Aula 9 – Decomposição LU

Considere o seguinte sistema de equações:

$$\begin{cases} 2x_1+6x_2+2x_3=1 \\ -3x_1-8x_2=1 \\ 4x_1+9x_2+2x_3=-1 \end{cases}$$

$$\begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$Ax=b$$
$$LU=b \qquad L \rightarrow inferior \quad e \quad U \rightarrow superior$$
$$Ux=y(2) \qquad Ly=b \quad (1)$$

Como obter L e U?

$$\begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} \begin{matrix} L_1 \\ L_2=l_2-(\frac{3}{2})\cdot L_1 \\ L_3=l_3-(2)\cdot L_1 \end{matrix}$$

$$\begin{bmatrix} 2 & 6 & 2 \\ 0 & 1 & 3 \\ 0 & -3 & -2 \end{bmatrix} \begin{matrix} L_1 \\ L_2 \\ L_3=L_3-(-3)\cdot L_2 \end{matrix}$$

$$\begin{bmatrix} 2 & 6 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix} = U(matriz\ superior)$$

$$L(matriz\ inferior) = \begin{bmatrix} 1 & 0 & 0 \\ -(\frac{3}{2}) & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix}$$

$$L\cdot y=b$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -(\frac{3}{2}) & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$y_1=1$$

$$-(\frac{3}{2})y_1+y_2=1 \rightarrow y_2=1+\frac{3}{2} \rightarrow y_2=\frac{5}{2}$$

$$2y_1-3y_2+y_3=-1 \rightarrow 2-\frac{15}{2}+y_3=-1 \rightarrow y_3=-3+\frac{15}{2} \rightarrow y_3=\frac{9}{2}$$

Agora, temos:

$$\begin{bmatrix} 2 & 6 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5/2 \\ 9/2 \end{bmatrix}$$

$$7x_3 = \frac{9}{2} \rightarrow x_3 = \left(\frac{9}{2}\right)/7 \rightarrow x_3 = \frac{9}{14}$$

$$x_2 + 3x_3 = \frac{5}{2} \rightarrow x_2 + \frac{27}{14} = \frac{5}{2} \rightarrow x_2 = \frac{5}{2} - \frac{27}{14} \rightarrow x_2 = \frac{(35-27)}{14} \rightarrow x_2 = \frac{8}{14} \rightarrow x_2 = \frac{4}{7}$$

$$2x_1 + 6x_2 + 2x_3 = 1 \rightarrow 2x_1 + \frac{24}{7} + \frac{9}{7} = 1 \rightarrow 2x_1 = 1 - \frac{33}{7} \rightarrow 2x_1 = \frac{(7-33)}{7} \rightarrow x_1 = -\left(\frac{26}{7}\right)/2 \rightarrow x_1 = -\left(\frac{13}{7}\right)$$