"CHAPTER-6 APPLICATIONS OF STATIC FIELDS"

Exercise 6.1 $L = 5\,cm$, $z = 0.5\,cm$, $m_e = 9.1\times10^{-31}\,kg$, $e = -1.6\times10^{-19}C$

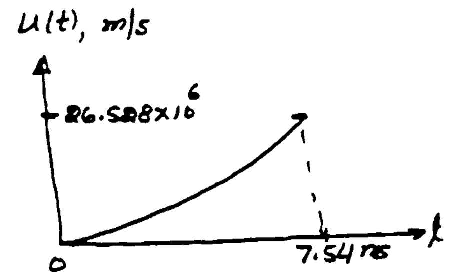
$u_x = 26.528\times10^6\,m/s$, $d = 20\,cm$

From (6.6), $V_o = -\frac{2m_eLz}{e}\left(\frac{u_x}{d}\right)^2 = 50V$

$T = \frac{d}{u_x} = 7.54\,ns$

$a_z = -\frac{eV_o}{mL} = 175.82\times10^{12}\,m/s^2$

$u(t) = \sqrt{u_x^2 + (a_z t)^2}$



Exercise 6.2 $z = 2\,cm$, $V_o = 15\,kV$, $L = 10\,cm$

$$d = \sqrt{\frac{-2Lz\,m_e}{e\,V_o}} = 32.66\,mm,\quad T = \frac{d}{u_x} = 1.232\,ns$$

$$a_z = -\frac{eV_o}{m_eL} = 52.747\times10^{15}\,m/s^2,\quad u_z = a_zT = 64.985\times10^6\,m/s$$

$$u = \sqrt{u_x^2 + u_z^2} = 70.188\times10^6\,m/s$$

Exercise 6.3 $z = 5\,cm$, $T = 12.5\,ns$, since $z = \frac{1}{2}a_z t^2$

at $z = 5\,cm$, $t \to T \Rightarrow a_z = \frac{2\times5\times10^{-2}}{T^2} = 6.4\times10^{14}\,m/s^2$, $u_z = a_zT = 8\times10^6\,m/s$

Also $|e|E = m_e a_z \Rightarrow E = \frac{m_e}{|e|}a_z = 3.64\times10^3\,V/m$, $V_o = 0.05E = 182\,V$

Exercise 6.4 $\vec{E} = 100\times10^3\,\vec{a}_z$ V/m $u_z = 6\times10^7\,m/s$

$$a_z = \frac{|e|E}{m_e} = 1.758\times10^6\,m/s^2,\quad T = \frac{u_z}{a_z} = 3.413\,ns$$

$$z = u_zT - \frac{1}{2}a_zT^2 = 102.375\,mm,$$

Let at $t = T_1$, the final energy be 20% of its initial energy. Then

$$u_1 = u_z\sqrt{0.2} = 2.683\times10^7\,m/s,\quad T_1 = \frac{u_1 - u_z}{a_z} = 1.886\,ns$$

$$z_1 = u_zT_1 - \frac{1}{2}a_zT_1^2 = 81.9\,mm$$

Exercise 6.5 $m_d = 8.38 \times 10^{-12}$ kg

Time to travel between the two plates: $T_0 = \dfrac{2\,mm}{25} = 80\,\mu s$

Time to travel to the paper after exit: $T_1 = \dfrac{8\times10^{-3}}{25} = 320\,\mu s$

Total time: $T = T_0 + T_1 = 400\,\mu s$, $q = -20\times10^{-12}\,C$, $V_0 = 2000\,V$

$d = 2mm$

$$u_z = \frac{-q\,V_0 T_0}{2\,d\,m_d} = 95.465\ m/s,\quad u = \sqrt{25^2 + u_z^2} = 98.685\ m/s$$

Exercise 6.6 $V_0 = 200V$, $L = 5mm$ $d = 1.5mm$, $D = 12\,mm$

$m = \frac{4\pi}{3}(0.01\times10^{-3})^3(2\times10^3) = 1.047\times10^{-12}\,kg$, $q = -2\times10^{-12}\,C$, $u_x = 20\,m/s$

$$z = \frac{-q\,d\,V_0}{mL}\cdot\frac{1}{u_x^2}(0.5d + D) = 3.653\ mm$$

Exercise 6.7 $m = 1.2\times10^{-3}$ kg, $q = -100\times10^{-9}\,C$, $V_0 = 5000V$, $d = 1.5m$

When $x = d$, $z = L_2$ From (6.18), $L_2 = \sqrt{\dfrac{|q|V_0 d}{9.81\,m}} = 25.241$ cm

$L = 2L_2 = 50.482$ cm. $T = \sqrt{\dfrac{2d}{9.81}} = 0.553\,s$, $u_x = 9.81T = 5.425\ m/s$

$u_z = a_z T = \dfrac{|q|}{mL}V_0 T = 0.456\ m/s$

Exercise 6.8 $m = 0.5\times10^{-3}\,kg$, $q = -120\times10^{-9}C$

$$L_2 = \sqrt{\frac{|q|\,V_0 d}{9.81\,m}} = 42.84\ cm,\qquad L = 2L_2 \cong 86\ cm$$

Exercise 6.9 $E_r = 0.3$ MV/m, $V_0 = 240\,kV$, $R = \dfrac{V_0}{E_r} = 0.8\,m$ or 80 cm

$$Q = 4\pi\epsilon_0 R V_0 = 21.33\,\mu C$$

Exercise 6.10

$r = 1$ cm, $R = 1m$

$q = 10\times10^{-9}C$, $Q = 10\times10^{-6}C$

$$V_{rR} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right] = 8.91\ kV$$

Exercise 6.11 $\theta = 30^\circ$, $V_o = 100\,V$, $\tau = 1.5$ n.m/rad

Then, $\frac{dC}{d\theta} = \frac{2\theta\tau}{V_o^2} = 1.571\times10^{-4}$ F/rad.

Exercise 6.12 $\theta = 60^\circ$, $V_o = \sqrt{\frac{2\theta\tau}{\frac{dc}{d\theta}}} = 141.421\,V$

Exercise 6.13 $m\frac{du_x}{dt}\vec{a}_x + m\frac{du_y}{dt}\vec{a}_y + m\frac{du_z}{dt}\vec{a}_z = qB[u_y\vec{a}_x - u_x\vec{a}_y]$

Let $\omega = \frac{qB}{m}$, then $\frac{du_x}{dt} = \omega u_y$ and $\frac{du_y}{dt} = -\omega u_x$

$\frac{d^2u_x}{dt^2} + \omega^2 u_x = 0 \quad \frac{d^2u_y}{dt^2} + \omega^2 u_y = 0 \Rightarrow u_x = u_o\cos\omega t,\ u_y = -u_o\sin\omega t$

$\frac{dx}{dt} = u_x \Rightarrow x = \frac{u_o}{\omega}\sin\omega t$, $\frac{dy}{dt} = u_y \Rightarrow y = \frac{u_o}{\omega}\cos\omega t - \frac{u_o}{\omega}$

Hence, $x^2 + (y + \frac{u_o}{\omega})^2 = (\frac{u_o}{\omega})^2 \Rightarrow R = \frac{u_o}{\omega} = \frac{u_o m}{qB}$

Exercise 6.14 Kinetic energy: $k = 5\times1.6\times10^{-19}\,J$. $m = 9.1\times10^{-31}\,kg$

Since $k = \frac{1}{2}mu^2$, $u = \sqrt{\frac{2k}{m}} = 1.326\times10^6$ m/s $\quad B = 1.2\times10^{-3}\,T$

$R = \frac{mu}{eB} = 6.285$ mm $\quad \omega = \frac{eB}{m} = 2.11\times10^8$ rad/s

$T = \frac{2\pi}{\omega} = 29.78$ ns

Exercise 6.15 $V_o = 20\times10^3\,V \Rightarrow u = \sqrt{\frac{2eV_o}{m}} = 8.386\times10^7$ m/s

$B = 50\times10^{-3}\,T$, $R = \frac{m}{eB}u = 9.539$ mm. $\quad D = \pi R = 29.968$ mm

Time $= \frac{D}{u} = 0.357$ ns, $\quad 2R = 19.08$ mm

Exercise 6.16 $R = 0.75$ m $\quad f = 10\times10^6$ Hz $\quad m = 1.7\times10^{-27}$ kg

$q = 1.6\times10^{-19}\,C \Rightarrow B = \frac{2\pi f m}{q} = 0.668\,T$

kinetic energy: $k = \frac{(qBR)^2}{2m} = 1.888\times10^{-12}\,J$ or $\frac{k}{e} = 11.797$ MeV

Exercise 6.17 $R = 0.25\,m$, $B = 1.2\times10^{-3}\,T$, $m = 9.1\times10^{-31}\,kg$

$$u = \frac{eBR}{m} = 5.275\times10^{7}\,m/s, \quad k = \frac{1}{2}mu^2 = 1.266\times10^{-15}\,J \text{ or } 7.91\,keV$$

$$f = \frac{eB}{2\pi m} = 33.58\,MHz$$

Exercise 6.18 $m_1 = 26.72\times10^{-27}\,kg$ $R_1 = 0.1\,m$ $R_2 = 0.11\,m$

$$m_2 = m_1\frac{R_2}{R_1} = 2.94\times10^{-26}\,kg$$

Exercise 6.19 $B = 0.5\,T$, $E = 1\times10^{6}\,V/m$

$$u_o = \frac{E}{B} = 2\times10^{6}\,m/s, \quad k = \frac{1}{2}mu_o^2 = \frac{1}{2}\times1.67\times10^{-27}u_o^2$$
$$= 3.34\times10^{-15}\,J \text{ or}$$
$$= 20.88\,keV$$

Exercise 6.20 $$R = \frac{mu_o}{eB} = \frac{1.67\times10^{-27}\times2\times10^{6}}{1.6\times10^{-19}\times0.5} = 4.175\,cm$$

Distance : $2R = 8.35\,cm$

$$\text{Time} = \frac{\pi R}{u_o} = 65.58\,ns$$

Exercise 6.21

$$n = \frac{BIW}{qAV_{ba}} \quad \text{and} \quad \frac{V_{ba}}{W} = E_H$$

$$\frac{I}{A} = J, \quad \text{Thus,} \quad n = \frac{JB}{qE_H}$$

Exercise 6.22

$$E = \frac{I}{A\sigma} \qquad E_H = \frac{V_{ba}}{W}$$

Thus.

$$\frac{E_H}{E} = \frac{BI}{qAn}\cdot\frac{A\sigma}{I} \Rightarrow \frac{E_H}{E} = \frac{B\sigma}{nq}$$

Exercise 6.23 $W = 50\,cm$ $B = 1.6\,T$ $R = 1150\,\Omega$ $I = 2\,A$

$$V_o = IR = 2300\,V \qquad u_o = \frac{V_o}{BW} = 2875\,m/s$$

Exercise 6.24 $B = 0.5\,T$ $r = 10\,cm$ $V_o = 0.25\,V$

$$u_o = \frac{V_o}{rB} = 5\ m/s$$

Exercise 6.25 $L = 2.54\ cm$ $d = 5.08\ cm$ $N = 1500$ Turns

$B = 1.5\,T$ $I = 12.5\,A$ Area : $A = dL = 12.903\ cm^2$

$$T = \frac{NIAB}{\pi} = 11.55\ N\text{-}m$$

Exercise 6.26 max. force experienced by a conductor,

$$F_m = BIL = 476.25 \times 10^{-3}\ N$$

Thus, the force on a conductor: $F = F_m \sin\theta$

The torque on each conductor:

$$T(\theta) = rF = \frac{5.08 \times 10^{-2}}{2} \times 476.25 \times 10^{-3} \sin\theta = 12.097 \sin\theta\ \ mN{\cdot}m$$

Average torque experienced by a conductor

$$T_{avg} = \frac{1}{2\pi}\int_0^{\pi} T(\theta)\,d\theta = 3.851 \times 10^{-3}\ N{\cdot}m$$

Torque developed by the motor: $T = 2N\,T_{avg} \cong 11.55\ N{\cdot}m$

Problem 6.1 $L = 5\,cm$ $d = 10\,cm$ $V_0 = 10\,kV$ $m_p = 1.67 \times 10^{-27}\,kg.$

$$u_x = \sqrt{\frac{2 \times 2000 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}} = 6.191 \times 10^{5}\ m/s \qquad T = \frac{d}{u_x} = 161.536\ ns$$

$$a_z = \frac{eV_0}{m_p L} = 1.916 \times 10^{13}\ m/s^2, \quad u_z = a_z T = 3.095 \times 10^{6}\ m/s$$

$$u = \sqrt{u_x^2 + u_z^2} = 3.157 \times 10^{6}\ m/s, \quad k = \frac{1}{2} m_p [u^2 - u_x^2] = 8 \times 10^{-15}\,J$$

or $50\,keV$.

Problem 6.2 $u = u_0 - q\frac{E}{m}t$ and $x = u_0 t - \frac{1}{2}\frac{qE}{m}t^2$ $m = 3.4 \times 10^{-27}\,kg$

$E = 50\,kV/m$ $u_0 = 2 \times 10^{6}\,m/s$

When $t = 0$, $u = 0 \Rightarrow$ $T = \frac{m u_0}{eE} = 850\,ns$

$$x = u_0 T - \frac{1}{2}\frac{eE}{m}T^2 = 85\,cm$$

$$u_{0_2} = \sqrt{0.5}\,u_0 = 1.414 \times 10^{6}\ m/s, \quad t_1 = \frac{u_0 - u_{02}}{eE}m = 248.959\ ns$$

$$x_1 = u_0 t_1 - \frac{1}{2}\frac{eE}{m}t_1^2 = 42.5\ cm$$

Problem 6.3 $m = 9.1 \times 10^{-31}\,kg$ $E = 10\,kV/m$ $u_0 = 0.8 \times 10^{6}\,m/s$

$$a = \frac{eE}{2m} = 8.791 \times 10^{14}\ m/s^2, \quad \text{since } x = at^2 + u_0 t \Rightarrow \text{when } x = 3cm,\ t = 5.4\,ns$$

$$u = \frac{eE}{m}t + u_0 = 1.029 \times 10^{7}\ m/s$$

Problem 6.4 $L = 2\,cm$ $V_0 = 200\,V$, $E = \frac{V_0}{L} = 10\,kV/m$

$$a_z = \frac{eE}{m} = 1.758 \times 10^{15}\ m/s^2, \quad u = a_z t \Rightarrow x = \frac{1}{2}a_z t^2$$

When $x = L$, $t = T \Rightarrow$ $T = \sqrt{\frac{2L}{a_z}} = 4.77\,ns$

$$u = a_z T = 8.385 \times 10^{6}\ m/s, \quad k = \frac{1}{2}mu^2 = 31.99 \times 10^{-18}\,J$$

$= 200\,eV$

Problem 6.5 $u_0 = 2\times10^6$ m/s $\quad u = \frac{eE}{m}t + u_0 \quad x = \frac{1}{2}\frac{eE}{m}t^2 + u_0 t$

$a = \frac{eE}{m} = 17.582\times10^{14}$ m/s^2 when $x = 2$ cm $\quad t = 7.54$ ns

$u = at + u_0 = 15.257\times10^6$ m/s

Energy gain : $\frac{1}{2}m[u^2 - u_0^2] = 1.04\times10^{-16}$ J or 650.58 eV

Problem 6.6 $u_{x0} = 100$ m/s $\quad u_{y0} = 100$ m/s $\quad \vec{E} = 200\times10^3\,\vec{a}_x$ V/m

$q = 100$ nC $\quad m = 2\times10^{-3}$ kg $\quad u_x = \frac{qE}{m}t + u_{x0} = 200$ m/s

$u = \sqrt{u_x^2 + u_{y0}^2} = 223.607$ m/s , $y = u_{y0}t$

and $x = \frac{1}{2}\frac{qE}{m}t^2 + u_{x0}t$ when $t = 10$ s, $x = 1500$ m and $y = 1000$ m

Problem 6.7 $\vec{E} = 150\times10^3\,\vec{a}_x$ V/m $\quad \vec{u}_0 = -32.48\times10^6$ m/s $\vec{a}_x$

$m = 1.67\times10^{-27}$ kg

$a_x = \frac{eE}{m} = 1.437\times10^{13}$ m/s^2

$u_x = u_{0x} + a_x t$ and $x = \frac{1}{2}a_x t^2 + u_{0x}t$

When $t = T$, $u_x = 0 \Rightarrow T = -\frac{u_{0x}}{a_x} = 2.26\,\mu$s and $x = -36.7$ m

$k_i = \frac{1}{2}m u_{0x}^2 = 8.809\times10^{-13}$ J or 5.51 MeV

Problem 6.8 $\quad m = \frac{4\pi}{3}\left(\frac{0.085}{2}\times10^{-3}\right)^2\times2\times10^3 = 1.636\times10^{-11}$ kg

$u_x = 10$ m/s $\quad D = 5$ mm $\quad L = 2.5$ cm $\quad d = 1.5$ mm $\quad E = 100$ kV/m

$V_0 = EL = 2.5$ kV $\quad q = -0.25$ PC

$$z = \frac{-qd}{mL}V_0\left(\frac{1}{u_x}\right)^2(0.5d + D) = 0.132\text{ mm}$$

Problem 6.9 $\quad z = 5$ mm $\quad$ since $E = \frac{V_0}{L}$

$$E = \frac{zm}{-qd}u_x^2\,\frac{1}{0.5d + D} = 3.794\times10^6\text{ V/m}$$

$$= 3.794\text{ MV/m}$$

Problem 6.10 Let T_1 and T_2 be the times to travel distances d and D.

Then $T_1 = \frac{d}{u_x} = 150\,\mu s \qquad T_2 = \frac{D}{u_x} = 500\,\mu s, \quad T = T_1 + T_2 = 650\,\mu s$

$$u_z = \frac{-qV_0}{mL} T_1 = 0.229 \text{ m/s} \qquad u = \sqrt{u_x^2 + u_z^2} \cong 10 \text{ m/s}$$

Problem 6.11 $m = 1.2 \times 10^{-3}$ kg, $q = -50 \times 10^{-9}$ C, $V_0 = 15 \times 10^3$ V, $d = 1$ m

$g = 9.81 \text{ m/s}^2 \qquad T = \sqrt{\frac{2d}{g}} = 0.452 s \qquad\qquad L = 40$ cm

$a_z = \frac{q}{mL} V_0 = -1.562 s$, $u_z = a_z T = -0.706$ m/s

$$z = \frac{1}{2} a_z T^2 = -15.93 \text{ cm}$$

Problem 6.12 Time to travel a distance of 3m: $T_3 = \sqrt{\frac{2 \times 3}{9.81}} = 0.782 s$

Time to travel 2m after exit: $T_2 = T_3 - T = 0.331 s$

When the particle hits the ground: $u_x = gT_3 = 7.672$ m/s

u_z does not change. Thus, $u = \sqrt{u_x^2 + u_z^2} = 7.704$ m/s

$$z_1 = z + u_z T_2 = -0.392 \text{ m}$$

Problem 6.13 $R = 25$ cm $\qquad V_0 = 200$ kV $\qquad E = \frac{V_0}{R} = 800$ kV/m

$$q = 4\pi\epsilon_0 R V_0 = 5.56\,\mu C$$

E can exist because breakdown E is 3 MV/m

Problem 6.14 $r = 2$ cm $\qquad R = 20$ cm $\qquad V_0 = 500$ V

$$q = 4\pi\epsilon_0 V_0 \left[\frac{rR}{R-r} \right] = 1.235 \text{ nC}$$

Problem 6.15 The kinetic energy must be equal to potential energy.

$$k = \frac{qQ}{4\pi\epsilon r} \Rightarrow r = \frac{qQ}{4\pi\epsilon k}$$

Problem 6.16 $\theta = 30^\circ = 0.524$ rad $V_0 = 100V$ $\tau = 1.2$ N.m/rad

From (6.89) $b = \frac{2\tau\theta}{V_0^2} = 1.257\times10^{-4}$ $\left(\frac{dc}{d\theta} = b\right)$

$\theta = 45^\circ = 0.785$ rad $V_0 = \sqrt{\frac{2\tau\theta}{b}} = 122.47V$

Problem 6.17

(a) $k_1 = 100\times1.6\times10^{-19} = 1.6\times10^{-17}J$ b) $k_2 = 10\times10^{3}\times1.6\times10^{-19} = 1.6\times10^{-15}J$

$u_1 = \sqrt{\frac{2k_1}{m_e}} = 5.93\times10^{6}$ m/s $u_2 = \sqrt{\frac{2k_2}{m_e}} = 5.93\times10^{7}$ m/s.

$m_e = 9.1\times10^{-31}$ kg

$B = 1.5T$ $r_1 = \frac{m_e u_1}{eB} = 0.022$ mm $r_2 = \frac{m_e u_2}{eB} = 0.225$ mm

Problem 6.18 $f = 10$ MHz $\omega = 2\pi f = 62.832\times10^{6}$ rad/s $r = 10$cm

$u = r\omega = 6.283\times10^{6}$ m/s $m_p = 1.67\times10^{-27}$ kg $\Rightarrow$ $k = \frac{1}{2}m_p u^2 = 32.964\times10^{-15}J$
or 206 keV

$B = \frac{\omega m_p}{e} = 0.66$ T

Problem 6.19 $k = 20\times10^{3}\times1.6\times10^{-19} = 3.2\times10^{-15}J$ $B_x = 1.25T$

$u_z = \sqrt{\frac{2k}{m_e}} = 8.386\times10^{7}$ m/s

$\vec{F} = -e\,\vec{u}\times\vec{B} = -eu_zB_x\,\vec{a}_y = -1.677\times10^{-11}\,\vec{a}_y$ N

$r = \frac{m_e u_z}{e B_x} = 0.382$ mm, Distance: $2r = 0.764$ mm

Problem 6.20 $B = 1.2$ T $I = 500$ A $W = 20$ cm Thickness $t = 0.2$ cm

Area: $A = wt = 4$ cm^2 $V_{ba} = \frac{BIW}{e.A.n} = 22.06\ \mu V$

$n = 8.5\times10^{28}$

Problem 6.21 $E_z = 20$ kV/m $B_x = 0.5T$, $\vec{u}$ must be in y-direction

$u_y = \frac{E_z}{B_x} = 4\times10^{4}$ m/s $k = \frac{1}{2}\times1.67\times10^{-27}\times(4\times10^{4})^2$

$= 1.336\times10^{-18}J$ or 8.35 eV

Problem 6.22 $r = 12cm$ $B = 1.5T$

$p = m_p u$ but $m_p u = eBr \Rightarrow p = eBr = 1.6\times10^{-19}\times1.5\times0.12$
$= 2.88\times10^{-20}$ kg-m/s

$k = \dfrac{p^2}{2m_p} = 2.483\times10^{-13} J$ or 1.55 MeV $\qquad m_p = 1.67\times10^{-27} kg$

Problem 6.23 $k = 8\times10^{6}\times1.6\times10^{-19} = 1.28\times10^{-12} J$ $\qquad r = 0.5 m$

$B = \dfrac{m_p u}{e r} = 0.817 T$ $\qquad u = \sqrt{\dfrac{2k}{m_p}} = 3.915\times10^{7} m/s$

$\omega = \dfrac{eB}{m_p} = 78.275\times10^{6}$ rad/s or $f = 12.46$ MHz

Problem 6.24 $r = 0.5 m$ $\quad f = 10$ MHz $\quad \omega = 2\pi f = 62.832\times10^{6}$ rad/s

$m_d = 3.4\times10^{-27} kg$, $B = \dfrac{\omega m_d}{e} = 1.335 T$

$k = \dfrac{(eBr)^2}{2m_d} = 1.678\times10^{-12} J$ or 10.5 MeV

Problem 6.25 Dia $= 12$ cm , $L = 30$ cm $\Rightarrow$ Area: $A = 360\times10^{-4} m^2$

$N = 1200$ Turns $\quad B = 0.8T$, $I = 120 A$

$T = \frac{1}{\pi} N I A B = 1.32\times10^{3}$ N.m

Problem 6.26 $N = 25$, $A = 10\times10^{-4} m^2$ $\quad I = 2A$ $\quad B = 0.5T$ $\quad \theta = 30^0$

$T = mB\sin\theta$ where $m = NIA \Rightarrow T = NIAB\sin\theta = 12.5\times10^{-3}$ N.m

Problem 6.27 Since $T = mB\sin\theta = NIAB\sin\theta = 0.025\sin\theta$

$$W = \int_0^{\pi} T\,d\theta = \int_0^{\pi} 0.025\sin\theta\,d\theta = 0.05\,J \text{ or } 50\,mJ$$