Huazhong University
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Electronic Circuit of Communications

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and Commnications

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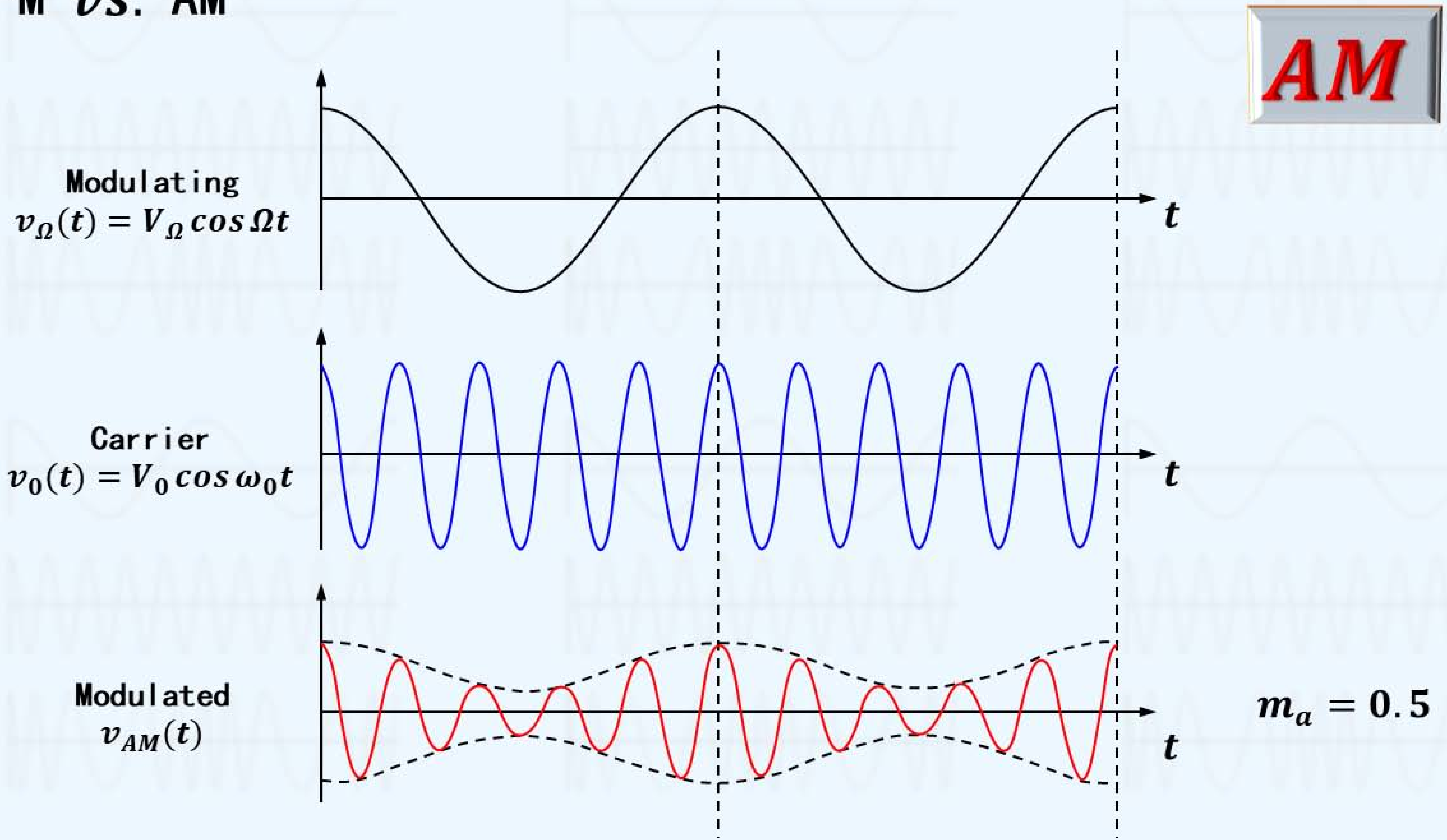
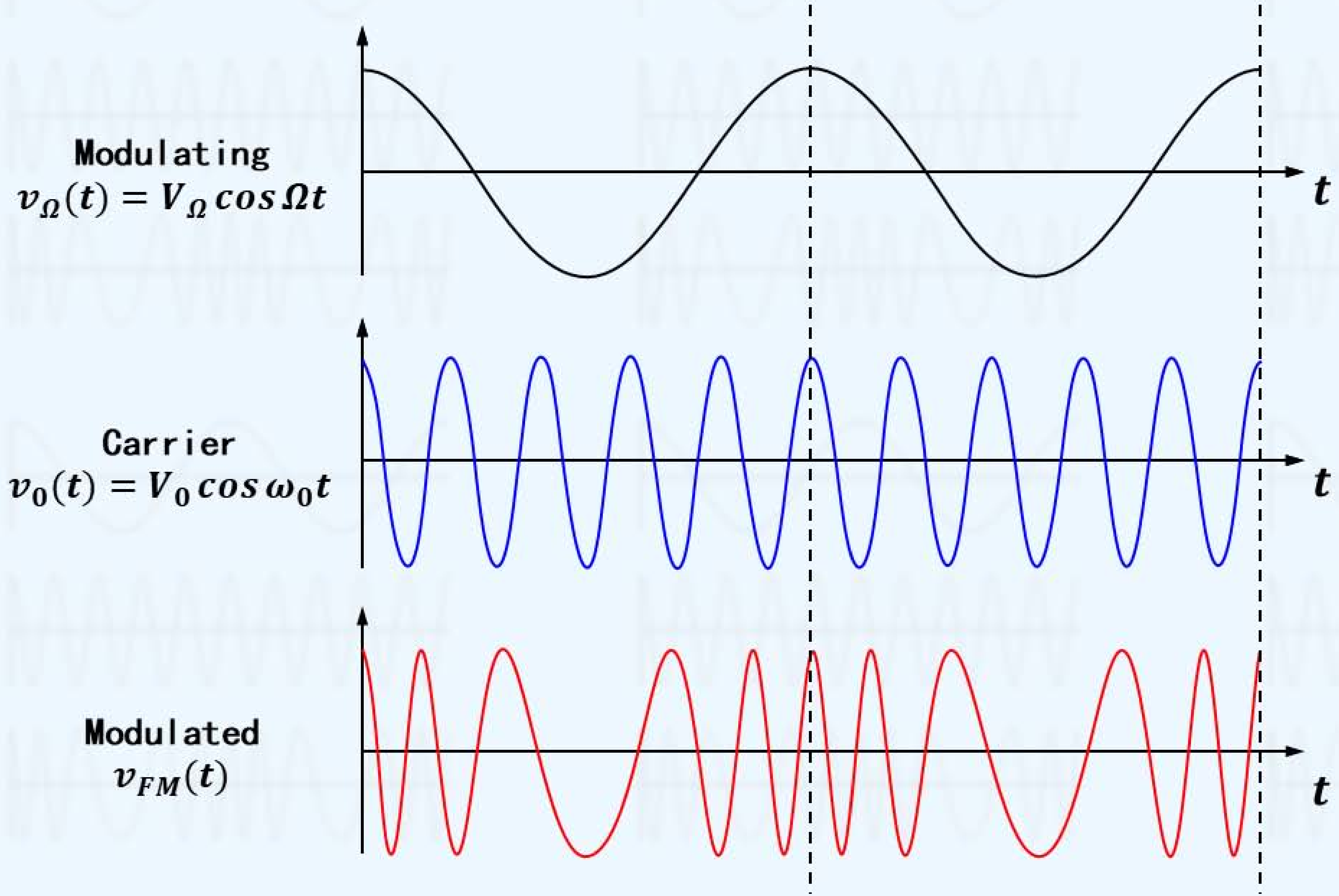
7 Angle Modulation

FM *vs.* AM



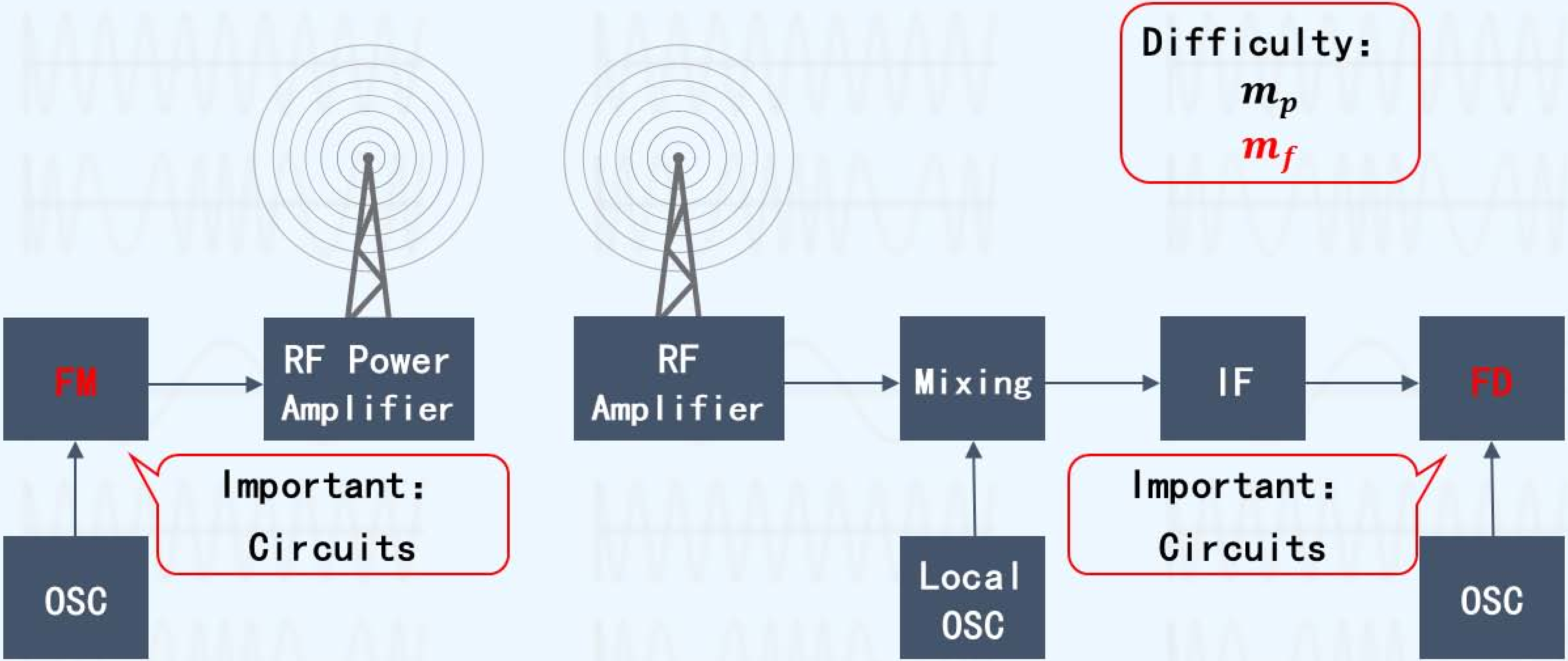
AM
Modulating
$v_\Omega(t) = V_\Omega \cos \Omega t$
t
Carrier
$v_0(t) = V_0 \cos \omega_0 t$
t
Modulated
$v_{AM}(t)$
t
$m_a = 0.5$

FM *vs.* AM


Modulating
$v_{\Omega}(t) = V_{\Omega} cos\, \Omega t$
t
Carrier
$v_0(t) = V_0 cos\, \omega_0 t$
t
Modulated
$v_{FM}(t)$
t


FM

Angle Modulation (PM/FM) —Important/Difficult Points

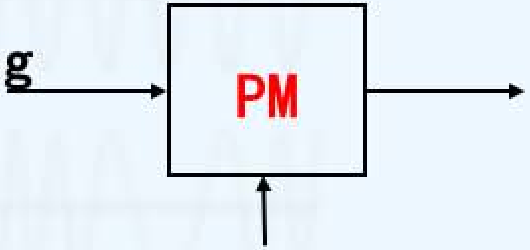


Phase Modulation

PM - Time Domain $\Delta\theta(t) = k_p v_\Omega(t)$

k_p, Unit: rad/V

Modulating $v_\Omega(t)$



Modulated $v_{PM}(t)$ $= V_0 cos[\omega_0 t + \quad k_p v_\Omega(t) \quad]$

Carrier $v_0(t) = V_0 \cos \omega_0 t$

$v_\Omega(t) = V_\Omega \cos \Omega t$
Single-tone PM

$$v_{PM}(t) = V_0 cos[\omega_0 t + k_p V_\Omega \cos \Omega t]$$
$$= V_0 cos[\omega_0 t + m_p \cos \Omega t]$$

PM Index m_p

Max Phase Shift $m_p = k_p |v_\Omega(t)|_{max} \xLeftrightarrow[\text{Single-tone}]{v_\Omega(t)=V_\Omega \cos \Omega t} m_p = k_p V_\Omega$

Max Frequency Shift $\Delta\omega_m = k_p \left|\frac{dv_\Omega(t)}{dt}\right|_{max} \xLeftrightarrow[\text{Single-tone}]{v_\Omega(t)=V_\Omega \cos \Omega t} \Delta\omega_m = k_p V_\Omega \Omega$

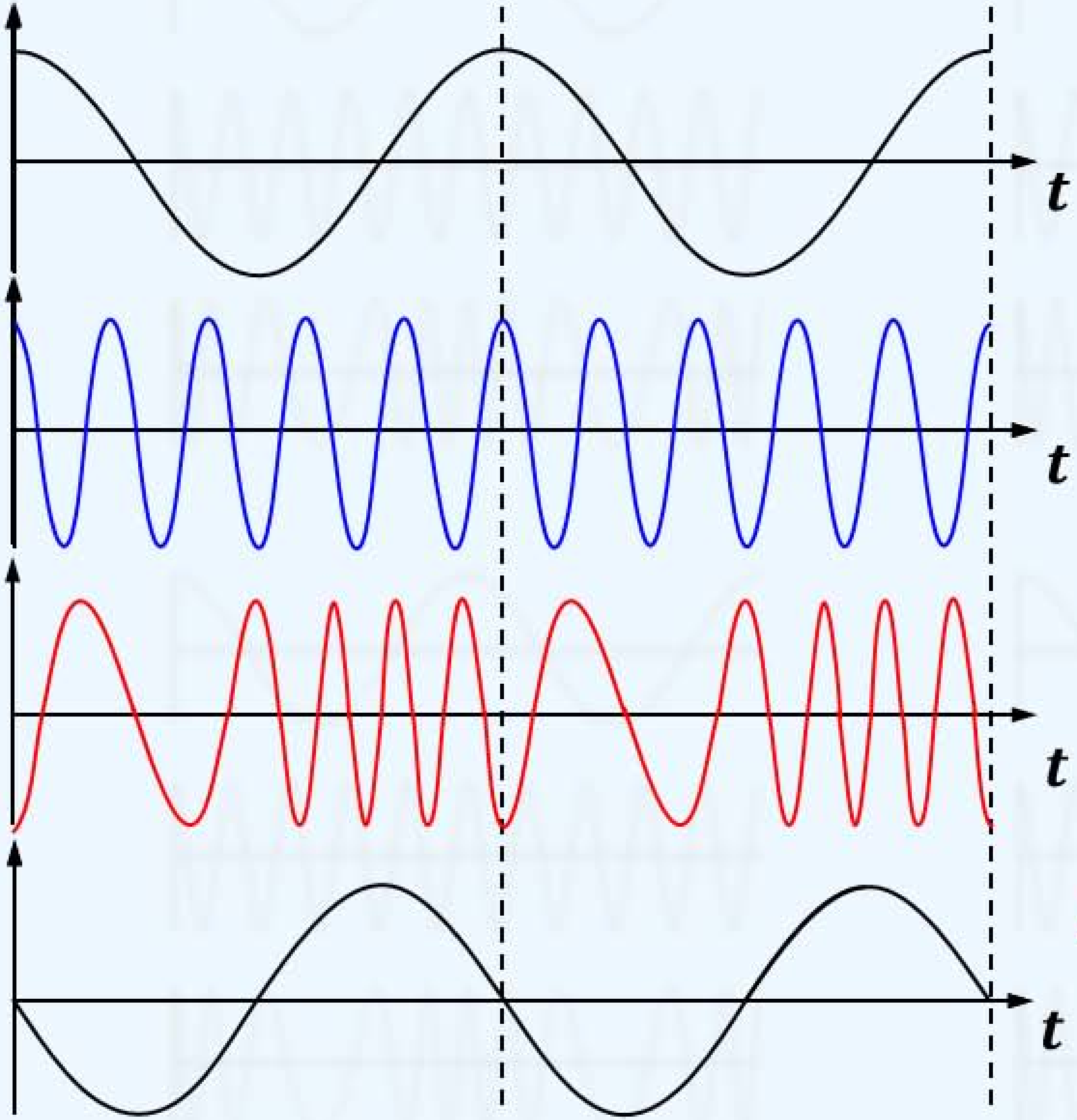
$$\Delta\omega_m = m_p \Omega$$
$$\Delta f_m = m_p F$$

PM - Time Domain (Single-tone Modulation)

Modulating
$v_\Omega(t) = V_\Omega \cos \Omega t$

Carrier
$v_0(t) = V_0 \cos \omega_0 t$

PM
$v_{PM}(t)$



$$\Delta\theta(t) = k_p v_\Omega(t)$$

$$\Delta\omega(t) = \frac{d\Delta\theta(t)}{dt} = k_p \frac{dv_\Omega(t)}{dt}$$

Summary – PM

- m_p

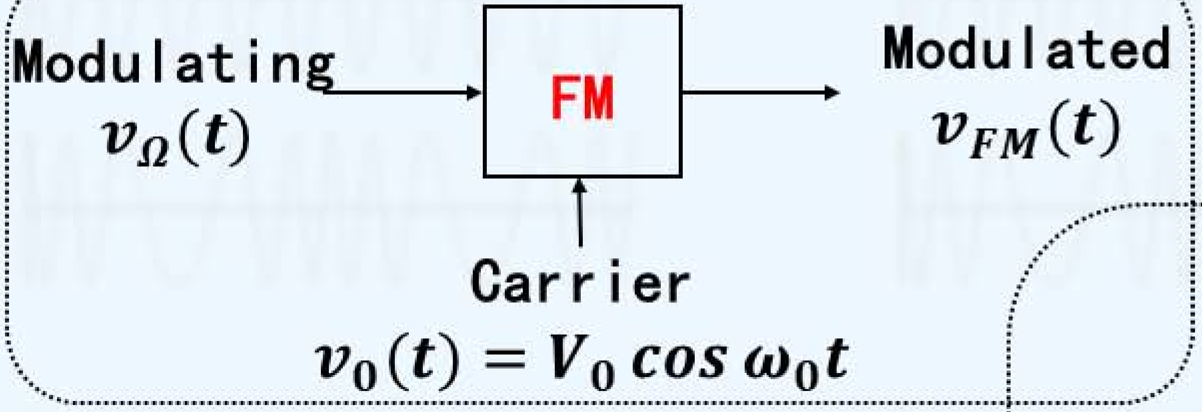
	PM	
	$\Delta\theta(t) = k_p v_\Omega(t)$	
	$v_{PM}(t) = V_0 cos[\omega_0 t + k_p v_\Omega(t)]$	
Max Phase Shift	$m_p = k_p \lvert v_\Omega(t) \rvert_{max}$	
Max Frequency Shift	$\Delta\omega_m = k_p \left\lvert \frac{dv_\Omega(t)}{dt} \right\rvert_{max}$	
	$v_\Omega(t) = V_\Omega \cos \Omega t$ (Single-tone Modulation)	
Max Phase Shift	$m_p = k_p V_\Omega$	
Max Frequency Shift	$\Delta\omega_m = k_p V_\Omega \Omega$	
	$v_{PM}(t) = V_0 cos[\omega_0 t + m_p \cos \Omega t]$	
	$\Delta\omega_m = m_p \Omega$ $\Delta f_m = m_p F$	

Frequency Modulation

FM - Time Domain

$\Delta\omega(t) = k_f v_\Omega(t)$ $\Rightarrow \Delta\theta(t) = k_f \int_0^t v_\Omega(t)dt$

k_f, Unit: rad/s·V



$$= V_0 cos[\omega_0 t + k_f \int_0^t v_\Omega(t)dt \ ]$$

$v_\Omega(t) = V_\Omega \cos \Omega t$
Single-tone

$$v_{FM}(t) = V_0 cos\left[\omega_0 t + \frac{k_f V_\Omega}{\Omega} \sin \Omega t\right]$$

$$= V_0 cos[\omega_0 t + m_f \sin \Omega t]$$

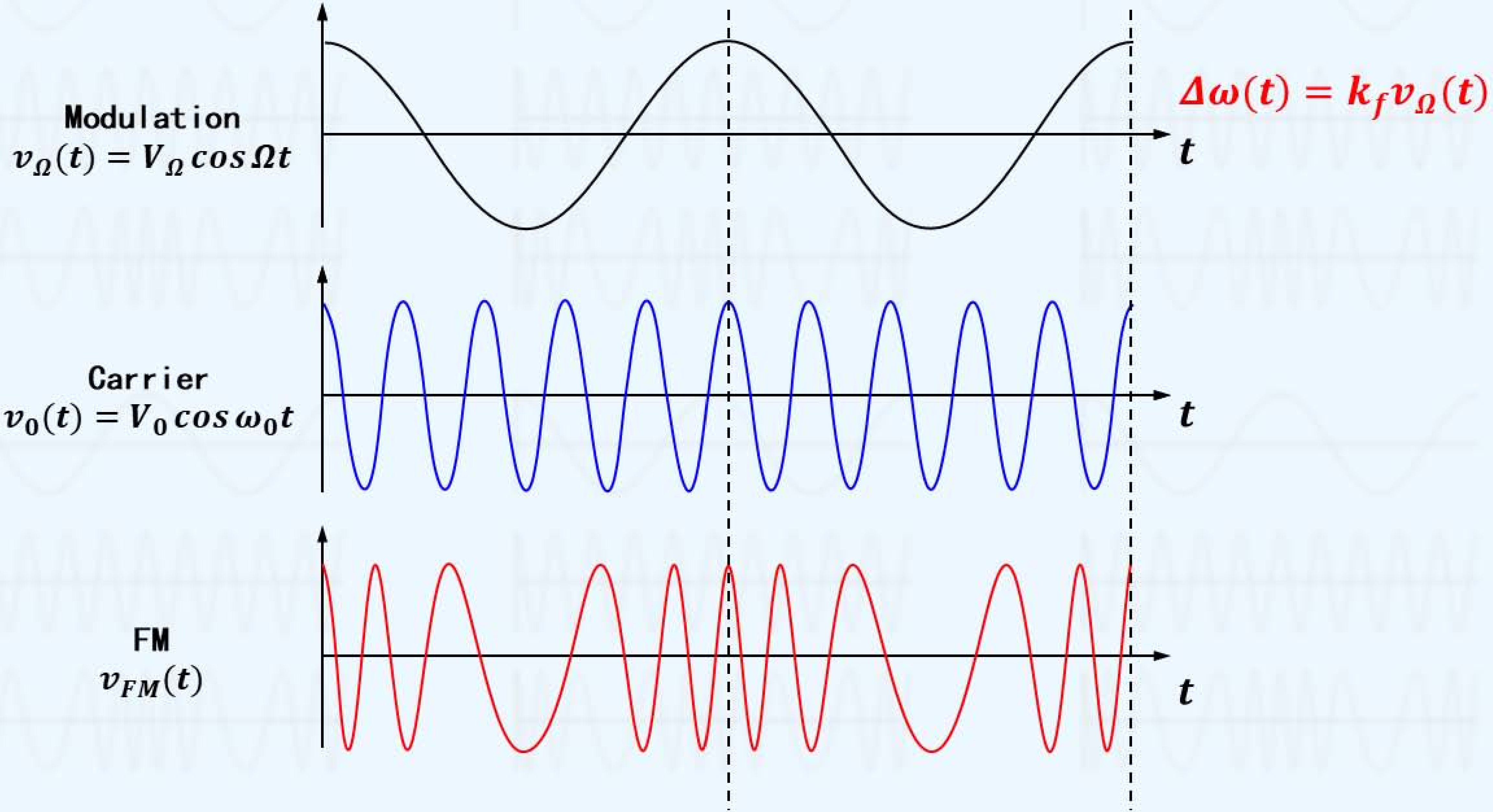
FM Index m_f

Max Phase Shift $m_f = k_f \left|\int_0^t v_\Omega(t)dt\right|_{max}$ $\xLeftrightarrow[\text{Single-tone}]{v_\Omega(t)=V_\Omega \cos \Omega t}$ $m_f = \frac{k_f V_\Omega}{\Omega}$

Max Frequency Shift $\Delta\omega_m = k_f|v_\Omega(t)|_{max}$ $\xLeftrightarrow[\text{Single-tone}]{v_\Omega(t)=V_\Omega \cos \Omega t}$ $\Delta\omega_m = k_f V_\Omega$

$$\Delta\omega_m = m_f \Omega$$
$$\Delta f_m = m_f F$$

FM - Time Domain (Single-tone Modulation)



Summary - FM

☆ m_f Essence

		FM
		$\Delta\omega(t) = k_f v_\Omega(t)$
		$\Delta\theta(t) = k_f \int_0^t v_\Omega(t)dt$
		$v_{FM}(t) = V_0 cos\left[\omega_0 t + k_f \int_0^t v_\Omega(t)dt\right]$
Max Phase Shift		$m_f = k_f \left\lvert \int_0^t v_\Omega(t)dt \right\rvert_{max}$
Max Frequency Shift		$\Delta\omega_m = k_f \lvert v_\Omega(t) \rvert_{max}$
	$v_\Omega(t) = V_\Omega\, cos\, \Omega t$ (Single-tone Modulation)	
Max Phase Shift		$m_f = \frac{k_f V_\Omega}{\Omega}$
Max Frequency Shift		$\Delta\omega_m = k_f V_\Omega$
		$v_{FM}(t) = V_0 cos[\omega_0 t + m_f\, sin\, \Omega t]$
		$\Delta\omega_m = m_f \Omega$ $\Delta f_m = m_f F$
	$\Delta f_m = m \cdot F$	

Summary - FM

☆ m_f Essence

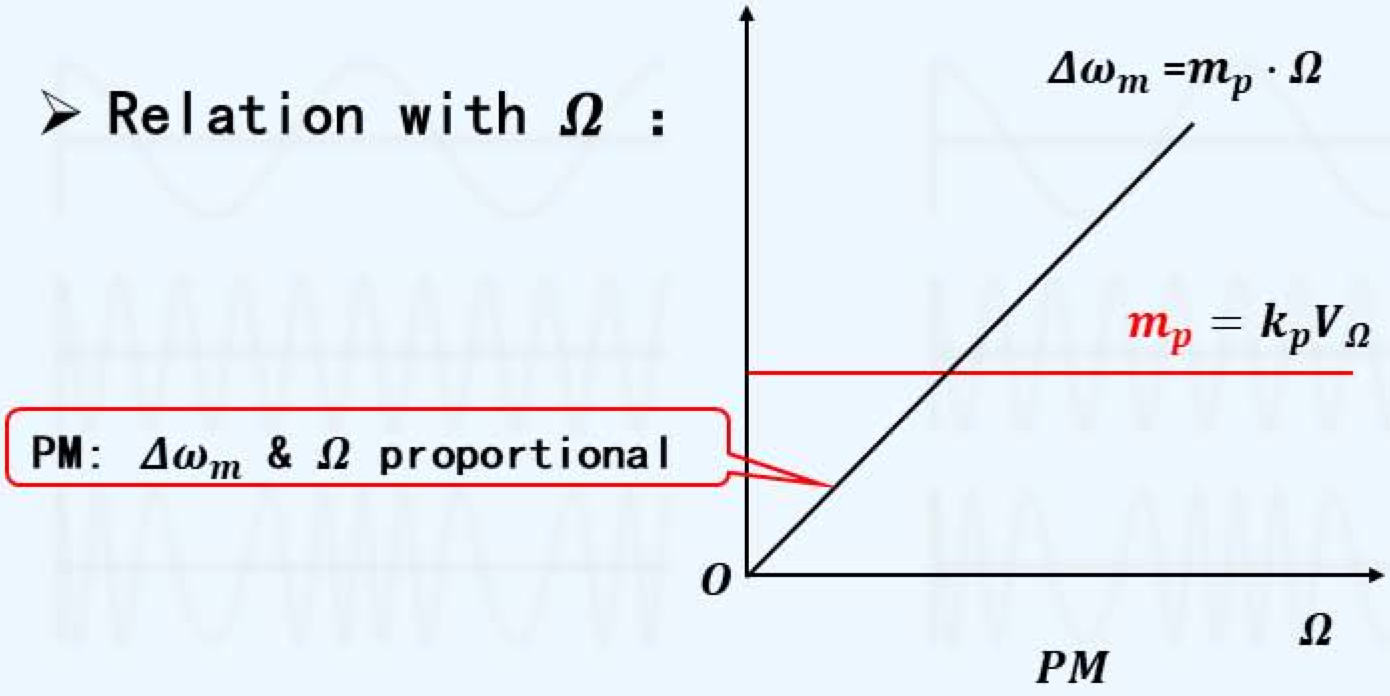
	PM	FM
		$\Delta\omega(t) = k_f v_\Omega(t)$
	$\Delta\theta(t) = k_p v_\Omega(t)$	$\Delta\theta(t) = k_f \int_0^t v_\Omega(t)dt$
	$v_{PM}(t) = V_0 cos[\omega_0 t + k_p v_\Omega(t)]$	$v_{FM}(t) = V_0 cos\left[\omega_0 t + k_f \int_0^t v_\Omega(t)dt\right]$
Max Phase Shift	$m_p = k_p \lvert v_\Omega(t)\rvert_{max}$	$m_f = k_f \left\lvert \int_0^t v_\Omega(t)dt \right\rvert_{max}$
Max Frequency Shift	$\Delta\omega_m = k_p \left\lvert \frac{dv_\Omega(t)}{dt} \right\rvert_{max}$	$\Delta\omega_m = k_f \lvert v_\Omega(t)\rvert_{max}$
	$v_\Omega(t) = V_\Omega\, cos\, \Omega t$ (Single-tone Modulation)	
Max Phase Shift	$m_p = k_p V_\Omega$	$m_f = \frac{k_f V_\Omega}{\Omega}$
Max Frequency Shift	$\Delta\omega_m = k_p V_\Omega \Omega$	$\Delta\omega_m = k_f V_\Omega$
	$v_{PM}(t) = V_0 cos[\omega_0 t + m_p\, cos\, \Omega t]$	$v_{FM}(t) = V_0 cos[\omega_0 t + m_f\, sin\, \Omega t]$
	$\Delta\omega_m = m_p\Omega$ $\Delta f_m = m_p F$	$\Delta\omega_m = m_f\Omega$ $\Delta f_m = m_f F$
	$\Delta f_m = m \cdot F$	

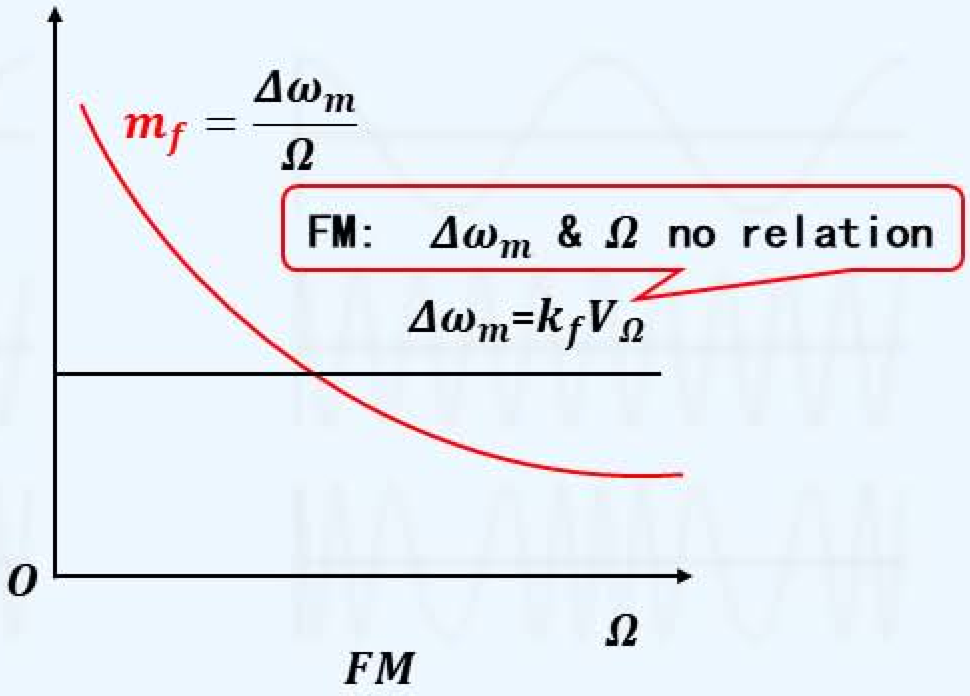
PM vs. FM

PM vs. FM

$v_\Omega(t) = V_\Omega \cos\Omega t$ (Single-tone Modulation)	
$m_p = k_p V_\Omega$	$m_f = \dfrac{k_f V_\Omega}{\Omega}$
$\Delta\omega_m = k_p V_\Omega \Omega$	$\Delta\omega_m = k_f V_\Omega$
$v_{PM}(t) = V_0 \cos[\omega_0 t + m_p \cos\Omega t]$	$v_{FM}(t) = V_0 \cos[\omega_0 t + m_f \sin\Omega t]$
$\Delta\omega_m = m_p\Omega$ $\Delta f_m = m_p F$	$\Delta\omega_m = m_f\Omega$ $\Delta f_m = m_f F$
$\Delta f_m = m \cdot F$	

➢ Relation with Ω :

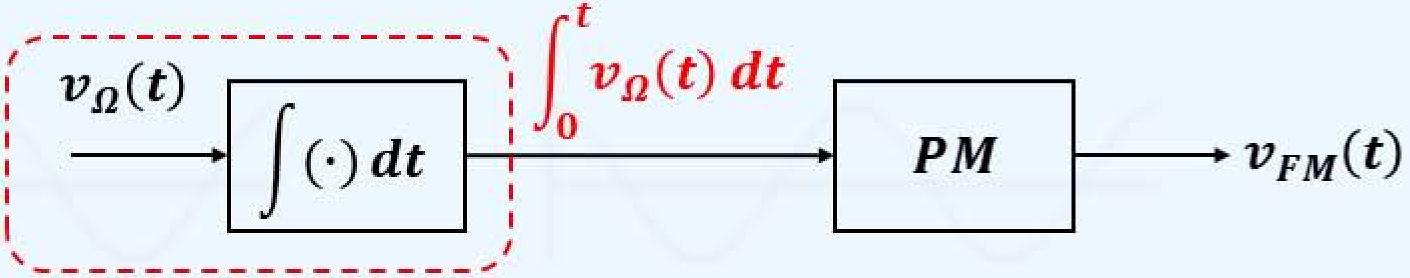




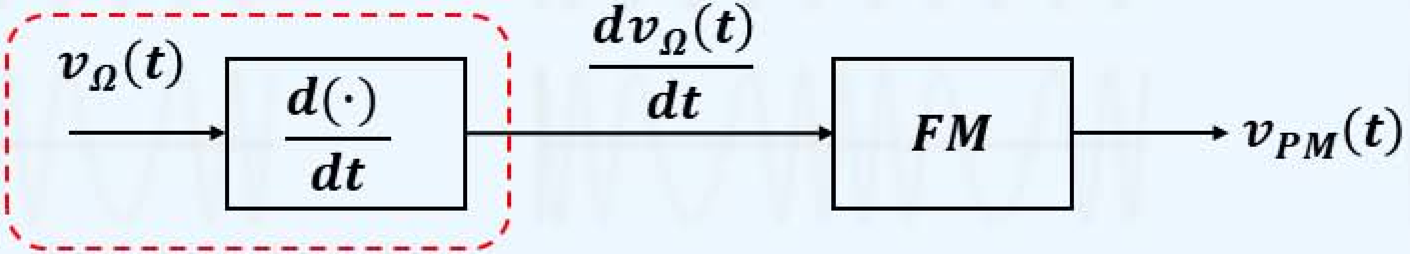
PM vs. FM

$$\begin{cases} \text{PM} & v_{PM}(t) = V_0 \cos\left[\omega_0 t + k_p v_\Omega(t)\right] \\ \text{FM} & v_{FM}(t) = V_0 \cos\left[\omega_0 t + k_f \int_0^t v_\Omega(t)\, dt\right] \end{cases}$$

- FM = PM of Modulating "$\int_0^t v_\Omega(t)\, dt$" Indirected FM



- PM = FM of Modulating "$\frac{dv_\Omega(t)}{dt}$"



FM Frequency Domain

Angle Modulation（FM Single-tone Modulation）

$J_n(m_f)$ Bessel function

$$\begin{cases} \cos(m_f \sin \Omega t) = J_0(m_f) + 2J_2(m_f)\cos 2\Omega t + 2J_4(m_f)\cos 4\Omega t + \cdots + 2J_n(m_f)\cos n\Omega t + \cdots & (n \text{ even}) \\ \sin(m_f \sin \Omega t) = \qquad +2J_1(m_f)\sin \Omega t + 2J_3(m_f)\sin 3\Omega t + \cdots + 2J_n(m_f)\sin n\Omega t + \cdots & (n \text{ odd}) \end{cases}$$

$$\begin{aligned} v_{FM}(t) &= V_0 \cos[\omega_0 t + m_f \sin \Omega t] \\ &= V_0[\boxed{\cos(m_f \sin \Omega t)}\cos\omega_0 t - \boxed{\sin(m_f \sin \Omega t)}\sin\omega_0 t] \\ &= V_0 J_0(m_f)\cos\omega_0 t \\ &\quad -V_0 J_1(m_f)[\cos(\omega_0 - \Omega)t - \cos(\omega_0 + \Omega)t] \\ &\quad +V_0 J_2(m_f)[\cos(\omega_0 - 2\Omega)t + \cos(\omega_0 + 2\Omega)t] \\ &\quad -V_0 J_3(m_f)[\cos(\omega_0 - 3\Omega)t - \cos(\omega_0 + 3\Omega)t] \\ &\quad + \cdots \end{aligned}$$

Amplitude by $J_n(m_f)$

Angle Modulation

$J_n(m_f)$ Bessel function

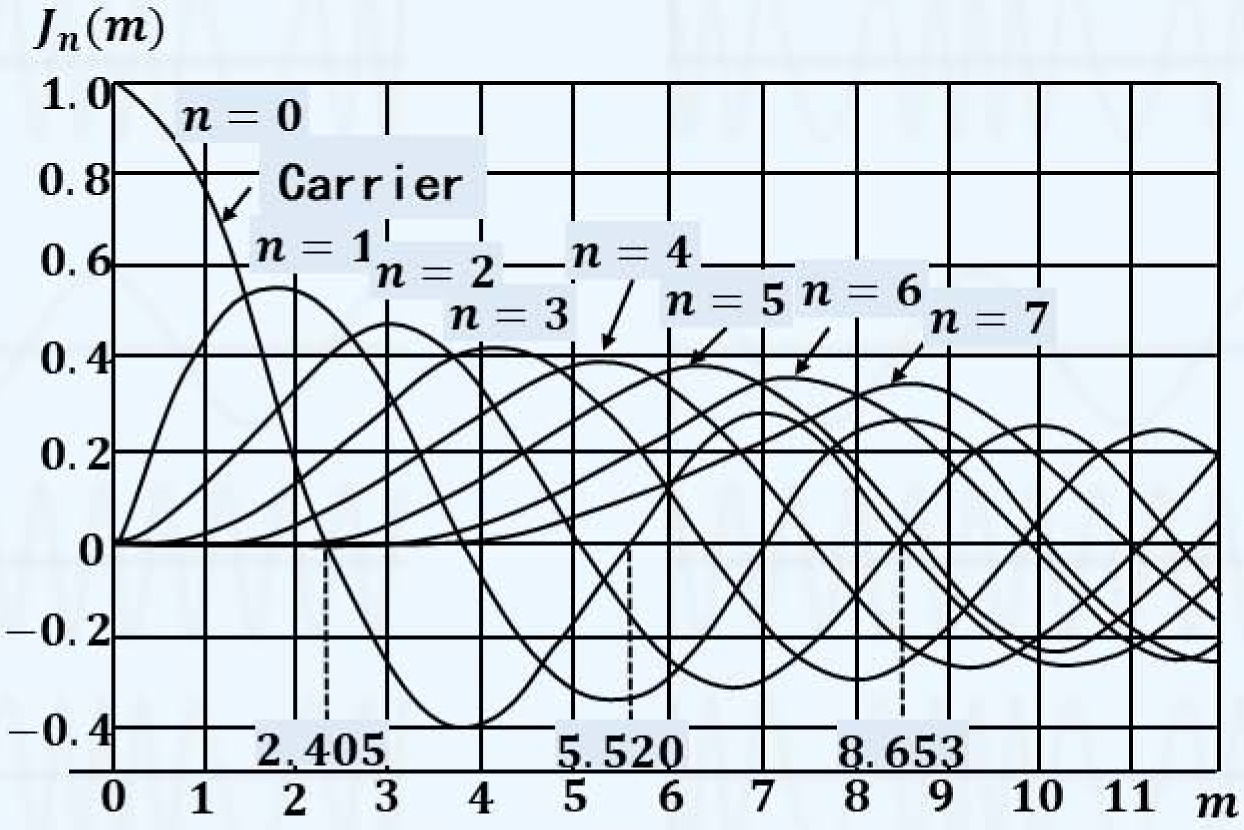
$$\begin{cases} \cos(m_f \sin\Omega t) = J_0(m_f) + 2J_2(m_f)\cos 2\Omega t + 2J_4(m_f)\cos 4\Omega t + \cdots + 2J_n(m_f)\cos n\Omega t + \cdots & (n \text{ even}) \\ \sin(m_f \sin\Omega t) = \qquad\qquad +2J_1(m_f)\sin \Omega t + 2J_3(m_f)\sin 3\Omega t + \cdots + 2J_n(m_f)\sin n\Omega t + \cdots & (n \text{ odd}) \end{cases}$$

- $n\uparrow\ m_f\uparrow,\ J_n(m_f)\downarrow$
- $J_n(m_f)$ may be positive or negative
- $J_n(m_f)$ may be 0

  Example: $m_f = 2.405,\ 5.520,\ 8.653$

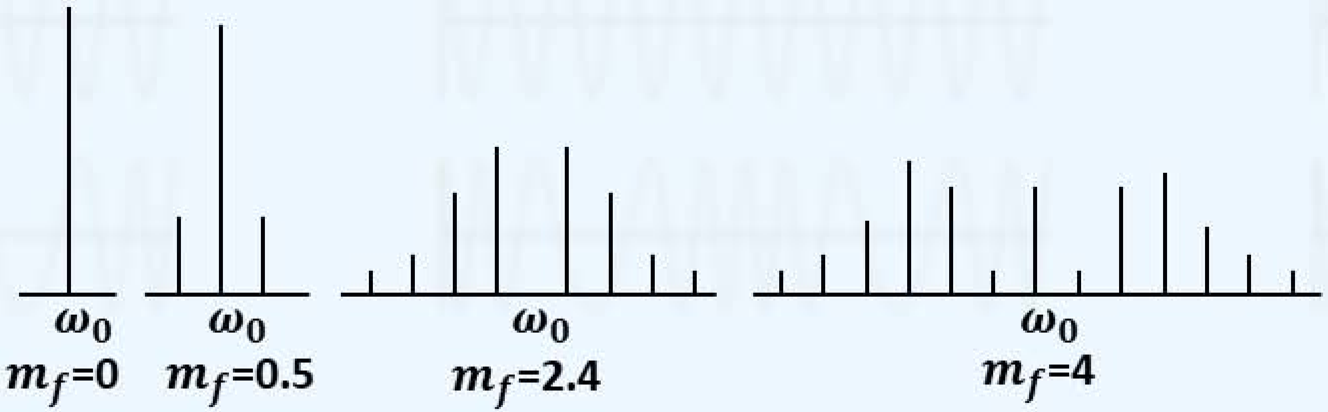
  $J_0(m_f) = 0$

To measure FM Index



Bessel Curve

Angle Modulation – Frequency Domain



FM Spectrum with Single-tone Modulation

- $m_f \uparrow$ Side frequency with high amplitude $\uparrow$
- $n \uparrow$ Amplitude $\downarrow$

Angle Modulation - Power

- Average Power (Single-tone)

≡1 (Bessel Function)

$$P = \frac{1}{2}\frac{V_0^2}{R_L}\{J_0^2(m) + 2[J_1^2(m) + J_2^2(m) + \cdots + J_n^2(m) + \cdots]\}$$

$$= \frac{1}{2}\frac{V_0^2}{R_L}$$

Befor Modulation

- Angle Modulation: Reallocate to side frequencies with fixed total Power

  ($vs.$ Amplitude Modulation)

- Major energy located around carrier

- $n > m$, $n\uparrow$ $J_n(m)\downarrow$

  $n > (m+1)$时, $|J_n(m)| < 0.1$

Effective side frequencies→Effective bandwidth (Neglect side frequencies whose amplitude<10%)

Angle Modulation : Cason Rule

- Effective side frequencies: $2(m+1)$
- Bandwidth

$$\Delta f_m = m \cdot F$$

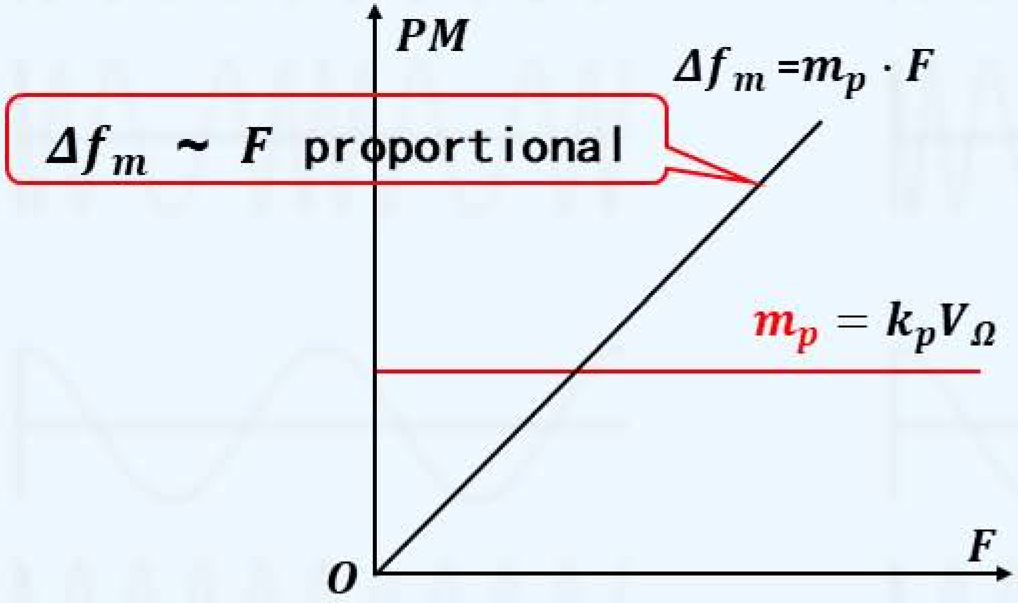
$$B_{PM/FM} = 2(m+1)F = 2(\Delta f_m + F)$$

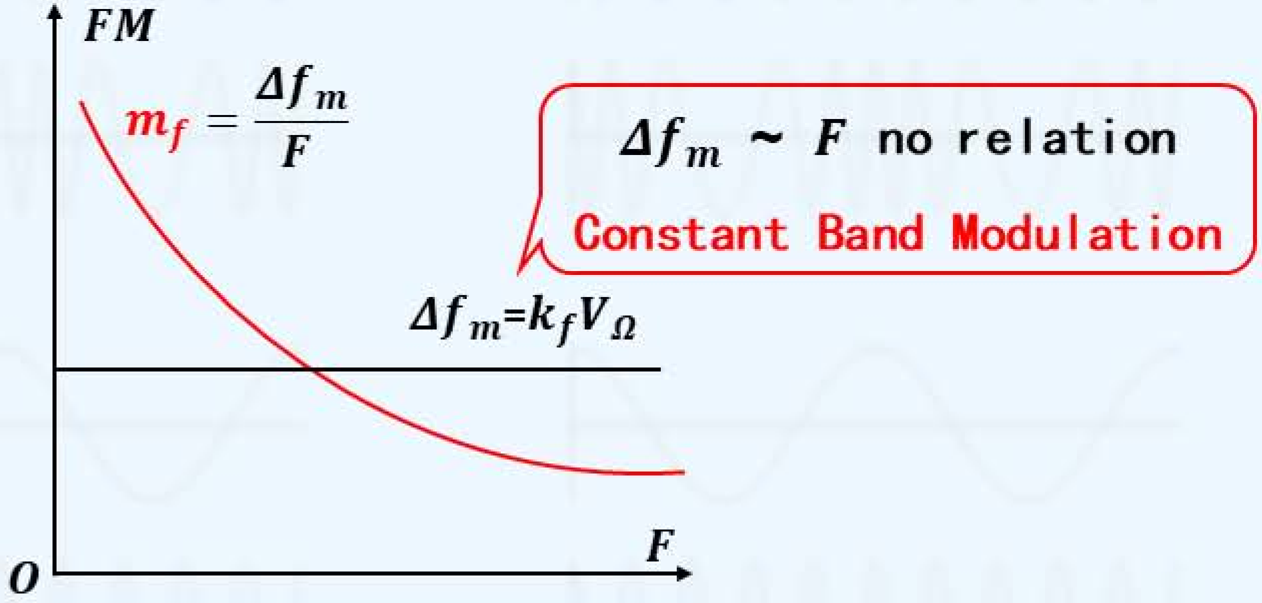
- Note: PM and FM use m_p and m_f, respectively

Angle Modulation : Cason Rule

$$B_{PM} = 2(m_p + 1)F$$

$$B_{FM} = 2(\Delta f_m + F)$$





PM:

m_p ~ F no relation

Δf_m & B_{PM} ~ F proportional

FM:

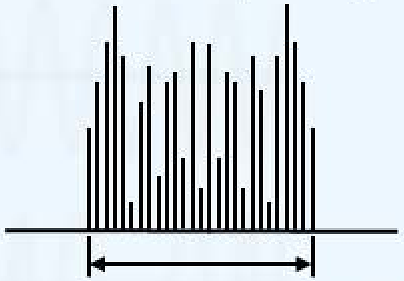
m_f ~ F reciprocal

Δf_m fixed, B_{FM} basically unchanged

Angle Modulation – Bandwidth

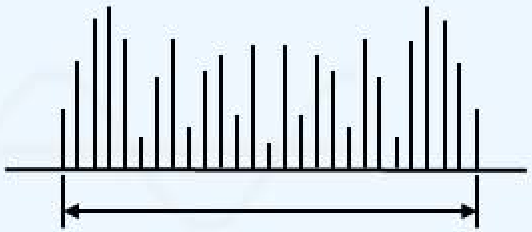
$$B_{PM/FM} = 2(m+1)F = 2(\Delta f_m + F)$$

PM Frequency Domain



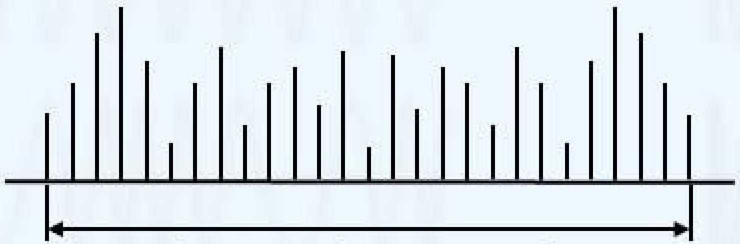
$2(m_p+1)F = 26kHz$

$F = 1kHz$
$m_p = 12$



$2(m_p+1)F = 52kHz$

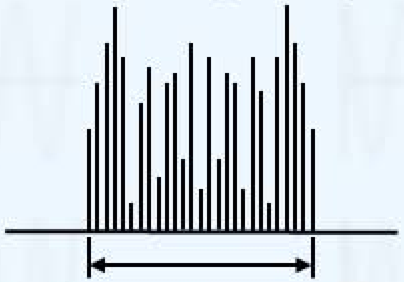
$F = 2kHz$
$m_p = 12$



$2(m_p+1)F = 104kHz$

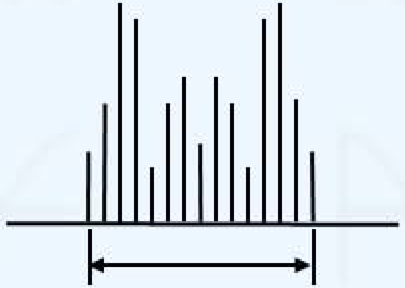
$F = 4kHz$
$m_p = 12$

FM Frequency Domain



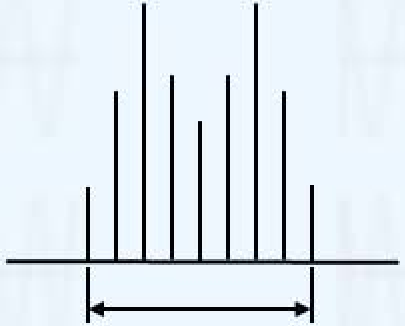
$2(\Delta f_m + F) = 26kHz$

$F = 1kHz$
$m_f = 12$



$2(\Delta f_m + F) = 28kHz$

$F = 2kHz$
$m_f = 6$



$2(\Delta f_m + F) = 32kHz$

$F = 4kHz$
$m_f = 3$