§ 2.12 the uniqueness theory

the uniqueness theory states that a vector field $\vec{F}$ is uniquely determined in a region if the following requirements are satisfied:

a)its divergence is specified throughout the region

b)its curl is specified throughout the region

c) its normal (tangent) component is specified on the closed surface bounding the region.

And the vector field can be expressed by an irrotational field($\vec{H} = -\nabla f$)and continuous field($\vec{G} = \nabla \times \vec{A}$) , that is,

$$\vec{F} = \vec{G} + \vec{H} = \nabla \times \vec{A} - \nabla f$$

Since

a) the divergence of a vector field at each point in a given region V is the rate of change of the field components along their directions in themselves. It is the flux density. It can specifies the flux intensity.

b) the curl of a vector field at each point in a given region V is the rate of change of the vector field components along their normal directions. It is t he maximum of the circulation of the vector field. It can specifies the intensity of the rotational field.

Namely,

The fact that the divergence and the curl are all specified can prove the fact that sources of flux and circulation In the given region.

At last, the boundary conditions (tangential or normal) can prove that sources on the surface b ounding the region is specified.

Thus , all sources have been specified. And th en the vector field is specified.

Exercises

Page 65, T2.24; T2.27;T2.32;T2.33;T2.36; T2.37;

T1:

$$\vec{A} = \vec{a}_x(3y^2 - 2x) + \vec{a}_y x^2 + \vec{a}_z 2z$$

is irrotational or continuous ?

find the distribution of the field.

page 69, T2.44,T2.45;T2.46, T2.47