§ 8.8 Polarization of a wave

❑ 1.concept

- the direction of the electric field commonly changes with time.
- A process or state in which the direction of the electric field at any given point in space changes with time is called polarization of the electromagnetic wave.
- the polarization of a wave is the locus of the tip of the electric field at a given point as a function of time.

•A wave is said to be linearly polarized when at some point in the medium the electric field oscillate along a straight line as a function of time.

•If the tip of the electric field traces a circle, the wave is said to be circularly polarized.

The wave is elliptically polarized when the electric field fellows an elliptical path.

An unpolarized wave, such as a light wave, is usually referred to as a randomly polarized wave.

2. A linearly polarized wave

A linearly polarized wave is that the tip of the electric field traces along a straight line as time progresses.

(1)a uniform plane wave only has the x component of the electric field.

For example, the electric field intensity of a uniform plane wave in a conducting medium can be written in the time domain as

$$\vec{E}(z,t) = \vec{\mathbf{a}}_x E_{xf} e^{-\alpha z} \cos(\omega t - \beta z + \varphi_{xf})$$

this represents a case that the electric field intensity is always in the x direction in a z=constant plane. When z=0 and φ_{xf}=0, the preceding equation can be rewritten as

$$\vec{E}(0,t) = \vec{\mathbf{a}}_x E_0 \cos(\omega t)$$

its plot as a function of time is given in figure 1

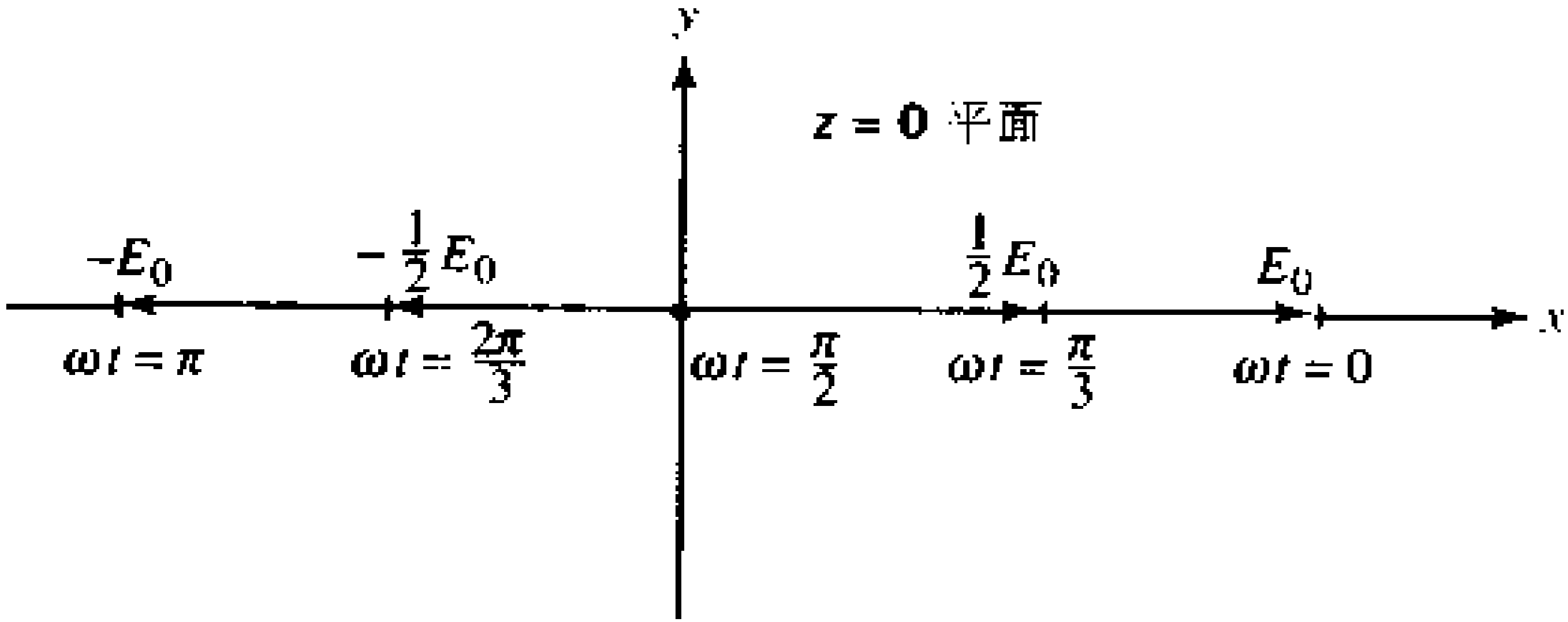

fig.1 linear polarization

(2) **a uniform plane wave has the following components of electric field intensity:**

$$E_x(z,t)=E_{0x}e^{-\alpha z}\cos(\omega t-\beta z+\varphi_x)$$

$$E_y(z,t)=E_{0y}e^{-\alpha z}\cos(\omega t-\beta z+\varphi_y)$$

At any point in a *z*=0 plane, these field components become

$$E_x(0,t)=E_{0x}\cos(\omega t+\varphi_x)$$

$$E_y(0,t)=E_{0y}\cos(\omega t+\varphi_y)$$

When the two components are in phase; i.e., $\varphi_x=\varphi_y=\varphi$**, then the preceding equations yield**

$$\frac{E_x(0,t)}{E_y(0,t)}=\frac{E_{0x}}{E_{0y}}$$

obviously, the equation describes a linear relationship between the two components, as sketched in fig.2

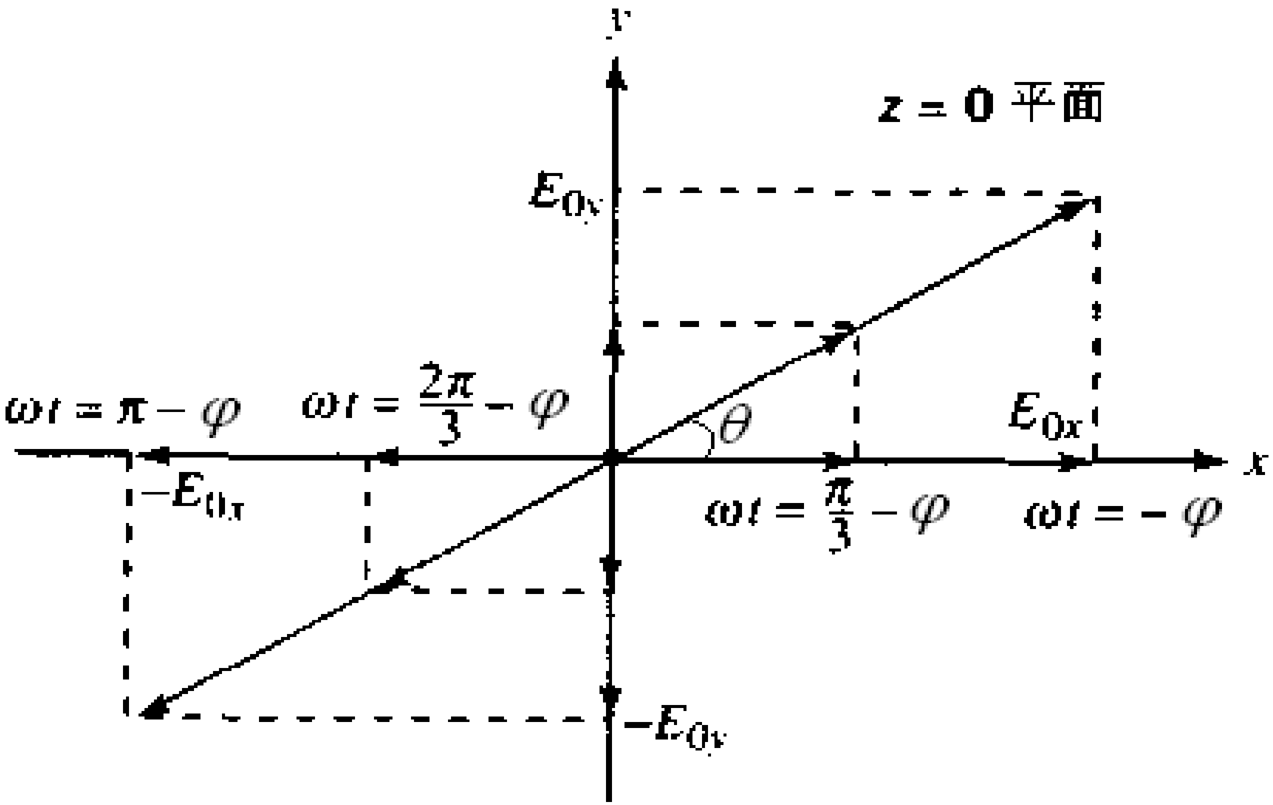

fig.2 Another example of linear polarization

the wave consisting of two components of the electric field is a linearly polarized wave when they are in phase.

The angle θ between the electric field intensity and the x direction is

$$\theta = \tan^{-1}\frac{E_y(0,t)}{E_x(0,t)} = \tan^{-1}\frac{E_{0y}}{E_{0x}}$$

when φ_x-φ_y=$\pm\pi$, these field components

$$E_x(0,t) = E_{0x}\cos(\omega t+\varphi_x)$$
$$E_y(0,t) = E_{0y}\cos(\omega t+\varphi_y)$$

become

$$E_x(0,t) = E_{0x}\cos(\omega t+\varphi_x)$$
$$E_y(0,t) = E_{0y}\cos(\omega t+\varphi_x\pm\pi)$$

Thus, we obatin

$$\frac{E_x(0,t)}{E_y(0,t)} = -\frac{E_{0x}}{E_{0y}}$$

the equation also represents a linearly polarized wave as shown by fig.3. The angle θ between the electric field intensity and the x direction is

$$\theta = -\tan^{-1}\frac{E_y(0,t)}{E_x(0,t)} = -\tan^{-1}\frac{E_{0y}}{E_{0x}}$$

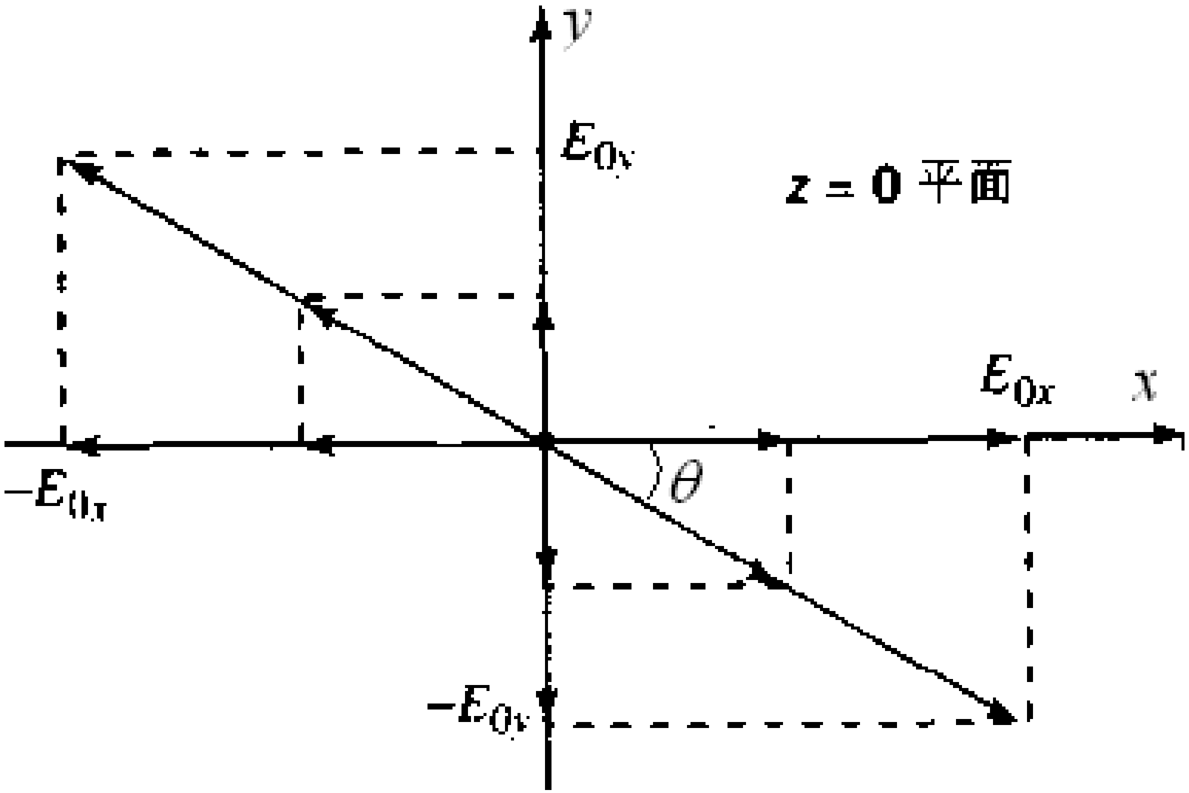

fig.3 a linearly polarized wave when the phase difference between the two components is $\pm\pi$

❖Summary :

When $\varphi_x-\varphi_y=\pm\pi$ or 0, the wave is a linearly polarized wave

❑3. A circularly polarized wave

(1)when $E_{0x}=E_{0y}=E_0$ and $\varphi_x-\varphi_y=\pi/2$, the two electric field components

$$E_x(z,\ t)\ =E_{0x}e^{-\alpha z}\cos(\omega t-\beta z+\varphi_x)$$

$$E_y(z,\ t)\ =E_{0y}e^{-\alpha z}\cos(\omega t-\beta z+\varphi_y)$$

In a z=0 plane become

$$E_x(0,\ t)=E_0\cos(\omega t+\varphi_x) \qquad (1)$$

$$E_y(0,\ t)\ =E_{0y}\cos(\omega t+\varphi_x-\pi/2)= E_0\sin(\omega t+\varphi_x) \qquad (2)$$

Which is an equation of a circle. By evaluating equation (1) and (2) for various values of *t*, we find that the wave is right handed circularly polarized, as shown in fig.4

Thus, by squaring and adding these equations we can obtain

$$E_x^2(0,\ t)+E_y^2(0,\ t)= E_0^2$$

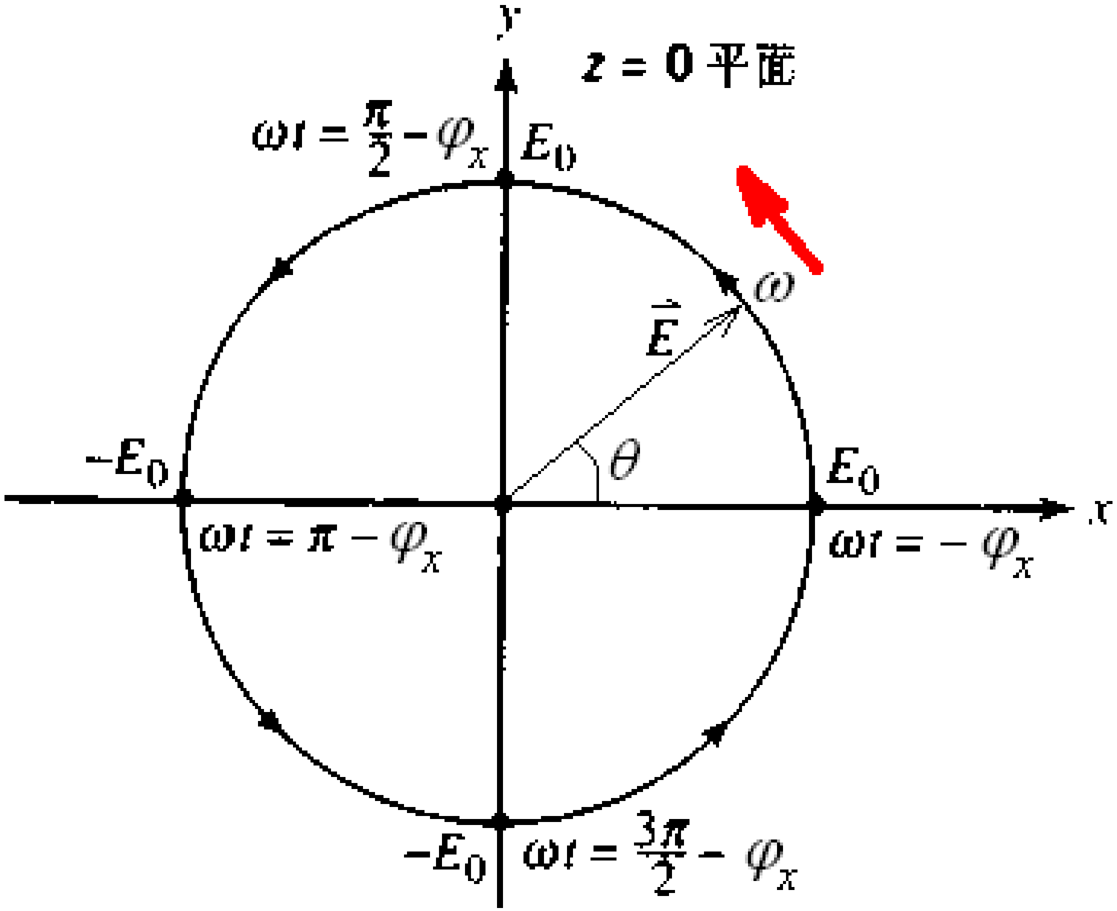

fig.4 a right-handed circularly polarized wave

$E_x(0, t) = E_0\cos(\omega t+\varphi_x)$ (1)

$E_y(0, t) = E_{0y}\cos(\omega t+\varphi_x-\pi/2) = E_0\sin(\omega t+\varphi_x)$ (2)

At any given time, the angle between the direction of the electric field and the *x* direction is

$$\theta = \tan^{-1}\left(\frac{\sin(\omega t + \varphi_x)}{\cos(\omega t + \varphi_x)}\right) = \omega t + \varphi_x$$

and

$$\frac{d\theta}{dt} = \omega$$

we find that the tip of the electric field rotates in a counterclockwise direction. Since the fingers of the right-hand curl in the direction of rotation when the thumb is extended in the direction of propagation(*z* direction),

the given electric field represents a right-handed circularly polarized wave.

(2) When $E_{0x}=E_{0y}=E_0$ and $\varphi_x-\varphi_y=-\pi/2$, the two electric field components

$$E_x(z,\ t)\ =E_{0x}e^{-\alpha z}\cos(\omega t-\beta z+\varphi_x)$$

$$E_y(z,\ t)\ =E_{0y}e^{-\alpha z}\cos(\omega t-\beta z+\varphi_y)$$

In a z=0 plane become

$$E_x(0,\ t)=E_0\cos(\omega t+\varphi_x) \qquad (3)$$

$$E_y(0,\ t)=E_{0y}\cos(\omega t+\varphi_x+\pi/2\ ) = -E_0\sin(\omega t+\varphi_x) \qquad (4)$$

Thus, by squaring and adding these equations we can obtain

$$E_x^2(0,\ t)+E_y^2(0,\ t)=E_0^2$$

Which is an equation of a circle. By evaluating equation (3) and (4) for various values of *t*, we find that the wave is left-handed circularly polarized, as shown in fig.5

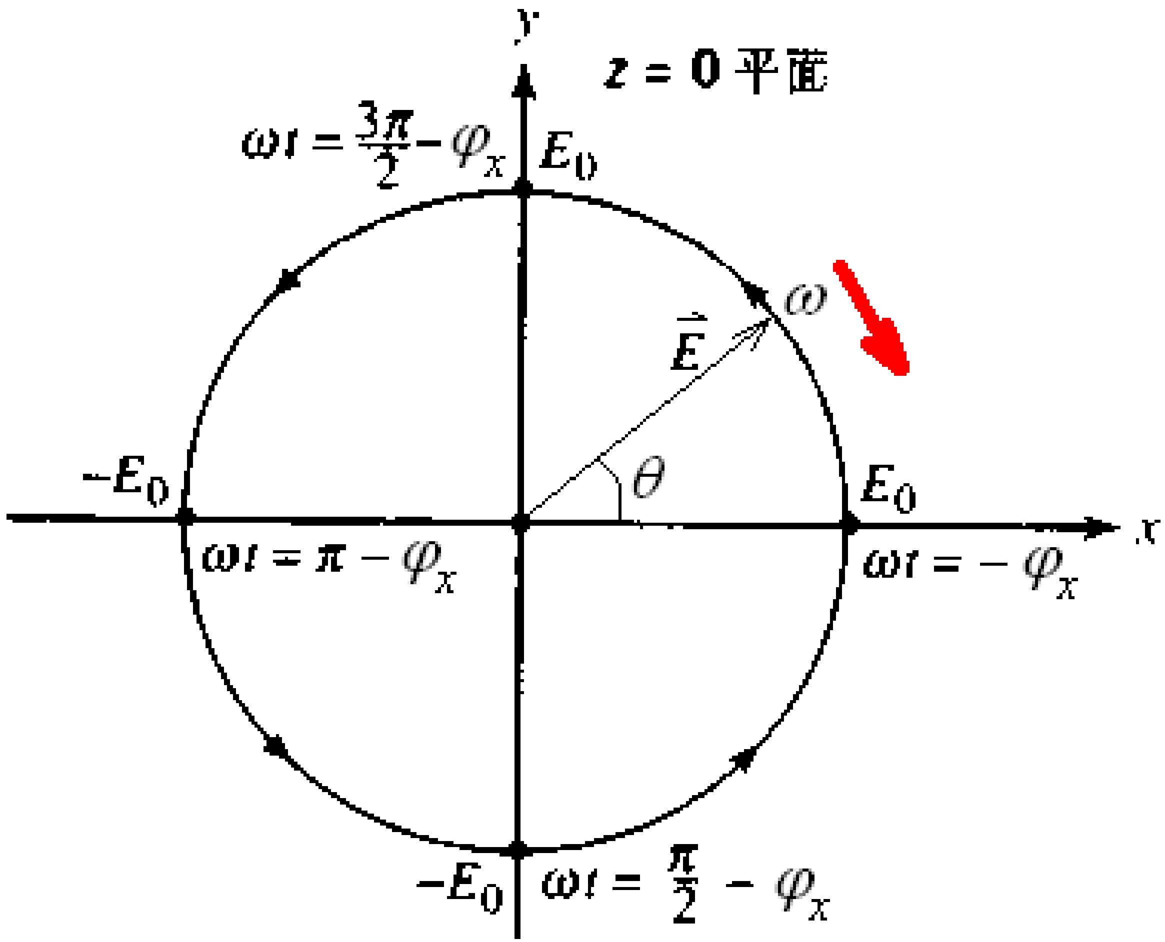

fig.5 a left-handed circularly polarized wave

$$E_x(0,\ t)=E_0\cos(\omega t+\varphi_x) \qquad (3)$$

$$E_y(0,\ t)=E_{0y}\cos(\omega t+\varphi_x+\pi/2) = -E_0\sin(\omega t+\varphi_x) \qquad (4)$$

At any given time, the angle between the direction of the electric field and the *x* direction is

$$\theta = \tan^{-1}\left(\frac{-\sin(\omega t + \varphi_x)}{\cos(\omega t + \varphi_x)}\right) = -\omega t + \varphi_x$$

and

$$\frac{d\theta}{dt} = -\omega$$

we find that the tip of the electric field rotates in a clockwise direction. Since the fingers of the left-hand curl in the direction of rotation when the thumb is extended in the direction of propagation (*z* direction), the given electric field represents a left-handed circularly polarized wave.

❖Summary:

When $E_{0x}=E_{0y}=E_0$ and $\varphi_x-\varphi_y=\pm\pi/2$, the given electric field represents a circularly polarized wave. **The electric field can be given in phasor form as**

$$\dot{E}_{0y} = j\dot{E}_{0x}$$ **Left-handed circularly polarized**

$$\dot{E}_{0y} = -j\dot{E}_{0x}$$ **Right-handed circularly polarized**

thus, Left-handed circularly polarized wave can be expressed by

$$\dot{\vec{E}} = \vec{a}_x\dot{E}_{0x}e^{-j\beta z} + \vec{a}_y\dot{E}_{0y}e^{-j\beta z} = \vec{a}_x\dot{E}_{0x}e^{-j\beta z} + \vec{a}_y j\dot{E}_{0x}e^{-j\beta z}$$

$$= (\vec{a}_x + j\vec{a}_y)\dot{E}_{0x}e^{-j\beta z}$$

thus, right-handed circularly polarized wave can be expressed by

$$\dot{\vec{E}} = \vec{\mathbf{a}}_x \dot{E}_{0x} e^{-j\beta z} + \vec{\mathbf{a}}_y \dot{E}_{0y} e^{-j\beta z} = \vec{\mathbf{a}}_x \dot{E}_{0x} e^{-j\beta z} - \vec{\mathbf{a}}_y j\dot{E}_{0x} e^{-j\beta z}$$

$$= (\vec{\mathbf{a}}_x - j\vec{\mathbf{a}}_y)\dot{E}_{0x} e^{-j\beta z}$$

❑4. An elliptically polarized wave

Now the two components of the electric field are arbitrary, that is,

$$E_x(z,\ t) = E_{0x} e^{-\alpha z}\cos(\omega t - \beta z + \varphi_x) \qquad (5)$$

$$E_y(z,\ t) = E_{0y} e^{-\alpha z}\cos(\omega t - \beta z + \varphi_y) \qquad (6)$$

Letting $\varphi = \varphi_x - \varphi_y$, in a z=0 plane, equation(5) and (6) yield

$$E_x(0,\ t) = E_{0x}\cos(\omega t + \varphi_x) \qquad (5)$$

$$E_y(0,\ t) = E_{0y}\cos(\omega t + \varphi_x - \varphi) \qquad (6)$$

Since equation(6) can be rewritten as

$$E_y(0,\ t) = E_{0y}\cos(\omega t + \varphi_x - \varphi)$$

Or $\quad E_y(0,\ t)/E_{0y} = \cos(\omega t + \varphi_x)\cos\varphi + \sin(\omega t + \varphi_x)\sin\varphi \qquad (7)$

We can obtain

$$\frac{E_y(0,t)}{E_{0y}} = \frac{E_x(0,t)}{E_{0x}}\cos\varphi \pm \sqrt{1 - \left(\frac{E_x(0,t)}{E_{0x}}\right)^2}\ \sin\varphi$$

$$\left(\frac{E_y(0,t)}{E_{0y}} - \frac{E_x(0,t)}{E_{0x}}\cos\varphi\right)^2 = \left(\pm\sqrt{1 - \left(\frac{E_x(0,t)}{E_{0x}}\right)^2}\ \sin\varphi\right)^2$$

$$\left(\frac{E_y(0,t)}{E_{0y}}\right)^2 - 2\frac{E_y(0,t)E_x(0,t)}{E_{0y}E_{0x}}\cos\varphi + \left(\frac{E_x(0,t)}{E_{0x}}\cos\varphi\right)^2$$

$$= \sin^2\varphi + \left(\frac{E_x(0,t)}{E_{0x}}\right)^2 \sin^2\varphi$$

$$\left(\frac{E_y(0,t)}{E_{0y}}\right)^2 - 2\frac{E_y(0,t)E_x(0,t)}{E_{0y}E_{0x}}\cos\varphi + \left(\frac{E_x(0,t)}{E_{0x}}\right)^2 = \sin^2\varphi \tag{8}$$

which describes an ellipse in a z=0 plane,

as shown in fig.6. The angle between the major axis and the *x* axis is

$$\psi = \frac{1}{2}\tan^{-1}\frac{2E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\varphi$$

The angle between the $\vec{E}$ **field and the *x* axis is**

$$\theta = \tan^{-1}\frac{E_{0y}\cos(\omega t + \varphi_y)}{E_{0x}\cos(\omega t + \varphi_x)}$$

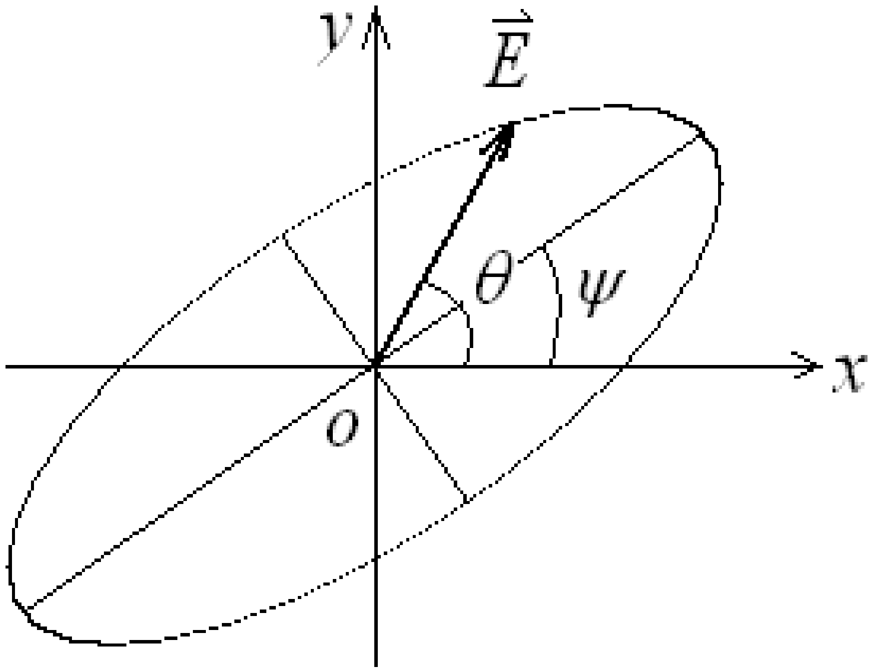

fig.6 an elliptically polarized wave

and

$$\frac{d\theta}{dt} = \frac{E_{0y}E_{0x}\omega\sin(\varphi_x - \varphi_y)}{E_{0x}^2\cos^2(\omega t + \varphi_x) + E_{0y}^2\cos^2(\omega t + \varphi_y)} \qquad (9)$$

❖discussion:

a) **when $\pi > \varphi_x - \varphi_y > 0$, $d\theta/dt > 0$, the tip of the electric field rotates in a counterclockwise direction. *We* extend our right hand in the direction of propagation of the wave (z direction), we find that the fingers of the right hand curl in the direction of rotation of the $\vec{E}$ field. Thus, equation(8) represents a right-handed elliptically polarized wave.**

b) when $-\pi<\varphi_x-\varphi_y<0$, $d\theta/dt<0$, **the tip of the electric field rotates in a clockwise direction.** We extend our left hand in the direction of propagation of the wave (z direction), we find that the fingers of the left hand curl in the direction of rotation of the $\vec{E}$ field. Thus, equation(8) represents a **left-handed elliptically polarized wave.**

c) Note the speed of rotation (or equation(9)) is a function of time, is not a constant.

d) When $\varphi=\varphi_x-\varphi_y=\pm\pi/2$, we get

$$\left(\frac{E_y(0,t)}{E_{0y}}\right)^2+\left(\frac{E_x(0,t)}{E_{0x}}\right)^2=1 \qquad (10)$$

and $\psi=0$

$$\left(\frac{E_y(0,t)}{E_{0y}}\right)^2-2\frac{E_y(0,t)E_x(0,t)}{E_{0y}E_{0x}}\cos\varphi+\left(\frac{E_x(0,t)}{E_{0x}}\right)^2=\sin^2\varphi$$

equation(10) represents an elliptically polarized wave and the major axis or the minor axis is along the *x* axis or the *y* axis, as shown in fig.7

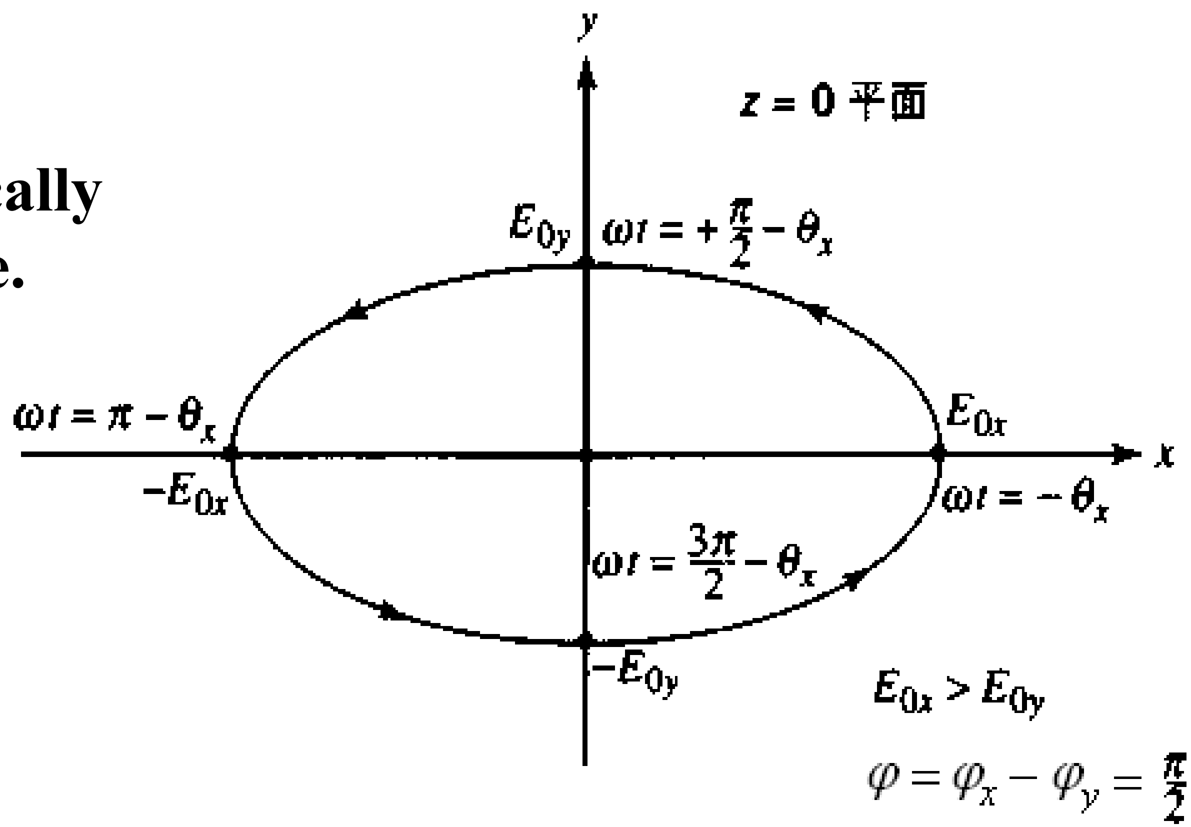

fig.7 a right-handed elliptically polarized wave.

e) When $\varphi = \varphi_x - \varphi_y = 0, \pm\pi$, equation (8)

$$\left(\frac{E_y(0,t)}{E_{0y}}\right)^2 - 2\frac{E_y(0,t)E_x(0,t)}{E_{0y}E_{0x}}\cos\varphi + \left(\frac{E_x(0,t)}{E_{0x}}\right)^2 = \sin^2\varphi$$

becomes

$$\left(\frac{E_y(0,t)}{E_{0y}}\right)^2 \pm 2\frac{E_y(0,t)E_x(0,t)}{E_{0y}E_{0x}} + \left(\frac{E_x(0,t)}{E_{0x}}\right)^2 = 0$$

$$\left(\frac{E_y(0,t)}{E_{0y}} \pm \frac{E_x(0,t)}{E_{0x}}\right)^2 = 0 \quad \text{or} \quad \frac{E_y(0,t)}{E_{0y}} = \pm\frac{E_x(0,t)}{E_{0x}}$$

which represents a linearly polarized wave.