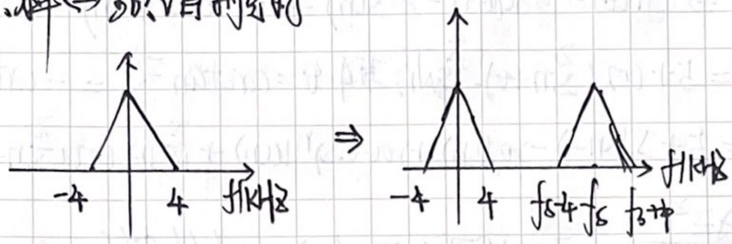
chapter 2

1. 采样⇔频谱搬移



$(f_s-4)-4=16\,kHz$

$\Rightarrow f_s=24\,kHz$

2. 因 $X(e^{j\omega})=FT[x(n)]=A(\omega)+jB(\omega)\Leftrightarrow A(\omega)=FT[x_e(n)],\ B(\omega)=FT[\frac{x_o(n)}{j}]$

$\Rightarrow Y(e^{j\omega})=FT[y(n)]=B(\omega)+A(\omega)e^{j\omega}=jFT[x_o(n)]+FT[x_e(n+1)]$; j

$\Rightarrow y(n)=x_o(n)+x_e(n+1)=-\frac{j}{2}[x(n)-x^*(-n)]+\frac{1}{2}[x(n+1)+x^*(-n-1)]$

3. $y(n)=x(n)-x(n-4)\Rightarrow H(z)=1-z^{-4}$ 当 $z=e^{j\omega}\Rightarrow H(e^{j\omega})=1-e^{-4j\omega}$

⇒ 想阻止直流，50Hz及其2,3,4等高次谐波 则需 $|H(e^{j\omega})|=0$

$\Rightarrow \omega=4\omega'=2\pi k\Leftrightarrow \omega'=\frac{2\pi k}{4}=\frac{k\pi}{2}$ 并且 $\omega'_{min}=50Hz\Rightarrow f_s=200Hz$

4. $Y(n)=x_1(n)*x_2(n)*x_1(n)$ 作Z变换有 $Y(z)=X_1^2(z)X_2(z)=z-2+2z^{-2}-z^{-3}$

并且 $X_1(z)=z-1\Rightarrow X_2(z)=\frac{z-2+2z^{-2}-z^{-3}}{(z-1)^2}=\frac{(z-1)^3(z+1)}{z^3(z-1)^2}=\frac{z^2-1}{z^3}=\frac{1}{z}-\frac{1}{z^3}$

$\Rightarrow X_2(z)=z^{-1}-z^{-3}$ 则有 $x_2(n)=\delta(n-1)-\delta(n-3)$

5. $h(n)$是因果序列，有 $h(n)=0,\ n<0$. 并且 $Re[X(e^{j\omega})]=\sum_{n=-\infty}^{+\infty}x_e(n)e^{-j\omega n}$

而且 $H_R(e^{j\omega})=1+\frac{1}{2}e^{j\omega}+\frac{1}{2}e^{-j\omega}\Rightarrow x_e(0)=1,\ x_e(-1)=\frac{1}{2}\ x_e(1)=\frac{1}{2}$

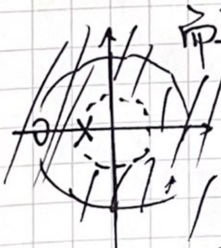
$x_e(n)=\frac{h(n)+h^*(-n)}{2}\Rightarrow h(0)=1,\ h(1)=1.\ h(-1)=0.\Rightarrow h(n)=\delta(n)+\delta(n-1)$

$\Rightarrow H(z)=1+z^{-1}\Rightarrow H(e^{j\omega})=H(z)|_{z=e^{j\omega}}=1+e^{-j\omega}$

6.(1) 系统A的系统函数为 $H_A(z)=1+z^{-1}$，系统B的系统函数为 $H_B(z)=\frac{1}{1+0.2z^{-1}}$

⇒ 二者级联有 $H_C(z)=H_A(z)\cdot H_B(z)=\frac{1+z^{-1}}{1+0.2z^{-1}}\Rightarrow H_C(z)$ 有零点 -1，极点 -0.2

而且 $H_C(z)$ 因果稳定 ⇒ 收敛域 $|z|>0.2$ (ROC)



(2) 当 $x(n)=3+\delta(n-1)+\cos(\frac{\pi n}{2})$ $H(e^{j\omega})=H(z)|_{z=e^{j\omega}}=\frac{1+e^{-j\omega}}{1+0.2e^{-j\omega}}$

当 $\omega=0$ 时 $H(e^{j0})=\frac{5}{3}$，当 $\omega=\frac{\pi}{2}$ 时 $H(e^{j\omega})=\frac{15-10j}{13}$

将 $x(n)=x_1(n)+x_2(n)$ 且 $x_1(n)=\delta(n-1)$，$x_2(n)=3+\cos(\frac{\pi}{2}n)$

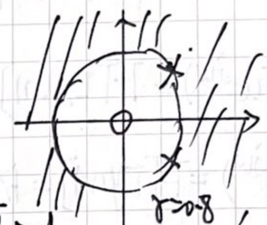
$x_1(n)$ 通过系统有 $Y_1(z) = X_1(z)\cdot H_c(z) = \frac{z^{-1}(1+z^{-1})}{1+0.2z^{-1}} = \frac{z+1}{z^2+0.2z} = \frac{5(z+1)}{5z^2+z}$

$\Rightarrow Y_1(z) = 5\cdot\frac{1}{z} - \frac{20}{5z+1} \Rightarrow y_1(n) = 5\delta(n-1) - 20\delta(n) + 20(-0.2)^n u(n)$

$x_2(n)$ 通过系统有 $y_2(n) = 5+\cos(\frac{\pi}{2}n+\theta)\cdot\frac{2}{13}\sqrt{13}$ 其中 $\theta = \arctan\frac{-2}{3} = -\arctan\frac{2}{3}$

由线性系统从而有 $y(n) = 5+5\delta(n-1)-20\delta(n)+20(-0.2)^n u(n)+\frac{2}{13}\sqrt{13}\cos(\frac{\pi}{2}n-\arctan\frac{2}{3})$

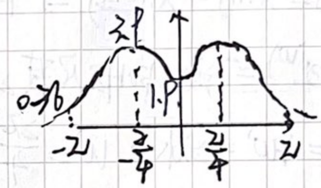
7.(1) 由题意得 $H(z) = \frac{Az^2}{(z-p_1)(z-p_2)}$ 而且是因果LTI系统



从而有 ROC: $|z|>0.8$，$H(\infty)=1) \Rightarrow A=1$

(2) $H(e^{j\omega}) = H(z)|_{z=e^{j\omega}} \Rightarrow |H(e^{j\omega})| = \frac{1}{|(e^{j\omega}-p_1)(e^{j\omega}-p_2)|}$

可以说明 $\omega=0$ 时有极值，为1.8，并且图关于 $x=0$ 对称。当 $\omega=\frac{\pi}{4}, -\frac{\pi}{4}$ 时有极大值为3.8，当 $\omega=\pi, -\pi$ 时有极小值为0.36



(3) 当 $\omega=\frac{\pi}{4}$ 时有 $H(e^{j\omega}) = 3.04-2.44j$

$\Rightarrow |H(e^{j\omega})| = 3.804$, $\arg(H(e^{j\omega})) = -0.21\pi$

从而有 $y(n) = |H(e^{j\omega})|\cdot\exp(j(\frac{\pi}{4}n+\frac{\pi}{2}-0.21\pi)) = 3.804\exp[j(\frac{\pi}{4}n+0.29\pi)]$

chapter 3

1. $x(n)=\mathrm{IDFT}[X(k)]=\frac{1}{N}\sum_{k=0}^{N-1}X(k)W_N^{-kn}$ 其中 $N=100$, $W_N^{-kn}=e^{+j\frac{2\pi}{N}kn}$

$\Rightarrow x(n)=\frac{1}{100}\sum_{k=0}^{99}e^{j\frac{\pi}{50}kn}+\frac{1}{4}=\frac{1}{100}\cdot\frac{1-e^{j\frac{\pi}{50}\times100kn}}{1-e^{j\frac{\pi}{50}n}}+\frac{1}{4}$

$=\delta(n)+\frac{1}{4}\Rightarrow (x(n))_N=\begin{cases}\frac{5}{4}, & n=0\\ \frac{1}{4}, & \text{others}\end{cases}$

2. $X(k)=\sum_{n=0}^{N-1}x(n)W_N^{nk}$ 并且有 $N=8$, 令 $m=2n$, $n=\frac{m}{2}$ 就有 $X(k)=\sum_{m=0}$

$W(k)=\sum_{n=0}^{2N-1}w(n)W_{2N}^{nk}=\sum_{m=0}^{N-1}x(m)W_{2N}^{2mk}=\sum_{m=0}^{N-1}x(m)W_N^{mk}=X(k)$

即 $W(k)=(X(k))_N$

3. $Y(k)=\sum_{n=0}^{\frac{N}{2}-1}y(n)W_{\frac{N}{2}}^{kn}=\sum_{n=0}^{\frac{N}{2}-1}x(2n)W_N^{2kn}=\sum_{m=0}^{N-1}x_e(m)W_N^{km}$

$=\sum_{n=0}^{N-1}\frac{1}{2}[x(n)+x(-n)]W_N^{kn}=\frac{1}{2}X(k)+\frac{1}{2}\sum_{n=0}^{N-1}x(-n)W_N^{(k+\frac{N}{2})(-n)}$

$=\frac{1}{2}X(k)+\frac{1}{2}X(\frac{N}{2}+k)=\frac{1}{2}[X(k)+X(k+\frac{N}{2})]$

4. (1) $[\frac{1}{8}\sum_{k=0}^{7}X(k)e^{j\frac{2\pi}{8}kn}]$ 当 $N=8$ 时有原式 $=\frac{1}{N}\sum_{k=0}^{N-1}X(k)W_N^{-kn}=(x(n))_N$

且当 $n=1$ 时 $(x(n))_N\big|_{n=1}=x(1)=x(1)$

(2) $w(n)=\frac{1}{N'}\sum_{k=0}^{N'-1}W(k)W_{N'}^{-kn}=\frac{1}{N'}\sum_{k=0}^{N'-1}X(k)W_{N'}^{-kn}+\frac{1}{N'}\sum_{k=0}^{N'-1}X(k+N')W_{N'}^{-kn}$

且令 $N=2N'$, $w(n)=\frac{1}{4}\sum_{k=0}^{3}X(k)W_4^{-kn}+\frac{1}{4}\sum_{k=0}^{3}X(k+4)W_4^{-kn}$ 展开 $X(k)$, $X(k+4)$

$\Rightarrow w(n)=\frac{1}{4}\sum_{k=0}^{3}\sum_{r=0}^{7}x(r)W_8^{rk}W_4^{-kn}+\frac{1}{4}\sum_{k=0}^{3}\sum_{r=0}^{7}x(r)W_8^{r(k+4)}W_4^{-kn}$

$=(\frac{1}{4}\sum_{r=0}^{7}x(r))[\sum_{k=0}^{3}W_8^{rk}W_8^{-2kn}+\sum_{k=0}^{3}W_8^{r(k+4)}W_8^{-2n(k+4)}\cdot W_8^{8n}]$

$=(\frac{1}{4}\sum_{r=0}^{7}x(r))\sum_{k=0}^{7}W_8^{k(r-2n)}=\frac{1}{4}\sum_{r=0}^{7}x(r)\cdot\delta(r-2n)\cdot 8=2x(2n)$

5. $y(n)=\frac{1}{N}\sum_{k=0}^{N-1}Y(k)W_N^{-kn}=\frac{1}{N}\sum_{k=0}^{N-1}e^{j\frac{2\pi}{5}k}X(k)W_N^{-kn}$

$=\frac{1}{N}\sum_{k=0}^{N-1}e^{j\frac{2\pi}{10}\cdot2k}\sum_{r=0}^{N-1}x(r)W_N^{kr}\cdot W_N^{-kn}=\frac{1}{N}\sum_{r=0}^{N-1}x(r)\cdot\sum_{k=0}^{N-1}W_N^{k(r-n-2)}$

$=\frac{1}{N}\sum_{r=0}^{N-1}x(r)\cdot\delta(r-(n+2))N=x(n+2)=2\delta(n-1)+\delta(n-3)+\delta(n-8)$

6. $q(n)=\frac{1}{4}\sum_{k=0}^{3}Q(k)W_4^{-kn}=\frac{1}{4}\sum_{k=0}^{3}X(2k)W_4^{-kn}=\frac{1}{4}\sum_{k=0}^{3}\sum_{r=0}^{7}x(r)W_8^{2kr}W_4^{-kn}$

$=\frac{1}{4}\sum_{r=0}^{7}x(r)\sum_{k=0}^{3}W_4^{r-n}=\frac{1}{4}\sum_{r=0}^{7}x(r)\cdot\delta(n-r)\cdot 4=x(n)+x(n+4),\ 0\le n\le 3$

7. (1) $X(z)=\sum_{n=0}^{5}z^{-n}$ 当 $z_k=e^{j\frac{2\pi}{5}k}$, $k=0,1,2,3,4$ 上对 $X(z)$ 采样有 $z_k=W_5^{-k}$

$\Rightarrow P(k)=X(z)|_{z=W_N^{-k}},\ k=0,1,2,3,4,\ N=5\Rightarrow p(n)=x((n))_5R_5(n)=2\delta(n)+\delta(n-1)$

$+\delta(n-2)+\delta(n-3)+\delta(n-4)$

(2) $Q(k)=X(z)|_{z=\frac{1}{2}W_4^{-k}}$ 而且 $X(k)=\sum_{n=0}^{5}x(n)z^{-n}$ 代入有

$Q(k)=\sum_{n=0}^{5}x(n)\cdot 2^nW_4^{nk}\Rightarrow Q(k)=W_4^0+2W_4^k+4W_4^{2k}+8W_4^{3k}+16W_4^{4k}+32W_4^{5k}$

$\Rightarrow Q(k)=17W_4^0+34W_4^k+4W_4^{2k}+8W_4^{3k}$

即 $q(n)=17\delta(n)+34\delta(n-1)+4\delta(n-2)+8\delta(n-3)$

chapter 4

1. $\begin{cases} V = x(n) - 2W \\ W = aV\cdot z^{-1} + bU \\ U = V\cdot z^{-2} + x(n) \\ y(n) = W + V\cdot z^{-2} \end{cases}$ $\Rightarrow$ $V = \dfrac{(1-2b)\,x(n)}{1+2az^{-1}+2bz^{-2}}$, $U = \dfrac{[(1-2b)z^{-2}+1]\,x(n)}{1+2az^{-1}+2bz^{-2}}$

$W = \dfrac{(a-2ab)z^{-1}+(b-2b^2)z^{-2}+b}{1+2az^{-1}+2bz^{-2}}\,x(n)$

$y(n) = \dfrac{b+(a-2ab)z^{-1}+(b+1-2b-2b^2)z^{-2}}{1+2az^{-1}+2bz^{-2}}\,x(n)$

$\Rightarrow H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{b+(a-2ab)z^{-1}+(1+b-2b-2b^2)z^{-2}}{1+2az^{-1}+2bz^{-2}}$

2. $H(z) = \dfrac{a(1+z^{-2})+bz^{-1}+z^{-2}}{1+a(1+z^{-2})+bz^{-1}} = \dfrac{a+bz^{-1}+(a+1)z^{-2}}{(a+1)+bz^{-1}+az^{-2}}$

$|H(e^{j\omega})| = \left|1+\dfrac{z^{-2}-1}{(a+1)+bz^{-1}+az^{-2}}\right|_{z=e^{j\omega}} = \sqrt{\dfrac{a^2+b^2+(a+1)^2}{(a+1)^2+b^2+a^2}} = 1$

3. 由图可得 $H_L(z) = \dfrac{1+2z^{-1}+z^{-2}}{1-Az^{-1}-Bz^{-2}}\cdot\dfrac{1+2z^{-1}+z^{-2}}{1-Cz^{-1}-Dz^{-2}}$

低通→高通有 $\alpha = \dfrac{\cos\left(\frac{\omega_p+\theta_p}{2}\right)}{\cos\left(\frac{\omega_p-\theta_p}{2}\right)} = -\dfrac{\cos\left(\frac{\omega_L+\omega_H}{2}\right)}{\cos\left(\frac{\omega_L-\omega_H}{2}\right)} = 0$ 则有 $v^{-1} = -z^{-1}$

所以有 $H_H(z) = \dfrac{1-2z^{-1}+z^{-2}}{1+Az^{-1}-Bz^{-2}}\cdot\dfrac{1-2z^{-1}+z^{-2}}{1+Cz^{-1}-Dz^{-2}}$

4. 当 $H(z) = 1+z^{-4}$ 出现峰值时有 $z^{-4}=1$，此时 $H(z) = A\cdot\dfrac{1+1}{1+a^4} = 2 \Rightarrow A = 1+a^4$

当用如图所示结构有 $H(z) = (1+B)\cdot\dfrac{1}{1+z^{-4}\cdot B}\cdot(1+z^{-4})$ 则有 $\begin{cases} a = B \\ 1+B = 1+a^4 \end{cases}$

$\Rightarrow B = a^4$

5. 1) 其它零点为 $z^* = -0.5j$, $\frac{1}{z} = -2j$, $\frac{1}{z^*} = 2j$, $\phi(0)=0$（线性）

2) $H(z) = A\cdot(z+0.5j)(z-0.5j)(z-2j)(z+2j)/z^4$ 并且 $z=1$ 时有 $|H(z)|_{z=1} = 25 \Rightarrow A=4$

$\Rightarrow H(z) = \dfrac{4}{z^4}(z^2+\frac{1}{4})(z^2+4) = 4+17z^{-2}+4z^{-4}$

$\Rightarrow H(n) = 4\delta(n)+17\delta(n-2)+4\delta(n-4)$

6. 1) Butterworth滤波器极点为 $s_k=\Omega_c e^{j(\frac{\pi}{2N}+\frac{k\pi}{N}+\frac{\pi}{2})}$ 而且极点关于虚轴对称

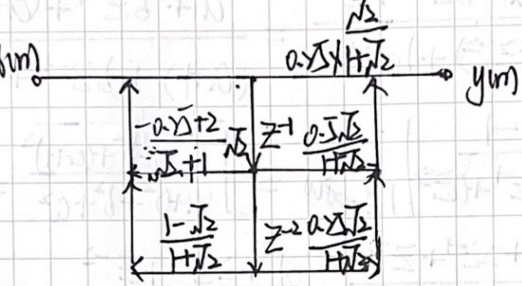
从而有 $\Omega_c=0.5$，左半平面极点为 $0.5e^{j\frac{3}{4}\pi}$ 和 $0.5e^{-j\frac{3}{4}\pi}$

$$\Rightarrow H_a(s)=\frac{0.25}{(s-0.5e^{-j\frac{3}{4}\pi})(s-0.5e^{j\frac{3}{4}\pi})}=\frac{0.25}{s^2+\frac{1}{\sqrt{2}}s+0.25}$$

2) 由双线性变换有 $s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \Rightarrow s=\frac{1-z^{-1}}{1+z^{-1}}$ 所以有

$$H(z)=\frac{z^2+2z+1}{4(z^2-1)^2+2\sqrt{2}(z^2-1)+z^2+2z+1}=\frac{0.25+0.5z^{-1}+0.25z^{-2}}{(1-\frac{1}{\sqrt{2}})z^{-2}+(0.5-2)z^{-1}+1+\frac{1}{\sqrt{2}}}$$

3) 直接II型框图



chapter 5

1.(1) $R_{xx}(m)=0.5^{|m|}, -\infty<m<+\infty$. $S_{xx}(z)=\sum_{m=-\infty}^{+\infty}0.5^{|m|}\cdot z^{-m}$

$\Rightarrow S_{xx}(z)=\sum_{m=0}^{+\infty}(2z)^{-m}+\sum_{m=-\infty}^{-1}(0.5z)^{-m}=\sum_{m=0}^{+\infty}(2z)^{-m}+\sum_{m=1}^{+\infty}(\frac{1}{2}z)^{m}$

$=\frac{1}{1-(2z)^{-1}}+\frac{\frac{1}{2}z}{1-\frac{1}{2}z}=\frac{2z}{2z-1}+\frac{z}{2-z}=\frac{3z}{(2z-1)(2-z)}$

$=\frac{0.75}{(1-0.5z)(1-0.5z^{-1})}$ 并且 $S_{xx}(z)=\sigma_w^2\cdot H(z)H(z^{-1})$

$\Rightarrow \sigma_w^2=0.75 \quad H(z)=\frac{1}{1-0.5z}$

(2) 对应差分方程为 $y(n)=x(n)+0.5y(n-1)$

2. $R_{xx}(m)=E[x(n)x(n+m)]=E[(A\sin(2\pi f_0n+\varphi)+w(n))(A\sin(2\pi f_0(n+m)+\varphi)+w(n+m))]$

$=A^2E[\sin(2\pi f_0n+\varphi)\sin(2\pi f_0(n+m)+\varphi)]+AE[w(n)]E[\sin(2\pi f_0(n+m)+\varphi)]$

$+AE[w(n+m)\sin(2\pi f_0n+\varphi)]+E[w(n)w(n+m)]$

$=-\frac{1}{2}A^2(E[\cos(2\pi f_0(2n+m)+2\varphi)]-E[\cos 2\pi f_0m])+\sigma_w^2\delta(m)$

$=\frac{1}{2}A^2E[\cos 2\pi f_0m]+\sigma_w^2\delta(m)$

即 $R_{xx}(m)=\frac{1}{2}A^2\cos 2\pi f_0m+\sigma_w^2\delta(m)$

3.(1) $E[x(n)]=E[y(n)-y(n-1)]=E[y(n)]-E[y(n-1)]=0$

$R_{xx}(m)=E[x(n)x(n+m)]=E[(y(n)-y(n-1))(y(n+m)-y(n+m-1))]$

$=E[y(n)y(n+m)]-E[y(n-1)y(n+m)]-E[y(n)y(n+m-1)]+E[y(n-1)y(n+m-1)]$

并且 $E[(y(n)-y(m))^2]=E[-2y(n)y(m)+y^2(n)+y^2(m)]=-2E[y(n)y(m)]=|n-m|$

$\Rightarrow E[y(n)y(m)]=-\frac{1}{2}|m-n|$ 代入原式有

$R_{xx}(m)=-\frac{1}{2}|m|+\frac{1}{2}|m-1|+\frac{1}{2}|m+1|-\frac{1}{2}|m|=\frac{1}{2}|m-1|+\frac{1}{2}|m+1|-|m|$

$\Rightarrow R_{xx}(m)=\begin{cases}1, & m=0\\ 0 & m\geq 1,\ m\leq -1\end{cases}$

4. $S_{XX}(e^{j\omega}) = S_{ww}(e^{j\omega}) H(e^{j\omega}) H(e^{-j\omega})$ 且 $S_{ww}(e^{j\omega}) = 1$

$$H(e^{j\omega}) H(e^{-j\omega}) = \frac{\omega^2+4}{\omega^4+10\omega^2+9} = \frac{(2-j\omega)(2+j\omega)}{(3+j\omega)(3-j\omega)(1+j\omega)(1-j\omega)}$$

令 $z = e^{j\omega}$，则 $\ln z = j\omega$. $H(z)H(z^{-1}) = \dfrac{(2+\ln z)(2-\ln z)}{(3+\ln z)(3-\ln z)(1+\ln z)(1-\ln z)}$

$H(z)$为稳定系统 $|z|<1 \Rightarrow H(z) = \dfrac{2+\ln z}{(3+\ln z)(1+\ln z)}$ 或 $\dfrac{2-\ln z}{(3-\ln z)(1-\ln z)}$

则有 $H(e^{j\omega}) = \dfrac{2+j\omega}{(3+j\omega)(1+j\omega)}$ 或 $\dfrac{2-j\omega}{(3-j\omega)(1-j\omega)}$

5. 1) $E[Z(n)] = E[X(n)] + E[Y(n)] = 0$

$R_Z(m) = E[Z(n)Z(n+m)] = E[X(n)X(n+m)] + E[X(n)Y(n+m)] + E[Y(n)]E[X(n+m)] + E[Y(n)Y(n+m)]$

$= E[A(n)A(n+m)]\cos\omega_0 n\cos\omega_0(n+m) + E[B(n)B(n+m)]\sin\omega_0 n\sin\omega_0(n+m)$

$= R(m)\cos\omega_0 m$

2) $R_Z(m) = R(m)\cos\omega_0 m$ 即 $R_Z(m)$ 只同 m 有关 ∴ 命题是平稳过程