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A Robust State Estimator With Adaptive Factor

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ABSTRACT This article proposes a robust state estimator with adaptively adjusted observation noise covariance for uncertain linear systems. Since the residuals between observation data and estimation data satisfying the normal distribution, the Mahalanobis distance are known to be Chi-square distributed. We use Chi-square tests to timely distinguish outliers that beyond the confidence interval and adjust the estimated observation noise covariance to a more likely value. Combined with the robust estimation method, an improved algorithm is derived to deal with uncertainties in both system parameters and observation noise covariance. Based on the proposed prediction form, we test the obtained robust state estimator with different deterioration conditions of observation noise covariance and compare it with the estimators without adaptive factor. The simulation results show that the derived state estimator may reduce the accumulation of estimation errors, smooth the estimated state curve, and the performance of the proposed estimator can be significantly improved as the deterioration enlarging.

INDEX TERMS Adaptive estimation, chi-square tests, robust state estimation.

I. INTRODUCTION

Applications like Free Space Optical Communication (FSO) need Acquisition, Tracking and Pointing (ATP) system to establish reliable channel between aircrafts and satellite earth stations. In consequence of the hardware and software's delay of ATP, to realize a precise optical alignment between laser maser and optical receiver, a target state estimator is introduced. Besides ensuring arithmetic speed, i.e. less algorithm complexity, the estimation algorithm for such scenarios needs to make up uncertainties in target motion model and handle unexpected observation noise surging due to platform's vibration, atmospheric attenuation and turbulence etc.

The state estimator based on Kalman filter theory which could minimize the regularized residual norm is widely applied [1], but the performance of the filter would deteriorate appreciably once the state-space model is inaccurate [2]. By contrast, the robust filter which aims at minimizing the worst-possible regularized residual norm has better estimation performance over admissible model uncertainties (one application in wireless sensor networks [3]), while its advantages are mainly showed when the model uncertainty is large. Several extensively used robust estimation methods such as well-known H_∞ approach [4], [5], set valued estimation discussion approach [6], [7], guaranteed cost paradigm [8], [9] are well studied. And many improvement tasks have been applied on those traditional robust filters, such as taking advantage of the Kalman filter and regularized least-squares to handle model errors in robust state estimator [10]; using sensitivity penalizing on estimation errors to differentiate it from model uncertainties and well compensates nonlinearly uncertainties in state space model [11]; combining the robust state estimator and the fuzzy rule to handle uncertainties in nonlinear model [12]; using *Pareto efficient* estimator to combine robust filter and Kalman filter by interpolating method, which aims to optimize the convex combination of the nominal regularized residue and the worst-case regularized residue [13]; one inspirational robust state-space estimator who has superiority in concision, i.e., similar algorithm form and parallel computational complexity as Kalman estimator, while it has difficulty in nonlinear estimation problem compared with adaptive fuzzy estimator or sliding mode estimator [14].

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Those improvement tasks effectively help to compensate target motion model uncertainties but they could not solve observation noise surging problem in proposed scenarios.

To realize fault-tolerant compensation control, one algorithm to against nonlinearity and actuators faults has been studied [15], in which additive and multiplicative faults are considered simultaneously and the realistic system is modeled by Markov jump systems. In [16], one sliding mode observer is introduced to continuous-time switched system. This algorithm could estimate the actuator faults directly and avoid the traditional sliding surface switching problem. And in [17], a novel linear descriptor reduced-order observer is created, which could simultaneously consider both the faults in actuator, sensor and output disturbances. These industrial practical algorithms could effectively compensate faults in observer but they are too complex and may have a weak pertinence to solve observation noise surging problem, thus we searched better method to deal with unknown observation noise. One adaptive Kalman filter based on *Maximum a Posteriori* estimation was proposed while it may not be efficient for sudden noise surging and uncertain motion model [18], [19]. One Kalman filter with Bayesian method for noise identification was proposed [20], [21]. It could be applied in nonlinear filtering problems by methods like extended Kalman filter but has not be extended in robust state estimator. The method about using statistical method to detect outliers was proposed in [22]. And in [23], [24], Kalman filter with adaptive factor who timely recognize the outliers that beyond the Chi-square tests confidence interval and adjust the observation noise covariance was applied. Though the improved Kalman filter could not solve uncertainties in target motion model, it could effectively handle unexpected observation noise surging and consume minor computation complexity.

In this article, we introduce adaptive factor into robust state estimator to modify the estimated observation noise covariance based on observed and estimated data. The contributions of our robust state estimator are threefold. (1) One Chi-square test based adaptive factor is designed to recognize outliers and adjust the observation noise covariance. (2) The additional computation time of the estimator with adaptive factor is enlarged no more than 1%, while the improved algorithm performs less accumulated error and smoother estimation curve in different deterioration situation than the one without adaptive factor. (3) The validity of the combination of robust estimator and adaptive factor is proved in both numerical and practical examples.

The rest of this article is sketched as follows. In Section II, we propose the state-space model together with the fundamentals of robust state estimator. In Section III, we introduce the adaptive factor theory and the recursion formula of prediction form estimator. Several simulation results are demonstrated in Section IV. And we draw the conclusion in Section V.

Notation: A^+ means the pseudo-inverse of the matrix A. $E(\cdot)$ represents the mathematical expectation of the matrix. For a column vector u and a positive-definite matrix Q, $\|u_i\|$ and $\|u_i\|_Q$ denote the Euclidean norm and its weighted version $\sqrt{u^T u}$ and $\sqrt{u^T Qu}$ respectively. $P(\cdot)$ is the probability of an event, α is the significance level.

II. PROBLEM STATEMENTS AND ROBUST STATE ESTIMATOR

A. PROBLEM STATEMENTS

Consider the following system:

$$\begin{aligned} x_{i+1} &= (F_i + \delta F_i)x_i + (G_i + \delta G_i)u_i, \quad i \geq 0, \\ y_i &= H_i x_i + v_i. \end{aligned} \tag{1}$$

In (1), x_0, u_i, and v_i are uncorrelated zero-mean random variables with a covariance matrix

$$E\left(\begin{bmatrix} x_0 \\ u_i \\ v_i \end{bmatrix}\begin{bmatrix} x_0 \\ u_i \\ v_i \end{bmatrix}^T\right) = \begin{bmatrix} \Pi_0 & & \\ & Q_i\delta_{ij} & \\ & & R_i\delta_{ij} \end{bmatrix} \tag{2}$$

In (2), Π_0, Q_i, and R_i are estimated positive definite matrices of the system. δ_{ij} is the Kronecher delta function which equals to 1 when $i = j$ and to 0 when $i \neq j$. The perturbations are modeled as

$$\begin{bmatrix} \delta F_i & \delta G_i \end{bmatrix} = M_i \Delta_i [\, E_{f,i} \;\; E_{g,i} \,]. \tag{3}$$

Here, Δ_i is an arbitrary contraction, that is $\|\Delta_i\| \leq 1$. M_i, $E_{f,i}$, $E_{g,i}$ are known matrices that could vary with time. In our simulations, three presupposed parameters are used, while in application, they are obtained through mechanism analyzing modeling or experimental modeling.

B. ROBUST STATE ESTIMATOR

A robust state estimator is proposed to compensate the effect of motion model uncertainties $[\delta F_i, \delta G_i]$. In each calculation recursion, we use the priori estimated $\hat{x}_{i|i-1}$, the estimated positive definite weighting matrix $P_{i|i-1}$ and the observed data y_{i+1} to update the estimate of x_i to $\hat{x}_{i|i}$ by solving the formula

$$\min_{\{x_i, u_i\}} \max_{\delta F_i, \delta G_i} \|x_i - \widehat{x}_{i|i}\|^2_{P^{-1}_{i|i}} + \|u_i\|^2_{Q^{-1}_i} + \|y_{i+1} - H_{i+1}x_{i+1}\|^2_{R^{-1}_{i+1}}. \tag{4}$$

Theorem 1 ([25]): Given the regularized least-squares problems with the form as follows,

$$\min_x [x^T Qx + (Ax - b)^T W(Ax - b)]. \tag{5}$$

Here, $x^T Qx$ is the regularized form, $Q = Q^T > 0$ and $W = W^T \geq 0$ are weighting matrix. The dimension of known matrix A is $N \times n$, b is $N \times 1$. Vector x is n-dimensional. Then, the optimal solution is

$$\widehat{x} = [Q + A^T WA]^{-1} A^T Wb. \tag{6}$$

◇

Due to model uncertainties and disturbances, model parameters $\{A, b\}$ ought to be written as $\{A + \delta A, b + \delta b\}$.

Thus, we introduced one robust form [14] to restrict the uncertainties in a certain range. Cost function $J(x, y)$ is defined as

$$J(x, y) = x^T Qx + R(x, y),$$
$$R(x, y) = (Ax - b + Hy)^T W(Ax - b + Hy).$$

H is one known $N \times m$ matrix which helps to restrict the $m \times 1$ unknown perturbation matrix y to certain range spaces. Assume that the Euclidean norm of y satisfies the inequality $\|y\| \leq \phi(x)$. $\phi(x)$ is one known (linear or nonlinear) nonnegative function that may depend on x.

The optimization objective equation expands to

$$\min_{x} \max_{\delta A, \delta b} [\|x\|_Q^2 + \|(A + \delta A)x - (b + \delta b)\|_W^2]. \tag{7}$$

Here, δA is $N \times n$ perturbation matrix appends to the nominal matrix A. And δb is $N \times 1$ perturbation vector appends to the nominal matrix b.

Assume that $\{\delta A, \delta b\}$ satisfy the form

$$[\,\delta A \;\; \delta b\,] = H\Delta[\,E_a \;\; E_b\,]. \tag{8}$$

In (8), Δ is unknown contraction that satisfies the range $\|\Delta\| \leq 1$, $\{H, E_a, E_b\}$ are known parameters of the system with applicable dimensions.

The optimization objective equation has one unique solution

$$\widehat{x} = [\widehat{Q} + A^T \widehat{W} A]^{-1}[A^T \widehat{W} b + \widehat{\lambda} E_a^T E_b]. \tag{9}$$

Here, the weighting matrix $\{\widehat{Q}, \widehat{W}\}$ are modified from $\{Q, W\}$ by

$$\widehat{Q} \triangleq Q + \widehat{\lambda} E_a^T E_a, \tag{10}$$
$$\widehat{W} \triangleq W + WH(\widehat{\lambda} I - H^T WH)^{+} H^T W. \tag{11}$$

And the function $G(\lambda)$ to determine non-negative scalar parameter $\widehat{\lambda}$ is defined as

$$\widehat{\lambda} = \arg \min_{\lambda \geq \|H^T WH\|} G(\lambda), \tag{12}$$
$$G(\lambda) = \|x(\lambda)\|_Q^2 + \lambda \|E_a x(\lambda) - E_b\|^2 + \|Ax(\lambda) - b\|_{W(\lambda)}^2, \tag{13}$$
$$W(\lambda) \triangleq W + WH(\lambda I - H^T WH)^{+} H^T W, \tag{14}$$
$$Q(\lambda) \triangleq Q + \lambda E_a^T E_a. \tag{15}$$

The solution of x ought to be written as

$$x(\lambda) \triangleq [Q(\lambda) + A^T W(\lambda) A]^{-1}[A^T W(\lambda) b + \lambda E_a^T E_b]\cdot \tag{16}$$

When we apply Theorem 1 into our system (1), the optimization formula (4) would be seen as a version of (7). Thus, we substitute parameters in our robust system by those parameters used in Theorem 1 as follows.

$$x \leftarrow col\{x_i - \widehat{x}_{i|i}\} \tag{17}$$
$$b \leftarrow y_{i+1} - H_{i+1} F_i \widehat{x}_{i|i} \tag{18}$$
$$A \leftarrow H_{i+1} \left[\, F_i \;\; G_i \,\right] \tag{19}$$
$$\delta A \leftarrow H_{i+1} M_i \Delta_i \left[\, E_{f,i} \;\; E_{g,i} \,\right] \tag{20}$$
$$\delta b \leftarrow -H_{i+1} M_i \Delta_i E_{f,i} \widehat{x}_{i|i} \tag{21}$$
$$Q \leftarrow (P_{i|i}^{-1} \oplus Q_i^{-1}) \tag{22}$$
$$W \leftarrow R_{i+1}^{-1} \tag{23}$$
$$H \leftarrow H_{i+1} M_i \tag{24}$$
$$E_a \leftarrow \left[\, E_{f,i} \;\; E_{g,i} \,\right] \tag{25}$$
$$E_b \leftarrow -E_{f,i} \widehat{x}_{i|i} \tag{26}$$
$$\Delta \leftarrow \Delta_i \tag{27}$$

We use relational expressions (12)-(16) in Theorem 1 to calculate $\widehat{\lambda}_i$. And modified parameters $\{P_{i|i}, \widehat{Q}_i, \widehat{R}_{i+1}, \widehat{F}_i, \widehat{G}_i\}$ are calculated as

$$P_{i|i} = (P_i^{-1} + H_i^T \widehat{R}_i^{-1} H_i^{-1})^{-1}, \tag{28}$$
$$P_{i|i} = P_i - P_i H_i^T (\widehat{R}_i + H_i P_i H_i^T)^{-1} H_i P_i, \tag{29}$$
$$\widehat{Q}_i^{-1} = Q_i^{-1} + \widehat{\lambda}_i E_{g,i}^T [I + \widehat{\lambda}_i E_{f,i} P_{i|i} E_{f,i}^T]^{-1} E_{g,i}, \tag{30}$$
$$\widehat{R}_{i+1} = R_{i+1} - \widehat{\lambda}_i^{-1} H_{i+1} M_i M_i^T H_{i+1}^T, \tag{31}$$
$$\widehat{G}_i = G_i - \widehat{\lambda}_i F_i P_{i|i} E_{f,i}^T E_{g,i}, \tag{32}$$
$$\widehat{F}_i = (F_i - \widehat{\lambda}_i \widehat{G}_i \widehat{Q}_i E_{g,i}^T E_{f,i}) \times (I - \widehat{\lambda}_i P_{i|i} E_{f,i}^T E_{f,i}). \tag{33}$$

Moreover, $\{\widehat{x}_i, P_i\}$are updated by following equations

$$\widehat{x}_{i+1} = \widehat{F}_i \widehat{x}_i + \widehat{F}_i P_{i|i} H_i^T \widehat{R}_i^{-1} (y_i - H_i \widehat{x}_i), \tag{34}$$
$$P_{i+1} = F_i P_i F_i^T - \overline{K}_i \overline{R}_{e,i}^{-1} \overline{K}_i^T + \widehat{G}_i \widehat{Q}_i \widehat{G}_i^T, \tag{35}$$
$$\overline{K} = F_i P_i \overline{H}_i^T, \; \overline{R}_{e,i} = I + \overline{H}_i P_i \overline{H}_i^T,$$
$$\overline{H}_i^T = [H_i^T \widehat{R}_i^{-T/2}, \sqrt{\widehat{\lambda}_i} E_{f,i}^T].$$

III. THE DESIGN OF ROBUST ESTIMATOR WITH ADAPTIVE FACTOR

A. ADAPTIVE FACTOR

As we mentioned in the introduction part, the performance of the observer could be appreciably influenced by platform's vibration, atmospheric attenuation and turbulence. The actual covariance value of measurement error may deteriorate from the presupposed one. Thus, we introduce one adaptive factor into the recursion, i.e. when the actual observation noise deviates from the expectation, the introduced algorithm will adaptively adjust the covariance R to a more receivable value. Set γ_i as the Mahalanobis distance between the observed data y_i and its estimated data $\widehat{y}_i$, their difference e_i is in accordance with normal distribution. $\widehat{R}_i + H_i P_i^T H_i^T$ is the priori estimated covariance of y_i, while P_i is the priori estimated covariance of x_i.

$$\gamma_i = e_i^T (\widehat{R}_i + H_i P_i^T H_i^T)^{-1} e_i, \tag{36}$$
$$e_i = y_i - \widehat{y}_i, \quad \widehat{y}_i = H_i \widehat{x}_i. \tag{37}$$

As e_i is in accordance with normal distribution, the Mahalanobis distance γ_i which has the same degree of freedom as the dimension of y_i, is Chi-square distribution. When the probability of the difference value is beyond the presupposed confidence bound $1-\alpha$, namely, $\gamma_i > \chi_\alpha^2$, the adapted covariance $\widetilde{R}_i$ will be introduced.

Proposition 1. Let $\gamma_i = {e_i}^T(P_e)^{-1}e_i$, and $P_e = \widehat{R}_i + H_iP_i^TH_i^T$. Assume that $\gamma_i > \chi_\alpha^2$. After the measurement noise covariance $\widehat{R}_i$ be adapted to $\widetilde{R}_i = \frac{e_ie_i^T}{\chi_\alpha^2} - H_iP_i^TH_i^T$, the Mahalanobis distance of y_i is in the given confidence interval, and the adapted covariance $\widetilde{R}_i$ could better describe the observation error.

As shown in (38)-(41), in order to satisfy $\gamma_i \leq \chi_\alpha^2$, we suppose $\gamma_i = \chi_\alpha^2$ to obtain the critical formula of $\widetilde{R}_i$. The derivative process of P_e is also listed in the following part, in which $H_i\widehat{x}_i$ and y_i are independent.

$$\chi_\alpha^2 = \widetilde{\gamma}_i = e_i^T(\widetilde{P}_e)^{-1}e_i, \tag{38}$$

$$\chi_\alpha^2 = e_i^T(\widetilde{R}_i + H_iP_i^TH_i^T)^{-1}e_i, \tag{39}$$

$$e_i^{-T}\chi_\alpha^2e_i^{-1} = (\widetilde{R}_i + H_iP_i^TH_i^T)^{-1}, \tag{40}$$

$$\widetilde{R}_i = \frac{e_ie_i^T}{\chi_\alpha^2} - H_iP_i^TH_i^T. \tag{41}$$

$$\begin{aligned} P_e &= E\left(e_i^Te_i\right) \\ &= E\{(y_i - H_i\widehat{x}_i)^T(y_i - H_i\widehat{x}_i)\} \\ &= E\{y_i^Ty_i - \widehat{x}_i^TH_i^Ty_i - y_i^TH_i\widehat{x}_i \\ &\quad + \widehat{x}_i^TH_i^TH_i\widehat{x}_i\} \\ &= E\{y_i^Ty_i\} - E\{\widehat{x}_i^TH_i^Ty_i\} \\ &\quad - E\{y_i^TH_i\widehat{x}_i\} + E\{H_i\widehat{x}_i\widehat{x}_i^TH_i^T\}^T \\ &= \widehat{R}_i + (H_iP_iH_i^T). \end{aligned}$$

B. RECURSION FORMULA OF THE ROBUST ESTIMATOR

The adaptive factor will be applied to the estimator when $\gamma_i > \chi_\alpha^2$. Based on the designed robust state estimator, we summarize the algorithm as follows to show the prediction form of our proposed estimator.

Assume the uncertain model perform as (1) and (3).

Initial conditions:

$$\widehat{x}_0 = 0, \quad P_0 = \Pi_0, \ \widehat{R}_0 = R_0.$$

Step 1, according to (28), calculate $P_{i|i}$;

Step 2, on the basis of (13)-(27), construct the cost function $G(\lambda_i)$ and get the value of $\widehat{\lambda}$ by minimize the cost function (12);

$$\widehat{\lambda}_i > \widehat{\lambda}_{l,i} \triangleq \left\| M_i^TH_{i+1}^TR_{i+1}^{-1}H_{i+1}M_i \right\|. \tag{42}$$

Step 3, determine $\{\widehat{Q}_i, \widehat{R}_{i+1}, \widehat{F}_i, \widehat{G}_i\}$ by (30)-(33);

Step 4, update $\{\widehat{x}_i, P_i\}$ to $\{\widehat{x}_{i+1}, P_{i+1}\}$ by (34)-(35);

Step 5, adjust $\widehat{R}_{i+1}$ to $\widetilde{R}_{i+1}$ according to (36)-(38).

The optimizated value $\widehat{\lambda}_i$ is close to its minimum value $\widehat{\lambda}_{l,i}$ and in actual applications, Step 2 causes massive calculation. Thus $\widehat{\lambda}_i$ is set to be $(1+\alpha)\widehat{\lambda}_{l,i}$ to simplify the time complexity [14], by which more than 97% of time is saved while the prediction outcomes deterioration is no more than 5% in our simulations.

IV. SIMULATION EXAMPLES

A. NUMERICAL EXAMPLE

For comparison purpose, we set $\widehat{\lambda}_i = 1.5\widehat{\lambda}_{l,i}$ and use following values as parameters of our two-dimensional time-invariant model (1) and (3):

$$F = \begin{bmatrix} 0.9802 & 0.0196 \\ 0 & 0.9802 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$H = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad M = \begin{bmatrix} 0.0198 \\ 0 \end{bmatrix},$$

$$Q = 0.02 * \begin{bmatrix} 1.9608 & 0.0195 \\ 0.0195 & 1.9605 \end{bmatrix},$$

$$R_0 = 5, E_f = \begin{bmatrix} 0 & 5 \end{bmatrix}, \quad E_g = \begin{bmatrix} 0 & 0 \end{bmatrix},$$

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad P_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

These parameters are set to be constant for simplicity. Model (1) and (3) can be simplified as follows, by which we simulate the actual state x_i.

$$\begin{aligned} x_i &= (F + M\Delta_iE_f)x_{i-1} + u_i, \\ y_i &= Hx_i + v_i. \end{aligned}$$

Here, u_i, v_i are randomly selected according to (2) and Δ_i is an arbitrary contraction with $\|\Delta_i\| \leq 1$. The actually used R in (2) is in accordance with the uniform distribution whose interval is $[R_0, (1+a)R_0]$. The result of simulations with $a = 0, 1, 2, 4, 9, 19, 49, 99$ are as Figures 1-8.

In each simulation, we calculate the estimated value $\widehat{x}_i$ for 1000 steps. For each given parameter a, we simulates 100 times to get the accumulation value of error. According to simulations, the adaptive factor helped to minimize 0.55% accumulated error when the proposed covariance R_0 is in accordance with the actual used R and minimize 5.60%, 12.22%, 17.99%, 28.52%, 43.68%, 58.32%, 68.59% of accumulated error when the actual used R is randomly produced from the interval $[R_0, 2R_0]$, $[R_0, 3R_0]$, $[R_0, 5R_0]$, $[R_0, 10R_0]$, $[R_0, 20R_0]$, $[R_0, 50R_0]$, $[R_0, 100R_0]$. We can see the performance of adaptive factor significantly increases as the actual value of R gradually deteriorate from the presupposed R_0. It means that the adaptive covariance $\widetilde{R}_i$ plays an increasingly important role as the actual observation noise covariance R differing larger from the presupposed R_0. One simulation result with actual R be randomly produced from the interval R_0 to $10R_0$ is as shown in Figures 9-11. Figure 9 shows the convergence and effectiveness of our algorithm. Figure 10 shows besides less average accumulated error from actual state to estimated state, the estimator with adaptive factor has significantly smoother curve than the estimator without adaptive factor, i.e. the blue line. And Figure 11 compares the error of estimators with or without adaptive factor.

In Table 1, with different produced interval of R, the evaluating indicators of error variance range, standard deviation, the ratios of adaptive factor reducing error are exhibited.

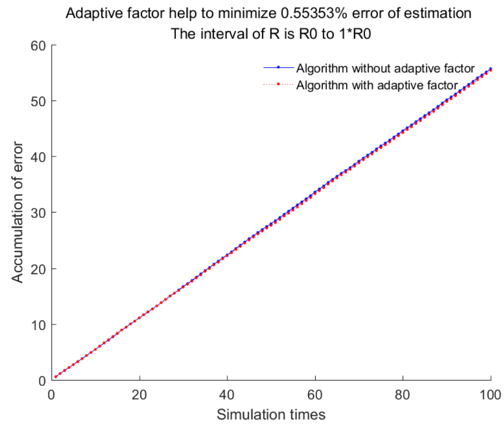

FIGURE 1. Simulation result with the maximum of R is R_0.

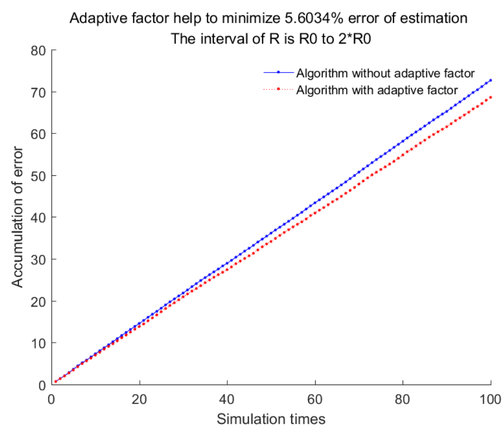

FIGURE 2. Simulation result with the maximum of R is $2R_0$.

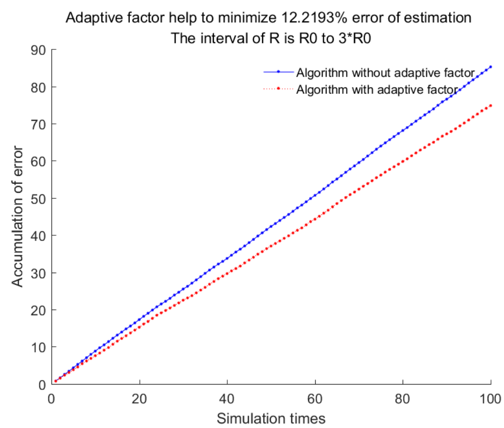

FIGURE 3. Simulation result with the maximum of R is $3R_0$.

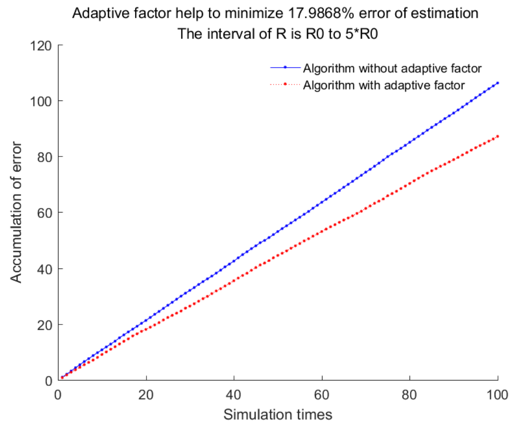

FIGURE 4. Simulation result with the maximum of R is $5R_0$.

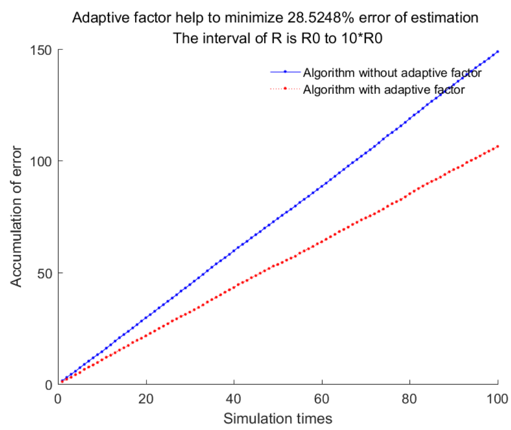

FIGURE 5. Simulation result with the maximum of R is $10R_0$.

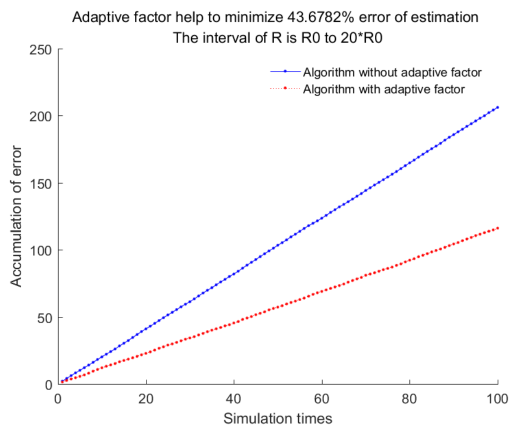

FIGURE 6. Simulation result with the maximum of R is $20R_0$.

The table demonstrates the adaptive factor may have significant smoothing effectiveness when confronting observation noise surging.

B. PRACTICAL EXAMPLE

We would give further introduction about our estimator with one practical example. Estimating the position and velocity state on x and y axes is familiar application in target tracking. With estimated state $X = [x_p, x_v, y_p, y_v]$, the model (1) and (3) with following parameters are proposed to be simulated.

$$F = \begin{bmatrix} 1 & 0.1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$x_0 = \begin{bmatrix} 0 \\ 3 \\ 0 \\ 3 \end{bmatrix}, \quad M = \begin{bmatrix} 0.0198 \\ 0 \\ 0.0198 \\ 0 \end{bmatrix},$$

TABLE 1. Smooth effectiveness.

Upper limit value of the producing of R	R_0	$2R_0$	$3R_0$	$5R_0$	$10R_0$	$20R_0$	$50R_0$	$100R_0$
Range of estimator without adaptive factor	5.14	6.26	8.10	9.80	12.36	18.20	31.27	42.17
Range of estimator with adaptive factor	4.58	5.73	6.78	8.27	8.39	10.56	9.62	12.42
Reducing ratio	10.96%	8.45%	16.26%	15.59%	32.11%	41.95%	69.24%	70.56%
Standard deviation of estimator without adaptive factor	0.69	0.89	1.08	1.34	1.83	2.60	4.00	5.68
Standard deviation of estimator with adaptive factor	0.67	0.83	0.95	1.09	1.26	1.43	1.57	1.77
Reducing ratio	2.04%	6.12%	12.01%	18.63%	31.05%	44.91%	60.69%	68.89%

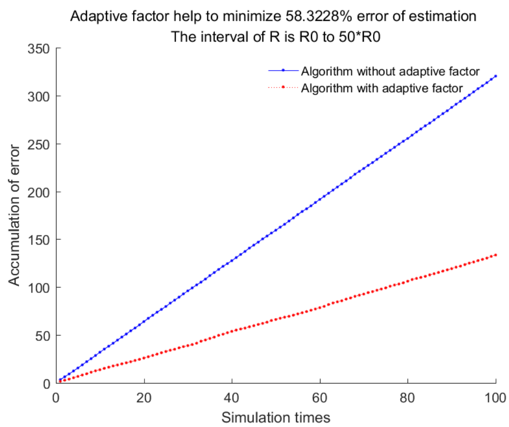

FIGURE 7. Simulation result with the maximum of R is $50R_0$.

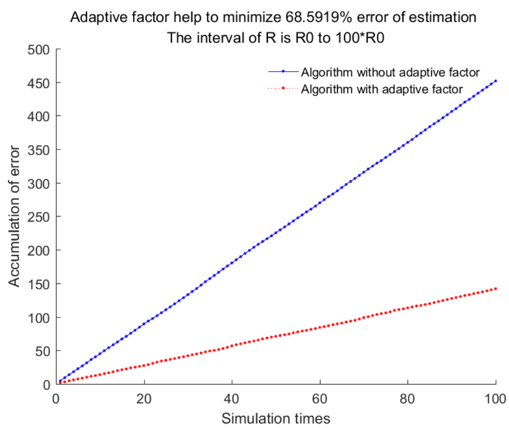

FIGURE 8. Simulation result with the maximum of R is $100R_0$.

$$H = \begin{bmatrix} 1\,0\,0\,0 \\ 0\,0\,1\,0 \end{bmatrix}, \quad R_0 = \begin{bmatrix} 5\;0 \\ 0\;5 \end{bmatrix},$$
$$G = diag(1, 1, 1, 1),$$
$$Q = 0.01 * diag(2, 2, 2, 2),$$
$$P_0 = diag(1, 1, 1, 1),$$
$$E_f = \begin{bmatrix} 0 \; 0.099 \; 0.099 \; 0 \end{bmatrix},$$
$$E_g = \begin{bmatrix} 0\;0\;0\;0 \end{bmatrix}.$$

In the simulations, the observation noise covariance is uniformly randomly selected from the interval R_0 to $10R_0$. The estimation result of robust estimators with or without adaptive

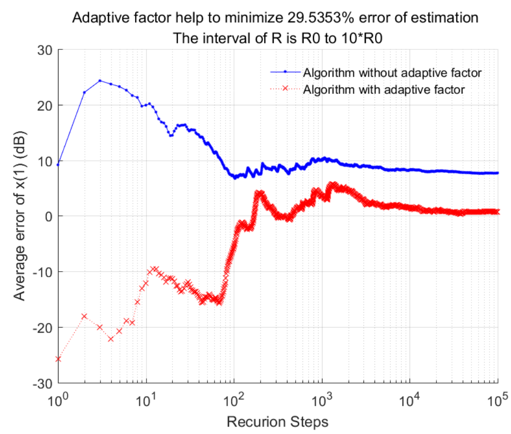

FIGURE 9. Simulation result with random interval of R: $[R_0, 10R_0]$.

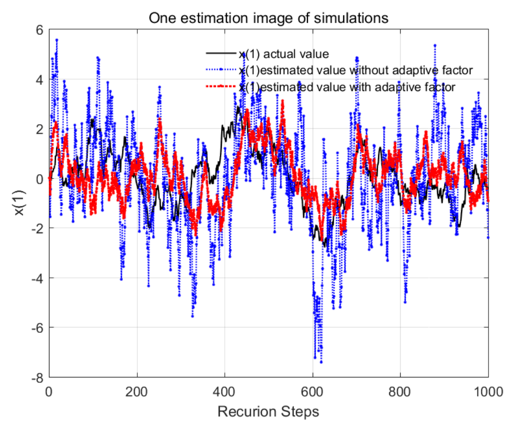

FIGURE 10. Simulation result with random interval of R: $[R_0, 10R_0]$.

factor is shown in Figure 12. The estimated trajectory may not able to demonstrate the effectiveness of adaptive factor. Thus as exhibited in Figure 13, we calculate the accumulation error of two estimators and the statistics show that 13.09% of error are reduced by our robust state estimator with adaptive factor. The accumulation error formula is designed as follows,

$$E_A = \sum_{i=1}^{n} \sqrt{(\hat{x}_{i,1} - x_{i,1})^2 + (\hat{x}_{i,3} - x_{i,3})^2},$$

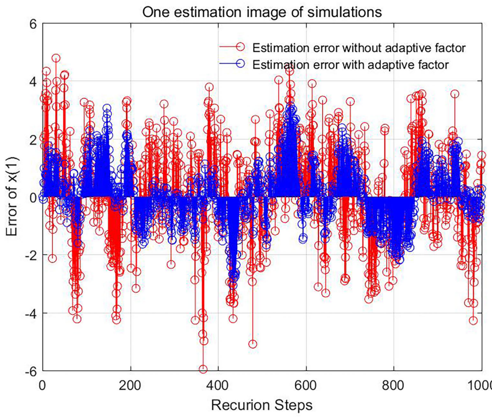

FIGURE 11. Comparison on error variance.

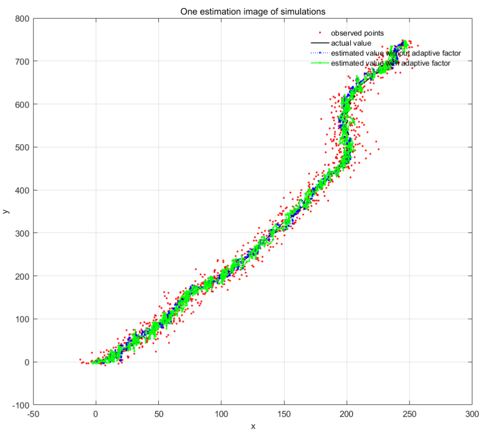

FIGURE 12. Estimated trajectory.

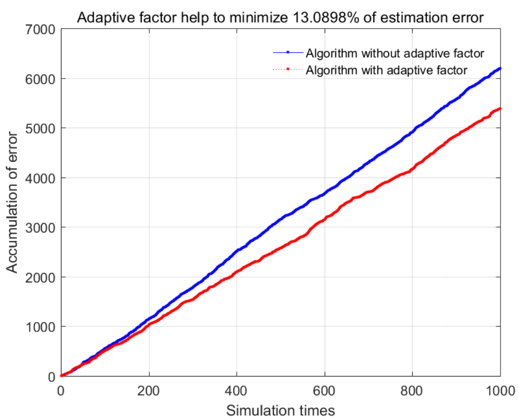

FIGURE 13. Accumulation error comparison of two estimator.

where $x_{i,1}, x_{i,3}$ are respectively the first and the third dimension of the actual state, while $\hat{x}_{i,1}, \hat{x}_{i,3}$ are respectively the first and the third dimension of estimated state of $\hat{x}_i$.

V. CONCLUSION

We introduce an adaptive factor in robust state estimator to solve uncertainties in target motion model and observation noise covariance. From the simulation study, we can see when the presupposed covariance R appreciably deteriorate from the actual noise, consuming a negligible additional computing resource, the adaptive factor shows a notable performance in minimizing estimation error. However, confronts large process noise, the estimator with adaptive observation noise covariance does not have significant improvement, which means the parameter being adaptively adjusted needs to be the principal factor of estimation error.

About further investigation issues, one of them is applying the adaptive factor to the process noise u_i to adaptively adjust estimation result. Other issues include combining the adaptive factor with guaranteed-cost filters, H_∞ filters, and set-valued estimation filters, applying the adaptive factor to nonlinear systems [26], extending our robust state estimator to multi-agent systems [27]–[29].

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