Information Suppression in Bayesian Persuasion

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Abstract

A sender is seeking approval from the receiver(s). He conducts experiments to two receivers with identical preferences sequentially. The first receiver can approve, reject, or delay the decision to the next receiver while the second receiver must approve or reject. Upon delay, the first receiver can communicate his information to the second receiver or hide it. This chance of information suppression creates the incentive to delay when the second receiver is naive – interpreting the absence of communication as the absence of information. Facing this incentive, the sender discloses more information to the first receiver to induce immediate action when delay is very costly, and discloses less information so that the first receiver may delay when delay is not so costly. And in the former possibility, the first receiver is better off than the static game and has a positive value of persuasion.

Keywords: Dynamic Bayesian persuasion, Information manipulation, Hiding information

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1. Introduction

Consider two agents, R_1 and R_2, who have identical preferences over possible outcomes. If R_1 has useful information pertaining to a decision problem that R_2 faces, then clearly, R_1 would share this information. However, if R_2 is engaged in a strategic interaction with another player S, and if R_1's sharing of information is observed by S, then R_1 may prefer not to share some types of information. Furthermore, if the information that R_1 gets is endogenous, and determined by S, this possibility may allow R_1 to get more information than he otherwise might, so that both R_1 and R_2 are better off.

I explore this possibility in the context of a dynamic persuasion problem where a single sender S would like her project to be approved by the receiver(s) and conducts experiments to this end. She faces a sequence of receivers, R_1 and R_2. The receivers have identical state-dependent preferences, and following the experiment conducted by S, R_1 may approve the project, reject it, or delay the decision so that it is made by R_2. If R_1 chooses to delay the decision, he may communicate his information to R_2, or hide it.

I assume, for the most part, that R_2 is *naive* – he interprets the absence of communication by R_1 as the absence of information.[2] This forces the sender S to conduct an additional experiment to convince R_2, which benefits the receivers. Anticipating this hiding behavior by R_1 and since delay is costly, the sender may disclose more information to R_1 so that he makes a more informed and immediate decision. In other words, possibility of hiding allows the receiver to get a *positive value of persuasion* – the value of persuasion to the receiver is always zero in the static problem. Alternatively, if delay is not so costly for the sender, she may choose to disclose less information to R_1, so that he does sometimes leave the decision to R_2. However, even in this case, the sender may be forced to disclose *some* information to R_1 to dissuade him from summarily

[2] I also consider the case where R_2 is sophisticated; in this case R_1 cannot benefit from hiding information from R_2

rejecting the project.

A natural application of our model is the relationship between a regulatory agency and a firm that seeks approval of its product or project. In the static context, the firm would conduct experiments that would convince the agency to the point where it is indifferent between approving and rejecting the project. However, regulatory agencies assign responsibilities to rotating cast of employees, and when one employee leaves his position, he may be selective in his briefing of his successor. Our model suggests that such a rotating cast may be beneficial for consumers. Besides the effect on consumers, the model can also explain the behaviors of decision makers facing term limits, like the shirking problem of legislators discussed by Lopez (2003); Carey et al. (2009); Klein and Sakurai (2015).

The organization of this paper is as follows. Section 2 provides the setup of the model. Section 3 illustrates the main results of this paper. Section 4 introduces some extensions of the model including multiple-period and multiple-state cases, and sophisticated receivers. Finally, Section 5 is the concluding remark.

Related Literature.

My paper relates to several literatures: Bayesian persuasion, bounded rationality and information manipulation.

Kamenica and Gentzkow (2011) initiated the study of Bayesian persuasion and showed that the value of persuasion to the receiver is zero when the sender has state-independent preferences and the receiver has binary actions – the receiver gets no additional payoff benefit from the experiment conducted by the sender. Dynamic models of Bayesian persuasion are studied by Ely (2017); Honryo (2018); Orlov et al. (2020); Smolin (2020); Bizzotto et al. (2021).

Bénabou and Tirole (2002) studied self-deception – an agent with time-inconsistent preferences can hide bad news from the future self who must decide whether or not to undertake a task. His incentive to manipulate information comes from the interest conflict between different selves. In contrast, I assume that two receivers share the same preferences, and the benefit of hiding information arises due to the strategic interaction between sender and receiver.

Fischer and Verrecchia (2000) study the incentives of the company manager to manipulate the information provided to the capital market. Goldman and Slezak (2006) study the design of contracts aimed at preventing information manipulation. Edmond (2013) discuss how politicians manipulate the information source to affect the citizens' decisions.

In my paper, the sender and the second period receiver may have different beliefs, due to the hiding of information. This relates to Bayesian persuasion with heterogeneous priors, as in Alonso and Câmara (2016). Also related are persuasion problems with a non-Bayesian agent, as in Levy et al. (2018), de Clippel and Zhang (2022) and Galperti (2019).

2. Model

2.1. Timing

I study a 2-period dynamic information design model. There are 3 players: one long-lived sender and 2 short-lived receivers (R_1 and R_2). $\omega \in \{g, b\} := \Omega$ is the unobserved payoff-relevant state. At period t, Receiver t has a prior $p_t = Pr(\omega = g)$ and the sender has a prior p_t^s. Though I assume that in period 1, $p_1 = p_1^s$, I distinguish players' priors since they can be different in period 2 due to information manipulation.

In each period, the sender chooses an experiment: $\pi_t = (\pi_t(.|\omega))_{\omega\in\Omega} \in \times_{\omega\in\Omega}\Delta(S) := \Pi$, where S is the signal space. I assume that the sender can choose any experiment he likes. I assume that outcomes of experiments in different periods are independent conditional on the state (i.e. $Pr(s_1, s_2|\omega) = \pi_1(s_1|\omega) \cdot \pi_2(s_2|\omega)$). This is an important assumption – if I were to allow experiments to be correlated across periods, the sender could credibly disclose the outcome of past experiments.

In period 1, observing the outcome s_1 of the experiment, beliefs are updated from p_1 and p_1^s to q_1 and q_1^s. Then R_1 chooses an action $a_1 \in \{yes,\ no,\ hide,\ reveal\} := A$. If *yes* or *no* is chosen, the game ends and payoffs are realized.

On the other hand, if *hide* or *reveal* is chosen, the game proceeds into the next period and the current receiver's payoff is decided by the action chosen by the future receiver. By revealing, R_1 verifiably discloses the experiment and the outcome in period 1 to the future receiver. And by hiding, no outcome or experiment is disclosed. In period 2, which is the deadline, the receiver must decide and only *yes* or *no* can be chosen.

This model can be extended to more than two periods, see Section 4.1.

2.2. Belief updating

R_1's hiding behavior can affect players' belief updating across periods. The sender can observe all experiments, outcomes, and R_1's action in the history, so his prior in period 2 is equal to his posterior in period 1 ($p_2^s = q_1^s$).

I firstly look at the naive R_2 who will assume that no experiment was conducted in the past period if the previous receiver did not disclose the information. He observes only the revealed experiment and outcome in period 1 and will ignore period 1 without the disclosure from R_1, so he may have a different update from the sender. In other words, he will only believe a previous experiment when it has verification from a previous receiver and will assume that the experiment is uninformative without the verification. As a result, without disclosed information from the last receiver, R_2's prior will stick to the prior in period 1 ($p_2 = p_1$). And with verifiable disclosure, the receiver's belief is updated in the same way as the sender ($p_2 = q_1$). This naive case is corresponding to the situation where interpreting past information without disclosure is too costly. Also, seeing the sender's choice of experiment in period 2, the naive R_2 does not update his belief yet, since he thinks there is no information in period 1. He will only update his belief after seeing the outcome of the experiment. Furthermore, R_2's prior will always be p_1 when R_1 hides the information. As a result, regardless of what q_1 is[3], when R_1 hides the information, the sender will choose the same experiment in period 2 since he is always persuading a receiver

[3]As long as it is between 0 and 1.

	yes	*no*
good	a	0
bad	0	b

Table 1: Receivers' payoff

with prior p_1.

As for a sophisticated receiver who will do Bayesian inference seeing no disclosure from the past receiver, I will deal with this case in Section 4.2.

2.3. Players' payoffs

When the game ends (*yes* or *no* is chosen), the current receiver gets a payoff depending on the state and the action: $u : \{yes, no\} \times \Omega \to \mathbb{R}$, which is summarized in Table 1. The previous receiver will have a discounted payoff by the discount factor δ. I assume that two receivers have the same preference u, but different receivers may have different expected payoffs since they may have different beliefs because of information manipulation. I also assume that receivers want to match the good state (g) with *yes* and the bad state (b) with *no*. A successful match benefits receivers but a failed match generates zero payoffs, i.e. $a, b > 0$. It can be understood as a policymaker seeking to approve a good project but turning down a bad project. Furthermore, with this preference, receivers' payoffs are always non-negative, and thus delay is costly to R_1 due to discount.

As for the sender, I assume a pure persuasion here, which means that the sender's payoff at the end of the game is assumed to be state-independent: $v : \{yes, no\} \to \mathbb{R}$. Without loss of generality, I assume that the sender always prefers *yes*, and normalize the payoff from *no* to 0 and *yes* to 1, i.e. $v(yes) = 1$, $v(no) = 0$.

2.4. Strategies and equilibria

The initial priors $p_1 = p_1^s$ are common knowledge. Suppose that we are in period 2,[4] the sender observes the whole history: (π_1, s_1) and R_1's action a_1. But R_2 only considers the experiments he believes: $(\pi_1, s_1) \times (\pi_2, s_2)$ if $a_1 = reveal$, and (π_2, s_2) if $a_1 = hide$. In other words, only the experiment verifiably revealed and the experiment performed in front of him are taken into consideration.

As for period 1, the sender only knows the priors $p_1 = p_1^s$ when conducting the experiment. And R_1 knows (π_1, s_1) when deciding a_1.

The sender's strategy maps the whole history into an experiment π_t in period t, and Receiver t's strategy maps the experiments in his consideration into A for $t = 1$, and into $\{yes, no\}$ for $t = 2$.

The equilibrium concept in this paper is perfect Bayesian equilibrium: players maximize their expected payoffs given other players' strategies and the beliefs generated by the Bayes rule if possible. Furthermore, as tie-breakers, I assume that receivers will choose the action preferred by the sender if they are indifferent between two actions, and the sender will choose the least informative experiment[5] when indifferent. The first tie-breaker can ensure the upper hemicontinuity. The second tie-breaker rules out the multiplicity caused by different experiments inducing the same outcome. And it makes the equilibrium a limit result of games with costly experiments—where the more informative experiment is more costly—as the cost goes to zero. Similar tie-breakers are also used in Bizzotto et al. (2021).

3. Main Results

3.1. Equilibrium results

Before I introduce the results, I define an important concept following Kamenica and Gentzkow (2011), *value of persuasion*, which describes what a player

[4]This would happen only if the game does not end at period 1, i.e. $a_1 \in \{hide,\ reveal\}$

[5]The informativeness here is the Blackwell order.

can get from the persuasion game.

Definition 1. The value of persuasion to a player is his highest expected payoff in an equilibrium minus his expected payoff without any experiment.

Based on this definition, I introduce the following useful lemma.

Lemma 1. *In static pure Bayesian persuasion with binary actions, the value of persuasion is zero to the receiver.*

Lemma 1 can be easily got from the results in Kamenica and Gentzkow (2011). This lemma is not only used in the proofs but also brings a comparison between the model and the benchmark—R_1 can have a positive value of persuasion in my model even with only binary concluding actions.

If the initial prior p_1 is making a receiver choose *yes* rather than *no*, the result is straightforward.

Proposition 1. *If* $p_1 \geq \frac{b}{a+b}$*, there is a unique equilibrium where the game ends in period 1.*

To see this, consider the sender choosing an uninformative experiment in period 1. In this case, R_1 will have the same belief as R_2 if he delays. So R_1's delaying payoff is exactly R_2's expected payoff from the experiment in period 2 with a discount. But in period 2, the situation is the same as a static Bayesian persuasion game and thus the equilibrium gives R_2 no value of persuasion, according to Lemma 1. As a result, R_1's payoff from delay without discount is the same as acting immediately.

So, R_1 will take action immediately. Notice that the concluding action preferred by R_1 at $p_1 \geq \frac{b}{a+b}$ is *yes*, which is also the preferred action by the sender, so the sender can get his highest possible payoff by choosing an uninformative experiment and will not deviate from that. The equilibrium here is that the sender chooses an uninformative experiment and R_1 chooses *yes* immediately. Trivially, this result can be extended to multiple-period cases.

If the initial prior p_1 is inducing *no* rather than *yes*, the result is more interesting.

Proposition 2. *For $p_1 < \frac{b}{a+b}$, there exists $\bar{\delta}$ such that:*
If $\delta \geq \bar{\delta}$, for cutoff points $\alpha_1(\delta)$ and $\beta_1(\delta)$ ($0 < \alpha_1(\delta) \leq \beta_1(\delta) < \frac{b}{a+b}$), the unique optimal experiment in period 1 induces only immediate actions when $p_1 \notin [\alpha_1(\delta), \beta_1(\delta)]$, and may induce delay when $p_1 \in [\alpha_1(\delta), \beta_1(\delta)]$.

There are two possibilities, one is that the sender chooses an experiment more informative than the static optimal experiment and R_1 always acts immediately; another case is that the sender chooses an experiment less informative than the static optimal experiment and R_1 may delay and hide information from R_2. In such situation, R_1 can benefit from hiding information from R_2 even if they share the same preference.

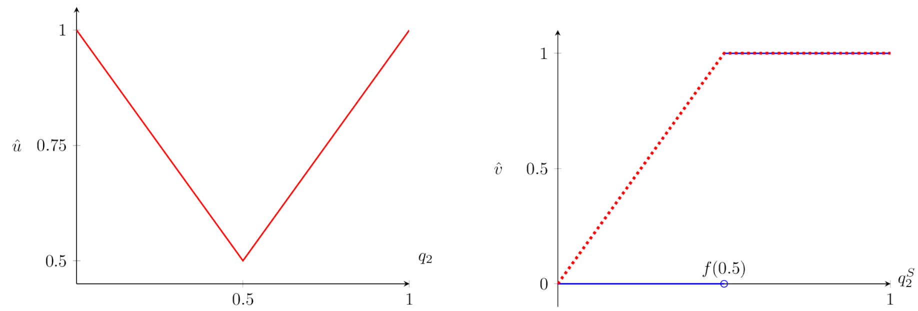
We can solve for the equilibrium by backward induction. In period 2, the situation is the same as a static Bayesian persuasion game since there is no choice of delay for R_2. The payoff functions in period 2 are plotted in Figure 1,[6] and we can easily get the optimal experiment in period 2 by the concave closure method as in Kamenica and Gentzkow (2011). The function $f(.)$ in the graph is the relationship between posteriors: $q_2^s = f(q_2)$. Two posteriors can be different due to the difference between two priors, though posteriors are generated by the same experiment. $f(.)$ is a strictly increasing function according to Alonso and Câmara (2016).

The strategy in period 1 is different from the static case since R_1 can delay—we need to compare R_1's payoffs from delay and immediate actions. Firstly we observe that *reveal* will not be chosen by R_1.

Lemma 2. *The action reveal is always suboptimal.*

If *reveal* is chosen, R_1 and R_2 share the same belief, so R_1's payoff is the same as R_2, which is the same as acting immediately due to Lemma 1. Then the discount factor makes *reveal* suboptimal.

[6] Figures in this section is plotted with parameter values $a = b = 1$ and serves the purpose of illustration.



(a) Receiver (b) Sender

Figure 1: Period 2

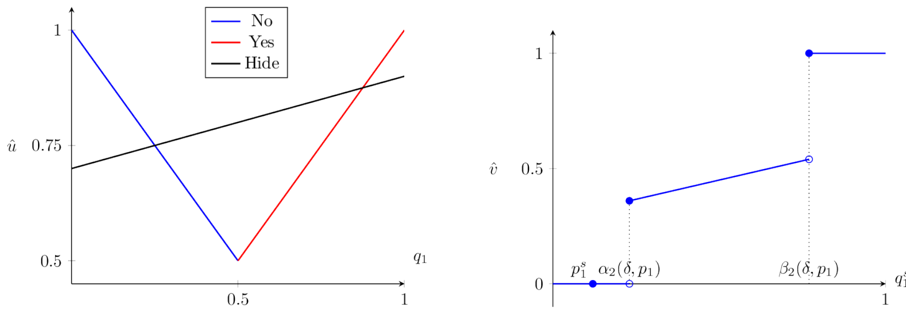
As a result, to solve for R_1's strategy, we only need to keep track of action *yes*, *no*, and *hide*. The payoffs from *yes* and *no* are easy. If *hide* is chosen, R_1 and R_2 can have different expected payoffs from the experiment in period 2 due to different priors. The sender in period 2 will choose a posterior split between 0 and $\frac{b}{a+b}$ for R_2, whose prior is $p_2 = p_1 < \frac{b}{a+b}$ due to hiding. If R_1's belief q_1 is larger than R_2's belief p_2, then the experiment induces a posterior split between 0 and $q > \frac{b}{a+b}$ for R_1. It is a more informative experiment than in the eyes of R_2. Similarly, with $q_1 < p_1$, the experiment in R_1's eyes is less informative than R_2.

I summarize R_1's strategy under certain p_1 and q_1 in the following lemma.

Lemma 3. *For discount factor $\delta \in (0,1)$ and initial belief $p_1 < \frac{b}{a+b}$, there exists $\hat{\delta}(p_1) \in (0,1)$:*
(1) If $\delta > \hat{\delta}(p_1)$, there exist cutoff points $\alpha_2(\delta, p_1), \beta_2(\delta, p_1)$ ($p_1 < \alpha_2(\delta, p_1) < \beta_2(\delta, p_1) < 1$) such that R_1 chooses yes immediately with $q_1 \geq \beta_2(\delta, p_1)$; chooses no immediately with $q_1 < \alpha_1(\delta, p_1)$; chooses hide with $\alpha_2(\delta, p_1) \leq q_1 < \beta_2(\delta, p_1)$.
(2) If $\delta \leq \hat{\delta}(p_1)$, R_1 always acts immediately.

We firstly look at the case with $\delta > \hat{\delta}(p_1)$. Given R_1's strategy, we have the payoff function for the sender, and period 1 can be summarized in Figure 2.

From Figure 2 (b), it is easy to see that since we have $p_1^s = p_1 < \alpha_2(\delta, p_1)$,



(a) Receiver

(b) Sender

Figure 2: Period 1

the optimal experiment in period 1 is either a posterior split between 0 and $\alpha_2(\delta, p_1)$ or between 0 and $\beta_2(\delta, p_1)$ in the eyes of the sender and R_1.

I denote the posterior split between 0 and $\alpha_2(\delta, p_1)$ as the *delaying experiment*, since R_1 chooses *hide* at $\alpha_2(\delta, p_1)$, and denote the split between 0 and $\beta_2(\delta, p_1)$ as the *immediate experiment*, since R_1 takes action immediately at both posteriors. Which one is the optimal experiment will depend on the values of the initial priors and the discount factor, which is summarized in Proposition 2.

When $\delta \leq \hat{\delta}(p_1)$, there are only immediate actions and the sender just chooses the same experiment as in a static case.

As δ goes up, we will have smaller $\alpha_2(\delta, p_1)$ and larger $\beta_2(\delta, p_1)$, which make R_1 more likely to delay. And this leads to a lower $\alpha_1(\delta)$ and a higher $\beta_1(\delta)$—more priors make the delaying experiment the optimal one. When δ is small enough, no prior can make the delaying experiment optimal.

The intuition is that higher δ makes the cost of delay lower, so delay is more attractive. With a more attractive option other than immediate actions, R_1 is harder to persuade. As a result, the sender will need a more informative experiment to persuade R_1, which reduces the sender's payoff from the immediate experiment and makes the sender tend to use the delaying experiment.

3.2. Value of persuasion

For the case with $p_1 \geq \frac{b}{a+b}$, in the equilibrium, there is an uninformative experiment from the sender and an immediate *yes* from R_1, which is the same as in a static Bayesian persuasion game. So the welfare is also the same.

For the case with $p_1 < \frac{b}{a+b}$, the welfare results are different between the delaying experiment and the immediate experiment.

When the immediate experiment[7] is optimal, R_1 gets a more informative experiment than the static optimal experiment and takes action immediately, so he is better off than the static case, where he has zero value of persuasion. We can easily see this from Figure 3, in which R_1's expected payoff from the immediate experiment is represented by point P. Here R_1 has a positive value of persuasion.

The delaying experiment[8] is less informative than the static optimal experiment, and R_1's expected payoff from it is represented by the point Q in Figure 3, which is the same as the expected payoff without any experiment. So when the delaying experiment is optimal, R_1 has zero value of persuasion as in the static case.

But for the sender, he is either forced to provide a more informative experiment or is forced to tolerate R_1's delay, and thus is always weakly worse off.

The above is the value of persuasion with both the manipulation power and naive R_2. If we remove the manipulation power, which means that a receiver always reveals the information, the game is simply the repetition of the identical Bayesian persuasions. Then there is no incentive to delay and the game always ends at period 1, leading to the same welfare result as the static case. And in Section 4.2, we will see that with sophisticated receivers who will do Bayesian inference instead of simply assuming no information when they do not observe the disclosure from the past receiver, delay is never an equilibrium choice either.

[7]The posterior split between 0 and the second cutoff point ($\beta_2(\delta, p_1)$).

[8]The posterior split between 0 and the second cutoff point ($\alpha_2(\delta, p_1)$).

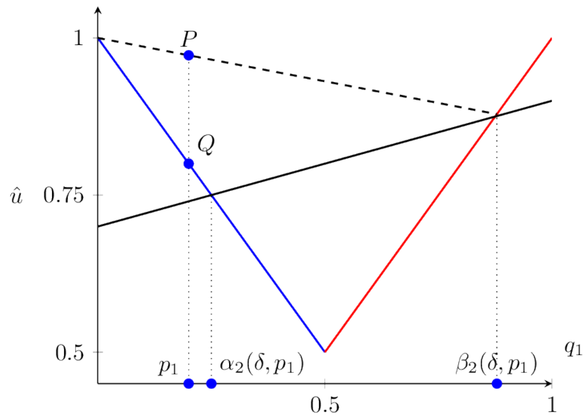


Figure 3: R_1's payoff

Considering our application of policymakers and firms, a delay only happens when policymakers can manipulate the information of naive successors. And they can benefit themselves by this trick at a cost to firms. This trick also threatens efficiency since it makes delay possible.

3.3. Comparison with benchmark

I compare the model to the benchmark where the receiver has no manipulation power and always discloses the information to see the effect of manipulation power.

In this benchmark, since there is no information manipulation, the game is just the repetition of the same Bayesian persuasions. In the last period, there is no choice of delay and thus players will have the same strategies as in a static persuasion. Recall our Lemma 1, since it is a pure persuasion with binary actions, R_2 gets zero value of persuasion from the sender's optimal experiment. So for R_1, who has the same belief as R_2 due to no manipulation, his payoff from delay is the same as acting immediately at any posterior. Then the discount factor excludes delay from R_1's equilibrium choice. The backward induction tells us that, even with more than 2 periods, no receiver will delay and the game will end at the very first period, generating the same result as a static persuasion.

Compared with the benchmark, manipulation power brings the incentive for R_1 to delay. With manipulation power and naive R_2, R_1 has the incentive to delay and hide the information from the following receiver to force the sender to provide a more informative experiment in the next period. Now for the greater good, the receiver may want to delay and manipulate the information to make the future receiver grow.

As discussed, there are two possible situations. In one situation, the sender chooses a less informative experiment in the first period than in the benchmark, and R_1 may delay in seek of more information. In the other situation, the sender chooses a more informative experiment than in the benchmark to induce an immediate action in period 1. In the first situation, R_1 gets some extra information in the following period at the cost of less information in period 1 and discount, which ends up with the same value of persuasion as the benchmark. When the manipulation power has made the receiver too hard to persuade, he can only ensure the same payoff as no manipulation power.

In the second situation, R_1 gets a more informative experiment than in the benchmark directly in period 1 and thus is better off than the benchmark. When the manipulation power increases the difficulty to persuade not too much, R_1 can benefit from it. But for the sender, in either case, he is weakly worse off, since in the benchmark he always gets his static optimal payoff in the first period, which is not happening in my model.

4. Extensions

4.1. Multiple periods

The model can be extended to any finite-period case and is still solvable by backward induction. Like the 2-period case, receivers still have the incentive to delay and may get a positive value of persuasion, though the conditions for different actions and experiments are much more complicated.

By induction, we can get the results for the case with more than 2 periods. The candidate optimal experiments can still be divided into *delaying experiments* and *immediate experiments*, depending on whether a delay is possible. But now the cutoff points for the receiver's strategy will be different.

Firstly, *reveal* is not always suboptimal. The next receiver may get an immediate experiment without hiding. When this happens, the next receiver gets a positive value of persuasion in the next period. And the current receiver, who shares the same expected payoff with the next receiver, can have a higher payoff than acting immediately by choosing *reveal*.

Furthermore, *yes* can be firstly better than *hide* and then worse than it as q_t goes up. This is because the optimal experiment in the next period will change from a delaying experiment to an immediate experiment at a point, which causes a jump in the receiver's payoff. I provide an example of the complicated strategy of the receiver in Appendix A2.

And how the discount factor δ affects the choice between the delaying experiment and the immediate experiment in multiple-period cases is also different from the 2-period case, where a higher δ makes the delaying experiment more likely to happen. Here in the multiple-period case, the effect of a higher δ is ambiguous. One effect is that a higher δ makes delay cost lower, which thus makes delay more attractive and makes the receiver harder to persuade. So this effect requires a more informative experiment to persuade the receiver and thus makes the delaying experiment more likely to be optimal. On the other hand, another effect is that a higher δ makes the delaying experiment more likely in the future. But from Section 3.2 we know that the receiver gets a higher payoff from the immediate experiment than from the delaying experiment, so this effect is reducing the payoff from delay and thus is making the receiver easier to persuade. In a 2-period case, we only have the former effect, so the overall effect of δ on the delaying experiment is clear. But in multiple-period cases, we have two effects of opposite directions and thus an ambiguous overall effect. An example illustrating this ambiguous effect can be found in Appendix A2.

4.2. Sophisticated receivers

In this section, I look at sophisticated receivers, who will do Bayesian inference seeing no information from a past period, given other players' strategies. In this case, to make things consistent with that the sender cannot verifiably reveal past information to R_2, I assume that if R_1 delays and hides at multiple posteriors, the sender chooses the same experiment for those posteriors in period 2 and maximizes the ex-ante expected payoff as if he is not informed of those posteriors. This assumption is consistent with the result in Koessler and Skreta (2021), which says that the ex-ante optimal experiment is also interim optimal in binary-action, one-receiver cases.

With sophisticated receivers, even in the case of multiple periods and multiple states, the current receiver cannot gain an informational advantage from following receivers, which removes the incentive for him to delay.

Proposition 3. *With sophisticated receivers, delay never happens in an equilibrium.*

Firstly in a 2-period case, given an experiment, the current receiver may see different signals. If he hides at only one signal in an equilibrium, then the following receiver will know for sure what the signal is seeing no information, which makes delay worse than an immediate action. If he hides at multiple signals in an equilibrium, the next receiver's belief will be the average of the posteriors induced by hidden signals. Among those posteriors, delay makes the current receiver better off than an immediate action at some posteriors but worse off at others. Since there always are some signals where the current receiver prefers to deviate to an immediate action, hiding at multiple signals cannot be an equilibrium choice either. In conclusion, hiding at only one signal has no effect, and hiding at multiple signals always includes points where an immediate action is better. Then the backward induction extends the result to multiple-state cases.

As we can see, only among naive receivers delay is possible. If the shirking problems we observe from legislators at their term limits are caused by information hiding, there must be a large cost in inferring missing messages, or the schedule is too tight for that, since informational advantage from hiding can only be gained from naive receivers.

4.3. Multiple states

Similar results as Proposition 1 and 2 hold in the model with more than two states, as long as the receivers' utility function satisfies the no-dominant-action requirement (no action is weakly preferred by receivers under all states). Let $a^*(q)$ be the action preferred by receivers among *yes* and *no* given belief q, we have the following proposition.

Proposition 4. *When* $\Omega = \{\omega_1, \omega_2, ..., \omega_M\}$ *and* $p_t \in \Delta(\Omega)$*, if receivers are naive:*

(1) If $a^*(p_1)$ *is preferred by the sender, the game ends in the first period in an equilibrium.*

(2) If $a^*(p_1)$ *is not preferred by the sender, whether there is delay in an equilibrium depends on the values of* δ *and* $p_1 = p_1^s$*.*

The result (1) is still straightforward, the sender can guarantee a sender-preferred action immediately by providing an uninformative experiment if the initial belief has already ensured the action preferred by the sender from R_1. If the sender has to provide an informative experiment in period 1 in pursuit of a higher payoff, he needs to trade off between more informative experiments that induce only immediate actions and less informative experiments that can induce delay, which is similar to the binary-state case. More complicated qualities of the project just lead to a more complicated representation of the cutoff points.

The result in Proposition 3 also holds unchanged with more than two states. The Bayesian inference done by the next receiver will remove the information advantage from manipulation.

Proposition 5. *When* $\Omega = \{\omega_1, \omega_2, ..., \omega_M\}$ *and* $p_t \in \Delta(\Omega)$*, if receivers are sophisticated, there is no delay in an equilibrium.*

5. Conclusion

I study a model of a sender seeking to persuade a series of manipulative receivers to take an action before a deadline. The optimal action for receivers depends on the state while the sender has a state-independent preference. The receiver can either choose the action by himself or leave the decision to his successor and take the chance to hide his information. If the receiver is naive in the sense that he will assume no experiment was conducted in the past period when the previous receiver hides the information, delay with hiding can be better than immediate actions for the previous receiver, even if he has the same preference as the next receiver. Thus, delay is possible in equilibrium. And with the naive receiver, the previous receiver may get a positive value of persuasion in the equilibrium, in contrast with the benchmark case without manipulation power where the receiver always gets zero value of persuasion from the sender's optimal experiment. Not like receivers, the sender is always weakly worse off from the receiver's ability to delay and hide the information. As for the sophisticated receiver who does Bayesian inference seeing no information from the past period given other players' strategies, the previous receiver is not able to gain an informational advantage by hiding and thus has no incentive to delay. So in this case, the equilibrium result is the same as the static persuasion.

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Appendix A. Mathematical Appendix

Appendix A.1. 2-period case

Appendix A.1.1. Proof for Proposition 1

In period 2, the game is a static Bayesian persuasion, so by the result in Kamenica and Gentzkow (2011), if $p_2 \geq \frac{b}{a+b}$, the sender chooses the uninformative experiment; if $0 < p_2 < \frac{b}{a+b}$, the optimal experiment is the posterior split of 0 and $\frac{b}{a+b}$ for R_2.

Back to period 1, R_1 will choose *yes* immediately with posterior $q_1 \geq \frac{b}{a+b}$, since the sender will choose uninformative experiment in period 2 if he delays, no matter whether he hides the information or not. With $q_1 < \frac{b}{a+b}$, R_1 will choose *no* immediately. This is because that if he delays and hides the information, the sender will choose an uninformative experiment in period 2. And if he delays and reveals the information, then his payoff from delay without discount is the same as acting immediately because of Lemma 1 and the fact that he has the same belief as R_2. As a result, the discount factor ensures that choosing *no* immediately is the only optimal action. Knowing R_1's strategy, we know that the sender is indifferent among posterior splits among $q_1 \in [\frac{b}{a+b}, 1]$ and will not choose a split involving $q_1 < \frac{b}{a+b}$. Finally, according to our tie-breaker, the sender chooses an uninformative experiment in period 1 and R_1 chooses *yes* immediately, which is the unique equilibrium here.

Appendix A.1.2. Proof for Lemma 2

If R_1 chooses to reveal the information, his belief will be the same as R_2 and thus he will have the same expected payoff as R_2 from delay. Easily, his expected payoff from *reveal* without discount is $a \cdot q_1$ if $q_1 \geq \frac{b}{a+b}$, and $(a \cdot \frac{b}{a+b}) \cdot \frac{q_1}{\frac{b}{a+b}} + b \cdot (1 - \frac{q_1}{\frac{b}{a+b}}) = b \cdot (1 - q_1)$ if $q_1 < \frac{b}{a+b}$. This payoff without discount is exactly the same as R_1's payoff from taking action immediately, so the discount factor δ makes *reveal* suboptimal.

Appendix A.1.3. Proof for Lemma 3

Firstly, with $q_1 = 0$ or 1, R_1 acts immediately.

We then calculate R_1's payoff from *hide* when the prior and the posterior are p_1 and $q_1 \in (0,1)$.

According to Alonso and Câmara (2016), if two players with different priors u_1 and u_2 ($\in (0,1)$) see the same experiment, their posteriors (u_1^p and u_2^p) have the following relationship:

$$u_1^p = \frac{u_2^p \frac{u_1}{u_2}}{u_2^p \frac{u_1}{u_2} + (1 - u_2^p)\frac{1-u_1}{1-u_2}}$$

Given hiding, when facing the experiment in period 2, R_1 has belief q_1 but R_2 has belief p_1. Notice that the sender's optimal experiment in period 2 is the split of 0 and $q_2 = \frac{b}{a+b}$ for R_2, so in R_1's eyes, the split is 0 and $q_2' = \frac{q_2 \frac{q_1}{p_1}}{q_2 \frac{q_1}{p_1} + (1-q_2)\frac{1-q_1}{1-p_1}}$. Moreover, the split in R_1's eyes is still Bayesian plausible, so the probability of q_2' is $\frac{q_2 \frac{q_1}{p_1} + (1-q_2)\frac{1-q_1}{1-p_1}}{\frac{q_2}{p_1}}$. Since R_2 chooses *no* at 0 and *yes* at q_2, R_1 gets $\frac{a \cdot q_2 \frac{q_1}{p_1}}{q_2 \frac{q_1}{p_1} + (1-q_2)\frac{1-q_1}{1-p_1}}$ by probability $\frac{q_2 \frac{q_1}{p_1} + (1-q_2)\frac{1-q_1}{1-p_1}}{\frac{q_2}{p_1}}$, and gets b by probability $1 - \frac{q_2 \frac{q_1}{p_1} + (1-q_2)\frac{1-q_1}{1-p_1}}{\frac{q_2}{p_1}}$. So, R_1's expected payoff from delay with hiding is $b + \frac{(a-b)\cdot q_2 \frac{q_1}{p_1} - b \cdot (1-q_2)\frac{1-q_1}{1-p_1}}{\frac{q_2}{p_1}} = b + (a-b)q_1 - a\frac{p_1}{1-p_1}(1-q_1)$:

$$\mathbf{E}u(hide) = \delta \cdot (b + (a-b)q_1 - a\frac{p_1}{1-p_1}(1-q_1))$$

As for acting immediately, R_1's expected payoff is:

$$\mathbf{E}u(act) = \begin{cases} b \cdot (1-q_1) & , q_1 < \dfrac{b}{a+b} \\ a \cdot q_1 & , q_1 \geq \dfrac{b}{a+b} \end{cases}$$

Comparing them, we can get that when $\delta > \frac{ab}{2ab - \frac{p_1}{1-p_1}a^2}$, there are $\alpha_2(\delta, p_1) = \frac{b(1-\delta) - p_1(b(1-\delta) - \delta a)}{\delta a + b(1-\delta) - b(1-\delta)p_1}$ and $\beta_2(\delta, p_1) = \frac{b\delta - \delta(a+b)p_1}{a(1-\delta) + b\delta - (a+b\delta)p_1}$, where $p_1 < \alpha_2(\delta, p_1) < \beta_2(\delta, p_1) < 1$, such that R_1 chooses *yes* with $q_1 \geq \beta_2(\delta, p_1)$; chooses *no* with $q_1 < \alpha_2(\delta, p_1)$; chooses *hide* in between.

When $\delta \leq \frac{ab}{2ab - \frac{p_1}{1-p_1}a^2}$, R_1 chooses *yes* with $q_1 \geq \frac{b}{a+b}$ and *no* with $q_1 < \frac{b}{a+b}$.

Appendix A.1.4. Proof for Proposition 2

Knowing R_1's strategy from Lemma 3, we can figure out the sender's optimal experiment in period 1.

When $\delta \leq \frac{ab}{2ab - \frac{p_1}{1-p_1}a^2}$, R_1 chooses *yes* with $q_1 \geq \frac{b}{a+b}$ and *no* with $q_1 < \frac{b}{a+b}$. Since $p_1 < \frac{b}{a+b}$, the optimal experiment is the split between 0 and $\frac{b}{a+b}$ in R_1's eyes and this experiment induces only immediate actions.

When $\delta > \frac{ab}{2ab - \frac{p_1}{1-p_1}a^2}$, we have two cutoff points in R_1's strategy $\alpha_2(\delta, p_1)$, $\beta(\delta, p_1)$. Since we have $p_1 < \alpha_2(\delta, p_1) < \beta_2(\delta, p_1)$, the optimal experiment is either the split between 0 and $\alpha_2(\delta, p_1)$ or between 0 and $\beta_2(\delta, p_1)$, where the former one induces *hide* at $\alpha_2(\delta, p_1)$ and the latter one only induces immediate actions. So, they are called the *delaying experiment* and the *immediate experiment* respectively. For the sender, his expected payoff from the immediate experiment is $\frac{p_1}{\beta_2(\delta,p_1)}$. As for the delaying experiment, at 0 he gets 0, at $\alpha_2(\delta, p_1)$ the game proceeds into the next period where he chooses an experiment making R_2 with prior p_1 believe that the split is 0 and $\frac{b}{a+b}$, so his expected payoff from the delaying experiment is $\delta \cdot (p_1 + \frac{a}{b}\frac{p_1}{1-p_1}\frac{1-\alpha_2(\delta,p_1)}{\alpha_2(\delta,p_1)}p_1)$.

Comparing the payoffs from two experiments, the delaying experiment is the optimal one if and only if:[9]

$$\frac{a}{b} \cdot \frac{\delta^2 a p_1}{b(1-\delta) - p_1(b(1-\delta) - \delta a)} - \frac{a(1-\delta)(1-p_1)}{b\delta - \delta \cdot p_1(a+b)} \geq 1 - \delta$$
$$\Leftrightarrow r_1 p_1^2 + r_2 p_1 + r_3 \geq 0 \tag{A.1}$$

Where:

$$r_1 = -\frac{\delta^3 a^2 (a+b)}{b} - a(1-\delta)(b(1-\delta) - \delta a)$$
$$-\delta(1-\delta)(b(1-\delta) - \delta a)(a+b)$$
$$r_2 = \delta^3 a^2 + a(1-\delta)(b(1-\delta) - \delta a) + ab(1-\delta)^2$$
$$+b\delta(1-\delta)^2(a+b) + (1-\delta)\delta b(b(1-\delta) - \delta a)$$
$$r_3 = -ab(1-\delta)^2 - b^2(1-\delta)^2\delta$$

[9] When indifferent between two experiments, the tie-breaker makes the sender choose the delaying experiment, which is less informative.

Notice that for $p_1 = 0$ and $p_1 = \frac{b}{a+b}$, LHS of (1) is always negative for $\delta \in (0,1)$. Moreover, when $\delta = 1$, we have $r_1 < 0$ and that the solution to (1) is $p_1 \in [0, \frac{b}{a+b}]$. So, when δ is close enough to 1, the solution to (1) would be $p_1 \in [\alpha_1(\delta), \beta_1(\delta)]$, where $0 < \alpha_1(\delta) \leq \beta_1(\delta) < \frac{b}{a+b}$.

Appendix A.2. Multiple-period case

The multiple-period case can be solved by the same method as in the 2-period case, with further backward induction.

Firstly consider the case where $p_1 \geq \frac{b}{a+b}$. According to the proof for Proposition 1, backward induction gives us that the game ends at the first period.

As for the case with $p_1 < \frac{b}{a+b}$, period N and period $N-1$ act in the same way as in the 2-period case.

The situation in period $t < N-1$ is more complicated. Firstly, if $p_t \geq \frac{b}{a+b}$, as proved in Proposition 1, the game ends at period t with an uninformative experiment. When $p_t < \frac{b}{a+b}$, we need to compare the payoffs from *yes*, *no*, *hide* and *reveal*.

yes and *no* always give us $a \cdot q_t$ and $b \cdot (1 - q_1)$ respectively.

Similar to the 2-period case, if *reveal* is chosen, Receiver t and Receiver $t+1$ share the same belief and the same expected payoff from an experiment. If *hide* is chosen, Receiver t and $t+1$ can have a different expected payoff due to different belief.

The calculation method is the same as in a 2-period case, but now the payoff in the next period involves immediate experiments and delaying experiments. According to Section 2.2, there is a jump in receivers' payoff when shifting from a delaying experiment to an immediate experiment, and immediate experiments provide positive value of persuasion. This will make the conditions of different optimal actions different from the 2-period case and make them more complicated. Now *reveal* may be better than hiding due to different types of optimal experiments in the next period. And the jump in the receiver's payoff can lead to back and forth between immediate actions and delay.

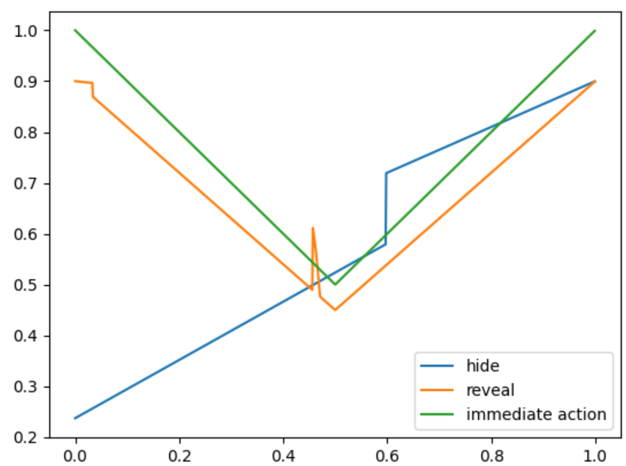
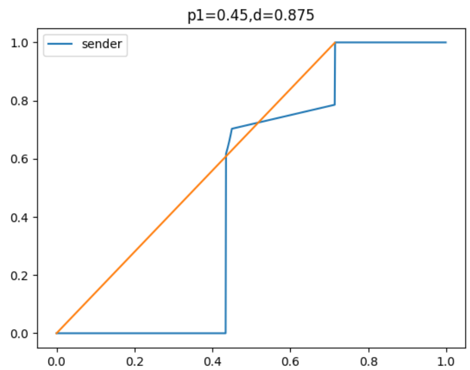


Figure A.4: R_1 in a 3-period example

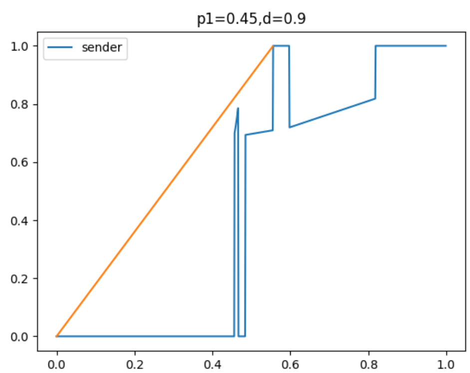
As an example, look at Figure A.4, which summarizes R_1's payoffs from different actions in a 3-period case with $a = b = 1$, $p_1 = p_1^s = 0.45$ and $\delta = 0.9$. In this example, R_1's optimal action is $no \to reveal \to no \to hide \to yes \to hide \to yes$, as q_1 goes up.

Such complicated choices of receivers' strategies naturally lead to different criteria in choosing the optimal experiment. And the effect of the discount factor on choosing the optimal experiment is ambiguous as discussed in the main text.

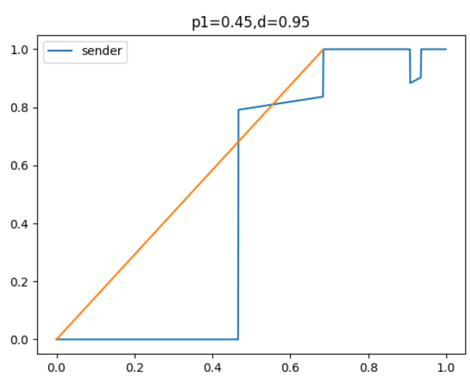
For example, in a 3-period case with $a = b = 1$ and $p_1 = p_1^s = 0.45$, $\delta = 0.875$ and 0.95 make the optimal experiment a delaying experiment, but $\delta = 0.9$, which is in between, makes the optimal experiment an immediate experiment. See Figure A.5 describing the sender's payoff in these three cases. The orange lines in (a) and (c) suggest that the immediate experiment is not the optimal one. And the orange line in (b) suggests that the sender will choose the immediate experiment.



(a) $\delta = 0.8$



(b) $\delta = 0.82$



(c) $\delta = 0.9$

Figure A.5: Ambiguous effect of δ

Appendix A.3. Sophisticated receivers

Appendix A.3.1. Proof for Proposition 3

I will directly prove this proposition under the case with M states and N periods, i.e. $\Omega = \{\omega_1, \omega_2, ..., \omega_M\}$.

By backward induction, we start from period N, which is simply a static persuasion.

As for period $N-1$, I will show that under any experiment, Receiver $N-1$ will not delay in an equilibrium.

Suppose under the equilibrium experiment in period $N-1$, Receiver $N-1$'s possible posteriors are $\mu_1, \mu_2, ..., \mu_G$ with probabilities $P_1, P_2, ..., P_G$.

If Receiver $N-1$ delays without hiding or delays and hides at only one outcome, Receiver N knows exactly what the signal is and has the same belief as Receiver $N-1$'s posterior. Now Receiver $N-1$ has the same expected payoff as Receiver N, but according to Lemma 1, Receiver N has no value of persuasion from the equilibrium and thus has the same expected payoff as no information. As a result, Receiver $N-1$'s payoff from delay is the same as acting immediately, and then the discount factor makes the deviation to an immediate action profitable. So these situations cannot happen in an equilibrium.

If Receiver $N-1$ delays and hides at multiple outcomes, say $\mu_1, \mu_2, ..., \mu_I$ ($I \leq G$). Now Receiver N does not know the signal but the sender knows the signal, so the problem becomes a Bayesian persuasion with an informed sender. We have assumed that the sender chooses the optimal experiment as if he is not informed. Since now the sender pools the same experiment for all signals hidden, Receiver N cannot know what the signal is from the experiment, and thus his belief seeing no signal is the weighted average $\bar{\mu} = \frac{\sum_{i=1}^{I} P_i \mu_i}{\sum_{i=1}^{I} P_i}$[10].

If $\bar{\mu}$ can directly induce *yes*, which is preferred by the sender, the sender chooses an uninformative experiment in period N, so clearly deviating to an immediate action can benefit Receiver $N-1$.

[10]Without loss, I assume $\bar{\mu}$ has full support.

If $\bar{\mu}$ induces *no*, the sender chooses an experiment inducing belief q_1' and q_2' from prior $\bar{\mu}$ with probabilities P_1' and P_2', since there are only two available actions. I also denote the actions induced by q_1' and q_2' as a_1' and a_2' respectively. According to Alonso and Câmara (2016), Receiver $N-1$ who has a prior μ_i ($i \leq I$) in period N will regard posterior q_j' as q_{ij}', where $q_{ij}'(\omega_0) = \frac{q_j'(\omega_0)\frac{\mu_i(\omega_0)}{\bar{\mu}(\omega_0)}}{\sum_{\omega\in\Omega} q_j'(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)}}$, and regards P_j' as $P_{ij}' = P_j' \sum_{\omega\in\Omega} q_j'(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)}$. So the payoff from delay for Receiver $N-1$ with posterior μ_i is $\delta u^i = \delta \sum_{j=1}^{2} P_{ij}' \sum_{\omega\in\Omega} u(a_j', \omega) q_{ij}'(\omega)$.

Now we have:

$$
\begin{aligned}
\frac{\sum_{i=1}^{I} P_i u^i}{\sum_{i=1}^{I} P_i} &= \frac{1}{\sum_{i=1}^{I} P_i} \sum_{j=1}^{2}\sum_{i=1}^{I} P_i P_{ij}' \sum_{\omega\in\Omega} u(a_j', \omega) q_{ij}'(\omega) \\
&= \frac{1}{\sum_{i=1}^{I} P_i} \sum_{j=1}^{2}\sum_{i=1}^{I} P_i (P_j' \sum_{\omega\in\Omega} q_j'(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)}) \sum_{\omega\in\Omega} u(a_j', \omega) q_{ij}'(\omega) \\
&= \sum_{j=1}^{2} P_j' \sum_{i=1}^{I} (\sum_{\omega\in\Omega} q_j'(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)} \frac{P_i}{\sum_{k=1}^{I} P_k}) \sum_{\omega\in\Omega} u(a_j', \omega) q_{ij}'(\omega) \\
&= \sum_{j=1}^{2} P_j' \sum_{\omega\in\Omega} u(a_j', \omega) \sum_{i=1}^{I} (\sum_{\omega\in\Omega} q_j'(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)} \frac{P_i}{\sum_{k=1}^{I} P_k}) q_{ij}'(\omega) \\
&= \sum_{j=1}^{2} P_j' \sum_{\omega\in\Omega} u(a_j', \omega) \sum_{i=1}^{I} (\sum_{\omega\in\Omega} q_j'(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)} \frac{P_i}{\sum_{k=1}^{I} P_k}) \frac{q_j'(\omega)\frac{\mu_i(\omega)}{\bar{\mu}(\omega)}}{\sum_{\omega'\in\Omega} q_j'(\omega')\frac{\mu_i(\omega')}{\bar{\mu}(\omega')}} \\
&= \sum_{j=1}^{2} P_j' \sum_{\omega\in\Omega} u(a_j', \omega) q_j'(\omega)
\end{aligned}
$$

Notice that $\sum_{j=1}^{2} P_j' \sum_{\omega\in\Omega} u(a_j', \omega) q_j'(\omega)$ is the expected payoff of Receiver N from the optimal experiment of the sender, so we have:

$$
\frac{\sum_{i=1}^{I} P_i u^i}{\sum_{i=1}^{I} P_i} = \sum_{j=1}^{2} P_j' \sum_{\omega\in\Omega} u(a_j', \omega) q_j'(\omega) := \hat{u}(\bar{\mu})
$$

Since period N is a static pure persuasion with binary actions, we have $\hat{u}(\bar{\mu}) = \max_a \sum_{\omega\in\Omega} u(a, \omega)\bar{\mu}(\omega)$, which means that $\hat{u}$ is convex. As a result, we have:

$$\frac{\sum_{i=1}^{I} P_i u^i}{\sum_{i=1}^{I} P_i} = \hat{u}(\bar{\mu}) \leq \sum_{i=1}^{I} \frac{P_i \hat{u}(\mu_i)}{\sum_{k=1}^{I} P_k} = \sum_{i=1}^{I} \frac{P_i \max_a \sum_{\omega \in \Omega} u(a,\omega)\mu_i(\omega)}{\sum_{k=1}^{I} P_k} \tag{A.2}$$

From (2), we can conclude that:

$$\exists\ i \leq I, u^i \leq \max_a \sum_{\omega \in \Omega} u(a,\omega)\mu_i(\omega) \Rightarrow \delta u^i < \max_a \sum_{\omega \in \Omega} u(a,\omega)\mu_i(\omega)$$

So, there must exist one hidden posterior where Receiver $N-1$ can benefit from deviating to an immediate action. And thus hiding at multiple posteriors cannot happen in an equilibrium either.

Then we can conclude that in an equilibrium, there is no delay in period $N-1$.

And we can construct an equilibrium in period $N-1$ where the sender chooses the static optimal experiment and Receiver $N-1$ always takes action immediately. For an static optimal experiment, there must be a posterior inducing the action preferred by the sender, the off-path belief of Receiver N seeing delay with hiding is assigned to this posterior[11]. If Receiver $N-1$ deviates to delay and hide the signal, since the off-path belief of Receiver N is assigned to the posterior inducing the action preferred by the sender, in period N the sender chooses an uninformative experiment and thus this deviation will not be profitable. If Receiver $N-1$ deviates to delay without hiding, by the fact that there is no value of persuasion in the next period, this deviation is not profitable either. As for the sender, if he deviates to another experiment, at any posterior of that experiment, Receiver $N-1$'s delay with hiding will be worse than an immediate action since the off-path belief makes the sender choose an uninformative experiment in the next period, and his delay without hiding is worse than an immediate action as well due to no value of persuasion in the next period. As a result, Receiver $N-1$ will choose an immediate action seeing

[11]Since delay without hiding verifiably reveals the information to Receiver N, that off-path belief of Receiver N is already decided.

any outcome of this deviation experiment, and thus this deviation will not give the sender a higher payoff than static optimal. So both the sender and Receiver $N-1$ do not deviate, it is an equilibrium. Also notice that since there will be no delay in an equilibrium, only the static optimal experiment can be the equilibrium choice of the sender in period $N-1$.

So we now have that the strategies in period $N-1$ will be the same as in period N, then further backward induction tells us that there is no delay in an equilibrium.

Appendix A.4. Multiple states

Appendix A.4.1. Proof for Proposition 4

(1) Firstly let us consider the case where $a^*(p_1)$ is preferred by the sender.

In period 2, players' strategies are just the same as in a static persuasion.

In period 1, when R_1 delays and hides the information, R_2 will have a prior $p_2 = p_1$. Notice that $a^*(p_1)$ is preferred by the sender, so the sender will choose an uninformative experiment in period 2, which makes delay with hiding worse than taking action immediately. When R_1 delays without hiding the information, he has the same belief as R_2 and thus the same expected payoff as R_2. According to Lemma 1, R_2's expected payoff from persuasion is the same as the expected payoff without any information, so R_1's payoff from delay without hiding is also worse than taking action immediately.

So we can see that R_1 will choose an immediate action, and thus the sender is indifferent between splits among posteriors that can induce *yes*. Then our tie-breaker will make the sender choose an uninformative experiment.

(2) Then suppose $a^*(p_1)$ is not preferred by the sender.

Again, in period 2, players' strategies are the same as in a static persuasion.

In period 1, without loss, I assume p_1 has full support. If R_1 has a posterior q_1 and chooses *yes* or *no*, his expected payoffs are $u^y(q_1) = \sum_{\omega\in\Omega} u(yes,\omega)q_1(\omega)$ and $u^n(q_1) = \sum_{\omega\in\Omega} u(no,\omega)q_1(\omega)$ respectively.

If R_1 chooses *reveal*, he has the same belief and the same expected payoff as R_2, and thus his payoff will be $\delta \max_{a\in\{yes,no\}} \sum_{\omega\in\Omega} u(a,\omega)q_1(\omega)$ according to Lemma 1. Clearly, delay without hiding will not be an equilibrium choice of R_1, since there is always an immediate action better than it.

If R_1 chooses to delay with hiding, R_2 has a prior $p_2 = p_1$. Since $a^*(p_1)$ is not preferred by the sender, so the sender will choose an informative experiment. Since there are only 2 actions in period 2, the optimal experiment can be the split of q^n and q^y with probabilities P^n and P^y respectively in the eyes of R_2, where $a^*(q^n) = no$ and $a^*(q^y) = yes$.[12] Moreover, by Bayesian plausibility, we have $P^n q^n + P^y q^y = p_1$. In R_1's eyes, the split is $q_1^n(\omega_0) = \frac{q^n(\omega_0)\frac{q_1(\omega_0)}{p_1(\omega_0)}}{\sum_{\omega\in\Omega} q^n(\omega)\frac{q_1(\omega)}{p_1(\omega)}}$ and $q_1^y(\omega_0) = \frac{q^y(\omega_0)\frac{q_1(\omega_0)}{p_1(\omega_0)}}{\sum_{\omega\in\Omega} q^y(\omega)\frac{q_1(\omega)}{p_1(\omega)}}$ with probabilities $P_1^n = P^n \sum_{\omega\in\Omega} q^n(\omega)\frac{q_1(\omega)}{p_1(\omega)}$ and $P_1^y = P^y \sum_{\omega\in\Omega} q^y(\omega)\frac{q_1(\omega)}{p_1(\omega)}$. Then R_1's payoff from delay with hiding is:

$$
\begin{aligned}
\delta u^h(q_1) =& \delta(P_1^n \sum_{\omega\in\Omega} u(no,\omega) q_1^n(\omega) + P_1^y \sum_{\omega\in\Omega} u(yes,\omega) q_1^y(\omega)) \\
=& \delta(P^n \sum_{\omega\in\Omega} u(no,\omega) q^n(\omega)\frac{q_1(\omega)}{p_1(\omega)} + P^y \sum_{\omega\in\Omega} u(yes,\omega) q^y(\omega)\frac{q_1(\omega)}{p_1(\omega)})
\end{aligned}
$$

Which action R_1 will choose at q_1 depends on $u^n(q_1)$, $u^y(q_1)$ and $\delta u^h(q_1)$, and the tie-breaker makes R_1 choose only one action.

Notice that at $q_1 = p_1$, $u^h(q_1) = u^n(q_1) > u^y(q_1)$.

If q^y has put some weight on states where *yes* is strictly preferred. Then we can pick a q_1 which only puts positive weight on those states, and thus have $u^n(q_1) < u^h(q_1) \leq u^y(q_1)$. So, when δ is large, there exist cutoff points where *yes* and *hide* are indifferent and cutoff points where *no* and *hide* are indifferent. And of course, there are cutoff points where *yes* and *no* are indifferent. It is easy to see that the optimal experiment by the sender will include at least one of these three kinds of cutoff points[13], so the optimal experiment can either induce delay or not[14], depending on parameters (δ, p_1 and receivers' preference u).

---

[12]Since it is an optimal experiment, *yes* and *no* are indifferent at belief q^y, otherwise we can shift two posteriors a little to make P^y larger while q^y still inducing *yes*.

[13]Otherwise the sender can change a posterior a little without changing the induced action to improve his payoff.

[14]If the optimal experiment includes a cutoff point where *no* and *hide* are indifferent (and

If q^y only puts positive weight on the states where *yes* and *no* are indifferent, for any q_1, we will have $u^h(q_1) = u^n(q_1)$. Then $\delta < 1$ makes *hide* suboptimal.

Appendix A.4.2. Proof for Proposition 5

See the proof for Proposition 3.

they are better than *yes*), then delay is possible. Otherwise, there is no delay in equilibrium.