Chapter 4 Steady electric current

Introduction

In the chapter, electric currents will be discussed.

1. currents

(a) **Class**

free currents: the motion of free charges is said to constitute free currents. It has two types of current:

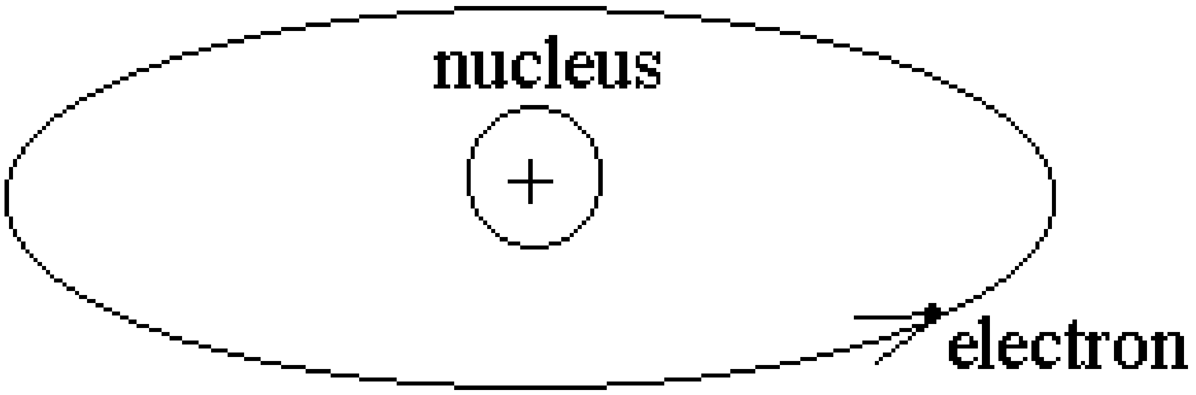
conduction and convection currents

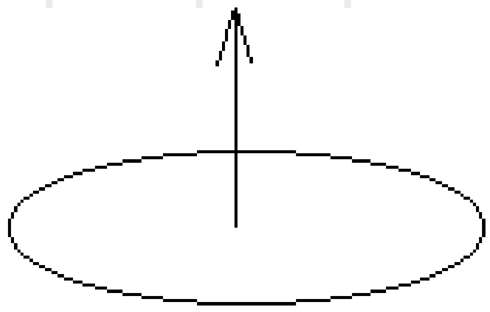
conduction current: a free electron may be considered as not being attached to any particular atom and has the capability of moving through a whole crystal lattice,

its movement contribute to the current in a conductor. The current in a metal conductor, called conductor current, is simply a flow of electrons.

convection current: the motion of charged particles in free space (vacuum) is said to constitute a convection current.

Bound currents: molecular current

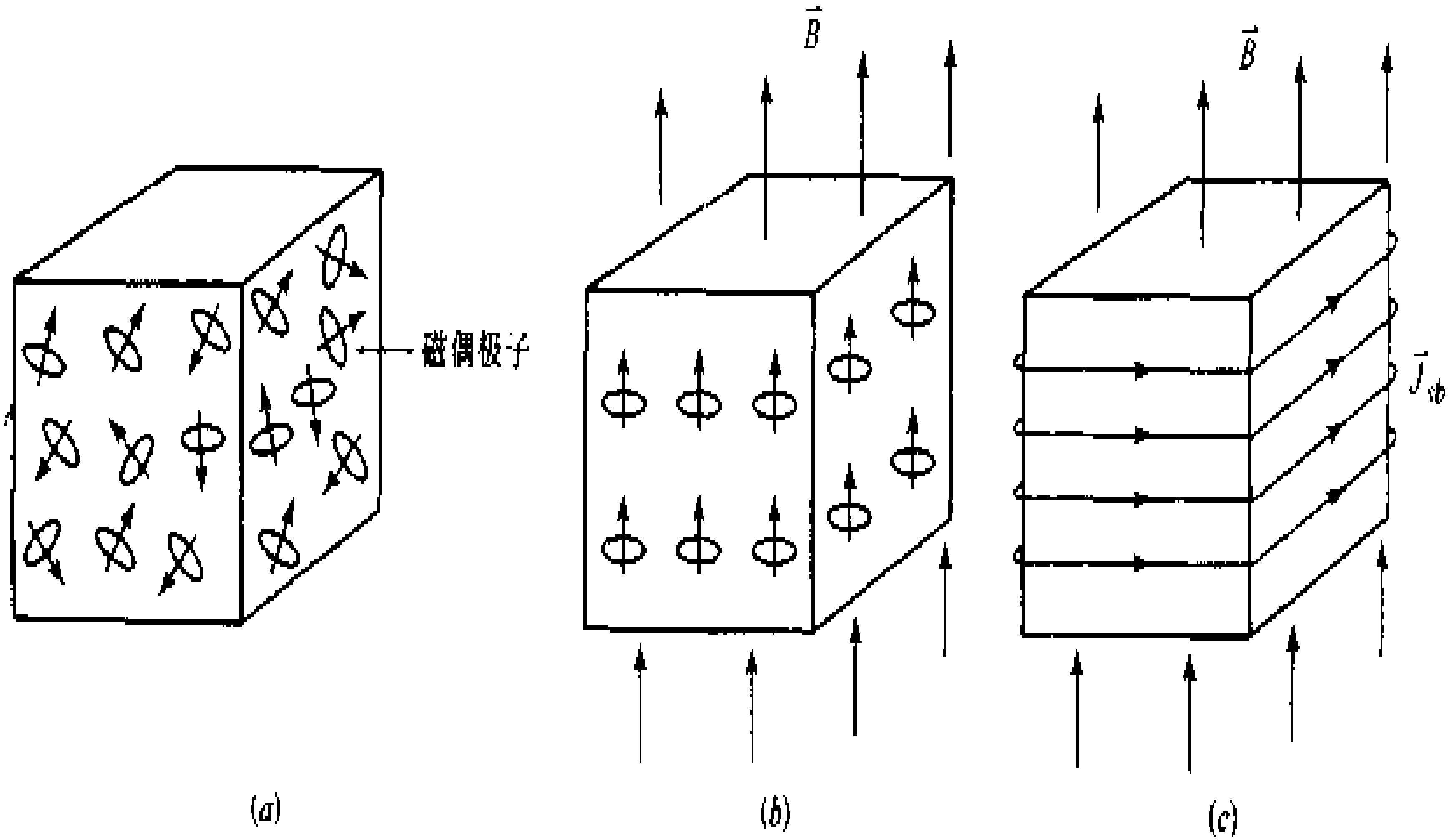


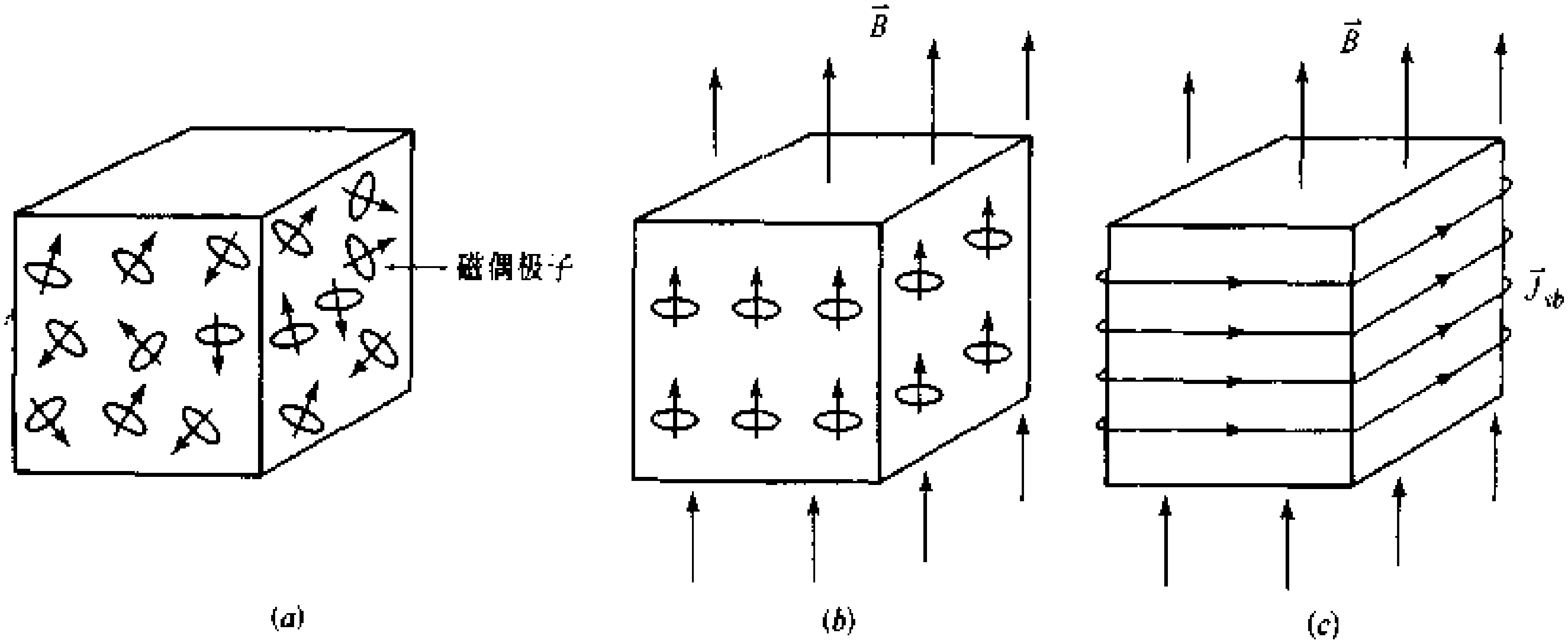


an electron moving in a circular orbit;

the electron is continually rotating(spinning) around its own axis (the spinning motion) is equivalent to a current carrying loop it is called magnetic dipole，in the absence of an extern magnetic field, the magnetic dipoles in a piece of material are oriented at random, the net magnetic moment is nearly zero.

In the presence of an extern magnetic field, each magnetic dipole experiences a torque that tends to align it with the magnetic field.





a) a piece of magnetic material with randomly oriented magnetic dipoles;

b) an extern magnetic field causes the magnetic dipoles to align with it;

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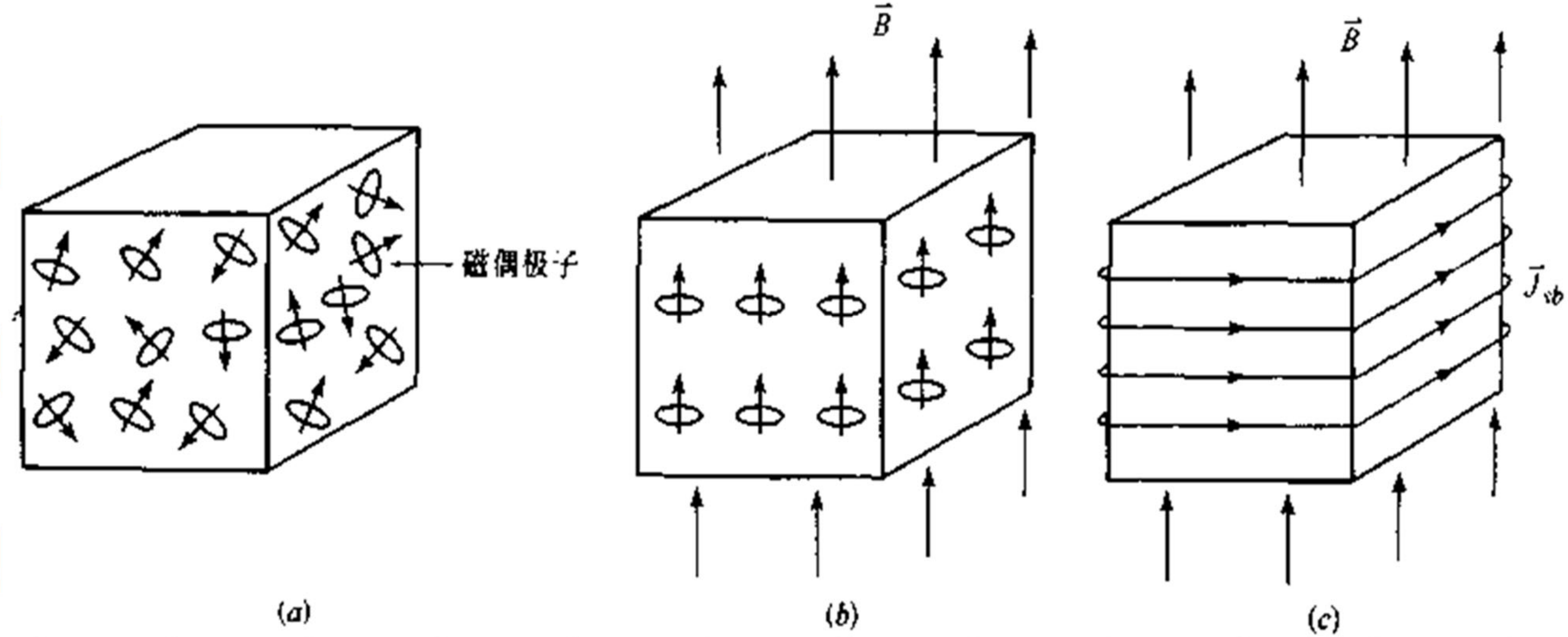
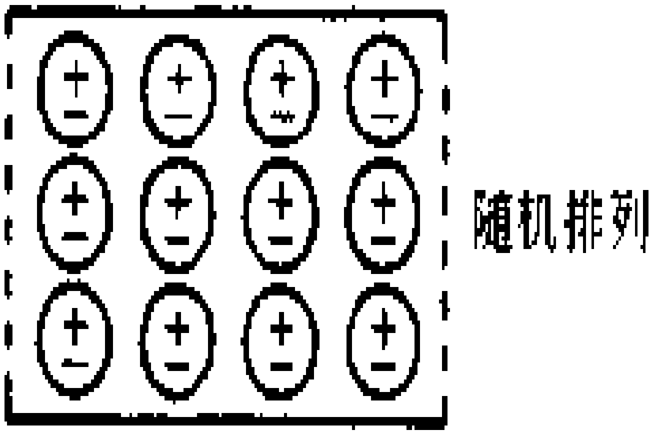
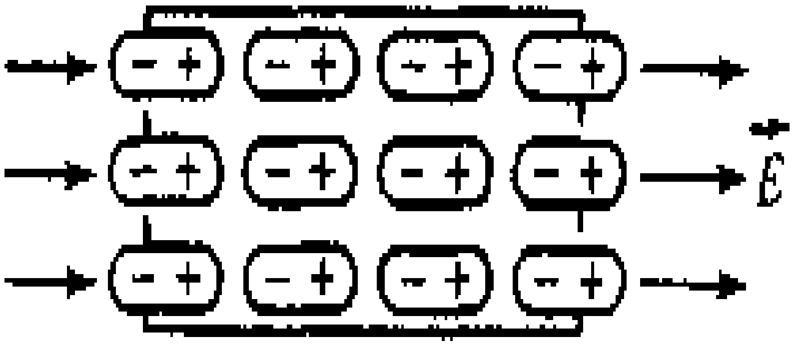
c) the small aligned current loops of (b) are equivalent to a current along the surface of the material.



Polarizing current: under the influence of a time-varying electric field, polar charges can constitute to polarizing current.



(a)



(b)

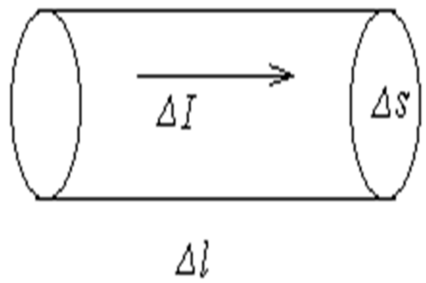
(b) current density

electric currents I is a macroscopical quantity, an integral. It can not describe the distribution of electric currents at each point in a given area (region)

① volume current distribution

volume current density

$$\vec{J}_v = \vec{a}_+ \left(\lim_{\Delta s \to 0} \frac{\Delta I}{\Delta s} \right)$$

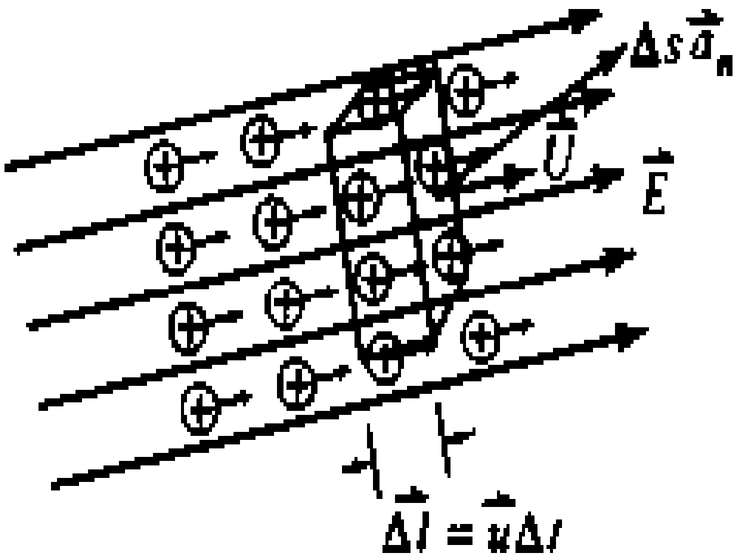


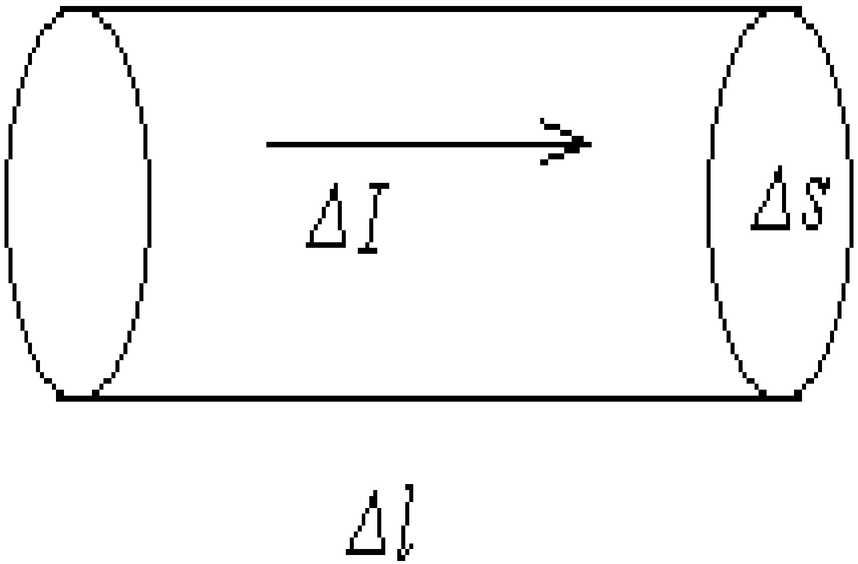
is a vector, it includes an idea of surface density, the current per unit area.

the current passing through a surface *s* is $I = \int_s \vec{J}_v \bullet d\vec{s}$

the surface area $\Delta\vec{s} = \Delta s\vec{a}_n$ normal to the drift velocity of positive charges

is the direction of the drift velocity of positive charges





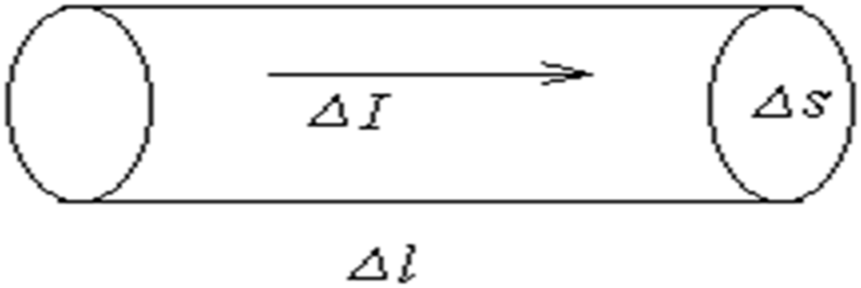
in order to describe the motion of the charges, let us consider a region with volume charge distribution density ρ_v in which the charges are moving under the influence of an electric field with an average velocity

in time Δt these charges will move a distance

such that $d\vec{l} = \vec{\upsilon}\Delta t$

the charge movement through the surface area

$\Delta\vec{s} = \Delta s\vec{a}_n$ **would be**



$$\Delta q = \rho_v \Delta v = \rho_v \Delta\vec{s} \bullet d\vec{l}$$

the current ΔI through the surface $\Delta\vec{s} = \Delta s\vec{a}_n$ **is**

$$\Delta I=\frac{\Delta q}{\Delta t}=\rho_v\Delta\vec{s}\bullet\left(\frac{d\vec{l}}{dt}\right)=\rho_v\Delta\vec{s}\bullet\vec{\upsilon}$$

from $\vec{J}_v=\vec{a}_+\left(\lim_{\Delta s\to 0}\frac{\Delta I}{\Delta s}\right)$ **, we can obtain**

$$\Delta I=\frac{\Delta q}{\Delta t}=\rho_v\Delta\vec{s}\bullet\left(\frac{d\vec{l}}{dt}\right)=\rho_v\Delta\vec{s}\bullet\vec{\upsilon}=\vec{J}_v\bullet\Delta\vec{s}$$

then

$$\vec{\boldsymbol{J}}_v=\rho_v\vec{\boldsymbol{\upsilon}}$$

and the current passing through a surface s is

$$I=\int_s\vec{\boldsymbol{J}}_v\bullet d\vec{\boldsymbol{s}}$$

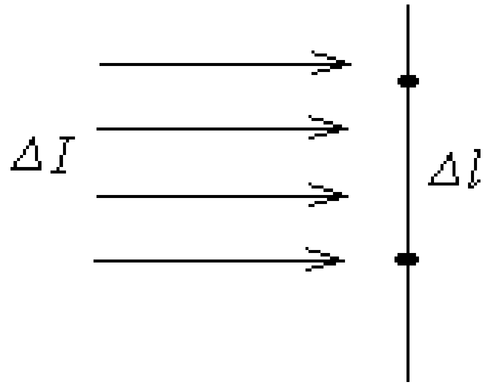
Volume current density $\vec{J}_v$ can be completely described By a vector point function.

Magnitude: the current per unit area which is normal to the direction of the drift velocity of positive charges

Direction: the direction of the drift velocity of positive charges

②surface current distribution

the oriented motion of the charges along a surface constitutes the current.



Let us consider an length element Δl normal to the direction of the current ΔI . we can define the surface current density as

$$\vec{J}_s = \vec{a}_+ \left(\lim_{\Delta l \to 0} \frac{\Delta I}{\Delta l} \right)$$

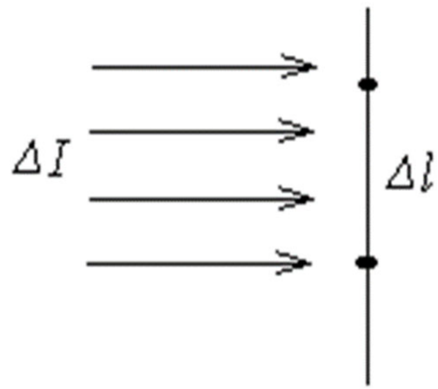
includes an idea of line density.

the current per unit length.

The surface current can be calculated by

$$\vec{J}_s = \vec{a}_+ \left(\lim_{\Delta l \to 0} \frac{\Delta I}{\Delta l} \right) \Leftrightarrow \Delta I = \vec{J}_s \bullet \vec{a}_N \Delta l$$

$$I = \int_l \vec{J}_s \bullet \vec{a}_N dl$$

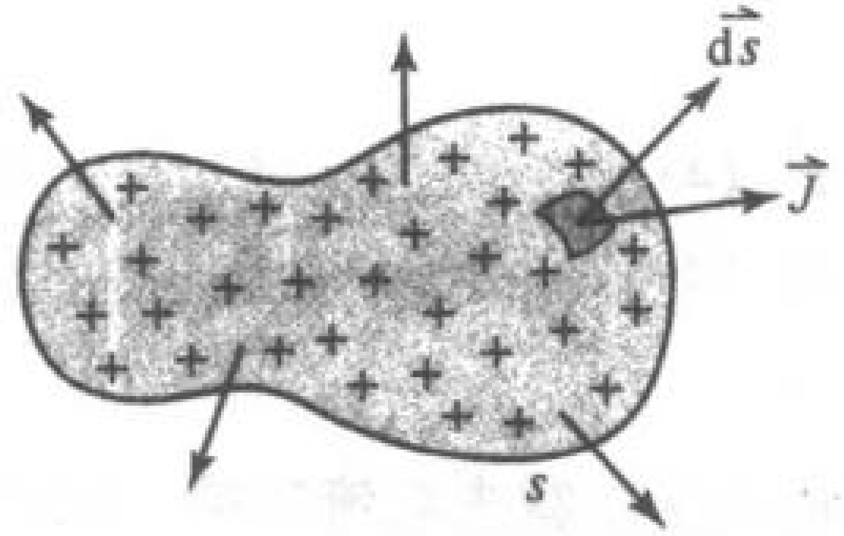


2. the equation of continuity(the principle of conservation of charge)

- (a) the integral form

the charge can neither be created nor destroyed, but merely transported.

A conducting region bounded by a closed surface s, we assume that the volume charge density in the region is ρ_v, and the current leaving the surface can be described in terms of the volume current density $\vec{J}_v$. At any time t,



the total current crossing the closed surface s in the outward direction is

$$I(t) = \int_s \vec{J}_v \bullet d\vec{s} \qquad \textbf{(4.1)}$$

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Since current is simply a flow of charge per second, an outward flow of charge must decrease the charge concentration by the same amount within the region bounded by *s*. Thus, the rate at which the charge is leaving the surface must be equal to the rate at which the charge is diminishing in the bounded region. Therefore, we can also express the current as

$$I(t) = -\frac{dq}{dt} \qquad (4.2)$$

where q is the total charge enclosed by the surface at any time t, we can write q in terms of the volume charge density ρ_v as

$$q = \int_V \rho_v dV \qquad (4.3)$$

where the integral is taken through the region enclosed by s. Combining (4.1), (4.2) and (4.3), we obtain

$$\oint_s \vec{J} \bullet d\vec{s} = -\frac{d}{dt}\int_V \rho_v dV \qquad (4.4)$$

Equation (4.4) is called the integral form of the equation of continuity and is a mathematical expression of the principle of conservation of charge.

It states that any change of charge in a region must be accompanied by a flow of charge across the surface bounding the region. the rate at which the charge is leaving the surface s (the flux of the vector field through the surface $\vec{s}$)

must be equal to the rate at which the charges diminishing in the bounded region *v*.

(b) the differential form

the closed surface integral on the left-hand side of (4.4) can be transformed into a volume integral by applying the divergence theorem. Since the volume under consideration is stationary, the differential with respect to time can be replaced by a partial derivative of volume charge density. We can now rewrite (4.4) as

$$\int_V \nabla \bullet \vec{J}\, dV = -\int_V \frac{\partial \rho_v}{\partial t} dV$$

or

$$\int_V \left(\nabla \bullet \vec{J} + \frac{\partial \rho_v}{\partial t} \right) dV = 0$$

since the volume under consideration is arbitrary, the only way for the preceding equation to be true in general is for the integral to vanish at each point. Hence,

$$\nabla \bullet \vec{J}_v + \frac{\partial \rho_v}{\partial t} = 0 \qquad (4.5)$$

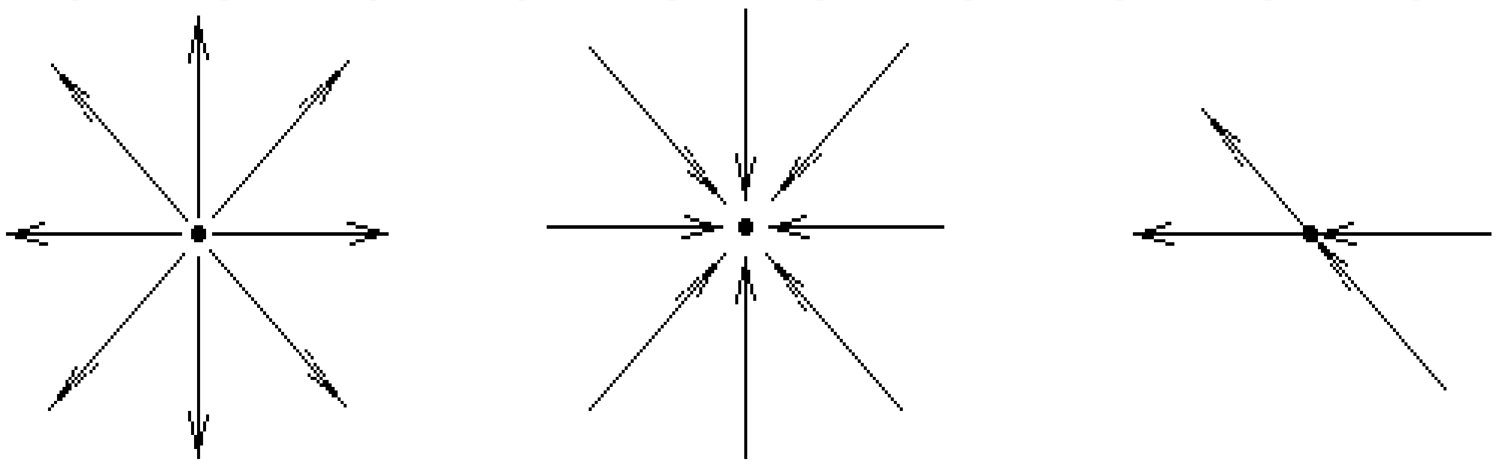
or

$$\nabla \bullet \vec{J}_v = -\frac{\partial \rho_v}{\partial t}$$

this is the differential (point) form of the equation of continuity.

The divergence of the vector $\vec{J}$ at a point is equal to the rate at which the charge is diminishing in the point.

Equation (4.5) states that the points of changing charge density ρ_v are sources of volume current density $\vec{J}_v$

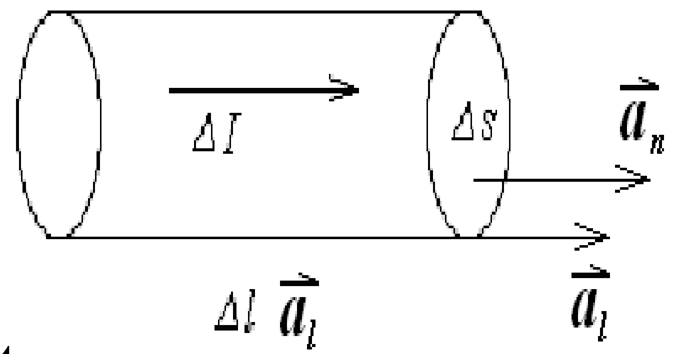


① $\nabla \bullet \vec{J} > 0,\ \dfrac{\partial \rho_v}{\partial t} < 0$

② $\nabla \bullet \vec{J} < 0,\ \dfrac{\partial \rho_v}{\partial t} > 0$

③ $\nabla \bullet \vec{J} = 0,\ \dfrac{\partial \rho_v}{\partial t} = 0$

3. Ohm's law



U=RI voltage=resistance×current

sectional area

$$\vec{E} \bullet \Delta\vec{l} = \frac{\Delta l}{\sigma \Delta s} \vec{J}_v \bullet \Delta\vec{S}$$

$$\vec{E} \bullet \vec{a}_l \Delta l = \frac{\Delta l}{\sigma \Delta s} \vec{J}_v \bullet \vec{a}_n \Delta S$$

$$\vec{E} \bullet \vec{a}_l = \frac{\vec{J}_v \bullet \vec{a}_n}{\sigma} \qquad \because \vec{a}_l = \vec{a}_n$$

$$\therefore \quad \vec{J}_v = \sigma \vec{E}$$

where σ is the conductivity of a medium (s/m)
σ ＝0， ideal dielectric (insulator)
σ ≠0, conductor
σ =∞ (infinity), ideal conductor

it is different from the convection current

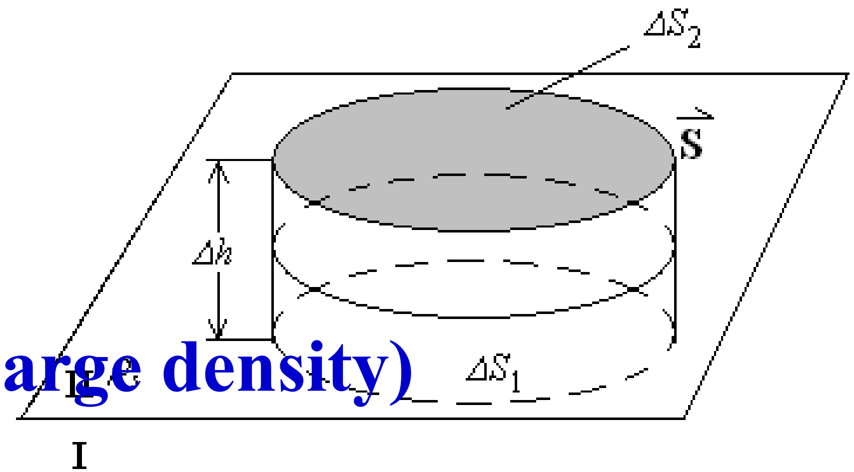
$$\Delta I = \frac{\Delta q}{\Delta t} = \rho_v \Delta \vec{s} \bullet \left(\frac{d\vec{l}}{dt} \right) = \rho_v \Delta \vec{s} \bullet \vec{\upsilon} = \vec{J}_v \bullet \Delta \vec{s}$$

$$\vec{J}_v = \rho_v \vec{\upsilon}$$

4. Boundary conditions for current density

- **(1) the normal component of $\vec{J}$ (page157).**
- **Let us construct a closed surface in the form of a pillbox. The height of the pillbox is so small that the contribution from the radial surface to the current can be neglected. computing the integral**

$$\int_s \vec{J}_v \bullet d\vec{s} = -\frac{d}{dt}\int_v \rho_v dv = 0$$



For a steady current

(no points of changing charge density)

Gaussian surface $\vec{S}$: a closed surface

- **Over the closed surface s of the pillbox when *h*→0, we find**

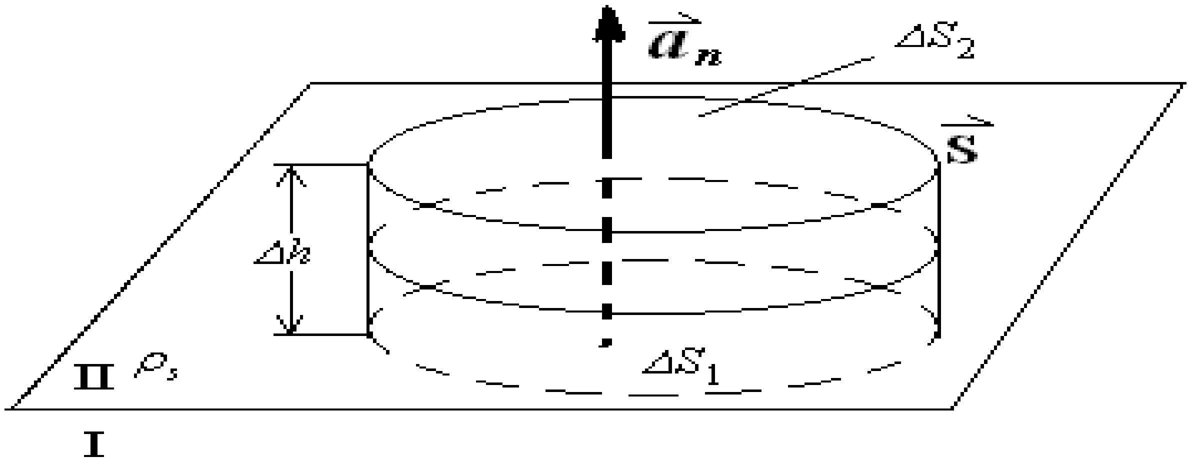
$$\oint_s \vec{J} \bullet d\vec{s} = \int_{\Delta S_1} \vec{J} \bullet d\vec{s} + \int_{\Delta S_2} \vec{J} \bullet d\vec{s} + \int_{cylinder} \vec{J} \bullet d\vec{s}$$

$$= \int_{\Delta S_1} \vec{J}_1 \bullet d\vec{s} + \int_{\Delta S_2} \vec{J}_2 \bullet d\vec{s} + \int_{cylinder} \vec{J} \bullet d\vec{s}$$

$$\int_{\Delta S_1} \vec{J}_1 \bullet d\vec{s} + \int_{\Delta S_2} \vec{J}_2 \bullet d\vec{s}$$

$$= \vec{J}_1 \bullet \Delta \vec{s}_1 + \vec{J}_2 \bullet \Delta \vec{s}_2$$

$$= \vec{J}_1 \bullet (-\vec{a}_n \Delta s_1) + \vec{J}_2 \bullet \vec{a}_n \Delta s_2$$



$$\lim_{\Delta h \to 0} \int_{cylinder} \vec{J} \bullet d\vec{s} = 0$$

$$\vec{J}_1 \bullet (-\vec{a}_n \Delta s) + \vec{J}_2 \bullet \vec{a}_n \Delta s = 0$$

or

$$\vec{a}_n \bullet (\vec{J}_2 - \vec{J}_1) = 0$$

or

$$J_{2n} - J_{1n} = 0$$

2. the tangential component of $\vec{J}$ (page157).

- **Since the tangential component of the electric field is continuous across the boundary, that is,**

$$\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0$$

$$\vec{a}_n \times \left(\frac{\vec{J}_2}{\sigma_2} - \frac{\vec{J}_1}{\sigma_1}\right) = 0$$

$$\frac{J_{2t}}{\sigma_2} = \frac{J_{1t}}{\sigma_1}$$

5. Analogy between $\vec{D}$ and $\vec{J}$

$$\nabla \cdot \vec{J} = 0$$

$$\nabla \cdot \vec{D} = 0$$

$$\vec{J} = \sigma \vec{E}$$

$$\vec{D} = \epsilon \vec{E}$$

自己阅读：

3.6 导体系统的电容；**3.7** 静电场的能量；

3.8 电场力；