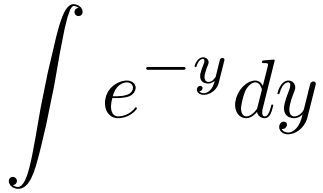
$$= \frac{g(x)}{f(y)}$$

$$\frac{dy}{dx} f(y)$$

$= g(x)$

$$\int f(y)dy = \int g(x)dx$$

$= e^{x+y} = e^x e^y$



$$= \int e^x dx$$

$= e^x + c$

$e^{x} + e^{-y}$

$= c$

c

y

$= c - e^x$

y

$$= -\ln(c - e^x)$$

$$= -\alpha(T - T_0)$$

$$\int \frac{dT}{T - T_0}$$

$$= -\alpha \int dt$$

$\ln(T - T_0)$

$= -\alpha t + c$

$T - T_0$

$= e^{-\alpha t+c} = Ae^{-\alpha t}$

$= e^{c}$

When t

$= 0$

$= 90 - 20 = 70$

$= 70$

$= 10$

T

$= T_0 + Ae^{-\alpha t}$

70

$= 20 + 70e^{-10\alpha}$

$e^{-10\alpha}$

$$= \frac{70 - 20}{70} = \frac{5}{7}$$

$= 20$

$= 20 + 70e^{-20\alpha}$

$\left(e^{-10\alpha}\right)^2$

$= e^{-20\alpha}$

$$= 20 + 70 \left(\frac{5}{7} \right)^2 = 55.7^\circ c$$

yt

x

xt

t

$$= f\left(\frac{y}{x}\right)$$

let v

$= \frac{y}{x}$ so $y = vx$

$$= \frac{d}{dx} vx$$

v

$$= v + x\frac{dv}{dx} = f(v)$$

$$\frac{dv}{dx}$$

$$= \frac{f(v) - v}{x}$$

$y(1) = 1$

$$= \frac{y}{x^2}(x-y) = \frac{y}{x} - \left(\frac{y}{x}\right)^2$$

$$= v + x\frac{dv}{dx} = v - v^2 \qquad \text{separable}$$

$$\int \frac{-dv}{v^2}$$

$$= \int \frac{dx}{x}$$

$$= \ln x + c = \frac{x}{y}$$

$$= \frac{x}{\ln x + c}$$

$y(1)$

$= 1$

1

$$= \frac{1}{\ln(1) + c}$$

c

$$\frac{dy}{dx} + y \cdot P(x) = Q(x)$$

$$Q(x) = 0$$

$$dy \cdot \frac{1}{y}$$

$= -P(x)dx$

$= Ae^{\int -P(x)dx} \quad \text{where } A = e^c$

$P(x) = 0$

$$\frac{d}{dx}(u \cdot v) = u'v + uv'$$

$\mu(x)$

μ

$$\mu' = P(x)\mu$$

$= P(x)\mu$


$P(x)$

$$= \frac{\mu'}{\mu} = \frac{d}{dx} \ln(\mu)$$

$$\int P(x)dx$$

$= \ln(\mu)$

μ

$= e^{\int P(x)dx}$

$$\cos(x)\frac{dy}{dx} + \sin(x)y$$

$$\frac{dy}{dx} + \tan(x)y$$

$= \sec(x)$ Linear ODE

$= e^{\int \tan(x)dx}$

$$\int \tan(x)dx$$

$= \ln(sec(x))$

$= \sec(x)$ Multiply ODE by μ

$$\frac{dy}{dx}\sec(x) + \sec(x)\tan(x)y$$

$= \sec^2(x)$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\sec(x)\cdot y$$

$$= \frac{dy}{dx}\sec(x) + \sec(x)\tan(x)y$$

$\sec(x) \cdot y$

$$= \int \sec^2(x) = \tan(x) + c$$

$$= \sin(x) + c \cdot \cos(x)$$

$f(x, y)$

$y = y(t), x = x(t)$

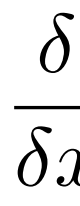
$$\frac{d}{dt} f(x,y) = \frac{\delta f}{\delta x}\frac{dx}{dt} + \frac{\delta f}{\delta y}\frac{dy}{dt}$$

$$x = x, y = y(x)$$

$$\frac{d}{dx} f(x,y) = \frac{\delta f}{\delta x} + \frac{\delta f}{\delta x}\frac{dy}{dx}$$

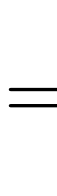
$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

$$\frac{\delta f}{\delta x} = M \quad \frac{\delta f}{\delta x} = N$$



$$= \frac{\delta^2 f}{\delta y \delta x}$$

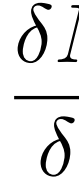

$$\frac{\delta M}{\delta x}$$



$$x + y^2 + 2xy\frac{dy}{dx}$$

$$M = x + y^2$$

$$N = 2xy$$



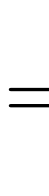
$= 2y$

M

$$\int M\,dx$$

$$= f = \frac{1}{2}x^2 + xy^2 + g(y)$$

N



$= f = xy^2 + h(x)$

$$\int M\,dx = \int N\,dy$$

$g(y) = 0, h(x) = \frac{1}{2}x^2$

$$f(x,y) = \frac{1}{2}x^2 + xy^2 + c = 0$$

$$m(x)\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y$$

$= r(x)$

$\neq 0$

$$y(x) = Ay_1(x) + By_2(x)$$

y_1

y_2

$$m\frac{d^2y}{dx^2} + b\frac{dy}{dx} + ky = F_0\cos(\omega t)$$

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$$

$y = e^{mx}$

$$\frac{dy}{dx} = me^{mx} \qquad \frac{d^2y}{dx^2} = m^2e^{mx}$$

e^{mx}

$(m^2 + am + b)$

$$(m - m_1)(m - m_2)$$

m_1

m_2

$m_1 \neq m_2$

$$y_1(x) = e^{m_1 x} \qquad y_2(x) = e^{m_2 x}$$

$$y(x) = Ae^{m_1 x} + Be^{m_2 x}$$

$$3\frac{d^2y}{dx^2} + -5\frac{dy}{dx} + -2y$$

$3m^2 - 5m - 2$

$(3m+1)(m-2)$

m_1

$$= \frac{-1}{3}$$

m_2

$= 2$

$y(x)$

$= Ae^{-x/3} + Be^{2x}$

$\text{let} \quad m_1$

$= \mu + i\lambda$

let m_2

$= \mu - i\lambda$

$$= Ae^{(\mu+i\lambda)x} + Be^{(\mu-i\lambda)x}$$

$$= e^{\mu x}[Ae^{i\lambda x} + Be^{-i\lambda x}]$$

$$= e^{\mu x}[A(\cos(\lambda x) + i\sin(\lambda x)) + B(\cos(\lambda x) - i\sin(\lambda x))]$$

$$= e^{\mu x}[(A+B)(\cos(\lambda x) + (A-B)i\sin(\lambda x))$$

$$= e^{\mu x}[(C)(\cos(\lambda x) + (D)\sin(\lambda x))$$

$$e^{\mu x}[(C)(\cos(\lambda x) + (D)\sin(\lambda x))$$

$$C = A + B$$

$$D = i(A - B)$$

$m_1 = m_2$

$y_1 = e^{m_1 x}$

$y_2(x)$

$= xe^{m_1 x}$

$= Ay_1(x) + By_2(x)$

$= (A + Bx)e^{m_1 x}$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y$$

$m^2 + 2m + 1$

$= (m+1)^2 = 0$

m

$= -1$

$y_1(x)$

$= e^{-x}$

$= xe^{-x}$

$= (A + Bx)e^{-x}$

$$= e^{-x} - xe^{-x} = (1-x)e^{-x}$$

$$\frac{d^2 y_2}{dx^2}$$

$$= -e^{-x} - e^{-x} + xe^{-x} = (x-2)e^{-x}$$

$$[(x-2)+2(1-x)+x]e^{-x}$$