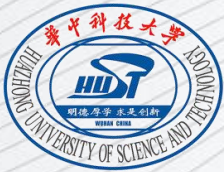


Huazhong University
of Science & Technology

Electronic Circuit of Communications

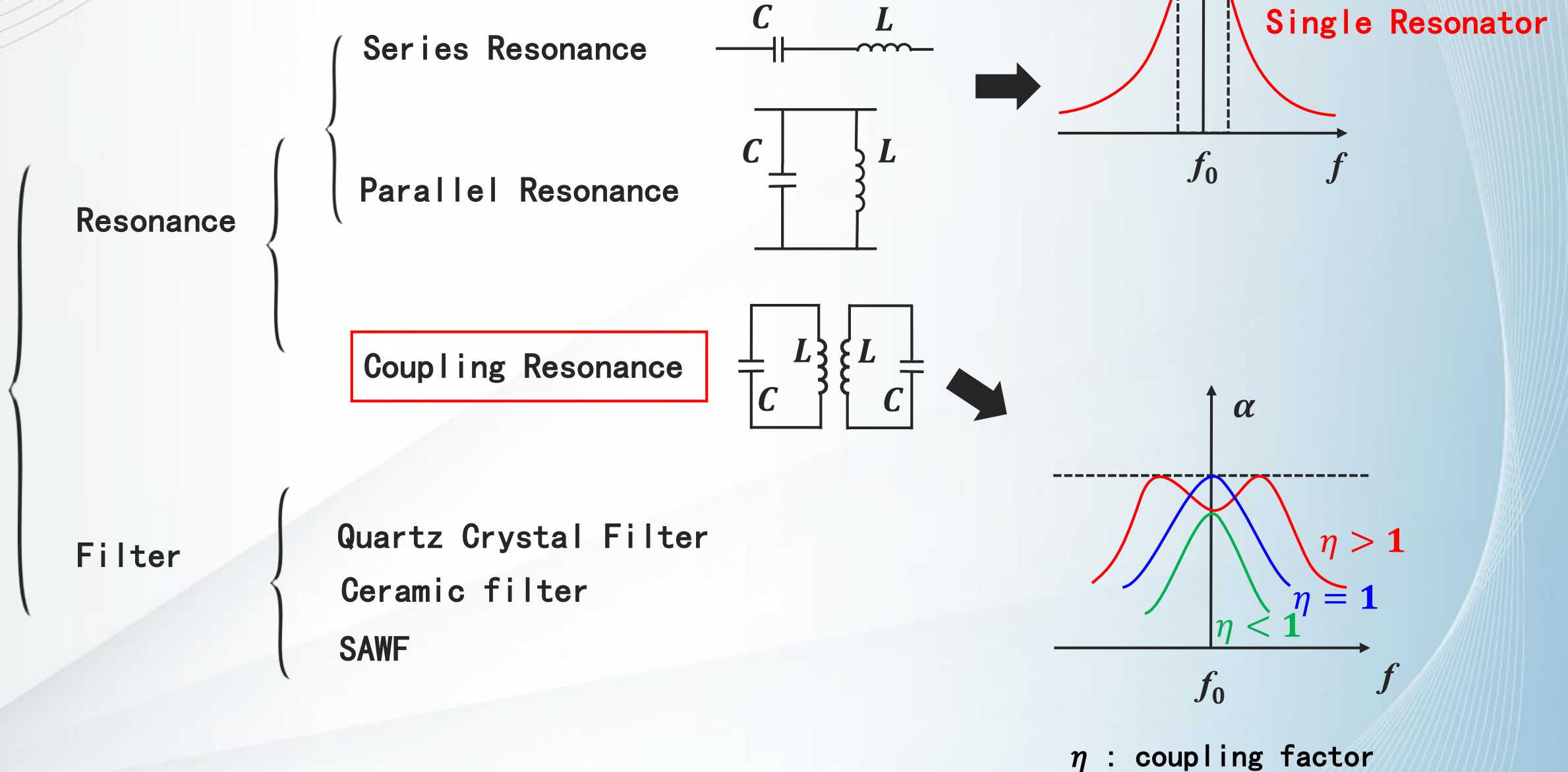
School of Electronic Information
and Communications

Jiaqing Huang

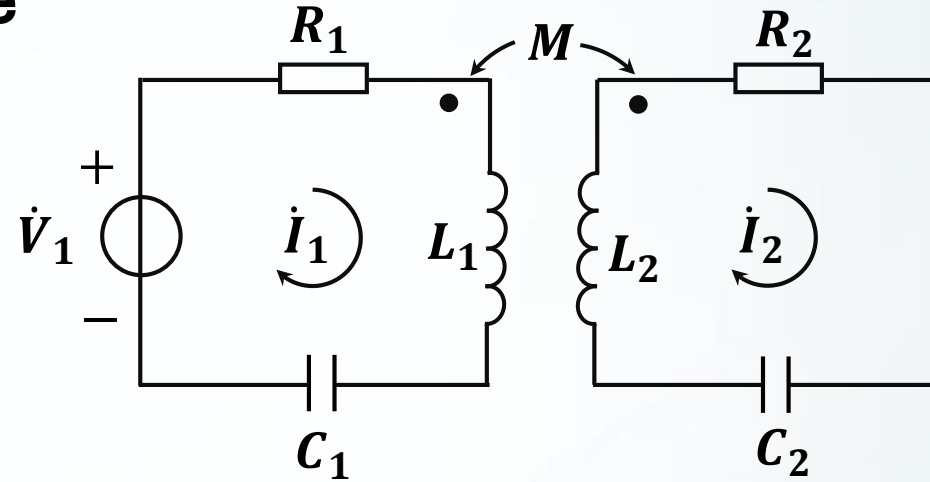


Coupling Circuits

Frequency Selective Circuits

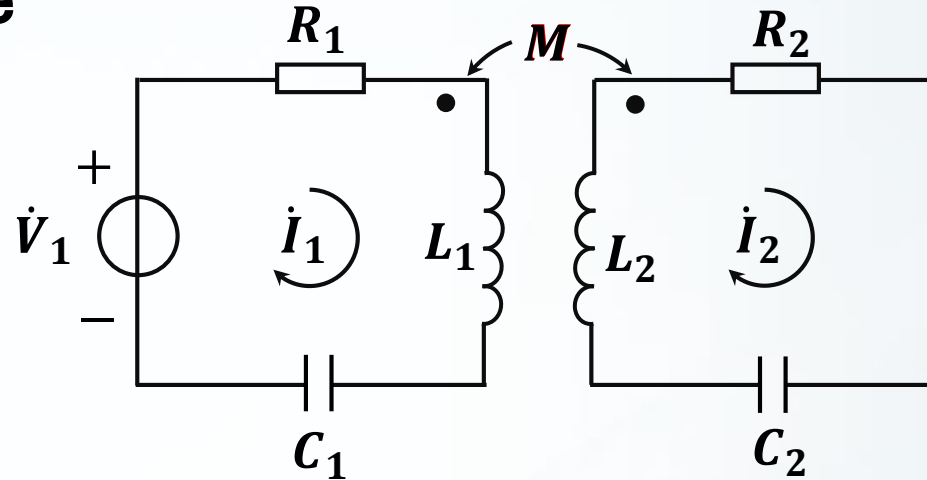


Coupling Resonance



Inductive Coupling Resonance

Coupling Resonance



Inductive Coupling Resonance

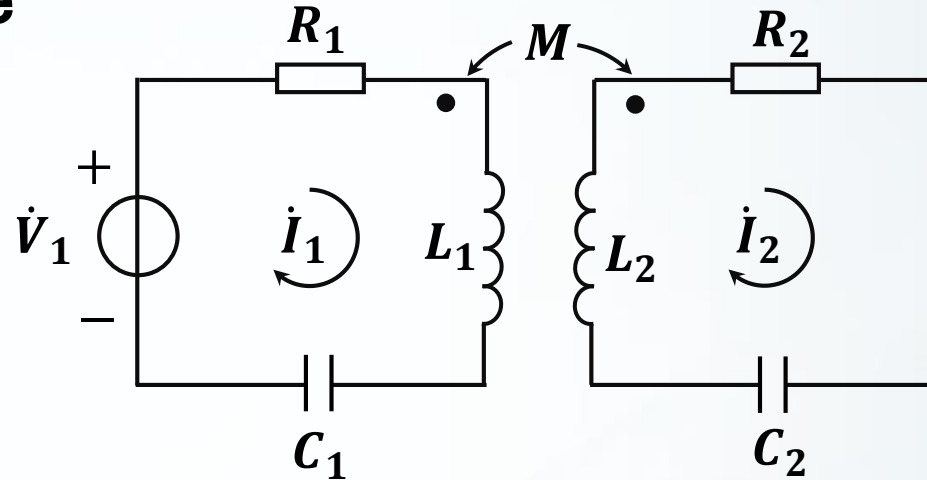
De-coupling



Primary Equivalent Circuit + Secondary Equivalent Circuit



Coupling Resonance



Primary Circuit

$$\dot{V}_1 = Z_{11}\dot{I}_1 - j\omega M\dot{I}_2$$

$$R_{11} + jX_{11} = Z_{11}$$

Primary Circuit

$$\dot{I}_1 = \frac{\dot{V}_1}{Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

Secondary Circuit

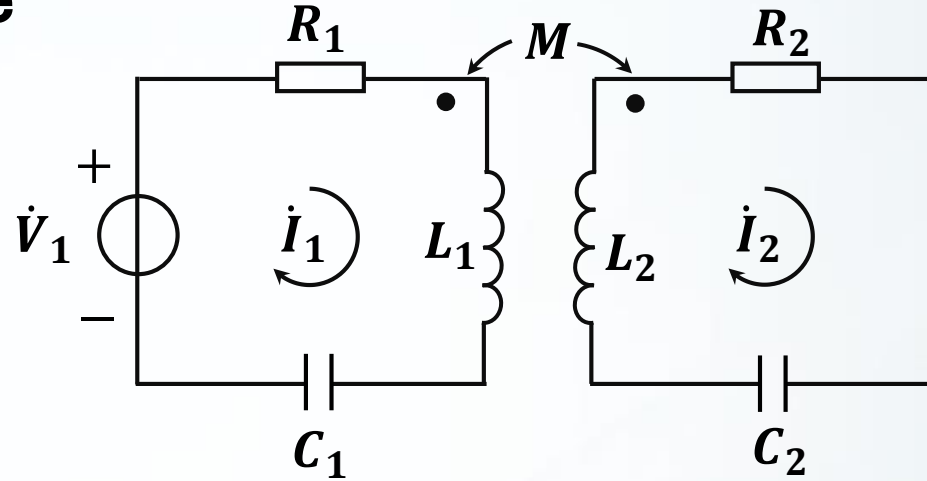
$$0 = Z_{22}\dot{I}_2 - j\omega M\dot{I}_1$$

$$Z_{22} = R_{22} + jX_{22}$$

Secondary Circuit

$$\dot{I}_2 = \frac{j\omega M \frac{\dot{V}_1}{Z_{11}}}{Z_{22} + \frac{(\omega M)^2}{Z_{11}}}$$

Coupling Resonance



Primary Circuit

$$\dot{V}_1 = Z_{11}\dot{I}_1 - j\omega M\dot{I}_2$$

Secondary Circuit

$$0 = Z_{22}\dot{I}_2 - j\omega M\dot{I}_1$$

$$\dot{I}_1 = \frac{\dot{V}_1}{Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

$$R_{11} + jX_{11} = Z_{11}$$

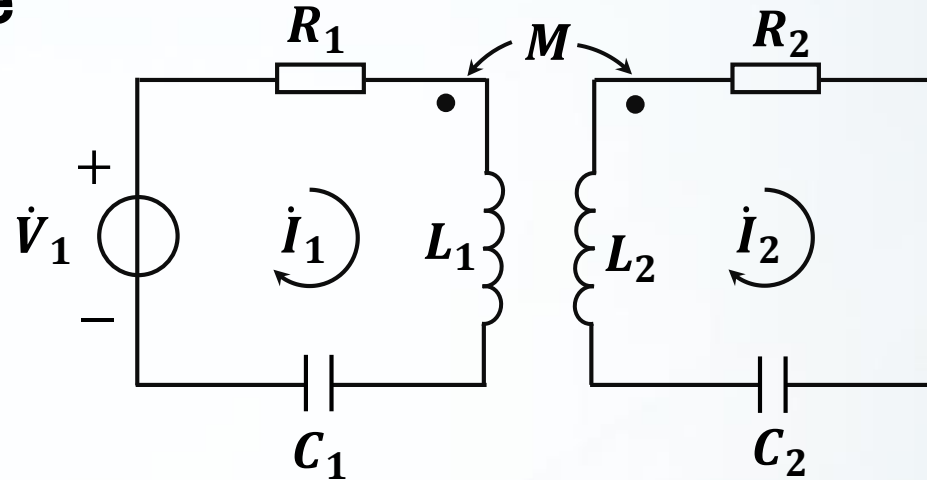
Primary Circuit

$$\dot{I}_2 = \frac{j\omega M \frac{\dot{V}_1}{Z_{11}}}{Z_{22} + \frac{(\omega M)^2}{Z_{11}}}$$

$$Z_{22} = R_{22} + jX_{22}$$

Secondary Circuit

Coupling Resonance



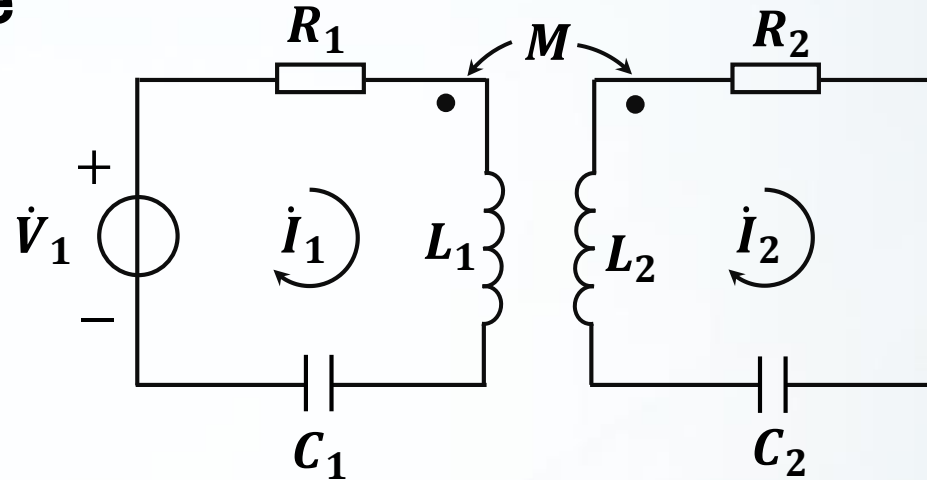
$$\dot{I}_1 = \frac{\dot{V}_1}{Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

$R_{11} + jX_{11} = Z_{11}$
Primary Circuit

$$\dot{I}_2 = \frac{j\omega M \frac{\dot{V}_1}{Z_{11}}}{Z_{22} + \frac{(\omega M)^2}{Z_{11}}}$$

$Z_{22} = R_{22} + jX_{22}$
Secondary Circuit

Coupling Resonance



$$\frac{(\omega M)^2}{Z_{22}} = Z_{f1} = R_{f1} + jX_{f1} \quad \text{Reflected Impedance}$$

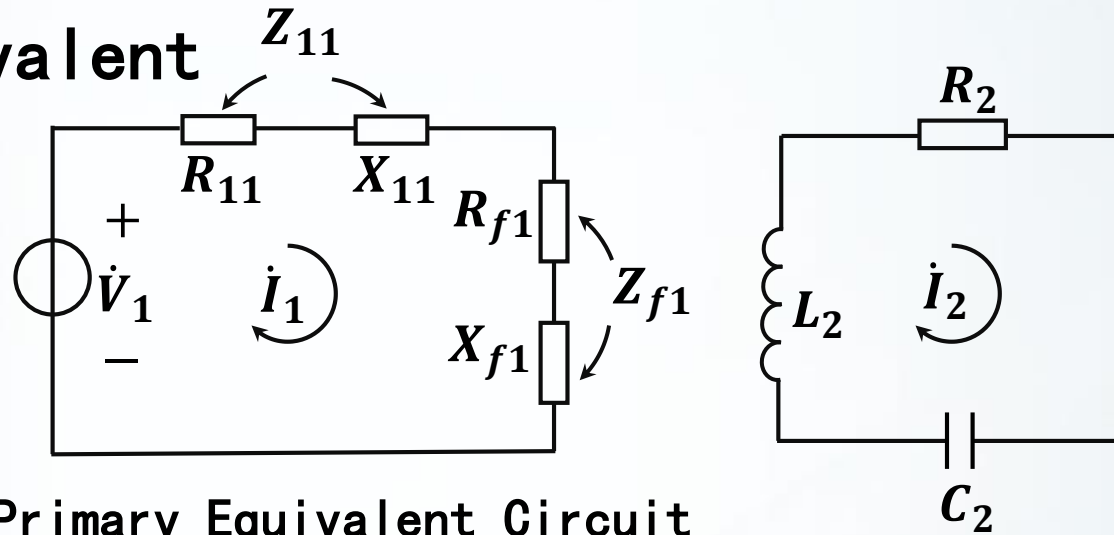
$$\dot{I}_1 = \frac{\dot{V}_1}{Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

$R_{11} + jX_{11} = Z_{11}$
Primary Circuit

$$\dot{I}_2 = \frac{j\omega M \frac{\dot{V}_1}{Z_{11}}}{Z_{22} + \frac{(\omega M)^2}{Z_{11}}}$$

$Z_{22} = R_{22} + jX_{22}$
Secondary Circuit

Primary Equivalent Circuit



(a) Primary Equivalent Circuit

$$\frac{(\omega M)^2}{Z_{22}} = Z_{f1} = R_{f1} + jX_{f1} \quad \text{Reflected Impedance}$$

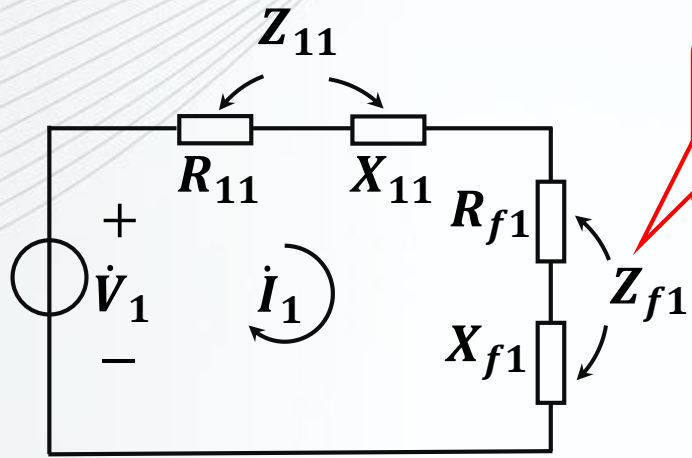
Note : Z_{f1} Not solid impedance, but effect

$$\dot{I}_1 = \frac{\dot{V}_1}{Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

$R_{11} + jX_{11} = Z_{11}$
Primary Circuit

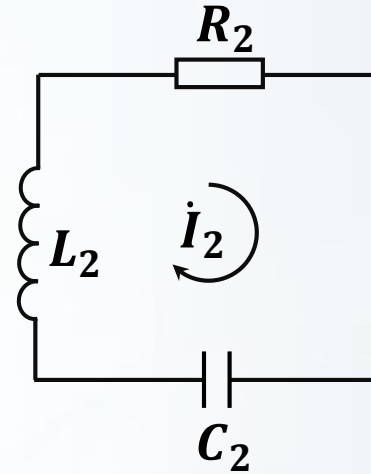
$$\dot{I}_2 = \frac{j\omega M \frac{\dot{V}_1}{Z_{11}}}{Z_{22} + \frac{(\omega M)^2}{Z_{11}}}$$

$Z_{22} = R_{22} + jX_{22}$
Secondary Circuit



(a) Primary Equivalent Circuit

$$Z_{f1} = \frac{(\omega M)^2}{Z_{22}}$$



$$Z_{f1} = \frac{(\omega M)^2}{Z_{22}}$$

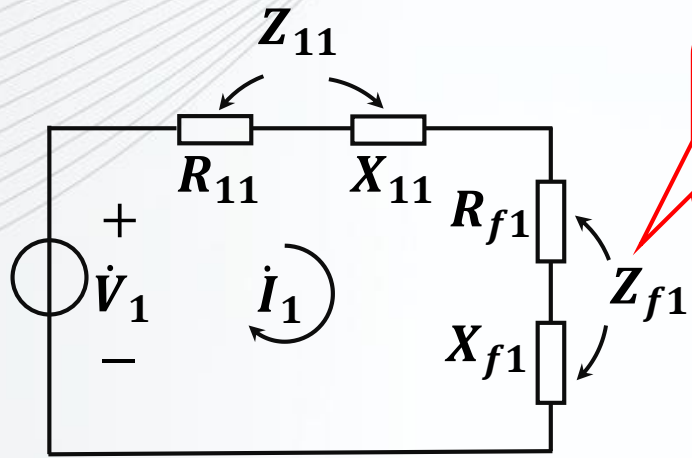
Reflected Impedance

$$\dot{I}_1 = \frac{\dot{V}_1}{Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

$$\dot{I}_2 = \frac{j\omega M \frac{\dot{V}_1}{Z_{11}}}{Z_{22} + \frac{(\omega M)^2}{Z_{11}}}$$

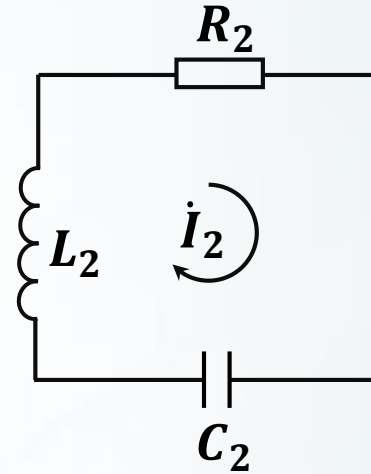
$$Z_{22} = R_{22} + jX_{22}$$

Secondary Circuit



(a) Primary Equivalent Circuit

$$Z_{f1} = \frac{(\omega M)^2}{Z_{22}}$$



$$\begin{cases} Z_{f1} = \frac{(\omega M)^2}{Z_{22}} \\ Z_{f2} = \frac{(\omega M)^2}{Z_{11}} \end{cases}$$

Reflected Impedance

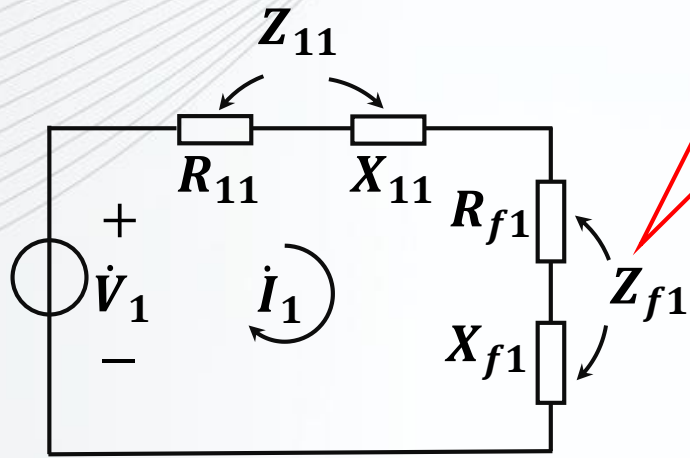
Reflected Impedance

$$\dot{I}_1 = \frac{\dot{V}_1}{Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

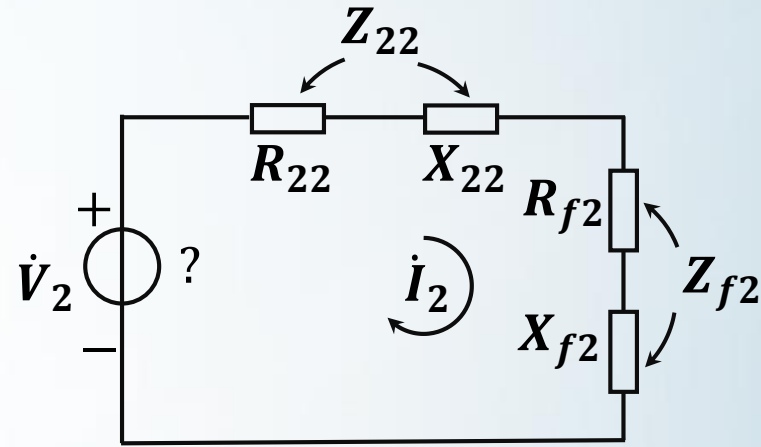
$$\dot{I}_2 = \frac{j\omega M \frac{\dot{V}_1}{Z_{11}}}{Z_{22} + \frac{(\omega M)^2}{Z_{11}}}$$

$$Z_{22} = R_{22} + jX_{22}$$

Secondary Circuit



(a) Primary Equivalent Circuit



(b) Secondary Equivalent Circuit

**Secondary
Equivalent
Circuit**

$$\begin{cases} Z_{f1} = \frac{(\omega M)^2}{Z_{22}} \\ Z_{f2} = \frac{(\omega M)^2}{Z_{11}} \end{cases}$$

Reflected Impedance

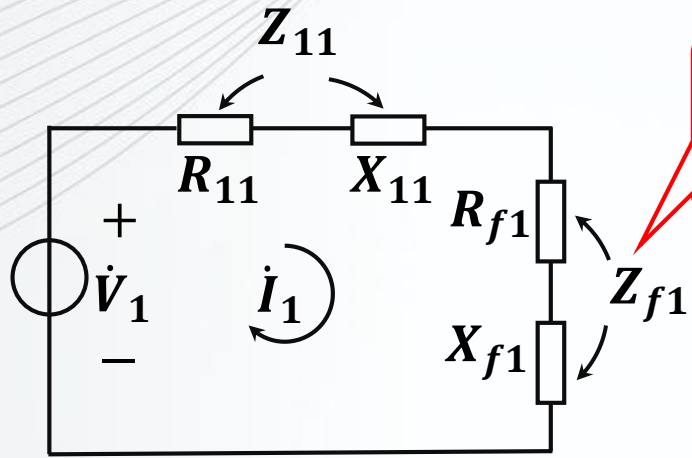
Reflected Impedance

$$\dot{I}_1 = \frac{\dot{V}_1}{Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

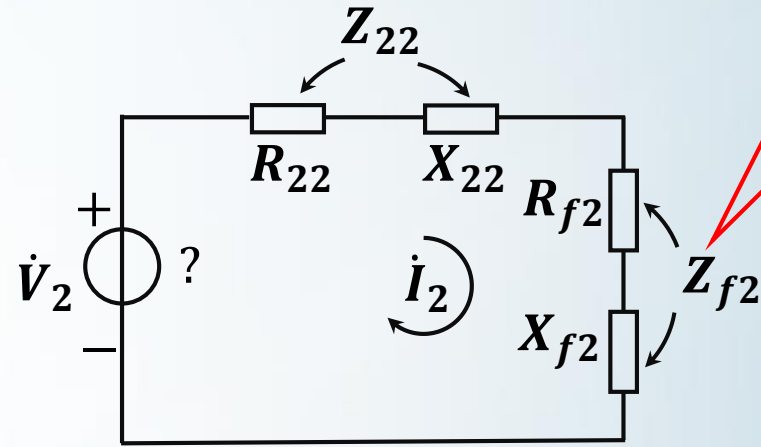
$$\dot{I}_2 = \frac{j\omega M \frac{\dot{V}_1}{Z_{11}}}{Z_{22} + \frac{(\omega M)^2}{Z_{11}}}$$

$$Z_{22} = R_{22} + jX_{22}$$

Secondary Circuit



(a) Primary Equivalent Circuit



(b) Secondary Equivalent Circuit

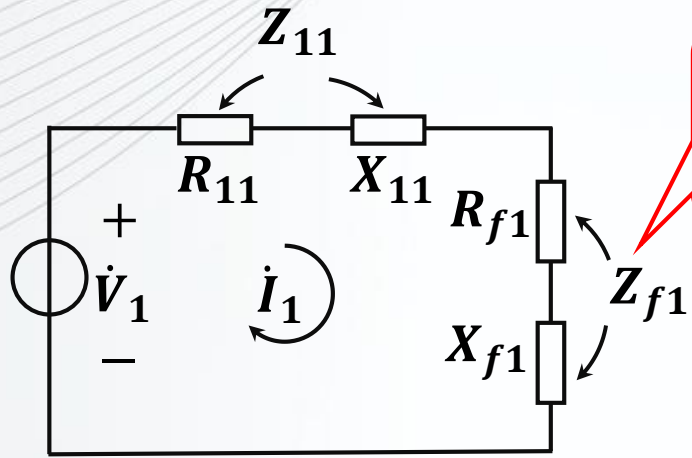
$$\begin{cases} Z_{f1} = \frac{(\omega M)^2}{Z_{22}} \\ Z_{f2} = \frac{(\omega M)^2}{Z_{11}} \end{cases}$$

Reflected Impedance

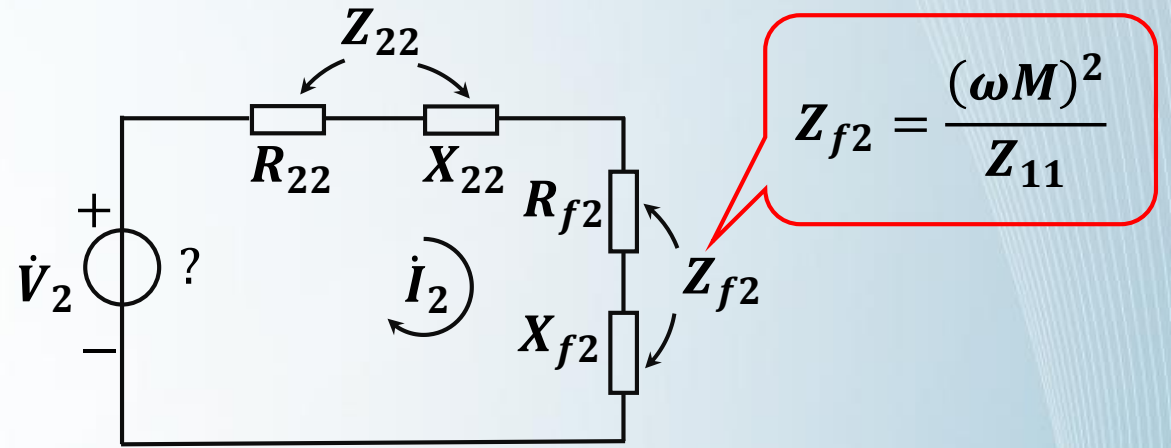
Reflected Impedance

$$\dot{I}_1 = \frac{\dot{V}_1}{Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

$$\dot{I}_2 = \frac{j\omega M \frac{\dot{V}_1}{Z_{11}}}{Z_{22} + \frac{(\omega M)^2}{Z_{11}}}$$



(a) Primary Equivalent Circuit



(b) Secondary Equivalent Circuit

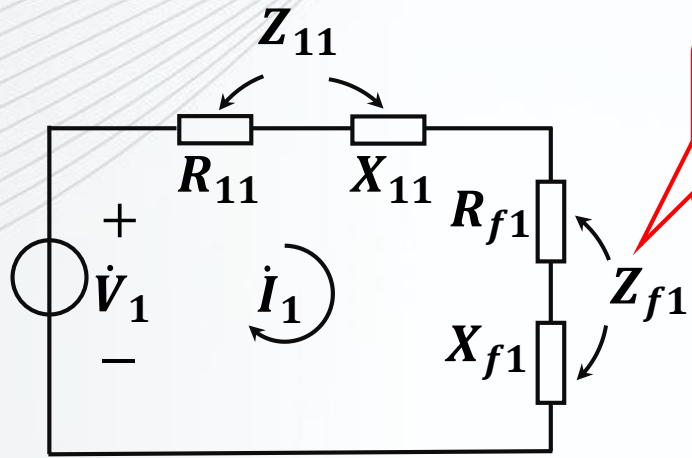
$\frac{\dot{V}_1}{Z_{11}}$ L_2 induced electromotive force

$\dot{V}_2 = j\omega M \frac{\dot{V}_1}{Z_{11}}$

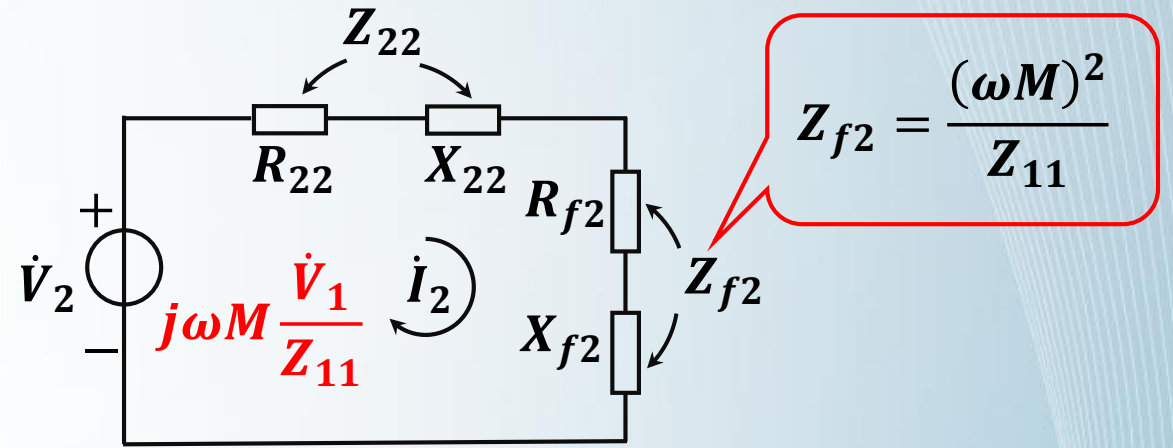
Secondary circuit is OPEN

$$\dot{I}_1 = \frac{\dot{V}_1}{Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

$$\dot{I}_2 = \frac{j\omega M \frac{\dot{V}_1}{Z_{11}}}{Z_{22} + \frac{(\omega M)^2}{Z_{11}}}$$



(a) Primary Equivalent Circuit



(b) Secondary Equivalent Circuit

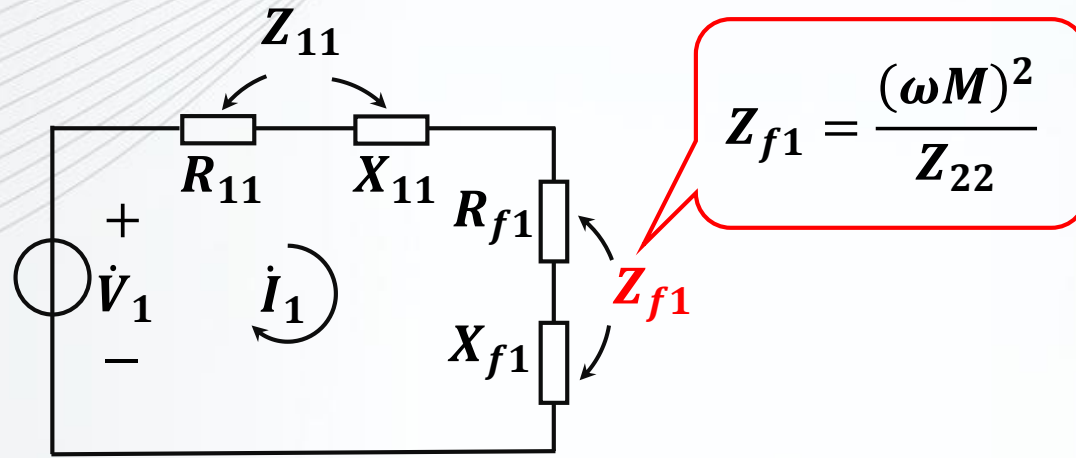
$\frac{\dot{V}_1}{Z_{11}}$ L_2 induced electromotive force

$$\dot{I}_1 = \frac{\dot{V}_1}{Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

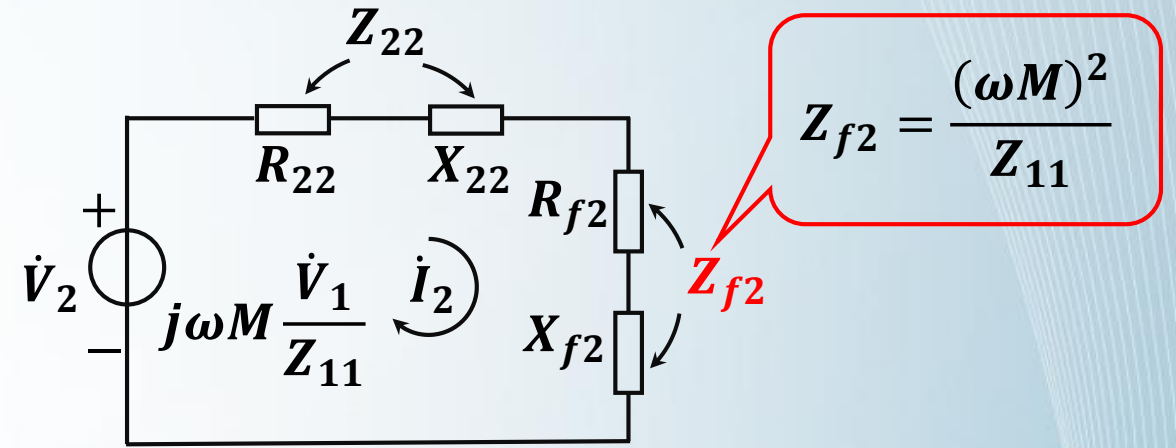
$$\dot{I}_2 = \frac{j\omega M \frac{\dot{V}_1}{Z_{11}}}{Z_{22} + \frac{(\omega M)^2}{Z_{11}}}$$

$$\dot{V}_2 = j\omega M \frac{\dot{V}_1}{Z_{11}}$$

Secondary circuit is OPEN



(a) Primary Equivalent Circuit



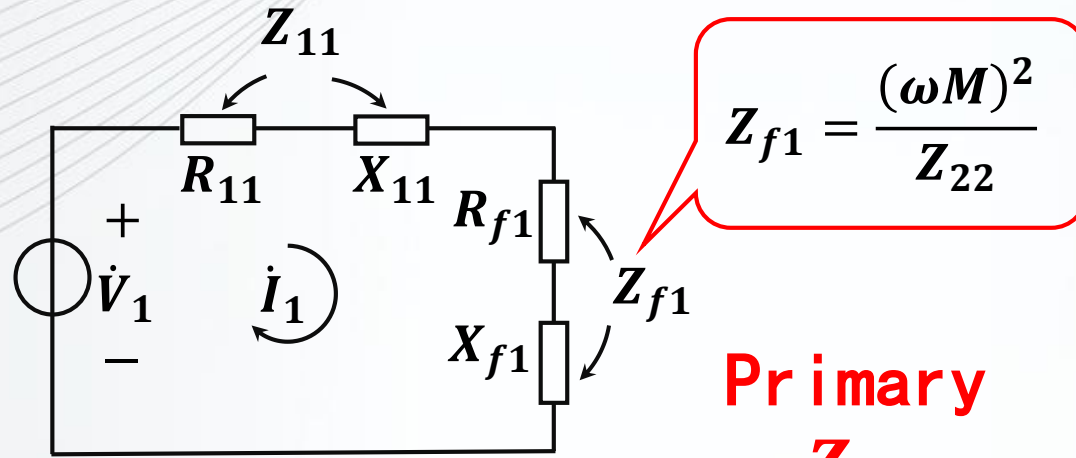
(b) Secondary Equivalent Circuit

Reflected Impedance :

Appear in **Equivalent** Circuits

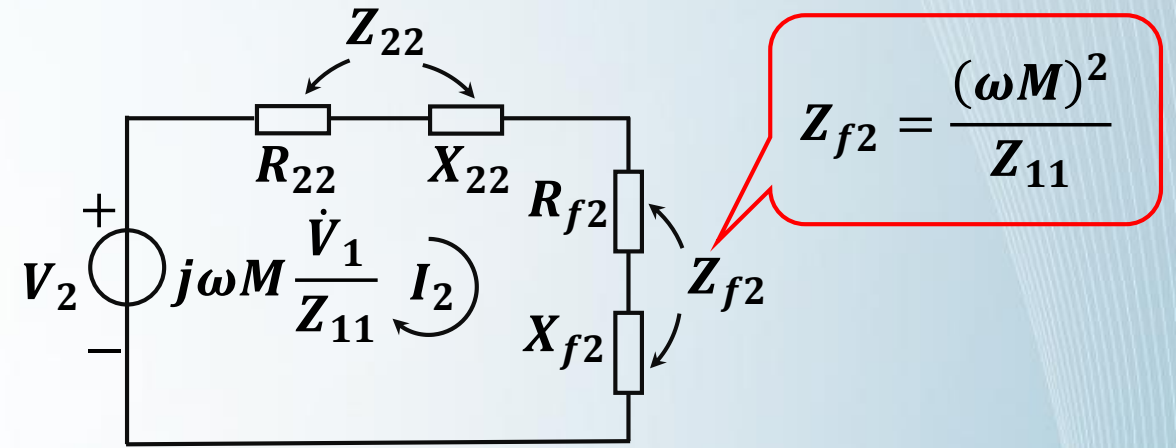
No solid impedance

Denote mutual effect



(a) Primary Equivalent Circuit

Primary
 Z_{f1}



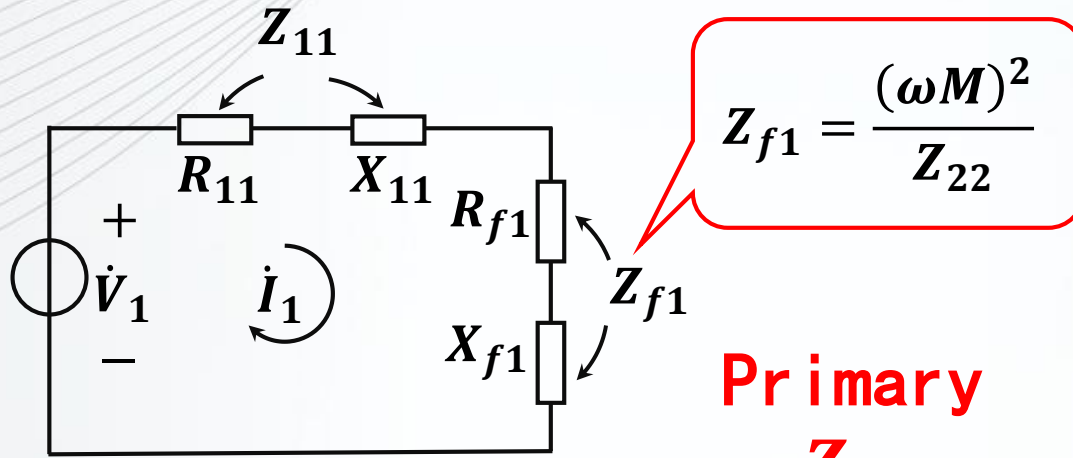
(b) Secondary Equivalent Circuit

$$Z_{f1} = R_{f1} + jX_{f1} = \frac{(\omega M)^2}{R_{22} + jX_{22}}$$

$$= \frac{(\omega M)^2}{R_{22}^2 + X_{22}^2} R_{22} + j \frac{-(\omega M)^2}{R_{22}^2 + X_{22}^2} X_{22}$$

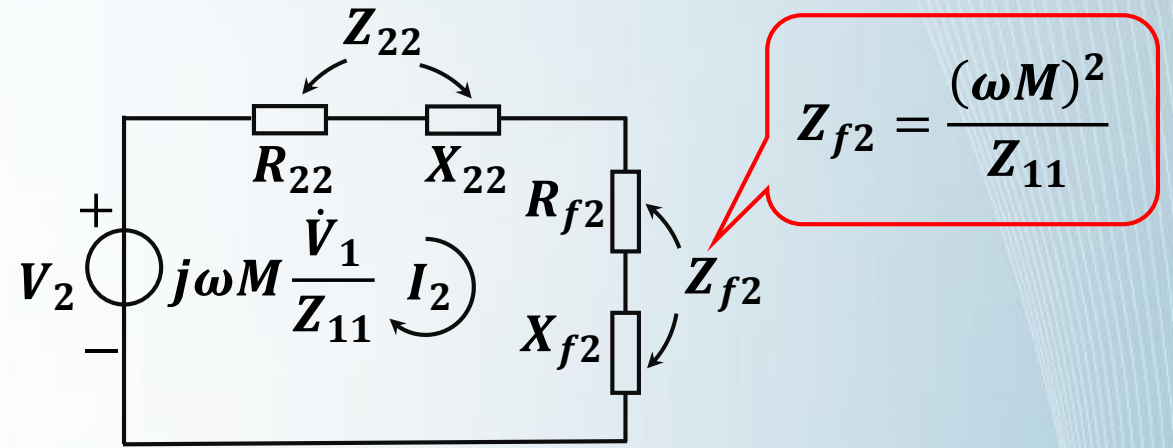
$$\begin{cases} R_{f1} = \frac{(\omega M)^2}{R_{22}^2 + X_{22}^2} R_{22} \\ X_{f1} = \frac{-(\omega M)^2}{R_{22}^2 + X_{22}^2} X_{22} \end{cases}$$

Negative



(a) Primary Equivalent Circuit

Primary
 Z_{f1}



(b) Secondary Equivalent Circuit

$$Z_{f1} = R_{f1} + jX_{f1} = \frac{(\omega M)^2}{R_{22} + jX_{22}}$$

$$= \frac{(\omega M)^2}{R_{22}^2 + X_{22}^2} R_{22} + j \frac{-(\omega M)^2}{R_{22}^2 + X_{22}^2} X_{22}$$

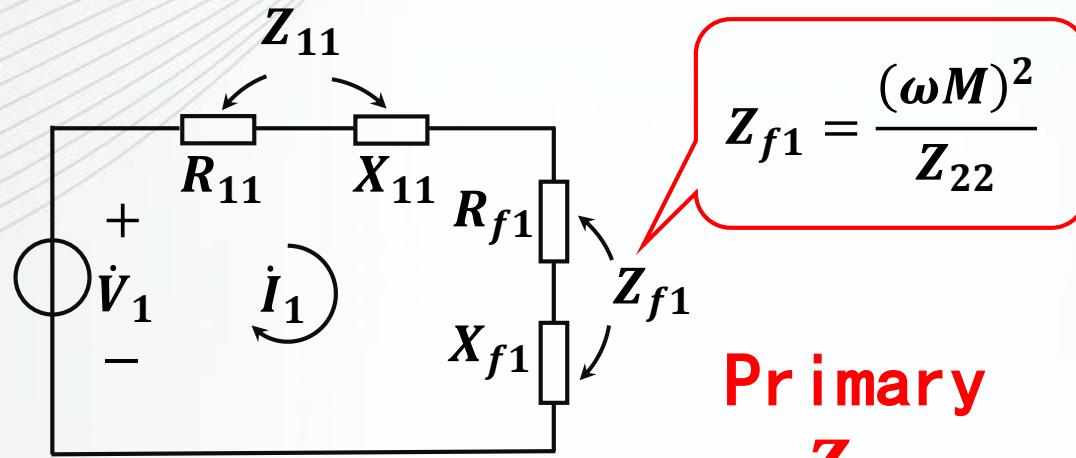
$$\begin{cases} R_{f1} = \frac{(\omega M)^2}{R_{22}^2 + X_{22}^2} R_{22} \\ X_{f1} = \frac{-(\omega M)^2}{R_{22}^2 + X_{22}^2} X_{22} \end{cases}$$

→ Positive (Energy Loss)

→ Opposite

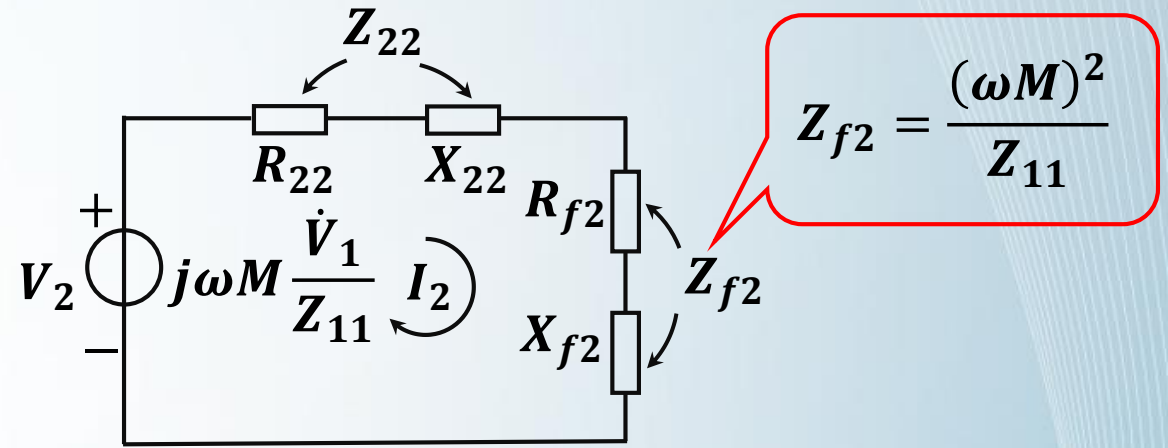
Exp: X_{22} Inductive ($X_{22} > 0$), X_{f1} Capacitive ($X_{f1} < 0$)

Negative



(a) Primary Equivalent Circuit

Primary
 Z_{f1}



(b) Secondary Equivalent Circuit

$$Z_{f1} = R_{f1} + jX_{f1} = \frac{(\omega M)^2}{R_{22} + jX_{22}}$$

$$= \frac{(\omega M)^2}{R_{22}^2 + X_{22}^2} R_{22} + j \frac{-(\omega M)^2}{R_{22}^2 + X_{22}^2} X_{22}$$

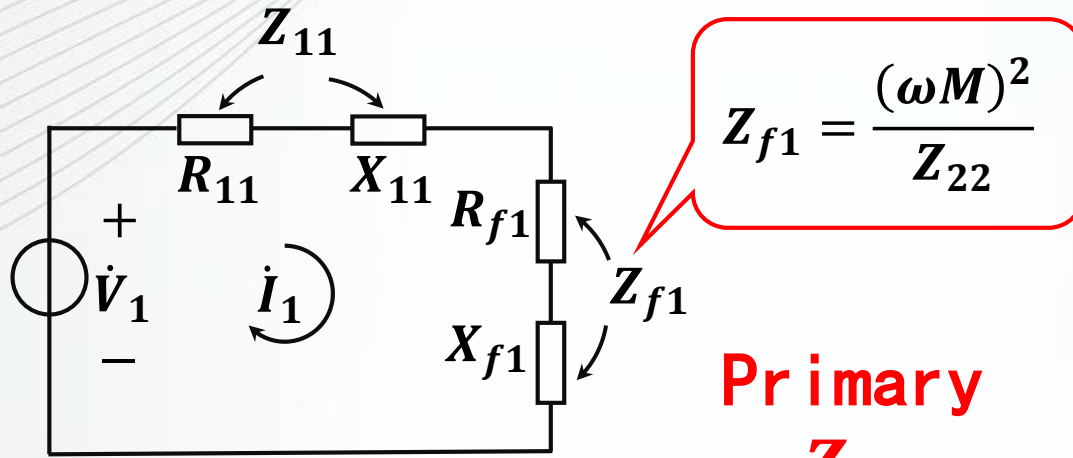
$$\begin{cases} R_{f1} = \frac{(\omega M)^2}{R_{22}^2 + X_{22}^2} R_{22} \\ X_{f1} = \frac{-(\omega M)^2}{R_{22}^2 + X_{22}^2} X_{22} \end{cases}$$

If $X_{11} = X_{22} = 0$

$$\Rightarrow R_{f1} = \frac{(\omega M)^2}{R_{22}}; \quad X_{f1} = 0$$

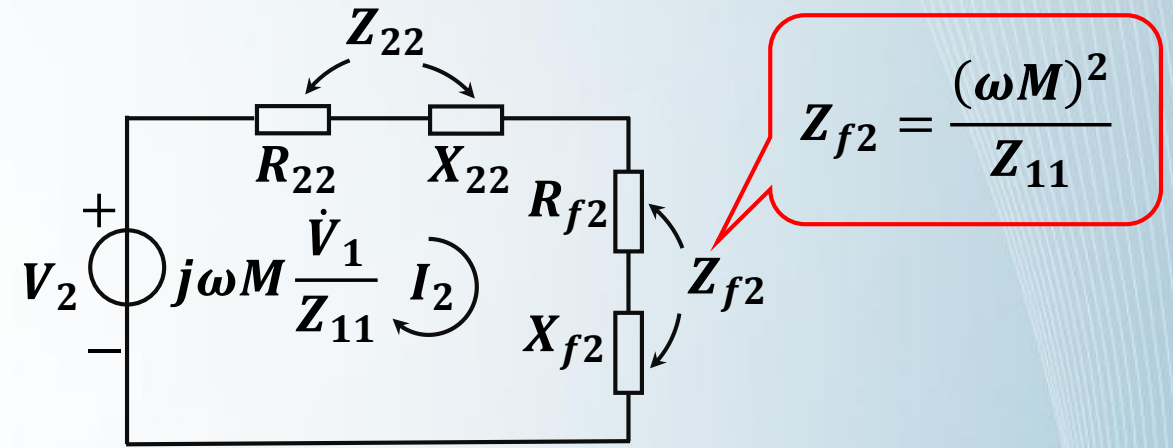
Pure resistance, R_{f1} & R_{22} are reciprocal

Negative



(a) Primary Equivalent Circuit

Primary
 Z_{f1}



(b) Secondary Equivalent Circuit

$$Z_{f1} = R_{f1} + jX_{f1} = \frac{(\omega M)^2}{R_{22} + jX_{22}}$$

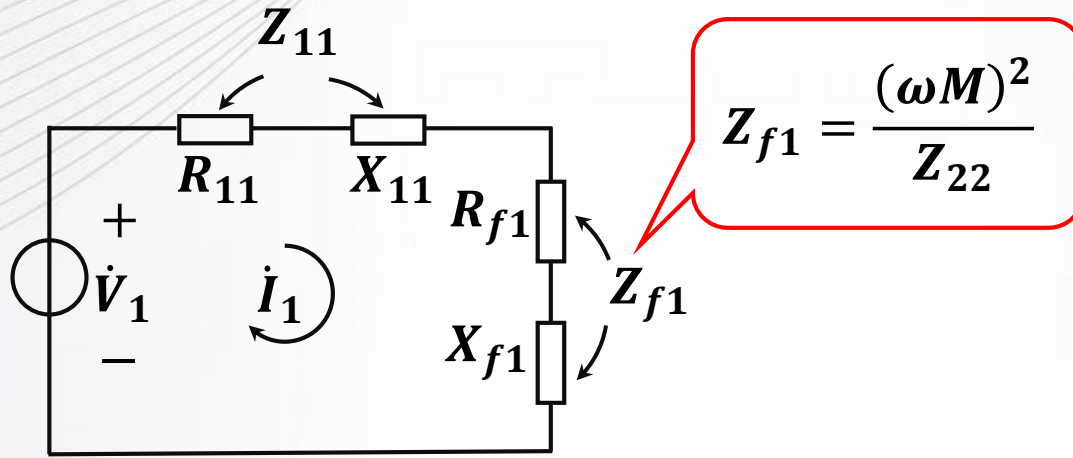
$$= \frac{(\omega M)^2}{R_{22}^2 + X_{22}^2} R_{22} + j \frac{-(\omega M)^2}{R_{22}^2 + X_{22}^2} X_{22}$$

$$\begin{cases} R_{f1} = \frac{(\omega M)^2}{R_{22}^2 + X_{22}^2} R_{22} \\ X_{f1} = \frac{-(\omega M)^2}{R_{22}^2 + X_{22}^2} X_{22} \end{cases}$$

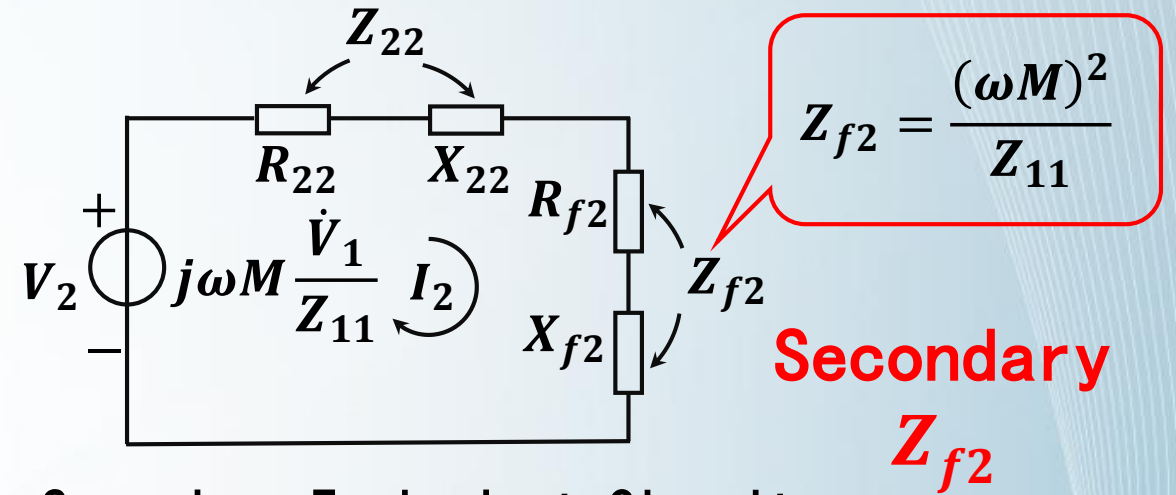
Proportional to $(\omega M)^2$

If $M = 0$, $Z_{f1} = 0$

Negative



(a) Primary Equivalent Circuit



(b) Secondary Equivalent Circuit

$$Z_{f2} = R_{f2} + jX_{f2} = \frac{(\omega M)^2}{R_{11} + jX_{11}}$$

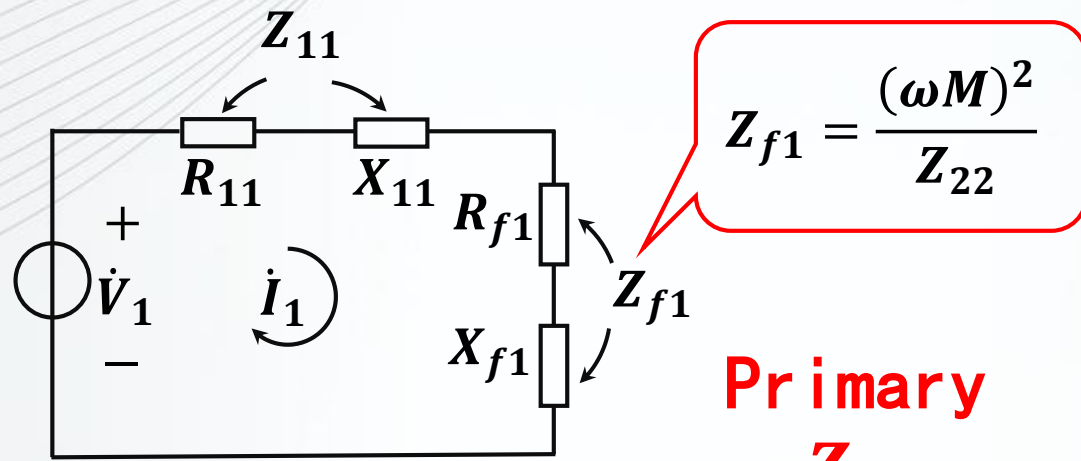
$$= \frac{(\omega M)^2}{R_{11}^2 + X_{11}^2} R_{11} + j \frac{-(\omega M)^2}{R_{11}^2 + X_{11}^2} X_{11}$$

Positive (Energy Loss) ←

Opposite ←

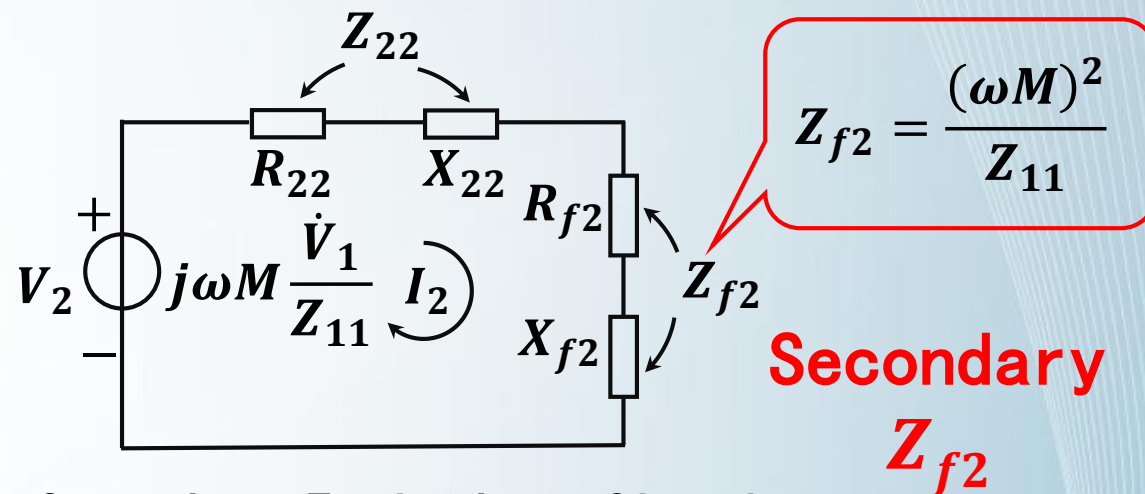
$$\begin{cases} R_{f2} = \frac{(\omega M)^2}{R_{11}^2 + X_{11}^2} R_{11} \\ X_{f2} = \frac{-(\omega M)^2}{R_{11}^2 + X_{11}^2} X_{11} \end{cases}$$

Exp: X_{11} Inductive ($X_{11} > 0$), X_{f2} Capacitive ($X_{f2} < 0$) **Negative**



(a) Primary Equivalent Circuit

Primary
 Z_{f1}



(b) Secondary Equivalent Circuit

Secondary
 Z_{f2}

Applicable to pure reactance coupling system

$$Z_{f2} = R_{f2} + jX_{f2} = \frac{(\omega M)^2}{R_{11} + jX_{11}}$$

$$= \frac{(\omega M)^2}{R_{11}^2 + X_{11}^2} R_{11} + j \frac{-(\omega M)^2}{R_{11}^2 + X_{11}^2} X_{11}$$

$$\begin{cases} R_{f1} = \frac{(\omega M)^2}{R_{22}^2 + X_{22}^2} R_{22} \\ X_{f1} = \frac{-(\omega M)^2}{R_{22}^2 + X_{22}^2} X_{22} \end{cases}$$

Negative

$$\begin{cases} R_{f2} = \frac{(\omega M)^2}{R_{11}^2 + X_{11}^2} R_{11} \\ X_{f2} = \frac{-(\omega M)^2}{R_{11}^2 + X_{11}^2} X_{11} \end{cases}$$

Negative

Coupling Circuit—Resonance

➤ Resonance: Reactance=0

{ Primary Resonance ($X_{11}=0$)

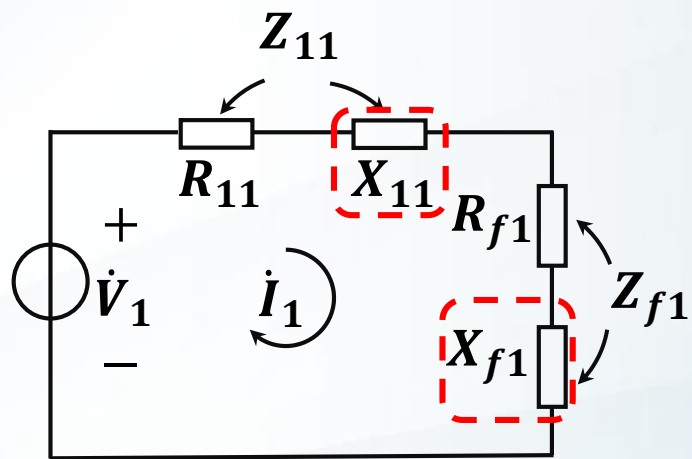
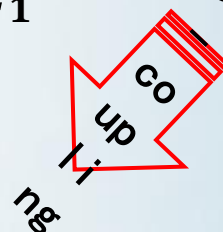
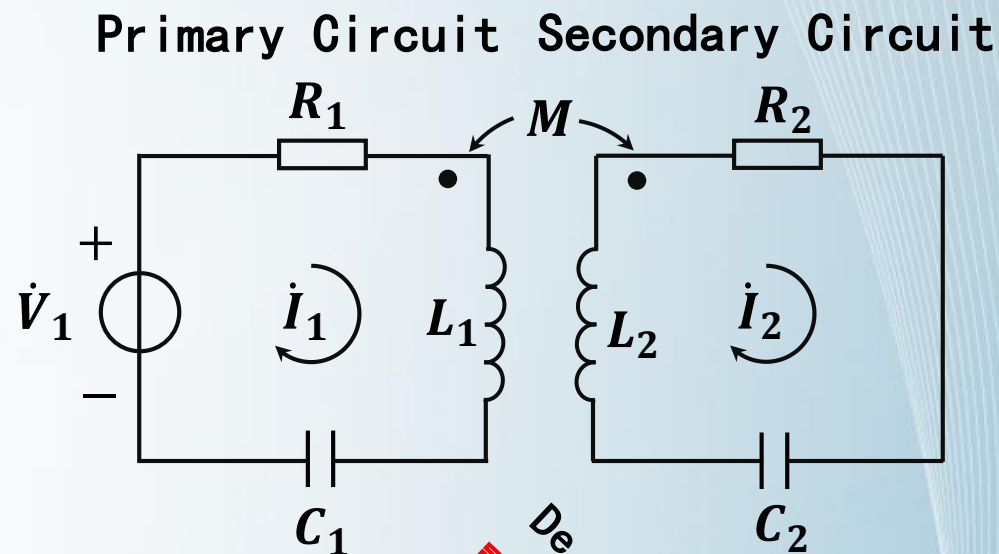
{ Primary **Equivalent** Resonance ($X_{11} + X_{f1} = 0$)

➤ Match: Optimal Power

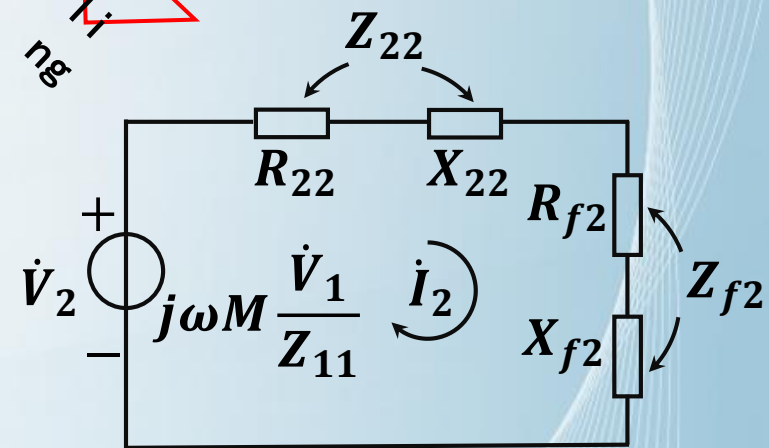
Transmission

$$(R_{11} = R_{f1})$$

$$X_{11} = \omega L_1 - \frac{1}{\omega C_1}$$

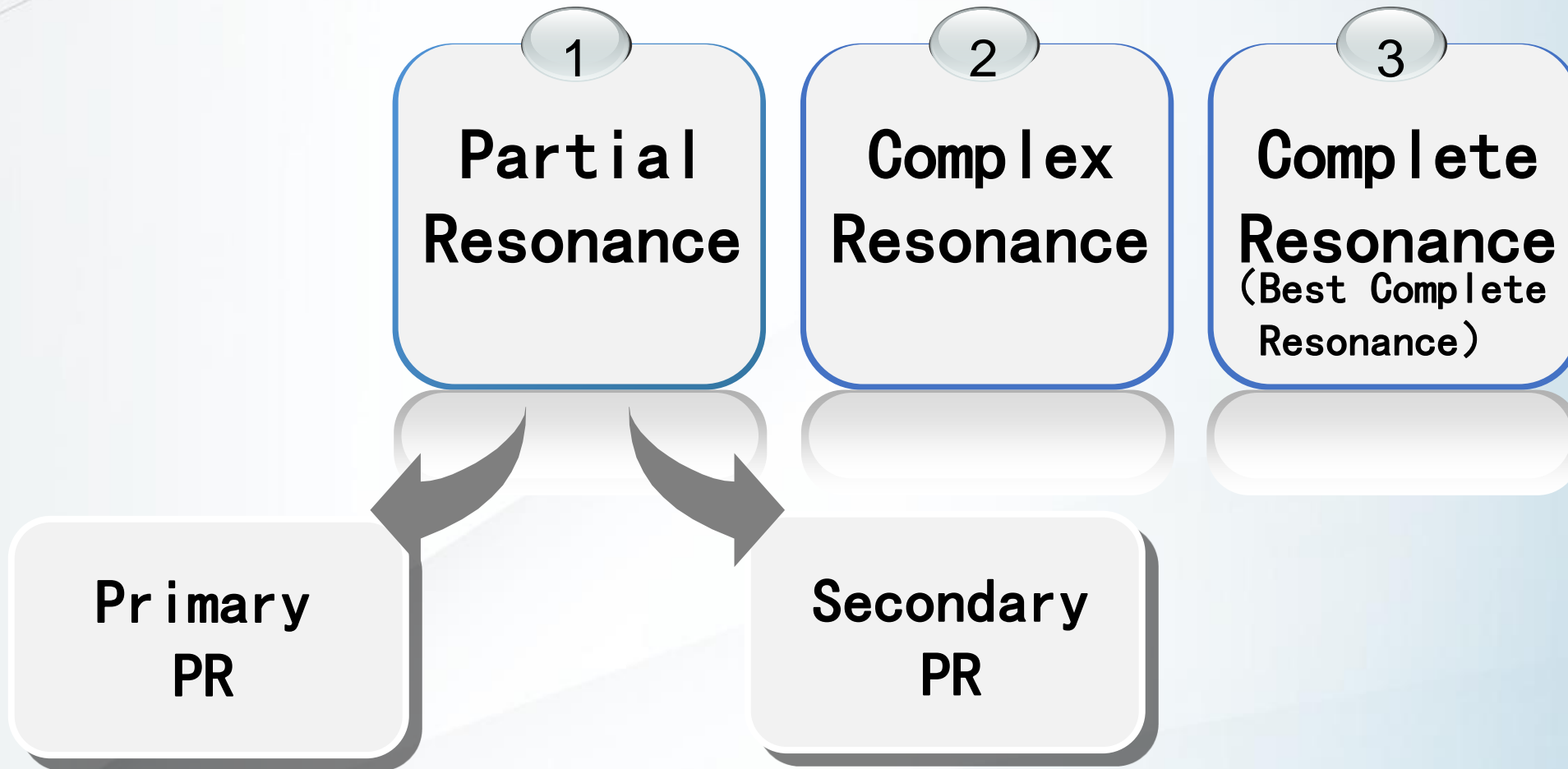


Primary **Equivalent** Circuit

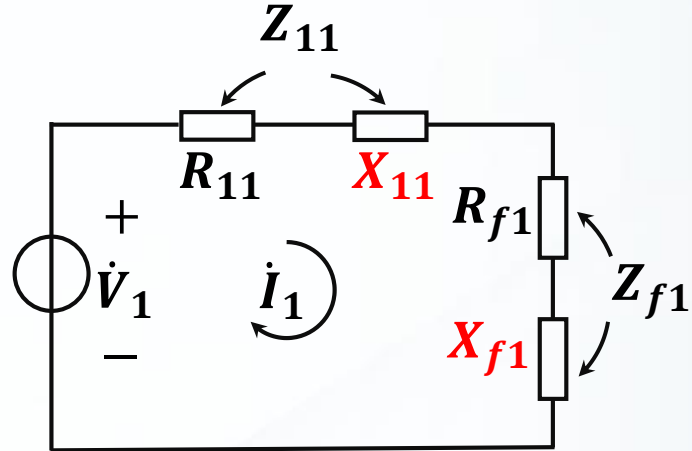
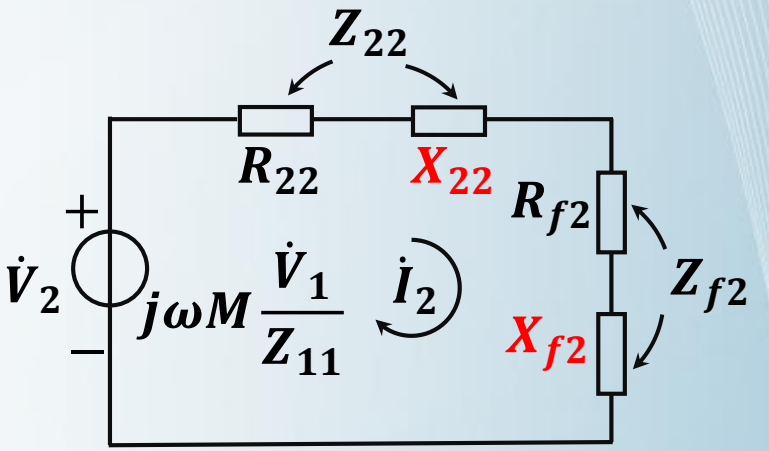


Secondary **Equivalent** Circuit

Coupling Circuit—Resonance (Classification)



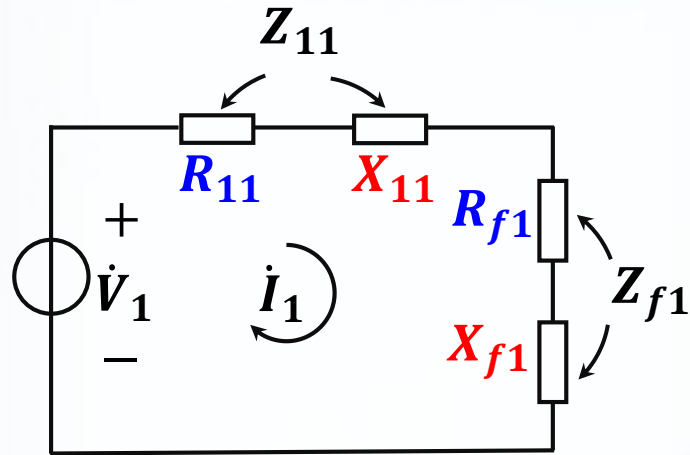
(1) Partial Resonance

	Primary Partial Resonance	Secondary Partial Resonance
Equivalent Circuit	 <p>Primary Equivalent Circuit</p>	 <p>Secondary Equivalent Circuit</p>
Definition	$X_{11} + X_{f1} = 0$	$X_{22} + X_{f2} = 0$
Physical Significance	Primary Equivalent Resonance \neq Primary Resonance	Secondary Equivalent Resonance \neq Secondary Resonance

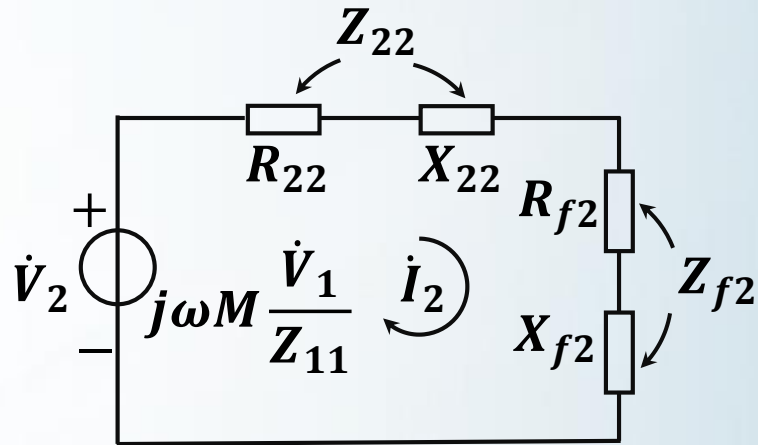
$$(X_{11} = 0)$$

$$(X_{22} = 0)$$

(2) Complex Resonance = Partial Resonance + Matching



(a) Primary **Equivalent** Circuit



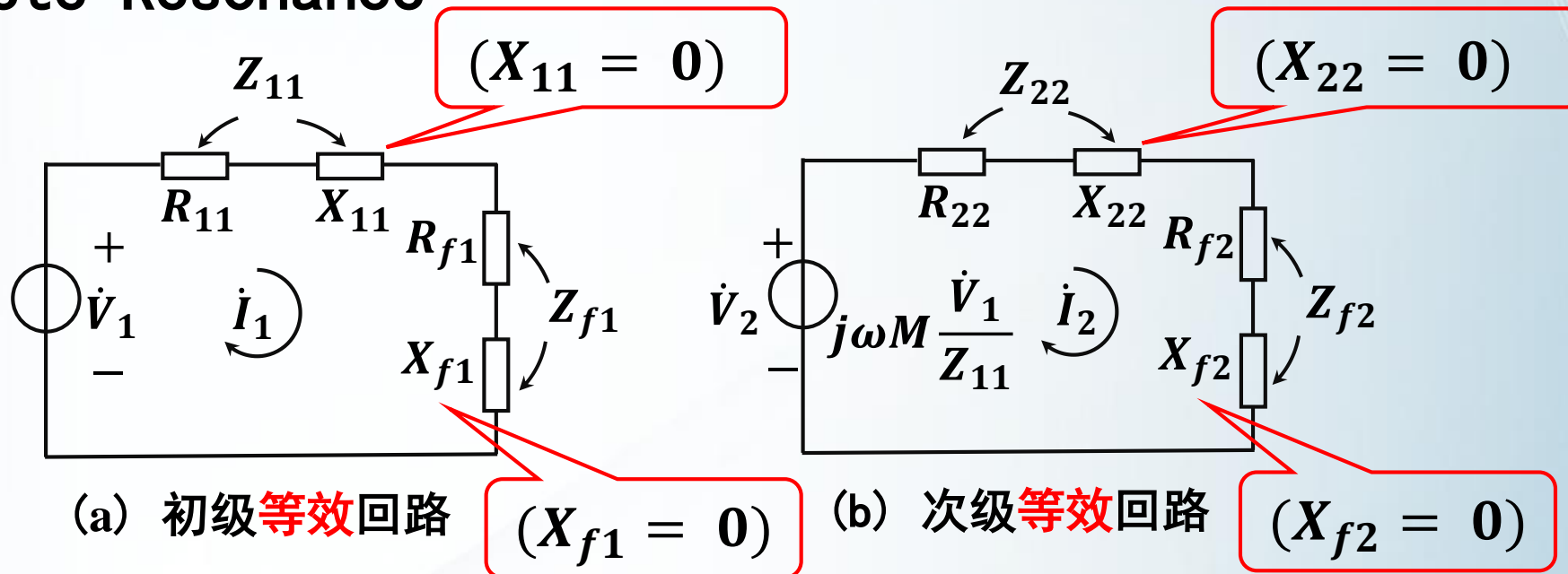
(b) Secondar **Equivalent** Circuit

Definition:

➤ Primary **Equivalent** Resonance ($X_{11} + X_{f1} = 0$) & **Matching** ($R_{11} = R_{f1}$)

≠ Primary Resonance ($X_{11} = 0$)

(3) Complete Resonance



➤ Complete Resonance \Leftrightarrow Primary Resonance $(X_{11} = 0)$ & Secondary Resonance $(X_{22} = 0)$

$$(X_{11} + X_{f1} = 0)$$

$$(X_{22} + X_{f2} = 0)$$

➤ Complete Resonance + Matching $(R_{11} = R_{f1}; R_{22} = R_{f2}) \Leftrightarrow$ Optimal Complete Resonance

Summary—Coupling Circuit (Resonance)

- Partial、Complex、Complete (Optimal Complete) Resonance
- Complex Resonance = Partial Resonance + Matching
- Optimal Complete Resonance = Complete Resonance + Matching (special case of complex resonance)

Q: Relationship

