Huazhong University
of Science & Technology

# Electronic Circuit of Communications

School of Electronic Information
and Commnications

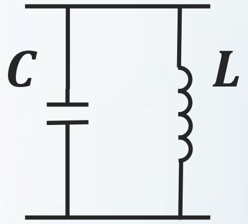
Jiaqing Huang

# Parallel Resonance

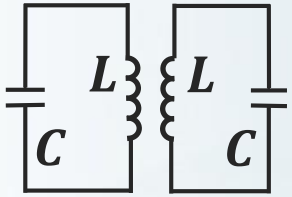
# Frequency Selective Circuits

- Resonance
  - Series Resonance


  - Parallel Resonance

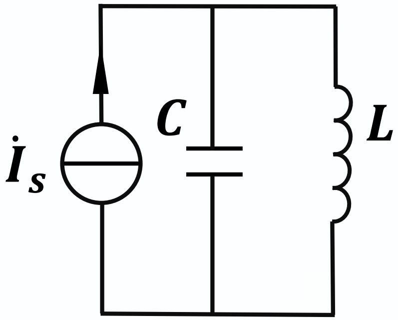


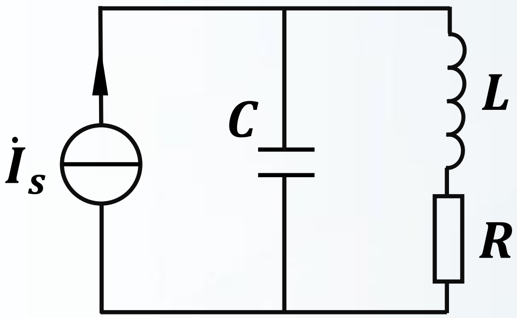

  - Coupling Resonance



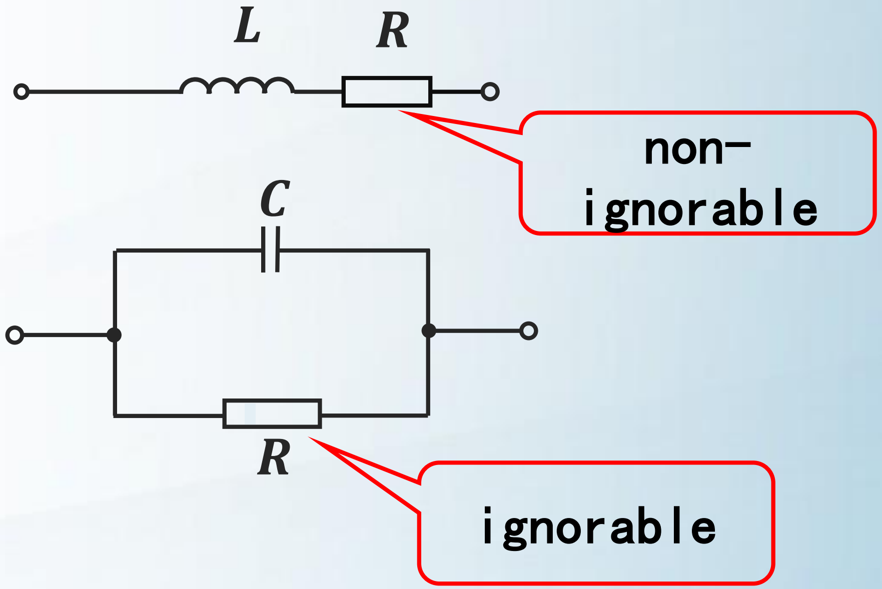

- Filter
  - Quartz Crystal Filter
  - Ceramic filter
  - SAWF

# Parallel Resonant Circuit




- Inductor $= L$ with loss resistance $R$
- Capacitor $= C$ with loss resistance $R$



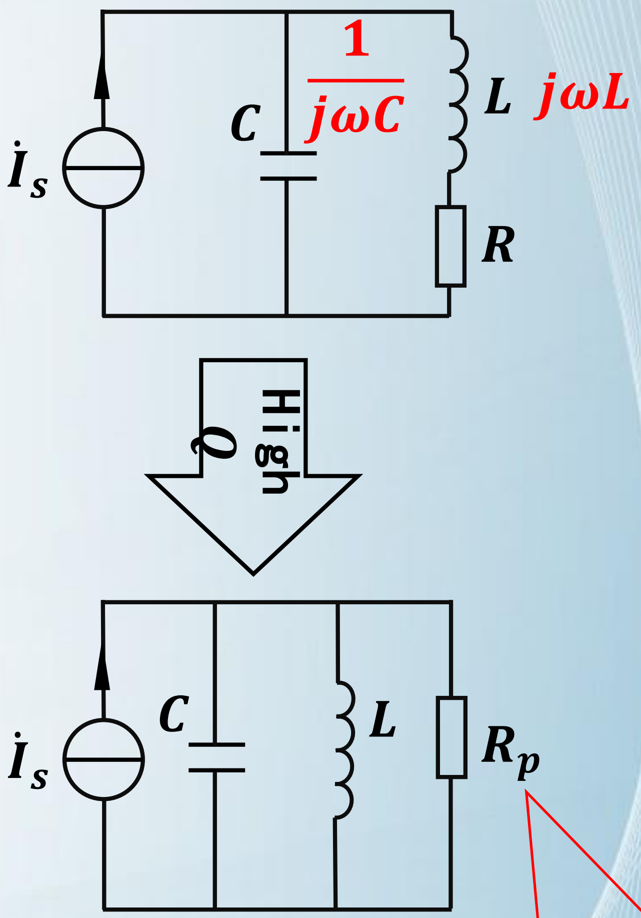
# Parallel Resonant Circuit—Impendance $Z$、Admittance $Y$

High Q, $\omega L \gg R$

Impendance $Z = \dfrac{(R+j\omega L)\dfrac{1}{j\omega C}}{(R+j\omega L)+\dfrac{1}{j\omega C}} = \dfrac{(\cancel{R}+j\omega L)\dfrac{1}{j\omega C}}{R+j(\omega L-\dfrac{1}{\omega C})}$

$$\approx \frac{\dfrac{L}{C}}{R+j(\omega L-\dfrac{1}{\omega C})} = \frac{1}{\dfrac{CR}{L}+j(\omega C-\dfrac{1}{\omega L})}$$

Admittance $Y = G + jB = \dfrac{CR}{L} + j\left(\omega C - \dfrac{1}{\omega L}\right)$

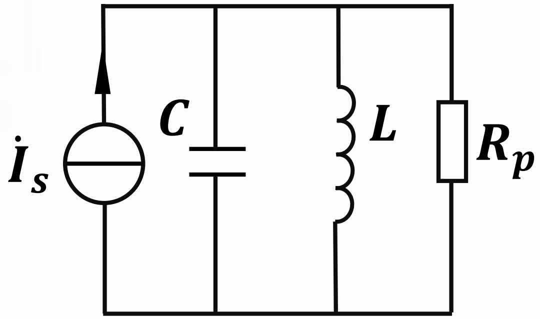


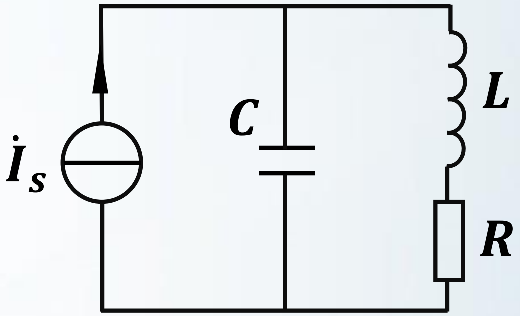

$R_p = \dfrac{1}{G} = \dfrac{L}{CR}$

Conductance $G = \dfrac{CR}{L}$

Susceptance $B = \omega C - \dfrac{1}{\omega L}$

# Parallel Resonant Circuit—$R_p$ vs $R$

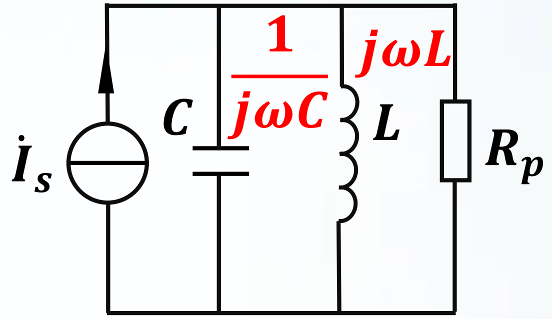


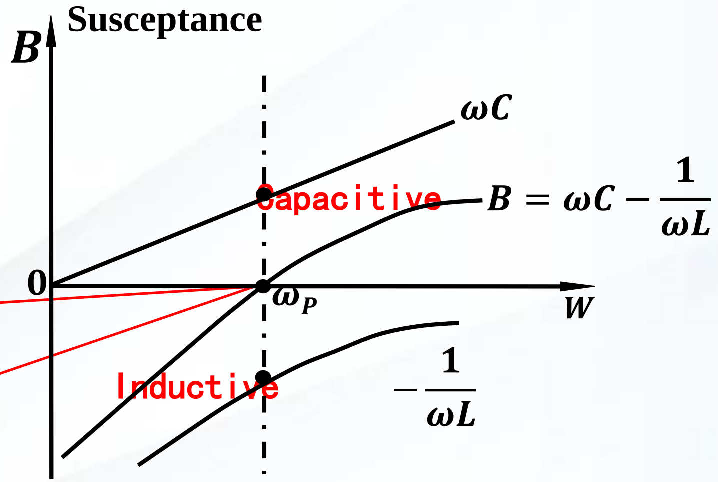

$$R_p = \frac{1}{G} = \frac{L}{CR}$$

**Relationship of $R_p$ & $R$**

# Parallel Resonant Circuit—Admittance $\boldsymbol{Y}$




Susceptance $B = \omega C - \frac{1}{\omega L} \Leftarrow Y = G + jB = \frac{CR}{L} + j\left(\omega C - \frac{1}{\omega L}\right)$



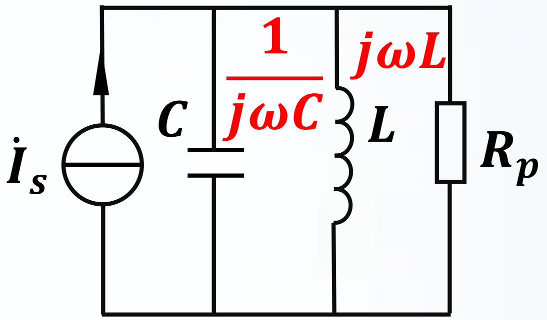

Resonant Freqency $\omega_p$ :

$$B = \omega_p C - \frac{1}{\omega_p L} = 0$$

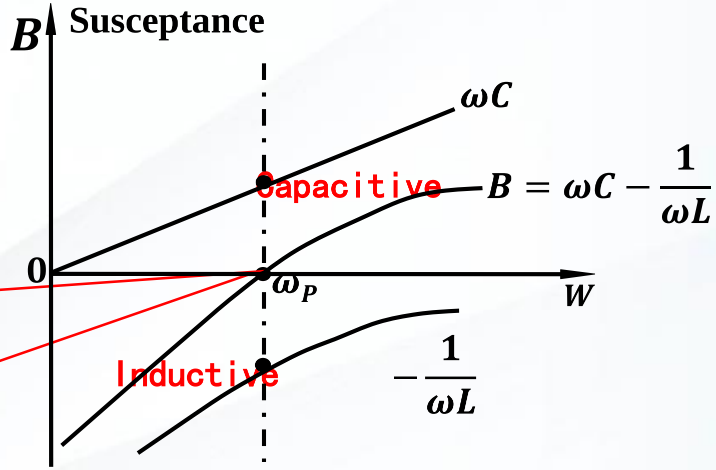
$$\Rightarrow \omega_p = \frac{1}{\sqrt{LC}}$$

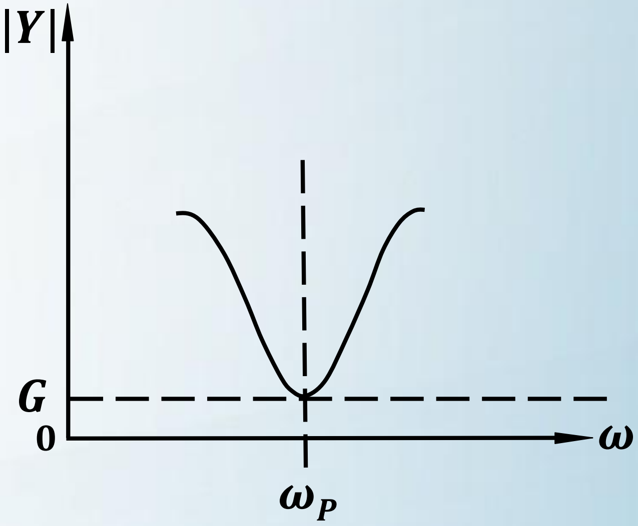
1) $\omega > \omega_p$, $B > 0$ Capacitive, ICE
2) $\omega < \omega_p$, $B < 0$ Indutive, ELI
3) $\omega = \omega_p$, $B = 0$ purely resistive

# Parallel Resonant Circuit—Admittance $Y$




Susceptance $B = \omega C - \frac{1}{\omega L} \Leftarrow Y = G + jB = \frac{CR}{L} + j\left(\omega C - \frac{1}{\omega L}\right)$



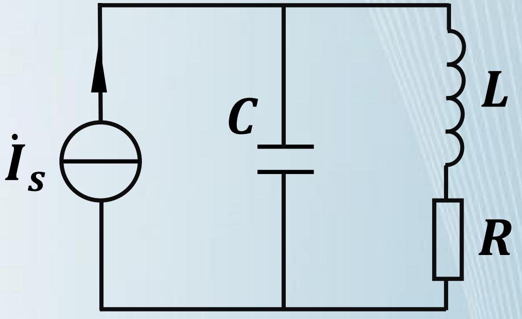




Resonant Freqency $\omega_p$ :

$$B = \omega_p C - \frac{1}{\omega_p L} = 0$$

$$\Rightarrow \omega_p = \frac{1}{\sqrt{LC}}$$

# Parallel Resonant Circuit— $Q$

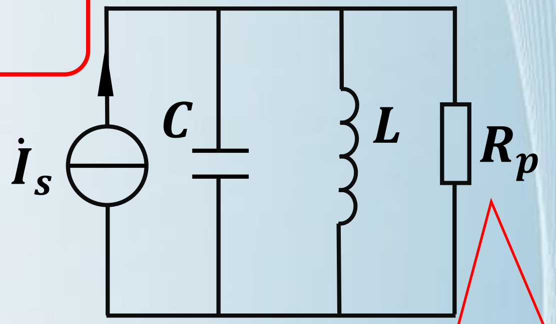



$$Q_p = \frac{\omega_p L}{R} = \frac{\frac{1}{\omega_p C}}{R} = \frac{\rho}{R} = \frac{(Reactance)X}{(Resistance)R}$$

$R$ version $Q_p$

$$Q_p = \frac{1}{\omega_p L} \cdot R_p = \omega_p C \cdot R_p$$

$R_p$ version $Q_p$



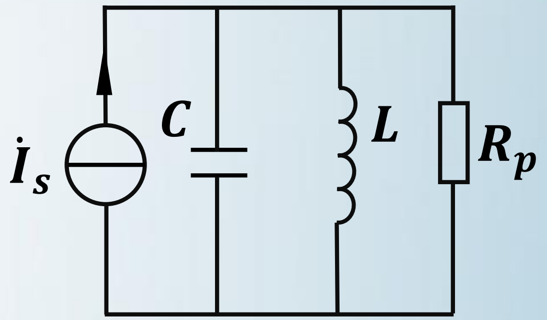

$$R_p = \frac{1}{G} = \frac{L}{CR}$$

$$Q_p = \frac{\frac{1}{\omega_p L}}{G} = \frac{\omega_p C}{G} = \frac{(Susceptance)B}{(Conductance)G}$$

$G$ version $Q_p$

Reason: $R$ and $R_p$

# Parallel Resonant Circuit— Current Resonance




Voltage

- Parallel Resonant

$$\begin{cases} \dot{I}_{Cp} = j\omega_p C \cdot R_p\dot{I}_s = j\omega_p C \cdot R_p\dot{I}_s = jQ_p\dot{I}_s \\ \dot{I}_{Lp} = \dfrac{1}{j\omega_p L} \cdot R_p\dot{I}_s = -j\dfrac{1}{\omega_p L}R_p\dot{I}_s = -jQ_p\dot{I}_s \end{cases}$$

$$Q_p = \frac{1}{\omega_p L} \cdot R_p = \omega_p C \cdot R_p$$

$\therefore\ \dot{I}_{Cp} = -\dot{I}_{Lp}$ same current value , $Q_p$ times of source current

# Parallel Resonant Circuit— Detuning Coefficient

$$Q_p = \frac{1}{\omega_p L} \cdot R_p = \omega_p C \cdot R_p$$

$$= \frac{\frac{1}{\omega_p L}}{G} = \frac{\omega_p C}{G}$$

Resonance , $Q_p = \frac{(\mathit{Susceptance})B}{(\mathit{Conductance})G}$

Detuning , $\xi = \frac{(\mathit{Susceptance\ sum})B}{(\mathit{Conductance})G} = \frac{\omega C - \frac{1}{\omega L}}{G}$

$\xi = 0$ denote resonance

$\omega \approx \omega_p$

$$= \frac{\omega_p C}{G}\left(\frac{\omega}{\omega_p} - \frac{\omega_p}{\omega}\right) = Q_p\left(\frac{\omega}{\omega_p} - \frac{\omega_p}{\omega}\right) = Q_p \frac{\cancel{(\omega+\omega_p)}(\omega-\omega_p)}{\omega_p \cancel{\omega}}$$

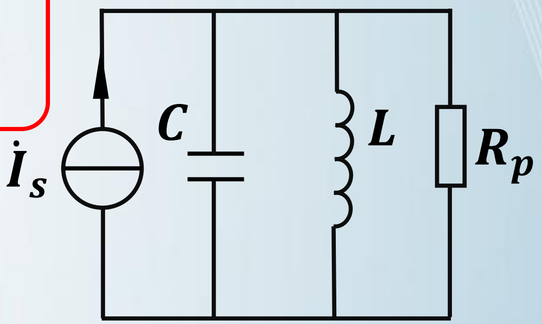
$$\xi \approx Q_p \frac{2(\omega-\omega_p)}{\omega_p}$$

$$\xi = Q_p \cdot \frac{2\Delta\omega}{\omega_p} \text{ 或 } \xi = Q_p \cdot \frac{2\Delta f}{f_p}$$

$\xi \neq 0$ dennote detuning value

# Parallel Resonant Circuit—Resonance Curve

$$Y=\frac{CR}{L}+j\left(\omega C-\frac{1}{\omega L}\right)=G_p+j\left(\omega C-\frac{1}{\omega L}\right)$$



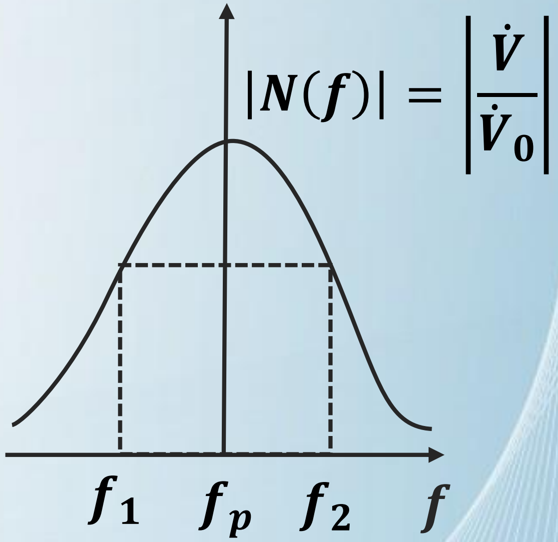

- Voltage: $\dot{V}=\frac{\dot{I}_s}{Y}=\frac{\dot{I}_s}{G_p+j\left(\omega C-\frac{1}{\omega L}\right)}\sim\ \omega$

- Resonance Curve :

$$N(f)=\frac{\dot{V}}{\dot{V}_0}=\frac{\frac{\dot{I}_s}{G_p+j\left(\omega C-\frac{1}{\omega L}\right)}}{\frac{\dot{I}_s}{G_p}}=\frac{G_p}{G_p+j(\omega C-\frac{1}{\omega L})}=\frac{1}{1+j(\frac{\omega C-\frac{1}{\omega L}}{G_p})}$$

$$\Rightarrow N(f)=\frac{1}{1+j\xi}$$

$$\xi=\frac{\omega C-\frac{1}{\omega L}}{G}$$




Amplitude-Frequency

# Parallel Resonant Circuit— Bandwidth

➢ Bandwidth: scope among $\dot{V}$ drop to 0.707 of $\dot{V}_0$

$$B = 2\Delta f_{0.7} = |f_2 - f_1|$$

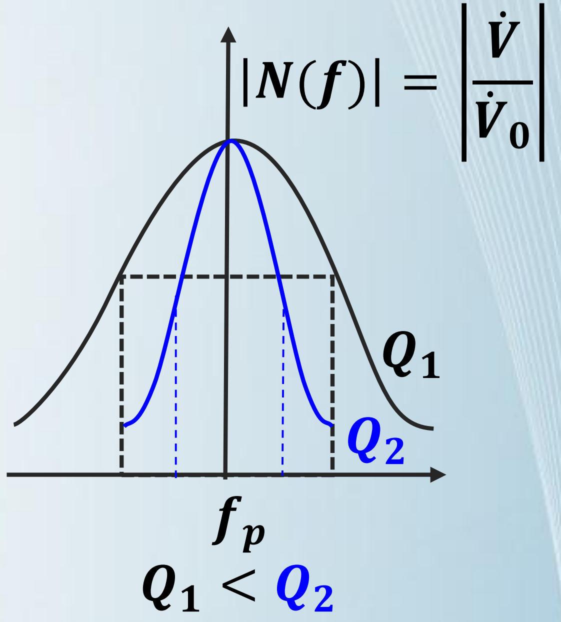
$$\left.\begin{aligned} &\text{Curve } N(f) = \frac{\dot{V}}{\dot{V}_0} = \frac{1}{1+j\xi} \\ &\text{AF: } |N(f)| = \frac{1}{\sqrt{1+\xi^2}} = \frac{1}{\sqrt{2}} \end{aligned}\right\} \Rightarrow \text{if } 2\Delta f_{0.7}$$

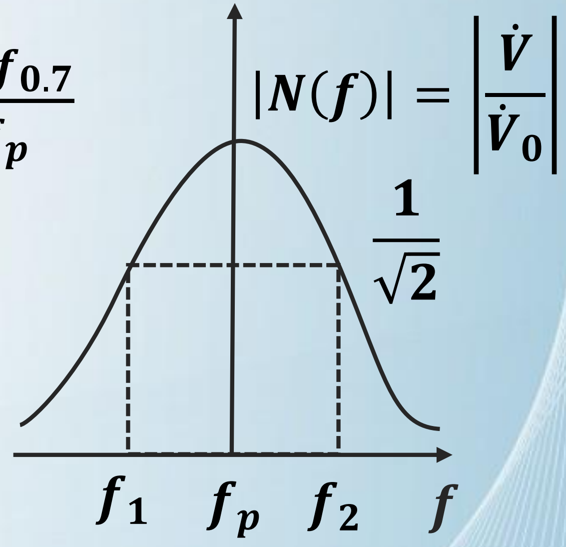
$$\left.\begin{aligned} &\xi = 1 \\ &\xi = Q_p \cdot \frac{2\Delta f}{f_p} \end{aligned}\right\} \Rightarrow 1 = Q_p \cdot \frac{2\Delta f_{0.7}}{f_p}$$

$$\boxed{2\Delta f = 2\Delta f_{0.7}}$$

$$1 = Q_p \cdot \frac{B}{f_p}$$

$$Q_p \cdot B = f_p$$



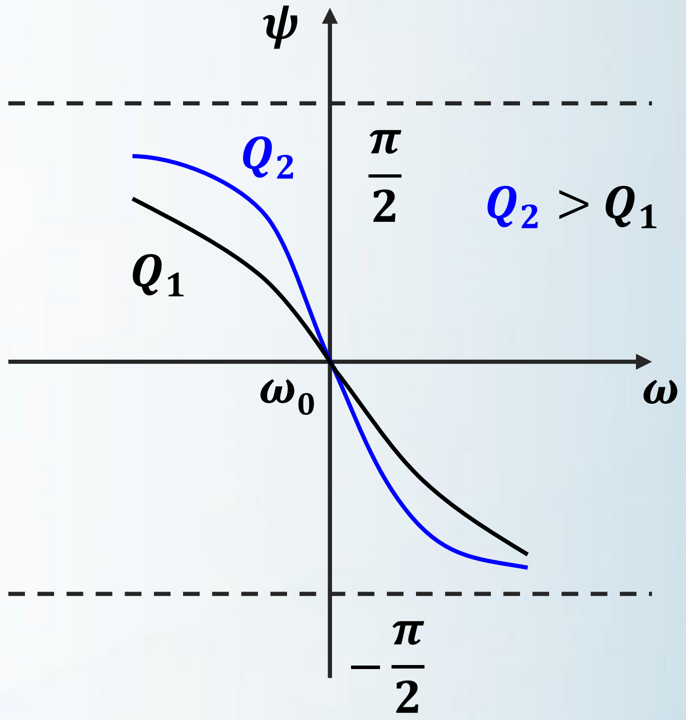




Amplitude-Frequency (AF)

# Parallel Resonant Circuit— Phase-Frequency (PF) Curve

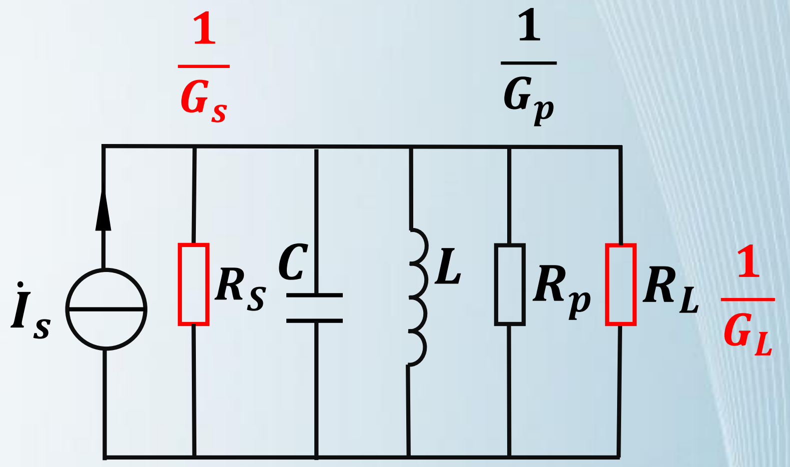
$$N(f) = \frac{\dot{V}}{\dot{V}_0} = \frac{1}{1 + j\xi}$$

- PF: $\psi = -arctg\xi$




$Q\uparrow$ linearality $\downarrow$

# Parallel Resonant Circuit— With Load




Unloaded $Q$:

$$Q_p = \frac{\omega_p L}{R} = \frac{1}{\omega_p L} \cdot R_p = \frac{1}{\omega_p L} \cdot \frac{1}{G_p}$$

Loaded $Q$:

$$Q_L = \frac{1}{\omega_p L} \cdot \frac{1}{G_p + G_s + G_L}$$

Consider source resistance & load resistance

$$Q_L \downarrow \Rightarrow B \uparrow$$

# Summary—Parallel Resonant Circuit

- Resonance Curve: $N(f) = \frac{\dot{V}}{\dot{V}_0} = \frac{1}{1+j\xi}$

$\because\ \rho = \omega_p L = \frac{1}{\omega_p C}$

Amplitude-Frequency $|N(f)| = \left|\frac{\dot{V}}{\dot{V}_0}\right|$

Resonance :

$$Q_p = \frac{\omega_p L}{R} = \frac{\frac{1}{\omega_p C}}{R} = \frac{\rho}{R}$$

$R$ version of $Q_p$

$$Q_p = \frac{1}{\omega_p L} \cdot R_p = \omega_p C \cdot R_p$$

$R_p$ version of $Q_p$

$$Q_p = \frac{(Susceptance)B}{(Conductance)G} = \frac{\frac{1}{\omega_p L}}{G} = \frac{\omega_p C}{G}$$

$G$ version of $Q_p$

Detunning :

$$\xi = \frac{(Susceptance\ sum)B}{(Conductance)G} = \frac{\omega C - \frac{1}{\omega L}}{G} \approx Q_p \cdot \frac{2\Delta f}{f_p} \quad \Rightarrow Q_p \cdot B = f_p$$

Phase-Frequency $\psi = -arctg\xi$