§ 8.6 Plane wave in a good conductor

1.Propagation characteristics

The total current in a conducting medium includes the conduction current and the displacement current. In terms of

$$\tan\delta = \frac{\sigma}{\omega\varepsilon} = \frac{\sigma\vec{E}}{\omega\varepsilon\vec{E}} = \frac{\left|\vec{J}_c\right|}{\left|\vec{J}_d\right|}$$

Any increase in the conduction current is accompanied by an increase in the loss tangent angle. However, for this case to happen either the conductivity σ of the medium is very high or the wave frequency is low. In either case, the conducting medium behaves as a good conductor (or a high-loss material) as long as

$$\sigma >> \omega\varepsilon \qquad (8.6.1)$$

the complex permittivity

In terms of (8.4.1), the definition of complex permittivity

$$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega} = \varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)$$ **can be rewritten as**

$$\varepsilon_c \approx \frac{\sigma}{j\omega} \qquad (8.6.2)$$

thus, the first term is neglected.

propagation constant

in terms of (8.6.2), the propagation constant can be rewritten as

$$\gamma = j\omega\sqrt{\mu\varepsilon_c} \approx j\omega\sqrt{\mu\frac{\sigma}{j\omega}} = \sqrt{j\omega\mu\sigma}$$

$$= \sqrt{\omega\mu\sigma}\ \angle 45^0$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}}(1+j)$$

$$= \alpha + j\beta$$

therefore, the attenuation constant is

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

and the phase constant is

$$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

we can also obtain the approximate equations of the intrinsic impedance, the phase speed, and the skin depth as

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} \approx \sqrt{\frac{\mu}{\sigma/(j\omega)}} = \sqrt{j\frac{\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\ \angle 45^0$$

$$= \sqrt{\frac{\omega\mu}{2\sigma}}(1+j)$$

$$\upsilon_p = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}} \qquad \delta_c = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

from these equations it is obvious that α, β, η_c and υ_p vary directly as $\omega^{1/2}$. Thus, the shape of a wave comprising many different frequencies will keep on changing as it progresses; that is, the signal is distorted by the time it reaches its destination.

A medium in which a signal becomes distorted is said to be a dispersive medium; a conducting medium, in general, is a dispersive medium.

For all practical purposes, the wave vanishes after traveling a distance of $5\delta c$ in a conducting medium. The skin depth δc =0.07mm for a copper at a frequency of 1 MHz. The amplitude of the wave becomes insignificant after penetrating a distance of 0.35mm.In good conductors, the wave attenuates very rapidly and the fields are confined to the region near the surface of the conductor.

The phenomenon is called the skin effect.

- **Surface resistance**

Let the electric field intensity in a good conductor be

$$\dot{\vec{\boldsymbol{E}}} = \dot{E}e^{-\gamma z}\vec{\boldsymbol{a}}_x$$

(a forward-travelling wave, the *x* direction)

Neglecting the displacement current density in a good conductor, the total current is

$$\dot{\vec{J}}_c = \sigma\dot{\vec{E}} = \sigma\dot{E}e^{-\gamma z}\vec{\mathbf{a}}_x$$

the average power dissipated (power loss) per unit volume is

$$W_{Lave} = \frac{1}{2}\dot{\vec{E}} \bullet \dot{\vec{J}}^* = \frac{1}{2}\sigma E^2 e^{-2\alpha z}$$

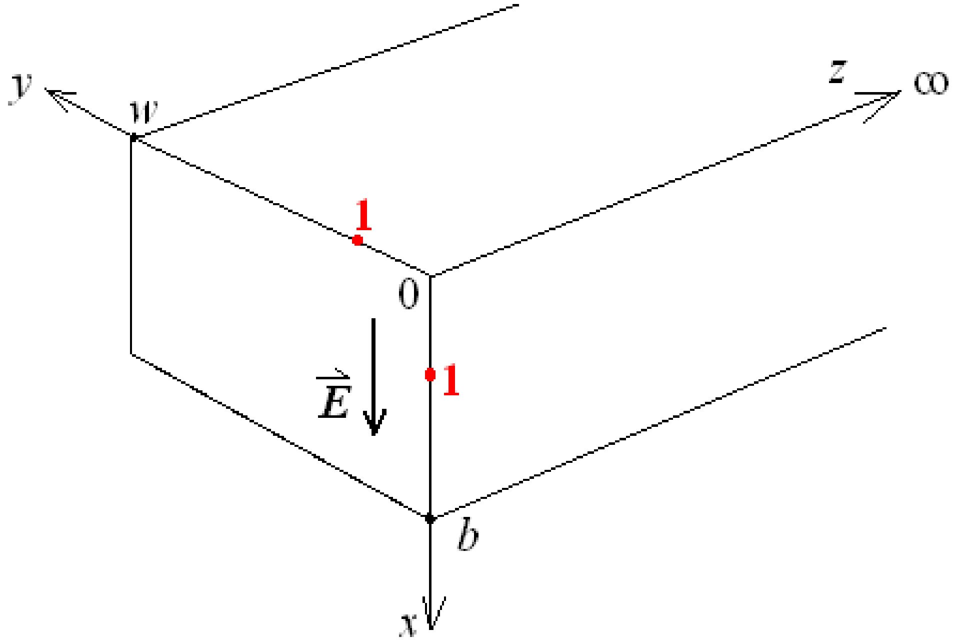
Let us concentrate on a region bounded by $0 \le x \le b$, $0 \le y \le w$, and $0 \le z \le \infty$. Then, the total power dissipated within this region is

$$P_{Lave} = \int_v W_{Lave} dv$$

$$= \frac{1}{2}\sigma E^2 \int_v e^{-2\alpha z} dv$$

$$= \frac{1}{2}\sigma E^2 \int_0^b dx \int_0^w dy \int_0^\infty e^{-2\alpha z} dz$$

$$= \frac{1}{4\alpha}\sigma E^2 bw \qquad \textbf{(8.6.3)}$$



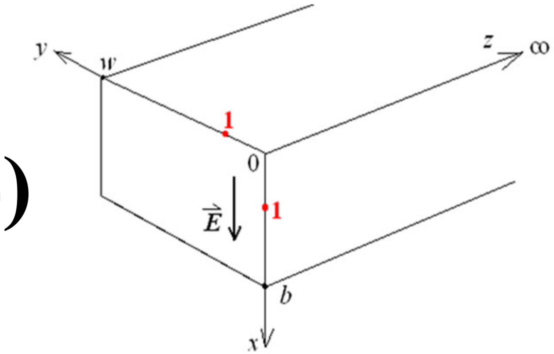
the total current along the *x* direction is

$$\dot{I} = \int_s \dot{\vec{J}}_c \bullet d\vec{s} = \int_s \dot{\vec{J}}_c \bullet \vec{a}_x ds_{yoz} = \sigma \dot{E} \int_0^w dy \int_0^\infty e^{-\gamma z} dz = \frac{\sigma \dot{E} w}{\gamma}$$

or $$I^2 = \dot{I} \bullet \dot{I}^* = \frac{\sigma \dot{E} w}{\gamma} \bullet \left(\frac{\sigma \dot{E} w}{\gamma} \right)^* = \left(\frac{\sigma E w}{\sqrt{2}\alpha} \right)^2$$

if *R* is the resistance of the block, then the power that it dissipates is

$$P_{Lave} = \frac{1}{2} I^2 R = \frac{1}{4} \left(\frac{\sigma E w}{\alpha} \right)^2 R \qquad \textbf{(8.6.4)}$$



comparing (8.6.3)

$$P_{Lave} = \int_v W_{Lave} dv = \frac{1}{2}\sigma E^2 \int_v e^{-2\alpha z} dv$$

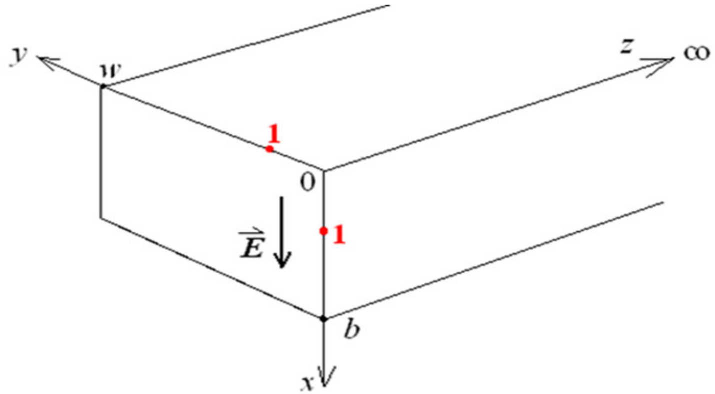
$$= \frac{1}{2}\sigma E^2 \int_0^b dx \int_0^w dy \int_0^\infty e^{-2\alpha z} dz \quad = \frac{1}{4\alpha}\sigma E^2 bw$$

and (8.6.4), that is

$$\frac{1}{4\alpha}\sigma E^2 bw = \frac{1}{4}\left(\frac{\sigma E w}{\alpha}\right)^2 R$$

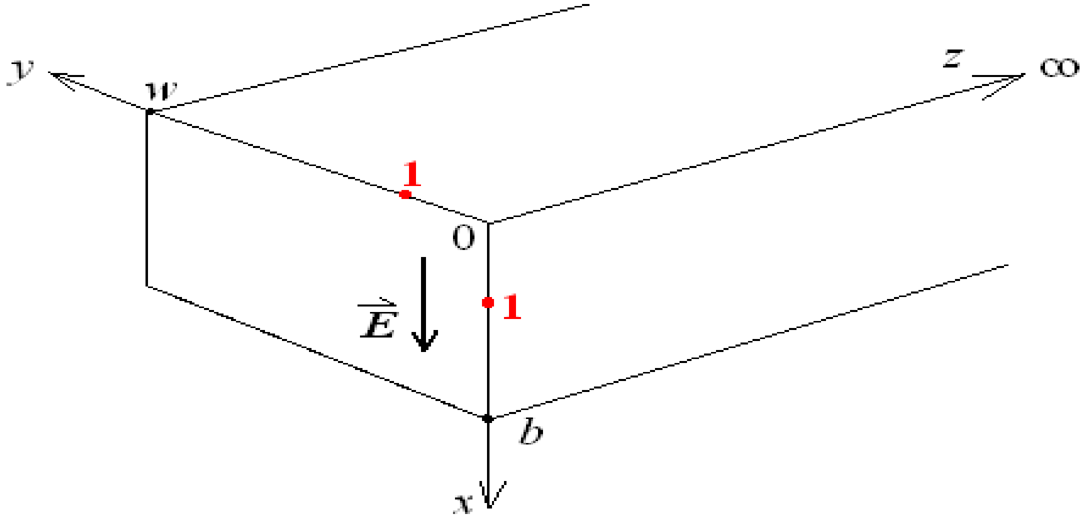
$$\frac{\alpha b}{\sigma w} = R$$

$$\frac{b}{\sigma w \delta_c} = R$$



we obtain the resistance of the block as

$$R = \frac{b}{\sigma w \delta_c}$$

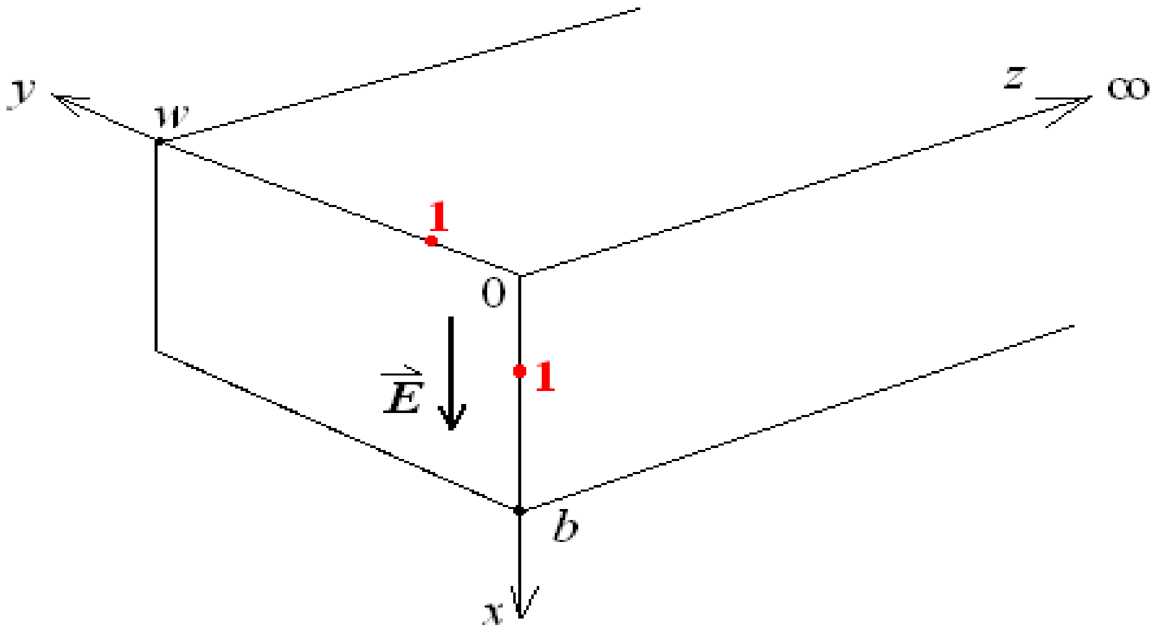


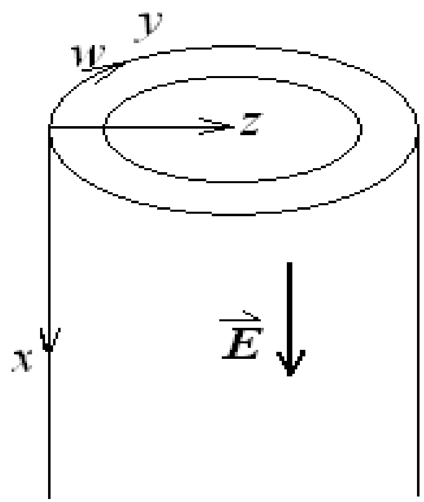
the skin resistance or surface resistivity is defined as the resistance of a plane conductor of unit length(b=1), unit width(w=1), and thickness δ_c. thus,

$$R_s = \frac{1}{\sigma \delta_c} \qquad \textbf{(8.6.5)}$$

equation(8.6.5)can also be used to obtain an approximate value of the skin resistance for a cylindrical conductor. When the current is along the length of a cylindrical conductor of radius *a* such that $a > \delta c$, the skin resistance per unit length is

$$R_s = \frac{1}{2\pi a \sigma \delta_c}$$





because $w=2\pi a$

§ 8.7 Plane wave in a good dielectric

A good dielectric is a conducting medium in which the displacement current dominates the conduction current. A poorly conducting medium may be viewed as a good dielectric as long as

$$\sigma << \omega\varepsilon \tag{1}$$

Note that this condition is satisfied when the conductivity of the medium is low or the wave frequency is very high.

Thus, the first-order approximation for $\sqrt{\varepsilon_c}$ in a good dielectric medium, using the binomial expansion, is

$$\sqrt{\varepsilon_c} = \sqrt{\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)} \approx \sqrt{\varepsilon}\left(1 - j\frac{\sigma}{2\omega\varepsilon}\right)$$

So the approximate expression for the propagation constant is

$$\gamma = j\omega\sqrt{\mu\varepsilon_c} = j\omega\sqrt{\mu\varepsilon}\left(1 - j\frac{\sigma}{2\omega\varepsilon}\right)$$

$$= \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} + j\omega\sqrt{\mu\varepsilon} = \alpha + j\beta$$

similarly, the attenuation constant and the phase constant in a good dielectric are

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]} \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]} \approx \omega\sqrt{\mu\varepsilon}$$

the wave impedance for a good dielectric becomes

$$\eta_c = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{j\omega\sqrt{\mu\varepsilon_c}} = \frac{\sqrt{\mu}}{\sqrt{\varepsilon_c}} \approx \frac{\sqrt{\mu}}{\sqrt{\varepsilon}\left(1 - j\dfrac{\sigma}{2\omega\varepsilon}\right)}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \frac{\left(1 + j\dfrac{\sigma}{2\omega\varepsilon}\right)}{\left[1 + \left(\dfrac{\sigma}{2\omega\varepsilon}\right)^2\right]} \approx \sqrt{\frac{\mu}{\varepsilon}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right)$$

$$\approx \sqrt{\frac{\mu}{\varepsilon}}$$

Example 8.4 you read by yourselves.
Example 8.5 you read by yourselves.

§7-3 沿任意方向传播的均匀平面波

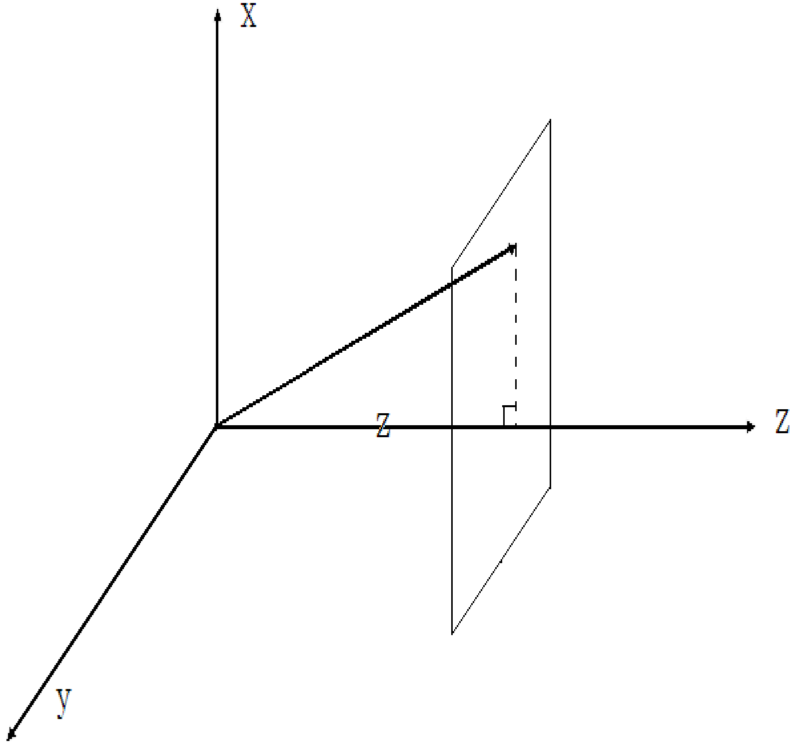
中文教材

§7-3 沿任意方向传播的均匀平面波

- 1.沿+z方向传播的波

$$\dot{\vec{E}} = \dot{\vec{E}}_0 e^{-jkz}$$

等相位面沿+z
方向移动如图示。



那么对于等相位面z处上任一点 $\vec{r}(x.y.z)$.

场量表达式如何表示?

§ 7-3 沿任意方向传播的均匀平面波

场量表达式为：

$$\dot{E}=\dot{E}_0 e^{-jkz}$$

$$=\dot{E}_0 e^{-jk\vec{a}_n \bullet \vec{r}}$$

$$\begin{cases} \vec{r}=\vec{a}_x x+\vec{a}_y y+\vec{a}_z z \\ \vec{a}_n \text{为等相位面方向（移动方向）} \\ \vec{a}_n=\vec{a}_z \end{cases}$$

此时 e^{-jkz} 中应注意

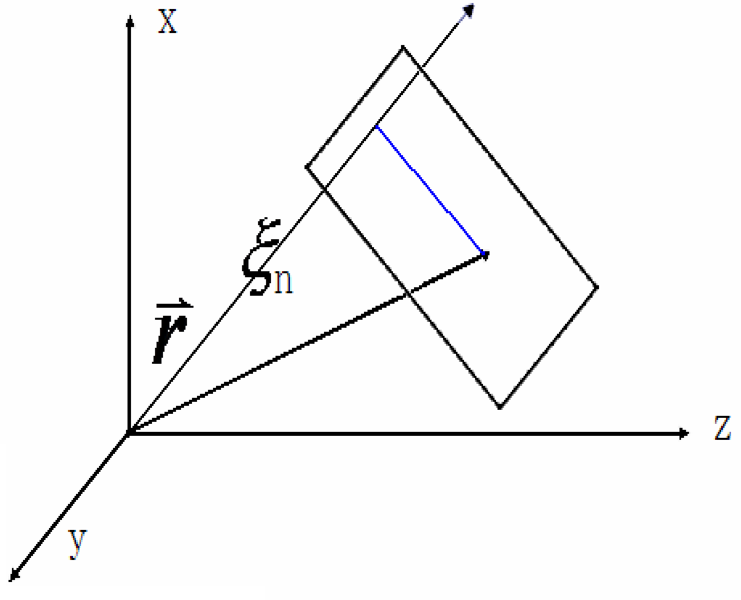
k 是波 z 方向传播的相位常数

z 是等相位面到原点的距离

等相位面沿 $\vec{a}_z=\vec{a}_n$ 方向传播

§7-3 沿任意方向传播的均匀平面波

- 2.沿任意方向传播的波



$$\dot{E}=\dot{E}_0 e^{-jk_n\xi_n}=\dot{E}_0 e^{-jk_n\vec{a}_n\cdot\vec{r}}$$

$$=\dot{E}_0 e^{-j\vec{k}\cdot\vec{r}} \qquad \vec{r}=\vec{a}_x x+\vec{a}_y y+\vec{a}_z z$$

$$\vec{k}=\vec{a}_x k_x+\vec{a}_y k_y+\vec{a}_z k_z$$

$$=\dot{E}_0 e^{-j(k_x x+k_y y+k_z z)}$$

$\vec{k}=k\vec{a}_n=\vec{a}_x k_x+\vec{a}_y k_y+\vec{a}_z k_z$ 代表波传播方向，称波矢量。

$k=\beta-j\alpha$ ----导电媒质

$k=\beta=\omega\sqrt{\mu\varepsilon}$ ----电介质 $\quad k=\sqrt{k_x^2+k_y^2+k_z^2}$

§7-3 沿任意方向传播的均匀平面波

- 3.电磁场之间关系

$$\left.\begin{aligned}\dot{\vec{E}} &= \eta \dot{\vec{H}} \times \vec{a}_z \\ \dot{\vec{H}} &= \frac{1}{\eta}\vec{a}_z \times \dot{\vec{E}}\end{aligned}\right\} \Longrightarrow \left\{\begin{aligned}\dot{\vec{E}} &= \eta \dot{\vec{H}} \times \vec{a}_n \\ \dot{\vec{H}} &= \frac{1}{\eta}\vec{a}_n \times \dot{\vec{E}}\end{aligned}\right.$$

$\dot{\vec{E}}, \dot{\vec{H}}, \vec{a}_n(\vec{k})$构成右手螺旋关系

- 4.有关计算

$$\nabla e^{-j\vec{k}\bullet\vec{r}} = \left(\vec{a}_x\frac{\partial}{\partial x} + \vec{a}_y\frac{\partial}{\partial y} + \vec{a}_z\frac{\partial}{\partial z}\right)e^{-j(k_x x + k_y y + k_z z)}$$

$$= \left(\vec{a}_x(-jk_x) + \vec{a}_y(-jk_y) + \vec{a}_z(-jk_z)\right)e^{-j(k_x x + k_y y + k_z z)}$$

$$= -j\vec{k}e^{-j\vec{k}\bullet\vec{r}}$$

§7-3 沿任意方向传播的均匀平面波

$$\nabla\bullet\vec{H}=\nabla\bullet(\vec{H}_0 e^{-j\vec{k}\bullet\vec{r}})=\nabla e^{-j\vec{k}\bullet\vec{r}}\bullet\vec{H}_0+e^{-j\vec{k}\bullet\vec{r}}\nabla\bullet\vec{H}_0$$

$$=\nabla e^{-j\vec{k}\bullet\vec{r}}\bullet\vec{H}_0+e^{-j\vec{k}\bullet\vec{r}}\nabla\bullet\vec{H}_0=-j\vec{k}\bullet\vec{H}_0=0$$

$$\nabla\times\vec{H}=\nabla\times(\vec{H}_0 e^{-j\vec{k}\bullet\vec{r}})=\nabla e^{-j\vec{k}\bullet\vec{r}}\times\vec{H}_0+e^{-j\vec{k}\bullet\vec{r}}\nabla\times\vec{H}_0$$

$$=-j\vec{k}e^{-j\vec{k}\bullet\vec{r}}\times\vec{H}_0=-j\vec{k}\times\vec{H}$$

例：已知空气中一均匀平面波的磁场强度复矢量为

$$\vec{H}=(-\vec{a}_x A+\vec{a}_z 4)e^{-j\pi(4x+3z)} \quad mA/m$$

求:(1)波的传播方向(单位矢量)

(2)参数A

(3)电场强度复矢量

(4) 波长及平均poynting矢量。

§7-3 沿任意方向传播的均匀平面波

$$\vec{r}=\vec{a}_x x+\vec{a}_y y+\vec{a}_z z$$

解：$\dot{\vec{H}}=\dot{\vec{H}}_0 e^{-j\vec{k}\bullet\vec{r}}=\dot{\vec{H}}_0 e^{-j(k_x x+k_y y+k_z z)}$

$$\vec{k}=\vec{a}_x k_x+\vec{a}_y k_y+\vec{a}_z k_z$$

(1)因为 $\vec{k}\bullet\vec{r}=k_x x+k_y y+k_z z=4\pi x+3\pi z$

所以 $k_x=4\pi, k_z=3\pi, k_y=0$. 于是 $k=(k_x{}^2+k_z{}^2)^{1/2}=5\pi$.

$$\vec{k}=\vec{a}_x k_x+\vec{a}_y k_y+\vec{a}_y k_z=\vec{a}_x 4\pi+\vec{a}_z 3\pi$$

$$\vec{a}_n=\vec{k}/k=(\vec{a}_x k_x+\vec{a}_y k_y+\vec{a}_y k_z)/k=\vec{a}_x\frac{4}{5}+\vec{a}_z\frac{3}{5}$$

(2)因为 $\nabla\bullet\vec{H}=-j\vec{k}\bullet\vec{H}=0$

所以 可求得**A=3.**

(3)因为

$$\dot{\vec{E}}=\eta\dot{\vec{H}}\times\vec{a}_n=377(-\vec{a}_x 3+\vec{a}_z 4)e^{-j\pi(4x+3y)}\times(\vec{a}_x 4/5+\vec{a}_z 3/5)$$

§7-4电磁波的极化

引言:平面波的电场方向可随时间按一定的规律变化.在空间任一固定点上电磁波的电场强度矢量$\vec{E}$的空间取向随时间变化的方式称电磁波的极化(物理学称为偏振)。

- 电磁波的极化可用$\vec{E}$的矢端轨迹来描述.
- 对于均匀平面波,在空间所有点上,波的极化状态都相同.
- 如果均匀平面波沿+Z方向传播, $\vec{E}.\vec{H}$ 均在XY平面内,一般$\vec{E}$有两个分量:

$$\begin{cases} E_x = E_{x0}\cos(\omega t - kz + \varphi_x) \\ E_y = E_{y0}\cos(\omega t - kz + \varphi_y) \end{cases}$$

- 不妨讨论Z=0平面,合成矢量 $\vec{E}$矢端的轨迹. →确定极化状态：线极化、圆极化、椭圆极化等。
- 待续