Bureaucratic Norms and Dynamic Bayesian Persuasion

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Abstract

A developer seeks to persuade a welfare-maximizing bureaucracy, with rotating officials, to award a larger fraction of a contract to her. Officials' decisions are subject to a bureaucratic norm, whereby a decision can be only based on evidence that is either recorded by her predecessor or is directly presented to her. Thus, Bayesian inference is *restricted* when a predecessor fails to record evidence, and bureaucrats can exploit this to induce the developer to conduct more informative experiments. In a class of information design problems where the static values of persuasion are zero to the bureaucracy and strictly positive for the developer, I show that there are two possibilities in the dynamic game. Either the developer conducts a more informative experiment and the official decides immediately, giving the bureaucracy a positive value; or there is delay and the gradual release of information. In either case, the developer is worse off compared to the static persuasion.

Keywords: Dynamic Bayesian persuasion; Restricted Bayesian inference; Information

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1 Introduction

The US Supreme Court decided, in *Miranda vs.Arizona*, that the Fifth Amendment guard against self-incrimination implies that the jury cannot draw a negative inference from the suspect's refusal to answer police questions. The jury must decide based on other evidence alone, i.e., it must behave as though no interrogation had taken place. Seidmann (2005) analyzes the effects of *Miranda*, using a model of *restricted Bayesian inference*: at the information set where the suspect has exercised his Miranda rights, the jury's decision is based only on the prior and upon witness statements. At other information sets, the inference is unrestricted. [1]

In this paper, I study a model of restricted Bayesian inference in the context of a bureaucracy with revolving bureaucrats. The bureaucracy has a norm whereby a new bureaucrat can base his decision only on the basis of evidence that is recorded by his predecessor. In other words, in the absence of any such record, he must infer nothing from the absence of the evidence. Consequently, his predecessor may strategically fail to record information, and this may serve the objectives of the organization.

As a leading example, I present a model with two short-tenure bureaucrats (he) who are being lobbied by an interest group, such as a local developer (she). Both bureaucrats have identical objectives and seek to maximize (discounted) social welfare. They must decide whether to award the contract to the developer or an outsider, and prior beliefs favor the outsider.[2] The developer seeks to persuade the first bureaucrat by conducting a Bayesian

[1] His main finding is that *Miranda* right can be socially beneficial: it reduces the wrongful conviction of innocent suspects. It also decreases the conviction rate and keeps the confession rate unchanged. See also Seidmann and Stein (2000).

[2] An alternative application: the bureaucrats must decide the location of a local public good, such as a park, and the lobby group favors location A over location B.

experiment. After observing the experiment, bureaucrat 1 must decide whether to award the contract to the outsider, to the local developer, or to postpone the decision so that it is made by bureaucrat 2. If he defers the decision, he may choose not to record the results of the experiment that he has observed, in which case bureaucrat 2 must decide based only on the prior and upon the experiment that he personally observes.

My main finding is that in equilibrium, the bureaucratic norm that restricts Bayesian inference may increase social welfare, the objective of the organization. When the value of persuasion to the bureaucrat (the bureaucracy) is zero in the static game, the norm may force the developer to conduct more informative experiments; if the developer conducts an experiment where the first bureaucrat only marginally prefers to award the contract to the developer after some signal realization, then the bureaucrat will fail to record this and defer the decision to his successor, and the developer will be forced to provide more information to convince him. To avoid this, the developer may provide more information to bureaucrat 1, so he is, after a positive signal, enthusiastic enough about the developer that she avoids costly delay. The reader may ask, why does the developer not defer all information provision to the second period? In this case, bureaucrat 1 correctly anticipates that the social value of second-period information will be zero, and thus he chooses the outside contractor since delay is costly.

The main results can be extended to a more general case with more than two actions. I assume that bureaucrats divide the project into several parts and decide how many parts to assign to the developer. The developer wants a larger share of the project. Bureaucrats want to assign the project to the developer in one state and to the outsider in the other, and they have a quadratic loss payoff depending on the shares assigned to the developer and the state.

I also assume that the developer's payoff is concave in the shares assigned to her to make the bureaucrat have zero value of persuasion in the static game. In this multiple-action case, we have a more complicated situation. Unlike the binary-action case, when the first bureaucrat delays and does not record the experiment, the optimal experiment in period 2 will change with the posterior in period 1. However, we still have a similar result as the leading example: when players are patient enough, in equilibrium, two scenarios arise. The first bureaucrat has a positive value of persuasion and chooses only immediate actions. Alternatively, he has a zero value of persuasion and delays when receiving a positive signal.

1.1 Related literature

Kamenica and Gentzkow (2011) initiated the study of Bayesian persuasion. Dynamics models of persuasion include Ely (2017), Honryo (2018), Orlov, Skrzypacz and Zryumov (2020), Smolin (2020), Bizzotto, Rüdiger and Vigier (2021).

Alonso and Câmara (2016) studied heterogeneous beliefs in Bayesian persuasion. Non-Bayesian updating is studied by Levy, de Barreda and Razin (2018), de Clippel and Zhang (2022), and Galperti (2019).

An alternative interpretation of the model is that bureaucrat 2 is naive in the sense of regarding the absence of communication as the absence of information. This interpretation is related to the self-deception problem studied by Bénabou and Tirole (2002). However, in their model, the incentive to manipulate the information comes from time-inconsistent preferences, while I assume identical preferences for two bureaucrats.

The organization of this paper is as follows. Section 2 discusses the implications of the bureaucratic norm via a leading example with binary actions. Section 3 analyzes the general case with multiple actions. Section 4 concludes.

2 Leading example

2.1 Setup

As a leading example, I present a two-period dynamic information design model with binary actions. There are three players: one long-lived local developer and 2 short-lived bureaucrats (B_1 and B_2). The local developer seeks to persuade the bureaucrat to contract with him instead of an outsider via Bayesian experiments.

Bureaucrats have the decision set $\mathcal{A} = \{1, 0\}$: $a = 1$ stands for contracting with the developer, and $a = 0$ stands for contracting with the outsider.

Let $\omega \in \{1, 0\} := \Omega$ denote the unobserved payoff-relevant state. Here $\omega = 1$ is the state where choosing the developer is more socially beneficial; $\omega = 0$ is the state where choosing the outsider is more socially beneficial. I need two requirements on the bureaucrats' preference, (1) they prefer $a = 1$ at state $\omega = 1$ and $a = 0$ at state $\omega = 0$, and (2) their payoffs are always non-negative, so that delay is costly. I assume quadratic-loss social welfare, which is also the payoffs of bureaucrats:

$$u(a, \omega) = C - (a - \omega)^2,$$

where C is a positive constant that keeps the payoff positive. The quadratic-loss social welfare satisfies the two requirements

Bureaucrats have the same objective, maximizing this discounted social welfare, while the developer has a state-independent payoff: $v : \mathcal{A} \to \mathbb{R}$. The developer prefers $a = 1$, and I normalize the payoff from $a = 0$ to 0 and $a = 1$ to 1.

At the beginning of the game, players share the same prior $p_0 = Pr(\omega = 1)$, but in period 2, the beliefs may be different between the developer and B_2, denoted as p_2 and p_2^d respectively. Moreover, to focus on the non-trivial case, I assume that the prior is in favor of the outsider ($p_0 \leqslant \frac{1}{2} - \epsilon$), where ϵ is a small positive number.[3] The belief $Pr(\omega = 1) = \frac{1}{2}$ is the point where the bureaucrat is indifferent between $a = 1$ and 0.

In each period, the developer chooses an experiment: $\pi_t = (\pi_t(.|\omega))_{\omega\in\Omega} \in \times_{\omega\in\Omega}\Delta(S) := \Pi$, where S is an unrestricted signal space. I assume that outcomes of experiments in different periods are independent conditional on the state (i.e. $Pr(s_1, s_2|\omega) = \pi_1(s_1|\omega) \cdot \pi_2(s_2|\omega)$). This is an important assumption – if I were to allow experiments to be correlated across periods, the developer could credibly disclose the outcome of the past experiment.

The assumption that the experiments presented to the two bureaucrats are independent conditional on the state is an important one and needs justification. One justification is as follows. Suppose that the developer has a facility that needs to be inspected by the bureaucrat in order to ascertain his suitability for fulfilling the contract. The developer may specify the length of the inspection, thereby determining the informativeness of the experiment, but the bureaucrat must choose a sample of aspects of the facility to inspect. It is plausible that the two bureaucrats independently select their samples, giving rise to experiments that are independent conditional on the state.

[3]If the initial belief p is in favor of $a = 1$, the result is trivial: the developer provides an uninformative experiment.

In period 1, observing the outcome of the experiment, beliefs are updated from p_0 to q_1. Then B_1 chooses an action from $\{1, 0\} \cup \{hide, record\}$. If $a = 1$ or 0 is chosen, the game ends and payoffs are realized.

On the other hand, if *hide* or *record* is chosen, the game proceeds into the next period, and the current bureaucrat's payoff is decided by the action chosen by the future bureaucrat. By recording, B_1 records and discloses the experiment and the outcome in period 1 to B_2, so B_2 has a prior $p_2 = q_1$. By hiding, no outcome or experiment is recorded, and B_2 observes nothing. I assume that B_2 is constrained by the bureaucratic norm that decisions can only depend on recorded evidence, so B_2 has the belief $p_2 = p_0$ observing no recorded evidence of the last experiment. The developer's prior in period 2 is $p_2^d = q_1$. Furthermore, in period 2, which is the deadline, the bureaucrat must choose from $\mathcal{A} = \{1, 0\}$.

The equilibrium concept in this paper is perfect Bayesian equilibrium: players maximize their expected payoffs given other players' strategies and the beliefs generated by the Bayes rule if possible. Notice that this PBE is with a restriction as in Seidmann (2005): if B_1 does not record the first experiment, B_2 can only base his decision on the prior and the experiment presented to him. Furthermore, as a tie-breaker, I assume that a bureaucrat will choose the action preferred by the developer if he is indifferent between two actions.

2.2 Equilibrium results

Before I introduce the results, I define an important concept, *the value of persuasion* (see Kamenica and Gentzkow (2011)).

Definition 1. *The value of persuasion to a player is his expected payoff in equilibrium minus* *his expected payoff without any experiment.*

Denote B_1's value of persuasion in the dynamic game as V_B and the developer's value of persuasion in the dynamic game as V_D.[4] And denote players' values of persuasion in the static game as $\bar{V}_B$ and $\bar{V}_D$. Based on this definition, I introduce the following useful lemma.

Lemma 1. *In static Bayesian persuasion with binary actions and a state-independent developer, $\bar{V}_B = 0$ and $\bar{V}_D > 0$ when the prior is in favor of the outsider.*

Lemma 1 can be obtained from the results in Kamenica and Gentzkow (2011).

Proposition 1. *There exists $\bar{\delta}$ such that for any $\delta > \bar{\delta}$, $\exists\ \alpha(\delta)$ and $\beta(\delta) \in (\alpha(\delta), \frac{1}{2})$ s.t. in the unique equilibrium:*

(1) if $p_0 \notin [\alpha(\delta), \beta(\delta)]$, $V_B > 0$ and the first bureaucrat always makes the decision;

(2) if $p_0 \in [\alpha(\delta), \beta(\delta)]$, $V_B = 0$ and the first bureaucrat delays at the positive signal and chooses $a = 0$ at the negative signal;

(3) V_D is strictly less than $\bar{V}_D$.

If the developer conducts an experiment where the first bureaucrat only marginally prefers to award the contract to the developer after some signal realization, B_1 will defer the decision to B_2. To avoid this costly delay, the developer may conduct a more informative experiment so that B_1 is enthusiastic enough about the developer after a positive signal and takes action immediately. B_1 strictly prefers $a = 1$ to $a = 0$ at the positive signal. In this case, the more informative experiment induces only immediate actions and gives B_1 a positive value of persuasion. As a result, the bureaucratic norm is socially beneficial here.

[4] Since in period 2, the game is the same as a static persuasion, I focus on B_1's value of persuasion in the dynamic game.

However, with other parameters, delay happens in equilibrium. The experiment in this case generates a positive signal and a negative signal, and B_1 delays at the positive signal. B_1 does not have a positive value of persuasion here.

This happens when the developer finds that providing a more informative experiment is so costly that it is better to provide a less informative experiment and let B_1 delay. With this less informative experiment, B_1 defers the decision to B_2 at the positive signal, and the developer provides a second experiment to B_2 in period 2. At the negative signal, B_1 immediately chooses $a = 0$. Though bureaucrats act on the basis of more information in this equilibrium, delay cost offsets the benefit. Thus, B_1 has $V_B = 0$ in this case.

We can solve for these equilibrium results by first looking at period 2. In period 2, the situation is the same as a static Bayesian persuasion game with possibly heterogeneous priors. When priors are heterogeneous, the equilibrium can still be solved by the concave closure according to Alonso and Câmara (2016).

As for period 1, firstly we observe that *record* will not be chosen by B_1.

Lemma 2. *B_1 will not choose record in equilibrium.*

When B_1 delays and records the information, he shares the same expected payoff as B_2. However, Lemma 1 says that B_2 has zero value of persuasion in period 2, so the extra experiment in period 2 does not improve B_1's payoff. Moreover, there is a discount factor δ, which makes an immediate action strictly better than *record* for B_1.

As a result, to solve for B_1's strategy, we only need to keep track of action $a = 1$, $a = 0$, and $a = hide$. The payoffs from $a = 1$ and $a = 0$ are $C - (1 - q_1)$ and $C - q_1$ respectively when the posterior is q_1. If $a = hide$ is chosen, the developer in period 2 chooses a posterior split between 0 and $\frac{1}{2}$ for B_2.

I summarize B_1's strategy under certain p_0 and q_1 in the following lemma.

Lemma 3. *For discount factor $\delta \in (0,1)$, there exists $\hat{\delta}(p_0) \in (0,1)$:*

(1) If $\delta > \hat{\delta}(p_0)$, there exist cutoff points $\hat{\alpha}(\delta, p_0), \hat{\beta}(\delta, p_0)$ ($p_0 < \hat{\alpha}(\delta, p_0) < \hat{\beta}_2(\delta, p_0) < 1$) such that B_1 chooses $a = 1$ immediately with $q_1 \geqslant \hat{\beta}_2(\delta, p_0)$; chooses $a = 0$ immediately with $q_1 < \alpha(\delta, p_0)$; chooses hide with $\hat{\alpha}(\delta, p_0) \leqslant q_1 < \hat{\beta}(\delta, p_0)$.

(2) If $\delta \leqslant \hat{\delta}(p_0)$, B_1 always acts immediately.

Proof for Lemma 3.

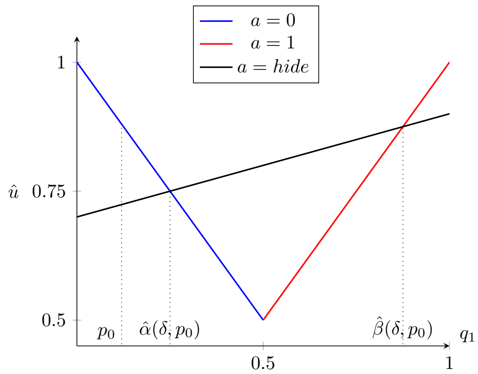
Firstly, with $q_1 = 0$ or 1, B_1 acts immediately.

We then calculate B_1's payoff from *hide* when the prior and the posterior are p_0 and q_1 $\in (0,1)$.

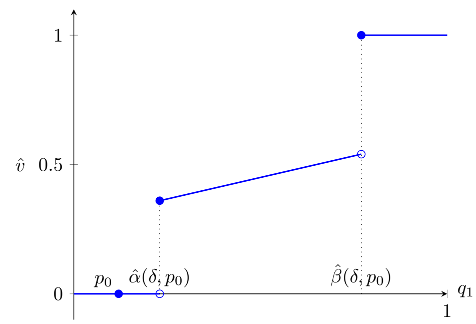
According to Alonso and Câmara (2016), if two players with different priors $u_1, u_2 \in (0,1)$ observe the same experiment, their posteriors u_1^p and u_2^p satisfy:

$$u_1^p = \frac{u_2^p \frac{u_1}{u_2}}{u_2^p \frac{u_1}{u_2} + (1 - u_2^p)\frac{1-u_1}{1-u_2}}$$

Given *hide*, when facing the experiment in period 2, B_1 has prior q_1 but B_2 has prior p_0. Notice that the developer's optimal experiment in period 2 is the split of 0 and $q_2 = \frac{1}{2}$ for B_2, so in B_1's eyes, the split is 0 and $q_2' = \frac{q_2 \frac{q_1}{p_0}}{q_2 \frac{q_1}{p_0} + (1-q_2)\frac{1-q_1}{1-p_0}}$. Moreover, the probability of q_2' is $\frac{q_2 \frac{q_1}{p_0} + (1-q_2)\frac{1-q_1}{1-p_0}}{\frac{q_2}{p_0}}$. Since B_2 chooses $a = 0$ at 0 and $a = 1$ at q_2, B_1's expected payoff from *hide* is:



(a) Bureaucrat's expected payoff



(b) Developer's expected payoff

Figure 1: Period 1

$$\mathbf{E}u(hide) = \delta \cdot (C - \frac{p_0}{1 - p_0}(1 - q_1))$$

As for acting immediately, B_1's expected payoff is:

$$\mathbf{E}u(act) = \begin{cases} C - q_1 & , q_1 < \dfrac{1}{2} \\ C - (1 - q_1) & , q_1 \geqslant \dfrac{1}{2} \end{cases}$$

Comparing them, we can get that when $\delta > \frac{2C-1}{2C - \frac{p_0}{1-p_0}}$, there are $\hat{\alpha}(\delta, p_0) = \frac{C(1-\delta) - p_0(C(1-\delta)-\delta)}{1-(1-\delta)p_0}$ and $\hat{\beta}(\delta, p_0) = \frac{1-C(1-\delta)-(1+\delta-C(1-\delta))p_0}{1-(1+\delta)p_0}$, where $p_0 < \hat{\alpha}(\delta, p_0) < \hat{\beta}(\delta, p_0) < 1$, and B_1 chooses $a = 1$ with $q_1 \geqslant \hat{\beta}(\delta, p_0)$; chooses $a = 0$ with $q_1 < \hat{\alpha}(\delta, p_0)$; chooses *hide* in between.

When $\delta \leqslant \frac{2C-1}{2C - \frac{p_0}{1-p_0}}$, B_1 chooses $a = 1$ with $q_1 \geqslant \frac{1}{2}$ and $a = 0$ with $q_1 < \frac{1}{2}$. ■

For $\delta > \hat{\delta}(p_0)$, two players' expected payoffs (y-$axis$) given different posteriors after the first experiment (x-$axis$) in period 1 can be summarized in Figure 1.[5]

Notice that Figure 1 is drawn for a specific prior p_0. When p_0 changes, two graphs in Figure 1 change. More specifically, as p_0 goes up, the payoff line from *hide* in Figure 1 (a)

[5]Figures in this section is plotted with parameter values $C = 1$, and serves the purpose of illustration.

moves downwards, making two cutoff points $\hat{\alpha}(\delta, p_0)$ and $\hat{\beta}_2(\delta, p_0)$ closer to each other.

From Figure 1 (b), it is easy to see that since we have $p_0 < \hat{\alpha}(\delta, p_0)$, the optimal experiment in period 1 has two possibilities: (1) the posterior split between 0 and $\hat{\alpha}(\delta, p_0)$ (*delaying experiment*), where B_1 chooses *hide* at $\hat{\alpha}(\delta, p_0)$; (2) the split between 0 and $\hat{\beta}_2(\delta, p_0)$ (*immediate experiment*), where B_1 takes action immediately at both posteriors. Which one is the optimal experiment will depend on the values of the initial priors and the discount factor, which is summarized in Proposition 1. Moreover, according to Figure 1 (a), *delaying experiment* gives $V_B = 0$ to B_1, while *immediate experiment* gives $V_B > 0$ to B_1. In other words, when *immediate experiment* is the optimal experiment, the bureaucratic norm is beneficial to bureaucrats who represent social welfare. However, this norm is not beneficial or harmful if *delaying experiment* is the optimal experiment.

Generically, the optimal experiment is unique — either *delaying experiment* or *immediate experiment*. The uniqueness is ensured when the origin and two cutoff points in Figure 1 (b) are not collinear. Only in the knife-edge cases where these three points are collinear, the optimal experiment is not unique. However, even in knife-edge cases, the equilibrium can still be unique with the help of the tie-breaker making the developer choose the less informative experiment when indifferent. A similar tie-breaker is also used in Bizzotto, Rüdiger and Vigier (2021).

When $\delta \leqslant \hat{\delta}(p_0)$, there are only immediate actions and the developer just chooses the same experiment as in the static case.

The discussion above shows how we get two possibilities of the unique equilibrium in Proposition 1. For the conditions governing these possibilities, see the proof below.

Proof for Proposition 1. Knowing B_1's strategy from Lemma 3, we can figure out the developer's optimal experiment in period 1.

When $\delta \leqslant \frac{2C-1}{2C-\frac{p_0}{1-p_0}}$, B_1 chooses $a=1$ with $q_1 \geqslant \frac{1}{2}$ and $a=0$ with $q_1 < \frac{1}{2}$. Since $p_0 < \frac{1}{2}$, the optimal experiment is the split between 0 and $\frac{1}{2}$ in B_1's eyes.

When $\delta > \frac{2C-1}{2C-\frac{p_0}{1-p_0}}$, we have two cutoff points in B_1's strategy $\hat{\alpha}(\delta, p_0)$, $\beta(\delta, p_0)$. Since we have $p_0 < \hat{\alpha}(\delta, p_0) < \hat{\beta}_2(\delta, p_0)$, the optimal experiment is either the split between 0 and $\hat{\alpha}(\delta, p_0)$ or the split between 0 and $\hat{\beta}_2(\delta, p_0)$. For the developer, her expected payoff from the latter one is $\frac{p_0}{\hat{\beta}_2(\delta, p_0)}$. As for the former one, at 0 she gets 0, at $\hat{\alpha}(\delta, p_0)$ the game proceeds into the next period, and her expected payoff is $\delta \cdot (p_0 + \frac{p_0}{1-p_0}\frac{1-\hat{\alpha}(\delta,p_0)}{\hat{\alpha}(\delta,p_0)}p_0)$.

Comparing the payoffs from two experiments, the delaying experiment is the optimal one if and only if:[6]

$$\frac{\delta(1-C(1-\delta))p_0}{C(1-\delta)-p_0(C(1-\delta)-\delta)} - \frac{C(1-\delta)(1-p_0)}{1-C(1-\delta)-p_0(1+\delta-C(1-\delta))} \geqslant 1-\delta$$

$$\Leftrightarrow r_1 p_0^2 + r_2 p_0 + r_3 \geqslant 0 \tag{1}$$

Where:

$$r_1 = -\delta(1-C(1-\delta))(1+\delta-C(1-\delta)) - C(1-\delta)(C(1-\delta)-\delta)$$
$$-(1-\delta)(C(1-\delta)-\delta)(1+\delta-C(1-\delta)),$$

$$r_2 = \delta(1-C(1-\delta))^2 + C(1-\delta)(C(1-\delta)-\delta) + C^2(1-\delta)^2$$
$$+C(1-\delta)^2(1+\delta-C(1-\delta)) + (1-\delta)(1-C(1-\delta))(C(1-\delta)-\delta),$$

[6] When indifferent between two experiments, the tie-breaker makes the developer choose the delaying experiment, which is less informative.

$$r_3 = -C^2(1-\delta)^2 - C(1-\delta)^2(1-C(1-\delta)).$$

Notice that for $p_0 = 0$ and $p_0 = \frac{1}{2}$, LHS of (1) is negative for δ close to 1. Moreover, when $\delta = 1$, we have $r_1 < 0$ and that the solution to (1) is $p_0 \in [0, \frac{1}{2}]$. So, when δ is close enough to 1, the solution to (1) would be $p_0 \in [\alpha(\delta), \beta(\delta)]$, where $0 < \alpha(\delta) < \beta(\delta) < \frac{1}{2}$.

It is easy to see that we can have $\bar{\delta} < 1$ such that for any $p_0 \leqslant \frac{1}{2} - \epsilon$ and $\delta > \bar{\delta}$, the optimal experiment choice depends on $p_0 \in [\alpha(\delta), \beta(\delta)]$ or not. ■

3 General result

The results in the example can be extended to a more general case, where bureaucrats have more than two actions.

Consider the situation where the government divides the project into N parts and decides to award how many parts to the local developer and the outsider. The decision set of bureaucrats is $\mathcal{A} := \{a_0, a_2, ..., a_N\}$ now, where a_n means awarding $\frac{n}{N}$ of the project to the developer and leaving the rest to the outsider.

The state space is still binary: $\Omega \in \{0, 1\}$. The state $\omega = 1$ means it is better to award the project to the developer, and $\omega = 0$ means it is better to award the project to the outsider. I still assume the same quadratic-loss social welfare:

$$u(a, \omega) = C - (a - \omega)^2$$

where C is a positive constant that keeps the payoff positive, so that delay will be costly for the bureaucrat.

The bureaucrat's optimal action among the decision set $\mathcal{A}$ is a_n if the belief $q = Pr(\omega = 1) \in [\frac{2n-1}{2N}, \frac{2n+1}{2N})$, $n = 1, 2, ..., N-1$. a_0 is optimal with belief $q \in [0, \frac{1}{2N})$, and a_N is optimal with belief $q \in [\frac{2N-1}{2N}, 1]$. The bureaucrat is indifferent between two adjacent actions at these cutoff points. Denote indifferent cutoffs as $\bar{q}_n = \frac{2n-1}{2N}$, $n = 1, 2, ..., N$, and $\bar{q}_0 = 0$, $\bar{q}_{N+1} = 1$.

The developer has a state-independent payoff: $v : \mathcal{A} \to \mathbb{R}$, and $v(a_i) < v(a_j)$, $\forall i < j$. She always prefers a larger share of the project. Moreover, the marginal benefit decreases in the share, i.e., $v(a_{n+1}) - v(a_n) < v(a_n) - v(a_{n-1})$. I also require $v(a_N) - v(a_{N-1}) < \frac{1}{2}\big(v(a_{N-1}) - v(a_{N-2})\big)$. With these assumptions, in the static game, $\bar{V}_B = 0$. Actually, $\bar{V}_B = 0$ in the static game is equivalent to that the static optimal experiment is inducing two adjacent actions, i.e., denote the initial prior as $p_0 = Pr(\omega = 1)$, if $p_0 \in [\bar{q}_{n-1}, \bar{q}_n)$, the static optimal experiment is the posterior split of $\bar{q}_{n-1}$ and $\bar{q}_n$. The assumption required here is to make the static optimal experiment always induce two adjacent actions, and thus $\bar{V}_B = 0$.

I also assume that the prior p_0 is bounded away from indifferent points $\bar{q}_n$ by a small positive number ϵ, i.e., $p_0 \notin \mathcal{B}_\epsilon(\bar{q}_n)$, $n = 0, 1, ..., N+1$, and $p_0 < \bar{q}_N$. [7]

Finally, I apply tie-breakers that the bureaucrat chooses the action preferred by the developer when indifferent, and the developer in period 2 chooses the experiment that is better for the developer in period 1 when indifferent.

3.1 Equilibrium in the multiple-action case

The result in the binary-action case does not directly apply to the multiple-action case. When B_1 chooses *hide* at posterior $q \neq p_0$, the game in period 2 is a Bayesian persuasion

[7]The developer will just choose an uninformative experiment when p_0 is equal to indifferent points or $p_0 \geqslant \bar{q}_N$.

with heterogeneous priors as in Alonso and Câmara (2016). In the binary-action case, no matter what q is, the optimal experiment in period 2 is always the posterior split of 0 and $\frac{1}{2}$ for prior p_0. As a result, B_1's payoff from *hide* is linear in his posterior q.

However, in the multiple-action case, we have more than one indifferent cutoff point ($\bar{q}_n$). When $q > p_0$, after considering the heterogeneous priors, the point that is originally on the concave closure of the developer's payoff function can become inside the closure, which makes the optimal experiment in period 2 different from the one with homogeneous priors (a more detailed argument is provided in the proof of Proposition 2).

Despite the complexity of the multiple-action case, under our assumptions, we have a similar result as the leading example when δ is large enough. In equilibrium, the developer is still worse off, and V_B can be zero or positive for B_1.

Proposition 2. $\exists\ \hat{\delta} < 1$ *such that,* $\forall \delta > \hat{\delta}$*, the unique equilibrium is either:*

(1) $V_B > 0$*, and the first bureaucrat always chooses immediate actions, or*

(2) $V_B = 0$*, and the first bureaucrat delays at the positive signal of the experiment.*

(3) V_D *is strictly less than* $\bar{V}_D$

Proof for Proposition 2. In this proof, I assume $p_0 \in (\bar{q}_n, \bar{q}_{n+1})$.

Firstly notice that *record* does not happen in the equilibrium, due to $\bar{V}_B = 0$ in the static game. We only need to look at immediate actions and *hide*.

For the following proof, define $\hat{v}(.)$ as the developer's payoff function in posteriors in the static game, i.e., $\hat{v}(q) = v(a_n)$ if $q \in [\bar{q}_n, \bar{q}_{n+1})$.

Lemma 4. *At any posterior* $q < p_0$*, hide is not the optimal action.*

Proof. Consider three points on the graph of the developer's payoff function in posteriors

$(t_1, \hat{v}(t_1))$, $(t_2, \hat{v}(t_2))$, $(t_3, \hat{v}(t_3))$, $t_1 < t_2 < t_3$. If point 2 is above the line connecting points 1 and 3, we have

$$\frac{\hat{v}(t_2) - \hat{v}(t_1)}{t_2 - t_1} > \frac{\hat{v}(t_3) - \hat{v}(t_1)}{t_3 - t_1} \Leftrightarrow \frac{t_3 - t_1}{t_2 - t_1} > \frac{\hat{v}(t_3) - \hat{v}(t_1)}{\hat{v}(t_2) - \hat{v}(t_1)}.$$

According to Alonso and Câmara (2016), when B_2 with prior p_0 has posterior t_i after an experiment, B_1 with prior q has posterior $t'_i = \frac{t_i \frac{q}{p_0}}{t_i \frac{q}{p_0} + (1-t_i)\frac{1-q}{1-p_0}}$. Because $q < p_0$, we have

$$\frac{t'_3 - t'_1}{t'_2 - t'_1} = \frac{t_3 - t_1}{t_2 - t_1} \frac{t_2 \frac{q}{p_0} + (1 - t_2)\frac{1-q}{1-p_0}}{t_3 \frac{q}{p_0} + (1 - t_3)\frac{1-q}{1-p_0}} > \frac{t_3 - t_1}{t_2 - t_1}$$

Thus, $\frac{t'_3 - t'_1}{t'_2 - t'_1} > \frac{\hat{v}(t_3) - \hat{v}(t_1)}{\hat{v}(t_2) - \hat{v}(t_1)}$, and $(t'_2, \hat{v}(t_2))$ is still above the line connecting $(t'_1, \hat{v}(t_1))$ and $(t'_3, \hat{v}(t_3))$.

Then we conclude that $\bar{q}_i$ $(i = 0, 1, ..., N)$, which is on the concave closure of the developer's utility $\hat{v}(.)$, is still on the concave closure after considering heterogeneous priors when $q < p_0$. Thus, when the developer holds prior q (same as B_1) and B_2 holds prior p_0 ($p_0 > q$ and $p_0 \in (\bar{q}_n, \bar{q}_{n+1})$), the optimal experiment in period 2 is the split of $\bar{q}_n$ and $\bar{q}_{n+1}$ for prior p_0.

When B_2 with prior p_0 has the posterior split $\bar{q}_n$ and $\bar{q}_{n+1}$, B_1 with prior q has the split of $\bar{q}'_n < \bar{q}_{n-1}$ and $\bar{q}'_{n+1} < \bar{q}_n$. Notice that B_2 chooses a_n and a_{n+1} at B_1's posteriors $\bar{q}'_n$ and $\bar{q}'_{n+1}$. But with belief $\bar{q}'_{n+1}$, bureaucrats prefers a_n to a_{n+1}. As a result, for B_1 with prior q, the expected payoff from the experiment is smaller than his expected payoff from choosing a_n at belief q. Thus, when the first experiment gives posterior $q < p_0$, B_1 will not choose *hide*, which is worse than choosing a_n. ■

By Lemma 4, B_1 always chooses immediate actions for $q < p_0$, and *hide* can only happen at $q > p_0$. Also notice that when B_1 chooses *hide* at q, the point of q and the developer's payoff from *hide* is always inside the concave closure of her static utility function $\hat{v}(.)$. Thus, if $p_0 \in [\bar{q}_n, \bar{q}_{n+1})$, for the optimal experiment chosen by the developer in period 1, which has two signals due to binary states, one signal is the point $\bar{q}_n$.

Then we need to look at the other posterior of the optimal experiment, which is larger than p_0. Denote this posterior as q_2.

When B_1 chooses *hide* at q, B_2 has prior p_0 and let the optimal experiment in period 2 be the split of $\bar{q}_i$ and $\bar{q}_j$ for him ($i < j$). The developer who has prior q will think the split is $\frac{\bar{q}_i \frac{q}{p_0}}{\bar{q}_i \frac{q}{p_0} + (1-\bar{q}_i)\frac{1-q}{1-p_0}}$ and $\frac{\bar{q}_j \frac{q}{p_0}}{\bar{q}_j \frac{q}{p_0} + (1-\bar{q}_j)\frac{1-q}{1-p_0}}$ instead, with B_2 choosing a_i and a_j at two posteriors. Her payoff from choosing *hide* (without discount) is

$$v_h(q) := \left[\frac{1-\bar{q}_i}{1-p_0} + \frac{\bar{q}_i - p_0}{p_0(1-p_0)} q\right] \frac{\bar{q}_j - p_0}{\bar{q}_j - \bar{q}_i} v(a_i) + \left[\frac{1-\bar{q}_j}{1-p_0} + \frac{\bar{q}_j - p_0}{p_0(1-p_0)} q\right] \frac{p_0 - \bar{q}_1}{\bar{q}_j - \bar{q}_i} v(a_j)$$

$$\Rightarrow v_h'(q) = [v(a_j) - v(a_i)] \frac{\bar{q}_j - p_0}{p_0(1-p_0)} \frac{p_0 - \bar{q}_i}{\bar{q}_j - \bar{q}_i}$$

When the experiment chosen in period 2 stays unchanged, the developer's payoff when B_1 chooses *hide* is linear in posterior q. However, when q goes up, the experiment chosen in period 2 can change – the points originally on the concave closure of $\hat{v}(.)$ can be inside the closure when $q > p_0$ (the opposite way compared to $q < p_0$).

By the similar argument as in Lemma 4, when $q > p_0$, if one point on the graph of developer's payoff $\hat{v}(.)$ is below a line connecting two other points on the graph, it is still below the line after considering heterogeneous priors. Furthermore, as q grows even larger, the point still stays below the line. Also notice that the point that is originally above the line can become below the line as q goes up, by the similar argument as in Lemma 4. As a result, as q goes up, the new experiment chosen by the developer in period 2 is a mean-preserving spread of the previous experiment. In the new experiment, we will have larger $\bar{q}_j, v(a_j)$ and/or smaller $\bar{q}_i, v(a_i)$. Thus, $v'_h(q)$ can be larger at larger q, and $v_h(q)$ is a convex function in q when $q > p_0$. By the convexity of the developer's payoff from *hide*, we have the following claim.

Claim 1. *If we have three posteriors s_1, s_2, s_3 where B_1 chooses hide and $s_3 > s_2 > s_1 > p_0$, the second point of the developer's optimal experiment in period 1 cannot be s_2.*

With the claim, I will prove that when the second point of the optimal experiment in period 1 is $q_2 > \bar{q}_{n+1}$, B_1 does not choose *hide* at q_2. Without loss, let $q_2 \in (\bar{q}_{n+1}, \bar{q}_{n+2})$.

Suppose B_1 chooses *hide* at $q_2 \in (\bar{q}_{n+1}, \bar{q}_{n+2})$, which is the second posterior of the optimal experiment. Then B_1 must chooses *hide* at $\bar{q}_{n+1}$ to make q_2 the optimal choice, because $\bar{q}_{n+1}$ is a better choice for the developer than q_2 in period 1 if B_1 chooses the immediate action a_{n+1} at $\bar{q}_{n+1}$. This is because the point $(\bar{q}_{n+1}, v(a_{n+1}))$ is on the concave closure of the developer's payoff function in posteriors $\hat{v}(.)$.

If B_1 also chooses *hide* in $\mathcal{B}_\epsilon(q_2)$, the claim says q_2 is not in the optimal experiment, contradiction.

If B_1 chooses *hide* at $q = q_2 - \epsilon$ and a_{n+1} at $q_2 + \epsilon$ for any small ϵ, we need to have $v_h(q_2) > v(a_{n+1})$ to keep q_2 optimal for the developer. That means when the posterior is q_2 in period 1 and B_1 chooses *hide*, in the optimal experiment in period 2, one of the posterior must induce a_k s.t. $k > n+1$, to make $v_h(q_2) > v(a_{n+1})$. As a result, B_1 must choose *hide* at $\bar{q}_{n+2}$. Otherwise B_1 chooses the immediate action a_{n+2} at $\bar{q}_{n+2}$, and the split of $\bar{q}_n$ and

$\bar{q}_{n+2}$ is a better experiment than the split of $\bar{q}_n$ and q_2 in period 1 (since $(\bar{q}_{n+2}, v(a_{n+2}))$ is on the concave closure of $\hat{v}(.)$). Now we have $\bar{q}_{n+2} > q_2$ and $\bar{q}_{n+1} < q_2$ choosing *hide*, so q_2 is not the optimal choice of the second posterior if B_1 chooses *hide* at q_2 according to the claim, contradiction.

If B_1 chooses *hide* at $q = q_2 + \epsilon$ and a_{n+1} at $q_2 - \epsilon$ for any small ϵ, we have $\bar{q}_{n+1} < q_2$ and $q_2 + \epsilon > q_2$ where B_1 chooses *hide*, so the claim says q_2 is not the optimal choice, contradiction.

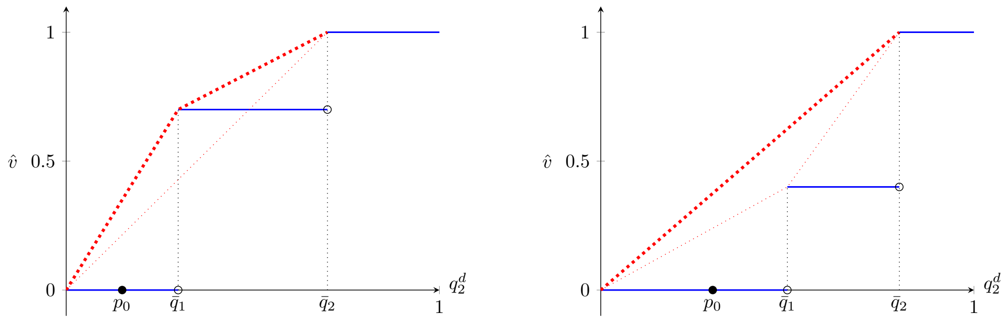
Thus, we conclude that when $q_2 \in (\bar{q}_{n+1}, \bar{q}_{n+2})$ is in the optimal experiment in period 1, B_1 does not choose *hide* at q_2. The conclusion extends to $q_2 \geqslant \bar{q}_{n+2}$.

So, only when the second posterior of the optimal experiment in period 1 is $q_2 < \bar{q}_{n+1}$, *hide* possibly happens in the optimal experiment. When (1) δ is large enough, (2) $q_2 < \bar{q}_{n+1}$ is in the optimal experiment, and (3) B_1 chooses *hide* at q_2, B_1 is indifferent between choosing *hide* and a_n at q_2. Notice that at the first posterior of the optimal experiment $\bar{q}_n$, B_1 chooses a_n. Thus, in this optimal experiment where *hide* happens at $q_2 < \bar{q}_{n+1}$, B_1 has zero value of persuasion.

Also notice that when the second posterior of the optimal experiment is $q_2 < \bar{q}_{n+1}$, B_1 does not choose an immediate action at q_2. This is because if B_1 chooses an immediate action at such q_2, the action will be a_n, and the developer's expected payoff from this experiment is $v(a_n)$. But the split of $\bar{q}_n$ and 1 is a better choice.

When the second posterior of the optimal experiment is $q_2 > \bar{q}_{n+1}$, B_1 will choose an immediate action at q_2 and this gives B_1 positive value of persuasion.

In conclusion, if δ is large enough, when the second posterior of the optimal experiment $q_2 < \bar{q}_{n+1}$, B_1 chooses *hide* at q_2 and he has $V_B = 0$. When $q_2 > \bar{q}_{n+1}$, B_1 chooses an



(a) Static optimal experiment, $V_B = 0$ (b) Static optimal experiment, $V_B > 0$

Figure 2: The developer's payoff in the static game

immediate action at q_2 and he has $V_B > 0$. ■

3.2 Convex utility of the developer

In the binary-action case, when the prior favors the outsider, the only possible optimal experiment in the static game is the posterior split between 0 and the posterior where the bureaucrat is indifferent between two actions. At two points, the bureaucrat chooses two actions respectively. We always have $\bar{V}_B = 0$ in the static game.

However, in the multiple-action cases, we can have static optimal experiments inducing adjacent actions and extreme actions. For example, consider a three-action case with $\mathcal{A} = \{a_0, a_1, a_2\}$ and prior $p_0 \in [0, \bar{q}_1)$. With different preferences of players, the optimal static experiment can be the split of 0 and $\bar{q}_1$, or the split of 0 and $\bar{q}_2$. The former case is shown in Figure 2 (a), where the static optimal experiment induces adjacent actions. The latter case is shown in Figure 2 (b), where the static optimal experiment induces extreme actions. Notice that when the static optimal experiment induces adjacent actions, $\bar{V}_B = 0$; when the static optimal experiment induces extreme actions, $\bar{V}_B > 0$.

When we have a strictly concave utility for the developer as I assumed, the optimal static experiment will be the case in Figure 2 (a). However, if we have a convex utility for the developer instead, the optimal static experiment will be the case in Figure 2 (b).

In this convex utility case, our analysis will change. In an example of the three-action case, the optimal experiment in period 1 can be uninformative when δ is large. B_1 has the incentive to delay when the experiment is uninformative in period 1 because the value of persuasion in period 2 is positive. This allows the developer to conduct an uninformative experiment in period 1 when players are patient. With other values of δ and p_0, the optimal experiment in period 1 can also induce only immediate actions, or induce *hide* at the positive signal and *record* at the negative signal.

4 Conclusion

This paper studies a model of a developer seeking to persuade a series of bureaucrats to take a specific action before a deadline. The optimal action for bureaucrats depends on the state, while the developer has a state-independent preference. The bureaucrat can either choose the action by himself or leave the decision to his successor and take the chance to hide his information. If the second bureaucrat is restricted by the bureaucratic norm and cannot update his belief when the first bureaucrat records no evidence from period 1, delay with hiding can be better than immediate actions for the first bureaucrat, even if he has the same preference as the second bureaucrat. Thus, he has the incentive to delay. The incentive to delay can make the first bureaucrat have a positive value of persuasion by forcing the developer to provide more information. With other parameters, the bureaucrat still has zero value of persuasion. Unlike bureaucrats, the developer is always worse off from the bureaucratic norm.

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