Suppose we're solving such a problem, where the beam is uniform (that is, H_{33}^{c} remains unchanged along the whole beam), and it is subjected to both distributed but uniform transverse load and concentrated loads, in the x_2 direction. Then we have the following propositions:

1. $\bar{u}_2$ and ϕ_3 are continuous along the whole beam, by its geometry.
2. M_3 is continuous along the whole beam, by the moment equilibrium equation.
3. For every place, x_0 for example, $-V_2(x_0^-) + V_2(x_0^+) + F(x_0) = 0$, by the force equilibrium equation, where $F(x_0)$ is the concentrated force applied on that place. And we can rewrite the equation as
$$V_2\big|_{x_0^-}^{x_0^+} + F(x_0) = 0. \tag{1}$$
4. $\dfrac{\mathrm{d}V_2}{\mathrm{d}x_1} + p_2 = 0$ holds along the whole beam, or at least for places where no concentrated forces are applied.

Of course there are some other boundary conditions, but we'll talk about that later.

Then, with those propositions, and assumptions we have made, we can construct a function $\hat{p}(x_1)$, which is
$$\hat{p}(x_1) = \frac{p_2}{H_{33}^{\mathrm{c}}} + \sum_i \frac{F_i}{H_{33}^{\mathrm{c}}}\delta(x_1 - x_i). \tag{2}$$
Note that this function is valid in dimension, since the dimension of the Dirac δ function is /m[1].

We integrate it by once and get
$$\frac{p_2}{H_{33}^{\mathrm{c}}}x_1 + \sum_i \frac{F_i}{H_{33}^{\mathrm{c}}}u(x_1 - x_i) + C_1.$$
We multiply it by $-H_{33}^{\mathrm{c}}$, and call it $\hat{V}_2$, which is
$$\hat{V}_2(x_1) = -p_2 x_1 - \sum_i F_i u(x_1 - x_i) - H_{33}^{\mathrm{c}} C_1, \tag{3}$$
and we surprisingly find that $\hat{V}_2$ satisfies equation 1. So we integrate equation 3 and add a minus sign to obtain so-called $\hat{M}_3$, which is
$$\hat{M}_3(x_1) = \frac{p_2}{2}x_1^2 + \sum_i F_i(x_1 - x_i)u(x_1 - x_i) + H_{33}^{\mathrm{c}} C_1 x_1 + C_2, \tag{4}$$
which is indeed continuous. In a similar fashion, we divide equation 4 by M_{33}^{c} and integrate it by once and twice respectively, we can obtain
$$\hat{\phi}_3(x_1) = \frac{p_2}{6H_{33}^{\mathrm{c}}}x_1^3 + \sum_i \frac{F_i}{2H_{33}^{\mathrm{c}}}(x_1 - x_i)^2 u(x_1 - x_i) + \frac{C_1}{2}x_1^2 + C_2 x_1 + C_3, \tag{5}$$
$$\hat{\bar{u}}_2(x_1) = \frac{p_2}{24H_{33}^{\mathrm{c}}}x_1^4 + \sum_i \frac{F_i}{6H_{33}^{\mathrm{c}}}(x_1 - x_i)^3 u(x_1 - x_i) + \frac{C_1}{6}x_1^3 + \frac{C_2}{2}x_1^2 + C_3 x_1 + C_4, \tag{6}$$
where there are still four constants to be determined. So we'll use the boundary conditions. For example, for a simply supported beam, the displacement $\bar{u}_2$ and bending moment M_3 are 0, and we can use equation 6 and 4 respectively to obtain those constants. What's more, we explicitly give equation 2 instead of some function that satisfies condition 4, because equation 6 can be obtained by simply integrate equation 2 by four times.

[1] Usually we use that function in signals, where the variable is usually t, and the dimension of $\delta(t - t_0)$ is s^{-1}.