. Solid line: numerical simulations of the autocorrelation of the filtered pulse. The experiment was conducted with 60–ps pulses in a 2-cm-long KTP crystal cut for SHG at 1064 nm. (Adapted from [44].)

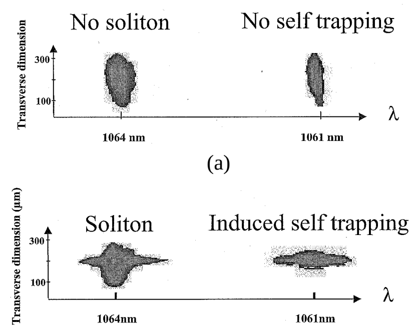
A related effect was employed recently by Simos *et al.* to demonstrate the use of spatial soliton generation for performing a temporal reshaping of laser pulses [44]. These authors aimed at the compression of high-energy pulses, along a similar line investigated by Liu *et al.* using cascading [45], but now employing spatial soliton generation together with spatial filtering. By means of spatial soliton generation combined with a spatial filtering of the output beam, pulses can be cleaned up from pedestal of low intensity and may be shortened in time. Such type of processing is interesting for high-bit-rate optical communication links where it could achieve all-optical reshaping of distorted pulses at very high speeds (100 Gbits/s). Fig. 9 shows the reshaping of the pulses measured after nonlinear propagation and filtering of the soliton part, in comparison with the free-propagation autocorrelation trace.

Many of the basic features of soliton excitation in geometries where the crystal anisotropy plays a significant role were demonstrated in the last few years and are well understood today. For example, Torruellas *et al.* showed experimentally in early experiments in SHG in bulk KTP that the beam locking associated to the excitation of walking solitons could be employed to implement all-optical switching functions [46], and Schiek *et al.* conducted experiments of temperature-tuning SHG in LN planar waveguides to observe directly the dependence of the transverse velocity of one-dimensional walking solitons as a function of the wave vector mismatch, hence the soliton energy sharing between the fundamental frequency and second-harmonic beams [47].

However, new concepts and schemes that exploit the crystal anisotropy in novel ways have recently revealed beautiful surprises. For example, Couderc *et al.* recently explored soliton generation under conditions of collinear type-II phase matching, where two fundamental frequency waves interact with their second harmonic, but under conditions of noncollinear pump beams, where the two pump beams are launched at the plane orthogonal to the phase-matching plane [48]. Experiments were conducted in KTP and the off-plane angular separation between the two input pump beams was about one degree. Fuerst *et al.* predicted a few years ago that in such scheme, under collinear pumping soliton generation is accompanied by amplification of the input power imbalance between the two fundamental beams [49]. Now, Couderc *et al.* discovered that under noncollinear pumping, identical input signals either generate a single soliton or self-split into two solitons, depending on the sign of the existing phase mismatch. They also found that at phase matching, the number and location of the excited solitons can be controlled by varying the input imbalance. New important anisotropy-induced possibilities have been also discovered recently by Polyakov *et al.*, whose observation of multiple soliton generation in a potassium niobate crystal cut for noncritical phase matching at 983 nm at room temperature, was attributed to the presence of spatial anisotropic diffraction [50], [51], assuming perfect symmetry of the input beams. Such anisotropic diffraction is linked to the longitudinal components of the electric field of the propagating light beams, and thus constitutes a significant step forward the common description of the wave evolution in quadratic crystals.

Light guiding light induced by quadratic solitons was investigated by Kivshar *et al.* in a scheme called multiple-step cascading [52]. Trapping of a weak probe through coupling with a quadratic soliton has been demonstrated recently by Couderc *et al.* in a related yet simpler system [53], which consisted of a weak field that is spatially and temporally superimposed to a strong input field at the fundamental frequency. Both fields were launched in a 2-cm KTP crystal cut for SHG at 1064 nm. In this scheme, a single phase-matching condition has to be fulfilled and a single quadratic nonlinear coefficient is involved. It was demonstrated experimentally that a weak beam can be guided by a quadratic soliton provided that the beam is slightly detuned in frequency $(\omega + \delta\omega)$ from the soliton low-frequency component (ω). The scheme is a new opportunity for the processing of low-level signals thanks to their coupling with more powerful quadratic solitary waves. Fig. 10 illustrates the salient point of the observations.

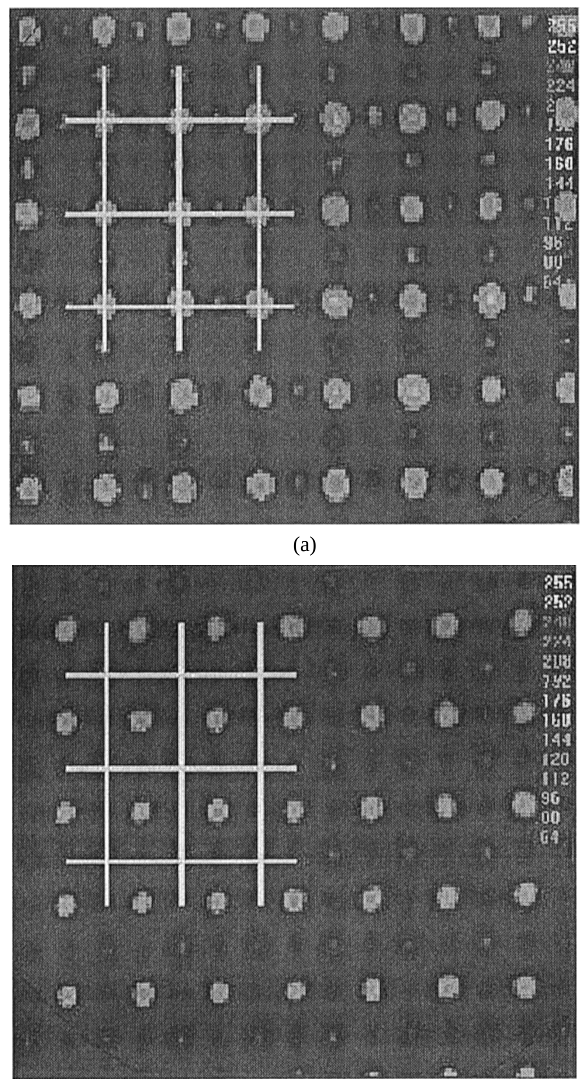
Complex patterns of solitons can be used to demonstrate the proof-of-principle of potential future parallel signal processing



(a)

(b)

Fig. 10. Trapping of a weak signal by a quadratic soliton analyzed by an imaging stroboscope. Horizontal scale: wavelength. Vertical scale: transverse dimension on the output pattern. (a) $\mathrm{I_{pump}} = 2\,\mathrm{GW/cm^2}$. (b) $\mathrm{I_{pump}} = 10\,\mathrm{GW/cm^2}$. Left side: the fundamental frequency component of the soliton. Right side: the probe wave. The observation was performed in SHG in a 2-cm-long KTP crystal pumped at 1064 and a weak signal at 1061 nm. (Adapted from [53].)



(a)

(b)

Fig. 11. Fractional Talbot effect with solitons: spatial shift of 2-D patterns of light beams in an optical parametric amplifier operated at the soliton regime. (a) Output nonsoliton pattern. (b) Output soliton pattern. The pattern is shifted by half its transverse period, e.g., by switching on/off the seed of the OPG. The experiment was performed with 1-ps pulses in a 2.2-cm-long lithium triborate crystal cut for SHG at 1055 nm. (Adapted from [54].)

or contrast-enhanced imaging devices. A beautiful example was demonstrated recently by Minardi *et al.*, who employed a fractional Talbot effect to shift by half of its transverse period a whole 2-D patterns of beams [54]. The experiments were performed with 1-ps pulses in a 2.2-cm-long lithium triborate crystal cut for SHG at 1055 nm, and the input pattern consisting of about one thousand beams was produced by passing the pump beam through a microlenses array. The system was operated under conditions of optical parametric amplification with input pump peak powers where solitons are excited when the process is seeded by a temporally overlapped pulse. Thus, the spatial shift of the light pattern can be controlled, e.g., by switching on and off the seed pulse. The half-period shift of the pattern induced by the generation of matrices of solitons that was observed is shown in Fig. 11.

V. Concluding Remarks

After several years of intense research pursued by different groups around the world, the basic properties of quadratic solitons are reasonably well-known today. However, many challenges still lay ahead. For example, the unique opportunities offered by quasiphase-matching engineering to control the solitons properties are awaiting experimental demonstration and exploitation. This area includes not only simple periodically poled structures, but also complex nonlinear photonic crystals (see [55]–[57] and references therein).

Elucidation of suitable settings that allow the experimental generation of gap, and stabilized dark and vortex solitons of different sorts, and the experimental implementation of competing nonlinearities are important lines that remain open. The possibility to experimentally generate competing nonlinearities might open the door to fascinating possibilities, such as the generation of stable fully-3-D spinning solitons [58], or the formation of robust metastable soliton clusters [59].

The formation of quadratic discrete solitons, and hence the exploitation of their potential to the investigation of the concept of *diffraction management* [60], [61] to the exploration of discrete features in soliton systems, and to the implementation of all-optical switching and routing operations in waveguide arrays [62]–[64] is a fascinating avenue that is under current experimental investigation in the framework of the ROSA Project funded by the European Union.

The formation of fully self-confined light packets, or light bullets, constitutes one of the fascinating frontiers that dates back to the early days of nonlinear optics, and that remains open. The known theoretical existence of stable 3-D quadratic solitons constitutes an important milestone along this path [65]–[68]. Novel engineered tandem material structures [69] and pumping techniques are currently investigated today with such goal in mind. The so-called X-pulse techniques, pioneered in this context by Di Trapani, Trillo, and coworkers constitutes an important spin-off of this program [33], [34], which might find wide applications in several areas of nonlinear optics.

Quadratic solitons form not only by the parametric mixing of optical waves. They also form when one of the signals belongs to the microwave or terahertz frequency band, and when optical signals interact with static fields through optical rectification [70]–[73]. Soliton formation under such conditions in real systems, and its actual implications are little understood today. Deeper exploration of this area is likely to introduce important new concepts and opportunities. For example, Torres *et al.* have predicted recently that cascaded optical rectification in the framework of the full local-field equations can be employed to generate different nonlinear contributions [74]. Thus, for example, acting on the shape and polarization of the pump light beam and on the geometric arrangement of the nonlinear crystal allows tuning the sign, the strength, and the type of the induced nonlinearities, opening the door to the exploration of a variety of tunable self- and cross-phase modulations.

Last, but not least, the concepts explored with quadratic solitons are expected to have important implications to other branches of nonlinear science and engineering. For example, Bose–Einstein condensates provide the matter analogy to laser light, and the nonlinear interactions in the BEC provide the atomic analogy to nonlinear optics. The exploration of atomic-molecular coherent mixing in BEC have already been suggested and explored theoretically (see [75]–[79] and references therein) and experimentally (see [80], [81] and references therein). The experimental confirmation of such prospects will yield a fascinating new playground for quadratic solitons.

Acknowledgment

The authors thank their many collaborators and colleagues for continuously enlightening discussions and in particular for sharing with them the results of their research presented in this overview.

References

[1] W. E. Torruellas, Z. Wang, D. J. Hagan, E. W. VanStryland, G. I. Stegeman, L. Torner, and C. R. Menyuk, "Observation of two-dimensional spatial solitary waves in a quadratic medium," *Phys. Rev. Lett.*, vol. 74, pp. 5036–5039, 1995.

[2] R. Schiek, Y. Baek, and G. I. Stegeman, "One-dimensional spatial solitary waves due to cascaded second-order nonlinearities in planar waveguides," *Phys. Rev. E*, vol. 53, pp. 1138–1141, 1996.

[3] Yu. N. Karamzin and A. P. Sukhorukov, "Mutual focusing of high-power light beams in media with quadratic nonlinearity," *Sov. Phys. JETP*, vol. 41, pp. 414–420, 1976.

[4] G. I. Stegeman, M. Sheik-Bahae, E. W. VanStryland, and G. Assanto, "Large nonlinear phase shifts in second-order nonlinear-optical processes," *Opt. Lett.*, vol. 18, pp. 13–15, 1993.

[5] G. I. Stegeman, D. J. Hagan, and L. Torner, "X(2) cascading phenomena and their applications to all-optical signal processing, mode-locking, pulse compression and solitons," *Opt. Quantum Electron.*, vol. 28, pp. 1691–1740, 1996.

[6] L. Torner, "Spatial solitons in quadratic nonlinear media," in *Beam Shaping and Control With Nonlinear Optics*, F. Kajzar and R. Reinisch, Eds. New York: Plenum Press, 1998, pp. 229–258.

[7] Yu. S. Kivshar, "Quadratic solitons: Past, present and future," in *Advanced Photonics With Second-Order Optically Nonlinear Processes*, A. D. Boardman, L. Pavlov, and S. Panev, Eds. Dordrecht, The Netherlands: Kluwer, 1998, pp. 451–475.

[8] G. I. Stegeman, "Quadratic spatial optical solitons," *Photon. Sci. News*, vol. 5, no. 1, pp. 19–26, 1999.

[9] C. Etrich, F. Lederer, B. A. Malomed, T. Peschel, and U. Peschel, "Optical solitons in media with a quadratic nonlinearity," *Progress in Optics*, vol. 41, pp. 483–501, 2000.

[10] L. Torner and G. I. Stegeman, "Multicolor solitons," *Opt. Photon. News*, vol. 12, no. 6, pp. 36–39, 2001.

[11] W. Torruellas, Yu. S. Kivshar, and G. I. Stegeman, "Quadratic solitons," in *Spatial Solitons*, S. Trillo and W. Torruellas, Eds. Berlin, Germany: Springer-Verlag, 2001, pp. 127–168.

[12] G. Assanto and G. I. Stegeman, “The simple physics of quadratic spatial solitons,” *Opt. Exp.*, vol. 10, pp. 388–396, 2002.
[13] F. Wise and P. Di Trapani, “Spatiotemporal solitons,” *Opt. Photon. News*, vol. 13, no. 2, pp. 28–32, 2002.
[14] L. Torner and A. P. Sukhorukov, “Quadratic solitons,” *Opt. Photon. News*, vol. 13, no. 2, pp. 42–47, 2002.
[15] A. V. Buryak, P. Di Trapani, D. Skryabin, and S. Trillo, “Optical solitons due to quadratic nonlinearities: from basic physics to futuristic applications,” *Phys. Rep.*, vol. 370, pp. 63–235, 2002.
[16] A. V. Buryak, Yu. S. Kivshar, and V. V. Steblina, “Self-trapping of light beams and parametric solitons in diffractive quadratic media,” *Phys. Rev. A*, vol. 52, pp. 1670–1674, 1995.
[17] L. Torner, D. Mihalache, D. Mazilu, E. M. Wright, W. E. Torruellas, and G. I. Stegeman, “Stationary trapping of light beams in bulk second-order nonlinear media,” *Opt. Commun.*, vol. 121, pp. 149–155, 1995.
[18] L. Torner and E. W. Wright, “Soliton excitation and mutual locking of light beams in bulk quadratic nonlinear crystals,” *J. Opt. Soc. Amer. B*, vol. 13, pp. 864–875, 1996.
[19] A. V. Buryak, Yu. S. Kivshar, and S. Trillo, “Parametric spatial solitary waves due to type II second-harmonic generation,” *J. Opt. Soc. Amer. B*, vol. 14, pp. 3110–3118, 1997.
[20] L. Torner, J. P. Torres, D. Artigas, D. Mihalache, and D. Mazilu, “Soliton content with quadratic nonlinearities,” *Opt. Commun.*, vol. 164, pp. 153–159, 1999.
[21] S. Carrasco, L. Torner, J. P. Torres, D. Artigas, E. Lopez-Lago, V. Couderc, and A. Barthelemy, “Quadratic solitons: Existence versus excitation,” *IEEE J. Select. Topics Quantum Electron.*, vol. 8, pp. 497–505, 2002.
[22] L. Torner, “Stationary solitary waves with second-order nonlinearities,” *Opt. Commun.*, vol. 114, pp. 136–140, 1995.
[23] R. A. Fuerst, M. T. G. Canva, G. I. Stegeman, G. Leo, and G. Assanto, “Robust generation, properties and potential applications of quadratic spatial solitons generated by optical parametric amplification,” *Opt. Quantum Electron.*, vol. 30, pp. 907–921, 1998.
[24] P. Di Trapani, A. Bramati, S. Minardi, W. Chinaglia, C. Conti, S. Trillo, J. Kilius, and G. Valiulis, “Focusing versus defocusing nonlinearities due to parametric wave mixing,” *Phys. Rev. Lett.*, vol. 87, 0 183 902-1/4, 2001.
[25] L. Torner, D. Mazilu, and D. Mihalache, “Walking solitons in quadratic nonlinear media,” *Phys. Rev. Lett.*, vol. 77, pp. 2455–2458, 1996.
[26] C. Etrich, U. Peschel, F. Lederer, and B. A. Malomed, “Stability of temporal chirped solitary waves in quadratically nonlinear media,” *Phys. Rev. E*, vol. 54, pp. 6155–6161, 1997.
[27] D. Mihalache, D. Mazilu, L.-C. Crasovan, and L. Torner, “Walking solitons in type II second-harmonic generation,” *Phys. Rev. E*, vol. 56, pp. R6294–R6297, 1997.
[28] D. V. Skryabin and A. R. Champneys, “Walking cavity solitons,” *Phys. Rev. E*, vol. 63, 066 610-1/5, 2001.
[29] H. Kim, L. Jankovic, R. Malendevich, G. I. Stegeman, M. Katz, S. Carrasco, and L. Torner, “Quadratic spatial solitons in bulk PPKTP,” in *Proc. CLEO-Conf. Lasers and Electro-Optics 2002*, Long Beach, CA, paper CWL2.
[30] P. Di Trapani, D. Caironi, G. Valiulis, D. Danielius, and A. Piskarskas, “Observation of temporal solitons in second-harmonic generation with tilted pulses,” *Phys. Rev. Lett.*, vol. 81, pp. 570–573, 1998.
[31] X. Liu, L. J. Qian, and F. W. Wise, “Generation of optical spatiotemporal solitons,” *Phys. Rev. Lett.*, vol. 82, pp. 4631–4634, 1999.
[32] X. Liu, K. Beckwitt, and F. W. Wise, “Noncollinear generation of spatiotemporal solitons and application ultrafast digital logic,” *Phys. Rev. E*, vol. 61, pp. R4722–4725, 2000.
[33] G. Valiulis, J. Kilius, O. Jedrkiewicz, A. Bramati, S. Minardi, C. Conti, S. Trillo, A. Piskarskas, and P. Di Trapani, “Space–time nonlinear compression and three dimensional complex trapping in normal dispersion,” in *Proc. QELS-Quantum Electronics and Laser Science Conf. 2001*, Baltimore, MD, QPD10-1.
[34] C. Conti, P. Di Trapani, S. Trillo, G. Valiulis, S. Minardi, and O. Jedrkiewicz, “Nonlinear X-waves: Light bullets in normally dispersive media,” in *Proc. QELS-Quantum Electronics and Laser Science Conf. 2002*, Long Beach, CA, paper QTuJ6.
[35] E. Lopez-Lago, C. Simos, V. Couderc, A. Barthelemy, D. Artigas, and L. Torner, “Efficiency of quadratic soliton generation,” *Opt. Lett.*, vol. 26, pp. 1277–1279, 2001.
[36] S. Carrasco, J. P. Torres, L. Torner, and R. Schiek, “Engineerable generation of quadratic solitons in synthetic phase matching,” *Opt. Lett.*, vol. 25, pp. 1273–1275, 2001.
[37] M. A. Karpierz, “Coupled solitons in waveguides with second- and third-order nonlinearities,” *Opt. Lett.*, vol. 20, pp. 1677–1679, 1995.
[38] A. V. Buryak, Yu. S. Kivshar, and S. Trillo, “Optical solitons supported by competing nonlinearities,” *Opt. Lett.*, vol. 20, pp. 1961–1963, 1995.
[39] O. Bang, Yu. S. Kivshar, A. V. Buryak, A. De Rossi, and S. Trillo, “Two-dimensional solitary-waves in media with quadratic and cubic nonlinearity,” *Phys. Rev. E*, vol. 58, pp. 5057–5069, 1998.
[40] S. Carrasco, H. Kim, G. I. Stegeman, P. Loza-Alvarez, and L. Torner, “Observation of self-focusing of light mediated by cubic nonlinearities in potassium titanyl phosphate,” *Opt. Lett.*, vol. 27, pp. 2016–2018, 2002.
[41] J. P. Torres, S. L. Palacios, L. Torner, L.-C. Crasovan, D. Mihalache, and I. Biaggio, “Method to generate solitons sustained by competing nonlinearities using optical rectification,” *Opt. Lett.*, vol. 27, pp. 1631–1633, 2002.
[42] S. Carrasco, J. P. Torres, D. Artigas, and L. Torner, “Generation of multicolor spatial solitons with pulsed light,” *Opt. Commun.*, vol. 192, pp. 347–355, 2001.
[43] P. Pioger, V. Couderc, L. Lefort, A. Barthelemy, F. Baronio, C. De Angelis, Y. Min, V. Quiring, and W. Sohler, “Spatial trapping of short pulses in Ti : PPLN waveguides,” *Opt. Lett.*, vol. 27, no. 24, Dec. 2002.
[44] C. Simos, V. Couderc, and A. Barthelemy, “Temporal reshaping of optical pulses using quadratic spatial soliton generation and spatial filtering,” *IEEE Photon. Technol. Lett.*, vol. 14, pp. 636–638, 2002.
[45] X. Liu, L. Qian, and F. W. Wise, “High-energy pulse compression using negative phase shifts produced by the cascade X(2) : X(2) nonlinearity,” *Opt. Lett.*, vol. 24, pp. 1777–1779, 1999.
[46] W. E. Torruellas, G. Assanto, B. L. Lawrence, R. A. Fuerst, and G. I. Stegeman, “All-optical switching by spatial walkoff compensation and solitary-wave locking,” *Appl. Phys. Lett.*, vol. 68, pp. 1449–1451, 1996.
[47] R. Schiek, Y. Baek, G. I. Stegeman, and W. Sohler, “One-dimensional quadratic walking solitons,” *Opt. Lett.*, vol. 24, pp. 83–85, 1999.
[48] V. Couderc, E. Lopez-Lago, C. Simos, and A. Barthelemy, “Experiments in quadratic spatial soliton generation and steering in a noncollinear geometry,” *Opt. Lett.*, vol. 26, pp. 905–907, 2001.
[49] R. A. Fuerst, M. T. G. Canva, D. Baboiu, and G. I. Stegeman, “Properties of type II quadratic solitons excited by imbalanced fundamental waves,” *Opt. Lett.*, vol. 22, pp. 1748–1750, 1997.
[50] R. Malendevich, L. Jankovic, S. Polyakov, R. Fuerst, G. I. Stegeman, C. Bosshard, and P. Gunter, “Two-dimensional type I quadratic spatial solitons in KNbO3 near noncritical phase matching,” *Opt. Lett.*, vol. 27, pp. 631–633, 2002.
[51] S. Polyakov, R. Malendevich, L. Jankovic, G. I. Stegeman, C. Bosshard, and P. Gunter, “Effects of anisotropic diffraction on quadratic multisoliton excitation in noncritically phase-matched crystals,” *Opt. Lett.*, vol. 27, pp. 1049–1051, 2002.
[52] Yu. S. Kivshar, A. A. Sukhorukov, and S. M. Saltiel, “Two-color multistep cascading and parametric soliton-induced waveguides,” *Phys. Rev. E*, vol. 60, pp. R5056–R5059, 1999.
[53] V. Couderc, E. Lopez-Lago, A. Barthelemy, C. De Angelis, and F. Gringoli, “Trapping of a weak probe through coupling with a two-color quadratic spatial soliton,” *Opt. Commun.*, vol. 203, pp. 421–425, 2002.
[54] S. Minardi, G. Arrighi, P. Di Trapani, A. Varanavicius, and A. Piskarskas, “A solitonic all-optical switch based on the fractional Talbot effect,” *Opt. Lett.*, vol. 27, pp. 2097–2099, 2002.
[55] A. A. Sukhorukov, Yu. S. Kivshar, O. Bang, and C. M. Soukoulis, “Parametric localized modes in quadratic nonlinear photonic structures,” *Phys. Rev. E*, vol. 63, 016 615-1/9, 2001.
[56] J. F. Corney and O. Bang, “Solitons in quadratic nonlinear photonic crystals,” *Phys. Rev. E*, vol. 64, 047 601-1/4, 2001.
[57] S. F. Mingaleev and Yu. S. Kivshar, “Nonlinear photonic crystals: Toward all-optical technologies,” *Opt. Photon. News*, vol. 13, no. 7, pp. 48–51, 2002.
[58] D. Mihalache, D. Mazilu, L.-C. Crasovan, I. Towers, B. A. Malomed, A. V. Buryak, L. Torner, and F. Lederer, “Stable three-dimensional spinning solitons supported by competing quadratic and cubic nonlinearities,” *Phys. Rev. E*, vol. 66, 016 613-1/7, 2002.
[59] Y. V. Kartashov, L.-C. Crasovan, D. Mihalache, and L. Torner, “Robust propagation of two-color soliton clusters supported by competing nonlinearities,” *Phys. Rev. Lett.*, to be published.
[60] H. S. Eisenberg, Y. Silberberg, R. Morandotti, and J. S. Aitchison, “Diffraction management,” *Phys. Rev. Lett.*, vol. 85, pp. 1863–1866, 2000.
[61] T. Pertsch, T. Zentgraf, U. Peschel, A. Brauer, and F. Lederer, “Anomalous refraction and diffraction in discrete optical systems,” *Phys. Rev. Lett.*, vol. 88, 093 901-1/4, 2002.
[62] T. Peschel, U. Peschel, and F. Lederer, “Discrete bright solitary waves in quadratically nonlinear media,” *Phys. Rev. E*, vol. 57, pp. 1127–1133, 1998.

[63] F. Lederer, S. Darmanyan, and A. Kobyakov, "Discrete solitons," in *Spatial Solitons*, S. Trillo and W. Torruellas, Eds. Berlin, Germany: Springer-Verlag, 2001.
[64] F. Lederer and Y. Silberberg, "Discrete solitons," *Opt. Photon. News*, vol. 13, no. 2, pp. 48–53, 2002.
[65] A. A. Kanashov and A. M. Rubenchik, "On diffraction and dispersion effect on three wave interaction," *Physica D*, vol. 4, pp. 122–134, 1981.
[66] B. A. Malomed, P. Drummond, H. He, A. Berntson, D. Anderson, and M. Lisak, "Spatiotemporal solitons in multidimensional optical media with a quadratic nonlinearity," *Phys. Rev. E*, vol. 56, pp. 4725–4735, 1997.
[67] D. V. Skryabin and W. J. Firth, "Generation and stability of optical bullets in quadratic nonlinear media," *Opt. Commun.*, vol. 148, pp. 79–84, 1998.
[68] D. Mihalache, D. Mazilu, B. A. Malomed, and L. Torner, "Asymmetric spatio–temporal solitons in quadratic nonlinear media," *Opt. Commun.*, vol. 152, pp. 365–370, 1998.
[69] L. Torner, S. Carrasco, J. P. Torres, L.-C. Crasovan, and D. Mihalache, "Tandem light bullets," *Opt. Commun.*, vol. 199, pp. 277–281, 2001.
[70] Ch. Bosshard, R. Spreiter, M. Zgonik, and P. Gunter, "Kerr nonlinearity via cascaded optical rectification and the linear electro-optic effect," *Phys. Rev. Lett.*, vol. 74, pp. 2816–2819, 1995.
[71] M. J. Ablowitz, G. Biondini, and S. Blair, "Multidimensional pulse propagation in nonresonant X(2) materials," *Phys. Lett. A*, vol. 236, pp. 520–525, 1997.
[72] V. V. Steblina, A. V. Buryak, R. Sammut, D. Zhou, M. Segev, and P. Prucnal, "Stable self-guided propagation of two optical harmonics coupled by a microwave or a terahertz wave," *J. Opt. Soc. Amer. B*, vol. 17, pp. 2026–2031, 2000.
[73] U. Peschel, F. Lederer, K. Bubke, and D. C. Hutchings, "Temporal solitary waves due to optical rectification and the electro-optic effect," *Opt. Lett.*, vol. 26, pp. 1090–1093, 2001.
[74] J. P. Torres, L. Torner, I. Biaggio, and M. Segev, "Tunable self-action of light in optical rectification," *Opt. Commun.*, vol. 213, pp. 351–356, 2002.
[75] P. S. Julienne, K. Burnett, Y. B. Band, and W. C. Stwalley, "Stimulated Raman molecule production in Bose–Einstein condensates," *Phys. Rev. A*, vol. 58, pp. R797–R800, 1998.
[76] P. D. Drummond, K. V. Kheruntsyan, and H. He, "Coherent molecular solitons in Bose–Einstein condensates," *Phys. Rev. Lett.*, vol. 81, pp. 3055–3058, 1998.
[77] J. Javanainen and M. Mackie, "Coherent photoassociation of a Bose–Einstein condensate," *Phys. Rev. A*, vol. 59, pp. R3186–R3189, 1998.
[78] B. J. Cusack, T. J. Alexander, E. A. Ostrovskaya, and Yu. S. Kivshar, "'Existence and stability of coupled atomic-molecular Bose–Einstein condensates," *Phys. Rev. A*, vol. 65, 013 609/1-4, 2001.
[79] T. J. Alexander, E. A. Ostrovskaya, Yu. S. Kivshar, and P. S. Julienne, "Vortices in atomic-molecular Bose–Einstein condensates," *J. Opt. B: Quantum Semicl. Opt.*, vol. 4, pp. S33–S38, 2002.
[80] R. Wynar, R. S. Freeland, D. J. Han, C. Ryu, and D. J. Heinzen, "Molecules in a Bose–Einstein condensate," *Science*, vol. 287, pp. 1016–1019, 2000.
[81] E. A. Donley, N. R. Claussen, S. T. Thompson, and C. E. Weiman, "Atom-molecule coherence in a Bose–Einstein condensate," *Nature*, vol. 417, pp. 529–533, 2002.

Lluís Torner (M'92–SM'00) received the B.Sc. and M.Sc. degrees in physics from the Universitat Autonoma de Barcelona, Spain, in 1985 and 1986, respectively, and the Ph.D. degree from the Universitat Politecnica de Catalunya, Barcelona, Spain, in 1989 (with highest honors).

He was a postdoctoral researcher at the Center for Research and Education in Optics and Lasers, University of Central Florida, Orlando, and at the Optical Sciences Center, University of Arizona. He is a Professor at the Universitat Politecnica de Catalunya, Barcelona, Spain, where he is conducting research and education in photonics, with emphasis on nonlinear optics and its applications. He is the co-author of approximately 150 scientific journal publications.

Dr. Torner is a Fellow of the Optical Society of America, and is currently serving as the Director of the Institute of Photonic Sciences recently launched in Barcelona, Spain.

Alain Barthélémy received the Ph.D. degree in high-frequency electronics and optoelectronics in 1982 and the Doctorat es Sciences Physiques in 1986 from the University of Limoges, Limoges, France.

His research has included fiber optics, nonlinear propagation, solitons in optical fibers, shaping and analysis of ultrashort laser pulses, laser cavity design, modelocking of lasers, transverse phenomena during nonlinear propagation and parametric processes, ultrafast photonic switching based on spatial solitons, optical probing of semiconductors, and the use of optics for the generation, control, and measurements of microwaves. He has published more than 100 publications and communications in various international journals and conferences and holds three patents.

Dr. Barthelemy received the Bronze medal from CNRS in 1986 and the Fabry de Gramont Prize awarded by Société Française d'Optique in 1987. He is a member of the Societe Francaise d'Optique and of the Optical Society of America.