1. (a) $A \triangle B = (A \setminus B) \cup (B \setminus A)$

$= (A \cap B^c) \cup (B \cap A^c)$

$= (A \cup B) \cap (A \cup A^c) \cap (B^c \cup B) \cap (B^c \cup A^c)$

$= (A \cup B) \cap (A \cap B)^c$

$= (A \cup B) \setminus (A \cap B)$

(b) $A \triangle \emptyset = (A \setminus \emptyset) \cup (\emptyset \setminus A) = A$

$A \triangle A = (A \setminus A) \cup (A \setminus A) = \emptyset$.

(c) $(A \triangle B) \triangle C = ((AB^c) \cup (A^cB)) \triangle C$

$= [[(AB^c) \cup (A^cB)] \cap C^c] \cup [[(AB^c) \cup (A^cB)]^c \cap C]$

$= AB^cC^c \cup A^cBC^c \cup [[(A^cB^c) \cup (AB)] \cap C]$

$= AB^cC^c \cup A^cBC^c \cup A^cB^cC \cup ABC$

$= A \triangle (B \triangle C)$.

(d) $A \cap (B \triangle C) = A \cap (BC^c \cup B^cC)$

$= ABC^c \cup AB^cC$

$(A \cap B) \triangle (A \cap C) = [AB(A^c \cup C^c)] \cup [(A^c \cup B^c)AC]$

$= ABC^c \cup AB^cC$

$= A \cap (B \triangle C)$

(e) $A \triangle (A \setminus B) = [A \setminus (A \setminus B)] \cup [(A \setminus B) \setminus A] = A \cap B$

$B \triangle (A \setminus B) = (A \setminus B \cup B) \setminus [(A \setminus B) \cap B] =$

3. $f$单：若 $f(x_1) = f(x_2)$，$x_1 = gf(x_1) = gf(x_2) = x_2$

$g$满．$\forall y \in Y$．$gf(y) = y$　$f(y) \in X$．

6. "$\Longrightarrow$" $\forall y \in Y$．$f$满 $\Longrightarrow \exists x \in X$．s.t. $f(x) = y$．

$g(y) = gf(x) = hf(x) = h(y) \Longrightarrow g = h$．

"$\Leftarrow$" 若 $\exists y_0 \in Y \setminus f(X)$. 定义 $g: Y \longrightarrow \{0,1\}$, $h: Y \longrightarrow \{0,1\}$.

$$y \longmapsto 0 \qquad\qquad y \longmapsto \begin{cases} 1 & y = y_0 \\ 0 & y \neq y_0 \end{cases}$$

则 $g \neq h$ 但 $h \circ f = g \circ f = 0_X$

7.(a) "$\Rightarrow$" 若 $x \in A \Rightarrow x \in f^{-1}(f(A))$. $A \subset f^{-1}(f(A))$.

若 $x \in f^{-1}(f(A))$, $f(x) \in f(A)$, $f(x) = f(x_0)$, $x_0 \in A$, $f$单, $x_0 = x \in A$.

$A = f^{-1}(f(A))$

"$\Leftarrow$" 若 $f(x_1) = f(x_2)$, 则 $x_2 \in f^{-1}(f(\{x_1\})) = \{x_1\}$. $x_2 = x_1$. $f$单.

(b) "$\Rightarrow$" 若 $y \in B$. 由$f$满, $\exists x \in X$, $f(x) = y$, $x \in f^{-1}(B)$, $y \in f(f^{-1}(B))$ $B \subset f(f^{-1}(B))$.

若 $y \in f(f^{-1}(B))$. 则 $\exists x \in f^{-1}(B)$, $y = f(x) \in B$. $B = f(f^{-1}(B))$.

"$\Leftarrow$" $\forall y \in B$, $y \in f(f^{-1}(y)) \Rightarrow \exists x \in f^{-1}(y)$. $f(x) = y$. $f$满

11.(a) "$\Rightarrow$" $\forall x \in X$, $(x, f(x)) \in f_*$, $(f(x), x) \in f^*$. $\Rightarrow (x,x) \in f^* \circ f_*$

若 $(x_1, x_2) \in f^* \circ f_*$. $\exists y \in Y$. $(x_1, y) \in f_*$, $(y, x_2) \in f^*$.

$f(x_1) = y = f(x_2)$, 由单 $x_1 = x_2$, $(x_1, x_2) \in Id_X$.

$\Rightarrow Id_X = f^* \circ f_*$

"$\Leftarrow$" 若 $f(x_1) = f(x_2)$. 则 $(x_1, x_2) \in f^* \circ f_* = Id_X \Rightarrow x_1 = x_2$.

(b) "$\Rightarrow$" $\forall y \in Y$. 由$f$满 $\exists x \in X$, $f(x) = y$, $(y, f(x)) \in f_*$, $(f(x), y) \in f^* \Rightarrow (y,y) \in f_* \circ f^*$

若 $(y_1, y_2) \in f^* \circ f_*$, 则 $\exists x \in X$. $(y_1, x) \in f^*$, $(x, y_2) \in f_* \Rightarrow y_1 = f(x) = y_2$.

$(y_1, y_2) \in Id_Y$.

$\Rightarrow Id_Y = f_* \circ f^*$

"$\Leftarrow$" $\forall y \in Y$. $(y,y) \in Id_Y = f_* \circ f^*$, $\exists x \in X$. $(x, y) \in f_*$ $(y, x) \in f^*$.

$y = f(x)$. $f$满.

公理I (域公理) $\begin{cases} \text{Ia. 加法abel群} \\ \text{Ib 乘法, 幺半群} \\ \text{Ic. 分配律.} \end{cases}$ Id $x = 0$、则 $x^{-1} x = 1$.

公理Ⅱ（序公理） $\leq$

(Ⅱa) $x \leq x$

(Ⅱb) $x \leq y$ 且 $y \leq z \Rightarrow x \leq z$（传递）

(Ⅱc) $x \geq y$ or $x \leq y$ $\forall x, y \in \mathbb{R}$ （全序）

(Ⅱd) $x \leq y$, $z \in \mathbb{R}$, then $x+z \leq y+z$ } 加法

(Ⅱf) $0 \leq x$, $y \leq z$, 则 $x \cdot y \leq x \cdot z$ } 乘法，相容.

公理Ⅲ 最小上界性.

1.2

1. (a) 若 $x \neq 0$. 则 $0 = x^{-1} x y = y$.

(b) 若 $0 < 1$, 则 $0 \geq 1$ (Ⅱd) $-1 \geq 0$ (Ⅱe).

$(-1)\cdot(-1) \geq 0$ (Ⅱf).

$1 \geq 0 \Rightarrow 0 = 1$ (Ⅱc) 矛盾.

(c) $x < y \Rightarrow x \leq y$, $y \leq z \Rightarrow x \leq z$. (Ⅱb).

若 $x = z$. 则 $z < y$ 且 $y \leq z \Rightarrow z \leq y$ 且 $y \leq z \Rightarrow y = z$. 矛盾.

(d) 若 $-y = -x$, 则 $x = y$, $\Rightarrow -y \neq -x$.

$x < y \Rightarrow x \leq y \Rightarrow -y \leq -x \Rightarrow -y < -x$.

(e) $(-x)\cdot y + (xy) = (-x+x)\cdot y = 0 \cdot y = 0$.

$(-x) y = -(xy)$

4. 若 $a$, 与 $a+x$ 为有理, 则 $(a+x) - a = x \in \mathbb{Q}$, 矛盾.

若 $a$ 与 $ax$ 为有理, $a \neq 0$. 则 $ax/a = x \in \mathbb{Q}$, 矛盾

5. 由 $B$ 非空, 取 $b_0 \in B$. 则 $\forall a \in A$. $a < b_0$. $A$ 有上界. $A$ 非空. 从而有上确界. $\sup A = c \in \mathbb{R}$.

$\forall a < c$. 若 $a \in B$. 则 $a$ 是 $A$ 上界, 矛盾. 从而 $a \in A$. 又 $c$ 为 $A$ 上界. $A \subset (-\infty, c]$

$\forall b > c$. 若 $b \in A$. 则与 $c$ 是上界矛盾. 从而 $b \in B$. $B \subset [c, +\infty)$

因此 $A=(-\infty,c)$，$B=[c,\infty)$.

或 $A=(-\infty,c]$，$B=(c,\infty)$.