# CHAPTER 10

## WAVEGUIDES AND CAVITY RESONATORS

### Exercise 10.1

$\ell = 2\,m$, $a = 2\,cm$, $b = 1\,cm$, $TM_{11}$, $\hat{\gamma}_{11} = j200$

$$f_{c_{11}} = \frac{3\times10^8}{2}\sqrt{\frac{1}{0.02^2}+\frac{1}{0.01^2}} = 6.71\times10^9\ Hz$$

$$\beta_{11} = \frac{\omega}{u_p}\sqrt{1-\left(\frac{f_{c11}}{f}\right)^2}$$

$$200 = \frac{2\pi f}{3\times10^8}\sqrt{1-\left(\frac{6.71\times10^9}{f}\right)^2}$$

Solving for "$f$" yields $f = 1.167\times10^{10}$ Hz

### Exercise 10.2

At $z = 0$ $E_{zm} = 2\,kV/m$, $\hat{\gamma}_{11} = j200$, $f = 1.167\times10^{10}$ Hz, $\mu=\mu_0$, $\epsilon=\epsilon_0$

$$M = \frac{\pi}{2\times10^{-2}} = 50\pi, \quad N = \frac{\pi}{10^{-2}} = 100\pi$$

From Eqs. (10.16) and (10.17),

$$\tilde{E}_z = 2000\sin(50\pi x)\sin(100\pi y)\,e^{-j200z}$$

$$E_z(t) = 2000\sin(50\pi x)\sin(100\pi y)\cos(2\pi\times1.167\times10^{10}t - 200z)$$

$$\tilde{E}_x = \frac{-j200}{(50\pi)^2+(100\pi)^2}\,50\pi\times2000\cos(50\pi x)\sin(100\pi y)\,e^{-j200z}$$

$$\tilde{E}_x = -j5.093\times10^2\cos(50\pi x)\sin(100\pi y)\,e^{-j200z}$$

$$E_x(t) = 509.3\cos(50\pi x)\sin(100\pi y)\cos\left(2\pi\times1.167\times10^{10}t - 200z - \frac{\pi}{2}\right)$$

$$\tilde{E}_y = -\frac{j200}{(50\pi)^2+(100\pi)^2}(100\pi)(2000)\sin(50\pi x)\cos(100\pi y)\,e^{-j200z}$$

$$\tilde{E}_y = -j1018.59\sin(50\pi x)\cos(100\pi y)e^{-j200z}$$

$$E_y(t) = 1018.59\sin(50\pi x)\cos(100\pi y)\cos(2\pi\times1.167\times10^{10}t - 200z - \frac{\pi}{2})$$

$$\tilde{H}_x = \frac{j1.167\times10^{10}\times2\pi\times8.85\times10^{-12}}{(50\pi)^2+(100\pi)^2}\,100\pi\times2000\sin(50\pi x)\cos(100\pi y)\,e^{-j200z}$$

$$\tilde{H}_x = j3.30\sin(50\pi x)\cos(100\pi y)\,e^{-j200z}$$

$$H_x(t) = 3.30\sin(50\pi x)\cos(100\pi y)\cos(2\pi\times1.167\times10^{10}t - 200z + \frac{\pi}{2})$$

$$\tilde{H}_y = -\frac{j2\pi\times1.167\times10^{10}\times8.85\times10^{-12}}{(50\pi)^2+(100\pi)^2}\,50\pi\times2000\cos(50\pi x)\sin(100\pi y)\,e^{-j200z}$$

$$\tilde{H}_y = -j1.65\cos(50\pi x)\sin(100\pi y)\,e^{-j200z}$$

$$H_y(t) = 1.65\cos(50\pi x)\sin(100\pi y)\cos(2\pi\times1.167\times10^{10}t - 200z - \frac{\pi}{2})$$

## Exercise 10.3

$a = 1\text{ cm}$, $b = 0.5\text{ cm}$, $TM_{11}$, $f = 9\text{ GHz}$, $E_{zm} = 1.5\text{ kV/m}$ at $z=0$

$\epsilon = 2.5\epsilon_0$ $\qquad u_p = \dfrac{1}{\sqrt{2.5\epsilon_0\mu_0}} = 1.897\times10^{8}\text{ m/s}$

a) $$f_{c_{11}} = \frac{1.897\times10^8}{2}\sqrt{\left(\frac{1}{10^{-2}}\right)^2+\left(\frac{1}{5\times10^{-3}}\right)^2} = 8.48\times10^{9}\text{ Hz}$$

b) $$\beta_{11} = \frac{2\pi\times9\times10^9}{1.897\times10^8}\sqrt{1-\left(\frac{8.48\times10^9}{9\times10^9}\right)^2} = 99.52\text{ rad/m}$$

$$\hat{\gamma}_{11} = j\,99.52$$

c) $u_{p_{11}} = \dfrac{\omega}{\beta_{11}} = \dfrac{2\pi \times 9\times10^9}{99.52} = 5.68\times10^8$ m/s

$$u_{g_{11}} = \frac{(1.897\times10^8)^2}{5.68\times10^8} = 6.333\times10^7 \text{ m/s}$$

d) $\eta = \sqrt{\dfrac{4\pi\times10^{-7}}{2.5\times8.85\times10^{-12}}} = 238.32\ \Omega$

$$\eta_{11} = 238.32\sqrt{1-\left(\frac{8.48\times10^9}{9\times10^9}\right)^2} = 79.83\ \Omega$$

e) From Eq.(10.50),

$$\langle P_{11}\rangle = \frac{99.52^2\times(10^{-2})^3(5\times10^{-3})^3}{8\pi^2\times79.83\left[(10^{-2})^2+(5\times10^{-3})^2\right]}\times1500^2$$

$\langle P_{11}\rangle = 3.53\times10^{-3}$ W or 3.53 mW

Exercise 10.4

$b = 1$ cm, $f = 12$ GHz, $\beta_{10} = 150$ rad/m TE$_{10}$.

$$150 = \frac{2\pi\times12\times10^9}{3\times10^8}\sqrt{1-\left(\frac{f_{c10}}{12}\right)^2} \Rightarrow f_{c10} = 9.63\text{ GHz}$$

$$f_{c10} = \frac{u_p}{2a} \Rightarrow a = \frac{3\times10^8}{2\times9.63\times10^9} = 1.557\times10^{-2}\text{ m or } a = 1.557\text{cm}$$

Exercise 10.5

$a = 1$ cm, $b = 1.5$ cm, $\beta_{01} = 100$ rad/m

$$f_{c01} = \frac{3\times10^8}{2\times0.015} = 10^{10}\text{ Hz}$$

$$100 = \frac{2\pi f}{3\times10^8}\sqrt{1-\left(\frac{10^{10}}{f}\right)^2} \Rightarrow f = 1.108\times10^{10}\text{ Hz}$$

a)

From Eq. (10.62) for $TE_{01}$,

$$\tilde{H}_x = 0 \quad , \tilde{E}_y = 0 , \quad \tilde{E}_z = 0$$

$$\tilde{E}_x = j \frac{\omega \mu b}{\pi} \hat{H}_{zm} \sin\left(\frac{\pi}{b} y\right) e^{-j\beta_{01} z}$$

or
$$\tilde{E}_x = \hat{E}_{xm} \sin\left(\frac{\pi}{b} y\right) e^{-j\beta_{01} z}$$

$$\therefore \quad \hat{E}_{xm} = j \frac{\omega \mu b}{\pi} \hat{H}_{zm}$$

$$500 = j \frac{2\pi \times 1.108 \times 10^{10} \times 4\pi \times 10^{-7} \times 1.5 \times 10^{-2} \ \hat{H}_{zm}}{\pi}$$

$$\hat{H}_{zm} = -j 1.197 \ \text{A/m}$$

$$\tilde{E}_x = 500 \sin\left(\frac{\pi}{1.5 \times 10^{-2}} y\right) e^{-j100z} = 500 \sin(209.43y) e^{-j100z}$$

$$E_x(t) = 500 \sin(209.43y) \cos(2\pi \times 1.108 \times 10^{10} t - 100z)$$

$$\tilde{H}_y = j \frac{100 \times 1.5 \times 10^{-2}}{\pi} (-j1.197) \sin(209.43y) e^{-j100z}$$

$$\tilde{H}_y = 0.5725 \sin(209.43y) e^{-j100z}$$

$$H_y(t) = 0.572 \sin(209.43y) \cos(2\pi \times 1.108 \times 10^{10} t - 100z)$$

$$\tilde{H}_z = -j1.197 \cos(209.43y) e^{-j100z}$$

$$H_z(t) = 1.197 \cos(209.43y) \cos\left(2\pi \times 1.108 \times 10^{10} t - 100z - \frac{\pi}{2}\right)$$

b) $$\eta_{01}^{TE} = \frac{377}{\sqrt{1 - \left(\frac{10^{10}}{1.108 \times 10^{10}}\right)^2}} = 875.45 \ \Omega$$

From Eq. (10.72)

$$\langle P_{01} \rangle = 875.45 \left[ \frac{(100)^2 \times (10^{-2})^3 (1.5\times10^{-2})^3 \times 1.197^2}{8\pi^2 (10^{-2})^2} \right]$$

$$\langle P_{01} \rangle = 5.36 \times 10^{-3}\ W \quad \text{or} \quad 5.36\ mW$$

Exercise 10.6

$a = 2\,cm$ , $b = 1\,cm$ , $TE_{10}$ , $f = 9\,GHz$ , $E = 20\,V/cm$ at $z=0$

a) $$f_{c10} = \frac{3\times10^8}{2\times0.02} = 7.5\times10^9\ Hz \quad \text{or} \quad 7.5\ GHz$$

b) $$\beta_{10} = \frac{2\pi\times9\times10^9}{3\times10^8}\sqrt{1-\left(\frac{7.5}{9}\right)^2} = 104.19\ rad/m$$

$$\hat{\gamma}_{10} = j104.19$$

c) Phase velocity : $$u_{p10} = \frac{3\times10^8}{\sqrt{1-\left(\frac{7.5}{9}\right)^2}} = 5.43\times10^8\ m/s$$

Group velocity : $$u_{g10} = 3\times10^8\sqrt{1-\left(\frac{7.5}{9}\right)^2} = 1.66\times10^8\ m/s$$

d) $$\eta_{10}^{TE} = \frac{377}{\sqrt{1-\left(\frac{7.5}{9}\right)^2}} = 682.02\ \Omega$$

e) $$\langle P_{10} \rangle = 682.02\ \frac{104.19^2\times(2\times10^{-2})^3\times(10^{-2})^3\ 4.42^2}{8\pi^2\times(10^{-2})^2} = 0.147\ W$$

$$H_{zm} = \frac{20\times10^2\pi}{2\pi\times9\times10^9\times4\pi\times10^{-7}\times2\times10^{-2}}$$

$$H_{zm} = 4.42\ A/m$$

Exercise 10.7

$\epsilon_r = 2.5 \quad , \mu_r = 1$

$$u_p = \frac{1}{\sqrt{2.5\epsilon_0 \mu_0}} = 1.897 \times 10^8 \text{ m/s}$$

a) $$f_{c_{10}} = \frac{1.897 \times 10^8}{2 \times 0.02} = 4.74 \times 10^9 \text{ Hz}$$

b) $$\beta_{10} = \frac{2\pi \times 9 \times 10^9}{1.897 \times 10^8} \sqrt{1 - \left(\frac{4.74}{9}\right)^2} = 253.4 \text{ rad/m}$$

$$\hat{\gamma}_{10} = j\,253.4$$

c) $$u_{p_{10}} = \frac{1.897 \times 10^8}{\sqrt{1 - \left(\frac{4.74}{9}\right)^2}} = 2.23 \times 10^8 \text{ m/s}$$

$$u_{g_{10}} = 1.897 \times 10^8 \sqrt{1 - \left(\frac{4.74}{9}\right)^2} = 1.613 \times 10^8 \text{ m/s}$$

d) $$\eta = \sqrt{\frac{4\pi \times 10^{-7}}{2.5 \times 8.85 \times 10^{-12}}} = 238.32\ \Omega$$

$$\eta_{10}^{TE} = \frac{238.32}{\sqrt{1 - \left(\frac{4.74}{9}\right)^2}} = 280.35\ \Omega$$

e) $$\langle P_{10} \rangle = 280.35\ \frac{253.4^2 \times (2 \times 10^{-2})^3 (10^{-2})^3\, 4.42^2}{8\pi^2 (10^{-2})^2} = 0.356\ W$$

## Exercise 10.8

$\epsilon_r = 2.5$, $\tan\delta = 10^{-13}$, $f = 4\text{ GHz}$, $\mu = \mu_0$

$$u_p = \frac{1}{\sqrt{2.5\epsilon_0 \mu_0}} = 1.897\times10^8 \text{ m/s} \qquad \eta = \sqrt{\frac{\mu_0}{2.5\epsilon_0}} = 238.32\,\Omega$$

$$f_{c_{10}} = \frac{1.897\times10^8}{2\times0.03} = 3.16\times10^9 \text{ Hz}$$

Skin depth: $$\delta_c = \frac{1}{\sqrt{5.76\times10^7\times4\pi\times10^{-7}\times4\times10^9\pi}} = 1.05\times10^{-6}\text{ m}$$

Conductivity of polyethylene:

$$\sigma_d = 2\pi\times4\times10^9\times10^{-13}\times2.5\times8.85\times10^{-12} = 5.56\times10^{-14}\text{ s/m}$$

$$\alpha_{c_{10}} = \frac{\left[1+\frac{2\times0.02}{0.03}\left(\frac{3.16}{4}\right)^2\right]}{5.76\times10^7\times1.05\times10^{-6}\times238.32\times0.02\sqrt{1-\left(\frac{3.16}{4}\right)^2}} = 1.037\times10^{-2}\text{ Np/m}$$

$$\alpha_{d_{10}} = \frac{1}{2}\times5.56\times10^{-14}\times238.32\sqrt{1-\left(\frac{3.16}{4}\right)^2} = 4.06\times10^{-12}\text{ Np/m}$$

## Exercise 10.9

$$\delta_c = \frac{1}{\sqrt{\pi\times6.25\times10^9\times3.55\times10^7\times4\pi\times10^{-7}}} = 1.068\times10^{-6}\text{ m}$$

for $\sigma_c = 3.55\times10^7$ s/m $\qquad Q \cong 4577$

Exercise 10.10

$a = 2\,cm$ , $b = 1\,cm$, $c = 4\,cm$ , $TM_{101}$

$$f_{101} = \frac{1}{2\sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{1}{0.02}\right)^2 + \left(\frac{1}{0.04}\right)^2} = 8.385 \times 10^9 \text{ Hz}$$

$$\delta_c = \frac{1}{\sqrt{5.8 \times 10^7 \times 4\pi \times 10^{-7} \times 8.385 \times 10^9 \,\pi}} = 7.217 \times 10^{-7} \text{ m}$$

From Eq. (10.124)

$Q = 7299$

From Eq. (10.123)

$P_d = 4.539 \times 10^{-5} H_{zm}^2$ W

# Problem 10.5

$a = 4$, $b = 3\text{cm}$, $f = 20\text{GHz}$, $E_{zm} = 600 \text{ V/m}$ $\quad TM_{11}$

a) $$f_{c_{11}} = \frac{3\times10^{8}}{2}\sqrt{\left(\frac{1}{0.04}\right)^2 + \left(\frac{1}{0.03}\right)^2} = 6.25\times10^{9} \text{ Hz}$$

$$\beta_{11} = \frac{2\pi\times20\times10^{9}}{3\times10^{8}}\sqrt{1-\left(\frac{6.25}{20}\right)^2} = 397.9 \text{ rad/m}$$

b) $x = 1$ cm, $y = 1.5$ cm, $z = 50$ cm

$$\tilde{E}_z = 600 \sin\left(\frac{\pi}{4}\times1\right)\sin\left(\frac{\pi}{3}\times1.5\right)e^{-397.9\times0.5}$$

$$\tilde{E}_z = \sqrt{2}\times30\, e^{-j198.95} \quad \text{V/m}$$

$$\tilde{E}_y = -\frac{j397.9\pi\times600}{\left[\left(\frac{\pi}{0.04}\right)^2+\left(\frac{\pi}{0.03}\right)^2\right]0.03}\sin\left(\frac{\pi}{4}\times1\right)\cos\left(\frac{\pi}{3}\times1.5\right)e^{-j397.9\times0.5}$$

$$\tilde{E}_y = 0$$

$$\tilde{E}_x = -\frac{j397.9\pi\times600}{\left[\left(\frac{\pi}{0.04}\right)^2+\left(\frac{\pi}{0.03}\right)^2\right]0.04}\cos\left(\frac{\pi}{4}\times1\right)\sin\left(\frac{\pi}{3}\times1.5\right)e^{-j397.9\times0.5}$$

$$\tilde{E}_x = 14.93\, e^{-j(198.95+\frac{\pi}{2})} \quad \text{V/m}$$

$$\tilde{H}_x = j\frac{2\pi\times20\times10^{9}\pi\times600\times\epsilon_0}{\left[\left(\frac{\pi}{0.04}\right)^2+\left(\frac{\pi}{0.03}\right)^2\right]0.03}\sin\left(\frac{\pi}{4}\times1\right)\cancelto{0}{\cos\left(\frac{\pi}{3}\times1.5\right)}e^{-j397.9\times\frac{1}{2}}$$

$$\tilde{H}_x = 0$$

$$\tilde{H}_y = -j \frac{2\pi \times 20\times 10^9 \pi \times 600\, \epsilon_0}{\left[\left(\frac{\pi}{0.04}\right)^2 + \left(\frac{\pi}{0.03}\right)^2\right] 0.04} \cos\left(\frac{\pi}{4}\times 1\right) \sin\left(\frac{\pi}{3}\times 1.5\right) e^{-397.9\times 0.5}$$

$$\tilde{H}_y = 2.16\, e^{-j(198.95 + \frac{\pi}{2})} \quad A/m$$

$$\tilde{H}_z = 0$$

Problem 10.6

$$a = 2\,cm, \quad f = 12\ GHz, \quad TM_{11}$$

a) $$f_{c_{11}} = \frac{3\times 10^8}{2} \sqrt{\left(\frac{1}{0.02}\right)^2 + \left(\frac{1}{0.02}\right)^2} = 10.61\ GHz$$

b) $$\lambda = \frac{3\times 10^8}{12\times 10^9} = 0.25\times 10^{-1}\ m$$

$$\lambda_{c_{11}} = \frac{3\times 10^8}{10.61\times 10^9} = 2.83\times 10^{-2}\ m$$

c)

$$\lambda_{11} = \frac{2.5\times 10^{-2}}{\sqrt{1 - \left(\frac{10.61}{12}\right)^2}} = 0.0535\ m \quad \text{or} \quad 5.35\ cm$$

d) $$u_{p_{11}} = \frac{3\times 10^8}{\sqrt{1 - \left(\frac{10.61}{12}\right)^2}} = 6.42\times 10^8\ m/s$$

e) $$u_{g_{11}} = 3\times 10^8 \sqrt{1 - \left(\frac{10.61}{12}\right)^2} = 1.4\times 10^8\ m/s$$

$$\beta_{11} = \frac{2\pi\times 12\times 10^9}{3\times 10^8} \sqrt{1 - \left(\frac{10.61}{12}\right)^2} = 117.4\ rad/m$$

$$M = N = \frac{\pi}{2\times 10^{-2}} = 50\pi$$

$$\tilde{E}_z = \hat{E}_{zm} \sin(50\pi x)\sin(50\pi y)\, e^{-j117.4z}$$

$$\tilde{E}_x = \frac{-j\,117.4}{2\times(50\pi)^2}\, 50\pi\, \hat{E}_{zm} \cos(50\pi x)\sin(50\pi y)\, e^{-j117.4z}$$

$$\tilde{E}_x = -j\,0.374\, \hat{E}_{zm} \cos(50\pi x)\sin(50\pi y)\, e^{-j117.4z}$$

$$\tilde{H}_x = \frac{j\,2\pi\times 12\times10^9\times 8.85\times10^{-12}}{2\times(50\pi)^2}\, 50\pi\, \hat{E}_{zm} \sin(50\pi x)\cos(50\pi y)\, e^{-j117.4z}$$

$$\tilde{H}_x = j\,2.124\times10^{-3}\, \hat{E}_{zm} \sin(50\pi x)\cos(50\pi y)\, e^{-j117.4z}$$

$$\tilde{H}_y = -\frac{j\,2\pi\times 12\times10^9\times 8.85\times10^{-12}}{2\times(50\pi)^2}\, 50\pi\, \hat{E}_{zm} \cos(50\pi x)\sin(50\pi y)\, e^{-j117.4z}$$

$$\tilde{H}_y = -j\,2.124\times10^{-3}\, \hat{E}_{zm} \quad (50\pi x)\sin(50\pi y)\, e^{-j117.4z}$$

## Problem 10.7

$$f = 3\,\text{GHz}, \quad u_g = 2\times10^8\ \text{m/s}, \quad \epsilon = 2\epsilon_0$$

$$u_p = \frac{1}{\sqrt{2\epsilon_0\mu_0}} = 2.12\times10^8\ \text{m/s}$$

$$2\times10^8 = 2.12\times10^8 \sqrt{1-\left(\frac{f_{c_{10}}}{3\times10^9}\right)^2} \Rightarrow f_{c_{10}} = 9.95\times10^8\ \text{Hz}$$

$$f_{c_{10}} = \frac{2.12\times10^8}{2a} = 9.95\times10^8 \Rightarrow a = 0.1065\ \text{m} \qquad a = 10.65\,\text{cm}$$

## Problem 10.8

$$b = 1\text{ cm} \quad , \quad \tilde{E}_y = -j100 \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_{10} z}$$

$$\tilde{H}_x = j0.1 \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_{10} z}$$

$$f = 10\text{ GHz} \quad , \quad TE_{10}$$

$$\eta_{TE} = -\frac{\tilde{E}_y}{\tilde{H}_x} = -\frac{-j100}{j0.1} = 1000\,\Omega$$

$$1000 = \frac{377}{\sqrt{1 - \left(\frac{f_{c10}}{f}\right)^2}} \Rightarrow f_{c10} = 9.26\text{ GHz}$$

$$f_{c10} = \frac{u_p}{2a} \qquad a = \frac{3\times10^8}{2\times 9.26\times10^9} = 0.0162\text{ m} \qquad a = 1.62\text{ cm}$$

$$\beta_{10} = \frac{2\times\pi\times10^{10}}{3\times10^8}\sqrt{1 - \left(\frac{9.26}{10}\right)^2} = 79.07\text{ rad/m}$$

$$\frac{\beta_{10} a H_{zm}}{\pi} = H_{xm} \Rightarrow H_{zm} = \frac{0.1\,\pi}{79.07\times0.0162} = 0.245 \text{ A/m}$$

$$\tilde{H}_z = 0.245\cos\left(\frac{\pi x}{a}\right) e^{-j\beta_{10} z} \quad , \tilde{H}_y = 0 \quad , \tilde{E}_x = 0 \quad , \quad \tilde{E}_z = 0$$

$$E_y(t) = 100\sin(193.92x)\cos\left(2\pi\, 10^{10} t - 79.07z - \frac{\pi}{2}\right)$$

$$H_x(t) = 0.1\sin(193.92x)\cos\left(2\pi\, 10^{10} t - 79.07z + \frac{\pi}{2}\right)$$

$$H_z(t) = 0.245\cos(193.92x)\cos\left(2\pi\, 10^{10} t - 79.07z\right)$$

$$\langle P_{10}\rangle_{z=0} = \frac{2\pi\times10^{10}\times4\pi\times10^{-7}\, 0.0162^3\times0.01}{4\pi^2}\, 79.07\times0.245^2 = 4.04\times10^{-4}\text{ W}$$

## Problem 10.9

$\beta_{21} = 165 \text{ rad/m} \qquad TM_{21} \qquad f = 1.1\, f_{c21}$

$$165 = \frac{2\pi \times 1.1\, f_{c21}}{3\times 10^8}\sqrt{1-\left(\frac{f_{c21}}{1.1\, f_{c21}}\right)^2} \quad \Rightarrow \quad f_{c21} = 17.17\ \text{GHz}$$

$f = 1.1 \times 17.17 = 18.89$ GHz

$$\lambda_{21} = \frac{2\pi \times 3\times 10^8}{2\pi \times 18.89\times 10^9}\,\frac{1}{\sqrt{1-\left(\frac{17.17}{18.89}\right)^2}} = 6.616\times 10^{-3}\ \text{m}$$

$\lambda_{21} = 6.616$ mm

## Problem 10.10

$E = 500\ \text{V/m}\ ,\quad TE_{10}\ ,\quad f = 10\ \text{GHz}\ ,\quad l = 2\text{m},\quad a = 3\text{cm},\quad b = 2\text{cm}$

$$f_{c_{10}} = \frac{3\times 10^8}{2\times 0.03} = 5\times 10^9\ \text{Hz}$$

$$\eta_{TE_{10}} = \frac{377}{\sqrt{1-\left(\frac{5}{10}\right)^2}} = 435.32\ \Omega$$

$$\langle S_{10}\rangle = \frac{1}{2}\,\frac{500^2 \sin^2\!\left(\frac{\pi}{a}x\right)}{435.32}\,\bar{a}_z = 287.15 \sin^2\!\left(\frac{\pi}{a}x\right) \qquad \text{W/m}^2$$

$$\langle P_{10}\rangle = \int_0^b \int_0^a 287.15 \sin^2\!\left(\frac{\pi}{a}x\right) dx\, dy \;=\; 287.15\, \frac{1}{2}\, a b$$

$$= \frac{1}{2}\times 287.15 \times 0.03 \times 0.02 = 8.614\times 10^{-2}\ \text{W}$$

$\langle P_{10}\rangle = 86.14$ mW

$$\tilde{H}_y = 0 \quad , \tilde{E}_x = 0 \ , \ \tilde{E}_z = 0 \qquad \beta_{10} = \frac{2\pi \times 10^{10}}{3\times10^8}\sqrt{1-\left(\frac{5}{10}\right)^2} = 181.38 \ \text{rad/m}$$

$$\tilde{E}_y = 500 \sin\left(\frac{\pi}{0.03}x\right) e^{-j181.38z}$$

$$\tilde{E}_y = 500 \sin(33.33\pi x)\, e^{-j181.38z}$$

$$E_y(t) = 500\sin(33.33\pi x)\cos(2\pi\times10^{10}t - 181.38z) \quad V/m$$

$$-j\,\frac{2\pi\times10^{10}\times4\pi\times10^{-7}\times0.03}{\pi}\,\hat{H}_{zm} = 500$$

$$\hat{H}_{zm} = \frac{500\pi}{-j\,2\pi\times10^{10}\times4\pi\times10^{-7}\times0.03} = j\,0.663 \quad A/m$$

$$\tilde{H}_z = j\,0.663\cos(33.33\pi x)\, e^{-j181.38z}$$

$$H_z(t) = 0.663\cos(33.33\pi x)\cos\left(2\pi\times10^{10}t - 181.38z + \frac{\pi}{2}\right) \quad A/m$$

$$\tilde{H}_x = j\,\frac{181.38\times0.03}{\pi}\,(j\,0.663)\sin(33.33\pi x)\, e^{-j181.38z}$$

$$\tilde{H}_x = -1.15\sin(33.33\pi x)\, e^{-j181.38z}$$

$$H_x(t) = 1.15\sin(33.33\pi x)\cos(2\pi\times10^{10}t - 181.38z + \pi) \quad A/m$$

<u>Problem 10.11</u>

$$a = 2\,cm, \quad b = 1\,cm \qquad f_{c10} = \frac{3\times10^8}{2\times2\times10^{-2}} = 7.5\times10^9\,Hz$$

$$f_{c11} = \frac{3\times10^8}{2}\sqrt{\frac{1}{0.02^2}+\frac{1}{0.01^2}} = 16.77\times10^9\,Hz$$

Frequency range for $TE_{10}$ : $7.5\,GHz < f < 16.77\,GHz$

Problem 10.12

$a = 2\text{cm}, \quad b = 3\text{ cm} \quad \epsilon_r = 3 \quad f = 50\text{ GHz} \quad TE_{22}$

$$u_p = \frac{1}{\sqrt{3\epsilon_0 \mu_0}} = \frac{3\times10^8}{\sqrt{3}} = \sqrt{3}\times10^8 \text{ m/s}$$

$$f_{c22} = \frac{\sqrt{3}\times10^8}{2}\sqrt{\left(\frac{2}{0.02}\right)^2 + \left(\frac{2}{0.03}\right)^2} = 10.4 \text{ GHz}$$

$$\lambda_{22} = \frac{2\pi\sqrt{3}\times10^8}{2\pi\times50\times10^9}\frac{1}{\sqrt{1-\left(\frac{10.4}{50}\right)^2}} = 3.54\times10^{-3}\text{ m} \quad \text{or} \quad 3.54\text{ mm}$$

$$\beta_{22} = \frac{2\pi\times50\times10^9}{\sqrt{3}\times10^8}\sqrt{1-\left(\frac{10.4}{50}\right)^2} = 1774.13 \text{ rad/m}$$

$$u_{p22} = \frac{\sqrt{3}\times10^8}{\sqrt{1-\left(\frac{10.4}{50}\right)^2}} = 1.77\times10^8 \text{ m/s}$$

$$u_{g22} = \sqrt{3}\times10^8\sqrt{1-\left(\frac{10.4}{50}\right)^2} = 1.694\times10^8 \text{ m/s}$$

$$\eta^{TE}_{22} = \frac{\sqrt{\frac{4\pi\times10^{-7}}{3\times8.85\times10^{-12}}}}{\sqrt{1-\left(\frac{10.4}{50}\right)^2}} = 222.42\ \Omega$$

$$M = \frac{2\pi}{0.02} = 100\pi \qquad N = \frac{2\pi}{0.03} = 66.66\pi$$

$$\tilde{H}_x = \frac{j1774.13}{(100\pi)^2+(66.66\pi)^2}\times100\pi\,\hat{H}_{zm}\sin(100\pi x)\cos(66.66\pi y)\,e^{-j1774.13z}$$

$$\tilde{H}_x = j3.91\,\hat{H}_{zm}\sin(100\pi x)\cos(66.66\pi y)\,e^{-j1774.13z}$$

$$H_x(t) = 3.91\,H_{zm}\sin(100\pi x)\cos(66.66\pi y)\cos\left(2\pi\times50\times10^9 t - 1774.13z + \theta_H + \frac{\pi}{2}\right)$$

$$\tilde{H}_y = \frac{j1774.13}{(100\pi)^2 + (66.66\pi)^2} \times 66.66\pi\, \hat{H}_{zm} \cos(100\pi x) \sin(66.66\pi y)\, e^{-j1774.13z}$$

$$\tilde{H}_y = j2.61\, \hat{H}_{zm} \cos(100\pi x) \sin(66.66\pi y)\, e^{-j1774.13z}$$

$$H_y(t) = 2.61\, H_{zm} \cos(100\pi x) \sin(66.66\pi y) \cos(2\pi \times 5\times10^{10} t - 1774.13z + \theta_H + \tfrac{\pi}{2})$$

$$\tilde{E}_x = \frac{j2\pi \times 5\times10^{10} \times 4\pi \times 10^{-7}}{(100\pi)^2 + (66.66\pi)^2}\, 66.66\pi\, \hat{H}_{zm} \cos(100\pi x) \sin(66.66\pi y)\, e^{-j1774.13z}$$

$$\tilde{E}_x = j579.96\, \hat{H}_{zm} \cos(100\pi x) \sin(66.66\pi y)\, e^{-j1774.13z}$$

$$E_x(t) = 579.96\, H_{zm} \cos(100\pi x) \sin(66.66\pi y) \cos(2\pi \times 5\times10^{10} t - 1774.13z + \theta_H + \tfrac{\pi}{2})$$

$$\tilde{E}_y = -\frac{j2\pi \times 5\times10^{10} \times 4\pi \times 10^{-7}}{(100\pi)^2 + (66.66\pi)^2}(100\pi)\, \hat{H}_{zm} \sin(100\pi x) \cos(66.66\pi y)\, e^{-j1774.13z}$$

$$\tilde{E}_y = -j870.03\, \hat{H}_{zm} \sin(100\pi x) \cos(66.66\pi y)\, e^{-j1774.13z}$$

$$E_y(t) = 870.03\, H_{zm} \sin(100\pi x) \cos(66.66\pi y) \cos(2\pi \times 5\times10^{10} t - 1774.13z + \theta_H - \tfrac{\pi}{2})$$

$$\tilde{E}_z = 0$$

## Problem 10.13

$a = 2\,\text{cm}$, $b = 1\,\text{cm}$, $TE_{10}$, $f = 15\,\text{GHz}$, $\langle P_{10} \rangle = 1\,\text{kW}$

$$\langle P_{10} \rangle = \frac{1}{2} \frac{E^2}{\eta_{10}}\, a\, b \quad , \quad f_{c_{10}} = \frac{3\times10^8}{2\times0.02} = 0.75\times10^{10}\ \text{Hz}$$

$$f_{c_{10}} = 7.5\ \text{GHz}$$

$$\eta_{10} = \frac{377}{\sqrt{1 - \left(\frac{7.5}{15}\right)^2}} = 435.32\ \Omega$$

$$E^2 = \eta_{10}\ 2\times1000\ \frac{1}{2\times10^{-2}\times10^{-2}} \Rightarrow E = \sqrt{10^7 \eta_{10}}$$

$$E = \sqrt{10^7 \times 435.32} = 65.978\ kV/m$$

$$H = \frac{65.978}{435.32} = 151.56\ A/m \qquad H_{zm} = \frac{151.56 \times \pi}{272.07 \times 0.02} = 87.5\ A/m$$

$$\tilde{H}_y = 0\ ,\ \tilde{E}_x = 0\ ,\ \tilde{E}_z = 0$$

$$\beta_{10} = \frac{2\pi \times 15 \times 10^9}{3 \times 10^8} \sqrt{1 - \left(\frac{7.5}{15}\right)^2} = 272.07\ rad/m$$

$$\tilde{H}_x = j\,151.56 \angle\theta_H \sin\left(\frac{\pi}{0.02} x\right) e^{-j272.07z}$$

$$\tilde{H}_x = j\ 151.56 \sin(50\pi x)\ e^{-j272.07z}\ e^{j\theta_H}$$

$$H_x(t) = 151.56 \sin(50\pi x) \cos(2\pi \times 15 \times 10^9 t - 272.07z + \tfrac{\pi}{2} + \theta_H)$$

$$\tilde{H}_z = 87.5 \cos(50\pi x)\ e^{-j272.07z} \cdot e^{j\theta_H}$$

$$H_z(t) = 87.5 \cos(50\pi x) \cos(2\pi \times 15 \times 10^9 t - 272.07z + \theta_H)$$

$$\tilde{E}_y = -j\,65{,}978 \sin(50\pi x)\ e^{-j272.07z} \cdot e^{j\theta_E}$$

$$E_y(t) = 65{,}978 \sin(50\pi x) \cos(2\pi \times 15 \times 10^9 t - 272.07z + \theta_E)$$

## Problem 10.14

$a = 3\,cm\ ,\ b = 1\,cm\ ,\ l = 1\,m\ ,\ f = 12\,GHz,\ TE_{10}$

$\tan\delta = 10^{-4}\ ,\ \sigma = 5.8 \times 10^7\ S/m$

$$f_{c10} = \frac{3 \times 10^8}{2 \times 0.03} = 5 \times 10^9\ Hz$$

$$\delta_c = \frac{1}{\sqrt{5.8\times10^7 \times 4\pi\times10^{-7}\times 12\times10^9\pi}} = 6.03\times10^{-7}\ m$$

$$\sigma_d = \omega\epsilon \tan\delta = 2\pi\times12\times10^9\times 8.85\times10^{-12}\times10^{-4} = 6.67\times10^{-5}\ S/m$$

$$\alpha_{c_{10}} = \frac{1 + \frac{2\times0.01}{0.03}\left(\frac{5}{12}\right)^2}{5.8\times10^7\times6.03\times10^{-7}\times377\times0.01\sqrt{1-\left(\frac{5}{12}\right)^2}} = 9.308\times10^{-3}\ Np/m$$

$$\alpha_{d_{10}} = \frac{6.67\times10^{-5}}{2}\times377\sqrt{1-\left(\frac{5}{12}\right)^2} = 1.143\times10^{-2}\ Np/m$$

## Problem 10.15

$$E_m = 800\ V/m \quad , \quad f = 12\ GHz$$

$$\langle P_d \rangle = \frac{H_{zm}^2}{\sigma_c\,\delta_c}\left[b + \frac{a}{2}\left(\frac{f}{f_{c_{10}}}\right)^2\right] e^{-2\alpha_{10} z}$$

$$\alpha_{10} = \alpha_{c_{10}} + \alpha_{d_{10}} = 9.308\times10^{-3} + 1.143\times10^{-2} = 2.074\times10^{-2}\ Np/m$$

$$E_{y_m} = \frac{a\omega\mu H_{zm}}{\pi} \Rightarrow H_{zm} = \frac{800\pi}{2\pi\times12\times10^9\times4\pi\times10^{-7}\times0.03} = 0.884\ A/m$$

$$\langle P_d \rangle = \frac{0.884^2}{5.8\times10^7\times6.03\times10^{-7}}\left[0.01 + \frac{0.03}{2}\left(\frac{12}{5}\right)^2\right] e^{-2\times2.074\times10^{-2}\times z}$$

$$P_{dT} = \int_0^l P_d\, e^{-2\alpha_{10} z}\, dz = \frac{2.15\times10^{-3}}{2\times2.074\times10^{-2}}\left(1 - e^{-2\times2.074\times10^{-2}\times1}\right)$$

$$P_{dT} = 2.1\times10^{-3}\ W \quad \text{or} \quad 2.1\ mW$$

At $x=0$ wall :

$$\tilde{\vec{J_s}} = -H_{zm}\, e^{-j\beta_{10}z}\, e^{-\alpha_{c_{10}}z}\, \vec{a}_y$$

$$\beta_{10} = \frac{2\pi \times 12\times10^9}{3\times10^8}\sqrt{1-\left(\frac{5}{12}\right)^2} = 228.47 \text{ rad/m}$$

$$\tilde{\vec{J_s}} = -0.884 \times e^{-j228.47z}\, e^{-9.308\times10^{-3}z}\, \vec{a}_y$$

$$\vec{J_s} = -0.884$$

At $y=0$ wall :

$$\tilde{\vec{J_s}} = -j\,\frac{228.47\times0.03}{\pi}\,0.884\,\sin\left(\frac{\pi}{0.03}x\right)e^{-j228.47z}\cdot e^{-9.308\times10^{-3}z}\,\vec{a}_z$$

$$+\ 0.884\cos\left(\frac{\pi}{0.03}x\right)e^{-j228.47z}\, e^{-9.308\times10^{-3}z}\,\vec{a}_x$$

$$\tilde{\vec{J_s}} = -j\,1.93\sin(33.33\pi x)\,e^{-j228.47z}\cdot e^{-9.308\times10^{-3}z}\,\vec{a}_z$$

$$+\ 0.884\cos(33.33\pi x)\,e^{-j228.47z}\cdot e^{-9.308\times10^{-3}z}\,\vec{a}_x$$

At $x=0$ wall : $\tilde{\rho}_s = 0$

At $y=0$ wall : $\tilde{\rho}_s = -j\,\dfrac{\omega\mu a}{\pi}\,\epsilon_0\, H_{zm}\sin(33.33\pi x)\,e^{-j228.47z}\,e^{-\alpha_{c_{10}}z}$

$$\tilde{\rho}_s = -j\,\frac{2\pi\times12\times10^9\times4\pi\times10^{-7}\times0.03}{\pi}\times 8.85\times10^{-12}\times0.884\,\sin(33.33\pi x)\,e^{-j228.47z}\cdot e^{-9.308\times10^{-3}z}$$

$$\tilde{\rho}_s = -j\,7.08\times10^{-9}\sin(33.33\pi x)\,e^{-j228.47z}\cdot e^{-9.308\times10^{-3}z} \quad \text{C/m}^2$$

Problem 10.16

$l = 10m$ , $a = 4\,cm$ , $b = 3\,cm$ , $TE_{10}$ , $f = 4\,GHz$ , $E = 1000\,V/m$

$\tan\delta = 10^{-4}$ , $\sigma = 5.8\times 10^{7}\ S/m$

$$f_{c_{10}} = \frac{3\times 10^{8}}{2\times 0.04} = 3.75\times 10^{9}\ Hz \qquad f_{c_{10}} = 3.75\ GHz$$

$$\delta_c = \frac{1}{\sqrt{5.8\times 10^{7}\times 4\pi\times 10^{-7}\times 4\times 10^{9}\pi}} = 1.045\times 10^{-6}\ m$$

$$\alpha_{c_{10}} = \frac{1}{5.8\times 10^{7}\times 1.045\times 10^{-6}\times 377\times 0.03}\ \frac{\left[1+\frac{2\times 0.03}{0.04}\left(\frac{3.75}{4}\right)^{2}\right]}{\sqrt{1-\left(\frac{3.75}{4}\right)^{2}}}$$

$$\alpha_{c_{10}} = 9.718\times 10^{-3}\ Np/m$$

$$\sigma_d = \omega\epsilon\tan\delta = 2\pi\times 4\times 10^{9}\times 8.85\times 10^{-12}\times 10^{-4}$$

$$\sigma_d = 2.224\times 10^{-5}\ S/m$$

$$\alpha_{d_{10}} = \frac{2.224\times 10^{-5}}{2}\times 377\sqrt{1-\left(\frac{3.75}{4}\right)^{2}} = 1.459\times 10^{-3}\ Np/m$$

$$\alpha_{10} = \alpha_{c_{10}} + \alpha_{d_{10}} = 9.718\times 10^{-3} + 1.459\times 10^{-3} = 11.18\times 10^{-3}\ Np/m$$

$$H_{zm} = \frac{1000\,\pi}{2\pi\times 4\times 10^{9}\times 4\pi\times 10^{-7}\times 0.04} = 2.49\ A/m$$

$$\beta_{10} = \frac{2\pi\times 4\times 10^{9}}{3\times 10^{8}}\sqrt{1-\left(\frac{3.75}{4}\right)^{2}} = 29.15\ rad/m$$

$$\langle P_{10}\rangle_z = \frac{2\pi\times 4\times 10^{9}\times 4\pi\times 10^{-7}\times 0.04^{3}\times 0.03}{4\pi^{2}}\ 29.15\times 2.49^{2}\ e^{-2\times 11.18\times 10^{-3}\times 10}$$

$$\langle P_{10}\rangle_z = 0.222\ W$$

## Problem 10.17

$E = 800\ V/m \quad , \quad TE_{10} \quad , \quad f = 4\ GHz$

$$\langle P_d \rangle = P_d\, e^{-2\alpha_{10} z}$$

$$P_{dT} = \int_0^{\ell} P_d\, e^{-2\alpha_{10} z}\, dz = P_d \frac{1}{2\alpha_{10}} \left(1 - e^{-2\alpha_{10}\ell}\right)$$

$$H_{zm} = \frac{800\pi}{2\pi \times 4\times10^{9} \times 4\pi\times10^{-7} \times 0.04} = 1.99\ A/m$$

$$P_d = \frac{1.99^2}{5.8\times10^{7} \times 1.045\times10^{-6}} \left[ 0.03 + \frac{0.04}{2} \left(\frac{4}{3.75}\right)^2 \right] = 3.44\times10^{-3}\ W/m$$

$$P_{dT} = 3.44\times10^{-3} \frac{1}{2\times11.18\times10^{-3}} \left(1 - e^{-2\times11.18\times10^{-3}\times10}\right)$$

$$P_{dT} = 3.09\times10^{-2}\ W$$

## Problem 10.18

$a = 5\ cm, \quad b = 2\ cm, \quad \ell = 7\ cm \quad , TM_{110}$

$$f_{110} = \frac{1}{2\sqrt{4\pi\times10^{-7} \times 8.85\times10^{-12}}} \sqrt{\left(\frac{1}{0.05}\right)^2 + \left(\frac{1}{0.02}\right)^2}$$

$$f_{110} = 8.07\times10^{9}\ Hz \quad \text{or} \quad 8.07\ GHz.$$

## Problem 10.19

$f = 9\,\text{GHz}$ , $TE_{101}$

$$f_{101} = \frac{1}{2\sqrt{4\pi\times10^{-7}\times 8.85\times10^{-12}}}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{a}\right)^2} = 9\times10^9$$

$a = 2.36\times10^{-2}\,m$ or $2.36\,cm$

## Problem 10.20

$a = 3\,cm$, $b = 1\,cm$, $l = 5\,cm$, $\sigma = 5.8\times10^7\,S/m$, $TE_{101}$

a) $$f_{101} = \frac{1}{2\sqrt{4\pi\times10^{-7}\times8.85\times10^{-12}}}\sqrt{\frac{1}{0.03^2}+\frac{1}{0.05^2}} = 5.83\times10^9\,Hz$$

b) $$\delta_c = \frac{1}{\sqrt{5.8\times10^7\times4\pi\times10^{-7}\times5.83\times10^9\pi}} = 8.66\times10^{-7}\,m$$

$$Q = \frac{4\pi(5.83\times10^9)^3(0.05)^3(0.03)^3(4\pi\times10^{-7})^2(8.85\times10^{-12})(0.01)(5.8\times10^{-7})\delta_c}{2(0.03)^3(0.01)+(0.03)^3(0.05)+(0.03)(0.05)^3+2\times0.01\times0.05^3}$$

$Q = 7243$

c) $$W = 2\times8.85\times10^{-12}\times0.01\times0.05\times0.03^3\times(5.83\times10^9)^2(4\pi\times10^{-7})^2\times2^2$$

$W = 5.13\times10^{-11}\,J$

## Problem 10.21

$C = 1pF \qquad Q = \frac{\omega L}{R} \qquad \omega = \frac{1}{\sqrt{LC}}$

$$L = \frac{1}{(2\pi \times 5.83\times10^{9})^2 \times 10^{-12}} = 7.45\times10^{-10}\,H \qquad L = 0.745\,nH$$

$$7243 = \frac{2\pi \times 5.83\times10^{9} \times 7.45\times10^{-10}}{R} \Rightarrow R = 3.77\times10^{-3}\,\Omega \text{ or } 3.77\,m\Omega$$

It is almost impractical to realize such a lumped circuit.

## Problem 10.22

$f = 10\,GHz$ , $TE_{101}$

$$10\times10^{9} = \frac{3\times10^{8}}{2}\sqrt{2\,\frac{1}{a^2}} \Rightarrow a = 2.12\times10^{-2}\,m$$

$a = 2.12$ cm

## Problem 10.23

$a = 2$ cm , $b = 3$ cm, $l = 5$ cm , $TE_{101}$

$\sigma_{cu} = 5.8\times10^{7}$ S/m $\qquad \sigma_{Al} = 3.5\times10^{7}$ S/m

$$f_{101} = \frac{3\times10^{8}}{2}\sqrt{\left(\frac{1}{0.02}\right)^2 + \left(\frac{1}{0.05}\right)^2} = 8.08\times10^{9}\,Hz$$

For Copper:

$$\delta_c = \frac{1}{\sqrt{5.8\times10^{7} \times 4\pi\times10^{-7} \times 8.08\times10^{9}\pi}} = 7.35\times10^{-7}\,m$$

$$Q = \frac{4\pi(8.08\times10^{9})^{3}(0.05)^{3}(0.02)^{3}(4\pi\times10^{-7})^{2}(8.85\times10^{-12})(0.03)(5.8\times10^{7})(7.35\times10^{-7})}{2(0.02)^{3}(0.03)+(0.02)^{3}(0.05)+(0.02)(0.05)^{3}+2\times0.03\times0.05^{3}}$$

$$Q = 10890$$

$$W = 2\times8.85\times10^{-12}\times0.03\times0.05\times0.02^{3}\times(8.08\times10^{9})^{2}(4\pi\times10^{-7})^{2}\times H_{zm}^{2}$$

$$W = 2.19\times10^{-11}\ H_{zm}^{2}$$

For aluminum:

$$\delta_c = \frac{1}{\sqrt{3.5\times10^{7}\times4\pi\times10^{-7}\times8.08\times10^{9}\pi}} = 9.46\times10^{-7}\ m$$

$$Q = \frac{4\pi(8.08\times10^{9})^{3}(0.05)^{3}(0.02)^{3}(4\pi\times10^{-7})^{2}(8.85\times10^{-12})(0.03)(\quad\times10^{7})(9.46\times10^{-7})}{2(0.02)^{3}(0.03)+(0.02)^{3}(0.05)+(0.02)(0.05)^{3}+2\times0.03\times0.05^{3}}$$

$$Q = 8462 \qquad W = 2.19\times10^{-11}\ H_{zm}^{2}$$

## Problem 10.24

$f_{101} = 20\ GHz \qquad \epsilon_r = 2.5$

$$20\times10^{9} = \frac{1}{2\sqrt{2.5\times8.85\times10^{-12}\times4\pi\times10^{-7}}}\sqrt{2\ \frac{1}{a^{2}}}$$

$$a = 6.7\times10^{-3}\ m \qquad a = 6.7\ mm$$