Handwritten Digits Recognition with Multilayer Backpropagation Neural Networks

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1 Abstract

In this report, we improve the handwritten digits recognition with multi-layer back-propagation neural networks. We correctly derived and implemented back-propagation algorithm and many tricks of the trade to improve the performance of learning. Furthermore, we try out different network topologies to select the best model based on the accuracy on validation set. The results are quite satisfying: using all the tricks of the trade and model selection strategy, we successfully achieve 97.51% on the final test set.

2 Classification

2.1 Mini-batch gradient descent

In this section, we use mini-batch gradient descent to classify the MNIST dataset. We split the 60000 images in the training set into two parts: the first 50000 images are used to train the model, the last 10000 images are used as validation set to do early stopping. We stop the training procedure once the loss on the validation set goes up and we save the weights that achieves the minimum loss on the validation set. And there are 10000 images in the test set.

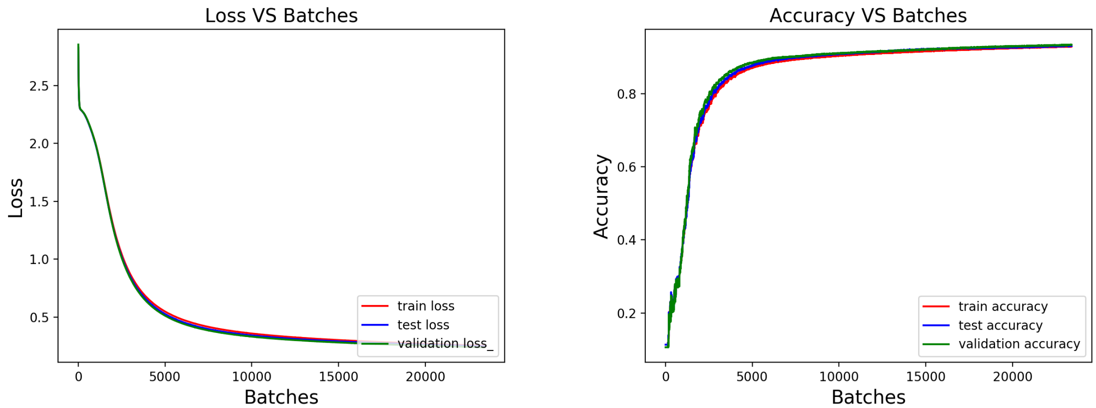
We use one hidden layer of 64 nodes, and the mini-batch size is 128. We use a learning rate of 0.01 and sigmoid activation function. We use standard normal distribution to initialize the weights and biases. For the weights, we multiply 0.01 to prevent large initialized values. We run the network for 60 epoches.

We report the accuracy and loss on the training set, test se and validation set every batch. The following graphs show the accuracy and loss over each batch on different sets. Without any tricks, after 60 epoches, the accuracy on the test set is 0.9308%. This is no early stopping occurs, the possible reason is that the choice of learning is small

2.2 Gradient checking

To verify the correctness of implementation of back-propagation, we compute the slope with respect to one weight using the numerical approximation: $\frac{\partial E^n}{\partial w_{ij}} \approx \frac{E^n(w_{ij}+\epsilon)-E^n(w_{ij}-\epsilon)}{2\epsilon}$ where we compute the numerical gradient for every weight and bias and for every example. Here we choose $\epsilon = 10^{-2}$ and according to the numerical theory, the difference between the gradients should be within $O(\epsilon^2)$ so that we expect the gradients to agree within 10^{-4}.

After successfully verifying that our back-propagation implementation is correct, we turn off the numerical gradient checking when learning since it is way slower than back-propagation.



(a) (b)

Figure 1: The loss and accuacy of different sets over the batches without any tricks.

3 Adding the Tricks of the Trade

In this section, we follow Yann LeCun's paper [1] to understand and implement several tricks of the trade. In the next section we will add more tricks at the same time when we try out different network topologies.

3.1 Training data shuffling

According to [1], we should shuffle the training set so that successive training example rarely belong to the same class and present input examples that produce a large error more frequently than examples that produce a small error. Here we use mini-batches and we shuffle the examples after each epoch.

3.2 Activation function

The activation function highly affects the learning process including speed and ability of representation such as non-linearity so it is critical to set it correct. According to [1], we should use symmetric sigmoids such as hyperbolic tangent since it often converge faster than the standard logistic function. Here we use the recommended sigmoid $f(x) = 1.7159 * \tanh(2x/3)$ since it has several good properties: (a) $f(1) = 1, f(-1) = -1$, (b) the second derivative is maximum at $x = 1$ which can make good use of non-linearity, (c) the effective gain is close to 1.

3.3 Weight initialization

In order to improve and speed up the learning process, it is better to make the outputs of each node have mean zero and a standard deviation of approximately one. Assuming that the training set has been normalized and we use the previous modified sigmoid function, then we can derive that weights should be randomly drawn from a distribution with zero mean and standard deviation as $\sigma = m^{-1/2}$ where m is the number of connections feeding into the node.

3.4 Momentum

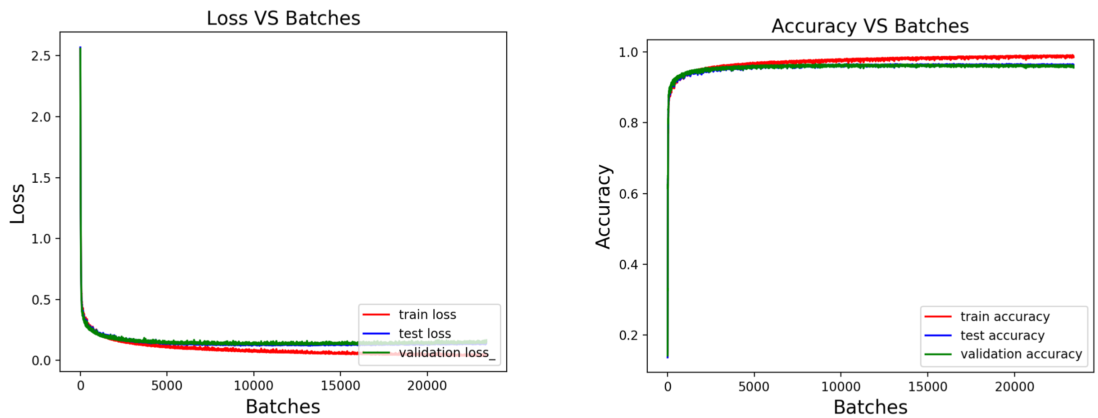
Momentum ($\Delta w(t+1) = \eta \frac{\partial E_{t+1}}{\partial w} + \mu \Delta w(t)$) can increase speed when the cost surface is highly non-spherical since it damps the size of steps along directions of high curvature thus yielding a larger effective learning rate along the directions of low curvature.

4 Experiment with Network Topology

4.1 Experiments with differnet hidden units

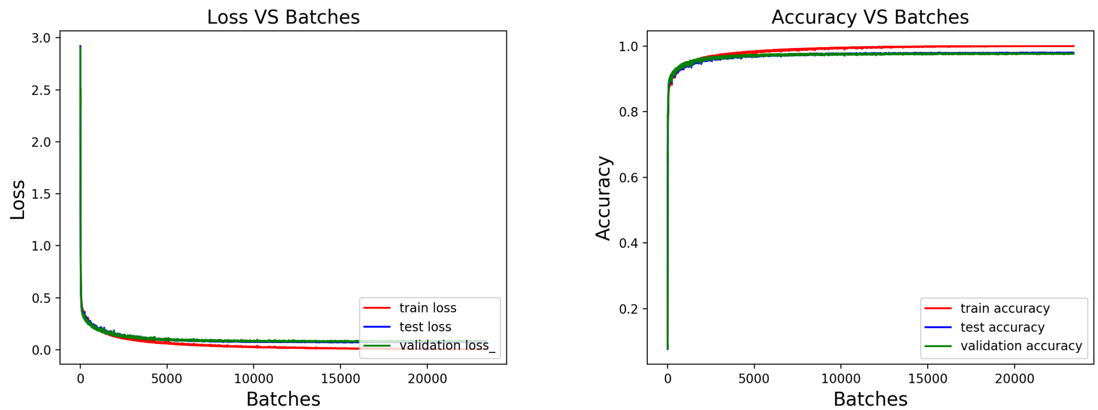
We use a momentum of 0.9 and use the sigmoid in Section 4.4 of "lecun98efficient.pdf". The initialization method of weights are as described in 4 (c) in Programming assignment 2. Learning rate is 0.01.

First, we half the hidden units. Now we have a network of 3 layers with a hidden layer with 32 nodes. We run the network for 60 epoches except early stopping occurs. After 60 epoches, the accuracy on the test set is 0.9635%. This is no early stopping occurs, the possible reason is that the choice of learning is small enough.



(a) (b)

Figure 2: The loss and accuacy of different sets over the batches with a hidden layer of 32 hidden nodes.



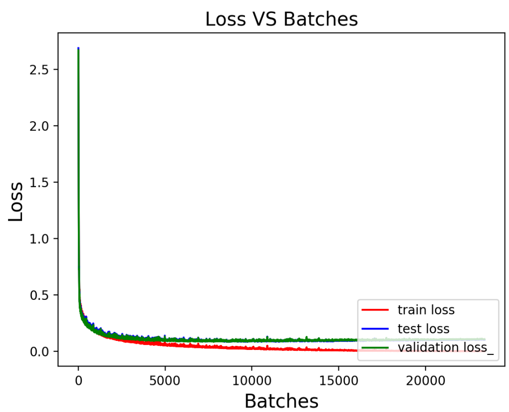
(a) (b)

Figure 3: The loss and accuacy of different sets over the batches with a hidden layer of 128 hidden nodes.

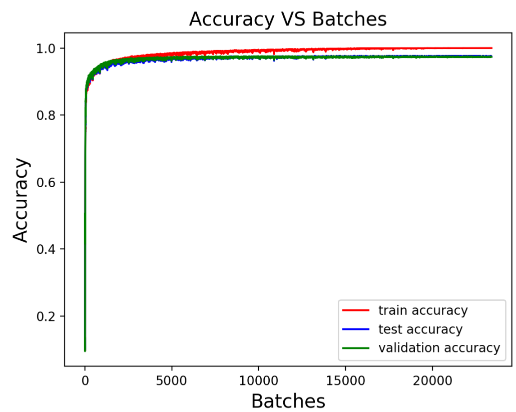
Then, we double the hidden units. Now we have a network of 3 layers with a hidden layer with 128 nodes. We run the network for 60 epoches except early stopping occurs. After 60 epoches, the accuracy on the test set is 0.9787%. This is no early stopping occurs, the possible reason is that the choice of learning is small enough.

4.2 Doubling the hidden layers

For a network with one hidden of 64 nodes, there are approximately about 50890 parameters. If we increase the hidden layers while keep the same number of parameters, there will be 58 hidden nodes in each hidden layer. We use a momentum of 0.9 and use the sigmoid in Section 4.4 of "lecun98efficient.pdf". The initialization method of weights are as described in 4 (c) in Programming assignment 2. Learning rate is 0.01. After 60 epoches, the accuracy on the test set is 0.9766%. This is no early stopping occurs, the possible reason is that the choice of learning is small enough.



(a)



(b)

Figure 4: The loss and accuacy of different sets over the batches with two hidden layers.

4.3 More tricks

In this section, in order to improve the preformance of the network, we consider the following tricks. In our experiments, we found that the network can achieve fast convergence and higher test accuracy with the tricks.

4.3.1 ReLU

We consider using ReLU as the activation function. The ReLU function can be

$$\mathrm{ReLU}(x) = \begin{cases} x & \text{if x} > 0, \\ 0 & \text{otherwise,} \end{cases}$$

The gradient of ReLU can be caculated as,

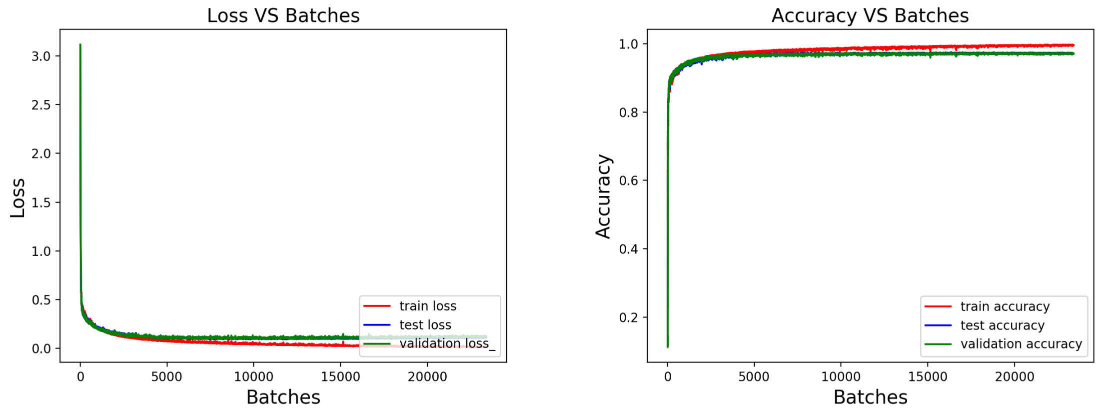
$$\mathrm{dReLU}(x) = \begin{cases} 1 & \text{if x} > 0, \\ 0 & \text{otherwise,} \end{cases}$$

We use a three layer network with a hidden layer with 64 nodes. We use a momentum of 0.9 and use ReLU as activation function. The initialization method of weights are as described in 4 (c) in Programming assignment 2. Learning rate is 0.01.

4.3.2 Leaky ReLU

We consider using leaky ReLU as the activation function. The leaky ReLU function can be

$$\mathrm{LeakyReLU}(x) = \begin{cases} x & \text{if x} > 0, \\ 0.01x & \text{otherwise,} \end{cases}$$



(a) (b)

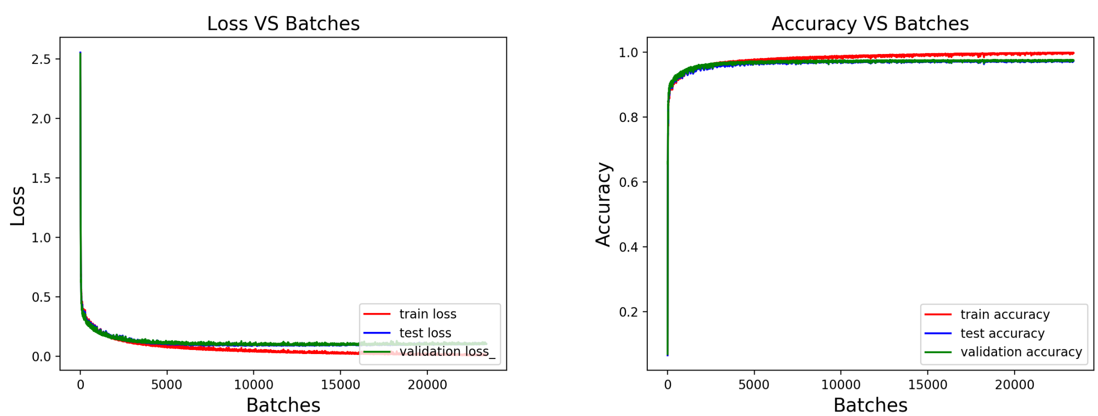
Figure 5: The loss and accuacy of different sets over the batches with ReLU.

The gradient of leaky ReLU can be caculated as,

$$\text{dLeakyReLU}(x) = \begin{cases} 1 & \text{if x} > 0, \\ 0.01 & \text{otherwise,} \end{cases}$$

We use a three layer network with a hidden layer with 64 nodes. We use a momentum of 0.9 and use leaky ReLU as activation function. The initialization method of weights are as described in 4 (c) in Programming assignment 2. Learning rate is 0.01.

After 60 epoches, the accuracy on the test set is 0.9723%.

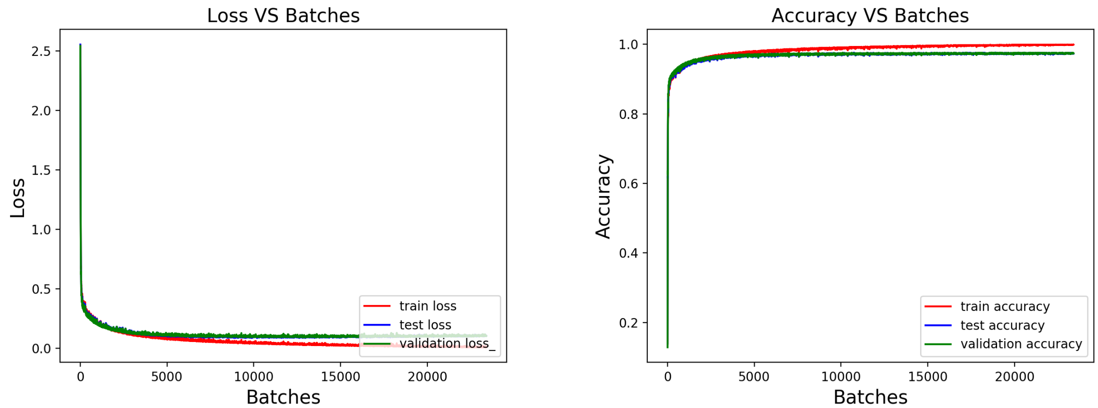


(a) (b)

Figure 6: The loss and accuacy of different sets over the batches with leaky ReLU.

4.3.3 Nesterov momentum

We consider using Nesterov momentum. We use a three layer network with a hidden layer with 64 nodes. We use a momentum of 0.9 and use leaky ReLU as activation function. The initialization method of weights are as described in 4 (c) in Programming assignment 2. Learning rate is 0.01. After 60 epoches, the accuracy on the test set is 0.9734%.



(a) (b)

Figure 7: The loss and accuacy of different sets over the batches with Nesterov momentum.

4.3.4 Xavier initializtion

We consider using Xavier initializtion. We use a three layer network with a hidden layer with 64 nodes. We use a nesterov momentum of 0.9 and use leaky ReLU as activation function. The initialization method of weights are as described in 4 (c) in Programming assignment 2. Learning rate is 0.01.

0.9751%.

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5 Summary

6 Contributions

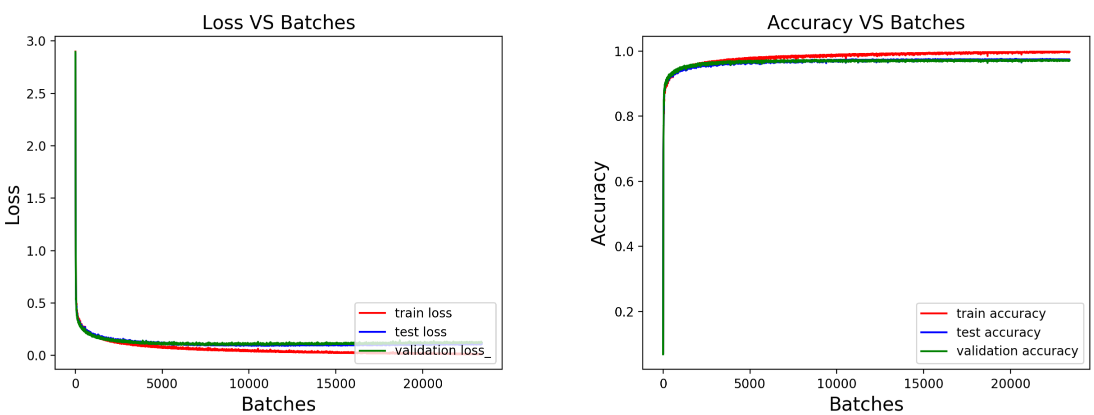
Acknowledgments

We would like to thank Prof. Gary Cottrell and all TAs' efforts in preparing and grading this assignment.

References

[1] LeCun, Y., Bottou, L., Orr, G. B., Mller, K. R. (1998). Efficient backprop. In Neural networks: Tricks of the trade (pp. 9-50). Springer, Berlin, Heidelberg. =======

¿¿¿¿¿¿¿ origin/master



(a) (b)

Figure 8: The loss and accuacy of different sets over the batches with Xavier initializtion.