"CHAPTER- 11 — ANTENNAS"

Exercise 11.1 $\nabla\cdot\vec{D} = \rho \quad \vec{D} = \epsilon\vec{E} \Rightarrow \nabla\cdot\vec{E} = \rho/\epsilon$

But $\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$. Thus $\nabla\cdot(\nabla V + \frac{\partial \vec{A}}{\partial t}) = -\frac{\rho}{\epsilon}$

Hence, $\nabla^2 V + \frac{\partial}{\partial t}(\nabla\cdot\vec{A}) = -\frac{\rho}{\epsilon}$

Since $\nabla\cdot\vec{A} = -\mu\epsilon\frac{\partial V}{\partial t}$, we have $\nabla^2 V - \mu\epsilon\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon}$

Exercise 11.2

$$\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} = -\mu J_x$$

$$\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} = -\mu J_y$$

$$\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} = -\mu J_z$$

Exercise 11.3

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}\left(\frac{\tilde{G}}{r}\right)\right] + \beta^2\frac{\tilde{G}}{r} = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(\frac{1}{r}\frac{\partial \tilde{G}}{\partial r} - \frac{1}{r^2}\tilde{G}\right)\right] + \beta^2\frac{\tilde{G}}{r} = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r\frac{\partial \tilde{G}}{\partial r} - \tilde{G}\right] + \beta^2\frac{\tilde{G}}{r} = 0$$

$$\frac{1}{r^2}\left[r\frac{\partial^2 \tilde{G}}{\partial r^2} + \frac{\partial \tilde{G}}{\partial r} - \frac{\partial \tilde{G}}{\partial r}\right] + \beta^2\frac{\tilde{G}}{r} = 0$$

Thus, if $r \neq 0$, $\quad \frac{\partial^2 \tilde{G}}{\partial r^2} + \beta^2\tilde{G} = 0$

Exercise 11.4 From (11.35) $\quad \frac{1}{\beta r} = 1$

Since $\beta = \frac{2\pi}{\lambda}$, $\quad r = \frac{1}{\beta} = \frac{\lambda}{2\pi} \simeq \frac{\lambda}{6}$

Exercise 11.5 $\quad \tilde{A} = \dfrac{\mu \tilde{I} l}{4\pi r} e^{-j\beta r}\left[\cos\theta\, \vec{a}_r - \sin\theta\, \vec{a}_\theta\right]$

$$\tilde{B} = \nabla\times\tilde{A} \;\Rightarrow\; \tilde{H} = \frac{1}{\mu}(\nabla\times\tilde{A}) = j\,\frac{\beta\tilde{I}l}{4\pi r}\left(1+\frac{1}{j\beta r}\right)\sin\theta\, e^{-j\beta r}\,\vec{a}_\phi$$

$$\tilde{E} = \frac{1}{j\omega\epsilon}(\nabla\times\tilde{H}) = \frac{\beta\tilde{I}l}{4\pi\omega\epsilon r^2}\left(1+\frac{1}{j\beta r}\right)(2\cos\theta)\, e^{-j\beta r}\,\vec{a}_r$$

$$+\; \frac{j\beta\tilde{I}l}{4\pi r\epsilon\omega}\,\beta\left[1+\frac{1}{j\beta r}-\frac{1}{\beta^2 r^2}\right]\sin\theta\, e^{-j\beta r}\,\vec{a}_\theta$$

substituting $\dfrac{\beta}{\omega\epsilon} = \eta$, we obtain the desired equation.

Exercise 11.6 $\quad \tilde{H} = \dfrac{j\beta\tilde{I}l}{4\pi r}\sin\theta\, e^{-j\beta r}\,\vec{a}_\theta\,, \quad \nabla\cdot\tilde{B}=0,\ \nabla\cdot\tilde{H}=0$ satisfied

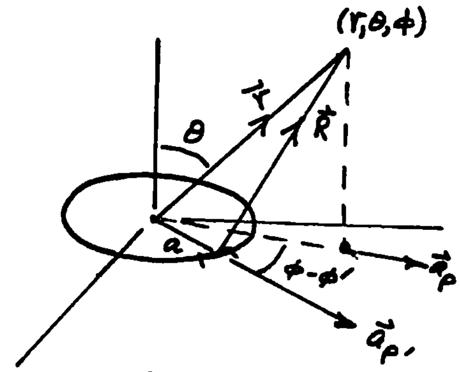
$\tilde{E} = \dfrac{j\beta\tilde{I}l}{4\pi r}\,\eta\,\sin\theta\, e^{-j\beta r}\,\vec{a}_\theta\,.\quad$ For $\nabla\cdot\tilde{D}=0$, $\nabla\cdot\tilde{E}$ must be zero.

However, $\nabla\cdot\tilde{E} = \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial\theta}(\sin\theta\,\tilde{E}_\theta)$ is not zero.

Exercise 11.7

$$\vec{a}_\rho\cdot\vec{a}_r = \sin\theta$$

$$(\vec{a}_\rho\cdot\vec{a}_r)\,\vec{a}_\rho = \sin\theta\,\vec{a}_\rho$$



$\vec{a}_r\cdot\vec{a}_{\rho'} = \sin\theta\cos(\phi-\phi')$

$a\vec{a}_{\rho'} + \vec{R} = r\,\vec{a}_r \;\Rightarrow\; \vec{R} = r\,\vec{a}_r - a\,\vec{a}_{\rho'}$

$R^2 = r^2 + a^2 - 2ar\,\vec{a}_r\cdot\vec{a}_{\rho'}$

$\quad = r^2 + a^2 - 2ar\sin\theta\cos(\phi-\phi')$

For $r \gg a$, $\quad R^2 \simeq r^2 - 2ar\sin\theta\cos(\phi-\phi')$

or $\quad R = r\left[1-\dfrac{2ar}{r^2}\sin\theta\cos(\phi-\phi')\right]^{1/2} = r - a\sin\theta\cos(\phi-\phi')$ [11.53]

$$\int_0^{2\pi}\left[1+j\beta a\sin\theta\cos(\phi-\phi')\right]\cos(\phi-\phi')\,d\phi'$$

$$= \int_0^{2\pi}\cancelto{0}{\cos(\phi-\phi')\,d\phi'} + j\beta a\sin\theta\int_0^{2\pi}\cancelto{\pi}{\cos^2(\phi-\phi')\,d\phi'}$$

$$= j\beta a\pi\sin\theta \qquad (11.57)$$

Exercise 11.8

$\omega = 3\times10^6$ rad/s $\quad \tilde{I} = 100\, e^{-j\pi/6}$ A, $\quad M = \pi a^2 \tilde{I} = 3.142\, e^{-j\pi/6}$

$\beta_o = \frac{\omega}{c} = 0.01$, $\quad \eta_o = 120\pi \quad \lambda_o = \frac{2\pi}{\beta_o} = 628.32\,m$

$\frac{\omega\mu_o\beta_o}{4\pi}\tilde{M} = 9.426\times10^{-3}\, e^{-j\pi/6} \qquad R_{rad} = \frac{\pi}{6}\times120\pi\,(0.01\times0.1)^4 \cong 197\ p\Omega$

$\tilde{E}_\phi = \frac{9.426}{r}\sin\theta\, e^{-j0.01r}\, e^{-j\pi/6}$ mV/m

$\tilde{H}_\theta = -\frac{25}{r}\sin\theta\, e^{-j0.01r}\, e^{-j\pi/6}\ \mu A/m$

when $\omega = 30\times10^6$ rad/s $\quad \beta = 0.1$ rad/m $\quad \lambda_o = 62.83\,m$ and $R_{rad} = 1.97\ \mu\Omega$

Exercise 11.9

$$\int_{-l/2}^{l/2}(1-2z/l)(1+j\beta z\cos\theta)\,dz = 2\int_0^{l/2}\left(1-\frac{2z}{l}+j\beta z\cos\theta - j\frac{2\beta}{l}z^2\cos\theta\right)dz$$

$$= 2\left[z - \frac{z^2}{2} + j\beta\frac{z^2}{2}\cos\theta - j\frac{2\beta}{l}\frac{z^3}{3}\cos\theta\right]_0^{l/2}$$

$$= \frac{l}{2} + j\frac{\beta l}{12}\cos\theta$$

Thus, $\quad \tilde{A}_z = \frac{\mu}{8\pi r}\tilde{I}l\left(1+j\frac{\beta l}{6}\cos\theta\right)e^{-j\beta r} \qquad (11.70)$

$$\tilde{H}_\phi = \frac{j\beta}{\mu}\sin\theta\,\tilde{A}_z = \frac{j\beta\tilde{I}l}{8\pi r}\left(1+j\frac{\beta l}{6}\cos\theta\right)\sin\theta\, e^{-j\beta r} \qquad (11.71)$$

and $\quad \tilde{E}_\theta = \eta\tilde{H}_\phi$

Exercise 11.10

$P_{rad}\big|_{mono} = \frac{1}{2}P_{rad}\big|_{dipole} \Rightarrow$

$$\langle\hat{S}\rangle_{mono} = \frac{I^2l^2\beta^2\eta}{128\pi^2r^2}\sin^2\theta\,\vec{a}_r$$

$$P_{rad} = \int_0^{\pi}\frac{I^2l^2\beta^2\eta}{\pi^2\cdot128}\sin^3\theta\,d\theta\int_0^{2\pi}d\phi$$

$$= \frac{I^2l^2\beta^2\eta}{96\pi}$$

$$\int_0^{\pi/2}\sin^3\theta\,d\theta = \left[\frac{1}{3}\cos^3\theta - \cos\theta\right]_0^{\pi/2} = \frac{2}{3}$$

Finally,

$$R_{rad} = \frac{l^2\beta^2\eta}{48\pi} = \frac{\pi}{12}\eta\left(\frac{l}{\lambda}\right)^2\ \Omega$$

Exercise 11.11

$$G = \frac{4\pi r^2 \langle \hat{S} \rangle}{P_{rad}} = \frac{4\pi r^2 \eta I_0^2}{8\pi^2 r^2} \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta} \cdot \frac{4\pi}{1.219\, \eta I_0^2}$$

$$= 1.64 \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}$$

when $\theta \to 90^\circ$, $G \to D$ $\Rightarrow$ $D = 1.64$ or $2.15\,dB$

Exercise 11.12

$f = 30\,MHz$, $\omega = 2\pi f = 1.885\times10^8$ rad/s $\quad \beta = \frac{\omega}{c} = 0.628$ rad/m

$\lambda = \frac{2\pi}{\beta} = 10m$, $\frac{\lambda}{2} = 5\,m$ $\quad \theta = \frac{\pi}{6}$ $\quad r = 5000\,m$

From (11.77) $\quad I_0 \cong 2A \quad \tilde{E} = \frac{j120}{r} e^{-j\beta r} \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \vec{a}_\theta$

$$\tilde{H}_\phi = \frac{\tilde{E}_\theta}{\eta} = \frac{j}{\pi r} e^{-j\beta r} \cos(\tfrac{\pi}{2}\cos\theta)/\sin\theta$$

$R_{rad} = 73.14\,\Omega \qquad P_{rad} = \frac{1}{2} I_0^2 R_{rad} = 145.5\,W$

Exercise 11.13

Math CAD:

$\phi = 0, \frac{\pi}{500} .. 2\cdot\pi$

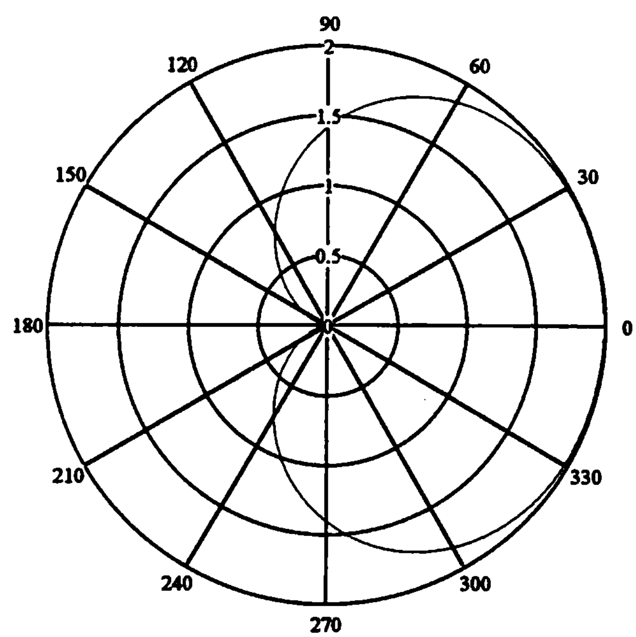
$k = 1 \quad \theta = \frac{\pi}{2} \quad \beta d = \frac{\pi}{2}$

$\delta = -\frac{\pi}{2}$

$\alpha(\phi) = \beta d \cdot \sin(\theta) \cdot \cos(\phi) + \delta$

$F(\theta) = 1$

$$G(\phi) = \sqrt{(1 + k\cdot\cos(\alpha(\phi)))^2 + (k\cdot\sin(\alpha(\phi)))^2}$$



Plot is $F\theta \cdot G\phi$ as a function of ϕ

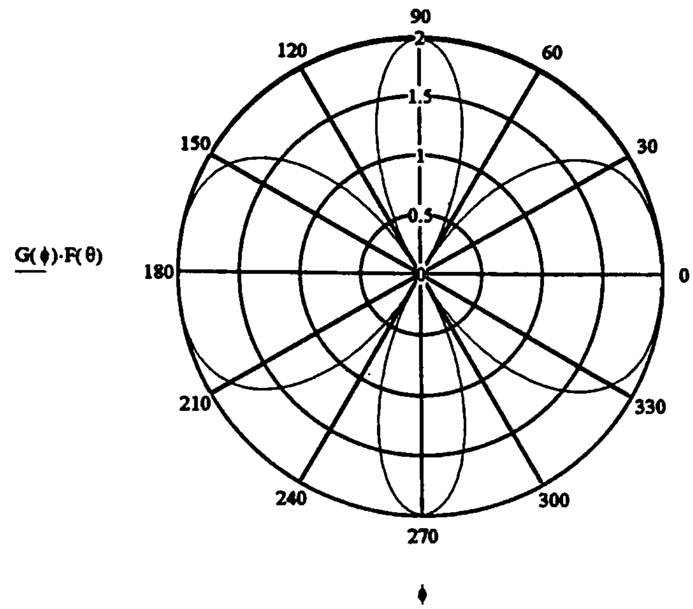
ϕ

Exercise 11.14

$\phi := 0, \frac{\pi}{500} .. 2\cdot\pi$ $\quad k := 1 \quad \theta := \frac{\pi}{2} \quad \beta d := 2\cdot\pi \quad \delta := 0$

$\alpha(\phi) := \beta d\cdot\sin(\theta)\cdot\cos(\phi) + \delta$

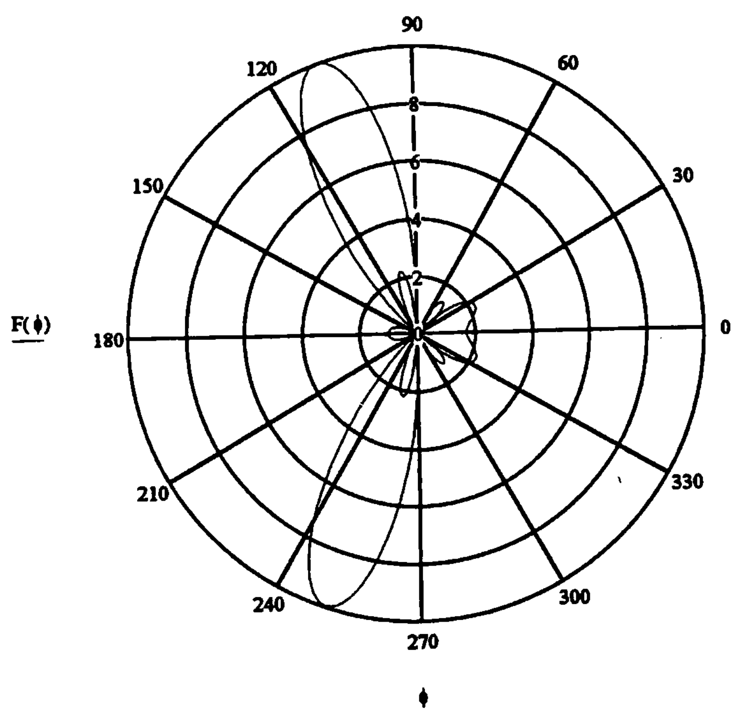
Note that $\quad F(\theta) := 1 \qquad G(\phi) := \sqrt{(1 + k\cdot\cos(\alpha(\phi)))^2 + (k\cdot\sin(\alpha(\phi)))^2}$



Exercise 11.15

$\phi := 0, \frac{\pi}{500} .. 2\cdot\pi \qquad \beta d := \frac{\pi}{2} \qquad \delta := \frac{\pi}{6} \qquad n := 10 \qquad \alpha(\phi) := \beta d\cdot\cos(\phi) + \delta$

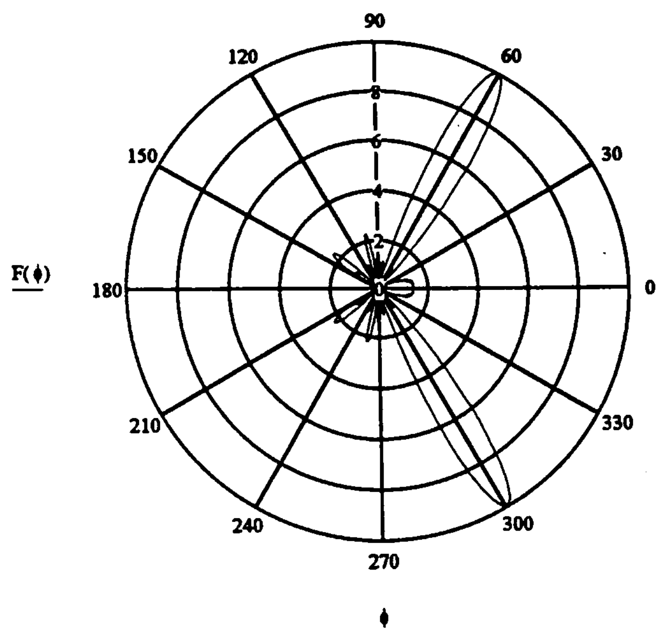
$$F(\phi) := \frac{\sin(n\cdot\alpha(\phi)\cdot 0.5)}{\sin(\alpha(\phi)\cdot 0.5)}$$



Exercise 11.16

$\phi := 0, \frac{\pi}{500} .. 2\cdot\pi$ $\quad \beta d := \pi$ $\quad \delta := -\left(\frac{\pi}{2}\right)$ $\quad n := 10$ $\quad \alpha(\phi) := \beta d\cdot\cos(\phi) + \delta$

$$F(\phi) := \frac{\sin(n\cdot\alpha(\phi)\cdot 0.5)}{\sin(\alpha(\phi)\cdot 0.5)}$$



Exercise 11.17

$f = 300\,MHz \qquad \omega = 2\pi f = 1.885\times10^{9}$ rad/s

$\mu_0 = 4\pi\times10^{-7}$ H/m $\qquad \sigma_{cu} = 5.8\times10^{7}$ S/m $\qquad a = 406.5\ \mu m$

$\alpha = \sqrt{\pi f \mu_0 \sigma_{cu}} = 262.092\times10^{3}$ NP/m $\Rightarrow \delta = \frac{1}{\alpha} = 3.815\ \mu m$

since $a \gg \delta$, $A_{cu} = 2\pi a\delta \qquad \ell = \frac{\lambda}{10} \qquad \lambda = \frac{2\pi}{\beta} = \frac{c}{f} = 1$

Hence $R_c = \dfrac{\lambda/10}{2\pi a \delta \sigma_{cu}} = 0.177\Omega$

Short. antenna: $R_{rad} = \frac{2\pi}{12}\eta\left(\frac{\ell}{\lambda}\right)^2 = 1.974\,\Omega$

$$\eta = \frac{R_{rad}}{R_{rad}+R_c} = 0.9177 \text{ or } 91.77\%$$

Exercise 11.18 $f = 600\,MHz$ $\omega = 2\pi f$ $\lambda = \frac{3}{f} = 0.5\,m$ $\ell = \frac{\lambda}{2} = 0.25\,m$

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{1}{\pi f \mu_o \sigma_{cu}}} = 2.698\,\mu m, \quad a = 406.5\,\mu m \quad a \gg \delta \quad A_{cu} = 2\pi a \delta$$

$$R_c = \frac{\ell}{2\pi a \delta \sigma_{cu}} = 0.626\,\Omega \qquad R_{rad} = 73.14\,\Omega \qquad \eta = \frac{73.14}{73.14 + 0.626} = 0.9915$$

or 99.15%

Exercise 11.19

$$A_{et} = \frac{\lambda^2}{4\pi} G_t = \frac{3^2}{4\pi} \times 1.64 = 1.175 \qquad P_R = 10 \times 10^3 \times (1.175)^2 \left(\frac{1}{3 \times 25000}\right)^2$$

$$A_{er} = \frac{\lambda^2}{4\pi} G_R = \frac{3^2}{4\pi} \times 1.64 = 1.175 \qquad = 2.45\,\mu W$$

Exercise 11.20 $D_r = 10^{1.2} = 15.85$ $D_t = 10^2 = 100$

Since $\theta = 90^o$ $G_R = D_r = 15.85$ $G_t = D_t = 100$ $R = 100\lambda$ $P_r = 10\,\mu W$

From (11.112), $$P_{rad} = \frac{P_R}{G_t G_R \, (\lambda/4\pi R)^2} = 9.96\,mW$$

Exercise 11.21 $$P_R = \frac{1}{4\pi}\left(\frac{G\lambda}{4\pi R^2}\right)^2 A_{eo} P_{rad}$$

$$A_e = \frac{\lambda^2}{4\pi} G \Rightarrow G = \frac{4\pi A_e}{\lambda^2}. \text{ Hence, } P_R = \frac{1}{4\pi}\left(\frac{A_e}{\lambda R^2}\right)^2 A_{eo} P_{rad}$$

Exercise 11.22 From Example 11.8,

$$P_R = \frac{1}{4\pi}\left[\frac{100 \times 0.1}{4\pi \times 2000^2}\right]^2 4 \times 100 \times 10^3 = 1.26\,nW$$

Problem 11.1.

$$\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} \Rightarrow \quad \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta E_\phi) - \frac{\partial E_\theta}{\partial\phi}\right] = -\mu\frac{\partial H_r}{\partial t}$$

$$\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\phi}E_r\right] - \frac{1}{r}\frac{\partial}{\partial r}(rE_\phi) = -\mu\frac{\partial H_\theta}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta) - \frac{1}{r}\frac{\partial}{\partial\theta}E_r = -\mu\frac{\partial H_\phi}{\partial t}$$

$$\nabla\cdot\vec{B}=0 \Rightarrow \nabla\cdot\vec{H}=0 \Rightarrow \quad \frac{1}{r^2}\frac{\partial}{\partial r}(r^2H_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, H_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}H_\phi = 0$$

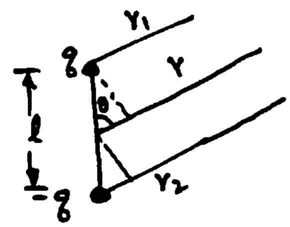
$$\nabla\times\vec{H} = \epsilon\frac{\partial\vec{E}}{\partial t} \Rightarrow \quad \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta H_\phi) - \frac{\partial H_\theta}{\partial\phi}\right] = \epsilon\frac{\partial E_r}{\partial t}$$

$$\frac{1}{r\sin\theta}\cdot\frac{\partial H_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(rH_\theta) = \epsilon\frac{\partial E_\theta}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH_\theta) - \frac{1}{r}\frac{\partial}{\partial\theta}H_r = \epsilon\frac{\partial E_\phi}{\partial t}$$

$$\nabla\cdot\vec{D}=0 \Rightarrow \nabla\cdot\vec{E}=0 \Rightarrow \quad \frac{1}{r^2}\frac{\partial}{\partial r}(r^2E_r) + \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta E_\theta) + \frac{\partial E_\phi}{\partial\phi}\right] = 0$$

Problem 11.2



$$\tilde{V} = \frac{q}{4\pi\epsilon}\left[\frac{e^{-j\beta r_1}}{r_1} - \frac{e^{-j\beta r_2}}{r_2}\right]$$

$$r_1 \cong r - \frac{l}{2}\cos\theta, \quad r_2 = r + \frac{l}{2}\cos\theta$$

$$r_1 r_2 \cong r^2 - \left(\frac{l}{2}\cos\theta\right)^2 \cong r^2$$

$$\tilde{V} = \frac{q}{4\pi\epsilon}e^{-j\beta r}\left[\frac{e^{j\frac{\beta l}{2}\cos\theta}}{r - \frac{l}{2}\cos\theta} - \frac{e^{-j\frac{\beta l}{2}\cos\theta}}{r + \frac{l}{2}\cos\theta}\right]$$

Thus, $$\tilde{V} = \frac{q}{4\pi\epsilon r^2}e^{-j\beta r}\left[\left(r + \frac{l}{2}\cos\theta\right)e^{j\frac{\beta l}{2}\cos\theta} - \left(r - \frac{l}{2}\cos\theta\right)e^{-j\frac{\beta l}{2}\cos\theta}\right]$$

$$= \frac{q}{4\pi\epsilon r^2}e^{-j\beta r}\left[j2r\sin\left(\frac{\beta l}{2}\cos\theta\right) + l\cos\theta\cos\left(\frac{\beta l}{2}\cos\theta\right)\right]$$

when $\beta l \ll r$, $$\tilde{V} = \frac{q}{4\pi\epsilon r^2}e^{-j\beta r}\left[j\beta l r\cos\theta + l\cos\theta\right]$$

$[\sin\theta \cong \theta]$

Finally, $$\tilde{V} = \frac{ql}{4\pi\epsilon}\left[\frac{1}{r^2} + j\frac{\beta}{r}\right]\cos\theta\, e^{-j\beta r}$$

Problem 11.3 $\tilde{E} = -j\omega\tilde{A} - \nabla\tilde{V}$ But $\nabla\cdot\tilde{A} = -j\omega\mu\epsilon\tilde{V}$

Hence $\tilde{E} = -j\omega\tilde{A} + \dfrac{\nabla(\nabla\cdot\tilde{A})}{j\omega\mu\epsilon} = -j\omega\tilde{A} - j\dfrac{\omega}{\beta^2}\nabla(\nabla\cdot\tilde{A})$ $\beta = \omega\sqrt{\mu\epsilon}$

Thus: $\tilde{E} = -j\omega\left[\tilde{A} + \dfrac{1}{\beta^2}\nabla(\nabla\cdot\tilde{A})\right]$

Problem 11.4 $\tilde{A} = \sin\beta y\ \tilde{a}_x$, The wave equation in a source-free medium is $\nabla^2\tilde{A} + \omega^2\mu\epsilon\tilde{A} = 0$ $\nabla^2\tilde{A} = \cancel{\dfrac{\partial^2\tilde{A}_x}{\partial x^2}}^{\,0} + \dfrac{\partial^2\tilde{A}_x}{\partial y^2} + \cancel{\dfrac{\partial^2\tilde{A}_x}{\partial z^2}}^{\,0} = -\beta^2\tilde{A}_x$

Thus, $\beta^2 = \omega^2\mu\epsilon$

$\tilde{B} = \nabla\times\tilde{A} = -\beta\cos\beta y\ \vec{a}_z \Rightarrow \tilde{H}_z = -\dfrac{\beta}{\mu}\cos\beta y$

$\nabla\times\tilde{H} = j\omega\epsilon\tilde{E} \Rightarrow \tilde{E} = \dfrac{\beta^2}{j\omega\mu\epsilon}\sin\beta y = -j\omega\sin\beta y$

Problem 11.5 From (11.30) and (11.41), $\dfrac{\tilde{H}_\phi}{\tilde{A}_z} = \dfrac{j\beta}{\mu}\sin\theta \Rightarrow \tilde{H}_\phi = \dfrac{j\beta}{\mu}\sin\theta\ \tilde{A}_z$

Problem 11.6 $R_{rad} = 80\pi^2(0.1)^2 = 7.9\,\Omega$ $P_{rad} = 500\,W$

$P_{rad} = \frac{1}{2}I^2 R_{rad} \Rightarrow I = \sqrt{\dfrac{2\times 500}{7.9}} = 11.25\,A$

Problem 11.7 From (11.42), $\dfrac{\beta I \ell \eta}{4\pi} = 6\times10^{-3}\times 10\times10^{3} = 60$ $r = 10\,km$

$\eta = 120\pi$

$\tilde{E}_\theta = j\dfrac{60}{r}\sin\theta\, e^{-j\beta r}$ $\langle\hat{S}_r\rangle = \frac{1}{2}Re[\tilde{E}\times\tilde{H}^*]\cdot\vec{a}_r$

$\tilde{H}_\phi = j\dfrac{1}{2\pi r}\sin\theta\, e^{-j\beta r}$ $= \dfrac{15}{\pi r^2}\sin^2\theta$ W/m^2

$P_r = \dfrac{15}{\pi}\int_0^\pi \sin^3\theta\, d\theta \int_0^{2\pi} d\phi = 40\,W$

Problem 11.8 $\omega = 300\times10^6$ rad/s $\beta = \omega/c = 1$ rad/m $\tilde{I} = 10A$ (max)

$\tilde{E}_\theta = 15.5\, e^{-j77.34^\circ}$ mV/m at $r = 3000m$ $\lambda = 2\pi$ $r = 3000m$

$\tilde{H}_\phi = 41.11\, e^{-j77.34^\circ}$ $\mu A/m$ " $\langle\hat{S}_r\rangle = 0.32$ $\mu W/m^2$

Problem 11.9

$$\int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \left(e^{j\beta(1+\cos\theta)z} + e^{-j\beta(1-\cos\theta)z}\right)dz = \left[\frac{e^{j\beta z}e^{j\beta z\cos\theta}}{j\beta(1+\cos\theta)} - \frac{e^{-j\beta z}e^{j\beta z\cos\theta}}{j\beta(1-\cos\theta)}\right]_{-\frac{\lambda}{4}}^{\lambda/4}$$

$$= \frac{j e^{j\frac{\pi}{2}\cos\theta} + j e^{-j\frac{\pi}{2}\cos\theta}}{j\beta(1+\cos\theta)} - \frac{-j e^{j\frac{\pi}{2}\cos\theta} - j e^{-j\frac{\pi}{2}\cos\theta}}{j\beta(1-\cos\theta)} \qquad \text{let } \frac{\pi}{2}\cos\theta = \alpha$$

Then

$$= \frac{e^{j\alpha} + e^{-j\alpha} - \cos\theta\, e^{j\alpha} - \cos\theta\, e^{-j\alpha} + e^{j\alpha} + e^{-j\alpha} + \cos\theta\, e^{j\alpha} + \cos\theta\, e^{-j\alpha}}{\beta(1-\cos^2\theta)}$$

$$= \frac{4\cos\alpha}{\beta\sin^2\theta} = \frac{4\cos(\frac{\pi}{2}\cos\theta)}{\beta\sin^2\theta}$$

Problem 11.10 Sum up the fields due to many Hertzian dipoles stacked upon each other. Thus. $l = dz$

$$\tilde{H}_\phi = \frac{j\beta}{4\pi}\sin\theta \int_{-\lambda/4}^{\lambda/4} \frac{I_0}{2R}\left(e^{j\beta z} + e^{-j\beta z}\right)e^{j\beta R}dz, \quad \text{where } R = r - z\cos\theta$$

$$= \frac{j\beta I_0}{8\pi r}\sin\theta \int_{-\lambda/4}^{\lambda/4} \left(e^{j\beta z} + e^{-j\beta z}\right)e^{j\beta z\cos\theta}\, e^{-j\beta r}dz$$

Problem 11.9 : $$\int_{-\lambda/4}^{\lambda/4} \left(e^{j\beta z} + e^{-j\beta z}\right)e^{j\beta z\cos\theta}dz = \frac{4\cos(\frac{\pi}{2}\cos\theta)}{\beta\sin^2\theta}$$

Hence, $$\tilde{H}_\phi = j\frac{I_0}{2\pi r}e^{-j\beta r}\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}$$

Problem 11.11 $\tilde{E}_\theta$ is given in (11.77), $\tilde{H}_\phi$ in (11.76b) and $\langle \hat{S} \rangle$ in (11.78). These equations are true for a $\lambda/4$ monopole above $z \geq 0$ plane. However, power radiated will be $\frac{1}{2}$ of that the dipole.

Hence $P_{rad}\big|_{mono} = \frac{1.219}{8\pi}\eta I_0^2$ and $R_{rad} = \frac{1.219}{4\pi}\eta = 36.57\Omega$ in free space

Problem 11.12 $f = 50\,MHz$ $\omega = 2\pi f = 3.142\times10^8$ rad/s $\beta = \frac{\omega}{c} = 1.047$ m

$\lambda = 2\pi/\beta = 6m \Rightarrow l = 3m$ $\eta = 120\pi\ \Omega$ $I_o = 5A$

$$\tilde{E}_\theta = \frac{j\eta}{2\pi r} I_o e^{-j\beta r} \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} = j\frac{300}{r} e^{-j1.047r} \frac{\cos(\pi/2\cos\theta)}{\sin\theta}$$

$$\tilde{H}_\phi = j\,\frac{0.796}{r}\, e^{-j1.047r} \cos(\pi/2 \cos\theta)/\sin\theta\ , \quad R_{rad} = 73.14\Omega$$

$$P_{rad} = \frac{1}{2}\times 5^2 \times 73.14 = 914.25\,W$$

Problem 11.13 $\tilde{J}_s = \frac{I_o}{2\pi b}\vec{a}_z$ A/m

$$dP = \frac{1}{2}I_o^2 \frac{dz}{2\pi b\sigma\delta} \Rightarrow P = \frac{1}{2}I_o^2 \int_{-\lambda/4}^{\lambda/4} \frac{dz}{2\pi b\sigma\delta} = \frac{1}{2}I_o^2\left[\frac{\lambda}{4\pi b\sigma\delta}\right] = \frac{1}{2}I_o^2 R$$

where $R = \dfrac{\lambda/2}{2\pi b\sigma\delta}$

Problem 11.14 Continue from Problem 11.13

$$dP = \frac{1}{2}I_o^2 \cos^2\beta z \frac{dz}{2\pi b\sigma\delta}$$

$$P = \frac{1}{2}I_o^2 \frac{1}{2\pi b\sigma\delta}\int_{-\lambda/4}^{\lambda/4} \cos^2\beta z\, dz = \frac{1}{2}I_o^2\left[\frac{\lambda/4}{2\pi b\sigma\delta}\right] \Rightarrow R = \frac{\lambda/4}{2\pi b\sigma\delta}$$

Problem 11.15 Set $\theta = 90^\circ$, from (11.77)

$$|E| = \frac{\eta I_o}{2\pi r} \Rightarrow \frac{120\pi I_o}{2\pi\times100\times10^3} = 25\times10^{-3} \Rightarrow I_o = 41.667\,A$$

From (11.79), $P_{rad} = \dfrac{1.219}{4\pi}\times120\pi\times41.667^2 = 63.5\,kW$

Problem 11.16 $|E|$ still the same as in Problem 11.15.

$I_o = 41.667A$ and $P_{rad} = \frac{1}{2}\left[P_{rad}\big|_{Half\text{-}wave}\right] = 31.75\,kW$

Problem 11.17 $\frac{l}{\lambda}=0.1 \quad \beta l = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{10} = \pi/5$

$P_{rad} = \frac{1}{2}I_0^2 R_{rad} = \frac{1}{2}I_0^2 \frac{2\pi}{3}\times 120\pi \times \left(\frac{1}{10}\right)^2 \Rightarrow$ when $P_{rad}=100\,W$, $I_0 = 10.07A$

$$|E| = \frac{\beta I_0 l}{8\pi r}\eta = \frac{\pi}{5}\cdot\frac{10.07}{8\pi}\cdot\frac{120\pi}{10\times 10^3} = 9.49 \text{ mV/m}$$

Problem 11.18 $I_0 = \sqrt{100\times\frac{4\pi}{1.219}\times\frac{1}{120\pi}} = 1.654A$

$$|E| = \frac{\eta I_0}{2\pi r} = \frac{120\pi\times 1.654}{2\pi\times 10\times 10^3} = 9.92 \text{ mV/m}$$

Problem 11.19 Since $P_{rad}|_{mono} = \frac{1}{2}P_{rad}|_{Half\text{-}wave}$, $I_0 = \sqrt{2}\times 1.654 = 2.34A$

Thus, $|E| = \sqrt{2}\times 9.92 = 14.03$ mV/m

Problem 11.20 $f = 100$ MHz, $\omega = 2\pi f$, $\lambda = c/f = 3m$ $\beta\lambda = 2\pi \Rightarrow \beta = 2\pi/3$

From (11.63a): $100 = \frac{4}{3}\pi^3 \times 120\pi\times\left(\frac{M}{\lambda}\right)^2 \Rightarrow M = 0.28$ A-m^2

$|E| = \frac{\omega\mu_0\beta}{4\pi r}M = 368$ mV/m. $\quad r = 10$ kM

Problem 11.21

$$\tilde{A}_z = \frac{\mu I_0}{2\pi r}e^{-j\beta r}\left[\int_0^{l/2}\sin\left(\frac{\beta l}{2}-\beta z\right)e^{j\beta z\cos\theta}dz + \int_{-l/2}^{0}\sin\left(\frac{\beta l}{2}+\beta z\right)e^{j\beta z\cos\theta}dz\right]$$

$$= \frac{\mu I_0}{2\pi r}e^{-j\beta r}\frac{\cos\left(\frac{\beta l}{2}\cos\theta\right)-\cos(\beta l/2)}{\beta\sin^2\theta}$$

$$\tilde{H}_\phi = \frac{j\beta}{\mu}\sin\theta\tilde{A}_z = \frac{jI_0 e^{-j\beta r}}{2\pi r\sin\theta}\left[\cos\left(\frac{\beta l}{2}\cos\theta\right)-\cos\left(\frac{\beta l}{2}\right)\right]$$

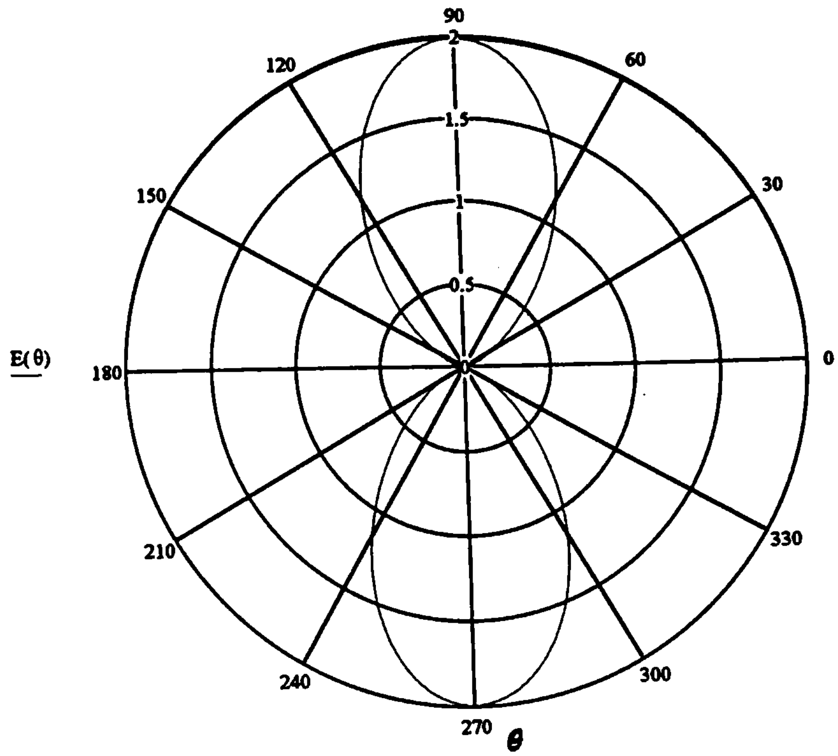
$$\tilde{E}_\theta = \eta\tilde{H}_\phi = \frac{jI_0\eta}{2\pi r}\frac{e^{-j\beta r}}{\sin\theta}\left[\cos\left(\frac{\beta l}{2}\cos\theta\right)-\cos\left(\frac{\beta l}{2}\right)\right]$$

$$\langle\hat{S}_r\rangle = \frac{1}{2\eta}E_\theta^2 = \frac{I_0^2\eta}{8\pi^2 r^2}\left[\frac{\cos\left(\frac{\beta l}{2}\cos\theta\right)-\cos(\beta l/2)}{\sin\theta}\right]^2$$

Problem 11.22

$$\theta := 0, \frac{\pi}{500} .. 2 \cdot \pi \qquad E(\theta) := \left| \frac{\cos(\pi \cdot \cos(\theta)) + 1}{\sin(\theta)} \right|$$

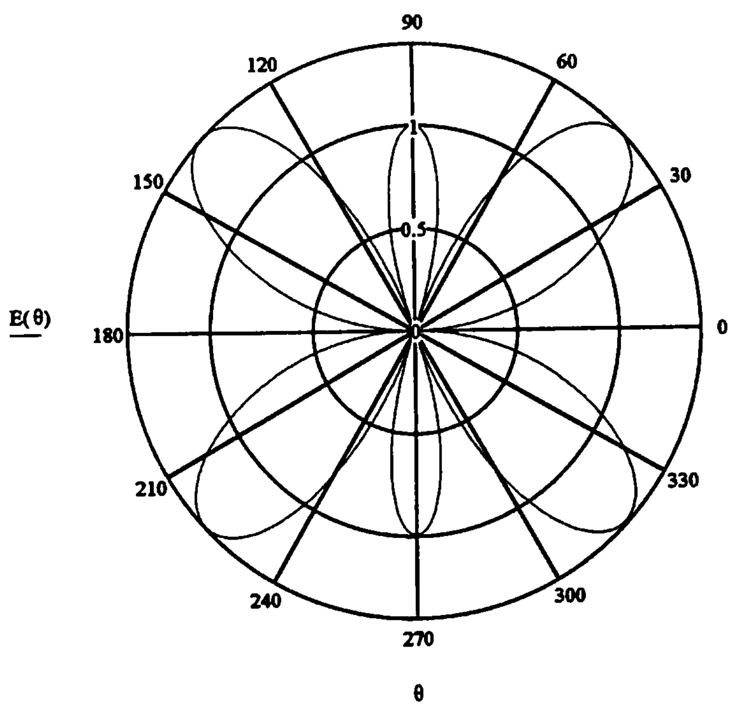
Field plot of a Full-wave Antenna



Problem 11.23

$$\theta := 0, \frac{\pi}{500} .. 2 \cdot \pi \qquad E(\theta) := \left| \frac{\cos(\pi \cdot 1.5 \cdot \cos(\theta))}{\sin(\theta + 0.0001)} \right|$$

Field plot of a one-and-a-half wavelength antenna

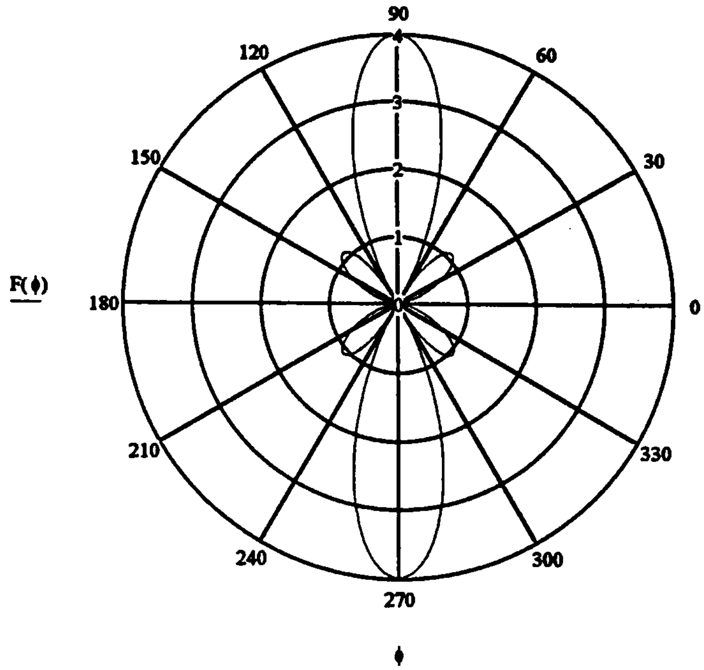


$\phi := 0, \frac{\pi}{500} .. 2\cdot\pi$ $\quad \beta d := \pi \quad \delta := 0 \quad n := 4 \quad \alpha(\phi) := \beta d\cdot\cos(\phi) + \delta$

$$F(\phi) := \left|\frac{\sin(n\cdot\alpha(\phi)\cdot 0.5)}{\sin(\alpha(\phi)\cdot 0.5)}\right|$$

Broadside array pattern of a 4-element half-wave dipole array in the xy-plane when currents are in-phase and spacing is half-wavelength.

Problem 11.24

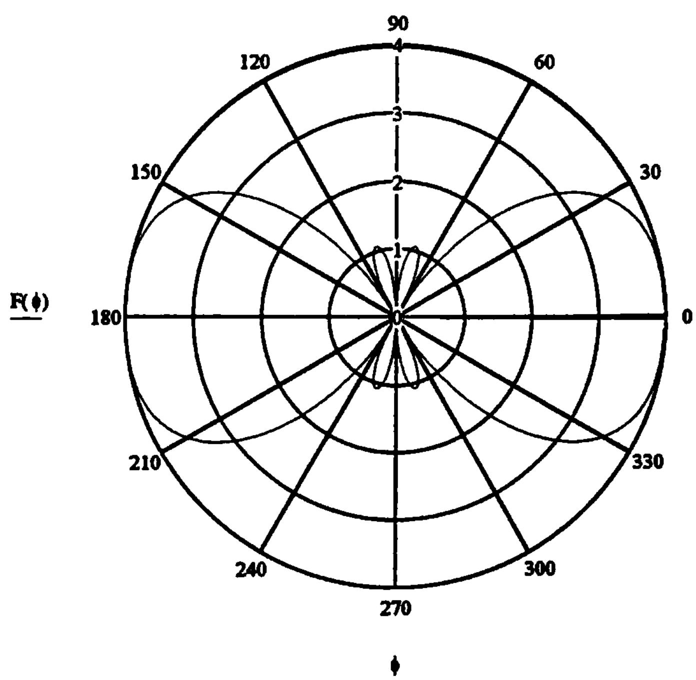


$\phi := 0, \frac{\pi}{500} .. 2\cdot\pi$ $\quad \beta d := \pi \quad \delta := -\pi \quad n := 4 \quad \alpha(\phi) := \beta d\cdot\cos(\phi) + \delta$

$$F(\phi) := \left|\frac{\sin(n\cdot\alpha(\phi)\cdot 0.5)}{\sin(\alpha(\phi)\cdot 0.5)}\right|$$

Endfire array pattern of a 4-element half-wave dipole array in the xy-plane when currents are -180° out of phase and spacing is half-wavelength.

Problem 11.25

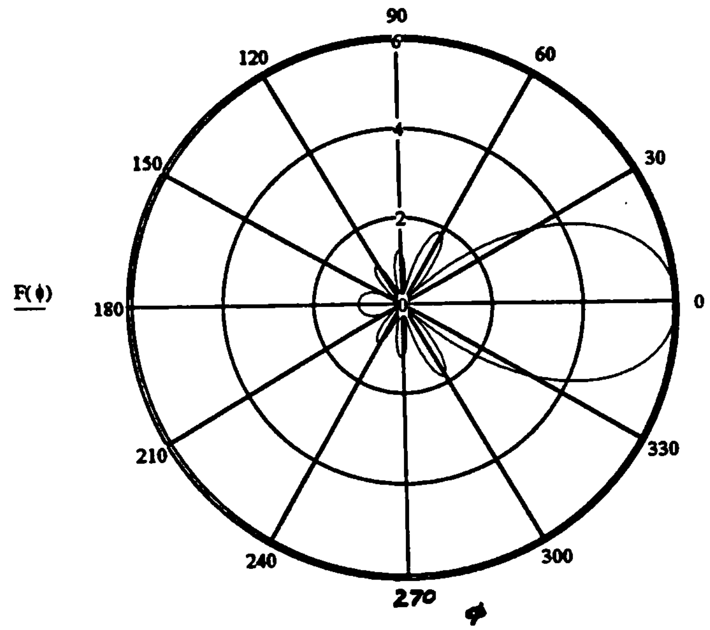


$\phi := 0, \frac{\pi}{500} .. 2\cdot\pi$ $\beta d := \frac{\pi}{2}$ $\delta := -0.6\cdot\pi$ $n := 8$ $\alpha(\phi) := \beta d\cdot\cos(\phi) + \delta$

$$F(\phi) := \left|\frac{\sin(n\cdot\alpha(\phi)\cdot 0.5)}{\sin(\alpha(\phi)\cdot 0.5)}\right|$$

Endfire array pattern of an 8-element half-wave dipole array in the xy-plane when currents are -108° out of phase and spacing is quarter-wavelength.

Problem 11.26

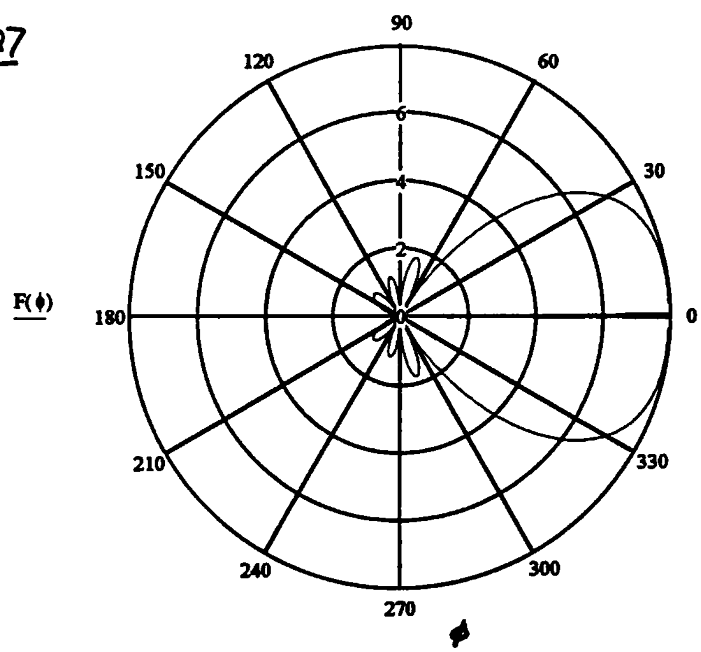


$\phi := 0, \frac{\pi}{500} .. 2\cdot\pi$ $\beta d := \frac{\pi}{2}$ $\delta := -\frac{\pi}{2}$ $n := 8$ $\alpha(\phi) := \beta d\cdot\cos(\phi) + \delta$

$$F(\phi) := \left|\frac{\sin(n\cdot\alpha(\phi)\cdot 0.5)}{\sin(\alpha(\phi)\cdot 0.5)}\right|$$

Endfire array pattern of an 8-element half-wave dipole array in the xy-plane when currents are -90° out of phase and spacing is quarter-wavelength.

Problem 11.27



Problem 11.28 $\tilde{E}_\theta = \frac{15}{r} I_o$ V/m, $\tilde{H}_\phi = \frac{15}{\eta_o r} I_o$ A/m, $\langle \hat{S}_r \rangle = \frac{225}{\eta_o r^2} I_o^2$ W/m^2

$$P_{rad} = \frac{225}{\eta_o} I_o^2 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi = 7.5 I_o^2, \quad \text{For } P_{rad} = 75\text{kW},\ I_o = 100\text{ A}$$

$\eta_o = 120\pi\ \Omega$ Omni directional antenna.

Problem 11.29

$\tilde{E}_\theta = \frac{15}{r} I_o \sin\theta$ V/m, $\tilde{H}_\phi = \frac{15}{\eta_o r} I_o \sin\theta$ A/m, $\langle \hat{S}_r \rangle = \frac{225}{\eta_o r^2} I_o^2 \sin^2\theta$ W/m^2

$$P_{rad} = \frac{225}{\eta_o} I_o^2 \int_0^{\pi} \sin^3\theta\, d\theta \int_0^{2\pi} d\phi = 5 I_o^2 \qquad \int_0^{\pi} \sin^3\theta\, d\theta = \frac{4}{3}$$

For $P_{rad} = 75$ kW, $I_o = 122.47$ A

Problem 11.30 $f = 3$ MHz $\quad \lambda = \frac{c}{f} = 100$ m $\quad \beta = \frac{2\pi}{\lambda} = \frac{2\pi}{100}$ rad/m

$\sigma = 5.8\times 10^7$ S/m $\quad \delta = \sqrt{\frac{1}{\pi f \mu_0 \sigma}} = 38.15\ \mu$m

$b = 5$ mm

Since $b \gg \delta$, $R_c = \frac{2\pi \times 0.5}{5.8\times10^7 \times 2\pi \times 5\times10^{-3} \times 38.15} \times 10^6 = 45.2$ mΩ

From (11.65b), $R_{rad} = \frac{\pi}{6} \times 120\pi \times \left(\frac{2\pi}{100}\times 0.5\right)^4 = 192.28\ \mu\Omega$

$$\%\eta_e = \frac{R_{rad}\times 100}{R_{rad} + R_c} = 0.42\ \%$$

Problem 11.31 From Prob.(11.21) $\langle \hat{S}_r \rangle = \frac{\eta I_o^2}{8\pi^2 r^2} \left[\frac{\cos(\frac{\beta l}{2}\cos\theta) - \cos(\frac{\beta l}{2})}{\sin\theta}\right]^2$

$$P_{rad} = \frac{I_o^2 \eta}{8\pi^2} \int_0^{\pi} \frac{[\cos(\beta l/2 \cos\theta) - \cos(\beta l/2)]^2}{\sin\theta} d\theta \int_0^{2\pi} d\phi$$

or $$R_{rad} = \frac{\eta}{2\pi} \int_0^{\pi} \frac{1}{\sin\theta} \left[\cos(\tfrac{\beta l}{2}\cos\theta) - \cos(\tfrac{\beta l}{2})\right]^2 d\theta$$

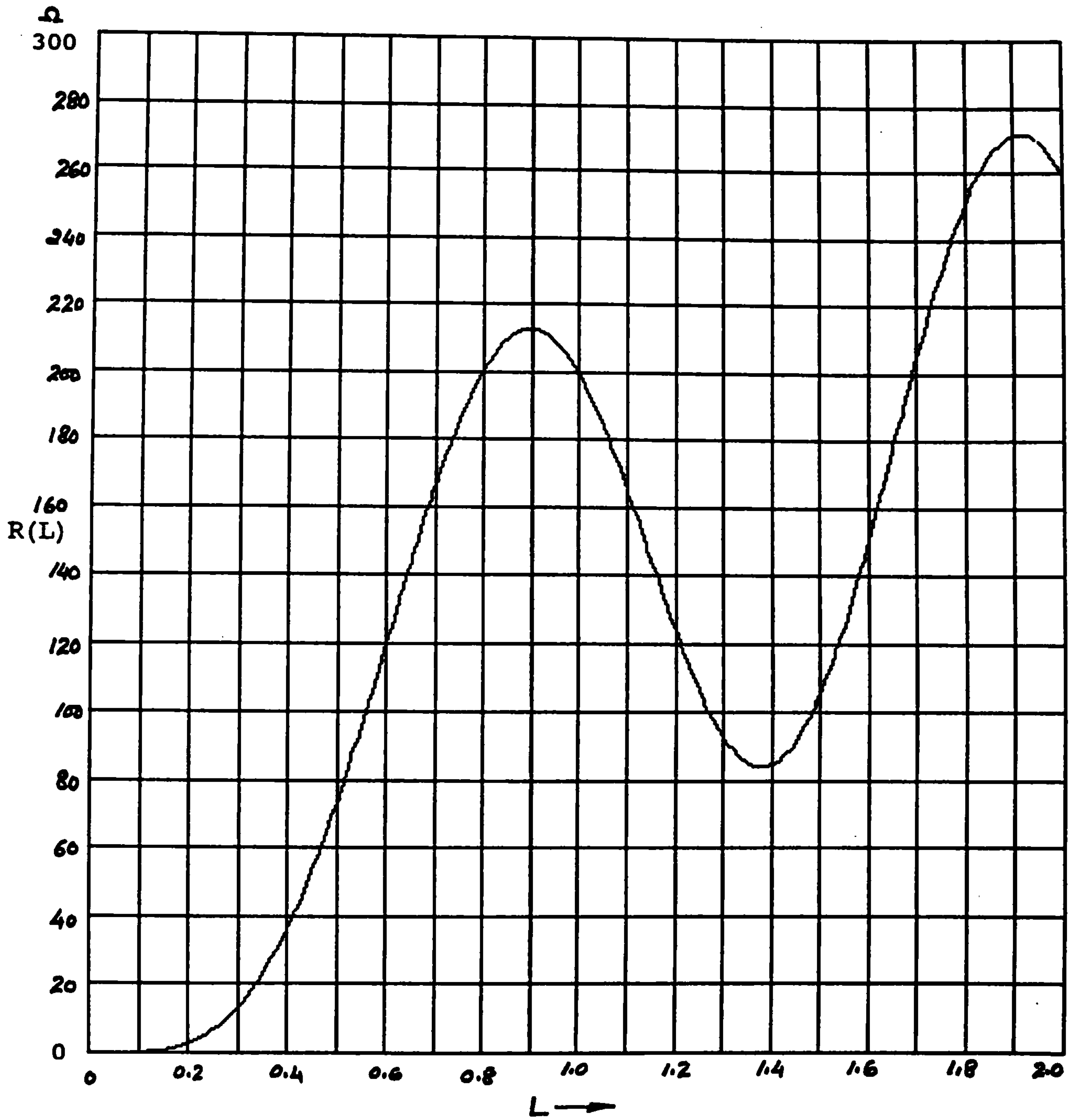
When $l = \lambda$ $\quad R_{rad} = \frac{3.318}{2\pi}\eta \Rightarrow R_{rad} \cong 200\,\Omega$ in free space

When $l = 2\lambda$ $\quad R_{rad} = \frac{4.327}{2\pi}\eta \Rightarrow R_{rad} = 259.62\,\Omega$ "

When $l = 1.5\lambda$ $\quad R_{rad} = \frac{1.758}{2\pi}\eta \Rightarrow R_{rad} \cong 105.5\,\Omega$ "

$L := .1, .11 \,..\, 2$ where L is the length in wavelength $\eta := 120 \cdot \pi$

$$R(L) := 60 \cdot \left[\int_{0.001}^{\pi - 0.001} \left[\frac{(\cos(\pi \cdot L \cdot \cos(\theta)) - \cos(\pi \cdot L))^2}{\sin(\theta)} \right] d\theta \right]$$



Problem 11.33 $G_t = G_R = 10^2 = 100$ $f = 100\,MHz \Rightarrow \lambda = \frac{c}{f} = 3m$

$R = \frac{300}{3} = 100\lambda$ $P_{rad} = \dfrac{P_r}{G_t G_R \left(\frac{\lambda}{4\pi R}\right)^2}$ $\left(\frac{\lambda}{4\pi R}\right)^2 = \left(\frac{1}{400\pi}\right)^2$

$$= \frac{10\times 10^{-3}}{100\times 100}\times(400\pi)^2 = 1.58\,W$$

Problem 11.34 $G_t = 1.64$ $G_R = 1.64$

$$P_{rad} = \frac{10\times 10^{-3}\times(400\pi)^2}{1.64\times 1.64} = 5.87\,kW$$

Problem 11.35 Since $P_r \propto \frac{1}{r^2}$, thus

$$(10\times 10^{-3})(500)^2 = (10\times 10^{-6})\, r_{max}^2 \Rightarrow r_{max} = 15.811\ km$$

Problem 11.36 $P_{rad} = 10\,kW$ $P_r = 3\times 10^{-12}\,W$

$\lambda = \dfrac{3\times 10^8}{5\times 10^9} = 0.06\,m$ $G = 10^3 = 1000$

Hence
$$r = \left[\frac{0.06^2\times 1000^2\times 1.5\times 10\times 10^3}{4\pi^3\times 3\times 10^{-2}}\right]^{1/4} \cong 9.8\ km$$