

Since the plain stress matrix is symmetric, there's a set of eigenvectors, \bar{v}_1 and \bar{v}_2 , which form a orthonormal basis. So for all vector \bar{n} , $\bar{n}^T \underline{\underline{\sigma}} \bar{n}$ can be rewritten as

$$\sigma_n = \bar{n}^T \underline{\underline{\sigma}} \bar{n} = (k_1 \bar{n} + k_2 \bar{n})^T \underline{\underline{\sigma}} (k_1 \bar{n} + k_2 \bar{n}) = k_1^2 \lambda_1 + k_2^2 \lambda_2 \in [\lambda_2, \lambda_1],$$

where λ_1 and λ_2 are eigenvalues of \bar{v}_1 and \bar{v}_2 respectively, and $\lambda_1 \geq \lambda_2$. And σ_n reaches its maximum or minimum, if and only if \bar{n} is in the direction of \bar{v}_1 or \bar{v}_2 , respectively, which is equivalent to the vanish of shear stresses.