

## Assignment #2 — Due: Tuesday, October 20, 2020, by 6:00 p.m.

Use R to answer the questions. Your solution should include relevant commands and outputs but not the data set.

1. [20 points] Given are seven observations for two variables  $x$  and  $y$ :

|   |   |   |   |   |    |    |    |
|---|---|---|---|---|----|----|----|
| x | 1 | 2 | 3 | 3 | 4  | 5  | 5  |
| y | 3 | 7 | 5 | 8 | 11 | 14 | 12 |

Suppose that we fitted the linear model:  $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$  ( $i = 1, \dots, 7$ ),  $\epsilon_i \sim N(0, \sigma^2)$ .

- (a) Write the fitted regression equation.
  - (b) Find a p-value for testing  $H_0 : \beta_1 \leq 2.0$  vs  $H_a : \beta_1 > 2.0$ .
  - (c) Find the 95% confidence interval for  $\beta_0$ , and use the confidence interval to test  $H_0 : \beta_0 = 1$ .
  - (d) Calculate the p-value of  $F$  statistics for the hypothesis  $H_0 : \beta_1 = 0$ . (First, find the value of  $F$  statistic and then calculate the p-value by your own coding.)
  - (e) Find the 90% confidence interval for  $\hat{\beta}_0 + 4\hat{\beta}_1$ .
2. [24 points] We are interested in developing a model that describes the gas mileage (in mpg). We will use engine size (in cubic cm), horsepower and weight of the car (in pounds) as explanatory variables. Download `HW2(data).RData` from the *blackboard*.
- (a) Write the estimated regression equation using `mpg` as a response variable and the others as explanatory variables.
  - (b) Interpret the estimated coefficient of `weight`.
  - (c) Conduct the  $F$ -test for the overall fit of the regression. Comment on the results.
  - (d) Test each of the regression coefficients. Do the results indicate that any of the explanatory variables should be removed from the model?
  - (e) Find the regression model with the explanatory variable(s) identified in part (d) removed.
  - (f) Conduct an  $F$  test to choose a model between the full model in (a) and reduced model in (e).
3. [26 points, 2 points for each question] The following tables are regression output when  $Y$  is regressed on  $X_1$  for 20 observations. Complete the 13 missing numbers. Provide some reasons of your answers. Correct answers without showing the calculation process will not get full credit.

Table 1: (i) ANOVA Table

| Source     | Sum of Squares | df  | Mean Square | F-statistic |
|------------|----------------|-----|-------------|-------------|
| Regression | 1848.76        | (a) | (b)         | (c)         |
| Residuals  | (d)            | (e) | (f)         |             |

Table 2: (ii) Coefficients Table

| Variable  | Coef        | SE            | $t$                  | $p$ -value |
|-----------|-------------|---------------|----------------------|------------|
| Intercept | -23.4325    | 12.74         | (g)                  | 0.0824     |
| $X_1$     | (h)         | 0.1528        | 8.32                 | < 0.0001   |
| $n =$ (i) | $R^2 =$ (j) | $R_a^2 =$ (k) | $\hat{\sigma} =$ (l) | df = (m)   |