Basic Concepts

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Outlines

- Resistor、Inductor、Capacitor： Lead vs. Lag
- Impedance vs. Admittance
- Quality Factor（Q）of Inductor
  - vs. Quality Factor（Q）of resonance circuit

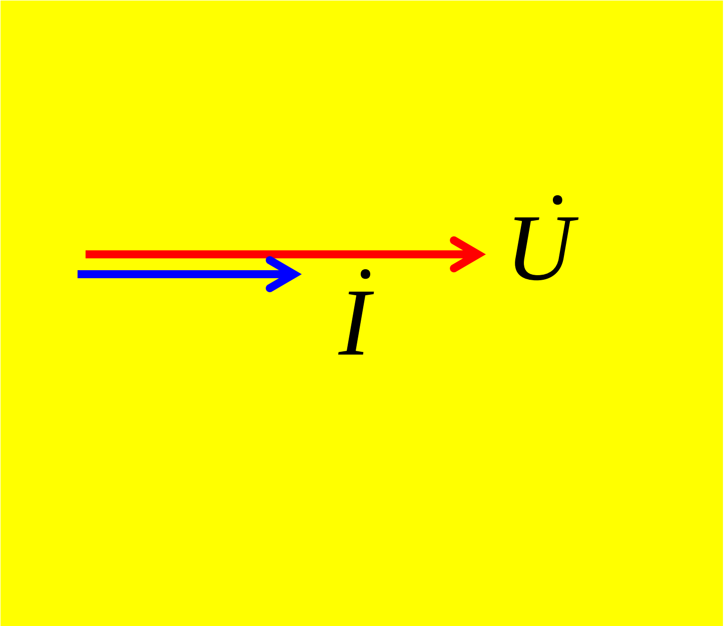
1 Resistor

$$u = \sqrt{2}U\sin\omega t$$

$$i = \frac{u}{R} = \sqrt{2}\frac{U}{R}\sin\omega t = \sqrt{2}I\sin\omega t$$

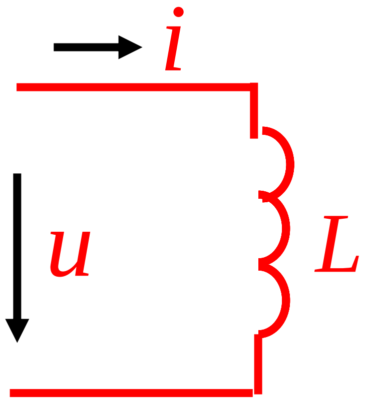
$$U = IR$$

Vector relation



2 Inductor

$$u = L\frac{\mathrm{d}i}{\mathrm{d}t}$$



If $i = \sqrt{2}I\sin\omega t$

Then $u = L\dfrac{\mathrm{d}i}{\mathrm{d}t} = \sqrt{2}I\cdot\omega L\cos\omega t$

$$= \sqrt{2}\,I\,\omega\,L\sin(\omega t + 90^\circ)$$

$$= \sqrt{2}U\sin(\omega t + 90^\circ)$$

2 Inductor Voltage leads Current

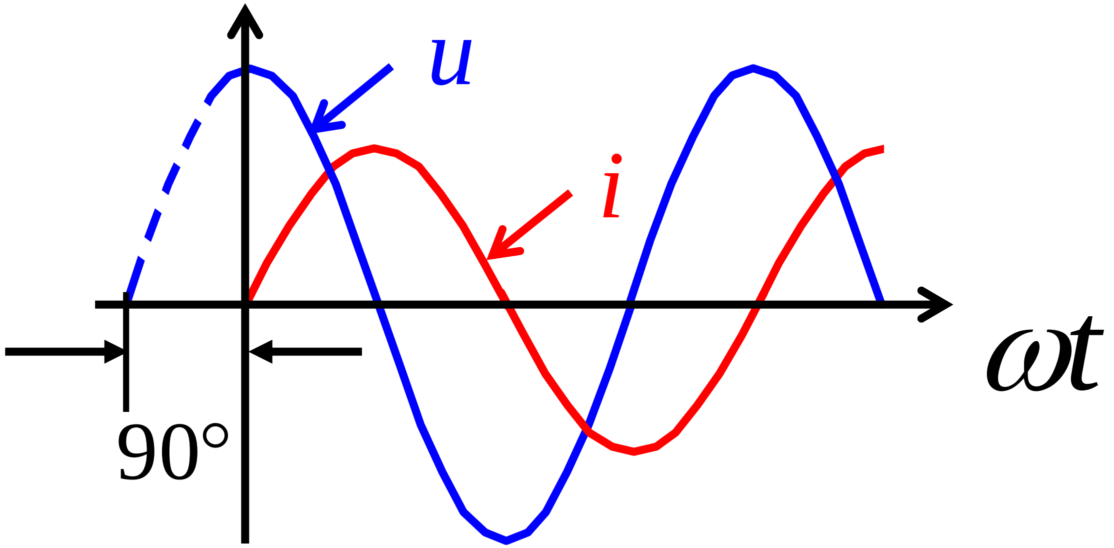
$$i = \sqrt{2} I \sin \omega t$$

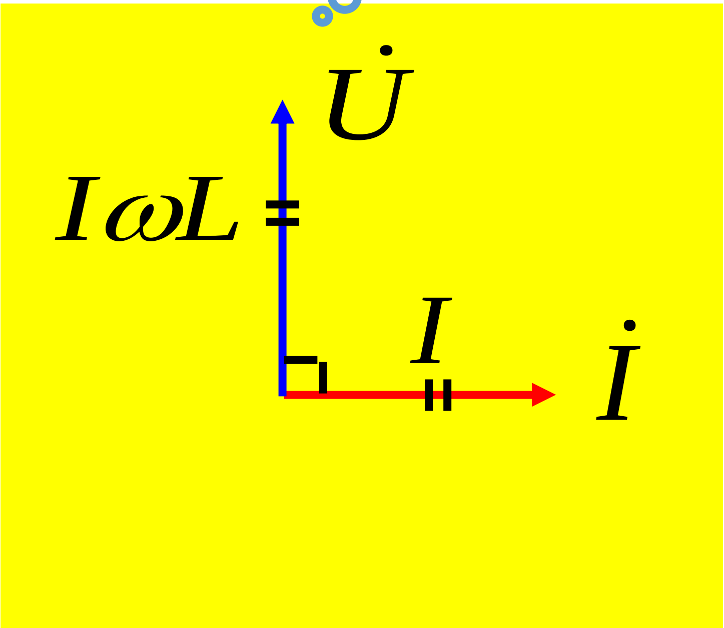
$$u = \sqrt{2}\, \underline{I \omega L} \sin(\omega t + 90^\circ)$$

$$= \sqrt{2}\, \underline{U} \sin(\omega t + 90^\circ)$$

1. Frequency is same
2. Phase diff **90°** （$\boldsymbol{u}$ leads $\boldsymbol{i}$ **90°** ）

$\dot{U}$ **Lead!**





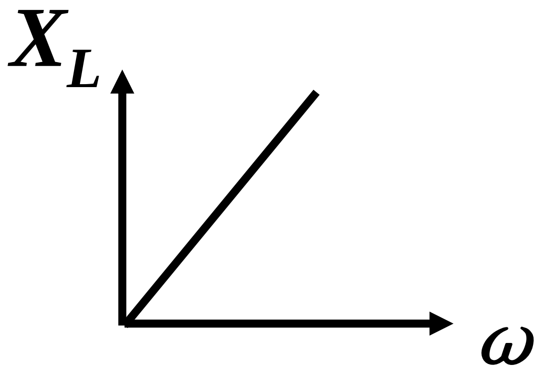
Inductance

$$u = \sqrt{2}\, I \omega L \sin(\omega t + 90^\circ)$$

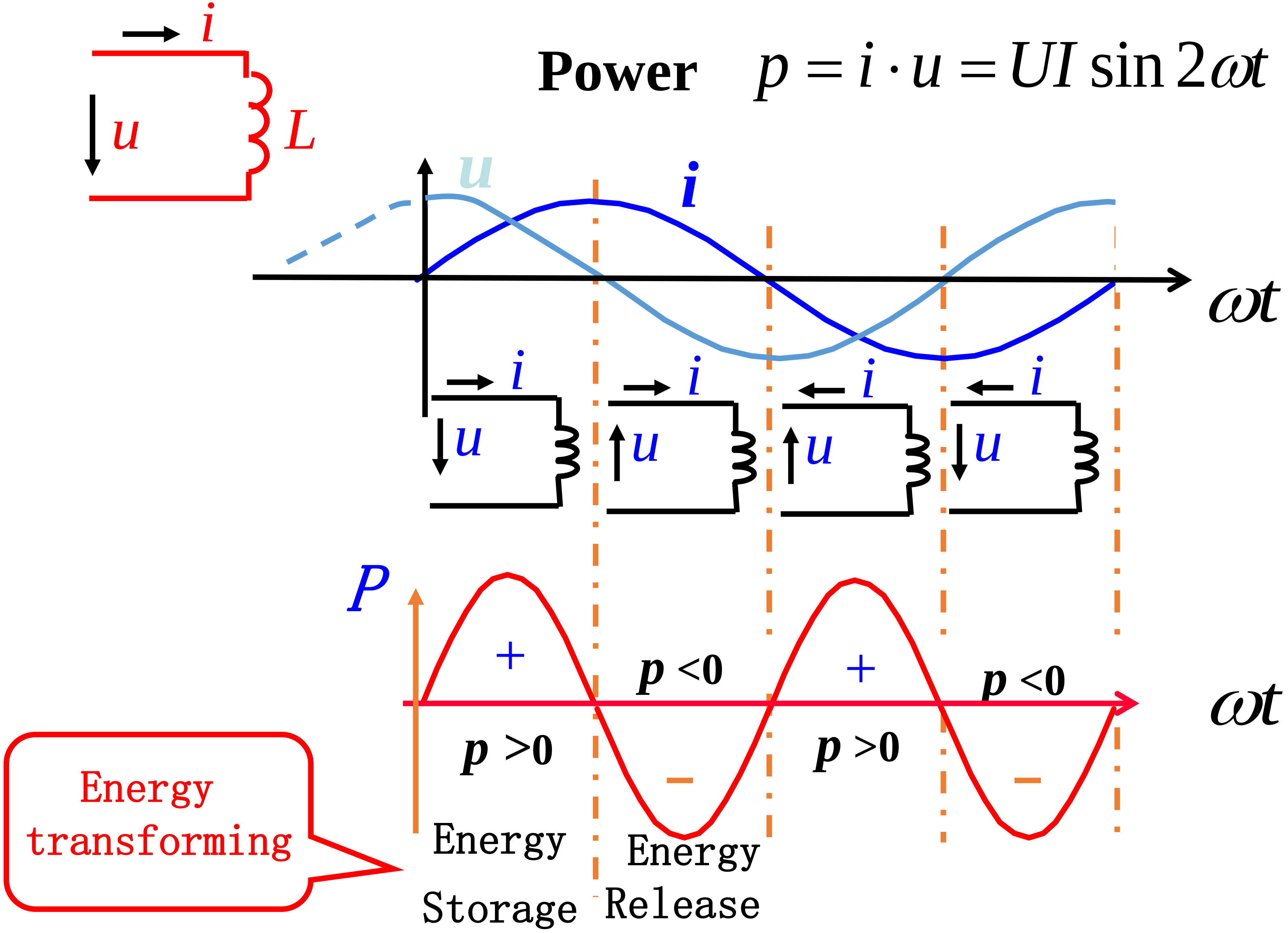
$$= \sqrt{2}\, U \sin(\omega t + 90^\circ)$$

Root-Mean-Square(RMS) $U = I\omega L$

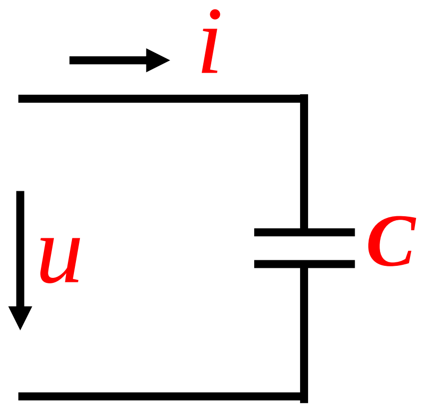
Def: $\boxed{X_L = \omega L}$ （Ω）



Power $p = i \cdot u = UI \sin 2\omega t$



3 Capacitor



$$i = C\frac{\mathrm{d}u}{\mathrm{d}t}$$

If $u = \sqrt{2}U\sin\omega t$

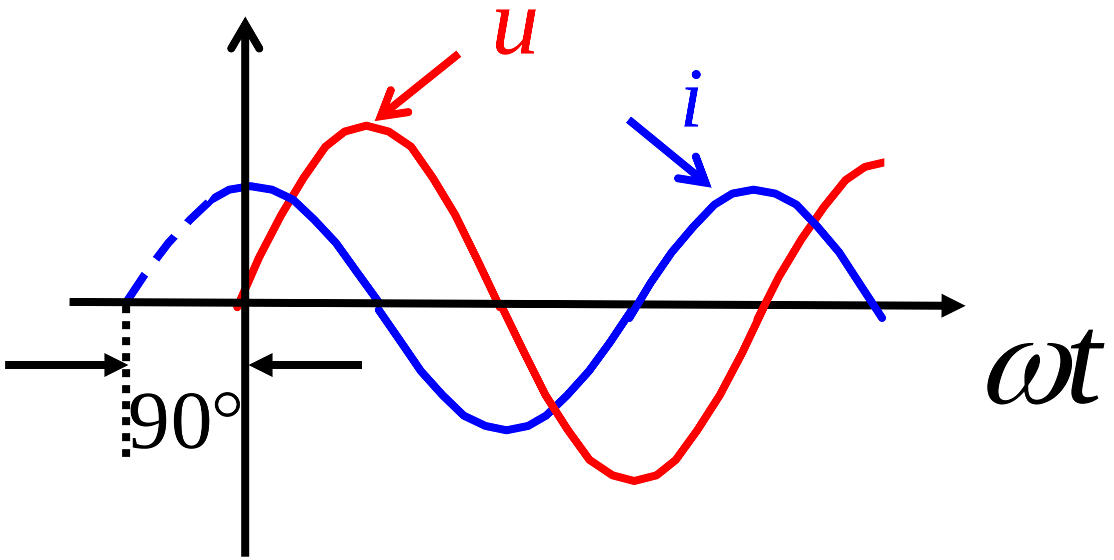
Then

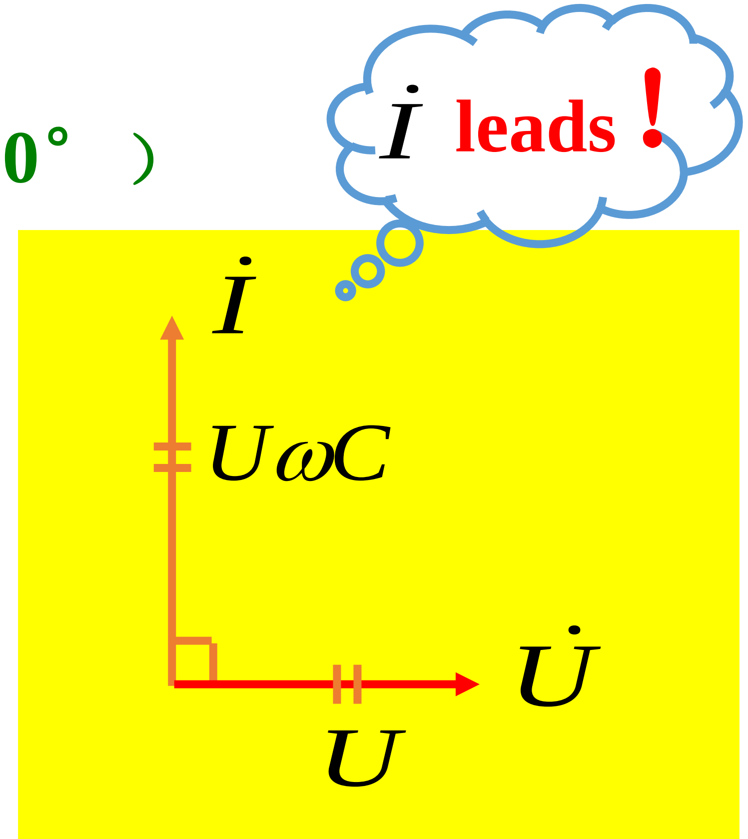
$$\boxed{\begin{aligned} i &= C\frac{\mathrm{d}u}{\mathrm{d}t} = \sqrt{2}UC\omega\cos\omega t \\ &= \sqrt{2}U\omega C\cdot\sin(\omega t + 90^\circ) \end{aligned}}$$

3 Capacitor Voltage lags Current

$$\begin{cases} u = \sqrt{2}U\sin\omega t \\ i = \sqrt{2}\underbrace{U\omega C}_{I}\cdot\sin(\omega t + 90^\circ) \end{cases}$$

1. Frequency is same
2. Phase diff 90° （u lags i 90° ）





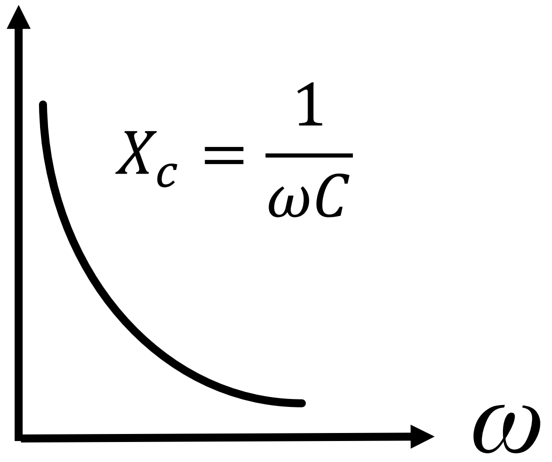
Captance

$$\begin{cases} u = \sqrt{2}U \sin \omega t \\ i = \sqrt{2}\underbrace{U\omega C}_{I} \cdot \sin(\omega t + 90^\circ) \end{cases}$$

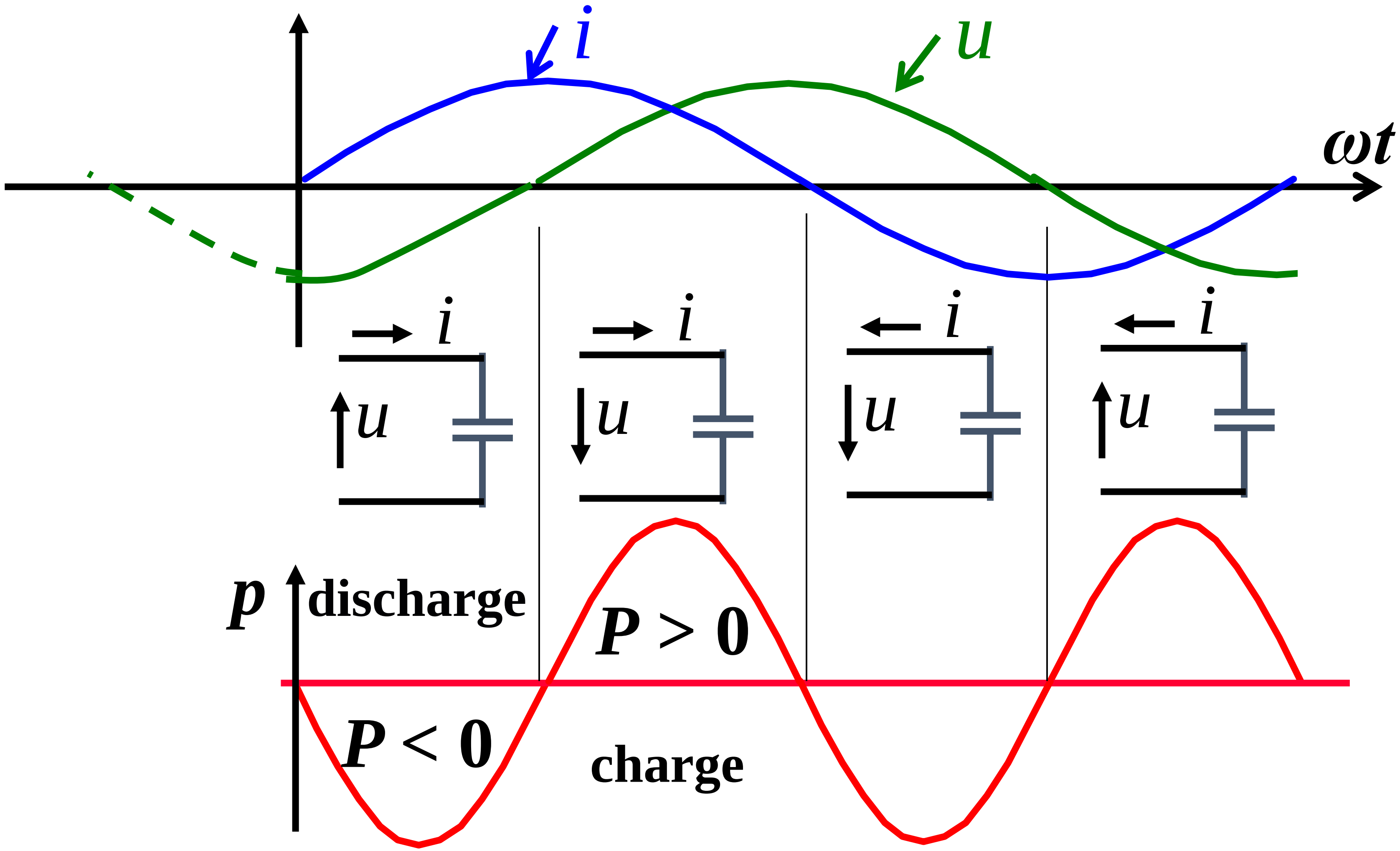
Root-Mean-Square(RMS) $U = \frac{1}{\omega C} I$

Def:

$$X_C = \frac{1}{\omega C} \quad (\Omega)$$

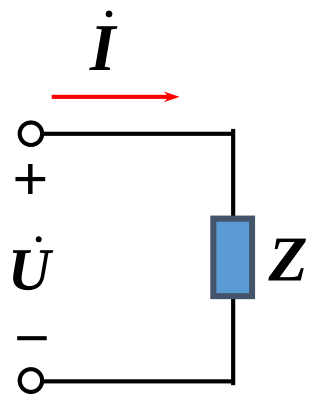


Power $p = i \cdot u = -UI\sin 2\omega t$



Impedance vs. Admittance

Impedance vs. Admittance

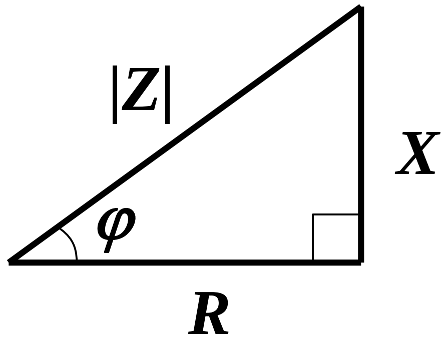


Impedance $Z = \dfrac{\dot{U}}{\dot{I}} = R + \mathrm{j}X$

Reactance

Resistance

Unit: Ω



Impedance Triangle

Impedance vs. Admittance

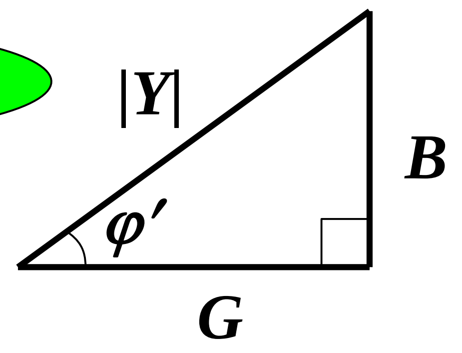
Admittance $Y = \dfrac{\dot{I}}{\dot{U}} = G + \mathrm{j}B$

Susceptance

Conductance

Unit: S

$$Y = \frac{1}{Z}$$



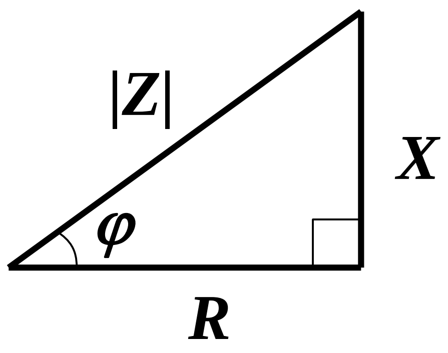
Admittance Triangle

Impedance $Z = \dfrac{\dot{U}}{\dot{I}} = R + \mathrm{j}X$

Reactance

Resistance

Unit: Ω

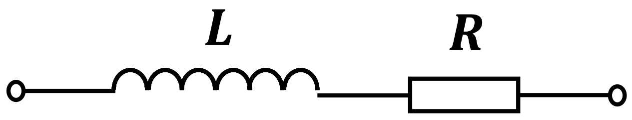


Impedance Triangle

Inductor Q vs. Resonator Q

Inductor Quality Factor (Q)

➢Definition



$$Q = \frac{\omega L}{R}$$

Physical Significance

- Q reflects energy loss of inductor
- The bigger Q, the less loss, the higher efficiency

- Compared with resonator Q (see Chapter 2)

Q & A