§ 8.9 Normal Incidence of uniform Plane Waves

❑1. General case

we now consider a monochromatic uniform plane wave that travels through one medium and then enters another medium of infinite extent.

We further presume that

a) the incident wave is propagating in the z direction which is normal to the interface between the two media.

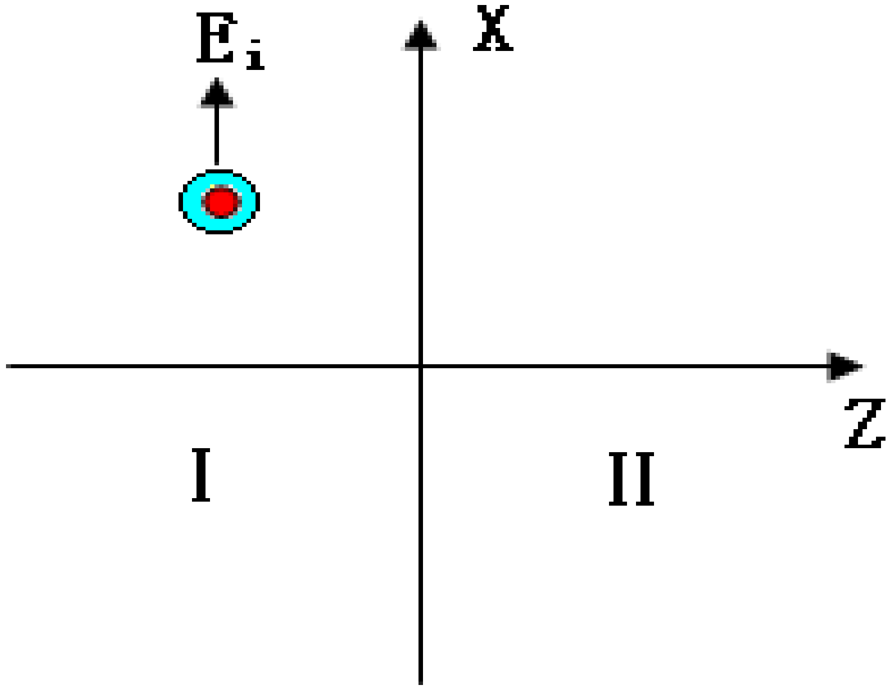
b)the interface is an infinite plane at z=0 c)the region to the right of the interface is medium 1($z < 0$), the region to the left of the interface is medium 2($z > 0$).

d) The wave penetrate the boundary and continues its propagation in medium 2. the wave is referred to as the transmittedwave.

e) The remainder of the wave is reflected at the interface and then propagates in the negative z direction, the wave is called the reflected wave.

Reflection coefficient and transmission coefficient

$$\begin{cases} \dot{\vec{E}}_i = \vec{a}_x \dot{E}_{i0} e^{-\gamma_1 z} \\ \dot{\vec{H}}_i = \vec{a}_y \dfrac{\dot{E}_{i0}}{\eta_{c1}} e^{-\gamma_1 z} \end{cases}$$



$$\begin{cases} \dot{\vec{E}}_r = \vec{a}_x \dot{R} \dot{E}_{i0} e^{\gamma_1 z} \\ \dot{\vec{H}}_r = -\vec{a}_y \dfrac{\dot{R}\dot{E}_{i0}}{\eta_{c1}} e^{\gamma_1 z} \end{cases}$$

$$\dot{\vec{E}}_t(z) = \dot{T}\dot{E}_0 e^{-\gamma_2 z} \vec{a}_x$$

$$\dot{\vec{H}}_t(z) = \frac{1}{\eta_{c2}} \dot{T}\dot{E}_0 e^{-\gamma_2 z} \vec{a}_y$$

If the electric field and the magnetic of the Incident wave are

$$\dot{\vec{E}}_i(z) = \dot{E}_0 e^{-\gamma_1 z} \vec{\mathbf{a}}_{\mathbf{x}}$$

$$\dot{\vec{H}}_i(z) = \frac{1}{\eta_{c1}} \dot{E}_0 e^{-\gamma_1 z} \vec{\mathbf{a}}_{\mathbf{y}}$$

where the subscript *i* stands for the incident wave. Subscripts *r* and *t* are used for the reflected and transmitted waves, respectively.

By defining, at the interface, a complex quantity known as the reflection coefficient:

$$\dot{R} = \frac{\dot{\vec{E}}_r(0)}{\dot{\vec{E}}_i(0)}$$

we can write the reflected fields as

$$\dot{\vec{E}}_r(z) = \dot{R}\dot{E}_0 e^{\gamma_1 z}\vec{\mathbf{a}}_\mathbf{x} \qquad \dot{\vec{H}}_r(z) = -\frac{1}{\eta_{c1}}\dot{R}\dot{E}_0 e^{\gamma_1 z}\vec{\mathbf{a}}_\mathbf{y}$$

the negative sign for the magnetic field is in accordance with the flow of energy in the negative z direction. If we define another complex quantity

$$\dot{T} = \frac{\dot{\vec{E}}_t(0)}{\dot{\vec{E}}_i(0)}$$

as the coefficient of transmission, then the transmitted fields are

$$\dot{\vec{E}}_t(z) = \dot{T}\dot{E}_0 e^{-\gamma_2 z}\vec{a}_x$$

$$\dot{\vec{H}}_t(z) = \frac{1}{\eta_{c2}}\dot{T}\dot{E}_0 e^{-\gamma_2 z}\vec{a}_y$$

where η_{c2} and γ_2 are the propagation constant and the intrinsic impedance in medium 2, respectively.

Therefore, supplying the boundary conditions at z=0, the continuity of the tangential components of the electromagnetic fields (the surface current density is neglected), we obtain

$$\dot{\vec{E}}_i(0) + \dot{\vec{E}}_r(0) = \dot{\vec{E}}_t(0)$$

$$(1 + \dot{R})\dot{E}_0\vec{\mathbf{a}}_x = \dot{T}\dot{E}_0\vec{\mathbf{a}}_x$$

$$\dot{\vec{H}}_i(0) + \dot{\vec{H}}_r(0) = \dot{\vec{H}}_t(0)$$

$$\frac{1}{\eta_{c1}}(1 - \dot{R})\dot{E}_0\vec{\mathbf{a}}_y = \frac{1}{\eta_{c2}}\dot{T}\dot{E}_0\vec{\mathbf{a}}_y$$

thus, we have

$$1 + \dot{R} = \dot{T}$$

$$\frac{1}{\eta_{c1}}(1-\dot{R}) = \frac{1}{\eta_{c2}}\dot{T}$$

manipulating the two equations, we get

$$\dot{R} = \frac{\eta_{c2} - \eta_{c1}}{\eta_{c2} + \eta_{c1}} \quad \text{and} \quad \dot{T} = \frac{2\eta_{c2}}{\eta_{c2} + \eta_{c1}}$$

as the reflection and transmission coefficients, respectively.

The average power density of the transmitted wave in medium 2 is

$$\vec{S}_{tave} = \frac{1}{2}\mathrm{Re}[\dot{\vec{E}}_t(z)\times\dot{\vec{H}}_t^{\,*}(z)]$$

$$= \frac{1}{2}\mathrm{Re}[\dot{T}\dot{E}_0 e^{-\alpha_2 z - j\beta_2 z}\vec{\mathbf{a}}_x \times \frac{1}{\eta_{c2}^{\,*}}\dot{T}^*\dot{E}_0^{\,*} e^{-\alpha_2 z + j\beta_2 z}\vec{\mathbf{a}}_y]$$

$$= \vec{\mathbf{a}}_z \frac{1}{2\left|\eta_{c2}^{\,*}\right|}\left|\dot{T}\right|^2\left|\dot{E}_0\right|^2 e^{-2\alpha_2 z}\cos\theta_{\eta_{c2}}$$

The average power density of the incident wave in medium 1 is

$$\vec{S}_{iave} = \frac{1}{2}\mathrm{Re}[\dot{\vec{E}}_i(z)\times\dot{\vec{H}}_i^{\,*}(z)]$$

$$= \frac{1}{2}\mathrm{Re}[\dot{E}_0 e^{-\alpha_1 z - j\beta_1 z}\vec{\mathbf{a}}_x \times \frac{1}{\eta_{c1}^{\,*}}\dot{E}_0^{\,*} e^{-\alpha_1 z + j\beta_1 z}\vec{\mathbf{a}}_y]$$

$$= \vec{\mathbf{a}}_z \frac{1}{2\left|\eta_{c1}{}^*\right|}\left|\dot{E}_0\right|^2 e^{-2\alpha_1 z} \cos\theta_{\eta_{c1}}$$

the average power density of the reflected wave is

$$\vec{S}_{rave} = \frac{1}{2}\mathrm{Re}[\dot{\vec{E}}_r(z) \times \dot{\vec{H}}_r{}^*(z)]$$

$$= -\frac{1}{2}\mathrm{Re}[\dot{R}\dot{E}_0 e^{\alpha_1 z + j\beta_1 z}\vec{\mathbf{a}}_x \times \frac{1}{\eta_{c1}{}^*}\dot{R}^*\dot{E}_0{}^* e^{\alpha_1 z - j\beta_1 z}\vec{\mathbf{a}}_y]$$

$$= -\vec{\mathbf{a}}_z \frac{1}{2\left|\eta_{c1}{}^*\right|}\left|\dot{R}\right|^2\left|\dot{E}_0\right|^2 e^{-2\alpha_1 z} \cos\theta_{\eta_{c1}}$$

the average power density due to the cross-coupling of the incident and the reflected waves is

$$\vec{S}_{irave} = \frac{1}{2}\mathrm{Re}[\dot{\vec{E}}_i(z)\times\dot{\vec{H}}_r^{\,*}(z)+\dot{\vec{E}}_r(z)\times\dot{\vec{H}}_i^{\,*}(z)]$$

$$= -\vec{\mathbf{a}}_z\frac{1}{\left|\eta_{c1}^{\,*}\right|}\left|\dot{R}\right|\left|\dot{E}_0\right|^2\sin(2\beta_1 z+\theta_{\dot{R}})\sin\theta_{\eta_1}$$

Example 8.7 you read by yourself.

- ❑2. special cases
  - ❑Dielectric-Dielectric Interface

When both media are lossless ($\sigma 1 = 0$, $\sigma 2 = 0$), the intrinsic impedance of each medium is a real quantity. Accordingly, the transmission and the reflection coefficients are real quantities. That is,

$$R = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \text{and} \quad T = \frac{2\eta_2}{\eta_2 + \eta_1}$$

where

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \qquad and \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

the propagation constants are

$$\gamma_1 = j\omega(\mu_1\varepsilon_1)^{1/2} = j\beta_1 \quad and \quad \gamma_2 = j\omega(\mu_2\varepsilon_2)^{1/2} = j\beta_2,$$

we can express the incident, reflected, and transmitted fields as

$$\dot{\vec{E}}_i(z) = \dot{E}_0 e^{-j\beta_1 z}\vec{\boldsymbol{a}}_x$$

$$\dot{\vec{H}}_i(z) = \frac{1}{\eta_1}\dot{E}_0 e^{-j\beta_1 z}\vec{\boldsymbol{a}}_y$$

$$\dot{\vec{E}}_r(z) = R\dot{E}_0 e^{j\beta_1 z}\vec{\mathbf{a}}_{\mathbf{x}}$$

$$\dot{\vec{H}}_r(z) = -\frac{1}{\eta_1}R\dot{E}_0 e^{j\beta_1 z}\vec{\mathbf{a}}_{\mathbf{y}}$$

$$\dot{\vec{E}}_t(z) = T\dot{E}_0 e^{-j\beta_2 z}\vec{a}_x$$

$$\dot{\vec{H}}_t(z) = \frac{1}{\eta_2}T\dot{E}_0 e^{-j\beta_2 z}\vec{a}_y$$

the average power densities of the incident, reflected, and transmitted waves are

$$
\begin{aligned}
\vec{S}_{iave} &= \frac{1}{2}\mathrm{Re}[\dot{\vec{E}}_i(z)\times\dot{\vec{H}}_i^{\,*}(z)] \\
&= \frac{1}{2}\mathrm{Re}[\dot{E}_0 e^{-j\beta_1 z}\vec{\mathbf{a}}_x \times \frac{1}{\eta_1^{\,*}}\dot{E}_0^{\,*} e^{+j\beta_1 z}\vec{\mathbf{a}}_y] \\
&= \vec{\mathbf{a}}_z \frac{1}{2\eta_1}\left|\dot{E}_0\right|^2
\end{aligned}
$$

$$\vec{S}_{rave} = \frac{1}{2}\mathrm{Re}[\dot{\vec{E}}_r(z)\times\dot{\vec{H}}_r^{\,*}(z)]$$

$$= -\frac{1}{2}\mathrm{Re}[R\dot{E}_0 e^{+j\beta_1 z}\vec{\mathbf{a}}_x \times \frac{1}{\eta_1^{\,*}} R^*\dot{E}_0^{\,*} e^{-j\beta_1 z}\vec{\mathbf{a}}_y]$$

$$= -\vec{\mathbf{a}}_z \frac{1}{2\eta_1} R^2 \left|\dot{E}_0\right|^2$$

and $$\vec{S}_{tave} = \frac{1}{2}\mathrm{Re}[\dot{\vec{E}}_t(z)\times\dot{\vec{H}}_t^{\,*}(z)]$$

$$= \frac{1}{2}\mathrm{Re}[T\dot{E}_0 e^{-j\beta_2 z}\vec{\mathbf{a}}_x \times \frac{1}{\eta_2^{\,*}} T^*\dot{E}_0^{\,*} e^{+j\beta_2 z}\vec{\mathbf{a}}_y]$$

$$= \vec{\mathbf{a}}_z \frac{1}{2\eta_2} T^2 \left|\dot{E}_0\right|^2$$

the total fields in medium 1 are (we assumed that *E*0 is the maximum value of the incident electric field at the interface, that is,

$\dot{E}_0$ **= *E*0 is a real quantity, initial phase is zero.)**

$$\dot{\vec{E}}_1(z) = \dot{\vec{E}}_i(z) + \dot{\vec{E}}_r(z)$$
$$= (1 + Re^{2j\beta_1 z})\dot{E}_0 e^{-j\beta_1 z}\vec{\mathbf{a}}_{\mathbf{x}}$$

$$\dot{\vec{H}}_1(z) = \dot{\vec{H}}_i(z) + \dot{\vec{H}}_r(z)$$
$$= (1 - Re^{2j\beta_1 z})\frac{1}{\eta_1}\dot{E}_0 e^{-j\beta_1 z}\vec{\mathbf{a}}_{\mathbf{y}}$$

since $(1 \pm Re^{2j\beta_1 z})E_0$

$$= \left[1 \pm R\left(j\sin(2\beta_1 z) + \cos(2\beta_1 z)\right)\right]E_0$$

$$= \sqrt{1 \pm 2R\cos(2\beta_1 z) + R^2}\, e^{j\varphi_{E(H)}} E_0$$

the amplitudes of the electromagnetic field in medium are

$$\left|\dot{\vec{E}}_1(z)\right| = \sqrt{1 + 2R\cos(2\beta_1 z) + R^2}\, E_0$$

$$\left|\dot{\vec{H}}_1(z)\right| = \sqrt{1 - 2R\cos(2\beta_1 z) + R^2}\, E_0 / \eta_1$$

- Discussion:

$$\dot{\vec{E}}_1(z) = \dot{\vec{E}}_i(z) + \dot{\vec{E}}_r(z) = (1 + Re^{2j\beta_1 z})E_0 e^{-j\beta_1 z}\vec{\mathbf{a}}_{\mathbf{x}}$$

$$= \left|\dot{\vec{E}}_1(z)\right| e^{-j\beta_1 z + j\varphi_E}\vec{\mathbf{a}}_{\mathbf{x}}$$

$$\dot{\vec{H}}_1(z) = \dot{\vec{H}}_i(z) + \dot{\vec{H}}_r(z) = (1 - Re^{2j\beta_1 z})\frac{1}{\eta_1}E_0 e^{-j\beta_1 z}\vec{\mathbf{a}}_{\mathbf{y}}$$

$$= \left|\dot{\vec{H}}_1(z)\right| e^{-j\beta_1 z + j\varphi_H}\vec{\mathbf{a}}_y$$

$$\left|\dot{\vec{E}}_1(z)\right| = \sqrt{1 + 2R\cos(2\beta_1 z) + R^2}\, E_0$$

$$\left|\dot{\vec{H}}_1(z)\right| = \sqrt{1 - 2R\cos(2\beta_1 z) + R^2}\, E_0 / \eta_1$$

- **i) the traveling-wave still exists in medium 1 since the factor** $e^{-j\beta_1 z + j\varphi_{E(H)}}$

ii)the amplitudes of the electromagnetic fields are functions of z. the space period length is λ/2.

when $\eta_2>\eta_1$, $R>0$

Note that $2\beta_1 z= -2n\pi$, n=0,1,2,…

$$\left|\dot{\vec{E}}_1(z)\right|_{\max} = \sqrt{1+2R+R^2}E_0 = (1+R)E_0$$

$$\left|\dot{\vec{H}}_1(z)\right|_{\min} = \sqrt{1-2R+R^2}E_0/\eta_1 \quad = (1-R)E_0/\eta_1$$

and

$2\beta_1 z= -(2n+1)\pi$, n=0,1,2,…

$$\left|\dot{\vec{E}}_1(z)\right|_{\min} = \sqrt{1-2R+R^2}E_0 = (1-R)E_0$$

$$\left|\dot{\vec{H}}_1(z)\right|_{\max} = \sqrt{1+2R+R^2}E_0/\eta_1 = (1+R)E_0/\eta_1$$

Obviously, the distance between the maximum and the minimum value of the amplitude of the electric field is $\lambda_1/4$: $2\beta_1|Z_{max}-Z_{min}| = \pi$

$$2(2\pi/\lambda_1)\ |Z_{max}-Z_{min}| = \pi \qquad |Z_{max}-Z_{min}| = \lambda_1/4$$

❑(2) Dielectri-Perfect conductor interface

Now let us consider the case when a wave traveling in a dielectric medium (medium 1) impinges normally upon a perfectly conducting medium (medium 2).

As the electromagnetic fields cannot exist inside a perfect conductor, that is,

$\sigma_2 = \infty$,

$$\eta_{c2} = \sqrt{\frac{\mu_2}{\varepsilon_{c2}}} = \sqrt{\frac{\mu_2}{\varepsilon(1 - j\frac{\sigma}{\omega})}} = 0$$

thus, we obtain

$$\dot{R} = \frac{\eta_{c2} - \eta_{c1}}{\eta_{c2} + \eta_{c1}} = -1 \quad \text{and} \quad \dot{T} = \frac{2\eta_{c2}}{\eta_{c2} + \eta_{c1}} = 0$$

in other words, the incident wave is totally reflected from the boundary.

The fields in medium 2(perfect conductor) are

$$\dot{\vec{E}}_t(z) = \dot{T}\dot{E}_0 e^{-j\beta_2 z}\vec{a}_x = 0$$

$$\dot{\vec{H}}_t(z) = \frac{1}{\eta_2}\dot{T}\dot{E}_0 e^{-j\beta_2 z}\vec{a}_y = 0$$

the fields in medium 1 are

$$\dot{\vec{E}}_i(z) = \dot{E}_0 e^{-j\beta_1 z}\vec{\boldsymbol{a}}_x$$

$$\dot{\vec{H}}_i(z) = \frac{1}{\eta_1}\dot{E}_0 e^{-j\beta_1 z}\vec{\boldsymbol{a}}_y$$

$$\dot{\vec{E}}_r(z) = \dot{R}\dot{E}_0 e^{j\beta_1 z}\vec{\mathbf{a}}_\mathbf{x} = -\dot{E}_0 e^{j\beta_1 z}\vec{\mathbf{a}}_\mathbf{x}$$

$$\dot{\vec{H}}_r(z) = -\frac{1}{\eta_1}\dot{R}\dot{E}_0 e^{j\beta_1 z}\vec{\mathbf{a}}_\mathbf{y} = \frac{1}{\eta_1}\dot{E}_0 e^{j\beta_1 z}\vec{\mathbf{a}}_\mathbf{y}$$

thus

$$\dot{\vec{E}}_1(z) = \dot{\vec{E}}_i(z) + \dot{\vec{E}}_r(z) \quad = (e^{-j\beta_1 z} - e^{j\beta_1 z})\dot{E}_0 \vec{\mathbf{a}}_{\mathbf{x}}$$

$$= -j2\sin(\beta_1 z)\dot{E}_0 \vec{\mathbf{a}}_{\mathbf{x}}$$

$$\dot{\vec{H}}_1(z) = \dot{\vec{H}}_i(z) + \dot{\vec{H}}_r(z) \quad = (e^{-j\beta_1 z} + e^{j\beta_1 z})\frac{1}{\eta_1}\dot{E}_0 \vec{\mathbf{a}}_{\mathbf{y}}$$

$$= 2\cos(\beta_1 z)\frac{1}{\eta_1}\dot{E}_0 \vec{\mathbf{a}}_{\mathbf{y}}$$

where

$$\beta_1 = \omega\sqrt{\mu_1 \varepsilon_1}$$

and $$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

- **discussion:**

i) the fields in medium can be rewritten in the time domain as

$$\vec{E}_1(z,t) = \mathrm{Re}[\dot{\vec{E}}_1(z)e^{j\omega t}]$$

$$= 2E_0 \sin(\beta_1 z)\sin(\omega t)\vec{\mathbf{a}}_{\mathbf{x}}$$

letting $\dot{E}_0 = E_0$

$$\vec{H}_1(z,t) = \mathrm{Re}[\dot{\vec{H}}_1(z)e^{j\omega t}]$$

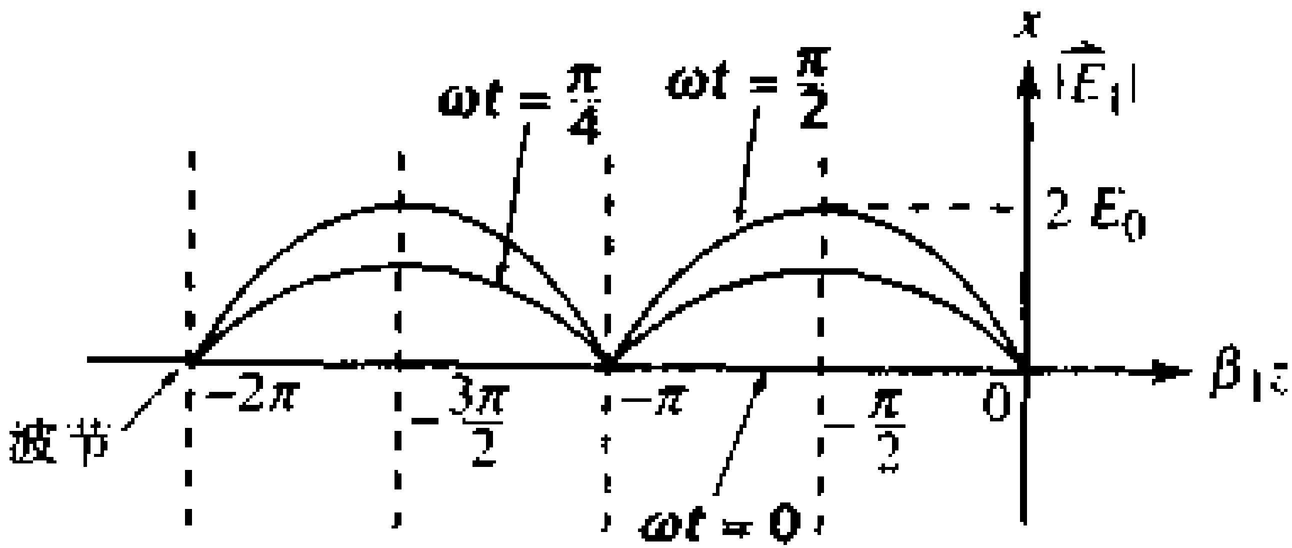
$$= \frac{2}{\eta_1} E_0 \cos(\beta_1 z)\cos(\omega t)\vec{\mathbf{a}}_{\mathrm{y}}$$

the amplitudes of the fields are

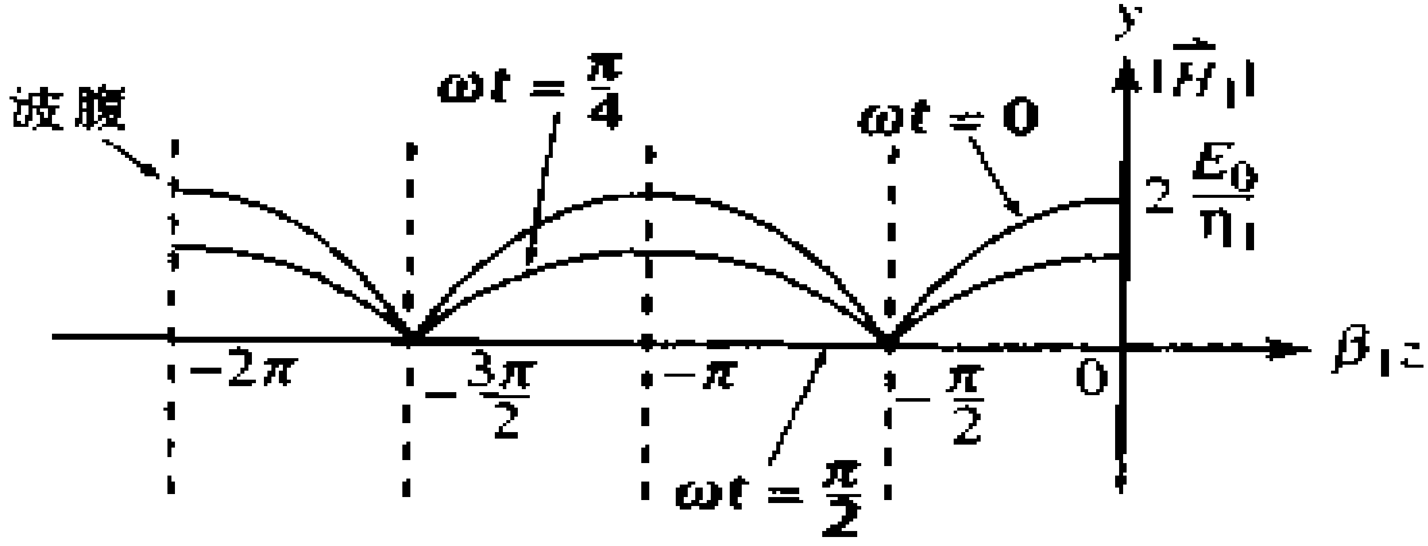
$$\left|\vec{E}_1(z)\right| = \left|2E_0 \sin(\beta_1 z)\sin(\omega t)\right|$$

$$\left|\vec{H}_1(z)\right| = \left|\frac{2}{\eta_1} E_0 \cos(\beta_1 z)\cos(\omega t)\right|$$

At any time, the magnitudes of the fields are functions of space (*z*)



$$\vec{E}_1(z) = 2E_0 \sin(\beta_1 z) \sin \omega t \, \vec{a}_x$$



$$\vec{H}_1(z) = \frac{2}{\eta_1} E_0 \cos(\beta_1 z) \cos \omega t \, \vec{a}_y$$

the magnitude of the electric field is maximum when

$$\sin(\beta_1 z) = \pm 1$$

the point at which the field is maximum is called a loop----loop point. The magnitude of the electric field is zero(minimum) when

$$\sin(\beta_1 z) = 0$$

the point at which the field is minimum is called a node----node point

Note that where there are loops of the electric field there are nodes of the magnetic field.

The waves is called pure standing waves. The standing waves are 90^0 out of time and space phase.

- **ii)the surface current density**

$$\dot{\vec{J}}_s = \vec{a}_n \times (\dot{\vec{H}}_2 - \dot{\vec{H}}_1) = -\vec{a}_z \times \dot{\vec{H}}_1(0) = \frac{2}{\eta_1} E_0 \vec{a}_x$$

in the time domain

$$\vec{J}_s = \frac{2}{\eta_1} E_0 \cos(\omega t)\vec{\mathbf{a}}_{\mathbf{x}}$$

To be continued