We start from the following constitutive law.

Proposition 1 (The constitutive law of three-dimensional isotropic meterial)**.**

$$\sigma^{ij} = \lambda \epsilon^{kk} \delta^{ij} + 2G\epsilon^{ij}, \tag{1}$$

where λ and G is defined as

$$\begin{aligned} \lambda &:= \frac{\nu E}{(1+\nu)(1-2\nu)}, \\ G &:= \frac{E}{2(1+\nu)}. \end{aligned} \tag{2}$$

See Theory of Elasticity, Timoshenko, for reference.
The governing equation.

Proposition 2 (The governing equation of material in statics)**.**

$$\frac{\partial \sigma^{ij}}{\partial x^j} + \rho f^i = 0. \tag{3}$$

And the kinematic equation.

Definition 3 (The kinematic description of material with infinitesimal deformation)**.**

$$\epsilon^{ij} := \frac{1}{2}\left(\frac{\partial u^i}{\partial x^j} + \frac{\partial u^j}{\partial x^i}\right). \tag{4}$$

A substitution gives

$$(\lambda + G)\frac{\partial}{\partial x^i}\left(\frac{\partial u^j}{\partial x^j}\right) + G\frac{\partial u^i}{\partial x^j \partial x^j} + \rho f^i = 0, \tag{5}$$

which is the three-dimensional Lamé-Naiver equation, in statics.

Now consider a plane stress problem, where $\sigma^{i3} = 0$, and we have

$$\rho f^3 = 0. \tag{6}$$

Meanwhile, we have

$$\sigma^{33} = \lambda \frac{\partial u^j}{\partial x^j} + 2G\frac{\partial u^3}{\partial x^3} = 0,$$

which gives

$$\frac{\partial u^3}{\partial x^3} = -\frac{\nu}{1-2\nu}\frac{\partial u^j}{\partial x^j} \qquad \text{and} \qquad \frac{\partial u^\beta}{\partial x^\beta} = \frac{1-\nu}{1-2\nu}\frac{\partial u^j}{\partial x^j}.$$

Also, we have

$$\sigma^{\alpha 3} = G\left(\frac{\partial u^\alpha}{\partial x^3} + \frac{\partial u^3}{\partial x^\alpha}\right) = 0,$$

which gives

$$\frac{\partial u^\alpha}{\partial x^3 \partial x^3} = -\frac{\partial}{\partial x^\alpha}\frac{\partial u^3}{\partial x^3}.$$

Hence, for a plane stress problem, we have

$$\left(\frac{1-2\nu}{1-\nu}(\lambda + G) - \frac{\nu}{1-\nu}G\right)\frac{\partial}{\partial x^\alpha}\frac{\partial u^\beta}{\partial x^\beta} + G\frac{\partial^2 x^\alpha}{\partial x^\beta x^\beta} + \rho f^\alpha = 0,$$

or

$$\frac{E}{2(1-\nu)}\frac{\partial}{\partial x^\alpha}\frac{\partial u^\beta}{\partial x^\beta} + G\frac{\partial^2 x^\alpha}{\partial x^\beta x^\beta} + \rho f^\alpha = 0. \tag{7}$$

By using the transform from a plane stress problem to a plane strain one, we have

$$\frac{E}{2(1+\nu)(1-2\nu)}\frac{\partial}{\partial x^\alpha}\frac{\partial u^\beta}{\partial x^\beta} + G\frac{\partial^2 x^\alpha}{\partial x^\beta x^\beta} + \rho f^\alpha = 0. \tag{8}$$