Fundamentals of Information Theory

Homework 4

Problem 1 *(10 points)*
Draw the channel diagram of the following discrete channels. Their channel matrix are shown as follows.

(a) A Z channel

$X\backslash Y$	0	1
0	1	0
1	s	$1-s$

(b) A binary erasure channel

$X\backslash Y$	0	E	1
0	$1-s_1-s_2$	s_1	s_2
1	s_2	s_1	$1-s_1-s_2$

(c) A non-symmetric channel

$X\backslash Y$	0	1
0	$\frac{1}{2}$	$\frac{1}{2}$
1	$\frac{1}{4}$	$\frac{3}{4}$

(d) A semi-symmetric channel

$X\backslash Y$	0	1	2	3
0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$
1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$

Problem 2 *(10 points) Find the channel capacity of the following discrete memoryless channel, where* $Pr\{Z=0\} = Pr\{Z=a\} = \frac{1}{2}$. *The alphabet for* x *is* $X = \{0,1\}$. *Assume that* Z *is independent of* X. *Observe that the channel capacity depends on the value of* a.

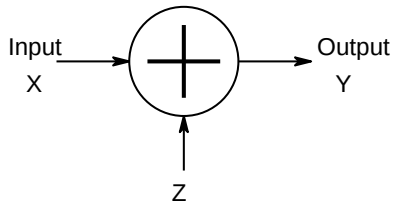

Figure 1: An additive noise channel in Problem 7.2.

Problem 3 *(10 points)* ***Erasures and errors in a binary channel.*** *Consider a channel with binary inputs that has both erasures and errors. Let the probability of error be* ϵ *and the probability of erasure be* α, *so that the channel is illustrated as below:*

(a) Find the capacity of this channel.

(b) Specialize to the case of the binary symmetric channel ($\alpha = 0$).

(c) Specialize to the case of the binary erasure channel ($\epsilon = 0$).

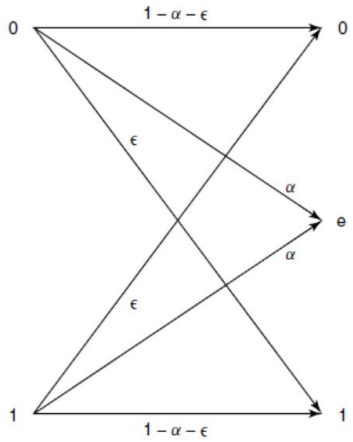

Figure 2: A binary channel with erasures and errors.