$$\frac{dy}{dx}$$

$$= \frac{g(x)}{f(y)}$$

$$\frac{dy}{dx}f(y)$$

$$= g(x)$$

$$\int f(y)dy = \int g(x)dx$$

$$= e^{x+y} = e^x e^y$$

$$\int e^{-y} dy$$

$$= \int e^x dx$$

$$-e^{-y}$$

$$= e^x + c$$

$$e^{x} + e^{-y}$$

$$= c$$

c

y

$$e^{-y}$$

$$= c - e^x$$

y

$$= -\ln(c - e^{x})$$

O

0

$$\frac{dT}{dt}$$

$$= -\alpha(T - T_0)$$

$$\int \frac{dT}{T - T_0}$$

$$= -\alpha \int dt$$

$$\ln(T - T_0)$$

$$= -\alpha t + c$$

$$T - T_0$$

$$= e^{-\alpha t + c} = Ae^{-\alpha t}$$

A

$$= e^c$$

When t

$$= 0$$

$$= 90 - 20 = 70$$

$$= 70$$

$$= 10$$

T

$$= T_0 + Ae^{-\alpha t}$$

70

$$= 20 + 70e^{-10\alpha}$$

$$e^{-10\alpha}$$

$$= \frac{70 - 20}{70} = \frac{5}{7}$$

$$= 20$$

$$= 20 + 70e^{-20\alpha}$$

$$(e^{-10\alpha})^2$$

$$= e^{-20\alpha}$$

$$= 20 + 70 \left(\frac{5}{7} \right)^2 = 55.7^\circ c$$

yt

x

xt

t

$$= f\left(\frac{y}{x}\right)$$

let v

$$= \frac{y}{x} \text{ so } y = vx$$

$$= \frac{d}{dx} vx$$

v

$$= v + x\frac{dv}{dx} = f(v)$$

$$\frac{dv}{dx}$$

$$= \frac{f(v) - v}{x}$$

$$y(1) = 1$$

$$= \frac{y}{x^2}(x - y) = \frac{y}{x} - \left(\frac{y}{x}\right)^2$$

$$= v + x\frac{dv}{dx} = v - v^2 \quad \text{separable}$$

$$\int \frac{-dv}{v^2}$$

$$= \int \frac{dx}{x}$$

$$\frac{1}{v}$$

$$= lnx + c = \frac{x}{y}$$

$$= \frac{x}{lnx + c}$$

$y(1)$

$$= 1$$

1

$$= \frac{1}{\ln(1) + c}$$

c

$$\frac{dy}{dx} + y \cdot P(x) = Q(x)$$

$$Q(x) = 0$$

$$dy \cdot \frac{1}{y}$$

$$= -P(x)dx$$

$$= Ae^{\int -P(x)dx} \quad \text{where } A = e^c$$

$$P(x) = 0$$

$$\frac{d}{dx}(u \cdot v) = u'v + uv'$$

$\mu(x)$

$\boldsymbol{\mu}$

$$\mu' = P(x)\mu$$

μ'

$$= P(x)\mu$$

$P(x)$

$$= \frac{\mu'}{\mu} = \frac{d}{dx}\ln(\mu)$$

$$\int P(x)dx$$

$$= \ln(\mu)$$

μ

$$= e^{\int P(x)dx}$$

$$cos(x)\frac{dy}{dx} + sin(x)y$$

$$\frac{dy}{dx} + tan(x)y$$

$= sec(x) \quad \text{Linear ODE}$

$$= e^{\int tan(x)dx}$$

$$\int tan(x)dx$$

$$= \ln(sec(x))$$

$= sec(x)$ Multiply ODE by μ

$$\frac{dy}{dx}sec(x) + sec(x)tan(x)y$$

$$= sec^2(x)$$

$$\frac{d}{dx} sec(x) = sec(x) tan(x)$$

$$\frac{d}{dx} sec(x) \cdot y$$

$$= \frac{dy}{dx} sec(x) + sec(x) tan(x) y$$

$$sec(x) \cdot y$$

$$= \int sec^2(x) = tan(x) + c$$

$$= \sin(x) + c \cdot cos(x)$$

$f(x, y)$

$$y = y(t), x = x(t)$$

$$\frac{d}{dt}f(x,y) = \frac{\delta f}{\delta x}\frac{dx}{dt} + \frac{\delta f}{\delta y}\frac{dy}{dt}$$

$$x = x, y = y(x)$$

$$\frac{d}{dx}f(x,y) = \frac{\delta f}{\delta x} + \frac{\delta f}{\delta x}\frac{dy}{dx}$$

$$M(x, y) + N(x, y)\frac{dy}{dx} = 0$$

$$\frac{\delta f}{\delta x} = M \quad \frac{\delta f}{\delta x} = N$$

$$\frac{\delta^2 f}{\delta x \delta y}$$

$$= \frac{\delta^2 f}{\delta y \delta x}$$

$$\frac{\delta M}{\delta x}$$

$$= \frac{\delta N}{\delta y}$$

$$x + y^2 + 2xy\frac{dy}{dx}$$

$$M = x + y^2$$

$$N = 2xy$$

$$\frac{\delta M}{\delta y}$$

$$= 2y$$

$$\frac{\delta N}{\delta x}$$

M

$$= \frac{\delta f}{\delta x}$$

$$\int M dx$$

$$= f = \frac{1}{2}x^2 + xy^2 + g(y)$$

N

$$= \frac{\delta f}{\delta y}$$

$$\int N dy$$

$$= f = xy^2 + h(x)$$

$$\int M dx = \int N dy$$

$$g(y) = 0, h(x) = \frac{1}{2}x^2$$

$$f(x, y) = \frac{1}{2}x^2 + xy^2 + c = 0$$