A derivation to the vanished couple

As we have assumed, the force applied to the surface is

$$\underline{F}_n = \iint_D \mathrm{d}\underline{F}_n = \iint_D \underline{\tau}_n \,\mathrm{d}A_n.$$

Similarly, the couple applied to it, with respect to some point P is

$$\underline{M}_{n,P} = \iint_D \underline{r}_{n,P} \times \mathrm{d}\underline{F}_n, \tag{1}$$

and if the 'stress couple' applied to $\mathrm{d}A_n$ is denoted as $\underline{\omega}_n$, the couple can be also written as

$$\underline{M}_{n,P} = \iint_D \underline{\omega}_n \,\mathrm{d}A_n + \iint_D \underline{r}_{n,P} \times \mathrm{d}\underline{F}_n. \tag{2}$$

Conparing with Eq. (1) and (2), and they give

$$\iint_D \underline{\omega}_n \,\mathrm{d}A_n = \underline{0}.$$

Since the closed region D can be chosen arbitarily, and $\underline{\omega}_n$ is supposed to be continuous over D, $\underline{\omega}_n$ has to be $\underline{0}$ everywhere.

Or, we can use some bruteforce to prove that:

$$\begin{aligned}
\underline{\omega}_n &= \lim_{A_n\to 0} \frac{\underline{M}_n}{A_n} = \lim_{A_n\to 0} \frac{\iint_D \underline{r}_n \times \mathrm{d}\underline{F}_n}{A_n} = \lim_{A_n\to 0} \frac{\iint_D \underline{r}_n \times \underline{\tau}_n \,\mathrm{d}A_n}{A_n},\\
\text{so}\quad \|\underline{\omega}_n\| &\le \lim_{A_n\to 0} \frac{1}{A_n} \iint_D \|\underline{r}_n\| \|\underline{\tau}_n\| \,\mathrm{d}A_n\\
&\le \lim_{A_n\to 0} \|\underline{r}_n\|_{\max} \|\underline{\tau}_n\|_{\max} = \underline{0}, \quad \text{which is equivalent to} \quad \underline{\omega}_n = \underline{0},
\end{aligned}$$

as long as we restrict the shape of D so that $\|\underline{r}_n\|_{\max}$ converges to 0 as A_n does.