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Electronic Circuit of Communications

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and Commnications

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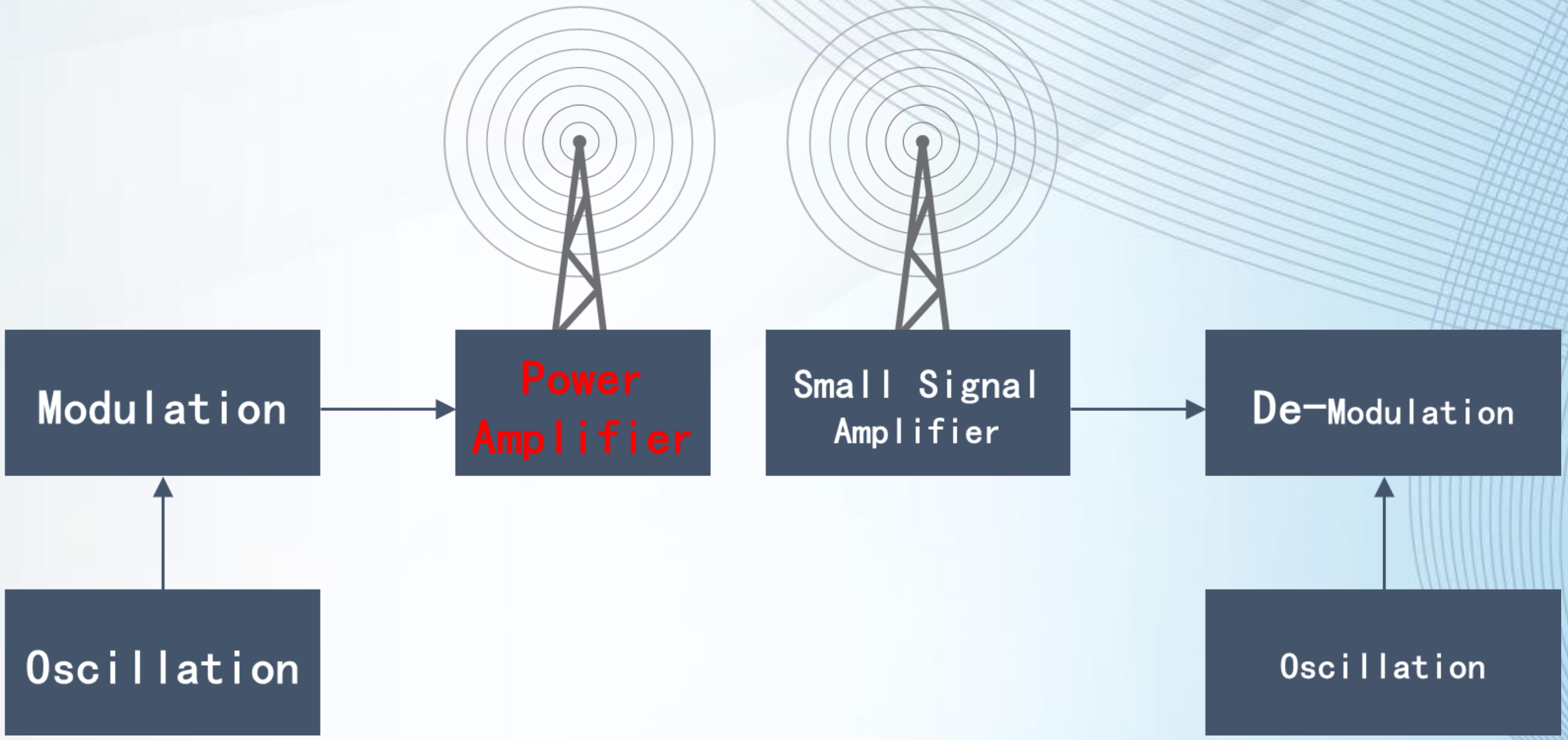
RF Power Amplifier

Radio Frequency Power Amplifier

- Objectives

  ① $P_o \uparrow$

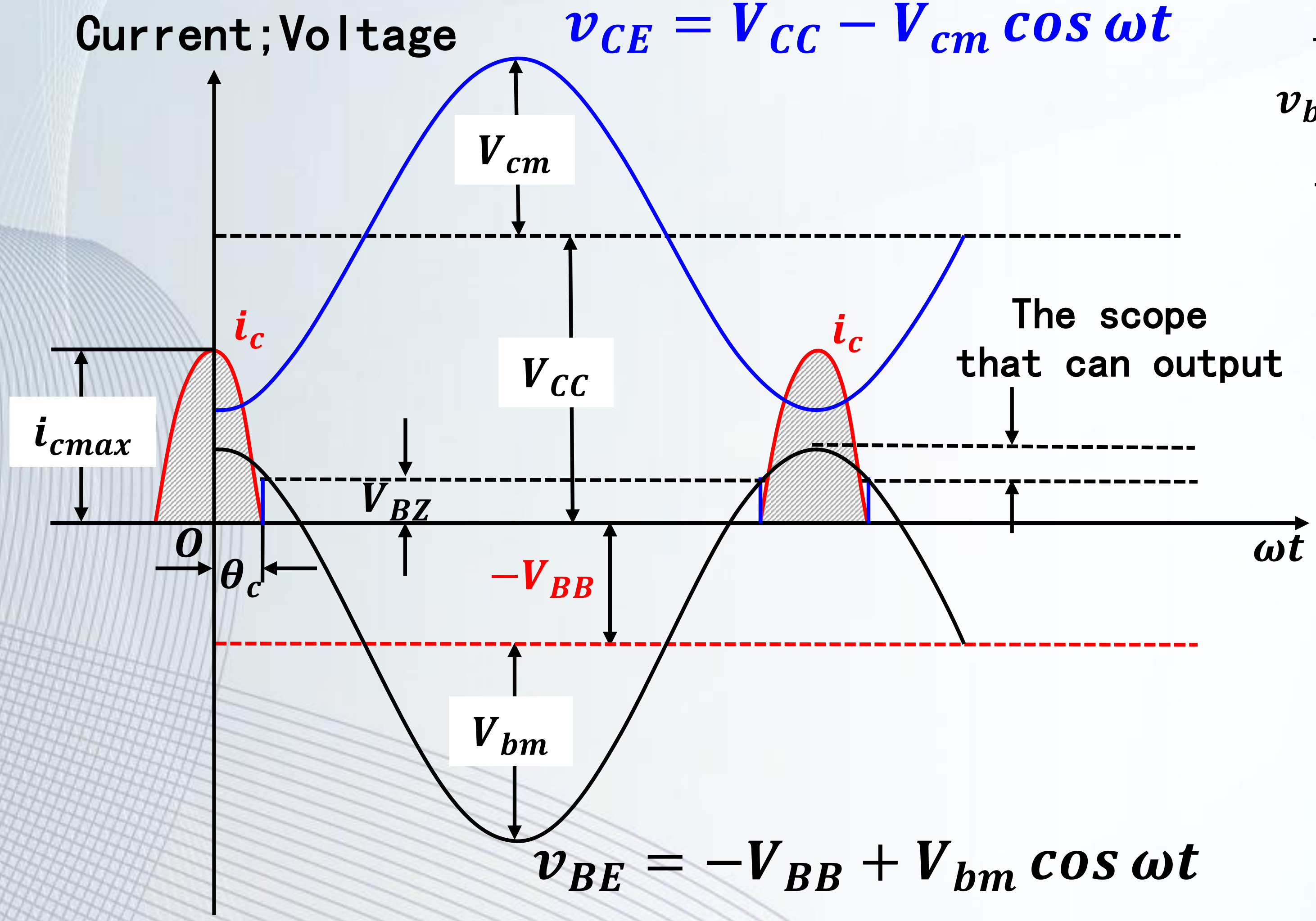
  ② $\eta_c \uparrow$

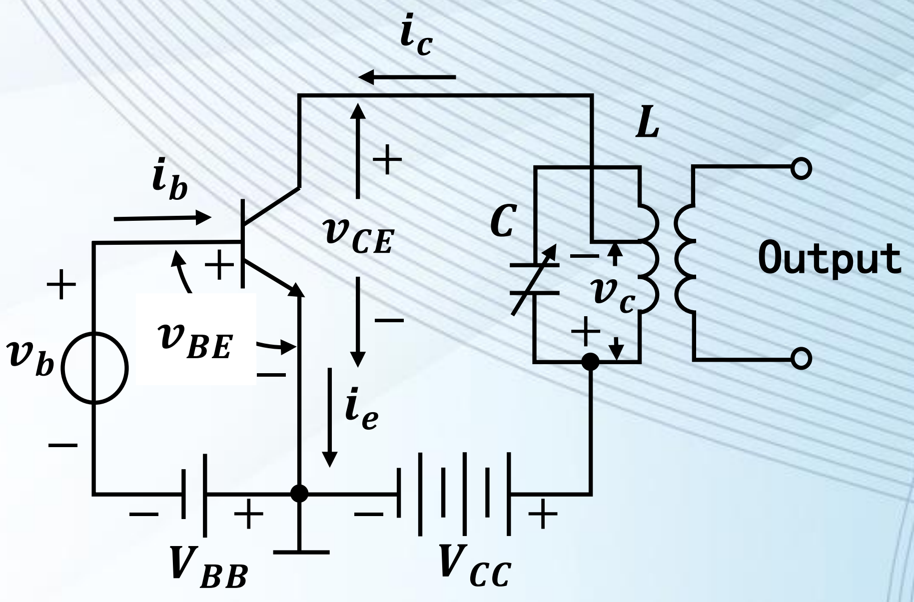


- Comparison

  - RF Power Amplifier *vs.* RF Small Signal Amplifier?
  - RF Power Amplifier *vs.* Audio Power Amplifier?

RF Power Amplifier — Principle

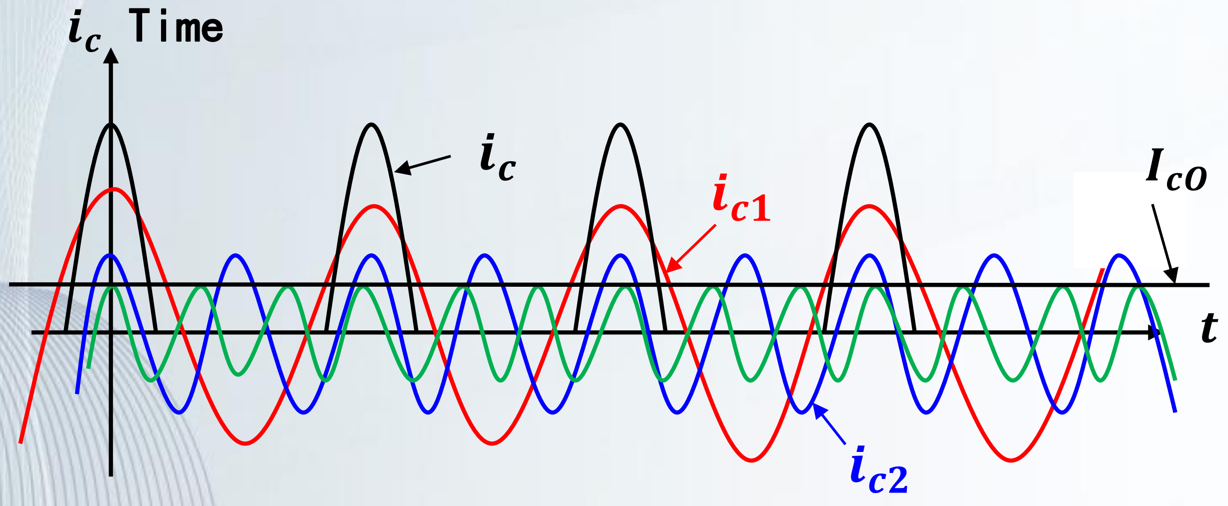


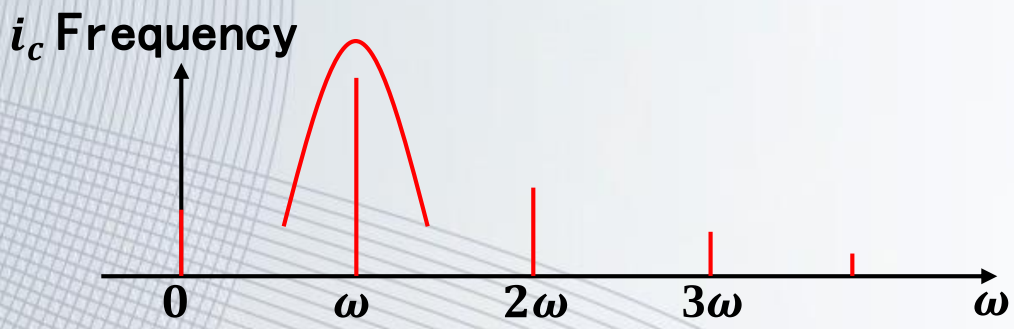


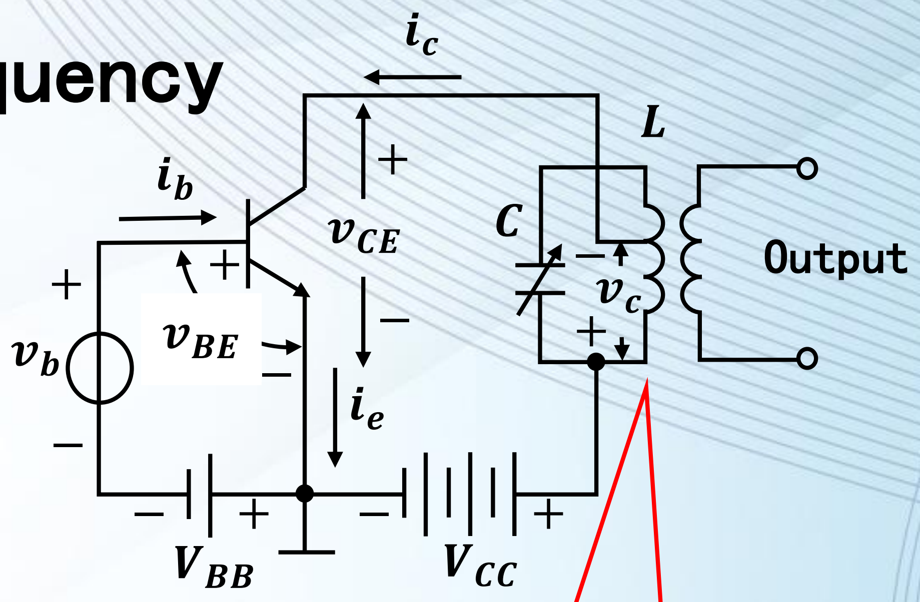
Key Concepts

- Reversely Biased
- $\theta_c < 90^\circ$ (Class C)
- Cosine Pulses i_c
- Nonlinear+Linear

RF Power Amplifier — Time & Frequency





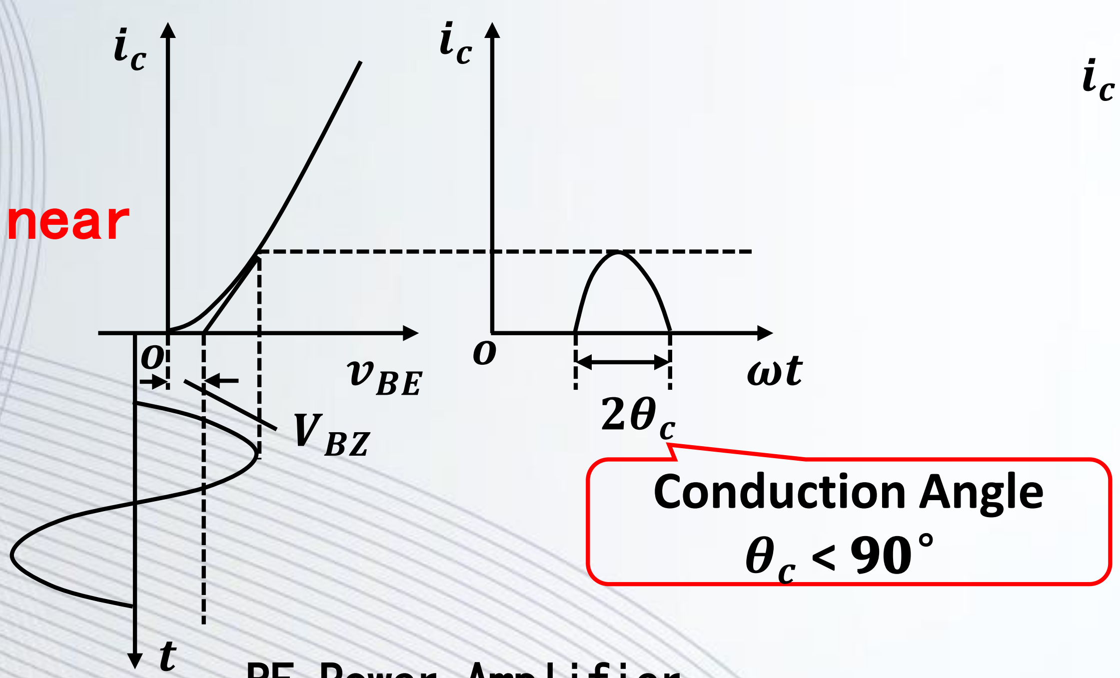


$$
\begin{aligned}
v_c &= i_{c1} \cdot R_P \\
&= I_{cm1} \cos \omega t \cdot R_P \\
&= V_{cm} \cos \omega t
\end{aligned}
$$

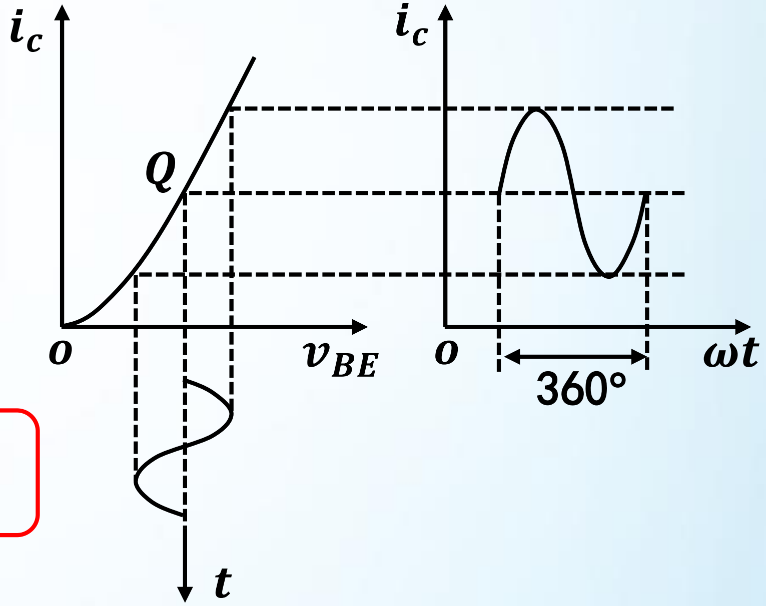
RF Power Amplifier *vs.* RF Small Signal Amplifier

- Samilarity
  - Radio frequency
  - Load is resonance
- Difference
  - Amplitude of input
  - Quiescent point of Transistor
  - Dynamic range

Non-linear



RF Power Amplifier



Linear

RF Small Signal Amplifier

RF Power Amplifier *vs.* Audio Power Amplifier

- Similarity
  - High Power
  - High Efficiency

- Difference

	RF Power Amplifier	Audio Power Amplifier
Load	Parallel Resonance	Resistance
Status	Class C	Class A / B
Relative Bandwidth	Radio Narrowband	Audio Wideband

Output Power and Efficiency

RF Power Amplifier — Power & Efficiency

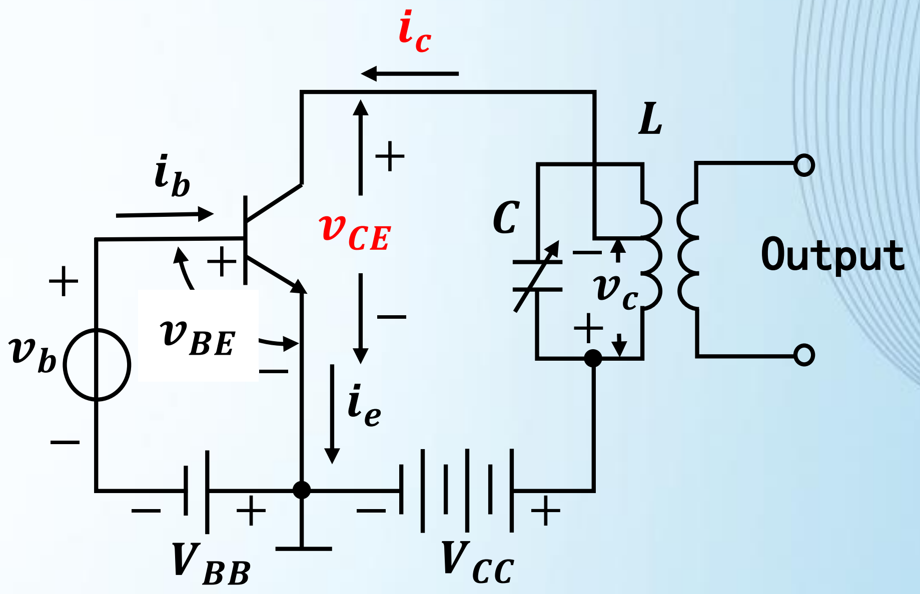
Power: $\boxed{\text{Output Power } P_o \uparrow} = \boxed{\text{DC Power } P_=} - \boxed{\text{Collector Dissipation Power } P_c \downarrow}$

Efficiency: Collector $\eta_c \uparrow = \dfrac{P_o}{P_=} = \dfrac{P_o}{P_o + P_c \downarrow}$

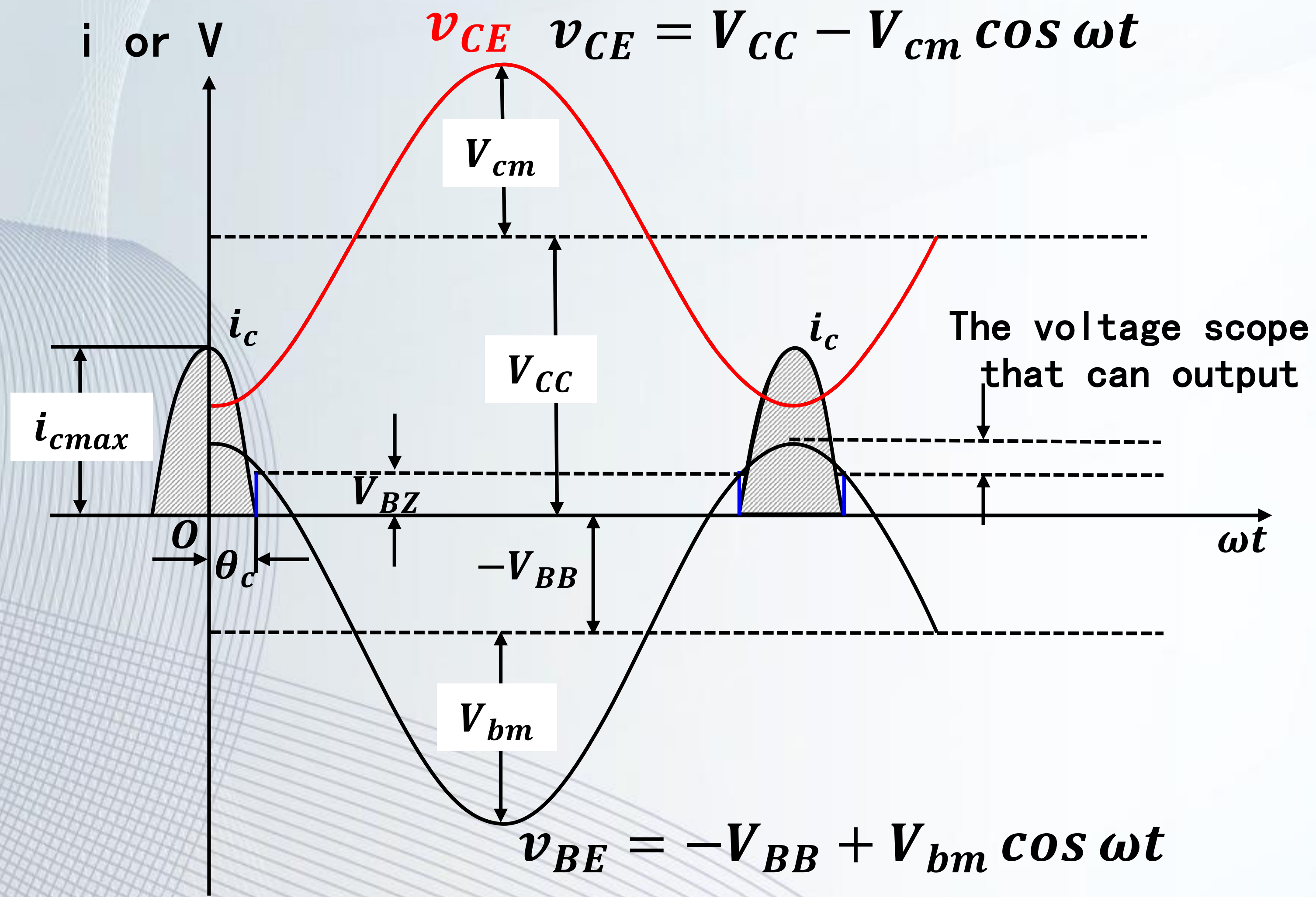
$$P_c = \frac{1}{T}\int_0^T i_C \cdot v_{CE}\, dt$$

- $P_c \downarrow$ $P_o \uparrow$ $\eta_c \uparrow$

  How to decrease P_c?



RF Power Amplifier — Decrease P_c



$$P_c = \frac{1}{T}\int_0^T i_C \cdot v_{CE}\,dt$$

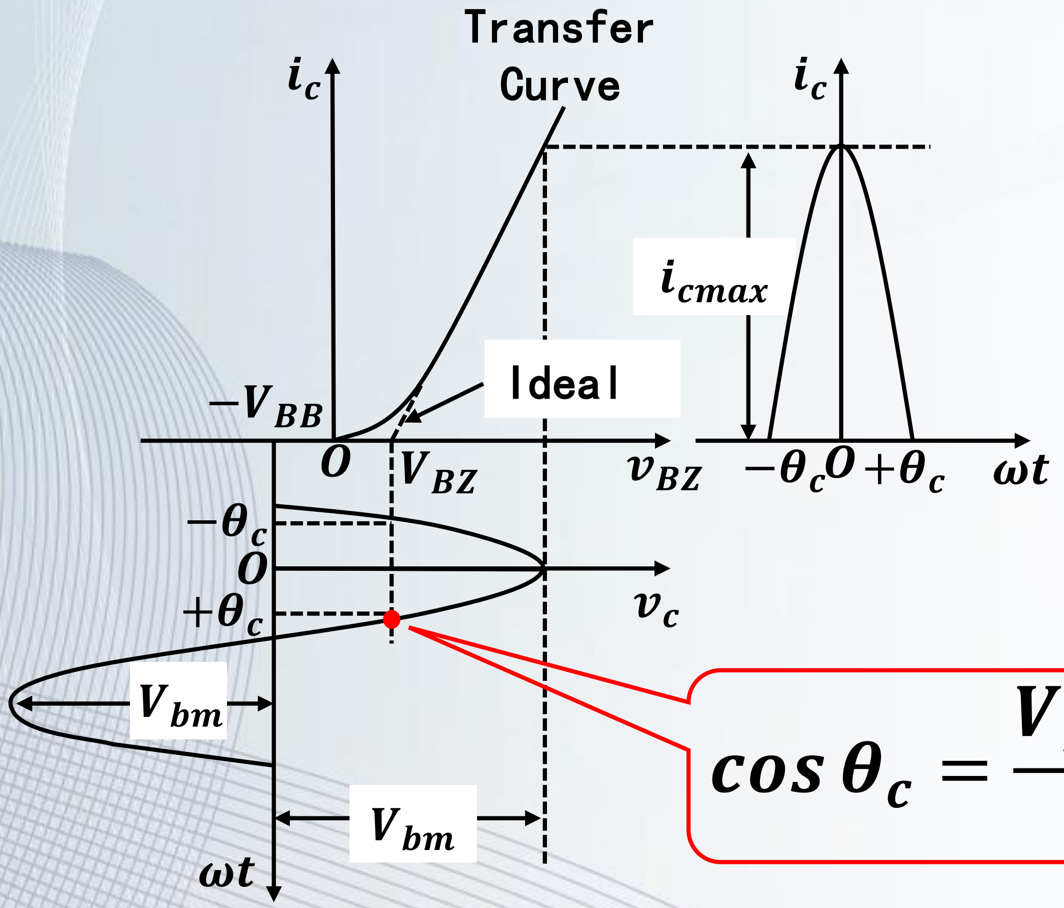
How to decrease P_c:

- *LC* Resonant on ω

  $i_{cmax} \Leftrightarrow v_{CEmin}$

- Decrease θ_c

RF Power Amplifier — Decrease P_c



$$\cos\theta_c = \frac{V_{BB}+V_{BZ}}{V_{bm}}$$

$$P_c = \frac{1}{T}\int_0^T i_C \cdot v_{CE}\, dt$$

How to decrease P_c:

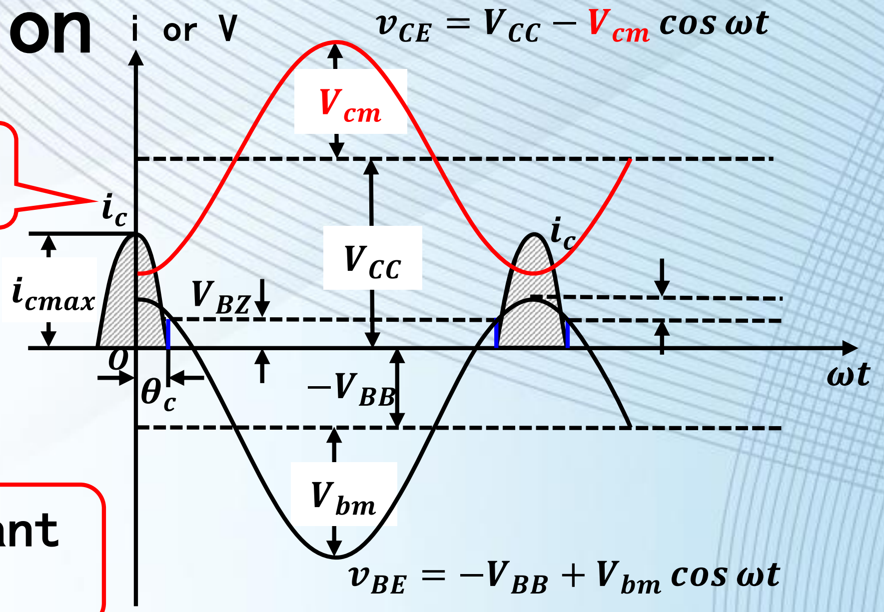
- LC Resonant on ω

  $i_{cmax} \Leftrightarrow v_{CEmin}$

- Decrease θ_c

RF Power Amplifier—Detailed Definition

$$i_c = I_{c0} + I_{cm1}\cos\omega t + I_{cm2}\cos 2\omega t + \cdots + I_{cmn}\cos n\omega t + \cdots$$



Output Power $$P_o = \frac{1}{2}V_{cm}\cdot I_{cm1} = \frac{1}{2}\frac{V_{cm}^2}{R_p} = \frac{1}{2}I_{cm1}^2 R_p$$

Parallel Resonant Resistance

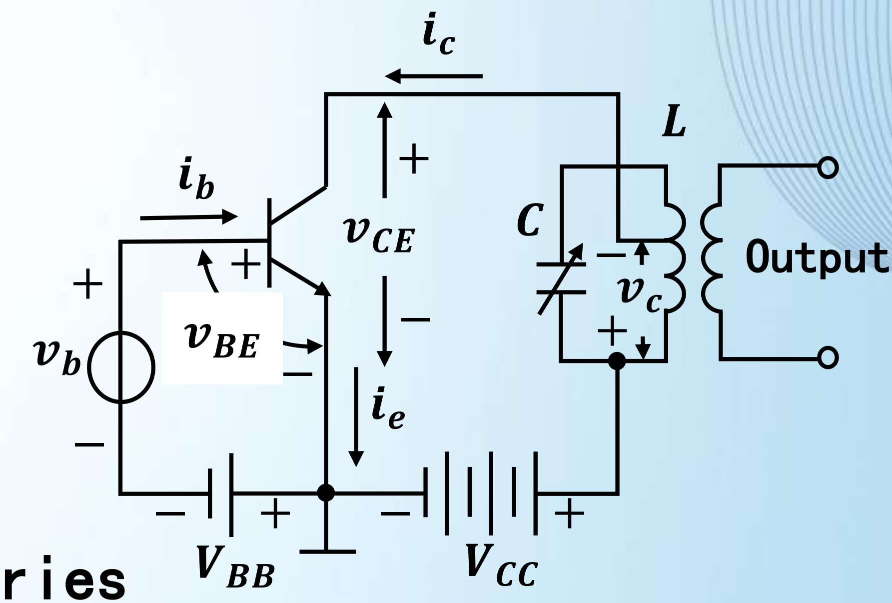
Voltage Utilization Factor ξ

Collector Efficiency $$\eta_c = \frac{P_o}{P_=} = \frac{\frac{1}{2}V_{cm}\cdot I_{cm1}}{V_{CC}\cdot I_{c0}} = \frac{1}{2}\xi g_1(\theta_c)$$

DC Power

$P_= = V_{CC}\cdot I_{c0}$

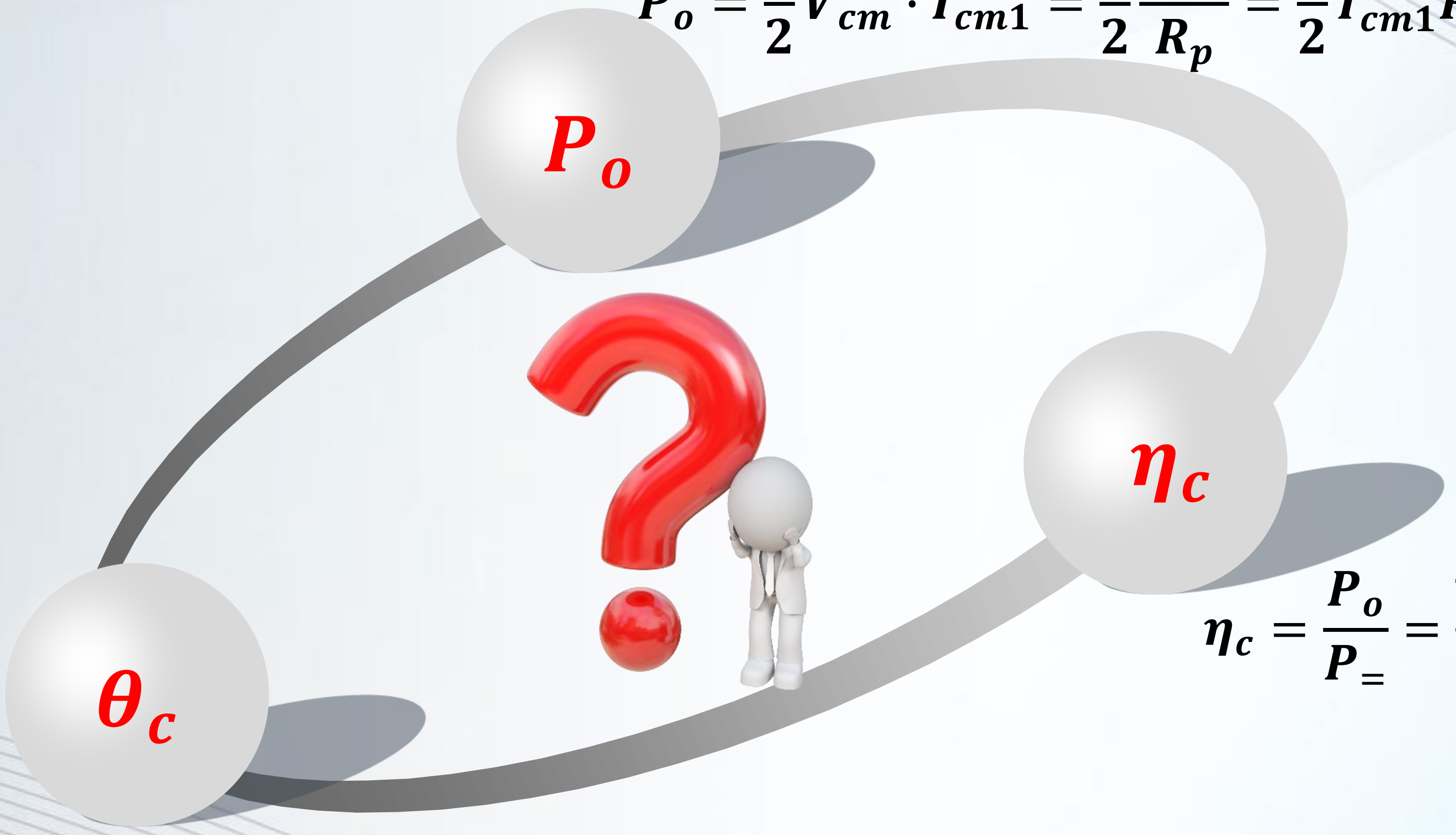
Current Utilization Factor $g_1(\theta_c)$



☆Compute I_{cm1} & I_{c0} ⇐ Analyze i_c by Fourier Series

Summary—Output Power & Efficiency

$$P_o = \frac{1}{2}V_{cm} \cdot I_{cm1} = \frac{1}{2}\frac{V_{cm}^2}{R_p} = \frac{1}{2}I_{cm1}^2 R_p$$

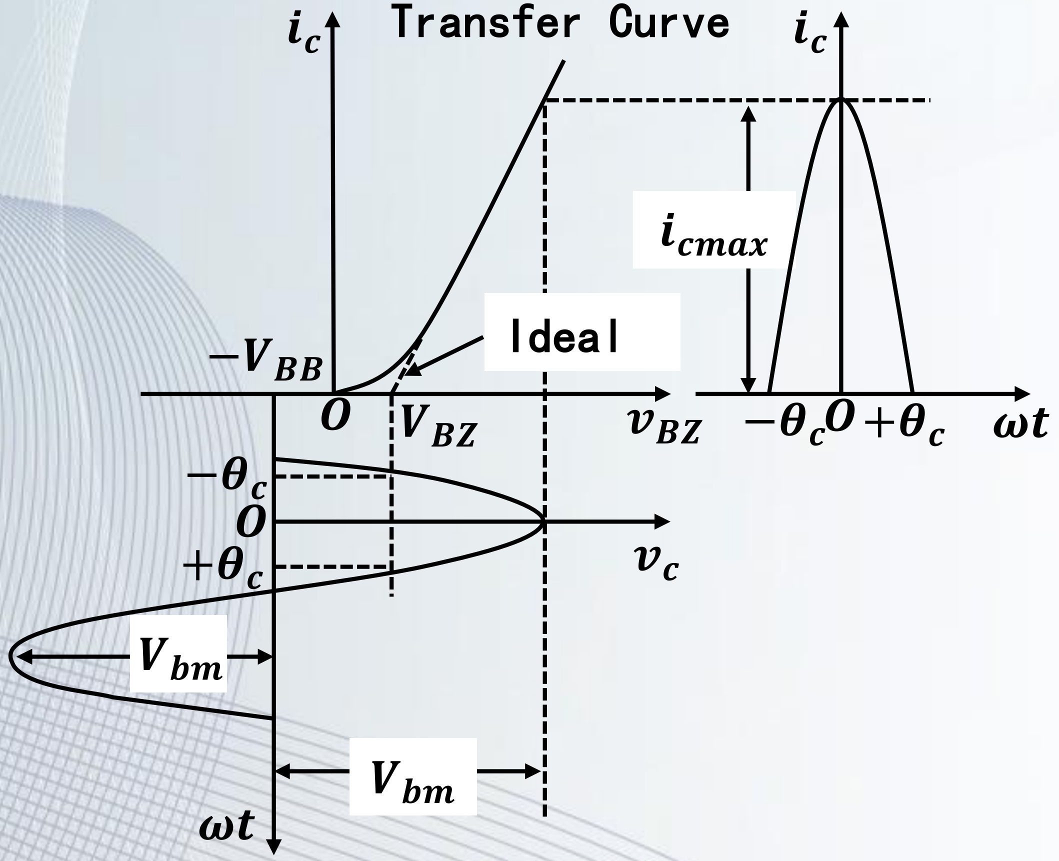


$$\eta_c = \frac{P_o}{P_=} = \frac{\frac{1}{2}V_{cm} \cdot I_{cm1}}{V_{CC} \cdot I_{c0}} = \frac{1}{2}\xi g_1(\theta_c)$$

RF Power Amplifier Computation

RF Power Amplifier — Clue

$$\begin{cases} P_o = \frac{1}{2}V_{cm}\cdot I_{cm1} = \frac{V_{cm}^2}{2R_p} = \frac{1}{2}I_{cm1}^2 R_p \\ \eta_c = \frac{P_o}{P_=} = \frac{\frac{1}{2}V_{cm}\cdot I_{cm1}}{V_{CC}\cdot I_{c0}} = \frac{1}{2}\xi g_1(\theta_c) \end{cases}$$



- Key: compute I_{c0} & I_{cm1}

$\Leftarrow$ Analyze Cosine Pulse i_c

- $i_c = ?$ ($\theta_c = ?$ $i_{cmax} = ?$)

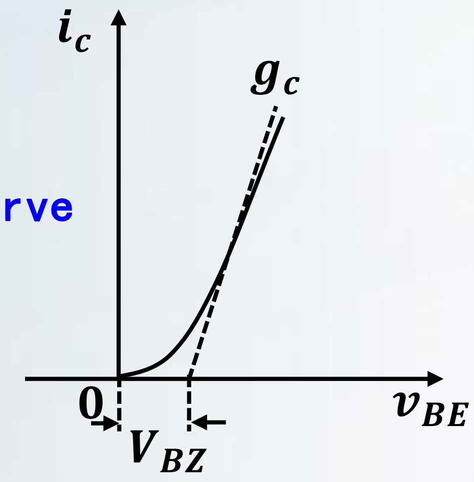
- By Fourier Series

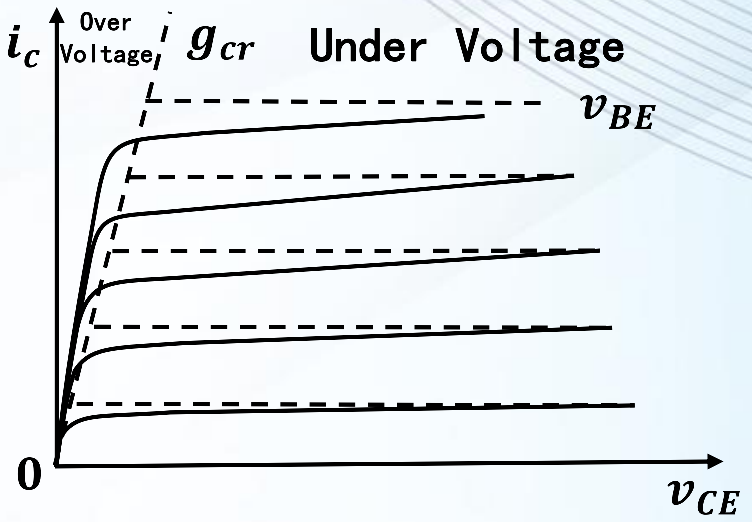
$\Leftarrow$ Piecewise Linear Approximation

(Transfer Curve、Output Curve)

RF Power Amplifier — Piecewise Linear Approximation

Transfer Curve





Output Curve

Trans-conductance

Transfer Equation: $i_c = g_c(v_{BE} - V_{BZ}) \quad (v_{BE} > V_{BZ})$ (1)

Critical Line Equation: $i_c = g_{cr} v_{CE}$ (2)

Critical Line Gradient

Periodic Cosine Pulses

Transfer Equation: $i_c = g_c(v_{BE} - V_{BZ})$ (1)

Base Equation : $v_{BE} = -V_{BB} + V_{bm}\cos\omega t$ (3)

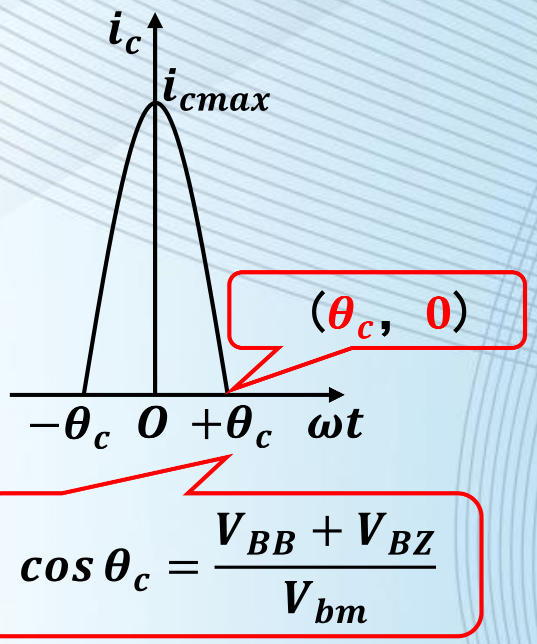
Substitute (3) into (1),

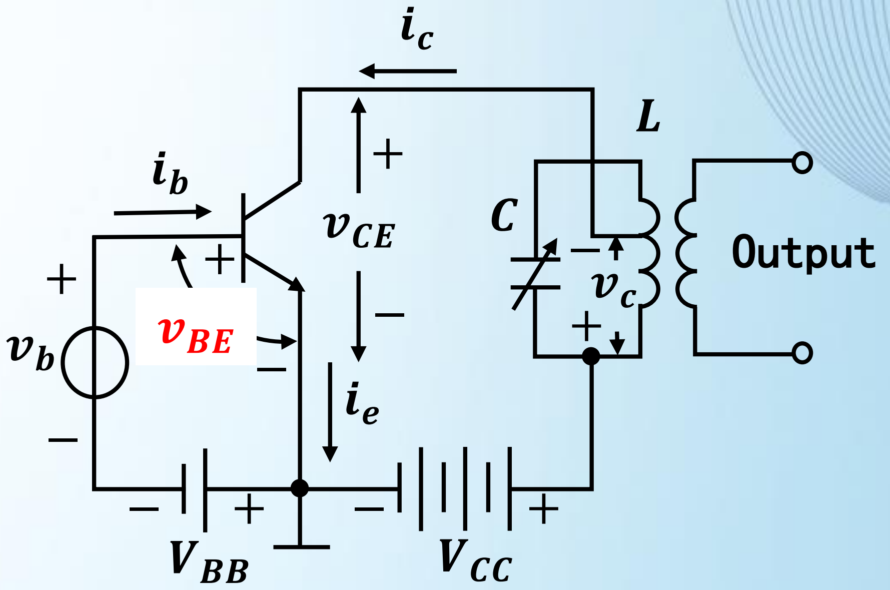
$$i_c = g_c(-V_{BB} + V_{bm}\cos\omega t - V_{BZ}) \quad (4)$$

If $\omega t = \theta_c$, $i_c = 0$, Substitute into (4),

$$0 = g_c(-V_{BB} + V_{bm}\cos\theta_c - V_{BZ}) \quad (5)$$

Obtain $\cos\theta_c = \frac{V_{BB}+V_{BZ}}{V_{bm}}$





Periodic Cosine Pulses

$$\begin{cases} i_c = g_c(-V_{BB} + V_{bm}\cos\omega t - V_{BZ}) & (4) \\ 0 = g_c(-V_{BB} + V_{bm}\cos\theta_c - V_{BZ}) & (5) \end{cases}$$

(4) − (5)，得

$$i_c = g_c V_{bm}(\cos\omega t - \cos\theta_c) \quad (6)$$

If $\omega t = 0$，$i_c = i_{cmax}$，Substitute into (6)

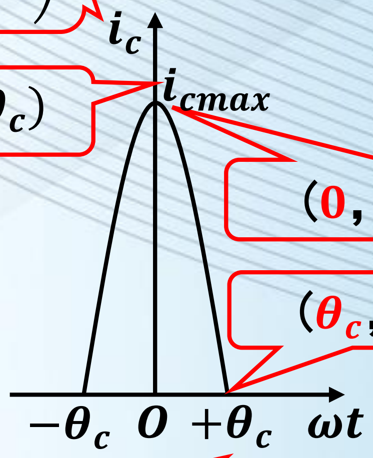
$$i_{cmax} = g_c V_{bm}(1 - \cos\theta_c) \quad (7)$$

(6) ÷ (7)，$i_c = i_{cmax}\left(\frac{\cos\omega t - \cos\theta_c}{1 - \cos\theta_c}\right)$

Cosine Pulse Formula

$$i_c = i_{cmax}\left(\frac{\cos\omega t - \cos\theta_c}{1 - \cos\theta_c}\right)$$

$$i_{cmax} = g_c V_{bm}(1 - \cos\theta_c)$$



$(0,\ i_{cmax})$

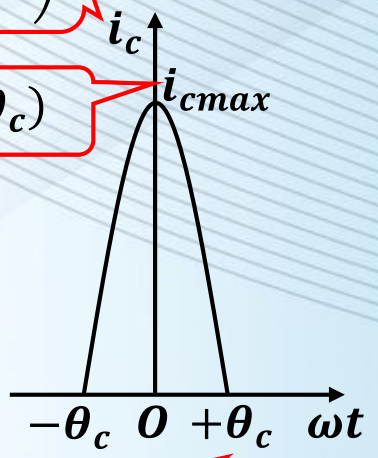
$(\theta_c,\ 0)$

$$\cos\theta_c = \frac{V_{BB} + V_{BZ}}{V_{bm}}$$

Periodic Cosine Pulses—Decompose

$$i_c = i_{cmax}\left(\frac{\cos\omega t - \cos\theta_c}{1-\cos\theta_c}\right)$$

$$i_{cmax} = g_c V_{bm}(1-\cos\theta_c)$$



$$\cos\theta_c = \frac{V_{BB}+V_{BZ}}{V_{bm}}$$

- By Fourier Series

$$i_c = I_{c0} + I_{cm1}\cos\omega t + I_{cm2}\cos 2\omega t + \cdots + I_{cmn}\cos n\omega t + \cdots$$

- Compute Coefficents

DC Power

$$I_{c0} = \frac{1}{2\pi}\int_{-\theta_c}^{+\theta_c} i_c\, d\omega t = i_{cmax}\cdot\alpha_0(\theta_c)$$

$$\alpha_0(\theta_c) = \frac{\sin\theta_c - \theta_c\cos\theta_c}{\pi(1-\cos\theta_c)}$$

Output Power

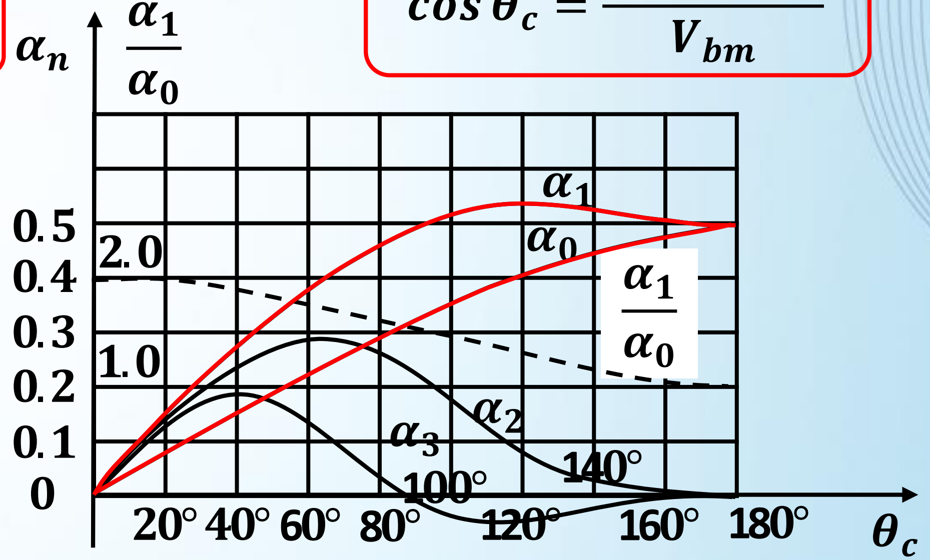
$$I_{cm1} = \frac{1}{2\pi}\int_{-\theta_c}^{+\theta_c} i_c\cos\omega t\, d\omega t = i_{cmax}\cdot\alpha_1(\theta_c)$$

$$\alpha_1(\theta_c) = \frac{\theta_c - \cos\theta_c\sin\theta_c}{\pi(1-\cos\theta_c)}$$

$$\vdots$$

$$I_{cmn} = \frac{1}{2\pi}\int_{-\theta_c}^{+\theta_c} i_c\cos n\omega t\, d\omega t = i_{cmax}\cdot\alpha_n(\theta_c)$$

$$\alpha_n(\theta_c) = \frac{2}{\pi}\cdot\frac{\sin n\theta_c\cos\theta_c - n\cos n\theta_c\sin\theta_c}{n(n^2-1)(1-\cos\theta_c)}$$

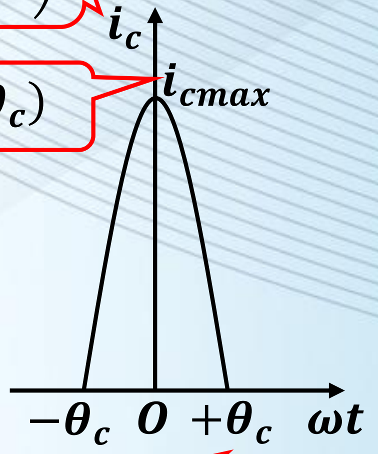


Fourier Coefficients

Periodic Cosine Pulses—Decompose

$$i_c = i_{cmax}\left(\frac{\cos\omega t - \cos\theta_c}{1-\cos\theta_c}\right)$$

$$i_{cmax} = g_c V_{bm}(1-\cos\theta_c)$$



$$\cos\theta_c = \frac{V_{BB}+V_{BZ}}{V_{bm}}$$

- By Fourier Series

$$i_c = I_{c0} + I_{cm1}\cos\omega t + I_{cm2}\cos 2\omega t + \cdots + I_{cmn}\cos n\omega t + \cdots$$

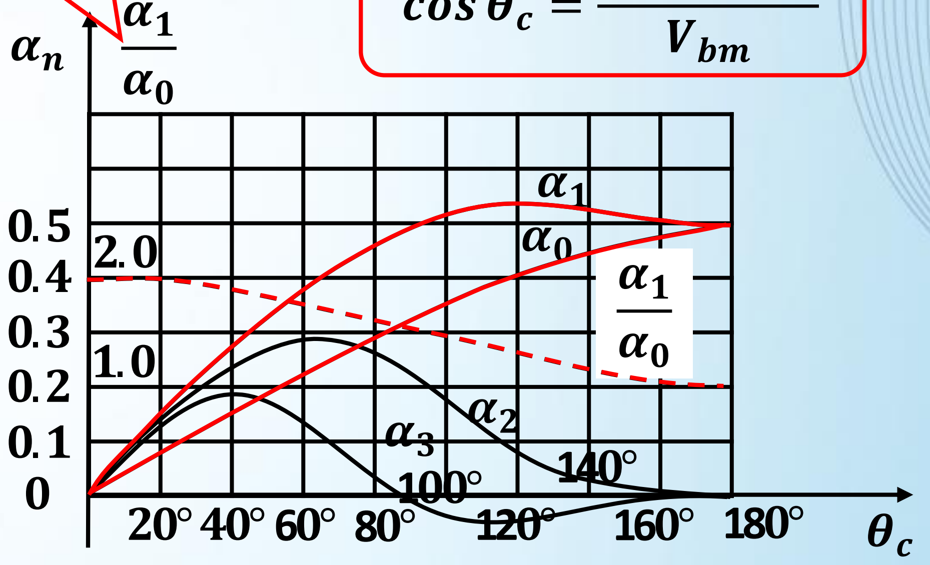
- Compute Coefficents

$$\begin{cases} I_{c0} = \dfrac{1}{2\pi}\displaystyle\int_{-\theta_c}^{+\theta_c} i_c\, d\omega t = i_{cmax}\cdot\alpha_0(\theta_c) \\ I_{cm1} = \dfrac{1}{2\pi}\displaystyle\int_{-\theta_c}^{+\theta_c} i_c \cos\omega t\, d\omega t = i_{cmax}\cdot\alpha_1(\theta_c) \end{cases}$$

$$\eta_c = \frac{P_o}{P_=} = \frac{\frac{1}{2}V_{cm}\cdot I_{cm1}}{V_{CC}\cdot I_{c0}} = \frac{1}{2}\xi\cdot g_1(\theta_c)$$

$$g_1(\theta_c) = \frac{I_{cm1}}{I_{c0}} = \frac{\alpha_1(\theta_c)}{\alpha_0(\theta_c)}$$

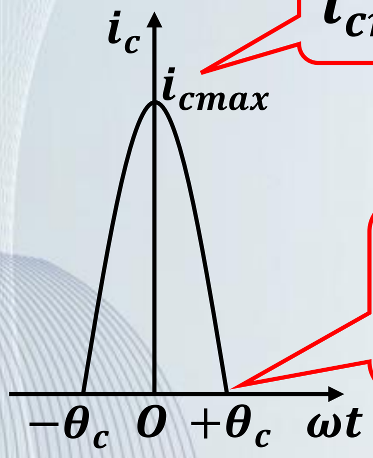
Efficiency



Fourier Coefficients

- Tradeoff between Power & Efficiency, Optimal θ_c around 70°

Summary—Computation of RF Power Amplifier



$$i_{cmax} = g_c V_{bm}(1 - \cos\theta_c)$$

$$\cos\theta_c = \frac{V_{BB} + V_{BZ}}{V_{bm}}$$

$$\begin{cases} I_{c0} = i_{cmax} \cdot \alpha_0(\theta_c) \\ I_{cm1} = i_{cmax} \cdot \alpha_1(\theta_c) \end{cases}$$

$$\begin{cases} P_o = \dfrac{1}{2} V_{cm} \cdot I_{cm1} = \dfrac{1}{2}\dfrac{V_{cm}^2}{R_p} = \dfrac{1}{2} I_{cm1}^2 R_p \\ \eta_c = \dfrac{P_o}{P_=} = \dfrac{\frac{1}{2} V_{cm} \cdot I_{cm1}}{V_{CC} \cdot I_{c0}} = \dfrac{1}{2}\xi \cdot g_1(\theta_c) \end{cases}$$

$$g_1(\theta_c) = \frac{\alpha_1(\theta_c)}{\alpha_0(\theta_c)}$$

Exp 4-1 The transfer characteristic of a RF power amplifier is as figure. The parameters of transistor are : $f_T \geq 150MHz$, $A_p \geq 13dB$, $I_{cmax} = 3A$, $P_{cmax} = 5W$. $V_{BZ} = 0.6V$, $V_{BB} = 1.4V$, $\theta_c = 70°$, $V_{CC} = 24V$, $\xi = 0.9$. Compute all metrics.

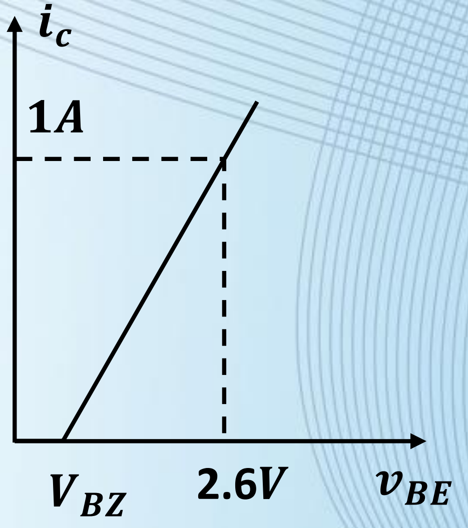


Figure $\Rightarrow g_c = \frac{1\text{A}}{(2.6-0.6)\text{V}} = 0.5\text{A/V}$

$\cos\theta_c = \frac{V_{BB}+V_{BZ}}{V_{bm}}$, $\Rightarrow V_{bm} = \frac{1.4+0.6}{\cos 70^\circ} = 5.8\,V$

Solution: $\because i_{cmax} = g_c V_{bm}(1-\cos\theta_c)$, Compute I_{cm1}、I_{c0}

$$i_{cmax} = \frac{1}{2} \times 5.8 \times \left(1-\cos 70^\circ\right) = 2A < I_{cmax} \text{ (Safe)}$$

$$\Rightarrow \begin{cases} I_{cm1} = i_{cmax} \cdot \alpha_1\left(70^\circ\right) = 2 \times 0.436 = 0.872A \\ I_{c0} = i_{cmax} \cdot \alpha_0\left(70^\circ\right) = 2 \times 0.253 = 0.506A \end{cases}$$

$$\Rightarrow \begin{cases} P_o = \frac{1}{2} I_{cm1} \cdot V_{cm} = \frac{1}{2} I_{cm1} \cdot (\xi V_{CC}) = \frac{1}{2} \times 0.872 \times 0.9 \times 24 = 9.4\text{W} \\ P_= = V_{CC} \cdot I_{c0} = 24 \times 0.506 = 12\text{W} \quad \Rightarrow P_c = P_= - P_o = 2.6\text{W} < P_{cmax} \text{ (Safe)} \end{cases}$$

$$\Rightarrow \quad \eta_c = \frac{P_o}{P_=} = \frac{9.4}{12} = 78\%$$