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COM221

States S: { Sunny, Cloudy }

Actions a: { Go to School, Stay at Home }

Discount $\gamma = 0.9$

	Go to School	Stay at Home
☀	+ 5	- 5
☁	+ 3	+ 1

Step 1:

① For Sunny:

$$r_\pi = [0.5 \times (5)] + [0.5 \times (-5)]$$
$$= 2.5 + (-2.5)$$

$$\boxed{r_\pi = 0}$$

② For Cloudy:

$$r_\pi = [0.5 \times (3)] + [0.5 \times (1)]$$
$$= 1.5 + 0.5$$

$$\boxed{r_\pi = 2}$$

③ r_π matrix $= \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

Step 2:

Go to School

	☀	☁
☀	0.8	0.2
☁	0.4	0.6

Stay at Home

	☀	☁
☀	0.9	0.1
☁	0.3	0.7

(Sunny)

$$P_\pi(1,1) = (0.5 \times 0.8) + (0.5 \times 0.9)$$
$$= 0.4 + 0.45$$

$$\boxed{P_\pi(1,1) = 0.85}$$

$$P_\pi(1,2) = (0.5 \times 0.2) + (0.5 \times 0.1)$$
$$= 0.1 + 0.05$$

$$\boxed{P_\pi(1,2) = 0.15}$$

(Cloud)

$$P_\pi(2,1) = (0.5 \times 0.4) + (0.5 \times 0.3)$$
$$= 0.2 + 0.15$$

$$\boxed{P_\pi(2,1) = 0.35}$$

$$P_\pi(2,2) = (0.5 \times 0.6) + (0.5 \times 0.7)$$
$$= 0.3 + 0.35$$

$$\boxed{P_\pi(2,2) = 0.65}$$

④ P_π matrix $= \begin{bmatrix} 0.85 & 0.15 \\ 0.35 & 0.65 \end{bmatrix}$

Step 3:

(5.) $V_1 = 0 + 0.9(0.85 v_1 + 0.15 v_2)$

$V_1 = 0 + 0.765 v_1 + 0.135 v_2$

$V_1 - 0.765 v_1 - 0.135 v_2 = 0$

$\boxed{0.235 v_1 - 0.135 v_2 = 0}$

(6.) $V_2 = 2 + 0.9(0.35 v_1 + 0.65 v_2)$

$V_2 = 2 + 0.315 v_1 + 0.585 v_2$

$V_2 - 0.315 v_1 - 0.585 v_2 = 2$

$0.415 v_2 - 0.315 v_1 = 2$

$\boxed{-0.315 v_1 + 0.415 v_2 = 2}$

Step 4:

$0.235 v_1 - 0.135 v_2 = 0$

$\dfrac{0.235 v_1}{0.235} = \dfrac{0.135 v_2}{0.235}$

$\boxed{V_1 = \dfrac{0 + 0.135 v_2}{0.235}}$

$-0.315 v_1 + 0.415 v_2 = 2$

$\left[-0.315\left(\dfrac{0}{0.235}\right)\right] + \left[-0.315\left(\dfrac{0.135 v_2}{0.235}\right)\right] + 0.415 v_2 = 2$

$0 + 0.1809 v_2 + 0.415 v_2 = 2$

$0 + 0.5959 v_2 = 2$

$\dfrac{0.5959 v_2}{0.5959} = \dfrac{2}{0.5959}$

7. $\boxed{V_{\pi} \text{ cloudy} \mid V_2 = 3.3563}$

$V_1 = \dfrac{0 + 0.135 v_2}{0.235}$

$= \dfrac{0 + 0.135(3.3563)}{0.235}$

8. $\boxed{V_{\pi} \text{ sunny} \mid V_1 = 1.9281}$

Step 5:

(9.) $V_x (\text{sunny}) = 5 + 0.9(0.8 v_1 + 0.2 v_2)$

$= 5 + 0.72 v_1 + 0.18 v_2$

$\boxed{-0.72 v_1 - 0.18 v_2 = 5}$

(10.) $V_x (\text{cloudy}) = 3 + 0.9(0.4 v_1 + 0.6 v_2)$

$= 3 + 0.36 v_1 + 0.54 v_2$

$\boxed{-0.36 v_1 - 0.54 v_2 = 3}$

Step 6:

(11) V_* (cloudy)

$-0.72v_1 - 0.18v_2 = 5$ $\qquad$ $-0.36v_1 - 0.54v_2 = 3$

$\dfrac{0.72v_1 := -5 - 0.18v_2}{0.72 \qquad 0.72}$ $\qquad$ $-0.36\left(\dfrac{-5 - 0.18v_2}{0.72}\right) - 0.54v_2 = 3$

$V_1 = \dfrac{-5 - 0.18v_2}{0.72}$

$\left[-0.36 \times \left(\dfrac{-5}{0.72}\right)\right] + \left[-0.36 \times \left(\dfrac{-0.18v_2}{0.72}\right)\right] - 0.54v_2 = 3$

$2.5 + 0.09v_2 - 0.54v_2 = 3$

$2.5 - 0.45v_2 = 3$

$\dfrac{0.45v_2 = -3 + 2.5}{0.45 \qquad 0.45}$

$\boxed{V_* \text{ cloudy} / V_2 = -1.11}$

(12) V_* Sunny

$V_1 = \dfrac{-5 - 0.18v_2}{0.72} = \dfrac{-5 - 0.18(-1.11)}{0.72}$

$\left(\cancel{\dfrac{-5 + 0.72}{0.72}}\right) = \dfrac{-5 + 0.1998}{0.72}$

$\boxed{V_* \text{ Sunny} / V_1 = -6.6669}$

STEP 7:

(13) $q(1, \text{School}) = 5 + 0.9(0.8v_1 + 0.2v_2) = -6.6669$

(14) $q(1, \text{Home}) = -5 + 0.9(0.9v_1 + 0.1v_2) = 1.9281$

(15) $q(2, \text{School}) = 3 + 0.9(0.4v_1 + 0.6v_2) = -1.11$

(16) $q(2, \text{Home}) = 1 + 0.9(0.3v_1 + 0.7v_2) = 3.3563$

VICTORY