Huazhong University
of Science & Technology

Electronic Circuit of Communications

School of Electronic Information
and Commnications

Jiaqing Huang

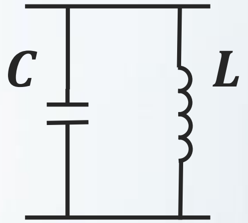
Parallel Resonance

Frequency Selective Circuits

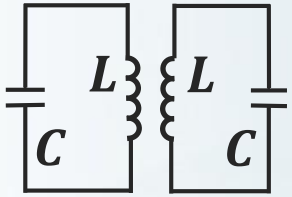
- Resonance
  - Series Resonance

  - Parallel Resonance

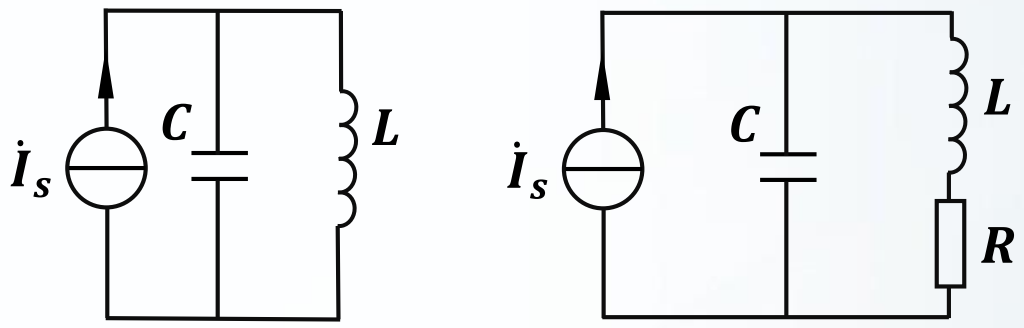
    

  - Coupling Resonance

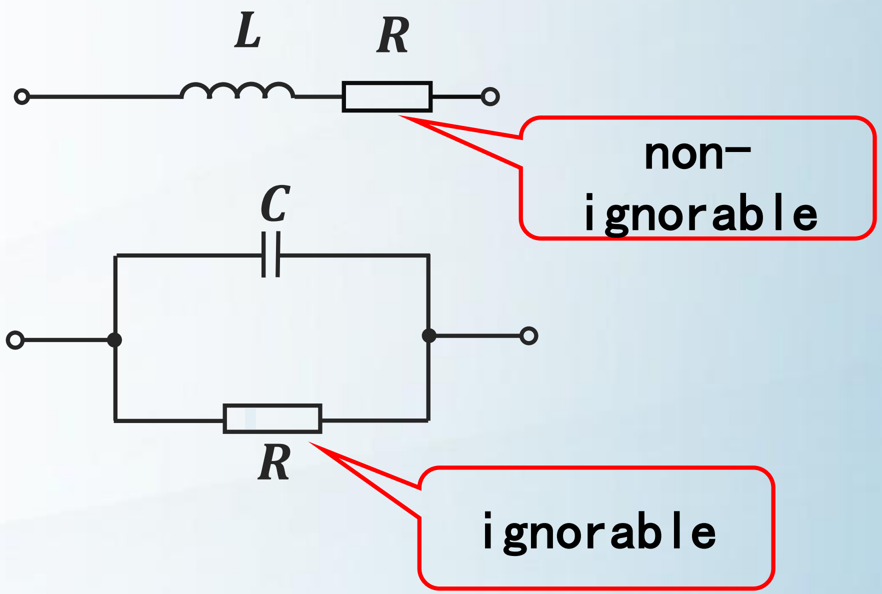
    

- Filter
  - Quartz Crystal Filter
  - Ceramic filter
  - SAWF

Parallel Resonant Circuit



- Inductor $= L$ with loss resistance R
- Capacitor $= C$ with loss resistance R



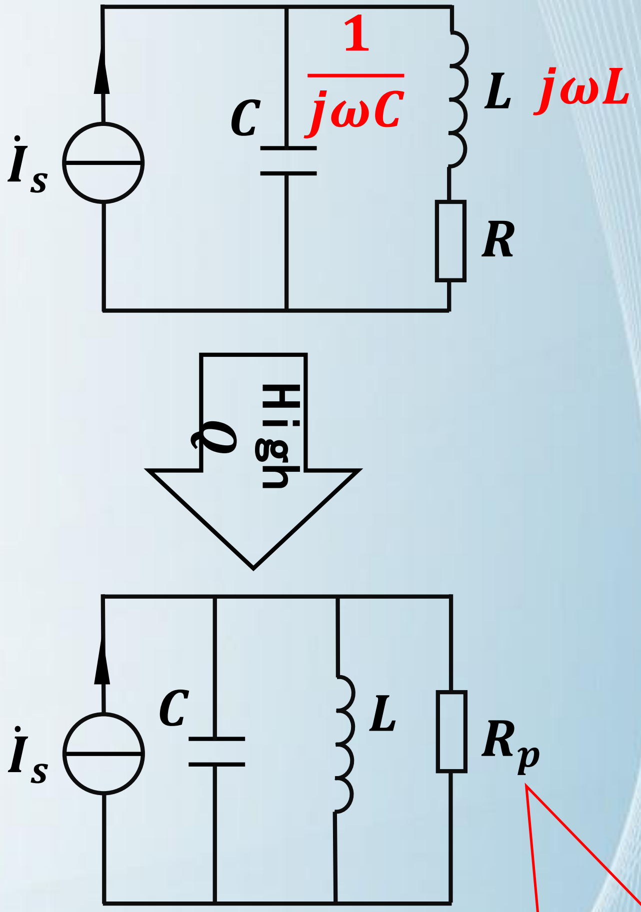
Parallel Resonant Circuit—Impendance Z、Admittance Y

High Q, $\omega L \gg R$

Impendance $$Z=\frac{(R+j\omega L)\frac{1}{j\omega C}}{(R+j\omega L)+\frac{1}{j\omega C}}=\frac{(\cancel{R}+j\omega L)\frac{1}{j\omega C}}{R+j(\omega L-\frac{1}{\omega C})}$$

$$\approx\frac{\frac{L}{C}}{R+j(\omega L-\frac{1}{\omega C})}=\frac{1}{\frac{CR}{L}+j(\omega C-\frac{1}{\omega L})}$$

Admittance $$Y=G+jB=\frac{CR}{L}+j\left(\omega C-\frac{1}{\omega L}\right)$$

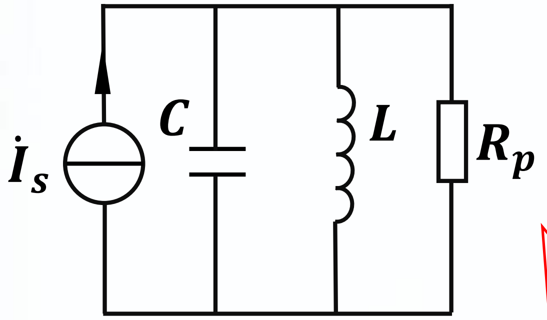


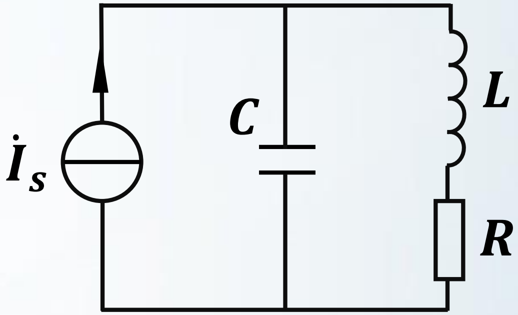
$$R_p=\frac{1}{G}=\frac{L}{CR}$$

Conductance $\quad G=\frac{CR}{L}$

Susceptance $\quad B=\omega C-\frac{1}{\omega L}$

Parallel Resonant Circuit—R_p vs R

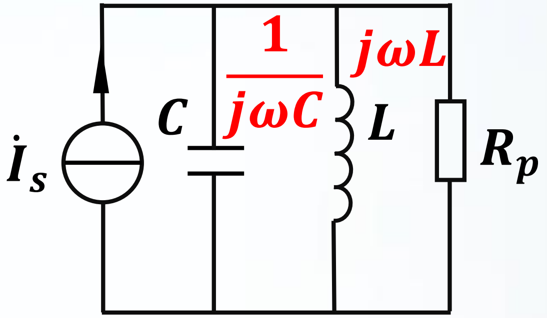




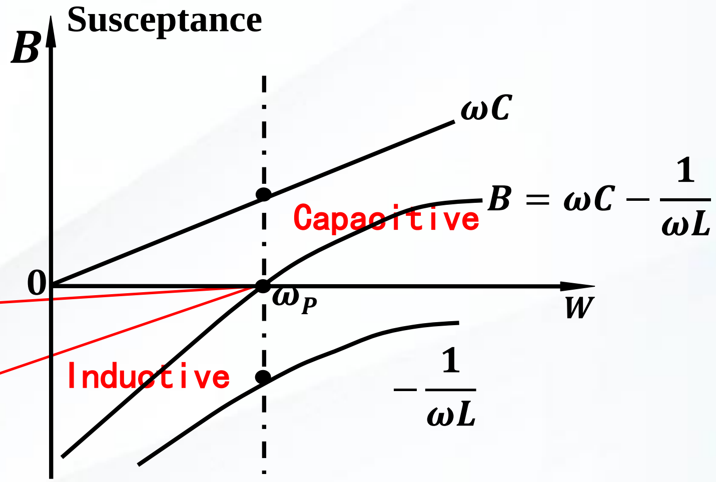
$$R_p = \frac{1}{G} = \frac{L}{CR}$$

Relationship of R_p & R

Parallel Resonant Circuit—Admittance Y



Susceptance $B = \omega C - \frac{1}{\omega L} \Leftarrow Y = G + jB = \frac{CR}{L} + j\left(\omega C - \frac{1}{\omega L}\right)$



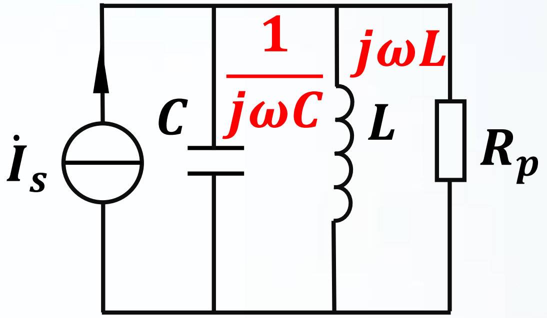
Resonant Freqency ω_p:

$$B = \omega_p C - \frac{1}{\omega_p L} = 0$$

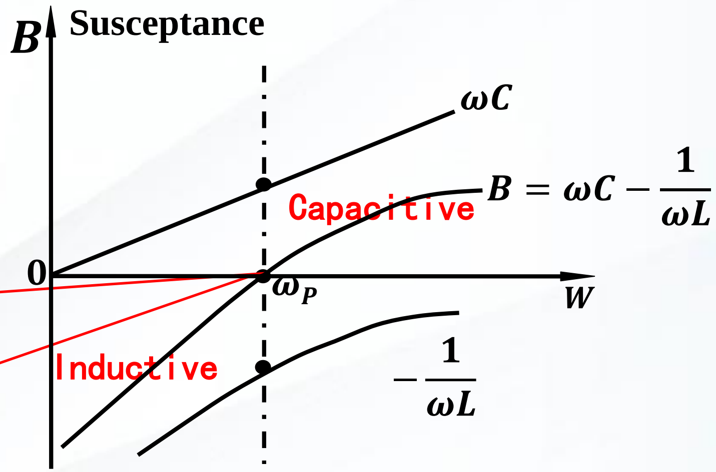
$$\Rightarrow \omega_p = \frac{1}{\sqrt{LC}}$$

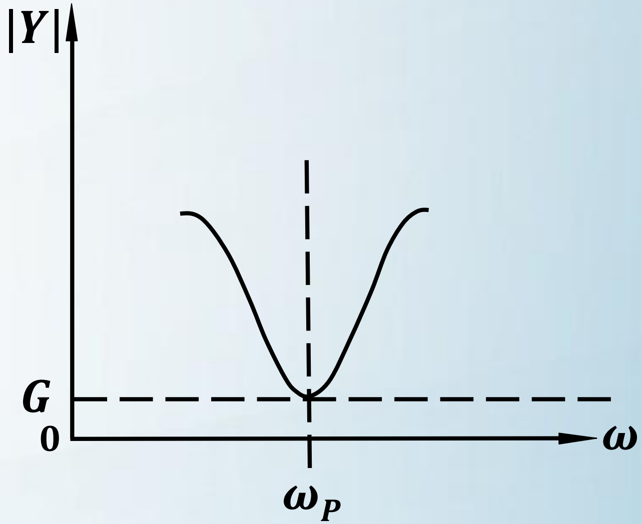
1) $\omega > \omega_p$, $B > 0$ Capacitive, ICE
2) $\omega < \omega_p$, $B < 0$ Indutive, ELI
3) $\omega = \omega_p$, $B = 0$ purely resistive

Parallel Resonant Circuit—Admittance Y



Susceptance $B = \omega C - \frac{1}{\omega L} \Leftarrow Y = G + jB = \frac{CR}{L} + j\left(\omega C - \frac{1}{\omega L}\right)$



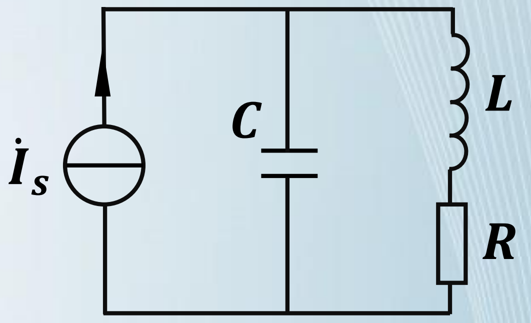


Resonant Freqency ω_p:

$$B = \omega_p C - \frac{1}{\omega_p L} = 0$$

$$\Rightarrow \omega_p = \frac{1}{\sqrt{LC}}$$

Parallel Resonant Circuit— Q

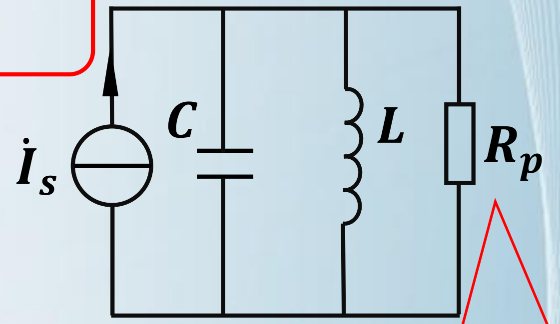


$$Q_p = \frac{\omega_p L}{R} = \frac{\frac{1}{\omega_p C}}{R} = \frac{\rho}{R} = \frac{(Reactance)X}{(Resistance)R}$$

R version Q_p

$$Q_p = \frac{1}{\omega_p L} \cdot R_p = \omega_p C \cdot R_p$$

R_p version Q_p



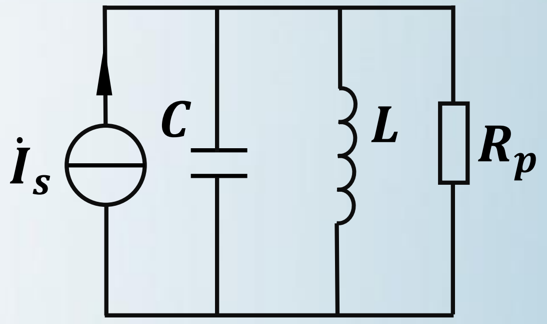
$$R_p = \frac{1}{G} = \frac{L}{CR}$$

$$Q_p = \frac{\frac{1}{\omega_p L}}{G} = \frac{\omega_p C}{G} = \frac{(Susceptance)B}{(Conductance)G}$$

G version Q_p

Reason: R and R_p

Parallel Resonant Circuit— Current Resonance



Voltage

- Parallel Resonant

$$\begin{cases} \dot{I}_{Cp} = j\omega_p C \cdot R_p \dot{I}_s = j\omega_p C \cdot R_p \dot{I}_s = jQ_p \dot{I}_s \\ \dot{I}_{Lp} = \dfrac{1}{j\omega_p L} \cdot R_p \dot{I}_s = -j\dfrac{1}{\omega_p L} R_p \dot{I}_s = -jQ_p \dot{I}_s \end{cases}$$

$$Q_p = \frac{1}{\omega_p L} \cdot R_p = \omega_p C \cdot R_p$$

$\therefore\ \dot{I}_{Cp} = -\dot{I}_{Lp}$ same current value , Q_p times of source current

Parallel Resonant Circuit— Detuning Coefficient

$$Q_p = \frac{1}{\omega_p L} \cdot R_p = \omega_p C \cdot R_p$$

$$= \frac{\frac{1}{\omega_p L}}{G} = \frac{\omega_p C}{G}$$

Resonance , $Q_p = \dfrac{(Susceptance)B}{(Conductance)G}$

Detuning , $\xi = \dfrac{(Susceptance\ sum)B}{(Conductance)G} = \dfrac{\omega C - \frac{1}{\omega L}}{G}$

$\xi = 0$ denote resonance

$\omega \approx \omega_p$

$$= \frac{\omega_p C}{G}\left(\frac{\omega}{\omega_p} - \frac{\omega_p}{\omega}\right) = Q_p\left(\frac{\omega}{\omega_p} - \frac{\omega_p}{\omega}\right) = Q_p \frac{\cancel{(\omega+\omega_p)}(\omega-\omega_p)}{\omega_p \cancel{\omega}}$$

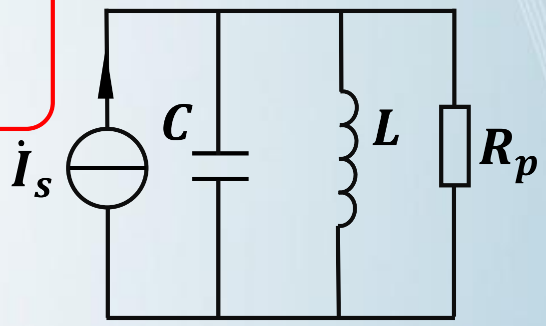
$$\xi \approx Q_p \frac{2(\omega-\omega_p)}{\omega_p}$$

$$\xi = Q_p \cdot \frac{2\Delta\omega}{\omega_p} \text{ 或 } \xi = Q_p \cdot \frac{2\Delta f}{f_p}$$

$\xi \neq 0$ dennote detuning value

Parallel Resonant Circuit—Resonance Curve

$$Y = \frac{CR}{L} + j\left(\omega C - \frac{1}{\omega L}\right) = G_p + j\left(\omega C - \frac{1}{\omega L}\right)$$



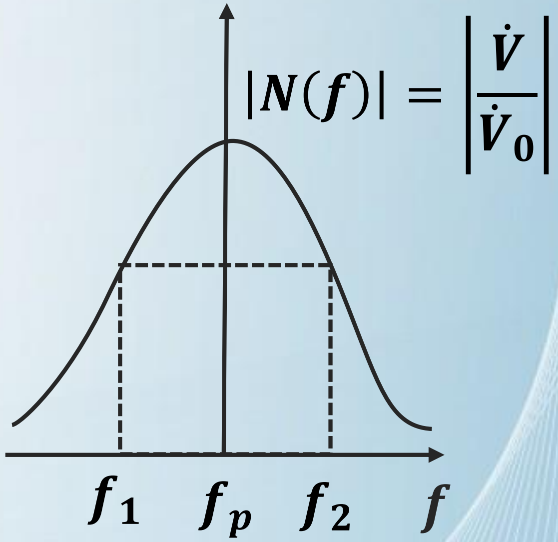
- Voltage: $\dot{V} = \frac{\dot{I}_s}{Y} = \frac{\dot{I}_s}{G_p + j\left(\omega C - \frac{1}{\omega L}\right)} \sim \omega$

- Resonance Curve :

$$N(f) = \frac{\dot{V}}{\dot{V}_0} = \frac{\frac{\dot{I}_s}{G_p + j\left(\omega C - \frac{1}{\omega L}\right)}}{\frac{\dot{I}_s}{G_p}} = \frac{G_p}{G_p + j\left(\omega C - \frac{1}{\omega L}\right)} = \frac{1}{1 + j\left(\frac{\omega C - \frac{1}{\omega L}}{G_p}\right)}$$

$$\Rightarrow N(f) = \frac{1}{1 + j\xi}$$

$$\xi = \frac{\omega C - \frac{1}{\omega L}}{G}$$



Amplitude-Frequency

Parallel Resonant Circuit— Bandwidth

- Bandwidth: scope among $\dot{V}$ drop to 0.707 of $\dot{V}_0$

$$B = 2\Delta f_{0.7} = |f_2 - f_1|$$

Curve $N(f) = \dfrac{\dot{V}}{\dot{V}_0} = \dfrac{1}{1+j\xi}$

AF: $|N(f)| = \dfrac{1}{\sqrt{1+\xi^2}} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow$ if $2\Delta f_{0.7}$

$\xi = 1$

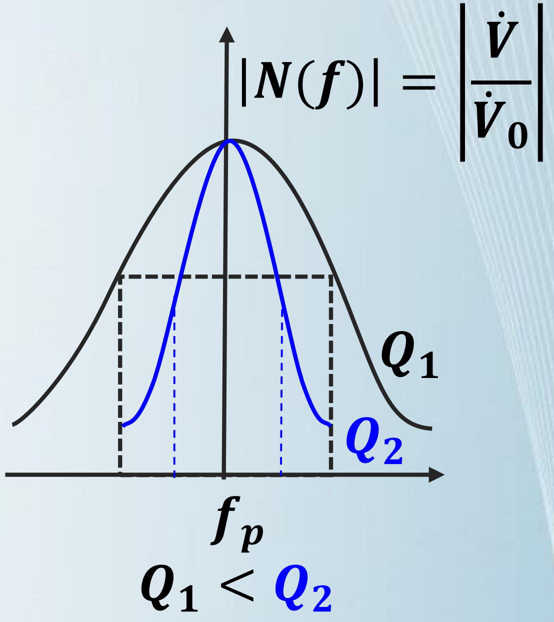
$\xi = Q_p \cdot \dfrac{2\Delta f}{f_p}$

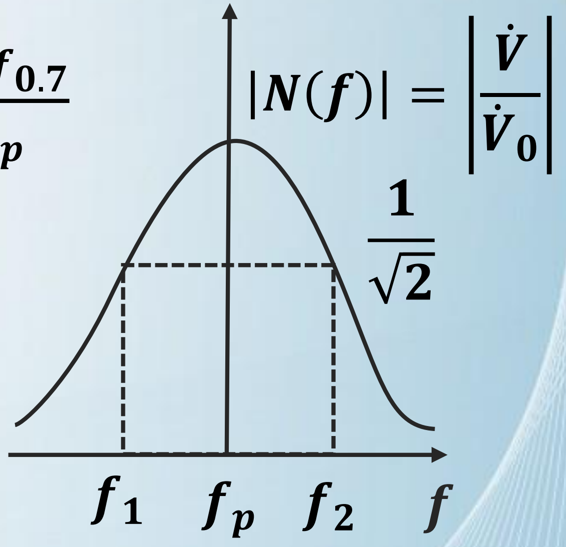
$$\boxed{2\Delta f = 2\Delta f_{0.7}}$$

$$\Rightarrow 1 = Q_p \cdot \frac{2\Delta f_{0.7}}{f_p}$$

$$1 = Q_p \cdot \frac{B}{f_p}$$

$$Q_p \cdot B = f_p$$





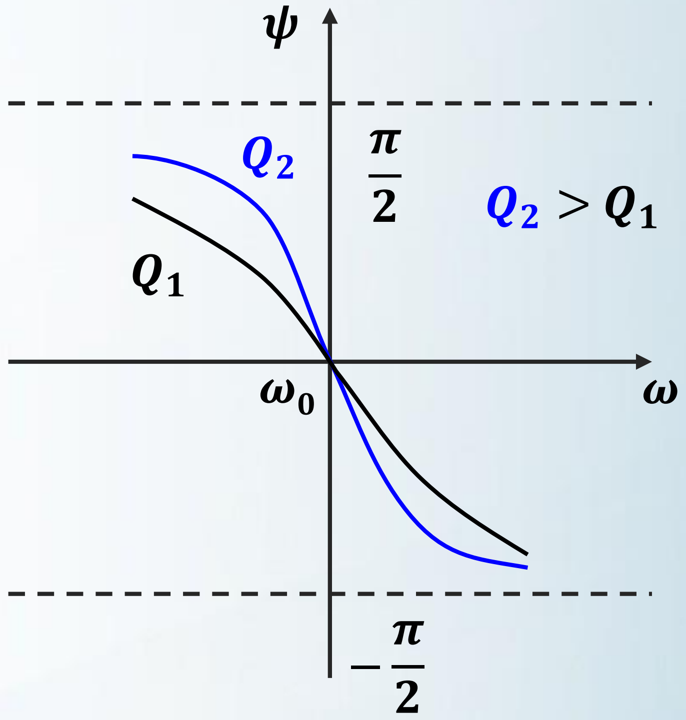
Amplitude-Frequency (AF)

Parallel Resonant Circuit— Phase-Frequency (PF) Curve

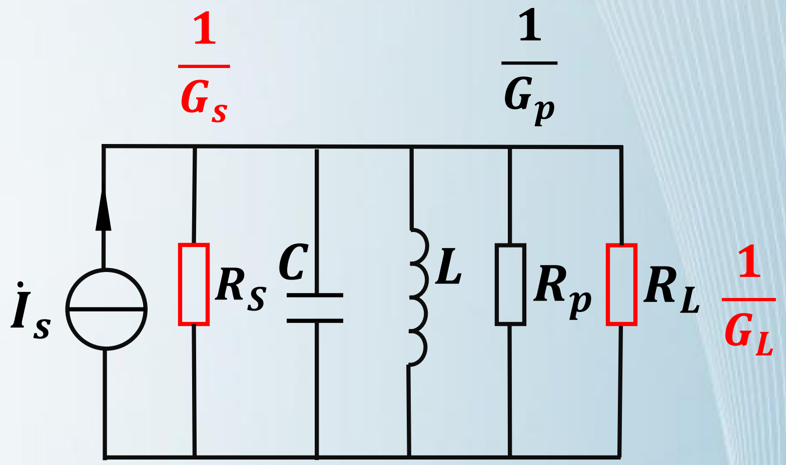
$$N(f) = \frac{\dot{V}}{\dot{V}_0} = \frac{1}{1 + j\xi}$$

- PF: $\psi = -arctg\xi$

$Q \uparrow$ linearality $\downarrow$



Parallel Resonant Circuit— With Load



Unloaded Q:

$$Q_p = \frac{\omega_p L}{R} = \frac{1}{\omega_p L} \cdot R_p = \frac{1}{\omega_p L} \cdot \frac{1}{G_p}$$

Loaded Q:

$$Q_L = \frac{\frac{1}{\omega_p L}}{G_p + G_s + G_L}$$

Consider source resistance & load resistance

$$Q_L \downarrow \Rightarrow B \uparrow$$

Summary—Parallel Resonant Circuit

- Resonance Curve: $N(f) = \frac{\dot{V}}{\dot{V}_0} = \frac{1}{1+j\xi}$

$\because\ \rho = \omega_p L = \frac{1}{\omega_p C}$

Amplitude-Frequency $|N(f)| = \left|\frac{\dot{V}}{\dot{V}_0}\right|$

Resonance : $Q_p = \frac{\omega_p L}{R} = \frac{\frac{1}{\omega_p C}}{R} = \frac{\rho}{R}$ — R version of Q_p

$Q_p = \frac{1}{\omega_p L} \cdot R_p = \omega_p C \cdot R_p$ — R_p version of Q_p

$Q_p = \frac{(\textit{Susceptance})B}{(\textit{Conductance})G} = \frac{\frac{1}{\omega_p L}}{G} = \frac{\omega_p C}{G}$ — G version of Q_p

Detunning : $\xi = \frac{(\textit{Susceptance sum})B}{(\textit{Conductance})G} = \frac{\omega C - \frac{1}{\omega L}}{G} \approx Q_p \cdot \frac{2\Delta f}{f_p} \quad \Rightarrow Q_p \cdot B = f_p$

Phase-Frequency $\psi = -arctg\xi$

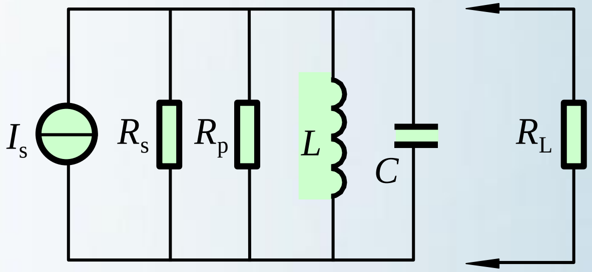
Example:

Parallel resonant circuit,

$Q_p = 80$, $R_p = 25\text{k}\Omega$, $f_p = 30\text{MHz}$, $I_s = 0.1\text{mA}$

(1) If $R_s = 10\text{k}\Omega$, and without R_L, Compute B and V_o

(2) If $R_s = 6\text{k}\Omega$, $R_L = 2\text{k}\Omega$, Compute B and V_o



Solve: **(1)** $\because R_s = 10\text{k}\Omega$,

$$\therefore v_o = I_s \cdot \frac{R_s \times R_p}{R_s + R_p} = 0.1\text{mA} \times \frac{10 \times 25}{10+25}\text{k}\Omega = 0.72\text{V}$$

$$Q_L = \frac{Q_o}{1+\dfrac{R_p}{R_s}} = \frac{80}{1+\dfrac{25}{10}} \approx 23$$

$$\therefore B = \frac{f_o}{Q_L} = \frac{30}{23} = 1.3\text{MHz}$$

$Q_L \downarrow$, B$\uparrow$

(2) $\because R_s = 6\text{K}\Omega \quad R_L = 2\text{K}\Omega$

$$\therefore V_o = I_s \cdot \frac{1}{\dfrac{1}{R_p}+\dfrac{1}{R_s}+\dfrac{1}{R_L}} = 0.1 \times \frac{1}{\dfrac{1}{25}+\dfrac{1}{6}+\dfrac{1}{2}} \approx 0.14\text{V}$$

$$\because Q_L = \frac{Q_o}{1+\dfrac{R_p}{R_s}+\dfrac{R_p}{R_L}} = \frac{80}{1+\dfrac{25}{6}+\dfrac{25}{2}} \approx 4.5$$

$$\therefore B = \frac{f_o}{Q_L} = \frac{30}{4.5} = 6.7\text{MHz}$$