Consider the following constitutive equation

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix} = \frac{1}{E} \begin{pmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix}, \tag{2.4a, b, and c}$$

if we assume a plane stress problem, then σ_3 vanishes, and the equation becomes

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = \frac{1}{E} \begin{pmatrix} 1 & -\nu \\ -\nu & 1 \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}.$$

However, if we assume a plane strain problem, by the last equation

$$0 = \epsilon_3 = \frac{1}{E}(-\nu\sigma_1 - \nu\sigma_2 + \sigma_3),$$

we have $\sigma_3 = \nu(\sigma_1 + \sigma_2)$, and substitute it into the first two equations, which gives

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = \frac{1}{E} \begin{pmatrix} (1-\nu^2) & -(\nu+\nu^2) \\ -(\nu+\nu^2) & (1-\nu^2) \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \frac{1-\nu^2}{E} \begin{pmatrix} 1 & -\frac{\nu}{1-\nu} \\ -\frac{\nu}{1-\nu} & 1 \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix},$$

and we find a surprising conclusion: if we substitute E with $\dfrac{E}{1-\nu^2}$, and ν with $\dfrac{\nu}{1-\nu}$, we can transfer any equation that is valid for a plane stress problem, to a equation that is valid for a plane strain problem.

For example, the governing equation for a plane stress problem is

$$\nabla^4\phi = -(1-\nu)\nabla^2 V, \tag{3.29}$$

where ϕ is the Airy stress function, and V is the force potential. Then using the rule mentioned above, we have

$$\nabla^4\phi = -(1 - \frac{\nu}{1-\nu})\nabla^2 V = -\frac{1-2\nu}{1-\nu}\nabla^2 V, \tag{3.25}$$

which is exactly the governing equation for a plane strain problem.