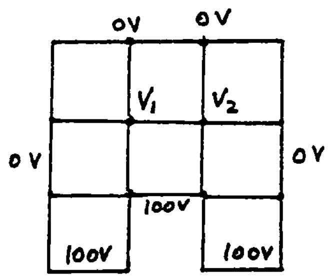
CHAPTER 12

COMPUTER-AIDED ANALYSIS OF ELECTROMAGNETIC FIELDS

Exercise 12.1



$$0 + 0 + 100 + V_2 - 4V_1 = 0$$
$$0 + V_1 + 100 + 0 - 4V_2 = 0$$

$$-4V_1 + V_2 = -100$$
$$V_1 - 4V_2 = -100$$

$$\underbrace{\begin{bmatrix} -4 & 1 \\ 1 & -4 \end{bmatrix}}_{A \,:\, \text{coefficient matrix}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -100 \\ -100 \end{bmatrix}$$

From the above equations $V_1 = 33.33\,V$, $V_2 = 33.33\,V$

Exercise 12.2

Let $V_1^{(0)} = V_2^{(0)} = 50$

Iteration 1 :

$$V_1^{(1)} = 50 + \frac{1}{4}(0 + 0 + 100 + 50 - 4 \times 50) = 37.5$$

$$|V_1^{(1)} - V_1^{(0)}| = |37.5 - 50| = 12.5$$

$$V_2^{(1)} = 50 + \frac{1}{4}(0 + 37.5 + 100 + 0 - 4 \times 50) = 34.38$$

$$|V_2^{(1)} - V_2^{(0)}| = |34.38 - 50| = 15.62$$

Iteration 2:

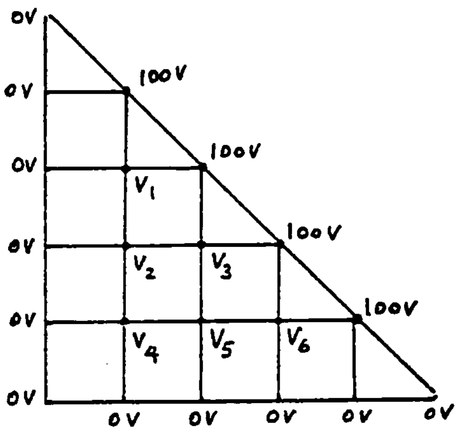
$$V_1^{(2)} = 37.5 + \frac{1}{4}\left(0 + 0 + 100 + 34.38 - 4 \times 37.5\right) = 33.6$$

$$\left| V_1^{(2)} - V_1^{(1)} \right| = \left| 33.6 - 37.5 \right| = 3.9$$

$$V_2^{(2)} = 34.38 + \frac{1}{4}\left(0 + 33.6 + 100 + 0 - 4 \times 34.38\right) = 33.4$$

$$\left| V_2^{(2)} - V_1^{(2)} \right| = \left| 33.4 - 34.38 \right| = 0.98$$

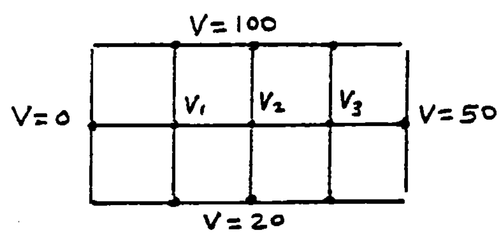
Problem 12.1



$$\begin{aligned}
-4V_1 + V_2 \quad &= -200 \\
V_1 - 4V_2 + V_3 + V_4 \quad &= 0 \\
V_2 - 4V_3 \quad + V_5 \quad &= -200 \\
V_2 \quad - 4V_4 + V_5 \quad &= 0 \\
V_3 + V_4 - 4V_5 + V_6 &= 0 \\
V_5 - 4V_6 &= -200
\end{aligned}$$

$$
\begin{bmatrix}
-4 & 1 & 0 & 0 & 0 & 0 \\
1 & -4 & 1 & 1 & 0 & 0 \\
0 & 1 & -4 & 0 & 1 & 0 \\
0 & 1 & 0 & -4 & 1 & 0 \\
0 & 0 & 1 & 1 & -4 & 1 \\
0 & 0 & 0 & 0 & 1 & -4
\end{bmatrix}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}
=
\begin{bmatrix} -200 \\ 0 \\ -200 \\ 0 \\ 0 \\ -200 \end{bmatrix}
$$

Problem 12.3



$$100 + 0 + 20 + V_2 - 4V_1 = 0$$
$$100 + V_1 + 20 + V_3 - 4V_2 = 0$$
$$100 + V_2 + 20 + 50 - 4V_3 = 0$$

$$
\begin{bmatrix}
-4 & 1 & 0 \\
1 & -4 & 1 \\
0 & 1 & -4
\end{bmatrix}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}
=
\begin{bmatrix} -120 \\ -120 \\ -170 \end{bmatrix}
$$

$V_1 = 43.75\ V$, $V_2 = 55\ V$, $V_3 = 56.25\ V$

APPENDIX A

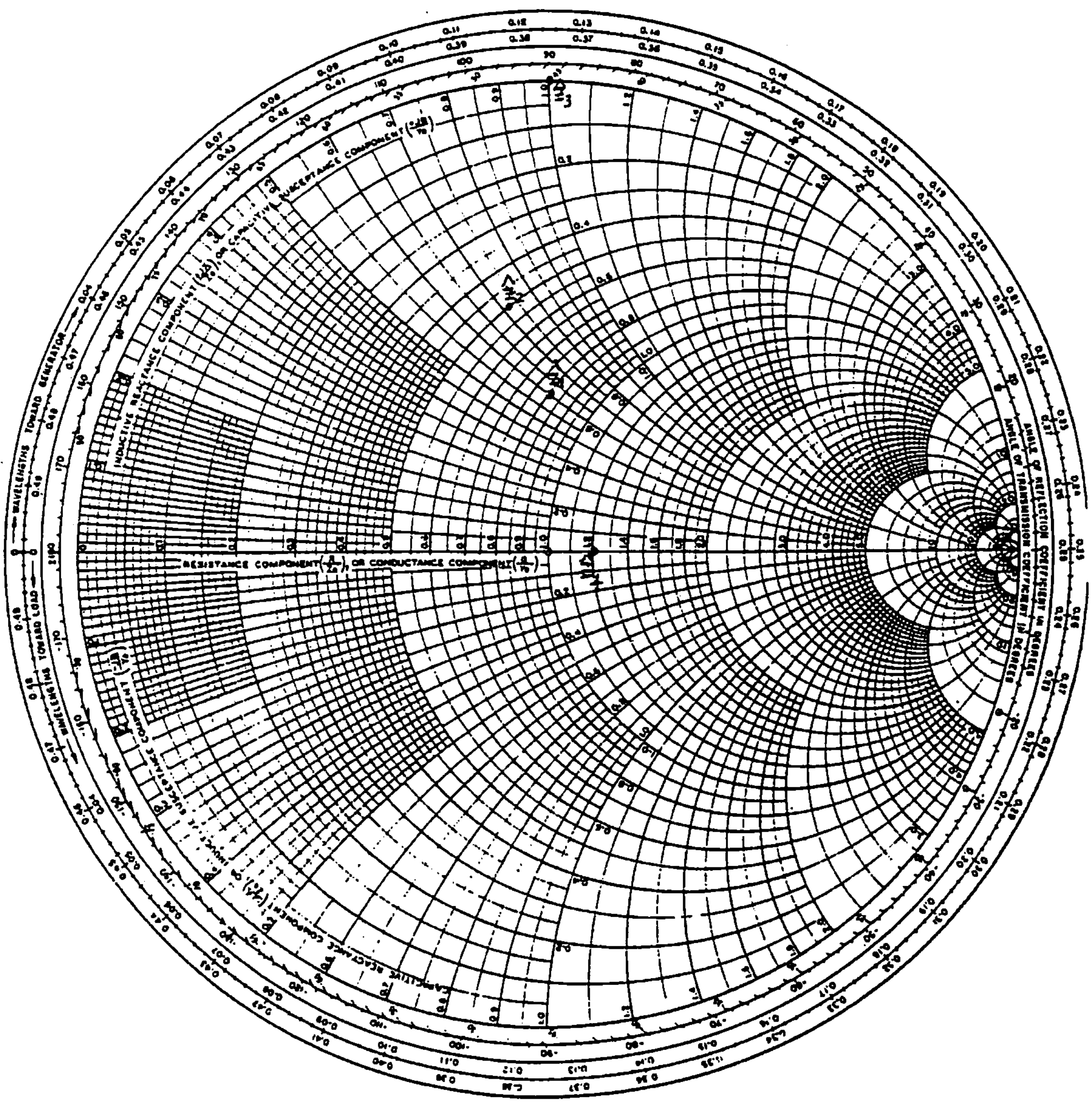
SMITH CHART AND ITS APPLICATIONS

Problem A.1

$$\hat{z}_1 = \frac{40+j30}{50} = 0.8+j0.6 \quad , \quad \hat{z}_2 = \frac{25-j36}{50} = 0.5-j0.72$$

$$\hat{z}_3 = \frac{j50}{50} = j1 \quad , \quad \hat{z}_4 = \frac{60}{50} = 1.2$$

IMPEDANCE OR ADMITTANCE COORDINATES



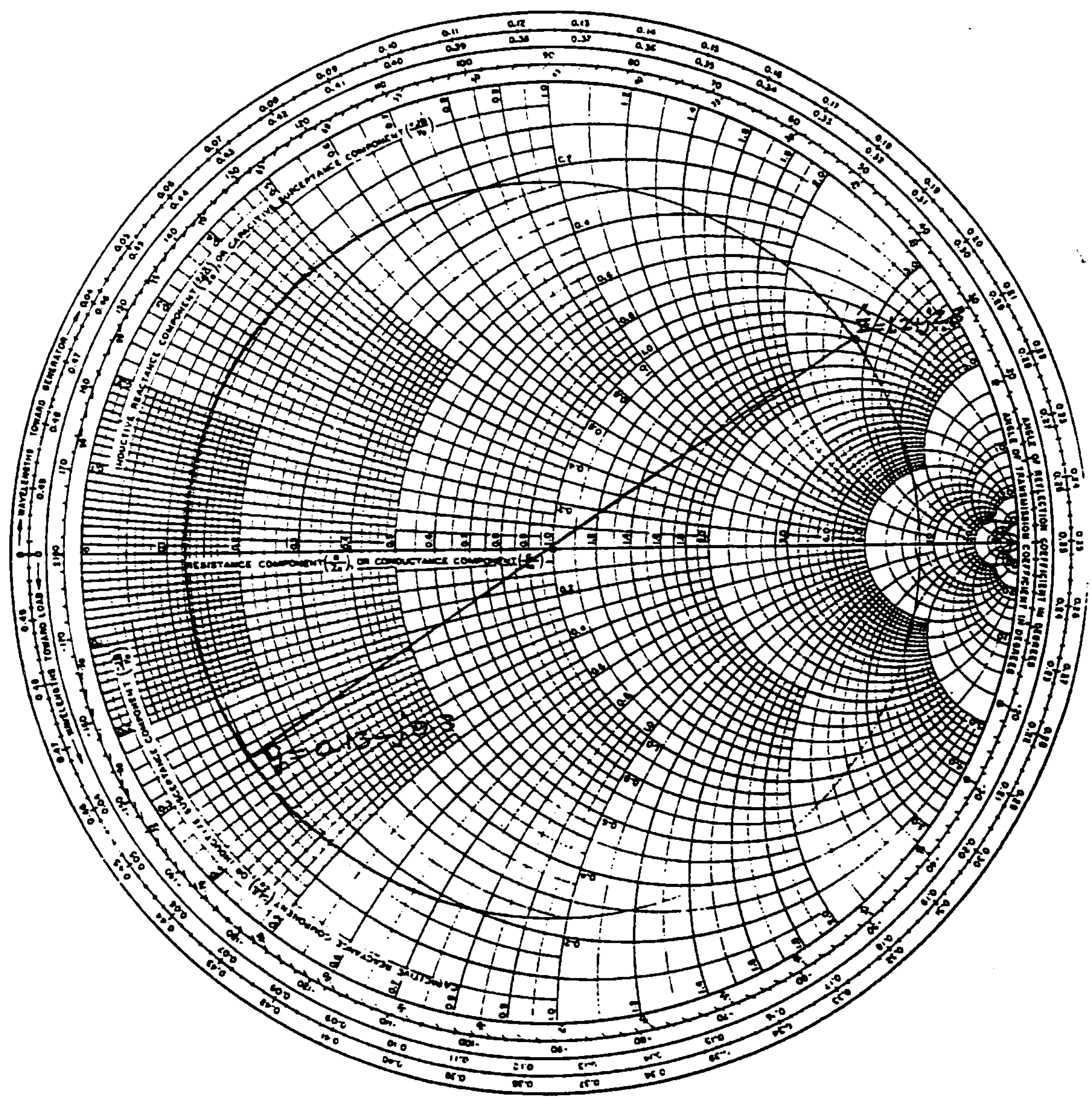
Problem A.2

$\hat{Z} = 120 + j280\,\Omega$, $\hat{Z}_c = 100\,\Omega$

$\hat{z} = \dfrac{120 + j280}{100} = 1.2 + j2.8$

From the Smith chart, $\hat{y} = 0.13 - j0.3$ or $\hat{Y} = 1.3\times10^{-3} - j3\times10^{-3}\,S$

IMPEDANCE OR ADMITTANCE COORDINATES



Problem A.3

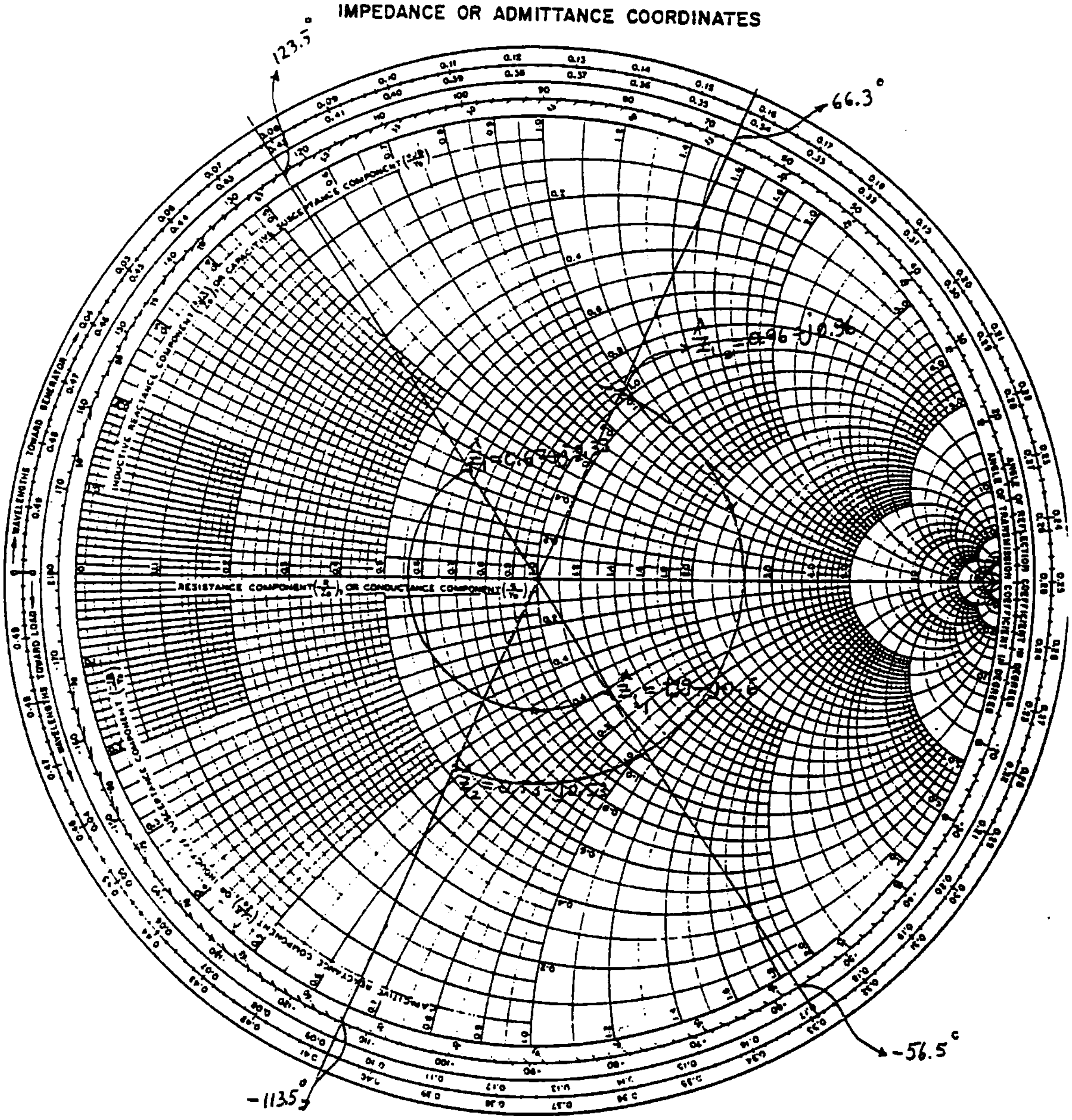
$l = \lambda/4$, $R_c = 75\,\Omega$ $\hat{Z}_1 = 50 + j25\,\Omega$, $\hat{Z}_2 = 40 - j40\,\Omega$

$$\hat{z}_1 = \frac{50 + j25}{75} = 0.67 + j\,0.33 \quad , \quad \hat{z}_2 = \frac{40 - j40}{75} = 0.53 - j\,0.53$$

From Smith chart $\hat{z}_{L_1} = 1.19 - j0.6$ or $\hat{Z}_{L_1} = 89.25 - j\,45\,\Omega$

$\hat{\rho}_{R_1} = 0.278\angle{-56.5^\circ}$ $\quad \hat{\rho}_{s_1} = 0.278\angle{123.5^\circ}$

$\hat{z}_{L_2} = 0.96 + j0.96$ or $\hat{Z}_{L_2} = 72 + j\,72\,\Omega$, $\rho_{R_2} = 0.44\angle{66.3^\circ}$, $\rho_{s_2} = 0.44\angle{-113.5^\circ}$



Problem A.4

$R_c = 50\,\Omega$, $l = 25\,m$, $\hat{Z}_{in} = 45 + j60\,\Omega$, $z = 12\,m$, $u_p = 2.6 \times 10^8\,m/s$

$f = 5\,MHz$

$$\beta = \frac{2\pi \times 5 \times 10^6}{2.6 \times 10^8} = 0.121 \text{ rad/m} \qquad \lambda = \frac{2\pi}{0.121} = 52\,m.$$

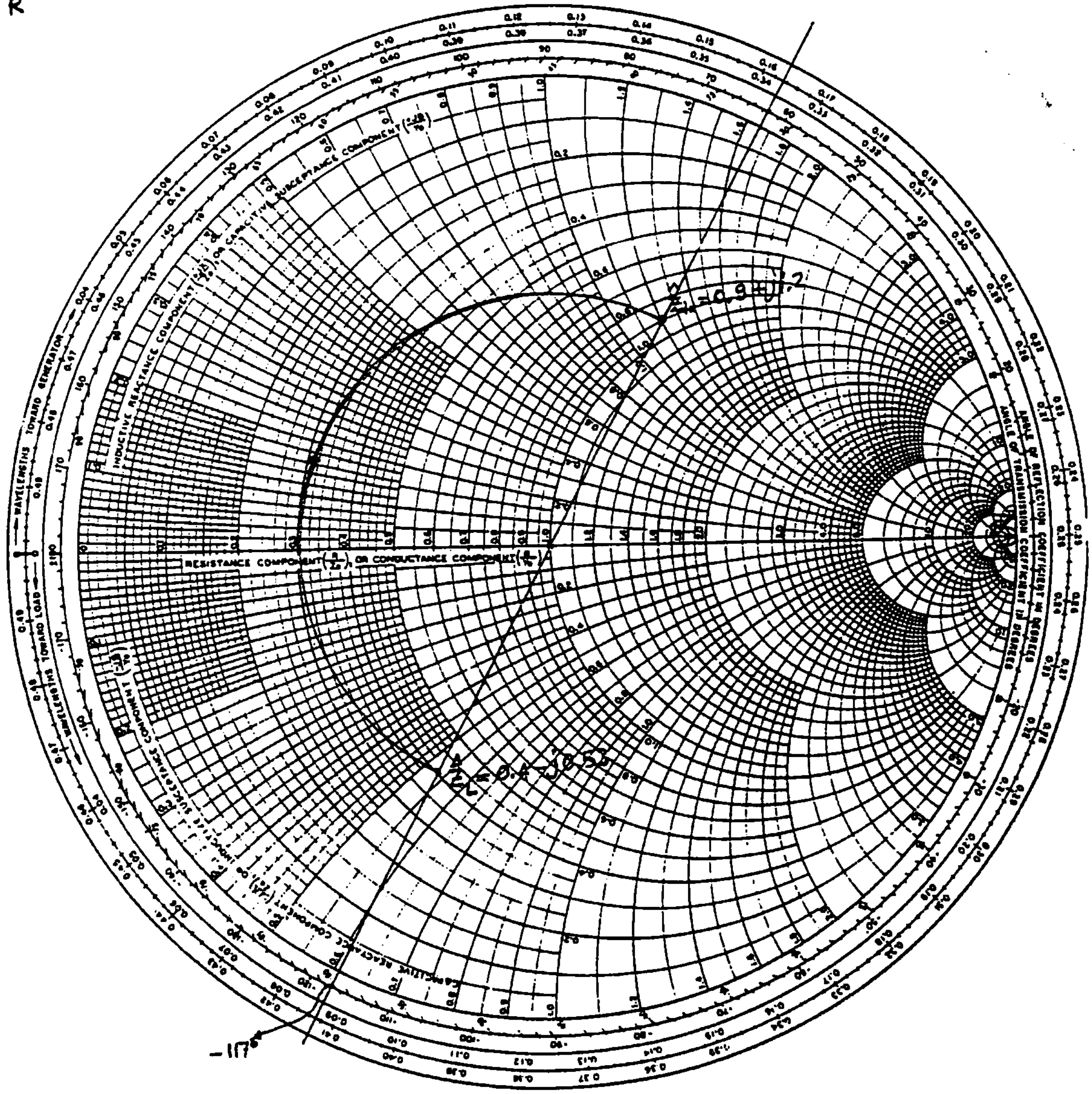
$$\frac{l - z}{\lambda} = \frac{25 - 12}{52} \doteq 0.25 \quad \text{or} \quad l - z = \frac{\lambda}{4}$$

$$\hat{Z}_{in} = \frac{45 + j60}{50} = 0.9 + j1.2$$

From Smith chart $\hat{Z}_L = 0.4 - j0.53$ or $\hat{Z}_L = 20 - j26.5\,\Omega$

$\hat{\rho}_R = 0.544\angle{-117^\circ}$

IMPEDANCE OR ADMITTANCE COORDINATES



Problem A.5

$l = 30m$, $R_c = 90\Omega$, $u_p = 2.8\times10^8\ m/s$, $f = 10\,MHz$

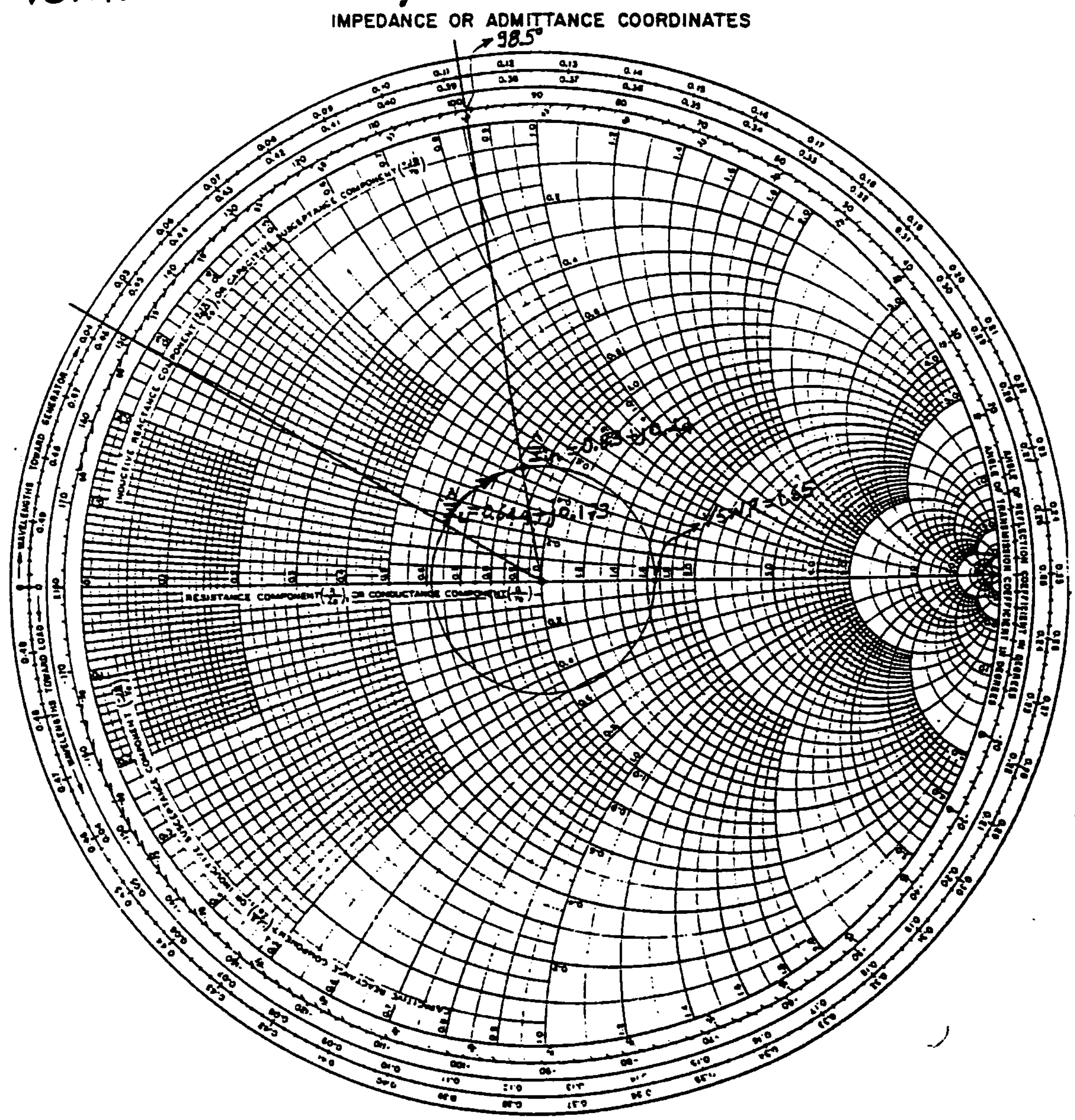
$\hat{Z}_L = 60\angle 15^\circ\ \Omega$, $\hat{z}_L = \dfrac{60\angle 15^\circ}{90} = 0.67\angle 15^\circ = 0.644 + j\,0.173$

$\beta = \dfrac{2\pi\times10^7}{2.8\times10^8} = 0.224\ rad/m$ $\qquad \lambda = \dfrac{2\pi}{0.224} = 28\,m$

$\dfrac{l}{\lambda} = \dfrac{30}{28} = 1.07 \Rightarrow l = 1.07\lambda$

From Smith chart $\hat{z}_{in} = 0.83 + j0.44$ or $\hat{Z}_{in} = 74.7 + j\,39.6\ \Omega$

VSWR $= 1.65$ and $\hat{\rho}_s = 0.25\angle 98.5^\circ$



Problem A.6

$l = 2\,m$, $\hat{Z}_c = 75\,\Omega$, $u_p = 2.6\times10^{8}\,m/s$, $\hat{Z}_L = 120 + j90\,\Omega$

$v_R(t) = 150\cos(1.26\times10^{8} t)\ V$

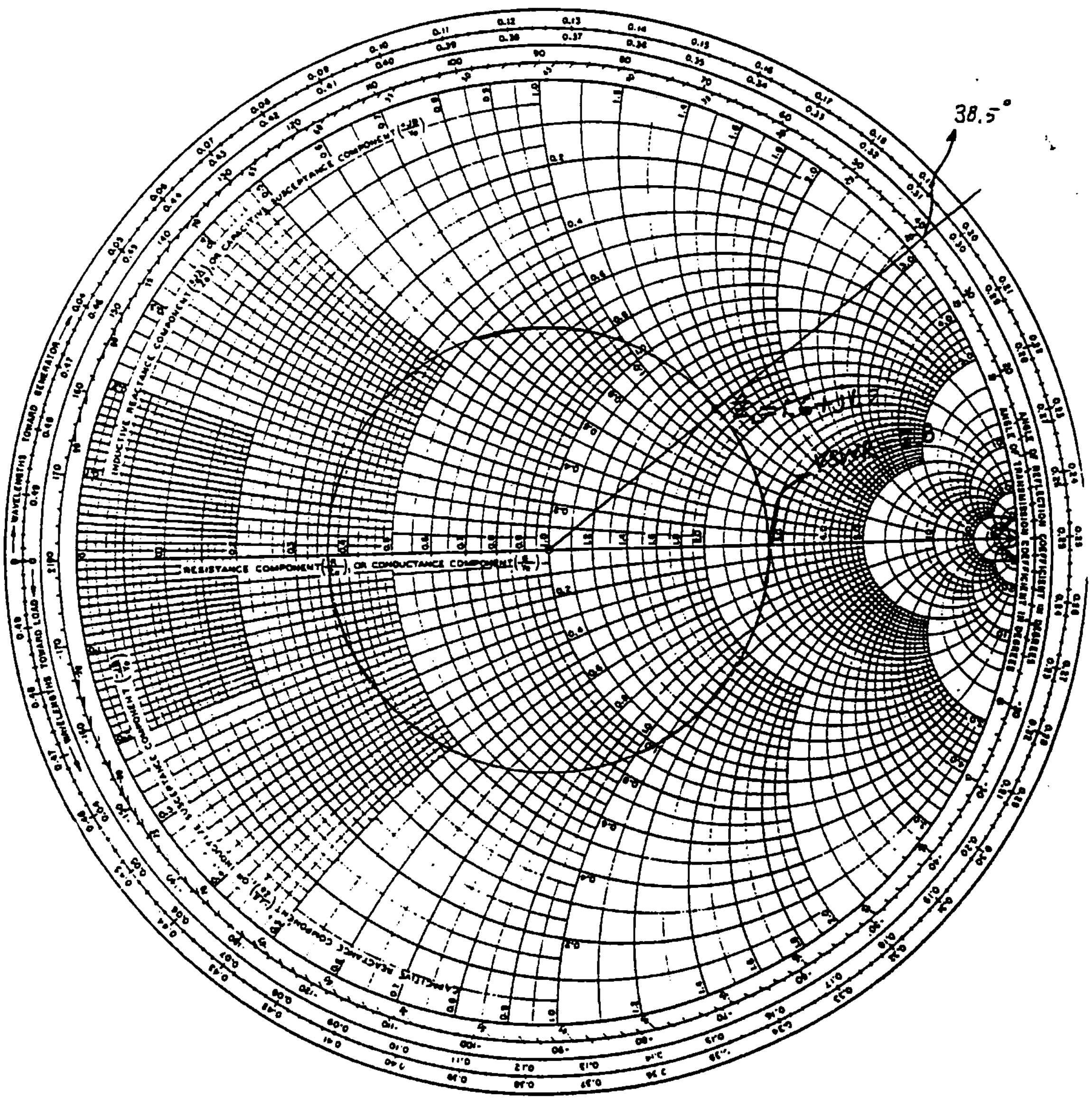
$$\beta = \frac{1.26\times10^{8}}{2.6\times10^{8}} = 0.485\ rad/m\ , \quad \lambda = \frac{2\pi}{0.L}$$

$$\hat{z}_L = \frac{120 + j90}{75} = 1.6 + j1.2$$

From Smith chart, a) $\hat{\rho}_R = 0.47\,\underline{|38.5^\circ}$, $\hat{\rho}(z) = 0.47\, e^{j38.5^\circ} \cdot e^{-j0.97(2-z)}$

b) VSWR = 2.8

IMPEDANCE OR ADMITTANCE COORDINATES



Problem A.7

$l = 50\,m \quad L_l = 0.5\,\mu H/m\,, \; C_l = 50\,pF/m\,, \; v_s(t) = 280\cos(6.28\times10^{7}t)\;V$

$\hat{Z}_L = 250\,\Omega \qquad \hat{Z}_c = \sqrt{\dfrac{0.5\times10^{-6}}{50\times10^{-12}}} = 100\,\Omega\,,$

$\beta = 6.28\times10^{7}\sqrt{0.5\times10^{-6}\times50\times10^{-12}} = 0.314\,\text{rad/m} \qquad \lambda = \dfrac{2\pi}{0.314} = 20\,m$

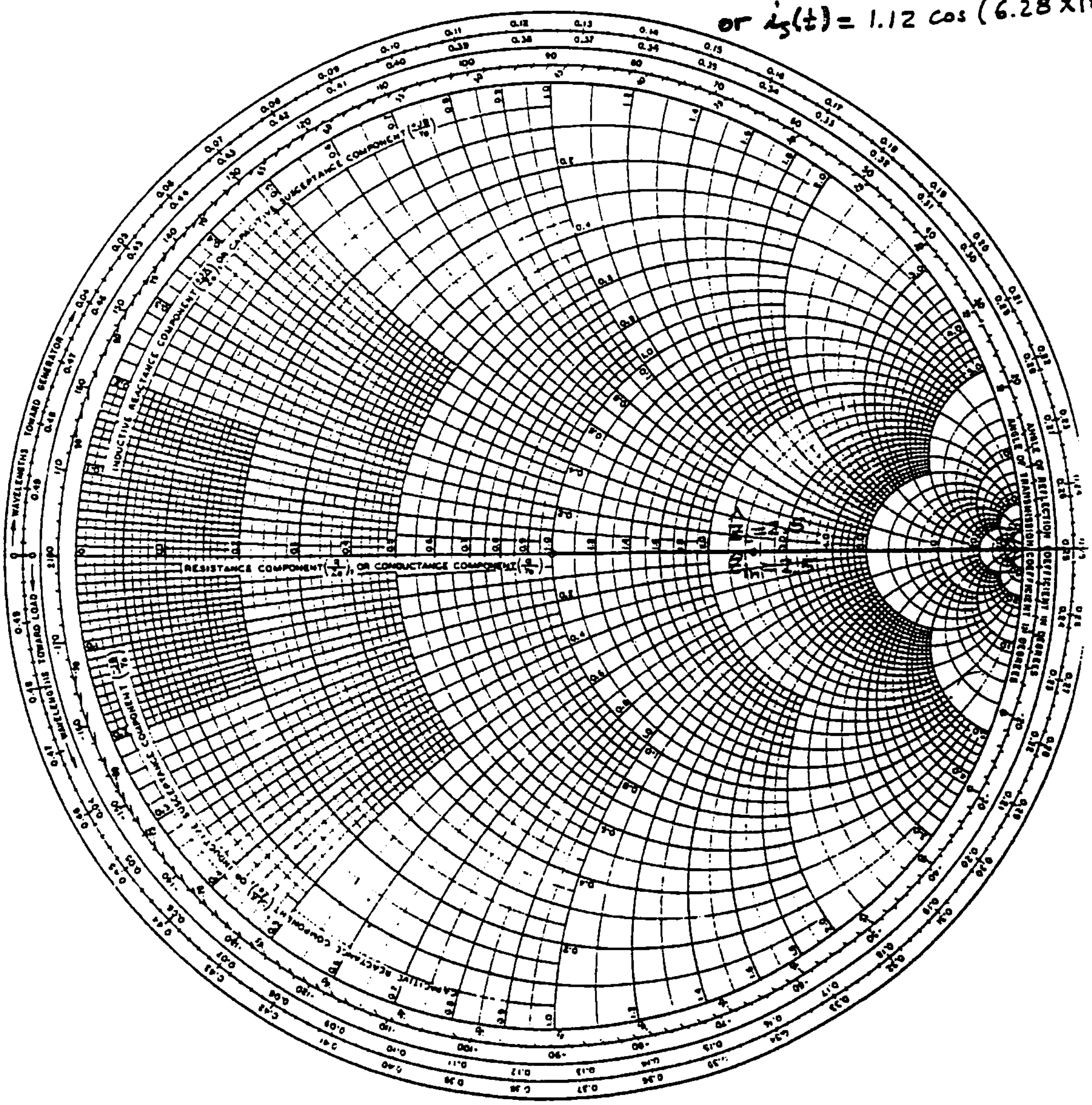
$l = \dfrac{50}{20}\lambda \quad l = 2.5\lambda \quad \hat{z}_L = \dfrac{250}{100} = 2.5$

From Smith chart a) $\hat{\rho}_R = 0.43\angle 0^\circ$, b) $= \hat{Z}_{in} = 250\,\Omega$

$i_s(t) = \dfrac{280}{250}\cos(6.28\times10^{7}t)$

or $i_s(t) = 1.12\cos(6.28\times10^{7}t)\;A$

IMPEDANCE OR ADMITTANCE COORDINATES



Problem A.8

$\hat{Z}_c = 75\,\Omega$, $v_s(t) = 50\cos(10^7 t)$ V , $i_s(t) = 0.5\cos(10^7 t - 36.8^\circ)$ A

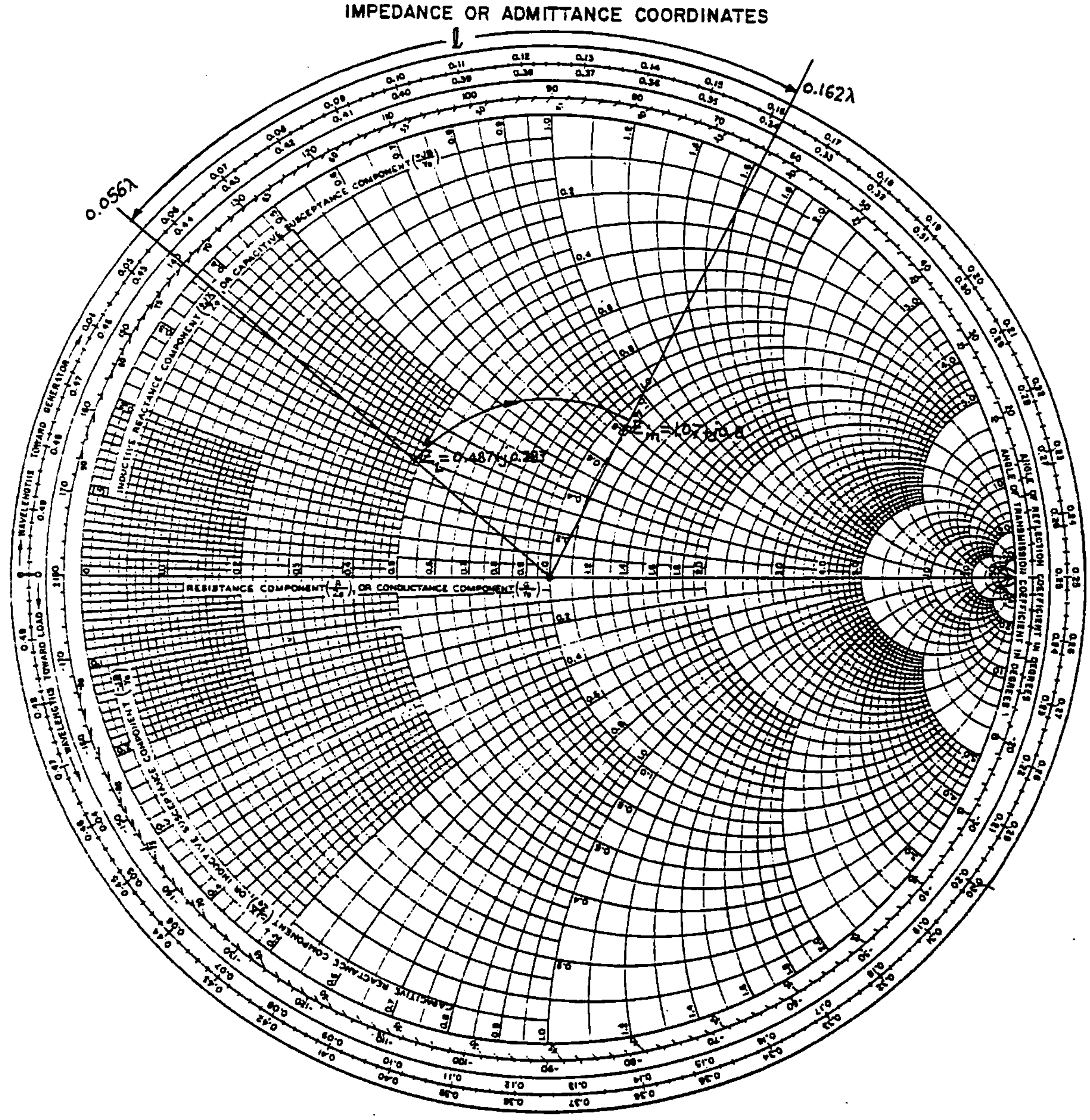
$\hat{Z}_L = 36.5 + j21.25\,\Omega$ $\quad u_p = 2.5\times10^8$ m/s

$$\hat{Z}_{in} = \frac{50\angle 0^\circ}{0.5\angle -36.8^\circ} = 100\angle 36.8^\circ\,\Omega = 80 + j60\,\Omega \qquad \hat{z}_{in} = \frac{80 + j60}{75} = 1.07 + j0.8$$

$$\hat{z}_L = \frac{36.5 + j21.25}{75} = 0.487 + j0.283 \qquad \beta = \frac{10^7}{2.5\times10^8} = 0.04 \text{ rad/m} \quad \lambda = \frac{2\pi}{0.04} = 157\,\text{m}$$

From Smith Chart $\ell = (0.162 - 0.056)\lambda = 0.106\lambda$, $\ell = 0.106\lambda = 0.106\times157 = 16.64\,\text{m}$

IMPEDANCE OR ADMITTANCE COORDINATES



Problem A.9

$\hat{Z}_L = 73 + j42.5\,\Omega$, $l = 50\,m$, $\hat{Z}_c = 50\,\Omega$, $\tilde{V}_s = 100\angle 0^\circ$ (rms) , $t_t = 0.2\,\mu s$

$f = 950\,kHz$

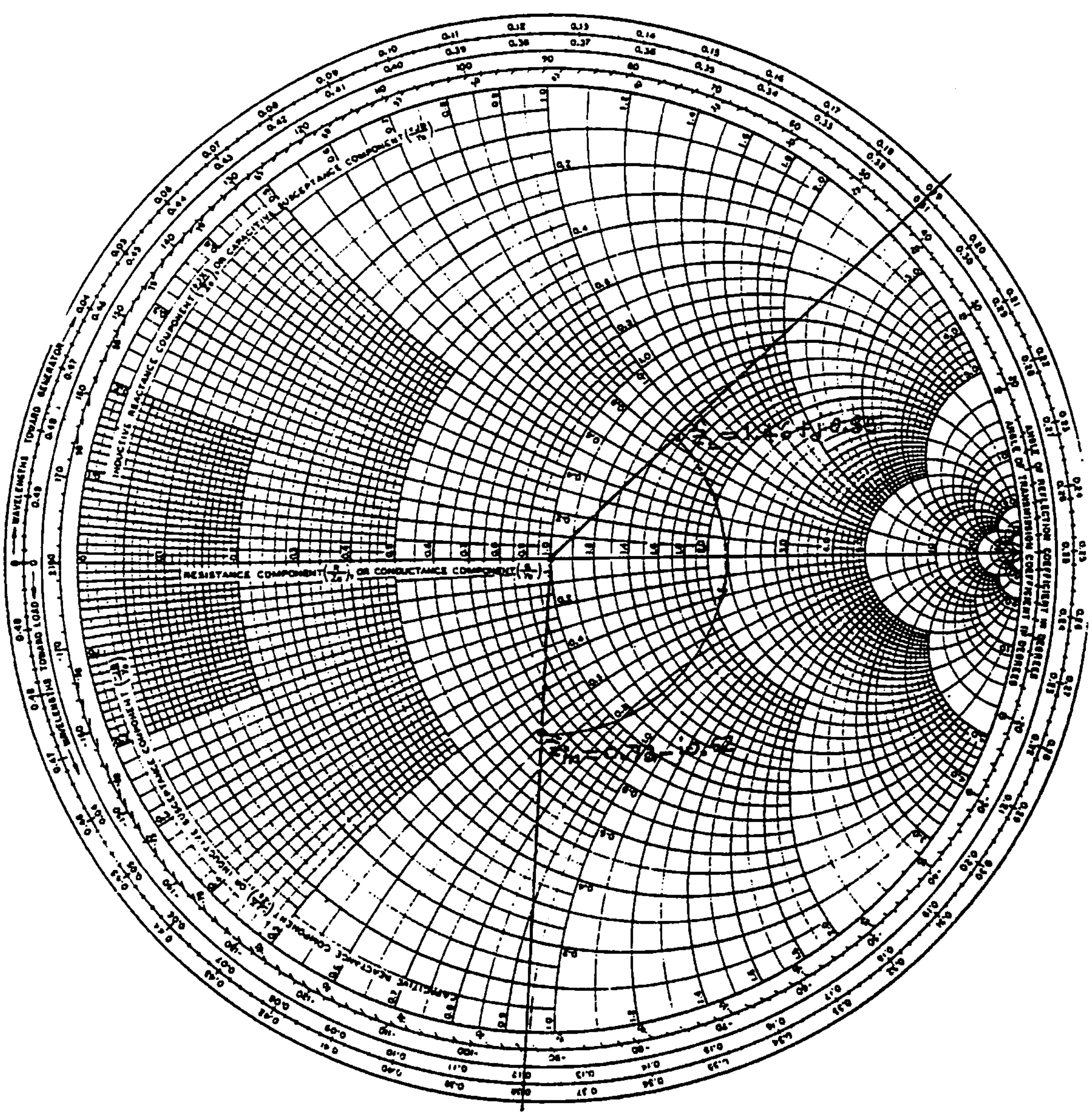
$u_p = \dfrac{50}{0.2 \times 10^{-6}} = 2.5 \times 10^8\, m/s$, $\beta = \dfrac{2\pi \times 950 \times 10^3}{2.5 \times 10^8} = 0.024\, rad/m$,

$\lambda = \dfrac{2\pi}{0.024} = 261.8\, m$, $l = \dfrac{50}{261.8}\lambda \Rightarrow l = 0.19\lambda$, $\hat{z}_L = \dfrac{73 + j42.5}{50} = 1.46 + j0.85$

From Smith chart , $\hat{z}_{in} = 0.73 - j0.62$ or $\hat{Z}_{in} = 36.5 - j31\,\Omega = 47.89\angle -40.34^\circ\,\Omega$

$\tilde{I}_s = (100\angle 0^\circ)/(47.89\angle -40.34^\circ) = 2.088\angle 40.34^\circ$ A (rms)

IMPEDANCE OR ADMITTANCE COORDINATES



Problem A.10

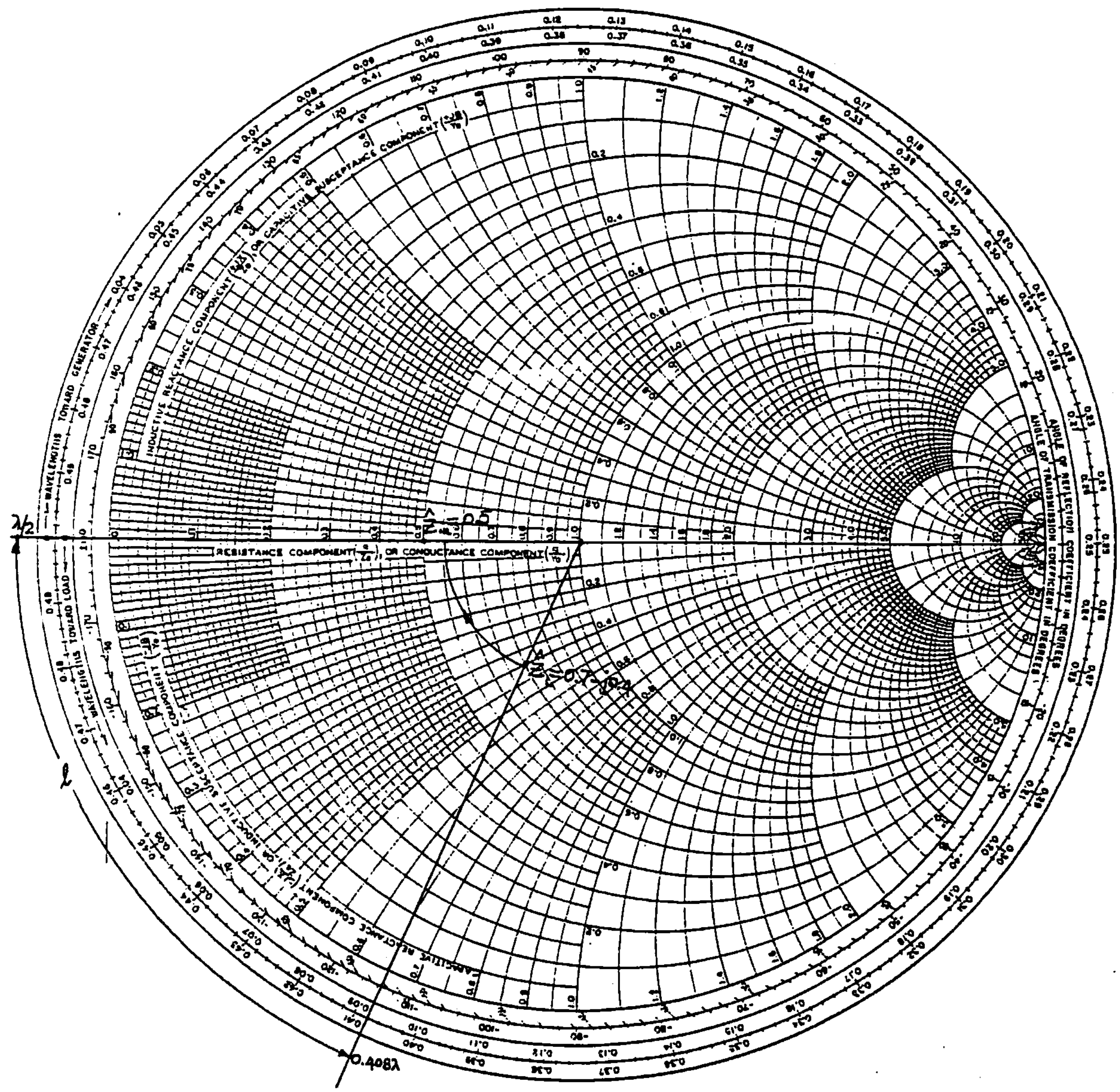
$\hat{Z}_c = 50\,\Omega$, $R_G = 25\,\Omega$, $\hat{Z}_L = 35 - j20\,\Omega$

$\hat{z}_L = \dfrac{35 - j20}{50} = 0.7 - j\,0.4$, $\hat{Z}_{in} = R_G = 25\,\Omega$ for the maximum power transfer.

$\hat{z}_{in} = \dfrac{25}{50} = 0.5$

From Smith Chart $l = (0.5 - 0.408)\lambda = 0.092\,\lambda$

IMPEDANCE OR ADMITTANCE COORDINATES



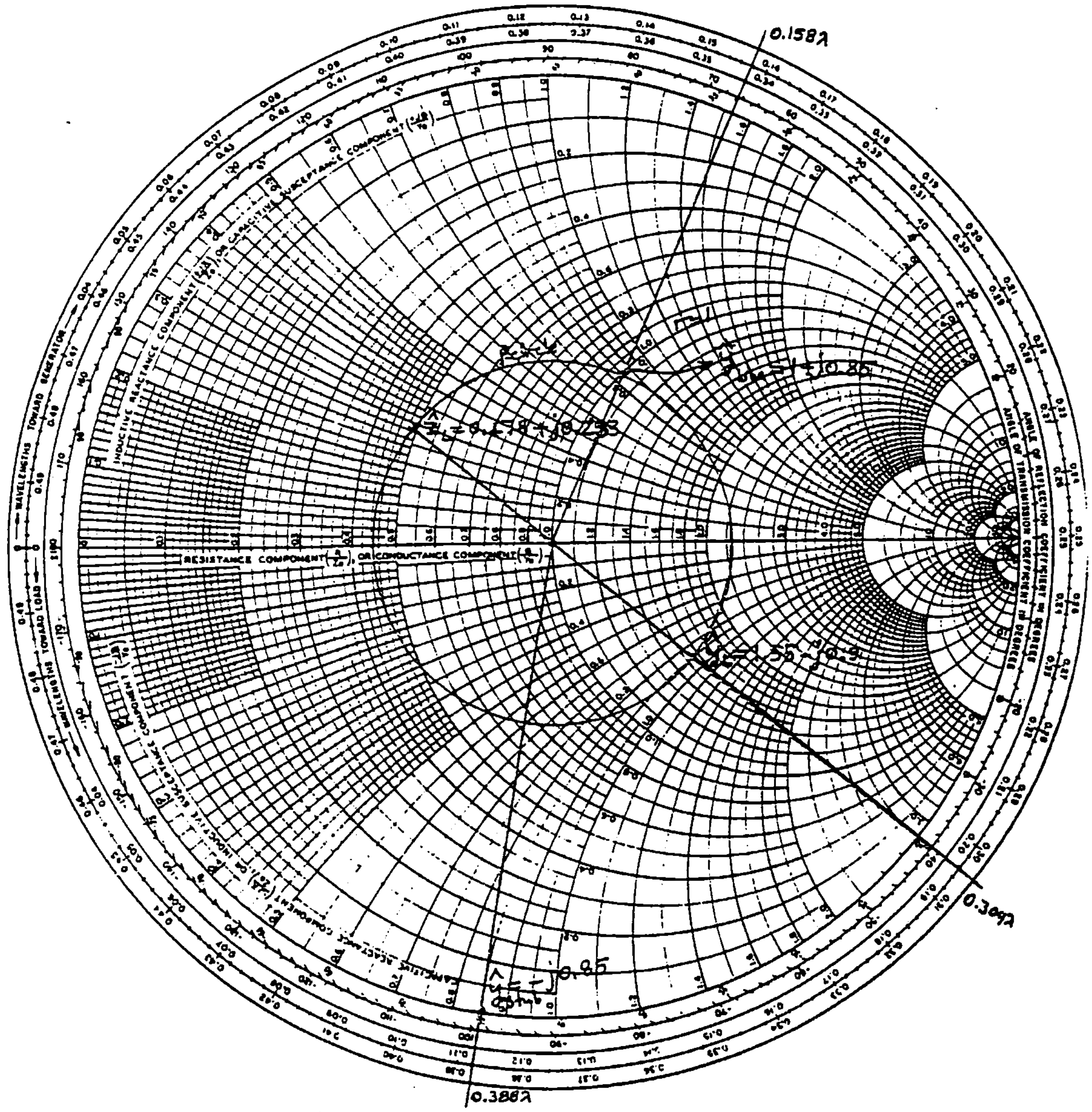
Problem A.11

From Smith chart $d = (0.5 - 0.304)\lambda + 0.158\lambda = 0.354\lambda$

$d = 0.354 \times 157 = 55.58\ m$

$l_s = (0.388 - 0.25)\lambda = 0.138\lambda \qquad l_s = 0.138 \times 157 = 21.67\ m$

IMPEDANCE OR ADMITTANCE COORDINATES



Problem A.12

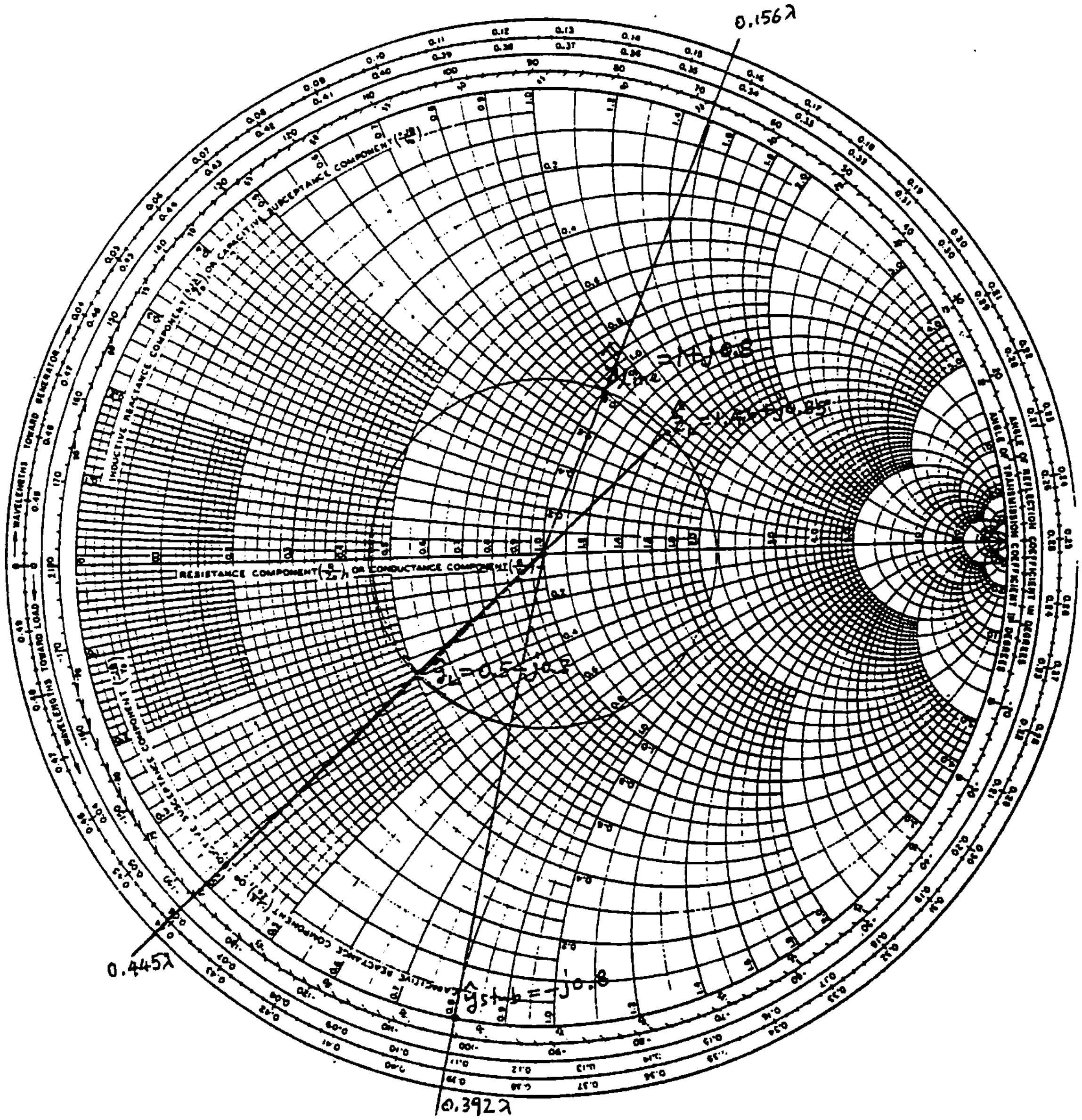
It is not perfectly matched.

From Smith chart $d = (0.5 - 0.445)\lambda + 0.156\lambda = 0.211\lambda$

$l_s = (0.392 - 0.25)\lambda \quad l_s = 0.142\lambda$

$d = 0.211 \times 261.8 = 55.24\,m \;,\; l_s = 0.142 \times 261.8 = 37.18\,m$

IMPEDANCE OR ADMITTANCE COORDINATES



Problem A.13

$\hat{Z}_c = 100\,\Omega$, $l = 15\,m$ $u_p = 2.8\times10^8\,m/s$ $\hat{Z}_L = 250\,\Omega$, $v_G = 35\cos(8\times10^7 t)\ V$,

$\hat{Z}_G = 20 - j10\,\Omega$

$\beta = \dfrac{8\times10^7}{2.8\times10^8} = 0.286\ rad/m$, $\lambda = \dfrac{2\pi}{0.286} = 21.99\ m$, $l = \dfrac{15\lambda}{21.99} = 0.682\lambda$

$\hat{z}_L = \dfrac{250}{100} = 2.5$

a) From Smith chart $\hat{z}_S = 0.46 - j0.37$ or $\hat{Z}_S = 46 - j37\,\Omega$

$\tilde{I}_S = \dfrac{35}{20 - j10 + 46 - j37} \cong 0.353 + j0.247 = 0.43\angle 34.98^\circ\ A$

$\tilde{V}_S = \hat{Z}_S \tilde{I}_S = 25.51\angle -3.17^\circ\ V$

$\hat{S}_S = \frac{1}{2}(25.51\angle -3.17^\circ)(0.43\angle -34.98^\circ) = 4.313 - j3.388\ VA$

b) $\tilde{V}_R = \tilde{V}_S\cos(\beta l) - j\hat{Z}_c\tilde{I}_S\sin(\beta l) = 46.484\angle 135.17^\circ\ V$

$\tilde{I}_R = \dfrac{\tilde{V}_R}{\hat{Z}_L} = 0.186\angle 135.17^\circ\ A$

$\hat{S}_R = \frac{1}{2}\tilde{V}_R\tilde{I}_R^* = 4.323\angle 0^\circ\ VA$ $P_R = 4.323\ W$

c) $z = 5m$ from the supply $z = \dfrac{5}{21.99}\lambda = 0.2273\lambda$

From Smith chart $\hat{z}(5m) = 1.8 + j1$ or $\hat{Z}(5m) = 180 + j100\,\Omega$

$\tilde{V}(5m) = \tilde{V}_S\cos[\beta(l-5)] - j\hat{Z}_c\tilde{I}_S\sin[\beta(l-5)] = 19.531\angle -154.35^\circ\ V$

$\tilde{I}(5m) = -j\dfrac{1}{\hat{Z}_c}V_S\sin[\beta(l-z)] + \tilde{I}_S\cos[\beta(l-z)] = 0.461\angle -138.079^\circ\ A$

$\hat{S}(5m) = \frac{1}{2}\tilde{V}(5m)\tilde{I}^*(5m) = 4.32 - j1.261\ VA$

d) $z = 10\,m$ from the supply $\quad z = \frac{10}{21.99}\lambda = 0.455\lambda$

From Smith chart $\hat{Z}(10m) = 0.405 - j0.12$ or $\hat{Z}(10m) = 40.5 - j\,12\ \Omega$

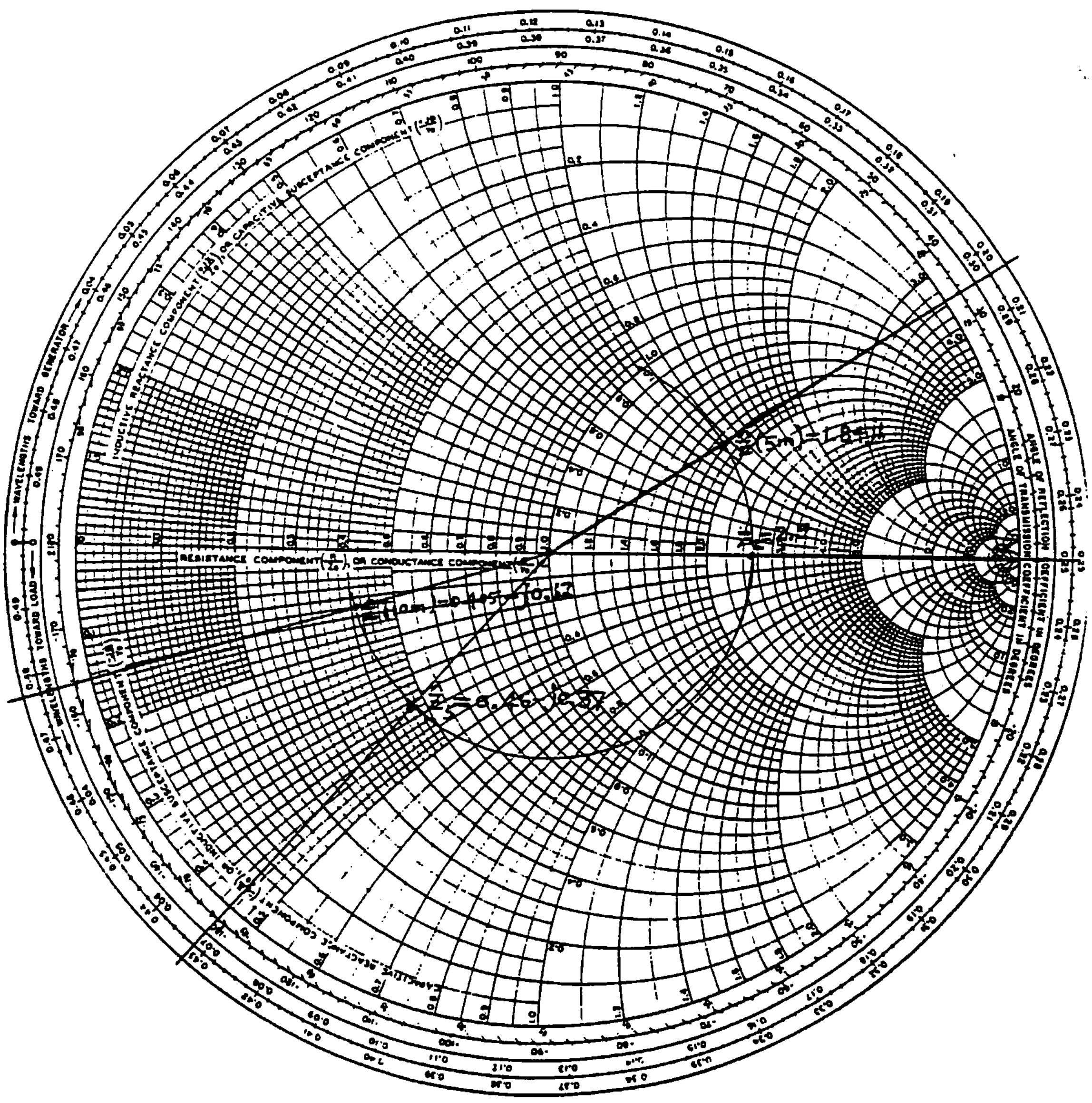
$\tilde{V}(10m) = \tilde{V}_s \cos[\beta(l-10)] - j\hat{Z}_c\tilde{I}_s \sin[\beta(l-10)] = 44.951 \underline{|-51.435^\circ}\ V$

$\tilde{I}(10m) = -j\frac{1}{\hat{Z}_c} V_s \sin[\beta(l-10)] + \tilde{I}_s \cos[\beta(\lambda-10)] = 0.22 \underline{|-80.71^\circ}\ A$

$\hat{S}(10m) = \frac{1}{2}\tilde{V}(10m)\,\tilde{I}^*(10m) = 4.32 + j\,2.42\ \ VA$

e) $\Delta V = V_s - V_R = 25.51 - 46.484 = -20.974$

IMPEDANCE OR ADMITTANCE COORDINATES



Problem A.14

$\hat{Z}_C = 50\,\Omega$, $l = 100\,m$, $\hat{Z}_L = 40 - j100\,\Omega$, $t_L = 0.5\,\mu s$, $f = 20\,MHz$

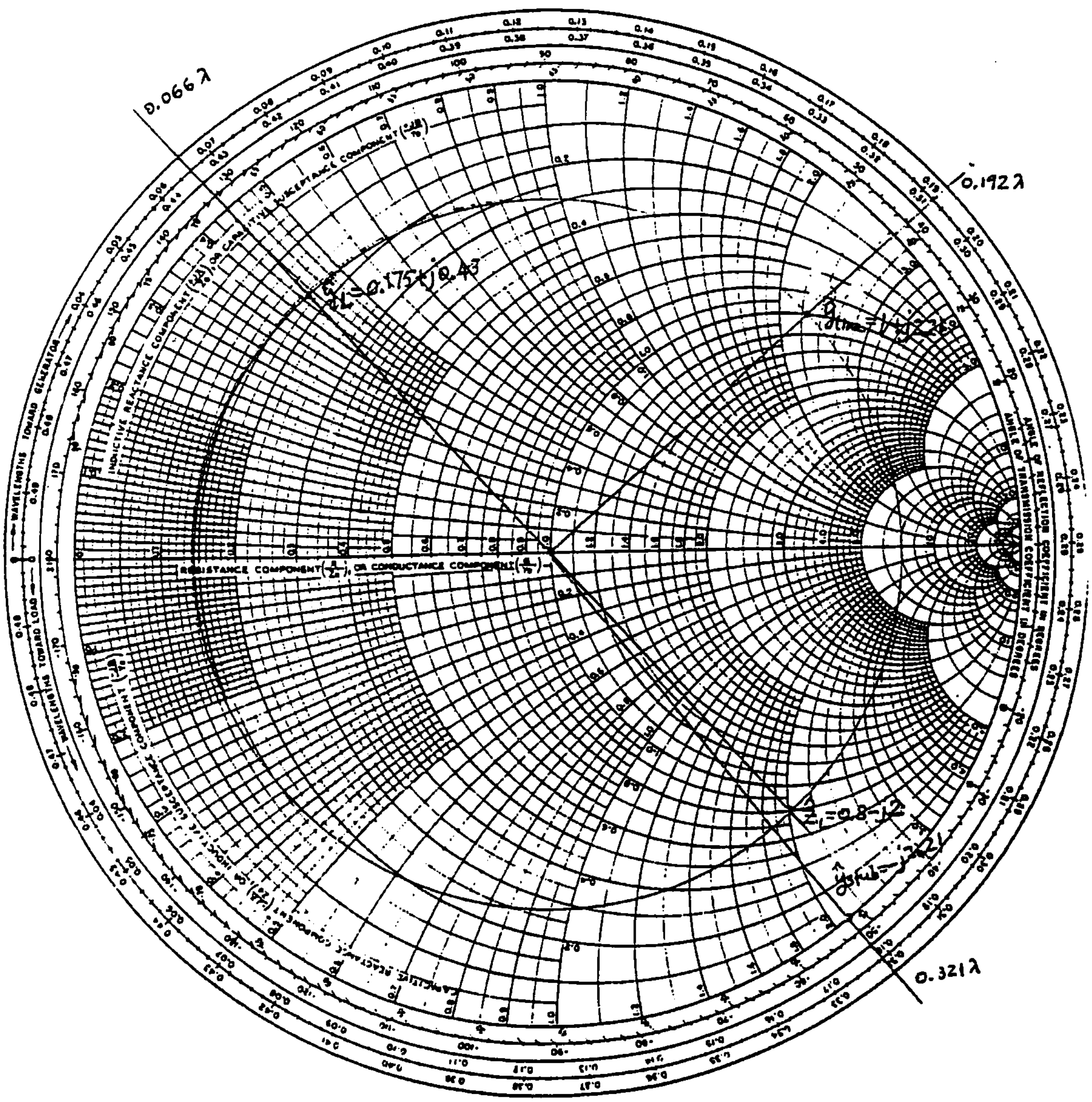
$$\beta = \frac{2\pi \times 20 \times 10^6}{(100/0.5\times10^{-6})} = 0.628\ rad/m \quad , \quad \lambda = \frac{2\pi}{0.628} = 10\,m \qquad l = \frac{100}{10}\lambda = 10\lambda$$

$$\hat{z}_L = \frac{40 - j100}{50} = 0.8 - j2$$

From Smith chart $d = (0.192 - 0.066)\lambda = 0.126\lambda$, $d = 0.126 \times 10 = 1.26\,m$

$l_s = (0.321 - 0.25)\lambda = 0.071\lambda$, $l_s = 0.071 \times 10 = 0.71\,m$

IMPEDANCE OR ADMITTANCE COORDINATES



Problem A.15

$l = 15\,m$, $f = 125\,MHz$, $\hat{Z}_L = 150 + j225\,\Omega$, $d_i = 2.5\,mm$, $d_o = 6\,mm$

$$L_l = \frac{4\pi\times10^{-7}}{2\pi}\ln\frac{3}{1.25} = 1.75\times10^{-7}\,H/m, \quad C_l = \frac{2\pi\times8.85\times10^{-12}}{\ln\frac{3}{1.25}} = 6.35\times10^{-11}\,F/m$$

$$Z_c = \sqrt{\frac{1.75\times10^{-7}}{6.35\times10^{-11}}} = 52.5\,\Omega \qquad u_p = \frac{1}{\sqrt{1.75\times10^{-7}\times6.35\times10^{-11}}} = 3\times10^{8}\,m/s$$

$$\beta = \frac{2\pi\times125\times10^{6}}{3\times10^{8}} = 2.62\,rad/m \qquad \lambda = \frac{2\pi}{2.62} = 2.4\,m \qquad l = \frac{15}{2.4}\lambda = 6.25\lambda$$

$$\hat{z}_L = \frac{150 + j225}{52.5} = 2.86 + j4.28$$

From Smith chart $\quad d = (0.5 - 0.474)\lambda + 0.2\lambda = 0.226\lambda$

$d = 0.226\times2.4 = 0.5424\,m$

$l_s = (0.305 - 0.25)\lambda = 0.055\lambda \qquad l_s = 0.055\times2.4 = 0.132\,m$

IMPEDANCE OR ADMITTANCE COORDINATES