Lambda Semantics

Giuseppe Lomurno

1 TYPING RULES

$$\frac{}{\Gamma \vdash \mathbb{Z} : \textbf{LInt}}\ \textsc{IntE} \qquad \frac{}{\Gamma \vdash \mathbb{B} : \textbf{LBool}}\ \textsc{BoolE} \qquad \frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash \langle e_1, e_2 \rangle : \textbf{LPair}\ \tau_1\ \tau_2}\ \textsc{Pair}$$

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau}\ \textsc{Var} \qquad \frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x : \tau_1 . e : \tau_1 \to \tau_2}\ \textsc{Lambda}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \to \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1\ e_2 : \tau_2}\ \textsc{App} \qquad \frac{\Gamma \vdash e : \tau \to \tau}{\Gamma \vdash \textbf{fix}\ e : \tau}\ \textsc{Fix}$$

$$\frac{\textbf{binop} : \tau_1 \to \tau_1 \to \tau_2 \quad \Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1\ \textbf{binop}\ e_2 : \tau_2}\ \textsc{PrimBinOp}$$

$$\frac{\textbf{op} : \tau_1 \to \tau_2 \quad \Gamma \vdash e : \tau_1}{\Gamma \vdash \textbf{op}\ e : \tau_2}\ \textsc{PrimOp}$$

$$\frac{\Gamma \vdash c : \textbf{LBool} \quad \Gamma \vdash e_1 : \tau \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash \textbf{if}\ c\ \textbf{then}\ e_1\ \textbf{else}\ e_2 : \tau}\ \textsc{Cond}$$

binop	τ_1	τ_2
PrimAdd	LInt	LInt
PrimSub	LInt	LInt
PrimMul	LInt	LInt
PrimDiv	LInt	LInt
PrimIntEq	LInt	LBool
PrimBoolEq	LBool	LBool
PrimAnd	LBool	LBool
PrimOr	LBool	LBool

op	τ_1	τ_2
PrimNeg	LInt	LInt
PrimNot	LBool	LBool
PrimFst	LPair f s	f
PrimSnd	LPair f s	s

2 BIG-STEP SEMANTICS

$$\frac{}{v \Downarrow v}\ \textsc{IntE/BoolE} \qquad \frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{\langle e_1, e_2\rangle \Downarrow \langle v_1, v_2\rangle}\ \textsc{Pair}$$

$$\frac{}{e \Downarrow \lambda x : \tau.\, e'}\ \textsc{Lambda}$$

$$\frac{e_1 \Downarrow \lambda x : \tau.\, e \quad e_2 \Downarrow v_2 \quad e[v_2/x] \Downarrow v}{e_1\ e_2 \Downarrow v}\ \textsc{App} \qquad \frac{e \Downarrow \lambda x : \tau.\, e' \quad e'[\textbf{fix}\ e/x] \Downarrow v}{\textbf{fix}\ e \Downarrow v}\ \textsc{Fix}$$

$$\frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2 \quad v_1\ \textbf{binop}\ v_2 \Downarrow v}{e_1\ \textbf{binop}\ e_2 \Downarrow v}\ \textsc{PrimBinOp}$$

$$\frac{e \Downarrow v \quad \textbf{op}\ v \Downarrow v'}{\textbf{op}\ e \Downarrow v'}\ \textsc{PrimOp}$$

$$\frac{c \Downarrow \textbf{True} \quad e_1 \Downarrow v}{\textbf{if}\ c\ \textbf{then}\ e_1\ \textbf{else}\ e_2 \Downarrow v}\ \textsc{CondTrue} \qquad \frac{c \Downarrow \textbf{False} \quad e_2 \Downarrow v}{\textbf{if}\ c\ \textbf{then}\ e_1\ \textbf{else}\ e_2 \Downarrow v}\ \textsc{CondFalse}$$

binop		
PrimAdd	$+$	$\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$
PrimSub	$-$	$\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$
PrimMul	$*$	$\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$
PrimDiv	$/$	$\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$
PrimIntEq	$=_{\mathbb{Z}}$	$\mathbb{Z} \times \mathbb{Z} \to \mathbb{B}$
PrimBoolEq	$=_{\mathbb{B}}$	$\mathbb{B} \times \mathbb{B} \to \mathbb{B}$
PrimAnd	$\wedge$	$\mathbb{B} \times \mathbb{B} \to \mathbb{B}$
PrimOr	$\vee$	$\mathbb{B} \times \mathbb{B} \to \mathbb{B}$

op		
PrimNeg	$-$	$\mathbb{Z} \to \mathbb{Z}$
PrimNot	$\neg$	$\mathbb{B} \to \mathbb{B}$
PrimFst	fst	$\alpha \times \beta \to \alpha$
PrimSnd	snd	$\alpha \times \beta \to \beta$

3 SMALL-STEP SEMANTICS

$$\frac{}{v \longrightarrow v}\ \textsc{IntE/BoolE} \qquad \frac{}{e \longrightarrow \lambda x : \tau.\, e'}\ \textsc{Lambda}$$

$$\frac{e_1 \longrightarrow e_1'}{\langle e_1, e_2\rangle \longrightarrow \langle e_1', e_2\rangle}\ \textsc{PairA1} \qquad \frac{e_2 \longrightarrow e_2'}{\langle e_1, e_2\rangle \longrightarrow \langle e_1, e_2'\rangle}\ \textsc{PairA2}$$

$$\frac{e_1 \longrightarrow e_1'}{e_1\ e_2 \longrightarrow e_1'\ e_2}\ \textsc{AppF} \qquad \frac{e_2 \longrightarrow e_2'}{(\lambda x : \tau.\, e_1)\ e_2 \longrightarrow (\lambda x : \tau.\, e_1)\ e_2'}\ \textsc{AppV}$$

$$\frac{e[v/x] \longrightarrow v'}{(\lambda x : \tau.\, e)\ v \longrightarrow v'}\ \textsc{App} \qquad \frac{e \longrightarrow e'}{\textbf{fix}\ e \longrightarrow \textbf{fix}\ e'}\ \textsc{FixF} \qquad \frac{e[\textbf{fix}\ (\lambda x : \tau.\, e)/x] \to e'}{\textbf{fix}\ (\lambda x : \tau.\, e) \to e'}\ \textsc{FixV}$$

$$\frac{c \longrightarrow c'}{\textbf{if}\ c\ \textbf{then}\ e_1\ \textbf{else}\ e_2 \longrightarrow \textbf{if}\ c'\ \textbf{then}\ e_1\ \textbf{else}\ e_2}\ \textsc{CondC}$$

$$\frac{e_1 \longrightarrow e_1'}{\textbf{if}\ v\ \textbf{then}\ e_1\ \textbf{else}\ e_2 \longrightarrow \textbf{if}\ v\ \textbf{then}\ e_1'\ \textbf{else}\ e_2}\ \textsc{CondA1}$$

$$\frac{e_2 \longrightarrow e_2'}{\textbf{if}\ v\ \textbf{then}\ e_1\ \textbf{else}\ e_2 \longrightarrow \textbf{if}\ v\ \textbf{then}\ e_1\ \textbf{else}\ e_2'}\ \textsc{CondA2}$$

$$\frac{v \longrightarrow \text{True}}{\textbf{if}\ v\ \textbf{then}\ e_1\ \textbf{else}\ e_2 \longrightarrow e_1}\ \textsc{CondTrue} \qquad \frac{v \longrightarrow \text{False}}{\textbf{if}\ v\ \textbf{then}\ e_1\ \textbf{else}\ e_2 \longrightarrow e_2}\ \textsc{CondFalse}$$

$$\frac{e_1 \longrightarrow e_1'}{e_1\ \textbf{binop}\ e_2 \longrightarrow e_1'\ \textbf{binop}\ e_2}\ \textsc{PrimBinOpA1}$$

$$\frac{e_2 \longrightarrow e_2'}{e_1\ \textbf{binop}\ e_2 \longrightarrow e_1\ \textbf{binop}\ e_2'}\ \textsc{PrimBinOpA1}$$

$$\frac{}{v_1\ \textbf{binop}\ v_2 \longrightarrow v}\ \textsc{PrimBinOp} \qquad \frac{e \longrightarrow e'}{\textbf{op}\ e \longrightarrow \textbf{op}\ e'}\ \textsc{PrimOpA} \qquad \frac{}{\textbf{op}\ v \longrightarrow v'}\ \textsc{PrimOp}$$