1. Prove formally that function is .

Ans: We know that if . Thus is for and .

2. Prove formally that function is . Yes, this seems to contradict exercise 1. Give an explanation for this.

Ans: We know that if . Thus is for and .

This conclusion is reasonable. As shown in the formal definition of Big-O, the terms including higher order of n is growing faster than those with lower order of n. Thus, the higher Big-O expression will cover the lower Big-O expression. However, we usually choose the lowest possible order Big-O expression to precisely express the asymptotic complexity of programs.

3. Prove formally that function is .

Ans: We know that if . Thus is for and .