Homework 3

Your Name: **TA sample solutions**

Student ID:

Using Case Study 1 data, Case1_Student Grades_Large.csv
Using student demographic info and learning behaviors (weekly hrs in different categories) to predict "Grade"

Note: exclude nominal variables, and student performance variables (e.g., Exam) from the list of x variables
Using hold-out evaluation only, 80% as training
Use feature selection to build multiple models, and compare the models based on RMSE

- Backward method using p-value in t-test as metric
- Backward method using AIC as metric
- Forward method using AIC as metric
- Stepwise method using ACI as metric
- Best subset method using Adj-R2 as metric

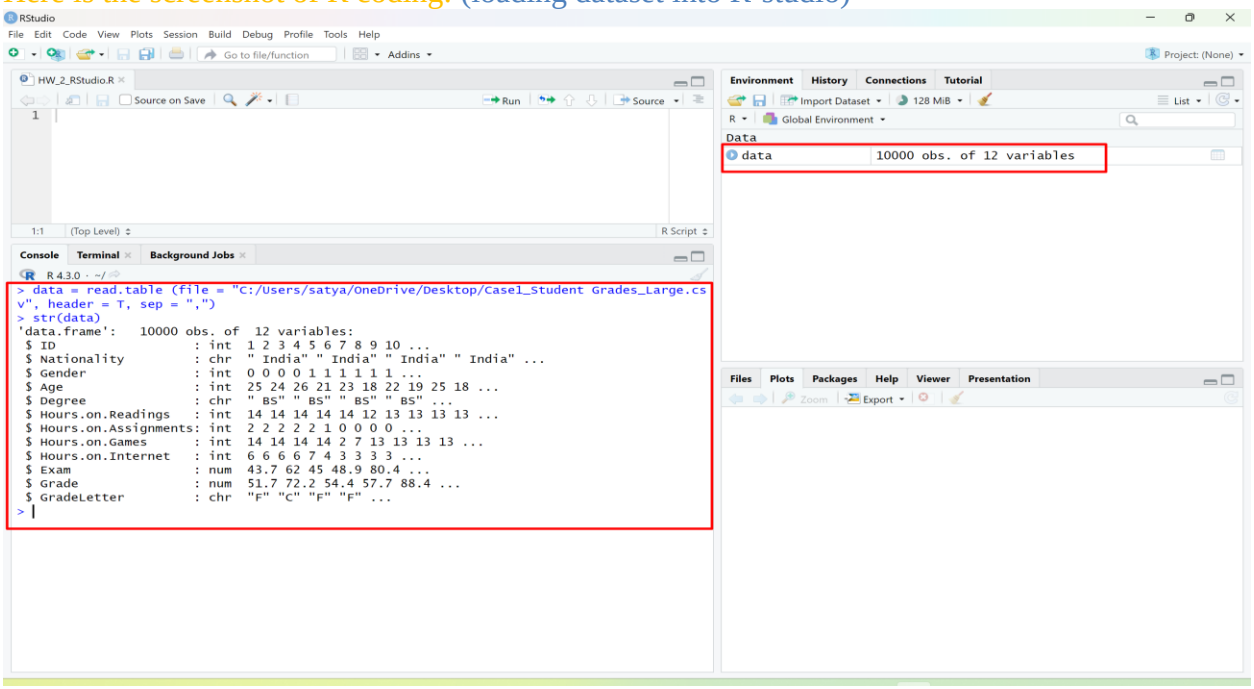
Q1 Show the R coding, outputs, and <u>your explanations</u> for each step in linear regression.

ANSWERS:

Importing the data into R- studio:

- First, we need to import the dataset into R-studio. Here we used "Case1_Student Grades_Large.csv".

Here is the screenshot of R coding: (loading dataset into R-studio)



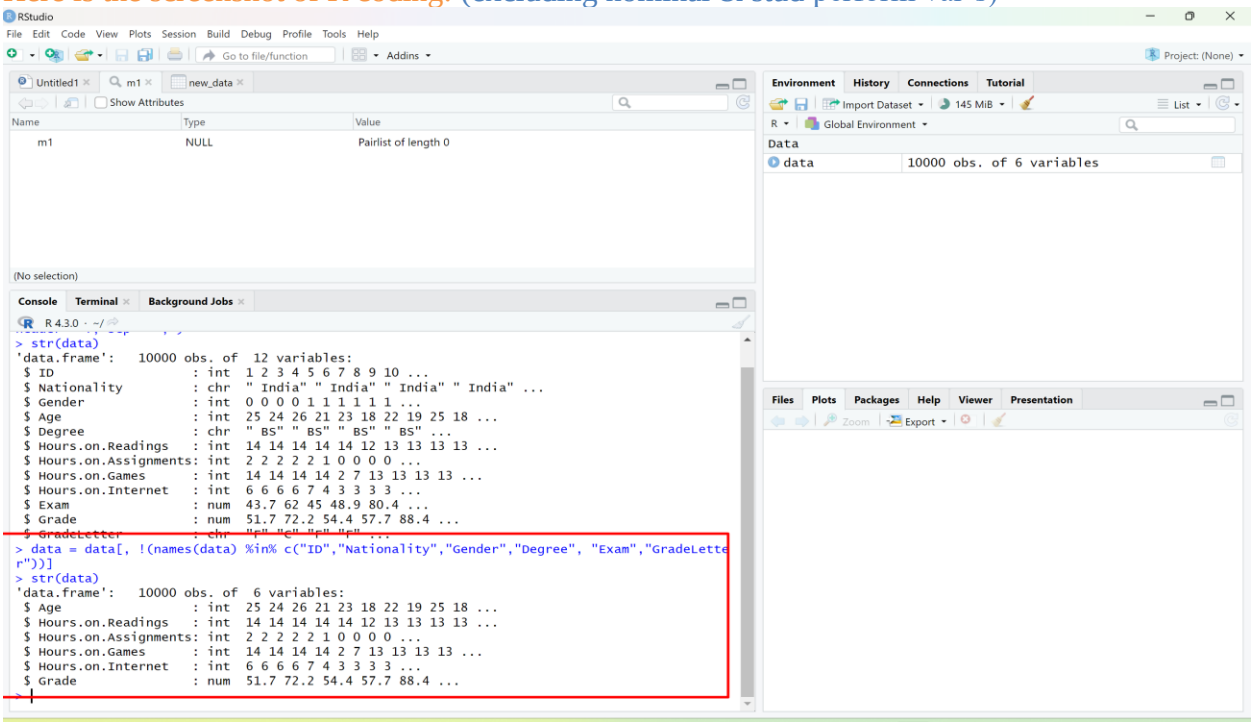
Step 1: Understand the data & well define independent variables & dependent variables:

- Here, X variables represents the predictor variables, and Y variable represents the target variable.
    - Y variable (target variable/ Dependent variable): Grade
    - X variables (Predictors/ Factors/Independent variables): ID, Nationality, Gender, Age, Degree, Hours on Readings, Hours on Assignments, Hours on Games, Hours on Internet, Exam, Grade Letter)

Excluding nominal & student performance variables (Exam) from the list of X-variables to perform linear regression:

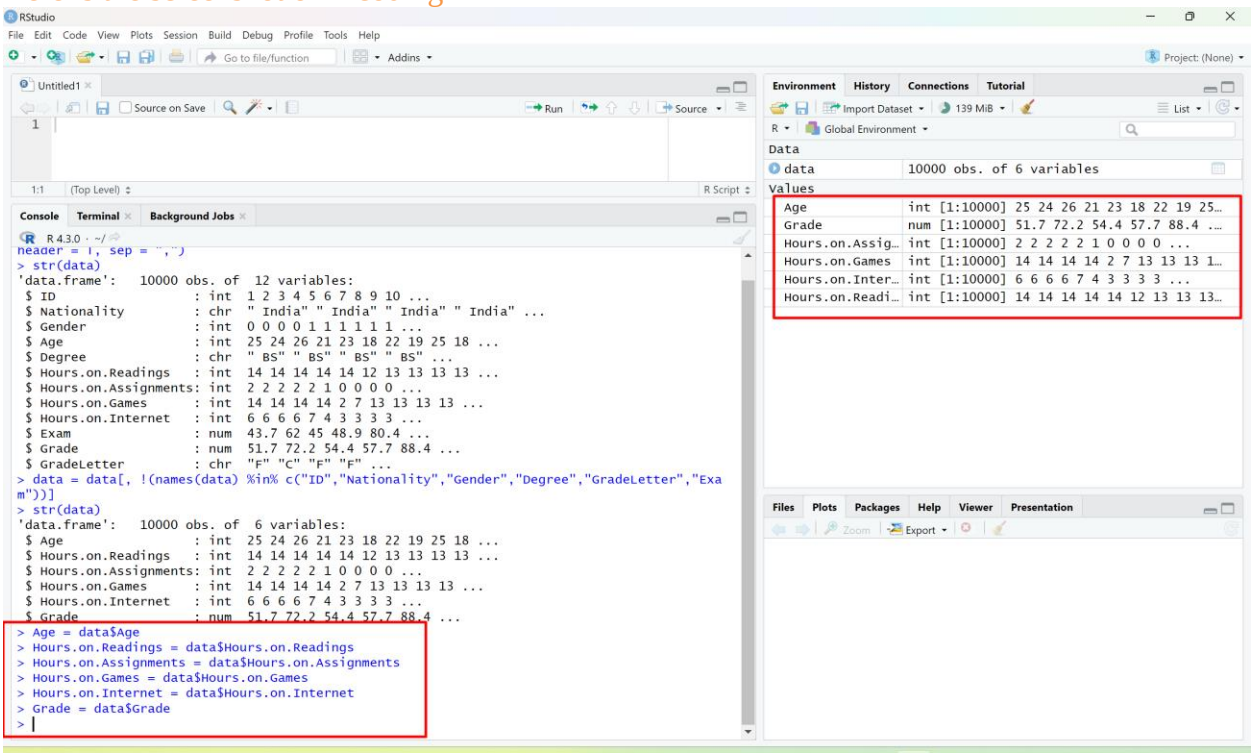
- As per the requirement in the question, we need to exclude nominal variables, and student performance variables (e.g., Exam) from the list of x variables to perform linear regression.
- So, the variables need to be excluded from the list of x-variables are:
    - ID
    - Nationality
    - Gender
    - Degree
    - Exam
    - GradeLetter
- So, the final X-variable list contains the following variables:
    - Age
    - Hours on Readings
    - Hours on Assignments
    - Hours on Games
    - Hours on Internet

Here is the screenshot of R coding: (excluding nominal & stud perform var's)



Storing X and Y variables:

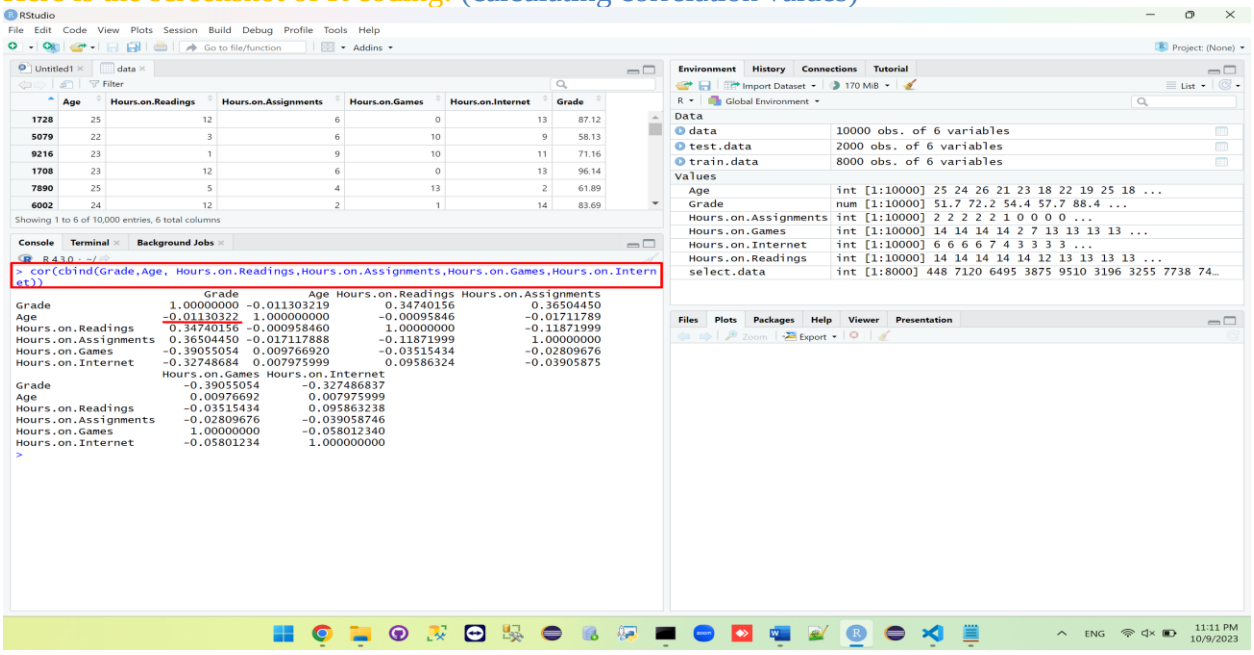
Here is the screenshot of R coding



Step 2: Examine the linear relationship between x and y variables.

- We can examine linearity by 2 methods.
    1. Produce a scatter plot for each x & y variable / produce a single plot with every pair of variables.
    2. Calculate correlation values for all the x variables with y variable.
- As the plot method is not clear & reliable all the time, we can perform 2nd method and calculate the correlation values.

Here is the screenshot of R coding: (calculating correlation values)

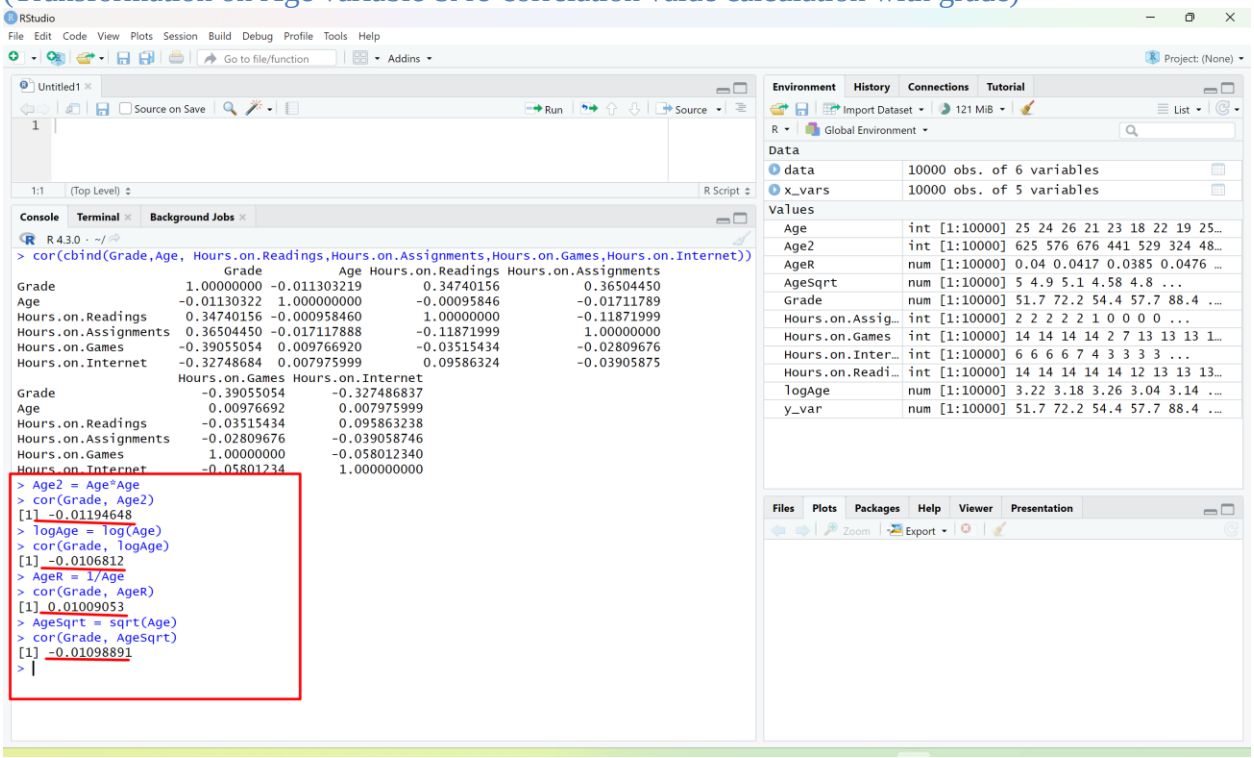


- We noticed that the correlation values are as follows:
  - Grade & Hours on readings = 0.34740156 - weak Correlation - +ve correlation
  - Grade & Hours on Assignments = 0.36504450 - weak Correlation - +ve correlation
  - Grade & Hours on games = -0.39055054 - weak Correlation – -ve correlation
  - Grade & Hours on internet = -0.32748684 - weak Correlation – -ve correlation

  - Grade & Age = -0.01130322 – no Correlation – Try transformation to improve correlation.
- As Age has no correlation with Grade(Y-variable), we can try transformation on this Age (x- variable) and re-calculate the correlation with Grade (y-variable) again.

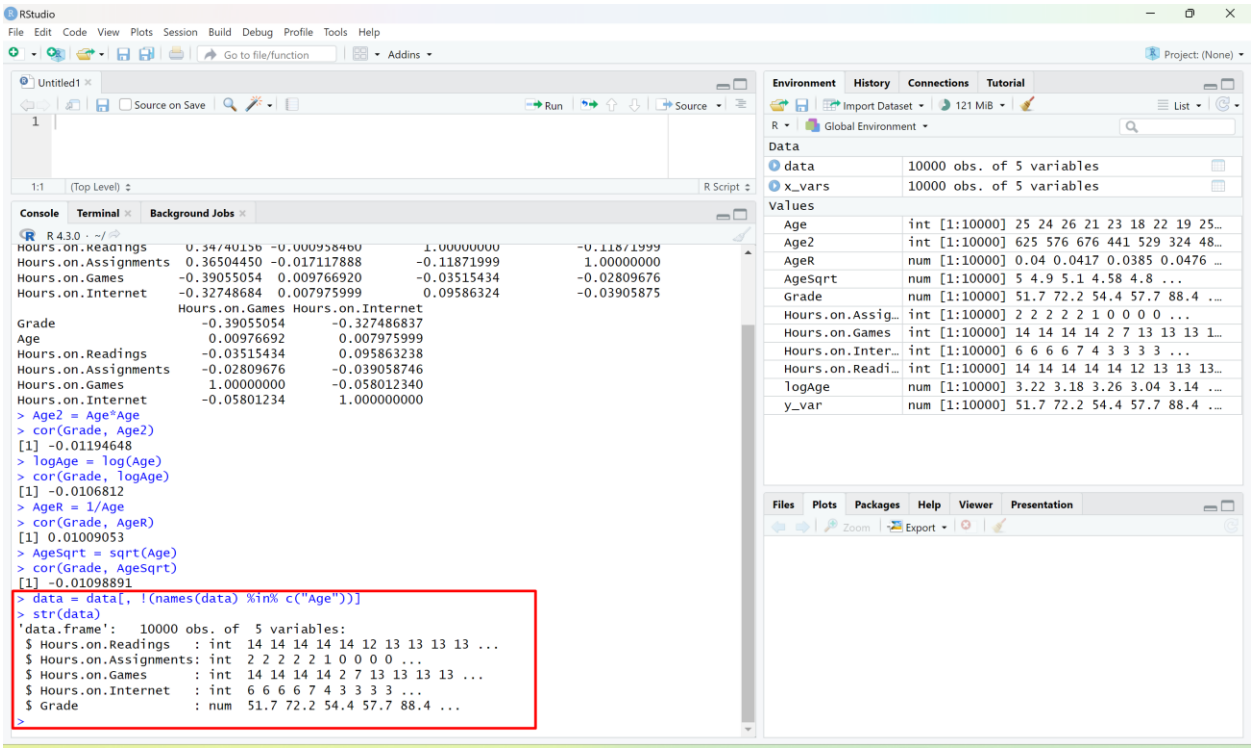
    - Square transformation: X' = X * X
    - Log transformation: X' = logX
    - Inversion transformation: X' = 1/X
    - Square root transformation: X' = sqrt(X)

Here is the screenshots of R coding:
(Transformation on Age variable & re-correlation value calculation with grade)



- As the correlation didn't improve after transformation on 'Age' variable, we decided to drop the age variable.

Here Is the screenshot of r coding: (dropping the age variable)

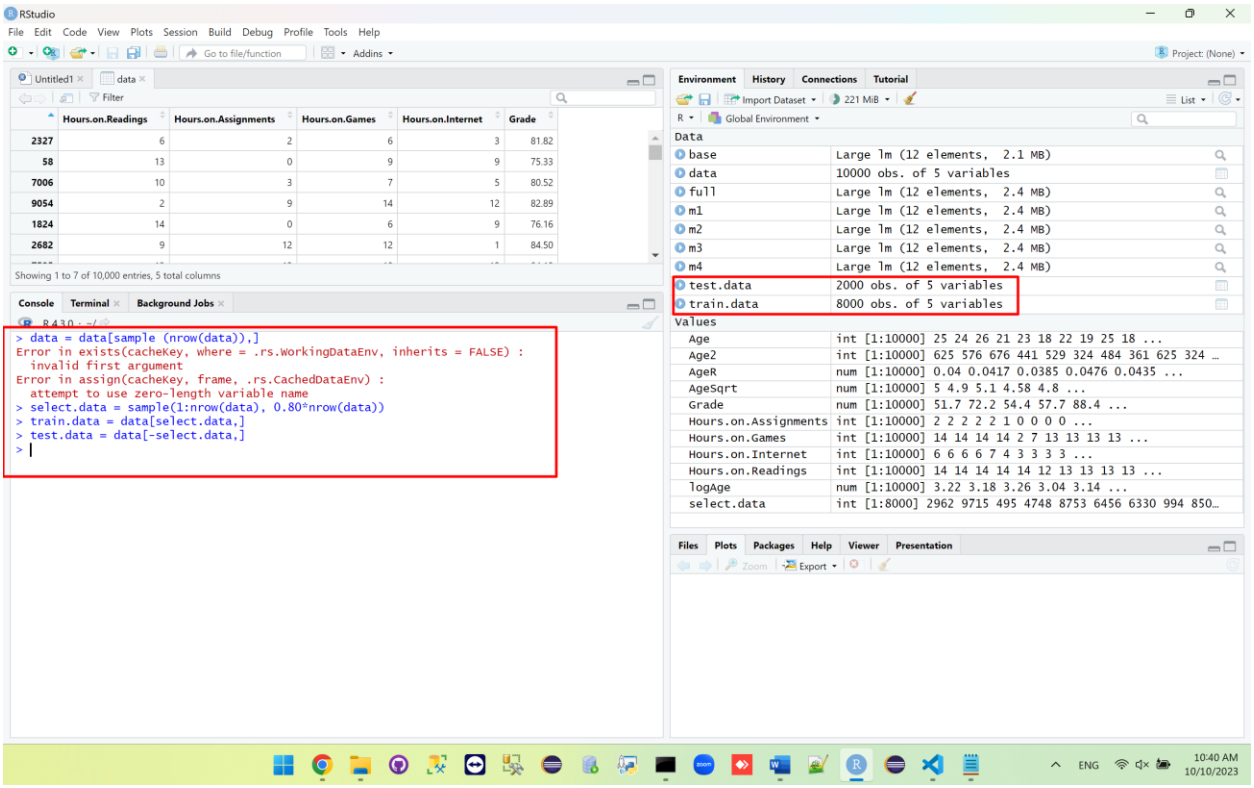


- Now the list of X-variables (Age is removed) (5 variables)
    - Hours.on.Readings
    - Hours.on.Assignments
    - Hours.on.Games
    - Hours.on.Internet
- Y-variable:
    - Grade

Step 3: Decision on evaluation strategy and data splits:

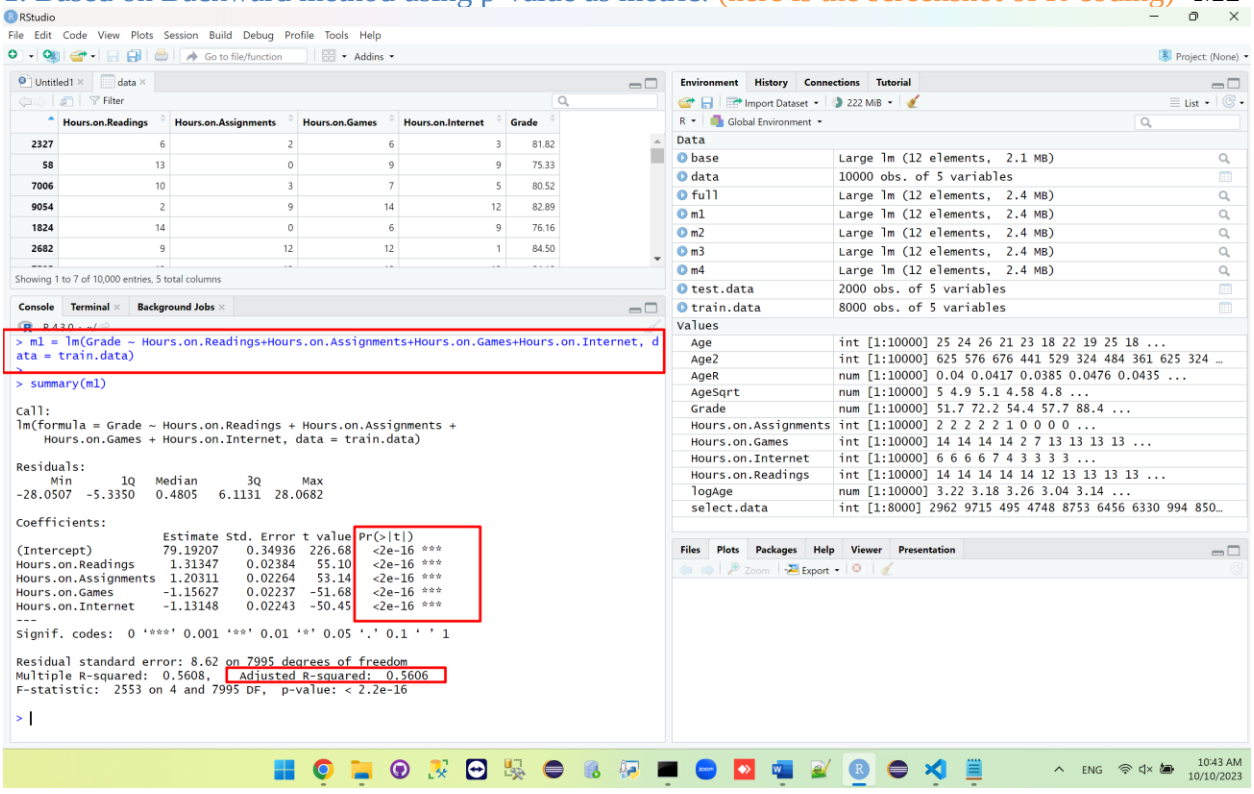
- As mentioned in the question, we need to use hold out evaluation for this data split.
- Note: In general, this data split depends on the size of the data.
    - If the data is large (more than 1 million rows) then hold-out evaluation is used.
    - If the data is small (1 million rows /smaller) then N- fold cross evaluation is used.
    - If your computer is powerful enough, choose N- fold cross evaluation even for the larger data.
    - If the computer is not powerful, then the process may take 5-6 hours, so use hold-out evaluation for the larger data.
    - If your data is small, then definitely choose N-fold cross evaluation.
- Using hold-out evaluation only, **80% as training set and 20% as testing set.**
- First, we need to shuffle the rows. **********important point

Here is the screenshot of R coding:



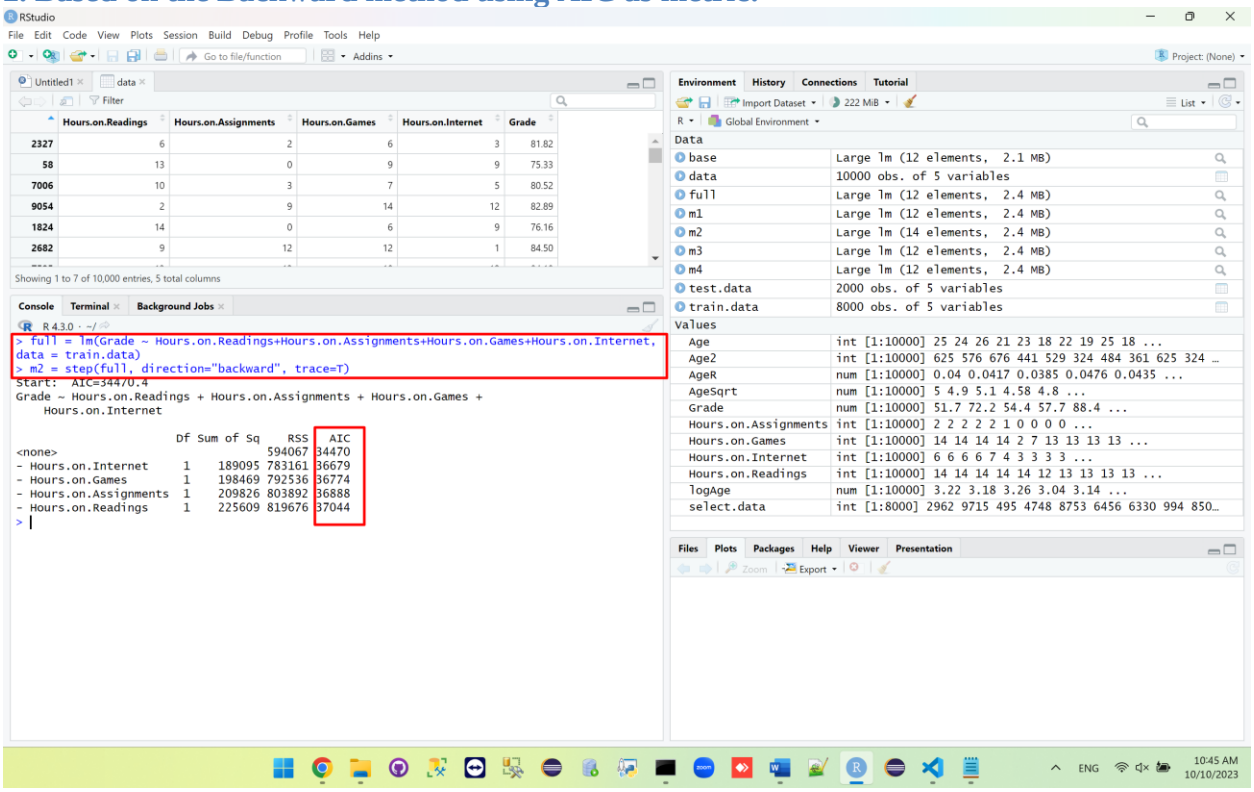
Step 4: Building Multiple linear regression models using 'feature selection' process:

1. Based on Backward method using p-value as metric: (here is the screenshot of R-coding)- **M1**

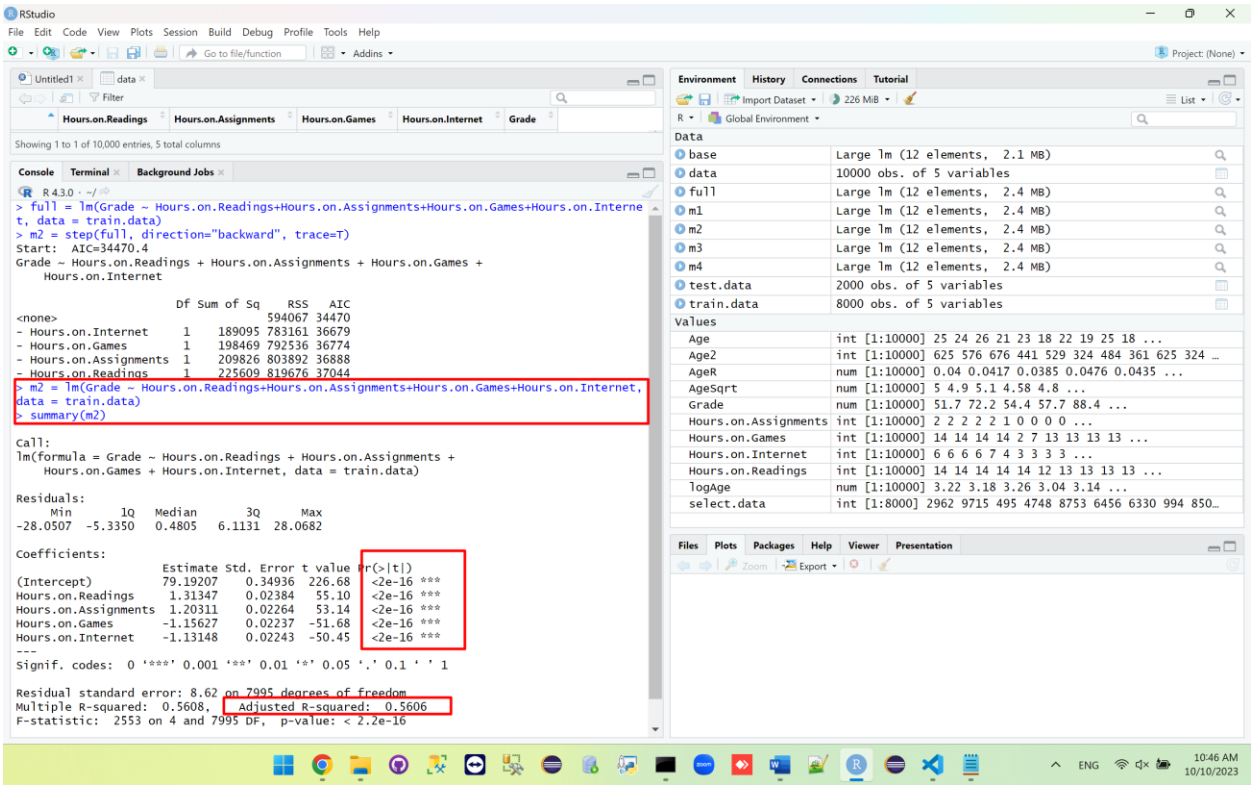


- As all the x'variables have smaller p values than alpha (Assuming using 95% confidence level), all the x-var's are useful to make prediction on Y-var (Grade).
- As no variable is having larger p-value than alpha, there is no need to eliminate any variable 1 by 1 here manually.

2. Based on the Backward method using AIC as metric:

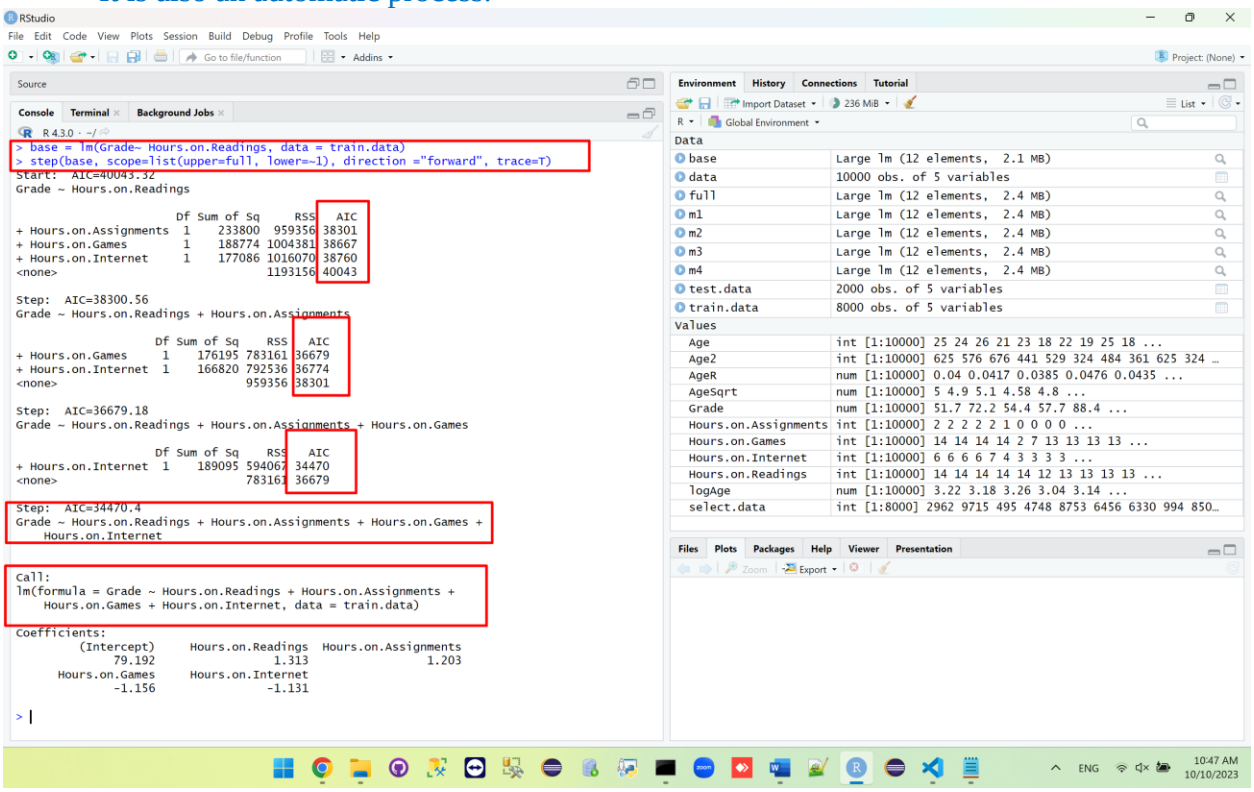


- All the x-variables became useful, so no x-variables got eliminated automatically.
- This is an automatic process.
- So, we got just 1 final step in the output.
- In this output, we can't see p-value, adj r2.
- So, build another model with all the var's that became useful to see adj r2, p-value. **M2**

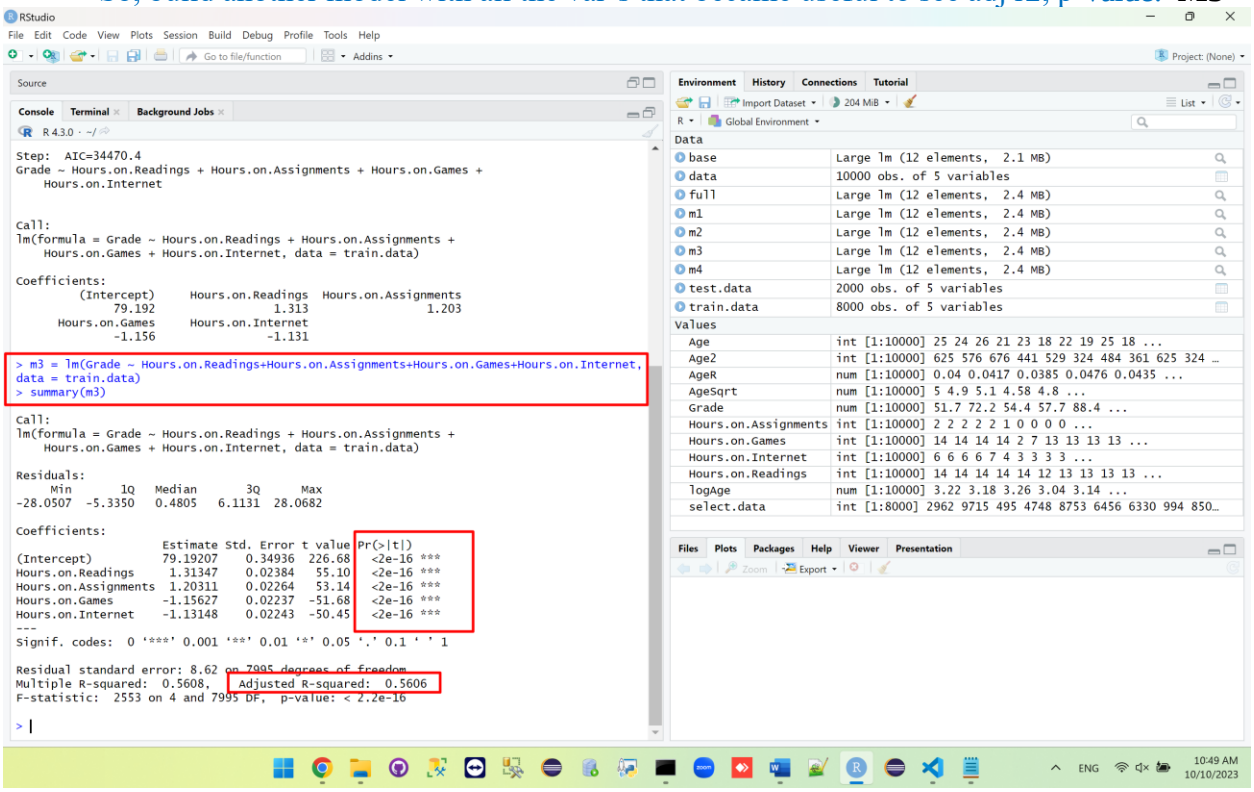


3. Based on Forward method using AIC as metric.

- First build a base model with just 1 X-var and y-var, then use step () function to automatically add remaining variables which are useful one by one.
- It is also an automatic process.

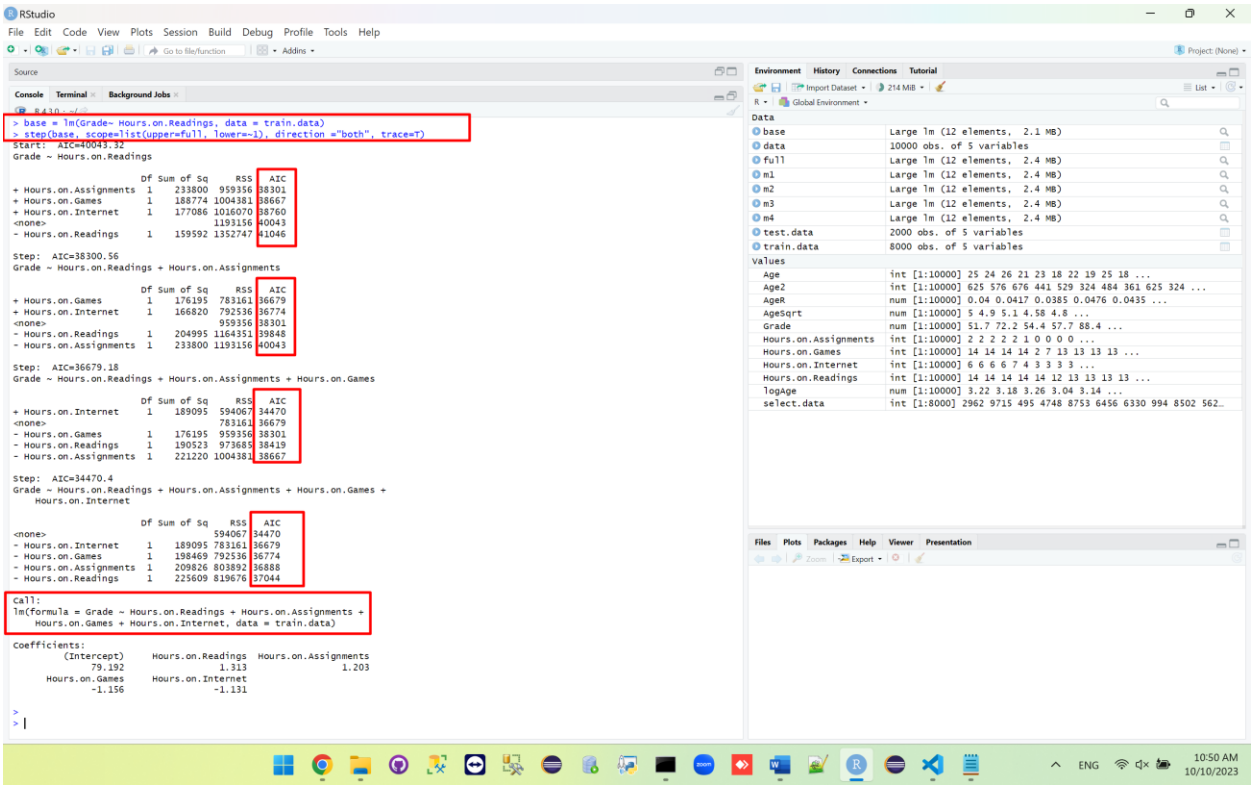


- In this output, we can't see p-value, adj r2.
- So, build another model with all the var's that became useful to see adj r2, p-value. **M3**

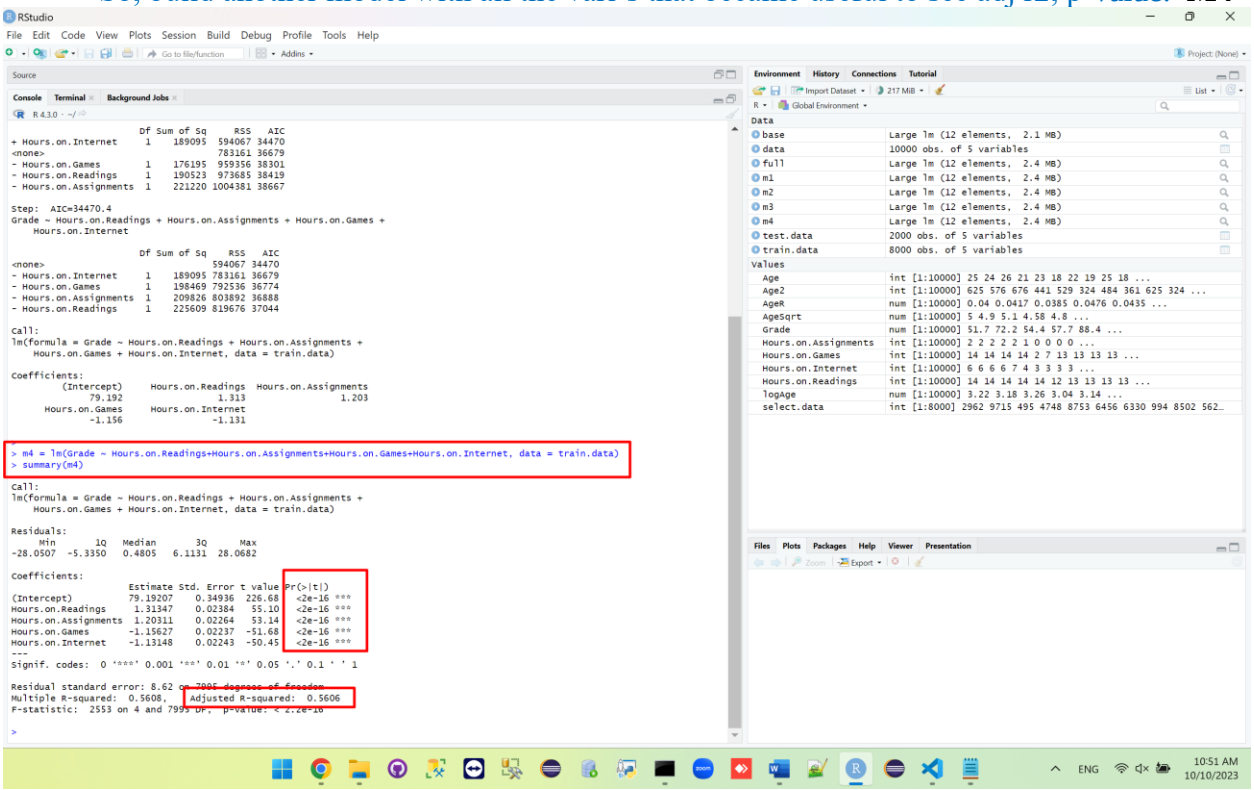


4. Based on Stepwise method using ACI as metric.

- First build a base model with just 1 X-var and y-var, then use step () function to automatically add remaining variables which are useful one by one.
- Set the direction as "both".
- It is also an automatic process.

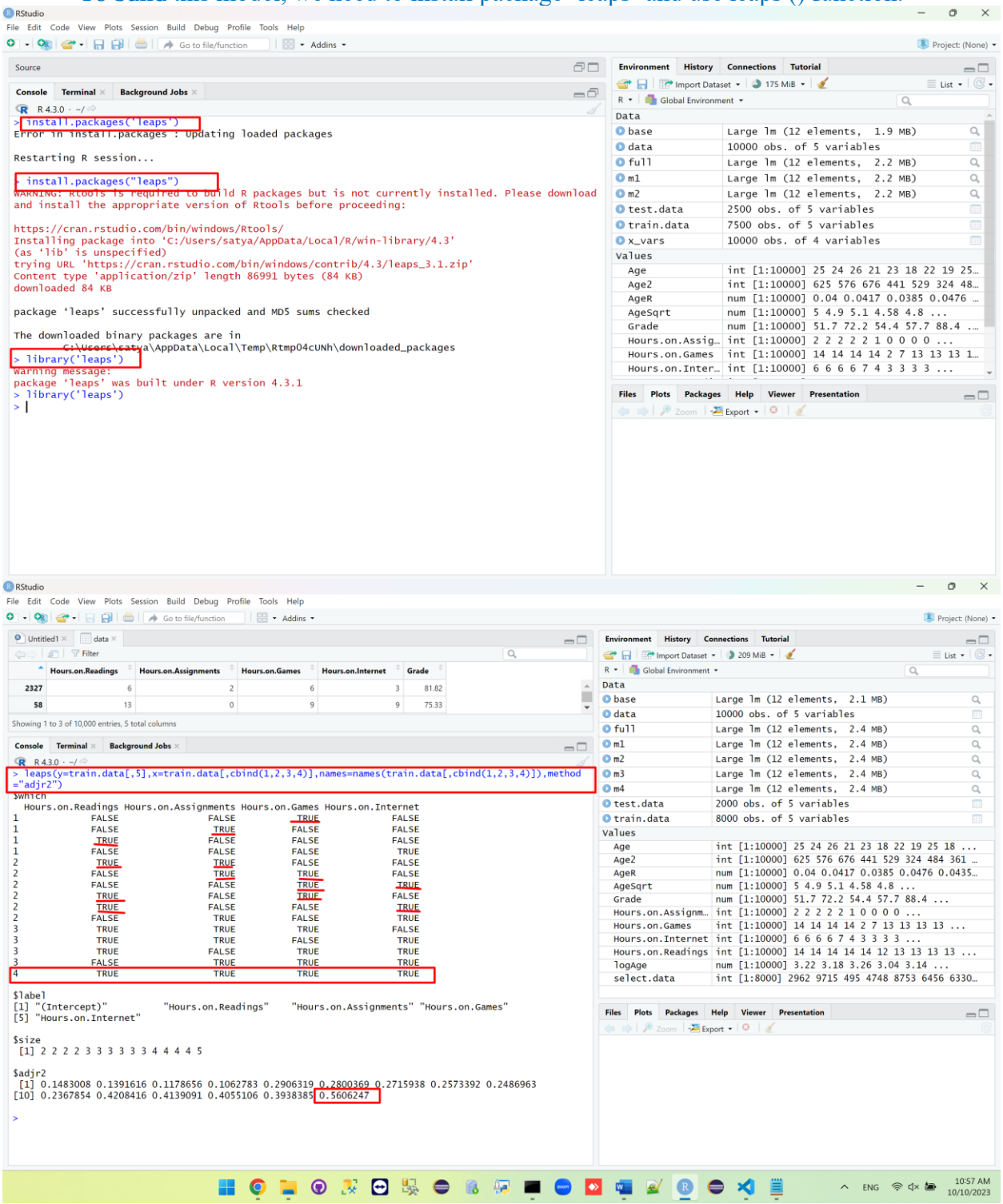


- In this output, we can't see p-value, adj r2.
- So, build another model with all the vari's that became useful to see adj r2, p-value. **M4**



5. Based on Best subset method using Adj-R2 as metric:

- It gives you all the combination of x-variables, then we are going to build a model by calculating a specific metric like adjr2.
- To build this model, we need to install package ‘leaps’ and use leaps () function.



Interpretation of the output:

Subset of x-variables selected are: (true / false output)

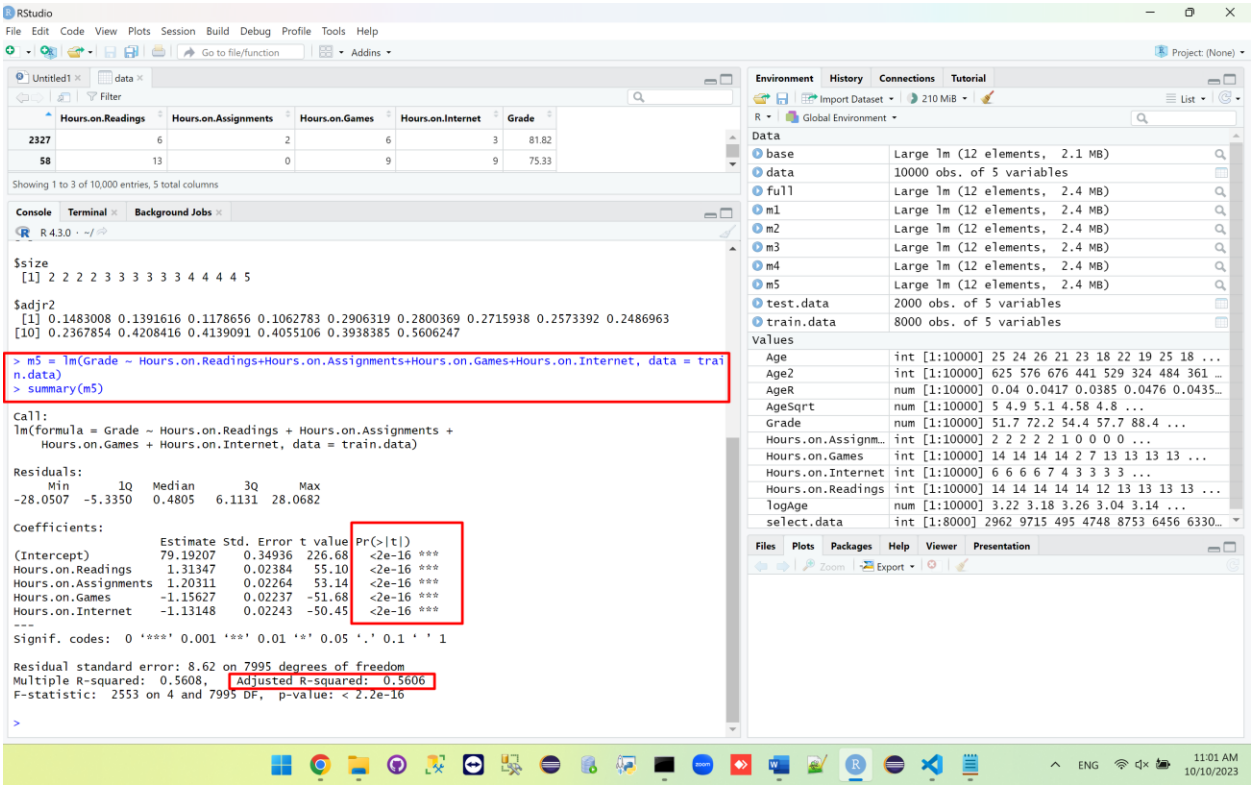
- 1- true -Hours.on.games
- 1 -true- hours.on.assignments
- ..
- ……. selected the variable which has true value in each row.
- ..
- 2 – true – hours on readings & hours on assignments
- ..
- ..
- ……..selected the subset of 2 variables which has true values in each row.
- ..
- ..

Adj r2 output explanation:

- We need to find the best and largest value in adj r2 output.
- The best value is = 0.5608489
- The index of this model is = 15. That means this adj r2 value belongs to 15th model.
- Then we need to look in to the true/false chart and go to the 15th row and check which variables are selected true.

- As all the variables are selected as true, all the variables are useful.
- So, build a model with all these x-var's and y-var Grade and look for the adj r2 in the output.

M5



Now we got 5 models : M1, M2, M3, M4,M5 (same results)

MODEL
M1 : Based on Backward method using p-value as metric:
M2: Based on Backward method using AIC as metric:
M3: Based on Forward method using AIC as metric.
M4: Based on Stepwise method using ACI as metric:
M5: Based on Best subset method using Adj-R2 as metric

Step5 : Model diagnosis: We need to perform model diagnosis for all the models above and compare and see which model is qualified.

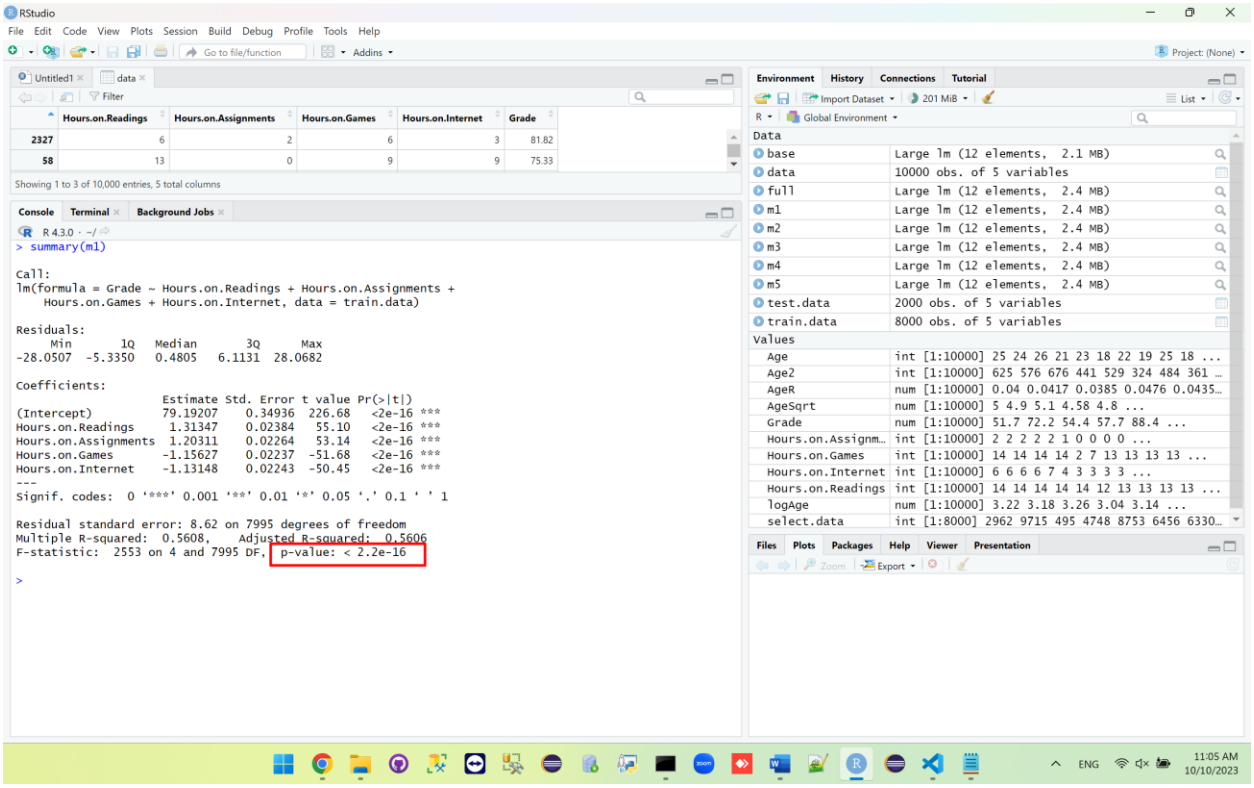
But pls make note that all models are same we can pick any 1 model and perform model diagnosis… Lets pick M1 and go ahead*******

There are 2 components in model diagnosis:

1. F-test (goodness of fit test)
2. Residual analysis

F-test: is a statistical test for hypothesis testing. We need to write down null hypothesis and alternative hypothesis:

F-test on M1:



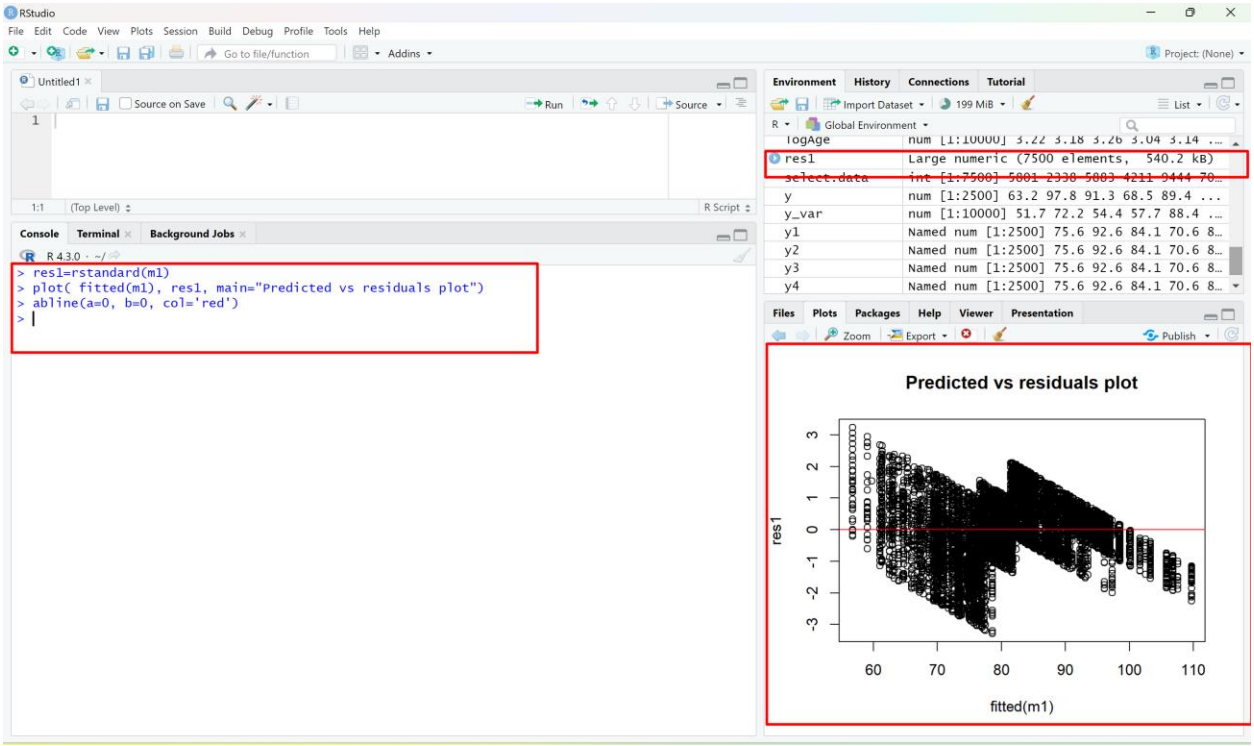
Residual analysis on M1:

- As we built 5 models above through the feature selection process(M1, M2, M3, M4, M5), we need to perform residual analysis for all the models.

**** but. Please note that all the above models are same, So, we can perform residual analysis on any 1 model. As we already picked M1 for model diagnosis and performed f-test on it, now lets perform residual analysis for the same model M1 ****

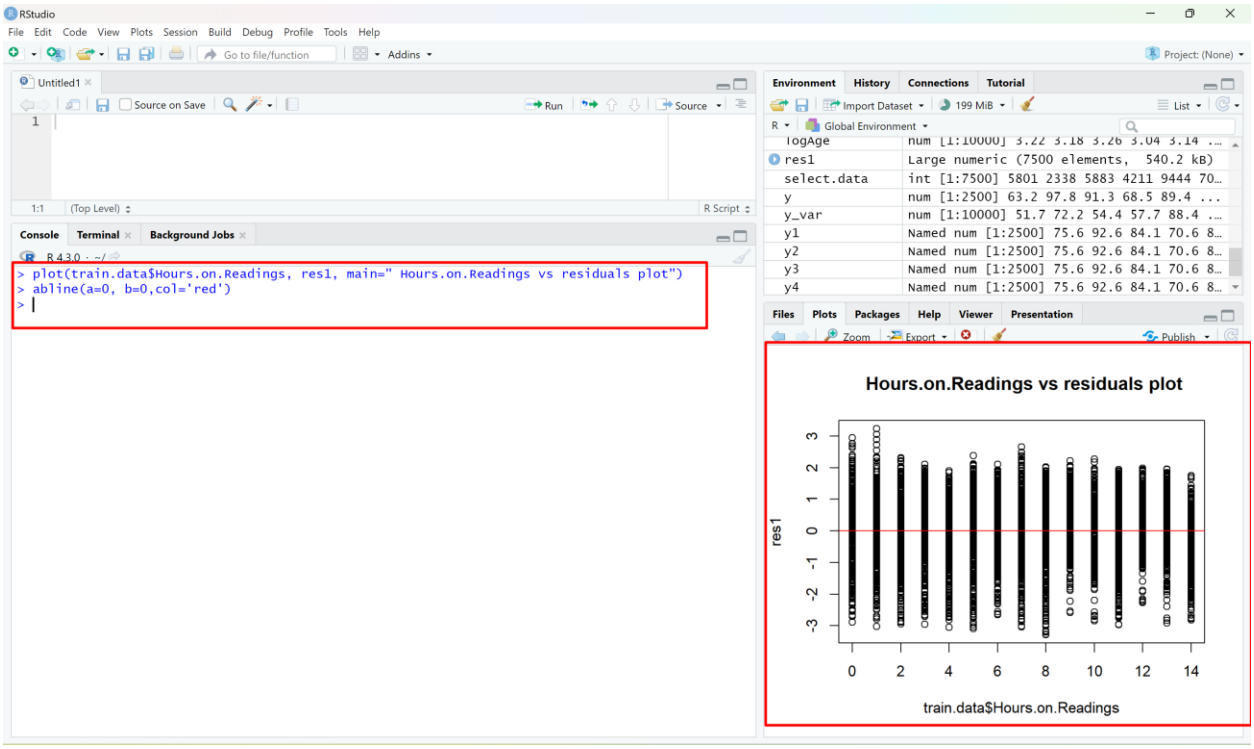
Residual analysis on M1:

1. Validate the constant variance: Plot residuals vs predicted values: To check constant variance for the residuals

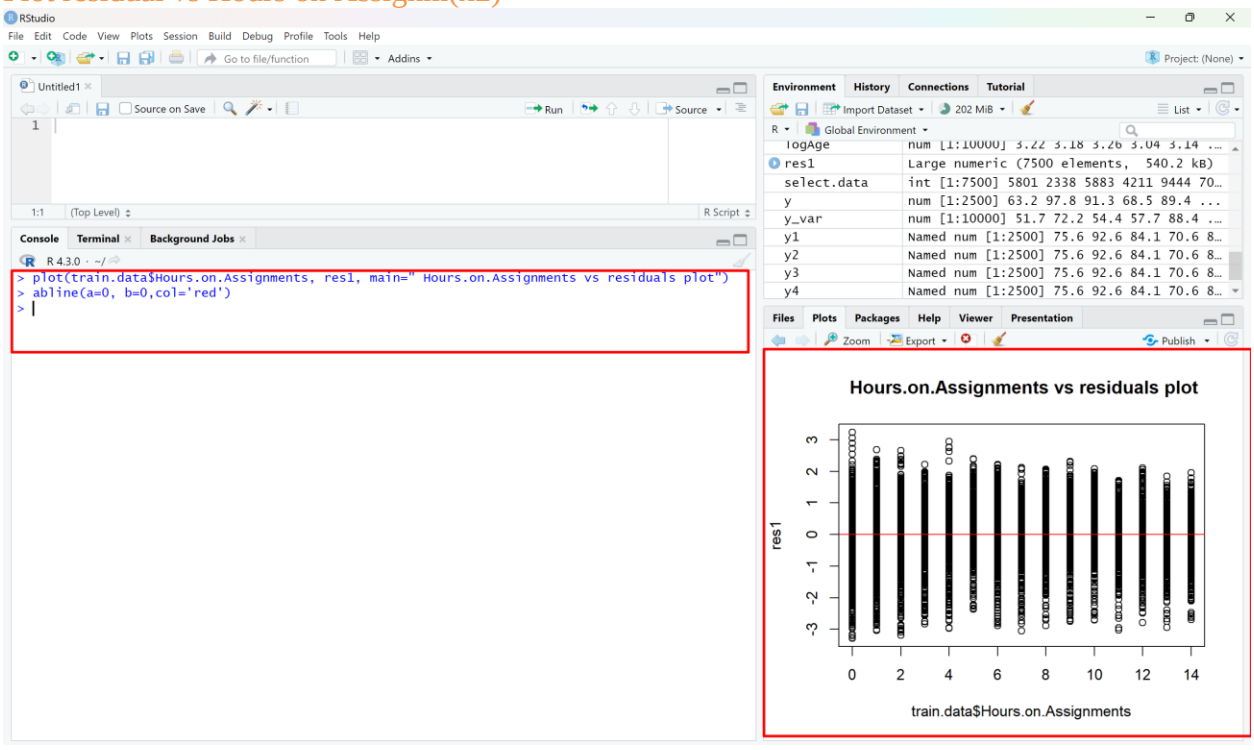


2. Validate the linearity relationship: Plot residuals vs each x-variable: To check linearity assumptions for Y and the x-variable.

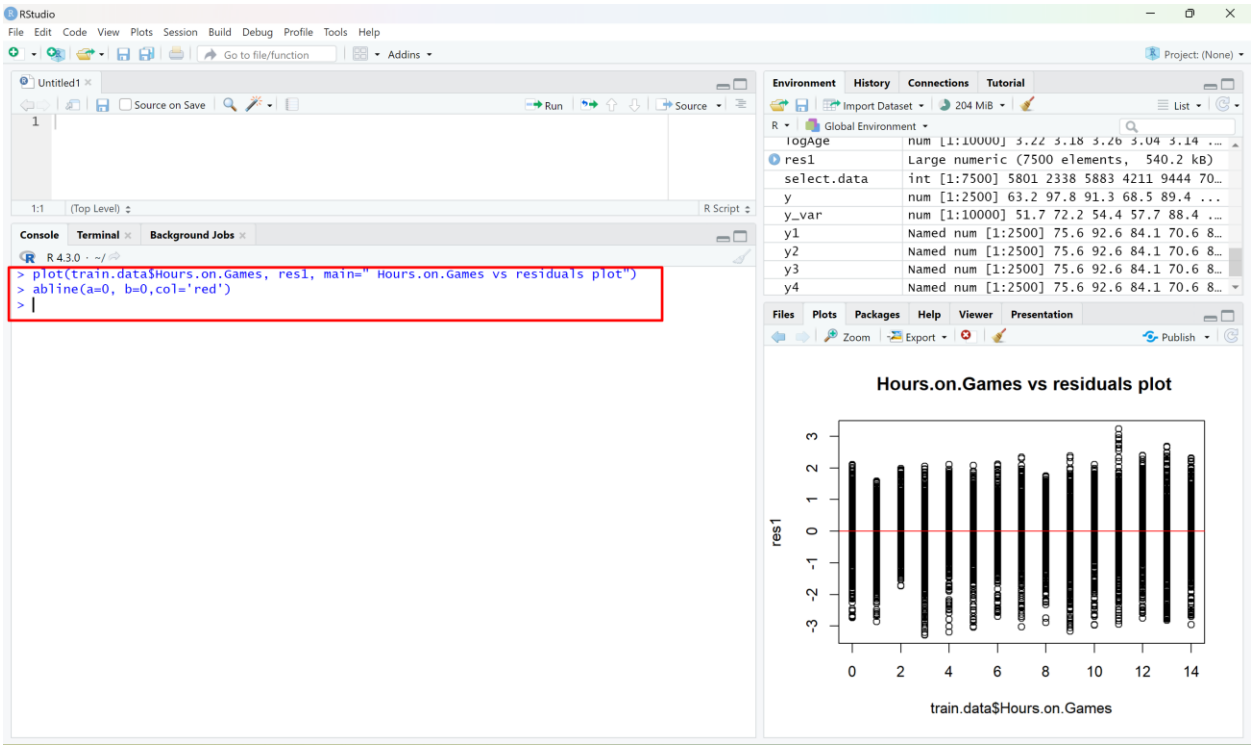
Plot residual vs Hours on Readings(x1)



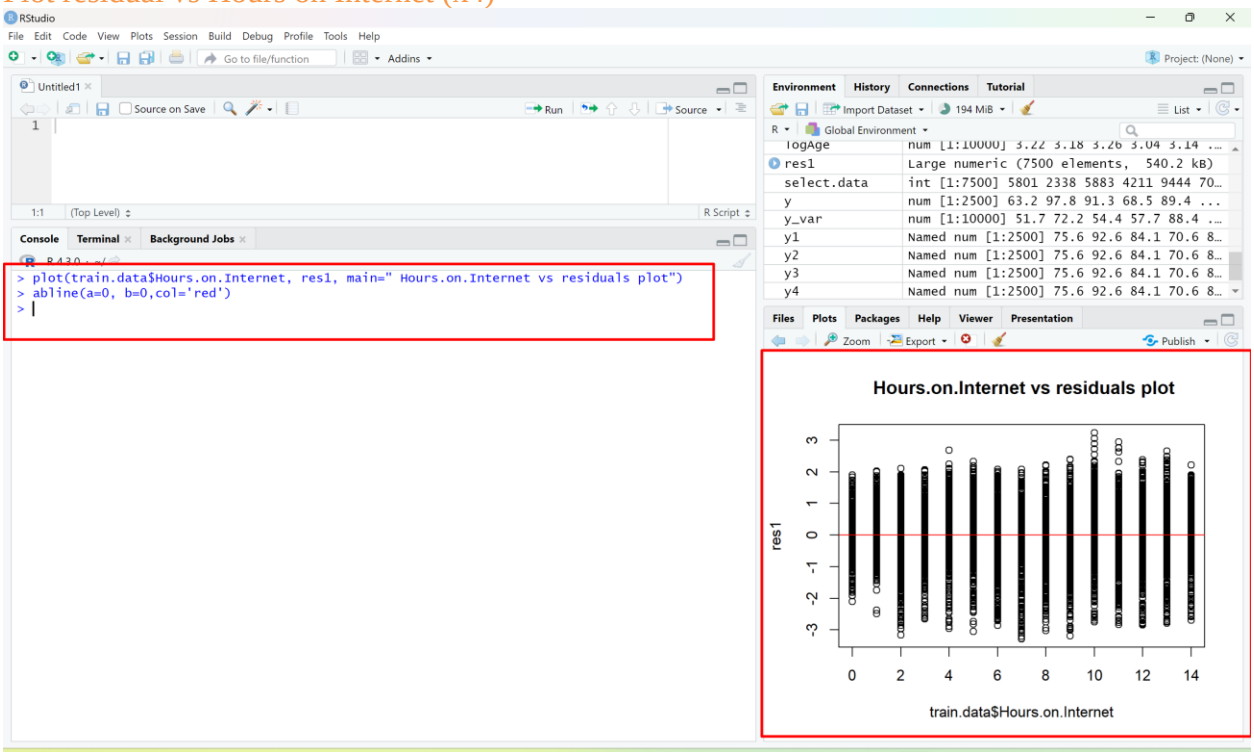
Plot residual vs Hours on Assignm(x2)



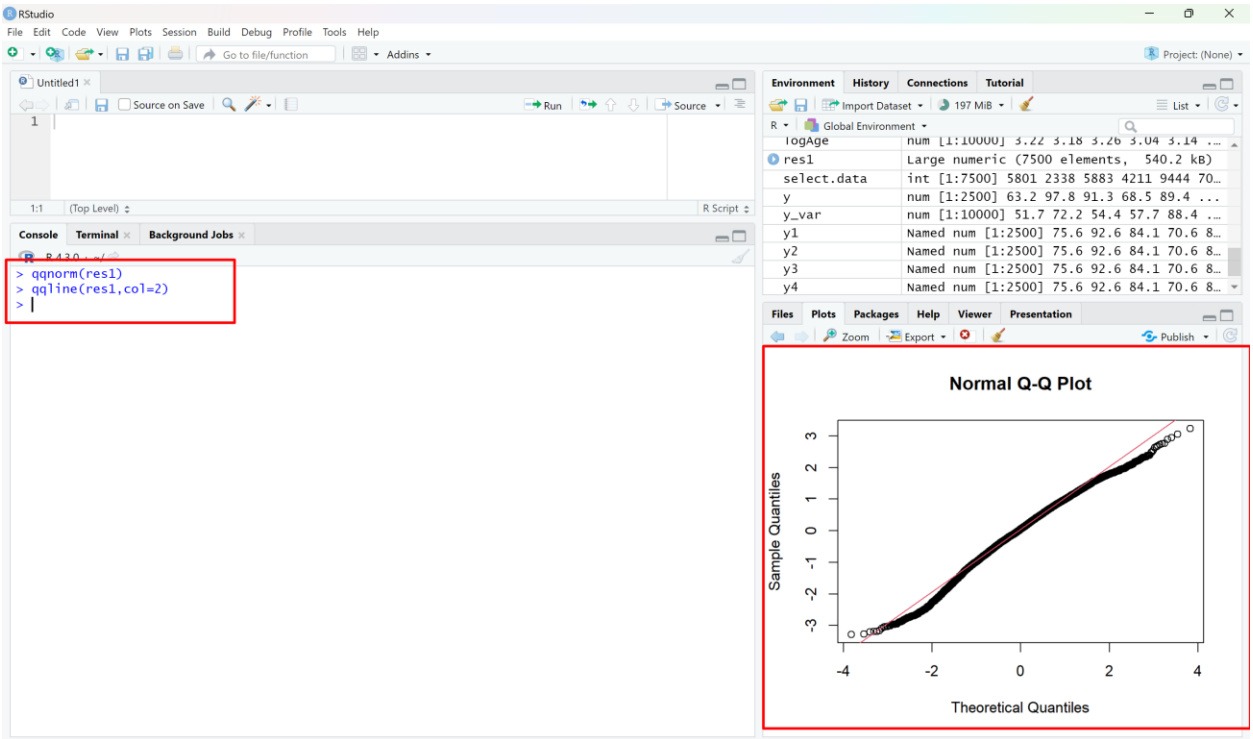
Plot residual vs Hours on Games(x3)



Plot residual vs Hours on Internet (x4)



3. Validate normal distribution of residuals: Draw normal probability plot of residuals: To check normality assumption for the error terms; if points lie close to a line, the errors can be assumed to be approximately normal. Otherwise the assumption of normality is not satisfied.



Conclusion on residual analysis: We can see there is **an issue in the constant variance (step1),** It is Constance in the beginning, but variance became smaller at the end. We can try to apply log transformation on y-variable to see whether we can get a better model.

**********We will solve this issue in Quest 2.**********

Q2 Particularly, you should answer the following questions (at the end of your homework submisson) by using your R coding and outputs.

****Note: As I performed all the steps in linear regression and pasted r-coding screenshots above already, I am using same screenshots here again to answer the below questions*****

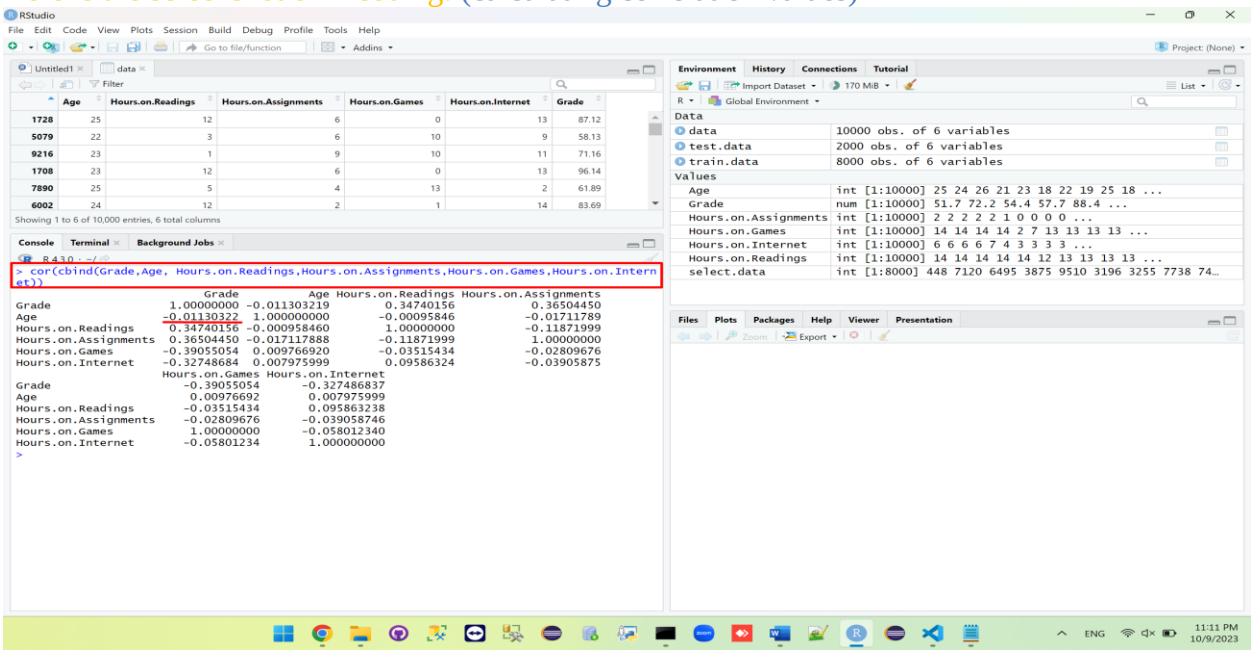
1. Do all x variables have linear relationship with y?

No, 'Age' has no linear relationship with Y. Here is the process of examining & solving the issue:

Examine the linear relationship between x and y variables.

- We can examine linearity by 2 methods.
    3. Produce a scatter plot for each x & y variable / produce a single plot with every pair of variables.
    4. Calculate correlation values for all the x variables with y variable.
- As the plot method is not clear & reliable all the time, we can perform 2nd method and calculate the correlation values.

Here is the screenshot of R coding: (calculating correlation values)

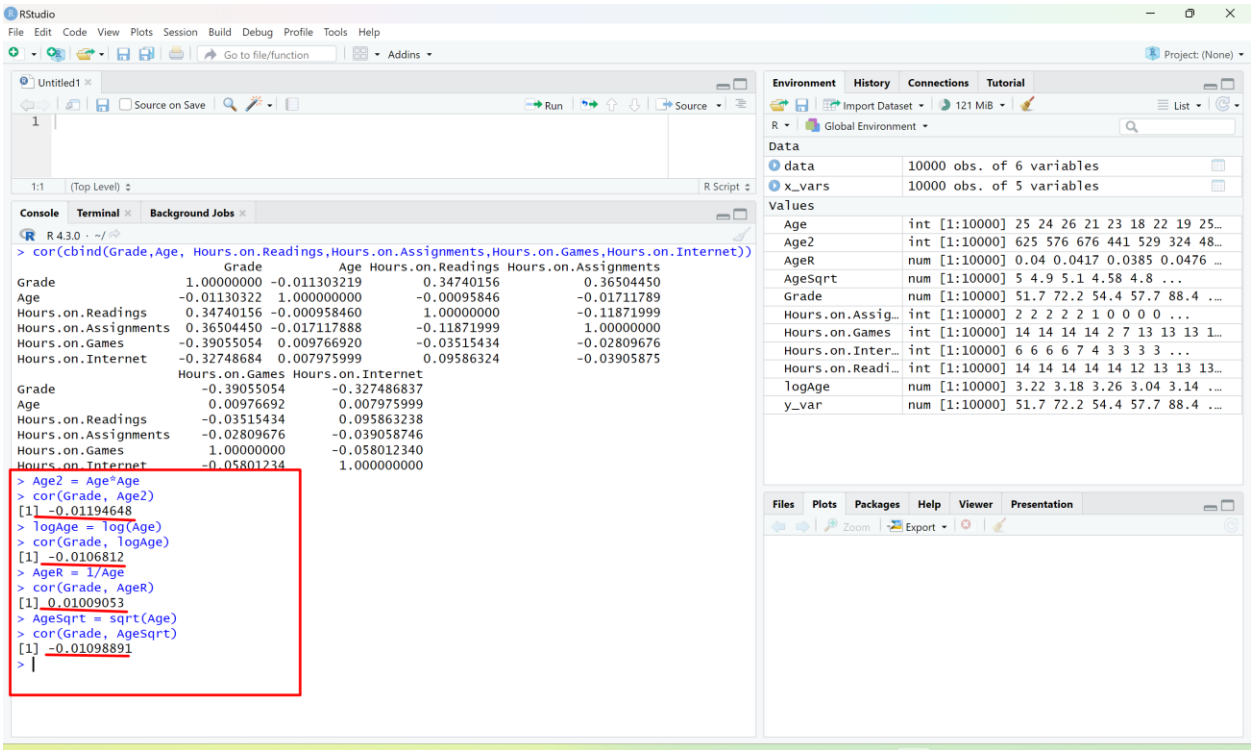


- We noticed that the correlation values are as follows:
  - Grade & Hours on readings = 0.34740156 - weak Correlation - +ve correlation
  - Grade & Hours on Assignments = 0.36504450 - weak Correlation - +ve correlation
  - Grade & Hours on games = -0.39055054 - weak Correlation – -ve correlation
  - Grade & Hours on internet = -0.32748684 - weak Correlation – -ve correlation

  - Grade & Age = -0.01130322 – no Correlation – Try transformation to improve correlation.
- As Age has no correlation with Grade(Y-variable), we can try transformation on this Age (x- variable) and re-calculate the correlation with Grade (y-variable) again.

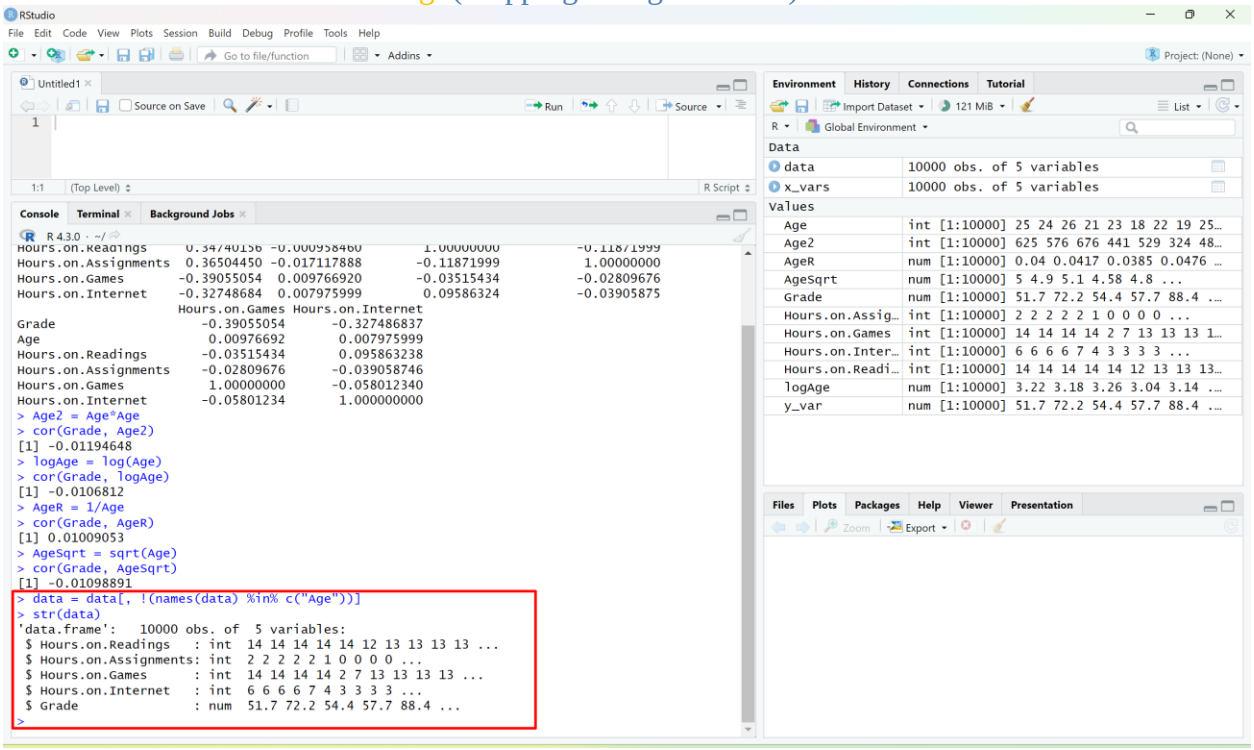
    - Square transformation: X’ = X * X
    - Log transformation: X’ = logX
    - Inversion transformation: X’ = 1/X
    - Square root transformation: X’ = sqrt(X)

Here is the screenshots of R coding:
(Transformation on Age variable & re-correlation value calculation with grade)



- As the correlation didn't improve after transformation on 'Age' variable, we decided to drop the age variable.

Here Is the screenshot of r coding: (dropping the age variable)



2. Write down the null and alternative hypothesis of F-test, use your outputs to draw conclusions of F-test.

ANSWER:

F-test: is a statistical test for hypothesis testing. We need to write down null hypothesis and alternative hypothesis:

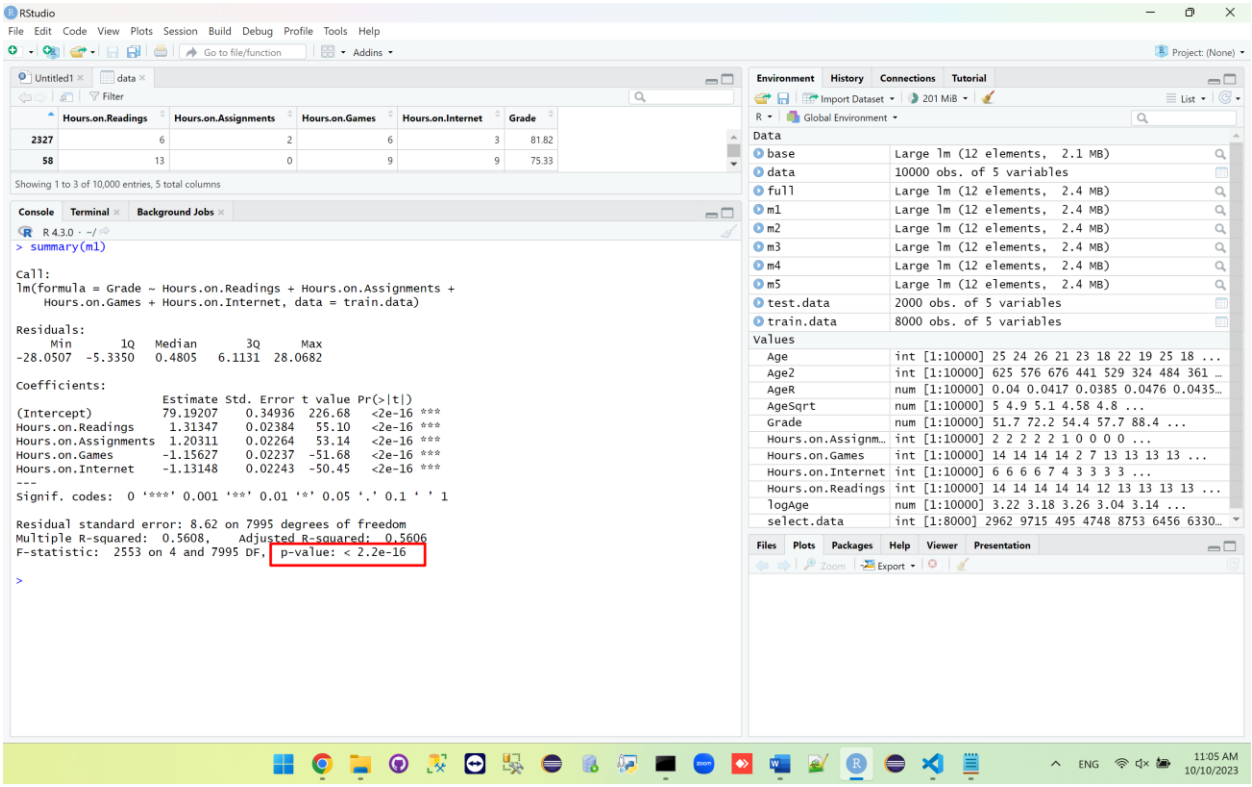
- Null hypothesis : (H0): The coefficients of all x-variables are zero and there is no linear relationship with Grade.
- $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$

- Alternative Hypothesis: At least one of the coefficients of the x-variables (Hours.on.Readings, Hours.on.Assignments, Hours.on.Games, Hours.on.Internet) is not zero and can affect Grade.
- $H_a : \beta_j \neq 0$

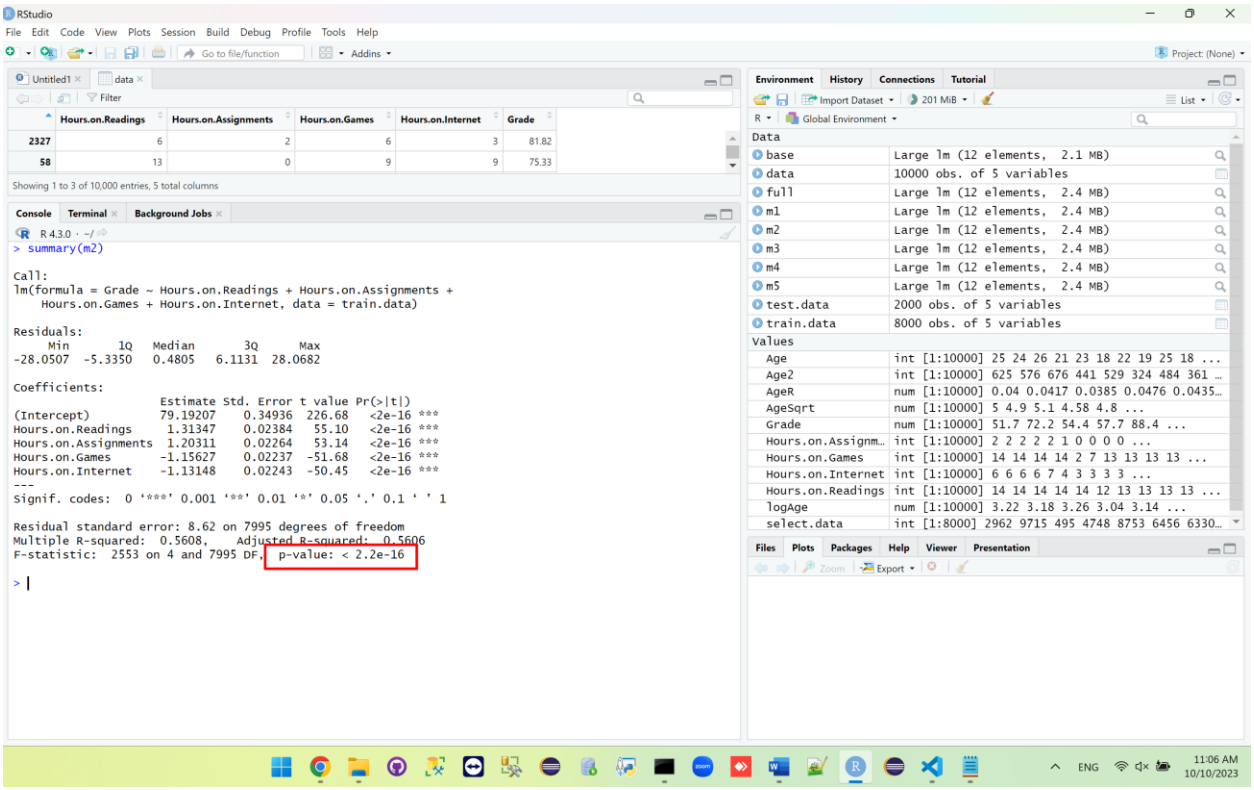
  Note : In these hypotheses, β1 represents the coefficient of "Hours.on.Readings," β2 represents the coefficient of "Hours.on.Assignments," β3 represents the coefficient of "Hours.on.Games," and β4 represents the coefficient of "Hours.on.Internet."

- As we built multiple models above through the feature selection process (M1, M2, M3, M4, M5), we need to do the f- test for all the models. But as all the models are same we can pick any 1 model and perform F-test. Lets pick M1:

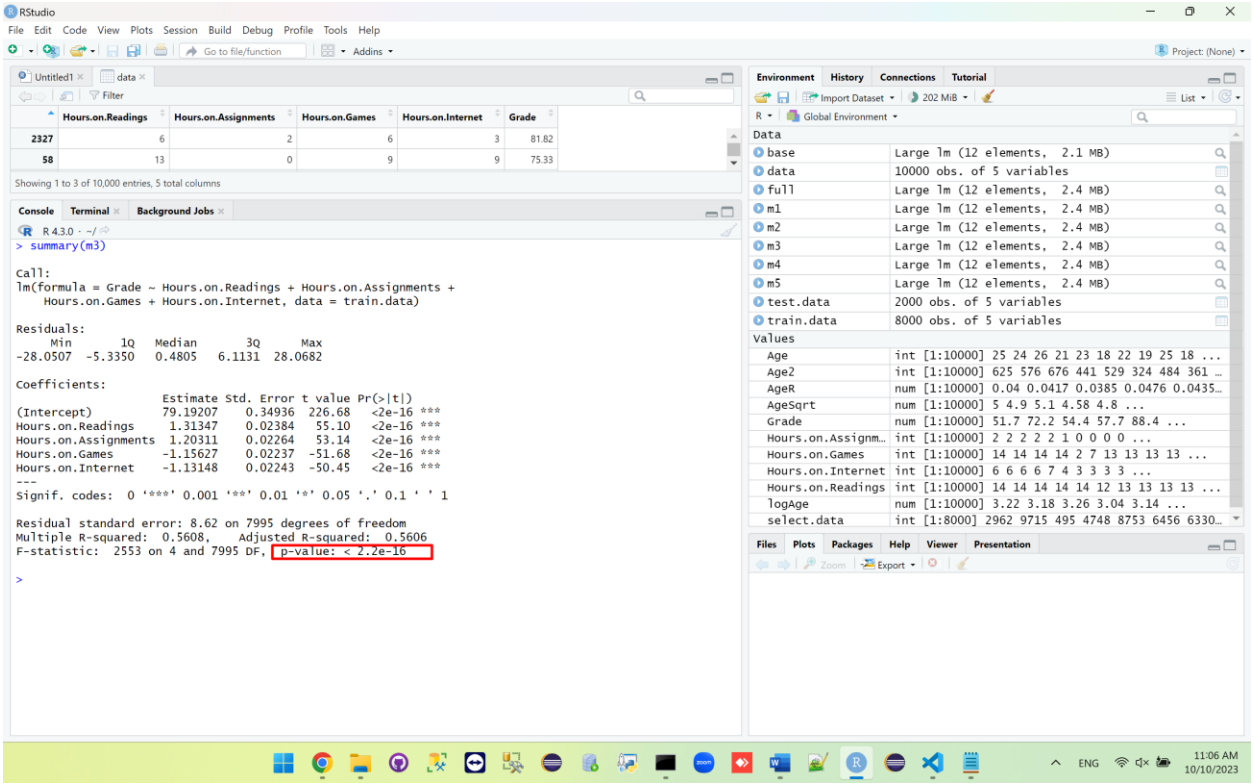
F-test on M1:



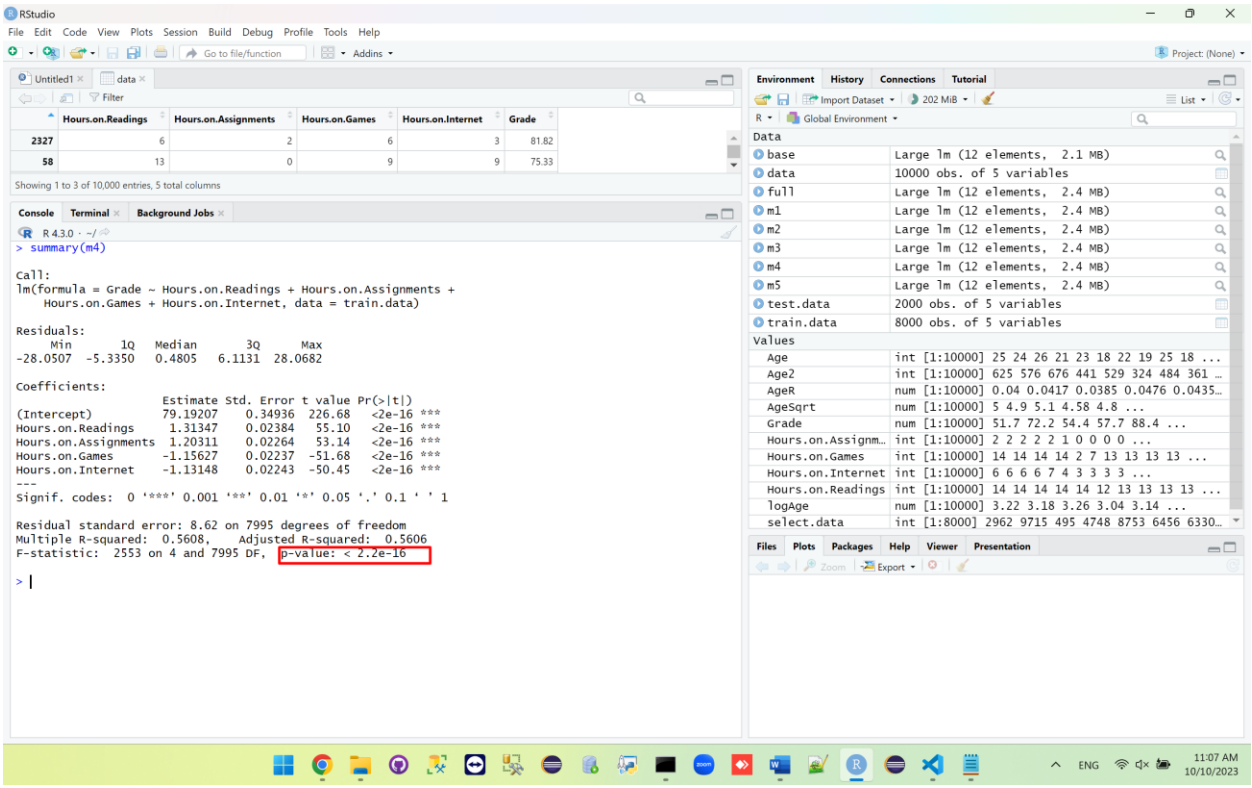
F-test on M2:



F-test on M3:

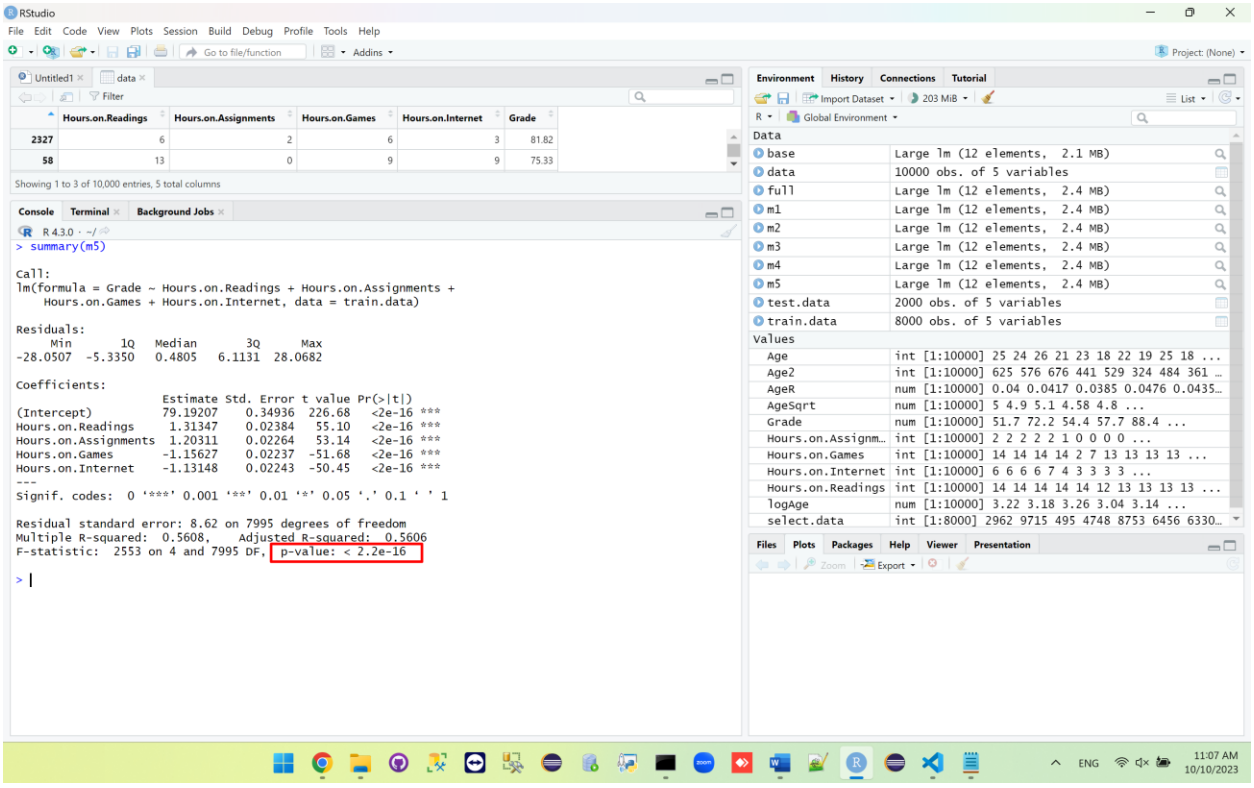


F-test on M4:


RStudio
File Edit Code View Plots Session Build Debug Profile Tools Help
Untitled1
data
Hours.on.Readings Hours.on.Assignments Hours.on.Games Hours.on.Internet Grade
2327 6 2 6 3 81.82
58 13 0 9 9 75.33
Showing 1 to 3 of 10,000 entries, 5 total columns
Console Terminal Background Jobs
R 4.3.0 · ~/
> summary(m4)
Call:
lm(formula = Grade ~ Hours.on.Readings + Hours.on.Assignments +
Hours.on.Games + Hours.on.Internet, data = train.data)
Residuals:
Min 1Q Median 3Q Max
-28.0507 -5.3350 0.4805 6.1131 28.0682
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 79.19207 0.34936 226.68 <2e-16 ***
Hours.on.Readings 1.31347 0.02384 55.10 <2e-16 ***
Hours.on.Assignments 1.20311 0.02264 53.14 <2e-16 ***
Hours.on.Games -1.15627 0.02237 -51.68 <2e-16 ***
Hours.on.Internet -1.13148 0.02243 -50.45 <2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 8.62 on 7995 degrees of freedom
Multiple R-squared: 0.5608, Adjusted R-squared: 0.5606
F-statistic: 2553 on 4 and 7995 DF, p-value: < 2.2e-16
Environment History Connections Tutorial
Import Dataset 202 MiB
Global Environment
Data
base Large lm (12 elements, 2.1 MB)
data 10000 obs. of 5 variables
full Large lm (12 elements, 2.4 MB)
m1 Large lm (12 elements, 2.4 MB)
m2 Large lm (12 elements, 2.4 MB)
m3 Large lm (12 elements, 2.4 MB)
m4 Large lm (12 elements, 2.4 MB)
m5 Large lm (12 elements, 2.4 MB)
test.data 2000 obs. of 5 variables
train.data 8000 obs. of 5 variables
Values
Age int [1:10000] 25 24 26 21 23 18 22 19 25 18 ...
Age2 int [1:10000] 625 576 676 441 529 324 484 361 ...
AgeR num [1:10000] 0.04 0.0417 0.0385 0.0476 0.0435...
AgeSqrt num [1:10000] 5 4.9 5.1 4.58 4.8 ...
Grade num [1:10000] 51.7 72.2 54.4 57.7 88.4 ...
Hours.on.Assignm... int [1:10000] 2 2 2 2 2 1 0 0 0 0 ...
Hours.on.Games int [1:10000] 14 14 14 14 2 7 13 13 13 13 ...
Hours.on.Internet int [1:10000] 6 6 6 6 7 4 3 3 3 3 ...
Hours.on.Readings int [1:10000] 14 14 14 14 14 12 13 13 13 13 ...
logAge num [1:10000] 3.22 3.18 3.26 3.04 3.14 ...
select.data int [1:8000] 2962 9715 495 4748 8753 6456 6330...
Files Plots Packages Help Viewer Presentation
Zoom Export
ENG
11:07 AM
10/10/2023

F-test on M5:


RStudio
File Edit Code View Plots Session Build Debug Profile Tools Help
Untitled1
data
Hours.on.Readings Hours.on.Assignments Hours.on.Games Hours.on.Internet Grade
2327 6 2 6 3 81.82
58 13 0 9 9 75.33
Showing 1 to 3 of 10,000 entries, 5 total columns
Console Terminal Background Jobs
R 4.3.0 · ~/
> summary(m5)
Call:
lm(formula = Grade ~ Hours.on.Readings + Hours.on.Assignments +
Hours.on.Games + Hours.on.Internet, data = train.data)
Residuals:
Min 1Q Median 3Q Max
-28.0507 -5.3350 0.4805 6.1131 28.0682
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 79.19207 0.34936 226.68 <2e-16 ***
Hours.on.Readings 1.31347 0.02384 55.10 <2e-16 ***
Hours.on.Assignments 1.20311 0.02264 53.14 <2e-16 ***
Hours.on.Games -1.15627 0.02237 -51.68 <2e-16 ***
Hours.on.Internet -1.13148 0.02243 -50.45 <2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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F-statistic: 2553 on 4 and 7995 DF, p-value: < 2.2e-16
Environment History Connections Tutorial
Import Dataset 203 MiB
Global Environment
Data
base Large lm (12 elements, 2.1 MB)
data 10000 obs. of 5 variables
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m1 Large lm (12 elements, 2.4 MB)
m2 Large lm (12 elements, 2.4 MB)
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m5 Large lm (12 elements, 2.4 MB)
test.data 2000 obs. of 5 variables
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Values
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Age2 int [1:10000] 625 576 676 441 529 324 484 361 ...
AgeR num [1:10000] 0.04 0.0417 0.0385 0.0476 0.0435...
AgeSqrt num [1:10000] 5 4.9 5.1 4.58 4.8 ...
Grade num [1:10000] 51.7 72.2 54.4 57.7 88.4 ...
Hours.on.Assignm... int [1:10000] 2 2 2 2 2 1 0 0 0 0 ...
Hours.on.Games int [1:10000] 14 14 14 14 2 7 13 13 13 13 ...
Hours.on.Internet int [1:10000] 6 6 6 6 7 4 3 3 3 3 ...
Hours.on.Readings int [1:10000] 14 14 14 14 14 12 13 13 13 13 ...
logAge num [1:10000] 3.22 3.18 3.26 3.04 3.14 ...
select.data int [1:8000] 2962 9715 495 4748 8753 6456 6330...
Files Plots Packages Help Viewer Presentation
Zoom Export
ENG
11:07 AM
10/10/2023

MODEL	F-test -P-VALUE
M1 : Based on Backward method using p-value as metric:	2.2e-16
M2: Based on Backward method using AIC as metric:	2.2e-16
M3: Based on Forward method using AIC as metric.	2.2e-16
M4: Based on Stepwise method using ACI as metric:	2.2e-16
M5: Based on Best subset method using Adj-R2 as metric	2.2e-16

Conclusion on F-test: p-value < 0.05 for all the models

- At 95% confidence level, we can say that at least 1 x variables among ((Hours.on.Readings, Hours.on.Assignments, Hours.on.Games, Hours.on.Internet) has significant linear relationship with Grade and can affect the value of y-variable ~ Grade.

  ******By conducting an F-test, we can observe that there is sufficient evidence to reject the null hypothesis and conclude that the independent variables have a significant impact on the dependent variable.*********

3. Which model is the best in terms of Adj-R2?

From the screenshots above:

MODEL	Adj r2
M1 : Based on Backward method using p-value as metric:	56.06%
M2: Based on Backward method using AIC as metric:	56.06%
M3: Based on Forward method using AIC as metric.	56.06%
M4: Based on Stepwise method using ACI as metric:	56.06%
M5: Based on Best subset method using Adj-R2 as metric	56.06%

Conclusion:

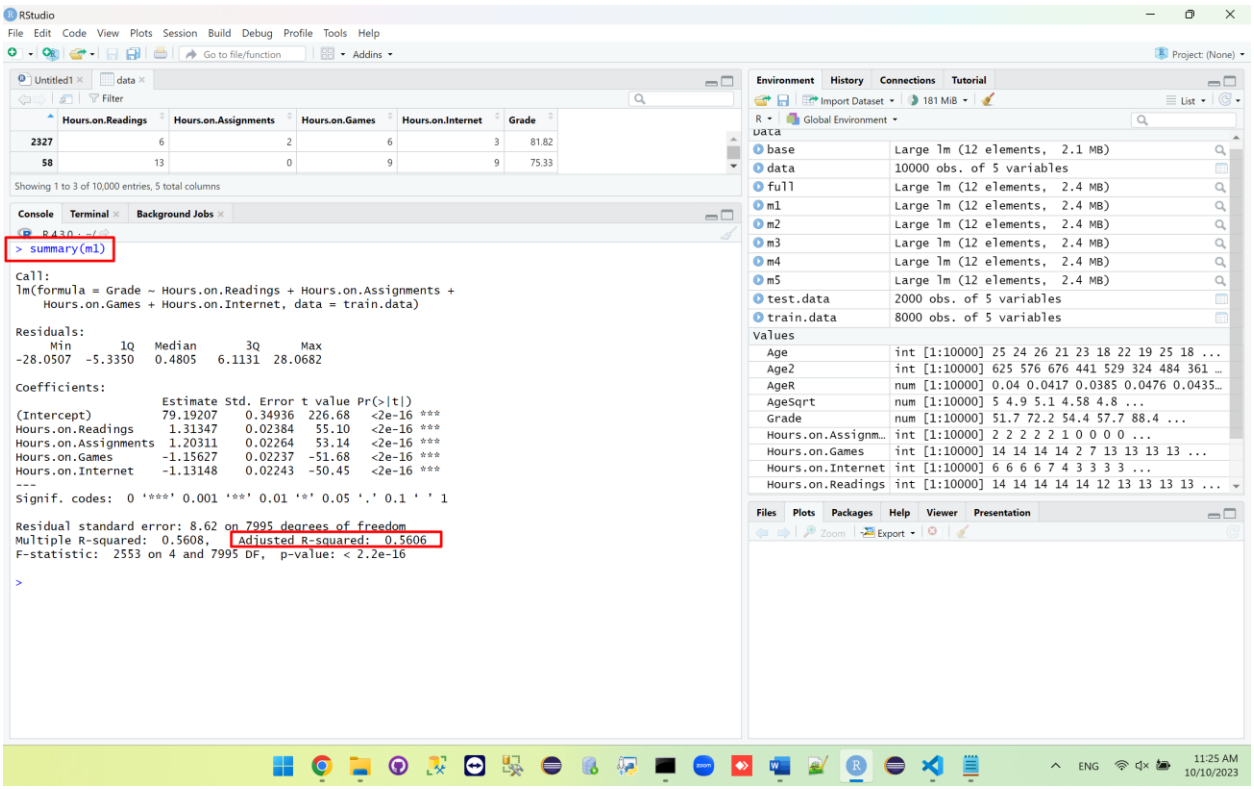
- All models have the same adj r2 values and used same set of x-variables, so all models are best.

*******Note: But Adj r2 is not the accurate, we need to evaluate based on testing data set by RMSE*****

4. Interpret the Adj-R2 in the best model above.

ANSWER:

- Since all models have the same adj r2 values and use same set of x-variables, so all models are best. So, we can interpret any model's adj r2. Lets take M1 and interpret adj r2:



Interpretation of adj r2 of m1:

- 56.06% variation in Grade can be explained by the variations in X variables (Hours.on.Readings+Hours.on.Assignments+Hours.on.Games+Hours.on.Internet) based on our fitted regression model

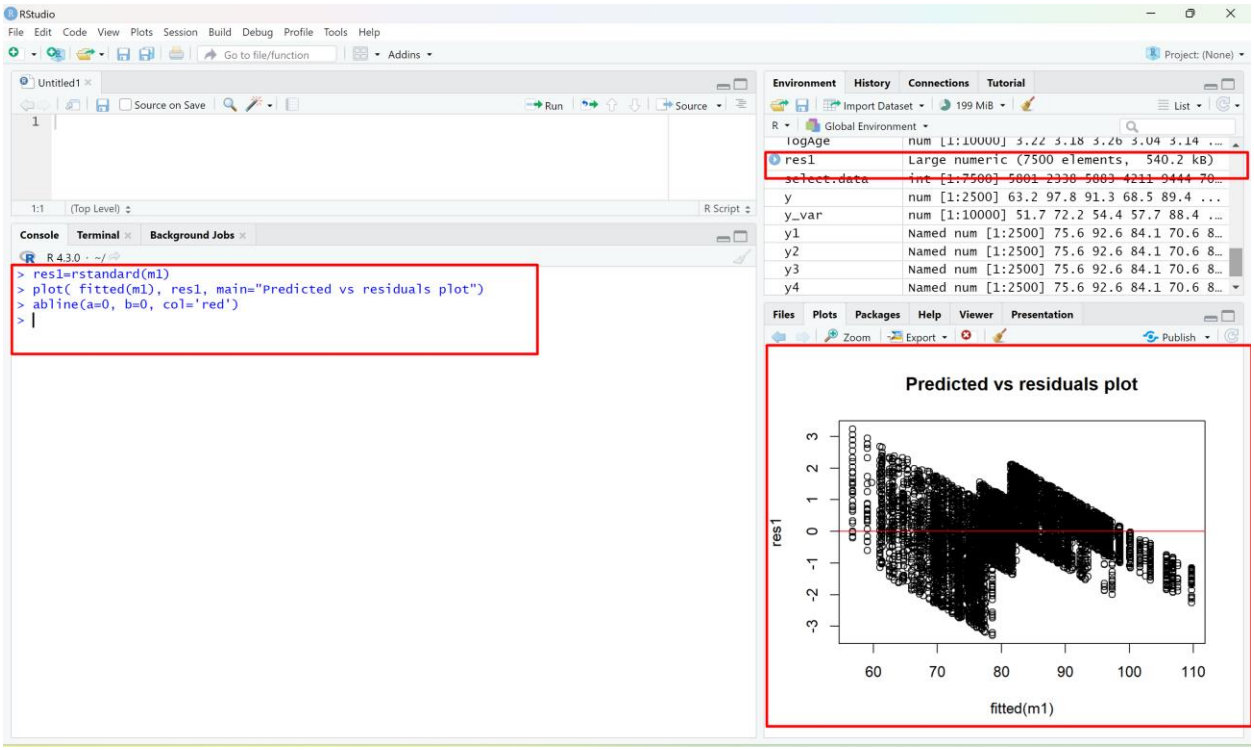
5. Any issues in the residual analysis for the model above?

ANSWER:

Yes, there is an issue about constant variance. As all models are same lets take m1 to check:

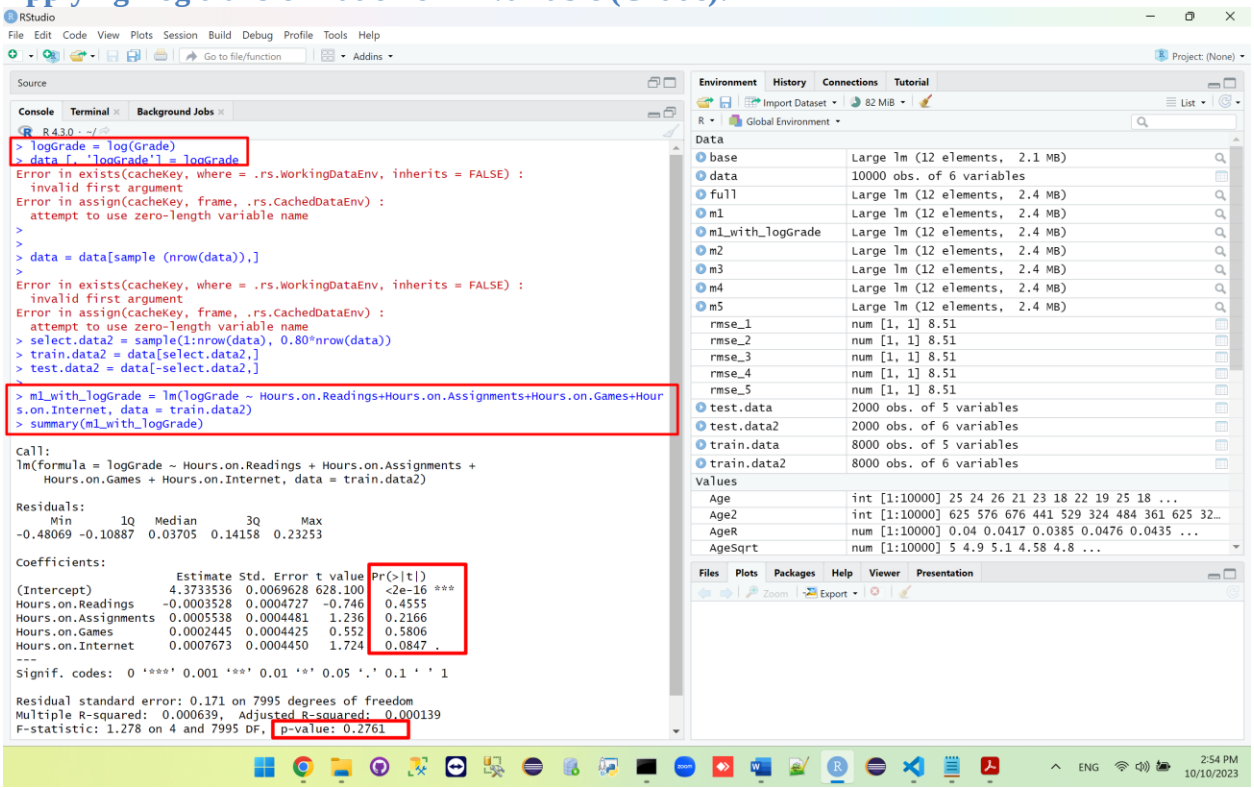
Residual analysis on m1:

1. Validate the constant variance: Plot residuals vs predicted values: To check constant variance for the residuals



It is Constance in the beginning, but variance became smaller at the end. We can try to apply log transformation on y-variable(Grade) to see whether we can get a better model.

Applying Log transformation on Y-variable (Grade):



After applying the log transformation on the variable Grade, I rebuilt the model M1 with logGrade, but it didn't pass the Goodness of fit test as the p - value (0.2761) is greater than 95%(0.05). From this observation I would like to say that we can stick to the old model M1.

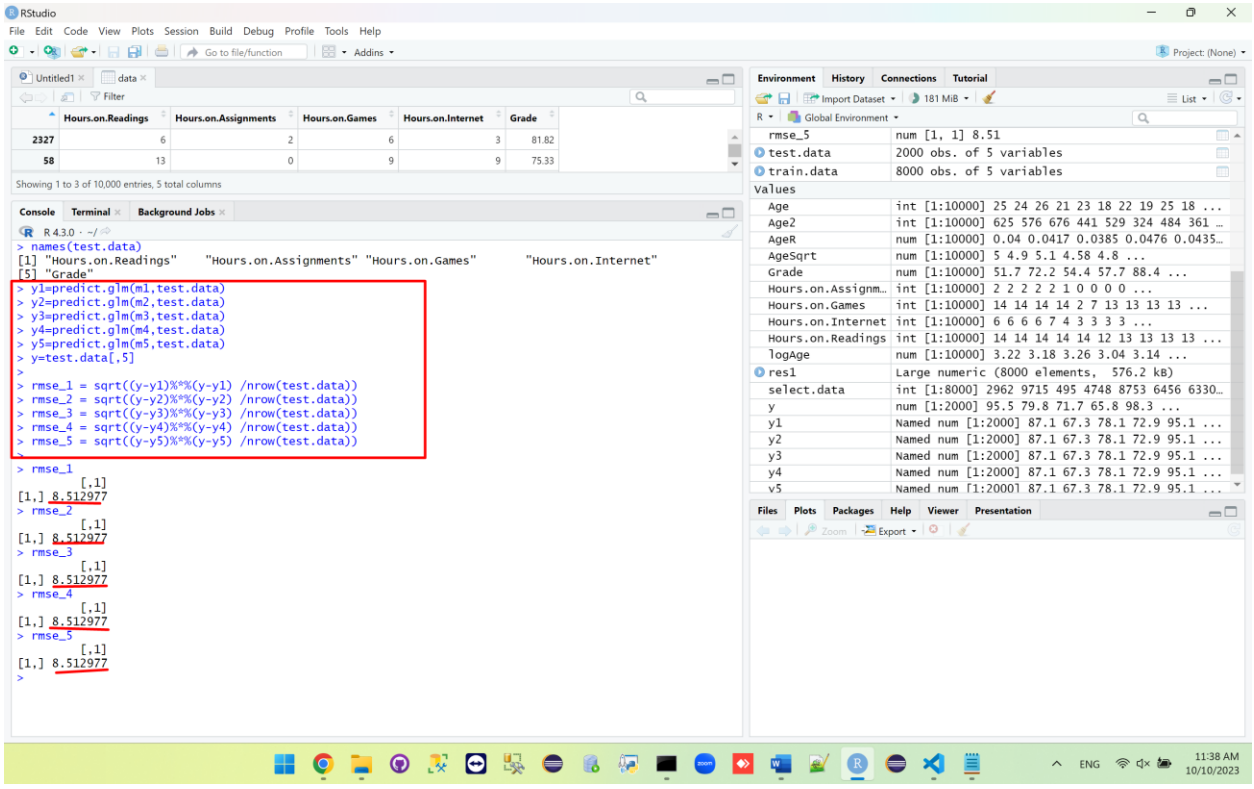
6. Which model is the best in terms of RMSE?

ANSWER:

Evaluate based on testing data set based on RMSE:

- To evaluate based on testing data set , we need to calculate the RMSE for all the models

Based on RMSE we need to compare all the models and make a conclusion about which model is best.



MODEL	RMSE
M1 : Based on Backward method using p-value as metric:	8.512
M2: Based on Backward method using AIC as metric:	8.512
M3: Based on Forward method using AIC as metric.	8.512
M4: Based on Stepwise method using ACI as metric:	8.512

M5: Based on Best subset method using Adj-R2 as metric	8.512

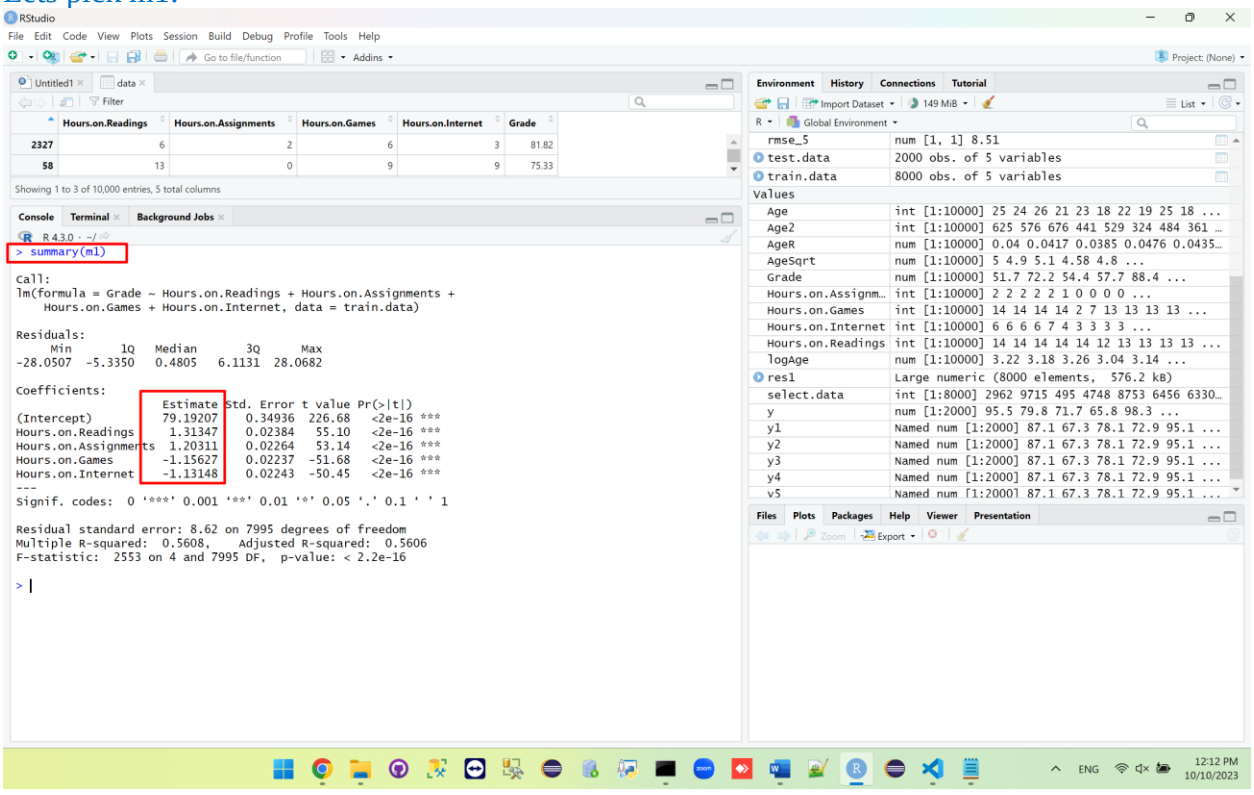
Conclusion: As the RMSE is same for all the models, any model Is best. We can pick any model and write the final model.

7. Write down the best model (identified by RMSE), and explain the intercepts and coefficients in the model.

ANSWER:

As the RMSE is same for all the models, Any model Is best
So, we can pick any 1 model and write down the best model by RMSE

Lets pick m1:



Y = 79.1920 + 1.31347 X1 + 1.20311 X2 + (-1.15627) X3 + (-1.13148) X4

Whereas :
Y = Grade
X1 = Hours.on.Readings
X2 = Hours.on.Assignments
X3 = Hours.on.Games
X4 = Hours.on.Internet

Explanation of co-efficient and Intercept (explaining effect):

β0 ~
Intercept: 79.1920:

The intercept, represented by 79.1920, is the value of Grade when X1, X2, X3, X4 (all the x-var's) is equal to zero. The intercept represents a constant or baseline for grade when all the x var's are at 0.

β_1 ~ It is the co-efficient of X1 variable: (Hours.on.Readings)

1.31347 (β_1) measures the changes in Grade variable for a unit increase of the variable Hours.on.Readings , (Assuming other variables are held constant).

β_2 ~ It is the co-efficient of X2 variable: (Hours.on.Assignments)

1.20311 (β_2) measures the changes in Grade variable for a unit increase of the variable Hours.on.Assignments, (Assuming other variables are held constant).

β_3 ~ It is the co-efficient of X3 variable: (Hours.on.Games)

-1.15627 (β_3) measures the changes in Grade variable for a unit increase of the variable Hours.on.Games. The negative sign suggests a concave-down relationship, meaning the Grade initially increases at a decreasing rate and eventually decreases as X3 increases. (Assuming other variables are held constant).

β_4 ~ It is the co-efficient of X4 variable: (Hours.on.Internet)

(-1.13148 (β_4) measures the changes in Grade variable for a unit increase of the variable Hours.on.Internet. The negative sign suggests a concave-down relationship, meaning the Grade initially increases at a decreasing rate and eventually decreases as X3 increases. (Assuming other variables are held constant).