"CHAPTER - 8 PLANE - WAVE PROPAGATION"

Exercise 8.1 $\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$ and $\nabla\times\vec{H} = \sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t}$

$\nabla\times\nabla\times\vec{H} = \sigma\,\nabla\times\vec{E} + \epsilon\frac{\partial}{\partial t}(\nabla\times\vec{E}) \Rightarrow$

$\nabla(\nabla\cdot\vec{H}) - \nabla^2\vec{H} = -\mu\sigma\frac{\partial\vec{H}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2}$ or $\nabla^2\vec{H} = \mu\sigma\frac{\partial\vec{H}}{\partial t} + \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2}$

Exercise 8.2

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu\sigma\frac{\partial E_x}{\partial t} + \mu\epsilon\frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu\sigma\frac{\partial E_y}{\partial t} + \mu\epsilon\frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \mu\sigma\frac{\partial E_z}{\partial t} + \mu\epsilon\frac{\partial^2 E_z}{\partial t^2}$$

$$\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} = \mu\sigma\frac{\partial H_x}{\partial t} + \mu\epsilon\frac{\partial^2 H_x}{\partial t^2}$$

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + \frac{\partial^2 H_y}{\partial z^2} = \mu\sigma\frac{\partial H_y}{\partial t} + \mu\epsilon\frac{\partial^2 H_y}{\partial t^2}$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = \mu\sigma\frac{\partial H_z}{\partial t} + \mu\epsilon\frac{\partial^2 H_z}{\partial t^2}$$

Exercise 8.3

$\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} \Rightarrow -\mu\frac{\partial\vec{H}}{\partial t} = -\frac{\partial E_y}{\partial z}\vec{a}_x + \frac{\partial E_x}{\partial z}\vec{a}_y$

For $E_x = F_x(t - z/u)$ and $E_y = F_y(t - z/u)$, $d/dt \Rightarrow 1$ and $\frac{d}{dz} \Rightarrow -\frac{1}{u}$

Thus

$-\mu H_x = \frac{1}{u}E_y \Rightarrow H_x = -\frac{\sqrt{\mu\epsilon}}{\mu}E_y = -\sqrt{\frac{\epsilon}{\mu}}\,E_y = -\frac{E_y}{\eta}$ $\qquad u = \frac{1}{\sqrt{\mu\epsilon}}$

$-\mu H_y = -\frac{1}{u}E_x \Rightarrow H_y = \sqrt{\frac{\epsilon}{\mu}}\,E_x = \frac{E_x}{\eta}$

Thus, $\eta\,[H_x\vec{a}_x + H_y\vec{a}_y] = -E_y\vec{a}_x + E_x\vec{a}_y$

or $\eta\vec{H} = \vec{a}_z\times\vec{E}$

Exercise 8.4 $\quad \vec{E} = 100 \sin(10^8 t + x/\sqrt{3})\,\vec{a}_z$ V/m

Verify $\nabla^2 \vec{E} = \mu\epsilon \dfrac{\partial^2 \vec{E}}{\partial t^2}$ ① $\qquad \nabla^2 \vec{E} = \dfrac{\partial^2 E_z}{\partial x^2} = -\dfrac{100}{3}\sin(10^8 t + x/\sqrt{3})$

and $\dfrac{\partial^2 E_z}{\partial t^2} = -100 \times 10^{16} \sin(10^8 t + x/\sqrt{3})$

From ① $\quad \mu\epsilon = \dfrac{10^{-16}}{3} \Rightarrow \epsilon_r = \dfrac{10^{-16}}{3} \times (3\times 10^8)^2 = 3$ — Hence $\epsilon_r = 3$

(b) $10^8 t + x/\sqrt{3} = \text{Const} \Rightarrow \dfrac{dx}{dt} = -\sqrt{3}\times 10^8$ Hence $\vec{u} = -\sqrt{3}\times 10^8\,\vec{a}_x$ m/s

$\nabla \times \vec{E} = -\mu \dfrac{\partial \vec{H}}{\partial t} \Rightarrow \mu \dfrac{\partial \vec{H}}{\partial t} = \dfrac{\partial E_z}{\partial x}\vec{a}_y$ or $H_y = 0.4594 \sin(10^8 t + x/\sqrt{3})$ A/m

$\vec{S} = \vec{E} \times \vec{H} = -45.94 \sin^2(10^8 t + x/\sqrt{3})\,\vec{a}_x$ W/m^2

$\langle \vec{S} \rangle = \dfrac{1}{T}\displaystyle\int_0^T \vec{S}\,dt = -22.97\,\vec{a}_x$ W/m^2

Exercise 8.5 $\quad \vec{E} \perp \vec{H}$ if $\vec{E}\cdot\vec{H} = 0 \qquad \vec{H} = \dfrac{1}{\eta}(\vec{a}_z \times \vec{E})$

Using vector identity $\quad \vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A})$

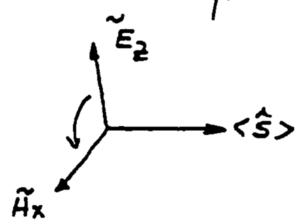
$\vec{E}\cdot\vec{H} = \dfrac{1}{\eta}[\vec{E}\cdot(\vec{a}_z \times \vec{E})] = \dfrac{1}{\eta}[\vec{a}_z\cdot(\vec{E}\times\vec{E})] = 0$

Exercise 8.6 $\quad \omega = 2\pi\times 10^9$ rad/s $\quad \beta_0 = \dfrac{\omega}{c} = 20.94$ rad/m $\quad \eta_0 \cong 377\Omega$

$\vec{E} = 120\, e^{-j20.94y}\,\vec{a}_z$ V/m

For y-directed propagation,

$\vec{H} = 0.318\, e^{-j20.94y}\,\vec{a}_x$ A/m



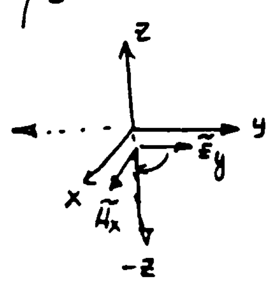
$\langle \hat{S} \rangle = \dfrac{1}{2}\,\mathrm{Re}[\tilde{E}\times\tilde{H}^*] = 19.1\,\vec{a}_y$ W/m^2

Exercise 8.7 $\quad \omega = 200\pi\times 10^6$ rad/s $\quad \beta_0 = \dfrac{\omega}{c} = \dfrac{2}{3}\pi$ rad/m, $\eta_0 = 377\Omega$

$\vec{u}_p = -3\times 10^8\,\vec{a}_z$ m/s $\qquad \lambda_0 = \dfrac{2\pi}{\beta_0} = 3$ m

$\tilde{H} = 0.1\, e^{j\beta_0 z}\,\vec{a}_x$ A/m

$\vec{E} = 37.7\, e^{j\beta_0 z}\,\vec{a}_y$ V/m



$\tilde{J}_d = j\omega\epsilon_0 \tilde{E} = j\,0.209\, e^{j\beta_0 z}\,\vec{a}_y$ A/m^2

$\langle \hat{S} \rangle = \dfrac{1}{2}\,\mathrm{Re}[\tilde{E}\times\tilde{H}^*] = -1.885\,\vec{a}_z$ W/m^2

Exercise 8.8 $\nabla\times\tilde{\vec{E}} = -j\omega\mu\tilde{\vec{H}}$, $\nabla\times\tilde{\vec{H}} = (\sigma+j\omega\epsilon)\tilde{\vec{E}} = j\omega\hat{\epsilon}\tilde{\vec{E}}$ $\hat{\epsilon} = \epsilon[1-j\frac{\sigma}{\omega\epsilon}]$

$\nabla\cdot\tilde{\vec{B}} = 0 \Rightarrow \nabla\cdot\tilde{\vec{H}} = 0$ $\nabla\cdot\tilde{\vec{D}} = 0 \Rightarrow \nabla\cdot\tilde{\vec{E}} = 0$

$\nabla\times\nabla\times\tilde{\vec{E}} = -j\omega\mu(\nabla\times\tilde{\vec{H}}) \Rightarrow \nabla(\nabla\cdot\tilde{\vec{E}}) - \nabla^2\tilde{\vec{E}} = -\omega^2\mu\hat{\epsilon}\tilde{\vec{E}}$

$\nabla^2\tilde{\vec{E}} = \hat{\gamma}^2\tilde{\vec{E}}$ where $\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}}$. similarly, $\nabla^2\tilde{\vec{H}} = \hat{\gamma}^2\tilde{\vec{H}}$

Exercise 8.9 $\hat{\epsilon} = \epsilon[1-j\tan\phi) = \epsilon\sec\phi\,\angle{-\phi}$

$\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = j\omega\sqrt{\mu\epsilon\sec\phi}\,\angle{-\phi/2} = \omega\sqrt{\mu\epsilon\sec\phi}\,[\sin\phi/2 + j\cos\phi/2]$

Thus $\alpha = \omega\sqrt{\mu\epsilon\sec\phi}\,\sin\phi/2$ and $\beta = \omega\sqrt{\mu\epsilon\sec\phi}\,\cos\phi/2$

Exercise 8.10 $f = 100$ MHz, $\omega = 6.28\times10^8$ rad/m, $\sigma = 9.375$ mS/m, $\epsilon_r = 2.25$

$\tan\phi = \frac{\sigma}{\omega\epsilon} = 0.75 \Rightarrow \hat{\epsilon} = \epsilon[1-j0.75]$ $\mu_r = 1$

$\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = j\omega\sqrt{\mu_0\epsilon_0}\sqrt{\mu_r\epsilon_r}\sqrt{1-j0.75} = 1.11 + j3.33 \Rightarrow \alpha = 1.11$ Np/m

$\beta = 3.33$ rad/m

$\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{\frac{\mu_r}{\epsilon_r}}\frac{1}{\sqrt{1-j0.75}} = 224.79\,\angle 18.43^\circ\,\Omega$ $\delta = \frac{1}{\alpha} = 900$ mm

$\tilde{E}_y = 125\,e^{-1.11z}\,e^{-j3.33z}$ V/m $u_p = \frac{\omega}{\beta} = 1.89\times10^8$ m/s

$\tilde{H}_x = -\frac{125}{224.79}\,e^{-1.11z}\,e^{-j3.33z}\,e^{-j18.43^\circ}$ A/m

$\langle\hat{S}\rangle = \frac{1}{2}\mathrm{Re}[\tilde{\vec{E}}\times\tilde{\vec{H}}^*] = \frac{1}{2}\times125\times\frac{125}{224.79}\cos18.43^\circ\,e^{-2.22z}\,\vec{a}_z$

$= 32.97\,e^{-2.22z}\,\vec{a}_z$ W/m^2

Exercise 8.11 $f = 2.4\times10^9$ Hz, $\omega = 1.508\times10^{10}$ rad/s

$\sigma = 6.1\times10^7$ S/m, $\epsilon_r = 1$, $\mu_r = 1$ $\tan\phi = \frac{\sigma}{\omega\epsilon} = 4.575\times10^8 \Rightarrow$ Very good Conductor

$\hat{\epsilon} = -j\frac{\sigma}{\omega} = -j\,0.004$, $\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = (7.602 + j7.602)10^5$

$\alpha = 7.602\times10^5$ Np/m

$R_s = \frac{1}{\sigma\delta} = 0.012$ or 12 mΩ/m^2

$\beta = 7.602\times10^5$ rad/m

$\delta = \frac{1}{\alpha} = 1.315\ \mu$m $\Rightarrow$ Thickness $= 5\delta \cong 1.32\ \mu$m

Exercise 8.12 @ 60 Hz: $\omega = 2\pi f = 120\pi$ rad/s

$\sigma = 3.5\times10^{7}$ S/m $\mu_r = 1$ $\epsilon_r = 1$ $\tan\phi = \frac{\sigma}{\omega\epsilon} = 1.05\times10^{16}$ Very good Conductor

Thus, $\hat{\epsilon} = -j\frac{\sigma}{\omega} = -j9.284\times10^{4}$ $\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = 91.052 + j91.052$

$\delta = \frac{1}{\alpha} = 10.983$ mm. Since Thickness of 5 mm < δ, $R_s = \frac{100}{2\pi\times2.54\times10^{-2}\times\sigma\times5\times10^{-3}}$

$= 3.58\ m\Omega$

@ 60 MHz: $\tan\phi = \frac{\sigma}{\omega\epsilon} = 1.05\times10^{10}$ (Very good Conductor)

$\hat{\epsilon} = -j\frac{\sigma}{\omega} = -j0.093$

$\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = (9.105 + j9.105)10^{4} \Rightarrow \delta = \frac{1}{\alpha} = 0.011$ mm and $R_s = 1.63\,\Omega$

Exercise 8.13 $\sqrt{\hat{\epsilon}} = \sqrt{\epsilon}\left[1 - j\frac{\sigma}{\omega\epsilon}\right]^{1/2} = \sqrt{\epsilon}\left[1 - j\frac{\sigma}{2\omega\epsilon}\right]$ for $\frac{\sigma}{\omega\epsilon} \ll 1$

$$\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = j\omega\sqrt{\mu\epsilon}\left[1 - j\frac{\sigma}{2\omega\epsilon}\right] = j\omega\sqrt{\mu\epsilon} + \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$$

$$\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} = \sqrt{\frac{\mu}{\epsilon}}\left[1 - j\frac{\sigma}{2\omega\epsilon}\right]^{-1} = \sqrt{\frac{\mu}{\epsilon}}\left[1 + j\frac{\sigma}{2\omega\epsilon}\right] = \sqrt{\frac{\mu}{\epsilon}} + j\frac{\sigma}{2\omega\epsilon}\sqrt{\frac{\mu}{\epsilon}} \cong \sqrt{\frac{\mu}{\epsilon}}$$

Exercise 8.14 $f = 60$ MHz $\omega = 120\pi\times10^{6}$ rad/s $\sigma = 5\times10^{-3}$ S/m

$\tan\phi = \frac{\sigma}{\omega\epsilon} = 0.094$ (Poor Conductor) $\mu \to \mu_0$, $\epsilon \Rightarrow \epsilon_0(16)$

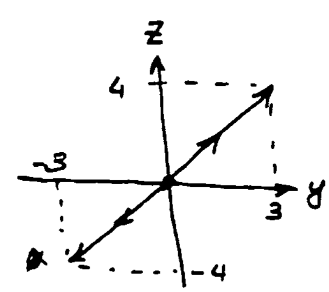
$\hat{\epsilon} = \epsilon\left[1 - j\frac{\sigma}{\omega\epsilon}\right] = \epsilon[1 - j0.094]$ $\beta = \omega\sqrt{\mu\epsilon} = 5.027$ rad/m

$\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = j\omega\sqrt{\mu\epsilon}\,[1 - j0.047]$ $\alpha = 0.047\beta = 0.236$ Np/m

If $\tilde{E}_x = E_0\, e^{-\alpha z} e^{-j\beta z}$ $\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} \cong \sqrt{\frac{\mu}{\epsilon}} = 94.25\,\Omega$

$$\left.\begin{array}{ll} \text{at } z=0 & \tilde{E}_x(0) = E_0 \\ \text{at } z=d & \tilde{E}_x(d) = E_0\, e^{-\alpha d} e^{-j\beta d} \end{array}\right\} \Rightarrow \begin{array}{l} E_0 e^{-\alpha d} = 0.1E_0 \text{ or } e^{\alpha d} = 10 \\ d = \frac{\ln 10}{\alpha} = 9.76\text{ m} \end{array}$$

Exercise 8.15 at $\beta x = 45^\circ$



$E_z = 4\sin\omega t$, $E_y = 3\sin\omega t$

$\frac{E_y}{E_z} = 0.75 \Rightarrow E_y = 0.75\,E_z$

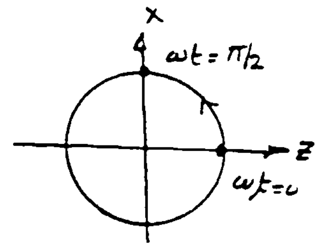
St. line relationship $\Rightarrow$ Linear Polarization

Exercise 8.16 $E_x = 25\, e^{-0.01y} \sin(\omega t - 120y)$ $E_z = 25\, e^{-0.01y} \cos(\omega t - 120y)$

at $y=0$, $E_x = 25 \sin\omega t$ $E_z = 25 \cos\omega t$

$E_x^2 + E_z^2 = 25^2$ [Circular]

$\left.\begin{array}{lll} \omega t = 0 & E_x = 0, & E_z = 25 \\ \omega t = \pi/2 & E_x = 25 & E_z = 0 \end{array}\right\}$ Right handed

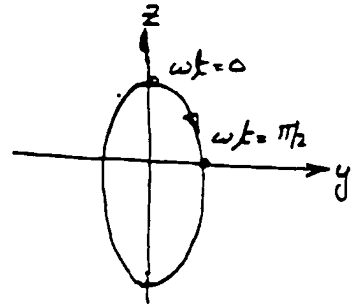


Exercise 8.17 Let $\beta x = 60^\circ$, then

$E_y = 30 \sin\omega t$ $E_z = 40 \cos\omega t$

$\left(\frac{E_y}{30}\right)^2 + \left(\frac{E_z}{40}\right)^2 = 1$ [Elliptical]

$\left.\begin{array}{lll} \omega t = 0 & E_y = 0 & E_z = 40 \\ \omega t = \pi/2 & E_y = 30 & E_z = 0 \end{array}\right\}$ Left-handed



Exercise 8.18

$\tilde{E}_i = 10\, e^{-0.92z}\, e^{-j4.29z}\, \vec{a}_x$ $\tilde{E}_r = 4.07\, e^{0.92z}\, e^{j4.29z}\, e^{j153.74^\circ}\, \vec{a}_x$

$\tilde{H}_i = \frac{1}{9}\, e^{-0.92z}\, e^{-j4.29z}\, e^{-j12.11^\circ}\, \vec{a}_y$ $\tilde{H}_r = -\frac{4.07}{90}\, e^{0.92z}\, e^{j4.29z}\, e^{+j141.63^\circ}\, \vec{a}_y$

$\tilde{E}_1 = \tilde{E}_i + \tilde{E}_r = \left[10\, e^{-0.92z}\, e^{-j4.29z} + 4.07\, e^{0.92z}\, e^{j4.29z}\, e^{j153.74^\circ}\right] \vec{a}_x$

$\tilde{H}_1 = \tilde{H}_i + \tilde{H}_r = \left[\frac{1}{9}\, e^{-0.92z}\, e^{-j4.29z}\, e^{-j12.11^\circ} - \frac{4.07}{90}\, e^{0.92z}\, e^{j4.29z}\, e^{j141.63^\circ}\right] \vec{a}_y$

$\tilde{E}_t = \tilde{E}_2 = 6.6\, e^{-5.3z}\, e^{-j7.45z}\, e^{j15.81^\circ}\, \vec{a}_x$

$\tilde{H}_t = \tilde{H}_2 = 0.153\, e^{-5.3z}\, e^{-j7.45z}\, e^{-j19.62^\circ}\, \vec{a}_y$

$\langle \hat{S}_i \rangle = \frac{1}{2} \mathrm{Re}\left[\tilde{E}_i \times \tilde{H}_i^*\right] = 0.543\, e^{-1.84z}\, \vec{a}_z$

$\langle \hat{S}_r \rangle = \frac{1}{2} \mathrm{Re}\left[\tilde{E}_r \times \tilde{H}_r^*\right] = -0.09\, e^{1.84z}\, \vec{a}_z$

$\langle \hat{S}_2 \rangle = \langle \hat{S}_t \rangle = 0.41\, e^{-10.6z}\, \vec{a}_z$

at $z=0$

$\langle \hat{S}_i \rangle = 0.543\, \vec{a}_z$

$\langle \hat{S}_r \rangle = -0.09\, \vec{a}_z$

$\langle \hat{S}_{ir} \rangle = -0.042\, \vec{a}_z$

$\langle \hat{S}_1 \rangle = \frac{1}{2} \mathrm{Re}\left[\tilde{E}_1 \times \tilde{H}_1^*\right] = \left[0.543\, e^{-1.84z} - 0.09\, e^{1.84z} \underbrace{+ 0.177 \cos(8.58z) - 0.219 \cos(8.58z)}_{\langle \hat{S}_{ir} \rangle}\right] \vec{a}_z$

at $z=0$ $\langle \hat{S}_1 \rangle = 0.411\, \vec{a}_z = \langle \hat{S}_2 \rangle$

Exercise 8.19

$f = 500\ MHz \qquad \omega = 3.14\times10^{9}\ rad/s$

$\epsilon_{r_1} = 16 \quad \sigma_1 = 0.02 \Rightarrow \hat{\epsilon}_1 = 1.41\times10^{-10} - j6.37\times10^{-12}$

$\hat{\gamma}_1 = j\omega\sqrt{\mu_0\hat{\epsilon}_1} = 41.91\angle 88.71^\circ = 0.94 + j41.9\ m^{-1} \quad \alpha_1 = 0.94\ Np/m,\ \beta_1 = 41.9\ rad/m$

$\hat{\eta}_1 = \sqrt{\frac{\mu_1}{\hat{\epsilon}_1}} = 94.18 + j2.12 = 94.2\angle 1.29^\circ\ \Omega$

$\epsilon_{r_2} = 25 \quad \sigma_2 = 0.2 \Rightarrow \hat{\epsilon}_2 = 2.21\times10^{-10} - j6.37\times10^{-11}$

$\hat{\gamma}_2 = j\omega\sqrt{\mu_0\hat{\epsilon}_2} = 53.41\angle 81.97^\circ = 7.46 + j52.89\ m^{-1} \qquad \hat{\eta}_2 = \sqrt{\frac{\mu_0}{\hat{\epsilon}_2}} = 73.91\angle 8.03^\circ\ \Omega$

$\hat{\rho} = \frac{\hat{\eta}_2 - \hat{\eta}_1}{\hat{\eta}_1 + \hat{\eta}_2} = 0.13\angle 154.38^\circ \qquad \hat{\tau} = \frac{2\hat{\eta}_2}{\hat{\eta}_1 + \hat{\eta}_2} = 0.88\angle 3.78^\circ$

Max. Value of the incidence field $= 10\ V/m$

$\tilde{E}_t = 8.8\, e^{-7.46z}\, e^{-j52.89z}\, e^{j3.78^\circ}\, \vec{a}_x\ V/m$

$\tilde{H}_t = \frac{8.8}{73.91}\, e^{-7.46z}\, e^{-j52.89z}\, e^{j3.78^\circ}\, e^{-j8.03^\circ}\, \vec{a}_y\ A/m$

$\langle \hat{S}_t\rangle = \frac{1}{2}Re[\tilde{E}_t \times \tilde{H}_t^*] = 0.52\, e^{-14.92z}\ \vec{a}_z\ W/m^2$

Exercise 8.20

$u_{p1} = \frac{\omega}{\beta_1} = 1\times10^{8}\ m/s \qquad \lambda_1 = \frac{2\pi}{\beta_1} = \frac{2\pi}{3}\ m$

$u_{p2} = \frac{\omega}{\beta_2} = 1.5\times10^{8}\ m/s \qquad \lambda_2 = \frac{2\pi}{\beta_2} = \pi\ m$

$\langle \hat{S}_i\rangle + \langle \hat{S}_r\rangle = [39.79 - 1.59]\,\vec{a}_z\ \mu W/m^2 = 38.2\,\vec{a}_z\ \mu W/m^2$

Exercise 8.21

$SWR = \frac{|E|_{max}}{|E|_{min}} \Rightarrow |E_{max}|$ at $z = 0 = E_0[1 + |\rho|]$

E_{min} at $z = \frac{\pi}{2\beta_1} = E_0[1 - |\rho|]$

$$SWR = \frac{1+|\rho|}{1-|\rho|}$$

Exercise 8.22

$\mu_1 = \mu_0,\ \epsilon_1 = 2.25\epsilon_0,\ \mu_2 = \mu_0,\ \epsilon_2 = 9\epsilon_0$

$\beta_1 = \omega\sqrt{\mu_0}\sqrt{\epsilon_0\epsilon_{r_1}} = 1.5\ rad/m$

$\beta_2 = \omega\sqrt{\mu_0\epsilon_0\epsilon_{r_2}} = 3\ rad/m$

$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{\frac{1}{\epsilon_{r_1}}} = 80\pi, \quad \eta_2 = \frac{120\pi}{3} = 40\pi$

$\rho = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} = -\frac{1}{3}, \quad \tau = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2}{3}$

$\left.\begin{array}{l}\tilde{E}_i = 0.25\, e^{-j1.5x}\,\vec{a}_z \\ \tilde{H}_i = -\frac{0.25}{80\pi}\, e^{-j1.5x}\,\vec{a}_y\end{array}\right\} \langle \hat{S}_i\rangle = 124.34\,\vec{a}_x\ \mu W/m^2$

$\left.\begin{array}{l}\tilde{E}_r = -\frac{0.25}{3}\, e^{j1.5x}\,\vec{a}_z \\ \tilde{H}_r = \frac{0.25}{240\pi}\, e^{j1.5x}\,\vec{a}_y\end{array}\right\} \langle \hat{S}_r\rangle = -13.82\,\vec{a}_x\ \mu W/m^2$

$\langle \hat{S}_i\rangle + \langle \hat{S}_r\rangle = 110.52\,\vec{a}_x\ \mu W/m^2$

$\left.\begin{array}{l}\tilde{E}_t = \frac{0.5}{3}\, e^{-j3x}\,\vec{a}_z \\ \tilde{H}_t = -\frac{0.5}{120\pi}\, e^{-j3x}\,\vec{a}_y\end{array}\right\} \langle \hat{S}_t\rangle = 110.52\,\vec{a}_x\ \mu W/m^2$

Exercise 8.23

$\omega = 600\times10^{6}$ rad/s $\quad \mu \rightarrow \mu_0 \quad \epsilon \rightarrow \epsilon_0(16)$

$\beta = \omega\sqrt{\mu\epsilon} = 8$ rad/m $\quad \eta = \frac{120\pi}{4} = 30\pi \quad u_p = \omega/\beta = 7.5\times10^{7}$ m/s $\quad \rho = -1$

$\tilde{\vec{E}}_i = 100\,e^{-j8}\,\vec{a}_x \qquad \tilde{\vec{E}}_r = -100\,e^{j8}\,\vec{a}_x \qquad \tilde{\vec{E}} = \tilde{\vec{E}}_i + \tilde{\vec{E}}_r = -j200\sin 8z\,\vec{a}_x$

$\tilde{\vec{H}}_i = \frac{100}{30\pi}e^{-j8}\,\vec{a}_y \qquad \tilde{\vec{H}}_r = \frac{100}{30\pi}e^{j8}\,\vec{a}_y \qquad \tilde{\vec{H}} = \tilde{\vec{H}}_i + \tilde{\vec{H}}_r = \frac{200}{30\pi}\cos 8z\,\vec{a}_y$

$\tilde{\vec{J}}_s(0)\Big|_{z=0} = -\vec{a}_z\times(\tilde{\vec{H}}_1 - \tilde{\vec{H}}_2) \Rightarrow \quad \tilde{\vec{J}}_s(0) = \frac{200}{30\pi}\,\vec{a}_x$

$\langle\vec{P}\rangle = \frac{1}{2}Re\left[\tilde{\vec{J}}_s(0)\times\tilde{\vec{B}}_i^{*}(0)\right] = \frac{1}{2}\left[\frac{200}{30\pi}\cdot\frac{100}{30\pi}(4\pi\times10^{-7})\right] = 1.414\,\vec{a}_z \quad \mu N/m^2$

$d = \frac{n\pi}{\beta} \quad$ For $n=1$, $d = \frac{\pi}{8} = 39.27$ cm

Exercise 8.24

$E_{ix} = 100\cos(\omega t - 8z) \qquad H_{iy} = \frac{100}{30\pi}\cos(\omega t - 8z)$

$E_{rx} = -100\cos(\omega t + 8z) \qquad H_{ry} = \frac{100}{30\pi}\cos(\omega t + 8z)$

$E_x = 200\sin 8z\sin\omega t \qquad E_y = \frac{200}{30\pi}\cos 8z\cos\omega t$

Exercise 8.25

$\omega = 96\times10^{6}$ rad/s $\quad \epsilon_1 = \epsilon_0 \quad \mu_1 = \mu_0 \quad \epsilon_2 = 81\epsilon_0 \quad \mu_2 = 1 \quad \sigma_2 = 4$ S/m

$\hat{\gamma}_1 = j\omega\sqrt{\mu_1\epsilon_1} = j0.32\ m^{-1} \Rightarrow \beta_1 = 0.32$ rad/m $\qquad \hat{\epsilon}_2 = \epsilon_2\left[1 - j\frac{\sigma_2}{\omega\epsilon_2}\right]$

$\eta_1 \cong 377\,\Omega \qquad \hat{\eta}_2 = \sqrt{\frac{\mu_0}{\hat{\epsilon}_2}} = 5.491\angle 44.51^\circ\,\Omega \qquad \hat{\gamma}_2 = j\omega\sqrt{\mu\hat{\epsilon}_2} = 15.4 + j15.667$

$\hat{\rho} = \frac{\hat{\eta}_2 - \eta_1}{\hat{\eta}_2 + \eta_1} = 0.979\angle 178.83^\circ$

$\hat{\tau} = 1 + \hat{\rho} = 0.029\angle 43.93^\circ$

$\tilde{\vec{E}}_i = 100\,e^{-j0.32z}\,\vec{a}_x$ V/m

$\tilde{\vec{H}}_i = \frac{100}{377}e^{-j0.32z}\,\vec{a}_y$ A/m

$\langle\hat{S}_i\rangle = \frac{1}{2}Re\left[\tilde{\vec{E}}_i\times\tilde{\vec{H}}_i^{*}\right] = 13.26\,\vec{a}_z$ W/m^2

$\tilde{\vec{E}}_r = 97.9\,e^{+j0.32z}\,e^{j178.83^\circ}\,\vec{a}_x$

$\tilde{\vec{H}}_r = -\frac{97.9}{377}e^{j0.32z}\,e^{j178.83^\circ}\,\vec{a}_y$

$\langle\hat{S}_r\rangle = -12.71\,\vec{a}_z$ W/m^2

$\tilde{\vec{E}}_t = 2.9\,e^{-15.4z}\,e^{-j15.667z}\,e^{j43.93^\circ}\,\vec{a}_x$

$\tilde{\vec{H}}_t = \frac{2.9}{5.491}e^{-15.4z}\,e^{-j15.667z}\,e^{j43.93^\circ}\,e^{-j44.51^\circ}\,\vec{a}_y$

$\langle\hat{S}_t\rangle_z = \frac{2.9^2}{2\times5.491}e^{-30.8z}\cos(44.51^\circ - 43.93^\circ)$

$\langle\hat{S}_t\rangle = 0.77\,e^{-30.8z}\,\vec{a}_z$ W/m^2

$\delta_2 = \frac{1}{\alpha_2} \Rightarrow \quad \delta_2 \cong 6.5$ cm

Problem 8.1 $\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$ $\quad\nabla\times\vec{H} = \epsilon\frac{\partial\vec{E}}{\partial t}$ $\quad\nabla\cdot\vec{B}=0$ $\quad\nabla\cdot\vec{D}=0 \Rightarrow \nabla\cdot\vec{E}=0$

$$\nabla\times\nabla\times\vec{E} = -\mu\frac{\partial}{\partial t}(\nabla\times\vec{H}) \Rightarrow \cancel{\nabla(\nabla\cdot\vec{E})}^{\,0} - \nabla^2\vec{E} = -\mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

Thus: $\nabla^2\vec{E} = \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$ $\quad$ Likewise $\quad\nabla^2\vec{H} = \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2}$

Problem 8.2 $\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$ $\quad\nabla\times\vec{H} = \sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t}$ $\quad\nabla\cdot\vec{B}=0$

$\nabla\cdot\vec{D}=\rho \Rightarrow$

$\nabla\cdot\vec{E} = \rho/\epsilon$

$$\nabla\times\nabla\times\vec{E} = -\mu\frac{\partial}{\partial t}(\nabla\times\vec{H}) \Rightarrow$$

$$\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} \quad \text{or}$$

$$\nabla^2\vec{E} - \mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} = \frac{1}{\epsilon}\nabla\rho \quad (1)$$

$$\nabla\times\nabla\times\vec{H} = \sigma\nabla\times\vec{E} + \epsilon\frac{\partial}{\partial t}(\nabla\times\vec{E}) \Rightarrow \nabla(\nabla\cdot\vec{H}) - \nabla^2\vec{H} = -\mu\sigma\frac{\partial\vec{H}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2}$$

or
$$\nabla^2\vec{H} - \mu\sigma\frac{\partial\vec{H}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2} = 0 \quad (2)$$

Problem 8.3 $\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$ $\quad\nabla\cdot\vec{B}=0$ $\quad\nabla\cdot\vec{D}=0 \Rightarrow \nabla\cdot\vec{E}=0,\ \nabla\times\vec{H}\cong\sigma\vec{E}$

$$\left.\begin{array}{l}\nabla\times\nabla\times\vec{E} = -\mu\frac{\partial}{\partial t}\nabla\times\vec{H} \Rightarrow \nabla^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t}\\ \nabla\times\nabla\times\vec{H} = \sigma\nabla\times\vec{E} \Rightarrow \nabla^2\vec{H} = \mu\sigma\frac{\partial\vec{H}}{\partial t}\end{array}\right\}\ \text{similar expressions}$$

Problem 8.4 $\vec{E} = E_0\cos\omega t\cos\beta z\ \vec{a}_x$

$$\frac{\partial^2\vec{E}}{\partial z^2} = -\beta^2E_0\cos\omega t\cos\beta z\ \vec{a}_x \qquad \frac{\partial^2\vec{E}}{\partial t^2} = -\omega^2E_0\cos\omega t\cos\beta z\ \vec{a}_x$$

For $\vec{E}$-field to exist: $\nabla^2\vec{E} = \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} \Rightarrow \beta^2 = \omega^2\mu\epsilon$ or $\beta = \pm\omega\sqrt{\mu\epsilon}$

$\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} \Rightarrow$

$$\mu\frac{\partial\vec{H}}{\partial t} = -\frac{\partial}{\partial z}E_x\,\vec{a}_y \quad\text{or}\quad \frac{\partial\vec{H}}{\partial t} = \frac{\beta E_0}{\mu}\cos\omega t\sin\beta z\ \vec{a}_y$$

Integrate w.r.t. t: $\vec{H} = \frac{\beta E_0}{\omega\mu}\sin\omega t\sin\beta z\ \vec{a}_y$

Thus:

$$\vec{E} = \frac{1}{2}\left[E_0\cos(\omega t-\beta z) + \cos(\omega t+\beta z)\right]\vec{a}_x$$

$$\vec{H} = \frac{1}{2}\frac{\beta}{\omega\mu}E_0\left[\cos(\omega t-\beta z) - \cos(\omega t+\beta z)\right)\vec{a}_y$$

Problem 8.5 $E_x = F_x(t + \sqrt{\mu\epsilon}\, z)$

$\nabla \times \vec{E} = \vec{a}_y \frac{\partial E_x}{\partial z} = \vec{a}_y \frac{\partial F_x}{\partial(t+\sqrt{\mu\epsilon}\, z)} \sqrt{\mu\epsilon}$

Since $\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \Rightarrow \frac{\partial \vec{H}}{\partial t} = -\sqrt{\frac{\epsilon}{\mu}} \frac{\partial F_x}{\partial(t+\sqrt{\mu\epsilon}\, z)} \vec{a}_y$ ①

However, $\frac{\partial \vec{H}}{\partial t} = \frac{\partial \vec{H}}{\partial(t+\sqrt{\mu\epsilon}\, z)}$ ②. From ① and ② $H_y = -\sqrt{\frac{\epsilon}{\mu}} F_x$

$\vec{S} = \vec{E} \times \vec{H} = -\sqrt{\frac{\epsilon}{\mu}}\, F_x^2\, \vec{a}_z$

Problem 8.6 Let there be an x-component of $\vec{E}$-field: $E_x = F(t - z/u)$

$\frac{\partial E_x}{\partial z} = -\frac{1}{u} \frac{\partial F}{\partial(t - z/u)}$ $\qquad \frac{\partial^2 E_x}{\partial z^2} = \frac{1}{u^2} \frac{\partial^2 F}{\partial(t - z/u)^2}$

$\frac{\partial E_x}{\partial t} = \frac{\partial F}{\partial(t - z/u)}$ $\qquad \frac{\partial^2 E_x}{\partial t^2} = \frac{\partial^2 F}{\partial(t - z/u)^2}$

$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$n = \sqrt{\mu_r \epsilon_r}$

For $\frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2} \Rightarrow \frac{1}{u^2} = \mu\epsilon$ or $u = \pm\sqrt{\frac{1}{\mu\epsilon}} = \pm\frac{1}{\sqrt{\mu_0\epsilon_0}} \frac{1}{\sqrt{\mu_r \epsilon_r}} = \pm \frac{c}{n}$

Problem 8.7 $\tilde{H} = 100\, e^{j\beta_0 z} \vec{a}_x$ A/m, $\quad \eta_0 = 120\pi \cong 377\,\Omega$

$\tilde{E} = 37{,}700\, e^{j\beta_0 z} \vec{a}_y$ V/m $\qquad \vec{u} = -3\times10^8 \vec{a}_z$ m/s $\qquad \omega = 30{,}000$ rad/s

$\beta_0 = \frac{\omega}{c} = 0.0001$ (rad/m) $\Rightarrow \lambda_0 = \frac{2\pi}{\beta_0} = 62.83$ km

$\langle \hat{S} \rangle = -\frac{1}{2} \times 37{,}700 \times 100\, \vec{a}_z$

$= 1.885\, \vec{a}_z$ MW/m^2

Problem 8.8 $f = 100$ MHz $\quad \omega = 2\pi f = 6.28\times10^8$ rad/s $\quad \mu_r = 1 \quad \epsilon_r = 4$

$\tilde{E}_x = 500\, e^{-j\beta y}$ V/m $\qquad u_p = \frac{c}{n} = 1.5\times10^8$ m/s $\qquad n = \sqrt{\mu_r \epsilon_r} = 2$

$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{\frac{\mu_r}{\epsilon_r}} = 60\pi\,\Omega$ $\qquad \beta = \frac{\omega}{u_p} = 4.19$ rad/m

$\tilde{H}_z = -\frac{500}{60\pi} e^{-j\beta y}$ A/m

$\langle \hat{S} \rangle = \frac{1}{2} \mathrm{Re}[\tilde{E} \times \tilde{H}^*] = 663.15\, \vec{a}_y$ W/m

$\langle P \rangle = \int_S \langle \hat{S} \rangle \cdot \vec{ds} = 1.06$ W

Problem 8.9 Assume $\tilde{E} = \tilde{E}_x \vec{a}_x$ and $\vec{u}_p = u_p \vec{a}_z$ $\qquad \mu_r = 1 \quad \epsilon_r = 2.5$

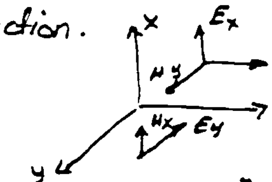
Then $\tilde{E} = 100\, e^{-j\beta z}\, \vec{a}_x$ kV/m $\qquad n = \sqrt{\mu_r \epsilon_r} = 1.581 \quad u_p = \frac{c}{n} = 1.897 \times 10^8$ m/s

$\tilde{H} = 0.42\, e^{-j\beta z}\, \vec{a}_y$ kA/m. $\qquad \beta = \frac{\omega}{u_p}, \; \eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{2.5}} = 238.43\ \Omega$

$\langle \hat{S} \rangle = \frac{1}{2} \times 100 \times 0.42\, \vec{a}_z = 21\, \vec{a}_z$ MW/m^2

Problem 8.10 Since $\vec{E} = E_m \cos(\omega t - \beta z)\vec{a}_x - E_m \sin(\omega t - \beta z)\vec{a}_y$,

the wave propagates in the z-direction.



Thus: $\vec{H} = \frac{E_m}{\eta}\cos(\omega t - \beta z)\vec{a}_y + \frac{E_m}{\eta}\sin(\omega t - \beta z)\vec{a}_x$

$$\vec{S} = \vec{E} \times \vec{H} = \left[\frac{E_m^2}{\eta}\cos^2(\omega t - \beta z) + \frac{E_m^2}{\eta}\sin^2(\omega t - \beta z)\right]\vec{a}_z = \frac{E_m^2}{\eta}\vec{a}_z \qquad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

Problem 8.11 $\nabla \times \tilde{E} = -j\omega\mu\tilde{H}, \quad \nabla \times \tilde{H} = j\omega\hat{\epsilon}\tilde{E} \qquad \hat{\epsilon} = \epsilon\left[1 - j\frac{\sigma}{\omega\epsilon}\right]$

$\nabla \times \nabla \times \tilde{E} = -j\omega\mu(\nabla \times \tilde{H})$ $\qquad \nabla \cdot \tilde{B} = 0 \Rightarrow \nabla \cdot \tilde{H} = 0$

$\nabla(\nabla \cdot \tilde{E}) - \nabla^2\tilde{E} = (-j\omega\mu)(j\omega\hat{\epsilon})\tilde{E}$ $\qquad \nabla \cdot \tilde{D} = 0 \Rightarrow \nabla \cdot \tilde{E} = 0$

$\nabla^2\tilde{E} = \hat{\gamma}^2\tilde{E} \qquad \hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}}$

Similarly, $\nabla^2\tilde{H} = \hat{\gamma}^2\tilde{H}$

Problem 8.12 $\nabla \times \tilde{H} = \sigma\tilde{E} + j\omega\epsilon\tilde{E} = \sigma\left[1 + j\frac{\omega\epsilon}{\sigma}\right]\tilde{E}$

In a highly conductive medium, $\omega\epsilon \ll \sigma$, we can write above equation as $\nabla \times \tilde{H} \cong \sigma\tilde{E}$

Thus, $\nabla \times \nabla \times \tilde{H} = \sigma(\nabla \times \tilde{E})$ $\qquad$ Since $\nabla \times \tilde{E} = -j\omega\mu\tilde{H}$

$\cancel{\nabla(\nabla \cdot \tilde{H})}^{\,0} - \nabla^2\tilde{H} = -j\omega\mu\sigma\tilde{H}$ $\qquad \nabla \cdot \tilde{B} = 0 \Rightarrow \nabla \cdot \tilde{H} = 0$

Thus, $\nabla^2\tilde{H} = j\omega\mu\sigma\tilde{H} = \hat{\gamma}^2\tilde{H}$ where $\hat{\gamma} = \sqrt{j\omega\mu\sigma}$

$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$ Np/m. $\qquad = \sqrt{\omega\mu\sigma}\,\angle 45^\circ$

$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$ rad/m. $\qquad = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$

$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$ m $\qquad u_p = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}$ m/s $\qquad = \alpha + j\beta$

Problem 8.13 $f = 50\,MHz$ $\omega = 2\pi f = 3.142 \times 10^8$ rad/s $\epsilon_r = 16$ $\sigma = 0.02$ S/m

$\hat{\epsilon} = \epsilon\left[1 - j\frac{\sigma}{\omega\epsilon}\right] = \epsilon[1 - j0.45]$ $\hat{\gamma} = j\omega\sqrt{\mu_0\epsilon_0}\sqrt{\mu_r\epsilon_r}\sqrt{1-j0.45} = j4.387\angle{-12.12^\circ}$

$= 4.387\angle 77.88^\circ = 0.921 + j4.289$

$\hat{\eta} = \sqrt{\frac{\mu_0}{\hat{\epsilon}}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{\frac{1}{\epsilon_r}}\sqrt{\frac{1}{1-j0.45}}$

$\alpha = 0.921$ Np/m $\beta = 4.289$ rad/m

$= 90\angle 12.12^\circ\ \Omega$

$\delta = \frac{1}{\alpha} = 1.086$ m

$u_p = \frac{\omega}{\beta} = 7.325 \times 10^7$ m/s

$\lambda = \frac{2\pi}{\beta} = 1.465$ m $e^{-\alpha d} = 0.1 \Rightarrow d = 2.5$ m

$\tilde{E}_x = 120\, e^{-\alpha z} e^{-j\beta z}$, $\tilde{H}_y = \frac{120}{90} e^{-\alpha z} e^{-j\beta z} e^{-j12.12^\circ}$

$\langle\hat{S}\rangle = \frac{1}{2}\ \frac{120^2}{90}\ e^{-2\alpha z} \cos(12.12^\circ)\, \vec{a}_z = 78.22\, e^{-2\alpha z}\, \vec{a}_z$ W/m²

Problem 8.14 $f = 10$ kHz $\omega = 2\pi f = 6.283 \times 10^4$ rad/s $\sigma = 0.01$ S/m $\epsilon_r = 9$, $\mu_r = 4$

$\frac{\sigma}{\omega\epsilon} = 2000$ very good conductor. $\hat{\epsilon} = \epsilon\left[1 - j\frac{\sigma}{\omega\epsilon}\right] = 7.958 \times 10^{-11} - j1.592 \times 10^{-7}$

$\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = 0.04 + j0.04$ $\alpha = 0.04$ Np/m $\beta = 0.04$ rad/m $\delta = \frac{1}{\alpha} = 25$ m

$\hat{\eta} = \sqrt{\mu/\hat{\epsilon}} = 5.62\angle 44.99^\circ\,\Omega$, $u_p = \omega/\beta = 1.57 \times 10^6$ m/s $\lambda = \frac{2\pi}{\beta} = 157.08$ m

$\tilde{E}_x = 100\, e^{-\alpha z} e^{-j\beta z}$ V/m, $\tilde{H}_y = \frac{100}{5.62} e^{-\alpha z} e^{-j\beta z} e^{-j44.99^\circ}$ A/m

$\langle\hat{S}\rangle = \frac{1}{2} \times \frac{100^2}{5.62}\ e^{-2\alpha z} \cos(44.99^\circ)\, \vec{a}_z = 629.27\, e^{-2\alpha z}\, \vec{a}_z$ W/m²

Problem 8.15 $\alpha = 77.485$ Np/m $\beta = 203.8$ rad/m, $\hat{\gamma} = 77.485 + j203.8$ m⁻¹

$\hat{\epsilon} = -\frac{(\alpha + j\beta)^2}{\omega^2\mu_0} = (7.162 - j6.366)\, 10^{-10}$ $\omega = 2\pi \times 10^9$ rad/s

$\epsilon = 7.162 \times 10^{-10} \Rightarrow \epsilon_r = \epsilon/\epsilon_0 \cong 81$ $\frac{\sigma}{\omega} = 6.366 \times 10^{-10} \Rightarrow \sigma = 4$ S/m

Propagation in y-direction: $\hat{\eta} = \sqrt{\frac{\mu_0}{\hat{\epsilon}}} = 36.213\angle 20.82^\circ\,\Omega$

$\left.\begin{array}{l} \tilde{H}_x = 0.1\, e^{-\alpha y} e^{-j\beta y} \text{ A/m} \\ \tilde{E}_z = 3.621\, e^{-\alpha y} e^{-j\beta y} e^{j20.82^\circ} \text{ V/m} \end{array}\right\} \Rightarrow \langle\hat{S}\rangle = \frac{1}{2} \times 0.1 \times 3.621\, e^{-2\alpha y} \cos(20.82^\circ)\, \vec{a}_y$

$= 0.169\, e^{-154.97y}\, \vec{a}_y$ W/m²

Problem 8.16 $f = 10\,kHz$ $\omega = 2\pi f = 6.283\times10^{4}$ rad/s $\mu_r = 1$, $\epsilon_r = 1$

$\sigma = 5.8\times10^{7}$ S/m $\frac{\sigma}{\omega\epsilon} = 1.044\times10^{14}$ $\hat{\epsilon} \cong -j\frac{\sigma}{\omega} = -j923.099$

$\hat{\gamma} = j\omega\sqrt{\mu_o\hat{\epsilon}} = 1513 + j1513\ m^{-1}$ $\alpha = 1513$ Np/m $\beta = 1513$ rad/m

$\hat{\eta} = \sqrt{\mu_o/\hat{\epsilon}} = 3.69\times10^{-5}\angle 45^\circ\ \Omega$ $\delta = 1/\alpha = 6.609\times10^{-4}$ m $\cong 0.661$ mm

$\tilde{E}_x = 100\, e^{-1513z} e^{-j1513z}$ V/m $\tilde{H}_y = 2.71\times10^{6}\, e^{-1513z} e^{-j1513z} e^{-j45^\circ}$ A/m

$\tilde{\vec{J}} = \sigma\tilde{\vec{E}} \Rightarrow \tilde{J}_x = 5.8\times10^{9}\, e^{-1513z} e^{-j1513z}$ A/m²

at $z = 0.2\delta$, $E_x = 81.87\cos(\omega t - 11.46^\circ)$ V/m. $J_x = 4.75\times10^{9}\cos(\omega t - 11.46^\circ)\ \frac{A}{m^2}$

$H_y = 2.219\times10^{6}\cos(\omega t - 56.46^\circ)$ A/m

Problem 8.17 $\tilde{H}_x = 0.1\, e^{-15z} e^{-j15z}$ A/m, $\alpha = \beta = 15 \Rightarrow$ Very good conductor ($\epsilon \to \epsilon_0$, $\mu \to \mu_o$)

$\omega = 2\pi\times10^{8}$ rad/s $\sigma/\omega\epsilon \gg 1 \Rightarrow \hat{\epsilon} \cong \frac{\sigma}{j\omega}$

$\hat{\gamma} = j\omega\sqrt{\mu_o\hat{\epsilon}} = \sqrt{j\omega\mu_o\sigma} \Rightarrow \alpha = \sqrt{\frac{\omega\mu_o\sigma}{2}}$ or $\sigma = \frac{2\alpha^2}{\omega\mu_o} = 0.57$ S/m, $\hat{\epsilon} = \frac{\sigma}{j\omega} = -j9.071\times10^{-10}$

$\hat{\eta} = \sqrt{\mu_o/\hat{\epsilon}} = 37.221\angle 45^\circ \Rightarrow \tilde{E}_y = -3.722\, e^{-15z} e^{-j15z} e^{j45^\circ}$ V/m

$\langle\hat{S}\rangle = \frac{1}{2}\times 3.722\times0.1\cos45^\circ\, e^{-30z}\,\vec{a}_z = 0.132\, e^{-30z}\,\vec{a}_z$ W/m²

at $z=0$ $\langle\hat{S}\rangle_{z=0} = 0.132\ \frac{W}{m^2}$, at $z=\delta$: or $z = 1/\alpha$ $\langle\hat{S}\rangle\big|_{z=\delta} = 0.018$ W/m²

Hence, power lost per unit area: $0.132 - 0.018 = 0.114$ W/m²

Problem 8.18 $\hat{\epsilon} = \epsilon[1 - j\tan\delta]$ where $\tan\delta = \frac{\sigma}{\omega\epsilon}$

$$\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} = \sqrt{\frac{\mu}{\epsilon}}\sqrt{\frac{1}{1-j\tan\delta}} \qquad 1 - j\tan\delta = \sqrt{1+\tan^2\delta}\,\angle -\delta$$

$$= \sqrt{\frac{\mu}{\epsilon\sec\delta}}\ \angle -\delta/2 \qquad\qquad = \sec\delta\,\angle -\delta$$

If $\hat{\eta} = \eta\angle\theta_\eta$, then $\eta = \sqrt{\frac{\mu}{\epsilon\sec\delta}}$ and $\theta_\eta = \frac{\delta}{2}$.

Problem 8.19

$f = 20\,MHz$, $\omega = 2\pi f = 40\pi \times 10^6$ rad/s

When $z=0$ $|\tilde{E}| = E_0$, when $z = 1\,m$, $|\tilde{E}| = 0.8E_0 \Rightarrow e^{-\alpha(1)} = 0.8 \Rightarrow \alpha = 0.223$ Np/m

$$\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} = \sqrt{\frac{\mu}{|\hat{\epsilon}|}}\angle 20^\circ \Rightarrow \hat{\epsilon} = |\hat{\epsilon}|\angle -40^\circ$$

$$\alpha = Re[\hat{\gamma}] = Re[j\omega\sqrt{\mu\hat{\epsilon}}] = Re[j\omega\sqrt{\mu|\hat{\epsilon}|}\angle -20^\circ]$$

$$= \omega\sqrt{\mu|\hat{\epsilon}|}\cos 70^\circ \Rightarrow \sqrt{|\hat{\epsilon}|} = \frac{0.223}{\cos 78\ \omega\sqrt{\mu_0}} \Rightarrow |\hat{\epsilon}| = 2.1423 \times 10^{-11}$$

$$\hat{\epsilon} = 2.1423 \times 10^{-11}\angle -40^\circ$$

$$= (1.641 - j\,1.377)10^{-11} \Rightarrow \epsilon_r = \frac{1.641 \times 10^{-11}}{\epsilon_0} = 1.856$$

$$\frac{\sigma}{\omega} = 1.377 \times 10^{-11} \Rightarrow \sigma = 1.73 \times 10^{-3}\ S/m$$

Problem 8.20

$\hat{\eta} = 60\pi\angle 30^\circ\ \Omega$ Thus, $\hat{\eta} = \sqrt{\frac{\mu_0}{\hat{\epsilon}}} \Rightarrow \hat{\epsilon} = \frac{\mu_0}{\hat{\eta}^2} = [1.768 - j3.063]\,10^{-11}$

$\mu = \mu_0$ $\quad \eta = 60\pi$

$$\hat{\gamma} = j\omega\sqrt{\mu_0\hat{\epsilon}} = j\frac{\omega\mu_0}{\hat{\eta}} = \frac{\omega\mu_0}{\eta}\cos 60^\circ + j\,\frac{\omega\mu_0}{\eta}\sin 60^\circ \qquad \epsilon_r = \frac{1.768 \times 10^{-11}}{\epsilon_0} = 2$$

since $\beta = 1.2$ rad/s, $\Rightarrow \omega = \frac{\beta\eta}{\mu_0 \sin 60^\circ} = 207.846 \times 10^6$ rad/s or $f = 33.08$ MHz

$\alpha = \frac{\omega\mu_0}{\eta}\cos 60^\circ = 0.693$ Np/m

Attenuation in dBs: $20\log_{10} e^{0.693} = 20 \times 0.693 \log_{10} e = 20 \times 0.693 \times 0.4343$
$= 6.02$ dB/m

Problem 8.21

$\mu_r = 1$ $\quad \epsilon_r = 16$ $\quad \sigma = 0.02$ S/m $\quad f = 500$ kHz

$\omega = 3.142 \times 10^6$ rad/s

$$\hat{\epsilon} = \epsilon\left[1 - j\frac{\sigma}{\omega\epsilon}\right] = 1.415 \times 10^{-10} - j\,6.366 \times 10^{-9}$$

$\hat{\gamma} = j\omega\sqrt{\mu\hat{\epsilon}} = 0.196 + j\,0.201\ m^{-1}$ $\quad \alpha = 0.196$ Np/m, $\beta = 0.201$ rad/m

$\delta = \frac{1}{\alpha} = 5.09\,m$

$u_p = \frac{\omega}{\beta} = 1.564 \times 10^7$ m/s

$\hat{\eta} = \sqrt{\frac{\mu}{\hat{\epsilon}}} = 14.048\angle 44.36^\circ\ \Omega$

$\tilde{E} = 120\,e^{-0.196z}\,e^{-j0.201z}\,\vec{a}_x$ V/m

$\tilde{H} = \frac{120}{14.048}\,e^{-0.196z}\,e^{-j0.201z}\,e^{-j44.36^\circ}\,\vec{a}_y$ A/m

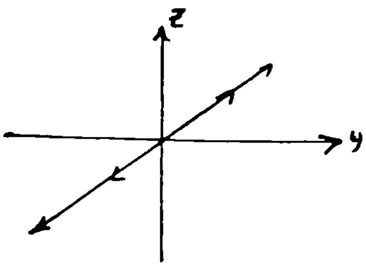
$\langle \vec{S} \rangle = \frac{1}{2}Re[\tilde{E} \times \tilde{H}^*] = 366.4\ e^{-0.392z}\ \vec{a}_z$ W/m^2

At $z = d$, only 10% is left $\Rightarrow$ $e^{-0.196d} = 0.1$ or $d = 11.75$ m

Problem 8.22 a) At $x=0$

$$\left.\begin{array}{l} E_y = 100\cos\omega t \\ E_z = 100\cos\omega t \end{array}\right\} \Rightarrow E_y = E_z$$

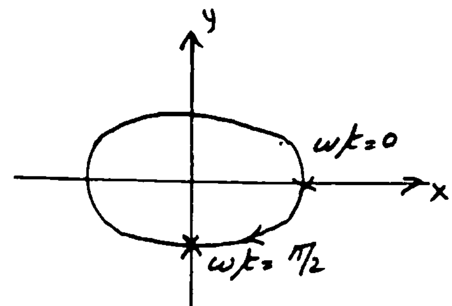
Linear polarization



b. When $100z = \pi/4$

$$\left.\begin{array}{l} E_x = 16\cos\omega t \\ E_y = -9\sin\omega t \end{array}\right\} \Rightarrow \left(\frac{E_x}{16}\right)^2 + \left(\frac{E_y}{9}\right)^2 = 1$$

(Elliptical)

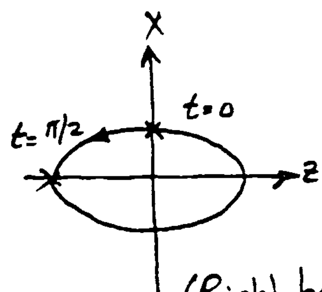


(Left-handed).

c) In a $y=0$ plane:

$$\left.\begin{array}{l} E_x = 3\cos t \\ E_z = -4\sin t \end{array}\right\} \Rightarrow \left(\frac{E_x}{3}\right)^2 + \left(\frac{E_z}{4}\right)^2 = 1$$

Elliptical



(Right-handed)

Problem 8.23

$\omega = 2\times10^8$ rad/s

$\epsilon_r = 2.5$

$\lambda = \frac{2\pi}{\beta} = 5.96\,m$

$\beta = \frac{\omega}{c}\sqrt{\epsilon_r} = 1.054$ rad/m

$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = 238.43\,\Omega$

Use Superposition Theorem:

$\tilde{E}_x = 12e^{-j\beta z} \Rightarrow \tilde{H}_y = \frac{12}{\eta}e^{-j\beta z}$

$\tilde{E}_y = j5e^{-j\beta z} \Rightarrow \tilde{H}_x = -j\frac{5}{\eta}e^{-j\beta z}$

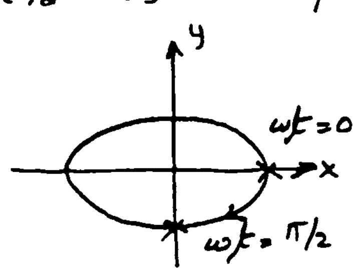
$\vec{\tilde{H}} = \frac{1}{\eta}\left[12e^{-j\beta z}\vec{a}_y - j5e^{-j\beta z}\vec{a}_x\right]$

$\vec{H} = 0.05\cos(\omega t - \beta z)\vec{a}_y$

$+ 0.02\sin(\omega t - \beta z)\vec{a}_x$ A/m

at $z=0$ $E_x = 12\cos\omega t$, $E_y = -5\sin\omega t$

$\left(\frac{E_x}{12}\right)^2 + \left(\frac{E_y}{5}\right)^2 = 1$ Elliptical.



Left-handed

Problem 8.24

$$\tilde{E} = E_0 e^{-j\beta z}\vec{a}_x + E_0 e^{-j\beta z}\vec{a}_y$$

$$= \left(a e^{-j\beta z}\vec{a}_x + ja\, e^{-j\beta z}\vec{a}_y\right) + \left(b e^{-j\beta z}\vec{a}_x - jb e^{-jbz}\vec{a}_y\right)$$

Hence: $a + b = E_0$ and $a - b = -jE_0 \Rightarrow a = \frac{E_0}{\sqrt{2}}\angle{-45^\circ}$ $\quad b = \frac{E_0}{\sqrt{2}}\angle 45^\circ$

Thus: $\vec{E} = \frac{E_0}{\sqrt{2}}\Big[\left(\cos(\omega t - \beta z - 45^\circ)\,\vec{a}_x + \cos(\omega t - \beta z + 45^\circ)\,\vec{a}_y\right)$

$$+ \left(\cos(\omega t - \beta z + 45^\circ)\vec{a}_x + \cos(\omega t - \beta z - 45^\circ)\,\vec{a}_y\right)\Big]$$

In a $\beta z = 45^\circ$ plane:

$$\vec{E} = \frac{E_0}{\sqrt{2}}\left[\sin\omega t\,\vec{a}_x + \cos\omega t\,\vec{a}_y\right] + \frac{E_0}{\sqrt{2}}\left[\cos\omega t\,\vec{a}_x + \sin\omega t\,\vec{a}_y\right]$$

Right-handed Circularly polarized wave $\quad$ Left-handed Circularly polarized

Problem 8.25

$$\tilde{E} = 3E_0 e^{-j\beta z}\vec{a}_x - j4E_0 e^{-j\beta z}\vec{a}_y$$

$$= a e^{-j\beta z}\vec{a}_x - ja\, e^{-j\beta z}\vec{a}_y + b e^{-j\beta z}\vec{a}_x + jb e^{-j\beta z}\vec{a}_y$$

Thus $\quad a + b = 3E_0$ and $a - b = 4E_0 \Rightarrow a = 3.5E_0$ and $b = -0.5E_0$

Time Domain:

$$\vec{E} = 3.5E_0\left[\cos(\omega t - \beta z)\,\vec{a}_x + \sin(\omega t - \beta z)\vec{a}_y\right]$$

$$+ 0.5E_0\left[-\cos(\omega t - \beta z)\,\vec{a}_x + \sin(\omega t - \beta z)\,\vec{a}_y\right]$$

In a $\beta z = 0$ plane:

$$\vec{E} = 3.5E_0 \cos\omega t\,\vec{a}_x + 3.5E_0 \sin\omega t\,\vec{a}_y$$ ← Right-handed Circularly polarized wave

$$+ 0.5E_0 \cos\omega t\,\vec{a}_x + 0.5E_0 \sin\omega t\,\vec{a}_y$$ ← Left-handed Circularly polarized wave

The two waves have different amplitudes.

Problem 8.26 $\quad f = 100\ MHz \quad \omega = 2\pi f = 6.283\times10^{8}\ rad/s$

$\epsilon_{r1} = 2.25 \quad \mu_{r1} = 1 \quad \sigma_1 = 2\ mS/m \quad \tan\delta_1 = \frac{\sigma_1}{\omega\epsilon_1} = 0.16$

$\epsilon_{r2} = 1 \quad \mu_{r2} = 1 \quad \sigma_2 = 20\ mS/m \quad \tan\delta_2 = \frac{\sigma_2}{\omega\epsilon_2} = 3.6$

$\hat{\gamma}_1 = j\frac{\omega}{c}\sqrt{\mu_{r1}\epsilon_{r1}}\sqrt{1-j\tan\delta_1} \quad 0.251 + j3.152\ m^{-1} \quad \alpha_1 = 0.251\ Np/m,\ \beta_1 = 3.152\ rad/m$

$\hat{\gamma}_2 = j\frac{\omega}{c}\sqrt{\mu_{r2}\epsilon_{r2}}\sqrt{1-j\tan\delta_2} \quad 2.45 + j3.223\ m^{-1},\ \alpha_2 = 2.45\ \frac{Np}{m},\ \beta_2 = 3.223\ rad/m$

$\hat{\eta}_1 = \sqrt{\frac{\mu_1}{\hat{\epsilon}_1}} = 120\pi\sqrt{\frac{\mu_{r1}}{\epsilon_{r1}}}\sqrt{\frac{1}{1-j\tan\delta_1}} = 249.744\angle 4.55^\circ\ \Omega,\quad \hat{\eta}_2 = 120\pi\sqrt{\frac{\mu_{r2}}{\epsilon_{r2}}}\sqrt{\frac{1}{1-j\tan\delta_2}} = 195.634\angle 37.24^\circ\ \Omega$

$\hat{\rho} = \frac{\hat{\eta}_2 - \hat{\eta}_1}{\hat{\eta}_2 + \hat{\eta}_1} = 0.318\angle 114.82^\circ, \quad \hat{\tau} = \frac{2\hat{\eta}_2}{\hat{\eta}_1 + \hat{\eta}_2} = 0.913\angle 18.41^\circ$

$\tilde{\vec{E}}_i = 10\, e^{-0.251z} e^{-j3.152z}\, \vec{a}_x\ V/m, \quad \tilde{\vec{H}}_i = \frac{10}{249.744} e^{-0.251z} e^{-j3.152z} e^{-j4.55^\circ}\vec{a}_y\ A/m$

$\tilde{\vec{E}}_r = 3.178\, e^{0.251z} e^{j3.152z} e^{j114.82^\circ}\vec{a}_x\ V/m, \quad \tilde{\vec{H}}_r = -\frac{3.178}{249.744} e^{0.251z} e^{j3.152z} e^{j114.82^\circ} e^{-j4.55^\circ}\vec{a}_y\ \frac{A}{m}$

$$\left.\begin{aligned} \tilde{\vec{E}}_t &= 9.133\, e^{-2.45z} e^{-j3.223z} e^{j18.41^\circ}\vec{a}_x\ V/m \\ \tilde{\vec{H}}_t &= \frac{9.133}{195.634} e^{-2.45z} e^{-j3.223z} e^{j18.41^\circ} e^{-j37.24^\circ}\vec{a}_y\ A/m \end{aligned}\right\} \rightarrow \langle\hat{S}_t\rangle = \frac{1}{2}Re[\tilde{\vec{E}}_t \times \tilde{\vec{H}}_t^*] = 0.17\, e^{-4.9z}\vec{a}_z\ W/m^2$$

$\langle\hat{S}_i\rangle = \frac{1}{2}Re[\tilde{\vec{E}}_i \times \tilde{\vec{H}}_i^*] = 0.2\, e^{-0.502z}\vec{a}_z\ W/m^2, \quad \langle\hat{S}_r\rangle = -0.02\, e^{0.502z}\vec{a}_z\ W/m^2$

Problem 8.27 $\quad f = 200\ MHz \quad \omega = 2\pi f = 1.257\times10^{9}\ rad/s$

$\epsilon_{r1} = 1 \quad \mu_{r1} = 1 \quad \sigma_1 = 0.04\ S/m,\ \epsilon_{r2} = 1,\ \mu_{r2} = 1 \quad \sigma_2 = 4\ S/m$

$\hat{\epsilon}_1 = \epsilon_0\left[1 - j\frac{\sigma_1}{\omega\epsilon_0}\right] = 8.842\times10^{-12} - j3.183\times10^{-11},\quad \hat{\epsilon}_2 = \epsilon_0\left[1 - j\frac{\sigma_2}{\omega\epsilon_0}\right] = 8.842\times10^{-12} - j3.183\times10^{-9}$

$\hat{\gamma}_1 = j\omega\sqrt{\mu_0\hat{\epsilon}_1} = 4.9 + j6.446\ m^{-1} \qquad \hat{\gamma}_2 = j\omega\sqrt{\mu_0\hat{\epsilon}_2} = 56.121 + j56.277\ m^{-1}$

$\hat{\eta}_1 = \sqrt{\mu_0/\hat{\epsilon}_1} = 195.634\angle 37.24^\circ\ \Omega \qquad \hat{\eta}_2 = \sqrt{\mu_0/\hat{\epsilon}_2} = 19.869\angle 44.92^\circ\ \Omega$

$\hat{\rho} = \frac{\hat{\eta}_2 - \hat{\eta}_1}{\hat{\eta}_2 - \hat{\eta}_1} = 0.817\angle 178.42^\circ \qquad \hat{\tau} = \frac{2\hat{\eta}_2}{\hat{\eta}_1 + \hat{\eta}_2} = 0.185\angle 6.97^\circ$

$\tilde{E}_i = 50\, e^{-4.9z}\, e^{-j6.446z}\, \vec{a}_x$ V/m

$\tilde{H}_i = \frac{50}{195.034}\, e^{-4.9z}\, e^{-j6.446z}\, e^{-j37.24^\circ}\, \vec{a}_y$ A/m

$\langle \hat{S}_i \rangle = \frac{1}{2} Re[\tilde{E}_i \times \tilde{H}_i^*] = 5.103\, e^{-9.8z}\, \vec{a}_z$ W/m^2

$\tilde{E}_r = 40.831\, e^{4.9z}\, e^{j6.446z}\, e^{j178.42^\circ}\, \vec{a}_x$ V/m

$\tilde{H}_r = -\frac{40.831}{195.034}\, e^{4.9z}\, e^{j6.446z}\, e^{j178.42^\circ}\, e^{-j37.24^\circ}\, \vec{a}_y$ A/m

$\langle \hat{S}_r \rangle = \frac{1}{2} Re[\tilde{E}_r \times \tilde{H}_r^*] = -3.403\, e^{9.8z}\, \vec{a}_z$ $\frac{W}{m^2}$

$\tilde{E}_t = 9.253\, e^{-56.121z}\, e^{-j56.277z}\, e^{j6.97^\circ}\, \vec{a}_x$ V/m

$\tilde{H}_t = \frac{9.253}{19.869}\, e^{-56.121z}\, e^{-j56.277z}\, e^{j6.97^\circ}\, e^{-j44.92^\circ}\, \vec{a}_y$ A/m,

$\langle \hat{S}_t \rangle = \frac{1}{2} Re[\tilde{E}_t \times \tilde{H}_t^*] = 1.525\, e^{-112.242z}\, \vec{a}_z$ W/m^2

$$\delta_2 = \frac{1}{\alpha_2} = 17.82 \text{ mm.}$$

<u>Problem 8.28</u> $\omega = 90 \times 10^6$ rad/s $\sigma_2 = 0.4$ S/m $\mu_2 = \mu_0$ $\epsilon_2 = \epsilon_0$

$\eta_1 = 377\Omega$ $\beta_1 = \frac{\omega}{c} = .3$ rad/m $\tan\delta_2 = \frac{\sigma_2}{\omega\epsilon_2} = 6.206$

$\hat{\gamma}_2 = j\frac{\omega}{c}\sqrt{\mu_{r2}\epsilon_{r2}}\sqrt{1 - j\tan\delta_2} = 4.389 + j5.153$ m^{-1} $\hat{\eta}_2 = 16.708\angle 40.42^\circ\,\Omega$

$\hat{\rho} = \frac{\hat{\eta}_2 - \eta_1}{\hat{\eta}_2 + \eta_1} = 0.935\angle 176.7^\circ$, $\hat{\tau} = \frac{2\hat{\eta}_2}{\eta_1 + \hat{\eta}_2} = 0.086\angle 38.83^\circ$

$\tilde{E}_i = [-j100\,\vec{a}_x + 200\,\vec{a}_y]\, e^{-j0.3z}$ V/m

$\tilde{H}_i = \frac{-200\,\vec{a}_x - j100\,\vec{a}_y}{377}\, e^{-j0.3z}$ A/m

$\langle \hat{S}_i \rangle = \frac{1}{2} Re[\tilde{E}_i \times \tilde{H}_i^*] = 66.31\, \vec{a}_z$ W/m^2

$\tilde{E}_r = [-j93.5\,\vec{a}_x + 187\,\vec{a}_y]\, e^{j0.3z}\, e^{j176.7^\circ}$ V/m.

$\tilde{H}_r = \frac{(187\,\vec{a}_x + j93.5\,\vec{a}_y)}{377}\, e^{j0.3z}\, e^{j176.7^\circ}$ A/m

$\langle \hat{S}_r \rangle = \frac{1}{2} Re[\tilde{E}_r \times \tilde{H}_r^*] = -57.97\, \vec{a}_z$ W/m^2

$\tilde{E}_t = (-j8.6\,\vec{a}_x + 17.2\,\vec{a}_y)\, e^{-4.389z}\, e^{-j5.153^\circ z}\, e^{j38.83^\circ}$ V/m

$\tilde{H}_t = \frac{-17.2\,\vec{a}_x + j8.6\,\vec{a}_y}{16.708}\, e^{-4.389z}\, e^{-j5.153^\circ}\, e^{j38.83^\circ}\, e^{-j40.42^\circ}$ A/m

$\langle \hat{S}_t \rangle = \frac{1}{2} Re[\tilde{E}_t \times \tilde{H}_t^*] = 8.4\, e^{-8.78z}\, \vec{a}_z$ W/m^2

At the Interface $z = 0$: $\langle \hat{S}_i \rangle + \langle \hat{S}_r \rangle = 8.34\, \vec{a}_z$ W/m^2 $= \langle \hat{S}_t \rangle$

Problem 8.29 $\omega = 60\times10^6$ rad/s $\mu_{r1}=1$, $\mu_{r2}=1$, $\epsilon_{r1}=9$, $\epsilon_{r2}=1$

$\beta_1 = \frac{\omega}{c}\sqrt{\epsilon_{r1}} = 0.6$ rad/m $\beta_2 = \frac{\omega}{c} = 0.2$ rad/m $c = 3\times10^8$ m/s

$\eta_1 = \frac{120\pi}{\sqrt{\epsilon_{r1}}} = 125.664\ \Omega$ $\eta_2 = 120\pi \cong 377\Omega$

$\rho = \frac{\eta_2-\eta_1}{\eta_1+\eta_2} = 0.5$ $\tau = \frac{2\eta_2}{\eta_1+\eta_2} = 1.5$

$\tilde{\vec{E}}_i = -j150\, e^{-j0.6z}\,\vec{a}_x$ V/m $\langle\hat{S}_i\rangle = \frac{1}{2}\,\mathrm{Re}[\tilde{\vec{E}}_i\times\tilde{\vec{H}}_i^*] = 89.5\,\vec{a}_z$ W/m^2

$\tilde{\vec{H}}_i = -j\frac{150}{\eta_1}\, e^{-0.6z}\,\vec{a}_y$ A/m

$\tilde{\vec{E}}_r = -j75\, e^{j0.6z}\,\vec{a}_x$, $\frac{V}{m}$ $\tilde{\vec{E}}_t = -j225\, e^{-j0.2z}\,\vec{a}_x$ V/m

$\tilde{\vec{H}}_r = +j\frac{75}{\eta_1}\, e^{j0.6z}\,\vec{a}_y$ A/m $\tilde{\vec{H}}_t = -j\frac{225}{377}\, e^{-j0.2z}\,\vec{a}_y$ A/m

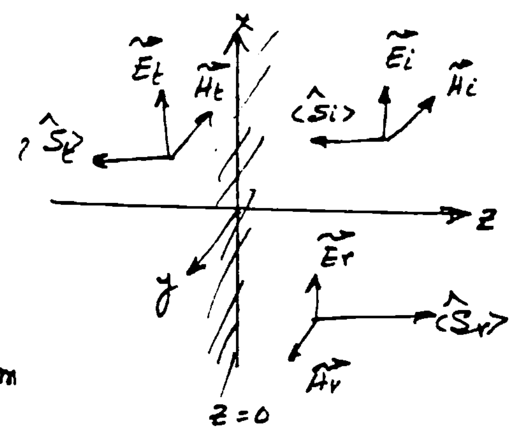
$\langle\hat{S}_r\rangle = -22.4\,\vec{a}_z$ W/m^2 $\langle\hat{S}_t\rangle = 67.1\,\vec{a}_z$ W/m^2

Problem 8.30 $\omega = 120\times10^6$ rad/s $c = 3\times10^8$ m/s, $\epsilon_{r1}=9$, $\epsilon_{r2}=1$, $\mu_{r1}=\mu_{r2}=1$

$\beta_1 = \frac{\omega}{c}\sqrt{\epsilon_{r1}} = 1.2$ rad/m, $\eta_1 = \frac{120\pi}{\sqrt{\epsilon_{r1}}} = 125.664\Omega$

$\beta_2 = \frac{\omega}{c} = 0.4$ rad/m, $\eta_2 = 120\pi \cong 377\Omega$

$\rho = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = 0.5$ $\tau = \frac{2\eta_2}{\eta_1+\eta_2} = 1.5$



$\tilde{\vec{E}}_i = -j50\, e^{j1.2z}\,\vec{a}_x$ V/m, $\tilde{\vec{H}}_i = j\frac{50}{\eta_1}e^{j1.2z}\,\vec{a}_y$ A/m.

$\tilde{\vec{E}}_r = -j25\, e^{-j1.2z}\,\vec{a}_x$ V/m, $\tilde{\vec{H}}_r = -j\frac{25}{\eta_1}\, e^{-j1.2z}\,\vec{a}_y$ A/m

$\tilde{\vec{E}}_t = -j75\, e^{j4z}\,\vec{a}_x$ V/m, $\tilde{\vec{H}}_t = j\frac{75}{377}\, e^{j4z}\,\vec{a}_y$ A/m

$\langle\hat{S}_i\rangle = \frac{1}{2}\mathrm{Re}[\tilde{E}_i\times\tilde{H}_i^*] = -9.95\,\vec{a}_z$ W/m^2

$\langle\hat{S}_r\rangle = \frac{1}{2}\mathrm{Re}[\tilde{E}_r\times\tilde{H}_r^*] = 2.49\,\vec{a}_z$ W/m^2

$\langle\hat{S}_t\rangle = \frac{1}{2}\mathrm{Re}[\tilde{E}_t\times\tilde{H}_t^*] = -7.46\,\vec{a}_z$ W/m^2

Problem 8.31

$\rho = 0.25 \Rightarrow \tau = 1.25$

Thus, $\frac{2\eta_2}{\eta_1+\eta_2} = 1.25$ or $0.75\eta_2 = 1.25\eta_1 \Rightarrow \frac{\eta_2}{\eta_1} = 1.667 = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \Rightarrow \frac{\epsilon_1}{\epsilon_2} = 2.278$

$\rho = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} = 0.25 \Rightarrow \frac{\eta_2/\eta_1 - 1}{\frac{\eta_2}{\eta_1} + 1} > 0$ ($\because$ $\rho > 0$). Thus, $\frac{\epsilon_1}{\epsilon_2} > 1$. or $\epsilon_1 > \epsilon_2$

If $\epsilon_2 = 1.25\epsilon_0$, Then $\epsilon_1 = 2.278\,\epsilon_2 = 3.47\epsilon_0$ Hence: $\epsilon_{r2} = 3.47$

Problem 8.32

$\rho = -0.75$ $\quad \tau = 0.25 = \frac{2\eta_2}{\eta_1+\eta_2} \Rightarrow \frac{\eta_2}{\eta_1} = 0.143 = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$

Hence $\frac{\epsilon_1}{\epsilon_2} = 0.02$

Since $\rho < 0$, $\frac{\eta_2}{\eta_1} < 1$ or $\sqrt{\frac{\epsilon_1}{\epsilon_2}} < 1 \Rightarrow \epsilon_2 > \epsilon_1$

Let $\epsilon_1 = 2.25\epsilon_0$, then $\epsilon_2 = 112.5\epsilon_0$.

Problem 8.33

$c = 3\times10^8$ m/s

$\beta_1\cos\theta_1 = 0.766$ $\quad \beta_1\sin\theta_1 = 0.643 \Rightarrow$

$\beta_1 = 1$ rad/m and $\theta_1 = 40^\circ$

$\omega = \beta_1 u_p = c\beta_1 = 3\times10^8$ rad/s

$\beta_2 = \frac{\omega}{c}\sqrt{\epsilon_{r2}} = 1.5$ rad/m.

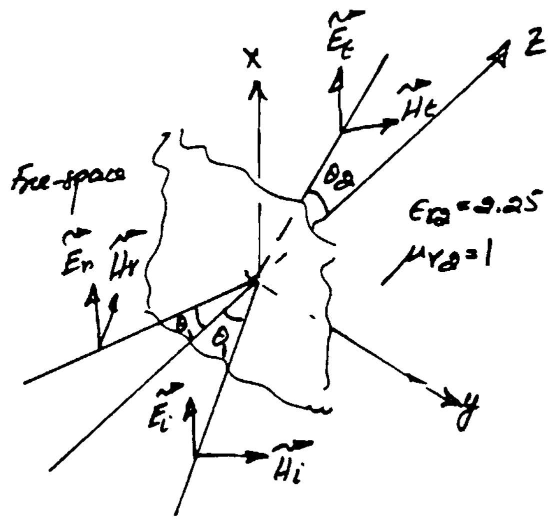
$\beta_1\sin\theta_1 = \beta_2\sin\theta_2 \Rightarrow \theta_2 = 25.38^\circ$

$\sin\theta_2 = 0.429$ $\quad \cos\theta_2 = 0.903$

$\eta_1 = 377\Omega$ $\quad \eta_2 = \frac{120\pi}{\sqrt{2.25}} = 251.327\Omega$

$$\rho_\eta = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2} = -0.278$$

$$\tau_n = \frac{2\eta_2\cos\theta_1}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2} = 0.732$$



$\tilde{E}_i = 50\, e^{-j0.766z}\, e^{j0.643y}\, \vec{a}_x$ V/m

From $\nabla\times\tilde{E} = -j\omega\mu_0\tilde{H}$,

$\tilde{H}_i = (0.102\,\vec{a}_y + 0.085\,\vec{a}_z)\, e^{-j0.766z}\, e^{j0.643y}$ A/m

$\langle \hat{S}_i \rangle = \frac{1}{2}\mathrm{Re}[\tilde{E}_i \times \tilde{H}_i^*]$

$= 2.55\,\vec{a}_z - 2.13\,\vec{a}_y$ W/m^2

$$\tilde{E}_r = -13.9\, e^{j0.766z}\, e^{j0.643y}\, \vec{a}_x \ \text{V/m} \qquad \langle \hat{S}_r \rangle = -0.19\, \vec{a}_z - 0.17\, \vec{a}_y \ \text{W/m}^2$$

$$\tilde{H}_r = (0.028\, \vec{a}_y - 0.024\, \vec{a}_z)\, e^{j0.766z}\, e^{j0.643y} \ \text{A/m}$$

$$\tilde{E}_t = 36.1\, e^{-j1.355z}\, e^{j0.643y}\, \vec{a}_x \ \text{V/m} \qquad \langle \hat{S}_t \rangle = 2.35\, \vec{a}_z - 1.12\, \vec{a}_y \ \text{W/m}^2$$

$$\tilde{H}_t = (0.13\, \vec{a}_y + 0.062\, \vec{a}_z)\, e^{-j1.355z}\, e^{j0.643y} \ \text{A/m}$$

<u>Problem 8.34</u> $\eta_1 = 377\Omega$ $\quad \eta_2 = 0$ $\quad \beta_1 = 1$ rad/m $\quad \omega = 3\times10^8$ rad/s $\quad \Gamma_n = -1$

$$\tilde{E}_i = 50\, e^{-j0.766z}\, e^{j0.643y}\, \vec{a}_x \ \text{V/m}$$

$$\tilde{E}_r = -50\, e^{j0.766z}\, e^{j0.643y}\, \vec{a}_x \ \text{V/m}$$

$$\tilde{E} = \tilde{E}_i + \tilde{E}_r = -j100 \sin(0.766z)\, e^{j0.643}\, \vec{a}_x \ \text{V/m}$$

$$\tilde{H}_i = (0.102\, \vec{a}_y + 0.085\, \vec{a}_z)\, e^{-j0.766z}\, e^{j0.643y} \ \text{A/m}$$

$$\tilde{H}_r = (0.102\, \vec{a}_y - 0.085\, \vec{a}_z)\, e^{j0.766z}\, e^{j0.643y} \ \text{A/m}$$

$$\tilde{H} = \tilde{H}_i + \tilde{H}_r = 0.204 \cos(0.766z)\, e^{j0.643y}\, \vec{a}_y$$
$$- j0.17 \sin(0.766z)\, e^{j0.43y}\, \vec{a}_z \ \text{A/m}$$

at $z=0$ $\quad \tilde{H}(0) = 0.204\, e^{j0.643y}\, \vec{a}_y$

$$\vec{a}_n \times \tilde{H}(0) = \tilde{J}_s(0) \Rightarrow$$

$$\tilde{J}_s(0) = 0.204\, e^{j0.643y}\, \vec{a}_x$$