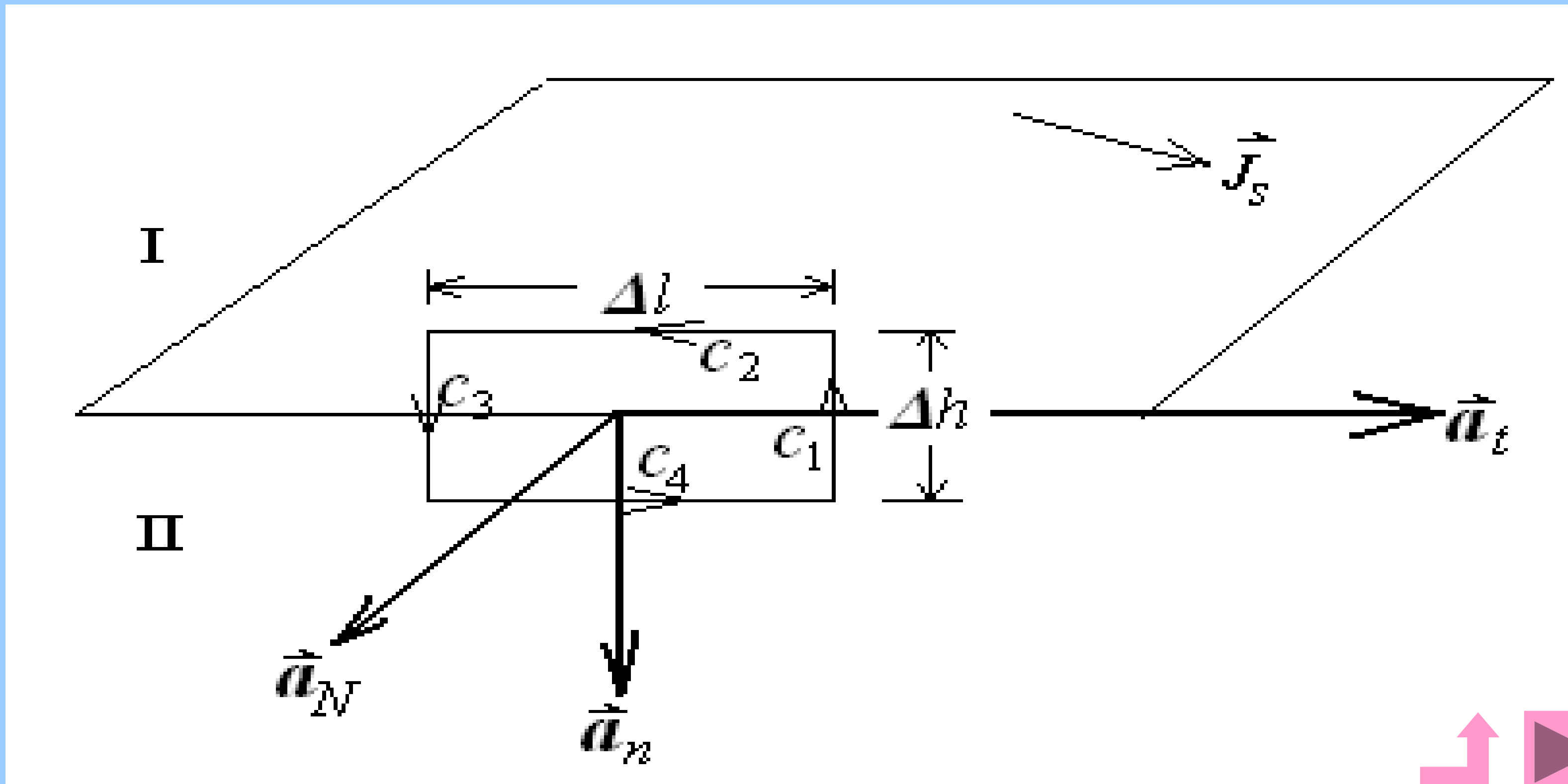
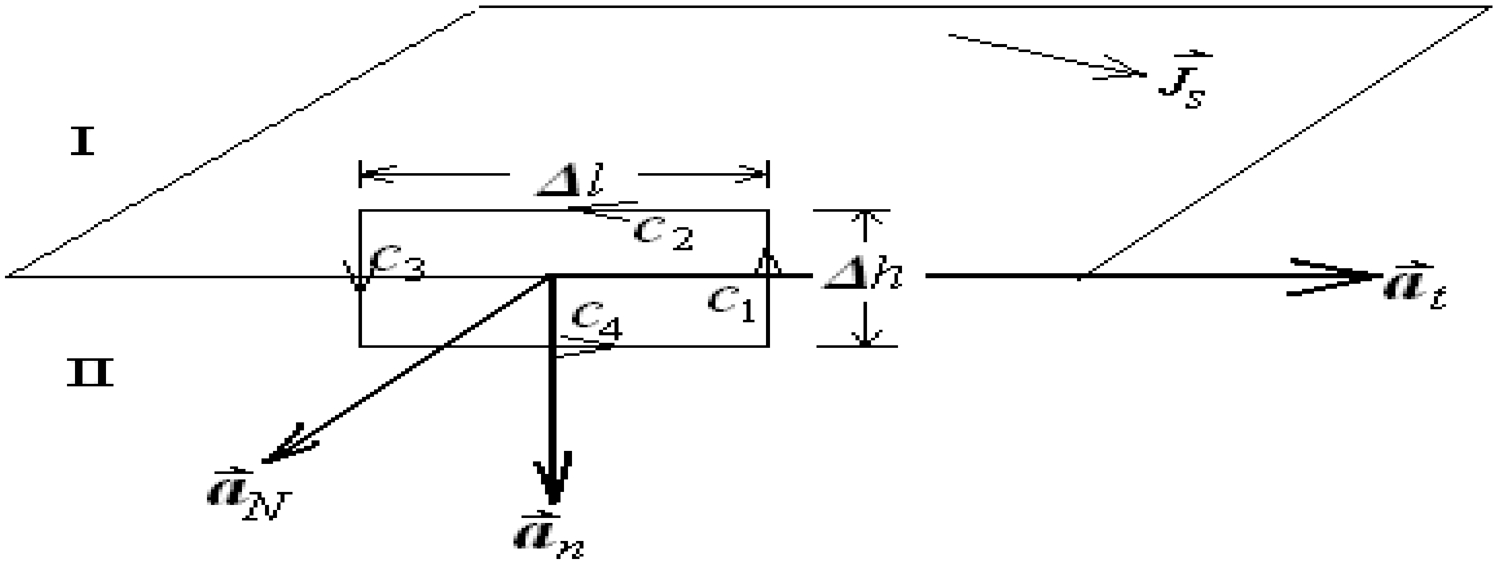
3. Boundary conditions for Tangential components of $\vec{H}$ field (page 212)

To obtain the Boundary conditions for Tangential components of $\vec{H}$ field, consider a closed path shown in the following figure.





where $\vec{a}_n$ is unit vector normal to the interface pointing from medium 1 to medium 2,

$\vec{\mathbf{a}}_N$ is unit vector normal to the surface bounded by the closed path *c*, which includes c_1, c_2, c_3 and c_4.

$\vec{a}_t$, $\vec{a}_N$ and $\vec{a}_n$ are three mutually perpendicular unit vectors as shown in the above figure. That is,

$$\vec{a}_t = \vec{a}_N \times \vec{a}_n$$

Applying Maxwell's equations to the closed path *c*, we obtain

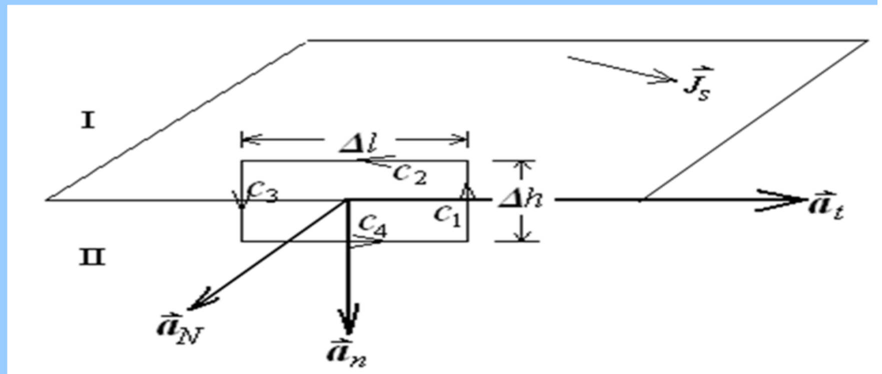
$$\oint_c \vec{H} \bullet d\vec{l} = \int_{c1} \vec{H} \bullet d\vec{l} + \int_{c2} \vec{H} \bullet d\vec{l} + \int_{c3} \vec{H} \bullet d\vec{l} + \int_{c4} \vec{H} \bullet d\vec{l}$$

$$= \int_s (\vec{J} + \frac{\partial \vec{D}}{\partial t}) \bullet d\vec{s} \qquad (2.7.6)$$

similarly, when $\Delta h \to 0$, we have

① $\lim\limits_{\Delta h \to 0} \int_{c1} \vec{H} \bullet d\vec{l} = 0, \quad \lim\limits_{\Delta h \to 0} \int_{c3} \vec{H} \bullet d\vec{l} = 0$

② $\int_s \frac{\partial \vec{D}}{\partial t} \bullet d\vec{\mathbf{s}} = 0$



because the area of the surface *S* bounded by the closed path *c* shrinks to zero while the time-varying electric field $\frac{\partial \vec{D}}{\partial t}$ is finite,

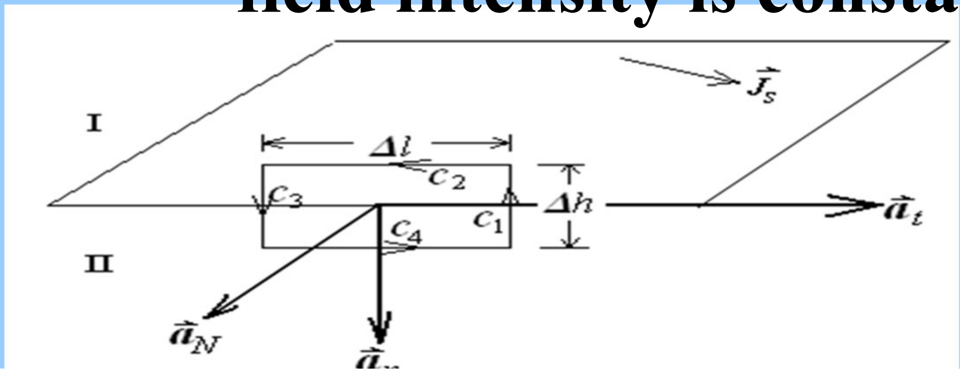
namely, $\lim\limits_{\Delta s \to 0} \int_{\Delta s} \frac{\partial \vec{\boldsymbol{D}}}{\partial t} \bullet d\vec{\boldsymbol{s}} = 0$

and

③ $\lim\limits_{\Delta h \to 0} \int_{\Delta s} \vec{J} \bullet d\vec{s} = \int_{\Delta l} \vec{J}_s \bullet \vec{a}_N dl$

Because Δl is so small that the magnetic field intensity is constant, we have

$$= \vec{J}_s \bullet \vec{a}_N \Delta l$$



④ $$\oint_l \vec{H} \bullet d\vec{l} = \int_{c_1} \vec{H}_1 \bullet d\vec{l} + \int_{c_2} \vec{H}_2 \bullet d\vec{l} + \int_{c_3} \vec{H} \bullet d\vec{l} + \int_{c_4} \vec{H} \bullet d\vec{l}$$

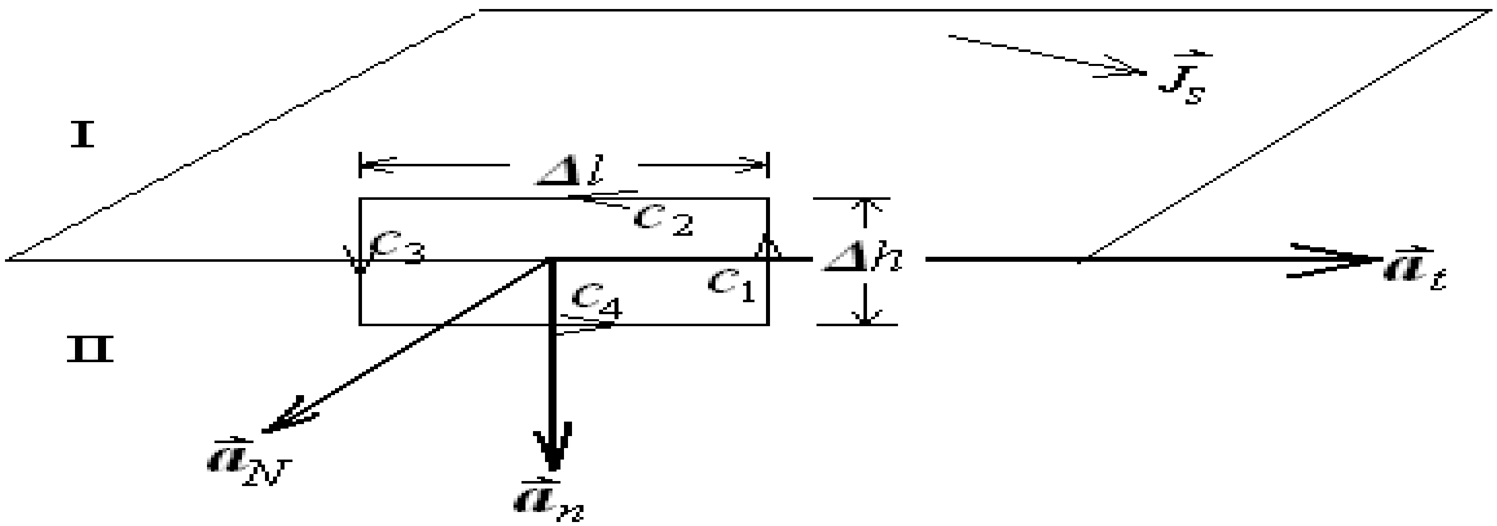
$$= \int_{c_2} \vec{H}_1 \bullet d\vec{l} + \int_{c_4} \vec{H}_2 \bullet d\vec{l} + 0 + 0$$

$$= \vec{H}_1 \bullet (-\vec{a}_t \Delta l) + \vec{H}_2 \bullet \vec{a}_t \Delta l$$

$$= \vec{a}_t \bullet (\vec{H}_2 - \vec{H}_1) \Delta l$$

therefore, we obtain

$$\vec{a}_t \bullet (\vec{H}_2 - \vec{H}_1) = \vec{J}_s \bullet \vec{a}_N \qquad (2.7.7a)$$



$$(\vec{a}_N \times \vec{a}_n) \bullet (\vec{H}_2 - \vec{H}_1) = \vec{J}_s \bullet \vec{a}_N$$

$$\vec{a}_N \bullet [\vec{a}_n \times (\vec{H}_2 - \vec{H}_1)] = \vec{J}_s \bullet \vec{a}_N$$

since $\vec{a}_N$ is arbitrary, we have

$$\boxed{\vec{a}_n \times (\vec{H}_2 - \vec{H}_1)] = \vec{J}_s} \qquad (2.7.7b)$$

which states that the tangential components of the magnetic field intensity are discontinuous. Note that the surface current density $\vec{J}_s$ results in the discontinuity of the tangential components of the magnetic field intensity

Equation(2.7.7) can be written in scalar form as

$$H_{2t} - H_{1t} = J_{sN}$$

❖**Discussion :**

when $\vec{J}_s = 0$**, we have**

$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1)] = 0$$

or $H_{2t} - H_{1t} = 0$

which states there is no conduction current over the interface.

If $\vec{B} = \mu \vec{H}$ **, we have**

$$\vec{a}_n \times \left(\frac{\vec{\boldsymbol{B}}_2}{\mu_2} - \frac{\vec{\boldsymbol{B}}_1}{\mu_1} \right) = 0$$

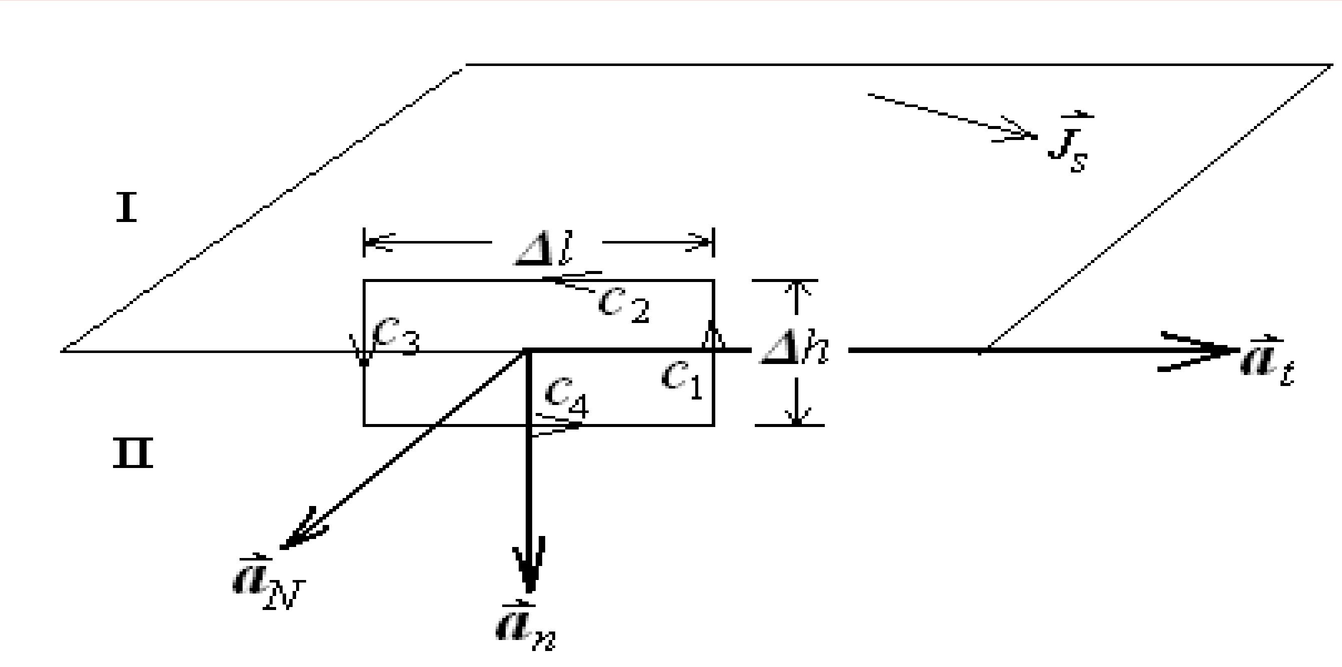
or

$$\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$$

when $\mu_1 \neq \mu_2$, *we have* $B_{1t} \neq B_{2t}$

- **4. Boundary conditions for tangential components of $\vec{E}$ Field page106**

To obtain the Boundary conditions for Tangential components of $\vec{E}$ field, consider a closed path shown in the following figure.



Applying Maxwell's equation

$$\oint_l \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}} = -\int_s \frac{\partial \vec{\mathbf{B}}}{\partial t} \bullet d\vec{\mathbf{s}}$$

we have

$$\oint_l \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}} = \int_{c1} \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}} + \int_{c2} \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}} + \int_{c3} \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}} + \int_{c4} \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}}$$

$$= -\int_s \frac{\partial \vec{\mathbf{B}}}{\partial t} \bullet d\vec{\mathbf{s}}$$

Similarly, consider $\varDelta h \to 0$, and also $\varDelta S = \varDelta h \cdot \varDelta l \to 0$, we can obtain

$$\int_{c1} \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}} = 0, \qquad \int_{c3} \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}} = 0$$

and

$$\lim_{\varDelta h \to 0} \int_s \frac{\partial \vec{\boldsymbol{B}}}{\partial t} \bullet d\vec{\boldsymbol{s}} = 0$$

therefore, $$\int_{c2} \vec{\mathbf{E}}_1 \bullet d\vec{\mathbf{l}} + \int_{c4} \vec{\mathbf{E}}_2 \bullet d\vec{\mathbf{l}} = 0$$

$$\vec{\mathbf{E}}_1 \bullet \Delta \vec{l}_2 + \vec{\mathbf{E}}_2 \bullet \Delta \vec{l}_4 = 0$$

$$\vec{\mathbf{E}}_1 \bullet (-\vec{\mathbf{a}}_t) \Delta l + \vec{\mathbf{E}}_2 \bullet \vec{\mathbf{a}}_t \Delta l = 0$$

$$\vec{a}_t \bullet (\vec{E}_2 - \vec{E}_1) = 0 \quad \Rightarrow \quad \boxed{\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0}$$

or

$$\boxed{E_{2t} = E_{1t}}$$

If $\vec{D} = \varepsilon \vec{E}$ **,we have**

$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$$

when $\varepsilon_1 \neq \varepsilon_2$***, we have*** $D_{2t} \neq D_{1t}$

❖Summary:

■**Normal components of electric fields**

$$\vec{a}_n \bullet (\vec{D}_2 - \vec{D}_1) = \rho_s \qquad D_{2n} - D_{1n} = \rho_s$$

■**Normal components of magnetic fields**

$$\vec{a}_n \bullet (\vec{B}_2 - \vec{B}_1) = 0 \qquad B_{2n} = B_{1n}$$

■**Tangential components of magnetic fields**

$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1)] = \vec{J}_s \qquad H_{2t} - H_{1t} = J_{sN}$$

■**Tangential components of electric fields**

$$\vec{a}_n \times (\vec{E}_2 - \vec{E}_1) = 0 \qquad E_{2t} = E_{1t}$$

❖Discussion:

Maxwell's equation (3) and (4) can be derived from Maxwell's equation (1) and (2), namely, Maxwell's equation (3) and (4) are dependent of Maxwell's equation (1) and (2), therefore, boundary conditions for normal components of electromagnetic fields is dependent of boundary conditions for tangential components of electro-magnetic fields.

For time-varying fields, as long as boundary conditions for tangential components of electric fields are met with, boundary conditions for normal components of magnetic fields will be met with. As long as boundary conditions for Tangential components of magnetic fields are met with, boundary conditions for normal components of electric fields.

- **1.The interface will be between a perfect dielectric and another perfect dielectric (ideal dielectric or lossless media $\rho_s=0$, $\vec{J}_s = 0$)**

- **Normal components of electric fields**

$$\vec{\boldsymbol{a}}_n \bullet (\vec{\boldsymbol{D}}_2 - \vec{\boldsymbol{D}}_1) = 0 \qquad D_{2n}=D_{1n}$$

- **Normal components of magnetic fields**

$$\vec{\boldsymbol{a}}_n \bullet (\vec{\boldsymbol{B}}_2 - \vec{\boldsymbol{B}}_1) = 0 \qquad B_{2n}=B_{1n}$$

- **Tangential components of magnetic fields**

$$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1)] = 0 \qquad H_{2t}=H_{1t}$$

- **Tangential components of electric fields**

$$\vec{\boldsymbol{a}}_n \times (\vec{\boldsymbol{E}}_2 - \vec{\boldsymbol{E}}_1) = 0 \qquad E_{2t}=E_{1t}$$

◆ **The fields are continuous at the boundary between the two ideal dielectric.**

2.The interface will be between a perfect Dielectric and an ideal conductor

Since, for an ideal conductor , its $\sigma \to \infty$, the electric field in the ideal conductor is zero.

(if the electric field is nonzero, say a finite value, the current density $\vec{J}_v = \sigma \vec{E}$ approaches infinite)

thus, time-varying magnetic fields do not exist in the ideal conductor (if not, time-varying magnetic fields will produce induced electric fields which produce infinite currents.)

All in all, in an ideal conductor,

$$\vec{\mathbf{E}} = 0, \qquad \vec{\mathbf{D}} = 0, \qquad \vec{\mathbf{B}} = 0, \qquad \vec{\mathbf{H}} = 0$$

If medium 1 is an ideal conductor, we can obtain

- **Normal components of electric fields**

$$\vec{\boldsymbol{a}}_n \bullet \vec{\boldsymbol{D}}_2 = \rho_s \qquad D_{2n} = \rho_s$$

- **Normal components of magnetic fields**

$$\vec{\boldsymbol{a}}_n \bullet \vec{\boldsymbol{B}}_2 = 0 \qquad B_{2n} = 0$$

- **Tangential components of magnetic fields**

$$\vec{a}_n \times \vec{H}_2 = \vec{J}_s \qquad H_{2t} = J_{sN}$$

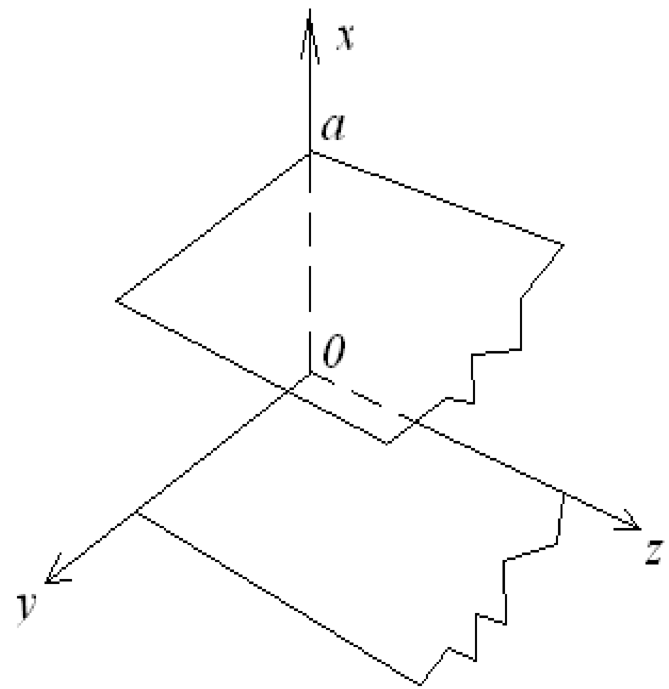
- **Tangential components of electric fields**

$$\vec{\boldsymbol{a}}_n \times \vec{\boldsymbol{E}}_2 = 0 \qquad E_{2t} = 0$$

The surface charge density ρ_s and the surface current J_{sN} are on the ideal conductor.

❖**Example:**

Two parallel conducting plates(σ→∞) are separated by an air gap shown in the figure. Between them the electric field intensity and the magnetic field intensity are given by



$$\vec{E} = \vec{a}_x E \cos(\omega t - \beta z)$$

$$\vec{H} = \vec{a}_y \frac{E}{\eta_0} \cos(\omega t - \beta z)$$

where E, β, η_0 and ω are constants. Find the surface charge/current density everywhere in these two conductors.

(1)the surface charge density everywhere in these two conductors.

(2) the surface current density everywhere in these two conductors.

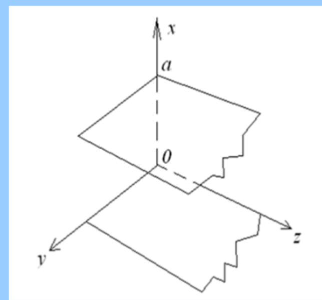
❖Solution:

Since σ→∞, charges and currents are distributed over the surfaces of these two conducting plates.

The charge distribution on the upper plate is

$$\rho_{upper} = \vec{\mathbf{a}}_n \bullet (\vec{\mathbf{D}}_{air} - \vec{\mathbf{D}}_{con}) = \vec{\mathbf{a}}_n \bullet (\vec{\mathbf{D}}_{air} - 0)$$

$$= -\vec{a}_x \bullet \varepsilon_0 \vec{\mathbf{E}} \quad = -\varepsilon_0 E \cos(\omega t - \beta z)$$



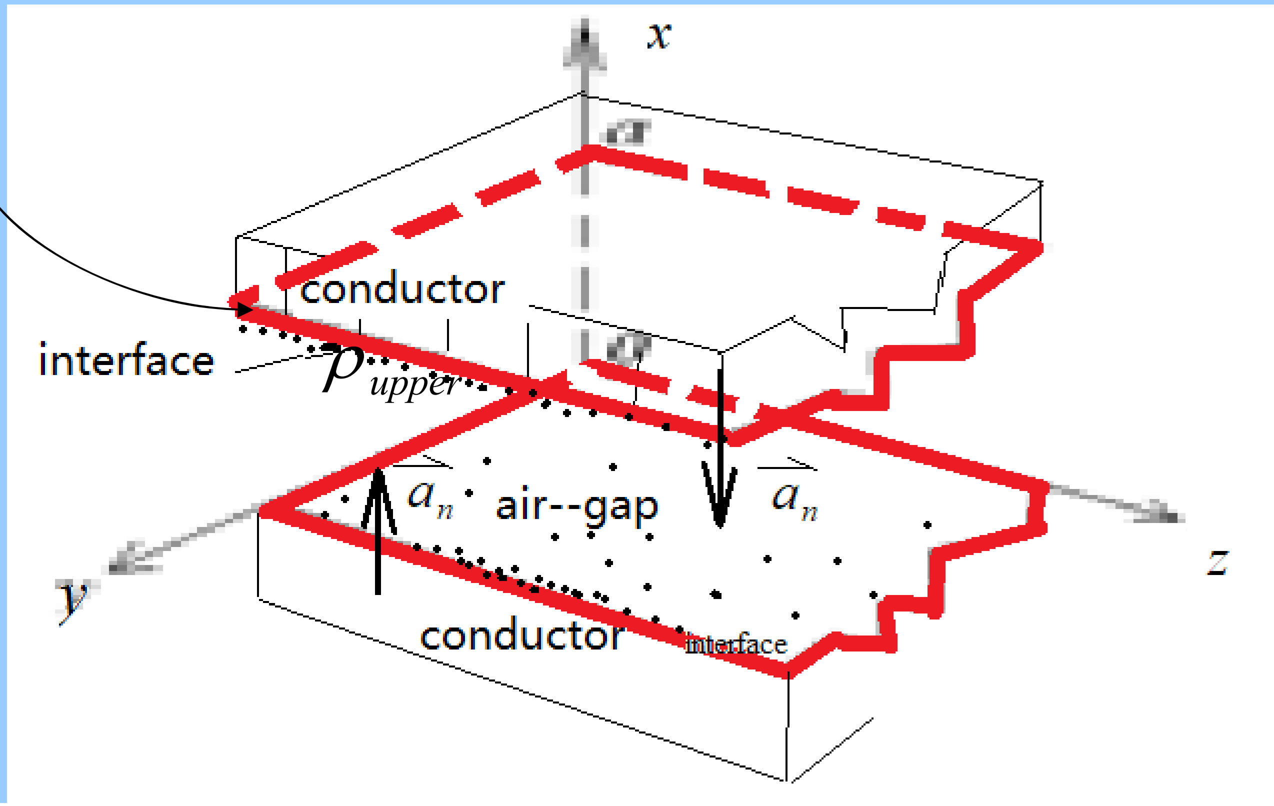
The current distribution on the upper plate is

The interface is between the upper plate and the air gap.

$$\rho_{upper} = \vec{\boldsymbol{a}}_n \bullet (\vec{\boldsymbol{D}}_{air} - \vec{\boldsymbol{D}}_{con}) = \vec{\boldsymbol{a}}_n \bullet (\vec{\boldsymbol{D}}_{air} - 0)$$

$$= -\vec{a}_x \bullet \varepsilon_0 \vec{\mathbf{E}} = -\varepsilon_0 E \cos(\omega t - \beta z)$$

The interface is between the upper plate and the air gap.



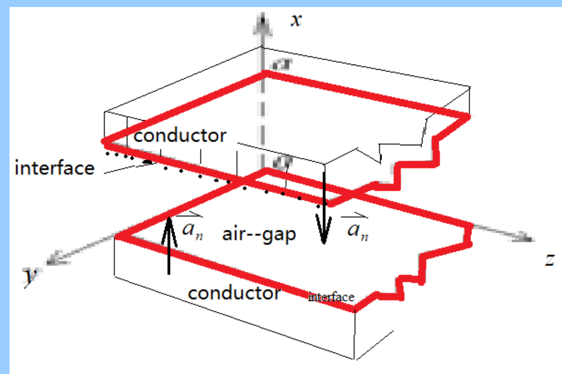
$$\vec{J}_{upper} = \vec{\mathbf{a}}_n \times (\vec{\mathbf{H}}_{air} - \vec{\mathbf{H}}_{con}) = \vec{\mathbf{a}}_n \times (\vec{\mathbf{H}}_{air} - 0)$$

$$= -\vec{a}_x \times \vec{a}_y \frac{E}{\eta_0} \cos(\omega t - \beta z)$$

$$= -\vec{a}_z \frac{E}{\eta_0} \cos(\omega t - \beta z)$$

where $\vec{a}_n$ is unit vector pointing from the upper conducting plate to the air.

The charge distribution on the lower plate is

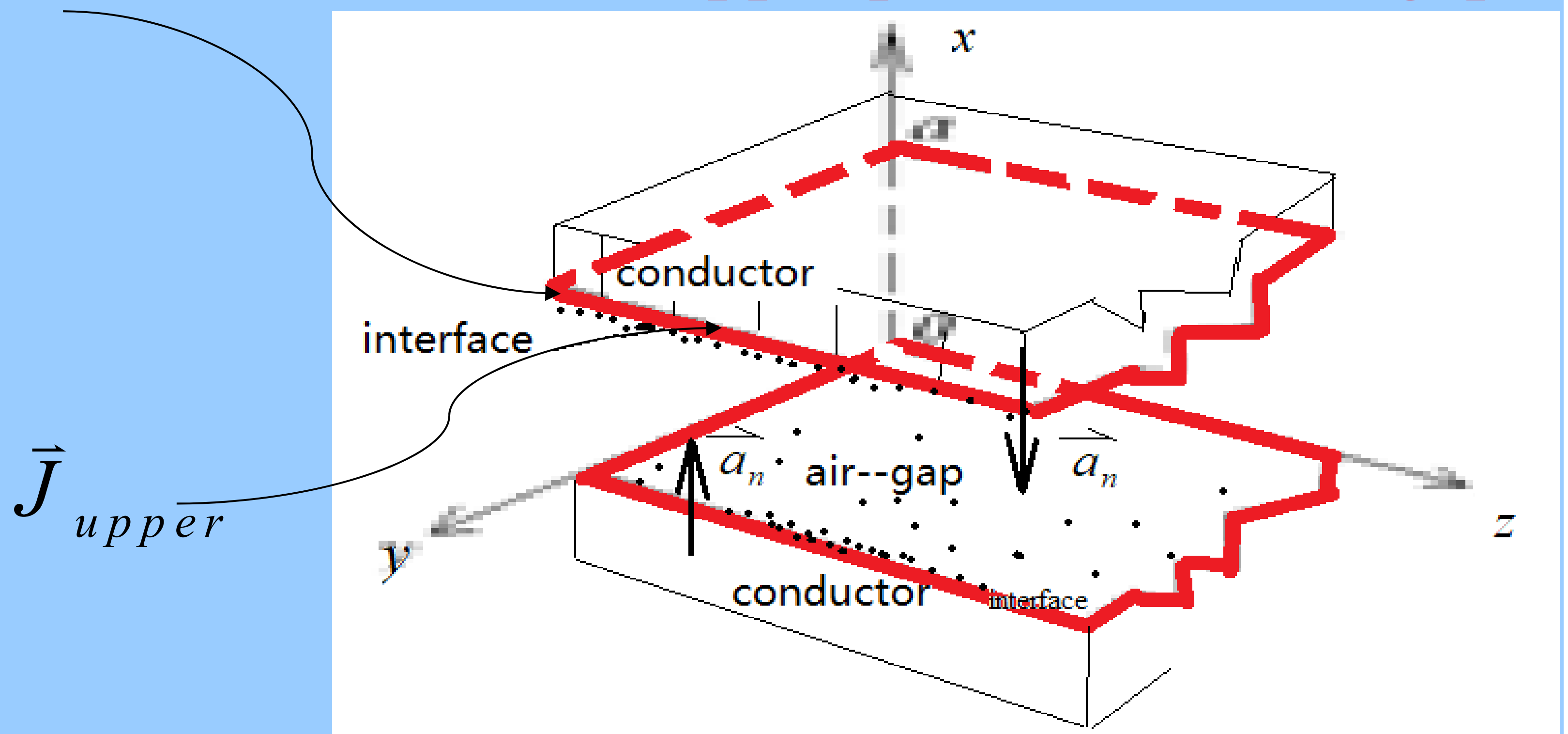


$$\vec{J}_{upper} = \vec{\boldsymbol{a}}_n \times (\vec{\boldsymbol{H}}_{air} - \vec{\boldsymbol{H}}_{con}) = \vec{\boldsymbol{a}}_n \times (\vec{\boldsymbol{H}}_{air} - 0)$$

$$= -\vec{a}_x \times \vec{a}_y \frac{E}{\eta_0} \cos(\omega t - \beta z)$$

$$= -\vec{a}_z \frac{E}{\eta_0} \cos(\omega t - \beta z)$$

The interface is between the upper plate and the air gap.



The charge distribution on the lower plate is

$$\rho_{lower} = \vec{\mathbf{a}}_n \bullet (\vec{\mathbf{D}}_{air} - \vec{\mathbf{D}}_{con}) = \vec{\mathbf{a}}_n \bullet (\vec{\mathbf{D}}_{air} - 0)$$

$$= \vec{a}_x \bullet \varepsilon_0 \vec{\mathbf{E}} = \varepsilon_0 E \cos(\omega t - \beta z)$$

The current distribution on the lower plate is

$$\vec{J}_{lower} = \vec{\mathbf{a}}_n \times (\vec{\mathbf{H}}_{air} - \vec{\mathbf{H}}_{con}) = \vec{\mathbf{a}}_n \times (\vec{\mathbf{H}}_{air} - 0)$$

$$= \vec{a}_x \times \vec{a}_y \frac{E}{\eta_0} \cos(\omega t - \beta z)$$

$$= \vec{a}_z \frac{E}{\eta_0} \cos(\omega t - \beta z)$$

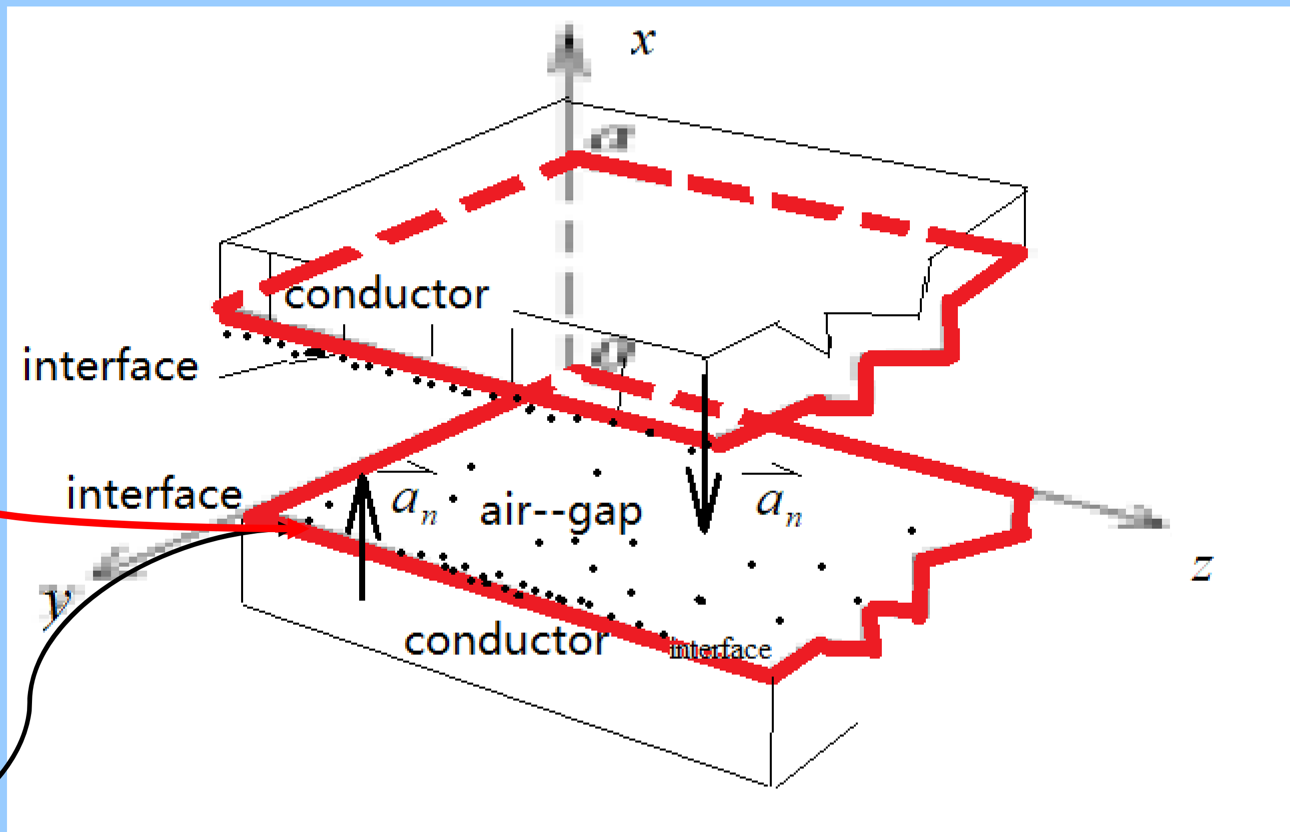
The interface is between the lower plate and the air gap.

where $\vec{a}_n$ is unit vector pointing from the lower conducting plate to the air.

$$\rho_{lower} = \vec{\mathbf{a}}_n \bullet (\vec{\mathbf{D}}_{air} - \vec{\mathbf{D}}_{con}) = \vec{\mathbf{a}}_n \bullet (\vec{\mathbf{D}}_{air} - 0)$$

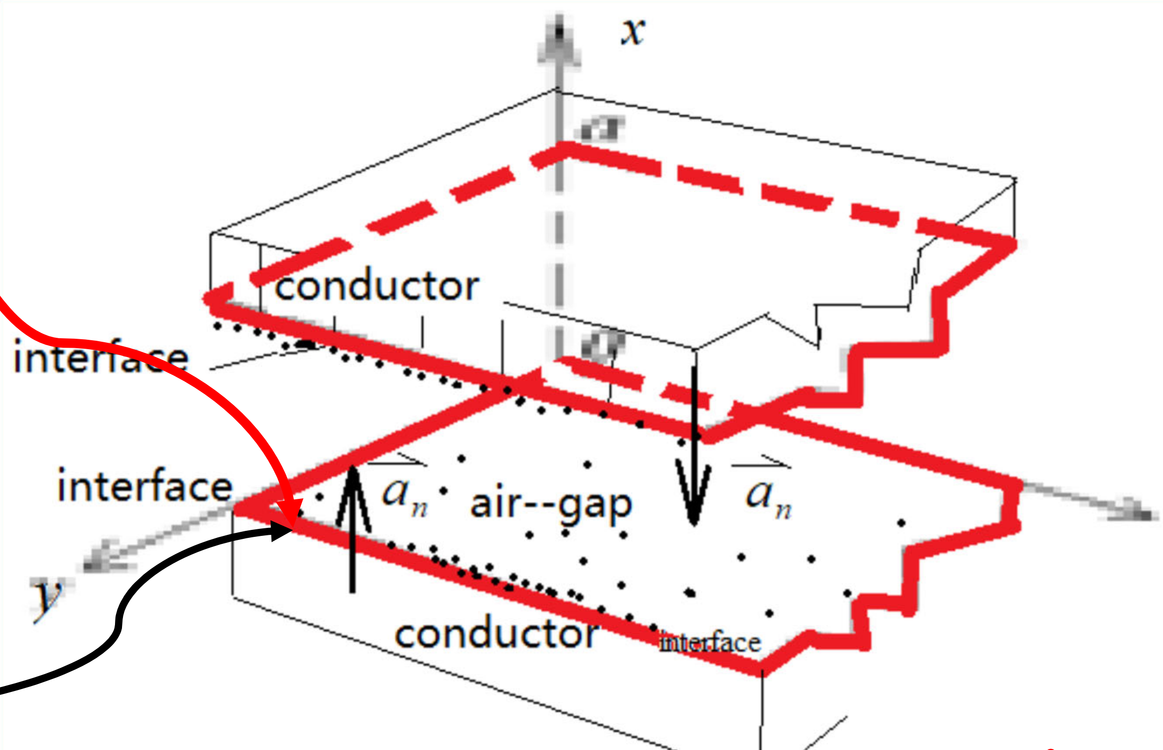
$$= \vec{a}_x \bullet \varepsilon_0 \vec{\mathbf{E}} = \varepsilon_0 E \cos(\omega t - \beta z)$$

ρ_{lower}



The interface is between the lower plate and the air gap.

$$
\begin{aligned}
\vec{J}_{lower} &= \vec{\boldsymbol{a}}_n \times (\vec{\boldsymbol{H}}_{air} - \vec{\boldsymbol{H}}_{con}) = \vec{\boldsymbol{a}}_n \times (\vec{\boldsymbol{H}}_{air} - 0) \\
&= \vec{a}_x \times \vec{a}_y \frac{E}{\eta_0} \cos(\omega t - \beta z) \\
&= \vec{a}_z \frac{E}{\eta_0} \cos(\omega t - \beta z)
\end{aligned}
$$



The interface is between the lower plate and the air gap.

作业：

自己阅读：6.8 时变电磁场的唯一性定理

P286-287：T6.1;T6.4;T6.5;T6.8;T6.11