Huazhong University
of Science & Technology

Electronic Circuit of Communications

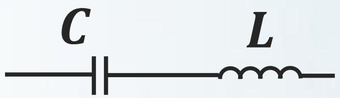
School of Electronic Information
and Commnications

Jiaqing Huang

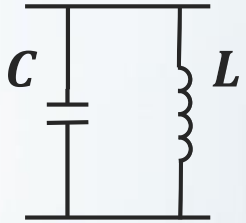
Series Resonance

Frequency Selective Circuits

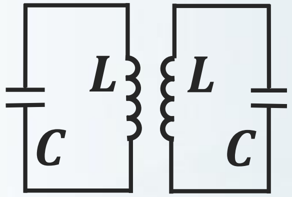
- Resonance
  - Series Resonance



  - Parallel Resonance

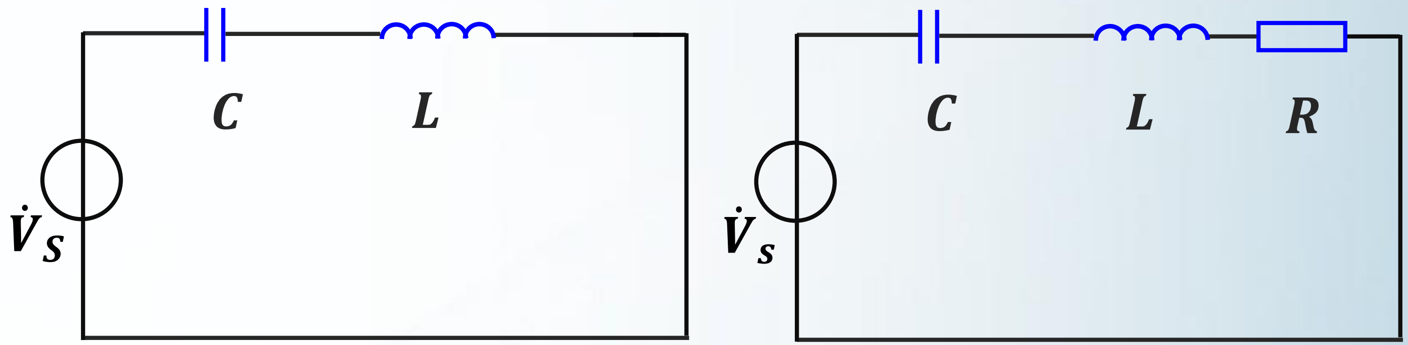


  - Coupling Resonance

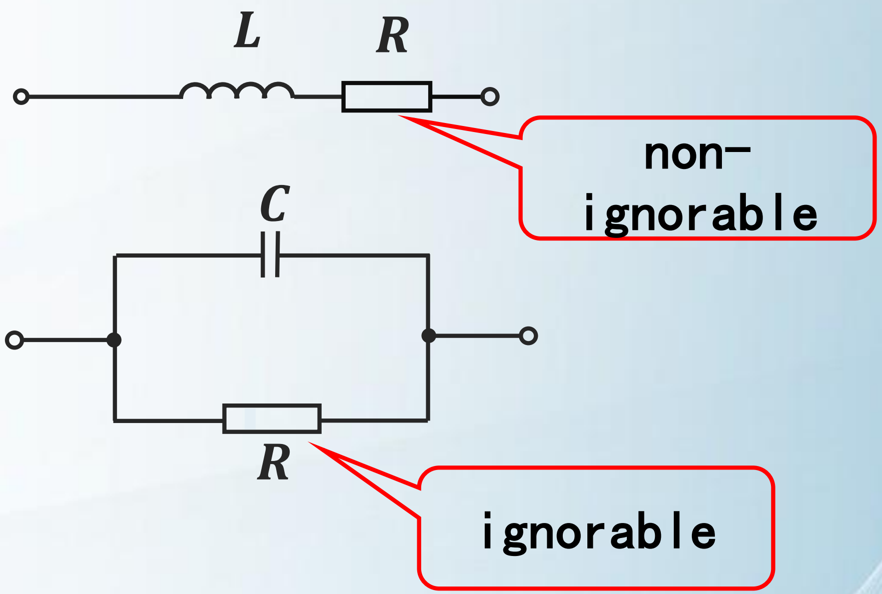


- Filter
  - Quartz Crystal Filter
  - Ceramic filter
  - SAWF

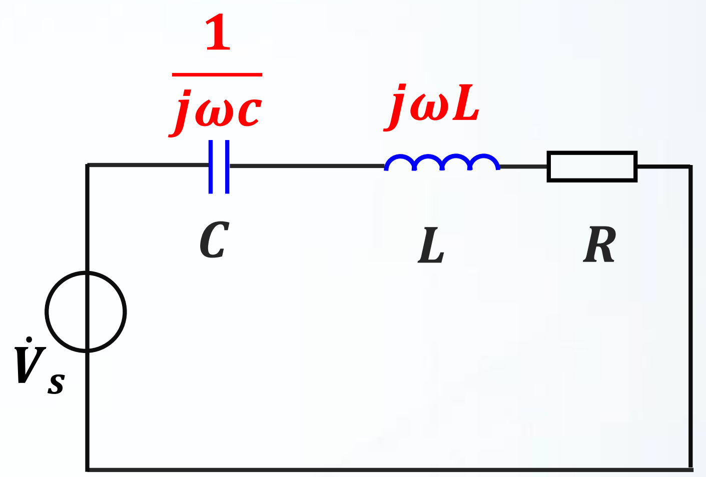
Series Resonant Circuit



- Inductor $= L$ with loss resistance R
- Capacitor $= C$ with loss resistance R



Series Resonant Circuit—Impendance $Z \sim \omega$

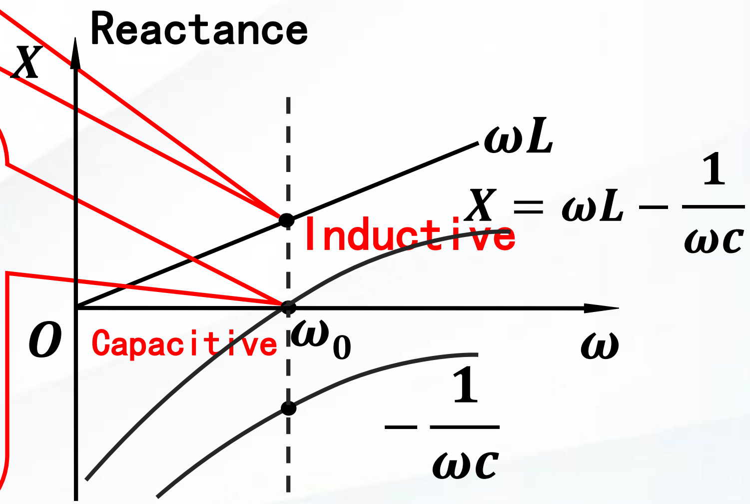


Characteristic Impedance ρ:

$$\rho = \omega_0 L = \frac{1}{\omega_0 C}$$

$$= \sqrt{\frac{L}{C}}$$

$$X = \omega L - \frac{1}{\omega C} \Rightarrow Z = R + jX = R + j(\omega L - \frac{1}{\omega C})$$



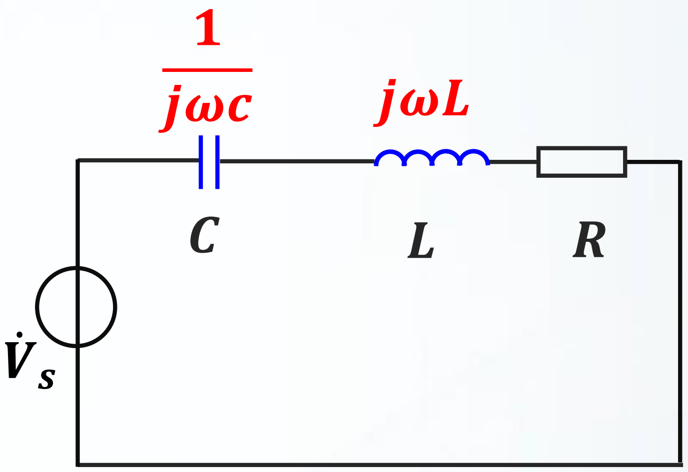
Resonant Freqency ω_0:

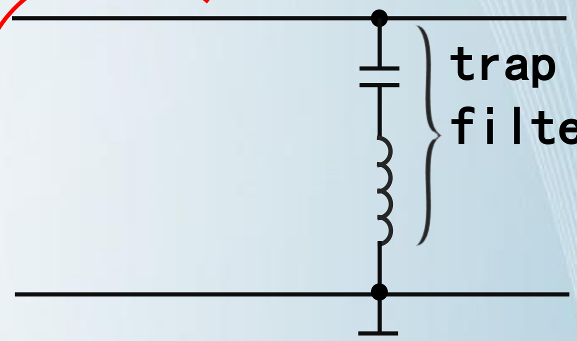
$$X = \omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

1) $\omega > \omega_0$, $X > 0$ Indutive, ELI
2) $\omega < \omega_0$, $X < 0$ Capacitive, ICE
3) $\omega = \omega_0$, $X = 0$ purely resistive

Series Resonant Circuit—Impendance $Z\sim\omega$





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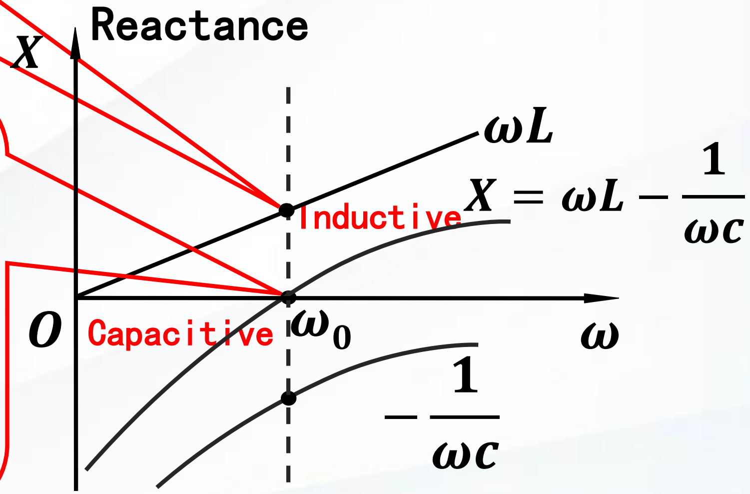

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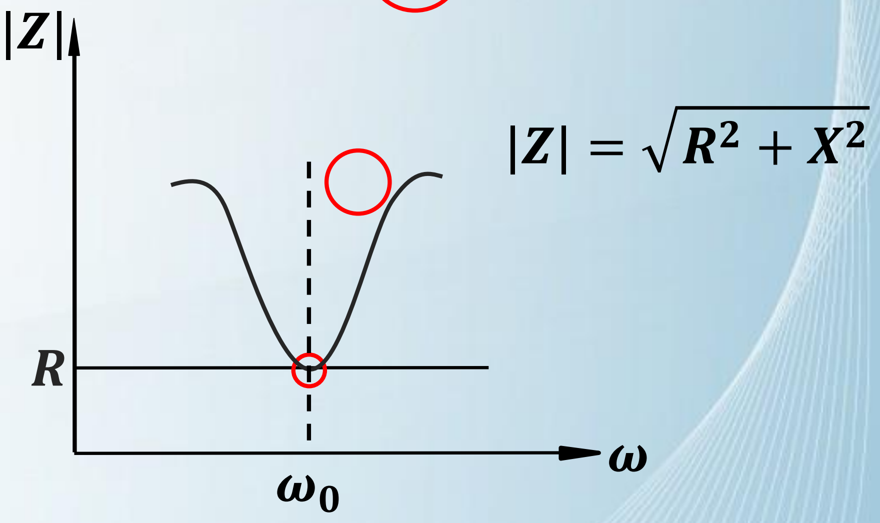


Series Resonant Circuit—Quality Factor (Q)

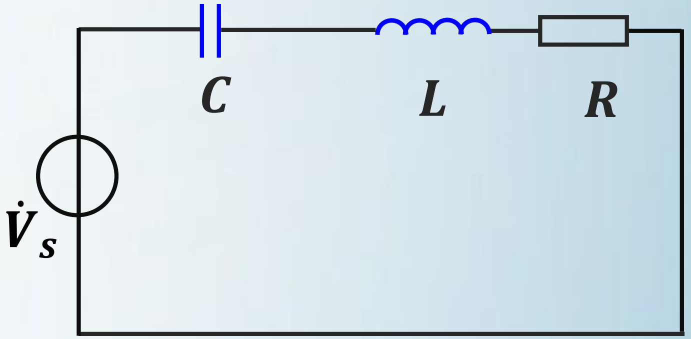
- Q of inductor

$$Q = \frac{\omega L}{R}$$

- Q of resonator

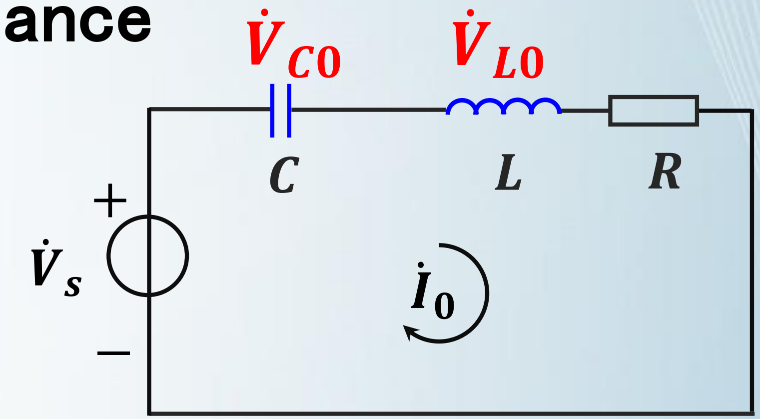
$$Q_0 = \frac{\omega_0 L}{R} = \frac{\frac{1}{\omega_0 C}}{R} = \frac{\rho}{R}$$

$$\because \ \rho = \omega_0 L = \frac{1}{\omega_0 C}$$



Difference: resonator Q_0 only for ω_0, inductor Q for ω
Similarity: energy loss (from R)

Series Resonant Circuit— Voltage Resonance



➢ Series Resonance

$$\begin{cases} \dot{V}_{L0} = \dot{I}_0 \cdot j\omega_0 L = \dfrac{\dot{V}_s}{R} j\omega_0 L = j\boxed{\dfrac{\omega_0 L}{R}}\dot{V}_s = jQ_0\dot{V}_s \\ \dot{V}_{C0} = \dot{I}_0 \cdot \dfrac{1}{j\omega_0 C} = \dfrac{\dot{V}_s}{R}\dfrac{1}{j\omega_0 C} = -j\boxed{\dfrac{\frac{1}{\omega_0 C}}{R}}\dot{V}_s = -jQ_0\dot{V}_s \end{cases}$$

$$\boxed{Q_0 = \frac{\omega_0 L}{R} = \frac{\frac{1}{\omega_0 C}}{R}}$$

Denote:

$\dot{V}_{L0} = -\dot{V}_{C0}$ same voltage value of, Q_0 times source voltage

★Note: withstand voltage of L and C (esp. C)

Series Resonant Circuit— Detuning Coefficient

$$Q_0 = \frac{\omega_0 L}{R} = \frac{\frac{1}{\omega_0 C}}{R} = \frac{\rho}{R}$$

$\xi = 0$ denote resonance

$\omega \approx \omega_0$

Resonance, $Q_0 = \dfrac{(Reactance)X}{(Resistance)R}$

Detuning, $\xi = \dfrac{(Reactance\ sum)X}{(Resistance)R} = \dfrac{\omega L - \frac{1}{\omega C}}{R}$

$$= \frac{\omega_0 L}{R}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = Q_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = Q_0 \frac{\cancel{(\omega+\omega_0)}(\omega-\omega_0)}{\omega_0 \cancel{\omega}}$$

$$\xi \approx Q_0 \frac{2(\omega-\omega_0)}{\omega_0}$$

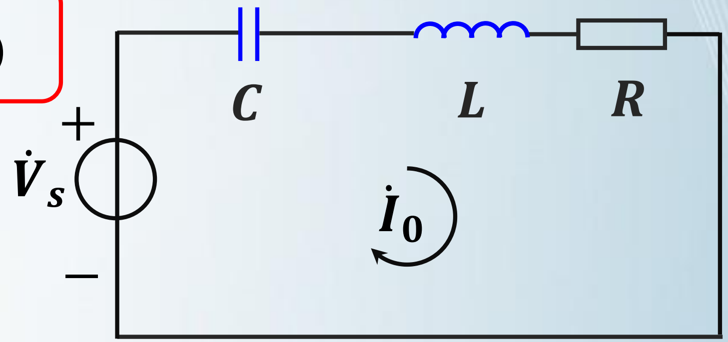
$\xi = Q_0 \cdot \dfrac{2\Delta\omega}{\omega_0}$ or $\xi = Q_0 \cdot \dfrac{2\Delta f}{f_0}$

$\xi \neq 0$ dennote detuning value

Series Resonant Circuit— Resonance Curve

$$Z = R + jX = R + j(\omega L - \frac{1}{\omega C})$$

- Current: $\dot{I} = \dfrac{\dot{V}_s}{R+j(\omega L-\frac{1}{\omega C})} \sim \omega$

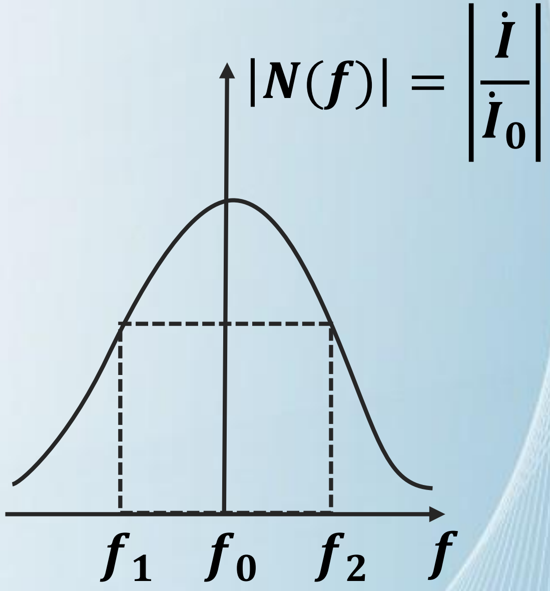


- Resonance Curve:

$$N(f) = \frac{\dot{I}}{\dot{I}_0} = \frac{\dfrac{\dot{V}_s}{R + j(\omega L - \frac{1}{\omega C})}}{\dfrac{\dot{V}_s}{R}} = \frac{R}{R + j(\omega L - \frac{1}{\omega C})} = \frac{1}{1 + j(\dfrac{\omega L - \frac{1}{\omega C}}{R})}$$

$$\Rightarrow N(f) = \frac{\dot{I}}{\dot{I}_0} = \frac{1}{1 + j\xi}$$

$$\xi = \frac{\omega L - \dfrac{1}{\omega C}}{R}$$



Amplitude-Frequency

Series Resonant Circuit— Bandwidth

➢ Bandwidth: scope among $\dot{I}$ drop to 0.707 of $\dot{I}_0$

$$B = 2\Delta f_{0.7} = |f_2 - f_1|$$

Curve $N(f) = \frac{\dot{I}}{\dot{I}_0} = \frac{1}{1+j\xi}$

AF: $|N(f)| = \frac{1}{\sqrt{1+\xi^2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow$ if $2\Delta f_{0.7}$

$\xi = \pm 1$

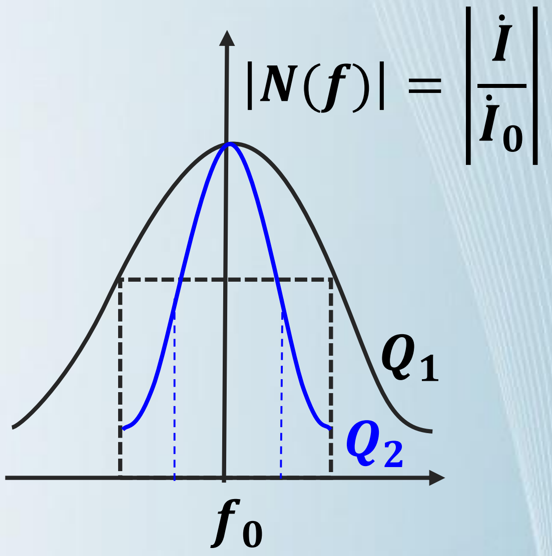
$$\xi = Q_0 \cdot \frac{2\Delta f}{f_0}$$

$2\Delta f = 2\Delta f_{0.7}$

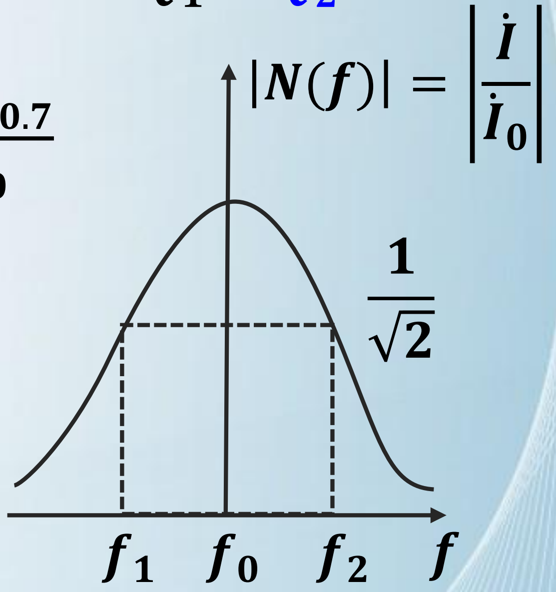
$$\Rightarrow 1 = Q_0 \cdot \frac{2\Delta f_{0.7}}{f_0}$$

$$1 = Q_0 \cdot \frac{B}{f_0}$$

$$Q_0 \cdot B = f_0$$



$Q_1 < Q_2$



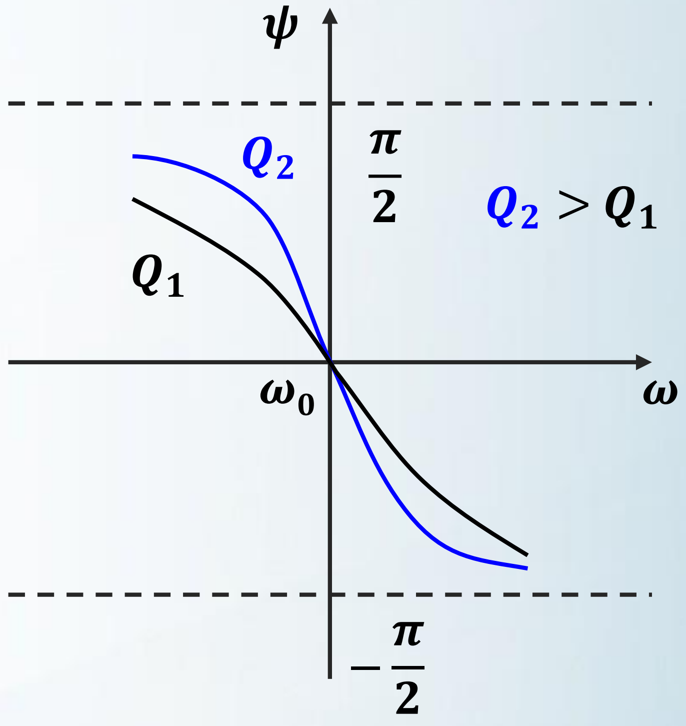
Amplitude-Frequency (AF)

Series Resonant Circuit— Phase-Frequency Curve

Resonance Curve $N(f) = \frac{\dot{I}}{\dot{I}_0} = \frac{1}{1+j\xi}$

- PF: $\psi = -arctg\xi$

$Q \uparrow$ linearality $\downarrow$

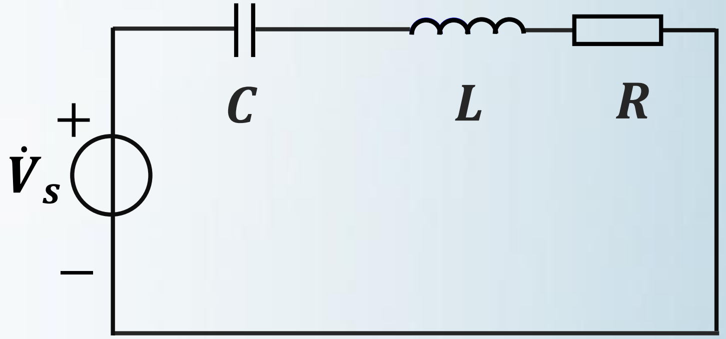


Phase-Frequency (PF)

Series Resonant Circuit— with load

- unloaded Q:

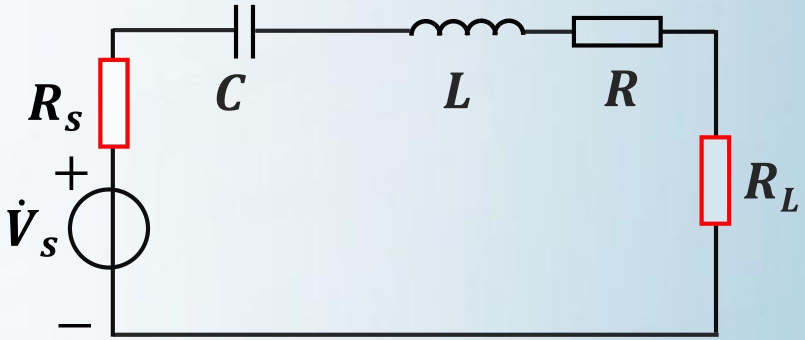
$$Q_0 = \frac{\omega_0 L}{R}$$



- loaded Q:

$$Q_L = \frac{\omega_0 L}{R+R_s+R_L}$$

Consider source resistance & load resistance



$$Q_L \downarrow \Rightarrow B \uparrow$$

Summary—Series Resonance

- Resonance Curve: $N(f) = \frac{\dot{I}}{\dot{I}_0} = \frac{1}{1+j\xi}$

cf. Inductor $Q = \frac{\omega L}{R}$

Amplitude-Frequency: $|N(f)| = \left|\frac{\dot{I}}{\dot{I}_0}\right|$

$\because \ \rho = \omega_0 L = \frac{1}{\omega_0 C}$

Resonance: $Q_0 = \frac{(Reactance)X}{(Resistance)R} = \frac{\omega_0 L}{R} = \frac{\frac{1}{\omega_0 C}}{R} = \frac{\rho}{R}$

Detunning: $\xi = \frac{(Reactance\ Sum)X}{(Resistance)R} = \frac{\omega L - \frac{1}{\omega C}}{R} \approx Q_0 \cdot \frac{2\Delta f}{f_0} \Rightarrow Q_0 \cdot B = f_0$

Phase-Frequency $\psi = -arctg\xi$