# Fastlink

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## Introduction

## Methodology

Let's first describe the canonical probabilistic model of record linkage to demonstrate its properties. Materials are from the

original papaer: https://imai.fas.harvard.edu/research/files/linkage-app.pdf

and its online supplementary: https://imai.fas.harvard.edu/research/files/linkage-app.pdf

Let a latent mixing variable  $M_{ij}$  to indicate whether a pair of records (for the ith record in the data set A and the jth record in the data set B). Notate  $\gamma(i,j)$  to be the distance between a pair, just as Fellegi and Sunter did[1], it's changebable but we just use Jaro-Winkler string distance here, which is commonly used[2]; and  $\delta(i,j)$  to the missing indicator, to deal with missing data, then generally, we are estimating  $\xi_{ij} := \Pr(M_{ij} = 1 \mid \delta(i,j), \gamma(i,j))$ . We can notice that, actually, without the missing indicator, Fastlink would be quite similar to Fellegi-Sunter. Even the assumptions are the same:

$$\gamma_k(i,j) \mid M_{ij} = m \stackrel{\text{indep.}}{\sim} \text{Discrete}(\pi_{km});$$

$$M_{ij} \stackrel{\text{i.i.d.}}{\sim} \text{Bernoulli}(\lambda)$$

But this can be actually relaxed, we will discuss it later. So addressing missing data would surely be a significant property of Fastlink. Theoretically, it relies on a missing at random assumption (i.e.  $\delta_k(i,j) \perp \gamma_k(i,j) \mid M_{ij}$ ), but it still works well under some other conditions. With some Bayes calculation (still similar to what Fellegi-Sunter did), actually we can obtain the exact probability to be:

$$\xi_{ij} = \frac{\lambda \prod_{k=1}^{K} \left( \prod_{\ell=0}^{L_k-1} \pi_{k\ell\ell}^{1\{\gamma_k(i,j)=\ell\}} \right)^{1-\delta_k(i,j)}}{\sum_{m=0}^{1} \lambda^m (1-\lambda)^{1-m} \prod_{k=1}^{K} \left( \prod_{\ell=0}^{L_k-1} \pi_{km\ell}^{1\{\gamma_\ell(i,j)=\ell\}} \right)^{1-\delta_k(i,j)}}$$

For the package, this is done by getPosterior(). This introduces us two main advantages:

Firstly, probabilistic models can quantify the uncertainty inherent in many merge procedures, offering a principled way to calibrate and account for false positives and false negatives. Also for post-merge analysis, such probability works as a good weight for merged variable, i.e.  $X_i^* = \sum_{j=1}^{N_B} \xi_{ij} X_j / \sum_{j=1}^{N_B} \xi_{ij}$ 

Secondly, this proviade an easy way to compute. Intuitively, we can plug in the maximum likelihood estimation of  $\lambda$  and  $\pi$  here, which is

$$L_{com}(\lambda, \pi \mid \gamma, \delta) \propto \prod_{i=1}^{N_A} \prod_{j=1}^{N_B} \prod_{m=0}^{1} \left\{ \lambda^m (1-\lambda)^{1-m} \prod_{k=1}^{K} \left( \prod_{\ell=0}^{L_k-1} \pi_{km\ell}^{\mathbf{1}\{\gamma_k(i,j)=\ell\}} \right)^{1-\delta_k(i,j)} \right\}^{\mathbf{1}\{M_{ij}=m\}}$$

which is hard to compute, but iteratively, we can apply EM (Expectation-Maximization) method[3] with

$$\lambda = \frac{1}{N_A N_B} \sum_{i=1}^{N_A} \sum_{j=1}^{N_B} \xi_{ij}$$

$$\pi_{km\ell} = \frac{\sum_{i=1}^{N_A} \sum_{j=1}^{N_B} \mathbf{1} \left\{ \gamma_k(i,j) = l \right\} \left( 1 - \delta_k(i,j) \right) \xi_{ij}^m \left( 1 - \xi_{ij} \right)^{1-m}}{\sum_{i=1}^{N_A} \sum_{j=1}^{N_B} \left( 1 - \delta_k(i,j) \right) \xi_{ij}^m \left( 1 - \xi_{ij} \right)^{1-m}}$$

together with the calculated  $\xi_{ij}$  above. You can find the codes for EM calculations in "emlinkMARmov.R", by codes you can find this package actually allows a prior for the hyperparameters, which is not shown above.

Actually we can find another function called "emlinklog.R" in the package, this is actually accommodating a more general pattern of interaction. This algorithm allows for the inclusion of interaction terms, that is what I said the assumptions can be relaxed, but without a prior, you can choose it by state "cond.indep = False" in the main function fastlink() according to your requirements, but we will not go through the details of this algorithm here.

### Model Framework and Structures

### Setup

The method involves merging two datasets, A and B, each containing NA and NB records respectively. They use K linkage variables for comparisons. The model defines an agreement vector g(i,j) for each record pair (i,j), where  $g_k(i,j)$  defines the similarity of the k-th variable between records i from A and j from B.

#### **Model Formulation**

- Linkage Variables: Uses Bernoulli random variables  $M_{ij}$  that identify whether a record pair (i, j) matches  $(M_{ij} = 1)$  or not  $(M_{ij} = 0)$ . In other words, The model uses simple yes/no variables, represented mathematically as Bernoulli random variables  $M_{ij}$ . These variables help decide whether a pair of records (i, j) from two different datasets is a match  $(M_{ij} = 1)$  or not  $(M_{ij} = 0)$ . Think of it as a sophisticated way of saying "these two records are talking about the same thing/person."
- Conditional Distributions: Assumes conditional independence among linkage variables given the match status  $M_{ij}$ . In other words, each variable's match status (like name, address) does not depend on each other after knowing whether the overall records match. This could enable the decomposition of the joint probability distribution into simpler, individual probabilities.
- Handling Missing Data: Utilizes a Missing At Random (MAR) framework to allow the omission
  of missing data in the probability calculations, which simplifies the likelihood function and enhances
  computational efficiency.

## Algorithm and Computation

## EM Algorithm

The parameter estimation is executed using the Expectation-Maximization (EM) algorithm. It starts with an initial guess, then repeatedly adjusts this guess aiming to improve the likelihood that the observed data came from the proposed model. This optimizes the observed-data likelihood function, which integrates over the probabilistic distributions of the linkage variables conditioned on the match hypotheses.

### Blocking and Filtering

To reduce computational demands: - **Blocking**: To avoid comparing every record in one dataset with every record in another, which can be overwhelmingly time-consuming with large datasets, the model groups records into blocks based on shared characteristics (like all people with the same birth year), which greatly cuts down on unnecessary comparisons. - **Filtering**: Eliminates highly unlikely pairs from consideration early in the process, using thresholds based on calculated probabilities.

### Scalability

The algorithm is designed to work efficiently even with very large datasets that contain millions of records. It uses parallel processing (splitting the work across multiple computer processors) and smart data structures to manage this, making it practical to run on a typical laptop without needing supercomputer resources.

## **Evaluation and Implementation**

#### Simulation Studies

The model's robustness is tested through simulations that mimic real-world problems like incomplete data or errors in the data (measurement errors). These simulations help verify that the model can handle different types of common data issues effectively. The model is compared to traditional methods (like exact match), showing that it can handle complex, imperfect data more effectively and efficiently.

### Package Realization

Repository: https://github.com/kosukeimai/fastLink

Example: https://imai.fas.harvard.edu/research/files/turnout.pdf

### Implementation

Dataset: https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/2NNA4L

## Statistical Analysis Post-Merging

### **Uncertainty Quantification**

The model quantifies the uncertainty in the merging process, allowing researchers to account for potential errors in subsequent analyses, which is critical for maintaining the integrity of research conclusions.

### Post-Merge Analysis

Discusses methodologies for incorporating the probabilities of matches into regression analyses and other statistical procedures to adjust for the uncertainty inherent in the linkage process.

### Contributions and Innovations

The model makes substantial contributions to the field of data management by providing: - A robust probabilistic framework that substantially outperforms traditional deterministic methods. Unlike older methods that just said 'yes' or 'no' to whether records match, this model calculates how likely it is that records match. This approach gives us a clearer picture and usually results in better performance. - Enhanced handling of missing data and the independence assumptions of linkage variables, which have been a significant limitation in earlier models. - Detailed documentation and an accessible implementation in R, which facilitates reproducible research and widespread adoption in the social sciences.

```
suppressMessages(require("fastLink"))
suppressMessages(require("plyr"))
data <- read.delim("cces2016voterval.tab")
summary(data)</pre>
```

```
##
         V101
                                        agreement_pattern
                                                             prob_match
                         merge_type
##
   Min.
          :222168628
                              :1.000
                                       Length: 64600
                                                           Min.
                                                                  :0.0000211
                       Min.
##
   1st Qu.:302801850
                        1st Qu.:1.000
                                        Class :character
                                                           1st Qu.:0.0173844
  Median :303320104
                       Median :1.000
                                       Mode :character
                                                           Median :1.0000000
##
## Mean
          :303452665
                       Mean :1.014
                                                           Mean
                                                                  :0.6659646
   3rd Qu.:303923982
                        3rd Qu.:1.000
                                                           3rd Qu.:1.0000000
```

```
##
            :307210331
                          Max.
                                 :2.000
                                                                       :1.0000000
    Max.
                                                                Max.
##
                                            vote2014
                                                               vote2012
    clerical_review
                          vote2016
##
    Min.
            :0.0000
                      Min.
                              :0.0000
                                         Min.
                                                 :0.0000
                                                           Min.
                                                                   :0.0000
    1st Qu.:0.0000
                      1st Qu.:0.0000
                                         1st Qu.:0.0000
                                                           1st Qu.:0.0000
##
##
    Median :1.0000
                      Median :1.0000
                                         Median :0.0000
                                                           Median :1.0000
    Mean
            :0.5859
                              :0.7162
                                         Mean
                                                 :0.4858
                                                                   :0.5961
##
                      Mean
                                                           Mean
    3rd Qu.:1.0000
                      3rd Qu.:1.0000
                                         3rd Qu.:1.0000
                                                           3rd Qu.:1.0000
##
##
    Max.
            :1.0000
                      Max.
                              :1.0000
                                         Max.
                                                 :1.0000
                                                           Max.
                                                                   :1.0000
##
    vote2016_prob
                      vote2014_prob
                                         vote2012_prob
                                                           vote2016 clerical
            :0.0000
                                                                   :0.0000
##
    Min.
                      Min.
                              :0.0000
                                         Min.
                                                 :0.0000
                                                           Min.
##
    1st Qu.:0.0000
                      1st Qu.:0.0000
                                         1st Qu.:0.0000
                                                           1st Qu.:0.0000
    Median :1.0000
                      Median :0.0000
                                         Median :0.1620
                                                           Median :1.0000
##
                                                 :0.4879
##
    Mean
            :0.5824
                              :0.4167
                                         Mean
                                                           Mean
                                                                   :0.5282
                      Mean
    3rd Qu.:1.0000
                      3rd Qu.:1.0000
                                                           3rd Qu.:1.0000
##
                                         3rd Qu.:1.0000
##
                              :1.0000
                                                 :1.0000
                                                                   :1.0000
    Max.
            :1.0000
                      Max.
                                         Max.
                                                           Max.
##
    vote2014_clerical vote2012_clerical
            :0.0000
##
    Min.
                       Min.
                               :0.0000
##
    1st Qu.:0.0000
                        1st Qu.:0.0000
    Median :0.0000
                       Median :0.0000
##
##
    Mean
            :0.3872
                       Mean
                               :0.4441
##
    3rd Qu.:1.0000
                       3rd Qu.:1.0000
##
    Max.
            :1.0000
                               :1.0000
                       Max.
```

### References

- [1] I. P. Fellegi and A. B. Sunter, "A theory for record linkage," *Journal of the American Statistical Association*, vol. 64, no. 328, pp. 1183–1210, 1969.
- [2] M. A. Jaro, "Advances in record-linkage methodology as applied to matching the 1985 census of tampa, florida," *Journal of the American Statistical Association*, vol. 84, no. 406, pp. 414–420, 1989.
- [3] W. E. Winkler, Using the EM algorithm for weight computation in the fellegi-sunter model of record linkage. US Bureau of the Census Washington, DC, 2000.