

# Hw1

## 題目

Ackermann's function  $A(m,n)$  is defined as follows:  $A(m,n) = \{[n+1, \text{ if } m = 0], [A(m-1, 1), \text{ if } n = 0], [A(m-1, A(m,n-1)), \text{ otherwise}]\}$  This function is studied because it grows very fast for small values of  $m$  and  $n$ . Write a recursive function for computing this function. Then write a nonrecursive algorithm for computing Ackermann's function.

## 說明

使用遞迴完成功能，依照題目要求撰寫，以下為該題虛擬碼。

```
A(int m, int n){  
    if(m == 0){return n+1;  
    }else if(n == 0){return A(m,n);  
    }else{return A(m-1,A(m,n-1));}  
}
```

## 效能分析

遞迴的深度與次數受  $m$  和  $n$  影響，尤其是  $m$ 。

當  $m = 0$ ，時間複雜度為  $O(1)$ 。

當  $m = 1$ ，時間複雜度為  $O(n)$ 。

當  $m = 2$ ，時間複雜度為  $O(n^2)$ 。

當  $m$  的值越大，所需的效能會越龐大。

## 執行結果

```
input (m n):2 3
Ackermann(2,3) = 9

-----
Process exited after 2.001 seconds with return value 0
請按任意鍵繼續 . . . |
```

## 範例計算(已排版)

$m = 2, n = 3$ 。

$$A(2,3) = A(1,A(2,2)) = A(1,7) = 9$$

$$A(2,2) = A(1,A(2,1)) = A(1,5) = 7$$

$$A(2,1) = A(1,A(2,0)) = A(1,3) = 5$$

$$A(1,7) = A(0,A(1,6)) = A(0,8) = 9$$

$$A(1,6) = A(0,A(1,5)) = A(0,7) = 8$$

$$A(1,5) = A(0,A(1,4)) = A(0,6) = 7$$

$$A(1,4) = A(0,A(1,3)) = A(0,5) = 6$$

$$A(1,3) = A(0,A(1,2)) = A(0,4) = 5$$

$$A(1,2) = A(0,A(1,1)) = A(0,3) = 4$$

$$A(1,1) = A(0,A(1,0)) = A(0,2) = 3$$

## 心得

Ackermann's function 是一個會快速增長且難以計算的函數，不易優化。

即便使用 C 或 C++ 撰寫，也容易出現效能瓶頸。