Aula 10 – Sistemas Lineares: Método da Inversa

Podemos escrever um sistema de equações na forma:

$$Ax=b \quad (Se\ Det(A)\neq 0)$$
$$A^{-1}\cdot Ax=A^{-1}\cdot b$$
$$Ix=A^{-1}\cdot b$$
$$x=A^{-1}\cdot b$$

Portanto:

$$A=\begin{bmatrix}2 & 6 & 2\\ -3 & -8 & 0\\ 4 & 9 & 2\end{bmatrix}\cdot\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}2a+6d+2a=1\\ -3a-8d=0\\ 4a+9d+2g=0\end{bmatrix}\rightarrow\begin{bmatrix}a=?\\ d=?\\ g=?\end{bmatrix}$$

$$\begin{bmatrix}2b+6e+2h=0\\ -3b-8e=1\\ 4b+9e+2h=0\end{bmatrix}\rightarrow\begin{bmatrix}b=?\\ e=?\\ h=?\end{bmatrix}$$

$$\begin{bmatrix}2c+6f+2i=0\\ -3c-8f=0\\ 4c+9f+2i=1\end{bmatrix}\rightarrow\begin{bmatrix}c=?\\ f=?\\ g=?\end{bmatrix}$$

$$\begin{bmatrix}2 & 6 & 2\vee 1 & 0 & 0\\ -3 & -8 & 0\vee 0 & 1 & 0\\ 4 & 9 & 2\vee 0 & 0 & 1\end{bmatrix}$$

Exemplo: use o método da inversa para encontrar a solução do sistema:

$$\begin{bmatrix}2x+6y+2z=1\\ -3x-8y=1\\ 4x+9y+2z=-1\end{bmatrix}$$

Do sistema temos a seguinte matriz

$$\begin{bmatrix}2 & 6 & 2\\ -3 & -8 & 0\\ 4 & 9 & 2\end{bmatrix}$$

Primeiramente, vamos calcular sua inversa:

$$\left[\begin{array}{ccc|cc} 2 & 6 & 2 \end{array}\right.\left.\begin{array}{cc} 1 & 0 & 0 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{array}\right.\left.\begin{array}{cc} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{array}\right] \begin{array}{c} L_1 = \frac{L_1}{2} \\ L_2 \\ L_3 \end{array}$$

$$\left[\begin{array}{ccc|cc} 1 & 3 & 1 \end{array}\right.\left.\begin{array}{cc} \frac{1}{2} & 0 & 0 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{array}\right.\left.\begin{array}{cc} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{array}\right] \begin{array}{c} L_1 \\ L_2 = L_2 + 3L_1 \\ L_3 = L_3 - 4L_1 \end{array}$$

$$\left[\begin{array}{ccc|cc} 1 & 3 & 1 \end{array}\right.\left.\begin{array}{cc} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 3 \\ 0 & -3 & -2 \end{array}\right.\left.\begin{array}{cc} 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ -2 & 0 & 1 \end{array}\right] \begin{array}{c} L_1 = L_1 - 3L_2 \\ L_2 \\ L_3 = L_3 + 3L_2 \end{array}$$

$$\left[\begin{array}{ccc|cc} 1 & 0 & -8 \end{array}\right.\left.\begin{array}{cc} -4 & -3 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{array}\right.\left.\begin{array}{cc} 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{5}{2} & 3 & 1 \end{array}\right] \begin{array}{c} L_1 \\ L_2 \\ L_3 = \frac{L_3}{7} \end{array}$$

$$\left[\begin{array}{ccc|cc} 1 & 0 & -8 \end{array}\right.\left.\begin{array}{cc} -4 & -3 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array}\right.\left.\begin{array}{cc} 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{5}{14} & \frac{3}{7} & \frac{1}{7} \end{array}\right] \begin{array}{c} L_1 = L_1 + 8L_3 \\ L_2 = L_2 - 3L_3 \\ L_3 \end{array}$$

$$\left[\begin{array}{ccc|cc} 1 & 0 & 0 \end{array}\right.\left.\begin{array}{cc} -\frac{8}{7} & \frac{3}{7} & \frac{8}{7} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right.\left.\begin{array}{cc} \frac{3}{7} & \frac{-2}{7} & \frac{-3}{7} \\ \frac{5}{14} & \frac{3}{7} & \frac{1}{7} \end{array}\right]$$

Portanto:

$$x = A^{-1} \cdot b$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-8}{7} & \frac{3}{7} & \frac{8}{7} \\ \frac{3}{7} & \frac{-2}{7} & \frac{-3}{7} \\ \frac{5}{14} & \frac{3}{7} & \frac{1}{7} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$x = \frac{(-8+3-8)}{7} = \frac{-13}{7}$$

$$y = \frac{(3-2+3)}{7} = \frac{4}{7}$$

$$z = \frac{(5+6-2)}{7} = \frac{9}{14}$$