Graphics Assignment No: 6

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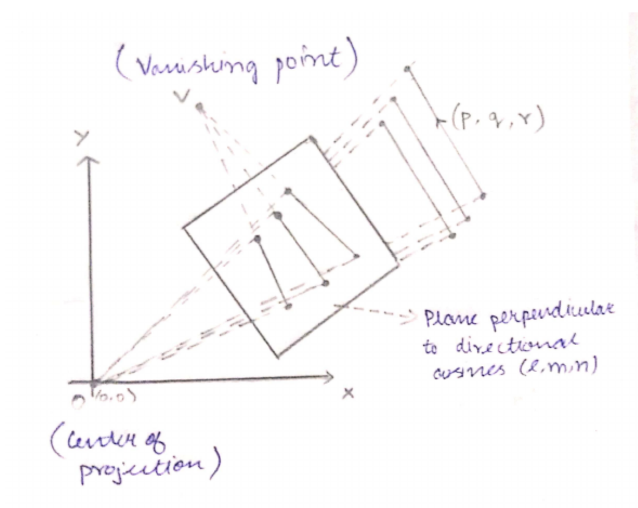
1 Assignment Description

Derive the general expression for the coordinates of the vanishing point. We consider a given plane with perpendicular with the directional cosines (l, m, n), with the origin as the centre of projection, and a set of parallel lines whose directional cosines are (p, q, r) to project.

2 Procedure

We consider the set of lines with direction cosines (p, q, r). The projection plane is perpendicular to the direction cosines (l, m, n), containing the point (x_0, y_0, z_0), with $O(0, 0, 0)$ as the centre of projection.

Vanishing Point



Consider any arbitrary point (a, b, c) on a parallel line. Any point (x, y, z) on the line is given by:

$$x = tp + a$$
$$y = tq + b$$
$$z = tr + c$$

In terms of the column vectors, the above equations can be rewritten as:

$$P^{'} = Dt + P_0$$

where P_0 is the new point coordinates (x, y, z), P_0 is the point (a, b, c) on the line, and D is the unit vector (p, q, r) in the direction of the parallel lines. The following transformation matrix T_{proj} gives the projection of any point from the origin on a plane, which is perpendicular to the unit vector (l, m, n) containing the point (x_0, y_0, z_0):

$$T_{proj} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ l & m & n & 0 \end{bmatrix}$$

where d is given by $lx_0 + my_0 + nz_0$.
Every point on the line follows the same transformation when projected on the plane, which yields another set of points that form a line. Let point p lie on the line then, p is given by:

$$p = \begin{bmatrix} tp + a \\ tq + b \\ tr + c \end{bmatrix}$$

Projecting the point, we have $p^{'} = T_{proj}p$

$$p^{'} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ l & m & n & 0 \end{bmatrix} \begin{bmatrix} tp + a \\ tq + b \\ tr + c \\ 1 \end{bmatrix}$$

$$p^{'} = \begin{bmatrix} dtp + da \\ dtq + db \\ dtr + dc \\ t(lp + mq + nr) + (la + mb + nc) \end{bmatrix}$$

The x coordinate of the point in cartesian coordinate system is given by:

$$x = \frac{dtp + da}{t(lp + mq + nr) + (la + mb + nc)}$$

Vanishing point is observed when the point tends to infinity, i.e. $lim_{t \to \infty} p$. When the similar limit is applied on the projection, we get the vanishing point as follows $p_{vp} = lim_{t \to \infty} p^{'}$, which yields:

$$x_{vp} = lim_{t \to \infty} \frac{dtp + da}{t(lp + mq + nr) + (la + mb + nc)} = \frac{t(dp)}{t(lp + mq + nr)} = \frac{dp}{lp + mq + nr}$$

similarly,

$$y_{vp} = \frac{dq}{lp + mq + nr}$$

$$z_{vp} = \frac{dr}{lp + mq + nr}$$

Vanishing point in the direction of X-axis can be given by substituting (p, q, r) with $(1, 0, 0)$:

$$x_{vp} = \frac{d}{l}$$

$$y_{vp} = 0$$

$$z_{vp} = 0$$

Vanishing point in the direction of Y-axis can be given by substituting (p, q, r) with $(0, 1, 0)$:

$$x_{vp} = 0$$

$$y_{vp} = \frac{d}{m}$$

$$z_{vp} = 0$$

Vanishing point in the direction of Z-axis can be given by substituting (p, q, r) with $(0, 0, 1)$:

$$x_{vp} = 0$$

$$y_{vp} = 0$$

$$z_{vp} = \frac{d}{n}$$