The Duckworth-Lewis-Stern Method

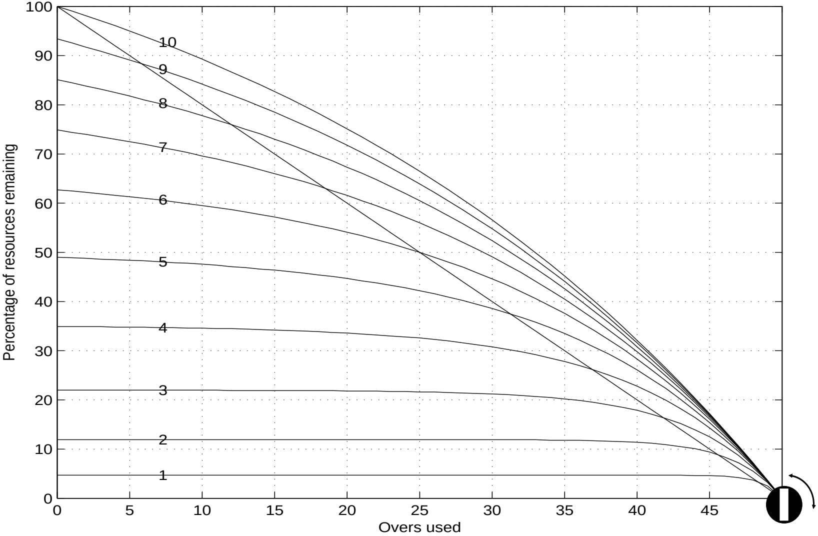
Data Analytics - DLS Lecture 3

The story thus far

- Revision of targets in shortened matches.
- D/L method:
  Quantify the resources of overs-to-go and wickets-in-hand.
  Resources = run scoring potential.
- Identify quantity of resources available with both teams.
  Set target by equating runs per resource.
- In this lecture: A discussion on the D/L method and new fixes.

Fix high first innings scores: D/L Professional Edition

Introduce dependence of curves on the first innings score.



The fix for high first innings scores

- Higher the first innings score, closer to a straight line.
- Resources remaining after 25 overs is lesser,
  R_2 is higher,
  so $T = SR_2$ is higher.
- Again, the choice of parameters based on data and is in the Professional Edition.

A discussion on the "relative positions" criterion

- The relative positions of the two teams before and after interruption should be the same.
- Think about the hypothetical IND-AUS game. What was your strategy? What did you try to optimise?
- One appealing criterion is *isoprobability*:
  The probability of winning before and after the interruption must be the same.
- Does D/L satisfy this?

An example to bring home the point

- 20 July 2003, Cambridge vs. Oxford, at the Lord's.
- Cambridge scored 190 off their 50 overs.
- Oxford were 162/1 off 31 overs when rain interrupted play.
  (29 to win off 19 overs)
- When rain stopped, 12 overs remained.
  But Oxford had already exceeded any target that D/L would set.
  Oxford was declared winner by D/L method.
- Before the rain, Oxford had a huge advantage, but cricket is a game of 'glorious uncertainties'.
  The probability of Oxford winning was not 1. Yet, after the interruption, Oxford was declared winner at resumption.
- Isoprobability criterion would have given Cambridge a positive chance to bowl Oxford out. Low probability, but still positive.
  Would spectators have preferred that? Players?

An insightful pair of comparable games

- Two adjacent grounds A and B hosting two matches.
  Both Teams 1 scored 250 off 50 overs.
  Both Teams 2 played 20 overs, lost 3 wickets, when it rained.
  10 overs lost due to rain, and now 20 overs remain.

- Team 2A: 120/3
  Team 2B: 50/3.

- Before the break, Teams 2A and 2B needed 131 and 201 (resp.) off 30 overs with 7 wickets in hand.

- But since both teams used up the same amount of resources, and get the same (reduced) resources at resumption, their D/L targets are identical: 221.
  - Team 2A must score 101 off 20
  - Team 2B must score 171 off 20.

- More difficult for Team 2B. D/L improved the advantage for the team that was ahead before the interruption.

Isoprobability criterion

- Isoprobability targets: Team 2A - 228, Team 2B - 216.
  - Team 2A must score 108 off 20. (7 runs more than D/L).
  - Team 2B must score 166 off 20. (5 runs less than D/L).

- A case of "from each according to his ability"?

Adding a third match (Carter and Guthrie)

Three adjacent grounds A, B, C hosting three matches.
Teams 1A and 1B scored 250 off 50 overs.
Team 1C scored 180 off 50 overs.
All Teams 2 played 20 overs, lost 3 wickets, when it rained.
10 overs lost due to rain. 20 overs remain.

	Team 1 score	Team 2 at int	Target at int	D/L target	D/L to-go	IsoP target	IsoP to-go
A	250	120/3	131 (in 30)	221	101 (20)	228	108 (20)
B	250	50/3	201 (in 30)	221	171 (20)	216	166 (20)
C	180	50/3	131 (in 30)	159	109 (20)	158	108 (20)

- Isoprobability: Teams 2A and 2C must go the same distance.
  D/L: Team 2A has it easier.
- Team 2A was the poorer bowling team. D/L gives it a discount.
  Again a case of 'socialism'?
- Which is the better criterion?

Incentive to alter strategy under D/L rule

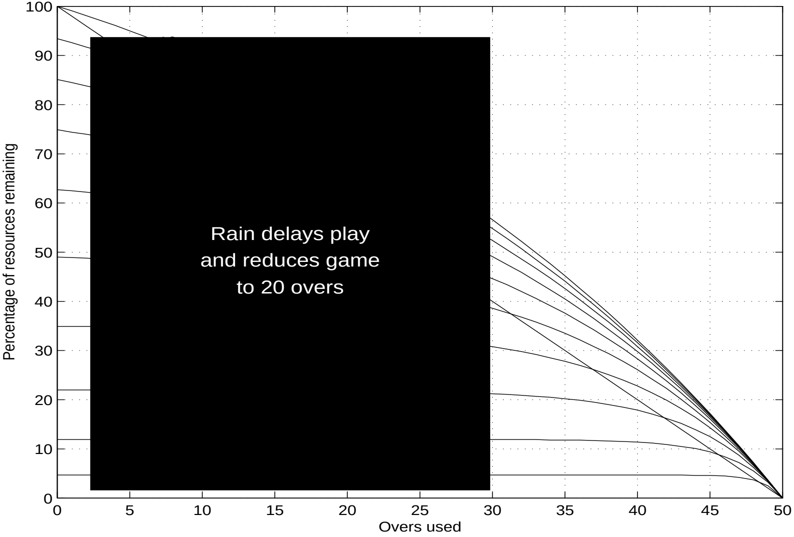
- Team 1 is at 160/4 in 39 overs.
  Rain is expected in the next over. Predicted duration of rain is such that their innings will be terminated, and Team 2 will have about 40 overs.

- Consider two options for Team 1:
  (i) bat carefully and lose no further wickets (Team 2 target 206)
  (ii) bat to maximise score but lose two wickets (Team 2 target 194).

- D/L assume that Team 1 will bat normally as in an uninterrupted match, and maximise their expected score.

- Will Team 1 do this?
  Team 1's sole objective is also maximising its chances of winning.
  D/L method is likely to distort their strategy.
  Is it ok if Team 1 plays (at that stage) contrary to maximising runs?
  What about ARR method?

Applying D/L to T20



Applying D/L to T20

- Curves are a lot flatter - D/L is now much closer to ARR method.
- ICC World T20: ENG-WI (May 2010).
- ENG 191/5 off 20. WI 30/0 in 2.2 overs. Rain stops play.
  At resumption, WI target reduced to 60 (in 6 overs).
- If rain had come before start of WI's play, RR method target = 58 (in 6 overs).
  But D/L target is only 66 (in 6 overs).
  Too much of an advantage for WI.
- But WI did much better. They consumed very little, and lost quite a bit of resources to rain.
  Revised target was much smaller.
- Is there a fix? ... Shrink the curves. Revised targets for shrink-the-curves method:
  Rain at start: 87 off 6 overs.
  Rain as in game: 69 (in 6 overs).
  VJD method apparently fares better. (A possible project).

Another issue with the D/L method

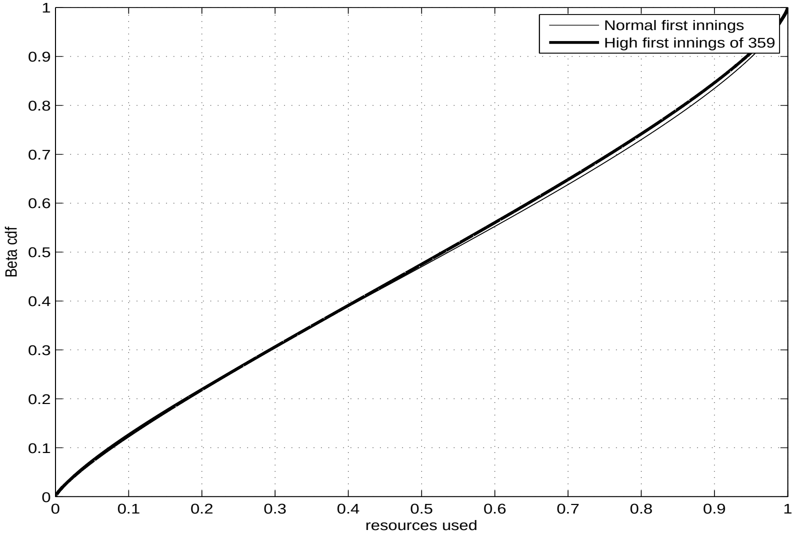
- Given u overs to go and w wickets in hand, D/L method assumes that both Team 1 and Team 2 will have the same scoring pattern.
- Run scoring potential at any stage mapped to remaining resources. For this Team 1 data alone suffices.
- But Team 2 is maximising probability of winning. It's pattern of scoring is perhaps different.
- Stern (2009) analysed the pattern of play in the second innings.

The typical pattern in successful chases ...

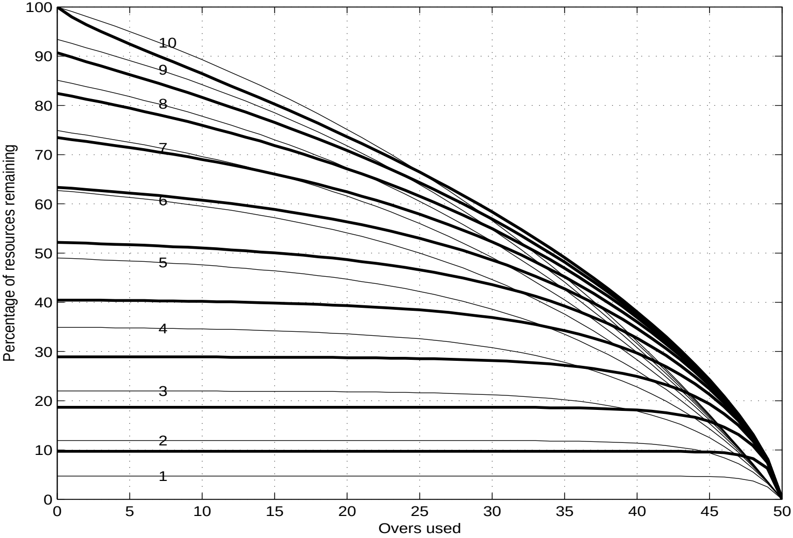
- If the first innings score is low, use lesser amount of resources.
- If the first innings score is average or high, use the full quota of overs and resources.
- Picture ...
- Also, a fast start, then slow middle overs, followed by acceleration near the end.

Stern's correction to resources used

$R_2'(u,w) = F(R_2(u,w))$: D/L Standard edition resources used R_2 transformed to Stern's resources used R_2'



Stern's correction to resources remaining



Lose overs where curve is steep → lose more resources → lower target.
Lose overs either initially or at the end → lower target.

Cricket as an Operations Research (OR) problem

- What is the 'optimal' batting strategy for Team 1 at any stage of the game?
  What is the 'optimal' batting strategy for Team 2?
- Suppose that there are n balls to go and w wickets in hand.
- The batsman can play a risky shot that fetches more runs or a safe shot with fewer runs.
  p_d = probability of losing wicket.
  p_x = probability of getting x runs, $x = 0, 1, 2, 3, 4, 5, 6$.
  Through his choice, the batsman can shape this probability vector.
- What is his best strategy at any stage of Team 1's innings?

Clarke's simplified model - Team 1

- Ignore bowler quality or varying quality of later batsmen.
- Also, let us assume that if batsman's run rate (per ball) is r then the probability of losing wicket is $p_d(r)$. Assume known (from historical data).
- $r = \sum_{x=0}^{6} x p_x$.
- Let $Z_b(n, w) =$ expected number of runs in n balls, with w wickets.
- Goal: Maximise $Z_b(300\ \mathit{balls}, 10\ \mathit{wickets})$.
- A recursive formula:

  $$Z_b(n, w) = \max_{p_0, \ldots, p_1} \left[p_d Z_b(n-1, w-1) + (1-p_d) Z_b(n-1, w) + \sum_x x p_x \right]$$

  This is called the dynamic programming equation or Bellman equation.
- Boundary conditions: $Z_b(0, w) = 0$ for all w, $Z_b(n, 0) = 0$ for all n. Enough information to determine $Z_b(n, w)$ for all relevant n, w.

Clarke's simplified model - Team 2

- Maximise probability of reaching the target.
- $q(s, n, w)$ = probability of scoring s with n balls to go and w wickets in hand.
- Again a dynamic programming equation:

$$q(s, n, w) = \max_{p_d, p_0, \ldots, p_6} \left[p_d q(s, n-1, w-1) + \sum_x p_x q(s-x, n-1, w) \right]$$

- Simplify even further.
  Assume batsman can score 0 or a runs only. He can choose p_a.
  This gives a run rate of ap_a.
  Suppose this fixes the p_d via $p_d(r)$. Then $p_0 = 1 - p_a - p_d$.
- Boundary conditions: $q(l, 0, w) = 0$ for all $l < s$ and all w.
  $q(l, n, 0) = 0$ for all $l < s$ and all n.
  $q(0, n, w) = 1$ for all $n \geq 0$ and all $w \geq 0$.
  Enough to determine $q(n, w)$ for all relevant n, w.

Your assignment

- Generate the production function (as a function of wickets) on 12 years of ODIs.
  Nonlinear regression to estimate the parameters.

- Projects you could take up:
  The OR problem for Team 1.
  The OR problem for Team 2.
  Stern's method to predict the probability of a win by Team 2.
  Jayadevan's method.

References

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