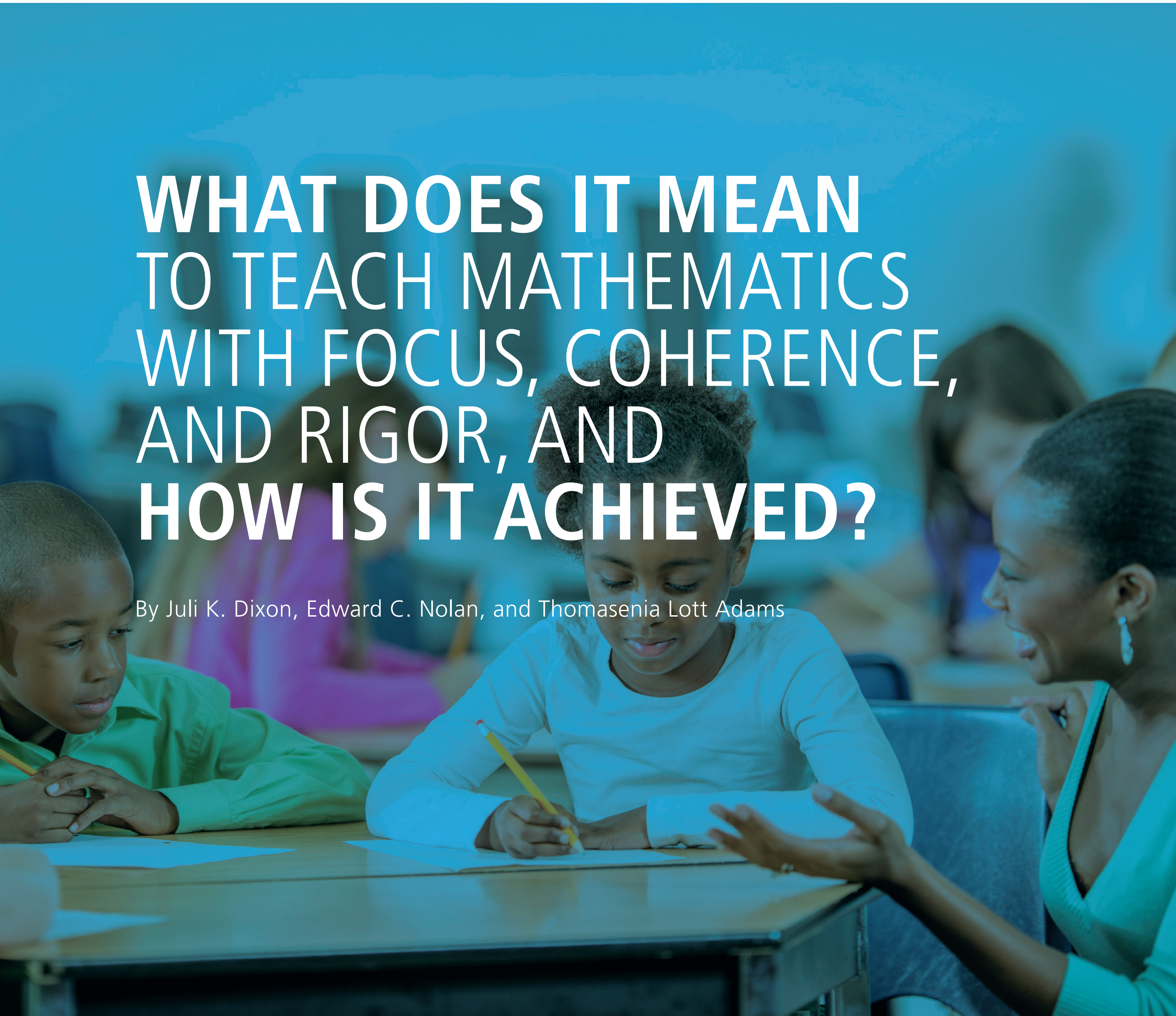
DIG DEEP
INTO CONTENT
DIXON · NOLAN · ADAMS
MATHEMATICS



WHAT DOES IT MEAN TO TEACH MATHEMATICS WITH FOCUS, COHERENCE, AND RIGOR, AND HOW IS IT ACHIEVED?

By Juli K. Dixon, Edward C. Nolan, and Thomasenia Lott Adams

WITH SO MUCH RHETORIC AROUND FOCUS, COHERENCE, AND RIGOR...

...we are in danger of losing sight of the importance of teaching mathematics with each of these goals in mind. Without a clear understanding of the intent of standards built on the foundation that focus, coherence, and rigor provide, achieving the intent will be out of reach. What follows are explicit examples of what it means to teach with focus, coherence, and rigor, along with a structure for addressing and a means for achieving these goals.

Focus, coherence, and rigor are best described by making sense of coherence first, then rigor, and finally focus. Coherence refers to making connection both within and across grades and courses, but what does this really mean in mathematics instruction? Consider the division problem 4 ÷ ⅕. You can likely determine the quotient using an algorithm. Have you already started chanting the familiar "keep, change, flip" or "invert and multiply" mantras reminding you to change the problem to 4 x 5⁄1? However, do you know what 4 ÷ ⅕ means? Could you represent it in context?

According to the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and numerous other rigorous standards, students in grade 5 must be able to create a story context for problems like this one. They should also build upon content addressed with division that started in grade 3. Making this connection supports teaching with coherence. In grade 3, students create story contexts for division with whole numbers. They might create a context for a problem like 12 ÷ 3.

Complete the story problem that begins, "I've got 12 cookies…" before reading further.

You likely completed the problem so that it would read something like this: "I've got 12 cookies. If I share them among 3 friends, how many cookies will each friend get?" This context correctly supports 12 ÷ 3, but how does it help to prepare students to create a context for 4 ÷ ⅕?

Using the same context and structure as the one created above for 12 ÷ 3, you would have something like this: "I've got four cookies. If I share these cookies among ⅕ of a friend…" This story problem obviously doesn't make sense. You cannot share something with ⅕ of a friend. A problem based on sharing division is appropriate when dividing by a whole number like three, but it is not always appropriate when dividing by a fraction. When dividing by a fraction, it is important to connect to a different meaning for division, also addressed in grade 3. A measurement division context for 12 ÷ 3 could be, "I've got 12 cookies. If I give 3 cookies to each friend, to how many friends can I give cookies?" The answer is still four, but the division is represented differently. Rather than sharing 12 cookies among 3 groups, you would make groups of 3 cookies each, using up all 12 cookies. For $4 \div \frac{1}{5}$ the problem would read: "I've got 4 cookies. If I give $\frac{1}{5}$ of a cookie to each friend, to how many friends can I give pieces of cookie?"

The effects of focusing on context to make sense of fraction division are immediate. Now consider the problem $2\frac{1}{2} \div \frac{1}{4}$. You can probably think of a story context for this division problem quite quickly. You are likely thinking of a context that would make sense for determining the number of $\frac{1}{4}$ of something there are in $2\frac{1}{2}$ of that thing. Students in grade 5 will be successful creating contexts like this if their instruction is coherent and they are helped to connect work with dividing fractions to earlier experiences dividing whole numbers using measurement division. Teachers will be more likely to make these connections explicit if they teach with the intent of supporting coherence.

How does this connect to rigor? "Rigor refers to the need to incorporate all forms of thinking about mathematics—including concepts, procedures, the language of mathematics, and applications—in the teaching and learning process" (Dixon, Nolan, Adams, Tobias, & Barmoha, 2016, p. 8). What does this mean for dividing fractions? Once you acknowledge that both concepts and procedures are necessary to achieve rigor, you should conclude that concepts must be taught first. Teaching procedures before concepts undermines the intent of teaching for rigor. As stated by the National Council of Teachers of Mathematics (2014), "conceptual understanding (i.e., the comprehension and selection of concepts, operations, and relations) establishes the foundation, and is necessary, for developing procedural fluency (i.e., the meaning and flexible use of procedures to solve problems)" (p. 7).

Reconsider $4 \div \frac{1}{5}$. How did knowing the algorithm ("keep, change, flip" or "invert and multiply") help you to create a context for $4 \div \frac{1}{5}$? It was likely no help whatsoever. However, once you were able to create a context for determining how many one-fifths are in four wholes, you can see how the algorithm would follow. It goes something like this:

How many one-fifths are in one whole?

Five.

How many wholes are there?

Four.

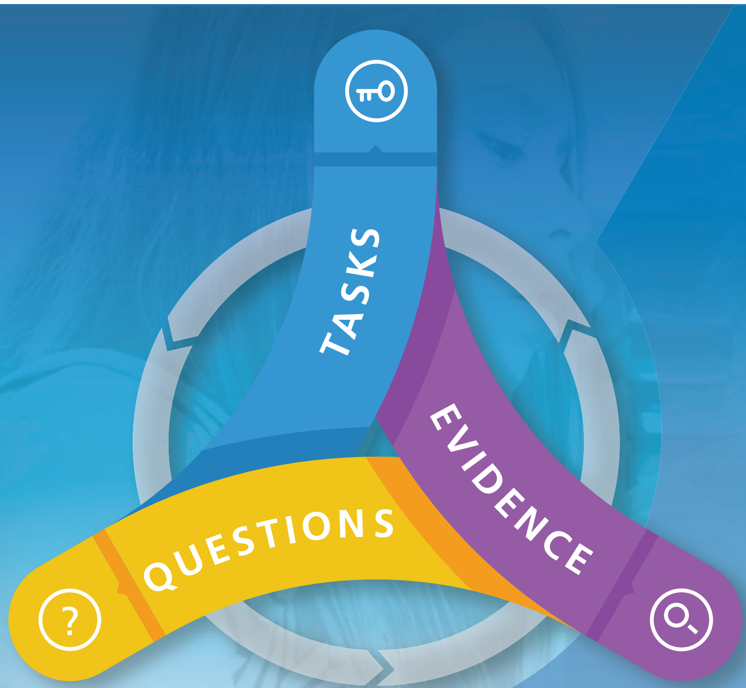
So how many one-fifths are in four wholes?

4 x 5.

This is the same is $4 \times \frac{5}{1}$. With teaching for rigor, both concepts and procedures are important and connected to one another. The key to success is for the concept to come first and for the concept (and procedure) to be presented with coherence. In this case, both the context and the algorithm for fraction division were connected to division with whole numbers.

TEACHING WITH COHERENCE AND RIGOR TAKES MORE TIME.

Where is that time found? The idea is that when students are taught with understanding, there will be less need to reteach concepts from year to year. Instead, content is revisited as connections are made to new content—first with concepts and then with procedures. This is accomplished through a focused curriculum. When fewer topics are addressed in a given grade or course, those topics can be taught coherently and with rigor. However, this will only occur if teachers have sufficient content knowledge for teaching so that they are able to select appropriate tasks to address the standards, support those tasks with productive and effective questioning, and collect meaningful evidence of students' conceptual understandings and misunderstandings to guide their instruction. We call this focus on tasks, questions, and evidence the TQE Process.



The **TQE Process**

TASKS

Select tasks that support identified learning goals.

QUESTIONS

Facilitate productive questioning during instruction to engage students in the mathematical practices and processes.

EVIDENCE

Collect and use evidence of student understanding in the formative assessment process to guide the delivery of instruction.

The **TQE Process**

Our newly published book series supporting making sense of mathematics for teaching is built upon the importance of the TQE Process in teaching a curriculum that is coherent, rigorous, and focused. An excerpt from this series further describing the TQE Process and how it is supported within the series, as well as the professional development we provide, is included here:

- "Teachers with a deep understanding of the content they teach select tasks that provide students with the opportunity to engage in practices that support learning concepts before procedures; they know that for deep learning to take place, students need to understand the procedures they use. Students who engage in mathematical tasks are also engaged in learning mathematics with understanding. Consider grade 3 students who are making sense of basic multiplication facts. The order of the presented facts is important to address a specific learning goal. For example, if the goal is for students to make sense of a doubling strategy as a means to determine a fact other than by rote, students might be presented with facts like 2 × 6 and 4 × 6 then 2 × 7 and 4 × 7. Students would then identify a pattern in the pairs of facts as a way to see the value in applying the doubling strategy. Once students identify the pattern, they can be led to see why doubling a factor results in doubling the product. These students are making sense of patterns in computation as a way of determining basic facts. Thus, this scenario provides insight into a classroom where carefully selected tasks support deeper learning.
- Teachers who have a deep understanding of the content they teach facilitate targeted and productive questioning strategies because they have a clear sense of how the content progresses within and across grades. For instance, in the grade 3 example, teachers would facilitate discussions around strategies for multiplying basic facts that focus on the application of properties of operations such as the distributive property of multiplication over addition when solving 7 × 8. Teacher questioning encourages students to use relational thinking strategies by thinking $7 \times 8 = (5 + 2) \times 8 = (5 \times 8) + (2 \times 8)$. They know that this is important for later work with multiplying multidigit numbers with understanding, foreshadowing concepts in algebra.
- Teachers who have a deep understanding of the content they teach use evidence gained from the formative assessment process to help them know where to linger in developing students' coherent understanding of mathematics. In the grade 3 example, teachers know that it is important for students to apply the associative and distributive properties as strategies and that these strategies are more valuable than a strategy based on skip counting. They look for evidence that students are using strategies based on properties of operations appropriately to make sense of basic facts.

The **TQE Process**

Throughout the book and the accompanying classroom videos, we share elements of the TQE process to help you as both a learner and a teacher of mathematics.

In addition, we ask that you try to answer three targeted questions as you watch each video. These questions are as follows:

1. How does the teacher prompt the students to make sense of the problem?

2. How do the students engage in the task; what tools or strategies are the students using to model the task?

3. How does the teacher use questioning to engage students in thinking about their thought processes?"

Dixon, Nolan, Adams, Tobias, & Barmoha, 2016, pp. 5–6.

The TQE Process, spanning grades K–12, has the potential to transform teaching and increase student achievement—especially if tasks are selected and questions and evidence are discussed within collaborative teacher teams focused on the practices of effective mathematics instruction. When teachers understand the mathematics they teach, they are more likely to select mathematically important tasks and to support them during instruction, using questions that garner important mathematical thinking and learning from students. These same teachers use evidence provided in student responses and discussions within the formative assessment process to support students to reach their full mathematical potential. This is the heart of the TQE Process.

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