§ 5.6 The magnetic vector potential

1.the magnetic vector potential (Page 194)

since $\nabla \bullet \vec{B} = 0$**, in terms of identity** $\nabla \bullet (\nabla \times \vec{A}) = 0$

(the divergence of the curl of a vector field is zero). we have $\vec{B} = \nabla \times \vec{A}$ **(5.24)**

($\vec{B}$ **can be expressed in terms of the curl of another vector field** $\vec{A}$**)**

where $\vec{A}$ **is called the magnetic vector potential and is expressed in webers per meter.**

- **We have known that the magnetic flux density at any point *P* (*x*, *y*, *z*)produced by a current-carrying conductor is**

$$\vec{\mathbf{B}} = \frac{\mu}{4\pi}\int_c \frac{Id\vec{l} \times \vec{\mathrm{R}}}{R^3}$$

where μ is the medium permeability, and

$$\vec{R} = \vec{a}_x(x-x') + \vec{a}_y(y-y') + \vec{a}_z(z-z')$$

The primed (x', y', z') stand for the source coordinates and the unprimed (*x*, *y*, *z*) stand for the field coordinates.

$$\vec{\mathbf{B}} = \frac{\mu}{4\pi}\int_c \frac{Id\vec{l} \times \vec{\mathbf{R}}}{R^3} = \frac{\mu}{4\pi}\int_c Id\vec{l} \times \frac{\vec{\mathbf{R}}}{R^3}$$

$$= \frac{\mu}{4\pi}\int_c Id\vec{l} \times \left(-\nabla\left(\frac{1}{R}\right)\right)$$

$$= \frac{\mu}{4\pi}\int_c \nabla\left(\frac{1}{R}\right) \times Id\vec{l} \qquad \textbf{(2.8.1)}$$

in terms of the vector identity,

$$\nabla\left(\frac{1}{R}\right) \times Id\vec{l} = \nabla \times \left[\frac{Id\vec{l}}{R}\right] - \frac{1}{R}\left[\nabla \times Id\vec{l}\right]$$

(2.8.1) can be rewritten as

$$\frac{\mu}{4\pi}\int_c \nabla\left(\frac{1}{R}\right)\times I d\vec{l} = \frac{\mu}{4\pi}\int_c \left(\nabla\times\left(\frac{I d\vec{l}}{R}\right) - \frac{1}{R}[\nabla\times I d\vec{l}\,]\right)$$

($\nabla\times I d\vec{l}$ **, because the curl operation is with respect to the unprimed coordinates of point** $P(x, y, z)$**,** $I d\vec{l}$ **is with respect to the primed coordinates of source point** $P'(x', y', z')$**,**

$$\nabla\times I d\vec{l} = 0 \;)$$

The preceding equation $= \dfrac{\mu}{4\pi}\displaystyle\int_c \nabla\times\left(\frac{I d\vec{l}}{R}\right)$

The integration and differentiation are with respect to two different sets of variables, so we can interchange the order and write the preceding equation as

$$\vec{\mathbf{B}} = \frac{\mu}{4\pi}\int_c \nabla \times \left(\frac{I d\vec{l}}{R}\right) = \nabla \times \frac{\mu}{4\pi}\int_c \frac{I d\vec{l}}{R} \quad \textbf{(5.26)}$$

therefore,

$$\vec{\mathbf{A}} = \frac{\mu}{4\pi}\int_c \frac{I d\vec{l}}{R} \qquad \textbf{(5.27}\boldsymbol{a}\textbf{)}$$

$$\vec{\mathbf{A}} = \frac{\mu}{4\pi}\int_c \frac{Id\vec{l}}{R} \qquad \textbf{(5.27 } \boldsymbol{a}\textbf{)}$$

is the magnetic vector potential. It states that the current-carrying conductor (or the current element $Id\vec{l}$) produces a magnetic vector potential at any field point in space (μ、 μ_0)

❖**discussion:**

i) if the current-carrying conductor forms a closed loop, (5.27*a*) becomes

$$\vec{\mathbf{A}} = \frac{\mu}{4\pi}\oint_c \frac{Id\vec{l}}{R} \qquad \textbf{(5.27}\boldsymbol{b}\textbf{)}$$

ii)in terms of the volume current density, (5.27*b*) can expressed by

$$\vec{\mathbf{A}} = \frac{\mu}{4\pi}\int_v \frac{\vec{\mathbf{J}}_v\, dv}{R} \qquad \textbf{(5.27}\boldsymbol{c}\textbf{)}$$

ii)in terms of the surface current density, (5.27*b*) can expressed by

$$\vec{\mathbf{A}} = \qquad \textbf{(5.27}\boldsymbol{d}\textbf{)}$$