Let A be a principal ideal domain. Then A is noetherian.

Let $(I_n)_{n\in\mathbb{N}}$ be an increasing chain of ideals of A. Then $I = \bigcup_{n\in\mathbb{N}} I_n$ is an ideal. Since A is a PID, we may write $I_n = (a_n)$ and $I = (a)$. There is some $n \in \mathbb{N}$ such that $a \in I_n$, that is $I_n = I$ and the result follows.