# HW1

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Use the data set in case study 2 and utilize R to answer the following questions.

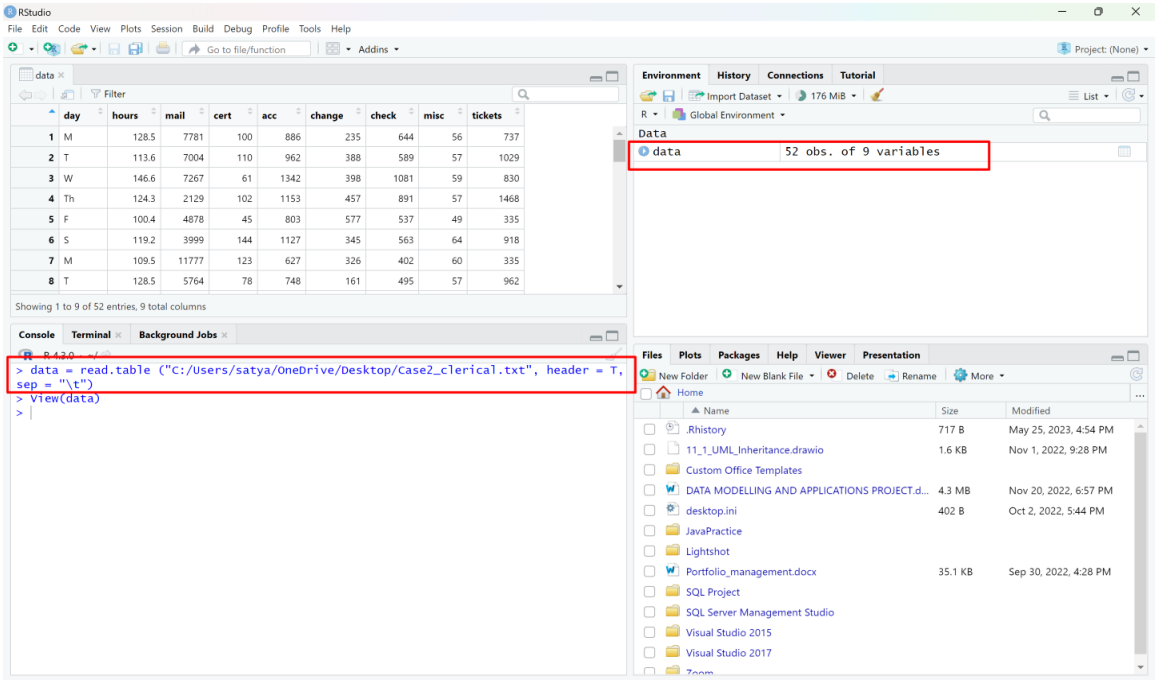
Note: you need to take snapshots of your R coding & outputs, also use your texts to answer the following questions (if necessary). Just like the example given in our slides. Upload a single PDF document as your submission.

1. Load dataset into R and show the column names, example of values, and the size of the data [10]

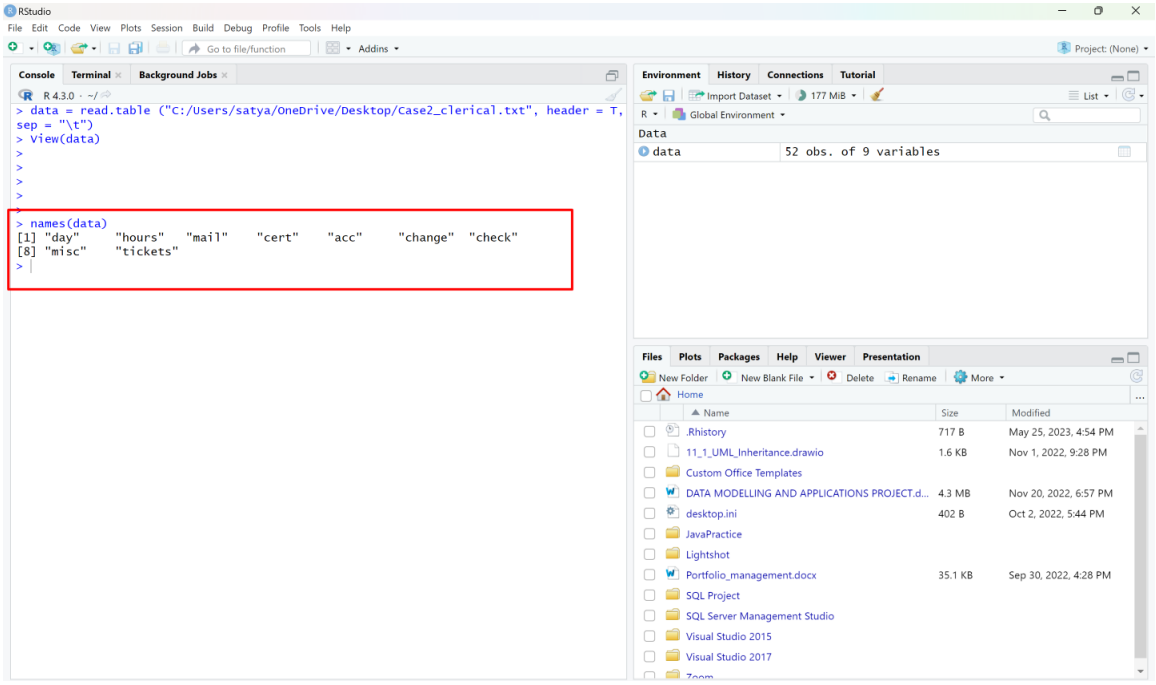
**ANSWER:**

**Loading data set:**

- Case study 2(Case2_clerical.txt) is loaded in R Studio by using the command:
- data = read.table ("C:/Users/satya/OneDrive/Desktop/Case2_clerical.txt", header = T, sep = "\t")
- **Note:** I used separator as "\t" , because I noticed that the separator in the text file you have given is not coma(,).
- The separator between the columns in the text file is space. So, we need to use \t as separator.



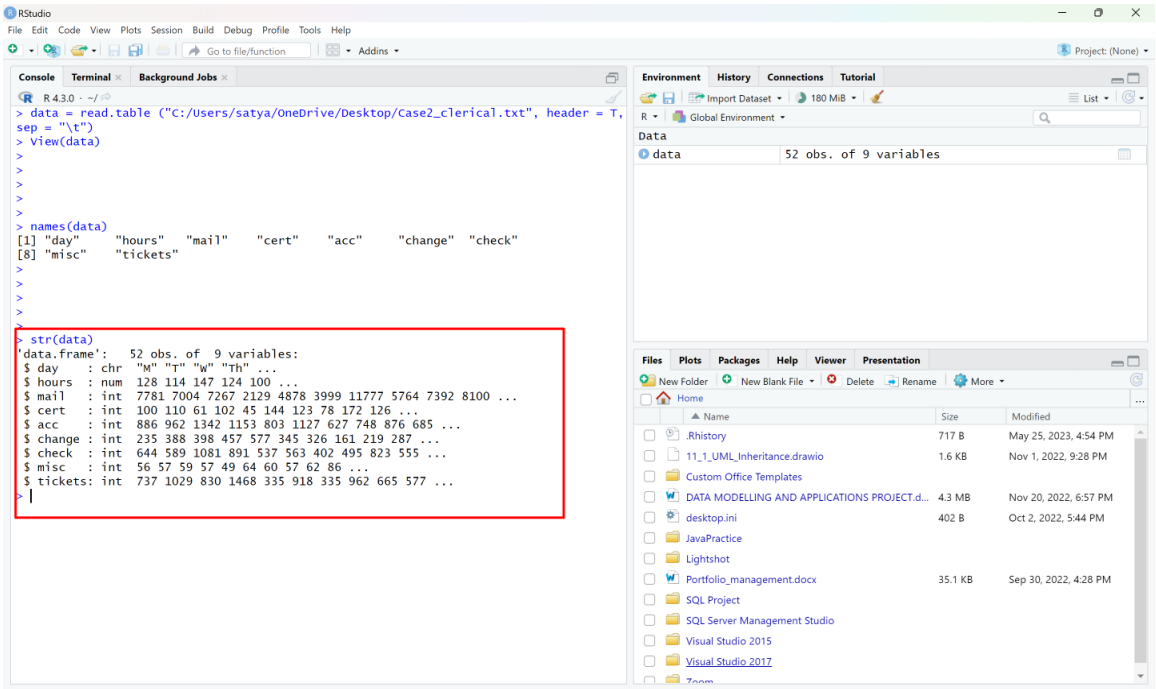
## Show the column names:



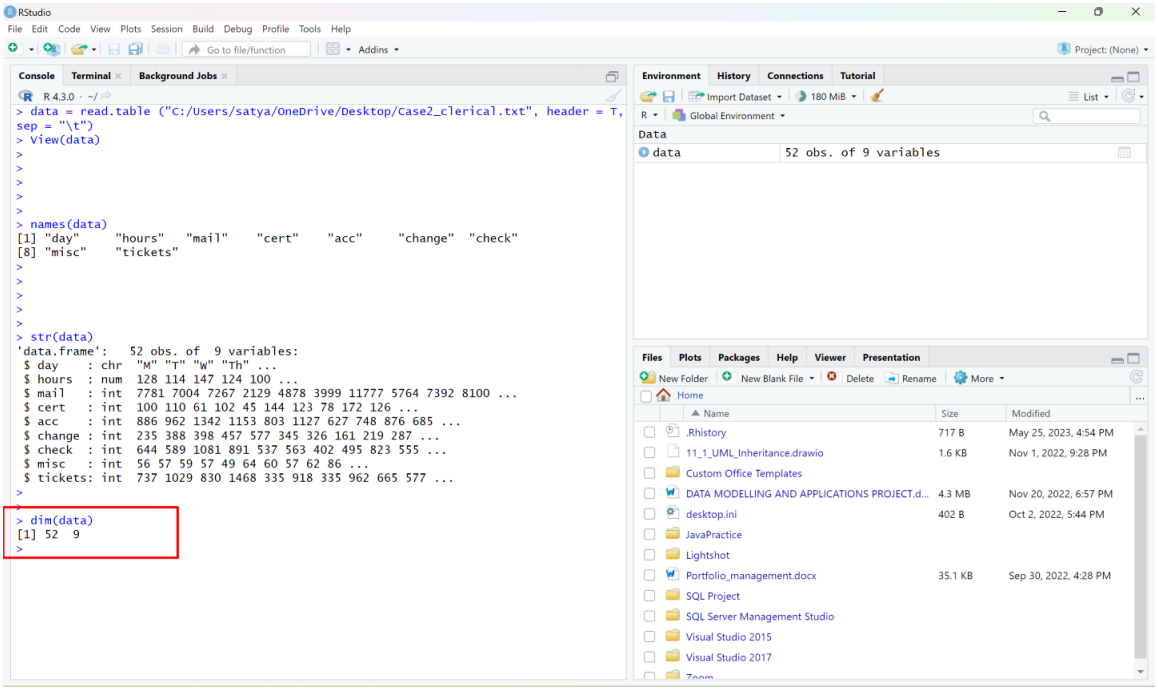

## Example of values:

The example of values here are : “M” “T”

128, 114 ……etc.,

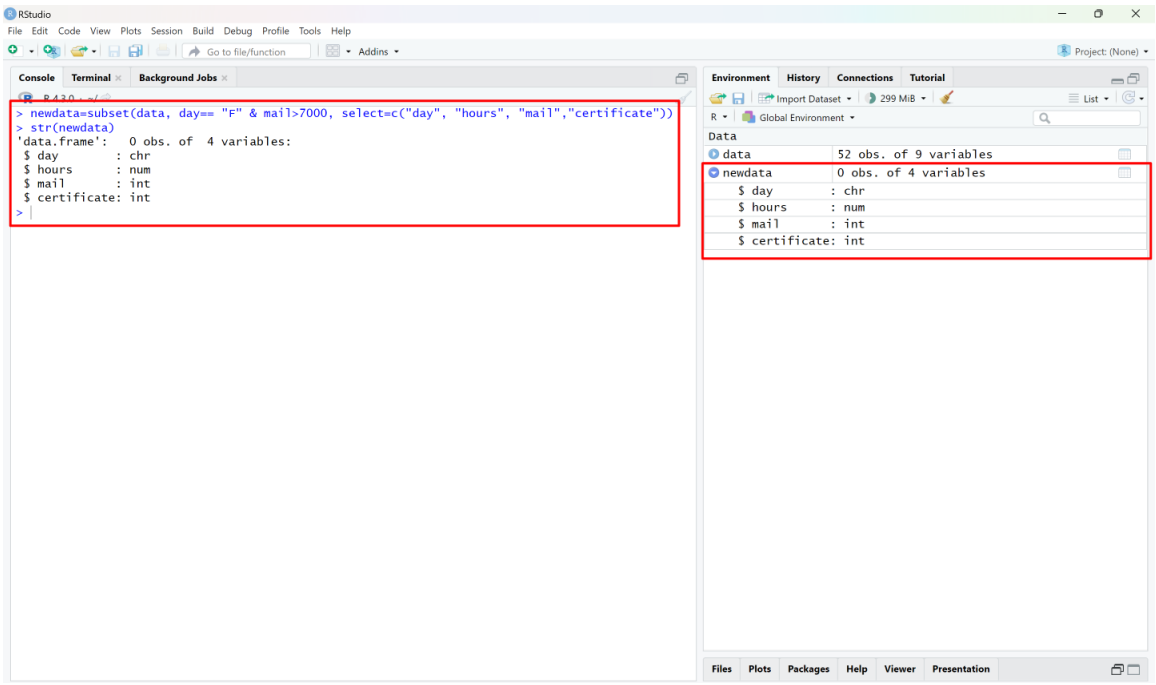


**Size of the data:** Size of the data can be known by using dim function. There are 52 observations/records & 9 variables/columns.




2. Return a list of records with columns <day, hours, mail, cert>, where day is Friday, and the number of mails is larger than 7000 [10]

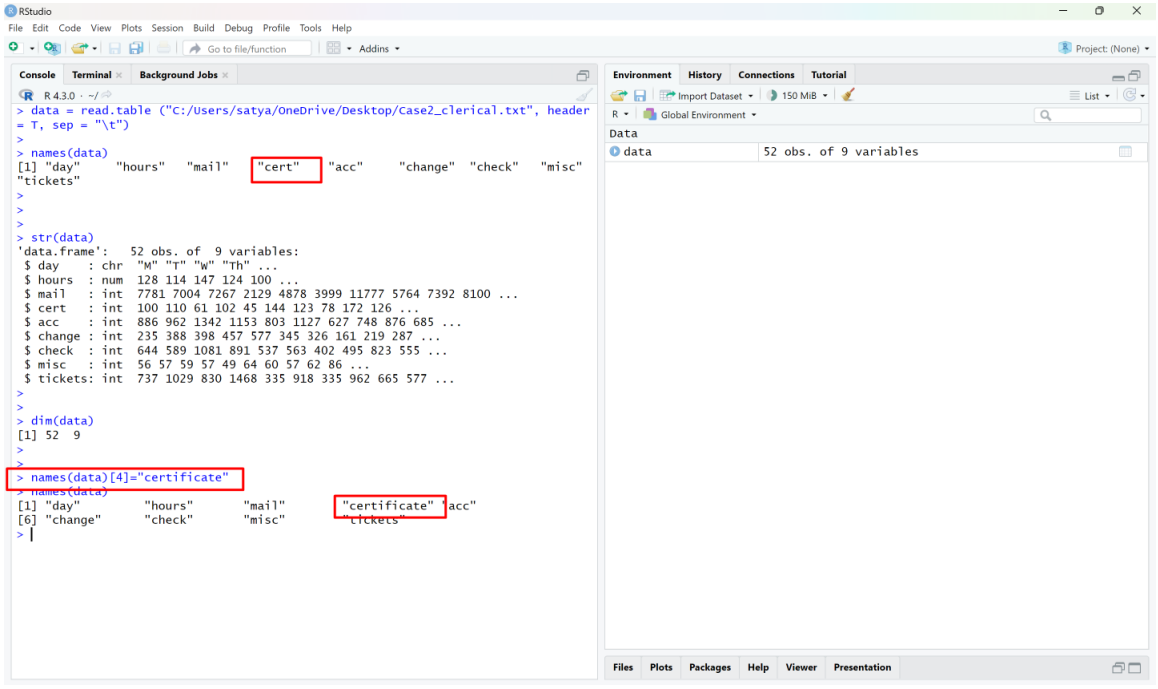
**ANSWER:** There are zero observations for the above constraints. Used Subset function.




3. Change the column name from “cert” to “certificate” [5]

## ANSWER:

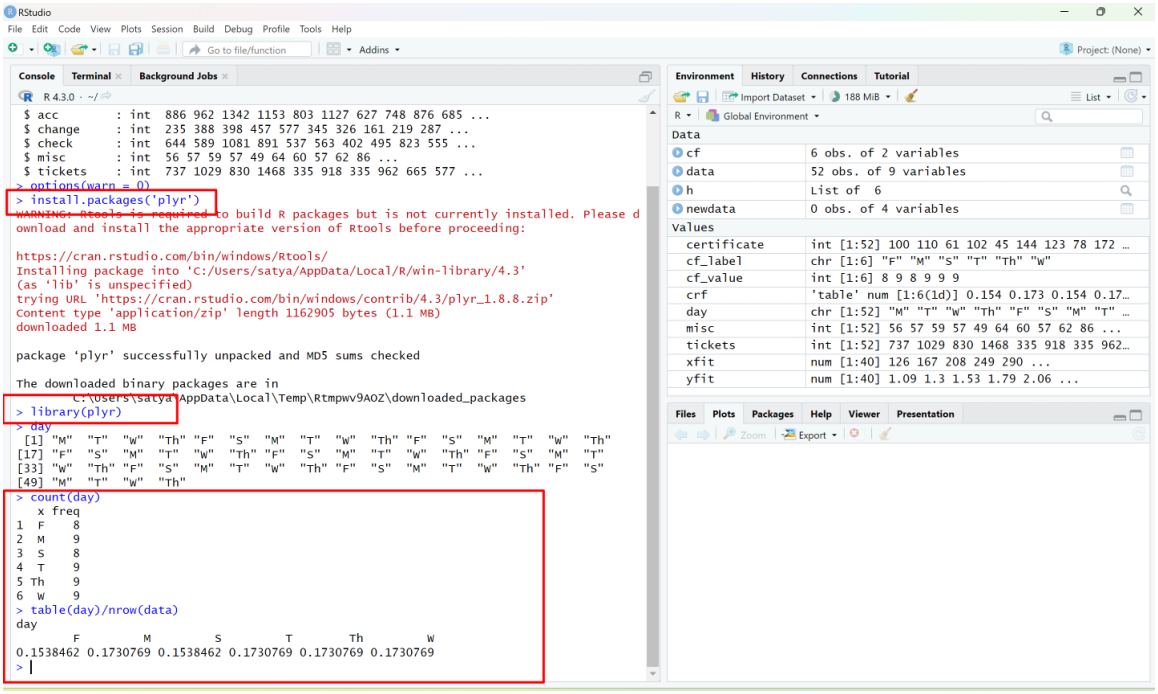
The column name changed from "cert” to "certificate” using index:



4. Use descriptive statistics to understand the column “day”. More specifically, return class frequency, and class relative frequency. Also visualize this column by using bar graph (use class relative frequency as y-axis) and pie chart [20]

**ANSWER:** Installed and loaded library ‘plyr’ to use and call the functions in it and to calculate the Class frequency and Class relative Frequency.

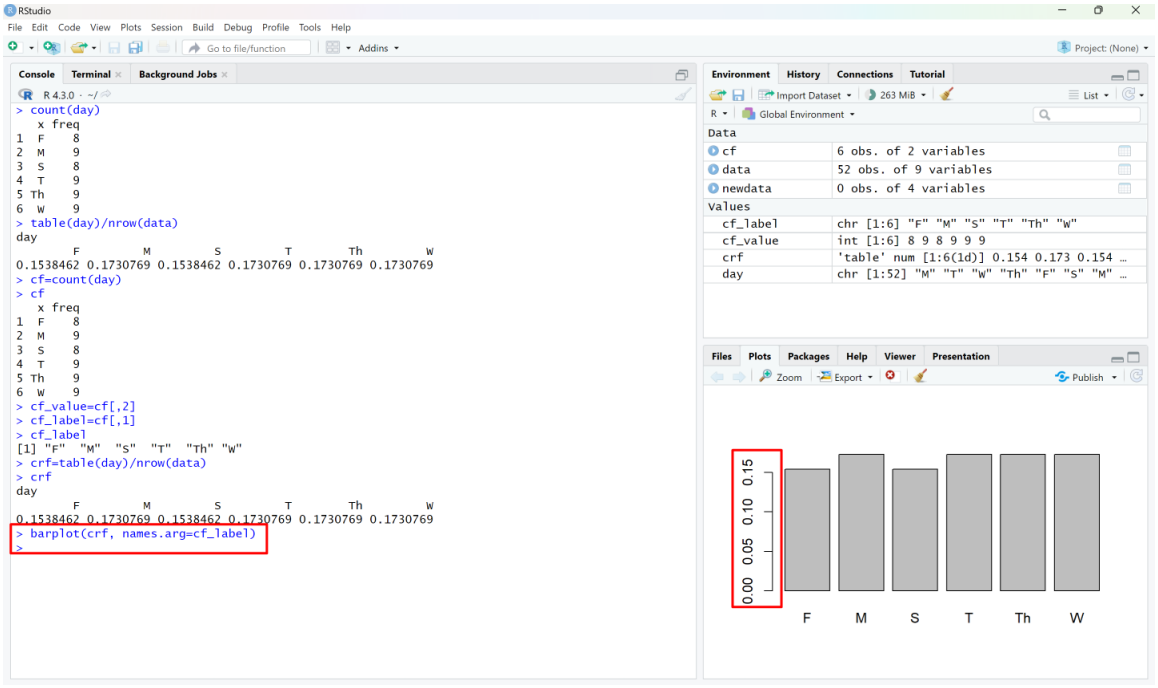
Class frequency (CF) of the column ‘day’ using count function & CRF is also calculated using R. Here is the screenshot.



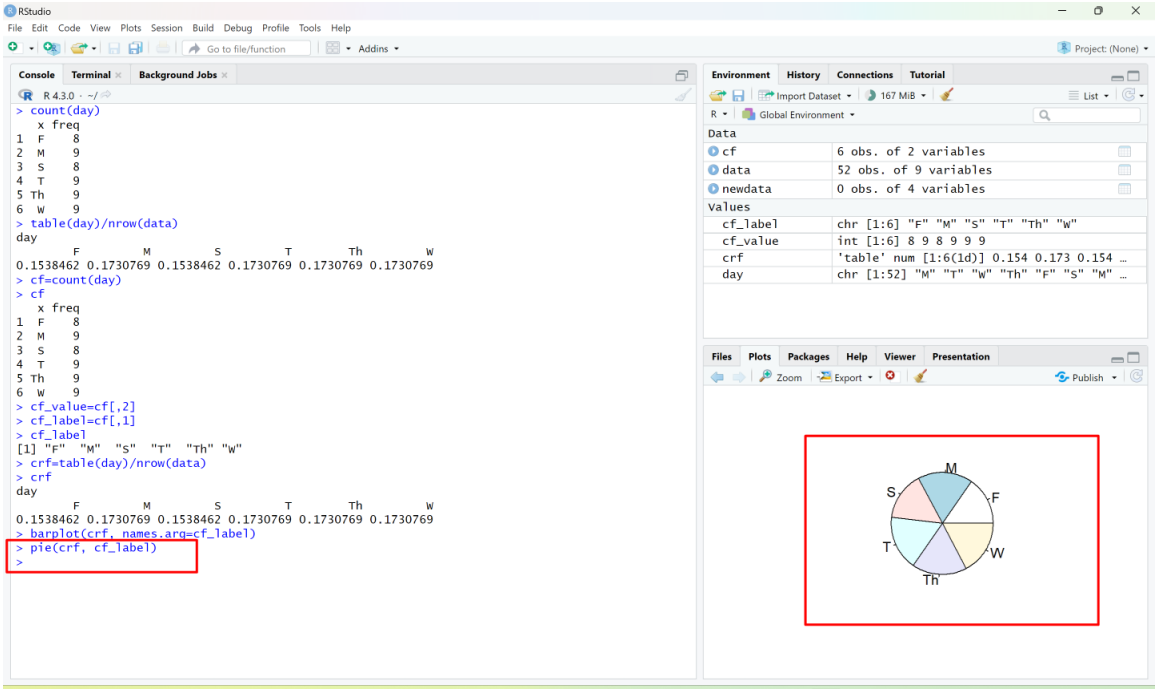

Class relative frequency of the column ‘day’: (CRF=CF/n) (manual calculation by using formula)

- CRF of F = CF/n = 8/52= 0.153846
- CRF of M = CF/n = 9/52= 0.1730769
- CRF of S = CF/n = 8/52= 0.153846
- CRF of T = CF/n = 9/52= 0.1730769
- CRF of Th = CF/n = 9/52= 0.1730769
- CRF of W = CF/n = 9/52= 0.1730769

## Visualizing the ‘day’ column by bar-graph using class relative frequency on y-axis:




## Visualizing the ‘day’ column by pie-chart using class relative frequency:



5. Use descriptive statistics to understand the column “certificate”. More specifically, we want to get q1, q2, q3, average value, variance value. Also, visualize this variable by using histogram, and interpret your histogram [20]

**ANSWER:**

To calculate the descriptive statistics asked, library ‘psych’ is installed and loaded. Q1, Q2, Q3, average value and variance value can be calculated by using 2 functions.

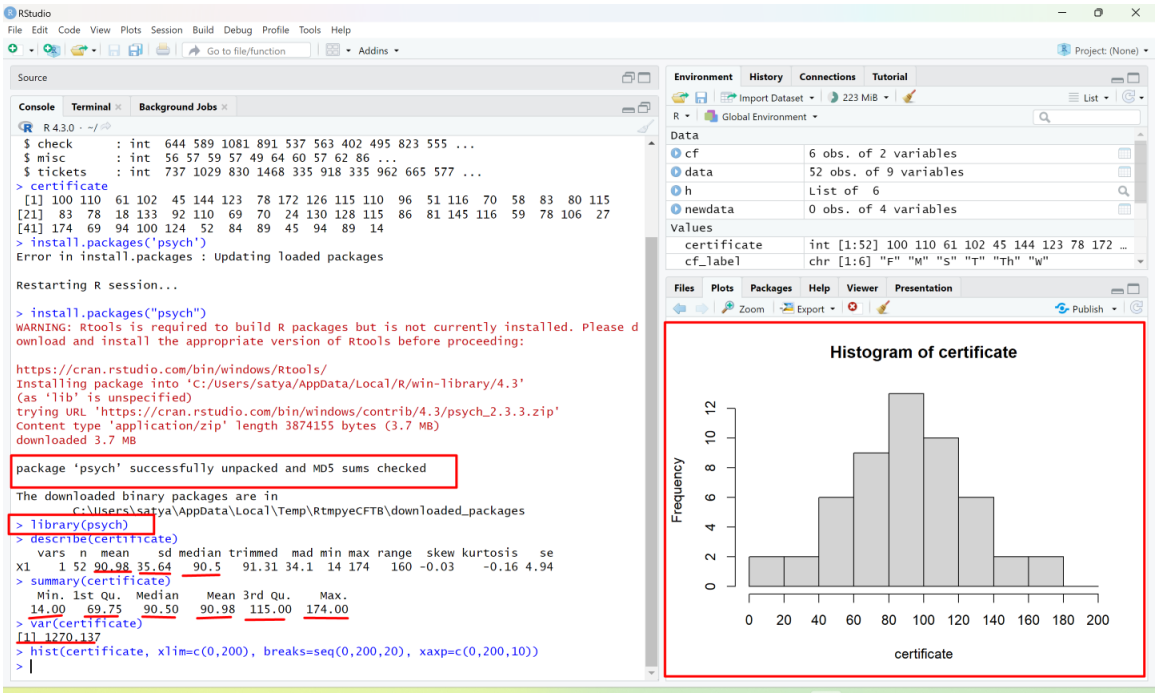
1. describe()

2. summary()

3. var()

The variable ‘certificate’ can be visualized by histogram by using hist() function.

**Here is the screenshot of the r coding calculating the q1, q2, q3, average value(mean) and variance value and the histogram of the variable ‘certificate’:**



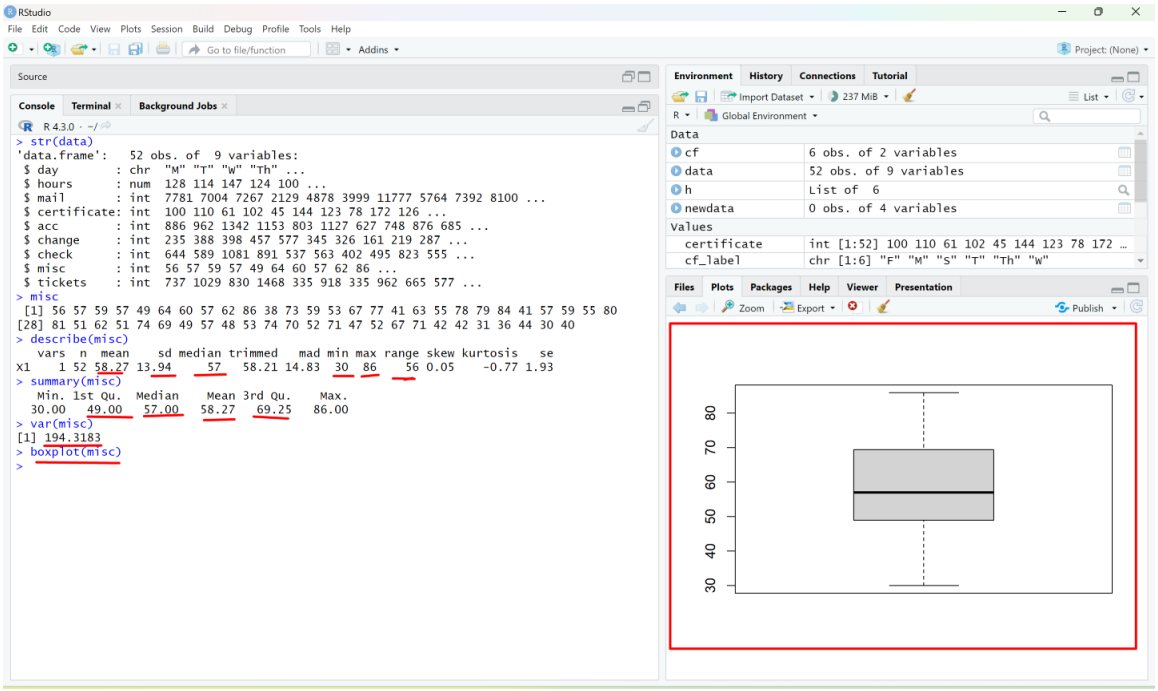

**Interpretation of the histogram of certificate:** (Analyzing the shape by skewness and outliers)

- **Distribution:** The distribution is “Normal distribution/ Symmetric”. There is no skew in data. Hence the data is evenly distributed around the center.
- **Variance:** Variance is nothing but the variation or the spread of data. The variance is large in this data.
  Calculation of variance. = 1270.137

- **Standard deviation:** 35.64. (it's the square root of variance)
- **Potential Outliers:** There are no potential outliers in this distribution of data.
- **Minimum value:** 14
- **Maximum value:** 174
- **1st Quartile(Q1):** 69.75
- **2nd Quartile(Q2):** 90.50(Note: Q2 is also called Median)
- **3rd Quartile(Q3):** 115
- **Range**: 160 (max-min)
- **Mean/ Average value of the data:** 90.98.
- **Median:** 90.50 (Note: Median is also called Q2)

6. Use descriptive statistics to understand the column "misc". Visualize it by using boxplot, and interpret your boxplot [20]

**ANSWER:**




**Interpretation of the boxplot of the variable 'misc':**

- **The distribution:** The box-plot follows "Normal distribution/ Symmetric". Hence the data is evenly distributed around the center. The median is in the middle.
- **Skewness:** There is no skewness in the boxplot. Because the median is exactly in the middle. So, it follows Normal distribution.
- **Potential Outliers:** There are no potential outliers.
- **The variance.:** Variance is nothing but the variation or the spread of data. The variance is small in the boxplot. The data is less
  Calculation of variance: 194.3183

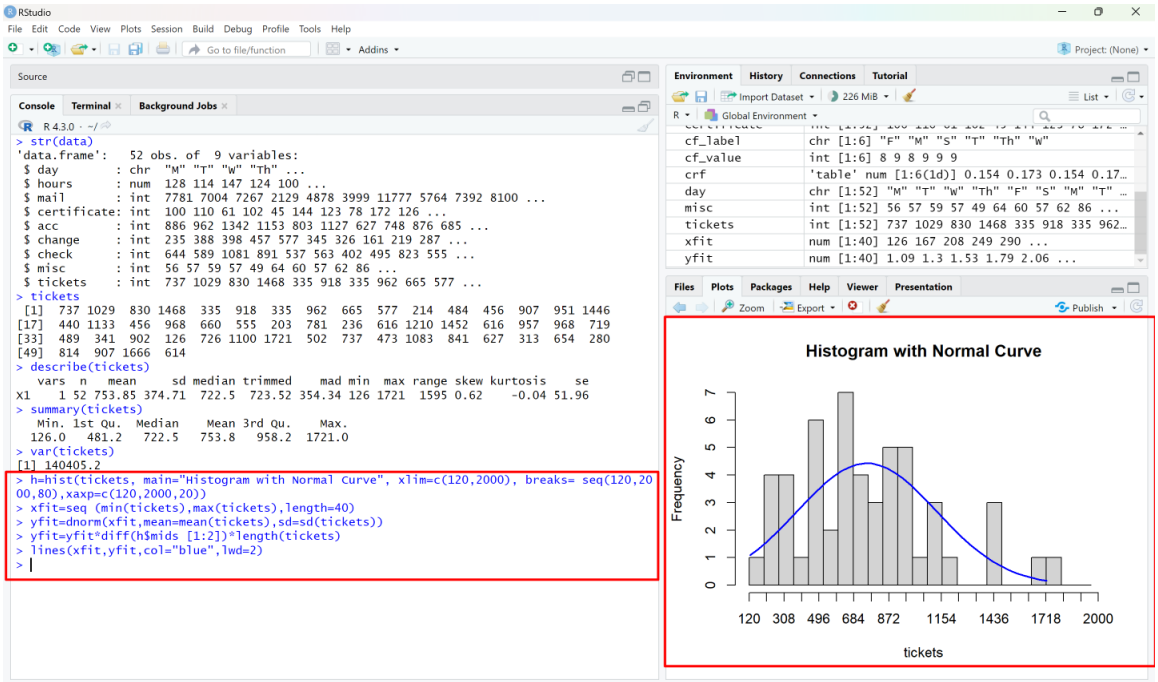
- **Standard deviation:** 13.94 (it's the square root of variance)
- **Minimum value:** 30
- **Maximum value:** 86
- **1st Quartile(Q1):** 49
- **2nd Quartile(Q2):** 57 (Note: Q2 is also called Median)
- **3rd Quartile(Q3):** 69.25
- **Range:** (maximum value – minimum value) = 56
- **Mean value/ Average value of the data:** 58.27
- **Median value:** 57 (Note: Median is also called Q2)

7. Visualize the column "tickets" by using probability curve, interpret it [15]

**ANSWER:**

To draw the probability curve, first we need to draw the histogram of the variable 'tickets' by using 'hist' function.

The process is as follows shown in the screenshot:




**Interpretation of the distribution curve:**

- **The distribution:** Its almost Normal/Symmetric distribution where the data is evenly distributed around the center. But the distribution is slightly skewed towards right.
  - ✓ Variable tickets follows normal distribution: tickets ~ N(mean, variance)
- **Skewness:** There is a slight rightly skewed/positive skewed distribution

- **Potential outliers:** There is 1 outlier towards the maximum value (1721)
    - The maximum value could be the outlier here, according to the formula: Q3 + 1.5(IQR) which calculates the upper boundary. Any data point above upper boundary / lower boundary falls under outliers. (where IQR = Interquartile Range)
    - Here the upper boundary is 1673.7 according to the formula.
    - So, the maximum value is 1721 above the upper boundary 1673.7, which is considered as an outlier in this distribution.
- **Variance:** Variance is nothing but the variation or the spread of data. The variance is large in this data.
    - Calculation of variance: 140405.2
- **Standard deviation:** 374.71 (it's the square root of variance)
- **Minimum value:** 126
- **Maximum value:** 1721
- **Range**: 1595 (max-min)
- **1st Quartile(Q1):** 481.2
- **2nd Quartile(Q2):** 722.5 (Note: Q2 is also called Median)
- **3rd Quartile(Q3):** 958.2
- **Mean value/ Average value of the data:** 753.8
- **Median value:** 722.5 (Note: Median is also called Q2)

Homework 2

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1. **(35 points) Manually solve the problem below (do not use R):**

Note, if you need either z value or t value, you can find them by using this tool:
http://www.mathcracker.com/z_critical_values.php
http://www.mathcracker.com/t_critical_values.php
https://www.socscistatistics.com/pvalues/

A bank branch located in a commercial district of a city has the business objective of improving the process for serving customers during the noon to 1 PM (lunch period). The waiting time (defined as the time the customer enters the line until he or she reaches the teller window) of a random sample of 15 customers is collected, and the results are organized and stored as below:

4.21, 5.55, 3.02, 5.13, 4.77, 2.34, 3.54, 3.20
4.50, 6.10, 0.38, 5.12, 6.46, 6.19, 3.79

a) Calculate the mean and standard deviation, and find q1, q3 from the values above. Is the distribution symmetric? Why? [10]

**ANSWER:**

- The sample size: n = 15
- Sample mean = $\sum_{i=0}^{n} xi/n$ (dividing the sum of all values in a data set by the number of values.)
  - Mean = (4.21 + 5.55 + 3.02 + 5.13 + 4.77 + 2.34 + 3.54 + 3.20 + 4.50 + 6.10 + 0.38 + 5.12 + 6.46 + 6.19 + 3.79) / 15 = 64.3/15 = 4.28
  - Mean ($\overline{x}$ = 4.28)

- Standard deviation (S) = square root of the variance. ($\sqrt{S^2}$. Where is $S^2$variance.)
- So, to calculate the Standard deviation, first we need to calculate the variance of the sample ($S^2$)
  - Variance = $S^2 = \sum_{i=1}^{n} \frac{(xi-\overline{x})\text{^}2}{n-1}$
  - First, we need to calculate the mean (mean is already calculated above i.e., 4.28)
  1. Subtract the mean from each data point and square the result:
     - (4.21 - 4.28) ^2 = 0.0049
     - (5.55 - 4.28) ^2 = 1.6129

- (3.02 - 4.28) ^2 = 1.5876
- (5.13 - 4.28) ^2 = 0.7225
- (4.77 - 4.28) ^2 = 0.2401
- (2.34 - 4.28) ^2 = 3.7636
- (3.54 - 4.28) ^2 = 0.5476
- (3.20 - 4.28) ^2 = 1.1664
- (4.50 - 4.28) ^2 = 0.0484
- (6.10 - 4.28) ^2 = 3.3124
- (0.38 - 4.28) ^2 = 15.21
- (5.12 - 4.28) ^2 = 0.7056
- (6.46 - 4.28) ^2 = 4.7524
- (6.19 - 4.28) ^2 = 3.6481
- (3.79 - 4.28) ^2 = 0.2401

2. Calculate the average of the squared differences:

(0.0049+1.6129+1.5876+0.7225+0.2401+3.7636+0.5476+1.1664+0.0484+3.3124+15.21+0.7056+4.7524+3.6481+0.2401) ÷ 15-1

= 37.5626÷ 14 = 2.68

Therefore, The variance of the given sample data set: $S^2$ = 2.68

- The standard deviation = square root of the variance ($\sqrt{S^2}$)

$\sqrt{(2.68)}$ = 1.63

- To calculate Q1 , Q2 , Q3 , First we need to re-arrange the sample dataset in Ascending order:
  - 0.38, 2.34, 3.02, 3.20, 3.54, 3.79, 4.21, 4.50, 4.77, 5.12, 5.13, 5.55, 6.10, 6.19,6.46
    - Q1: (Lower quartile) = 3.20
    - Q2: (Median/Middle value) = 4.50
    - Q3: (Upper quartile) = 5.55

- Is the distribution symmetric?
  - Based on the central limit theorem, If the sample size(n) is less than 30, then it will not follow normal distribution.
  - Here the sample size (n) = 15 which is less than 30, The distribution is not symmetric. It follows t-distribution.
  - To tell whether it is symmetric or not, we can say if the mean and median are equal, then its symmetric. Here the mean is 4.28 & the median is 4.50.
  - Mean and median are not equal, hence we can say the distribution is not symmetric and it is skewed.
  - To say its skewness in detail, we can use a formula:

Skewness = (3 * (Mean - Median)) / Standard Deviation

= (3 * (4.28 – 4.50)) / 1.63

= (3 * –0.22) / 1.63

= - 0.66 / 1.63
= -0.40

Hence, we can say it is slightly left-skewed/ negative-skewed.

b) As a customer walks into the branch office during the lunch period. She asks the branch manager how long she can expect to wait. If you are the manager, answer this question by using 95% confidence. [10]

**ANSWER:**

To answer this question, we need to follow the below steps one by one:

1. Calculate sample statistics such as 'sample mean' and 'sample standard deviation'.

Sample mean: already calculated in the above question.

$\bar{y}$ = 4.28

Sample standard deviation: already calculated in the above question.

S = 1.63

2. As sample size ( n = 15) is less than 30, it follows t- distribution, hence we need to use t-value in the margin of error.

3. Produce confidence interval [a,b] by using the formula :

a = sample estimate – margin of error (sample estimate i.e, sample mean)

$$\bar{y} - t\ \frac{\alpha}{2}\left(\frac{S}{\sqrt{n}}\right)$$

b = sample estimate + margin of error

$$\bar{y} + t\ \frac{\alpha}{2}\left(\frac{S}{\sqrt{n}}\right)$$

- To get the t- value we need to have 2 inputs:

1. alpha ($\alpha$) = 1 – confidence level
= 1- 95%

($\alpha$) = 0.05

2.degree of freedom (df) = n – 1
= 15 – 1
= 14

- http://www.mathcracker.com/t_critical_values.php

from this given tool, the t- value is calculated from the t- table and observed as $t\ \frac{\alpha}{2} = 1.761$

Now,

- Let's calculate a = $\bar{y} - t\ \frac{\alpha}{2}\left(\frac{S}{\sqrt{n}}\right)$

$= 4.28 - 1.761\left(\frac{1.63}{\sqrt{15}}\right)$

$= 4.28 - 1.761\left(\frac{1.63}{3.87}\right)$

$= 4.28 - 1.761\ (0.421)$

$= 4.28 - 0.741$ (0.741 is the margin of error)

$= 3.53$

- Let's calculate b = $\bar{y} + t\ \frac{\alpha}{2}\left(\frac{S}{\sqrt{n}}\right)$

$= 4.28 + 1.761\left(\frac{1.63}{\sqrt{15}}\right)$

$= 4.28 + 1.761\left(\frac{1.63}{3.87}\right)$

$= 4.28 + 1.761\ (0.421)$

$= 4.28 + 0.741$

$= 5.02$

Conclusion: I have 95% confidence to say that the average waiting time (population mean) will fall in the interval [3.53, 5.02].

c) We were told the average waiting minute will be 5 minutes. But we think it could be more than 5 minutes. By using 90% as confidence level, validate the hypothesis. Show your steps and calculations [15]

**ANSWER:**

To answer this question, we need to follow the below steps one by one:

1. Collect sample data & Calculate sample statistics such as 'sample mean' and 'sample standard deviation':

- ✓ n = 15
- ✓ Sample mean $\bar{X}$ = 4.28
- ✓ Sample standard deviation = S = 1.63

2. State null hypothesis and alternative hypothesis:

$$H_0 : \mu = 5$$
$$H_a : \mu > 5$$

3. Based on Ha, decide it is one-tailed or two-tailed test:

- Based on alternative hypothesis $H_a$, it is one- tail test.
- As alternative hypothesis is larger than $\mu$, the tail is on the right side. That means the reject region is always on the right.

4. Specify the desired level of significance:

α = 1 – confidence level.
= 1- 90%
= 1- 0.90
= 0.10

5. Determine the appropriate technique:

- ➢ σ is unknown and n = 15 which is less than 30.
- ➢ As sample size is less, this is a t- test.
- ➢ Hence use t- statistics to make the conclusion.

6. Calculate critical value:

- Since it is t- test, to calculate the t α / t- critical value, we need to have 2 inputs.
  - α = 0.10
  - df (degree of freedom) = n-1 = 15 – 1 = 14

- with the α value and df value available, the t- critical value/ t α value is calculated from the t- table and observed as follows:

$$t_c \; or \; t_\alpha = \; 1.345$$

7. Calculate the test statistic value (t-value) by using the formula: (t-score)

$$t_{STAT} = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$$

$$= \frac{4.28 - 5}{\frac{1.63}{\sqrt{15}}}$$

$$= \frac{-\ 0.72}{\frac{1.63}{3.87}}$$

$$= \frac{-0.72}{0.42}$$

$$= -1.714$$

8.Compare the test- statistic value with the critical value / t α value. 6. Is the test statistic in the rejection region?

$$t_{STAT} < t_c \; or \; t_\alpha$$

$$-\ 1.714 < 1.345$$

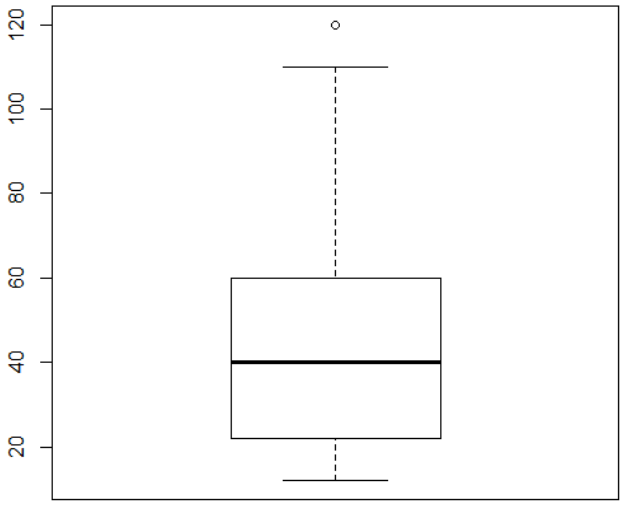
The test statistic is in the non-rejection region.

9. Draw the conclusion based on test statistic value:

- Since the test statistic falls in the non- rejection region, we fail to reject the null hypothesis. Therefore, we do not have enough evidence to conclude that the average waiting minute is more than 5 minutes with a 90% confidence level.
- In summary, based on the given data and calculations, we do not have enough evidence to support the hypothesis that the average waiting minute is more than 5 minutes with a 90% confidence level.

**2. (55 points)** Chicago Ventra Transit Card can be used on both CTA bus, metro and Pace buses. We are going to explore a resident's average monthly cost on CTA transportations. In this case, we performed a survey, and collect monthly cost on CTA transits from 30 people, their monthly cost can be listed as follows:
12, 12, 12, 15, 24, 35, 14, 12, 120, 55, 45, 30, 40, 40, 40, 60, 60, 40, 50, 22, 36, 28, 21, 50, 39, 60, 90, 100, 110, 100

1). [5] To further understand the distribution, we draw a boxplot as follows. Interpret the box plot.




**ANSWER:**
**Interpretation of the boxplot:**

- **The distribution:** The distribution is not symmetric; the data is not evenly distributed. The median is not exactly in the middle. From the visualization, the box plot clearly shows that it is positively skewed.

- **Skewness:** The box plot clearly follows a positive skew from the visualization.

    - To say its skewness in detail, we can use a formula:
      Skewness = (3 * (Mean - Median)) / Standard Deviation
      = (3 * (45.73 – 40)) / 30.18
      = (3 * 5.73) / 30.18
      = 17.19 / 30.18
      = 0.56
      Hence, we can say it is positively skewed / right skewed.

- **Potential Outliers:** From the visualization we can clearly see that there is a potential outlier towards the value 120.

    - To confirm the potential outliers, we need to follow the below steps:

- Calculate IQR (Interquartile range):
  - Q3 – Q1 = 60-22 = 38
- Upper bound value = Q3 + 1.5* IQR = 60 + (1.5 * 38) = 60 + 57 = 117

- Lower bound value = Q1 – 1.5* IQR = 22 – (1.5 * 38) = 22 – 57 = -35

❖ Any data values not falling between upper bound value and lower bound value are suspected to be potential outliers. Here 120 is the potential outlier that is not falling between the upper and lower bound values.

- **The variance.:** Variance is nothing but the variation or the spread of data. The variance in the boxplot is as follows.
  - ➢ Variance = $S^2 = \sum_{i=1}^{n} \frac{(xi-\overline{x})^2}{n-1}$

    Hence the variance from the above formula: $S^2 = 910.83$

- **Standard deviation:** 30.18 (it's the square root of variance- ($\sqrt{S^2}$)
- **Minimum value:** 12
- **Maximum value:** 110 (Since 120 is the outlier, it cannot be considered as the max value)
- **1st Quartile(Q1):** 22
- **2nd Quartile(Q2):** 40 (Note: Q2 is also called Median)
- **3rd Quartile(Q3):** 60
- **Range:** (maximum value – minimum value) excluding the outliers.
  = 110 -12 = 98
- **Mean value/ Average value of the data:** mean = $\sum_{i=0}^{n} xi/n$
  (Dividing the sum of all values in a data set by the number of values.)
  = 45.733

- **Median value:** Median is the middle value. If the size of the data is odd there is 1 median, If the data size is even, then the sum of middle 2 numbers divided by 2.

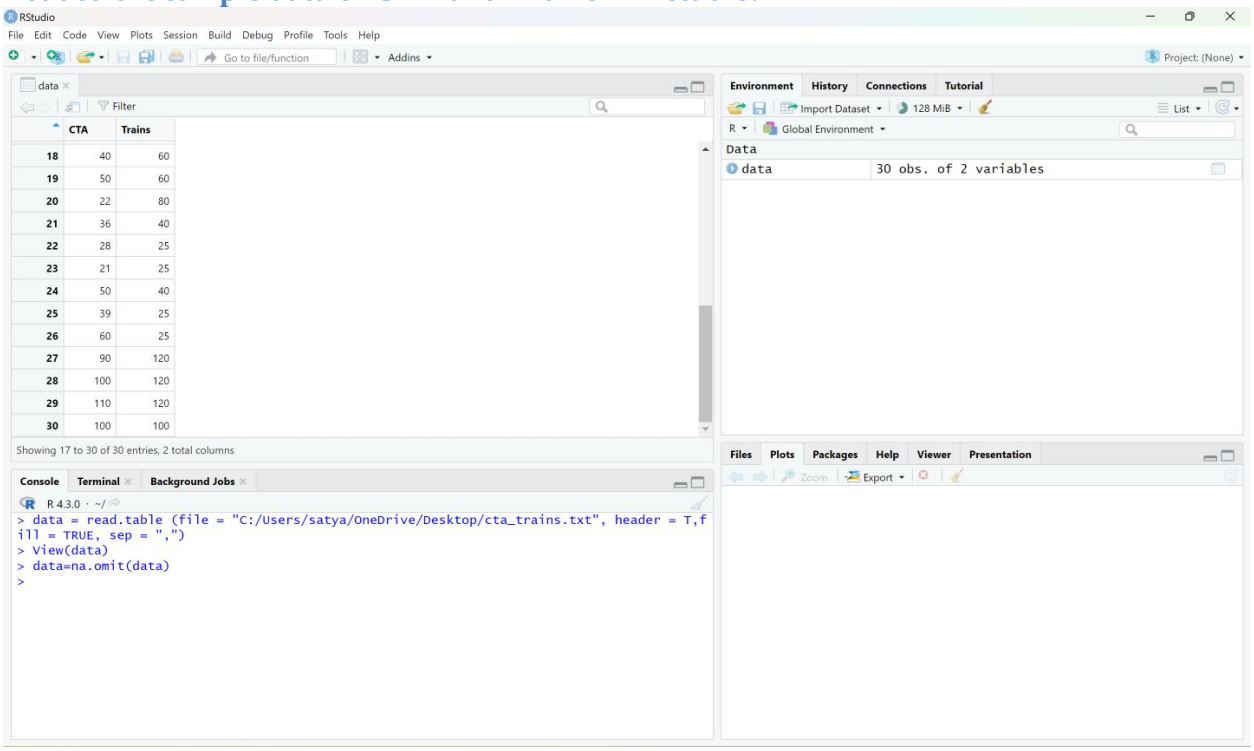
  Fist we need to arrange the data in Ascending order:
  12,12,12,12,14,15,21,22,24,28,30,35,36,39,40,40,40,40,45,50,50,55,60,60,60,90,100, 100,110,120

  Hence median = (40+40)/2 = 40 (Note: Median is also called Q2)

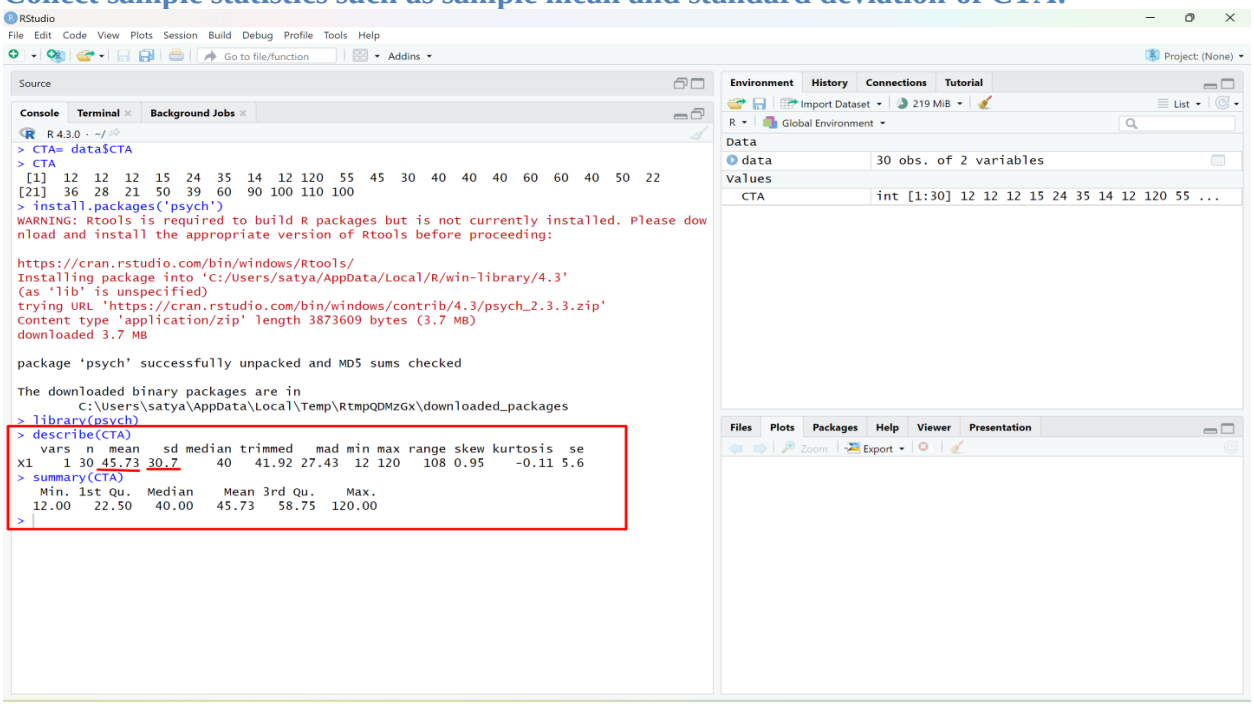
2). [15] Use the sample statistics to estimate the average monthly cost on CTA transits by Chicago residents by using 95% as the confidence level. Assume that we know the population variance is 4. Use R to solve the problem. paste your snapshot of R coding and outputs, also deliver your conclusions.

## ANSWER:

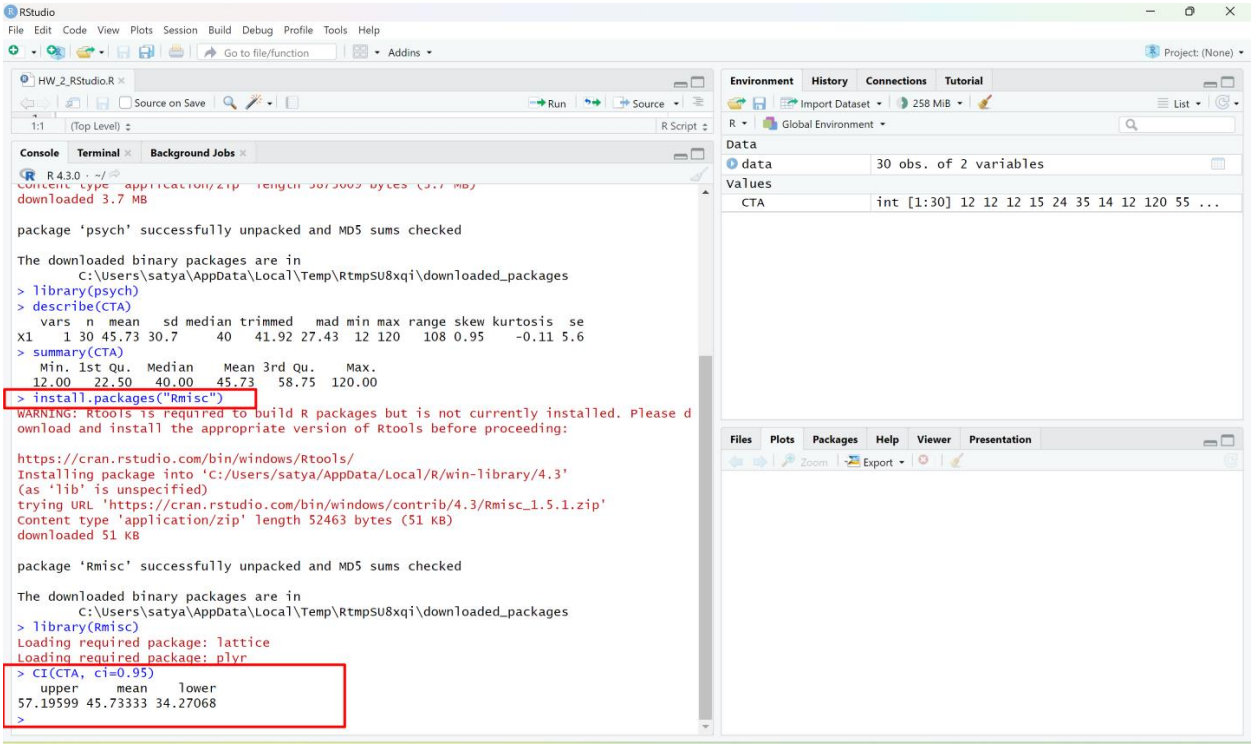
## Loaded the sample data of CTA and Trains in r-studio:




## Collect sample statistics such as sample mean and standard deviation of CTA:



**Produce confidence interval:** To Produce confidence interval, "Rmisc" package was installed & called to use the function CI in it.




Conclusion: I have 95% confidence to say that the average monthly cost on CTA transits by Chicago residents (population mean) will fall in the interval [34.27, 57.19].

4). In addition, they can use Chicago metro trains in their daily life. We ask the same group of 30 users to use Chicago metro trains only and record their monthly cost on trains. In this case, we get two groups of data as follows. We display it as two tables, since it is not able to put them on a single table. For each table, row 1 is the monthly cost by using CTA for each person, row2 is the monthly cost by using train for each person. Each column contains the costs by a same person but use different transportation (CTA vs Metro Trains) . Assume that we know they have the same population variance of 4.

| user | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| row1 | 12 | 12 | 12 | 15 | 24 | 35 | 14 | 12 | 120 | 55 | 45 | 30 | 40 | 40 | 40 |
| row2 | 10 | 16 | 13 | 14 | 28 | 41 | 16 | 10 | 80 | 40 | 75 | 25 | 41 | 29 | 40 |

| user | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| row1 | 60 | 60 | 40 | 50 | 22 | 36 | 28 | 21 | 50 | 39 | 60 | 90 | 100 | 110 | 100 |
| row2 | 50 | 50 | 60 | 60 | 80 | 40 | 25 | 25 | 40 | 25 | 25 | 120 | 120 | 120 | 100 |

It is told that there are no differences if they are going to use the CTA or Metro trains. However, we believe using CTA is more expensive than using Metro trains. We are going to use hypothesis testing to examine whether the costs by two different means should be the same or not. Assume we use 95% confidence level.

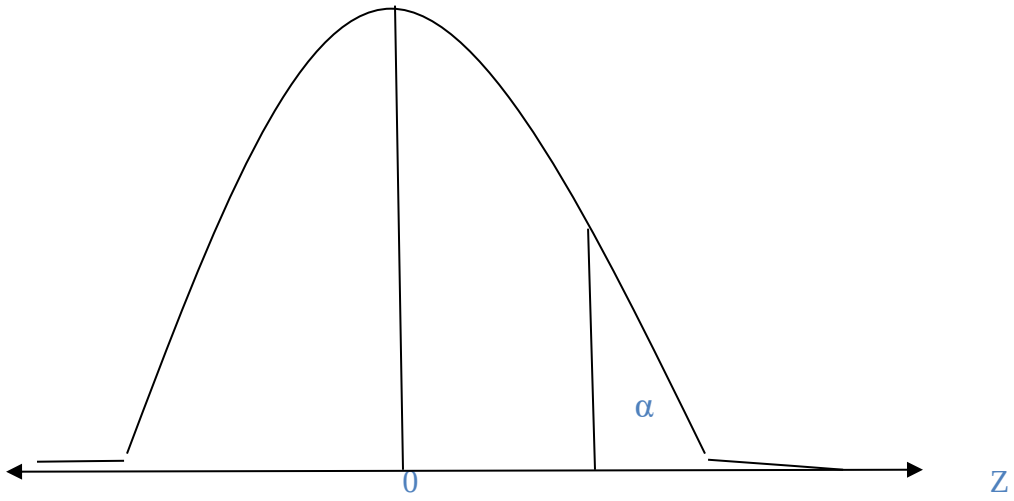
4.1), [15] write down your null and alternative hypothesis, and tell me is it a two-tailed or one-tailed test, and it is two independent or paired samples, why?

**ANSWER:**

-It is a 2- sample hypothesis testing. Sample 1: CTA $(\mu_1)$
Sample 2: Metro trains. $(\mu_2)$

Null & alternative Hypothesis representation:
$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 > \mu_2$

- It is one- tailed hypothesis test, because based on the alternative hypothesis ( $H_1$ *or* $H_a$), it says that, CTA is more expensive than Metro trains. So, we can observe there is only 1 reject region $(Z_\alpha)$ towards the right. Z test because the sample size is 30 (n = 30) and the population variance is known.




- It is two – paired samples because the user (same group) was measured for the monthly cost twice by CTA & Metro trains. So, the 2 samples are paired & dependent on each other.

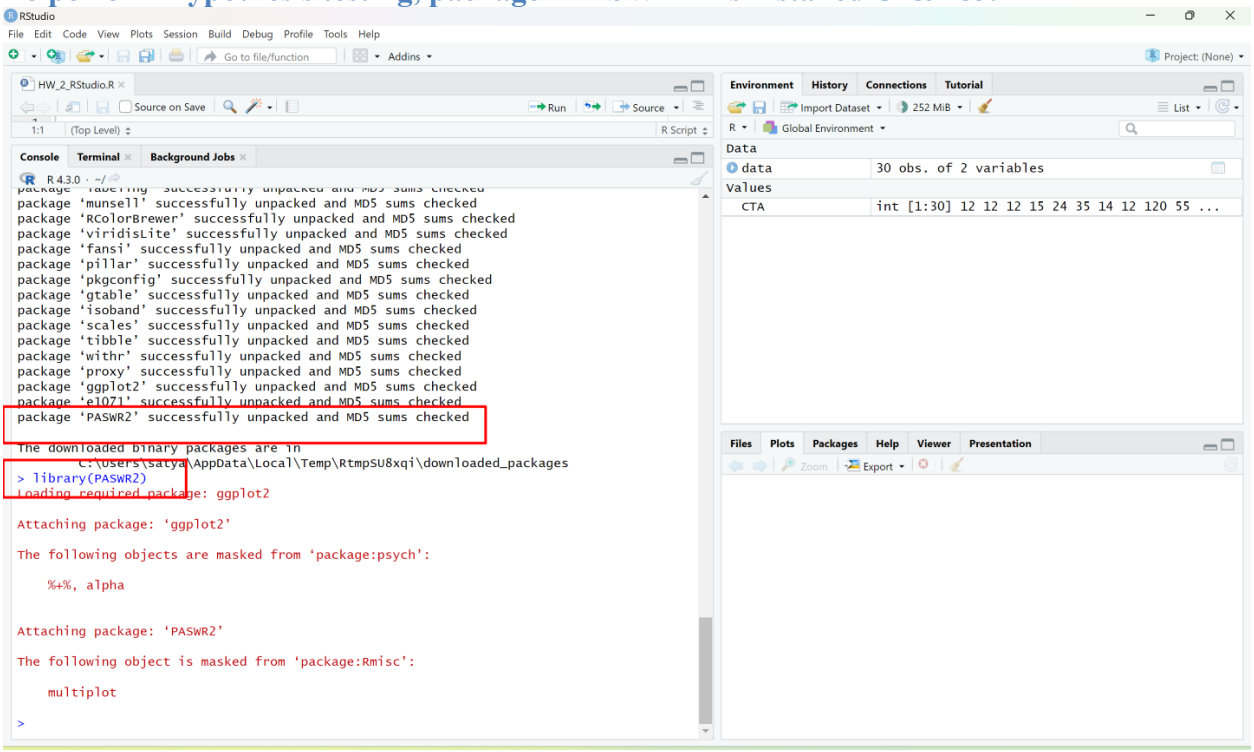
4.2), [20] Perform hypothesis testing to tell whether the costs by two different means are the same or not based on 95% confidence level, by using R. Again, give the R coding, snapshot, outputs, deliver your conclusions by referring to/explaining the outputs.

**ANSWER:** It is 2-sample paired hypothesis testing.

As it is 2- paired sample hypothesis, we can convert it into 1-sample and perform hypothesis testing. For that we need to calculate the difference between CTA & Trains columns & re-represent null and alternative hypothesis in 1- sample as follows:

$$\mu_{diff} = \mu_1 - \mu_2$$
$$H_0 : \mu_{diff} = 0$$
$$H_1 : \mu_{diff} > 0$$

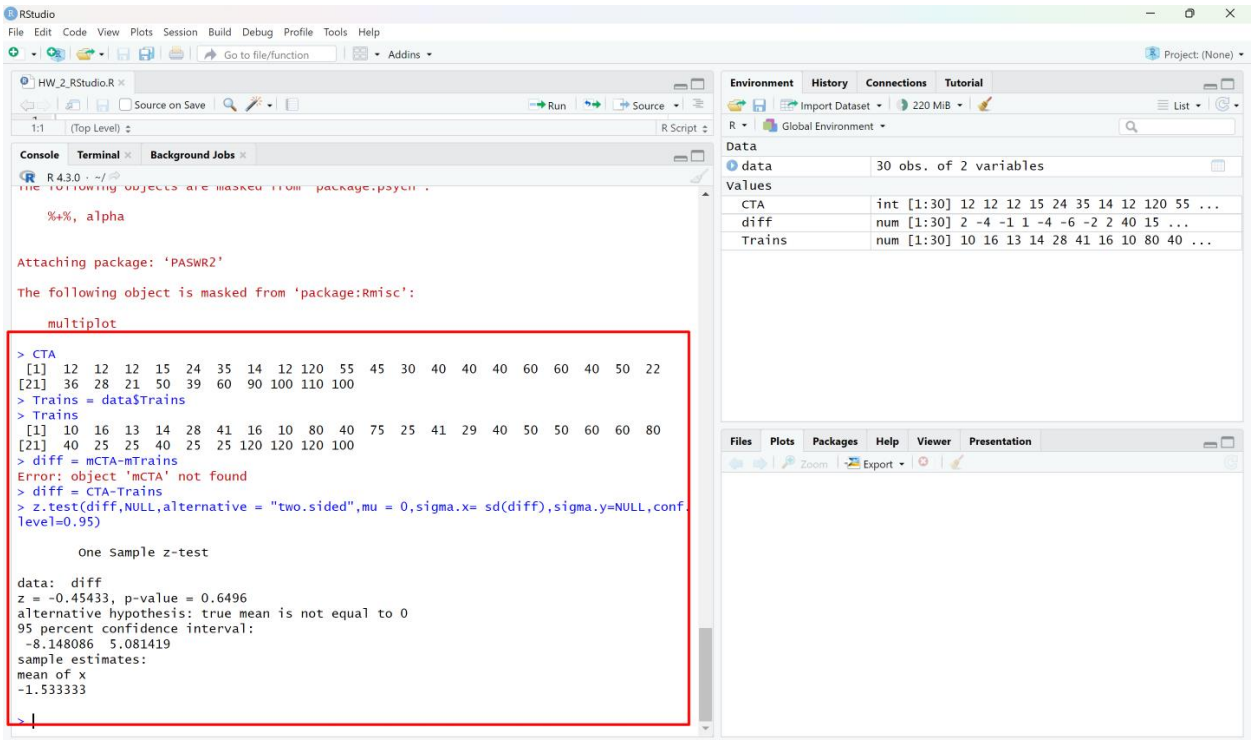
**To perform hypothesis testing, package "PASWR2" is installed & called:**




**Now, let's perform hypothesis testing:** as the sample size is 30 (n=30) which is large, we do z-test. We converted the 2-paired sample to 1-sample by calculating the difference between 2 columns.

Now we have only one column (1-sample) which is the 'diff' column. And we can perform hypothesis testing on that one 'diff' column,

Note: diff = CTA - Trains




## Conclusion:

Based on the paired z-test performed at a 95% confidence level, we fail to reject the null hypothesis. The p-value is 0.6496, which is greater than the significance level of 0.05. Therefore, we do not have sufficient evidence to conclude that there is a significant difference in the monthly costs between using CTA and metro trains. The 95% confidence interval for the difference in means is -8.1480 to 5.0814, which includes zero. This further supports the conclusion that the means of the two transportation methods are likely to be equal.

Hence, we do not reject the null hypothesis based on 95% confidence level.

**3. (10 points)** The z-test and t-test we used for hypothesis testing (including both one-sample and two-sample hypothesis testing) are also known as "Parametric Test". Alternatively, there are other statistical tests which can be used as alternatives, and they are called "Non-Parametric Test". Search online or find other learning materials to answer the questions below.

1). (5 points) what are the differences between Parametric Test and Non-Parametric Test?

**ANSWER:**

| Basis for comparison | Parametric Test | Non-Parametric Test |
|---|---|---|
| 1. Definition | The parametric test is a hypothesis test that offers generalizations for claiming something about the parent population's mean. A commonly employed t-test in this context is one that is based on Student's t-statistic. | The nonparametric test is characterized as a hypothesis test without underlying assumptions, i.e., it does not call for the population's distribution to be represented by parameters. |
| 2. Meaning | A parametric test is a statistical test in which assumptions about a population parameter are made. | Non-parametric test is a statistical test that is used when there are non-metric independent variables. |
| 3. Population Information | Completely known | Not known |
| 4. Applicability | Only on variables. | Both variables and attributes |
| 5. Central tendency measurement | In general, the measure of central tendency in the parametric test is mean, | - While in the case of the nonparametric test is median.<br>- As a result, it is also known as the distribution-free test. |
| 6. Correlation test | The parametric test makes use of Pearson's coefficient of correlation to assess the degree of relationship between two quantitative variables. | while spearman's rank correlation is used in the nonparametric test. |
| 7. Basis of test statistic | Distribution | Arbitrary |

2). (5 points) what are the requirements/conditions to use z-test or t-test? If our data does not meet these requirements, which non-parametric test is the alternative for hypothesis testing?

**ANSWER:**

**Requirements to use Z-test / t-test:**

1) If the sample size is large (n >= 30) & population standard deviation is known => Z-test
2) If Sample size is small (n < 30) and population STD is unknown => t-test
3) If Sample size is large (n >= 30) and population STD is unknown => t-test
4) Either a Z-test or a T-test can be used to compare the means of two independent samples, such as two separate groups.

5) A paired T-test is often employed when we need to compare the means of paired samples (for instance, before and after measurements).
6) Df (degree of freedom must be known to carry t-test

- You can use non-parametric tests for hypothesis testing if your data does not fulfill these conditions or if you would prefer a non-parametric option. Non-parametric tests don't rely on presumptions about how the data are distributed.

Equivalent alternative non-parametric tests for hypothesis testing:

| **Parametric Test** | **Non- Parametric Test** |
|---|---|
| 1. Independent Sample t Test | Mann-Whitney U test:<br><br>Utilizing this test, two independent groups' medians are compared. The independent samples t-test can be replaced with this method. |
| 2. Paired samples t test | Wilcoxon signed Rank test:<br><br>This test is used to determine if the median of one group deviates from a value that is hypothesized or to compare the medians of two groups that are related. It serves as a substitute for the paired samples t-test. |
| 3. One way Analysis of Variance (ANOVA) | Kruskal Wallis Test:<br><br>To compare the medians of three or more independent groups, apply this test. The one-way analysis of variance (ANOVA) test can be substituted with this method |
| 4. One-way repeated measures Analysis of Variance | Friedman's ANOVA:<br><br>The medians of three or more related groups are compared using this test. This test serves as an alternative to the repeated measures ANOVA. |

Homework 3

Your Name: **TA sample solutions**

Student ID:

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Using Case Study 1 data, Case1_Student Grades_Large.csv
Using student demographic info and learning behaviors (weekly hrs in different categories) to predict "Grade"

Note: exclude nominal variables, and student performance variables (e.g., Exam) from the list of x variables
Using hold-out evaluation only, 80% as training
Use feature selection to build multiple models, and compare the models based on RMSE

- Backward method using p-value in t-test as metric
- Backward method using AIC as metric
- Forward method using AIC as metric
- Stepwise method using ACI as metric
- Best subset method using Adj-R2 as metric

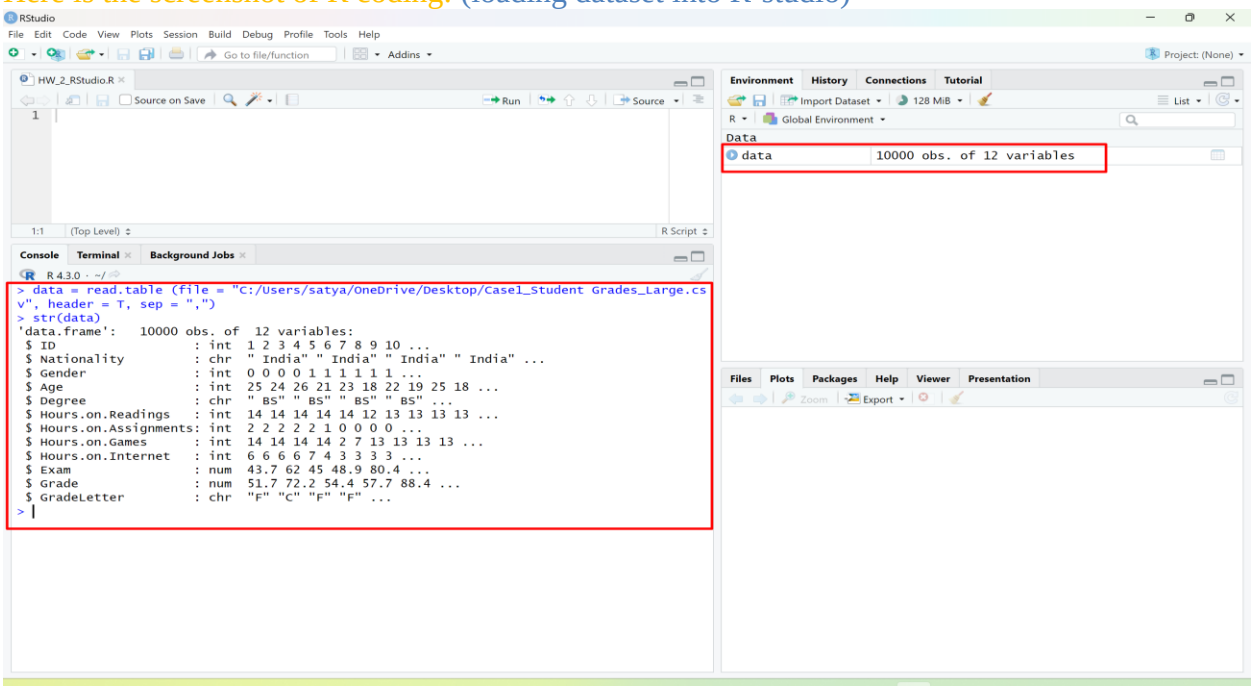
Q1 Show the R coding, outputs, and <u>your explanations</u> for each step in linear regression.

## ANSWERS:

### Importing the data into R- studio:

- First, we need to import the dataset into R-studio. Here we used "Case1_Student Grades_Large.csv".

Here is the screenshot of R coding: (loading dataset into R-studio)



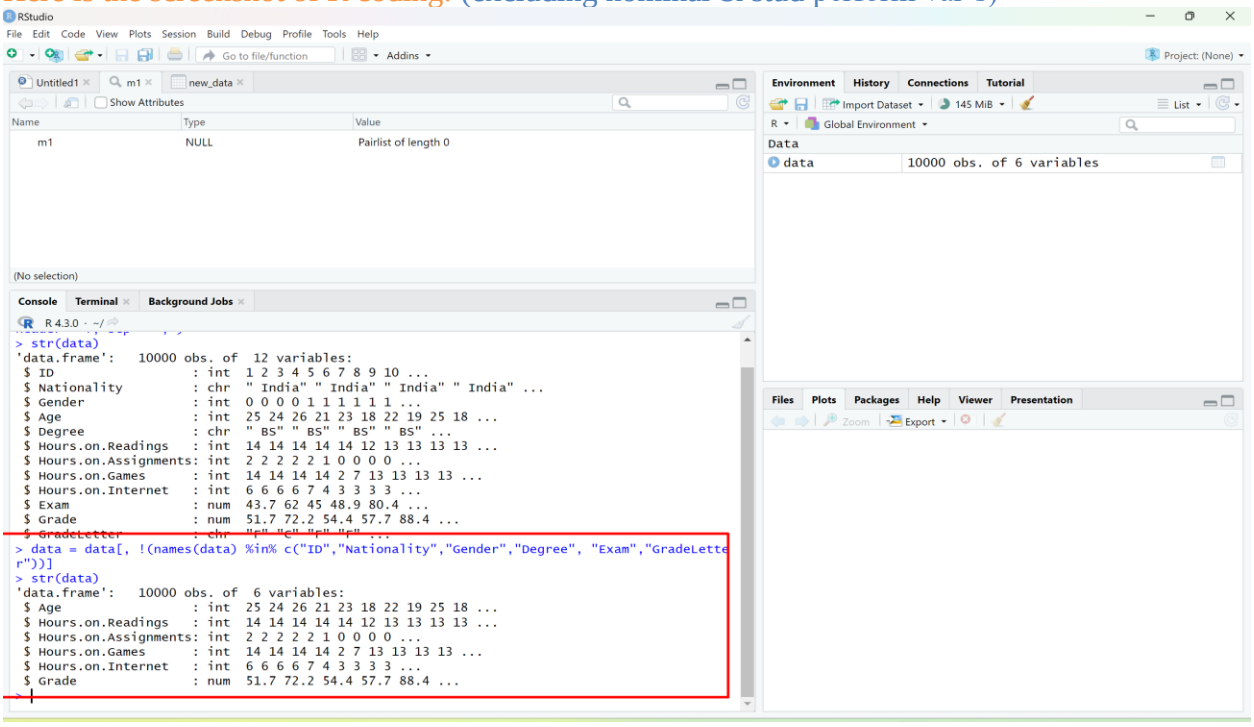
**Step 1: Understand the data & well define independent variables & dependent variables:**

- Here, X variables represents the predictor variables, and Y variable represents the target variable.
    - Y variable (target variable/ Dependent variable): Grade
    - X variables (Predictors/ Factors/Independent variables): ID, Nationality, Gender, Age, Degree, Hours on Readings, Hours on Assignments, Hours on Games, Hours on Internet, Exam, Grade Letter)

**Excluding nominal & student performance variables (Exam) from the list of X-variables to perform linear regression:**

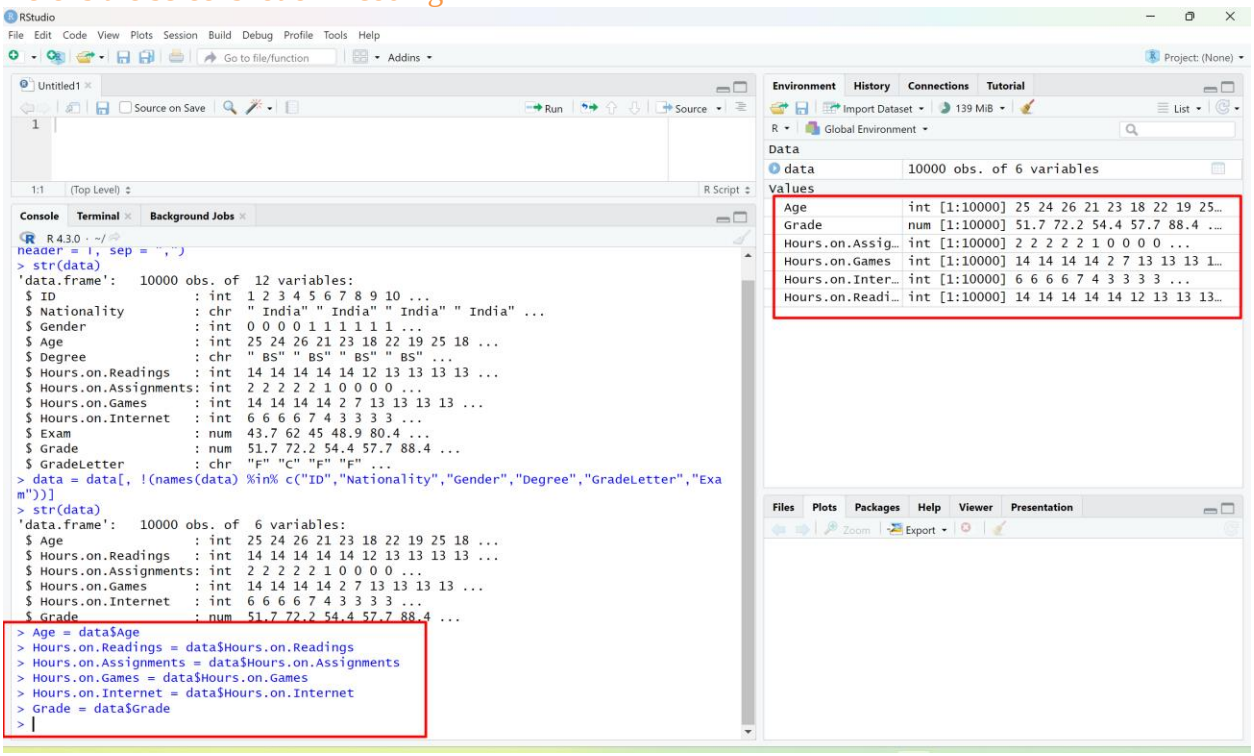
- As per the requirement in the question, we need to exclude nominal variables, and student performance variables (e.g., Exam) from the list of x variables to perform linear regression.
- So, the variables need to be excluded from the list of x-variables are:
    - ID
    - Nationality
    - Gender
    - Degree
    - Exam
    - GradeLetter
- So, the final X-variable list contains the following variables:
    - Age
    - Hours on Readings
    - Hours on Assignments
    - Hours on Games
    - Hours on Internet

Here is the screenshot of R coding: (excluding nominal & stud perform var's)




**Storing X and Y variables:**

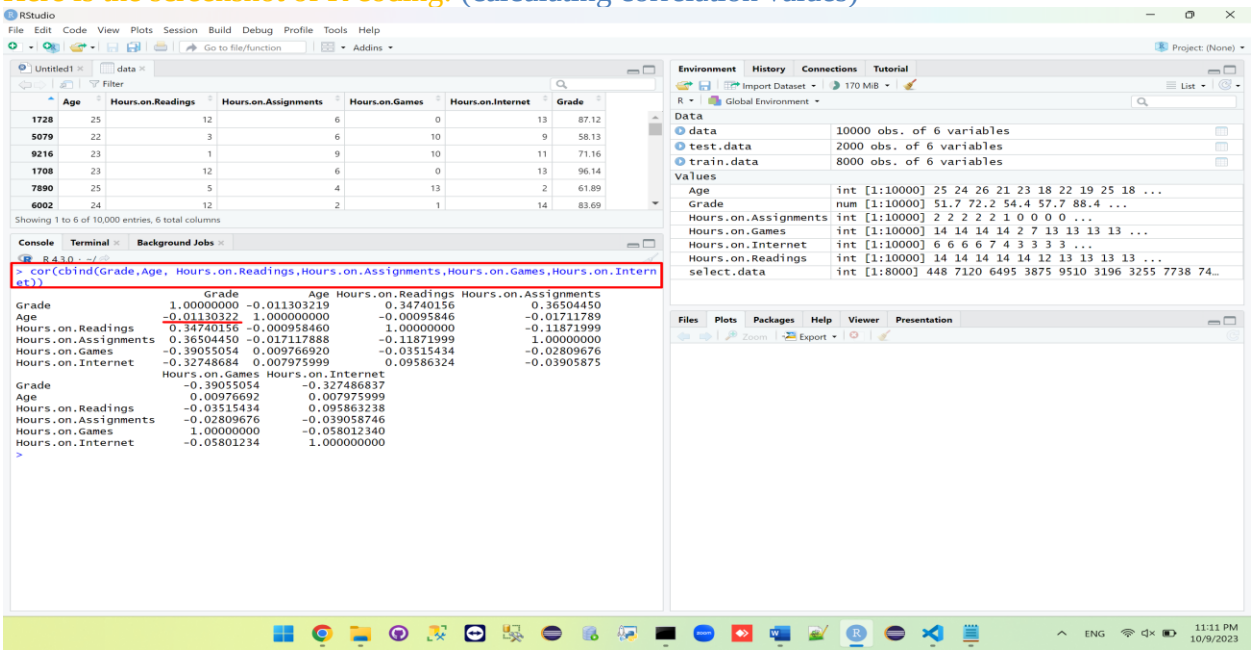
Here is the screenshot of R coding




## Step 2: Examine the linear relationship between x and y variables.

- We can examine linearity by 2 methods.
    1. Produce a scatter plot for each x & y variable / produce a single plot with every pair of variables.
    2. Calculate correlation values for all the x variables with y variable.
- As the plot method is not clear & reliable all the time, we can perform 2nd method and calculate the correlation values.

Here is the screenshot of R coding: (calculating correlation values)

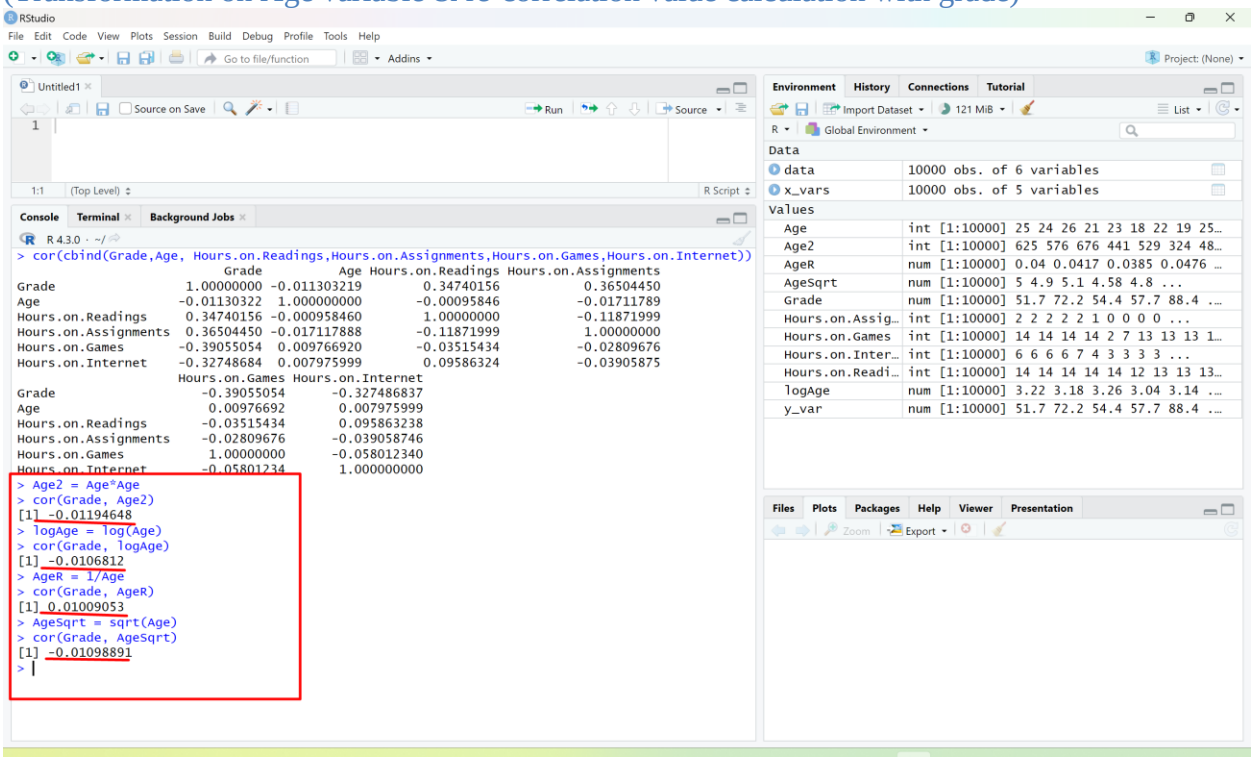


- We noticed that the correlation values are as follows:
  - Grade & Hours on readings = 0.34740156 - weak Correlation - +ve correlation
  - Grade & Hours on Assignments = 0.36504450 - weak Correlation - +ve correlation
  - Grade & Hours on games = -0.39055054 - weak Correlation – -ve correlation
  - Grade & Hours on internet = -0.32748684 - weak Correlation – -ve correlation

  - Grade & Age = -0.01130322 – no Correlation – Try transformation to improve correlation.
- As Age has no correlation with Grade(Y-variable), we can try transformation on this Age (x- variable) and re-calculate the correlation with Grade (y-variable) again.

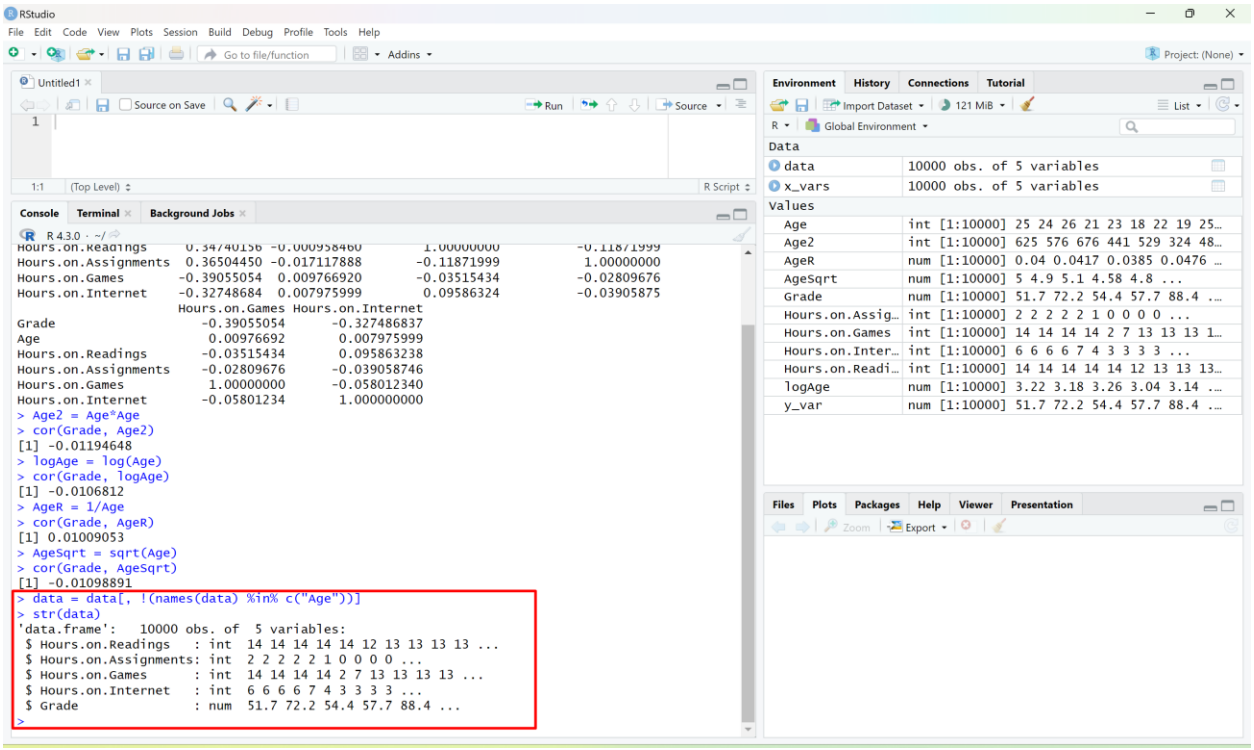
    - Square transformation: X' = X * X
    - Log transformation: X' = logX
    - Inversion transformation: X' = 1/X
    - Square root transformation: X' = sqrt(X)

Here is the screenshots of R coding:
(Transformation on Age variable & re-correlation value calculation with grade)




- As the correlation didn't improve after transformation on 'Age' variable, we decided to drop the age variable.

Here Is the screenshot of r coding: (dropping the age variable)

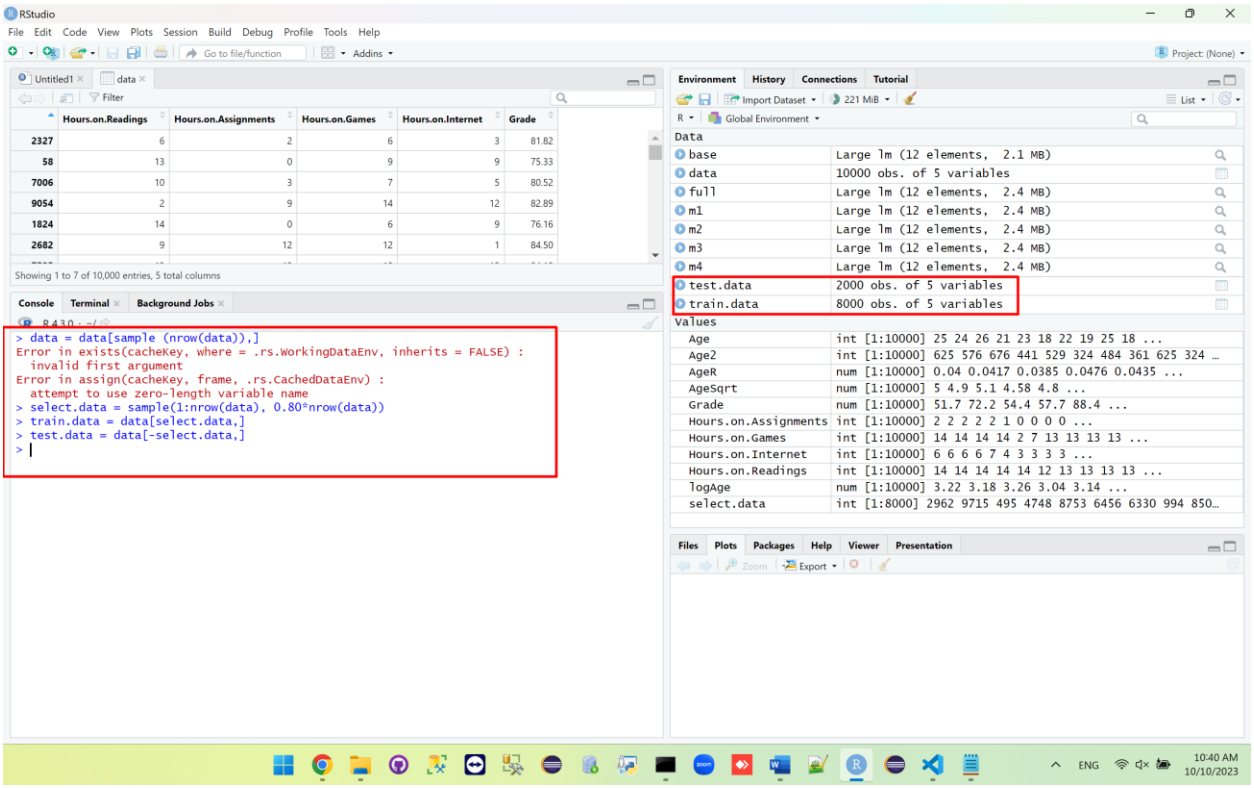



- Now the list of X-variables (Age is removed) (5 variables)
    - Hours.on.Readings
    - Hours.on.Assignments
    - Hours.on.Games
    - Hours.on.Internet
- Y-variable:
    - Grade

**Step 3: Decision on evaluation strategy and data splits:**

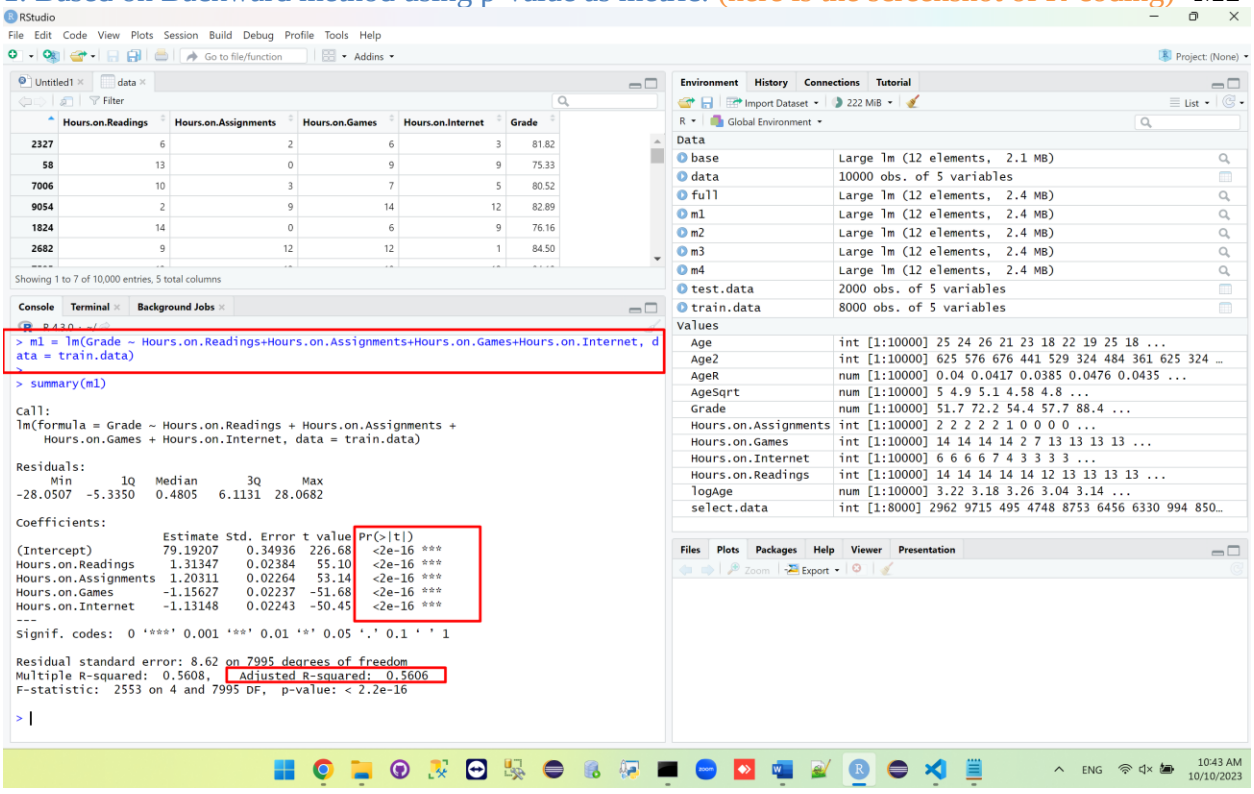
- As mentioned in the question, we need to use hold out evaluation for this data split.
- Note: In general, this data split depends on the size of the data.
    - If the data is large (more than 1 million rows) then hold-out evaluation is used.
    - If the data is small (1 million rows /smaller) then N- fold cross evaluation is used.
    - If your computer is powerful enough, choose N- fold cross evaluation even for the larger data.
    - If the computer is not powerful, then the process may take 5-6 hours, so use hold-out evaluation for the larger data.
    - If your data is small, then definitely choose N-fold cross evaluation.
- Using hold-out evaluation only, **80% as training set and 20% as testing set.**
- First, we need to shuffle the rows. **********important point

Here is the screenshot of R coding:



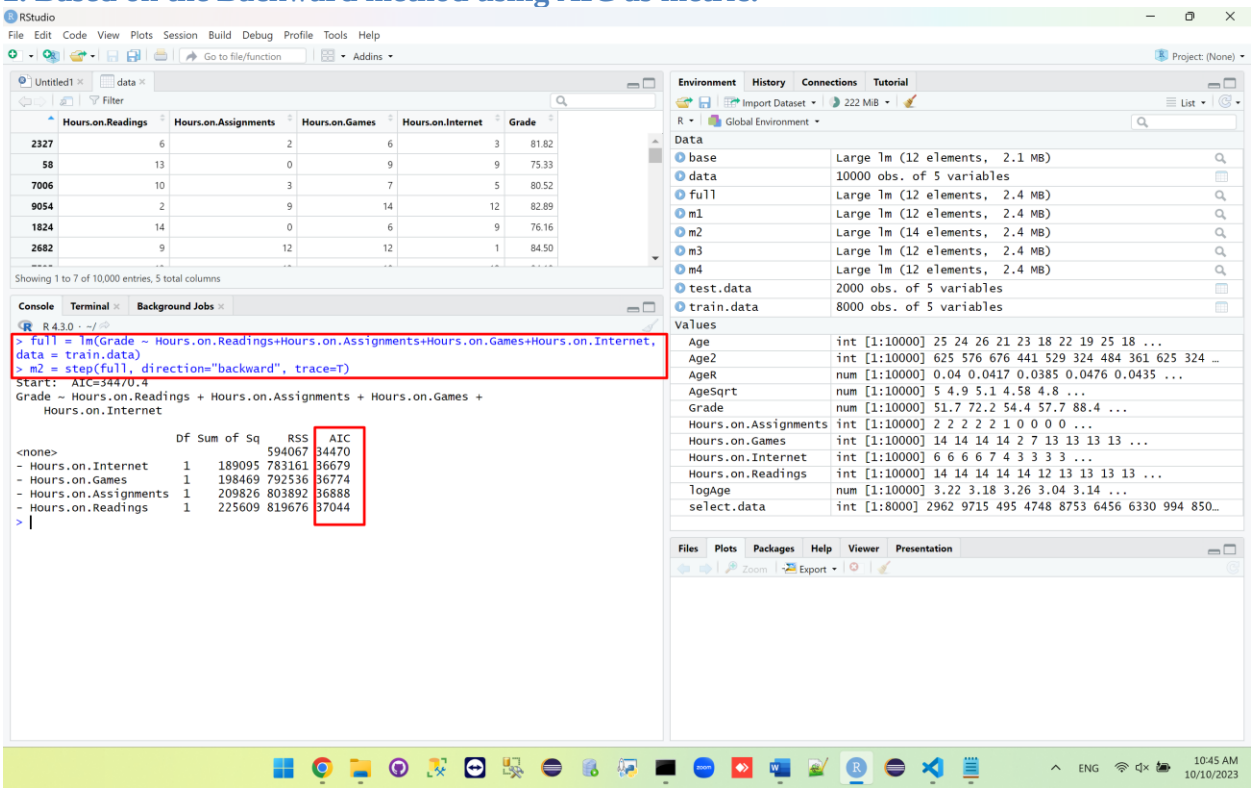

## Step 4: Building Multiple linear regression models using 'feature selection' process:

1. Based on Backward method using p-value as metric: (here is the screenshot of R-coding)- **M1**

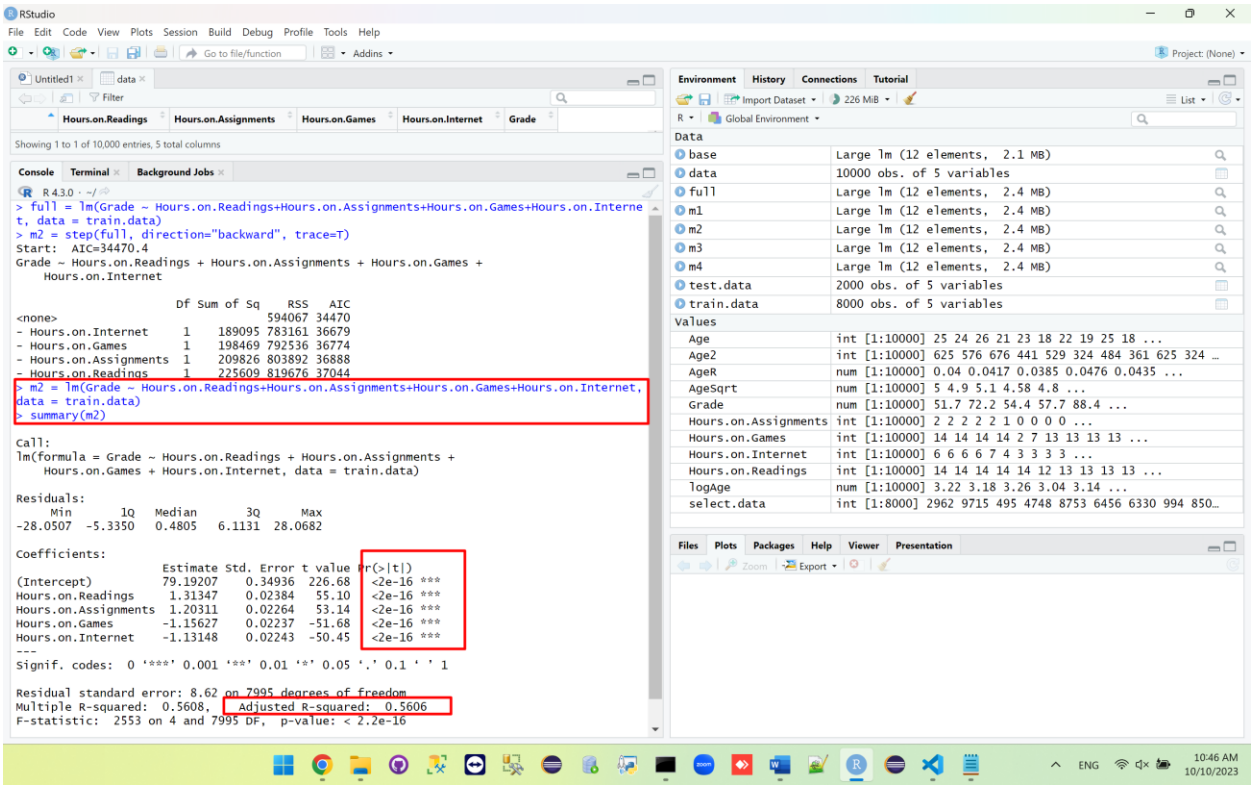


- As all the x'variables have smaller p values than alpha (Assuming using 95% confidence level), all the x-var's are useful to make prediction on Y-var (Grade).
- As no variable is having larger p-value than alpha, there is no need to eliminate any variable 1 by 1 here manually.

## 2. Based on the Backward method using AIC as metric:

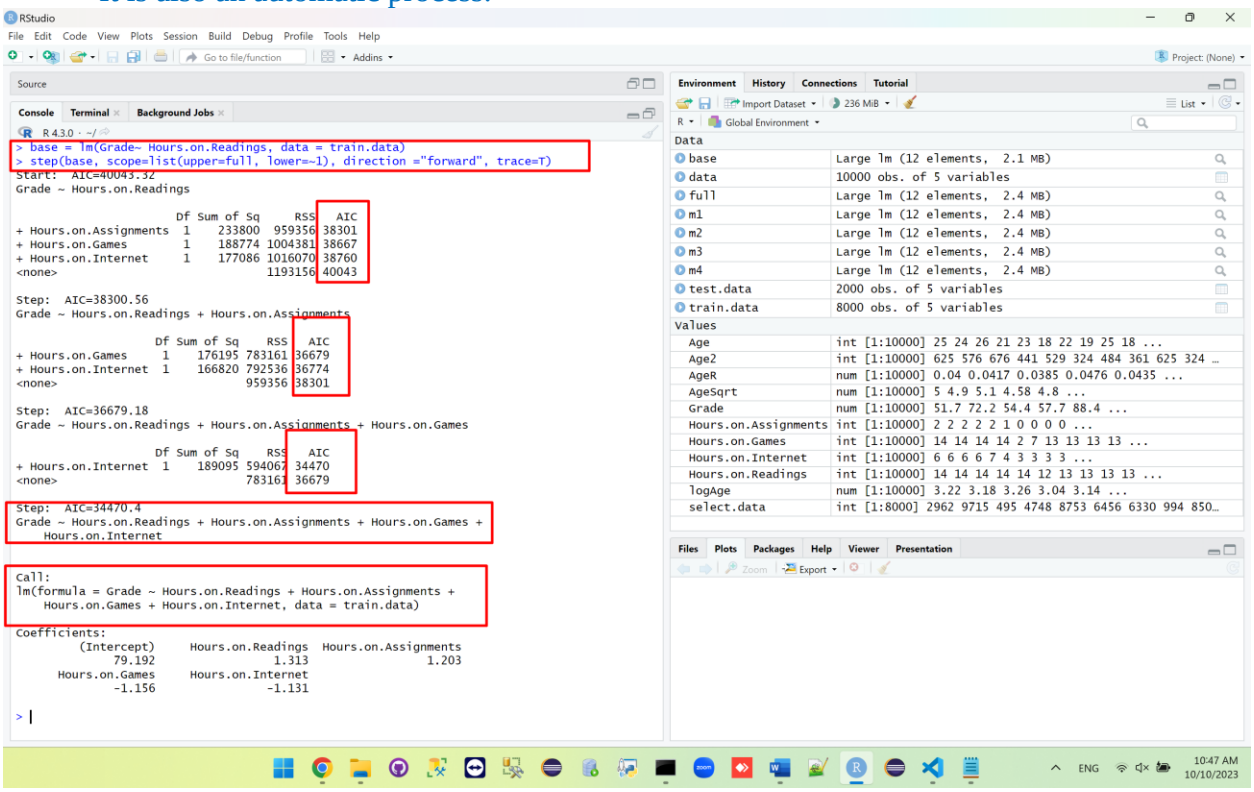



- All the x-variables became useful, so no x-variables got eliminated automatically.
- This is an automatic process.
- So, we got just 1 final step in the output.
- In this output, we can't see p-value, adj r2.
- So, build another model with all the var's that became useful to see adj r2, p-value. **M2**

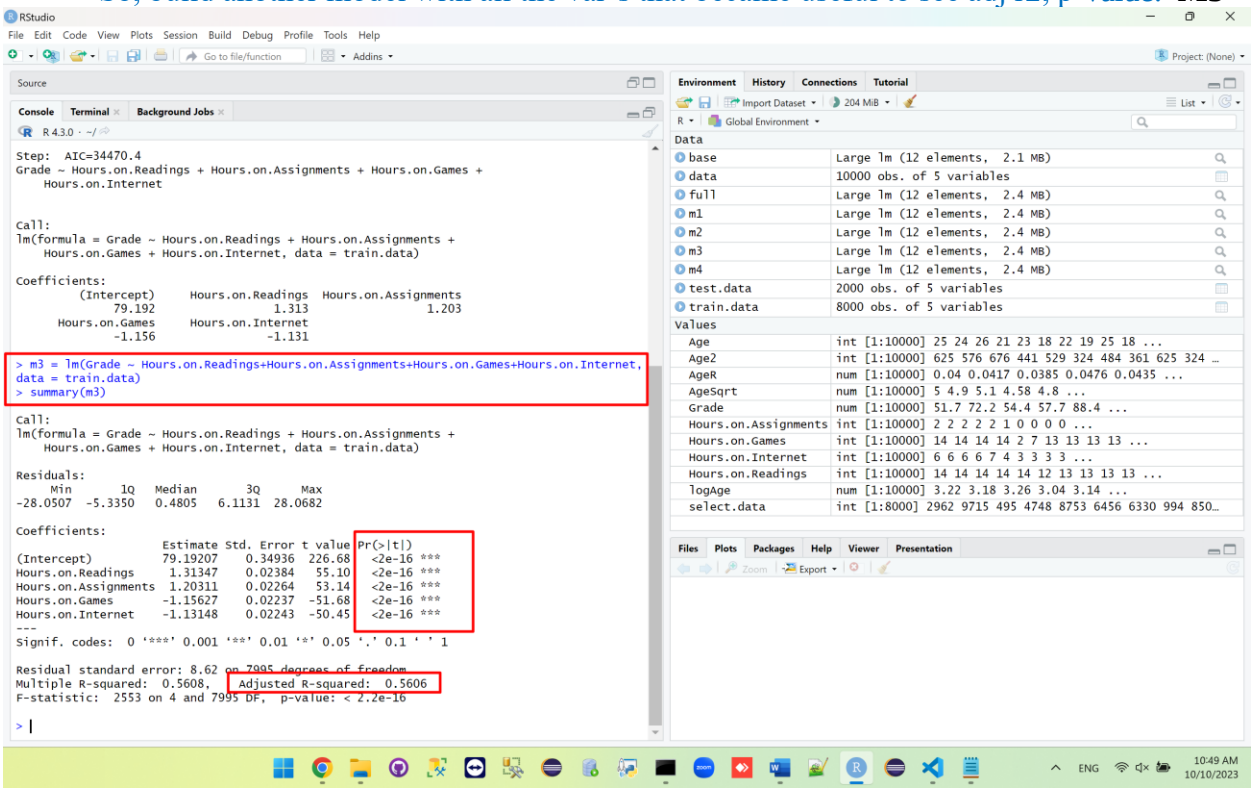



## 3. Based on Forward method using AIC as metric.

- First build a base model with just 1 X-var and y-var, then use step () function to automatically add remaining variables which are useful one by one.
- It is also an automatic process.

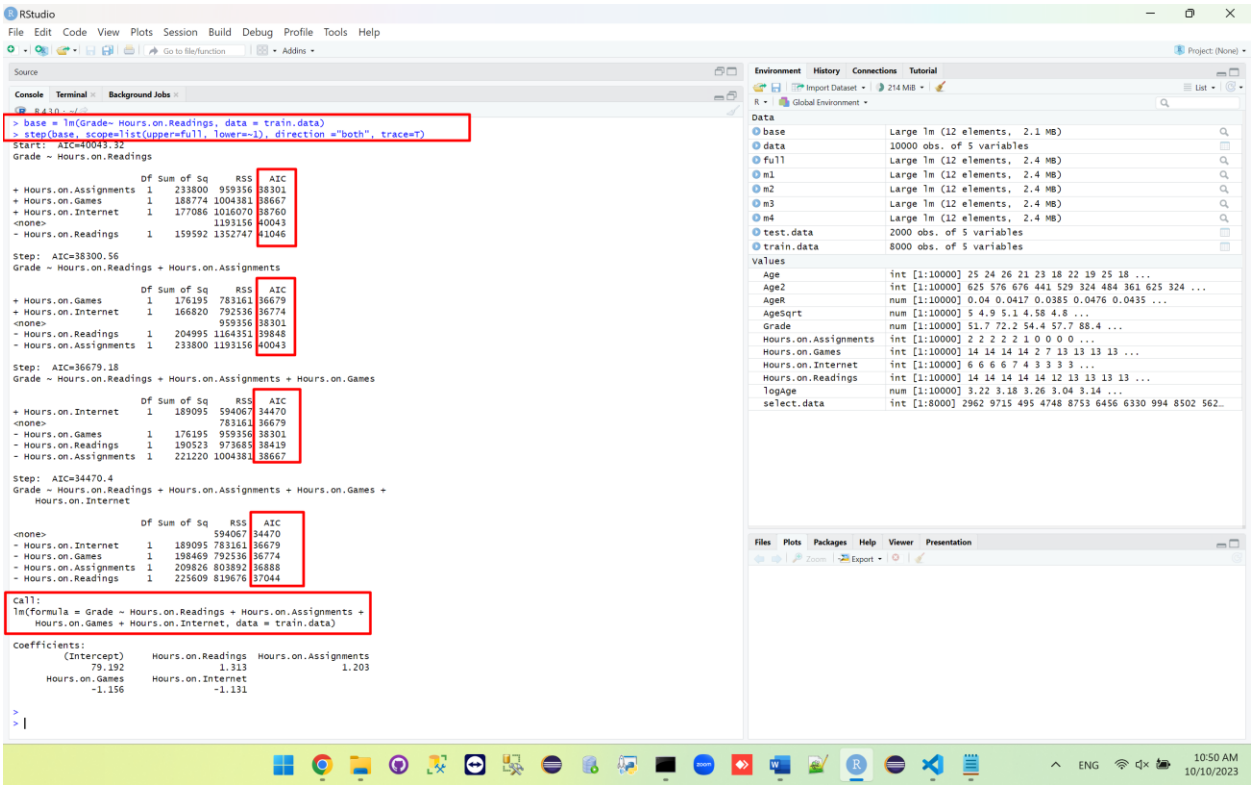


- In this output, we can't see p-value, adj r2.
- So, build another model with all the var's that became useful to see adj r2, p-value. **M3**

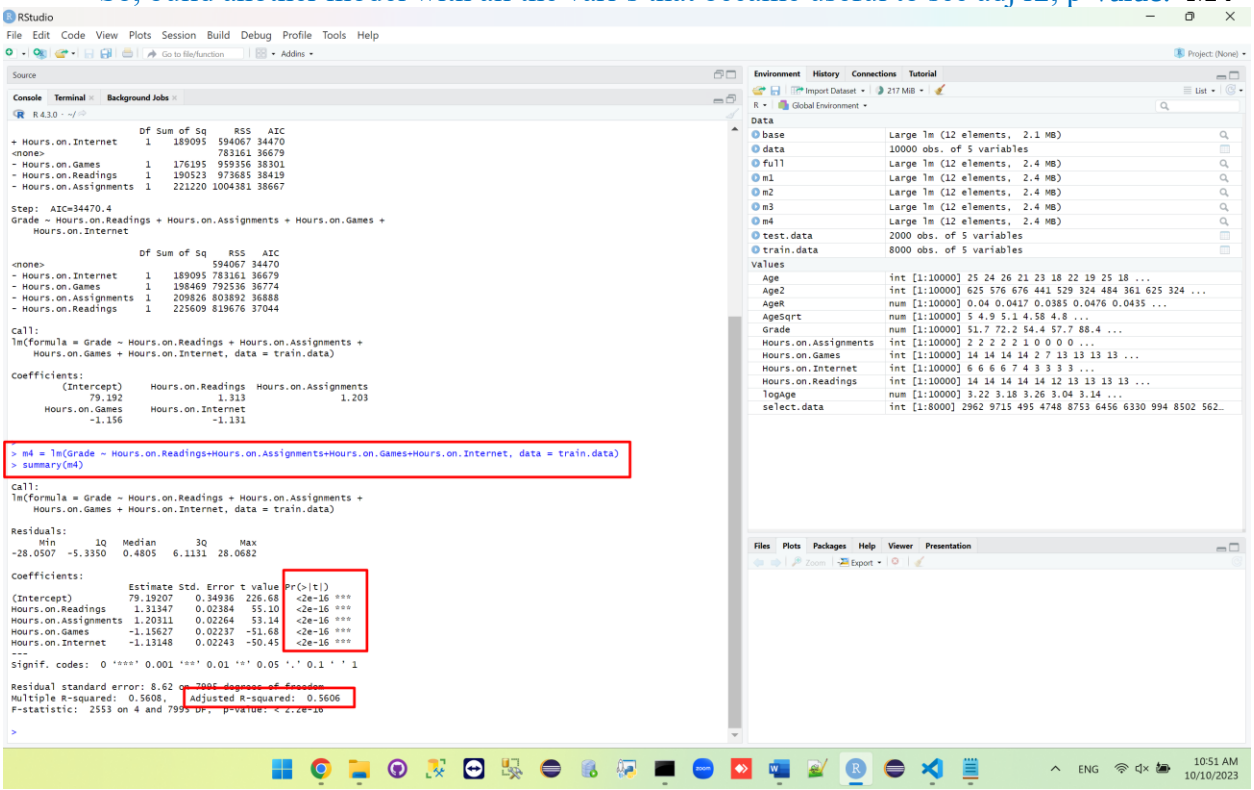



## 4. Based on Stepwise method using ACI as metric.

- First build a base model with just 1 X-var and y-var, then use step () function to automatically add remaining variables which are useful one by one.
- Set the direction as "both".
- It is also an automatic process.

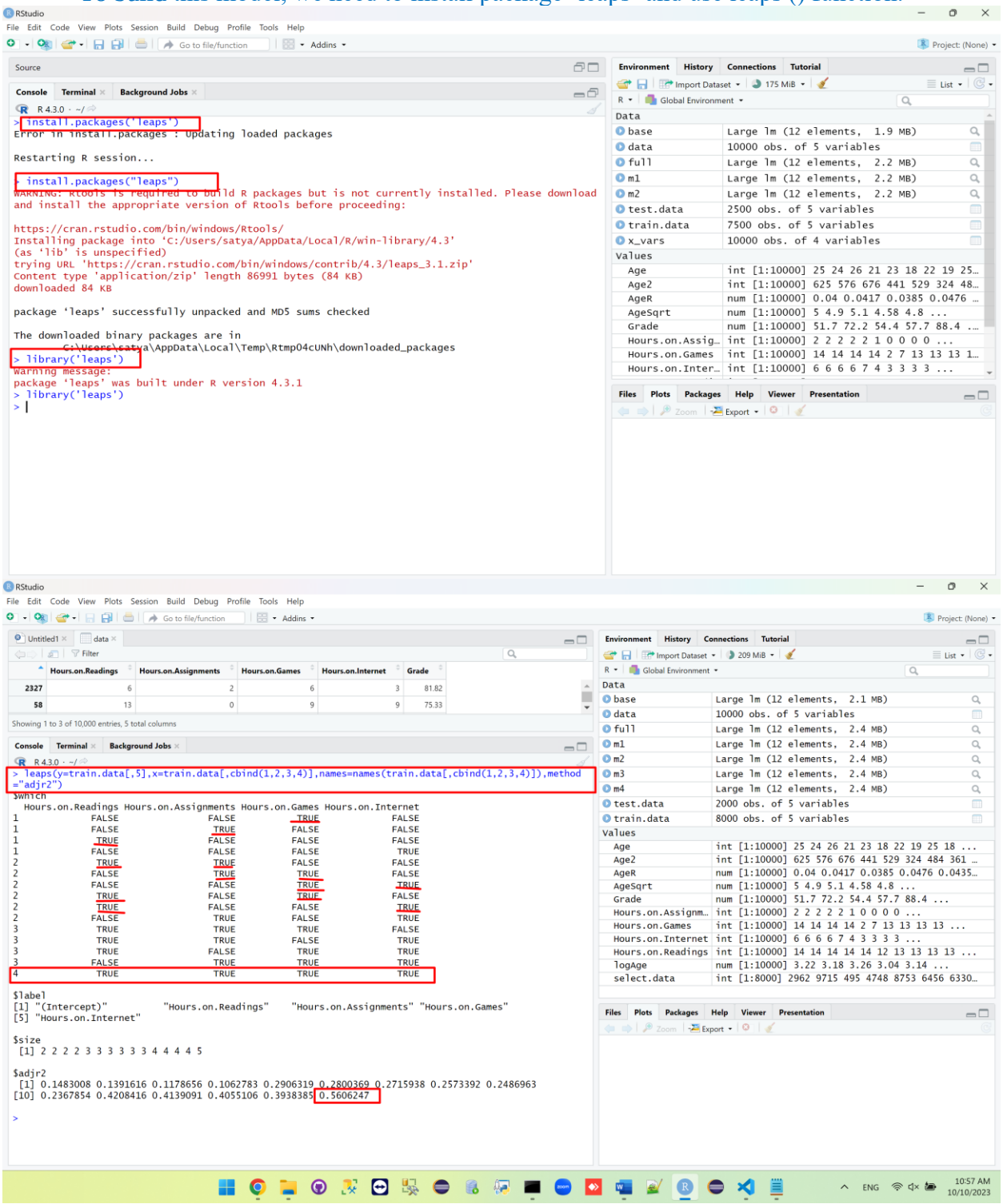



- In this output, we can't see p-value, adj r2.
- So, build another model with all the vari's that became useful to see adj r2, p-value. **M4**




## 5. Based on Best subset method using Adj-R2 as metric:

- It gives you all the combination of x-variables, then we are going to build a model by calculating a specific metric like adjr2.
- To build this model, we need to install package ‘leaps’ and use leaps () function.




Interpretation of the output:

Subset of x-variables selected are: (true / false output)

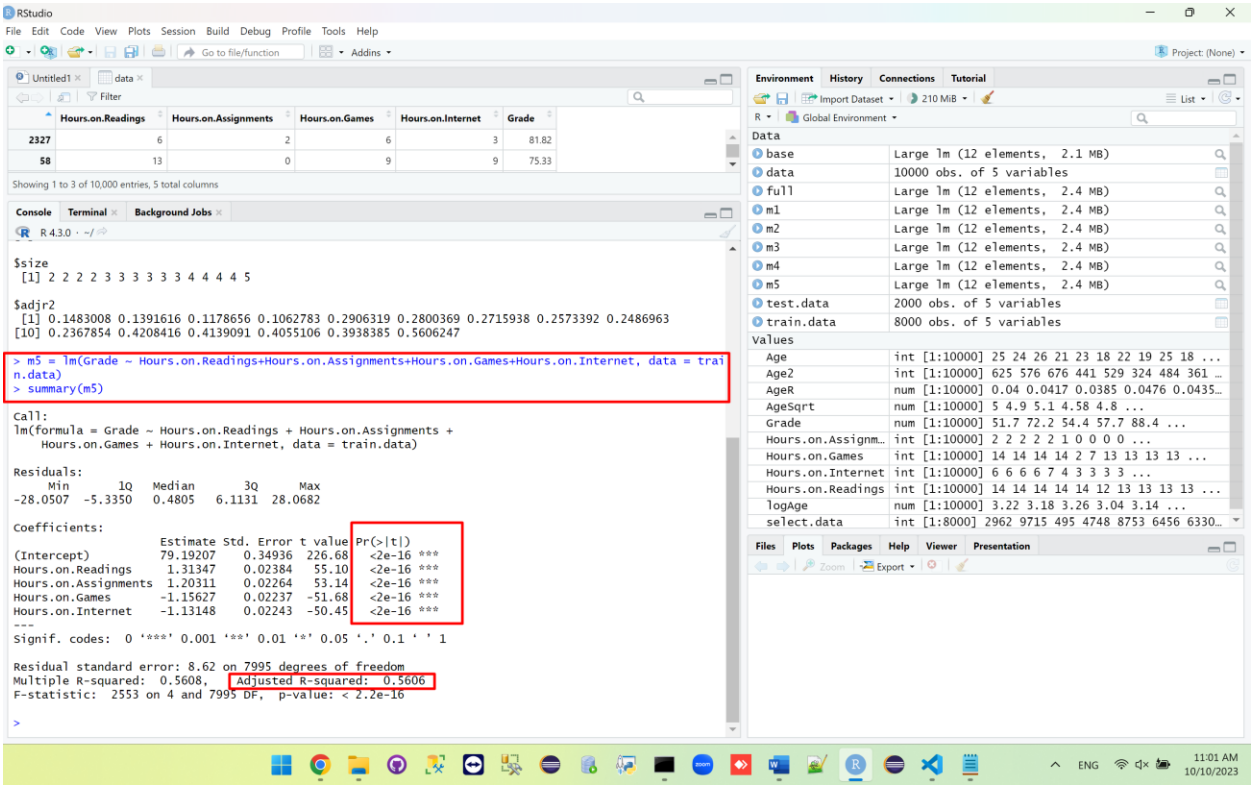
- 1- true -Hours.on.games
- 1 -true- hours.on.assignments
- ..
- ……. selected the variable which has true value in each row.
- ..
- 2 – true – hours on readings & hours on assignments
- ..
- ..
- ……..selected the subset of 2 variables which has true values in each row.
- ..
- ..

Adj r2 output explanation:

- We need to find the best and largest value in adj r2 output.
- The best value is = 0.5608489
- The index of this model is = 15. That means this adj r2 value belongs to 15th model.
- Then we need to look in to the true/false chart and go to the 15th row and check which variables are selected true.

- As all the variables are selected as true, all the variables are useful.
- So, build a model with all these x-var's and y-var Grade and look for the adj r2 in the output.

**M5**



Now we got 5 models : M1, M2, M3, M4,M5 (same results)

| MODEL |
| --- |
| M1 : Based on Backward method using p-value as metric: |
| M2: Based on Backward method using AIC as metric: |
| M3: Based on Forward method using AIC as metric. |
| M4: Based on Stepwise method using ACI as metric: |
| M5: Based on Best subset method using Adj-R2 as metric |

**Step5 : Model diagnosis:** We need to perform model diagnosis for all the models above and compare and see which model is qualified.

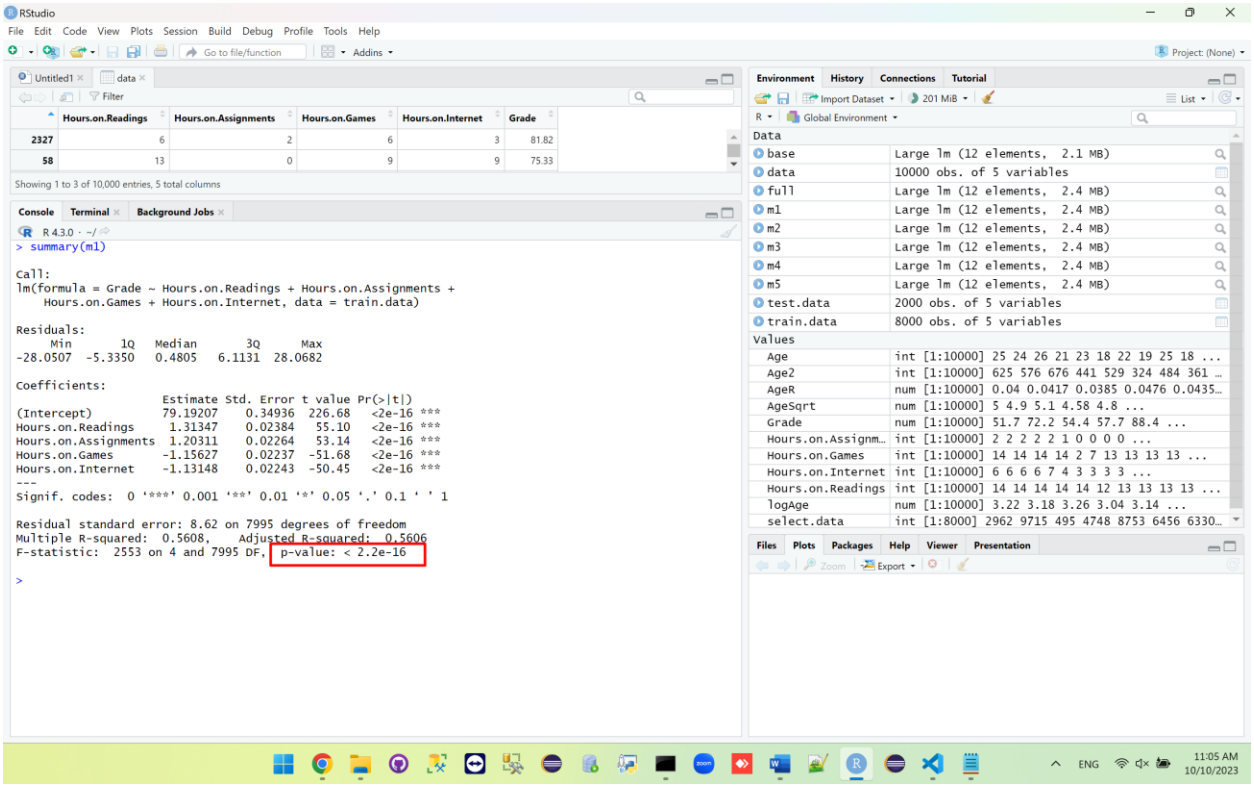
***But pls make note that all models are same we can pick any 1 model and perform model diagnosis… Lets pick M1 and go ahead**********

There are 2 components in model diagnosis:

1. F-test (goodness of fit test)
2. Residual analysis

**F-test:** is a statistical test for hypothesis testing. We need to write down null hypothesis and alternative hypothesis:

F-test on M1:



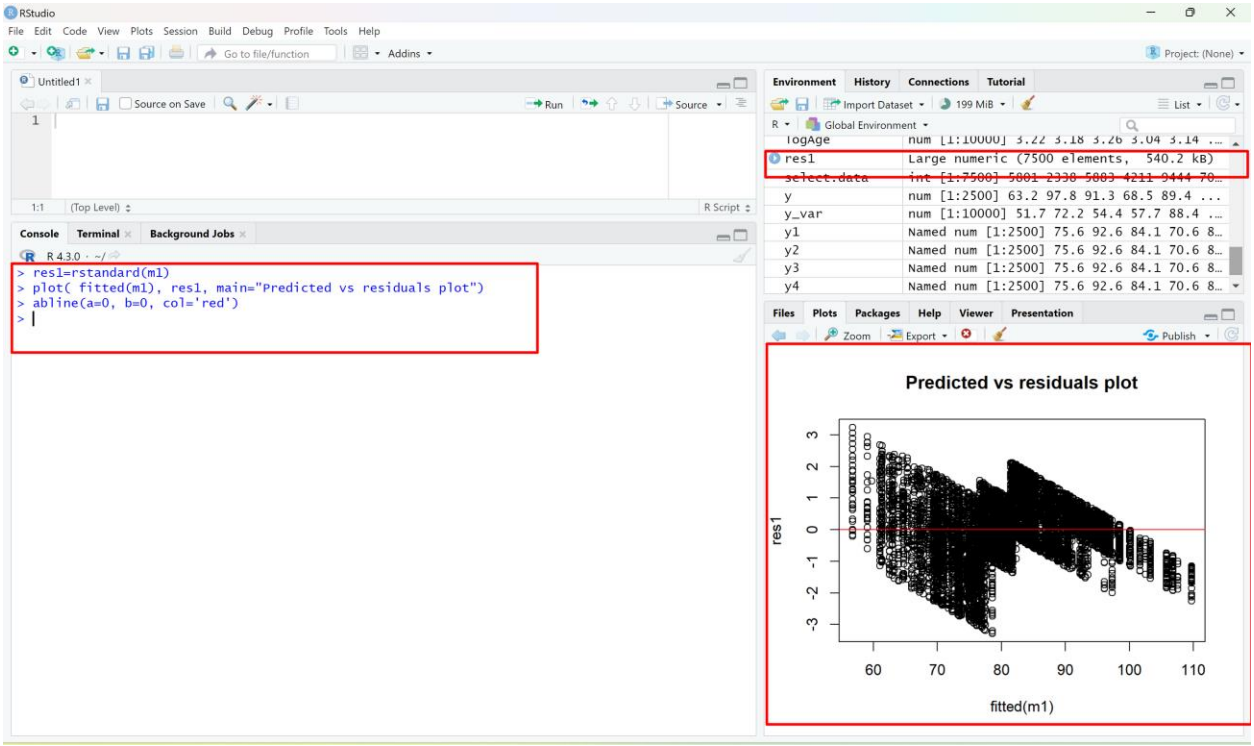
## Residual analysis on M1:

- As we built 5 models above through the feature selection process(M1, M2, M3, M4, M5), we need to perform residual analysis for all the models.

**** but. Please note that all the above models are same, So, we can perform residual analysis on any 1 model. As we already picked M1 for model diagnosis and performed f-test on it, now lets perform residual analysis for the same model M1 ****

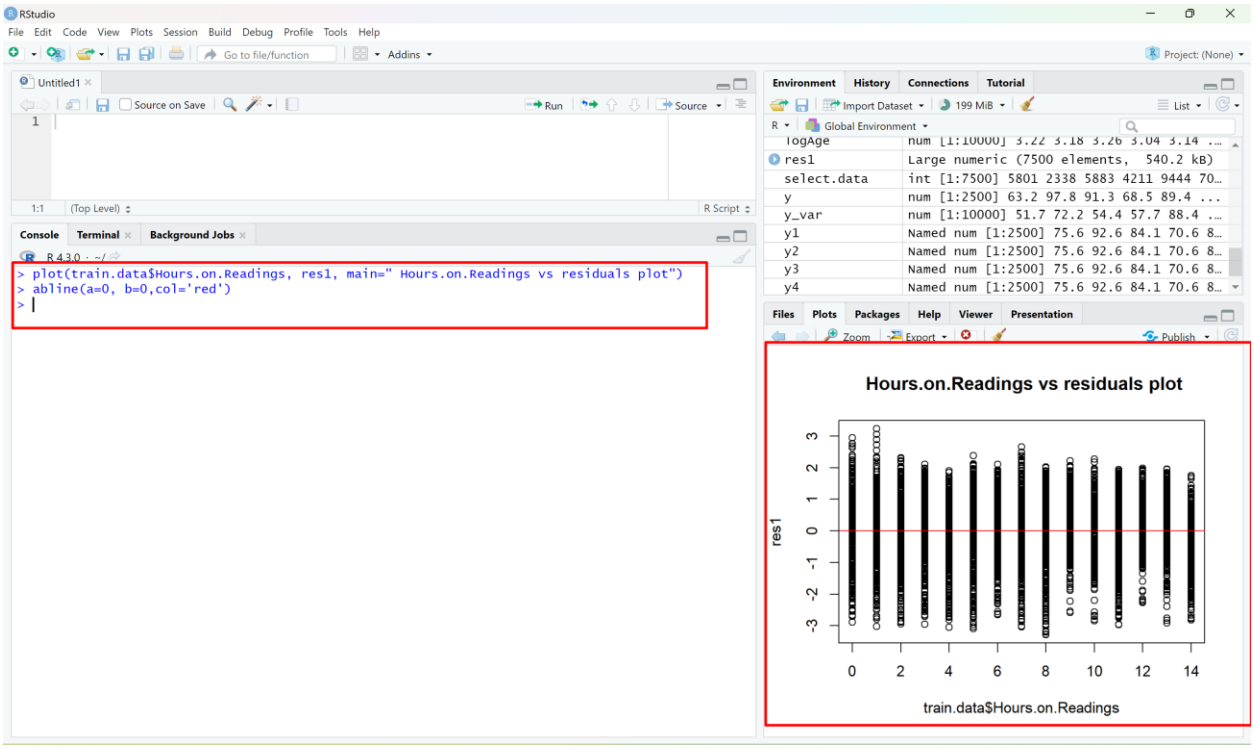
### Residual analysis on M1:

1. Validate the constant variance: Plot residuals vs predicted values: To check constant variance for the residuals

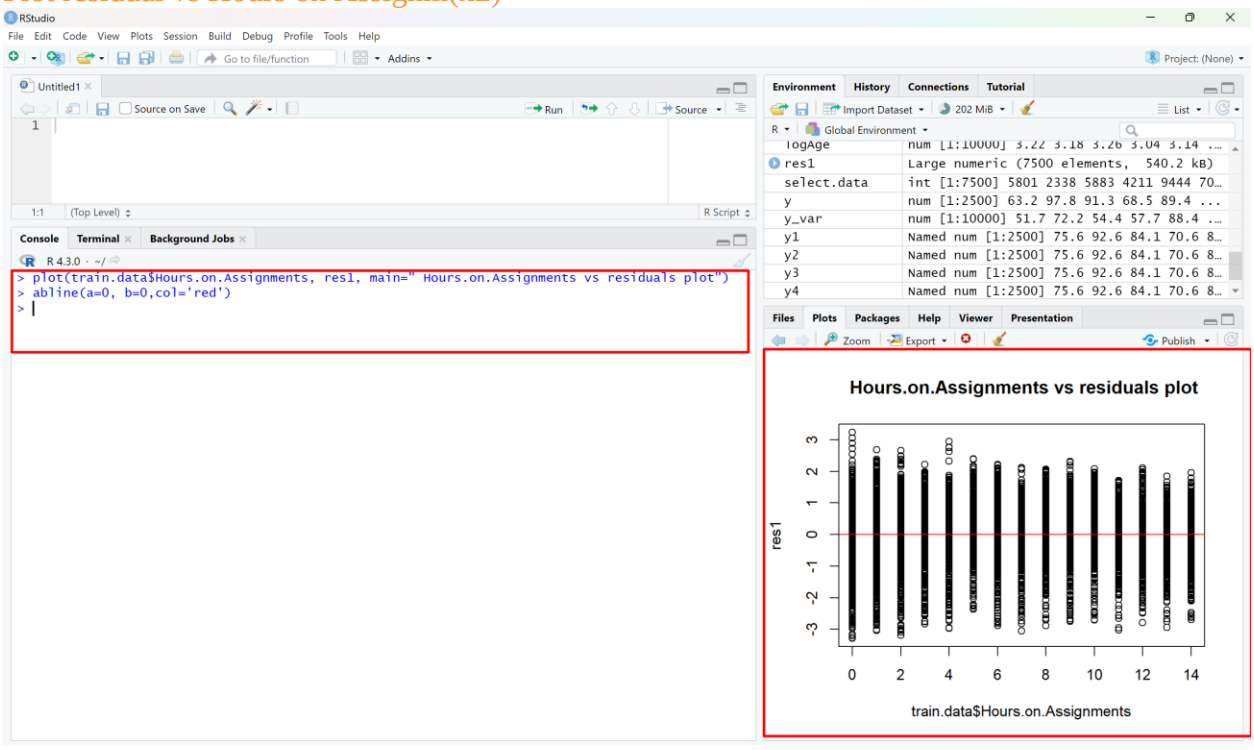



2. Validate the linearity relationship: Plot residuals vs each x-variable: To check linearity assumptions for Y and the x-variable.

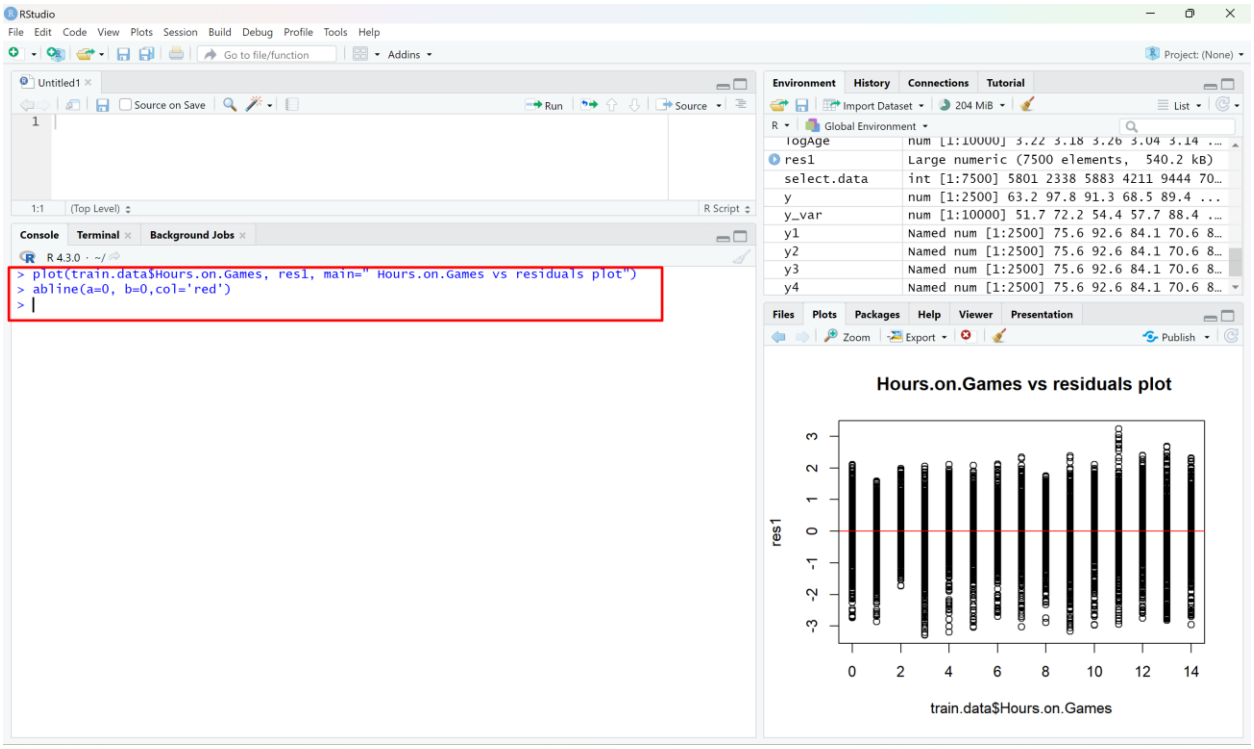
Plot residual vs Hours on Readings(x1)



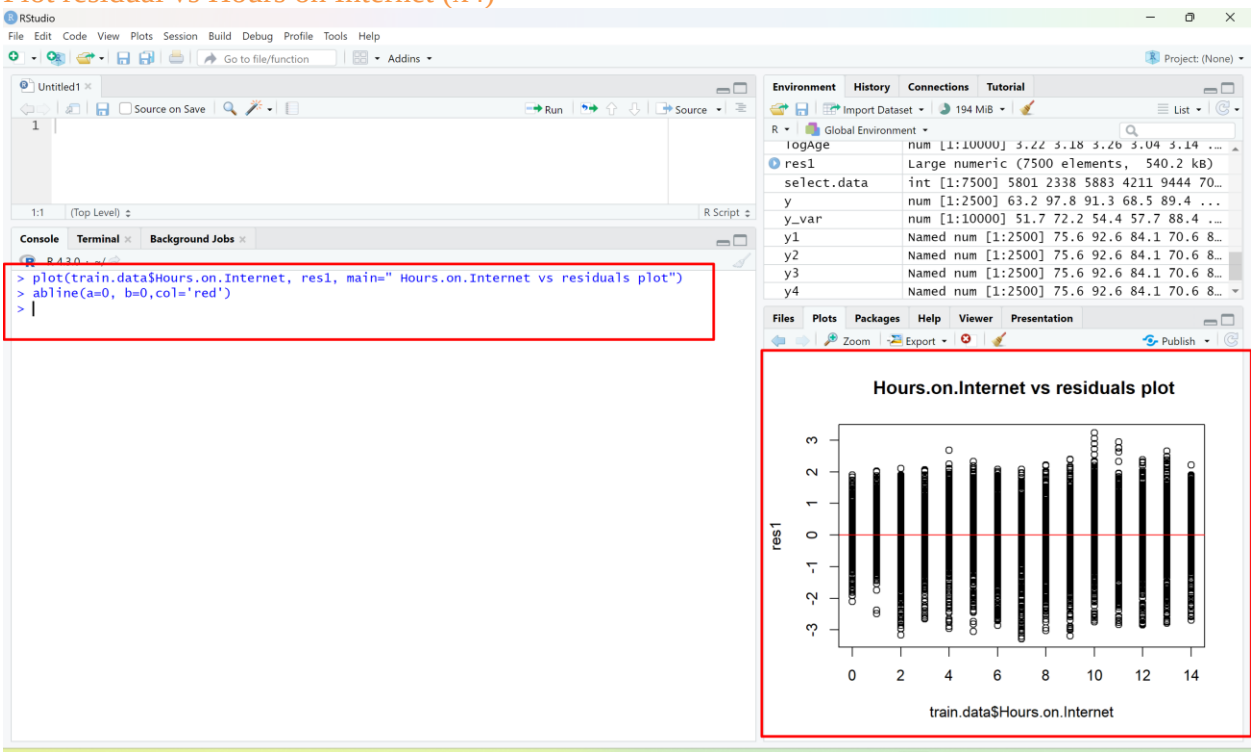

Plot residual vs Hours on Assignm(x2)



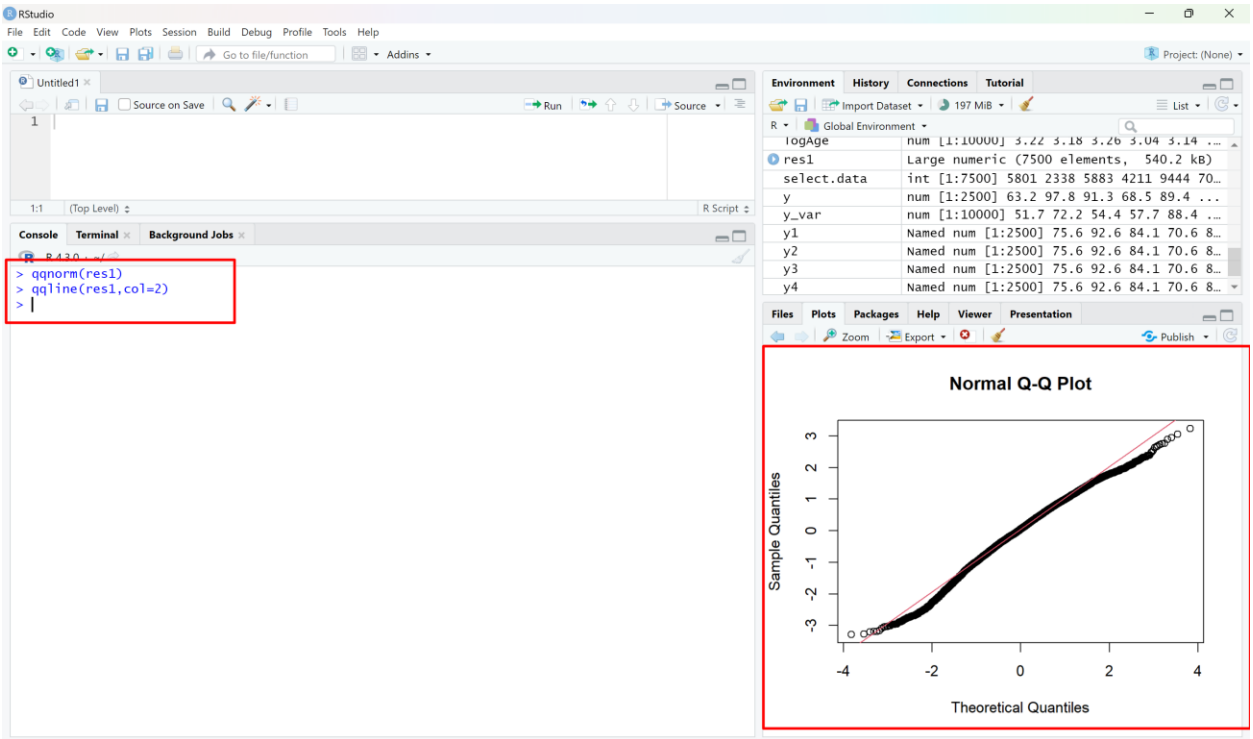

Plot residual vs Hours on Games(x3)




Plot residual vs Hours on Internet (x4)




3. Validate normal distribution of residuals: Draw normal probability plot of residuals: To check normality assumption for the error terms; if points lie close to a line, the errors can be assumed to be approximately normal. Otherwise the assumption of normality is not satisfied.




**Conclusion on residual analysis:** We can see there is **an issue in the constant variance (step1),** It is Constance in the beginning, but variance became smaller at the end. We can try to apply log transformation on y-variable to see whether we can get a better model.

**********We will solve this issue in Quest 2.**********

Q2 Particularly, you should answer the following questions (at the end of your homework submisson) by using your R coding and outputs.

****Note: As I performed all the steps in linear regression and pasted r-coding screenshots above already, I am using same screenshots here again to answer the below questions*****

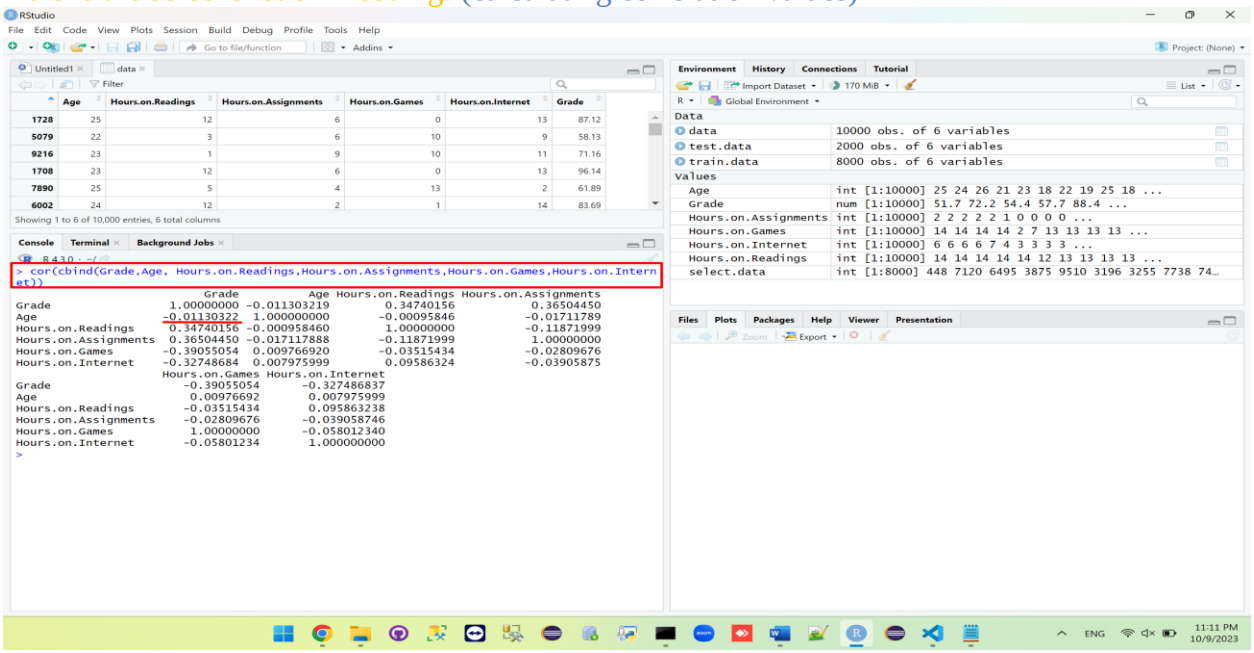
1. Do all x variables have linear relationship with y?

No, 'Age' has no linear relationship with Y. Here is the process of examining & solving the issue:

**Examine the linear relationship between x and y variables.**

- We can examine linearity by 2 methods.
    3. Produce a scatter plot for each x & y variable / produce a single plot with every pair of variables.
    4. Calculate correlation values for all the x variables with y variable.
- As the plot method is not clear & reliable all the time, we can perform 2nd method and calculate the correlation values.

Here is the screenshot of R coding: (calculating correlation values)

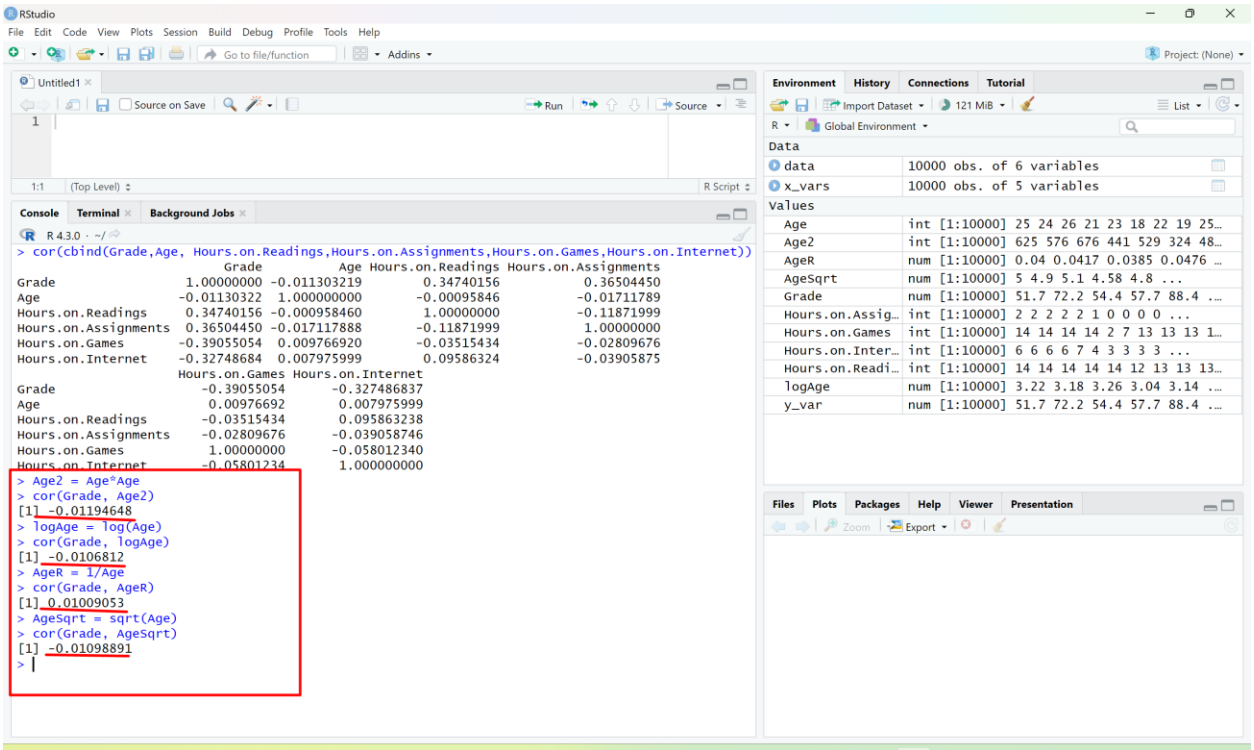



- We noticed that the correlation values are as follows:
  - Grade & Hours on readings = 0.34740156 - weak Correlation - +ve correlation
  - Grade & Hours on Assignments = 0.36504450 - weak Correlation - +ve correlation
  - Grade & Hours on games = -0.39055054 - weak Correlation – -ve correlation
  - Grade & Hours on internet = -0.32748684 - weak Correlation – -ve correlation

  - Grade & Age = -0.01130322 – no Correlation – Try transformation to improve correlation.
- As Age has no correlation with Grade(Y-variable), we can try transformation on this Age (x- variable) and re-calculate the correlation with Grade (y-variable) again.

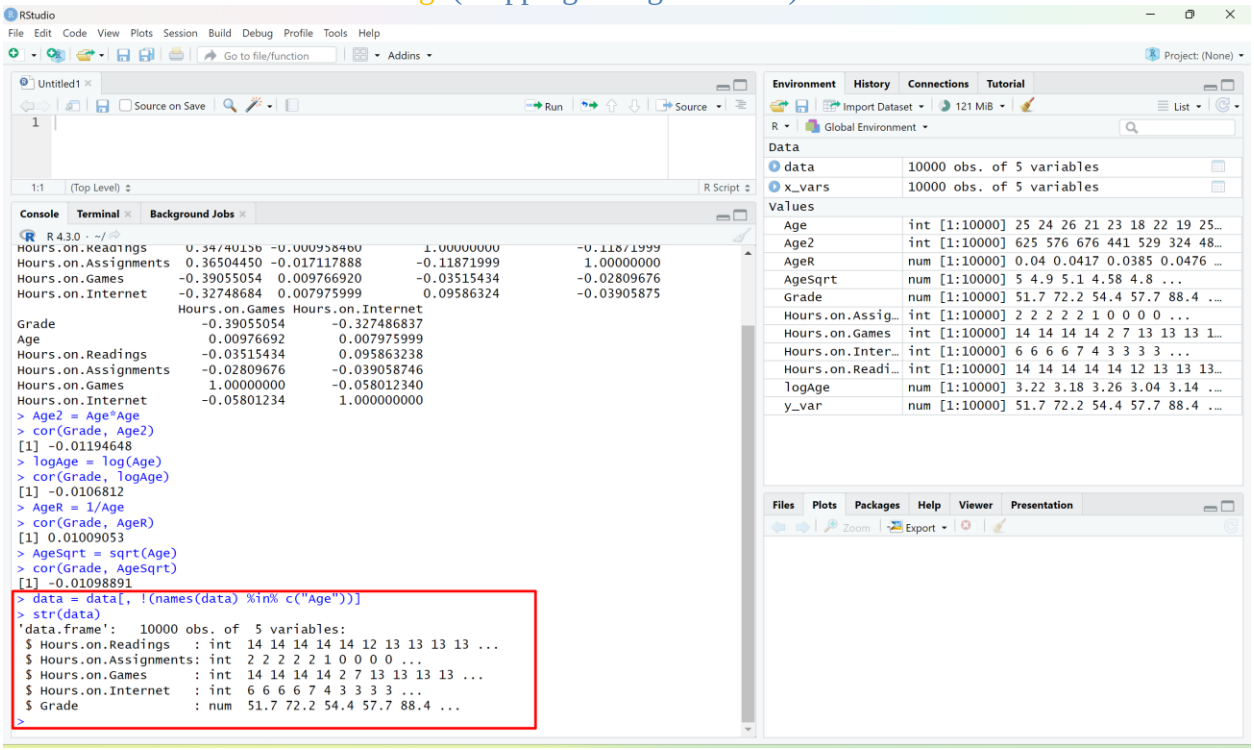
    - Square transformation: X’ = X * X
    - Log transformation: X’ = logX
    - Inversion transformation: X’ = 1/X
    - Square root transformation: X’ = sqrt(X)

Here is the screenshots of R coding:
(Transformation on Age variable & re-correlation value calculation with grade)




- As the correlation didn't improve after transformation on 'Age' variable, we decided to drop the age variable.

Here Is the screenshot of r coding: (dropping the age variable)




2. Write down the null and alternative hypothesis of F-test, use your outputs to draw conclusions of F-test.

ANSWER:

**F-test:** is a statistical test for hypothesis testing. We need to write down null hypothesis and alternative hypothesis:

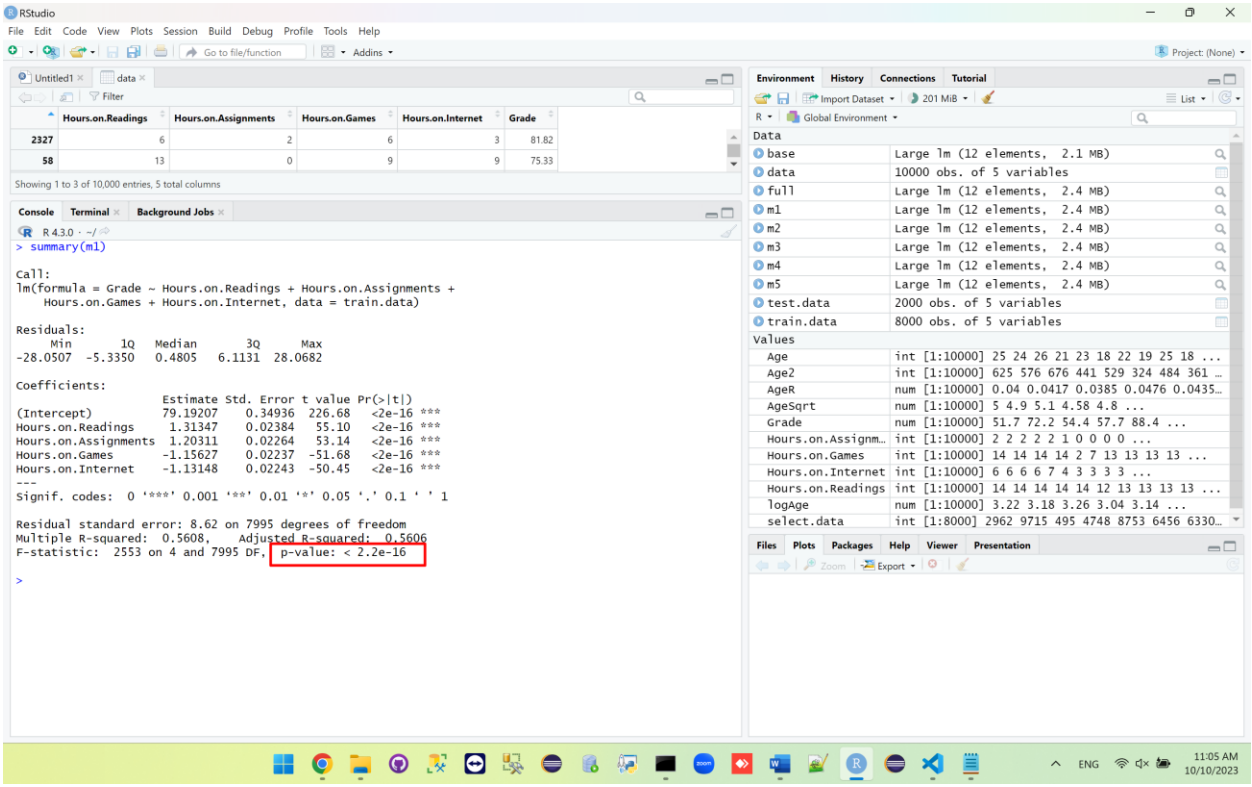
- Null hypothesis : (H0): The coefficients of all x-variables are zero and there is no linear relationship with Grade.
- $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$

- Alternative Hypothesis: At least one of the coefficients of the x-variables (Hours.on.Readings, Hours.on.Assignments, Hours.on.Games, Hours.on.Internet) is not zero and can affect Grade.
- $H_a : \beta_j \neq 0$

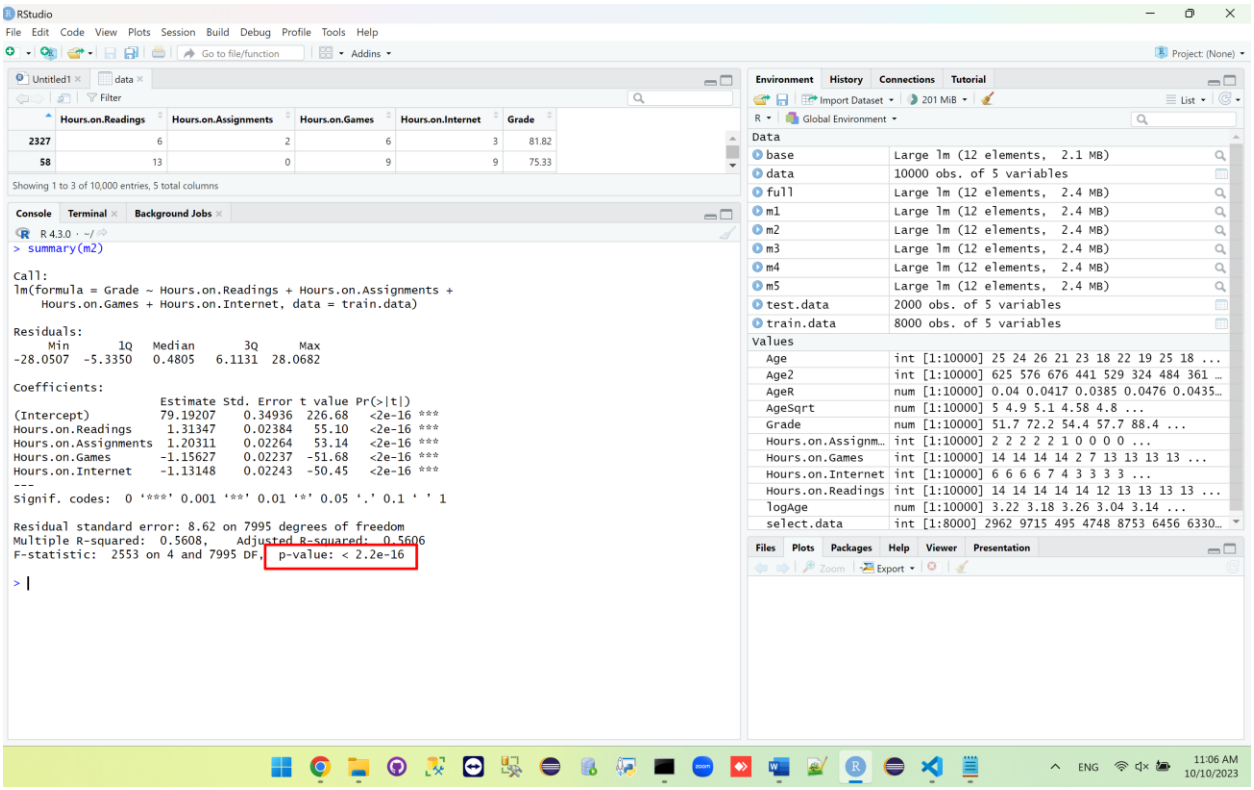
  Note : In these hypotheses, β1 represents the coefficient of "Hours.on.Readings," β2 represents the coefficient of "Hours.on.Assignments," β3 represents the coefficient of "Hours.on.Games," and β4 represents the coefficient of "Hours.on.Internet."

- As we built multiple models above through the feature selection process (M1, M2, M3, M4, M5), we need to do the f- test for all the models. But as all the models are same we can pick any 1 model and perform F-test. Lets pick M1:

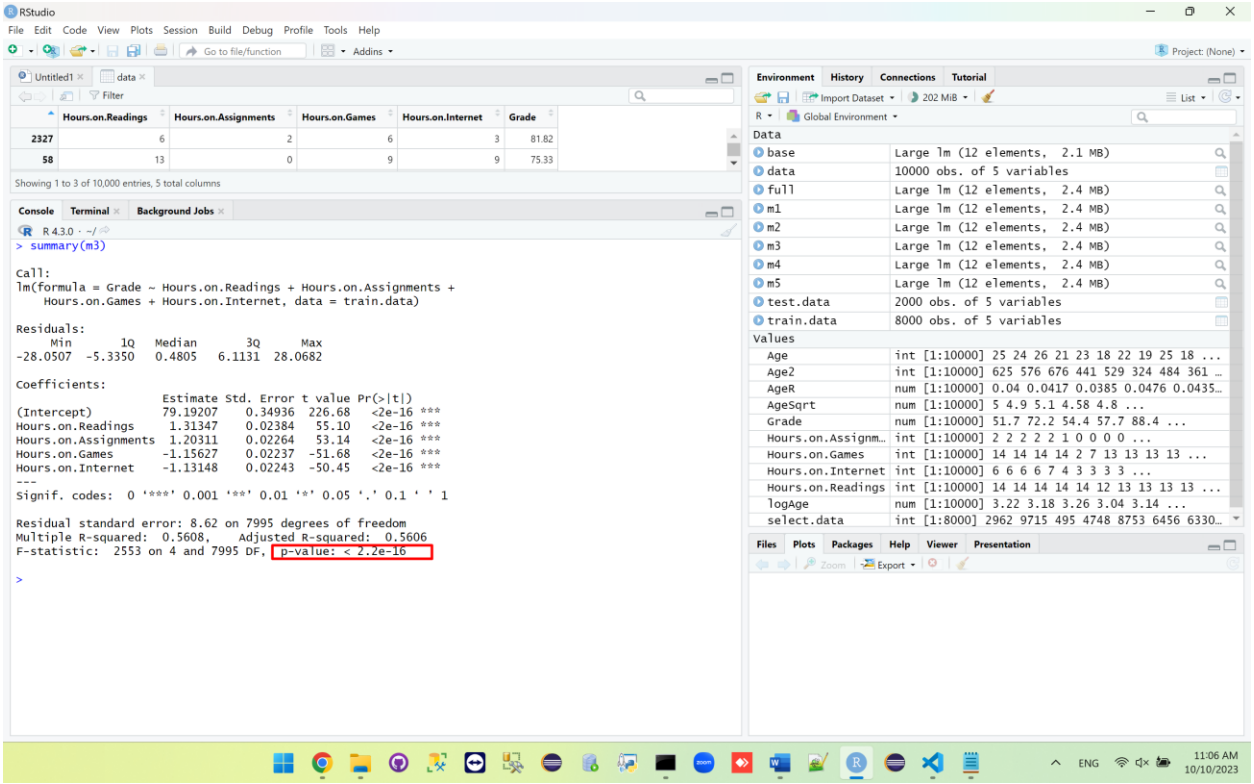
F-test on M1:



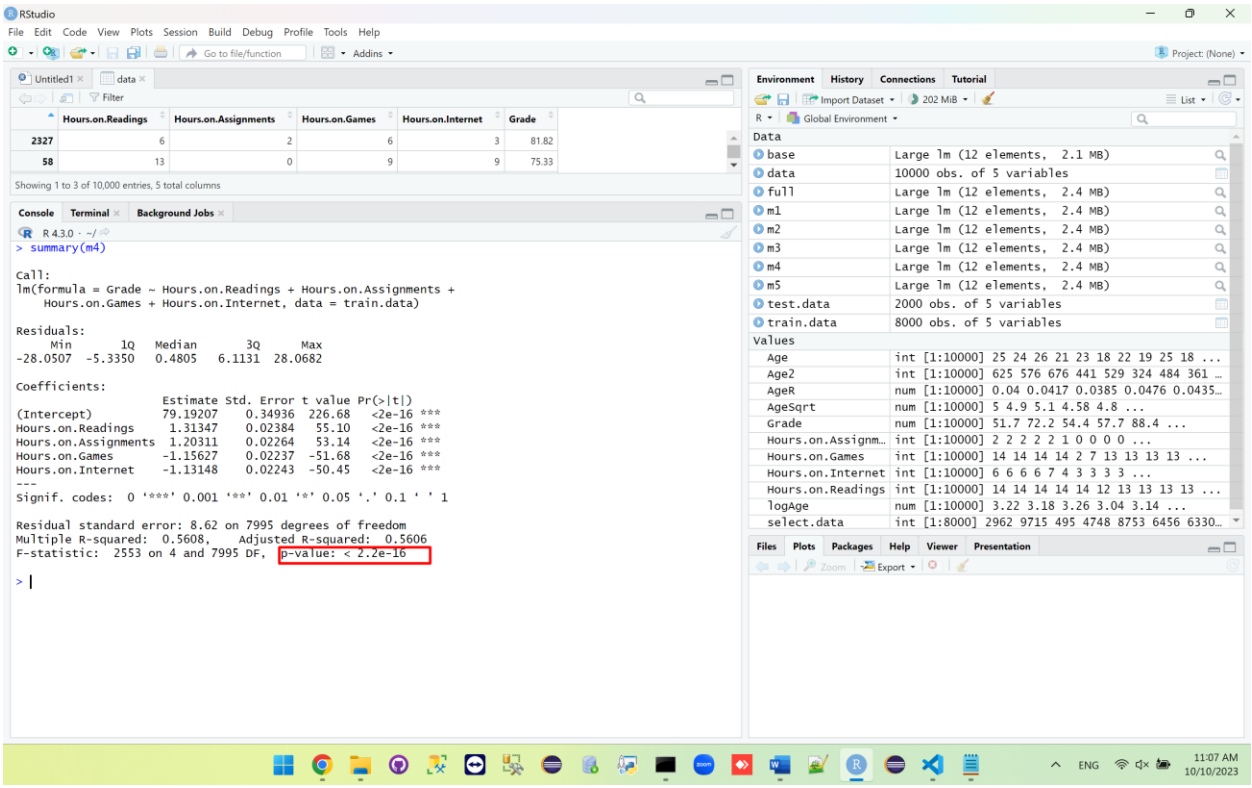

F-test on M2:



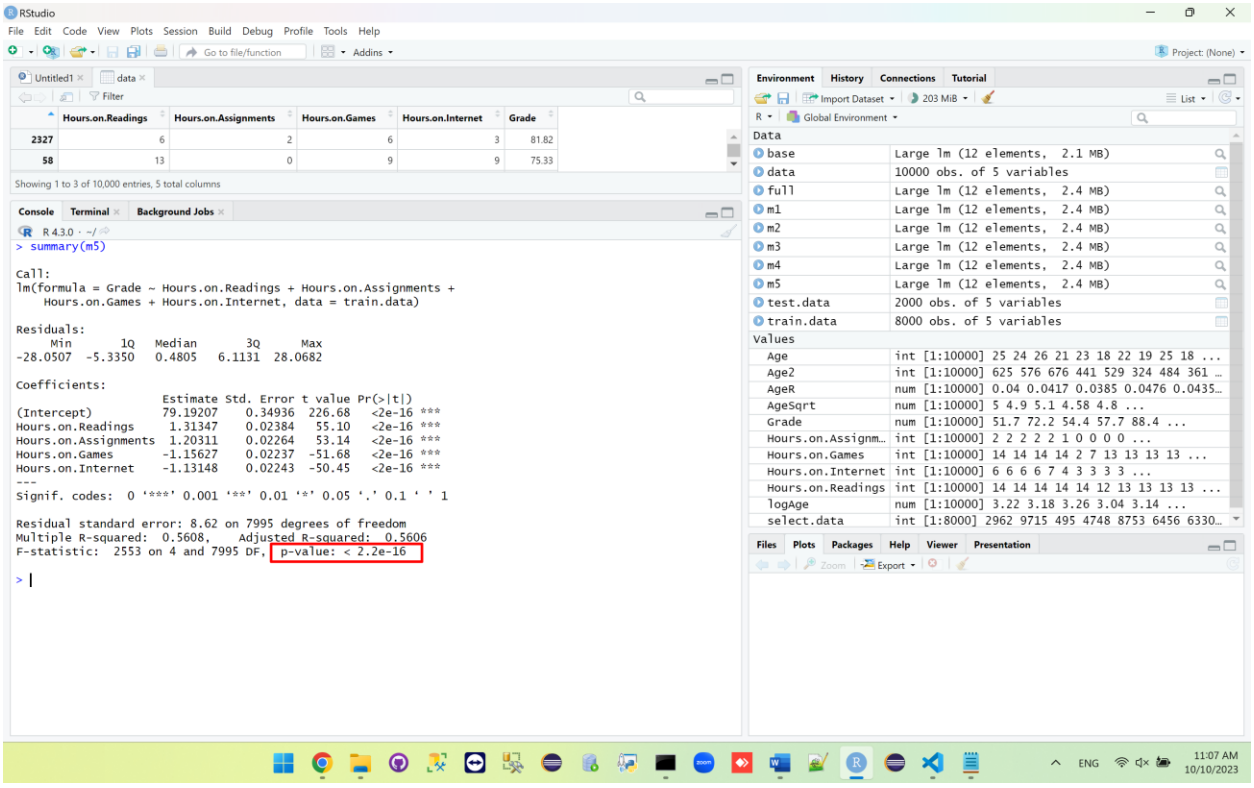

## F-test on M3:




## F-test on M4:


RStudio
File Edit Code View Plots Session Build Debug Profile Tools Help
Untitled1
data
Hours.on.Readings Hours.on.Assignments Hours.on.Games Hours.on.Internet Grade
2327 6 2 6 3 81.82
58 13 0 9 9 75.33
Showing 1 to 3 of 10,000 entries, 5 total columns
Console Terminal Background Jobs
R 4.3.0 · ~/
> summary(m4)
Call:
lm(formula = Grade ~ Hours.on.Readings + Hours.on.Assignments +
Hours.on.Games + Hours.on.Internet, data = train.data)
Residuals:
Min 1Q Median 3Q Max
-28.0507 -5.3350 0.4805 6.1131 28.0682
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 79.19207 0.34936 226.68 <2e-16 ***
Hours.on.Readings 1.31347 0.02384 55.10 <2e-16 ***
Hours.on.Assignments 1.20311 0.02264 53.14 <2e-16 ***
Hours.on.Games -1.15627 0.02237 -51.68 <2e-16 ***
Hours.on.Internet -1.13148 0.02243 -50.45 <2e-16 ***
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 8.62 on 7995 degrees of freedom
Multiple R-squared: 0.5608, Adjusted R-squared: 0.5606
F-statistic: 2553 on 4 and 7995 DF, p-value: < 2.2e-16
Environment History Connections Tutorial
Import Dataset 202 MiB
Global Environment
Data
base Large lm (12 elements, 2.1 MB)
data 10000 obs. of 5 variables
full Large lm (12 elements, 2.4 MB)
m1 Large lm (12 elements, 2.4 MB)
m2 Large lm (12 elements, 2.4 MB)
m3 Large lm (12 elements, 2.4 MB)
m4 Large lm (12 elements, 2.4 MB)
m5 Large lm (12 elements, 2.4 MB)
test.data 2000 obs. of 5 variables
train.data 8000 obs. of 5 variables
Values
Age int [1:10000] 25 24 26 21 23 18 22 19 25 18 ...
Age2 int [1:10000] 625 576 676 441 529 324 484 361 ...
AgeR num [1:10000] 0.04 0.0417 0.0385 0.0476 0.0435...
AgeSqrt num [1:10000] 5 4.9 5.1 4.58 4.8 ...
Grade num [1:10000] 51.7 72.2 54.4 57.7 88.4 ...
Hours.on.Assignm... int [1:10000] 2 2 2 2 2 1 0 0 0 0 ...
Hours.on.Games int [1:10000] 14 14 14 14 2 7 13 13 13 13 ...
Hours.on.Internet int [1:10000] 6 6 6 6 7 4 3 3 3 3 ...
Hours.on.Readings int [1:10000] 14 14 14 14 14 12 13 13 13 13 ...
logAge num [1:10000] 3.22 3.18 3.26 3.04 3.14 ...
select.data int [1:8000] 2962 9715 495 4748 8753 6456 6330...
Files Plots Packages Help Viewer Presentation
Zoom Export
ENG
11:07 AM
10/10/2023

## F-test on M5:


RStudio
File Edit Code View Plots Session Build Debug Profile Tools Help
Untitled1
data
Hours.on.Readings Hours.on.Assignments Hours.on.Games Hours.on.Internet Grade
2327 6 2 6 3 81.82
58 13 0 9 9 75.33
Showing 1 to 3 of 10,000 entries, 5 total columns
Console Terminal Background Jobs
R 4.3.0 · ~/
> summary(m5)
Call:
lm(formula = Grade ~ Hours.on.Readings + Hours.on.Assignments +
Hours.on.Games + Hours.on.Internet, data = train.data)
Residuals:
Min 1Q Median 3Q Max
-28.0507 -5.3350 0.4805 6.1131 28.0682
Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 79.19207 0.34936 226.68 <2e-16 ***
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Hours.on.Assignments 1.20311 0.02264 53.14 <2e-16 ***
Hours.on.Games -1.15627 0.02237 -51.68 <2e-16 ***
Hours.on.Internet -1.13148 0.02243 -50.45 <2e-16 ***
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 8.62 on 7995 degrees of freedom
Multiple R-squared: 0.5608, Adjusted R-squared: 0.5606
F-statistic: 2553 on 4 and 7995 DF, p-value: < 2.2e-16
Environment History Connections Tutorial
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Global Environment
Data
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data 10000 obs. of 5 variables
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m1 Large lm (12 elements, 2.4 MB)
m2 Large lm (12 elements, 2.4 MB)
m3 Large lm (12 elements, 2.4 MB)
m4 Large lm (12 elements, 2.4 MB)
m5 Large lm (12 elements, 2.4 MB)
test.data 2000 obs. of 5 variables
train.data 8000 obs. of 5 variables
Values
Age int [1:10000] 25 24 26 21 23 18 22 19 25 18 ...
Age2 int [1:10000] 625 576 676 441 529 324 484 361 ...
AgeR num [1:10000] 0.04 0.0417 0.0385 0.0476 0.0435...
AgeSqrt num [1:10000] 5 4.9 5.1 4.58 4.8 ...
Grade num [1:10000] 51.7 72.2 54.4 57.7 88.4 ...
Hours.on.Assignm... int [1:10000] 2 2 2 2 2 1 0 0 0 0 ...
Hours.on.Games int [1:10000] 14 14 14 14 2 7 13 13 13 13 ...
Hours.on.Internet int [1:10000] 6 6 6 6 7 4 3 3 3 3 ...
Hours.on.Readings int [1:10000] 14 14 14 14 14 12 13 13 13 13 ...
logAge num [1:10000] 3.22 3.18 3.26 3.04 3.14 ...
select.data int [1:8000] 2962 9715 495 4748 8753 6456 6330...
Files Plots Packages Help Viewer Presentation
Zoom Export
ENG
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10/10/2023

| MODEL | F-test -P-VALUE |
|---|---|
| M1 : Based on Backward method using p-value as metric: | 2.2e-16 |
| M2: Based on Backward method using AIC as metric: | 2.2e-16 |
| M3: Based on Forward method using AIC as metric. | 2.2e-16 |
| M4: Based on Stepwise method using ACI as metric: | 2.2e-16 |
| M5: Based on Best subset method using Adj-R2 as metric | 2.2e-16 |

Conclusion on F-test: p-value $< 0.05$ for all the models

- At 95% confidence level, we can say that at least 1 x variables among ((Hours.on.Readings, Hours.on.Assignments, Hours.on.Games, Hours.on.Internet) has significant linear relationship with Grade and can affect the value of y-variable ~ Grade.

  ******By conducting an F-test, we can observe that there is sufficient evidence to reject the null hypothesis and conclude that the independent variables have a significant impact on the dependent variable.*********

3. Which model is the best in terms of Adj-R2?

From the screenshots above:

| MODEL | Adj r2 |
|---|---|
| M1 : Based on Backward method using p-value as metric: | 56.06% |
| M2: Based on Backward method using AIC as metric: | 56.06% |
| M3: Based on Forward method using AIC as metric. | 56.06% |
| M4: Based on Stepwise method using ACI as metric: | 56.06% |
| M5: Based on Best subset method using Adj-R2 as metric | 56.06% |

Conclusion:

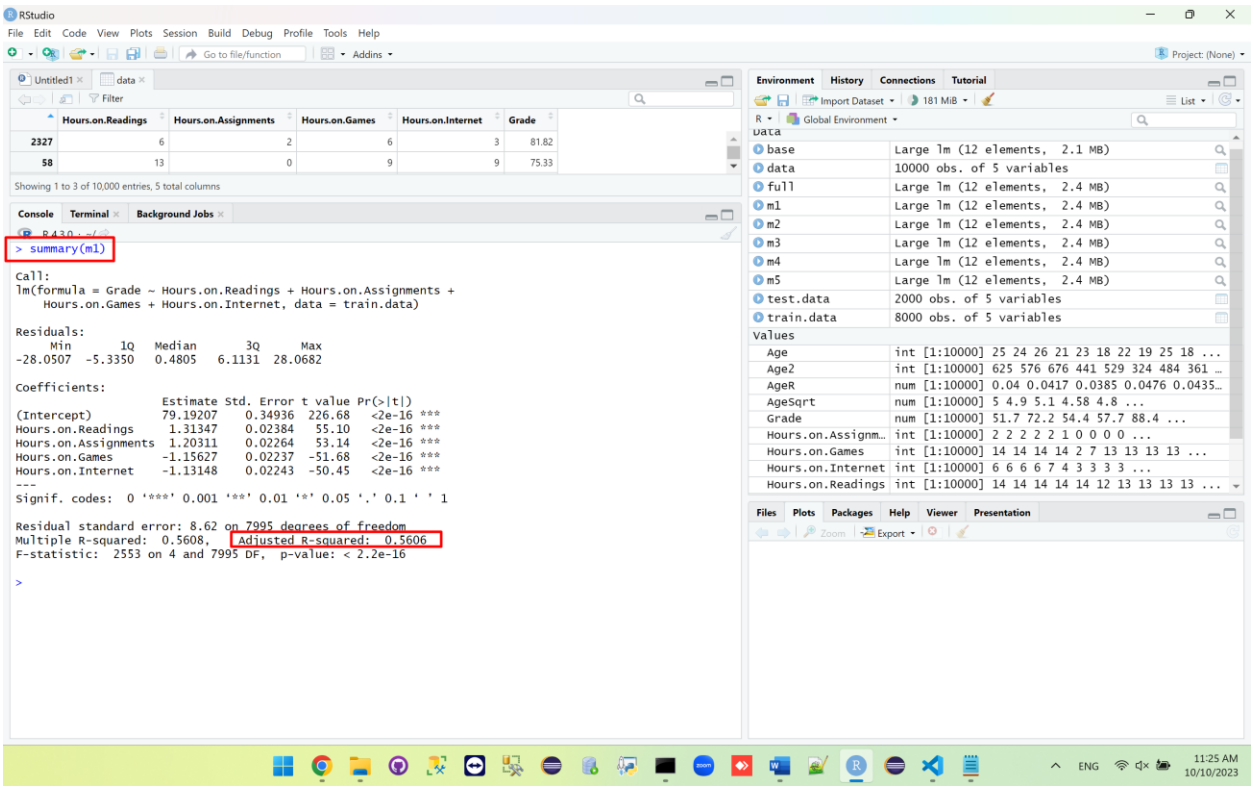
- All models have the same adj r2 values and used same set of x-variables, so all models are best.

*******Note: But Adj r2 is not the accurate, we need to evaluate based on testing data set by RMSE*****

4. Interpret the Adj-R2 in the best model above.

ANSWER:

- Since all models have the same adj r2 values and use same set of x-variables, so all models are best. So, we can interpret any model's adj r2. Lets take M1 and interpret adj r2:




Interpretation of adj r2 of m1:

- 56.06% variation in Grade can be explained by the variations in X variables (Hours.on.Readings+Hours.on.Assignments+Hours.on.Games+Hours.on.Internet) based on our fitted regression model

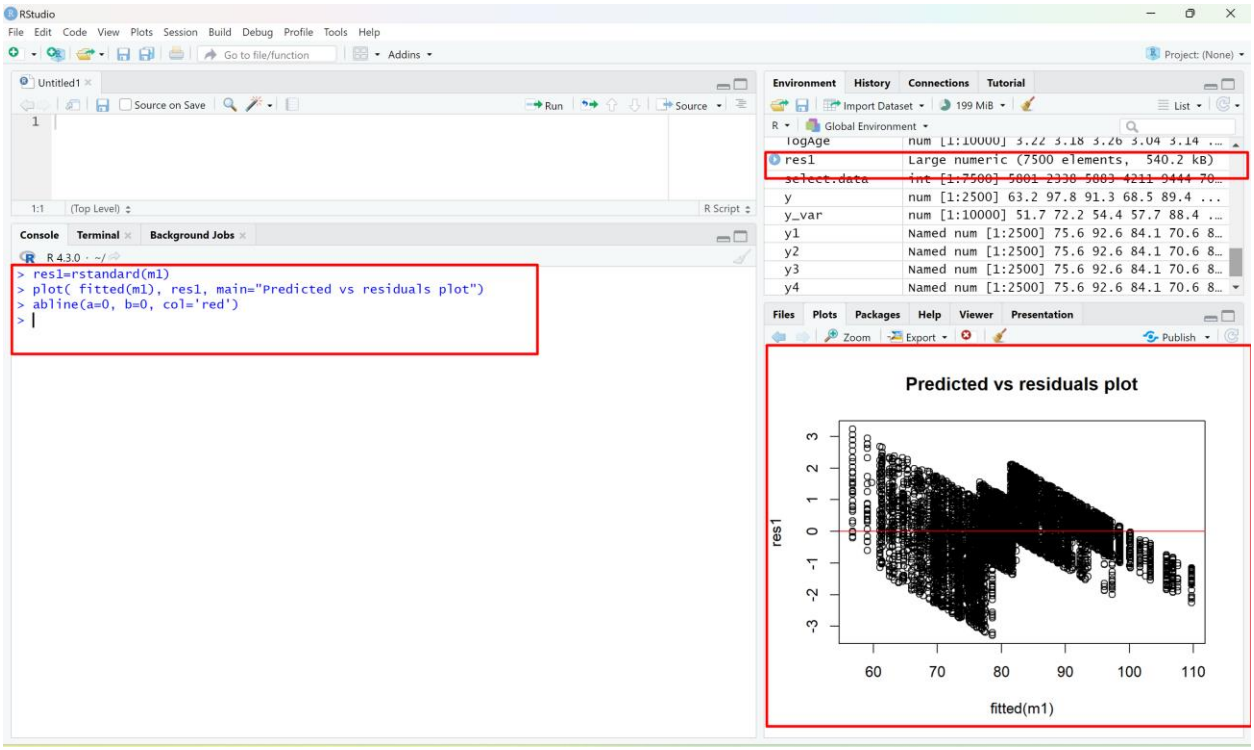
5. Any issues in the residual analysis for the model above?

**ANSWER:**

**Yes**, there is an issue about constant variance. As all models are same lets take m1 to check:

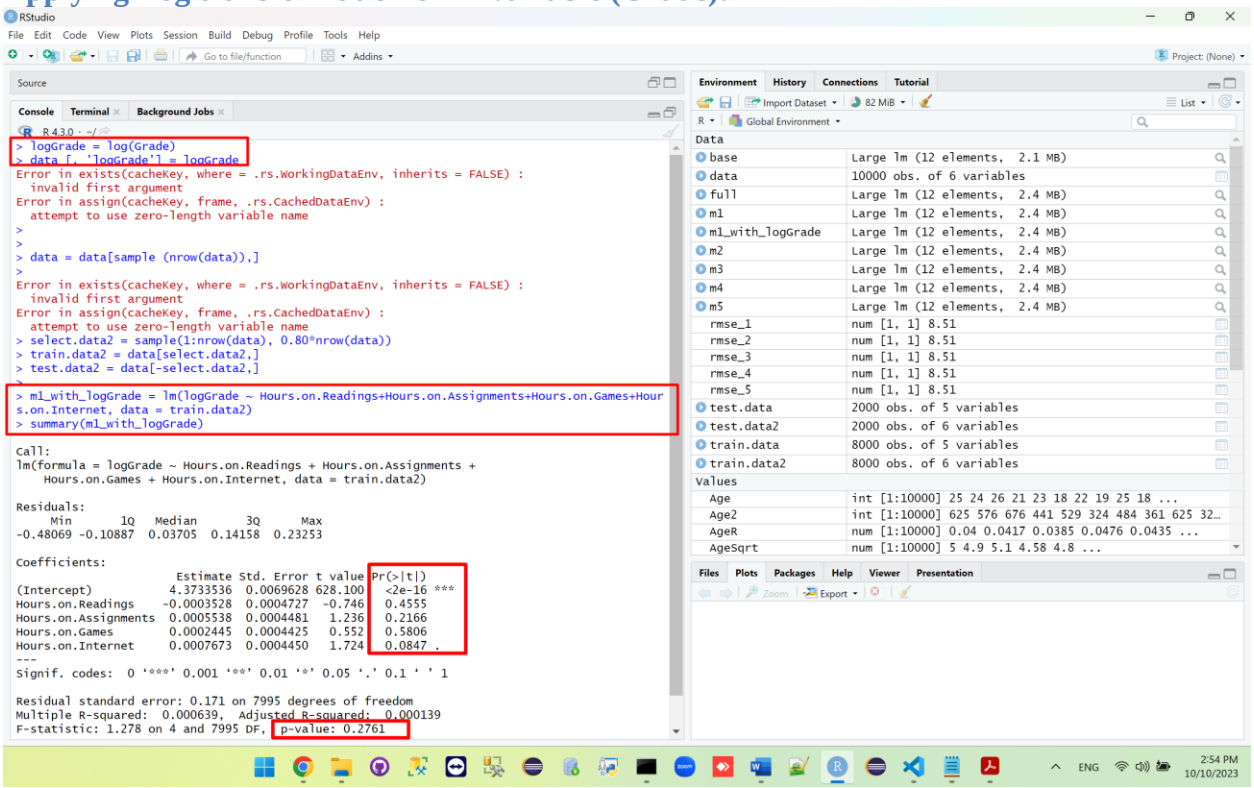
**Residual analysis on m1:**

1. Validate the constant variance: Plot residuals vs predicted values: To check constant variance for the residuals




It is Constance in the beginning, but variance became smaller at the end. We can try to apply log transformation on y-variable(Grade) to see whether we can get a better model.

## Applying Log transformation on Y-variable (Grade):



After applying the log transformation on the variable Grade, I rebuilt the model M1 with logGrade, but it didn't pass the Goodness of fit test as the p - value (0.2761) is greater than 95%(0.05). From this observation I would like to say that we can stick to the old model M1.

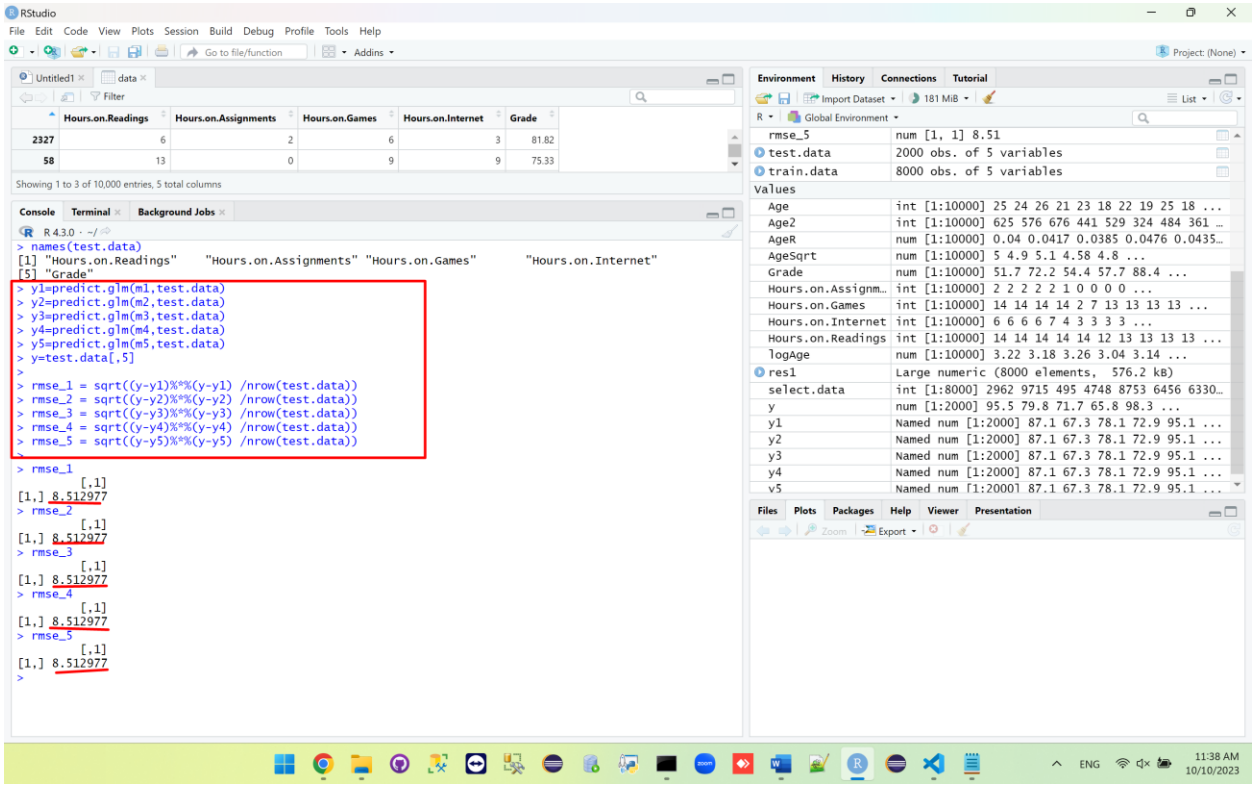
6. Which model is the best in terms of RMSE?

ANSWER:

**Evaluate based on testing data set based on RMSE:**

- To evaluate based on testing data set , we need to calculate the RMSE for all the models

Based on RMSE we need to compare all the models and make a conclusion about which model is best.




| MODEL | RMSE |
|---|---|
| M1 : Based on Backward method using p-value as metric: | 8.512 |
| M2: Based on Backward method using AIC as metric: | 8.512 |
| M3: Based on Forward method using AIC as metric. | 8.512 |
| M4: Based on Stepwise method using ACI as metric: | 8.512 |

| | |
|---|---|
| M5: Based on Best subset method using Adj-R2 as metric | 8.512 |

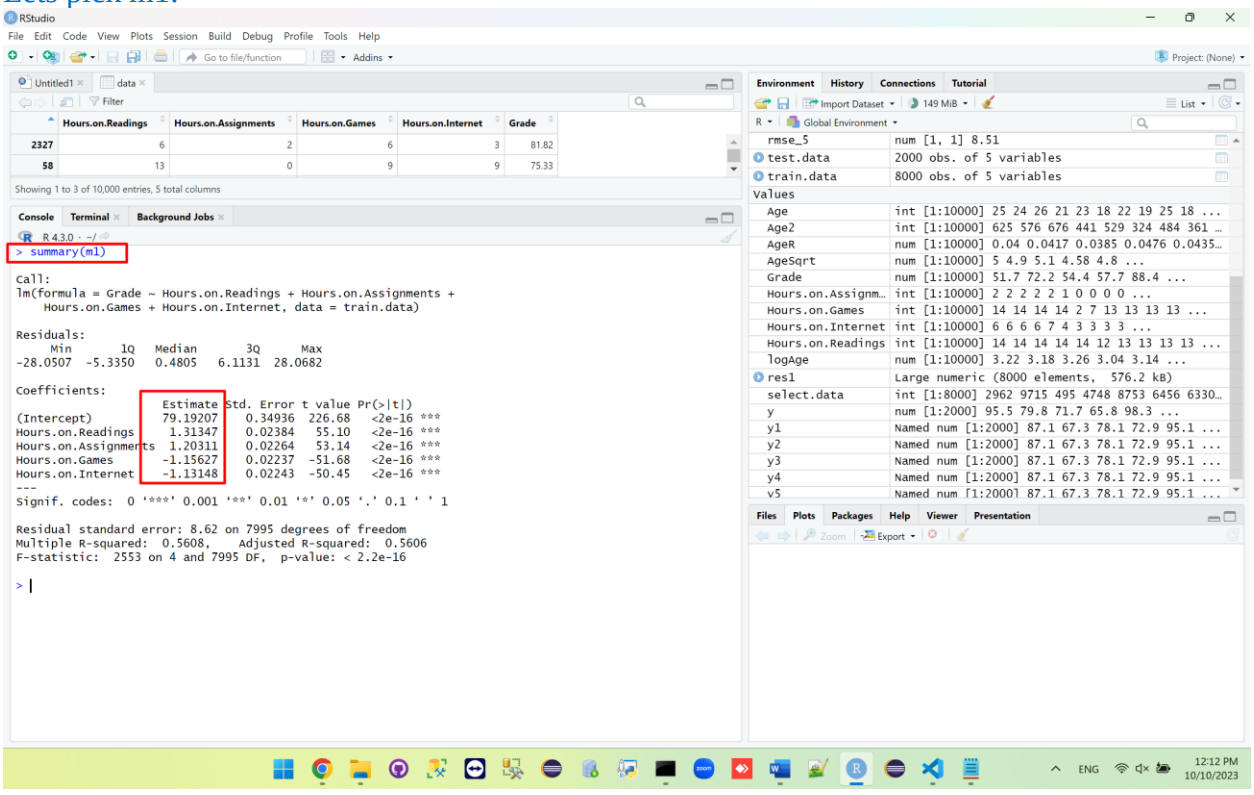
Conclusion: As the RMSE is same for all the models, any model Is best. We can pick any model and write the final model.

7. Write down the best model (identified by RMSE), and explain the intercepts and coefficients in the model.

ANSWER:

As the RMSE is same for all the models, Any model Is best
So, we can pick any 1 model and write down the best model by RMSE

Lets pick m1:




Y = 79.1920 + 1.31347 X1 + 1.20311 X2 + (-1.15627) X3 + (-1.13148) X4

Whereas :
Y = Grade
X1 = Hours.on.Readings
X2 = Hours.on.Assignments
X3 = Hours.on.Games
X4 = Hours.on.Internet

Explanation of co-efficient and Intercept (explaining effect):

β0 ~
Intercept: 79.1920:

The intercept, represented by 79.1920, is the value of Grade when X1, X2, X3, X4 (all the x-var's) is equal to zero. The intercept represents a constant or baseline for grade when all the x var's are at 0.

$\beta_1$ ~ It is the co-efficient of X1 variable: (Hours.on.Readings)

1.31347 ($\beta_1$ ) measures the changes in Grade variable for a unit increase of the variable Hours.on.Readings , (Assuming other variables are held constant).

$\beta_2$ ~ It is the co-efficient of X2 variable: (Hours.on.Assignments)

1.20311 ($\beta_2$ ) measures the changes in Grade variable for a unit increase of the variable Hours.on.Assignments, (Assuming other variables are held constant).

$\beta_3$ ~ It is the co-efficient of X3 variable: (Hours.on.Games)

-1.15627 ($\beta_3$ ) measures the changes in Grade variable for a unit increase of the variable Hours.on.Games. The negative sign suggests a concave-down relationship, meaning the Grade initially increases at a decreasing rate and eventually decreases as X3 increases. (Assuming other variables are held constant).

$\beta_4$ ~ It is the co-efficient of X4 variable: (Hours.on.Internet)

(-1.13148 ($\beta_4$ ) measures the changes in Grade variable for a unit increase of the variable Hours.on.Internet. The negative sign suggests a concave-down relationship, meaning the Grade initially increases at a decreasing rate and eventually decreases as X3 increases. (Assuming other variables are held constant).

**HW4: Multiple Linear Regression Analysis**

Note: every step you use R, you should provide the snapshots of your R commands and R outputs.

Note: do not split the data for this assignment, just use all the data to build the models

## Problem 1 [40]

A researcher is interested in evaluating the relationship between energy consumption by the homeowner and the difference between the internal and external temperatures. A sample of 30 homes was used in the study. During an extended period of time, the average temperature difference (in °F) (TEMPD) inside and outside the homes was recorded. The average energy consumption (ENERGY) was also recorded for each home. The data are stored in the energytemp.txt data file.

a) Create a scatterplot of ENERGY (y) versus TEMPD (x) to visualize the association between the two variables. Analyze the association displayed by the scatterplot.

b) Fit a cubic model $y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + e$ (*HINT: create two new variables TEMP2 and TEMP3:*

*In SAS DATA STEP use the code:*

```
tempd2 = tempd**2;
tempd3 = tempd**3;
```

*In R use the code:*

```
tempd2 = tempd^2;
tempd3 = tempd^3;
```

*Include the new variables in the regression model*)

a) Test the goodness of fit of the model at the 5% significance level.

b) Are all variables in the model significant?

c) Create the residual plots (residuals vs predicted; residuals vs x variable; and normal plot of residuals). Analyze residual plots to evaluate the normality and constant variance assumptions. Discuss your findings.

d) If you are satisfied with the fitted regression model, write down its expression.

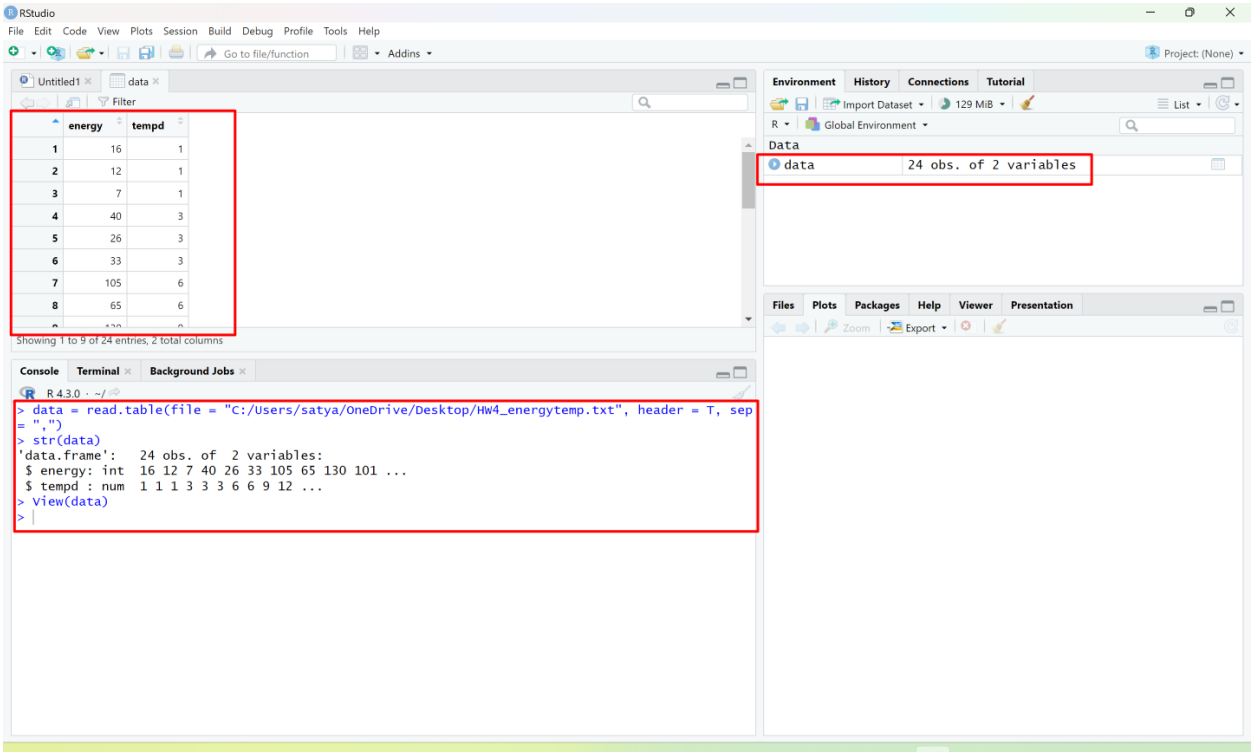
e) Use the fitted regression model to predict the average energy consumption for an average difference in temperature equal to TEMPD=10.

In R, you should use new = data.frame (tempd=c(10), tempd2=c(100), tempd3=c(1000)), and then use the predict() function in R to produce predictions and confidence interval

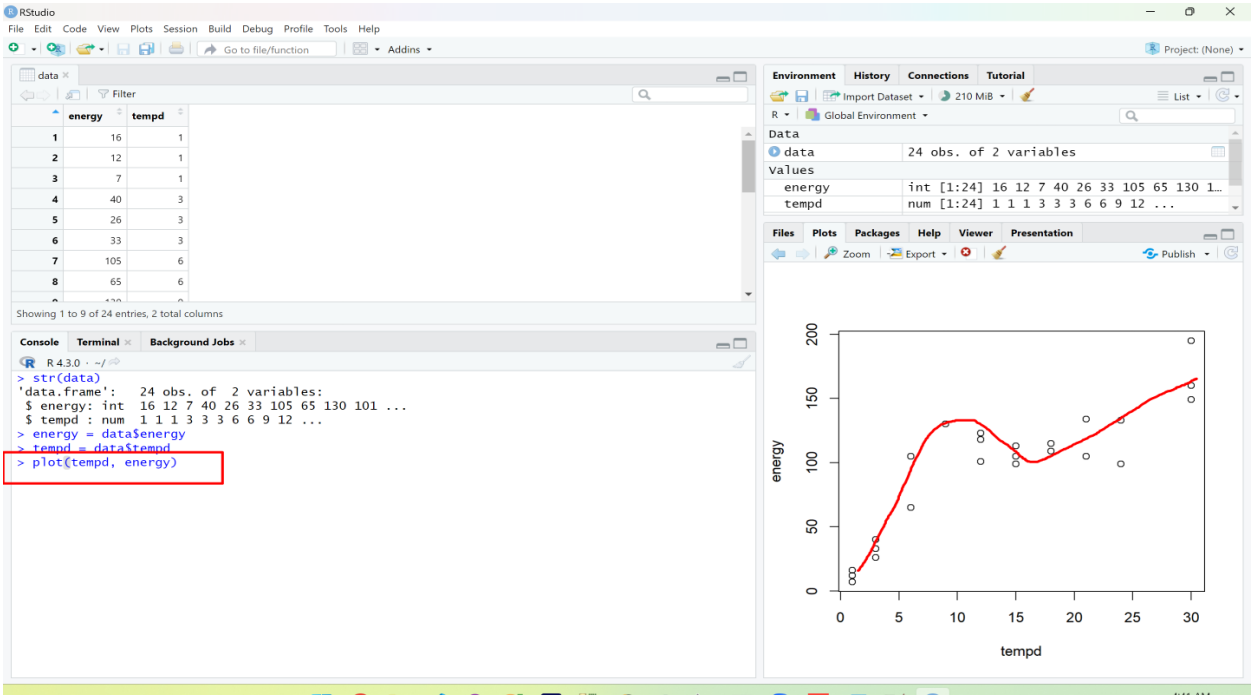
f) By using influence.measures() function to identify whether there are influential points that can affect your final model. Use cook's distance as the metric to identify the influential points.

## ANSWERS:

### Imported data into R-studio:




### a. Create a scatterplot of ENERGY (y) versus TEMPD (x) to visualize the association between the two variables. Analyze the association displayed by the scatterplot.

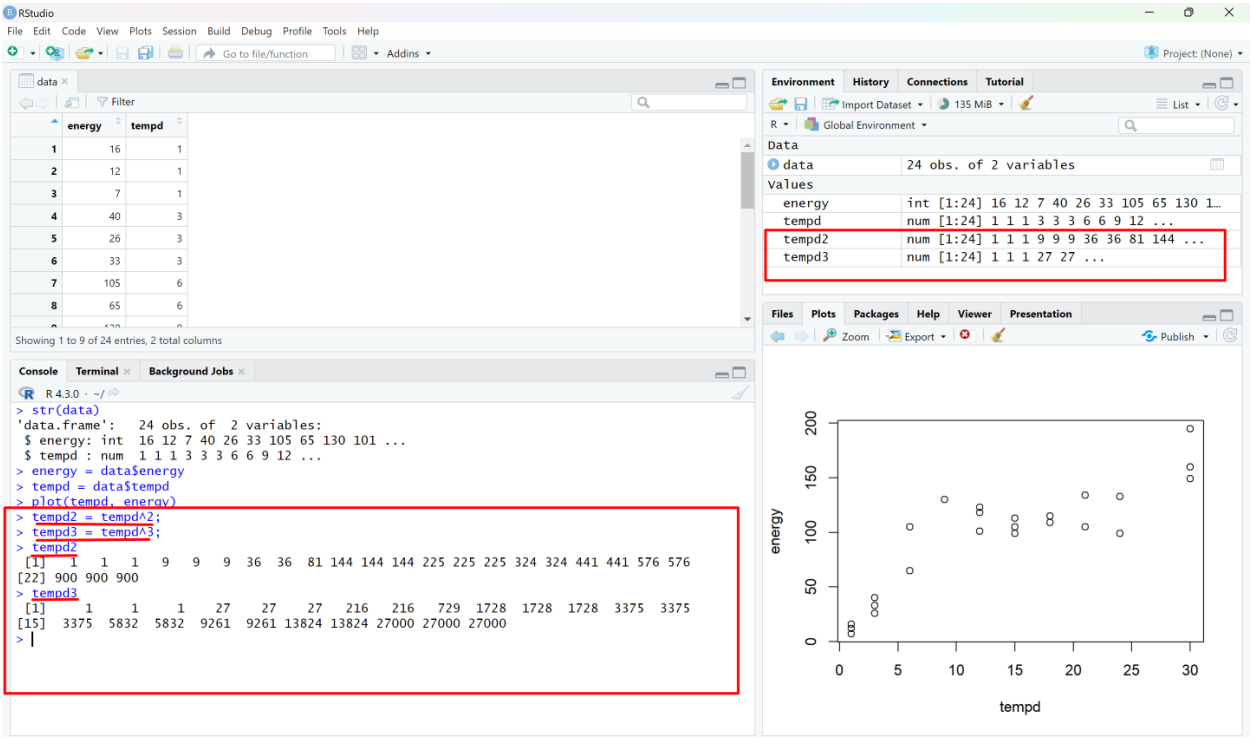


## Analyzing the association b/w the 2 variables:

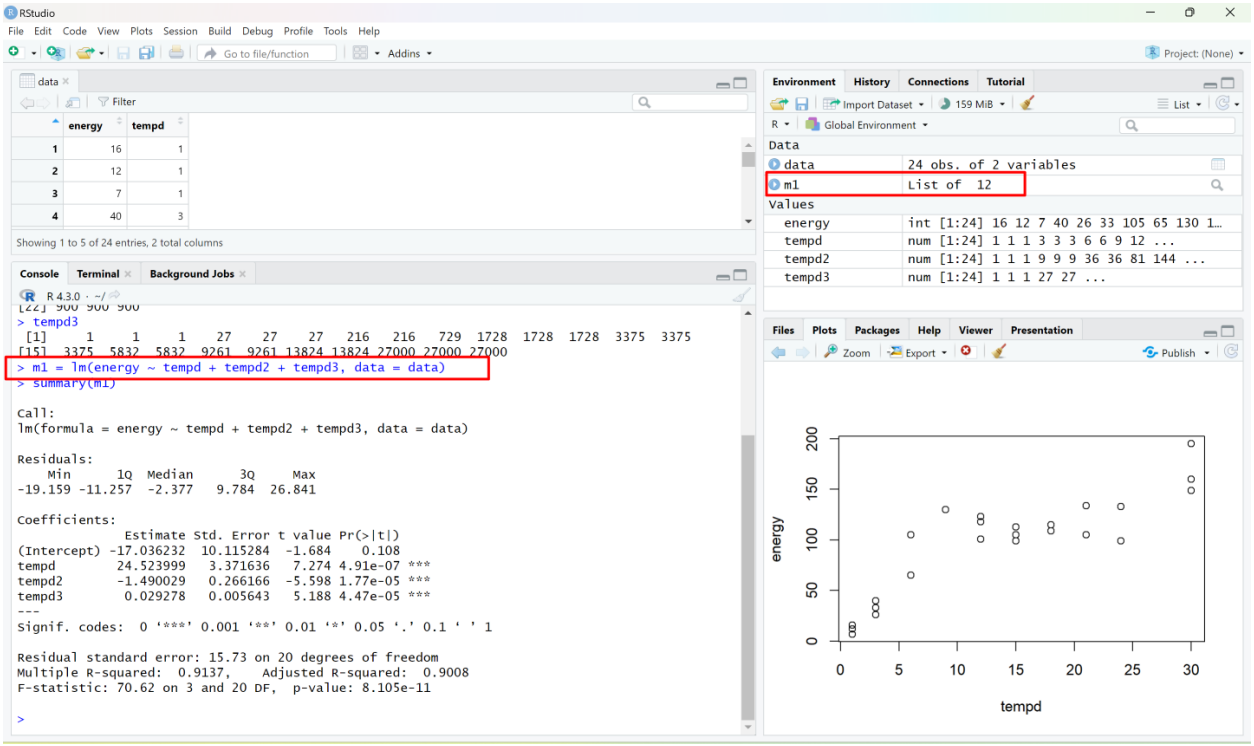
- From the scatterplot, we can observe that there is 1-top & 1-bottum in the curve, and the scatterplot produced S-shaped pattern. Hence, we can conclude that the association between energy (y) and tempd (X) is cubic association / cubic relation.
- As the scatterplot follows cubic relation, we need to add 3rd order term to build polynomial regression model (fit a cubic model)
- But we cannot add 3rd order term alone. We also need to add 2nd and 1st order terms also.
- So, we need to try power transformation on tempd variable & create 2 new variables.
- And add the 2 new variables to the model that we got after power transformation.
- As follows:

## b. Fitting a cubic model, and creating 2 new variable tempd2, tempd3 and adding the new 2 variables to the dataset:

creating 2 new variables: tempd2, tempd3:



## fitting a cubic regression model: M1




## a. Test the goodness of fit of the model at the 5% significance level:

In f-test, we need to write down null and alternative hypothesis for the model:

- Null hypothesis: (H0): The coefficients of all x-variables are zero and there is no linear relationship with energy.
- $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$

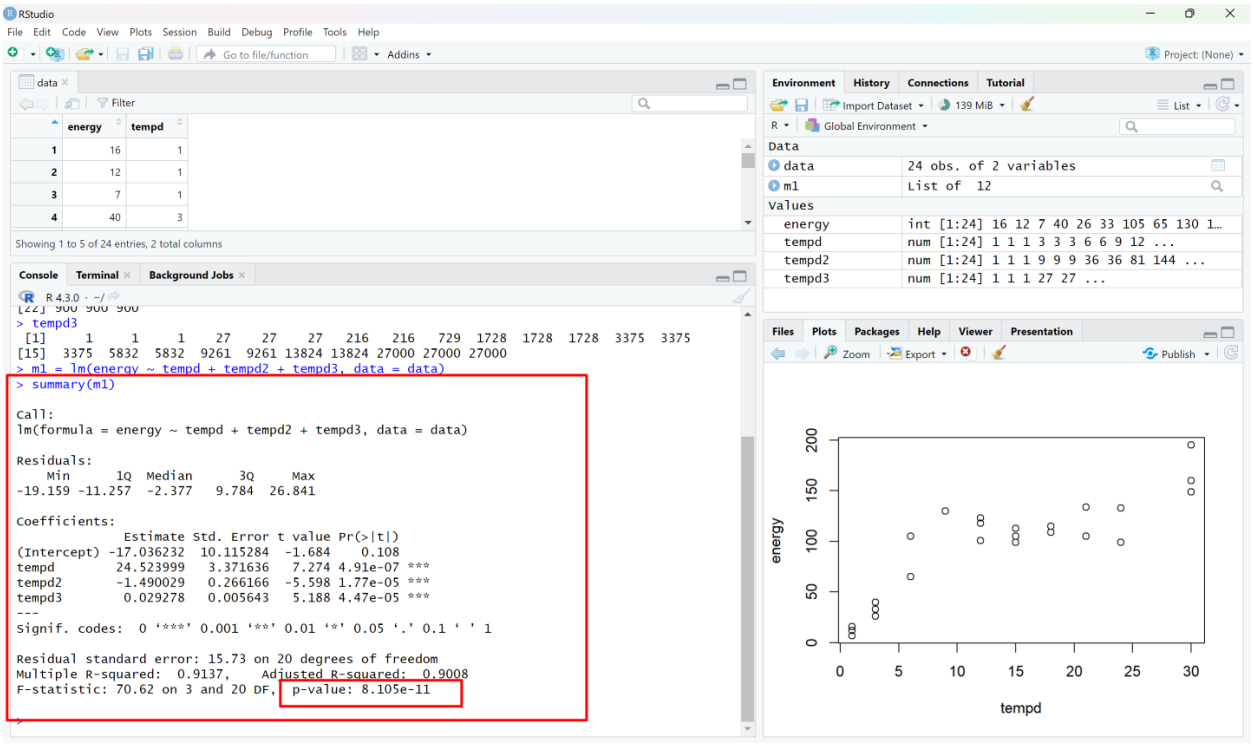
- Alternative Hypothesis: At least one of the coefficients of the x-variables (tempd, tempd2, tempd3) is not zero and can affect energy.
- $H_a : \beta_j \neq 0$

Note: In these hypotheses,

β1 represents the coefficient of "tempd"

β2 represents the coefficient of "tempd2."

β3 represents the coefficient of "tempd3."

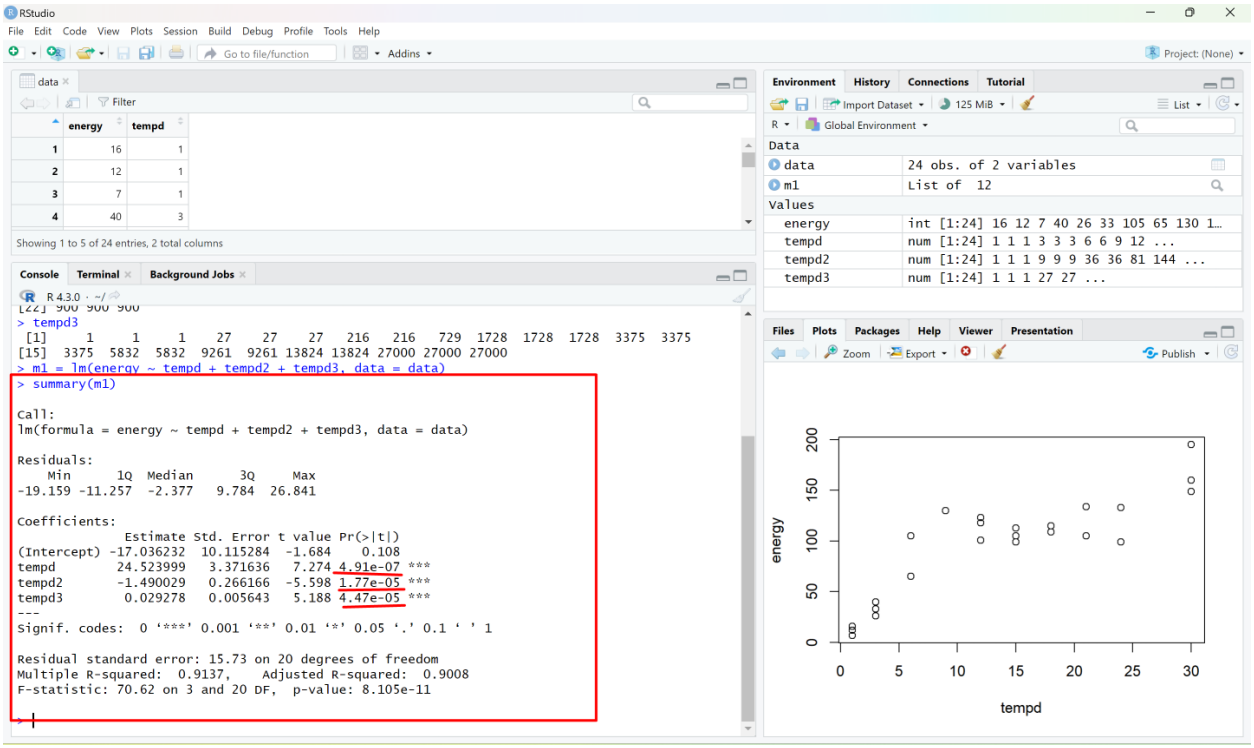



Conclusion on F-test: p-value $< 0.05$ for the model m1(fitted cubic regression model)

- At 5% signicance level(alpha = 5%, i.e., alpha = 0.05) we can say that at least 1 x variables among (tempd, tempd2, tempd3) has a significant linear relationship with energy and can affect the value of y-variable ~ energy.

******By conducting an F-test, we can observe that there is sufficient evidence to reject the null hypothesis and conclude that independent variables (tempd, tempd2, tempd3) have a significant impact on the dependent variable.(energy)*********

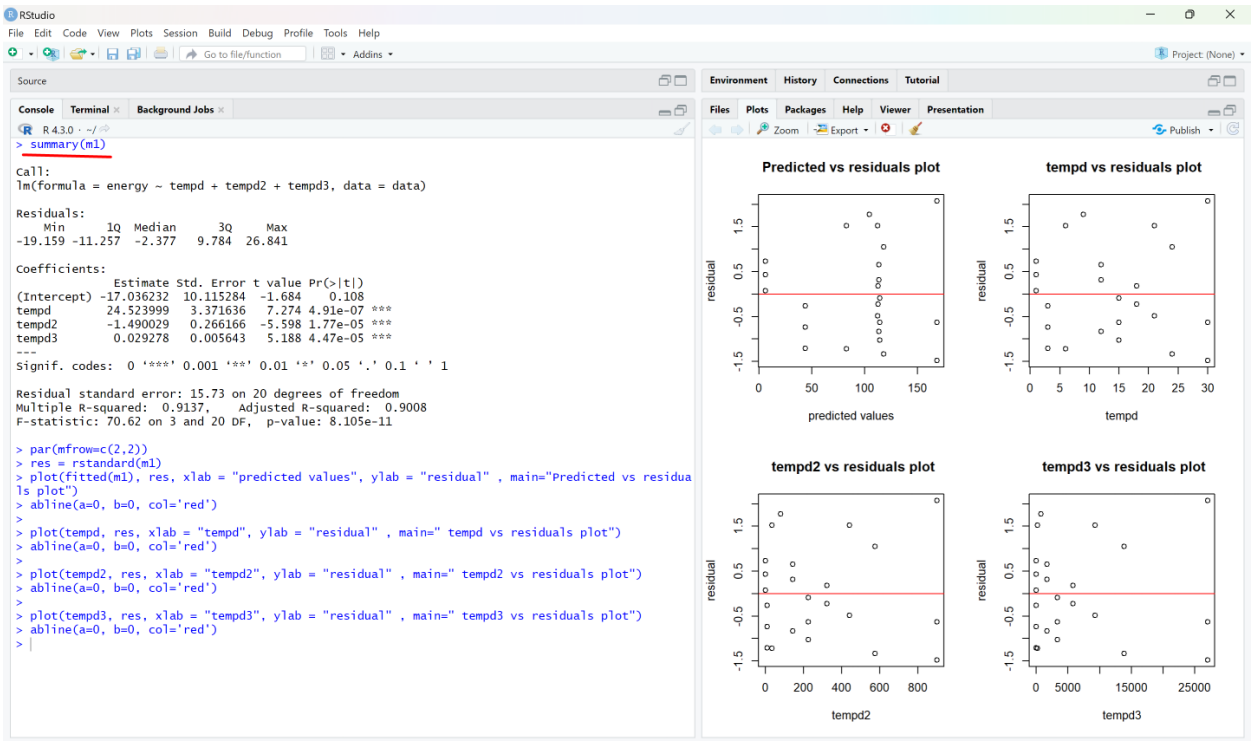
## b. Are all variables in the model significant?

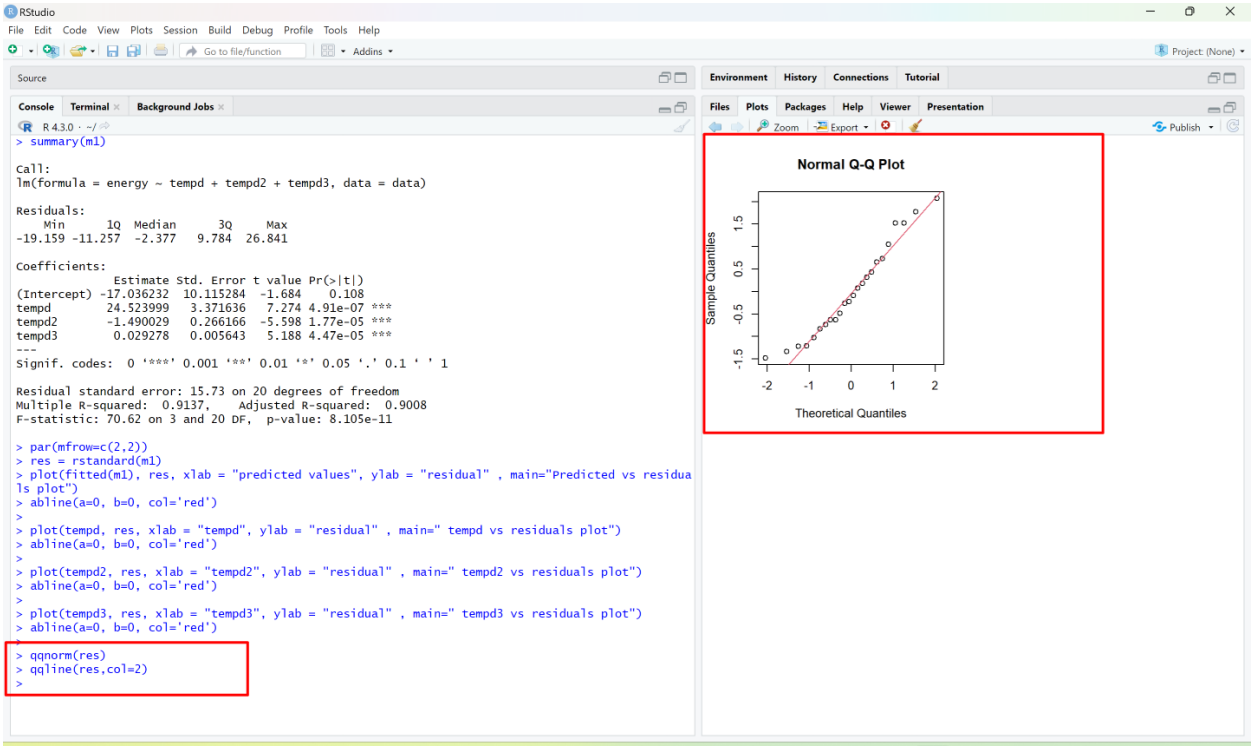



- To check whether all the variables are significant or not, we need to look at the individual t-test.
- As all the x-variables (tempd, tempd2,tempd3) have smaller p values than alpha 0.05(Assuming using 95% confidence level), all the x- variables in the model are significant.

## c. Perform residual analysis: create residual plots and analyze the plots & discuss findings:

1. Plot residuals vs predicted values plot: To check constant variance for the residuals:
2. Plot residuals vs each x-variable to validate the linearity relationship:
3. qqplot to validate normal distribution of residual:



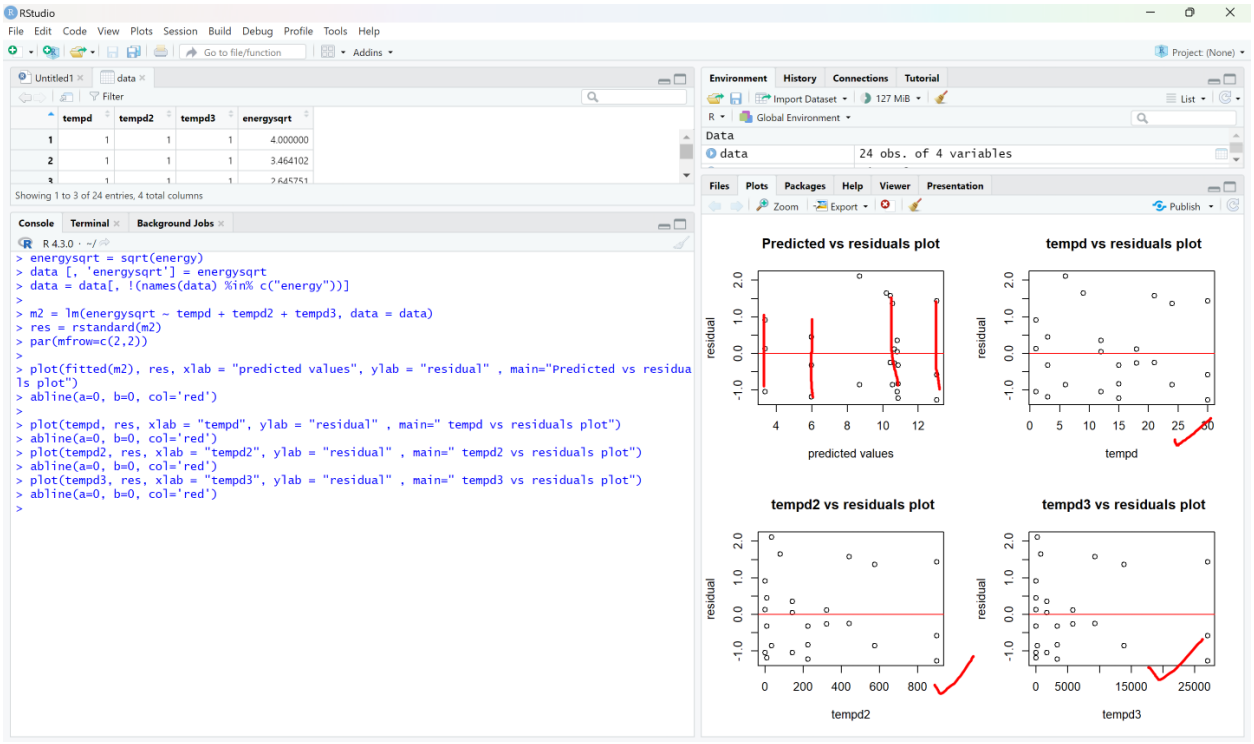



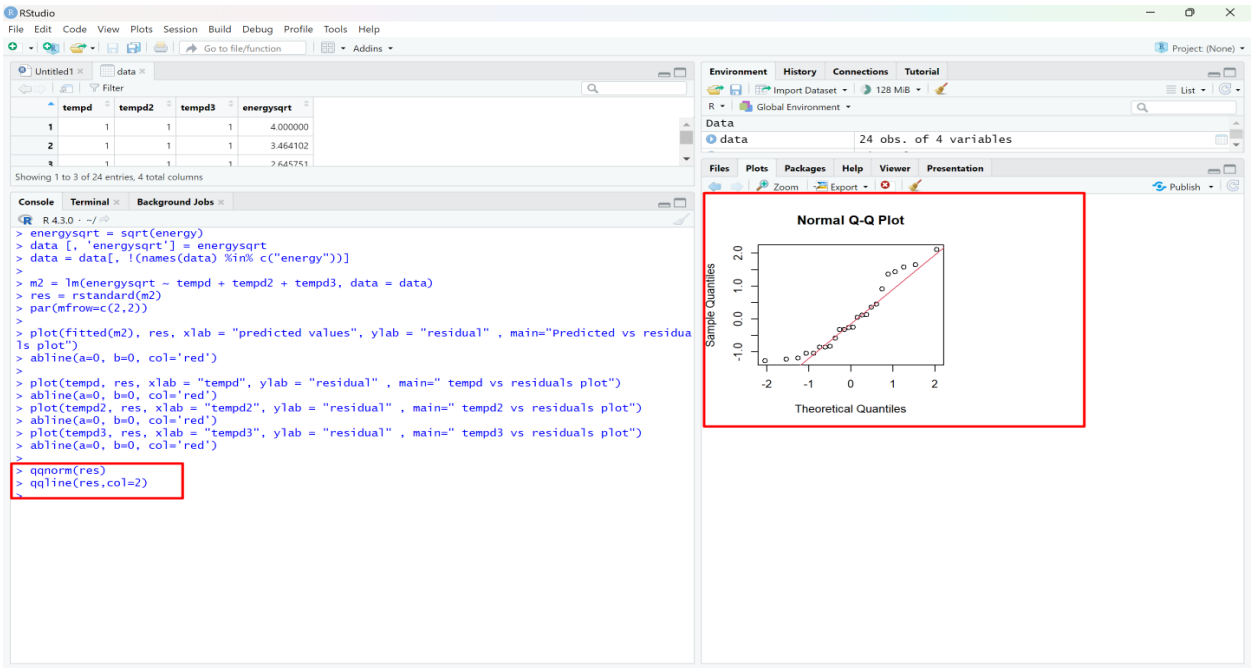

**Observation**: while performing res analysis for m1, we observed that

- In the res vs fitted values plot, we can observe that there is no constant variance, so we need to do transformation on y-var (energy)

- From the qqplot we can observe that residual follows normal distribution.

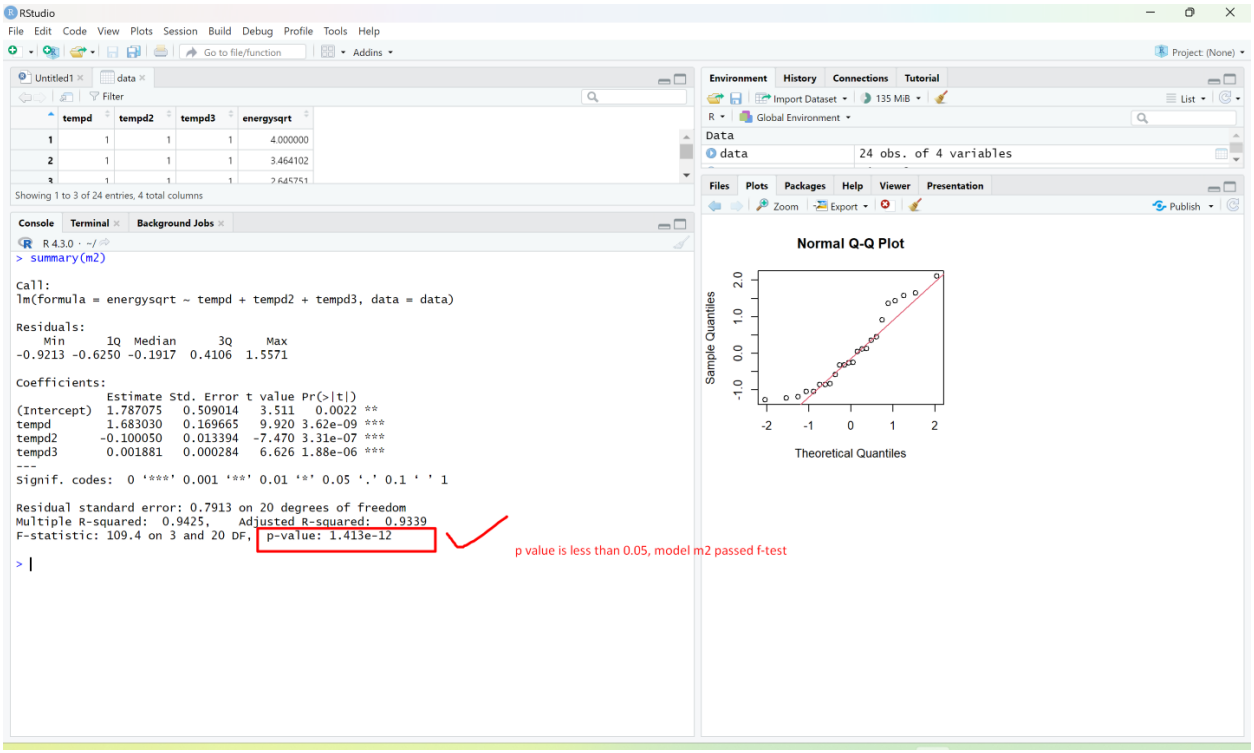
I tried log , reverse, sqrt on energy. I found sqrt transformation is better.
Sqrt tanformation on y-variable and built model m2 and performed residual analysis and f-test for **m2:**




Passed res analysis.

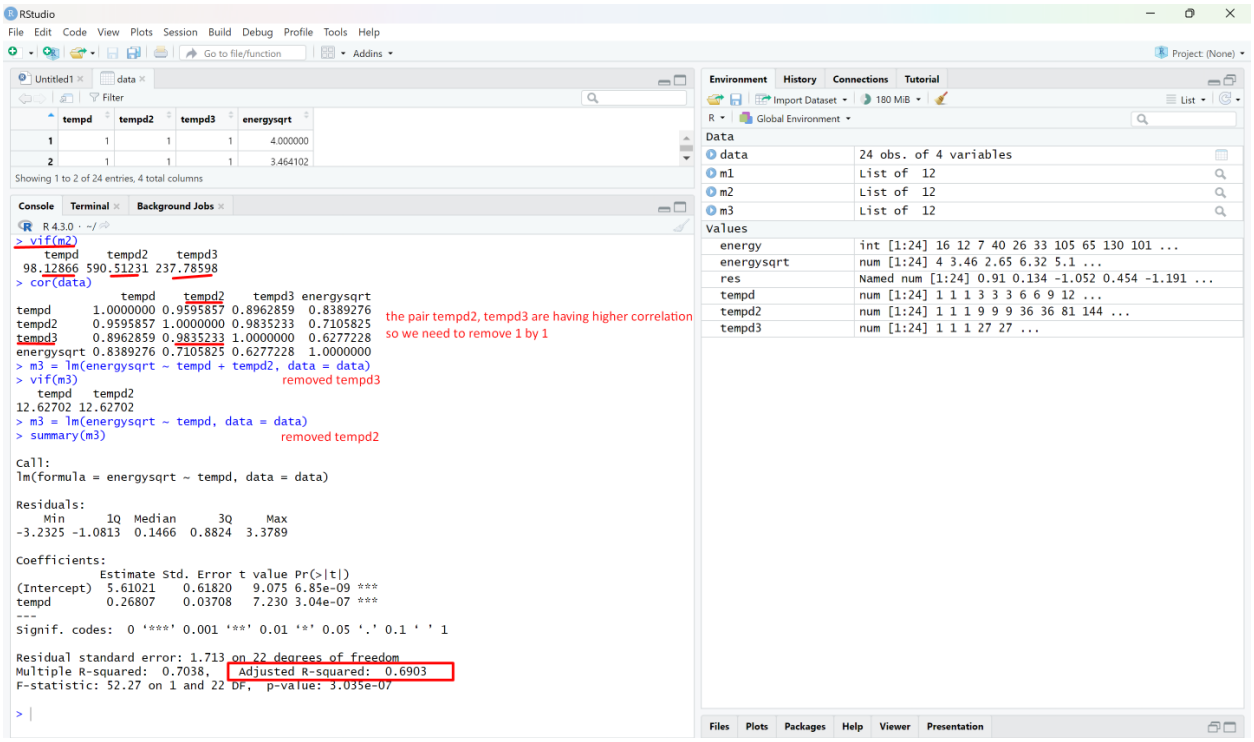
## f-test on m2: passed f-test




## Now we need check vif for m2: to remove multi-collinearity problem

- Using vif () function we can calculate the mcp of m1, library(car) is installed.
- We can observe there is hight vif value for all the x -variables.
- To know which pair of x-variables has high collinearity, we used cor(data)
- From cor(data) we observed that the pair tempd2 & tempd3 has high collinearity
- build new model m3 by removing tempd2 and tempd3 1 by 1.
- Now we got model m3 with variables energy and tempd

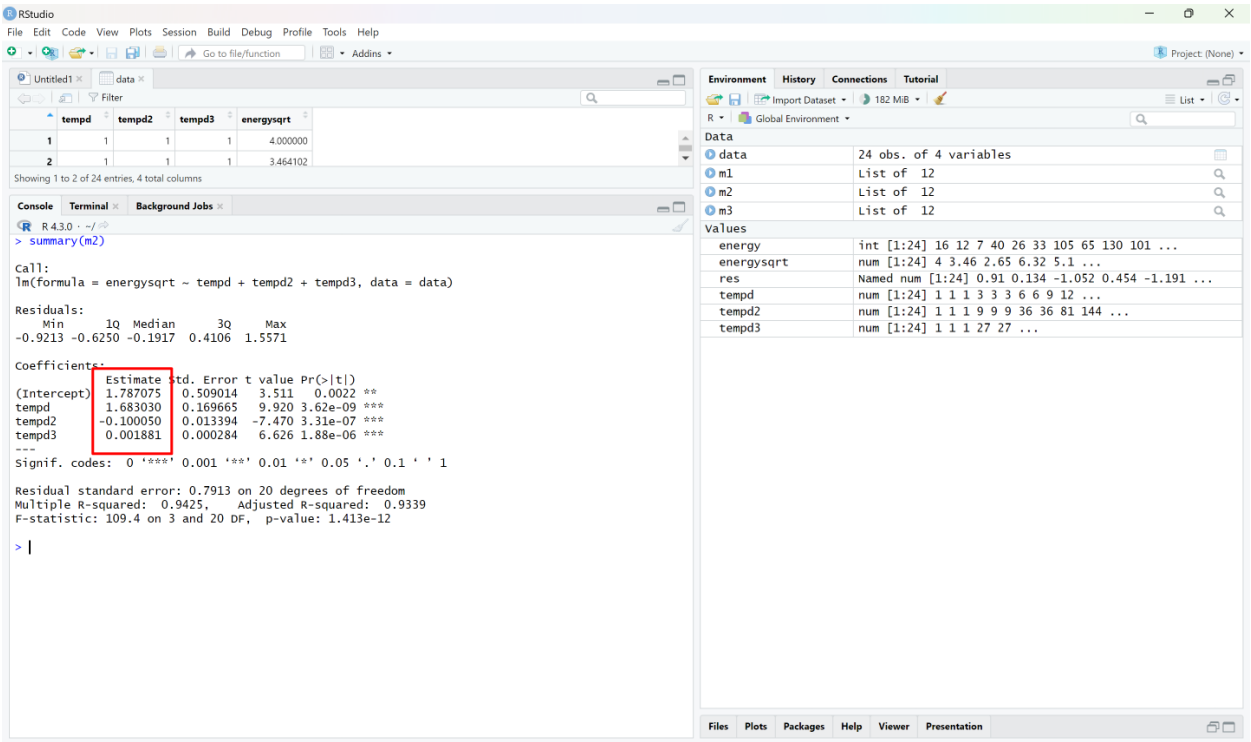
The entire process is carried out in R as follows in the screenshot: **M3**




## d. If you are satisfied with the fitted regression model, write down its expression:

Though we removed multicollinearity in m2 and built m3, it didn't improve the model. Because we observed that adjr2 is better for m2, we decided to **pick m2**. To do the prediction.

| MODEL | ADJ r2 | |
|---|---|---|
| M1 = initial model with energy~ tempd,tempd2,tempd3 | 90.08% | |
| **M2 = after sqrt tranf on energy variable (energysqrt~tempd,tempd2,tempd3)** | **93.39%** | **Passed f-test & res analysis.** |
| M3 = after removing mcp for m2 ( energysqrt~ tempd) | 69.03% | |




**From the coefficients of the fitted regression model m1, we can write the expression as follows:**

- ENERGYsqrt = β0 + β1 * TEMPD + β2 * TEMPD^2 + β3 * TEMPD^3 + e
- i.e.,

- SqrtEnergy = 1.78 + 1.683*tempd + (-0.10)*tempd2 + 0.001*tempd3 + e

Explaining the affect:

$\beta_0$ ~ explaining the intercept:

The intercept, represented by 1.78, is the value of energy when tempd is equal to zero. In other words, it indicates the sqrtenergy level when there is no temperature change (tempd = 0). The intercept represents a constant or baseline sqrtenergy level.

$\beta_1$ ~

The slope for the term 1.683*tempd represents the linear relationship between sqrtenergy and tempd. It indicates how energy changes per unit change in tempd. For every unit increase in tempd, the sqrtenergy increases by 1.683 (Assuming other variables are held constant).

$\beta_2$ ~

The slope for the term (-0.10)*tempd^2 represents the quadratic relationship between sqrtenergy and tempd. It indicates how the sqrtenergy changes as the square of tempd

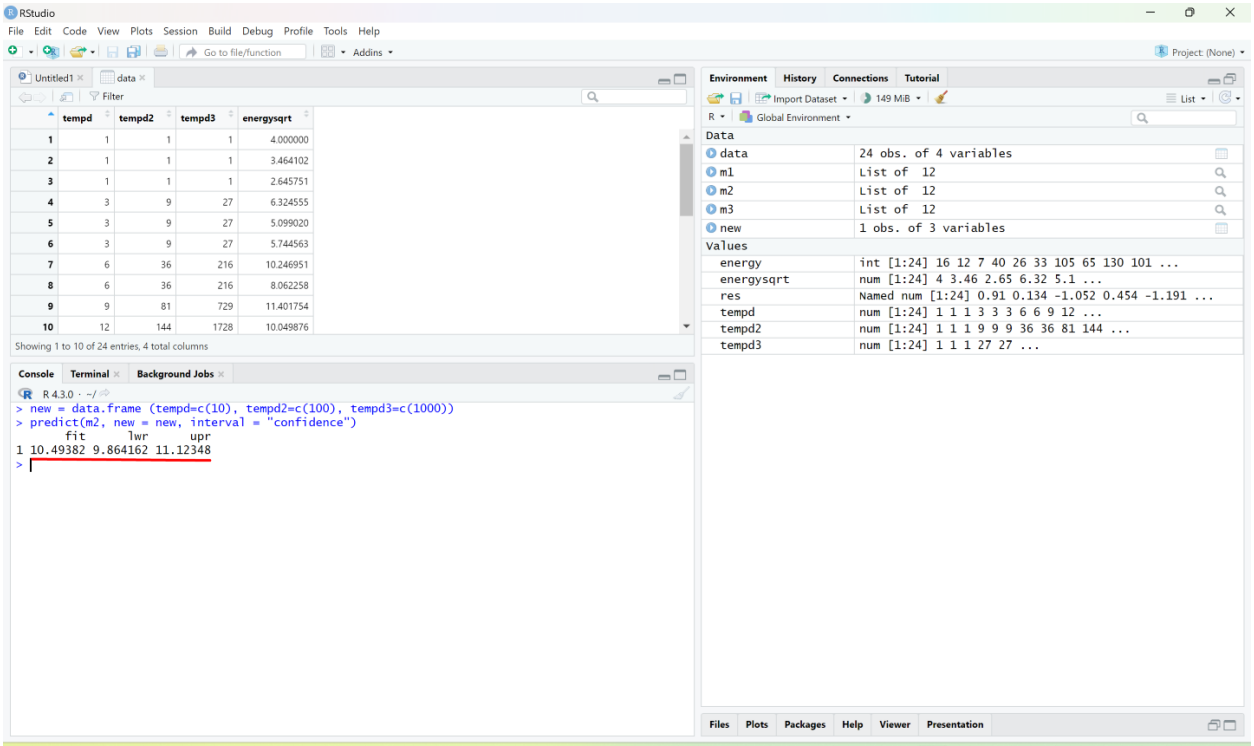
changes. The negative sign suggests a concave-down relationship, meaning the sqrtenergy initially increases at a decreasing rate and eventually decreases as tempd increases. (Assuming other variables are held constant).

$\beta_3$ ~

The slope for the term 0.001*tempd^3 represents the cubic relationship between sqrtenergy and tempd. It indicates how the sqrtenergy changes as the cube of tempd changes. The positive sign suggests a concave-up relationship, where the sqrtenergy increases as tempd increases. (Assuming other variables are held constant).

**e. Use the fitted regression model to predict the average energy consumption for an average difference in temperature equal to TEMPD=IO:**

**(In R, you should use new = data.frame (tempd=c(10), tempd2=c(100), tempd3=c(1000)), and then use the predict() function in R to produce predictions and confidence interval)**



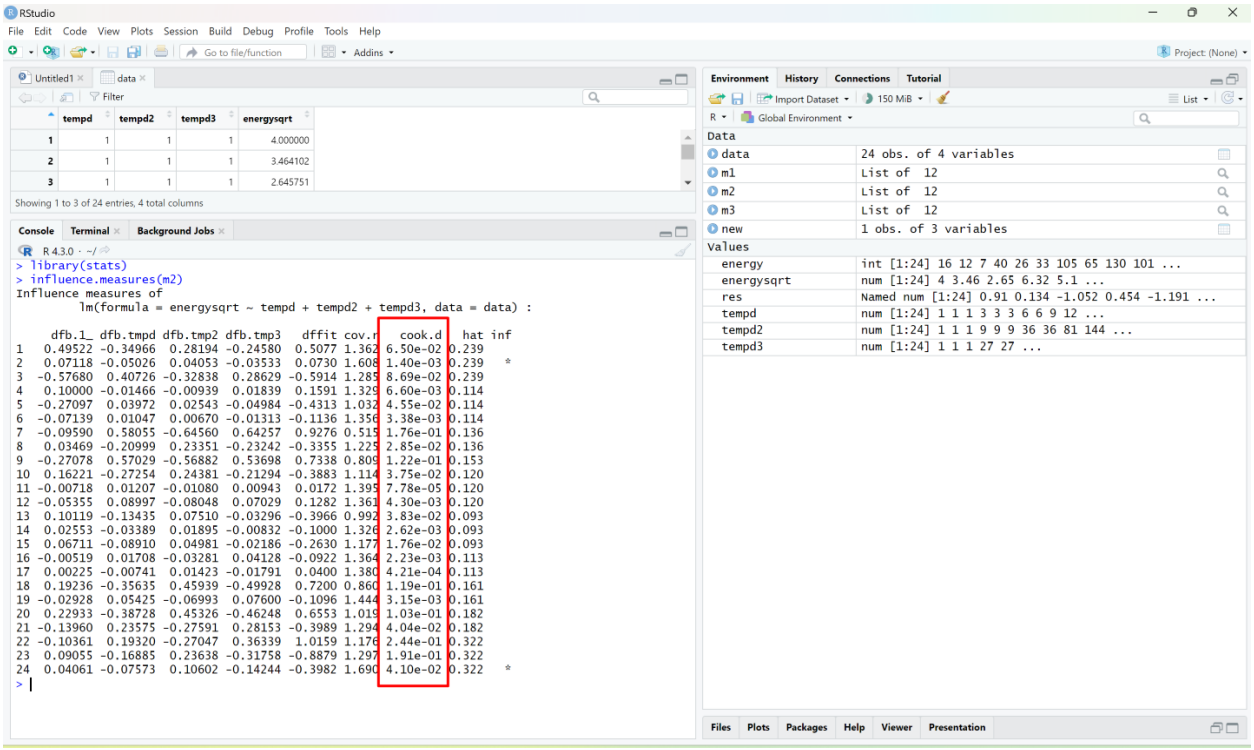

Observation:
From the output we can observe that
fit value is the predicted value: 10.49 (energy concumption) (energysqrt variable)
Confidence interval is lower ci = 9.86
Upper ci = 11.12

**f) By using influence.measures() function to identify whether there are influential points that can affect your final model. Use cook's distance as the metric to identify the influential points.**

Final model :m2

If we use cook distance, any points with cook distance > 4/n are influential points. We need to remove them. And re-build the model and check the adj r2. Here, n = data size = 24; 4/24 = 0.16

So, we need to remove any data points with values larger than 0.16:




From the output we observed that there are no data points has larger value than 0.16, Hence,

we can conclude that there are no influential points that can affect the final model m2.

## Problem 2 [50]

Variation in gasoline mileage among makes and models of automobiles is influenced substantially by the weight and horsepower of the vehicles. The data in the file *mileage.txt* were provided by the U.S. Environmental Protection Agency and report car models, miles per gallon (MPG), weight and horse power (HP) .

**a)** Fit a regression model to predict miles per gallon (MPG) from WEIGHT and horsepower (HP). Analyze residual plots to evaluate if the regression model is appropriate. Discuss if there is any evidence that the assumption of non-constant variance is satisfied by the data.

b) Apply a transformation to the Y variable (for instance you can try log(y), sqrt(y) or 1/y). Find the regression model that seems more appropriate for the analysis.

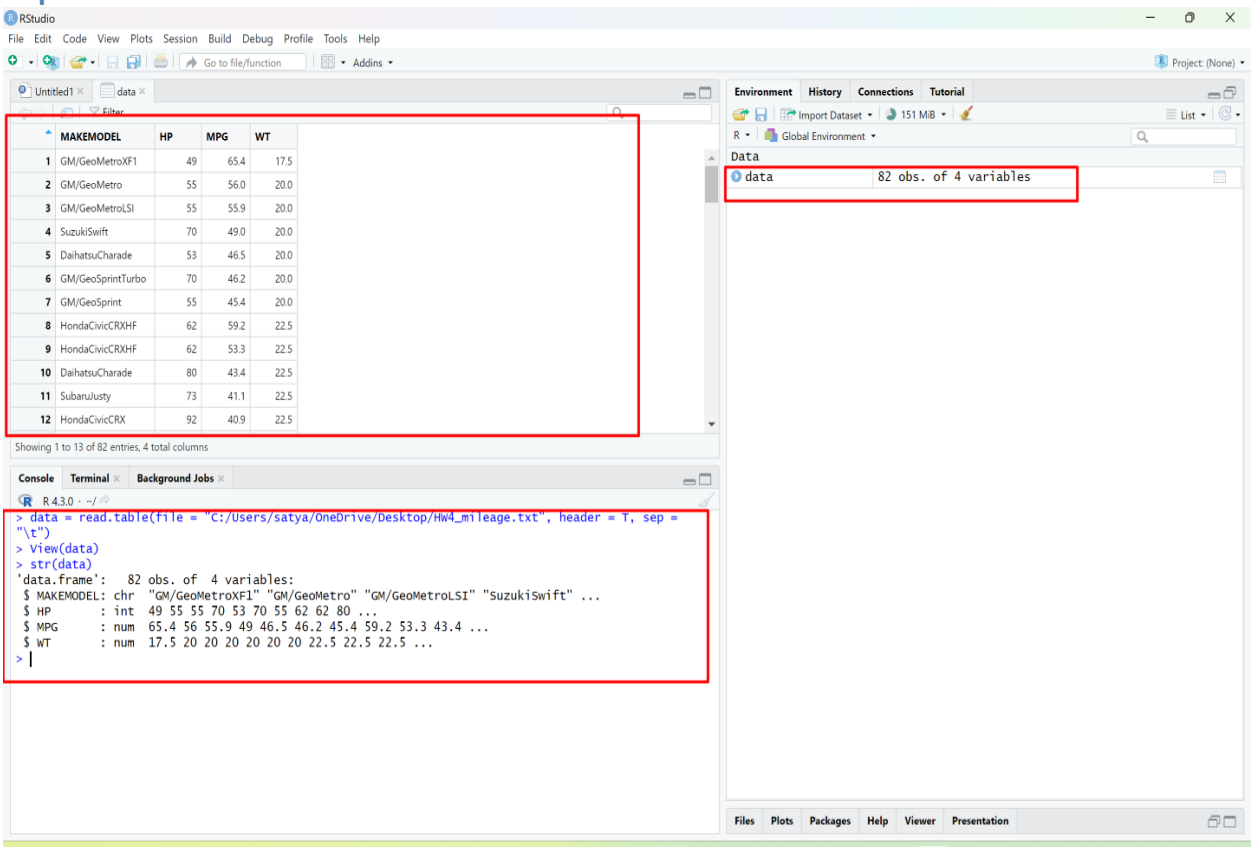
c) Analyze residual plots to evaluate if the regression model for the transformed Y variables is adequate. Do the plots show a deviation from the assumption of constant variance?

d) Write down the expression of the regression model and interpret the estimated values of the regression parameters.

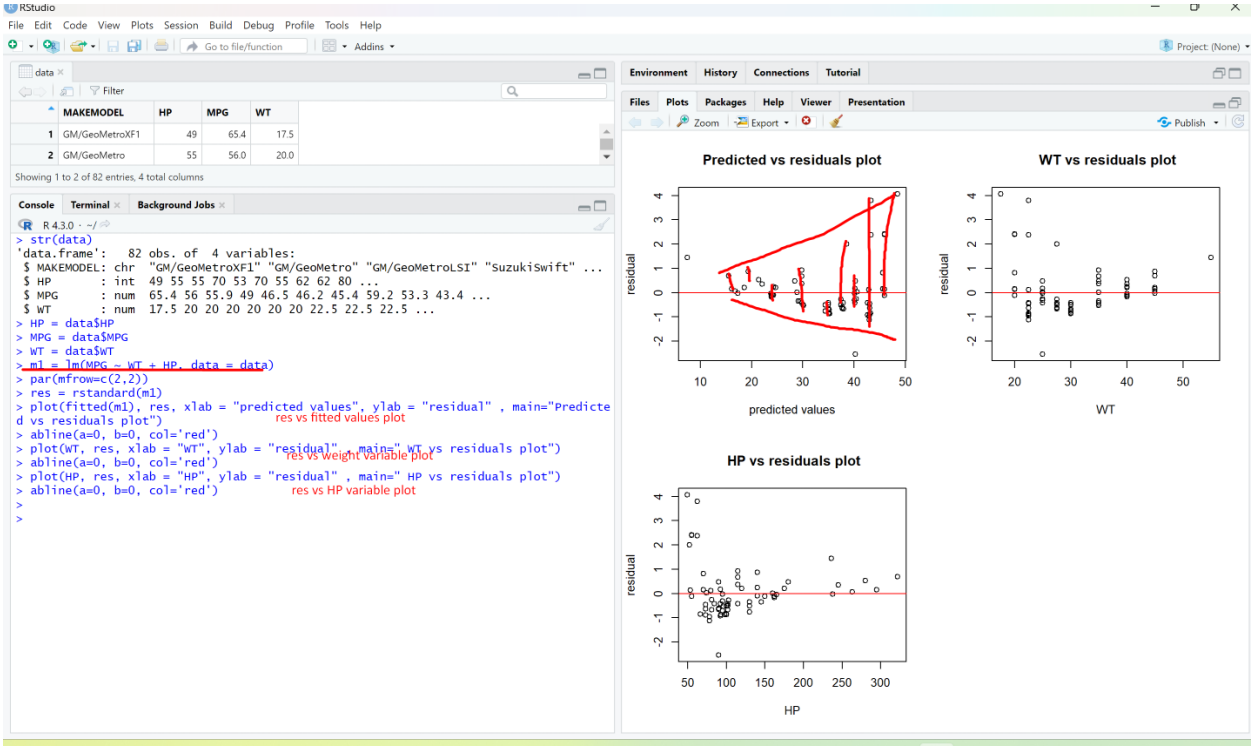
e). Use the step function to adopt both backward and stepwise forward selection to build a new model, compare the new model with the previous model in terms of the adj-R2 value.

### ANSWER:

### Imported data into r -studio:



**a) Fit a regression model to predict miles per gallon (MPG) from WEIGHT and horsepower (HP). Analyze residual plots to evaluate if the regression model is appropriate. Discuss if there is any evidence that the assumption of non-constant variance is satisfied by the data.**




We draw residual plots: (assumptions on residual)

1. res vs fitted value/pred values: To check if res has constant variance or not.
2. res vs wt variable: To check if res has linear r/n with Weight(x1-variable) or not.
3. res vs hp variable: To check if res has linear r/n with HP(x2-variable) or not.

- To say whether the regression model (m1) is appropriate or not, we need to look for patterns or deviations in the residual plots or is there any constant variance in the res vs pred values plot.
- Specifically, we are interested in assessing whether there is evidence of non-constant variance. If the residual plots exhibit a cone or fan-like shape, it suggests non-constant variance.

**EVIDENCE:**

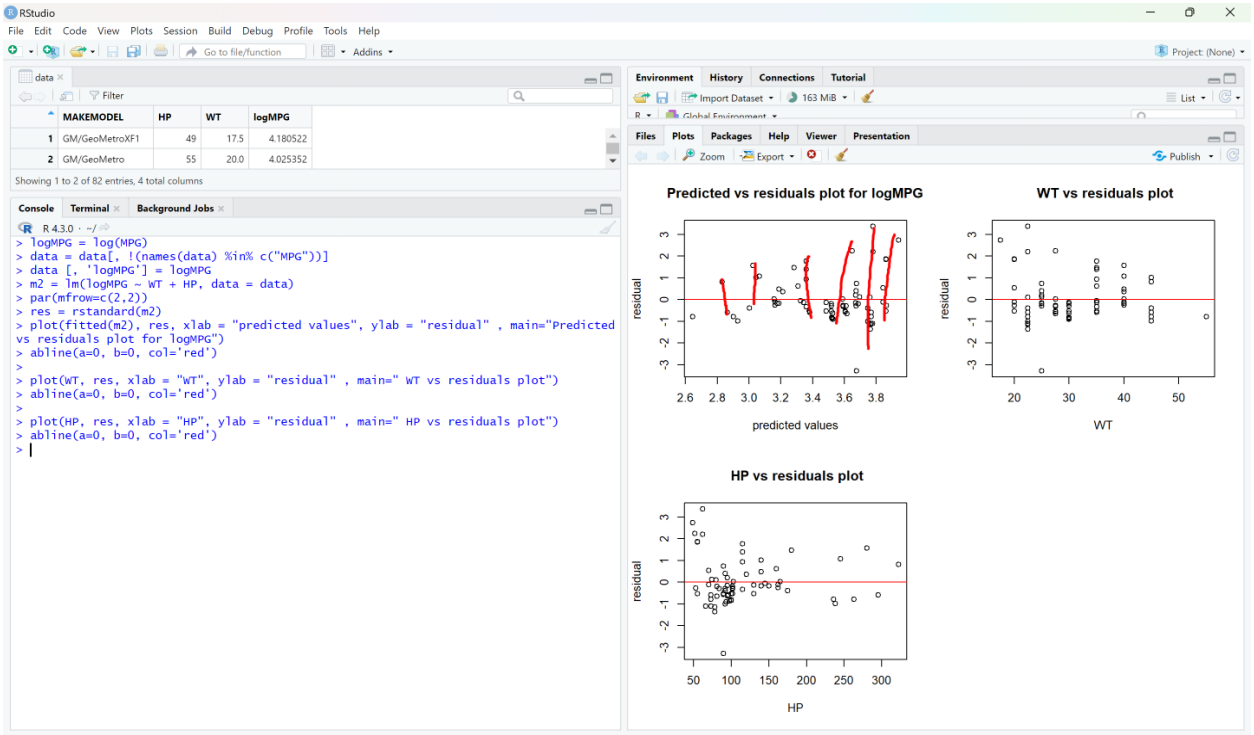
- From the res vs fitted values plot we can clearly observe that the variance is not constant. The variance on the left is small and on the right the variance is large.
- In other words, the variance increased from left to right and exhibits a fan shaped pattern which indicates that there is non- constant variance by the data.
- As the variance is not constant, we need to apply transformation on y-variable (MPG)

**ANALYSING THE MODEL IS APPROPRIATE OR NOT:**

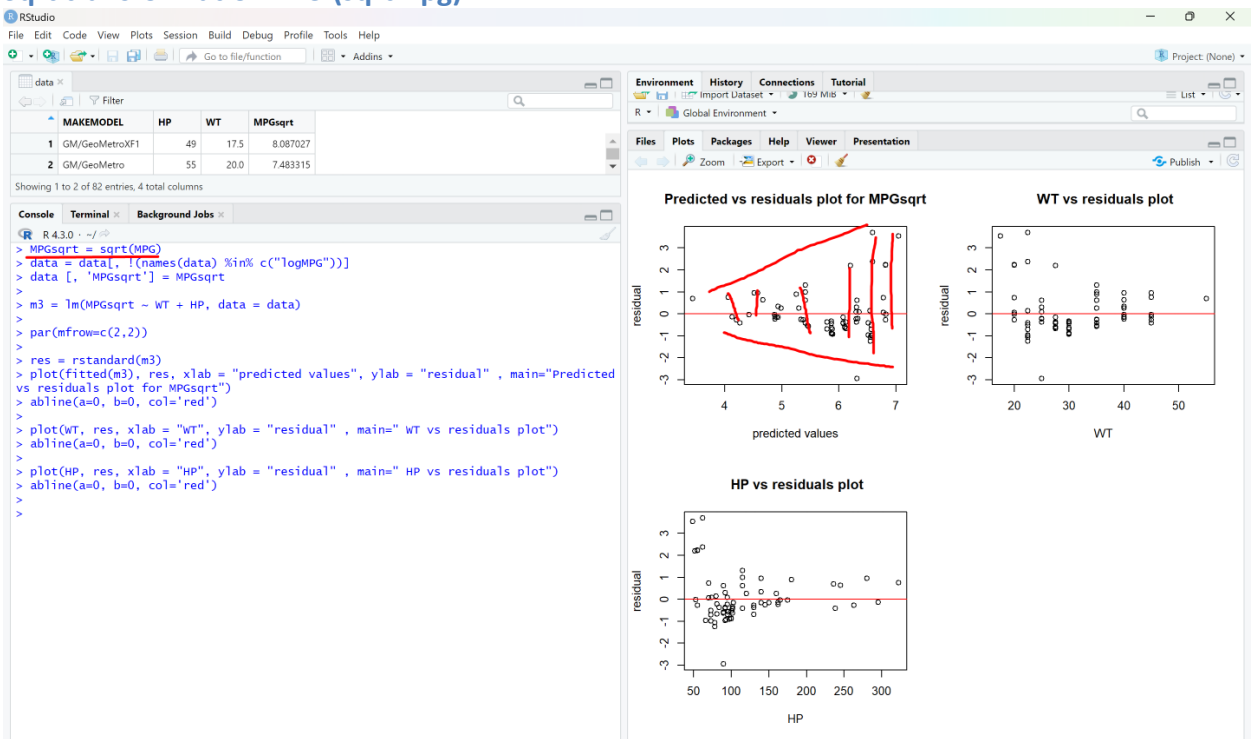
- As, there is no constant variance, **the model (m1) is not appropriate.**
- After transforming y-variable, we need to rebuild the model and check.

## b) Apply a transformation to the Y variable (for instance you can try log(y), sqrt(y) or l/y). Find the regression model that seems more appropriate for the analysis.

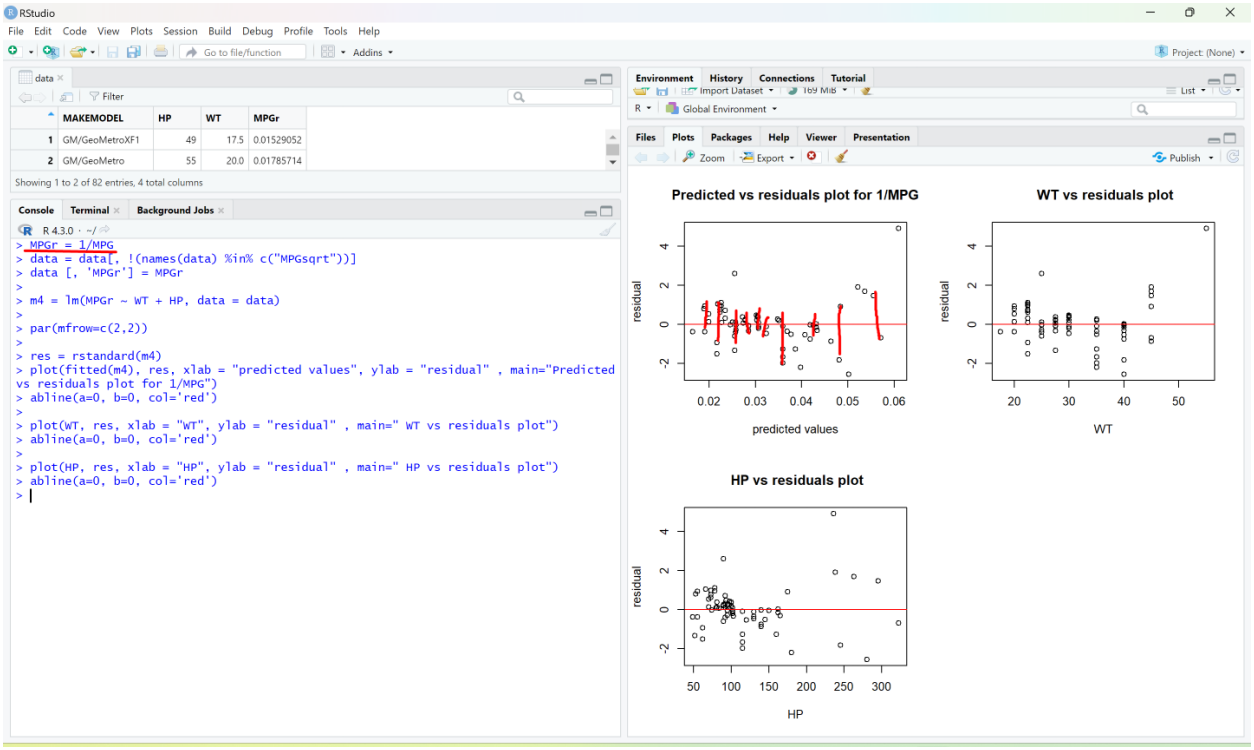
### Log transformation: m2 (logmpg)




### Sqrt transformation: m3 (sqrtmpg)



## Reverse transformation: m4 (1/mpg) mpgr




## Observation:

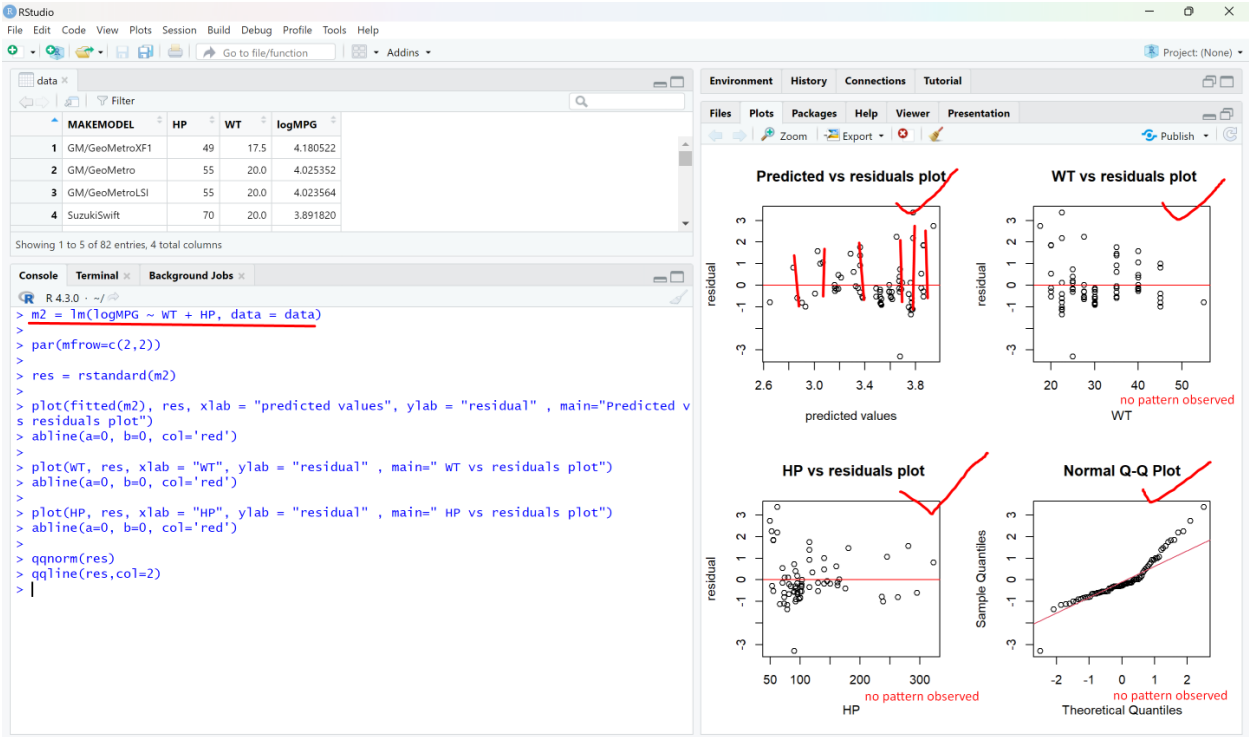
We did transformation on y-variable 'mpg' and built models. And analyzed res plots with all three transformed variables:

| | | | | |
|---|---|---|---|---|
| No-transormation(M1) | Original mpg | Model m1 (mpg~ wt+hp) | Res plot | |
| Log transformation (M2) | log mpg | model m2 (logmpg ~ wt + hp) | Res plot | **More appropriate** |
| Sqrt transformation (M3) | mpgsqrt | model m3 (mpgsqrt ~ wt + hp) | Res plot | |
| Rev transformation (M4) | 1/mpg(mpgr) | model m4 (mpgr ~ wt + hp) | Res plot | |

- We found that model m2 (log mpg model) is more appropriate in terms of res constant variance with x- variables and pred/fitted values & adj r2 and co-efficients interpretability.

**c) Analyze residual plots to evaluate if the regression model for the transformed Y variables is adequate. Do the plots show a deviation from the assumption of constant variance?**

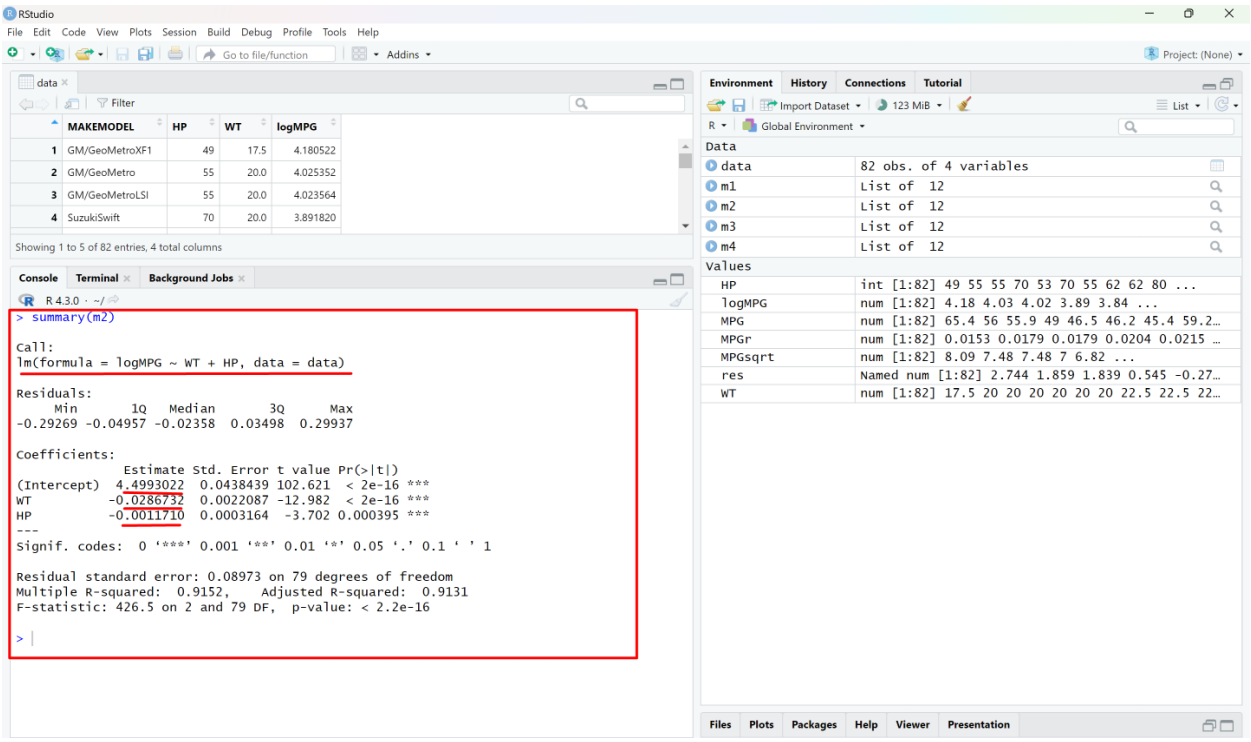
## Residual analysis for final model: m2 (logMPG)




No, the plots do not show deviation from constant variance.
** in the qq plot, the residual slightly follows normal distribution, but it depends on the size of the data. As our data size 24 rows, there is no effective solution. **********

**d) Write down the expression of the regression model and interpret the estimated values of the regression parameters.**

Final model we picked : M2




**From the coefficients of the regression model m2, we can write the expression as follows:**

- LogMPG = β0 + β1*WT + β2 * HP + e
- i.e.,

- LogMPG = 4.49 + (-0.02)*WT + (-0.0017)*HP + e

**Explaining the affect:**

$\beta_0$ ~ explaining the intercept:

The intercept, represented by 4.49, is the value of LogMPG when both Weight and Horsepower are equal to zero. In this context, the intercept represents the baseline or average LogMPG value when there is no weight or horsepower influencing the MPG. It is a constant term in the equation.

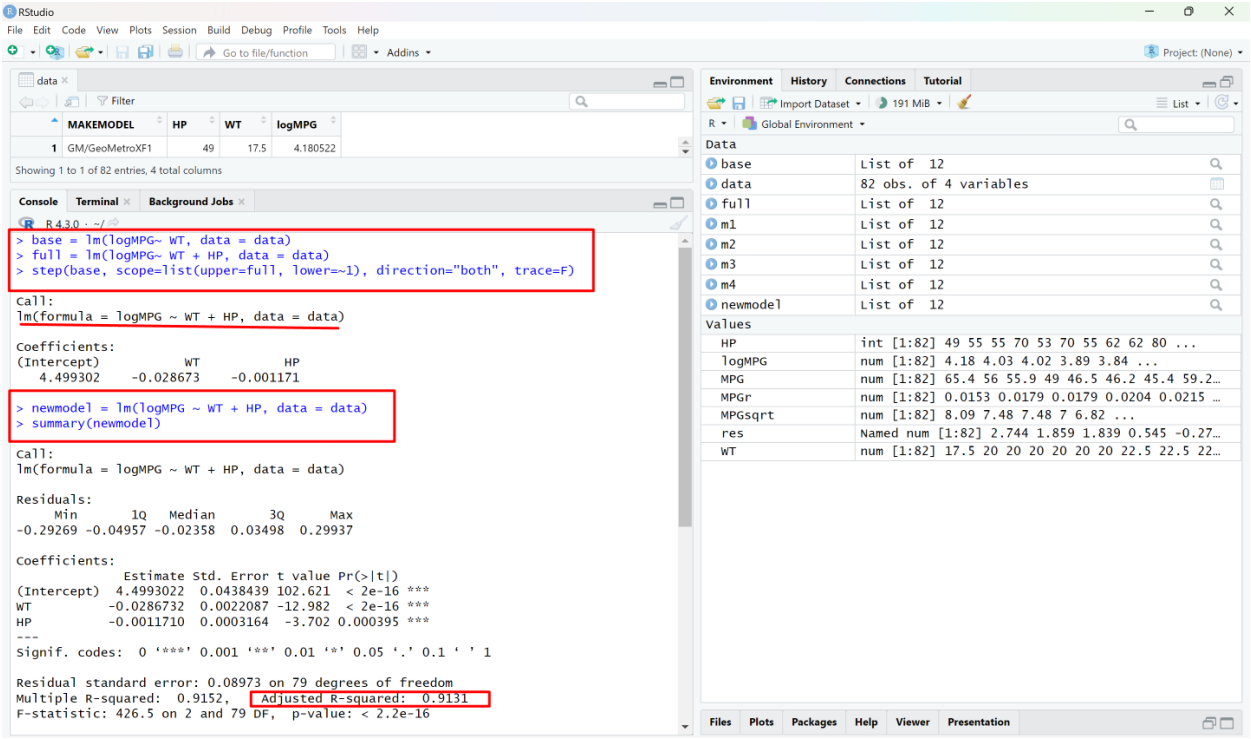
$\beta_1, \beta_2$ ~ The slopes represent how the logarithm of MPG changes with respect to the independent variables, Weight and Horsepower:

$\beta_1$ ~ The slope for the term (-0.02) *Weight indicates the change in LogMPG for every unit increase in Weight, **assuming other variables are held constant**. A negative slope suggests that as the weight of the vehicle increases, the LogMPG decreases. This implies that heavier vehicles tend to have lower fuel efficiency.

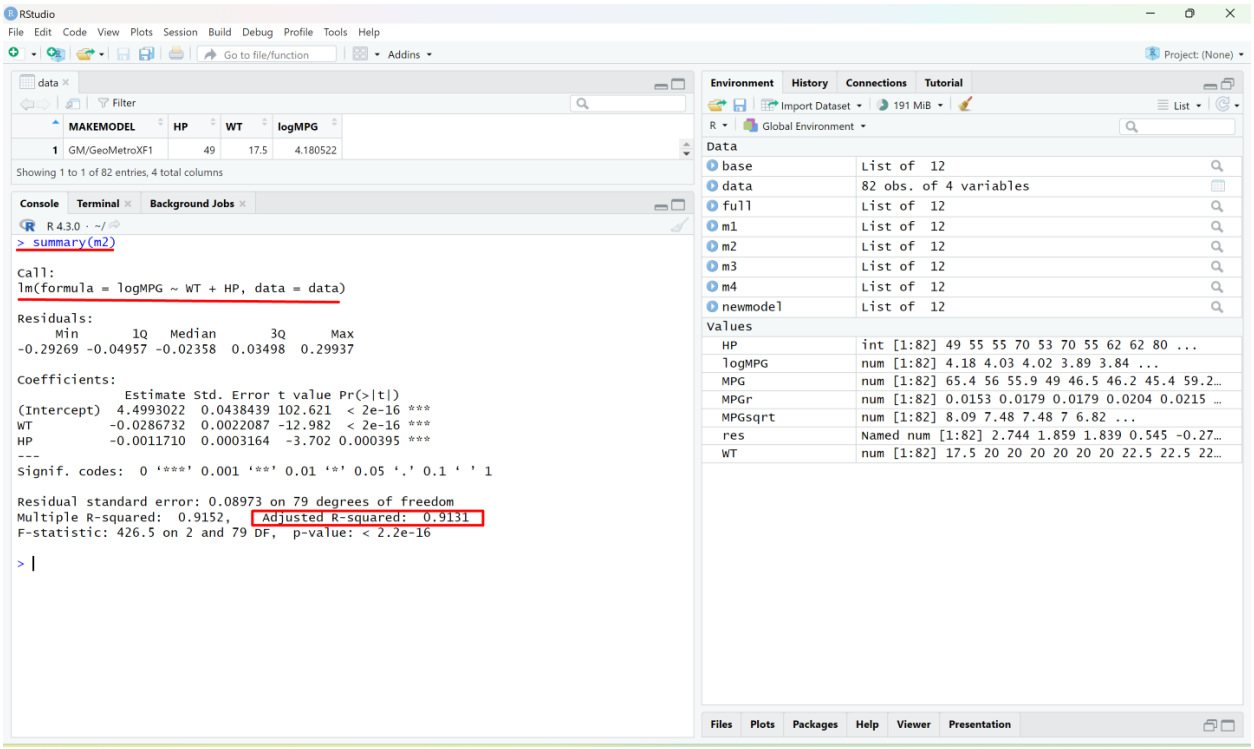
$\beta_2$ ~

The slope for the term (-0.0017)*Horse Power indicates the change in LogMPG for every unit increase in Horse Power, **assuming other variables are held constant**. A negative slope suggests that as the Horsepower of the vehicle increases, the LogMPG decreases. This implies that more powerful vehicles tend to have lower fuel efficiency.

**e) Use the step function to adopt both backward and stepwise forward selection to build a new model, compare the new model with the previous model in terms of the adj-R2 value.**




**Previous model: (m2) after tranformation on y-variable ~ logMPG:**




## Comparison:

| MODEL | ADJ r2 |
|---|---|
| M2 = after log transformation on y-variable(logMPG)<br>lm(logMPG ~WT + HP, data=data) | 91.31% |
| New model = after step(both)<br><br>lm(logMPG ~WT + HP, data=data) | 91.31% |

Both the new-model and previous model(m2) have same adj r2 values. So, both are the same.

**Problem 3 [10]**

Answer the following concept questions.

1). What are the differences between outliers and influential points in linear regression models.

ANSWER:

| Basis for comparison | outliers | Influential points |
|---|---|---|
| Definition | An outlier is a single data point that drastically deviates from the pattern of the data as a whole. These are observations that are a long way from the majority of the data. | Influential points are the outliers that heavily influence the estimated regression coefficients. They have a high leverage on the model parameters.<br><br>* You may have many outliers but not all of them are influential points. |
| Impact | The calculated regression coefficients and the overall model fit can be greatly impacted by outliers. They can change the slope and intercept of the regression line by bringing it closer or farther away from the bulk of the data points. | The slope, intercept, and standard errors of the coefficients may all be dramatically changed by influential points, which also impact the regression line. They have the ability to alter the model fit and the overall findings from the analysis.<br><br>* If we include influential points and built a model, most of the datapoints will not fall on the regression line.<br><br>*if we remove them, most of the data points will fall on the line. |
| Identification | Outliers can be identified by:<br>*Residual plot<br>* Residual vs pred values plot<br>* Res vs each x-variable plot.<br><br>If any data points goes beyond upper and lower bound values, they can be identified | They can be identified by:<br>*DFFITS ()<br>*DFBETAS ()<br>*covratio ()<br>*hatvalues ()<br>*cooks. distance ()<br>If cooks.distance is larger than 4/n, they can be identified as |

| | | |
|---|---|---|
| | as outliers. | influential points. (n = size of the data/no.of rows) |
| **Causes** | Outliers can occur due to measurement errors, data entry mistakes, natural variation, or genuine extreme observations. | Excessive values of the predictor variables are frequently linked to influential locations. A predictor's extreme value in an observation might have a disproportionately large impact on the calculated regression coefficients. |
| **Treatment** | Any data values not falling between upper bound value and lower bound value are suspected to be potential outliers and can be treated and removed. | Influential points can be treated by calculating cooks.distance. if cooks distance for any data point in a data is larger than 4/n, they those data points has to be removed. |

2). How to identify a 2nd order and 3rd order terms in linear regression.

ANSWER:

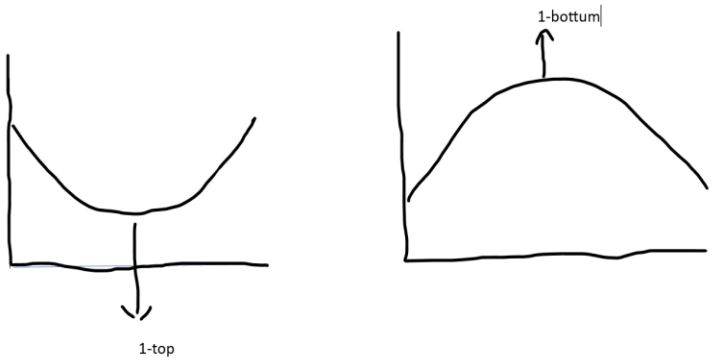
- 2nd & 3rd order terms are called higher order terms.
- First, to identify if the linear regression has higher order terms or not, we need to draw scatter plot with y-variable (dependent variable) and x-variable (predictor variable/independent variables)
- If the plot shows a curve pattern, there are higher order terms in the linear regression.

**2nd order term Identification:**

If the curve pattern is having 1-top or 1- bottom, we can conclude that the linear regression is having quadratic relation.

In other words, predictor variables might have a quadratic relationship with the response variable.

Hence, the linear regression needs to have 2nd order term in the model.

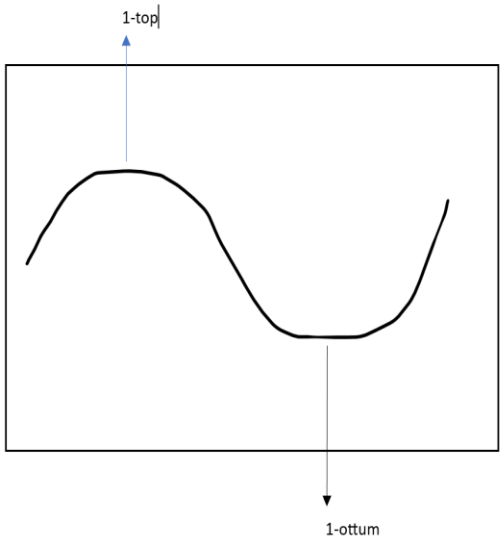



**3rd order term identification:**

If the curve pattern is having at least 1-top & at least 1- bottom, we can conclude that the linear regression is having cubic relation.

In other words, predictor variables might have a cubic relationship with the response variable.

Hence, the linear regression needs to have 3rd order term in the model.



**HW5**

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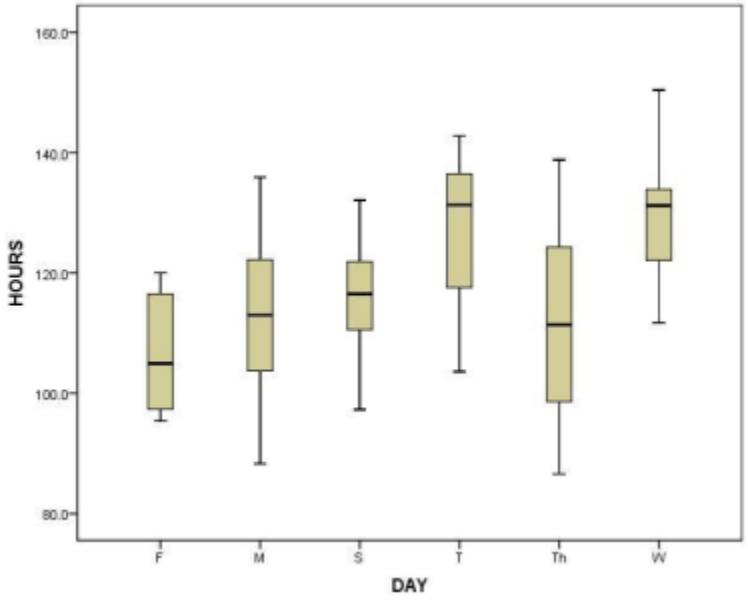
**Student ID: A20517818.**

let's use the "clerical_Q2.txt" data.
A store manager noticed that the busiest days for clerical staff are Wednesdays and Tuesdays. See enclosed box plot. The manage tries to compare the group means in hours by different days




a). [10] Observe the box plot. Can you confirm that the hours in Tuesday is the highest? Why?

**ANSWER:**

- From the boxplot, we can observe that there are 6 different groups with days variable on the x-axis and hours variable on the y-axis. For each group, we can observe a 1-box plot.
- From the visualization, we can clearly observe that the variation within the groups (boxplots) are not even/uniform. So, we cannot compare the groups based on their q2 values. Why? Because the within-group variation (IQR) is large. So, we cannot use q2 to compare the groups.
- So, we need to have additional information such as mean values for every group to compare them & give the conclusion.
- As the boxplot cannot provide mean values, we need to use ANOVA technique to say hours on Tuesday is the highest or not.
- So, with the variation difference among the groups & without knowing the mean values for every group, we cannot confirm that hours on Tuesday is the highest.

b). [20] Write down your hypothesis in the ANOVA to compare the group means in hours by different days

**ANSWER:**

**Null hypothesis:** All the groups have the same mean. In other words, the means of hours for different days are equal.

$H0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6$

- $\mu_1$ = average of hours on Monday group
- $\mu_2$ = average of hours on Tuesday group
- $\mu_3$ = average of hours on Wednesday group
- $\mu_4$ = average of hours on Thursday group
- $\mu_5$ = average of hours on Friday group
- $\mu_6$ = average of hours on Saturday group

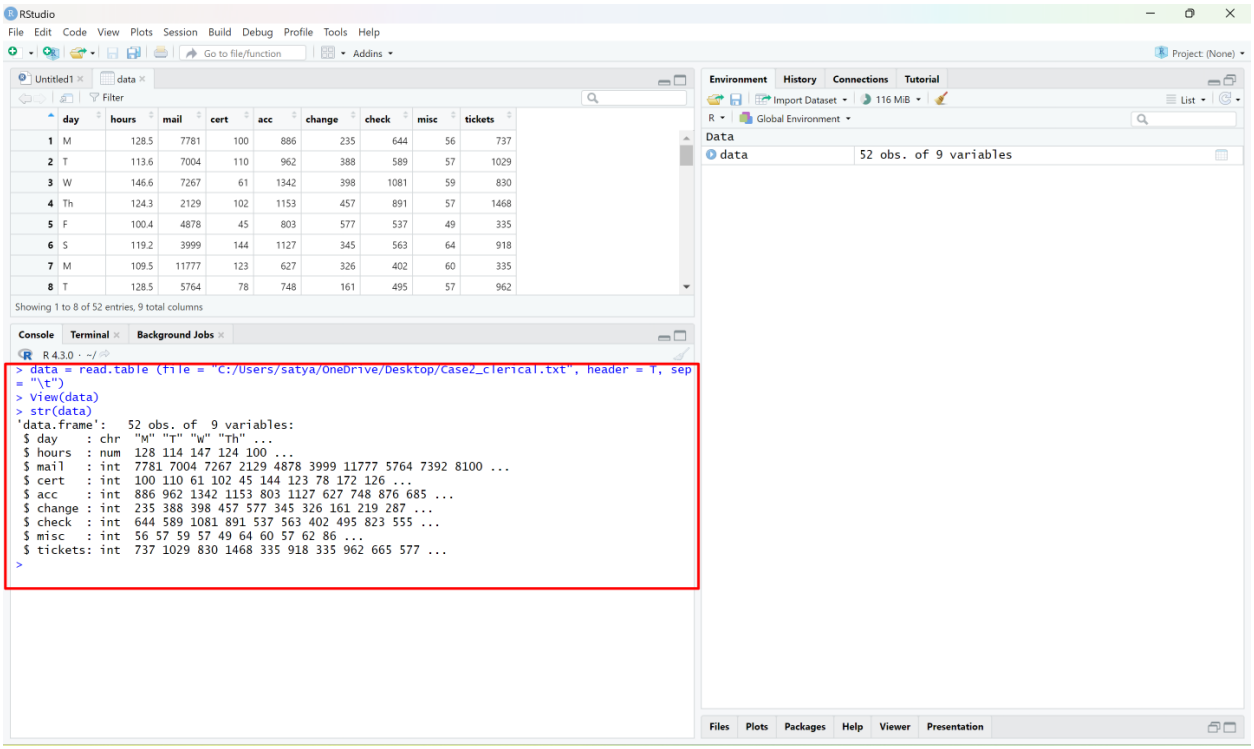
**Alternative hypothesis**: At least 2 groups (1-pair of the mean) have different means. In other words, at least 2 group's means among the means of hours for different days are not equal.

$H$a = Not all the $\mu_t$ are equal / $\mu_i \neq \mu_j$

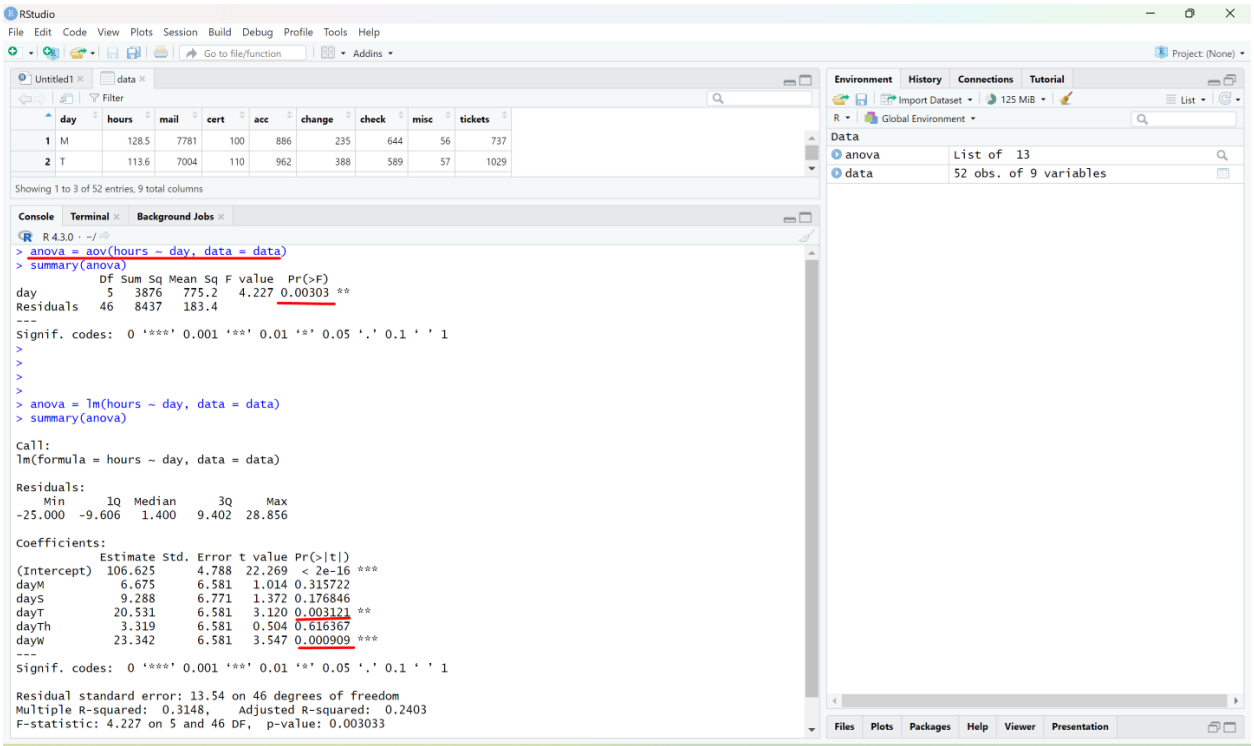
b). [30] Using R to build the ANOVA regression model and help the manager to make the decision whether the group means in hours or different days are the same or not.

**ANSWER:**

Importing clerical_Q2.txt data into the r-studio:




## Building anova regression model using aov () function:




## Decision/ conclusion:

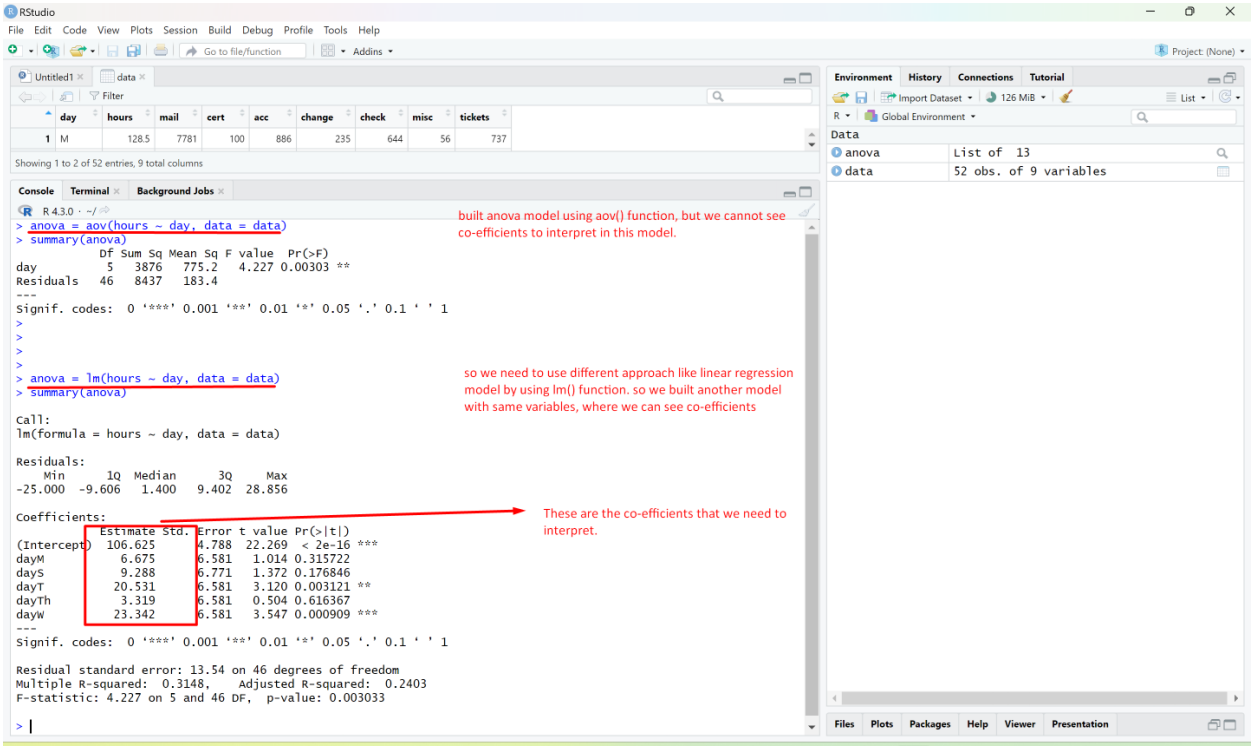
- Using 95% confidence level, as the p-value (0.00303) in F-test is smaller than alpha (p < 0.05%) we do have enough evidence to reject null hypothesis.
- In other words, we have enough evidence to reject that all group means in hours or different days are not same. At least 2 group means are different.

- To know which 2 groups means are different, we need to look at the t-test. We can clearly observe that Tuesday and Wednesday group means are not same.

c). [20] Try to interpret the coefficients you got in the ANOVA regression model from part b).

**ANSWER:**

We cannot see co-efficients in the model which we built using aov () function.
So we need to use other modeling approaches such as linear regression (e.g., lm())
So built model using lm() function with same variables to see the co-effeicients.




```
> anova = aov(hours ~ day, data = data)
> summary(anova)
            Df Sum Sq Mean Sq F value  Pr(>F)
day          5   3876   775.2   4.227 0.00303 **
Residuals   46   8437   183.4
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
>
>
>
> anova = lm(hours ~ day, data = data)
> summary(anova)

Call:
lm(formula = hours ~ day, data = data)

Residuals:
    Min      1Q  Median      3Q     Max
-25.000  -9.606   1.400   9.402  28.856

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  106.625      4.788  22.269  < 2e-16 ***
dayM           6.675      6.581   1.014 0.315722
dayS           9.288      6.771   1.372 0.176846
dayT          20.531      6.581   3.120 0.003121 **
dayTh          3.319      6.581   0.504 0.616367
dayW          23.342      6.581   3.547 0.000909 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.54 on 46 degrees of freedom
Multiple R-squared:  0.3148,    Adjusted R-squared:  0.2403
F-statistic: 4.227 on 5 and 46 DF,  p-value: 0.003033
```

Interpretation of co-efficients: **as there is no dayFriday, we are going to use Friday as baseline**.

hours = β0 + β1*X1 + β2 * X2 + β3 * X3 + β4 * X4 + β5 * X5 + e

- Intercept (+106.625): ~ β0

The intercept represents the estimated mean value for the reference group (Friday) when all other dummy variables are zero. In this case, it suggests that the estimated mean value for Friday is 106.625.

- dayM (+6.675): β1…….. X1= dayM

  The coefficient for "dayM" (+6.675) represents the difference in mean between Monday and Friday. The positive coefficient indicates that, on average, the mean for Monday is 6.675 units higher than the mean for Friday.

- dayS (+9.288): β2…….. X2= dayS

  The coefficient for "dayS" (+9.288) represents the difference in mean between Saturday and Friday. The positive coefficient indicates that, on average, the mean for Saturday is 9.288 units higher than the mean for Friday.

- dayT (+20.531): β3…….. X3= dayT

  The coefficient for "dayT" (+20.531) represents the difference in mean between Tuesday and Friday. The positive coefficient suggests that, on average, the mean for Tuesday is 20.531 units higher than the mean for Friday.

- dayTh (+3.319): β4…….. X4= dayTh

  The coefficient for "dayTh" (+3.319) represents the difference in mean between Thursday and Friday. The positive coefficient indicates that, on average, the mean for Thursday is 3.319 units higher than the mean for Friday.

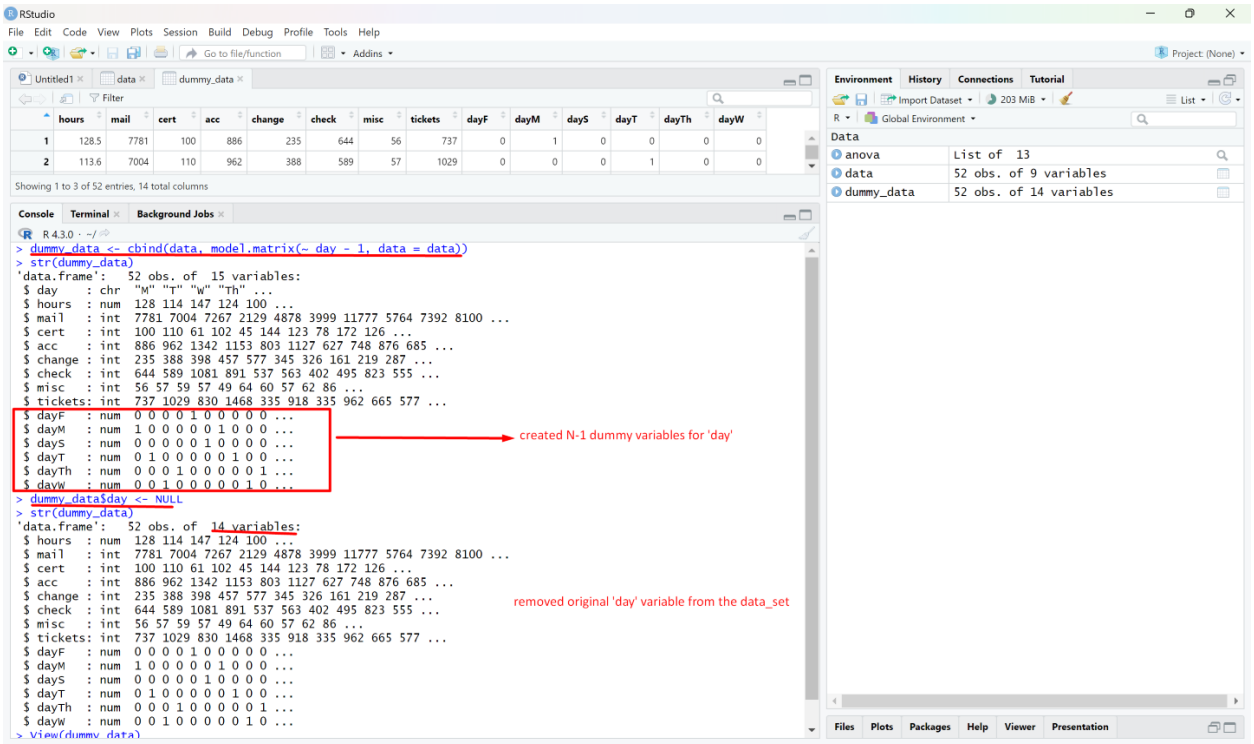
- dayW (+23.342): β5…….. X5= dayW

  The coefficient for "dayW" (+23.342) represents the difference in mean between Wednesday and Friday. The positive coefficient suggests that, on average, the mean for Wednesday is 23.342 units higher than the mean for Friday.

d). [20] practice for data preprocessing: create N-1 dummy variables for the variable 'DAY'. Convert the variable "mail" to nominal variable by creating 4 groups. Again, paste the codes and snapshots.

**ANSWER:**

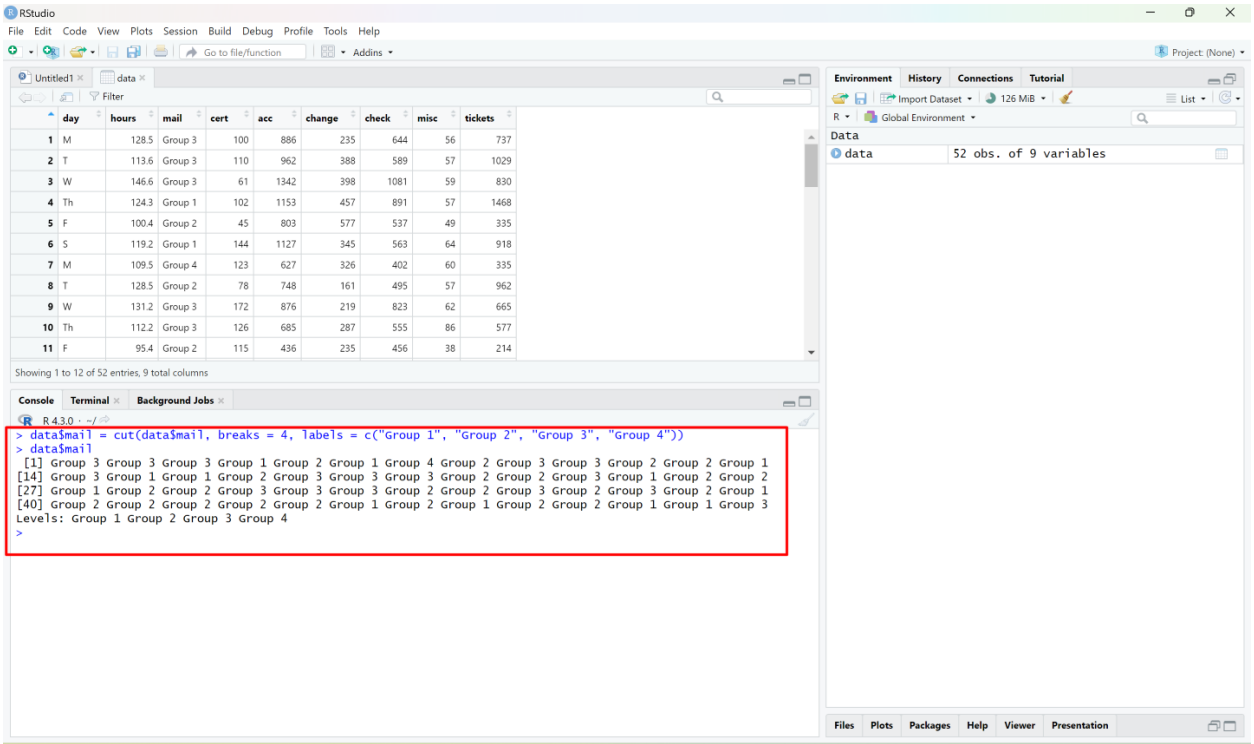
Created N-1 dummy variable for the variable "DAY":




Note: For creating dummy variable we need to use model.matrix() function , As dummies package was removed from the latest version of R.

**Convert the variable "mail" to nominal variable by creating 4 groups.:**

We need to use cut() function to cut the mail into 4 groups:



Note: as the group values came into scientific notation, I used labels to improve the readability.