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**2018**
**MCM/ICM**
**Summary Sheet**

# A High Frequency Radio Communication Model Over Oceans


**Summary**

High Frequency radio wave, defined to be 3 to 30MHz, is a popular communication method when the Ionosphere naturally prevents the wave from getting out of the Earth and serves as the propagation media. Therefore, it becomes a promising solution to marine communication. Under the circumstances that signals have to be propagated by reflecting off the ocean surface, the quality of signals when they hit the target becomes hard to predict.

To address this situation, our paper gives a full analysis of signals reflecting off both calm and turbulent ocean. The basic model plays an important role in our approach. It is an integration of several smaller models that together determine the direction and strength of signals. Moreover, two variants that slightly modify the situations such as a multi-hop condition and a moving target are proposed.

We first estimate the behavior of signals traversing the Ionosphere. By assuming a parabolic electron density distribution, we exam the escape condition, signal traveling trace and at last the attenuation factors of signals with different angle of incidence and frequencies.

Next, we analyze the ocean surface energy density using PM sea spectrum. With the knowledge that sea surface amplitude varies with speed of wind, we propose a 3D demo of the shape of sea surface with respect to different wind speed.

In order to describe the first hit of signals on turbulent ocean surface, we put our efforts on deciding signal reflection direction and strength. To decide the reflecting direction, we derive our sea surface slope model from the sea spectral model and we prove the Gaussian Distribution of sea slope.

In parallel we develop our reflection behavior analysis scheme and the reflection strength distribution model with respect to ocean surface. Together with the Ionosphere attenuation model, we gain full knowledge of signal strength as well as signal direction and finish our basic model. Based on the basic model, we compare the marine model to the ground model and discuss about the model adjustment.

In addition, aided by the basic model, two variants are proposed. The first variant is multi-hop model over calm ocean surface, where we use the signal attenuation together with Additive White Gaussian Noise to get a solution.

The second variant is a moving ship model that take the angle of divergence into consideration. Based on the multi-hop model and the basic model, we provide a projection area giving a target ship and simulate the power of signal received by the ship.

In the end, we discuss the strengths and weaknesses and make a conclusion.



**Keywords**: High Frequency, Shortwave Propagation, Sea Spectrum, Gaussian Distribution

# A High Frequency Radio Communication Model Over Oceans

February 12, 2018

## Contents

# 1 Introduction

## 1.1 Problem Restatement

High frequencies (HFs) are radio wave frequencies defined to be within 3 to 30MHz. HF radio wave can travel long distance by bouncing between ionosphere and the Earth until it hits the target.

Voyages are particularly in need of long distance communication scheme. One reason is that locating fixed base stations in the middle of oceans is expensive to locate. And another reason is that ships always need emergency channel to satisfy safety need. As a result, HF radio wave is promising with potentials of providing long range and even worldwide communications [1]. Besides marine communication, HF radio waves are also applicable to ground communications but is somehow less attractive.

However, the radio wave reflections suffer from multiple changes on ocean and ground surface. Unlike plain terrain surface, ocean surface varies with wind directions, weather conditions, salt distributions, etc. Therefore, under situation where waves take multiple hops in its travel, it is hard to predict the strength of signals reaching the destination and thus harm the quality of long distance communications.

Ground situations are relatively static comparing to oceans, but communication quality is still threatened by mountains and land architectures. In addition, reflections on ground are weak so signals may not be able to reach the target before it falls below the signal-to-noise(SNR) requirement.

Lastly, in order to ensure reliable long distance communication under different circumstances, a multi-hop model and a moving ship model is required to be investigated.

## 1.2 Our Work

In order to address the problem, we first divide the problem into two parts: determining the signal direction and signal strength the after the reflection.

First we build a sea spectral model based on PM sea spectrum. We generate the waves in a brand new way that we consider it as a randomized procedure. We prove that the slope of the ocean surface follows Gauss Distribution.

To get the signal strength, we assume that the wave strength follows a certain distribution.

The assumption is reasonable based on the fact that the ocean surface slope follows Gauss Distribution. We calculate the details of the distribution through a series of physics laws, such as Law of Refraction and the Fresnel Laws. Then we get the signal strength.

Finally, we build a multi-hop communication model. Then we use our model to solve a realistic problem successfully in which a ship is travelling on the ocean.

The rest of paper is organized as follows. In Section 2 , we state several basic assumptions; in Section 3 we give some notations used in our model; in Section 4, we will give details of our model; in Section 5 , we present the implementations and experiments based on our model; in Section 6, we analyze advantages and limitations of our model and do some sensitivity analysis. At last, in Section 7, we draw the final conclusion of our model.

# 2 Assumptions

Our model makes the following assumptions:

- The source is able to emit HF radio in extremely small angle, which means that during the HF communication the energy of HF radio wave does not dissipate in different directions. In the reflecting process on the ocean surface, we can model the wave as a light ray.

- When HF radio travels in ionosphere, it covers a horizontal distance. We assume that the distance can be ignored because it is short enough compared with the height of ionosphere. Besides, the scattering effect is so weak that we dont take it into consideration as well.
- The sea waves are not too strong. In this way, the white froth forms by violent sea waves or storming weathers can be ignored. The surface of the ocean is still transparent.

# 3 Notations

In this paper we use the Notations in Table 1 to describe our model. Other symbols that are used only once will be described later.

Table 1: Notations

| Symbol | Definition |
|---|---|
| $N_e$ | electron density in the Ionosphere |
| $n_i$ | refractive index of the $i^{th}$ layer |
| $\mu_i$ | magnetic permeability of the $i^{th}$ layer |
| $f$ | wave frequencies |
| $I$ | power density |
| $u_K$ | wind velocity at an altitude of K kilometres |
| $g$ | acceleration of gravity. |
| $I$ | signal strength. |
| $\omega$ | wave speed, $\omega = 2\pi f$ |
| $k_i$ | derivative of ocean waves at point i |
| $\sigma$ | the variance variable |
| $\theta^h$ | angle with respect to horizon |
| $\theta^o$ | angle with respect to ocean |
| $\Psi$ | the strength density with respect to angle |
| $\varepsilon$ | the angle of light weight |
| $S_{cover}$ | the area covered by signal |
| $S_l$ | the area of wave before projection |
| $S_{board}$ | the area of shipboard |
| $l$ | light traveling distance |

# 4 Statement of our Model

## 4.1 Model Summary

Figure 1 shows a brief summary of our model. We first set up the model of sea spectrum before we proof that the slope of sea surface follows Gauss Distribution. To get the strength of the signal, we regard the strength as a distribution and then we calculate the details of the strength distribution. Then we discuss the reflection properties off the ionosphere before we finally obtain the signal strength. After that, we get the multi-hop model after adding some descriptions of noise. Furthermore, we solve a real-world problem in which a ship is sailing on the oceans.

## 4.2 Behavior when Waves Hit the Ionosphere

The Ionosphere is the ionized part of Earth's upper atmosphere, from about 60km to 1000km altitude, a region that includes the thermosphere and parts of the mesosphere and exosphere [2]. It is indicates by previous work that the Ionosphere can be separated into region $D$, $E$, $F_1$ and $F_2$ as shown in figure 2a. Behavior of the Ionosphere is largely dependent on solar activity and thus it is

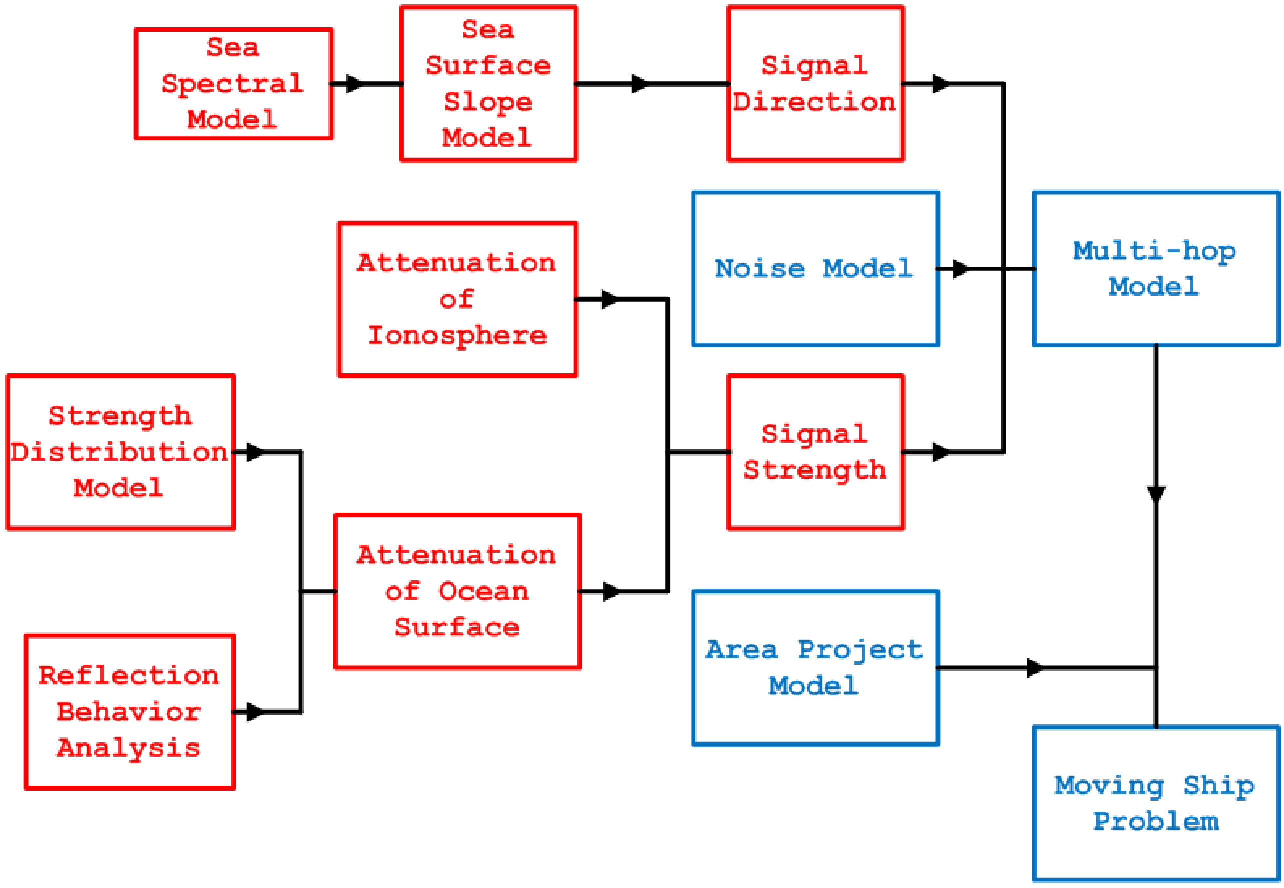



Figure 1: An Overview of our Model

distinguished between day and night. Figure 2b indicates the electron density that changes with time of the day and monthly median solar index(R).

#### 4.2.1 Trace Estimation

Based on the knowledge of the Ionosphere, we try to calculate the trace of a signal. For simplicity we use parabolic models to describe electron density [3]:

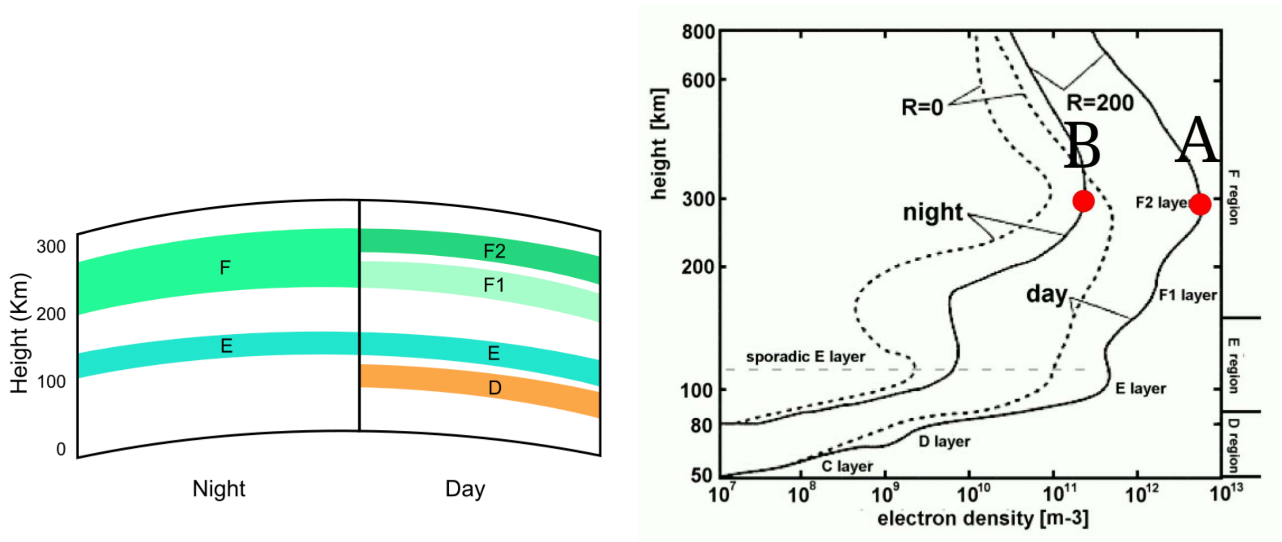
$$N_e = -\frac{(h - h_{max})^2}{h_{max}^2} N_0 + N_{max} \tag{1}$$

where $N_{max}$ denotes the maximum electron density as illustrated in Figure 2b point A and B; h denotes depth of wave entering into the Ionosphere; $h_{max}$ describes maximum height in the Ionosphere the wave travels. Together with the Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{2}$$

where $\theta_1$ denotes angle of incidence in layer 1 and $\theta_1$ denotes angle of angle of refraction in layer 2. We can get traces of signals traveling through the Ionosphere.

We show in Figure 3 signal traces with different frequencies but the same $45°$ angle of incidence. The x-coordinate represents horizontal traveling distance and y-coordination represents vertical distance. The origin is where waves enter the Ionosphere. It can be observed that signals travel further if their frequencies are higher, and all signals with frequency higher than the critical frequency value do not return back to Earth.




(a) The ionosphere layers (b) The ionosphere electron density

Figure 2: The ionosphere [2]

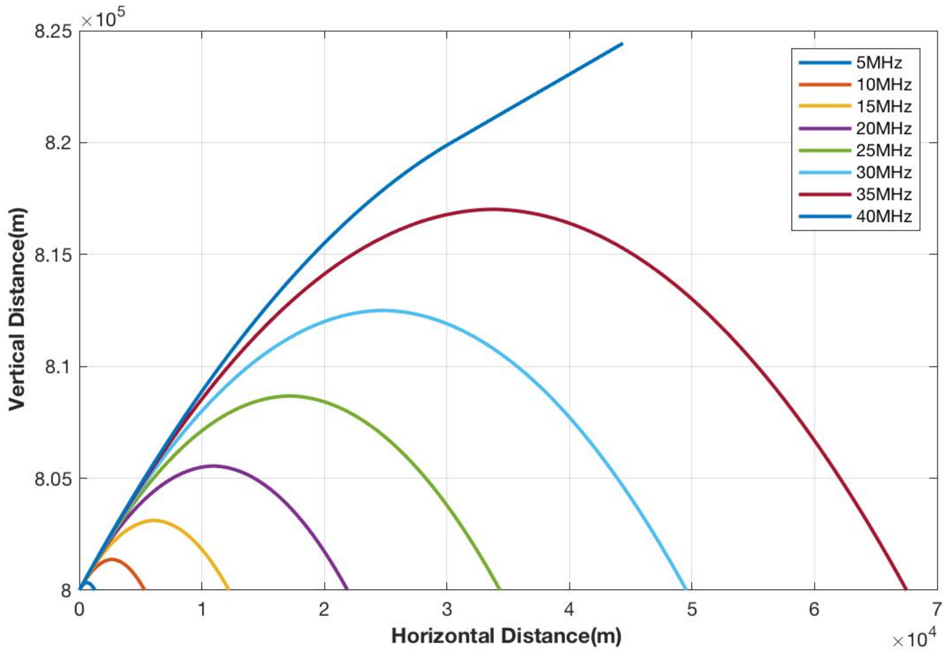



Figure 3: Signal traces in the Ionosphere

### 4.2.2 Attenuation

Energy will be absorbed when the waves reach D, E and F regions in the Ionosphere. A lot of literatures put effort on attenuations when waves traverse through the Ionosphere [3, 4] . It is indicated that the distribution and amount of electron in each layer decide energy attenuation.

For signals that return back to Earth, we calculate the energy attenuation based on Function (1).

At first we calculate the refraction index with:

$$n = \sqrt{1 - \frac{\delta N_e}{f^2}} \tag{3}$$

where $\delta$ = 8.08 $\times$ $10^{-5}$, is an empiric value [5]. Next, we calculate the power ratio of layer 1 to layer 2.

$$\frac{I_1}{I_2} = \frac{4\alpha\beta}{(\alpha+\beta)^2} \tag{4}$$

$$\alpha = \frac{\cos\theta_r}{\cos\theta_i} \tag{5}$$

$$\beta = \frac{\mu_1}{\mu_2}\frac{n_1}{n_2} \approx \frac{n_1}{n_2} \tag{6}$$

where $\theta_r$ denotes angle of refraction, $\theta_i$ denotes angle of incidence. The values of $\mu_i$ are all almost equal to $\mu_0$ in inorganic medium and thus $\frac{\mu_1}{\mu_2} \approx 1$.

Last, we approximate the power attenuation in an Ionosphere travel through integration.

$$\frac{I}{I_0} = \lim_{m\to\infty}\prod_{i=1}^{m-1}\frac{4\sqrt{n_i^2-\sin^2\theta_0}\sqrt{n_{i-1}^2-\sin^2\theta_0}}{(\frac{n_{i-1}}{n_i}\sqrt{n_i^2-\sin^2\theta_0}+\frac{n_i}{n_{i-1}}\sqrt{n_{i-1}^2-\sin^2\theta_0})^2} \tag{7}$$

where $\theta_0$ is the initial angle of instance and $I_0$ is the starting signal strength. The simulation results are shown in Figure 4.

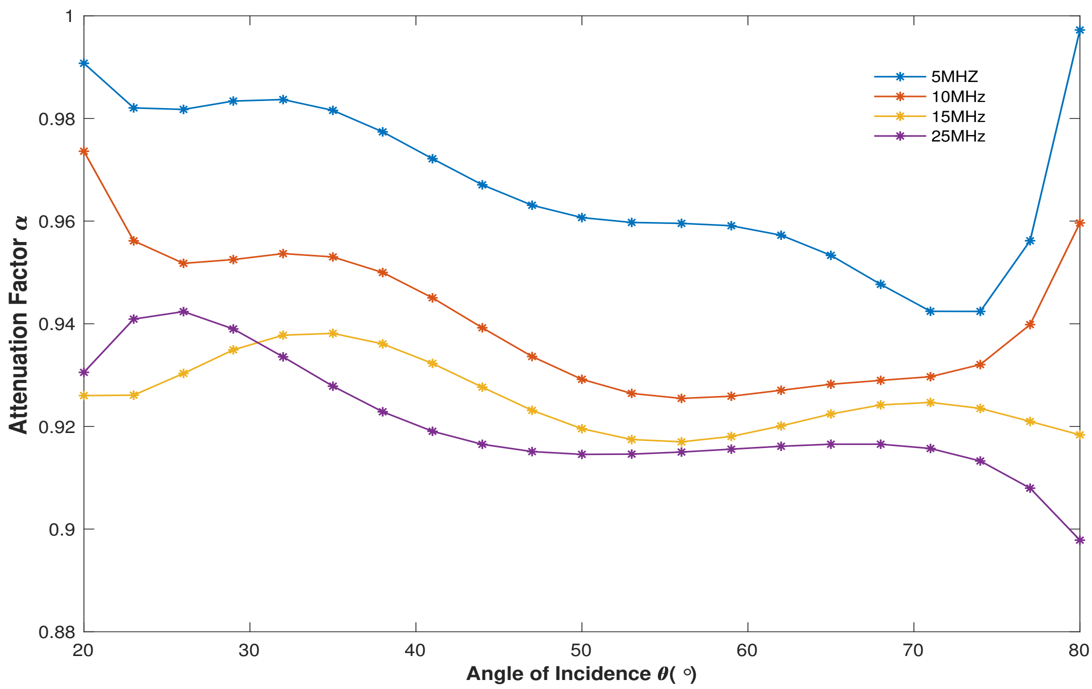



Figure 4: Attenuation factor in the Ionosphere

It can be observed that in general the attenuation factors are in the range of $[0.9, 1)$

### 4.3 The PM Sea Spectrum

After simulating the behavior of the signal in the Ionosphere, we further investigate the characteristic on ocean surface. We use the PM sea spectrum developed by Pierson and Moskowitz to simulate the ocean surface [6, 7].

We introduce the sea spectrum S($\omega$) which describes energy distribution.

$$S(\omega) = \frac{\partial E}{\partial \omega} \tag{8}$$

The height and shape of ocean wave are mainly dependent on wind speed. Friction between the wind and the ocean surface accelerates the wave. When they achieve a same speed, the system remains stable. In the PM model, it is considered that

$$S(\omega) = \frac{c}{\omega^5} e^{-\gamma(\frac{\omega_0}{\omega})^4} \tag{9}$$

where $c = 0.78$, $\gamma = 0.74$, $\omega_0 = \frac{g}{u_{19.5}}$ [7]. To further simulate waves traveling along different directions, a vibration term is attached to S($\omega$) and thus

$$S(\omega, \phi) = \frac{c}{\omega^5} e^{-\gamma(\frac{\omega_0}{\omega})^4} \cos^4(\frac{\phi}{2}) \tag{10}$$

where $\phi$ denotes the angle of between wind direction vector and target direction vector. Four images are plotted to show ocean surface under different wind speed. It is observed that both wave amplitude and frequency vary with wind speed.

# 5 Implementations

### 5.1 First Hit Comparisons under Different Conditions

In this section we discuss the first hit behavior of a signal when it is reflected off turbulent ocean surface.

We at first propose a reflection coefficient calculation scheme considering the angle of incidence, next we fully develop a signal reflection model based on the ocean surface that has been discussed in Section 4.3, and at last make a comparison between previous signal strength distributions and that after.

#### 5.1.1 The Reflection Behavior

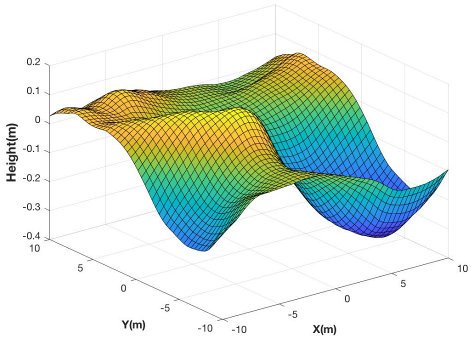
The reflection coefficient describes how much of an electromagnetic wave is reflected by a transmission medium [8]. It varies with the angle of incidence and refraction index of the media.

We have the reflection ratio function

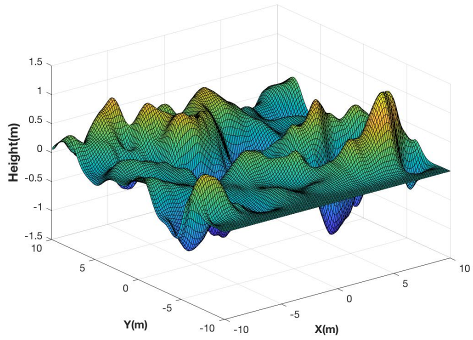
$$R = \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2 \tag{11}$$

together with (2) (5) (6) yields

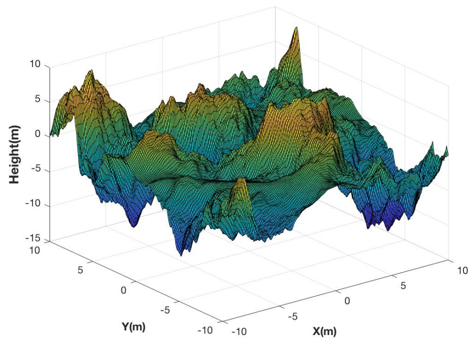
$$R = \left(\frac{\sqrt{n_2^2 - n_1^2 \sin^2\theta_i}\mu_2 n_1 - \mu_1 n_2^2 \cos\theta_i}{\sqrt{n_2^2 - n_1^2 \sin^2\theta_i}\mu_2 n_1 + \mu_1 n_2^2 \cos\theta_i}\right)^2 \tag{12}$$



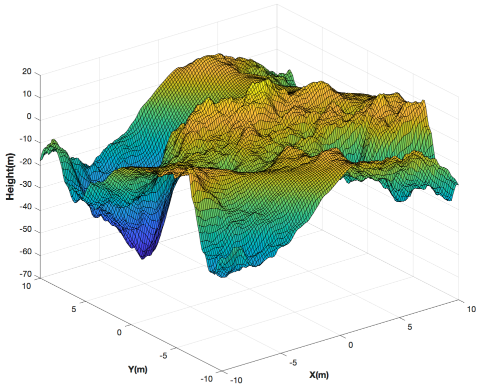

(a) ocean surface when wind speed equals 3m/s




(b) ocean surface when wind speed equals 6m/s




(c) ocean surface when wind speed equals 20m/s




(d) ocean surface when wind speed equals 35m/s

Figure 5: Simulation of ocean surface

#### 5.1.2 The Sea Surface Slope Variance

To discuss the signal interacting with ocean surface, we have to consider the normal vector of ocean surface. However, we argue that only some of the signal traveling directions are under our concern and thus the problem can be simplified as a one dimensional problem. A 3D light reflection problem can be reduced to 2D from a facet perspective.

In this section we develop a novel method of calculating sea surface slope variance. Although previous jobs have existing results [cite!!!!], we find out that they can be further simplified.

At first we will state the characteristic of a 2D wave profile. The ocean wave can be treated as an overlay of various sinusoid each with different amplitudes, frequencies and phases. Therefore, the shape of waves can be described with linear combinations of sine functions [FULIYE?? ].

Consider the derivative $k_i$ at arbitrary point i on the wave, it is a linear combination of various cosine functions.

And we have

$$\int_{-\pi}^{\pi} \cos(w_i x + \phi_i) = 0 \tag{13}$$

Therefore, the expectation of $k_i$ is zero.

According to **the Central Limit Theorem**, sum of large independent variables follows a Gaussian Distribution. Because each $k_i$ is the sum of countless harmonic waves, it follows a Gaussian Distribution $k \sim N(E, \sigma^2)$.

Because linear combination of Gaussian distribution yields Gaussian distribution, the slope k of a profile along any directions follows Gaussian distribution.

With basic knowledge of ocean waves in Section 4.3 , our challenge lies in calculating the variance of k. The procedure consists of four steps that we will discuss in details next.

**The Probability Distribution** First we aim to calculate the probability distribution f($k_i$) in a single cosine wave. At a fixed time t, the wave function can be presented as

$$h = \frac{4\pi^2 A_i}{\omega u}\cos(\frac{4\pi^2}{\omega u}x + \phi_i) \tag{14}$$

and we define $F(K)$ to be the probability that x $\leq K$. With simple integration we have:

$$F(K_i) = \begin{cases} \frac{1}{2} - \frac{1}{\pi}\arcsin(-\frac{K\omega_i u}{4\pi^2 A_i}) & K \in \left[-\frac{4\pi^2 A_i}{\omega u}, 0\right) \\ \frac{1}{2} + \frac{1}{\pi}\arcsin(\frac{K\omega_i u}{4\pi^2 A_i}) & K \in \left[0, \frac{4\pi^2 A_i}{\omega u}\right] \end{cases} \tag{15}$$

**The Probability Density Function** Function (15) instinctively yields the probability density function $f(K)$ by differentiating with respect to $K$.

$$f(K) = \frac{1}{\pi}\frac{1}{\sqrt{w_i^2 A_i^2 - K^2}}, \quad K \in \left[-\frac{4\pi^2 A_i}{\omega u}, \frac{4\pi^2 A_i}{\omega u}\right] \tag{16}$$

**Variance of each $k_i$** We derive the variance of $k_i$ using the standard equation $Var(X) = E(X^2) - E^2(X)$ and since $E(X) = 0$, we have

$$Var(k_i) = \int_{-\infty}^{+\infty} K^2 f(K)\, dx \tag{17}$$

which yields

$$Var(k_i) = \frac{8\pi^4 A_i^2}{w_i^2 u^2} \tag{18}$$

**Variance of k on Ocean Surface** We have argued at the beginning of this section that each point of the function can be replaced by a summation of multiple standard sine functions, and the slopes of the wave function are still linear and addable. Under the assumption that $k$ follows a Gaussian distribution, if

$$k = \sum_{i=1}^{m} k_i \tag{19}$$

we have

$$\sigma(k) = \sum_{i=1}^{m} \sigma^2(k_i) \tag{20}$$

Next we state two basic arguments:

- S($\omega$) is proportional to square of wave amplitude $A^2$. we can see from (8) that S($\omega$) $\propto$ E($\omega$) while E($\omega$) $\propto A^2$. Therefore, the argument holds.

- The wave profile can be regarded as an overlay of multiple waves. The projection function is

$$w_p = w_j \cos \varphi_{jp} \tag{21}$$

Figure 6 indicates the thinking on wave projection. A wave from 2D perspective is viewed as a summation of cylindrical wave that travels in different direction. Waves that do not travel in the same direction(Direction 1 and Direction 2) are stretched and the projected to the Main direction.

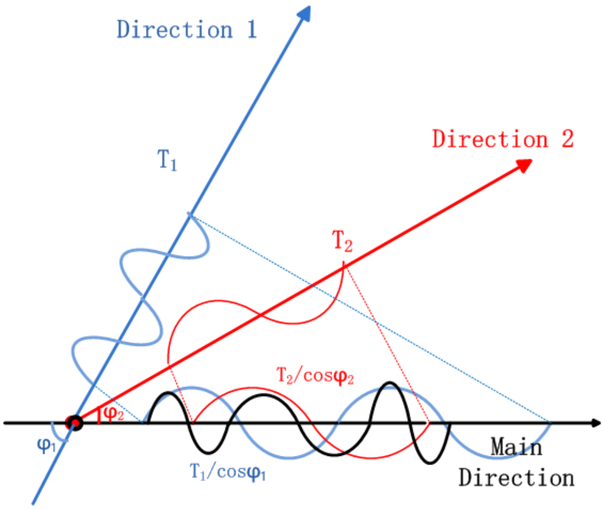



Figure 6: Wave Projection Illustration.

After clarifying the two arguments, we introduce a variable $\rho$ that has S($\omega$) = $\rho A^2$. Under the consideration that $\omega_i$ is continuous in the complex ocean system, we specify variance of k into

$$\sigma^2(k) = \sum_{i=1}^{m} Var(k_i) = \iint \frac{8\pi^4\rho}{w^2u^2} S(\omega, \phi) \cos^2 \phi \, dw \, d\phi \tag{22}$$

where the variance of $k_i$ is derived from (18) and integration is a continuous version of summation with respect to $\omega$ and $\phi$.

### 5.1.3 Signal Reflection Model

In this section we will discuss the light strength distribution after the first hit. We based our model on variance $\sigma$ calculated in the previous section.

In figure 7 we illustrate the reflection behavior on ocean profile where $\theta_i$ denotes angle of incidence , $\theta_r$ denotes angle of reflection and $\theta$ denotes ocean surface direction with respect to horizon. The figure directly yields

$$\theta_i - \theta = \theta_r + \theta \tag{23}$$

which yields

$$\theta = \frac{\theta_i + \theta_r}{2} \tag{24}$$

Remind that in section 5.1.1 we denoted R to be reflection ratio before and after the wave hits the ocean. The value of R depends on angle of incidence and refraction index, thus R($\theta$, n). Added that the refraction index of air is always 1, the denotation is modified to R($\theta$, $n_{air}$). Based on sea surface slope distribution, together with (24) we can calculate the reflection strength distribution

$$\Psi(\theta_r) = I_0 R\left(\frac{\theta_i + \theta_r}{2}\right) f(\theta = \theta_0) \tag{25}$$

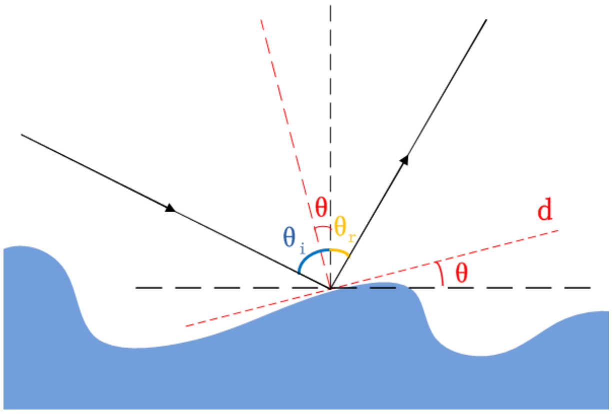



Figure 7: Signal Reflection Illustration

by taking tangent of both $\theta$ and $\theta_0$, we have

$$\Psi(\theta_r) = I_0 R\left(\frac{\theta_i + \theta_r}{2}\right) f(\tan\theta = \tan\theta_0) \tag{26}$$

because k = $\tan\theta$, together with (22) yields

$$\Psi(\theta_r) = I_0 R(\frac{\theta_i + \theta_r}{2})\frac{1}{\sqrt{2\pi}\sigma(k)} e^{-\frac{\tan^2(\frac{\theta_i - \theta_r}{2})}{2\sigma^2(k)}} \tag{27}$$

(27) formulates the strength distribution with respect to $\theta_r$ given $\theta_i$. In this function $\sigma(k)$ is the ocean surface variance and can be determined with (22) given a specific $\rho$. Moreover, (12) decides the value of R. As a result, (27) is solvable.

With (27) we can decide three important characteristics .

- **Signal Direction $\theta_r^*$ with maximum strength versus angle of incidence**
  To calculate the maximum $\Psi(\theta_r)$, we develop

$$\frac{d\Psi(\theta_r)}{d\theta_r} = \frac{I_0}{2\sqrt{2\pi}\sigma(k)} e^{-\frac{\tan^2\frac{\theta_i - \theta_r}{2}}{2\sigma^2}} \left[\frac{dR}{d\theta_r} - \frac{R}{\sigma^2}\frac{\tan\frac{\theta_i - \theta_r}{2}}{\cos^2\frac{\theta_i - \theta_r}{2}}\right] = 0 \tag{28}$$

  $\theta_r$ can therefore be calculated with (28). We exam the direction with maximum strength versus angle of incidence in Figure 8. It has to be made clear that we do not show angle less than $55^\circ$ because they vanish during reflection. This is explained by reflection coefficient R in figure 8.

- **Strength distribution $\frac{\Psi(\theta_r)}{I_0}$ for wave reflecting off turbulent ocean**
  Next we take a look at the relationship between the strength density and the angle of incidence $\theta_i$ on turbulent ocean in figure 9. Area under the given lines are all equal to 1. The taller the bell, the more centralized the energy. On the contrary, the flatter the 'bell', the more energy is diffused.

## 5.2 Multi hop Analysis off Calm Oceans

### 5.2.1 Additive White Gaussian Noise

Additive white Gaussian Noise(AWGN) is a basic noise model. It is regarded as a random process and the summation of the noise follows a Gaussian Distribution $\sim$ N(0, N). In our model, the noise power is set to remain unchanged. An initial decibel value is given and we calculate the maximum

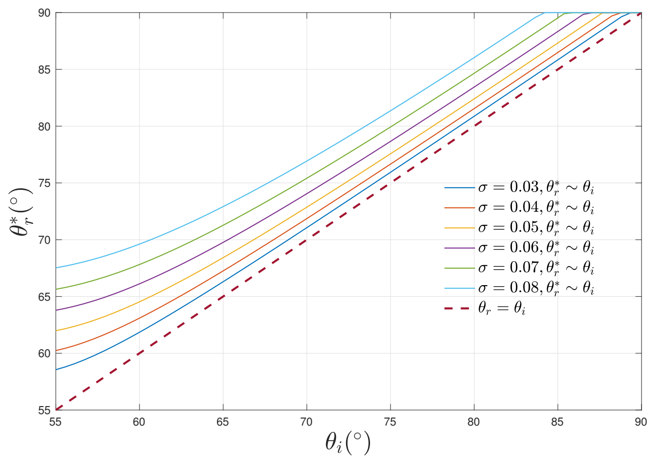



Figure 8: Angle of Reflection with Maximum Strength. Observation 1: $\theta_r^*$ deviates from $\theta_i$ because of coarse ocean surface. Observation 2: The larger the variance, the more angle of reflection is deviated

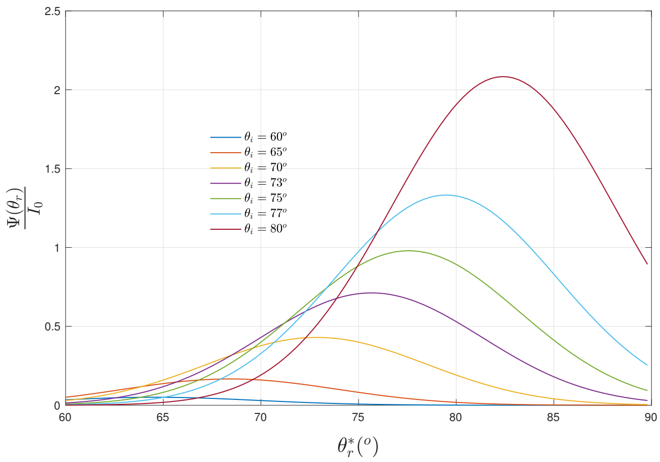



Figure 9: Strength Distribution versus different Angle of Incidence. Observation 1: There is a peak reflection strength value for each angle of incidence and the peak value $\theta_r$ is slightly larger than $\theta_i$. Observation 2: When the angle of incidence grows larger, the wave is more likely to preserve energy in original direction; when the angle of incidence is smaller, it becomes less able to preserve energy.

number of hops that can be taken before the signal strength falls below an usable range. We have the decibel equation [9]

$$D = 10\log\left(\frac{P}{P_{noise}}\right) \tag{29}$$

where D denotes the decibel value, P denotes the signal power and $P_{noise}$ denotes the noise power.

### 5.2.2 Determine the maximum number of hops

In this section we discuss the maximum number of hops given different angle of incidence. There are two basic knowledges before we go deep into the problem.

- Reflection coefficient of the Ionosphere is set to a constant 0.9. We have discussed the reflection coefficient in Section 4.2.2. It is shown that despite the variance in frequency and angle of incidence value, the reflection coefficient does not change much. For simplicity, we set the coefficient in the Ionosphere $r = 0.9$.

- The angle of incidence does not change with hops. This trivially holds because the Ionosphere and the ocean surface are concentric circles. The proof is ignored here.

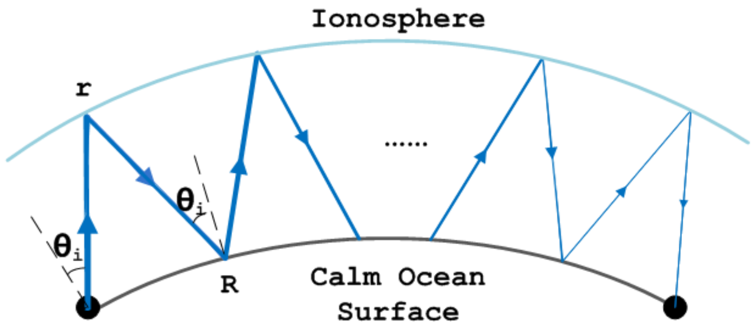



Figure 10: An illustration of Signal Multi-hop over the Ocean Surface

In figure 10 shows the signal multi-hop traces. The upper surface is the Ionosphere and the lower surface represents the ocean. Signals are emitted from the source and they travel up and down between the two surfaces. Each time they hit a surface some energy is detracted. If signal power falls below the usable range, it can no longer be received by the ship.

Assume the signal hits n times of the Ionosphere and n-1 times of the calm ocean. We also assume that initial decibel value is $D_i$. And n has to satisfy equation (30) and equation (31).

$$10\lg\left(\frac{P_0 r^n R^{n-1}(\theta_i)}{P_{noise}}\right) \geq 10 \tag{30}$$

$$10\lg\left(\frac{P_0}{P_{noise}}\right) = D_i \tag{31}$$

We find the largest n to be the maximum hops. Figure 11 shows the value setting different $D_i$.

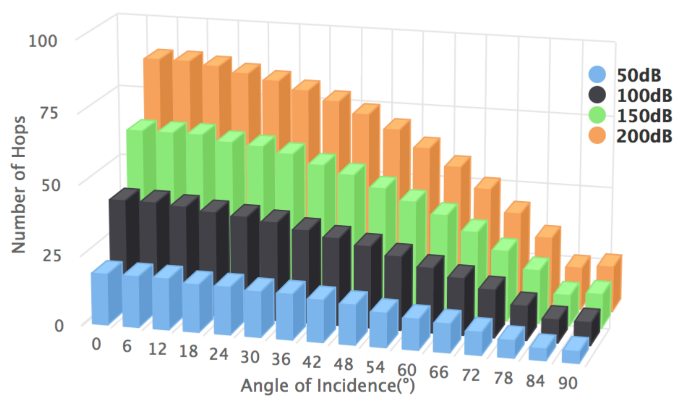



Figure 11: An illustration of Signal Multi-hop over the Ocean Surface. Observation 1: In general, the higher the initial decibel, the larger the number of hops. Observation 2: The number of hops increases as the angle of incidence increases.

### 5.3 Discussions on Marine versus Land Communications

After a full discussion on ocean shortwave propagation, in this section we discuss the difference between marine signal propagation and land signal propagation. We will discuss an adjustment to ocean model and make it applicable to the land condition.

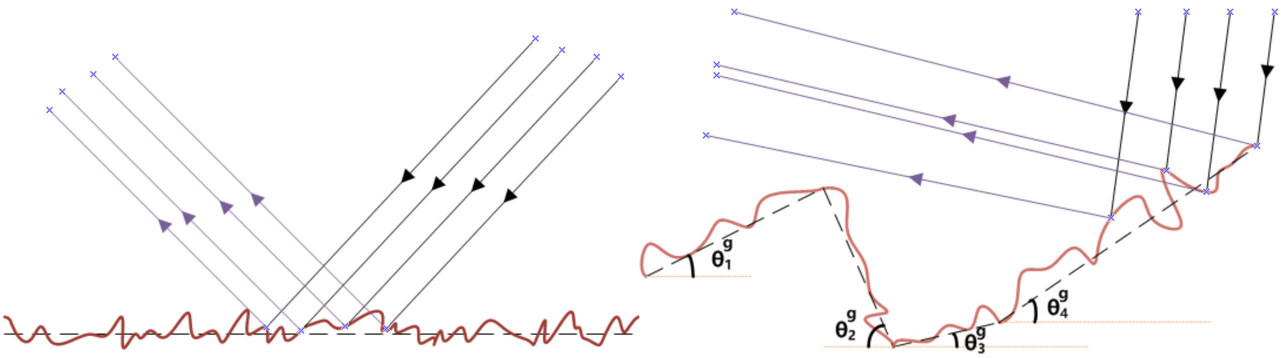
#### 5.3.1 A Comparison of Ocean and Land

Ocean and land are somehow similar to each other. The houses and mountains can be compared to turbulent ocean waves; the plain terrain can be compared to calm ocean surface; the.(more) . . .

We concentrate on the difference between marine signal propagation and land signal propagation and compare land solution with the marine solution.

| **Land Characteristics** | **Ocean Characteristics** |
|---|---|
| Surface is static and changes slowly over time | Surface moves all the time |
| Wave slope does not necessarily follow a Gaussian Distribution but is easy to calculate | Distribution is hard to predict |
| Wave ( buildings, mountains, etc. ) is very tall with an average of 113m (Paris) and 142.6m (New York) | Ocean wave is low and only in tsunami can waves become higher than 100m |
| The size of 'waves' are larger | The size of waves are much smaller |
| Refractive Index $n_l$ | Refractive Index $n_o$ |

Table 2: Comparison of Land and Ocean



(a) Mountainous or Rugged Terrain (b) Smooth Terrain

Figure 12: The Landscapes

#### 5.3.2 The Land Surface Model and its Behavior

We develop two types of model that can generally represent landscapes on Earth. Figure 12 shows the two landscapes. Figure 12a simulates a plain with some potholes on the road and Figure 12b simulates a rugged mountain with with several potholes.

Because the wavelength of HF (ranging from 10m to 100m) is much larger than the depth of potholes, they can be mirror reflected. As a result, the only thing remains is the difference in refractive index. In figure 13 we can find that

### 5.4 A Traveling Ship Model

In this section we discuss a traveling ship model. In the previous model the wave is viewed as a bunch of parallel light. Considering the ship movement on ocean surface, we modified our model

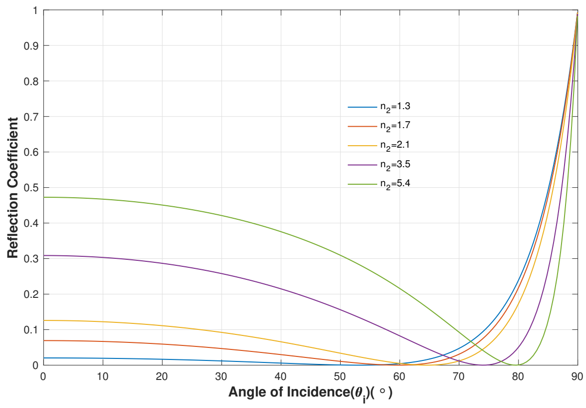


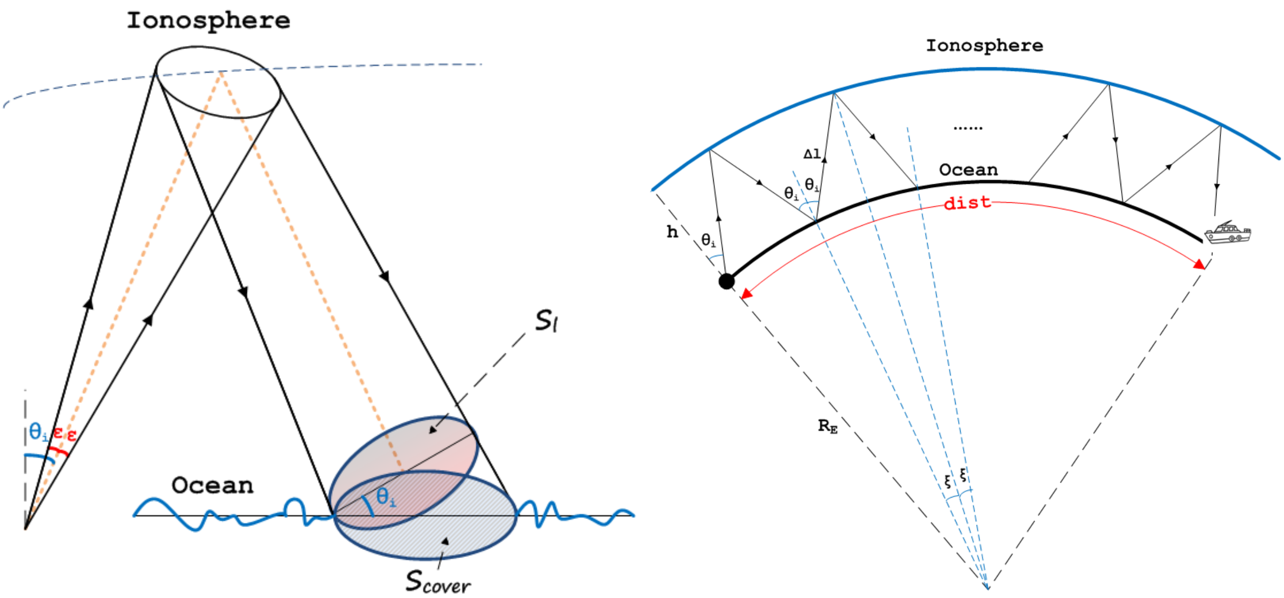

Figure 13: A Comparison among different Refractive Index. Observation 1: the critical angle value increases when the refractive index increases. Observation 2: when the refractive index increases, the angle below index value cannot be omitted anymore

and view the source wave to be a bunch of light having an angle of $\varepsilon$. Therefore the wave projects on ocean surface and covers the ship.

In Figure 14 we give a demonstration of how our model works. The variables are described on the figure where $\Delta l$ denotes a single traveling distance of wave, and n the hops taken by the signal. Based on definition have

$$l = 2n\Delta l \tag{32}$$

Denote $\Upsilon_E$ to be radius of the Earth, we derive $\xi$ to be




(a) Wave Projection Model (b) The

Figure 14: The Moving Ship Model Illustration

$$\xi = \frac{d}{2n\Upsilon_E} \tag{33}$$

Assume h to be height of the Ionosphere, we have

$$\Delta l^2 = \Upsilon_E^2 + (\Upsilon_E + h)^2 + 2\Upsilon_E(\Upsilon_E + h)\cos\left(\frac{dist}{2n\Upsilon_E}\right) \tag{34}$$

and the area of $S_l$ can be described with

$$S_l = \pi(l \tan\varepsilon)^2 \approx \pi l^2 \varepsilon^2 \tag{35}$$

where the approximation holds because $\varepsilon$ is closed to zero. According to the Sine Theorem, we have

$$\sin\theta_i = \frac{(\Upsilon_E + h)\sin\xi}{\Delta l} \tag{36}$$

and it directly yields

$$\cos\theta_i = \sqrt{1 - \frac{((\Upsilon_E + h)^2 \sin^2\xi}{\Delta l^2}} \tag{37}$$

Together with equation (32), (34), (35), (36), (38) yields

$$S_{cover} = \frac{S_l}{\cos\theta_i} = \frac{4n^2\pi\varepsilon^2\Delta l^3}{\sqrt{\Delta l^2 - (\Upsilon_E + h)^2 \sin^2\xi}} \tag{38}$$

We denote the initial power of the signal to be $P_0$, which equals 100w here. The final hop power P can be described with

$$P = P_0 r^n R^{n-1} \frac{S_{board}}{S_{cover}} \tag{39}$$

where $S_{board}$ is the area of shipboard and R is a function of $\theta_i$. Viewing $S_{cover}$ as an oval, we have

$$S_{cover} = \pi ab \tag{40}$$

and based on equation 35 we have

$$a = \frac{\sqrt{S_l}}{\pi\cos\theta_i} \tag{41}$$

assume the speed of ship is $v$, the time $t$ can be calculated with

$$t = \frac{a}{v} \tag{42}$$

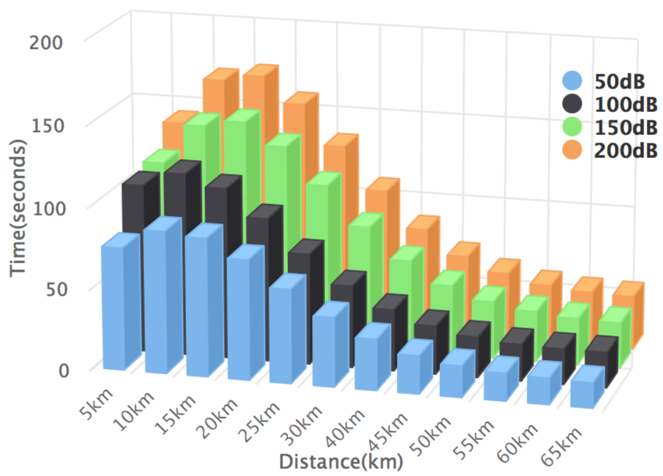



Figure 15: The Simulation Results. Observation 1:

In Figure 15 shows the simulation results with respect to different dB.

# 6 Advantages and Limitations

## 6.1 Advantages

- We give a rigorous proof of the Gauss Distribution of the ocean surface slope.
- We use a distribution to describe the wave strength reflected off turbulent ocean instead of a single strength value. In this way, the explanation of reflection strength is more precise.
- We give an overall experiment of our model

## 6.2 Limitations

- When the wind is so strong that a storm occurs, the ocean wave can not be precisely predicted. Our slope distribution model is not convincing enough to describe the reflection process.
- We do not take the electromagnetic noise on the ocean. We just regard the noise as a Gaussian white noise. Therefore, the calculation of SNR threshold can be more accurate.

# 7 Conclusions

HF is now popular and widely used in communication areas and the related technology becomes more and more practical. In our paper, we present a fairly integrated model to describe the properties of HF reflection on sea surface and the multi-hop communication.

When HF radio waves reflect off the ocean surface, it simply follows the Reflection Law. The strength of radio waves obeys Fresnel Formula at the same time. But we are first supposed to obtain the angle of incidence which is determined by the slope of the ocean surface. We set up an improved PM model to describe the sea spectrum. Then we give a rigorous proof that the slope of surface follows Gauss Distribution. After that, the strength distribution of reflected wave can be determined.

Multi-hop technique is used to travel the signal to longer distance. We establish an attenuation model to estimate the energy loss when HF experiences multi-hop reflection. The attenuation model mainly depends on the former reflection rules we get. The experimental evaluation we did shows that our results fit the real conditions quite well.

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# Appendices

## Appendix A Reduction Ratio vs. $\theta_r$ Calculation

```
sigma=[0.03,0.04,0.05,0.06,0.07,0.08];
ratio=zeros(size(sigma));% attenuation
theta_i=deg2rad([60,65,70,73,75,77,80]);%Different theta_i
alpha=1/120; %Coefficient considering
%windSeed->refractivity->local permittivity and permeability
for j=1:numel(theta_i)
    for i=1:numel(sigma)
        n2=4/3*(1+alpha*sigma(i));
        fun=@(r) -Psi(theta_i(j),r,sigma(i),n2);
        maxPoint=fminbnd(fun,0,pi/2);%get theta_R of max I

        fun=@(r) Psi(theta_i(j),r,sigma(i),n2);
        ratio(i)=integral(fun,maxPoint,maxPoint+3*sigma(i));
        %The value of integral around theta_r for tribble sigma
        %stands for attenuation
    end
    plot(sigma.^2,ratio,'-*','LineWidth',1);%Draw the graph
    hold on;
end

%Function Psi(theta_r)/I0 considering different params.
function I = Psi(i,r,sigma,n2)
reflection_loss=@(x) (((n2^2-sin(x).^2).^0.5-n2^2*cos(x))./...
    ((n2^2-sin(x).^2).^0.5+n2^2*cos(x))).^2;
I=1/((2*pi)^0.5*sigma)*reflection_loss((i+r)./2).*...
    exp(-tan((i-r)./2).^2./(2.*sigma.^2));
end
```

## Appendix B Trace in the Ionosphere Calculation

```
clc;
clear;
Nmax=10^11.95;
N0=10^9;
Xmax=20000;
theta0=deg2rad(45);
step=1;
alpha=8.08e2;

for k=0:7
    f=(k*5+5)*10^6;
    theta_list=zeros(1,numel(0:step:Xmax));
    n_list=zeros(1,numel(0:step:Xmax));
    theta_list(1)=theta0;
    n_list(1)=(1-alpha*N(0,Nmax,N0,Xmax)/f^2)^0.5;
    count=1;

    for h=0+step:step:Xmax
        count=count+1;
        p=N(h,Nmax,N0,Xmax);
        n=(1-alpha*p/(f^2))^0.5;
        sin_alpha=n_list(count-1)*sin(theta_list(count-1))/n;
        if sin_alpha>1
            break;
        end
        theta=asin(sin_alpha);
        theta_list(count)=theta;
        n_list(count)=n;
    end
    n_list(n_list==0)=[];
    theta_list(theta_list==0)=[];


    x_list=zeros(1,(numel(theta_list)));

    for i=2:numel(x_list)
        x_list(i)=x_list(i-1)+step/cot(theta_list(i-1));
    end
    h_list=1:(numel(theta_list));

    if sin_alpha>1

        x__list=2*x_list(end)-x_list;
        x__list=fliplr(x__list);
        x_list=[x_list,x__list];
        h__list=fliplr(h_list);
        h_list=[h_list,h__list];
        h_list=h_list+8*10e4;
        plot(x_list,h_list);
        hold on;
```

```
    else
        h_list=h_list+8*10e4;
        more_x=x_list(end):1000:4.5e4;
        more_y=h_list(end)+(more_x-x_list(end))*cot(theta_list(end));
        x_list=[x_list,more_x];
        h_list=[h_list,more_y];
        plot(x_list,h_list);
        hold on;
    end
    if sin_alpha>1
        enery=zeros(1,numel(theta_list)-1);
        for i=1:numel(theta_list)-1
            a=n_list(i);
            b=n_list(i+1);
            beta=b/a;
            alp=cos(theta_list(i+1))/cos(theta_list(i));
            enery(i)=4*alp*beta/(alp+beta)^2;
        end
        disp(prod(enery));
    end
end
x=-1e4:1000:0;
y=tan(theta0)*x;
y=y+8e5;
plot(x,y);
legend('5MHz','10MHz','15MHz','20MHz','25MHz','30MHz','35MHz','40MHz','Mixed HF wave');

function y=N(x,Nmax,N0,Xmax)
y=N0+((Nmax-N0)/Xmax)*x;
end
```