HW2 Pattern Recognition

2019150445 신백록

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0. Data Preprocessing & Visualization

In [1]:
```python
import numpy as np
import tensorflow as tf
from tensorflow.keras.datasets import mnist
import matplotlib.pyplot as plt

(x_train,y_train),(x_test,y_test)=mnist.load_data()
print(x_train.shape)
print(x_test.shape)
print(y_train.shape)
print(y_test.shape)
```

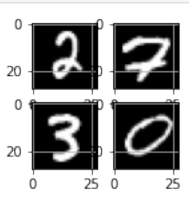
```
Init Plugin
Init Graph Optimizer
Init Kernel
(60000, 28, 28)
(10000, 28, 28)
(60000,)
(10000,)
```

In [2]:
```python
unique,counts=np.unique(y_train,return_counts=True)
print('Train labels:',dict(zip(unique,counts)))
unique,counts=np.unique(y_test,return_counts=True)
print('Test labels:',dict(zip(unique,counts)))
```

```
Train labels: {0: 5923, 1: 6742, 2: 5958, 3: 6131, 4: 5842, 5: 5421, 6: 5918, 7: 6265, 8: 5851, 9: 5949}
Test labels: {0: 980, 1: 1135, 2: 1032, 3: 1010, 4: 982, 5: 892, 6: 958, 7: 1028, 8: 974, 9: 1009}
```

In [3]:
```python
indices=np.random.randint(0,x_train.shape[0],size=4)
images=x_train[indices]
labels=y_train[indices]

plt.figure(figsize=(2,2))
for i in range(len(indices)):
    plt.subplot(2,2,i+1)
    image=images[i]
    plt.imshow(image, cmap='gray')
    plt.grid('off')
plt.show()
```



Randomly select 4 digits & plot

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1. PCA with MNIST data

In [4]:
```python
x_train=x_train.reshape(-1,28*28)
x_test=x_test.reshape(-1,28*28)
print(x_train.shape)
print(x_test.shape)
```

```
(60000, 784)
(10000, 784)
```

Reshape (28,28)->728

In [5]:
```python
from sklearn.preprocessing import StandardScaler
sc=StandardScaler()
sc.fit(x_train)
X_train_std=sc.transform(x_train)

print(np.mean(X_train_std))#Almost 0
print(np.var(X_train_std))#close to 1
```

```
-2.1974863349995617e-18
0.9145408163265558
```

Center the data at zero

In [6]:
```python
X_train_cov=np.matmul(X_train_std.T,X_train_std)
```

Compute the covariance matrix

In [7]:
```python
print(X_train_cov.shape)
```

```
(784, 784)
```

In [8]:
```python
eig=np.linalg.eig
eigen_value=eig(X_train_cov)[0]
eigen_vector=eig(X_train_cov)[1]

idx = eigen_value.argsort()[::-1]
eigen_value = eigen_value[idx]
eigen_vector = eigen_vector[:,idx]
```

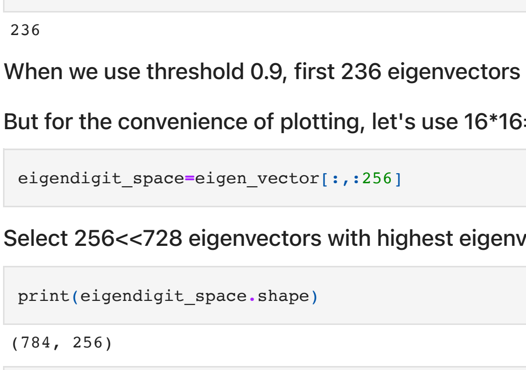
Find eigenvector & eigenvalue by covariance matrix & sort by descending way.

In [9]:
```python
print(eigen_value.shape)
print(eigen_vector.shape)
```

```
(784,)
(784, 784)
```

In [10]:
```python
tot=sum(eigen_value)
var_exp=[(i/tot) for i in eigen_value[:299]]
cum_var_exp=np.cumsum(var_exp)

import matplotlib.pyplot as plt
plt.bar(range(1,300),var_exp,alpha=0.5,align='center',label='individual explained variance')
plt.step(range(1,300),cum_var_exp,where='mid',label='cumulative explained variance')
plt.ylabel('Explained variance ratio')
plt.xlabel('Principal component index')
plt.legend(loc='best')
plt.show()
```



In [11]:
```python
for i in range(len(eigen_value)):
    if sum(eigen_value[:i])/sum(eigen_value)>0.9:
        print(i)
        break
```

```
236
```

When we use threshold 0.9, first 236 eigenvectors are used to explain 90% of total variance.

But for the convenience of plotting, let's use 16*16=256 eigenvectors

In [12]:
```python
eigendigit_space=eigen_vector[:,:256]
```

Select 256<<728 eigenvectors with highest eigenvalues

In [13]:
```python
print(eigendigit_space.shape)
```

```
(784, 256)
```

In [14]:
```python
projection=np.matmul(X_train_std,eigendigit_space)
```

Project data points to those eigenvectors

In [15]:
```python
print(projection.shape)
```

```
(60000, 256)
```

In [16]:
```python
print(projection[:10,:5])
```

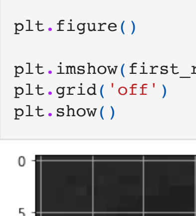
```
[[  0.92215881  -4.81479035  -0.06755984  -8.0513293   -0.9853473 ]
 [ -8.70897698  -7.75440302   3.44791044  -1.66832141   0.83479532]
 [ -2.32838932   9.43133817   6.18411405   1.72506609   4.09245604]
 [  6.58217331  -3.74631834  -3.69085127  -0.46104068   5.62732462]
 [  5.1832512    3.13329712   6.27794746   1.45965399  -1.60884235]
 [ -2.19840211  -3.06836644   0.2312572    2.75260056  -1.21688731]
 [  6.77334369   0.82307934  -2.93236971   0.98851194   1.46036522]
 [ -1.92690853  -5.40626434   0.16303186 -11.1530086   -5.0037197 ]
 [  7.82233526   0.19181     -1.10015127   1.55955849   4.22122044]
 [  3.22762585  -4.72642548   4.28898033   3.63120125  -0.71205431]]
```

In [17]:
```python
eigendigit=eigendigit_space.T.reshape(256,28,28) #For plotting eigendigits
print(eigendigit.shape)

indices=[0,1,254,255] #Top 2 eigendigits & Last 2 eigendigits
images=eigendigit[indices]
labels=y_train[indices]

plt.figure(figsize=(2,2))
for i in range(len(indices)):
    plt.subplot(2,2,i+1)
    image=images[i]
    plt.imshow(image, cmap='gray')
    plt.grid('off')
plt.show()
```

```
(256, 28, 28)
```



For the top 2 eigendigits, it seems that some digits are overlapped and that overlapping makes the feature.

But for the last 2 eigendigits, we cannot figure out any features through our eyes.

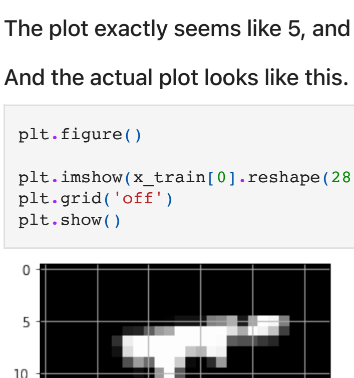
Next, let's reconstruct the first data for example.

In [18]:
```python
first_recon=np.mean(x_train)+np.matmul(projection,eigendigit_space.T)[0]
```

In [19]:
```python
first_recon=first_recon.reshape(28,28) #for plotting

plt.figure()

plt.imshow(first_recon, cmap='gray')
plt.grid('off')
plt.show()
```



In [20]:
```python
y_train[0]
```

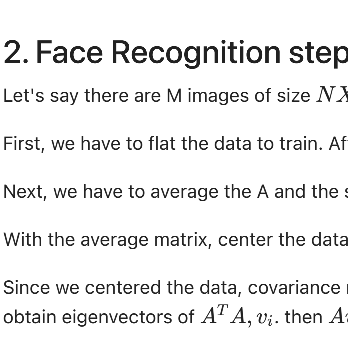
Out[20]:
```
5
```

The plot exactly seems like 5, and it's 5 actually.

And the actual plot looks like this. It's simillar each other.

In [21]:
```python
plt.figure()

plt.imshow(x_train[0].reshape(28,28), cmap='gray')
plt.grid('off')
plt.show()
```



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2. Face Recognition steps

Let's say there are M images of size NXN. Then the shape of the train set is (N, N, M). Let's call this train set as A.

First, we have to flat the data to train. After doing this, the size of the A would be (N^2, M).

Next, we have to average the A and the shape of average matrix will be $(N^2, 1)$. Let's say this as A_avg

With the average matrix, center the data by subtracting the A_avg from A. Let's say this as A_std

Since we centered the data, covariance matrix can be computed simply by AA^T and the size of this matrix would be (N^2, N^2). But it's too big to compute. So compute A^TA whose size is (M,M), and obtain eigenvectors of A^TA, v_i. then Av_i will be eigen-face space. Let's say that as U whose size is (N^2, M).

Select the number of eigenvectors with some criterion. Assume that we select $k << M$ eigenvectors. Then the size of U would be (N^2, k).

Then project the data in eigen-face space by simply computing U^TA_std and that will be the eigen-face coordinates. Let's say this projection as W whose size is (k,M).

We can reconstruct the train data by $A\_avg + \Sigma(W_k^{(train)}U_k)$.

With the test image, normalize the test image by subtracting the A_avg from train set. Next, project on eigen-space U to obtain the weights for the test image.

Then select the argmin i of the euclidean distance between the weights from the train data i & weights from the specific test image. And that test image is recognized as i'th sample from the training set.

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3. Psedo-code

1. Load the Face train data A that have M images & number of N^2 pixels.
2. If face data has shape of (N, N, M) then
   flatten the data to (N^2, M)
3. Compute the average of face data along the row axis, A_avg.
4. Compute $A - A\_avg$.
5. If there is many data (i.e. $N^2 << M$) then
   Just compute Covariance matrix by AA^T.
   Compute eigenvalue & eigenvector by covariance matrix.
   Select $k << M$ eigenvector by the criterion and call it U.
   else
   Compute A^TA.
   Compute eigenvector v_i of A^TA.
   Compute Av_i.
   Select $k << M$ eigenvector by the criterion and call it U.
6. Compute $U^T(A - A\_avg)$ which is the projection of image.
7. We can reconstruct the data by computing $A\_avg + U^T(A - A\_avg)U$.
8. Given an unknown face image, Compute $test - A\_avg$.
9. Compute weights for the test image in eigen-space by $U^T(test - A\_avg)$
10. Compute $dist(U^T(train_i - A\_avg), U^T(test - A\_avg))$ for all ith samples in train set.
11. Test image is recognized as face i which has the minimum distance.

---

4. Distinguish MNIST test set

This starts at step 8 of psedo-code because I already done step 1 to 7 already in Chapter 1.

In [22]:
```python
sc=StandardScaler()
sc.fit(x_train)
X_test_std=sc.transform(x_test)
```

Center the data by subtracting mean of the train set.

In [23]:
```python
weight_test=np.matmul(X_test_std,eigendigit_space)
print(weight_test.shape)
```

```
(10000, 256)
```

In [24]:
```python
weight_train=projection
print(weight_train.shape)
```

```
(60000, 256)
```

In [25]:
```python
index=[]
for i in range(len(weight_test)):
    dist=[]
    append=dist.append
    for j in range(len(weight_train)):
        append(np.linalg.norm(weight_train[j]-weight_test[i]))
    index.append(np.argmin(dist))
```

Compute all euclidean distances betweein weight_train & weight_test and select the minimum distance.

In [26]:
```python
predicted_y=y_train[index]
```

In [27]:
```python
sum(y_test==predicted_y)/len(predicted_y)
```

Out[27]:
```
0.9513
```

95.13% accuracy with the test set.