Chapter 2 Vector Analysis

Contents

2.1 Introduction

- Knowledge of vector algebra and vector calculus is essential in developing the concepts of electromagnetic field theory:

(1) The widespread acceptance of vectors in electromagnetic field theory is due in part to the fact that they provide compact mathematical representations of complicated phenomena and allow for easy visualization and manipulation.

(2) The ever-increasing number of textbooks on the subject are evidence of the popularity of vectors. As you will see in subsequent chapters, a single equation in vector form is sufficient to represent up to three scalar equations.

2.1 Introduction

(3) Although a complete discussion of vectors is not within the scope of this text. Some of the vector operations that will play a prominent role in our discussion of electromagnetic field theory are introduced in this chapter.

- We begin our discussion by defining scalar and vector quantities. Most of the quantities encountered in electromagnetic fields can easily be divided into two classes:
- Scalar and Vector quantities

2.2 Scalar and Vector quantities

1. Scalar quantities

- A physical quantity that can be completely described by its magnitude is called a scalar. Each of these quantities is completely described by a single number.
- some examples of scalar quantities are mass, time, temperature, work and electric charge.

  a temperature of 20^0C; a mass of 100 grams; a charge of 0.5 coulomb.
- In fact, all real numbers are scalars.

2.2 Scalar and Vector quantities

2. Vector quantities

- **A physical quantity having a magnitude as well as direction is called a vector.**
- **Some examples of vector quantities are Force, velocity, electric field intensity and acceleration.**

2.5 Scalar and Vector Fields

- A field is a function that describes a physical quantity at all points in space. A physical quantity can be either a scalar or a vector, thus, a field can also be a scalar field or a vector field.

1. Scalar fields

- A scalar field is specified by a single number(or a scalar quantity) at each point in space. These scalar functions represent a scalar field(s).

- Some well-known Examples of scalar fields include:

  Temperature of a gas, pressure of a gas, the altitude above sea level and electric potential.

2.5 Scalar and Vector Fields

- In the rectangular coordinate system, a scalar field can commonly be described by a scalar function $f(x, y, z, t)$.

2. Vector fields

(1) concept:

A vector field is specified by both a magnitude and a direction at each point in space.

- Some examples of vector fields:

The velocity and acceleration of a fluid, the gravitational force, the electric field within a coaxial cable. The electric field intensity within a parallel-plate capacitor is constant and is directed from the higher potential conductor toward the lower potential conductor.

2.5 Scalar and Vector Fields

(2)Representation

(*a*) a vector field can be described by a vector function $\vec{F}(x,y,z,t)$

$$\vec{\mathbf{F}}(x,y,z,t)=\vec{\mathbf{a}}_{\mathbf{F}}F(x,y,z,t)$$

$\vec{\mathbf{a}}_{\mathbf{F}}$ is unit vector, indicates the direction of the vector field $\vec{F}(x,y,z,t)$

$F(x,y,z,t)$ is the magnitude of the vector field $\vec{F}(x,y,z,t)$

In the rectangular coordinate system, each vector field can be expressed by three components of the vector along the *x*, *y* and *z* axes, respectively.

2.5 Scalar and Vector Fields

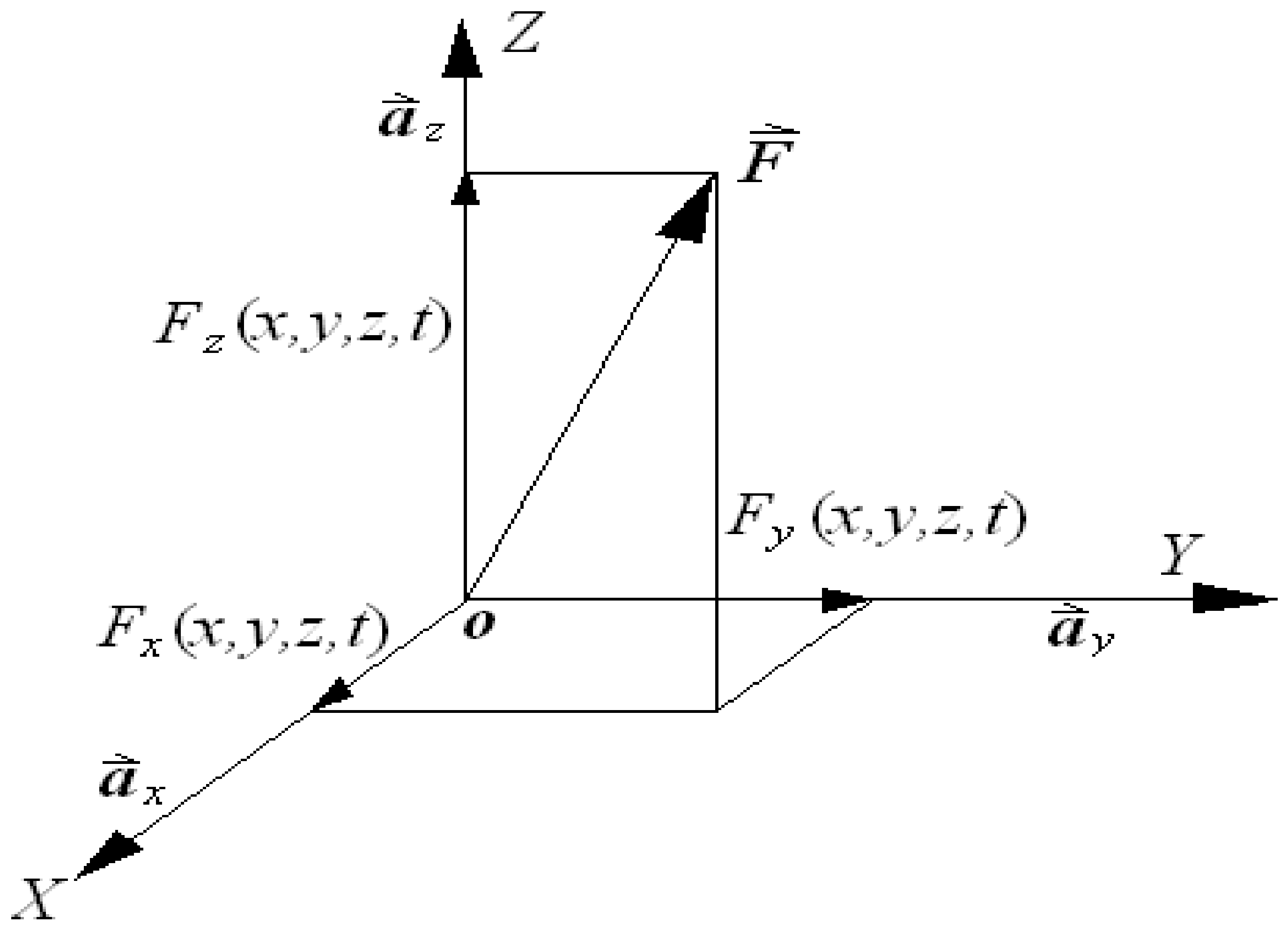
$$\vec{\mathbf{F}}(x, y, z, t) = \vec{\mathbf{a}}_x F_x(x, y, z, t) + \vec{\mathbf{a}}_y F_y(x, y, z, t) + \vec{\mathbf{a}}_z F_z(x, y, z, t)$$

these unit vectors $\vec{a}_x, \vec{a}_y, \vec{a}_z$ indicate the directions of the components of the vector field $\vec{F}(x, y, z, t)$ along the *x*, *y* and *z* axes, respectively.

Three scalar functions $F_x(x, y, z, t), F_y(x, y, z, t), F_z(x, y, z, t)$ are the scalar projections of $\vec{F}(x, y, z, t)$ on the *x*, *y* and *z* axes, respectively.

They are the scalar functions of time and space and indicate their magnitudes.

2.5 Scalar and Vector Fields



2.5 Scalar and Vector Fields

the three unit vectors are mutually orthogonal, the dot product yields

$$\vec{\boldsymbol{a}}_x \bullet \vec{\boldsymbol{a}}_x = 1, \quad \vec{\boldsymbol{a}}_y \bullet \vec{\boldsymbol{a}}_y = 1, \quad \vec{\boldsymbol{a}}_z \bullet \vec{\boldsymbol{a}}_z = 1$$

and

$$\vec{\mathbf{a}}_x \bullet \vec{\mathbf{a}}_y = 0, \quad \vec{\mathbf{a}}_y \bullet \vec{\mathbf{a}}_z = 0, \quad \vec{\mathbf{a}}_x \bullet \vec{\mathbf{a}}_z = 0$$

the three unit vectors are mutually orthogonal, the cross product yields

$$\vec{\boldsymbol{a}}_x \times \vec{\boldsymbol{a}}_x = 0, \quad \vec{\boldsymbol{a}}_y \times \vec{\boldsymbol{a}}_y = 0, \quad \vec{\boldsymbol{a}}_z \times \vec{\boldsymbol{a}}_z = 0$$

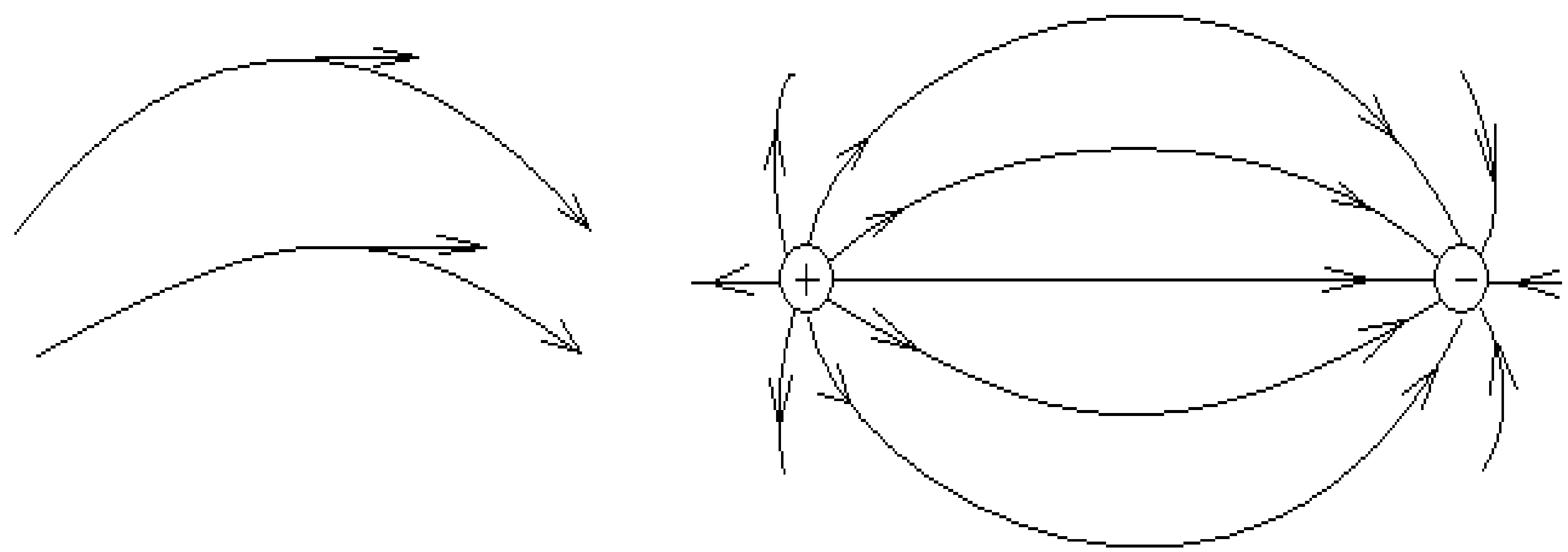
and

$$\vec{\boldsymbol{a}}_x \times \vec{\boldsymbol{a}}_y = \vec{\boldsymbol{a}}_z, \quad \vec{\boldsymbol{a}}_y \times \vec{\boldsymbol{a}}_z = \vec{\boldsymbol{a}}_x, \quad \vec{\boldsymbol{a}}_z \times \vec{\boldsymbol{a}}_x = \vec{\boldsymbol{a}}_y$$

right-handed screw rule , right-hand rule

2.5 Scalar and Vector Fields

(b) curve charts



they are called vector curves: the density of curves indicates the magnitude of the vector field, the direction of its tangent for each point in space stands for the direction of the vector field.

2.6 Differential elements of length, surface, and volume

- In our study of electromagnetism we will often be require to perform line, surface, and volume integrations. The evaluation of these integrals in a particular coordinate system requires the knowledge of differential elements of lengths, surface, and volume.

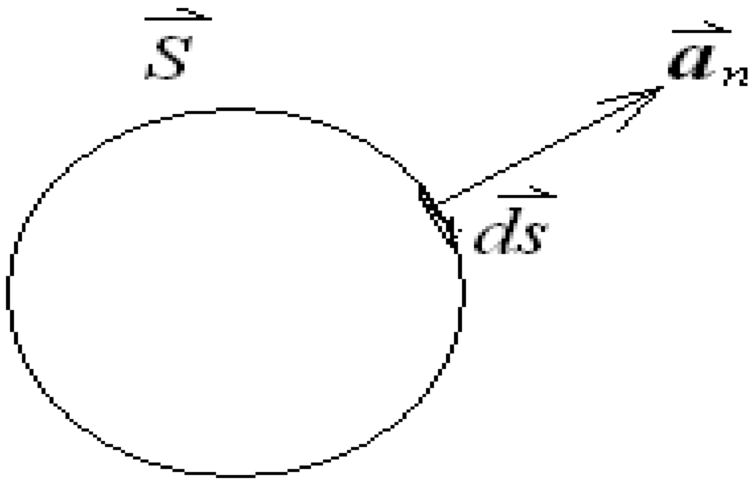
Requirement:
Some expressions in the rectangular coordinate system.

2.6 Differential Elements

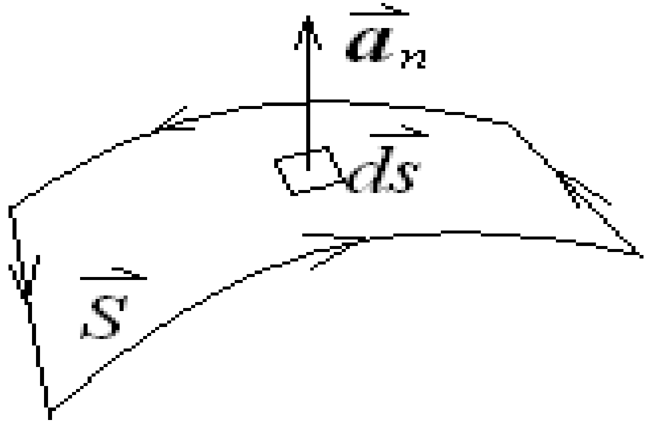
- differential surface element

a differential surface element can be written as

$$d\vec{s} = \vec{a}_n ds$$



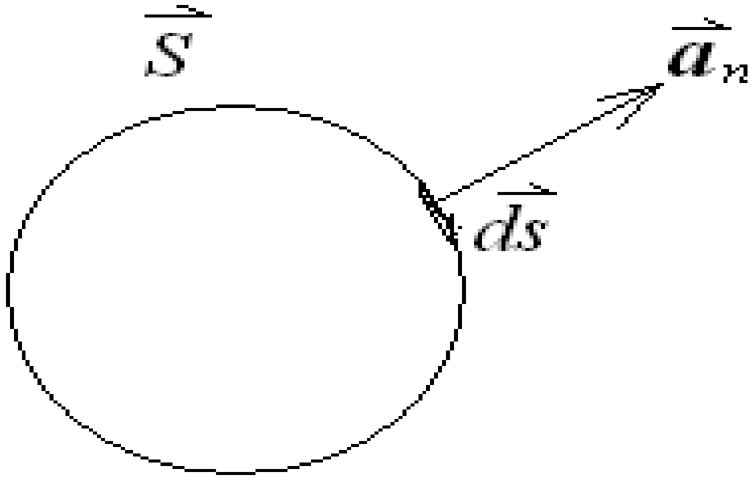
an enclosed surface



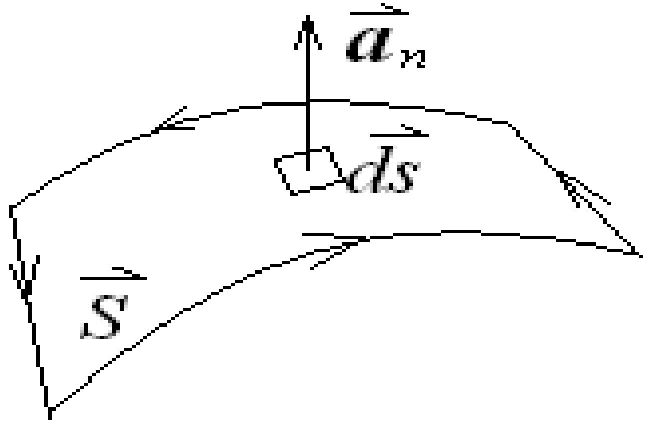
an opened surface

2.6 Differential Elements

$d\vec{s} = \vec{a}_n ds$ is a vector, its magnitude is *ds*; its direction is $\vec{a}_n$, which is normal to the surface *ds*. Generally, for an enclosed surface, the outward direction $\vec{\mathbf{a}}_n$ is positive; for an opened surface, right-hand rule Will define the direction of $\vec{a}_n$

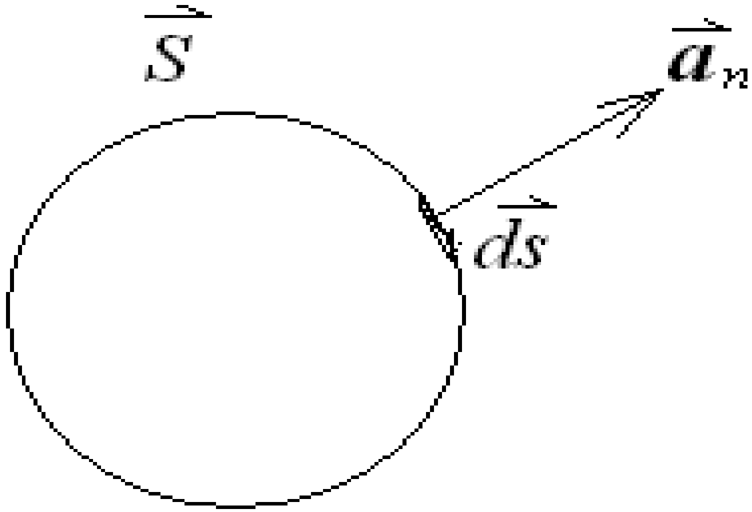


an enclosed surface

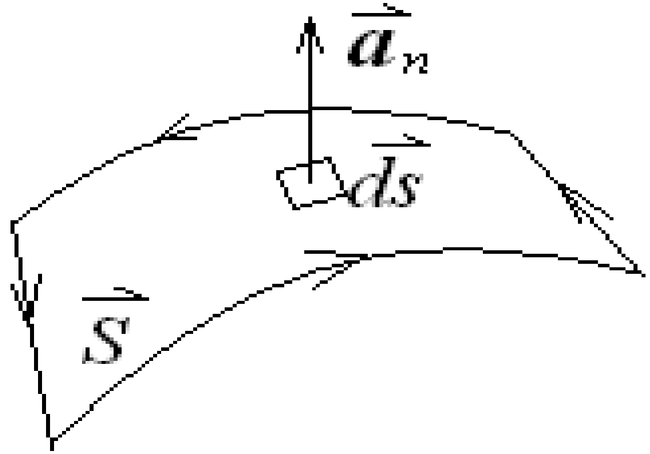


an opened surface

2.6 Differential Elements



an enclosed surface



an opened surface

2.6 Differential Elements

- **differential volume element**
- **A differential volume element in the rectangular coordinate system is generated by making differential changes *dx*, *dy* and *dz* along the unit vectors $\vec{a}_x$, $\vec{a}_y$ and $\vec{a}_z$**

the differential volume element is given by the expression

$$dv = dx\,dy\,dz$$

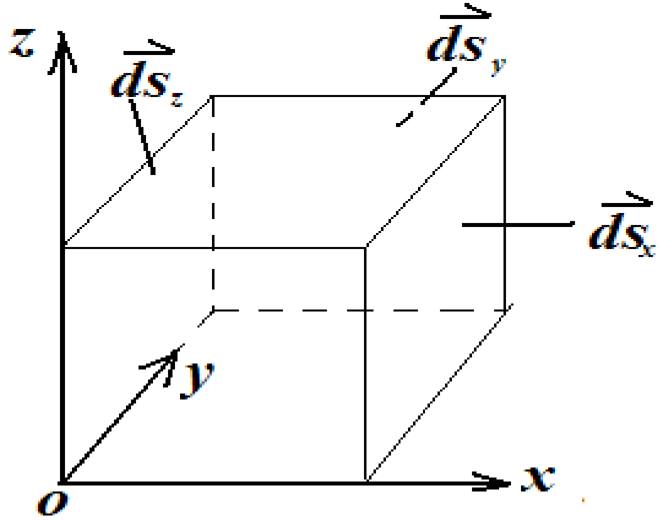
2.6 Differential Elements

- The volume is enclosed by six differential surfaces. Each surface is defined by a unit vector normal to that surface. Thus ,we can express the differential surfaces in the direction of positive unit vectors as

$$d\vec{s}_x = dydz\vec{a}_x$$

$$d\vec{s}_y = dxdz\vec{a}_y$$

$$d\vec{s}_z = dxdy\vec{a}_z$$



- The general differential length element from P to Q is

$$d\vec{l} = dx\vec{a}_x + dy\vec{a}_y + dz\vec{a}_z$$

2.8 The gradient of a scalar function

To be continued.

§ 2.8 The gradient and the directional derivative of a scalar field (function)

➢ **1.directional derivative**

(1) concept

Before we begin our discussion of our object, it is important to define the derivative of a function of one or more variables.

The derivative of a scalar function *f* (*x*) with respect to *x* is defined as

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} \qquad \textbf{(2.8-1)}$$

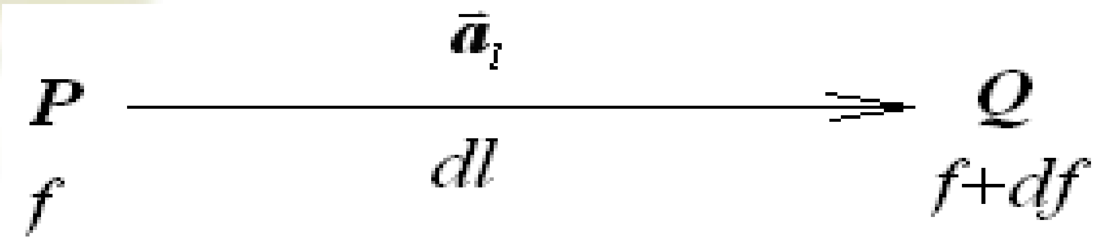
let $f(x, y, z)$ be a real-valued differentiable function of x, y, and z. The partial derivative of $f(x, y, z)$ from point P to M along the x axes(or along $\vec{a}_x$), can be written as

$$\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y, z) - f(x, y, z)}{\Delta x} \qquad (2.8\text{-}2)$$

The derivative of the function $f(l)$ from point P to Q along the direction of $d\vec{l}$(or $\vec{a}_l$), can be written as

$$\frac{df}{dl} = \lim_{\Delta l \to 0} \frac{f(l+\Delta l) - f(l)}{\Delta l} = \lim_{\Delta l \to 0} \frac{\Delta f}{\Delta l} \qquad (2.8\text{-}3)$$

$\frac{df}{dl}$ **is called the directional derivative of *f* along** $\vec{a}_l$



In fact, $\frac{df}{dl}$ **is the rate of change of the scalar function *f* in the direction of the unit vector** $\vec{a}_l$

Obviously,

$$P \xrightarrow[dl]{\vec{a}_l} Q \qquad f \;\to\; f+df$$

$$\frac{df}{dl} = \lim_{\Delta l \to 0} \frac{f(l+\Delta l) - f(l)}{\Delta l} = \lim_{\Delta l \to 0} \frac{\Delta f}{\Delta l}$$

$$\vec{a}_l = \vec{a}_x$$

$$P \xrightarrow[dx]{\vec{a}_x} Q \qquad f \;\to\; f+df$$

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$