Aula 10 – Sistemas Lineares: Método da Inversa

Podemos escrever um sistema de equações na forma:

$$\begin{gathered} Ax=b \quad (Se\ Det(A)\neq 0) \\ A^{-1}\cdot Ax=A^{-1}\cdot b \\ Ix=A^{-1}\cdot b \\ x=A^{-1}\cdot b \end{gathered}$$

Portanto:

$$A=\begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}\cdot\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2a+6d+2a=1 \\ -3a-8d=0 \\ 4a+9d+2g=0 \end{bmatrix} \rightarrow \begin{bmatrix} a=? \\ d=? \\ g=? \end{bmatrix}$$

$$\begin{bmatrix} 2b+6e+2h=0 \\ -3b-8e=1 \\ 4b+9e+2h=0 \end{bmatrix} \rightarrow \begin{bmatrix} b=? \\ e=? \\ h=? \end{bmatrix}$$

$$\begin{bmatrix} 2c+6f+2i=0 \\ -3c-8f=0 \\ 4c+9f+2i=1 \end{bmatrix} \rightarrow \begin{bmatrix} c=? \\ f=? \\ g=? \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 2\vee 1 & 0 & 0 \\ -3 & -8 & 0\vee 0 & 1 & 0 \\ 4 & 9 & 2\vee 0 & 0 & 1 \end{bmatrix}$$

Exemplo: use o método da inversa para encontrar a solução do sistema:

$$\begin{bmatrix} 2x+6y+2z=1 \\ -3x-8y=1 \\ 4x+9y+2z=-1 \end{bmatrix}$$

Do sistema temos a seguinte matriz

$$\begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$$

Primeiramente, vamos calcular sua inversa:

$$\left[\begin{matrix} 2 & 6 & 2!1 & 0 & 0 \\ -3 & -8 & 0!0 & 1 & 0 \\ 4 & 9 & 2!0 & 0 & 1 \end{matrix}\right] \begin{matrix} L_1 = \frac{L_1}{2} \\ L_2 \\ L_3 \end{matrix}$$

$$\left[\begin{matrix} 1 & 3 & 1!\frac{1}{2} & 0 & 0 \\ -3 & -8 & 0!0 & 1 & 0 \\ 4 & 9 & 2!0 & 0 & 1 \end{matrix}\right] \begin{matrix} L_1 \\ L_2 = L_2 + 3L_1 \\ L_3 = L_3 - 4L_1 \end{matrix}$$

$$\left[\begin{matrix} 1 & 3 & 1!\frac{1}{2} & 0 & 0 \\ 0 & 1 & 3!\frac{3}{2} & 1 & 0 \\ 0 & -3 & -2!-2 & 0 & 1 \end{matrix}\right] \begin{matrix} L_1 = L_1 - 3L_2 \\ L_2 \\ L_3 = L_3 + 3L_2 \end{matrix}$$

$$\left[\begin{matrix} 1 & 0 & -8!-4 & -3 & 0 \\ 0 & 1 & 3!\frac{3}{2} & 1 & 0 \\ 0 & 0 & 7!\frac{5}{2} & 3 & 1 \end{matrix}\right] \begin{matrix} L_1 \\ L_2 \\ L_3 = \frac{L_3}{7} \end{matrix}$$

$$\left[\begin{matrix} 1 & 0 & -8!-4 & -3 & 0 \\ 0 & 1 & 3!\frac{3}{2} & 1 & 0 \\ 0 & 0 & 1!\frac{5}{14} & \frac{3}{7} & \frac{1}{7} \end{matrix}\right] \begin{matrix} L_1 = L_1 + 8L_3 \\ L_2 = L_2 - 3L_3 \\ L_3 \end{matrix}$$

$$\left[\begin{matrix} 1 & 0 & 0!-\frac{8}{7} & \frac{3}{7} & \frac{8}{7} \\ 0 & 1 & 0!\frac{3}{7} & \frac{-2}{7} & \frac{-3}{7} \\ 0 & 0 & 1!\frac{5}{14} & \frac{3}{7} & \frac{1}{7} \end{matrix}\right]$$

Portanto:

$$x = A^{-1} \cdot b$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-8}{7} & \frac{3}{7} & \frac{8}{7} \\ \frac{3}{7} & \frac{-2}{7} & \frac{-3}{7} \\ \frac{5}{14} & \frac{3}{4} & \frac{1}{7} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$x = \frac{-13}{7}$$

$$y = \frac{4}{7}$$

$$z = \frac{9}{14}$$