"CHAPTER-7 TIME-VARYING EM FIELDS"

<u>Exercise 7.1</u> $L = 2m$ $\quad \vec{B} = 12.5\,\vec{a}_z$ mT $\quad \omega = 188$ rad/s

$$e_{ba} = \frac{1}{2} B\omega L^2 = 4.7V \qquad I_{ba} = e_{ba}/2 = 2.35\,A$$

$$P_{sup} = 4.7 \times 2.35 = 11.05\ W$$

$$\vec{dl} = d\rho\,\vec{a}_\rho \Rightarrow \vec{F}_m = \int_L I\,\vec{dl} \times \vec{B} = -IB\int_0^2 d\rho\,\vec{a}_\phi = -58.75\,\vec{a}_\phi \text{ mN}$$

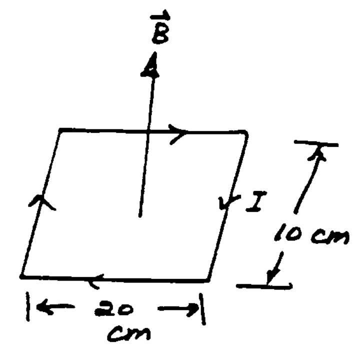
Thus, $\vec{F}_{ext} = 58.75\,\vec{a}_\phi$ mN

<u>Exercise 7.2</u> $L = 1m$, $e_{ba} = \frac{1}{2} \times 12.5 \times 10^{-3} \times 188 \times 1^2 = 1.175V$

For $R = 2\Omega$, $I_R = 0.5875\,A$, $I/\text{strip} = 293.75\,mA$, $P_{sup} = \frac{1.175^2}{2} = 0.69\,W$

<u>Exercise 7.3</u> $\ell = 20 + 10 + 20 + 10 = 60\,cm$

$r = 1.2mm$, $A = \pi r^2 = 4.524 \times 10^{-6}\,m^2$

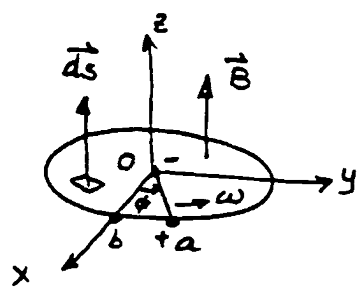


$$R = \frac{\ell}{\sigma A} = 371.51\,\mu\Omega, \text{ where } \sigma = 3.57 \times 10^7\ S/m$$

$$|e| = N\frac{d\Phi}{dt} = (+1)(200 \times 10^{-4})\frac{dB}{dt}$$

$$= 200 \times 10^{-4} \times 40 = 0.8V \Rightarrow I = 2153.4\,A \text{ as shown.}$$

<u>Exercise 7.4</u> $\vec{B} = 12.5\,\vec{a}_z$ mT



$$\vec{ds} = \rho\,d\rho\,d\phi\,\vec{a}_z$$

$$\Phi = \int_s \vec{B}\cdot\vec{ds} = \int_0^2 12.5 \times 10^{-3} \rho\,d\rho \int_0^\phi d\phi$$

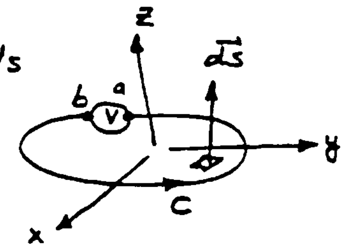
$$= 0.025\phi = 0.025\omega t \qquad \omega = 188 \text{ rad/s}$$

$$e = -N\frac{d\Phi}{dt} = -0.025\omega = -4.7V$$

As the flux passing thro' boa is increasing with time, a must be at a higher potential than b. Thus, $e_{ao} = 4.7V$

Exercise 7.5

$f = 200\ \text{kHz}$



Let $\vec{H} = 10\cos\omega t\ \vec{a}_z$ where $\omega = 2\pi f = 1.257\times10^{6}$ rad/s

$\vec{B} = \mu_0\vec{H} \Rightarrow \dfrac{\partial\vec{B}}{\partial t} = -15.8\sin\omega t\ \vec{a}_z$

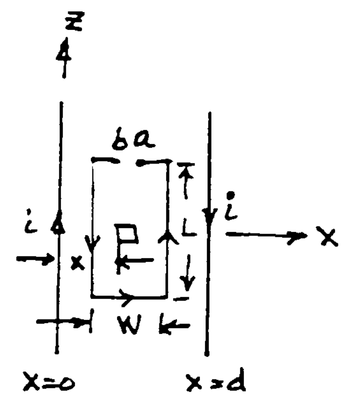
$\vec{ds} = \rho\, d\rho\, d\phi\ \vec{a}_z$

$$e_{ab} = -\int_s \frac{\partial\vec{B}}{\partial t}\cdot\vec{ds} = 15.8\sin\omega t\int_0^{0.1}\rho\, d\rho\int_0^{2\pi} d\phi$$

$$= 0.496\sin\omega t \Rightarrow \text{Voltmeter reading: } \frac{0.496}{\sqrt{2}} = 0.35\text{V}$$

Exercise 7.6

$i = I_m \sin\omega t$ A



$\vec{B} = \dfrac{\mu_0 I_m \sin\omega t}{2\pi}\left[\dfrac{1}{x} + \dfrac{1}{d-x}\right]\vec{a}_y \qquad \vec{ds} = dx\, dz\ \vec{a}_y$

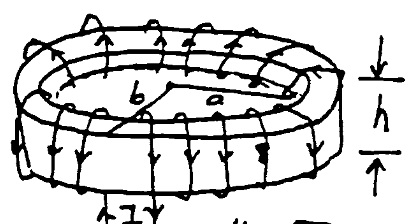
$$\Phi = \int_s \vec{B}\cdot\vec{ds} = \frac{\mu_0 I_m \sin\omega t}{2\pi}\int_{\frac{d-W}{2}}^{\frac{d+W}{2}}\left(\frac{1}{x} + \frac{1}{d-x}\right)dx\int_0^L dz$$

$$= \frac{\mu_0 L I_m}{\pi}\sin\omega t\ \ln\left[\frac{d+W}{d-W}\right]$$

$$e_{ab} = -N\frac{d\Phi}{dt} = -\frac{N\omega L\mu_0 I_m}{\pi}\cos\omega t\ \ln\left[\frac{d+W}{d-W}\right]$$

Exercise 7.7

$i = 2\sin 314t$ A



$\mu_r = 500$

$a = 20\text{cm},\ b = 25\text{cm}$

$h = 5\text{cm},\ N = 200$ Turns

$\vec{ds} = d\rho\, dz\ \vec{a}_\phi$

$a \le \rho \le b$: $\oint_c \vec{H}\cdot\vec{dl} = Ni \Rightarrow$

$H_\phi = \dfrac{Ni}{2\pi\rho}, \quad B_\phi = \dfrac{\mu Ni}{2\pi\rho}$

$$\Phi = \int_s \vec{B}\cdot\vec{ds} = \frac{\mu Ni}{2\pi}\int_a^b \frac{1}{\rho}d\rho\int_0^h dz$$

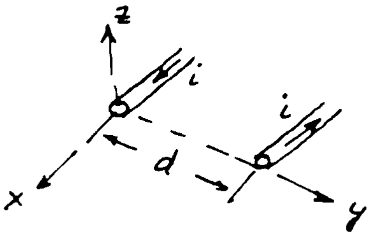
$$= \frac{\mu Ni}{2\pi}h\ \ln(b/a)$$

$L = \dfrac{\lambda}{i} = \dfrac{N\Phi}{i} = \dfrac{\mu N^2}{2\pi}h\ \ln(b/a)$ · Substitute values. $L = 44.63$ mH

$i = 2\sin 314t$, A , $\quad e = -L\dfrac{di}{dt} = -44.63\times10^{-3}\times2\times314\cos 314t$

$$= -28.03\cos 314t,\ \text{V}$$

Exercise 7.8



Assume $d \gg a$ & current is distributed over the surface ($\sigma \to$ very high)

$$B_z = \frac{\mu_0 I}{2\pi}\left[\frac{1}{y} + \frac{1}{d-y}\right]$$

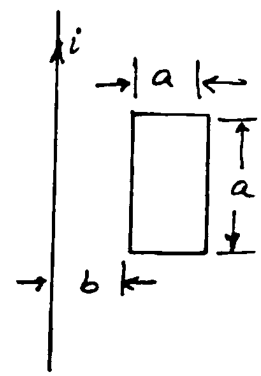
on a per-unit length basis: $\Phi = \frac{\mu_0 I}{2\pi}\int_a^{d-a}\left(\frac{1}{y} + \frac{1}{d-y}\right)dy = \frac{\mu_0 I}{\pi}\ln\left(\frac{d-a}{a}\right)$

$$\simeq \frac{\mu_0 I}{\pi}\ln\left(\frac{d}{a}\right) \quad \text{Wb/m}$$

Thus, $L = \frac{\mu_0}{\pi}\ln(d/a)$ H/m.

Exercise 7.9

$$B_\phi = \frac{\mu_0 i}{2\pi\rho}$$



$$\Phi = \frac{\mu_0 i}{2\pi}\int_b^{b+a}\frac{1}{\rho}d\rho\int_0^a dz = \frac{\mu_0 i}{2\pi}a\ln\left(\frac{b+a}{a}\right)$$

$$M = \frac{\mu_0}{2\pi}a\ln\left(\frac{b+a}{a}\right)$$

Exercise 7.10

$M = 16$ mH, $L_1 = 20$ mH, $L_2 = 80$ mH

$$M = k\sqrt{L_1L_2} \Rightarrow k = \frac{16}{\sqrt{20\times 80}} = 0.4$$

Exercise 7.11

Refer to Fig. 7.19 [Parallel aiding)

$$v(t) = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \quad (1)$$

$$v(t) = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \quad (2)$$

From (1) and (2): $\frac{di_2}{dt} = \frac{L_1 - M}{L_2 - M}\frac{di_1}{dt}$ (3)

Since $i = i_1 + i_2$, $\frac{di_2}{dt} = \frac{di}{dt} - \frac{di_1}{dt}$ (4)

From (3) and (4)

$$\frac{di}{dt} = \frac{L_1 + L_2 - 2M}{L_2 - M}\frac{di_1}{dt} \quad (5)$$

If L is the eqvt. inductance,

$$v(t) = L\frac{di}{dt} \quad (6)$$

From (1), (5) and (6)

$$L\frac{di}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \quad \text{or}$$

$$L\left[\frac{L_1 + L_2 - 2M}{L_2 - M}\right]\frac{di_1}{dt} = L_1\frac{di_1}{dt} + M\frac{L_1 - M}{L_2 - M}\frac{di_1}{dt}$$

Thus, $L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$

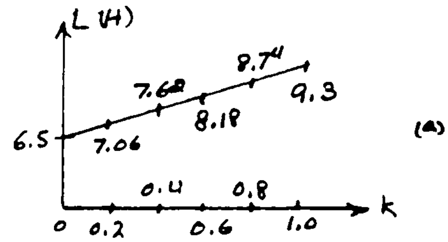
Similarly, you can show that

$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$ for parallel-opposing

Exercise 7.12 $L_1 = 1.6H$, $L_2 = 4.9H$

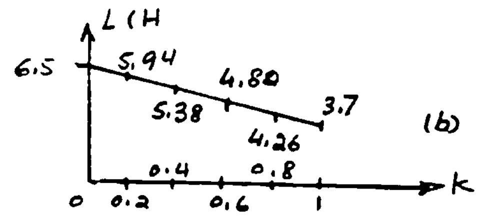
a) $L = L_1 + L_2 + k\sqrt{L_1 L_2}$

$= 6.5 + 2.8k$ (series-aiding)



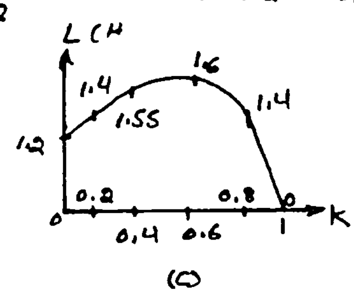
b) (Series-opposing)

$L = 6.5 - 2.8k$



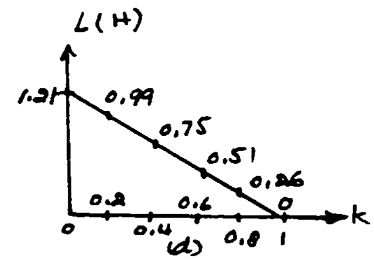
c) Parallel-aiding

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{7.84 - 7.84k^2}{6.5 - 5.6k}$$



d) Parallel-opposing

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{7.84 - 7.84k^2}{6.5 + 5.6k}$$



Exercise 7.13 $\vec{J} = J_o \, \vec{a}_z$ A/m^2 $\rho \le a$ $\Rightarrow$ $\oint \vec{H} \cdot d\vec{l} = I_{enc}$ or

$= 0$ $\rho \ge a$ $H_\phi = \frac{J_o \rho}{2}$ $\rho \le a$ (Inside)

$$w_m = \frac{1}{2}\mu_o H_\phi^2 = \frac{1}{8}\mu_o J_o^2 \rho^2 \Rightarrow W_m = \frac{1}{8}\mu_o J_o^2 \int_0^a \rho^3 d\rho \int_0^{2\pi} d\phi \int_0^{z=1} dz = \frac{\mu_o \pi}{16} a^4 J_o^4 \ \text{J/m}$$

Outside, $\rho \ge a$ $H_\phi = \frac{J_o a^2}{2\rho}$ $\Rightarrow$ $w_m = \frac{1}{8}\mu_o \left(\frac{J_o}{\rho}\right)^2 a^4$ J/m^3

Exercise 7.14 $\Phi = a\sqrt{i}$, $d\Phi = \frac{1}{2} a \frac{di}{\sqrt{i}}$, $dW = N i \, d\Phi = \frac{1}{2} a N \sqrt{i} \, di$

$$W = \int_0^i \frac{1}{2} a N \sqrt{i} \, di = \frac{1}{3} a N i^{3/2} = \frac{1}{3} N \Phi i \quad \text{(Non-linear medium)}$$

Exercise 7.15 $W_i = \frac{1}{2} \times 1.02 \times 2^2 = 2.04$ J

$W_f = \frac{1}{2} \times 1.02 \times 5^2 = 12.75$ J

Thus, $W = W_f - W_i = 10.71$ J

Exercise 7.16 $\vec{J} = J_0 \cos\omega t\, \vec{a}_z \Rightarrow \vec{E} = \frac{\vec{J}}{\sigma}$, $\vec{D} = \epsilon\vec{E} \Rightarrow \vec{D} = \frac{\epsilon}{\sigma} J_0 \cos\omega t\, \vec{a}_z$

$$\frac{\partial \vec{D}}{\partial t} = -\frac{\omega\epsilon}{\sigma} J_0 \sin\omega t\, \vec{a}_z \Rightarrow \frac{\left|\frac{\partial \vec{D}}{\partial t}\right|}{|\vec{J}|} = \frac{\omega\epsilon}{\sigma} \qquad \omega = 2\pi f$$

For Cu: $\epsilon \to \epsilon_0 = 10^{-9}/36\pi$ F/m, $\sigma = 5.8\times10^{7}$ S/m $\Rightarrow \frac{\omega\epsilon}{\sigma} = 9.58\times10^{-19} f$

Exercise 7.17 $\vec{E} = C\cos(\alpha x)\cos(\omega t - \beta z)\,\vec{a}_y$, since $\vec{D} = \epsilon\vec{E}$

$$\vec{D} = \epsilon C \cos(\alpha x)\cos(\omega t - \beta z)\,\vec{a}_y \quad \text{and} \quad \nabla\cdot\vec{D} = 0$$

From $\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$, we have

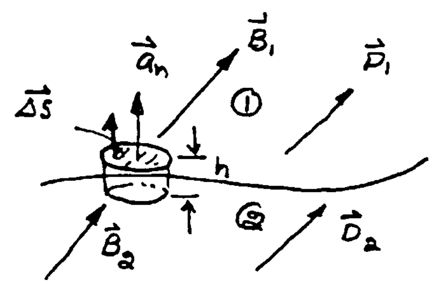
$$-\frac{\partial \vec{B}}{\partial t} = \beta C \cos(\alpha x)\sin(\omega t - \beta z)\,\vec{a}_x - \alpha C \sin(\alpha x)\cos(\omega t - \beta z)\,\vec{a}_z$$

or $$\vec{B} = -\frac{\beta C}{\omega}\cos(\alpha x)\cos(\omega t - \beta z)\,\vec{a}_x + \frac{\alpha C}{\omega}\sin(\alpha x)\sin(\omega t - \beta z)\,\vec{a}_z \quad \text{T}$$

Thus, $$\vec{H} = -\frac{\beta C}{\omega\mu}\cos(\alpha x)\sin(\omega t - \beta z)\,\vec{a}_x + \frac{\alpha C}{\omega\mu}\sin(\alpha x)\sin(\omega t - \beta z)\,\vec{a}_z \quad \text{A/m}$$

And $\nabla\cdot\vec{B} = 0$

Exercise 7.18



$\oint_S \vec{B}\cdot d\vec{S} = 0 \Rightarrow$ $\quad \vec{\Delta S} = \vec{a}_n \Delta S$

$$\lim_{h\to 0}\left[\vec{B}_1\cdot\vec{\Delta S} - \vec{B}_2\cdot\vec{\Delta S}\right] = 0$$

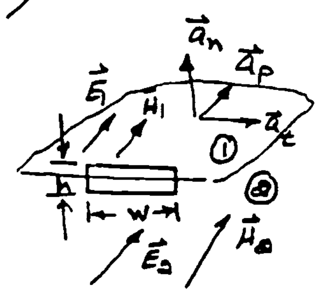
or $\vec{a}_n\cdot(\vec{B}_1 - \vec{B}_2) = 0$

Similarly, $\oint_S \vec{D}\cdot d\vec{S} = \int_v \rho_v\, dv$

$$\lim_{h\to 0}\left[\vec{D}_1\cdot\vec{\Delta S} - \vec{D}_2\cdot\vec{\Delta S}\right] = \rho_s \Delta S$$

$$\vec{a}_n\cdot(\vec{D}_1 - \vec{D}_2) = \rho_s$$

$\lim_{h\to 0} \rho_v = \rho_s$



$$\oint_C \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

as $h\to 0$ $dS \to 0 \Rightarrow \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S} \to 0$

Thus $\vec{E}_1\cdot\vec{a}_t w - \vec{E}_2\cdot\vec{a}_t w = 0$

or $\vec{a}_t\cdot(\vec{E}_1 - \vec{E}_2) = 0 \Rightarrow$

$$\vec{a}_n\times(\vec{E}_1 - \vec{E}_2) = 0$$

$$\oint_C \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{S} + \int \frac{\partial \vec{D}}{\partial t}\cdot d\vec{S}$$

as $h\to 0$ $\frac{\partial \vec{D}}{\partial t}\cdot d\vec{S} = 0$

and $\lim_{h\to 0}\vec{J} = \vec{J}_s$

Thus $\vec{a}_t\cdot(\vec{H}_1 - \vec{H}_2) = \vec{J}_s$ or

$$\vec{a}_n\times(\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

Exercise 7.19 Medium-2 is a perfect conductor $\Rightarrow \vec{E}_2=0, \vec{B}_2=0, \vec{D}_2=0, \vec{H}_2=0$

(a) Thus, $D_{n1} = \rho_s \quad B_{n1}=0 \quad E_{t1}=0 \quad H_{t1}=J_s$

(b) Two perfect dielectrics: $\rho_s=0 \quad J_s=0$

$$D_{n1}=D_{n2}, \quad B_{n1}=B_{n2} \quad E_{t1}=E_{t2} \quad H_{t1}=H_{t2}$$

(c) Medium 2 is a conductor, $J_s=0$

$$\vec{a}_n\cdot(\vec{D}_1-\vec{D}_2)=\rho_s \quad \vec{a}_n\cdot(\vec{B}_1-\vec{B}_2)=0 \quad \vec{a}_n\times(\vec{E}_1-\vec{E}_2)=0, \quad \vec{a}_n\times(\vec{H}_1-\vec{H}_2)=0$$

Exercise 7.20 $\vec{E}=C\cos(\alpha x)\cos(\omega t-\beta z)\vec{a}_y$, From Exercise 7.17,

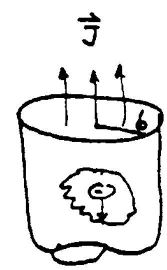
$$\vec{H} = -\frac{\beta C}{\omega\mu}\cos(\alpha x)\cos(\omega t-\beta z)\vec{a}_x + \frac{\alpha C}{\omega\mu}\sin(\alpha x)\sin(\omega t-\beta z)\vec{a}_z$$

However, $\nabla\times\vec{H}=\frac{\partial\vec{D}}{\partial t} \Rightarrow \vec{a}_y\left[\frac{\partial}{\partial z}H_x-\frac{\partial}{\partial x}H_z\right]=\frac{\partial}{\partial t}(\epsilon\vec{E})$

$$-\frac{\beta^2C}{\omega\mu}\cos(\alpha x)\sin(\omega t-\beta z)-\frac{\alpha^2C}{\omega\mu}\cos(\alpha x)\sin(\omega t-\beta z)=-\omega\epsilon C\cos(\alpha x)\sin(\omega t-\beta z)$$

or $\qquad \beta^2+\alpha^2=\omega^2\mu\epsilon$ condition for fields to exist.

Exercise 7.21 $\vec{J}=\frac{I}{\pi b^2}\vec{a}_z \quad A/m^2$



$$\vec{E}=\frac{\vec{J}}{\sigma}=\frac{I}{\sigma\pi b^2}\vec{a}_z \quad V/m$$

$$P=\int_v \vec{J}\cdot\vec{E}\,dv=\frac{I^2}{\sigma\pi^2b^4}\int_0^b\rho\,d\rho\int_0^{2\pi}d\phi\int_0^L dz$$

$$=I^2\frac{L}{\sigma\pi b^2}=I^2R \quad \text{where } R=\frac{L}{\sigma\pi b^2}\ \Omega$$

On the surface of the conductor: $\vec{H}=\frac{I}{2\pi b}\vec{a}_\phi$

Thus. $\vec{S}=\vec{E}\times\vec{H}=-\frac{I^2}{2\sigma\pi^2b^3}\vec{a}_\rho \quad W/m^3$

$$\int_s \vec{S}\cdot\vec{ds}=-\frac{I^2}{2\sigma\pi^2b^3}\int_0^{2\pi}b\,d\phi\int_0^L dz=-I^2\frac{L}{\sigma\pi b^2}=-I^2R$$

The minus sign indicates that the power is flowing into the volume v of the solid conductor.

Exercise 7.22 $\vec{E} = 10\cos(\omega t + ky)\,\vec{a}_x$ V/m $\quad T = 100\text{ns} \Rightarrow f = 10\text{MHz}$

$\vec{D} = 10\epsilon_0 \cos(\omega t + ky)\,\vec{a}_x$ C/m², $\nabla\cdot\vec{D} = 0$ $\quad \omega = 2\pi f$

$$-\frac{\partial \vec{B}}{\partial t} = \nabla\times\vec{E} = -\frac{\partial E_x}{\partial y}\vec{a}_z = 10k\sin(\omega t + ky)\,\vec{a}_z \Rightarrow \vec{B} = +\frac{10k}{\omega}\cos(\omega t + ky)\,\vec{a}_z$$

$\nabla\cdot\vec{B} = 0$

$$\frac{\partial \vec{D}}{\partial t} = \nabla\times\vec{H} \Rightarrow -10\omega\epsilon_0 \sin(\omega t + ky)\,\vec{a}_x = -\frac{10k^2}{\omega\mu_0}\sin(\omega t + ky)\,\vec{a}_x$$

or $\quad k^2 = \omega^2\mu_0\epsilon_0 \Rightarrow k = \pm\omega\sqrt{\mu_0\epsilon_0} = \pm 2\pi\times10\times10^6/3\times10^8 = \pm 0.209$ rad/m

Exercise 7.23 $\tilde{E}_x = E e^{-jkz} \Rightarrow \tilde{D}_x = \epsilon E e^{-jkz}$ and $\nabla\cdot\tilde{D} = 0$

$$\nabla\times\tilde{E} = \frac{\partial \tilde{E}_x}{\partial z}\vec{a}_y = -jkE e^{-jkz}\vec{a}_y$$

Thus, from $\nabla\times\tilde{E} = -j\omega\tilde{B} \Rightarrow \tilde{B} = \frac{kE}{\omega}e^{-jkz}\vec{a}_y$ and $\nabla\cdot\tilde{B} = 0$

$$\tilde{H}_y = \frac{kE}{\omega\mu}e^{-jkz} \Rightarrow H_y = \frac{kE}{\omega\mu}\cos(\omega t - kz)$$

$$\nabla\times\tilde{H} = -\frac{\partial}{\partial x}(\tilde{H}_y)\,\vec{a}_x = j\frac{k^2}{\omega\mu}e^{-jkz}$$

From $\nabla\times\tilde{H} = j\omega\tilde{D}$, we have $k^2 = \omega^2\mu\epsilon$

Finally, $\langle\hat{S}\rangle = \frac{1}{2}\text{Re}[\tilde{E}\times\tilde{H}^*] = \frac{1}{2}\frac{k}{\omega\mu}E^2\,\vec{a}_z$

Exercise 7.24 $\tilde{E}_x = 10e^{jky}$, $\tilde{D}_x = 10\epsilon_0 e^{jky} \Rightarrow \nabla\cdot\tilde{D} = 0$

$\nabla\times\tilde{E} = -\frac{\partial}{\partial y}\tilde{E}_x\,\vec{a}_z = -j10k\,e^{jky}\vec{a}_z$, since $\nabla\times\tilde{E} = -j\omega\tilde{B}$, $\Rightarrow \tilde{B}_z = \frac{10k}{\omega}e^{jky}$ and $\nabla\cdot\tilde{B} = 0$

$\tilde{H}_z = \frac{10k}{\omega\mu_0}e^{jky} \qquad \nabla\times\tilde{H} = \frac{\partial}{\partial y}\tilde{H}_z\,\vec{a}_x = j\frac{10}{\omega\mu_0}k^2 e^{jky}\vec{a}_x \qquad c = 3\times10^8$ m/s

From $\nabla\times\tilde{H} = j\omega\tilde{D}$, we have $k^2 = \omega^2\mu_0\epsilon_0$ or $k = \pm\omega\sqrt{\mu_0\epsilon_0} = \pm\frac{\omega}{c}$

$T = 100$ ns, $f = 10$ MHz and $k = \pm 0.209$ rad/m.

$$\langle\hat{S}\rangle = \frac{1}{2}\text{Re}[\tilde{E}\times\tilde{H}^*] = -\frac{50k}{\omega\mu_0}\vec{a}_y = -0.133\,\vec{a}_y \quad \text{W/m}^2$$

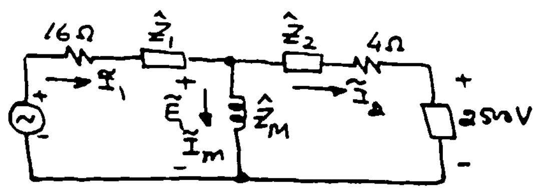
Exercise 7.25

$S = 10$ kVA, $V_1 = 500$ V $\quad a = 2 \Rightarrow V_2 = 500/2 = 250$V

$P_o = 8000\times0.8 = 6400$ W $\qquad I_2 = \frac{8000}{250} = 32$ A, $I_1 = \frac{I_2}{a} = 16$

$Pf = 0.8$ (lead) $\Rightarrow \theta = +36.87°$ $\quad \tilde{I}_2 = 32\underline{/36.87°}$ A, $\tilde{I}_1 = 16\underline{/36.87°}$ A

Exercise 7.26



$N_2/N_1 = 0.5$, $\omega = 377$ rad/s

$L_2 = \frac{N_2^2}{\mathcal{R}}$ $L_1 = \frac{N_1^2}{\mathcal{R}}$ $L_1 = 80\,mH$

$L_2 = \left(\frac{N_2}{N_1}\right)^2 L_1 = 20\,mH$

$k = 1 \Rightarrow M = \sqrt{L_1 L_2} = 40\,mH$

$\hat{z}_1 = j\omega(L_1 - M) = j15.08\Omega$, $\hat{z}_2 = j\omega(L_2 - M) = -j7.54\Omega$, $\hat{z}_M = j\omega M = j15.08\Omega$

$\tilde{V}_2 = 2500\angle 0^\circ$ V, $\tilde{I}_2 = 40\angle -45^\circ$ A

$\tilde{E}_1 = 2500 + 40\angle -45^\circ (4 - j7.54) = 2421.98\angle -7.74^\circ$ V

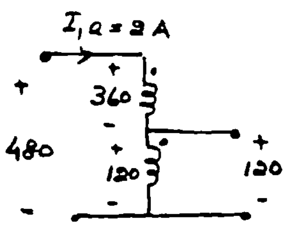
$\tilde{I}_M = \frac{\tilde{E}_1}{\hat{z}_M} = 160.61\angle -97.74^\circ$ A, $\tilde{I}_1 = \tilde{I}_M + \tilde{I}_2 = 187.54\angle -87.94^\circ$ A

$\tilde{V}_1 = \tilde{E}_1 + \tilde{I}_1(16 + \hat{z}_1) = 6231.84\angle -31.17^\circ$ V

$P_{in} = Re[\tilde{V}_1 \tilde{I}_1^*] = 640\,kW$ $P_o = Re[\tilde{V}_2 \tilde{I}_2^*] = 70.7\,kW$, $\eta = \frac{P_o}{P_{in}} = 0.11$ or 11%

Exercise 7.27

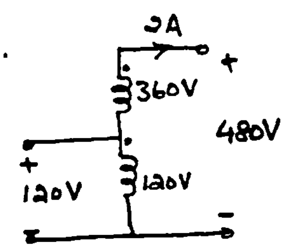
$I_1 = 720/360 = 2A$ $I_2 = 720/120 = 6A$



$S_a = 480 \times 2 = 960$ VA

$a_T = \frac{480}{120} = 4$

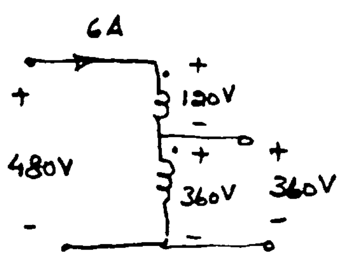
480/120 V, 960 VA



$a_T = 120/480 = 0.25$

$S_a = 480 \times 2 = 960$ VA

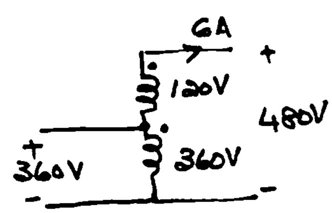
120/480 V, 960 VA



$a_T = 480/360 = 1.333$

$S_a = 480 \times 6 = 2880$ VA

480/360 V, 2880 VA.



$a_T = 360/480 = 0.75$

$S_a = 480 \times 6 = 2880$ VA

360/480 V, 2880 VA

Exercise 7.28

From Exercise 7.27,

$a_T = 4$, $\tilde{I}_{1a} = 2\angle{-36.87°}$ A

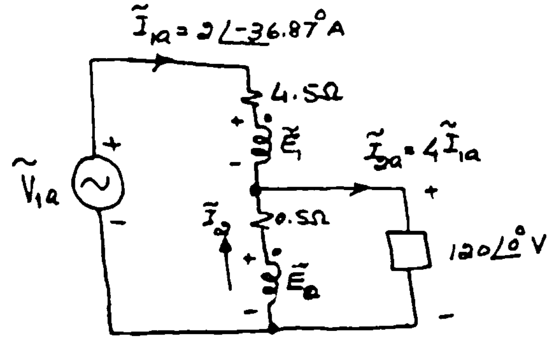
$\tilde{I}_{2a} = 4\tilde{I}_{1a} = 8\angle{36.87°}$

$\tilde{I}_2 = 6\angle{-36.87°}$ A

$P_o = 120 \times 8 \times 0.8 = 768$ W

$P_{in} = P_o + I_{1a}^2(4.5) + I_2^2(0.5) = 804$ W

$\eta = \frac{768}{804} = 0.9552$ or 95.52%



$\tilde{I}_{1a} = 6\angle{-36.87°}$ A , $\tilde{I}_1 = 2\angle{-36.87°}$ A

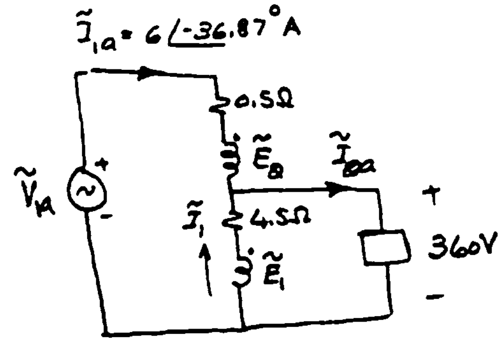
$\tilde{I}_{2a} = 8\angle{-36.87°}$ A

$P_o = 360 \times 8 \times 0.8 = 2304$ W

$P_{in} = P_o + I_{1a}^2(0.5) + I_1^2(4.5)$

$= 2340$ W

$\eta = \frac{P_o}{P_{in}} = 0.9846$ or 98.46%



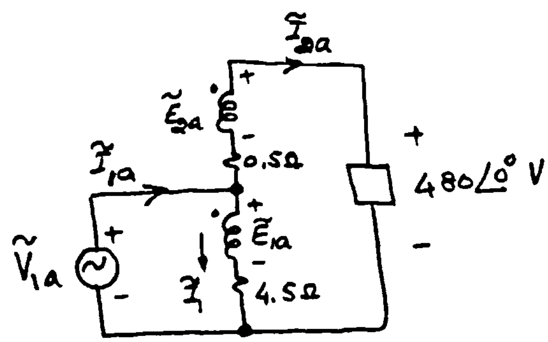
$\tilde{I}_{2a} = 6\angle{-36.87°}$ A

$\tilde{I}_1 = 2\angle{-36.87°}$ V

$P_o = 480 \times 6 \times 0.8 = 2304$ W

$P_{in} = 2304 + 2^2 \times 4.5 + 6^2 \times 0.5$

$= 2340$ W $\Rightarrow \eta = 98.46\%$



$\tilde{I}_{2a} = 2\angle{-36.87°}$ A

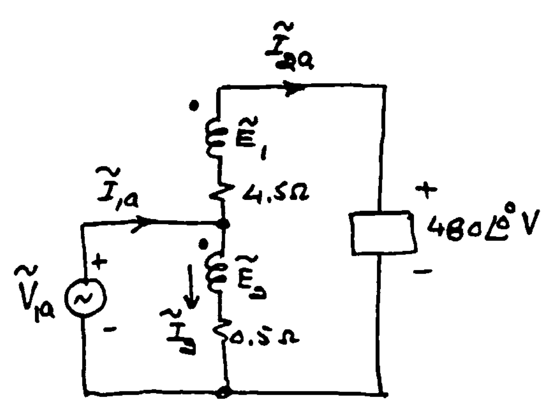
$\tilde{I}_{1a} = 8\angle{-36.87°}$ A, $\tilde{I}_2 = 6\angle{-36.87°}$ A

$P_o = 480 \times 2 \times 0.8 = 768$ W

$P_{in} = 768 + 6^2 \times 0.5 + 2^2 \times 4.5$

$= 804$ W

$\eta = 768/804 = 0.9552$ or 95.52%



Exercise 7.29 at $\rho = a$, $\vec{B} = B_m \sin \omega t \, \vec{a}_z$

From (7.153) $\vec{B}_o = 2\vec{B} = 2B_m \sin \omega t \, \vec{a}_z$

Thus, $\Phi = 2B_m \pi a^2 \sin \omega t$

$$e = -\frac{d\Phi}{dt} = -2\pi a^2 \omega B_m \cos\omega t$$

Hence, $\vec{E} = \frac{e}{2\pi a} = -a\omega B_m \cos\omega t \, \vec{a}_\phi$

Force: $\vec{F} = -e\vec{E} = ae\omega B_m \cos\omega t \, \vec{a}_\phi$

Work done per revolution: $W = 2\pi a F = 2\pi a^2 e\omega \cos\omega t$

Exercise 7.30 $B_m = 0.4\,T$ $a = 0.84\,m$ $f = 60\,Hz$, $\omega \simeq 377$ rad/s

$B_o = 0.8 \sin\omega t$ (T) $\Phi = 1.773 \sin\omega t$ (Wb)

$E_\phi = -126.67 \cos\omega t$ (V/m), $W = 1.07 \times 10^{-16} \cos\omega t$ J

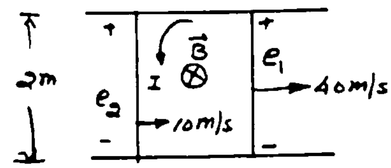
$$W_{avg} = \frac{2}{\pi}(1.07 \times 10^{-16}) = 6.81 \times 10^{-17}\,J \text{ or } 425.6 \text{ eV}$$

Problem 7.1

$e_1 = 0.8 \times 2 \times 40 = 64\text{V}$

$e_2 = 0.8 \times 2 \times 10 = 16\text{V}$

$I = \dfrac{64-16}{12} = 4\text{A}$



Problem 7.2

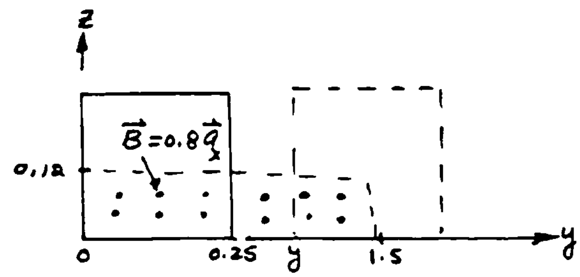
$0 \le y \le 1.25\text{m}$

$u_y = 100 \text{ m/s} \Rightarrow 0 \le t \le 12.5\text{ms}$

Flux Linking the loop is constant.

$\Phi = 0.8 \times 0.12 \times 0.25 = 24 \text{ mWb}$

$e = -\dfrac{d\Phi}{dt} = 0 \Rightarrow I = 0$



$1.25 \le y \le 1.5\text{m}$ or $12.5 \le t \le 15\text{ ms}$

$\Phi = 0.8 \times 0.12 \displaystyle\int_t^{15\text{ms}} 100\,dt = 0.144 - 9.6t \Rightarrow e = -\dfrac{d\Phi}{dt} = 9.6\text{V}, \quad I = \dfrac{9.6}{12} = 0.8\text{A}$

$y \ge 1.5\text{m}$ or $t \ge 15\text{ms}$, $\Phi = 0 \Rightarrow e = 0$ and $i = 0$

Problem 7.3 $L = 0.2\text{ m}, \quad \omega = \dfrac{2\pi \times 1200}{60} = 40\pi \qquad B_z = 250\cos 30^\circ = 216.5 \text{ mT}$

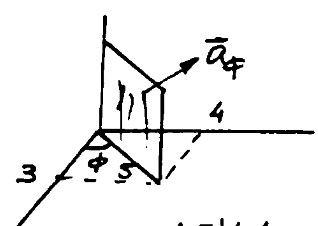
Hence, $e = \frac{1}{2} B_z \omega L^2 = 0.54\text{V}$

Problem 7.4 $n = 50{,}000$ Turns/unit length $\quad I = 12\text{ A}, \; L = 3\text{cm}$

$\omega = \dfrac{2\pi \times 3600}{60} = 120\pi$ rad/s $\qquad e = \frac{1}{2} B\omega L^2 = \frac{1}{2}(\mu_0 n I)\omega L^2 \cong 0.13\text{ V}$

Problem 7.5

$\vec{a}_\phi = -\vec{a}_x \sin 53.13^\circ + \vec{a}_y \cos 53.13^\circ$
$= -0.8\,\vec{a}_x + 0.6\,\vec{a}_y$



$\phi = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$

$\vec{ds} = d\rho\, dz\, \vec{a}_\phi$

$\vec{B} \cdot \vec{ds} = -2\sin 300t + 1.05\cos 300t \quad (\text{mWb})$

$\underset{(\text{mWb})}{\Phi} = \int \vec{B}\cdot\vec{ds} = \displaystyle\int_0^5 d\rho \int_0^4 (-2\sin 300t + 1.05\cos 300t)\, dz$

$= -40\sin 300t + 21\cos 300t$ mWb

$e = -\dfrac{d\Phi}{dt} = 12\cos 300t + 6.3\sin 300t \;(\text{V})$

$i = \dfrac{e}{2} = 6\cos 300t + 3.15\sin 300t$
$= 6.78\cos(300t - 27.67^\circ)$ A

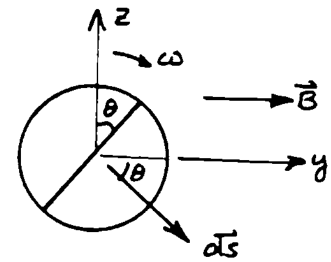
Problem 7.6 $\vec{B} = 0.8\,\vec{a}_y$ T

$\theta = \omega t$

Area: $A = 40\text{cm}^2$

$N = 200$

$\omega = 120$ rad/s



$$\Phi = \int_S \vec{B}\cdot\vec{ds} = 0.8A\cos\theta$$
$$= 3.2\cos\omega t \quad \text{mWb}$$

$$e = -N\frac{d\Phi}{dt} = 3.2N\omega\sin\omega t \quad (\text{mV})$$
$$= 76.8\sin 120t\text{, V}$$

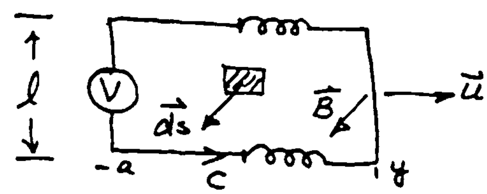
Problem 7.7 $\vec{B} = 0.8\sin 120t$, T $\quad \Phi = 0.8\sin(120t)\cos(120t)\int_S ds$

$$= 1.6\sin 240t \quad \text{mWb}$$

$$e = -N\frac{d\Phi}{dt} = -200(1.6\times10^{-3})\,240\cos 240t$$
$$= -76.8\cos 240t = 76.8\sin(240t - 90^\circ)\text{ V}$$

Problem 7.8

$$\vec{u} = u\cos\omega t\,\vec{a}_y \quad \text{m/s}$$
$$\vec{B} = B\cos\omega t\,\vec{a}_x \quad \text{T}$$



$$e_m = \int_C (\vec{u}\times\vec{B})\cdot\vec{dl} = -Bu\cos^2\omega t\int_0^l dz$$
$$= -Bul\cos^2\omega t$$

$$e_T = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot\vec{ds} = B\omega\sin\omega t\int_0^l dz\int_{-a}^{y} dy = B\omega l(y+a)\sin\omega t$$

But $\frac{dy}{dt} = u\cos\omega t \Rightarrow y = \frac{u}{\omega}\sin\omega t + \cancel{c}^{\,0}$ $(\because t=0, y=0)$

Thus, $e = e_m + e_T = -Bul\cos^2\omega t + Bul\sin^2\omega t + B\omega la\sin\omega t$
$$= B\omega la\sin\omega t - Blu\cos 2\omega t \quad (\text{V})$$

b) Faraday's law: $d\Phi = \vec{B}\cdot\vec{ds} = B\cos\omega t\,dy\,dz$

$$\Phi = B\cos\omega t\int_{-a}^{y} dy\int_0^l dz = Bly\cos\omega t + Bla\cos\omega t$$

$$e = -\frac{d\Phi}{dt} = Bly\omega\sin\omega t - Bl\cos\omega t\frac{dy}{dt} + Bla\omega\sin\omega t$$

Since $\frac{dy}{dt} = u\cos\omega t$ and $y = \frac{u}{\omega}\sin\omega t$

$$e = Bla\omega\sin\omega t + Blu\sin^2\omega t - Blu\cos^2\omega t$$
$$= Bla\omega\sin\omega t - Blu\cos 2\omega t\text{, V}$$

Problem 7.9 $\quad \vec{u} = -1000\,\vec{a}_y$ m/s $\quad \vec{B} = 0.2\,\vec{a}_z$ T

$$\vec{u}\times\vec{B} = -200\,\vec{a}_x \Rightarrow e = \int_{0.1}^{0} (\vec{u}\times\vec{B})\cdot d\vec{l} = 20\text{V}$$

Problem 7.10 $\quad \mathcal{F} = NI = 1200\times 0.75 = 900$ At

From Figure 5.37, $B = 1.05$T, $\quad \Phi = BA = 1.05\times 10^{-4}\times 4 = 4.2\times 10^{-4}$ Wb

$$L = \frac{N\Phi}{I} = \frac{1200\times 4.2\times 10^{-4}}{0.75} = 0.672\text{ H} \quad \text{or} \quad 672\text{ mH}$$

Problem 7.11

$$\Phi = \frac{LI}{N} = 20\times 10^{-3}\times 2.5/1000 = 50\,\mu\text{Wb}$$

Problem 7.12

(a) $$L = \frac{\mu_0}{2\pi}\ln(b/a) = \frac{4\pi\times 10^{-7}}{2\pi}\ln(4/2) = 138.63\text{ nH}$$

(b) $$L = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln(b/a) = 50 + 138.63 = 188.63\text{ nH}$$

Problem 7.13 $$\mathcal{R} = \frac{l}{\mu A} = \frac{80\times 10^{-2}}{4\pi\times 10^{-7}\times 500\times 40\times 10^{-4}} = 318{,}309.89 \quad (H^{-1})$$

$$L_{11} = \frac{N_1^2}{\mathcal{R}} = \frac{100^2}{\mathcal{R}} = 31.42\text{mH}, \quad L_{22} = \frac{N_2^2}{\mathcal{R}} = \frac{150^2}{\mathcal{R}} = 70.69\text{mH}, \quad L_{33} = \frac{N_3^2}{\mathcal{R}} = \frac{200^2}{\mathcal{R}} = 126\text{ mH}$$

$$L_{12} = L_{21} = \frac{N_1N_2}{\mathcal{R}} = 47.12\text{mH}, \quad L_{13} = L_{31} = \frac{N_1N_3}{\mathcal{R}} = 62.83\text{mH}, \quad L_{23} = L_{32} = \frac{N_2N_3}{\mathcal{R}} = 94.25\text{mH}$$

$$i_1 = 10\sin(800\pi t) \Rightarrow \frac{di_1}{dt} = 25132.74\cos(800\pi t)$$

$$e_1 = L_{11}\frac{di_1}{dt} = 789.67\cos(800\pi t)\text{ V}, \quad e_2 = L_{12}\frac{di_1}{dt} = 1184.3\cos(800\pi t)\text{ V}$$

$$e_3 = L_{13}\frac{di_1}{dt} = 1579.1\cos(800\pi t)\text{ V}$$

Problem 7.14

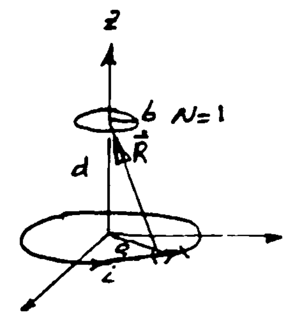
$$\left.\begin{aligned} L_1 + L_2 + 2M &= 3.28 \quad (\text{mH}) \\ L_1 + L_2 - 2M &= 0.72 \quad (\text{mH}) \end{aligned}\right\} = \quad M = 0.64\text{ mH}$$

Set $L_1 = 4L_2$, then $\quad 5L_2 + 2M = 3.28 \Rightarrow L_2 = 0.4\text{mH}$

$$L_1 = 1.6\text{ mH}$$

$$k = \frac{M}{\sqrt{L_1L_2}} = 0.8$$

Problem 7.15



$\vec{R} = -a\,\vec{a}_\rho + d\,\vec{a}_z$

$$\vec{B} = \frac{\mu_0 i}{4\pi}\int_0^{2\pi} \frac{a\,d\phi(\vec{a}_\phi \times \vec{a}_R)}{R^2}$$

$$= \frac{\mu_0 i}{4\pi}\int_0^{2\pi} \frac{a^2 d\phi}{R^3}\,\vec{a}_z + \cancel{\frac{\mu_0 i}{4\pi} a \int_0^{2\pi} \frac{d\phi}{R^3}\,\vec{a}_\rho} \rightarrow 0$$

$$= \frac{\mu_0 i}{2}\,\frac{a^2}{R^3} \qquad R = \sqrt{a^2+d^2}$$

$$\Phi = \int_S \vec{B}\cdot d\vec{s} = B_z \pi b^2 = \frac{\mu_0 i}{2}\,\frac{\pi b^2 a^2}{(a^2+d^2)^{3/2}}$$

$$L_{12} = L_{21} = M = \frac{N\Phi}{i} = \frac{\mu_0}{2}\,\frac{\pi a^2 b^2}{(a^2+d^2)^{3/2}}$$

Problem 7.16 Same as Problem 7.15 except $d=0$. Thus.

$$M = \frac{\mu_0}{2a}\pi b^2$$

Problem 7.17 $B_\phi = \dfrac{\mu_0 i}{2\pi\rho}$ but $\rho = r\sin\theta$ $\quad d\vec{s} = r\,dr\,d\theta\,\vec{a}_\phi$

$$\Phi = \frac{\mu_0 i}{2\pi}\int_a^b dr \int_{\pi/6}^{\pi/2} \frac{d\theta}{\sin\theta} = \frac{\mu_0 i}{2\pi}(b-a)\left[\ln\left(\tan\frac{\theta}{2}\right)\right]_{\pi/6}^{\pi/2}$$

$$= 0.21\,\mu_0 (b-a)\, i$$

Thus, $\quad M = 0.21\,\mu_0 (b-a)$

Problem 7.18 $\Phi = BA = \mu_0 N_1 i_1 A \Rightarrow M = \dfrac{N_2\Phi}{i_1} = N_1 N_2 \mu_0 A$

$$= 4\pi\times10^{-7}\times 400\times 4000\times \pi\times 10^{-4} = 631.65 \ \mu H/m$$

$$e = M\frac{di_1}{dt} = -631.65\times10^{-6}\times 0.5\times 200\sin 200t = -63.16\sin 200t,\ mV/m$$

Problem 7.19 $\Phi = a i^n \quad d\phi = a n\, i^{n-1} di$ $\quad \left[\begin{array}{l} p = vi = N\dfrac{d\Phi}{dt} i \\ dW = p\,dt = N i\, d\Phi \end{array}\right]$

$$dW = N i\, d\Phi = N a n\, i^n di$$

$$W = N a n\int_0^I i^n di = N a \left(\frac{n}{n+1}\right) I^{n+1} = N\left(\frac{n}{n+1}\right)\Phi I$$

Problem 7.20 $\Phi = a\ln(bi) \Rightarrow d\Phi = \frac{a}{i}di$

$$dW = Ni\,d\Phi = Na\,di \Rightarrow W = Na\int_0^I di = NaI$$

Problem 7.21 $\Phi = \dfrac{ai}{b+ci} \Rightarrow d\Phi = \dfrac{ba\,di}{(b+ci)^2}$

$$W = Nba\int_0^I \frac{i}{(b+ci)^2}di = \frac{Nba}{c^2}\left[\ln(b+ci) + \frac{b}{b+ci}\right]_0^I$$

$$= \frac{Nba}{c^2}\left[\ln\left(\frac{b+cI}{b}\right) - \frac{cI}{b+cI}\right]$$

Problem 7.22 $I = 200/10 = 20\,A$ $\quad W_m = \frac{1}{2}LI^2 = \frac{1}{2}\times 30\times 10^{-3}\times 20^2 = 6\,J$

$$\omega_m = \frac{6}{20\times 10^{-2}\times \pi\times 2.5^2\times 10^{-4}} = 15.28\,kJ/m^3$$

Since $\omega_m = \frac{1}{2}\frac{B^2}{\mu_0} \Rightarrow B = \sqrt{2\times 4\pi\times 10^{-7}\times 15.28\times 10^3} \cong 0.196\,T$

Problem 7.23 $H_\phi = \frac{I}{2\pi\rho} \Rightarrow \omega_m = \dfrac{\mu_0 I^2}{8\pi^2\rho^2} = \dfrac{4\pi\times 10^{-7}\times 1000^2}{8\pi^2\rho^2} = \dfrac{0.0159}{\rho^2}\,J/m^3$

$$W = 0.0159\int_{0.05}^{0.1}\frac{1}{\rho}d\rho\int_0^{2\pi}d\phi = 69.3\ \text{mJ/m}$$ [Energy per unit length]

Problem 7.24 $B = 0.04\,mT \Rightarrow \omega_m = \frac{1}{2}\dfrac{(0.04\times 10^{-3})^2}{4\pi\times 10^{-7}} = 636.62\,\mu J/m^3$

$$W = 636.62\times 10^{-6}\times \frac{4\pi}{3}\left[12.8^3 - 6.4^3\right]10^{12} = 4.89\times 10^{12}\,J$$

Problem 7.25 $R = 0.5\,\Omega,\quad L = 2\,H \qquad W = 6.4\,kJ$

$$W = \frac{1}{2}LI^2 \Rightarrow I = \sqrt{\frac{2\times 6400}{2}} = 80\,A$$

Thus, $P = I^2R = 3.2\,kW$

Problem 7.26 $N = 500$ Turns $a = 10$ cm, $b = 15$ cm, $\mu_r = 1000$ $I = 10A$

$$H_\phi = \frac{NI}{2\pi\rho} \Rightarrow w_m = \frac{\mu N^2 I^2}{8\pi^2\rho^2} = \frac{397.89}{\rho^2} \ J/m^3$$

$$W_m = 379.89 \int_{0.1}^{0.15} \frac{1}{\rho} d\rho \int_0^{2\pi} d\phi \int_0^{0.05} dz = 379.89(2\pi)0.05 \ln(1.5) = 50.68 \ J$$

$$\text{Since } W_m = \frac{1}{2} L I^2 \Rightarrow L = \frac{2 \times 50.68}{10^2} = 1.014 \ H$$

Problem 7.27 $\nabla \cdot \vec{J} + \frac{\partial \rho_v}{\partial t} = 0$ $\vec{J} = \sigma \vec{E} = \frac{\sigma}{\epsilon} \vec{D}$ $\nabla \cdot \vec{J} = \frac{\sigma}{\epsilon} \nabla \cdot \vec{D} = \frac{\sigma}{\epsilon} \rho_v$

$$\text{Hence} \quad \frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\epsilon} \rho_v = 0$$

Problem 7.28 $\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$ since $B = \nabla \times \vec{A}$,

$$\nabla \times \vec{E} + \frac{\partial}{\partial t}(\nabla \times \vec{A}) = 0 \Rightarrow \nabla \times \vec{E} + \nabla \times \left(\frac{\partial \vec{A}}{\partial t}\right) = 0$$

or $$\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0 \Rightarrow \oint_c \left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) \cdot d\vec{l} = 0 \quad [\text{Stokes' Thm}]$$

Problem 7.29 $\vec{E} = [E_o \sin(\alpha x - \omega t) + E_o \sin(\alpha x + \omega t)] \vec{a}_y = 2E_o \sin(\alpha x) \cos\omega t \ \vec{a}_y$

$$\vec{D} = 2\epsilon E_o \sin(\alpha x) \cos\omega t \ \vec{a}_y \Rightarrow \frac{\partial \vec{D}}{\partial t} = -2\omega\epsilon E_o \sin(\alpha x) \sin\omega t \ \vec{a}_y \quad A/m^2$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \frac{\partial \vec{B}}{\partial t} = -2\alpha E_o \cos\alpha x \cos\omega t \ \vec{a}_z$$

Thus, $$\vec{B} = -\frac{2\alpha}{\omega} E_o \cos\alpha x \sin\omega t \ \vec{a}_z \quad \text{and} \quad \vec{H} = -\frac{2\alpha}{\omega\mu} E_o \cos\alpha x \sin\omega t \ \vec{a}_z \ A/m$$

Problem 7.30 $\vec{H} = H_o[\cos(\alpha x - \omega t) + \cos(\alpha x + \omega t)] \vec{a}_z$

$$= 2H_o \cos(\alpha x) \cos\omega t \ \vec{a}_z \quad A/m$$

$$\frac{\partial \vec{D}}{\partial t} = \nabla \times \vec{H} = 2\alpha H_o \sin(\alpha x) \cos\omega t \ \vec{a}_y \Rightarrow$$

$$\vec{D} = \frac{2\alpha}{\omega} H_o \sin(\alpha x) \sin\omega t \ \vec{a}_y \quad C/m^2$$

and $$\vec{E} = \frac{2\alpha}{\omega\epsilon} H_o \sin(\alpha x) \sin\omega t \ \vec{a}_y \quad V/m$$

Problem 7.31 Refer to the solution of Problem 7.29

$\nabla \cdot \vec{B} = 0$ and $\nabla \cdot \vec{D} = 0$

$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \Rightarrow -\frac{\partial}{\partial x} H_z = \frac{\partial}{\partial t}(\epsilon E_y)$

or $\frac{2\alpha^2}{\omega\mu} E_o \sin\alpha x \sin\omega t = 2\omega\epsilon E_o \sin\alpha x \sin\omega t \Rightarrow \alpha^2 = \omega^2\mu\epsilon$

Problem 7.32 Refer to the solution of Problem 7.30

$\nabla \cdot \vec{D} = 0 \qquad \nabla \cdot \vec{B} = 0$

From $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$, we have $\frac{\partial}{\partial x} E_y = -\frac{\partial}{\partial t} B_z$ or

$\frac{2\alpha^2}{\omega\epsilon} H_o \cos\alpha x \sin\omega t = 2\omega\mu H_o \cos\alpha x \sin\omega t \Rightarrow \alpha^2 = \omega^2\mu\epsilon$

Problem 7.33 $\vec{E} = E_o \cos(\omega t - \beta z)\,\vec{a}_x$, $\vec{D} = \epsilon\vec{E} \Rightarrow \nabla \cdot \vec{D} = 0$

$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} = -\beta E_o \sin(\omega t - \beta z)\,\vec{a}_y \Rightarrow \vec{B} = \frac{\beta E_o}{\omega}\cos(\omega t - \beta z)\,\vec{a}_y$

$\vec{H} = \frac{\beta E_o}{\omega\mu}\cos(\omega t - \beta z)\,\vec{a}_y \qquad \nabla \cdot \vec{B} = 0$

From $\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$ we obtain $\beta^2 = \omega^2\mu\epsilon$

$w_e = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2}\epsilon E_o^2 \cos^2(\omega t - \beta z) \Rightarrow \langle w_e \rangle = \frac{1}{T}\int_0^T w_e\,dt = \frac{1}{4}\epsilon E_o^2 \;\; \mathrm{J/m^3}$

$w_m = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\mu H^2 = \frac{\beta^2}{2\omega^2\mu} E_o^2 \cos^2(\omega t - \beta z) \Rightarrow \langle w_m \rangle = \frac{1}{T}\int_0^T w_m\,dt = \frac{1}{4}\frac{\beta^2}{\omega^2\mu}E_o^2 = \frac{1}{4}\epsilon E_o^2$

$\langle \vec{S} \rangle = \frac{1}{T}\int_0^T (\vec{E}\times\vec{H})\,dt = \frac{\beta}{\omega\mu}E_o^2 \frac{1}{T}\int_0^T \cos^2(\omega t - \beta z)\,dt\;\vec{a}_z = \frac{1}{2}\frac{\beta}{\omega\mu}E_o^2\,\vec{a}_z \;\; \mathrm{W/m^2}$

Problem 7.34 $\vec{E} = 2E_o \sin\alpha x \cos\omega t\,\vec{a}_y$, $\vec{H} = -\frac{2\alpha}{\omega\mu}E_o \cos\alpha x \sin\omega t\,\vec{a}_z$

$w_e = \frac{1}{2}\epsilon E^2 = 2\epsilon E_o^2 \sin^2\alpha x \cos^2\omega t \Rightarrow \langle w_e \rangle = \frac{1}{T}\int_0^T w_e\,dt = \epsilon E_o^2 \sin^2\alpha x$

$w_m = \frac{1}{2}\mu H^2 = \frac{2\alpha^2}{\omega^2\mu}E_o^2 \cos^2\alpha x \sin^2\omega t \Rightarrow \langle w_m \rangle = \frac{\alpha^2}{\omega^2\mu}E_o^2 \cos^2\alpha x$

$= \epsilon E_o^2 \cos^2\alpha x \;\; (\because \alpha^2 = \omega^2\mu\epsilon)$

[Prob. 7.31]

Problem 7.35 $\vec{E} = \frac{2\alpha}{\omega\epsilon} H_0 \sin\alpha x \sin\omega t \; \vec{a}_y$

$\vec{H} = 2H_0 \cos\alpha x \cos\omega t \; \vec{a}_z$

From Problem 7.32 $\alpha^2 = \omega^2\mu\epsilon$

$$w_e = \frac{1}{2}\epsilon E^2 = \frac{2\alpha^2}{\omega^2\epsilon} H_0^2 \sin^2\alpha x \sin^2\omega t \Rightarrow \langle w_e \rangle = \mu H_0^2 \sin^2\alpha x \quad J/m^3$$

$$w_m = \frac{1}{2}\mu H^2 = 2\mu H_0^2 \cos^2\alpha x \cos^2\omega t \Rightarrow \langle w_m \rangle = \mu H_0^2 \cos^2\alpha x \quad J/m^3$$

Problem 7.36 $q = \int i\,dt = \frac{I_m}{\omega} \sin\omega t$

$$\oint_s \vec{D}\cdot\vec{ds} = q \Rightarrow D = \frac{q}{A} = \frac{I_m}{\omega A}\sin\omega t$$

$$J_d = \frac{\partial D}{\partial t} \Rightarrow J_d = \frac{I_m}{A}\cos\omega t, \text{ Thus, } i_d = J_d A = I_m \cos\omega t \quad A$$

Problem 7.37 $J_c = \sigma E$ $\quad J_d = \omega\epsilon E \Rightarrow J_d/J_c = \frac{\omega\epsilon}{\sigma}$

For $J_d/J_c = 1$, $\omega\epsilon = \sigma \Rightarrow f = \frac{\sigma}{2\pi\epsilon}$

Sea water: $\epsilon = 81\epsilon_0$ $\sigma = 0.4\times10^{-3}$ S/m $\Rightarrow f = 88.889$ kHz

when $f \ll 88.889$ kHz $\quad J_c \gg J_d$ conductor

and when $f \gg 88.889$ kHz, $J_d \gg J_c$ poor conductor

Problem 7.38 $$E_z = -\frac{V}{d} = \frac{141\sin10^9 t}{0.005} = -28.2\sin10^9 t \quad kV/m$$

$$\vec{J}_c = \sigma\vec{E} = -0.02\times28{,}200\sin10^9 t\,\vec{a}_z = -564\sin10^9 t\,\vec{a}_z \quad A/m^2$$

$$i_c = \int_s \vec{J}_c\cdot\vec{ds} = 564\times0.4\sin10^9 t = 225.6\sin10^9 t \quad A$$

$$\vec{J}_d = \epsilon\frac{\partial\vec{E}}{\partial t} = -\frac{4\times10^{-9}}{36\pi}\times28200\times10^9\cos10^9 t\,\vec{a}_z$$

$$= -997.37\cos10^9 t\;\vec{a}_z \quad A/m^2$$

$$i_d = \int_s \vec{J}_d\cdot\vec{ds} = 398.95\cos10^9 t \quad A$$

$$i = i_c + i_d = 225.6\sin10^9 t + 398.95\cos10^9 t \quad A$$

$$I_{rms} = \sqrt{\frac{225.6^2 + 398.95^2}{2}} = 324.08\,A$$

Problem 7.39 $\quad \vec{E} = E_0 \cos(\omega t - ax - kz)\,\vec{a}_y$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \Rightarrow \quad \frac{\partial B_x}{\partial t} = \frac{\partial}{\partial z}E_y = kE_0 \sin(\omega t - ax - kz) \Rightarrow$$

$$B_x = -\frac{kE_0}{\omega}\cos(\omega t - ax - kz)$$

and $\quad \frac{\partial}{\partial t}B_z = -\frac{\partial}{\partial x}E_y = -aE_0 \sin(\omega t - ax - kz) \Rightarrow B_z = \frac{aE_0}{\omega}\cos(\omega t - ax - kz)$

$\nabla\cdot\vec{B} = 0 \qquad \nabla\cdot\vec{D} = 0 \qquad$ Source-free : $\Rightarrow \vec{J} = 0$

$$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} \Rightarrow \quad \nabla\times\vec{B} = \mu\epsilon\frac{\partial\vec{E}}{\partial t} \Rightarrow$$

$$-(a^2+k^2)\frac{E_0}{\omega}\sin(\omega t - ax - kz)\,\vec{a}_y = -\mu\epsilon\omega E_0 \sin(\omega t - ax - kz)\,\vec{a}_y$$

or $\qquad \omega^2\mu\epsilon = a^2 + k^2 \qquad$ [Condition for the fields to exist]

$$w_e = \frac{1}{2}\epsilon E^2 = \frac{1}{2}\epsilon E_0^2\cos^2(\omega t - ax - kz) \Rightarrow \langle w_e\rangle = \frac{1}{4}\epsilon E_0^2$$

$$w_m = \frac{1}{2}\frac{B^2}{\mu} = \frac{E_0^2}{2\mu}\left(\frac{a^2+k^2}{\omega^2}\right)\cos^2(\omega t - ax - kz) \Rightarrow \langle w_m\rangle = \frac{E_0^2(a^2+k^2)}{4\mu\omega^2} = \frac{1}{4}\epsilon E_0^2$$

$$\vec{S} = \vec{E}\times\vec{H} = \frac{E_0^2}{\omega\mu}\cos^2(\omega t - ax - kz)\left[a\,\vec{a}_x + k\,\vec{a}_z\right]$$

$$\langle\vec{S}\rangle = \frac{E_0^2}{2\omega\mu}(a\,\vec{a}_x + k\,\vec{a}_z)$$

Problem 7.40

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = -\mu\frac{\partial\vec{H}}{\partial t} \quad (1)$$

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} = \sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t} \quad (2)$$

Take Curl of (1): $\qquad \nabla\times\nabla\times\vec{E} = -\mu\frac{\partial}{\partial t}(\nabla\times\vec{H}) \qquad$ Source-free : $\nabla\cdot\vec{D} = 0$

$$\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

Thus, $\qquad \nabla^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$

Problem 7.41 $\quad \nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} \; (1) \qquad \nabla\times\vec{H} = \sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t} \; (2)$

Take Curl of (2): $\quad \nabla\times\nabla\times\vec{H} = \sigma\,\nabla\times\vec{E} + \epsilon\frac{\partial}{\partial t}(\nabla\times\vec{E})$

$$\nabla\cancelto{0}{(\nabla\cdot\vec{H})} - \nabla^2\vec{H} = -\mu\sigma\frac{\partial\vec{H}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2} \qquad [\nabla\cdot\vec{B} = 0]$$

Thus, $\qquad \nabla^2\vec{H} = \mu\sigma\frac{\partial\vec{H}}{\partial t} + \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2}$

or $\qquad \nabla^2\vec{H} - \mu\sigma\frac{\partial\vec{H}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2} = 0$

Problem 7.42 For a dielectric Medium: Source free:

$$\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} \quad ① \qquad \nabla\times\vec{H} = \epsilon\frac{\partial\vec{E}}{\partial t} \qquad \nabla\cdot\vec{D}=0 \qquad \nabla\cdot\vec{B}=0$$

$$\nabla\times\nabla\times\vec{E} = -\mu\frac{\partial}{\partial t}(\nabla\times\vec{H}) \Rightarrow -\nabla^2\vec{E} + \nabla(\cancel{\nabla\cdot\vec{E}})^{0} = -\mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

or $$\nabla^2\vec{E} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

$$\nabla\times\nabla\times\vec{H} = \epsilon\frac{\partial}{\partial t}(\nabla\times\vec{E}) \Rightarrow \nabla(\nabla\cdot\vec{H}) - \nabla^2\vec{H} = -\mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2}$$

a $$\nabla^2\vec{H} = \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2}$$

Problem 7.43

$$(\nabla\times\tilde{E})\cdot\tilde{H}^* = -j\omega\tilde{B}\cdot\tilde{H}^* \quad ① \qquad (\nabla\times\tilde{H}^*)\cdot\tilde{E} = -j\omega\tilde{D}^*\cdot\tilde{E} \quad ②$$

However, $$\nabla\cdot(\tilde{E}\times\tilde{H}^*) = (\nabla\times\tilde{E})\cdot\tilde{H}^* - (\nabla\times\tilde{H}^*)\cdot\tilde{E}$$

Thus, $$= -j\omega\tilde{B}\cdot\tilde{H}^* + j\omega\tilde{D}^*\cdot\tilde{E}$$

Since $\hat{S} = \frac{1}{2}\tilde{E}\times\tilde{H}^*$, $$-\nabla\cdot\hat{S} = \frac{1}{2}j\omega[\tilde{B}\cdot\tilde{H}^* - \tilde{E}\cdot\tilde{D}^*]$$
$$= 2j\omega[\tfrac{1}{4}\tilde{B}\cdot\tilde{H}^* - \tfrac{1}{4}\tilde{E}\cdot\tilde{D}^*]$$

Problem 7.44 For a conductive region: $\vec{J} = \sigma\vec{E}$

Thus, $$\nabla\times\tilde{E} = -j\omega\mu\tilde{H} \quad \text{and} \quad \nabla\times\tilde{H} = \sigma\tilde{E} + j\omega\epsilon\tilde{E}$$

$$(\nabla\times\tilde{E})\cdot\tilde{H}^* = -j\omega\mu\,\tilde{H}\cdot\tilde{H}^* \qquad (\nabla\times\tilde{H}^*)\cdot\tilde{E} = \sigma\tilde{E}\cdot\tilde{E}^* - j\omega\epsilon\tilde{E}\cdot\tilde{E}^*$$

$$\nabla\cdot(\tilde{E}\times\tilde{H}^*) = (\nabla\times\tilde{E})\cdot\tilde{H}^* - (\nabla\times\tilde{H}^*)\cdot\tilde{E}$$
$$= -j\omega\mu\,\tilde{H}\cdot\tilde{H}^* - \sigma\tilde{E}\cdot\tilde{E}^* + j\omega\epsilon\,\tilde{E}\cdot\tilde{E}^*$$

Since $\tilde{H}\cdot\tilde{H}^* = H^2$ and $\tilde{E}\cdot\tilde{E}^* = E^2$,

$$\nabla\cdot(\tilde{E}\times\tilde{H}^*) = -\sigma E^2 - j\omega[\mu H^2 - \epsilon E^2)$$

$\because$ $\hat{S} = \frac{1}{2}(\tilde{E}\times\tilde{H}^*)$,

$$-\nabla\cdot\hat{S} = \frac{1}{2}\sigma E^2 + 2j\omega[\tfrac{1}{4}\mu H^2 - \tfrac{1}{4}\epsilon E^2]$$

Problem 7.45 $\quad \tilde{E}_y = E_o\, e^{-jax} e^{-jkz} \Rightarrow \tilde{D}_y = \epsilon E_o e^{-jax} e^{-jkz} \Rightarrow \nabla\cdot\tilde{D} = 0$

$$\nabla\times\vec{E} = -\frac{\partial}{\partial z}\tilde{E}_y\,\vec{a}_x + \frac{\partial}{\partial x}E_y\,\vec{a}_z = j(k\vec{a}_x - a\vec{a}_z)\,e^{-jax}e^{-jkz}E_o$$

Since $\nabla\times\vec{E} = -j\omega\mu\vec{H} \Rightarrow \vec{H} = \left(\frac{aE_o}{\omega\mu}\vec{a}_z - \frac{kE_o}{\omega\mu}\vec{a}_x\right)e^{-jax}e^{-jkz}$ A/m

$\nabla\cdot\vec{B} = 0 \Rightarrow \nabla\cdot\vec{H} = 0$ and $\nabla\cdot\vec{H} = \frac{\partial}{\partial x}\tilde{H}_x + \frac{\partial}{\partial z}\tilde{H}_z = +j\frac{kE_o a}{\omega\mu} - j\frac{aE_o k}{\omega\mu} = 0$

$$\nabla\times\vec{H} = j\frac{E_o}{\omega\mu}\left[k^2 + a^2\right]e^{-jax}e^{-jkz}\vec{a}_y$$

From $\nabla\times\vec{H} = j\omega\epsilon\vec{E}$, we obtain $k^2 + a^2 = \omega^2\mu\epsilon$ as the condition for fields to exist.

$$\langle w_e\rangle = \frac{1}{4}\vec{E}\cdot\vec{D}^* = \frac{1}{4}\epsilon E_o^2 \quad \text{J/m}^3$$

$$\langle W_m\rangle = \frac{1}{4}\vec{B}\cdot\vec{H}^* = \frac{1}{4}\mu H^2 = \frac{k^2+a^2}{4\omega^2\mu}E_o^2 = \frac{1}{4}\epsilon E_o^2 \quad \text{J/m}^3$$

$$\langle\hat{S}\rangle = \frac{1}{2}\mathrm{Re}\left[\vec{E}\times\vec{H}^*\right] = \frac{E^2}{2\omega\mu}\left[k\vec{a}_z + a\,\vec{a}_x\right] \quad \text{W/m}^2$$

Problem 7.46 $\quad \tilde{E}_\rho = \frac{V}{\rho\ln(b/a)}e^{-jkz}$ V/m, $\quad \tilde{H}_\phi = \frac{I}{2\pi\rho}e^{-jkz}$ A/m

$$\nabla\times\vec{E} = \frac{\partial}{\partial z}\tilde{E}_\rho\,\vec{a}_\phi = -j\frac{kV}{\rho\ln(b/a)}e^{-jkz}\vec{a}_\phi$$

Since $\nabla\times\vec{E} = -j\omega\mu_o\vec{H} \Rightarrow -j\frac{kV}{\rho\ln(b/a)}e^{-jkz} = -j\omega\mu_o\frac{I}{2\pi\rho}e^{-jkz}$, or $k = \frac{\omega\mu_o I}{2\pi V}\ln(b/a)$ ①

$$\nabla\times\vec{H} = \frac{1}{\rho}\left(-\frac{\partial}{\partial z}\rho\tilde{H}_\phi\right)\vec{a}_\rho = \frac{jkI}{2\pi\rho}e^{-jkz}\vec{a}_\rho$$

Since $\nabla\times\vec{H} = j\omega\epsilon_o\vec{E} \Rightarrow k = \frac{2\pi V\omega\epsilon_o}{I\ln(b/a)}$ ②

From ① and ②: $k^2 = \omega^2\mu_o\epsilon_o$

$$\langle\hat{S}\rangle = \frac{1}{2}\mathrm{Re}\left[\vec{E}\times\vec{H}^*\right] = \frac{VI}{4\pi\rho^2\ln(b/a)}\vec{a}_z$$

Finally, $\langle P\rangle = \int_s \langle\hat{S}\rangle\cdot d\vec{s} = \frac{VI}{4\pi\ln(b/a)}\int_a^b \frac{1}{\rho}d\rho\int_0^{2\pi}d\phi = \frac{1}{2}VI$ W

Note that V and I are the max. values.

Problem 7.47 $\quad \tilde{A} = \tilde{A}_r + j\tilde{A}_i \Rightarrow \vec{A}(t) = \vec{A}_r \cos\omega t - \vec{A}_i \sin\omega t$

$\tilde{B} = \tilde{B}_r + j\tilde{B}_i \Rightarrow \vec{B}(t) = \vec{B}_r \cos\omega t - \vec{B}_i \sin\omega t$

$\tilde{A}\cdot\tilde{B}^* = \tilde{A}_r\cdot\tilde{B}_r + \tilde{A}_i\cdot\tilde{B}_i - j(\tilde{A}_r\cdot\tilde{B}_i - \tilde{A}_i\cdot\tilde{B}_r) \quad (1)$

$\vec{A}\cdot\vec{B} = \vec{A}_r\cdot\vec{B}_r \cos^2\omega t + \vec{A}_i\cdot\vec{B}_i \sin^2\omega t - (\vec{A}_r\cdot\vec{B}_i + \vec{A}_i\cdot\vec{B}_r)\sin\omega t\cos\omega t$

Average: $\langle\vec{A}\cdot\vec{B}\rangle = \frac{1}{T}\int_0^T(\vec{A}\cdot\vec{B})dt = \frac{1}{2}(\vec{A}_r\cdot\vec{B}_r + \vec{A}_i\cdot\vec{B}_i) \quad (2)$

From (1) and (2): $\langle\vec{A}\cdot\vec{B}\rangle = \frac{1}{2}\mathrm{Re}[\tilde{A}\cdot\tilde{B}^*]$

Problem 7.48 Using the definitions given in Problem 7.47

$\tilde{A}\times\tilde{B}^* = \tilde{A}_r\times\tilde{B}_r + \tilde{A}_i\times\tilde{B}_i + j(\tilde{A}_i\times\tilde{B}_r - \tilde{A}_r\times\tilde{B}_i) \quad (1)$

$\vec{A}\times\vec{B} = (\vec{A}_r\times\vec{B}_r)\cos^2\omega t + (\vec{A}_i\times\vec{B}_i)\sin^2\omega t - (\vec{A}_r\times\vec{B}_i + \vec{A}_i\times\vec{B}_r)\sin\omega t\cos\omega t$

$\langle\vec{A}\times\vec{B}\rangle = \frac{1}{2\pi}\int_0^{2\pi}(\vec{A}\times\vec{B})\,d\omega t = \frac{1}{2}[\vec{A}_r\times\vec{B}_r] + \frac{1}{2}[\vec{A}_i\times\vec{B}_i] \quad (2)$

Comparing (1) and (2): $\langle\vec{A}\times\vec{B}\rangle = \frac{1}{2}\mathrm{Re}[\tilde{A}\times\tilde{B}^*]$

Problem 7.49 $\quad \nabla\times\tilde{E} = -j\omega\mu\tilde{H} \qquad \nabla\times\tilde{H} = (\sigma + j\omega\epsilon)\tilde{E}$

$\nabla\times\nabla\times\tilde{E} = -j\omega\mu\,\nabla\times\tilde{H} \Rightarrow \nabla(\nabla\cdot\tilde{E}) - \nabla^2\tilde{E} = -j\omega\mu(\sigma+j\omega\epsilon)\tilde{E}$

Source free: $\tilde{\rho}_v = 0 \Rightarrow \nabla\cdot\tilde{D} = 0$ or $\nabla\cdot\tilde{E} = 0$

Hence: $\quad \nabla^2\tilde{E} + \omega^2\mu\epsilon\tilde{E} - j\omega\mu\sigma\tilde{E} = 0$

Problem 7.50

$\nabla\times\tilde{E} = -j\omega\mu\tilde{H} \qquad \nabla\times\tilde{H} = \sigma\tilde{E} + j\omega\epsilon\tilde{E}$

$\nabla\times\nabla\times\tilde{H} = (\sigma+j\omega\epsilon)\nabla\times\tilde{E} \Rightarrow \nabla(\cancel{\nabla\cdot\tilde{H}}^{\,0}) - \nabla^2\tilde{H} = -j\omega\mu(\sigma+j\omega\epsilon)\tilde{H}$

Since $\nabla\cdot\tilde{B} = 0 \Rightarrow \nabla\cdot\tilde{H} = 0$

Hence:

$$\nabla^2\tilde{H} = j\omega\mu(\sigma + j\omega\epsilon)\tilde{H}$$

or

$$\nabla^2\tilde{H} + \omega^2\mu\epsilon\tilde{H} - j\omega\mu\sigma\tilde{H} = 0$$

Problem 7.51 $\quad \tilde{E}_z = 1000\, e^{-j\beta x}$ V/m $\qquad \tilde{H}_y = -\frac{1000}{\eta} e^{-j\beta x}$ A/m $\qquad \beta = \frac{\pi}{3}$ rad/m

$\nabla\times\vec{\tilde{E}} = j\beta 1000\, e^{-j\beta x}\, \vec{a}_y$, since $\nabla\times\vec{\tilde{E}} = -j\omega\mu_0\vec{\tilde{H}} \Rightarrow \eta = \frac{\omega\mu_0}{\beta}$ or $\beta\eta = \omega\mu_0$ ①

$\nabla\cdot\vec{\tilde{D}} = 0 \quad \nabla\cdot\vec{\tilde{B}} = \nabla\cdot\vec{\tilde{H}} = 0 \qquad \epsilon = 4\epsilon_0$

$\nabla\times\vec{\tilde{H}} = j\,\frac{1000\beta}{\eta}\, e^{-j\beta x}\, \vec{a}_z$, since $\nabla\times\vec{\tilde{H}} = j\omega\epsilon\vec{\tilde{E}} \Rightarrow \beta/\eta = 4\omega\epsilon_0$ ②

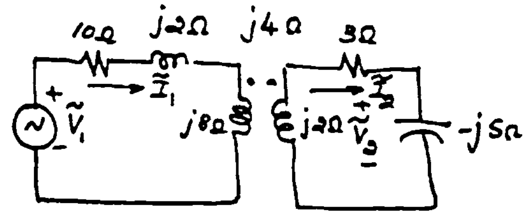
From ① and ②: $\quad \beta^2 = \omega^2 4\mu_0\epsilon_0 \Rightarrow \omega = \frac{\beta}{2\sqrt{\mu_0\epsilon_0}} = 0.5\pi\times10^8$ rad/s

$\omega = 2\pi f \Rightarrow f = 25$ MHz $\quad$ and $\quad \eta = \frac{\omega\mu_0}{\beta} = 60\pi\ \Omega$

$\langle\hat{S}\rangle = \frac{1}{2}\mathrm{Re}[\vec{\tilde{E}}\times\vec{\tilde{H}}^*] = \frac{1000^2}{\eta}\vec{a}_x$ W/m^2

$$\langle P\rangle = \int_S \langle\hat{S}\rangle\cdot\vec{ds} = \frac{1000^2}{\eta}\int_0^4 dy\int_0^{0.5y} dz = 21.22\ \text{kW}$$

Problem 7.52 $\quad \omega = 1000$ rad/s

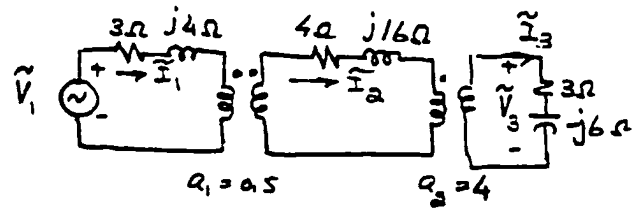


$j\omega L_1 = j8\Omega$, $\quad j\omega L_2 = j2\Omega \qquad -\frac{j}{\omega C} = -j5\Omega$

$j\omega M = \sqrt{j\omega L_1 \times j\omega L_2} = j4\Omega \qquad \tilde{V}_1 = 120\angle 0^\circ$ V

$$\left.\begin{aligned}(10+j10)\tilde{I}_1 - j4\tilde{I}_2 &= 120\\ -j4I_1 + (3-j3)\tilde{I}_2 &= 0\end{aligned}\right\} \quad \begin{aligned}\tilde{I}_1 &= 6.699\angle{-45^\circ}\ \text{A}\\ \tilde{I}_2 &= 6.316\angle 90^\circ\ \text{A}\end{aligned} \qquad \begin{aligned}\tilde{V}_2 &= [(-j5)(6.316\angle 90^\circ)]\\ &= 31.58\angle 0^\circ\ \text{V}\end{aligned}$$

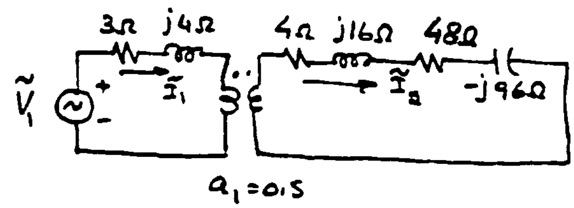
Problem 7.53 $\quad \tilde{V}_1 = 120\angle 30^\circ$ V (rms)



$\tilde{I}_1 = \frac{\tilde{V}_1}{\hat{Z}} = 5.303\angle 75^\circ$ A

$P_{in} = \mathrm{Re}[\tilde{V}_1\tilde{I}_1^*] = 450$ W

$\tilde{I}_2 = a_1\tilde{I}_1 = 2.6515\angle 75^\circ$ A

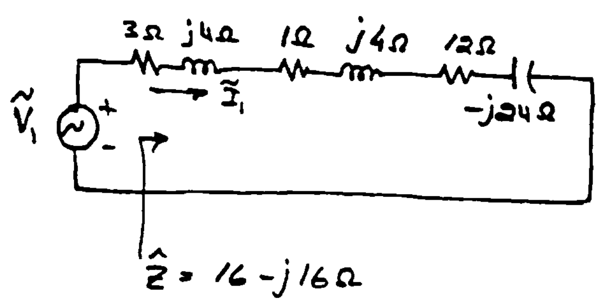


$\tilde{I}_3 = a_2\tilde{I}_2 = 10.616\angle 75^\circ$ A

Load Voltage: $\tilde{V}_3 = \tilde{I}_3[3-j6\Omega]$

$= 71.214\angle 11.57^\circ$ V

Power to load: $P_L = 10.616^2\times 3 = 338$ W



$\eta = \frac{P_L}{P_{in}} = 0.75$ or 75%

Power lost on Transmission Line:

$P_{TL} = 2.6515^2\times 4 = 28.12$ W

Problem 7.54 $a = 30/750 = 0.04$ $N_1 = 30$ $N_2 = 750$ $f = 50\,Hz$

$pf = 0.8$ lag $\Rightarrow \theta = -36.87^\circ$. Thus, $\tilde{I}_2 = 4\angle{-36.87^\circ}\,A$, $\tilde{I}_1 = \dfrac{\tilde{I}_2}{a} = 100\angle{-36.87^\circ}\,A$

From $E_1 = 4.44 f N_1 \Phi_m \Rightarrow \Phi_M = \dfrac{240}{4.44 \times 50 \times 30} \cong 36$ mWb. (Peak)

Problem 7.55 $\Phi_m = \sqrt{2}\,\Phi_{rms} = 1.414$ mWb (max value)

From $E = 4.44 f N \Phi_m$, $\Rightarrow N = \dfrac{230 \times 10^{+3}}{4.44 \times 60 \times 1.414} \cong 610$ Turns

Problem 7.56 $I_2 = \dfrac{1000}{120} = 8.333A$ $pf = 0.6$ lead $\Rightarrow \tilde{I}_2 = 8.333\angle{53.13^\circ}\,A$

Thus, $\hat{Z}_L = \dfrac{120}{\tilde{I}_2} = 14.4\angle{-53.13^\circ} = 8.64 - j11.52\,\Omega$

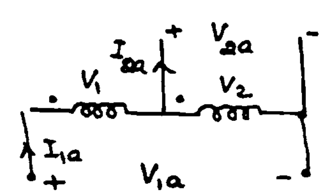
Problem 7.57 Two-winding Transformer: $V_1 = 120\,V$, $V_2 = 480V$, $S = 4800VA$

$I_1 = 40\,A$, $I_2 = 10\,A$ $a = 0.25$

a)
$V_{1a} = 600\,V$, $V_{2a} = 480V$

$I_{1a} = 40\,A$, $I_{2a} = 40 \times \frac{600}{480} = 50A$

$S = 480 \times 50 = 24\,kVA$

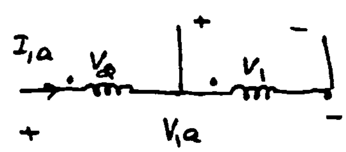


(a)

b) $V_{1a} = 600\,V$ $V_{2a} = 120V$

$I_{1a} = 10\,A$ $I_{2a} = 10 \times \frac{600}{120} = 50\,A$

$S = 120 \times 50 = 6\,kVA$

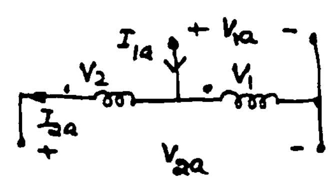


(b)

c) $V_{1a} = 120V$ $V_{2a} = 600V$

$I_{2a} = 10A$ $I_{1a} = 10 \times \frac{600}{120} = 50A$

$S = 600 \times 10 = 6\,kVA$

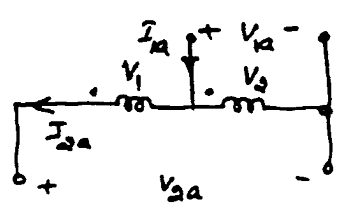


(c)

d) $V_{1a} = 480V$ $V_{2a} = 600V$

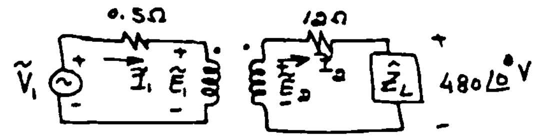
$I_{2a} = 40\,A$ $I_{1a} = 50A$

$S = 600 \times 40 = 24\,kVA$



(d)

Problem 7.58 $\tilde{I}_2 = 10\angle{-53.13^\circ}$ A, $\tilde{I}_1 = 40\angle{-53.13^\circ}$ A



$a = 0.25$

$P_o = 480 \times 10 \times 0.6 = 2880$ W

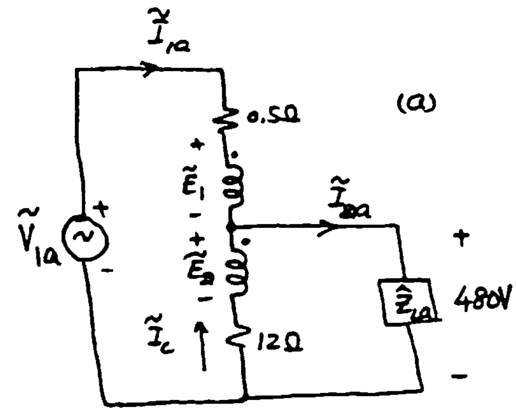
$\tilde{E}_2 = 480 + 12 \times 10\angle{-53.13^\circ} = 552 - j96$ V

$\tilde{E}_1 = a\tilde{E}_2 = 138 - j24$ V, $\quad \tilde{V}_1 = \tilde{E}_1 + 0.5\tilde{I}_1 = 150 - j40 = 155.242\angle{-14.93^\circ}$ V

$P_{in} = Re[\tilde{V}_1 \tilde{I}_1^*] = Re[155.242 \times 40\angle{53.13^\circ - 14.93^\circ}] \cong 4880$ W $\quad \eta = \frac{P_o}{P_{in}} = 0.59$ or 59%

Problem 7.59

(a)



$\tilde{I}_{1a} = \tilde{I}_1 = 40\angle{-53.13^\circ}$ A $\qquad a = \frac{120}{480} = 0.25$

$a_T = 600/480 = 1.25$

$\tilde{I}_{2a} = a_T \tilde{I}_{1a} = 50\angle{-53.13^\circ}$ A

$\tilde{I}_c = \tilde{I}_{2a} - \tilde{I}_{1a} = 10\angle{-53.13^\circ}$ A

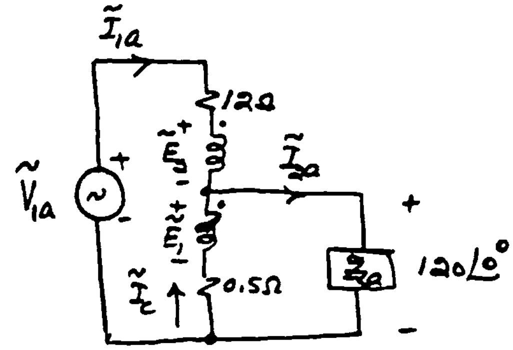
$\tilde{E}_2 = 480 + \tilde{I}_c(12) = 552 - j96$ V

$\tilde{E}_1 = a\tilde{E}_2 = 138 - j24\ \Omega$

$\tilde{V}_{1a} = 0.5\tilde{I}_{1a} + \tilde{E}_1 + 480 = 630 - j40 = 631.27\angle{-3.63^\circ}$ V

$P_o = Re[\tilde{V}_2 \tilde{I}_{2a}^*] = 14{,}400$ W $\quad P_{in} = Re[\tilde{V}_{1a} \tilde{I}_{1a}^*] \cong 16399$ W $\Rightarrow \eta = \frac{P_o}{P_{in}} = 0.878$ or 87.8%

(b) $\tilde{I}_{1a} = 10\angle{-53.13^\circ}$ A, $\quad a_T = 600/120 = 5$



$\tilde{I}_{2a} = 50\angle{-53.13^\circ}$ A $\quad , \quad \tilde{I}_c = 40\angle{-53.13^\circ}$ A

$\tilde{E}_1 = 120 + 0.5\tilde{I}_c = 132 - j16$

$\tilde{E}_2 = 528 - j64$ V

$\tilde{V}_{1a} = 12\tilde{I}_{1a} + \tilde{E}_2 + 120 = 737.56\angle{-12.53^\circ}$ V

$P_o = Re[120\, \tilde{I}_{2a}^*] = 3600$ W

$P_{in} = Re[\tilde{V}_{1a} \tilde{I}_{1a}^*] = 5600$ W

$\eta = \frac{P_o}{P_{in}} = 0.643$ or 64.3%

Another Method:

$P_{in} = P_o + P_{cu}$

$P_{cu} = I_c^2(0.5) + I_{1a}^2(12)$

$= 0.5 \times 40^2 + 12 \times 10^2 = 2000$ W

Thus, $P_{in} = 3600 + 2000 = 5600$ W

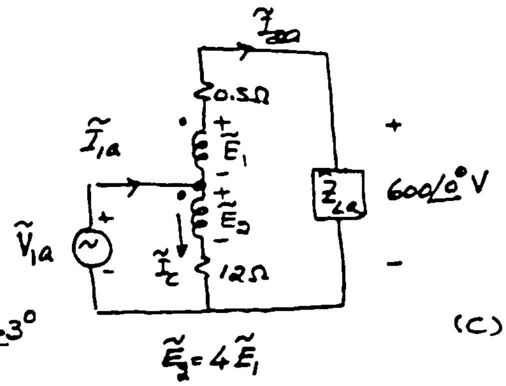
c) $\tilde{I}_{2a} = 40\angle{-53.13^\circ}$ A $\quad a_T = 480/600 = 0.8$

$\tilde{I}_{1a} = 50\angle{-53.13^\circ}$ A, $\quad \tilde{I}_c = 10\angle{-53.13^\circ}$

$\tilde{E}_1 + \tilde{E}_2 = 600 + 0.5\tilde{I}_{2a} - 12\tilde{I}_c$

$5\tilde{E}_1 = 540 + j80 \quad$ or $\quad \tilde{E}_1 = 108 + j16$

$\tilde{V}_{1a} = 12\tilde{I}_c + \tilde{E}_2 = 12\tilde{I}_c + 4\tilde{E}_1 = 505.015\angle{-3.63^\circ}$



(c)

$P_o = Re[600 \times 40\angle{53.13^\circ}] = 14{,}400$ W $\qquad \eta = \frac{P_o}{P_{in}} = 0.878$ or 87.8%

$P_{in} = Re[\tilde{V}_{1a}\tilde{I}_{1a}^*] = 16{,}399$ W

d) $\tilde{I}_{2a} = 10\angle{-53.13^\circ}$ A $\quad a_T = 120/600 = 0.2$

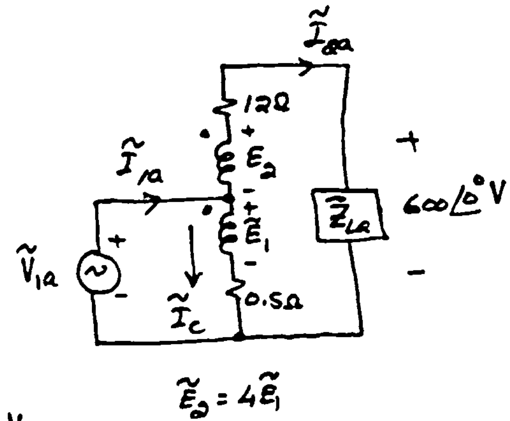
$\tilde{I}_{1a} = \frac{\tilde{I}_{2a}}{a_T} = 50\angle{-53.13^\circ}$ A

$\tilde{I}_c = 40\angle{-53.13^\circ}$ A

$\tilde{E}_1 + \tilde{E}_2 = 600 + 12\tilde{I}_{2a} - 0.5\tilde{I}_c = 660 - j80$

$\tilde{E}_1 = 132 - j16$

$\tilde{V}_{1a} = \tilde{E}_1 + 0.5\tilde{I}_c = 144 - j32 = 147.51\angle{-12.53^\circ}$ V



$P_o = Re[600\,\tilde{I}_{2a}^*] = 3600$ W $\qquad \eta = \frac{P_o}{P_{in}} = 0.643$ or 64.3%

$P_{in} = Re[\tilde{V}_{1a}\,\tilde{I}_{1a}^*] = 5600$ W

Problem 7.60

$\Phi = 1.5\sin(100\pi t)$ Wb $\quad a = 0.75$ m

$\vec{B} = \frac{\Phi}{\pi a^2}\vec{a}_z = 0.8488\sin(100\pi t)\vec{a}_z$ T

Thus, $B_m = 0.4244$ T

$\vec{E} = -a\omega B_m\cos(100\pi t)\vec{a}_\phi$

$= -100\cos(100\pi t)\vec{a}_\phi$

$\vec{F} = -e\vec{E} = 1.6\times10^{-17}\cos(100\pi t)\vec{a}_\phi$

$W = 2\pi aF = 7.54\times10^{-17}\cos(100\pi t)$ J

$\langle W\rangle = \frac{2}{\pi}W = 4.8\times10^{-17}$ J $= 300$ eV

Number of Trips:

$N = \frac{90\times10^6}{300} = 300{,}000$