CHAPTER 9

TRANSMISSION LINES

Exercise 9.1

$r_i = 3\ mm \quad , \quad r_o = 6\ mm \quad , \quad \epsilon = 2.5\epsilon_o$

$$C_\ell = \frac{2\pi \times 2.5\epsilon_o}{\ln \frac{6}{3}} = 2 \times 10^{-10}\ F/m \qquad C_\ell = 0.2\ nF/m$$

$$L_\ell = \frac{\mu_o}{2\pi} \ln \frac{6}{3} = 1.386 \times 10^{-7}\ H/m \qquad L_\ell = 0.1386\ \mu H/m$$

Exercise 9.2

$\ell = 600\ m \quad , \quad L_\ell = 0.4\ \mu H/m \quad , \quad C_\ell = 85\ pF/m \quad , \quad f = 100\ kHz$

a) $$u_p = \frac{1}{\sqrt{0.4 \times 10^{-6} \times 85 \times 10^{-12}}} = 1.715 \times 10^{8}\ m/s$$

$$\beta = \frac{\omega}{u_p} = \frac{2\pi \times 100 \times 10^{3}}{1.715 \times 10^{8}} = 3.664 \times 10^{-3}\ rad/m$$

b) $$\hat{Z}_c = \sqrt{\frac{L_\ell}{C_\ell}} = \sqrt{\frac{0.4 \times 10^{-6}}{85 \times 10^{-12}}} = 68.6\ \Omega$$

Exercise 9.3

$t_d = 100\ ns \quad , \quad L_\ell = 0.2\ \mu H/m \quad , \quad C_\ell = 60\ pF/m$

$$u_p = \frac{1}{\sqrt{0.20 \times 10^{-6} \times 60 \times 10^{-12}}} = 2.8867 \times 10^{8}\ m/s$$

$$\ell = u_p t_d = 2.8867 \times 10^{8} \times 100 \times 10^{-9} = 28.87\ m$$

Exercise 9.4

$\ell = 20\ m \quad , \quad L_\ell = 0.35\ \mu H/m \quad , \quad C_\ell = 45\ pF/m$

$P_R = 20\ W \quad , \quad V_R = 50\ V \quad , \quad f = 1\ MHz$

a) $$\hat{Z}_c = \sqrt{\frac{0.35 \times 10^{-6}}{45 \times 10^{-12}}} = 88.19\ \Omega$$

$$\beta = 2\pi \times 10^{6} \sqrt{0.35 \times 10^{-6} \times 45 \times 10^{-12}}$$

$$\beta = 2.49 \times 10^{-2}\ rad/m$$

b) $I_R = \frac{20}{50} = 0.4\ A \quad , \quad \tilde{V}_R = 50 \angle 0^\circ\ V \quad , \quad \tilde{I}_R = 0.4 \angle 0^\circ\ A$

$$\tilde{I}_s = j \frac{1}{88.19} \sin(2.49 \times 10^{-2} \times 20) \times 50 \angle 0^\circ + \cos(2.49 \times 10^{-2} \times 20) \times 0.4 \angle 0^\circ$$

$$\tilde{I}_s = 0.444 \angle 37.62^\circ\ A$$

$$\tilde{V}_s = \cos(2.49 \times 10^{-2} \times 20) \times 50 \angle 0^\circ + j\ 88.19 \times \sin(2.49 \times 10^{-2} \times 20) \times 0.4 \angle 0^\circ$$

$$\tilde{V}_s = 47.048 \angle 20.99^\circ\ V$$

c) $\hat{S}_s = \tilde{V}_s \tilde{I}_s^{*} = 20 - j\,5.975\ VA \Rightarrow P_s = 20\ W \ , \ Q_s = -5.975\ VAR$

Exercise 9.5

$l = 50\,m$, $L_l = 0.3\mu H/m$, $C_l = 40\,pF/m$, $\hat{S}_R = 10 + j2\ VA$

$V_R = 20V$, $f = 100\,kHz$

$$\tilde{I}_R^* = \frac{\hat{S}_R}{\tilde{V}_R} = \frac{10+j2}{20\angle 0^\circ} = 0.5 + j\,0.1 \Rightarrow \tilde{I}_R = 0.5 - j0.1\,A = 0.51\angle{-11.31^\circ}\,A$$

$$\hat{Z}_L = \frac{20\angle 0^\circ}{0.51\angle{-11.31^\circ}} = 39.22\angle 11.31^\circ\ \Omega$$

$$\beta = 2\pi \times 100 \times 10^3 \sqrt{0.3 \times 10^{-6} \times 40 \times 10^{-12}} = 2.18 \times 10^{-3}\,rad/m$$

$$\hat{Z}_c = \sqrt{\frac{0.3 \times 10^{-6}}{40 \times 10^{-12}}} = 86.6\,\Omega$$

$$\hat{Z}_{in}(0) = 86.6\,\frac{39.22\angle 11.31^\circ + j\,86.6 \tan(2.18 \times 10^{-3} \times 50)}{86.6 + j\,39.22\angle 11.31^\circ \tan(2.18 \times 10^{-3} \times 50)}$$

$$\hat{Z}_{in}(0) = 39.592 + j15.383\,\Omega = 42.475\angle 21.23^\circ\ \Omega$$

Exercise 9.6

$l = 2m$, $f = 10\,MHz$, $L_l = 0.251\mu H/m$, $C_l = 48.28\,pF/m$

$\hat{Z}_{in}(0) = 50 + j25\ \Omega$

$$\beta = 2\pi \times 10 \times 10^6 \sqrt{0.251 \times 10^{-6} \times 48.28 \times 10^{-12}} = 0.219\ rad/m$$

$$\hat{Z}_c = \sqrt{\frac{0.251 \times 10^{-6}}{48.28 \times 10^{-12}}} = 72.103\,\Omega$$

$$\hat{Z}_L = \hat{Z}_c \frac{\hat{Z}_{in} - j\hat{Z}_c \tan \beta l}{\hat{Z}_c - j\hat{Z}_{in} \tan \beta l}$$

$$\hat{Z}_L = 72.103 \frac{(50+j25) - j72.103 \tan(0.219\times 2)}{72.103 - j(50+j25)\tan(0.219\times 2)}$$

$$\hat{Z}_L = 41.858 + j4.177 \ \Omega$$

<u>Exercise 9.7</u>

$l = 3m$, $\quad v_s = 20\cos(3.14\times 10^8 t)\ V$, $\quad \hat{Z}_L = 100 + j20\Omega$

$L_l = 0.2\,\mu H/m$, $\quad C_l = 40pF/m$

$$\hat{Z}_c = \sqrt{\frac{0.2\times 10^{-6}}{40\times 10^{-12}}} = 70.71\ \Omega$$

$$\beta = 3.14\times 10^8 \sqrt{0.2\times 10^{-6} \times 40 \times 10^{-12}} = 0.605\ rad/m$$

$$\hat{Z}_{in}(0) = 70.71 \frac{(100+j20) + j70.71 \tan(0.605\times 3)}{70.71 + j(100+j20)\tan(0.605\times 3)} = 46.521 + j0.153\Omega$$

$$\tilde{I}_s = \frac{20\angle 0^\circ}{46.521 + j0.153} = 0.43\angle -0.188^\circ\ A$$

$$i_s = 0.43\cos(3.14\times 10^8 t - 0.188^\circ)$$

$$\hat{S}_s = \frac{1}{2}\tilde{V}_s\tilde{I}_s^* = 4.299 + j0.014\ VA$$

$$P_s = 4.299\ W$$

Exercise 9.8

$\ell = 10\ m$, $\hat{Z}_c = 75\ \Omega$, $\hat{Z}_L = 35 + j10\ \Omega$, $v_R = \sqrt{2} \times 50 \cos 10^8 t$ V

$v_S = \sqrt{2} \times 66 \cos(10^8 t + 31°)$ V

$\tilde{V}_R = \sqrt{2} \times 50 \angle 0°$ V , $\hat{Z}_L = 36.4 \angle 15.95°\ \Omega$

$$\tilde{I}_R = \frac{\tilde{V}_R}{\hat{Z}_L} = \sqrt{2} \times 1.374 \angle -15.95°\ A = \sqrt{2}\,(1.321 - j0.3776)\ A$$

In terms of "rms" voltages and currents, we can write,

$$66 \angle 31° = 50 \cos(10\beta) + j(1.321 \cdot 75 - j0.3776 \times 75) \sin(10\beta)$$

$$50 \cos(10\beta) + 0.3776 \times 75 \sin(10\beta) = 56.57$$

$$1.321 \times 75 \sin(10\beta) = 33.99$$

$$\beta = 0.035$$

$$C_\ell = \frac{\beta}{\omega Z_c} = \frac{0.035}{10^8 \times 75} = 4.67 \times 10^{-12}\ F/m$$

$$C_\ell = 4.67\ pF/m$$

$$L_\ell = C_\ell Z_c^2$$

$$L_\ell = 4.67 \times 10^{-12} \times 75^2 = 2.63 \times 10^{-8}\ H/m$$

$$L_\ell = 0.0263\ \mu H/m$$

Exercise 9.9

$\ell = 50\ m$, $\hat{\tau}_R = 0.75\,\underline{/9^\circ}$, $V_R = 20V$, $P_R = 10W$, $C_\ell = 75pF/m$

$\cos\theta = 0.97$ (inductive) , $f = 1kHz$

$$I_R = \frac{10/0.97}{20} = 0.515\ A \quad , \quad \tilde{I}_R = 0.515\,\underline{/-14.07^\circ}\ A$$

$$\hat{Z}_L = \frac{20}{0.515\,\underline{/-14.07^\circ}} = 38.8\,\underline{/14.07^\circ}\ \Omega$$

a) $\hat{\tau}_R = 1 + \hat{\rho}_R \Rightarrow \hat{\rho}_R = \hat{\tau}_R - 1 = 0.75\,\underline{/9^\circ} - 1 = 0.285\,\underline{/155.65^\circ}$

b) $$\hat{Z}_c = \hat{Z}_L \frac{1 - \hat{\rho}_R}{1 + \hat{\rho}_R} = \frac{1 - 0.285\,\underline{/155.65^\circ}}{1 + 0.285\,\underline{/155.65^\circ}} \times 38.8\,\underline{/14.07^\circ} =$$

$\hat{Z}_c = 65.426 - j0.289\ \Omega$ $\qquad \hat{Z}_c \cong 65.4\ \Omega$

c) $L_\ell = 65.4^2 C_\ell \Rightarrow L_\ell = 3.208 \times 10^{-7}\ H/m$

$$\beta = 2\pi \times 10^3 \sqrt{3.208 \times 10^{-7} \times 75 \times 10^{-12}} = 3.082 \times 10^{-5}\ rad/m$$

$\tilde{V}_s = \tilde{V}_R \cos\beta\ell + j\hat{Z}_c \tilde{I}_R \sin\beta\ell \qquad \tilde{V}_s = 20 + j0.052\ \ V$

$\tilde{I}_s = \frac{j}{\hat{Z}_c} \sin\beta\ell \cdot \tilde{V}_R + \cos\beta\ell \cdot \tilde{I}_R \qquad \tilde{I}_s = 0.515 + j4.71 \times 10^{-4}\ \ A$

$\hat{S}_s = \tilde{V}_s \tilde{I}_s^* = 10.3 + j0.017\ VA \qquad P_s \cong 10\ W$

Exercise 9.10

$\hat{Z}_c = 75\ \Omega$, $VSWR = 1.3$

$$VSWR = \frac{1 + |\rho_R|}{1 - |\rho_R|} = 1.3 \Rightarrow |\rho_R| = 0.13$$

$$\rho_R = \frac{R_L - 75}{R_L + 75} = 0.13 \Rightarrow R_L = 97.41\ \Omega$$

$$\rho_{R_m} = \frac{R_L // R_m - 75}{R_L // R_m + 75} = 0 \Rightarrow R_L // R_m = 75\ \Omega$$

$$\frac{R_L R_m}{R_L + R_m} = 75 \Rightarrow R_m = 326\ \Omega$$

Exercise 9.11

$\hat{Z}_c = 50\ \Omega$, $f = 1\,\text{MHz}$, $R_L = 100\ \Omega$, $L_L = 10\,\mu H$

$$\hat{Z}_L = 100 + j\, 2\pi \times 10^6 \times 10 \times 10^{-6} = 100 + j62.83\ \Omega$$

$$\hat{\rho}_R = \frac{100 + j62.83 - 50}{100 + j62.83 + 50} = 0.494 \angle 28.76^\circ \qquad |\hat{\rho}_R| = 0.494$$

$$VSWR = \frac{1 + 0.494}{1 - 0.494}$$

$$VSWR = 2.95$$

Exercise 9.12

$\hat{Z}_c = 75\,\Omega$, $l = 10\,m$, $f = 150\,MHz$, $\hat{Z}_L = 150 + j225\,\Omega$

$u_p = 2.95 \times 10^8$ m/s

$$\beta = \frac{2\pi \times 150 \times 10^6}{2.95 \times 10^8} = 3.195 \text{ rad/m}$$

$$\frac{1}{75}\,\frac{75 + j(150 + j225)\tan(3.195 d)}{(150 + j225) + j75\tan(3.195 d)} = \frac{1}{75} + j\,\frac{1}{75\tan(3.195\, l_s)}$$

Solving the above equation yields

$d = 0.449$ m and $l_s = 0.21$ m

Exercise 9.13

$\hat{Z}_c = 50\,\Omega$, $l = 2\,m$, $f = 60\,MHz$, $l_s = 0.5\,m$, $d = 0.6\,m$, $t_t = 7\,ns$

$$u_p = \frac{2}{7 \times 10^{-9}} = 2.86 \times 10^8 \text{ m/s} \qquad \beta = \frac{2\pi \times 60 \times 10^6}{2.86 \times 10^8} = 1.32 \text{ rad/m}$$

$$\frac{1}{50}\,\frac{50 + j(R_L + jX_L)\tan(1.32 \times 0.6)}{(R_L + jX_L) + j50\tan(1.32 \times 0.6)} = \frac{1}{50} + j\,\frac{1}{50\tan(1.32 \times 0.5)}$$

where $\hat{Z}_L = R_L + jX_L$

Solving the above equation yields

$R_L = 93.76\,\Omega$ and $X_L = -25.78\,\Omega$

Exercise 9.14

$i_s = 2 \sin(314 \times 10^3 t)$ A , $\ell = 10$m, $v_s = 50 \cos(314 \times 10^3 t)$ V

$R = 0.25\ \Omega$, $L = 6.5\mu H$, $G = 0$, $C = 320pF$

$$R_\ell = \frac{0.25}{10} = 0.025\ \Omega/m\ ,\quad L_\ell = \frac{6.5}{10} = 0.65\ \mu H/m\ ,\quad C_\ell = \frac{320}{10} = 32pF/m$$

$$\hat{Z}_\ell = 0.025 + j\,314 \times 10^3 \times 0.65 \times 10^{-6} = 0.025 + j0.204\ \Omega/m$$

$$\hat{Y}_\ell = j\,314 \times 10^3 \times 32 \times 10^{-12} = j1.005 \times 10^{-5}\ S/m$$

$$\hat{Z}_c = \sqrt{\frac{\hat{Z}_\ell}{\hat{Y}_\ell}} = \sqrt{\frac{0.025 + j0.204}{j1.005 \times 10^{-5}}} = 142.79 - j\,8.71\ \Omega$$

$$\hat{\gamma} = \sqrt{\hat{Z}_\ell \hat{Y}_\ell} = \sqrt{(0.025 + j0.204)(j1.005 \times 10^{-5})} = 8.75 \times 10^{-5} + j0.001435$$

$$\hat{Z}_{in}(0) = \frac{\tilde{V}_s}{\tilde{I}_s} = \frac{50\angle 0^\circ}{2\angle 0^\circ} = 25\angle 0^\circ\ \Omega$$

$$\hat{Z}_L = \frac{\hat{Z}_c \tanh\hat{\gamma}\ell - \hat{Z}_{in}(0)}{\frac{\hat{Z}_{in}(0)}{\hat{Z}_c}\tanh\hat{\gamma}\ell - 1} = 24.76 - j1.98\ \Omega$$

$$P_{in} = (50 \times 2)\frac{1}{2} = 50W$$

$$\tilde{V}_R = \cosh\hat{\gamma}\ell \cdot \tilde{V}_s - \hat{Z}_c \tilde{I}_s \sinh\hat{\gamma}\ell = 49.66\angle -4.714^\circ\ V$$

$$\tilde{I}_R = -\frac{1}{\hat{Z}_c}\sinh\hat{\gamma}\ell \cdot \tilde{V}_s + \cosh\hat{\gamma}\ell \cdot \tilde{I}_s = 2\angle -0.143^\circ\ A$$

$$\hat{S}_R = (\tilde{V}_R \tilde{I}_R^*)\frac{1}{2} = 49.5 - j3.96\ VA \qquad P_R = 49.5W$$

$$\eta = \frac{P_R}{P_s} = \frac{49.5}{50} = 0.99 \quad \text{or} \quad \eta = 99\%$$

Exercise 9.15

$\ell = 40\,m$, $\hat{Z}_c = 75\angle{-4^\circ}\ \Omega$, $\alpha_{dB} = 0.001\ dB/m$, $f = 2MHz$,

$u_p = 2.5\times10^8\ m/s$, $\tilde{V}_s = 60\angle{0^\circ}\ V$, $\tilde{I}_R = 0$

$$\alpha = \frac{\alpha_{dB}}{8.69} = \frac{0.001}{8.69} = 1.151\times10^{-4}\ Np/m \quad , \quad \beta = \frac{2\pi\times2\times10^6}{2.5\times10^8} = 0.05\ \frac{rad}{m}$$

$\hat{\gamma} = 1.151\times10^{-4} + j\,0.05 \qquad \cosh\hat{\gamma}\ell = 0.426\angle{179.44^\circ}$, $\sinh\hat{\gamma}\ell = 0.905\angle{90.12^\circ}$

a) $$\tilde{V}_R = \frac{\tilde{V}_s}{\cosh\hat{\gamma}\ell} = \frac{60\angle{0^\circ}}{0.426\angle{179.44^\circ}} = 140.91\angle{-179.44^\circ}\ V$$

b) $$\tilde{I}_s = \frac{1}{\hat{Z}_c}\sinh\hat{\gamma}\ell\cdot\tilde{V}_R = \frac{60\angle{0^\circ}}{75\angle{-4^\circ}}\left(0.905\angle{90.12^\circ}\right) = 1.7\angle{-85.32^\circ}\ A$$

Assuming V_s and I_s as the "rms" values yields the power

$\hat{S}_s = \tilde{V}_s\tilde{I}_s^{*} = 8.33 + j\,101.66\ \ VA$

$P_s = 8.33\ W$, $Q_s = 101.66\ VAR$

Exercise 9.16

$V_s = 5\,V$, $l = 10\,m$, $R_c = 50\,\Omega$, $R_L = 100\,\Omega$, $R_s = 0$

$u_p = 2.85 \times 10^8\ m/s$ $\qquad t = 5\,t_t$

$$\rho_R = \frac{100 - 50}{100 + 50} = \frac{1}{3} \quad , \quad \rho_s = \frac{0 - 50}{0 + 50} = -1$$

$$t_t = \frac{10}{2.85 \times 10^8} = 3.5 \times 10^{-8}\ s = 35\,ns$$

$$V_s(0) = 5V \quad , \quad I_s(0) = \frac{5}{50} = 0.1\ A$$

$$V_R(35ns) = 5 + \frac{5}{3} = \frac{20}{3}\ V \quad , \quad I_R(35ns) = 0.1 - \frac{0.1}{3} = \frac{0.2}{3}\ A$$

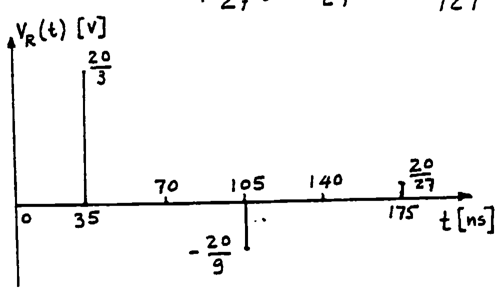
$$P_R(35ns) = V_R(35ns)\,I_R(35ns) = \frac{20}{3} \times \frac{0.2}{3} = \frac{4}{9}\ W$$

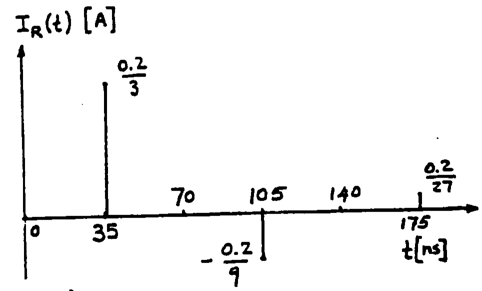
$$V_R(105ns) = -\frac{5}{3} - \frac{5}{9} = -\frac{20}{9}\ V \quad , \quad I_R(105ns) = -\frac{0.1}{3} + \frac{0.1}{9} = -\frac{0.2}{9}\ A$$

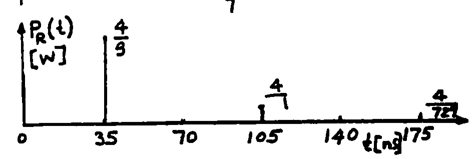
$$P_R(105ns) = \left(-\frac{20}{9}\right)\left(-\frac{0.2}{9}\right) = \frac{4}{81}\ W$$

$$V_R(175ns) = \frac{5}{9} + \frac{5}{27} = \frac{20}{27}\ V \quad , \quad I_R(175ns) = \frac{0.1}{9} - \frac{0.1}{27} = \frac{0.2}{27}\ A$$

$$P_R(175ns) = \left(\frac{20}{27}\right)\left(\frac{0.2}{27}\right) = \frac{4}{729}\ W$$







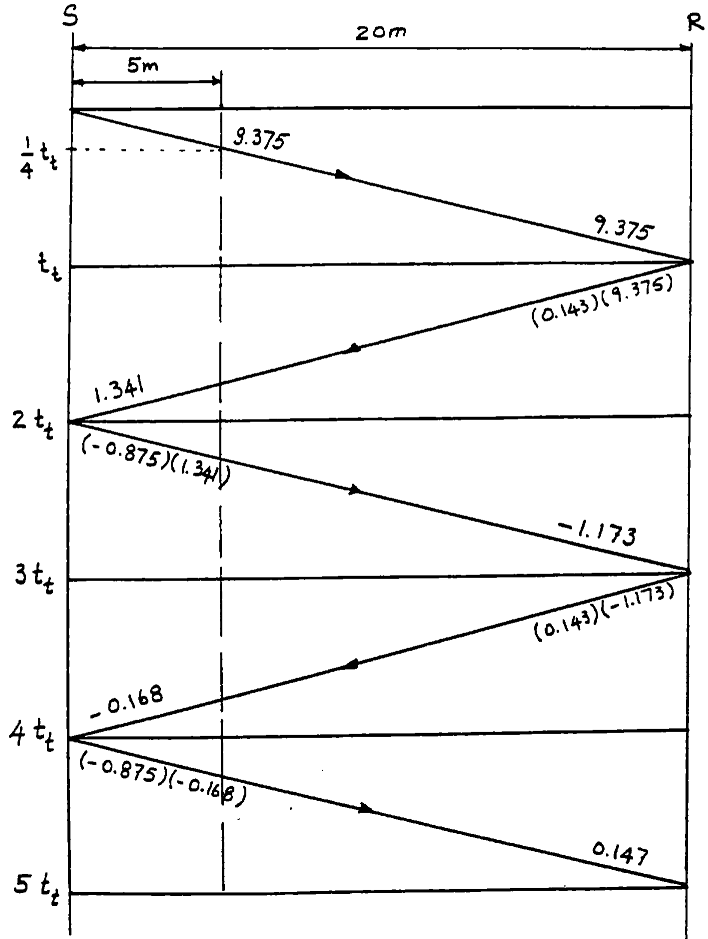
Exercise 9.17

$V_G = 10V$, $l = 20m$, $R_c = 75\Omega$, $R_G = 5\Omega$, $R_L = 100\Omega$

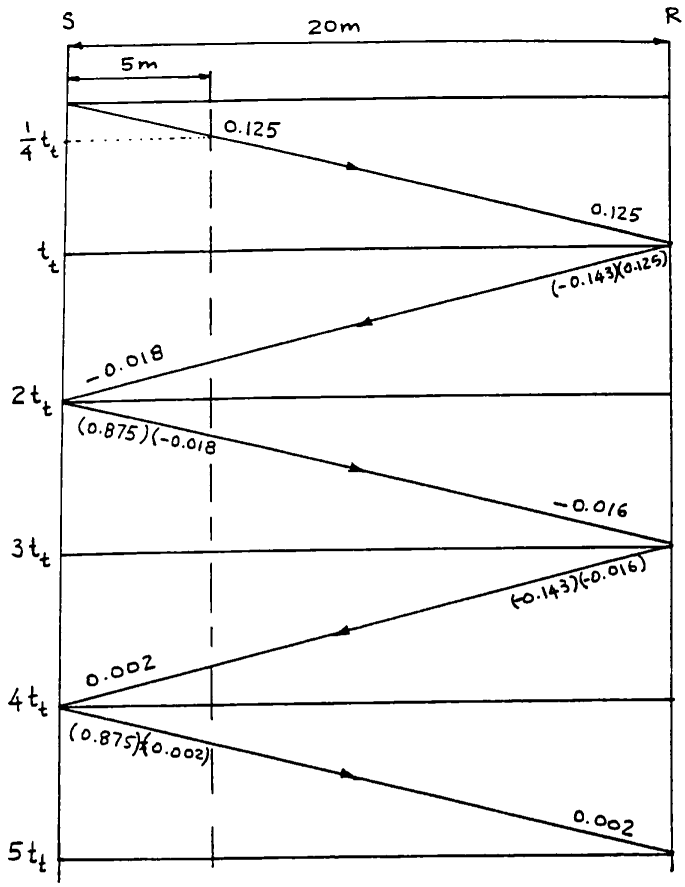
$u_p = 3 \times 10^8 \ m/s$, $z = 5m$

$$\rho_R = \frac{100 - 75}{100 + 75} = 0.143 \quad , \quad \rho_s = \frac{5 - 75}{5 + 75} = -0.875$$

$$t_t = \frac{20}{3 \times 10^8} = 6.67 \times 10^{-8} \ s \quad , \quad V_s = \frac{V_G}{R_G + R_c} R_c = \frac{10 \times 75}{5 + 75} = 9.375V$$



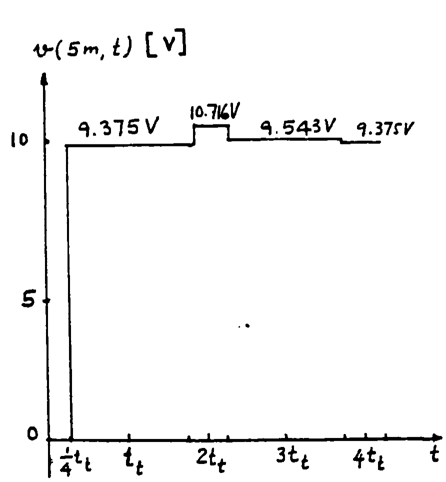
Voltage Lattice diagram.



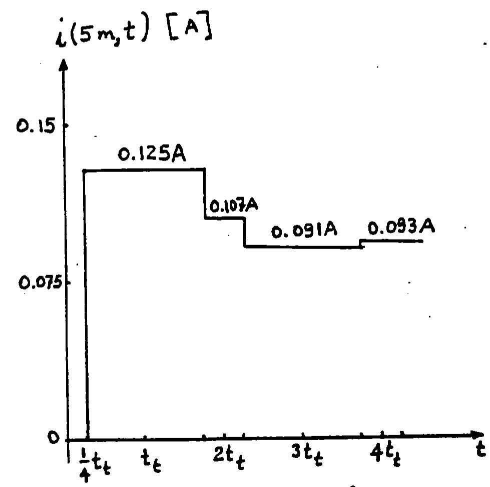
$$I_s = \frac{V_G}{R_G + R_c}$$

$$I_s = \frac{10}{5+75} = 0.125\text{A}$$

Current lattice diagram.



Voltage waveform.



Current waveform.

Exercise 9.18

$R_{c_1} = 75\,\Omega$, $R_{c_2} = 50\,\Omega$, $R_L = 30\,\Omega$, $V_G = 10\,V$, $R_G = 25\,\Omega$

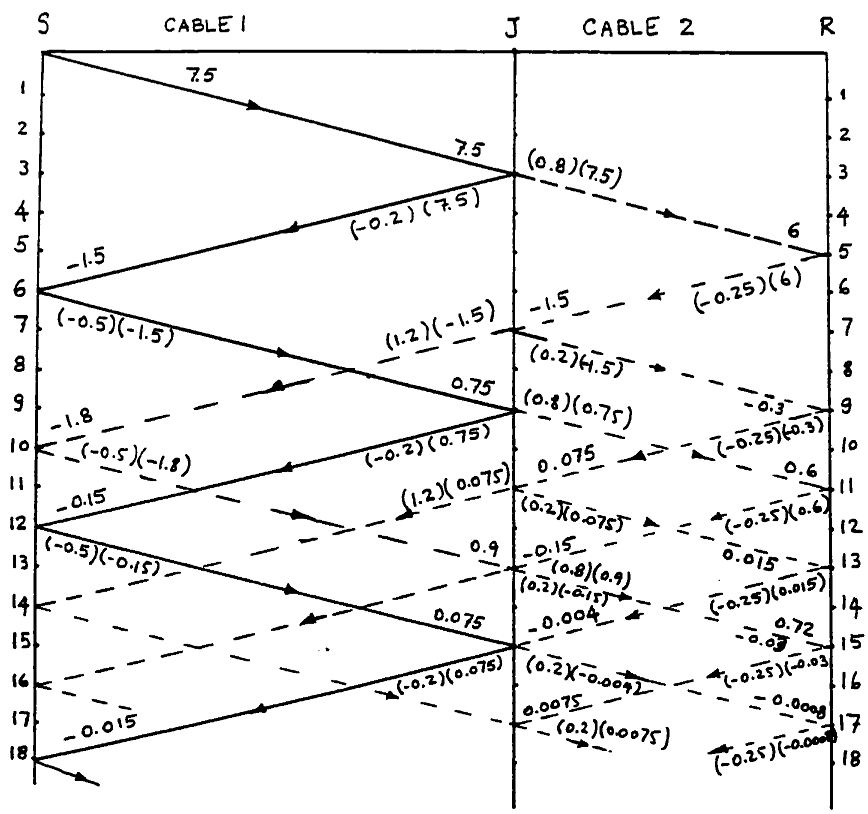
$t_{t_1} = 3\,\mu s$, $t_{t_2} = 2\,\mu s$

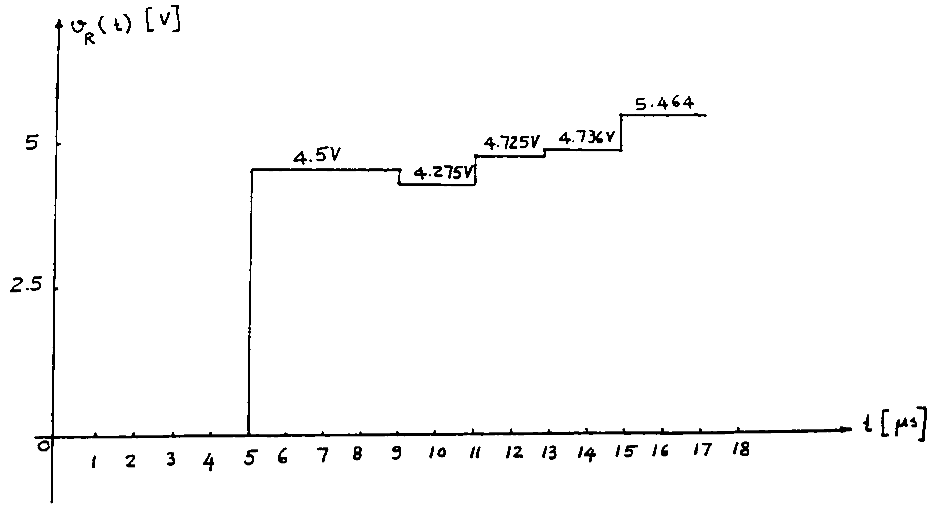
$\rho_S = \frac{25-75}{25+75} = -0.5$, $\rho_R = \frac{30-50}{30+50} = -0.25$,

$\rho_1 = \frac{50-75}{50+75} = -0.2$, $\tau_1 = 1 - 0.2 = 0.8$ Towards the second cable.

$\rho_2 = \frac{75-50}{50+75} = 0.2$, $\tau_2 = 1 + 0.2 = 1.2$ Towards the first cable.

$V_S = \frac{10}{25+75} 75 = 7.5\ V$





Voltage waveform at the receiving end.

Exercise 9.19

$r = 2\,mm$, $f = 60\,Hz$, $f = 1000\,Hz$, $f = 1\,MHz$, $\sigma = 3.55 \times 10^{7}\,S/m$

At $f = 60\,Hz$ $\quad \delta_c = \sqrt{\dfrac{2}{2\pi\, 60 \times 4\pi \times 10^{-7} \times 3.55 \times 10^{7}}} = 1.09 \times 10^{-2}\,m$

$\delta_c = 1.09\,cm$

At $f = 1000\,Hz$ $\quad \delta_c = \sqrt{\dfrac{2}{2\pi\; 10^{3} \times 4\pi \times 10^{-7} \times 3.55 \times 10^{7}}} = 2.67 \times 10^{-3}\,m$

$\delta_c = 2.67\,mm$

At $f = 1\,MHz$ $\quad \delta_c = \sqrt{\dfrac{2}{2\pi \times 10^{6} \times 4\pi \times 10^{-7} \times 3.55 \times 10^{7}}} = 8.45 \times 10^{-5}\,m$

$\delta_c = 0.0845\,mm$

Exercise 9.20

$a = \frac{10}{2} = 5\,mm \qquad f = 1\,kHz, \quad f = 1\,MHz, \quad \sigma = 3.55 \times 10^{7}\,S/m$

At $f = 1\,kHz$

$$\delta_c = \sqrt{\frac{2}{2\pi \times 10^{3} \times 3.55 \times 10^{7} \times 4\pi \times 10^{-7}}} = 1.09 \times 10^{-2}\,m$$

The internal resistance :

$$R_{li} = \frac{1}{2\pi a \sigma \delta_c} = \frac{1}{2\pi \times 5 \times 10^{-3} \times 3.55 \times 10^{7} \times 1.09 \times 10^{-2}} = 82.26 \times 10^{-6}\,\Omega/m$$

The internal inductance :

$$L_{li} = \frac{82.26 \times 10^{-6}}{2\pi \times 10^{3}} = 1.309 \times 10^{-8}\,H/m$$

At $f = 1\,MHz$

$$\delta_c = \sqrt{\frac{2}{2\pi \times 10^{6} \times 3.55 \times 10^{7} \times 4\pi \times 10^{-7}}} = 8.45 \times 10^{-5}\,m$$

The internal resistance:

$$R_{li} = \frac{1}{2\pi \times 5 \times 10^{-3} \times 3.55 \times 10^{7} \times 8.45 \times 10^{-5}} = 1.06 \times 10^{-2}\,\Omega/m$$

The internal inductance:

$$L_{li} = \frac{1.06 \times 10^{-2}}{2\pi \times 10^{6}} = 1.69 \times 10^{-9}\,H/m$$

Problem 9.2

$l = 20\,\text{km} \qquad \hat{Z}_c = 150\,\Omega \qquad u_p = 0.9c$

$$u_p = \frac{1}{\sqrt{L_\ell C_\ell}} = 0.9 \times 3 \times 10^8 \quad , \quad \hat{Z}_c = \sqrt{\frac{L_\ell}{C_\ell}} = 150\,\Omega$$

$$\frac{L_\ell}{C_\ell} = 150^2 \quad (1) \qquad , \qquad \frac{1}{L_\ell C_\ell} = (2.7 \times 10^8)^2 \quad (2)$$

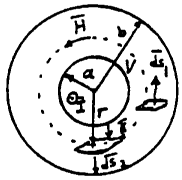
Solving (1) and (2) simultaneously yields,

$$L_\ell = 5.56 \times 10^{-7}\,\text{H/m} \ , \quad C_\ell = 2.47 \times 10^{-11}\,\text{F/m}$$

Total inductance $L = 20 \times 10^3 \times 5.56 \times 10^{-7} = 11.12 \times 10^{-3}\,H$ or $11.12\,mH$

Total capacitance $C = 20 \times 10^3 \times 2.47 \times 10^{-11} = 4.94 \times 10^{-7}\,F$ or $0.494\,\mu F$

Problem 9.3



Inductance:

$$\bar{H} = \frac{I}{2\pi r}\,\bar{a}_\phi \ [A/m] \ , \quad \bar{B} = \mu_o \frac{I}{2\pi r}\,\bar{a}_\phi \quad [T]$$

$$\bar{ds}_1 = dr\,dz\,\bar{a}_\phi$$

$$\Phi = \int_0^1 \int_a^b \mu_o \frac{I}{2\pi r}\,\bar{a}_\phi \cdot dr\,dz\,\bar{a}_\phi = \mu_o \frac{I}{2\pi} \ln \frac{b}{a} \quad [Wb/m]$$

$$\lambda = N\,\Phi \qquad N = 1 \qquad \lambda = \Phi = \mu_o \frac{I}{2\pi} \ln \frac{b}{a} \quad \left[\frac{Wb}{m}\right]$$

$$L = \frac{d\Phi}{dI} = \frac{\mu_o}{2\pi} \ln \frac{b}{a} \quad [H/m]$$

Capacitance:

$$\bar{E} = \frac{V}{r \ln \frac{b}{a}} \bar{a}_r \quad \left[\frac{V}{m}\right] \quad , \bar{D} = \epsilon \bar{E} = \epsilon \frac{V \bar{a}_r}{r \ln \frac{b}{a}} \left[C/m^2\right]$$

$$\bar{ds}_2 = r\, d\phi\, dz\, \bar{a}_r$$

$$Q = \int_0^1 \int_0^{2\pi} \epsilon \frac{V}{r \ln \frac{b}{a}} \bar{a}_r \cdot r\, d\phi\, dz\, \bar{a}_r = \frac{2\pi \epsilon V}{\ln \frac{b}{a}} \left[\frac{C}{m}\right]$$

$$C = \frac{dQ}{dV} = \frac{2\pi\epsilon}{\ln \frac{b}{a}} \quad \left[\frac{F}{m}\right]$$

Problem 9.4

$t_d = 14\,ns$ at $f = 1\,MHz$ $\quad l = 2m \quad \mu = \mu_o \quad \epsilon_r = ?$

$$t_d = \frac{l}{u_p} = l\sqrt{L_l C_l}$$

From Problem 9.3 $\quad L_l = \frac{\mu_o}{2\pi} \ln \frac{b}{a} \quad , \; C_l = 2\pi\epsilon_o / \ln \frac{b}{a}$

$$t_d = l \sqrt{\left(\frac{\mu_o}{2\pi} \ln \frac{b}{a}\right)\left(\frac{2\pi \epsilon_r \epsilon_o}{\ln \frac{b}{a}}\right)} = l\sqrt{\mu_o \epsilon_o \epsilon_r}$$

$$\epsilon_r = \frac{\left(\frac{14 \times 10^{-9}}{2}\right)^2}{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}} = 4.4$$

Problem 9.5

$l = 300\,m$, $\hat{Z}_c = 75\,\Omega$, $u_p = 220{,}000$ km/s , $f = 3$ MHz

$\hat{Z}_L = 150 + j400\,\Omega$, $\tilde{V}_R = 50\angle 0^\circ$ V (rms)

$$\tilde{I}_R = \frac{\tilde{V}_R}{\hat{Z}_L} = \frac{50\angle 0^\circ}{150 + j400} = 0.117\angle -69.44^\circ \text{ A}$$

$$\beta = \frac{\omega}{u_p} = \frac{2\pi \times 3\times 10^6}{220{,}000 \times 10^3} = 8.57\times 10^{-2} \text{ rad/m}$$

$$\tilde{V}_s = (50\angle 0^\circ)\cos(8.57\times 10^{-2}\times 300) + j75\times(0.117\angle -69.44^\circ)\sin(8.57\times 10^{-2}\times 300)$$

$$\tilde{V}_s = 46.412\angle 2.077^\circ \text{ V}$$

Problem 9.6

$l = 2\,m$, $f = 15MHz$, $\beta = 369.6\times 10^{-3}$ rad/m , $\hat{Z}_c = Z_c\angle 0^\circ$, $\hat{S}_R = 3.5 - j1.5$ VA,

$\tilde{V}_R = 50\angle 0^\circ$ V , $V_s = 34$ V

$$\tilde{I}_R^* = \frac{3.5 - j1.5}{50} = 0.07 - j003 \Rightarrow \tilde{I}_R = 0.0762\angle 23.2^\circ \text{ A}$$

$$34\angle\phi = 50\cos(369.6\times 10^{-3}\times 2) + jZ_c\, 0.0762\angle 23.2^\circ \sin(369.6\times 10^{-3}\times 2)$$

$$34\cos\phi + j34\sin\phi = 36.95 - 0.02Z_c + j0.047Z_c$$

$$34\cos\phi + 0.02Z_c = 36.95 \quad (1) \quad , \quad 34\sin\phi = 0.047Z_c \quad (2)$$

Solving (1) and (2) simultaneously yields,

$Z_c = 283.24\,\Omega$, $\phi = 23.05^\circ$

Problem 9.7

$\hat{S}_R = 100 + j\,30$ MVA , $l = 100$ km, $f = 60$ Hz, $V_R = 110$ kV

$L_l = 0.372\ \mu H/m$, $C_l = 76\ pF/m$

a) $$\hat{Z}_c = \sqrt{\frac{0.372 \times 10^{-6}}{76 \times 10^{-12}}} = 69.96\ \Omega$$

b) $$u_p = \frac{1}{\sqrt{0.372 \times 10^{-6} \times 76 \times 10^{-12}}} = 188{,}071\ \text{km/s}$$

$$\beta = \frac{2\pi \times 60}{188{,}071 \times 10^3} = 2 \times 10^{-6}\ \text{rad/m}$$

c) $$\tilde{I}_R^* = \frac{(100 + j30) \times 10^6}{110 \times 10^3} = 909.1 + j\,272.73$$

$$\tilde{I}_R = 949.13 \angle -16.7^\circ\ A$$

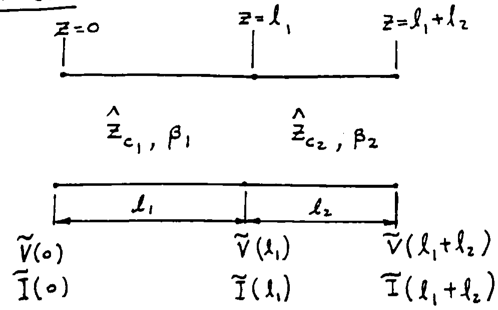
$$\tilde{V}_s = (110 \times 10^3 \angle 0^\circ) \cos(2 \times 10^{-6} \times 100 \times 10^3) + j(69.96 \angle 0^\circ)(949.13 \angle -16.7^\circ) \sin(2 \times 10^{-6} \times 10^5)$$

$$\tilde{V}_s = 112.31 \angle 6.46^\circ\ \text{kV}$$

d) $$\Delta V = V_s - V_R$$

$$\Delta V = 112.31 - 110 = 2.31\ \text{kV}$$

Problem 9.8



$$\begin{bmatrix} \tilde{V}(l_1) \\ \tilde{I}(l_1) \end{bmatrix} = \begin{bmatrix} \cos\beta_2 l_2 & j\hat{Z}_{c_2}\sin\beta_2 l_2 \\ j\frac{1}{\hat{Z}_c}\sin\beta_2 l_2 & \cos\beta_2 l_2 \end{bmatrix} \begin{bmatrix} \tilde{V}(l_1+l_2) \\ \tilde{I}(l_1+l_2) \end{bmatrix}$$

$$\begin{bmatrix} \tilde{V}(0) \\ \tilde{I}(0) \end{bmatrix} = \begin{bmatrix} \cos\beta_1 l_1 & j\hat{Z}_{c_1}\sin\beta_1 l_1 \\ j\frac{1}{\hat{Z}_{c_1}}\sin\beta_1 l_1 & \cos\beta_1 l_1 \end{bmatrix} \begin{bmatrix} \tilde{V}(l_1) \\ \tilde{I}(l_1) \end{bmatrix}$$

$$\begin{bmatrix} \tilde{V}(0) \\ \tilde{I}(0) \end{bmatrix} = \begin{bmatrix} \cos\beta_1 l_1 & j\sin\beta_1 l_1 \\ j\frac{1}{\hat{Z}_{c_1}}\sin\beta_1 l_1 & \cos\beta_1 l_1 \end{bmatrix} \begin{bmatrix} \cos\beta_2 l_2 & j\sin\beta_2 l_2 \\ j\frac{1}{\hat{Z}_{c_2}}\sin\beta_2 l_2 & \cos\beta_2 l_2 \end{bmatrix} \begin{bmatrix} \tilde{V}(l_1+l_2) \\ \tilde{I}(l_1+l_2) \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} \cos\beta_1 l_1 \cos\beta_2 l_2 - \frac{\hat{Z}_{c_1}}{\hat{Z}_{c_2}}\sin\beta_1 l_1 \sin\beta_2 l_2 & j\hat{Z}_{c_2}\sin\beta_2 l_2\cos\beta_1 l_1 + j\hat{Z}_{c_1}\sin\beta_1 l_1\cos\beta_2 l_2 \\ j\frac{1}{\hat{Z}_{c_1}}\sin\beta_1 l_1\cos\beta_2 l_2 + j\frac{1}{\hat{Z}_{c_2}}\sin\beta_2 l_2\cos\beta_1 l_1 & -\frac{\hat{Z}_{c_2}}{\hat{Z}_{c_1}}\sin\beta_1 l_1\sin\beta_2 l_2 + \cos\beta_1 l_1\cos\beta_2 l_2 \end{bmatrix}$$

Problem 9.9

$l = 20\,m$, $f = 10MHz$, $\hat{Z}_L = 100 + j\,60\,\Omega$, $L_l = 3\times10^{-7}\,H/m$, $C_l = 40\times10^{-12}\,F/m$

$$\hat{Z}_c = \sqrt{\frac{3\times10^{-7}}{40\times10^{-12}}} = 86.6\,\Omega$$

$$\beta = 2\pi\times10\times10^{6}\sqrt{3\times10^{-7}\times40\times10^{-12}} = 0.218\ \mathrm{rad/m}$$

$$\frac{l}{2} = \frac{20}{2} = 10\,m$$

$$\hat{Z}_{in}(0) = 86.6\,\frac{100+j60 + j\,86.6\tan[0.218(20-10)]}{86.6 + j(100+j60)\tan[0.218(20-10)]} = 45.81\angle 6.77^\circ\ \Omega$$

$$\hat{Z}_{in}(0) = 45.49 + j5.40\ \Omega$$

Problem 9.10

$l = 90\,m$, $\hat{Z}_c = 50\,\Omega$, $f = 500\,kHz$, $u_p = 2.8\times10^{8}\,m/s$

$\hat{Z}_{in}(0) = 60 - j\,20\,\Omega$,

$$\beta = \frac{2\pi\times500\times10^{3}}{2.8\times10^{8}} = 1.122\times10^{-2}\ \mathrm{rad/m}$$

$$\hat{Z}_L = \hat{Z}_c\,\frac{\hat{Z}_{in}(0) - j\,\hat{Z}_c\tan\beta l}{\hat{Z}_c - j\,\hat{Z}_{in}(0)\tan\beta l}$$

$$\hat{Z}_L = 50\,\frac{(60-j20) - j50\tan(1.122\times10^{-2}\times90)}{50 - j(60-j20)\tan(1.122\times10^{-2}\times90)}$$

$$\hat{Z}_L = 56.09 + j\,20.75\,\Omega$$

Problem 9.11

$l = 2\,m, \quad \hat{Z}_c = 75\,\Omega, \quad u_p = 2.6\times10^8\,m/s, \quad \hat{Z}_L = 120 + j90\,\Omega$

$v_R(t) = 150\cos(1.26\times10^8 t) \qquad \omega = 1.26\times10^8\,rad/s$

$$\beta = \frac{1.26\times10^8}{2.6\times10^8} = 4.85\times10^{-1}\,rad/m$$

a) $$\hat{\rho}_R = \frac{120+j90-75}{120+j90+75} = 0.469\angle 38.68^\circ = 0.366 + j0.293$$

$$\hat{\rho}(z) = (0.469\angle 38.68^\circ)(e^{-j2\times0.485(2-z)})$$

$$\hat{\rho}(z) = 0.469\,e^{-j(1.265-0.97z)}$$

b) $$150\angle 0^\circ = \hat{V}^+ e^{-j0.485\times2}(1 + 0.469\,e^{j0.675})$$

$$\hat{V}^+ = 107.36\angle 43.47^\circ\ V$$

$$\tilde{V}_f = (107.36\angle 43.47^\circ)(1\angle -0.485\times2\ rad) = 107.36\angle -12.08^\circ\ V$$

$$\tilde{V}_b = \hat{\rho}_R\tilde{V}_f = (0.469\angle 36.68^\circ)(107.36\angle -12.08^\circ) = 50.35\angle 26.60^\circ\ V$$

c) $$VSWR = \frac{1+\rho_R}{1-\rho_R} = \frac{1+0.469}{1-0.469} = 2.766$$

d) $$\hat{\rho}(0) = 0.469\,e^{-j1.265} = 0.469\angle -72.48^\circ$$

$$\tilde{V}(0) = 107.36\angle 43.67^\circ\,[1 + 0.469\angle -72.48^\circ]$$

$$\tilde{V}(0) = 131.59\angle 22.09^\circ\ V$$

$$\Delta V = V(0) - V(l) = 131.59 - 150 = -18.41\ V$$

e) $P_f(z) = \mathrm{Re}\left[(\hat{V}^+ e^{-j\beta z})\left(\frac{\hat{V}^{+*}}{\hat{Z}_c^*} e^{j\beta z} \right)\right]$

$$= \frac{V^{+2}}{Z_c} = \frac{107.36^2}{75} = 153.68\,W$$

$P_f(z) = 153.68\,W$

$$P_b(z) = \mathrm{Re}\left\{\left[\hat{V}^+ e^{-j\beta z}\, \hat{\rho}_R\, e^{-j2\beta(l-z)}\right]\left[-\frac{\hat{V}^{+*}}{\hat{Z}_c^*} e^{j\beta z}\, \hat{\rho}_R^*\, e^{j2\beta(l-z)}\right]\right\}$$

$$= -\frac{V^{+2}}{Z_c}\rho_R^2 = -\frac{107.36^2}{75}\,0.469^2 = -33.80\,W$$

$P_b(z) = -33.80\,W$

f) $P(0) = P_f(0) + P_b(0) = 153.68 - 33.80 = 119.88\,W$

$P(l) = P_f(l) + P_b(l) = 153.68 - 33.80 = 119.88\,W$

$$\eta = \frac{P(l)}{P(0)} = \frac{119.88}{119.88} = 100\%$$

Problem 9.12

$l = 50\,m$, $L_l = 0.5\,\mu H/m$, $C_l = 50\,pF/m$, $v_s(t) = 280\cos(6.28\times10^7 t)$

$\hat{Z}_L = 250\angle 0^\circ\ \Omega$

a) $\hat{Z}_c = \sqrt{\dfrac{0.5\times10^{-6}}{50\times10^{-12}}} = 100\,\Omega$ $\quad \rho_R = \dfrac{250-100}{250+100} = 0.43$

b) $\beta = 6.28\times10^7\sqrt{0.5\times10^{-6}\times50\times10^{-12}} = 0.314\,rad/m$

$$\hat{\rho}(0) = \hat{\rho}_R e^{-j2\beta l} = 0.43\, e^{-j2\times0.314\times50} = 0.43\, e^{-j31.4}$$

$$\hat{V}^+ = \frac{280\angle 0^\circ}{1 + 0.43\, e^{-j31.4}} = 195.81 \angle -0.274^\circ \ \ V$$

$$\tilde{V}_f(z) = \hat{V}^+ e^{-j\beta z} = (195.81 \angle -0.274^\circ)\, e^{-j0.314z} \qquad [V]$$

$$\tilde{V}_b(z) = \hat{\rho}(z)\, \hat{V}^+ e^{-j\beta z} = (195.81 \angle -0.274^\circ \ e^{-j0.314z})(0.43 e^{-j2\times 0.314(50-z)})$$

$$\tilde{V}_b(z) = (84.2 \angle -0.274^\circ)\, e^{j(0.314z - 31.4)} \qquad [V]$$

$$\tilde{I}_f(z) = \frac{195.81 \angle -0.274^\circ \ e^{-j0.314z}}{100} = 1.96 \angle -0.274^\circ \ e^{-j0.314z} \qquad [A]$$

$$\tilde{I}_b(z) = -\frac{84.2 \angle -0.274^\circ \ e^{j(0.314z - 31.4)}}{100}$$

$$\tilde{I}_b(z) = -0.842 \angle -0.274^\circ \ e^{j(0.314z - 31.4)} \qquad [A]$$

c) $P_f(z) = \mathrm{Re}[\tilde{V}_f \tilde{I}_f^*]$

$$= \mathrm{Re}\left[(195.81 \angle -0.274^\circ \ e^{-j0.314z})(1.96 \angle 0.274^\circ \ e^{j0.314z})\right]$$

$$P_f(z) = 383.79 \ W$$

$$P_b(z) = \mathrm{Re}[\tilde{V}_b \tilde{I}_b^*]$$

$$= \mathrm{Re}\left[(84.2 \angle -0.274^\circ \ e^{j(0.314z - 31.4)})(-0.842 \angle 0.274^\circ \ e^{-j(0.314z - 31.4)})\right]$$

$$P_b(z) = -70.9 \ W$$

Problem 9.13

$\ell = 0.42\,m$, $\ell = \frac{\lambda}{4}$, $\hat{Z}_L = 20 - j10\,\Omega$, $C = 50.78\,pF$

$i_s(t) = \sqrt{2}\cos(6\times10^8 t)$ A

$\omega = 6\times10^8$ rad/s , $\frac{\lambda}{4} = 0.42 \Rightarrow \lambda = 1.68\,m$, $\beta = \frac{2\pi}{1.68} = 3.74$ rad/m

$C_\ell = \frac{50.78\times10^{-12}}{0.42} = 1.209\times10^{-10}$ F/m , $L_\ell = \frac{\beta^2}{\omega^2 C_\ell} = \frac{3.74^2}{(6\times10^8)^2\times1.209\times10^{-10}}$

$L_\ell = 3.214\times10^{-7}$ H/m

$$\hat{Z}_c = \sqrt{\frac{3.214\times10^{-7}}{1.209\times10^{-10}}} = 51.56\,\Omega$$

$$\hat{Z}_{in}(0) = \frac{51.56^2}{20 - j10} = 118.87\angle 26.57^\circ\ \Omega$$

$I_{s_{rms}} = \frac{\sqrt{2}}{\sqrt{2}} = 1\,A$ $\qquad \tilde{I}_s = 1\angle 0^\circ$ A

$\tilde{V}_s = (1\angle 0^\circ)(118.87\angle 26.57^\circ) = 118.87\angle 26.57^\circ$ V

Problem 9.14

$\ell = 7\,m$, $\hat{Z}_G = 28\angle -20^\circ\ \Omega$, $\tilde{V}_R = 50\angle 0^\circ$ V , $\tilde{I}_R = 2\angle 0^\circ$ A (rms)

$\hat{Z}_L = \frac{50\angle 0^\circ}{2\angle 0^\circ} = 25\angle 0^\circ\ \Omega$ $\qquad$ For the maximum power transfer, $\hat{Z}_{in}(0) = \hat{Z}_G^*$

$\therefore\ \hat{Z}_{in}(0) = 28\angle 20^\circ\ \Omega$

$$28\angle 20^\circ = Z_c \frac{25 + jZ_c \tan 7\beta}{Z_c + j25 \tan 7\beta}$$

$$(26.31 + j\,9.58)(Z_c + j25 \tan 7\beta) = Z_c(25 + jZ_c \tan 7\beta)$$

$$26.31 Z_c - 23.95 \tan 7\beta = 25 Z_c \qquad (1)$$

$$657.75 \tan 7\beta + 9.58 Z_c = Z_c^2 \tan 7\beta \qquad (2)$$

Solving (1) and (2) simultaneously yields

$$Z_c = 28.86\,\Omega \quad \text{and} \quad \beta = 0.144 \text{ rad/m}$$

Problem 9.15

$$\hat{Z}_c = 75\,\Omega \quad , \quad \hat{Z}_L = 10 - j40\,\Omega$$

Voltage reflection coefficient : $\hat{\rho}_{RV} = \dfrac{10 - j40 - 75}{10 - j40 + 75} = 0.813\angle -123.2^\circ$

Current reflection coefficient : $\hat{\rho}_{RI} = -\hat{\rho}_{RV} = 0.813\angle 56.8^\circ$

Problem 9.16

$$\hat{Z}_c = 50\,\Omega \quad , f = 1\text{MHz}, \quad \hat{Z}_{osc} = 1\text{M}\Omega$$

$$\hat{Z}_L = \hat{Z}_m // \hat{Z}_{osc} = \frac{\hat{Z}_m \hat{Z}_{osc}}{\hat{Z}_m + \hat{Z}_{osc}}$$

No reflection at R yields $\hat{\rho}_R = 0$,

$$\hat{\rho}_R = \frac{\hat{Z}_L - \hat{Z}_c}{\hat{Z}_L + \hat{Z}_c} = \frac{\dfrac{\hat{Z}_m \hat{Z}_{osc}}{\hat{Z}_m + \hat{Z}_{osc}} - \hat{Z}_c}{\dfrac{\hat{Z}_m \hat{Z}_{osc}}{\hat{Z}_m + \hat{Z}_{osc}} + \hat{Z}_c} = 0$$

$$\hat{Z}_m 10^6 - 50(\hat{Z}_m + 10^6) = 0 \qquad \hat{Z}_m = \frac{50 \times 10^6}{10^6 - 50} \cong 50\,\Omega$$

Problem 9.17

$\ell = 1.2\,m$, $\hat{Z}_{in}(0) = 120 - j80\ \Omega$, $\lambda = 2\,m$, $f = 50\,MHz$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{2} = \pi \text{ rad/m}$$

$$120 - j80 = Z_c \frac{Z_L + jZ_c \tan(1.2\pi)}{Z_c + jZ_L \tan(1.2\pi)}$$

$$120 Z_c + 58.08 Z_L = Z_c Z_L \qquad (1)$$

$$87.12 Z_L - 80 Z_c = Z_c^2\, 0.726 \qquad (2)$$

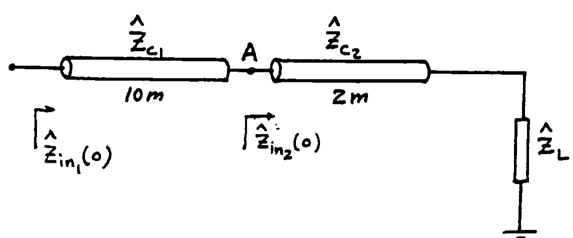
Solving (1) and (2) simultaneously yields,

$$Z_c = 120.51\,\Omega$$

Problem 9.18

$\hat{Z}_{c_1} = 50\,\Omega$, $\ell = 10\,m$, $f = 200\,kHz$, $\tilde{V} = 50\angle 0^\circ$ V (rms)

$\hat{Z}_{c_2} = 75\,\Omega$, $\ell = 2\,m$, $\hat{Z}_L = 120 - j200\ \Omega$



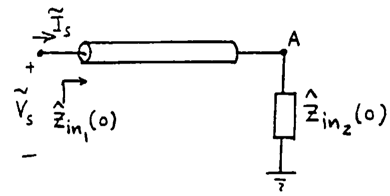
a) $\rho_A = \dfrac{75 - 50}{75 + 50} = 0.2 \qquad \tau_A = 1 + \rho_A = 1 + 0.2 = 1.2$

b) $\omega = 2\pi \times 200 \times 10^3 = 4\pi \times 10^5$ rad/s

$$u_{p_1} = \frac{10}{36\times10^{-9}} = 2.78\times10^8 \text{ m/s} \quad , \quad \beta_1 = \frac{4\pi\times10^5}{2.78\times10^8} = 4.52\times10^{-3} \text{ rad/m}$$

$$u_{p2} = \frac{2}{8\times10^{-9}} = 2.5\times10^8 \text{ m/s} \quad , \quad \beta_2 = \frac{4\pi\times10^5}{2.5\times10^8} = 5.03\times10^{-3} \text{ rad/m}$$

$$\hat{Z}_{in_2}(0) = 75\,\frac{(120-j200) + j75\tan(5.03\times10^{-3}\times2)}{75 + j(120-j200)\tan(5.03\times10^{-3}\times2)} = 226.49\angle{-59.84^\circ}\ \Omega$$



$$\hat{Z}_{in_1}(0) = \frac{226.49\angle{-59.84^\circ} + j50\tan(4.52\times10^{-3}\times10)}{50 + j(226.49\angle{-59.84^\circ})\tan(4.52\times10^{-3}\times10)} = 190.02\angle{-64.55^\circ}\ \Omega$$

$$\tilde{I}_s = \frac{50\angle{0^\circ}}{190.02\angle{-64.55^\circ}} = 0.263\angle{64.55^\circ}\ \text{A}$$

$$\tilde{V}_A = (50\angle{0^\circ})\cos(4.52\times10^{-3}\times10) - j(0.263\angle{64.55^\circ})\times50\times\sin(4.52\times10^{-3}\times10)$$

$$\tilde{V}_A = 50.48\angle{-0.29^\circ}\ \text{V}$$

$$\tilde{I}_A = -j\,\frac{1}{50}(50\angle{0^\circ})\sin(4.52\times10^{-3}\times10) + (0.263\angle{64.55^\circ})\cos(4.52\times10^{-3}\times10)$$

$$\tilde{I}_A = 0.223\angle{59.52^\circ}\ \text{A}$$

Problem 9.19

$\hat{Z}_c = 50\,\Omega \qquad VSWR = 1.5$

$$VSWR = \frac{1+\rho_R}{1-\rho_R} \qquad 1.5 = \frac{1+\rho_R}{1-\rho_R} \Rightarrow \rho_R = 0.2$$

$$\rho_R = \frac{Z_L - 50}{Z_L + 50} = 0.2 \qquad \Rightarrow Z_L = 75\,\Omega$$

With no standing waves $VSWR = 1$ or $\rho_R = 0$.

For $\rho_R = 0 \qquad Z_L' = \hat{Z}_c = 50\,\Omega$

$$Z_L' = Z_L // Z_M = 50 \quad \Rightarrow \quad \frac{1}{75} + \frac{1}{Z_M} = \frac{1}{50} \quad \Rightarrow Z_M = 150\,\Omega$$

Problem 9.20

$\ell = 10\,m$, $f = 50\,MHz$, $\hat{Z}_c = 80\,\Omega$, $\beta = 1.18\,rad/m$, $Z_L = 1500\,\Omega$, $V_R = 100\,V$

$$\tilde{I}_R = \frac{100\angle 0^\circ}{1500} = 0.067\angle 0^\circ\ A$$

$$\tilde{V}_S = (100\angle 0^\circ)\cos(1.18 \times 10) + j(80 \times 0.067\angle 0^\circ)\sin(1.18 \times 10)$$

$$\tilde{V}_S = 72.14\angle -2.94^\circ\ V$$

$$V_S = 72.14\,V\ (rms)$$

Problem 9.21

$$\rho_R = \frac{1500 - 80}{1500 + 80} = 0.899\angle 0^\circ$$

$$\sqrt{2}\ 72.14\angle -2.94^\circ = \hat{V}^+ \left(1 + 0.899\, e^{-j2\times 1.18\times 10} \right)$$

$$\hat{V}^+ = 57.91\angle -23.67^\circ \ \ V$$

$$V(z) = 57.91\sqrt{1 + 0.899^2 + 2\times 0.899 \cos\left[2\times 1.18\,(10 - z)\right]}$$

$$V(z) = 57.91\sqrt{1.808 + 1.798 \cos(23.6 - 2.36z)}$$

To determine the voltage peak $23.6 - 2.36z = 0 \Rightarrow z = 10\,m$
or $23.6 - 2.36z = 2\pi \Rightarrow z = 7.34\,m$

$$V_{max} = V(10\,m) = 57.91\sqrt{1.808 + 1.798} = 109.97\,V$$

This maximum occurs at every $\lambda/2 = \frac{\pi}{1.18} = 2.66\,m$

$$I(z) = \frac{57.91}{50}\sqrt{1 + 0.899^2 - 2\times 0.899 \cos\left[2\times 1.18\,(10 - z)\right]}$$

$$I(z) = 1.16\sqrt{1.808 - 1.798 \cos(23.6 - 2.36z)}$$

To determine the current peak $23.6 - 2.36z =$ $z = 8.67\,m$
or $23.6 - 2.36z = 3\pi$ $z = 6.0\,m$

$$I_{max} = I(8.67\,m) = 1.16\sqrt{1.808 + 1.798} = 2.2\,A$$

This maximum current occurs at every half wavelength,
i.e. $\frac{\lambda}{2} = \frac{\pi}{1.18} = 2.66\,m$

Problem 9.22

$Z_c = 75\,\Omega \quad , \quad VSWR = 2$

$$2 = \frac{1+\rho_R}{1-\rho_R} \Rightarrow \rho_R = \frac{1}{3}$$

$$Z_{max} = \frac{V_{max}}{I_{max}} = \frac{V^+(1+\rho_R)}{\frac{V^+(1-\rho_R)}{Z_c}} = Z_c \frac{1+\rho_R}{1-\rho_R} = 75 \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = 150\,\Omega$$

$$Z_{min} = \frac{V_{min}}{I_{min}} = \frac{V^+(1-\rho_R)}{\frac{V^+(1+\rho_R)}{Z_c}} = Z_c \frac{1-\rho_R}{1+\rho_R} = 75 \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = 37.5\,\Omega$$

Problem 9.23

$f = 10\,MHz \; , \; Z_c = 55\,\Omega \; , \; Z_L = (R_L = 20\,\Omega \,//\, C_L = 100\,pF)$

$$\hat{Z}_L = \frac{1}{\frac{1}{20} + j2\pi \times 10^7 \times 10^{-10}} = 19.69 - j2.47\,\Omega$$

$$\hat{\rho}_R = \frac{19.69 - j2.47 - 55}{19.69 - j2.47 + 55} = -0.471 - j0.049 \Rightarrow \rho_R = 0.474 \angle -3.039\,rad$$

$$\phi = -3.039\,rad$$

$$VSWR = \frac{1+0.474}{1-0.474} = 2.8$$

$2\beta(l - z) - \phi = 2n\pi \qquad n = 0, 1, 2, 3, \ldots\ldots$

For $n=0$: $2 \times 0.22(l-z) + 3.039 = 0 \Rightarrow l - z = -6.9\,m$

$n = 0$ is infeasible because $l - z < 0$

$n = 1$: $2 \times 0.22\,(l-z) + 3.039 = 2\pi$

$l - z = 7.37\,m$

Problem 9.24

$\hat{Z}_c = 50\,\Omega$, $\ell = 12\,m$, $u_p = 2.7\times10^8\,m/s$, $\hat{Z}_L = 150\,\Omega$

$v_G(t) = 25\cos(8\times10^5 t)$ V , $\hat{Z}_G = 10 - j5\,\Omega$

$\omega = 8\times10^5$ rad/s , $\beta = \dfrac{8\times10^5}{2.7\times10^8} = 2.96\times10^{-3}$ rad/m

a) $$\hat{Z}_{in}(0) = 50\,\frac{150 + j50\tan(2.96\times10^{-3}\times12)}{50 + j150\tan(2.96\times10^{-3}\times12)} = 148.5 - j14.05\,\Omega$$

$$\tilde{I}_s = \frac{25\angle 0^\circ}{10 - j5 + 148.5 - j14.05} = 0.157\angle 6.86^\circ \text{ A}$$

$$\tilde{V}_s = \hat{Z}_{in}(0)\,\tilde{I}_s = 23.359\angle 1.449^\circ \text{ V}$$

$$\hat{S}_s = \tilde{V}_s\,\tilde{I}_s^* = 3.642 - j0.345 \text{ VA}$$

$P_s = 3.642$ W , $Q_s = -0.345$ VAR

b) $\hat{Z}_{in}(12m) = \hat{Z}_L = 150\,\Omega$

$$\tilde{V}_R = (23.359\angle 1.449^\circ)\cos(2.96\times10^{-3}\times12) - j50\times(0.157\angle 6.86^\circ)\sin(2.96\times10^{-3}\times12)$$

$$\tilde{V}_R = 23.37\angle 0.77^\circ \text{ V}$$

$$\tilde{I}_R = \frac{V_R}{\hat{Z}_L} = \frac{23.37\angle 0.77^\circ}{150} = 0.156\angle 0.77^\circ \text{ A}$$

$$\hat{S}_R = \tilde{V}_R\,\tilde{I}_R^* = 3.642\,V$$

$P_R = 3.642$ W , $Q_R = 0$

c) $\hat{Z}_{in}(3m) = 50 \dfrac{150 + j50\tan[2.96\times10^{-3}(12-3)]}{50 + j150\tan[2.96\times10^{-3}(12-3)]} = 149.153 - j10.591\ \Omega$

$\tilde{V}(3m) = (23.359\angle 1.449^\circ)\cos[2.96\times10^{-3}(12-3)] - j\,50\times0.157\angle 6.855^\circ \sin(2.96\times10^{-3}\times9)$

$\tilde{V}(3m) = 23.368 + j0.383\ V$

$\tilde{I}(3m) = 23.35\angle 1.449^\circ (-j\frac{1}{50})\sin[2.96\times10^{-3}(12-3)]$

$+ 0.157\angle 6.855^\circ \cos(2.96\times10^{-3}(12-3))$

$\tilde{I}(3m) = 0.156 + j0.006\ A$

$\hat{S}(3m) = \tilde{V}(3m)\tilde{I}^*(3m) = 3.642 - j0.086\ VA$

$P(3m) = 3.642W$, $Q(3m) = -0.086\ VAR$

d) $\hat{Z}_{in}(9m) = 50 \dfrac{150 + j50\tan[2.96\times10^{-3}(12-9)]}{50 + j150\tan[2.96\times10^{-3}(12-9)]} = 149.905 - j3.55\ \Omega$

$\tilde{V}(9m) = 23.359 + j0.521\ V$

$\tilde{I}(9m) = 0.156 + j0.015\ A$

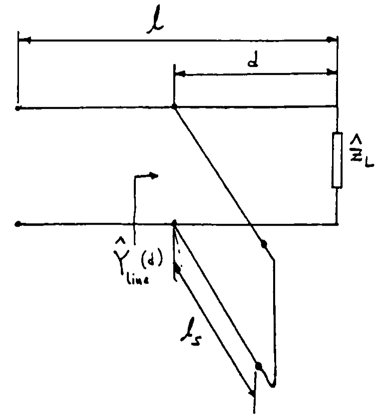
$\hat{S}(9m) = \tilde{V}(9m)\tilde{I}^*(9m) = 3.642 - j0.259\ VA$

$P(9m) = 3.642\ VA$, $Q = -0.259\ VAR$

e) $V_{drop} = V_S - V_R = 23.359 - 23.37 = -0.011V$

Problem 9.25

$R_c = 50\Omega$, $l = 100\text{m}$, $\hat{Z}_L = 40 - j100\Omega$, $t_d = 0.5\mu s$, $f = 20\text{MHz}$



$$u_p = \frac{100}{0.5 \times 10^{-6}} = 20 \times 10^{7}\ \text{m/s} \quad , \quad \beta = \frac{\omega}{u_p} = \frac{2\pi \times 20 \times 10^{6}}{2 \times 10^{8}} = 20\pi \times 10^{-2}\ \text{rad/m}$$

$\beta = 0.2\pi$ rad/m

$$\hat{Y}_{line}(d) = \frac{1}{50}\,\frac{50 + j(40 - j100)\tan(0.2\pi d)}{(40 - j100) + j50\tan(0.2\pi d)} = \frac{1}{50} + j\frac{1}{50\tan(0.2\pi l_s)}$$

Solving the above equation yields,

$d = 2.42\text{ m}$ or $d = 0.931\text{ m}$ and $l_s = 0.5\text{ m}$

Usually the shortest distance "d" to the load is preferred.

Problem 9.26

$l = 15\,m$, $f = 125\,MHz$, $\hat{Z}_L = 150 + j225\,\Omega$, $d_i = 2.5\,mm$, $d_o = 6\,mm$

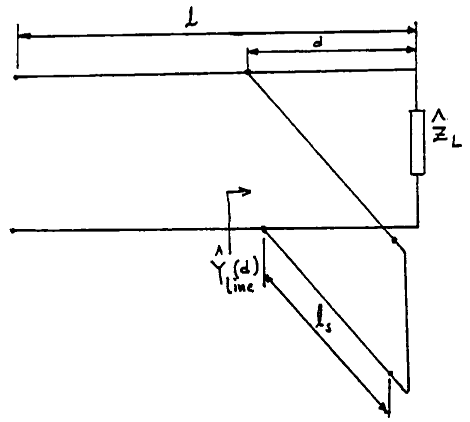
$\epsilon_r = 2.5$

$$L_l = \frac{4\pi \times 10^{-7}}{2\pi} \ln \frac{3}{1.25} = 1.75 \times 10^{-7}\ H/m$$

$$C_l = \frac{2\pi \times 2.5 \times 8.85 \times 10^{-12}}{\ln \frac{3}{1.25}} = 1.59 \times 10^{-10}\ F/m$$

$$Z_C = \sqrt{\frac{L_l}{C_l}} = \sqrt{\frac{1.75 \times 10^{-7}}{1.59 \times 10^{-10}}} = 33.18\,\Omega$$

$$\beta = 2\pi \times 125 \times 10^{6} \sqrt{1.75 \times 10^{-7} \times 1.59 \times 10^{-10}} = 4.14\ rad/m$$



$$\hat{Y}_{line}(d) = \frac{1}{33.18} \frac{33.18 + j(150 + j225)\tan(4.14d)}{(150 + j225) + j\,33.18\tan(4.14d)} = \frac{1}{33.18} + j\frac{1}{33.18\tan(4.14 l_s)}$$

Solving the above equation yields,

$d = 34.28\,cm$ and $l_s = 6.56\,cm$

Problem 9.27

$\ell = 100\,m$, $r = 5\,mm$, $L = 150\,\mu H$, $G = 0$, $C = 2000\,pF$

$L_\ell = \frac{150}{100} = 1.5\,\mu H/m$, $C_\ell = \frac{2000}{100} = 20\,pF/m$, $R_\ell(f) = \frac{1}{r}\sqrt{\frac{f\mu}{\pi\sigma}}$

$\hat{Z}_\ell(f) = R_\ell(f) + j\omega L_\ell$, $\hat{Y}_\ell(f) = j\omega C_\ell$

$$\hat{Z}_c = \sqrt{\frac{\hat{Z}_\ell(f)}{\hat{Y}_\ell(f)}} \quad , \quad \hat{\gamma} = \sqrt{\hat{Z}_\ell(f)\,\hat{Y}_\ell(f)} = \alpha(f) + j\beta(f)$$

$$u_p = \frac{\omega}{\beta(f)}$$

f [kHz]	R_ℓ [Ω]	$\hat{Z}_c$ [Ω]	$\hat{\gamma}$	α [Np/m]	β [rad/m]	u_p [m/s]
10	0.002	$273.872 - j\,2.413$	$3.032\times10^{-6} + j\,3.442\times10^{-4}$	3.032×10^{-6}	3.442×10^{-4}	1.83×10^{8}
10^2	0.005	$273.862 - j\,0.763$	$9.589\times10^{-6} + j\,0.003$	9.589×10^{-6}	0.003	1.83×10^{8}
10^3	0.017	$273.861 - j\,0.241$	$3.032\times10^{-5} + j\,0.034$	3.032×10^{-5}	0.034	1.83×10^{8}
10^4	0.053	$273.861 - j\,0.076$	$9.589\times10^{-5} + j\,0.344$	9.589×10^{-5}	0.344	1.83×10^{8}
10^5	0.166	$273.861 - j\,0.024$	$3.032\times10^{-4} + j\,3.44$	3.032×10^{-4}	3.44	1.83×10^{8}
10^6	0.525	$273.861 - j\,0.008$	$9.589\times10^{-4} + j\,34.414$	9.589×10^{-4}	34.414	1.83×10^{8}

Problem 9.28

$l = 100\,km$, $R_l = 34.63 \times 10^{-6}\,\Omega/m$, $L_l = 1500 \times 10^{-9}\,H/m$, $G=0$, $f = 60\,Hz$

$C_l = 55 \times 10^{-12}\,F/m$, $P_R = 100\,MW$, $\cos\theta = 0.9$ (lagging) , $V_R = 100{,}000\,V$

a) $I_R = \dfrac{100 \times 10^6}{10^5 \times 0.9} = 1111.11\,A$,

$$\hat{Z}_l = 34.63\times10^{-6} + j\,2\pi\times60\times1.5\times10^{-6} = 5.665\times10^{-4}\angle 86.5^\circ \;\Omega/m$$

$$\hat{Y}_l = j\,2\pi\times55\times10^{-12} = 2.073\times10^{-8}\angle 90^\circ \;S/m$$

$$\hat{Z}_c = \sqrt{\frac{\hat{Z}_l}{\hat{Y}_l}} = 165.3\angle -1.752^\circ\,\Omega \quad , \quad \hat{\gamma} = \sqrt{\hat{Z}_l\hat{Y}_l} = 1.048\times10^{-7} + j\,3.426\times10^{-6}$$

$$\tilde{V}_s = 100000\cosh\left[(1.048\times10^{-7}+j\,3.426\times10^{-6})\times100\times10^{3}\right] - 165.$$

$$-(165\angle -1.752^\circ)\times(1111.11\angle -25.84^\circ)\sinh\left[(1.048\times10^{-7}+j\,3.426\times10^{-6})\times100\times10^{3}\right]$$

$\tilde{V}_s = 1.093\times10^{5}\angle -34.097^\circ\;V$ or $\tilde{V}_s = 109.3\angle -34.097^\circ\;kV$

$$\tilde{I}_s = -\frac{1}{165\angle -1.752^\circ}\sinh(\hat{\gamma}l)\times100000 + (1111.11\angle -25.84^\circ)\cosh(\hat{\gamma}l)$$

$\tilde{I}_s = 1066\angle -10.785^\circ\;A$

$\hat{S}_s = \tilde{V}_s\tilde{I}_s^{*} = 1.07\times10^{8} - j\,4.609\times10^{7}\;VA$

$P_s = 107\,MW$ $Q_s = -46.09\,MVAR$

$\hat{S}_R = \tilde{V}_R\tilde{I}_R^{*} = 10^{8} + j\,4.843\times10^{7}\;VA$

$P_R = 100\,MW$, $Q_R = 48.43\,MVAR$

Efficiency: $\eta = \dfrac{P_R}{P_s} = \dfrac{100}{107}$

$\eta = 93.5\%$

Voltage regulation:

$\Delta V = \dfrac{|\tilde{V}_s| - |\tilde{V}_R|}{|\tilde{V}_R|} = \dfrac{109.3-100}{100}$

$\Delta V = 9.3\%$

Problem 9.29

$l = 200\,m$, $\hat{Z}_L = \hat{Z}_{in}^{dipole} = 74 + j\,42.5\,\Omega$, $f = 90\,MHz$

$R_l = 1.4\times10^{-3}\,\Omega/m$, $L_l = 220\times10^{-9}\,H/m$, $C_l = 177\times10^{-12}\,F/m$

$G_l = 0.1\,\mu S/m$

a)

$\omega = 2\pi f = 2\pi\times90\times10^{6} = 180\pi\times10^{6}\ rad/s$

$$\hat{Z}_C = \sqrt{\frac{1.4\times10^{-3} + j\,180\pi\times10^{6}\times220\times10^{-9}}{0.1\times10^{-6} + j\,180\pi\times10^{6}\times177\times10^{-12}}} = 35.255 - j\,1.808\times10^{-4}\ \Omega$$

$$\hat{\gamma} = \sqrt{(1.4\times10^{-3} + j\,180\pi\times10^{6}\times220\times10^{-9})(0.1\times10^{-6} + j\,180\pi\times10^{6}\times177\times10^{-12})}$$

$$\hat{\gamma} = 2.162\times10^{-5} + j\,3.529$$

$$\hat{Z}_{in}(0) = \hat{Z}_C\,\frac{\hat{Z}_L + \hat{Z}_C\tanh(\hat{\gamma}l)}{\hat{Z}_C + \hat{Z}_L\tanh(\hat{\gamma}l)} = 12.775 + j\,7.262\ \Omega$$

b) $\tilde{I}_s = 10\angle0^\circ\ A$

$$\tilde{V}_s = \hat{Z}_{in}(0)\,\tilde{I}_s = (12.775 + j\,7.262)(10\angle0^\circ) = 127.75 + j\,72.62\ V$$

$$\tilde{V}_s = 146.946\angle29.619^\circ\ V$$

$$\tilde{V}_R = \cosh(\hat{\gamma}l)\cdot\tilde{V}_s - \hat{Z}_C\tilde{I}_s\sinh(\hat{\gamma}l) \qquad \tilde{V}_R = 352.074\angle{-99.234^\circ}\ V$$

$$\tilde{I}_R = \frac{\tilde{V}_R}{\hat{Z}_L} = \frac{352.074\angle{-99.234^\circ}}{74 + j\,42.5} = -2.602 - j\,3.202\ A = 4.126\angle{-129.10^\circ}\ A$$

Power delivered to the cable: $\hat{S}_s = \tilde{V}_s\tilde{I}_s^{*} = 1277 + j\,726.239\ VA$

$P_s = 1277\,W$, $Q_s = 726.239\,VAR$

Power transmitted by the antenna: $\hat{S}_R = \tilde{V}_R\tilde{I}_R^{*} = 1260 + j\,723.423\ VA$

$P_R = 1260\,W$, $Q_R = 723.423\,VAR$

Problem 9.30

$l = 500\,m$, $\hat{Z}_c = 50\angle{-5^\circ}\ \Omega$, $\alpha = 50\times10^{-3}\ dB/m$, $f = 2.5\,MHz$

$u_p = 2.3\times10^5\ km/s$, $\hat{Z}_L = 200 - j300\ \Omega$ $\tilde{V}_s = 20\angle{0^\circ}\ V$

$$\alpha = \frac{50\times10^{-3}}{8.69} = 0.0058\ Np/m \qquad \beta = \frac{2\pi\times2.5\times10^6}{2.3\times10^8} = 0.0683\ rad/m$$

$$\hat{\gamma} = \alpha + j\beta = 0.0058 + j0.0683 = 0.0685\angle{85.15^\circ}$$

$$\hat{Z}_{in}(0) = (50\angle{-5^\circ})\,\frac{(200-j300) + (50\angle{-5^\circ})\tanh[(0.0685\angle{85.15^\circ})\times500]}{(50\angle{-5^\circ}) + (200-j300)\tanh[(0.0685\angle{85.15^\circ})\times500]}$$

$$\hat{Z}_{in}(0) = 50.211\angle{-4.834^\circ}\ \Omega$$

$$\tilde{I}_s = \frac{\tilde{V}_s}{\hat{Z}_{in}(0)} = \frac{20\angle{0^\circ}}{50.211\angle{-4.834^\circ}} = 0.398\angle{0^\circ}\ A$$

$$\tilde{I}_R = -\frac{1}{50\angle{-5^\circ}}(20\angle{0^\circ})\sinh[(0.0685\angle{85.15^\circ})\times500]$$

$$+(0.398\angle{0^\circ})\cosh[(0.0685\angle{85.15^\circ})\times500]$$

$$\tilde{I}_R = 0.006\angle{-105.983^\circ}\ A$$

$$\tilde{V}_R = \hat{Z}_L\tilde{I}_R = 2.011\angle{-162.293^\circ}\ V$$

Power delivered to the load:

$$\hat{S}_R = \tilde{V}_R\tilde{I}_R^* = 0.006 - j0.009\ VA$$

$$P_R = 6\,mW\ ,\quad Q_R = 9\,mVAR$$

Problem 9.32

$l = 50\,m$, $\hat{Z}_c = 40\angle{-5^\circ}\ \Omega$, $\hat{Z}_L = 280\ \Omega$, $v_G(t) = 20\cos(6\times10^6 t)\ V$

$v_s(t) = 18\cos(6\times10^6 t - 12^\circ)\ V$, $\hat{Z}_G = 30 + j40\ \Omega$

a) $\tilde{V}_s = 18\angle{-12^\circ}\ V$, $\tilde{V}_G = 20\angle{0^\circ}\ V$

$$\tilde{V}_s = \frac{\tilde{V}_G}{\hat{Z}_G + \hat{Z}_{in}(0)}\hat{Z}_{in}(0) \Rightarrow \hat{Z}_{in}(0) = 194.485 - j56.76\ \Omega$$

b)
The propagation constant:

From the input-impedance, at $z=0$,

$$\tanh(\hat{\gamma}l) = \frac{\hat{Z}_c[\hat{Z}_L - \hat{Z}_{in}(0)]}{\hat{Z}_{in}(0)\hat{Z}_L - \hat{Z}_c^2} = \frac{(40\angle{-5^\circ})[280 - (194.485 - j56.76)]}{(194.485 - j56.76)(280) - 40^2}$$

$$\tanh(\hat{\gamma}l) = 0.053 + j0.053 \Rightarrow \hat{\gamma}l = 0.053 + j0.053$$

$$\hat{\gamma} = \frac{\hat{\gamma}l}{l} = 1.06\times10^{-3} + j1.06\times10^{-3}$$

c) Reflection coefficient:

$$\hat{\rho}_R = \frac{\hat{Z}_L - \hat{Z}_c}{\hat{Z}_L + \hat{Z}_c} = \frac{280 - 40\angle{-5^\circ}}{280 + 40\angle{-5^\circ}} = 0.751\angle{1.456^\circ}$$

$$\tilde{I}_s = \frac{\tilde{V}_s}{\hat{Z}_{in}(0)} = \frac{18\angle{-12^\circ}}{194.485 - j56.76} = 0.089\angle{4.27^\circ}\ A$$

$$\tilde{V}_R = \cosh(\hat{\gamma}l)\cdot\tilde{V}_s - \hat{Z}_c\tilde{I}_s\sinh(\hat{\gamma}l) \Rightarrow \tilde{V}_R = 17.854\angle{-12.548^\circ}\ V$$

$$\tilde{I}_R = \frac{\tilde{V}_R}{\hat{Z}_L} = \frac{17.854\angle{-12.548^\circ}}{280} = 0.064\angle{-12.548^\circ}\ A$$

$v_R(t) = 17.854 \cos(6\times10^6 t - 12.548^\circ) \; V$

$i_R(t) = 0.064 \cos(6\times10^6 t - 12.548^\circ) \; A$

d) Average power due to forward wave:

$$\hat{V}^+ = \frac{\tilde{V}_S + \hat{Z}_C \tilde{I}_S}{2} = 10.748 \underline{/-10.148^\circ} \; V$$

$$P^+(z) = \frac{1}{2}\frac{(V^+)^2}{Z_c} e^{-2\alpha z} \cos\theta_{zc}$$

$$P^+(z) = \frac{1}{2}\frac{(10.748)^2}{40} e^{-2\times1.05\times10^{-3}z} \cos 5^\circ$$

$$P^+(z) = 1.438\, e^{-2.10\times10^{-3}z} \quad W$$

Average power due to backward wave:

$$\hat{V}^- = \tilde{V}_S - \hat{V}^+ = 7.266 \underline{/-14.74^\circ} \; V$$

$$P^-(z) = -\frac{1}{2}\frac{(V^-)^2}{Z_c} e^{2\alpha z} \cos\theta_{zc}$$

$$P^-(z) = -\frac{1}{2}\frac{(7.266)^2}{40} e^{2\times1.05\times10^{-3}z} \cos 5^\circ$$

$$P^-(z) = -0.657\, e^{2.10\times10^{-3}z} \quad W$$

e) $P_S = P^+(0) + P^-(0) = 1.438 - 0.657 = 0.781 \, W$

$P_R = P^+(50m) + P^-(50) = 1.295 - 0.729 = 0.566 \, W$

$$\eta = \frac{P_R}{P_S} = \frac{0.566}{0.781} = 0.7247 \quad \Rightarrow \quad \eta = 72.47\%$$

Problem 9.33

$\ell = 25m, \quad L_\ell = 0.4\,\mu H/m, \quad C_\ell = 45\,pF/m, \quad R_\ell = 8\,\Omega/m, \quad G_\ell = 0$

$v_s(t) = 60\cos(7\times10^6 t)$ V $, \quad \hat{Z}_L = 160\,\Omega$

$$\hat{Z}_c = \sqrt{\frac{8 + j7\times10^6\times0.4\times10^{-6}}{j7\times10^6\times45\times10^{-12}}} = 164.035\angle\ \ ^\circ\ \Omega$$

$$\hat{\gamma} = \sqrt{(8 + j7\times10^6\times0.4\times10^{-6})(j7\times10^6\times45\times10^{-12})} = 0.03 + j0.042$$

a) $$\hat{\rho}_R = \frac{\hat{Z}_L - \hat{Z}_c}{\hat{Z}_L + \hat{Z}_c} = \frac{160 - 164.035\angle{-21.81^\circ}}{160 + 164.035\angle{-21.81^\circ}} = 0.319\angle\overset{1.614\,rad}{92.48^\circ}$$

$$\hat{\rho}(z) = \rho_R\, e^{-[2\hat{\gamma}(\ell-z) - j\phi_R]}$$

$$\hat{\rho}(z) = 0.319\, e^{-[2(0.03+j0.042)(\ell-z) - j1.614]}$$

$$\hat{\rho}(z) = 0.319\, e^{-0.06(25-z)}\, e^{-j[0.084(25-z) - 1.614]}$$

b) $$\hat{Z}_{in}(0) = (164.035\angle{-21.81^\circ})\frac{160 + (164.035\angle{-21.81^\circ})\tanh[(0.03+j0.042)\times25]}{(164.035\angle{-21.81^\circ}) + 160\tanh[(0.03+j0.042)\times25]}$$

$$\hat{Z}_{in}(0) = 144.09 - j117.73\ \Omega$$

$$\tilde{I}_s = \frac{\tilde{V}_s}{\hat{Z}_{in}(0)} = \frac{60\angle0^\circ}{144.09 - j117.73} = 0.322\angle39.25^\circ\ A$$

$$\left.\begin{aligned}\tilde{V}_s &= \hat{V}^+ + \hat{V}^- \\ \tilde{I}_s &= \frac{\hat{V}^+}{\hat{Z}_c} - \frac{\hat{V}^-}{\hat{Z}_c}\end{aligned}\right\} \Rightarrow \begin{aligned}\hat{V}^+ &= 56.415\angle\overset{0.032\,rad}{1.83^\circ}\ V \\ \hat{V}^- &= 4.035\angle\overset{-0.461\,rad}{-26.438^\circ}\ V\end{aligned}$$

Forward voltage wave :

$$\tilde{V}_f(z) = 56.415\, e^{-(0.03+j0.042)z} \cdot e^{j0.032} = 56.415\, e^{-0.03z} \cdot e^{-j(0.042z-0.032)}$$

Backward voltage wave :

$$\tilde{V}_b(z) = 4.035\, e^{(0.03+j0.042)z} \cdot e^{-j0.461} = 4.035\, e^{0.03z} \cdot e^{j(0.042z-0.461)}$$

Forward current wave :

$$\tilde{I}_f(z) = 0.344\, e^{j0.032} \cdot e^{j0.617}\, e^{-(0.03+j0.042)z} = 0.344 e^{-0.03z} \cdot e^{j(0.649-0.042z)}$$

Backward current wave :

$$\tilde{I}_b(z) = -0.0246\, e^{0.03z} \cdot e^{j(0.649+0.042z)}$$

c) Average power due to forward wave:

$$P^+(z) = \frac{1}{2}\frac{(V^+)^2}{Z_c} e^{-2\alpha z} \cos\theta_{Z_c}$$

$$P^+(z) = \frac{1}{2}\frac{56.415^2}{164.035} e^{-2\times 0.03z} \cos 35.35^\circ$$

$$P^+(z) = 7.912\, e^{-0.06z}$$

Average power due to backward wave:

$$P^-(z) = -\frac{1}{2}\frac{(V^-)^2}{Z_c} e^{2\alpha z} \cos\theta_{Z_c}$$

$$P^-(z) = -\frac{1}{2}\frac{4.035^2}{164.035} e^{2\times 0.03z} \cos 35.35^\circ$$

$$P^-(z) = -0.0405\, e^{0.06z}$$

d) $P_s = 7.912 - 0.0405 = 7.87\,W$

$P_R = 7.912 e^{-0.06\times 25} - 0.0405 e^{0.06\times 25} = 1.58W$

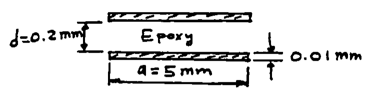
$\eta = \frac{1.58}{7.87} = 0.201 \quad \eta = 20.1\%$

e) $\tilde{V}_R = 56.415\, e^{-0.03\times 25}\, e^{-j(0.042\times 25 - 0.032)} + 4.035\, e^{0.03\times 25}\, e^{j(0.042\times 25 - 0.461)}$

$\tilde{V}_R = 27.688 \angle{-40.39^\circ}$ V

Voltage drop : $\Delta V = 60 - 27.688 = 32.31$ V

Problem 9.34



$l = 10\text{cm}$, $f = 100\text{MHz}$

$\epsilon_{epoxy} = 3.5\epsilon_0 \qquad \mu = \mu_0 \qquad \sigma_{cu} = 5.8\times 10^{7}\ \text{S/m}$

$$R_l = \frac{2}{a}\sqrt{\frac{\mu f \pi}{\sigma}} = \frac{2}{5\times 10^{-3}}\sqrt{\frac{4\pi\times 10^{-7}\times 100\times 10^{6}\,\pi}{5.8\times 10^{7}}} = 1.044\ \Omega/\text{m}$$

$$L_l = \frac{\mu d}{a} = \frac{4\pi\times 10^{-7}\times 0.2\times 10^{-3}}{5\times 10^{-3}} = 5.0265\times 10^{-8}\ \text{H/m}$$

$$C_l = \frac{\epsilon a}{d} = \frac{3.5\times 8.85\times 10^{-12}\times 5\times 10^{-3}}{0.2\times 10^{-3}} = 7.744\times 10^{-10}\ \text{F/m}$$

a) $$\hat{Z}_c = \sqrt{\frac{1.044 + j2\pi\times 10^{8}\times 5.0265\times 10^{-8}}{j2\pi\times 10^{8}\times 7.744\times 10^{-10}}} = 8.059 \angle{-0.947^\circ}\ \Omega$$

b) $$\hat{\gamma} = \sqrt{(1.044 + j2\pi\times 10^{8}\times 5.0265\times 10^{-8})(j2\pi\times 10^{8}\times 7.744\times 10^{-10})}$$

$\hat{\gamma} = 0.065 + j3.921 \quad 1/\text{m}$

c) $$\beta = \frac{\omega}{u_p} \Rightarrow u_p = \frac{\omega}{\beta} = \frac{2\pi\times 10^{8}}{3.921} = 1.602\times 10^{8}\ \text{m/s}$$

Problem 9.35

$v_s = 5\cos(6.28\times10^8 t)$, $\tilde{I}_R = 5\times10^{-3}\angle 0^\circ$ A

$\tilde{V}_s = 5\angle 0^\circ$ V

$$\tilde{I}_s = \frac{1}{\hat{Z}_c}\sinh\hat{\gamma}l \cdot \tilde{V}_R + \cosh\hat{\gamma}l \cdot \tilde{I}_R \qquad (1)$$

$$\tilde{V}_R = \cosh\hat{\gamma}l \cdot \tilde{V}_s - \hat{Z}_c \sinh\hat{\gamma}l \cdot \tilde{I}_s \qquad (2)$$

Substituting (2) in (1) yields

$$\tilde{I}_s = \frac{\frac{1}{\hat{Z}_c}\sinh\hat{\gamma}l\,\cosh\hat{\gamma}l \cdot \tilde{V}_s + \cosh\hat{\gamma}l \cdot \tilde{I}_R}{1+\sinh^2\hat{\gamma}l} \qquad (3)$$

$\tilde{I}_s = 0.604\angle 5.328^\circ$ A

$\hat{S}_s = \frac{1}{2}\tilde{V}_s\,\tilde{I}_s^* = 1.503 - j\,0.14$ VA

$P_s = 1.503$ W , $Q_s = -0.14$ VAR

Problem 9.36

$l = 2\,m$, $r_i = 1\,mm$, $r_o = 5\,mm$, $v_s(t) = 10\cos(5\times10^{10} t)$ V

$i_R(t) = 0.5\cos(5\times10^{10} t - 10^\circ)$ mA

$\sigma = 5.8\times10^7$ S/m , $\epsilon = 2.5\,\epsilon_o$, $\mu = \mu_o$

$$R_l = \frac{1}{2}\sqrt{\frac{f\mu}{\pi\sigma}}\left(\frac{1}{r_i}+\frac{1}{r_o}\right) = 4.445\,\Omega$$

$$L_l = \frac{\mu_o}{2\pi}\ln\frac{r_o}{r_i} = 3.219\times10^{-7}\,H/m, \quad C_l = \frac{2\pi\epsilon}{\ln\frac{r_o}{r_i}} = 8.638\times10^{-11}\,F/m$$

$$\hat{Z}_c = \sqrt{\frac{4.445 + j\,5\times10^{10}\times 3.219\times10^{-7}}{j\,8.638\times10^{-11}\times 5\times10^{10}}} = 61.046 \angle{-0.0075^\circ}\ \Omega$$

$$\hat{\gamma} = \sqrt{(4.445 + j\,5\times10^{10}\times 3.219\times10^{-7})(j\,8.638\times10^{-11}\times 5\times10^{10})}$$

$$\hat{\gamma} = 0.036 + j\,263.643 \quad 1/m$$

From (3) of Problem 9.35,

$$\tilde{I}_s = 0.091 \angle{-79.812^\circ} \ A$$

$$\hat{S}_s = \frac{1}{2}\tilde{V}_s\tilde{I}_s^* = 0.08 + j\,0.446 \quad VA$$

$$P_s = 0.08\,W$$

$$\tilde{V}_R = \cosh(\hat{\gamma}l)\cdot\tilde{V}_s - \hat{Z}_c \sinh\hat{\gamma}l \cdot \tilde{I}_s$$

$$\tilde{V}_R = 11.364 \angle{2.365^\circ} \ V$$

$$\hat{S}_R = \frac{1}{2}\tilde{V}_R\tilde{I}_R^* = 0.003 + j\,6.084\times10^{-4} \ VA$$

$$P_R = 0.003\,W, \quad Q_R = 6.084\times10^{-4}\ VAR$$

$$P_{loss} = P_s - P_R = 0.077\ W$$

Problem 9.37

$\ell = 350\ \text{km}$, $P_R = 150\text{MW}$, $V_R = 300\text{kV}$, $\cos\theta = 1$ $f = 60\text{Hz}$

$R_\ell = 0.1\Omega/\text{km}$, $L_\ell = 1.5\ \text{mH/km}$, $C_\ell = 7.9\ \text{nF/km}$, $G_\ell = 0$

a) $$\hat{Z}_\ell = \sqrt{\frac{0.1 + j\,2\pi \times 60 \times 1.5 \times 10^{-3}}{j\,2\pi \times 60 \times 7.9 \times 10^{-9}}} = 439.11 \angle{-5.01^\circ}\ \Omega$$

b) $$\hat{\gamma} = \sqrt{(0.1 + j\,2\pi \times 60 \times 1.5 \times 10^{-3})(j\,2\pi \times 60 \times 7.9 \times 10^{-9})}$$

$$\hat{\gamma} = 1.143 \times 10^{-4} + j\,1.303 \times 10^{-3}\ \ 1/\text{km}$$

c) $$u_p = \frac{\omega}{\beta} = \frac{2\pi \times 60}{1.303 \times 10^{-3}} = 2.893 \times 10^{5}\ \text{km/s}$$

d) $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{1.303 \times 10^{-3}} = 4822.09\ \text{km}$$

e) At $\ell = 350\,\text{km}$, $V_R = 300{,}000\ \text{V}$, $I_R = \dfrac{150 \times 10^6}{300 \times 10^3} = 500\,\text{A}$

$$300 \times 10^3 \angle{0^\circ} = \hat{V}^+ e^{-\hat{\gamma}350} + \hat{V}^- \cdot e^{\hat{\gamma}350} \qquad (1)$$

$$500 \angle{0^\circ} = \frac{\hat{V}^+ e^{-\hat{\gamma}350}}{439.11\angle{-5.01^\circ}} - \frac{\hat{V}^- e^{\hat{\gamma}350}}{439.11\angle{-5.01^\circ}} \qquad (2)$$

Solving (1) and (2) simultaneously yields

$$\hat{V}^+ = 2.701 \times 10^5 \angle{\overset{0.419\ \text{rad}}{24.007^\circ}}\ V, \quad \hat{V}^- = 4.012 \times 10^4 \angle{\overset{0.224\ \text{rad}}{-12.843^\circ}}\ V$$

$$\tilde{V}_f = 2.701 \times 10^5\ e^{j0.419}\ e^{-(1.143 \times 10^{-4} + j1.303 \times 10^{-3})z}$$

$$\tilde{V}_f = 2.701 \times 10^5\ e^{-1.143 \times 10^{-4} z} \cdot e^{-j(1.303 \times 10^{-3} z - 0.419)}\ \text{V}$$

where z is in km.

$$\tilde{V}_b = 4.012\times10^4\, e^{-j0.224}\, e^{(1.143\times10^{-4}+j1.303\times10^{-3})z}$$

$$\tilde{V}_b = 4.012\times10^4\, e^{1.143\times10^{-4}z}\, e^{j(1.303\times10^{-3}z-0.224)} \quad V$$

where z is in km.

f) $\tilde{V}_s = \tilde{V}_f(0) + \tilde{V}_b(0)$

$$\tilde{V}_s = 2.701\times10^5 \angle 24.007^\circ + 4.012 \angle -12.843^\circ$$

$$\tilde{V}_s = 3.032\times10^5 \angle 19.455^\circ \quad V \quad \Rightarrow \quad 303.2 \angle 19.455^\circ \ kV$$

g) $\tilde{I}_s = \dfrac{\tilde{V}_f(0)}{\hat{Z}_c} - \dfrac{\tilde{V}_b(0)}{\hat{Z}_c}$

$$\tilde{I}_s = 544.818 \angle 34.794^\circ \quad A$$

$$\hat{S}_s = \tilde{V}_s \tilde{I}_s^*$$

$$\hat{S}_s = 1.593\times10^8 - j4.369\times10^7 \quad VA$$

$P_s = 159.3$ MW , $Q_s = -43.69$ MVAR

h) $\eta = \dfrac{P_R}{P_s}$

$$\eta = \frac{150}{159.3} = 0.942$$

$$\eta = 94.2\ \%$$

Problem 9.38

For a distortionless line $R_\ell C_\ell = L_\ell G_\ell$ (1) .

$$\hat{Z}_c = \sqrt{\frac{R_\ell + j\omega L_\ell}{G_\ell + j\omega C_\ell}} = \sqrt{\left(\frac{L_\ell}{C_\ell}\right)\left(\frac{\frac{R_\ell}{L_\ell} + j\omega}{\frac{G_\ell}{C_\ell} + j\omega}\right)}$$

For a distortionless line using (1) yields

$$\hat{Z}_c = \sqrt{\frac{L_\ell}{C_\ell}}$$

$$\hat{\gamma} = \sqrt{(R_\ell + j\omega L_\ell)(G_\ell + j\omega C_\ell)} = \sqrt{\left(\sqrt{R_\ell^2 + \omega^2 L_\ell^2}\,\underline{/\tan^{-1}\frac{\omega L_\ell}{R_\ell}}\right)\left(\sqrt{G_\ell^2 + \omega^2 C_\ell^2}\,\underline{/\tan^{-1}\frac{\omega C_\ell}{G_\ell}}\right)}$$

$$\hat{\gamma} = \sqrt{\sqrt{(R_\ell^2 + \omega^2 L_\ell^2)(G_\ell^2 + \omega^2 C_\ell^2)}}\ \underline{/\tan^{-1}\frac{\omega L_\ell}{R_\ell}}$$ for a distortionless line.

$$\alpha = \sqrt{\sqrt{(R_\ell^2 + \omega^2 L_\ell^2)(G_\ell^2 + \omega^2 C_\ell^2)\frac{R_\ell^4}{(R_\ell^2 + \omega^2 L_\ell^2)^2}}} = \sqrt{\sqrt{\frac{R_\ell^4 G_\ell^2}{R_\ell^2}\,\frac{1 + \omega^2\frac{C_\ell^2}{G_\ell^2}}{1 + \omega^2\frac{L_\ell^2}{R_\ell^2}}}}$$

For a distortionless line using (1) yields, $\alpha = \sqrt{R_\ell G_\ell}$

$$\beta = \sqrt{\sqrt{(R_\ell^2 + \omega^2 L_\ell^2)(G_\ell^2 + \omega^2 L_\ell^2)\frac{\omega^4 L_\ell^4}{(R_\ell^2 + \omega^2 L_\ell^2)^2}}} = \sqrt{\sqrt{\frac{\omega^4 L_\ell^4\,\omega^2 C_\ell^2}{\omega^2 L_\ell^2}\,\frac{\frac{G_\ell^2}{\omega^2 C_\ell^2} + 1}{\frac{R_\ell^2}{\omega^2 L_\ell^2} + 1}}}\,.$$

For a distortionless line using (1) yields,

$$\beta = \omega\sqrt{L_\ell C_\ell}$$

Problem 9.39

$L_\ell = 0.4\mu H/m$, $C_\ell = 86 pF/m$, $R_\ell = 11 m\Omega/m$, $f = 95 MHz$

$$\hat{Z}_c = \sqrt{\frac{L_\ell}{C_\ell}} = \sqrt{\frac{0.4\times10^{-6}}{86\times10^{-12}}} = 68.2\ \Omega$$

$$\alpha = \sqrt{R_\ell G_\ell} \qquad G_\ell = \frac{R_\ell C_\ell}{L_\ell} = \frac{11\times10^{-3}\times 86\times10^{-12}}{0.4\times10^{-6}} = 2.365\times10^{-6} S/m$$

$$\alpha = \sqrt{11\times10^{-3}\times 2.365\times10^{-6}} = 2\times10^{-4}\ Np/m$$

$$\beta = 2\pi\times 95\times10^{6}\sqrt{0.4\times10^{-6}\times 86\times10^{-12}} = 3.5\ rad/m$$

$$\hat{\gamma} = 2\times10^{-4} + j\,3.5$$

$$u_p = \frac{2\pi\times 95\times10^{6}}{3.5} = 1.705\times10^{8}\ m/s$$

Problem 9.40

$\ell = 20 m$, $\hat{Z}_c = 75\Omega$ $\quad t_d = 90 ns$ $\quad (\alpha\ell)_{dB} = 0.1\ dB$

$$\alpha\ell = \frac{0.1}{8.69} = 0.0115\ Np \qquad \alpha = \frac{0.0115}{20} = 5.75\times10^{-4}\ Np/m$$

$$t_d = 90 ns \qquad u_p = \frac{20}{90\times10^{-9}} = 2.22\times10^{8}\ m/s$$

$$\frac{1}{\sqrt{L_\ell C_\ell}} = 2.22\times10^{8} \qquad 75 = \sqrt{\frac{L_\ell}{C_\ell}} \quad \Rightarrow \begin{array}{l} L_\ell = 3.378\times10^{-7}\ H/m \\ C_\ell = 6\times10^{-11}\ F/m \end{array}$$

$$R_\ell C_\ell = G_\ell L_\ell \Rightarrow G_\ell = \frac{R_\ell C_\ell}{L_\ell} \ , \quad \alpha^2 = R_\ell G_\ell \ , \quad R_\ell = \sqrt{\frac{\alpha^2 L_\ell}{C_\ell}} = \alpha\hat{Z}_c$$

$$R_\ell = 5.75\times10^{-4}\times 75 = 0.0432\ \Omega/m, \quad G_\ell = \frac{0.0432\times6\times10^{-11}}{3.378\times10^{-7}} = 7.67\times10^{-6} S/m$$

Problem 9.41

$f = 100\text{kHz}$, $r_i = 1.5\text{mm}$, $r_o = 3\text{mm}$, $\sigma = 5.8\times10^{7}\,S/m$, $\epsilon = 2.2\epsilon_o$

$$L_l = \frac{4\pi\times10^{-7}}{2\pi}\ln\frac{3}{1.5} = 1.39\times10^{-7}\ H/m$$

$$C_l = \frac{2\pi\times2.2\times8.85\times10^{-12}}{\ln\frac{3}{1.5}} = 8.82\times10^{-11}\,F/m$$

$$R_l = \frac{1}{2}\sqrt{\frac{100\times10^{3}\times4\pi\times10^{-7}}{\pi\times5.8\times10^{7}}}\left(\frac{1}{1.5\times10^{-3}}+\frac{1}{3\times10^{-3}}\right) = 1.313\times10^{-2}\,\Omega/m$$

$$G_l = \frac{1.313\times10^{-2}\times8.82\times10^{-11}}{1.39\times10^{-7}} = 8.33\times10^{-6}\ S/m$$

$$\hat{Z}_c = \sqrt{\frac{1.39\times10^{-7}}{8.02\times10^{-11}}} = 41.63\,\Omega$$

$$\alpha = \sqrt{R_l G_l} = \sqrt{1.313\times10^{-2}\times8.33\times10^{-6}} = 3.31\times10^{-4}\ Np/m$$

$$\beta = 2\pi\times100\times10^{3}\sqrt{1.39\times10^{-7}\times8.82\times10^{-11}} = 2.199\times10^{-3}\,rad/m$$

$$u_p = \frac{100\times10^{3}\times2\pi}{2.199\times10^{-3}} = 2.856\times10^{8}\ m/s$$

Problem 9.42

$l = 10m$, $L = 2\mu H$, $C = 2000pF$, $R_G = 10\Omega$, $R_L = 100\Omega$

$V_s = 10V$ $t_p = 1ns$

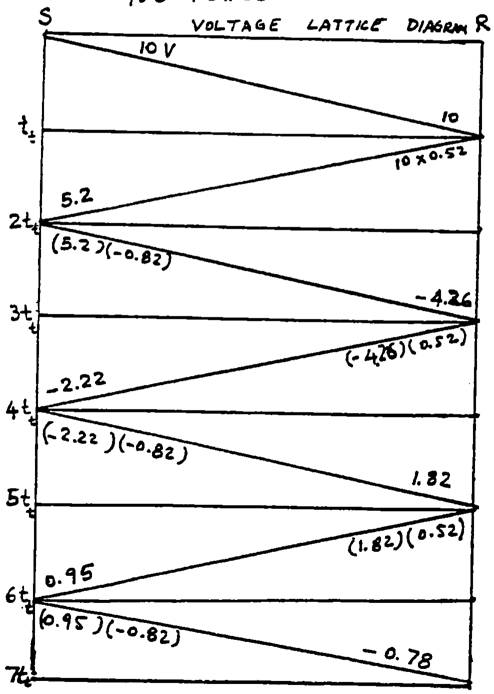
a) $L_l = \frac{2}{10} = 0.2\mu H/m$, $C_l = \frac{2000}{10} = 200pF/m$

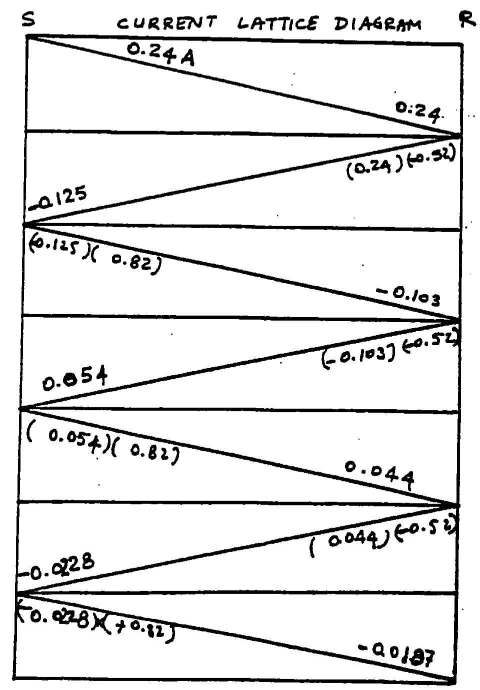
$$R_c = \sqrt{\frac{0.2\times10^{-6}}{200\times10^{-12}}} = 31.63\Omega \qquad I_s = \frac{10}{10+31.63} = 0.24A$$

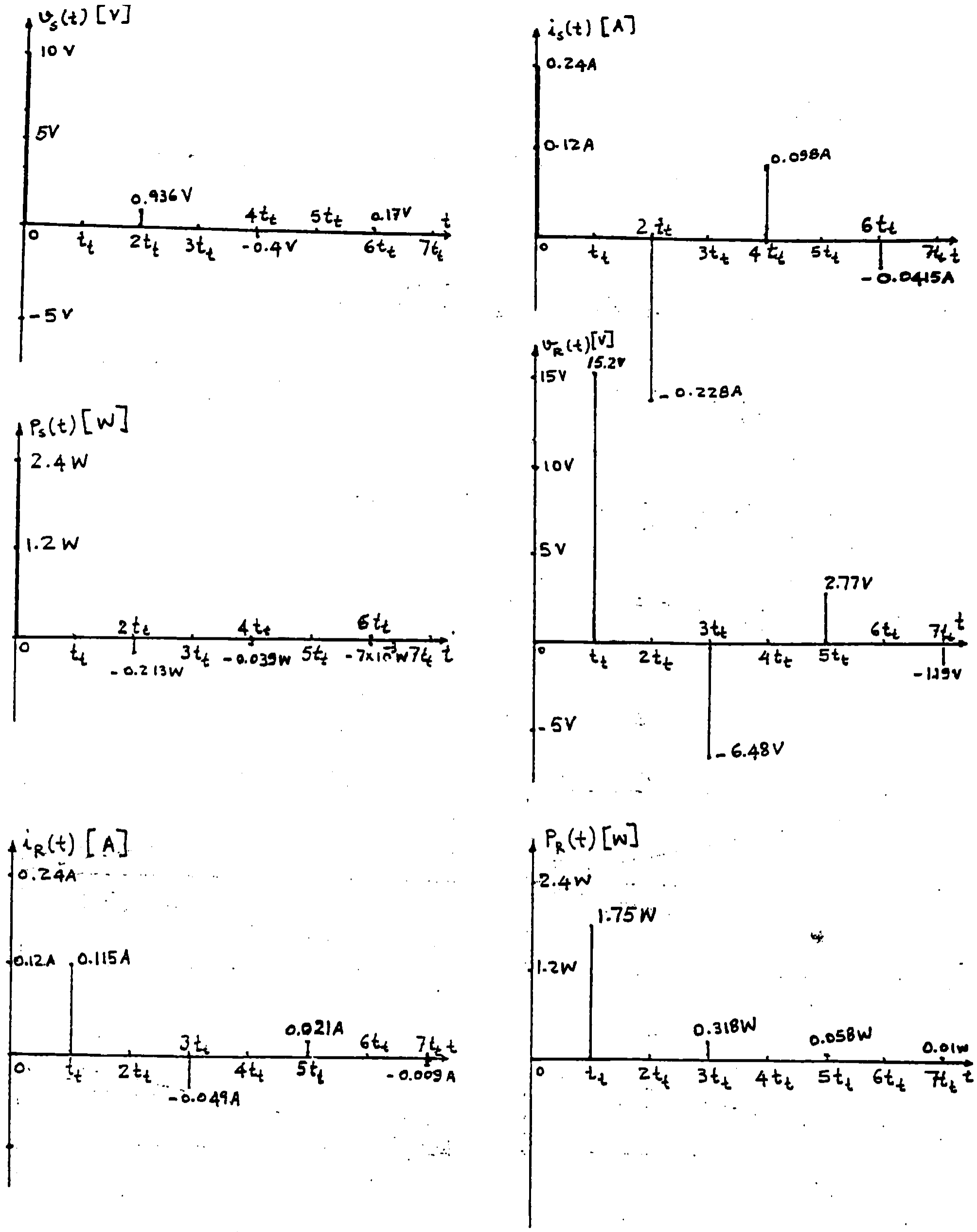
$$u_p = \frac{1}{\sqrt{0.2\times10^{-6}\times200\times10^{-12}}} = 1.58\times10^{8}\ m/s$$

$$t_t = \frac{10}{1.58\times10^{8}} = 6.32\times10^{-8}\ s \qquad t_t = 63.2ns$$

$$\rho_R = \frac{100-31.63}{100+31.63} = 0.52, \quad \rho_s = \frac{10-100}{10+100} = -0.82$$







vs(t) [V]
10 V
5V
-5 V
0.936 V
0.17V
-0.4V
is(t) [A]
0.24A
0.12A
0.098A
-0.228A
-0.0415A
Ps(t) [W]
2.4 W
1.2 W
-0.213W
-0.039W
vR(t) [V]
15.2V
15V
10V
5V
2.77V
-5V
-6.48V
-1.19V
iR(t) [A]
0.24A
0.12A
0.115A
0.021A
-0.049A
-0.009 A
PR(t) [W]
2.4W
1.75W
1.2W
0.318W
0.058W
0.01W

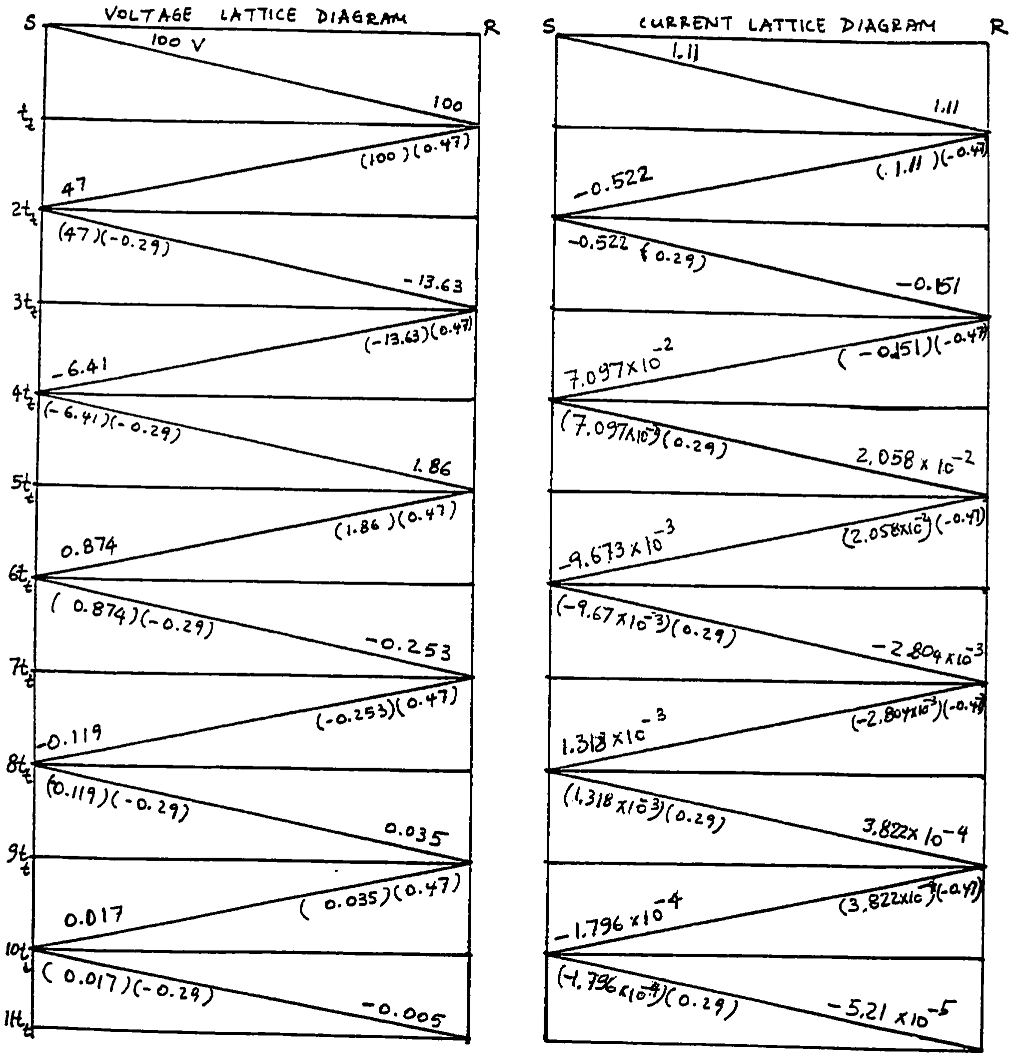
Problem 9.43

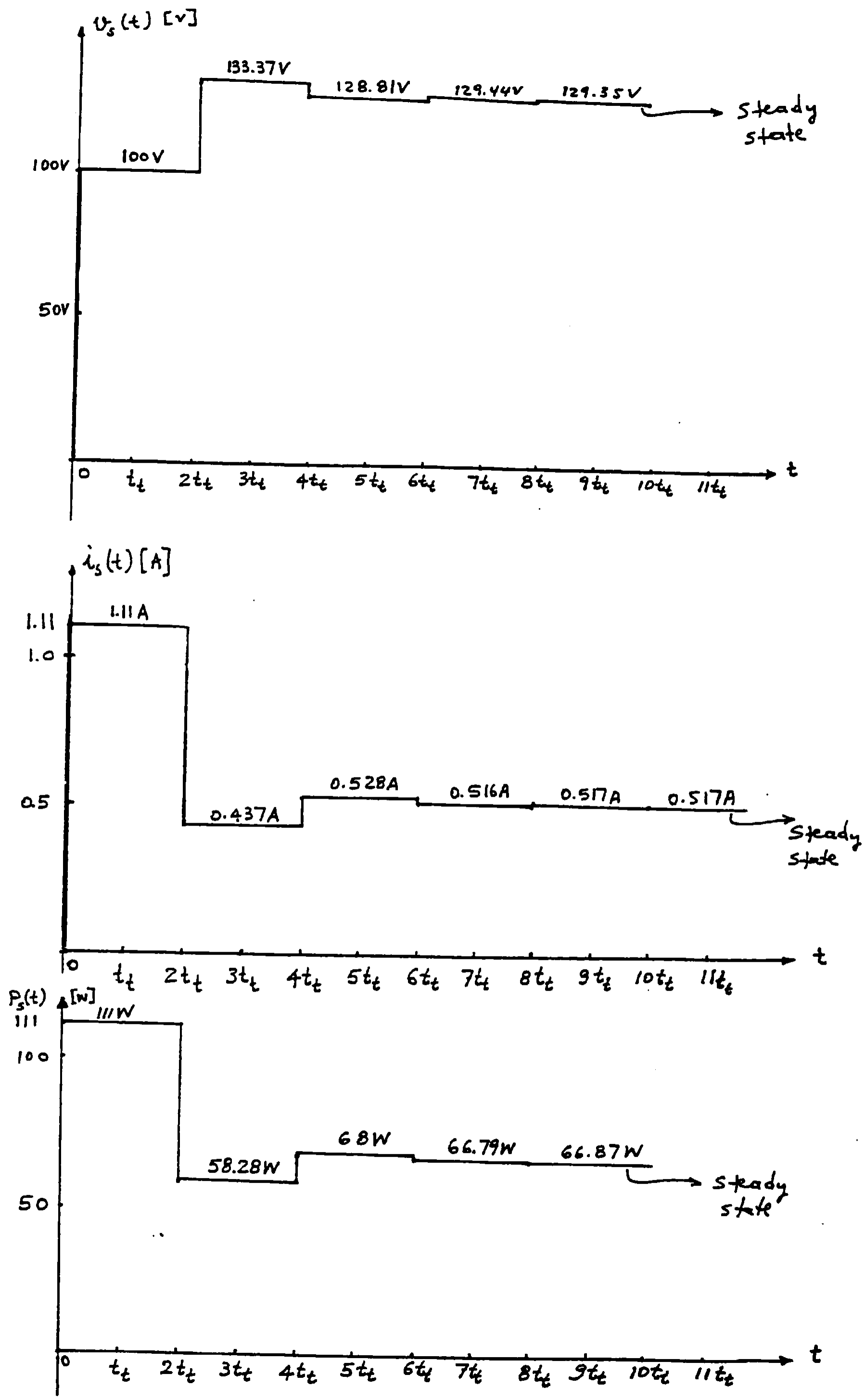
$V_s = 100\,V$, $l = 100\,m$, $R_c = 90\,\Omega$, $R_G = 50\,\Omega$, $R_L = 250\,\Omega$

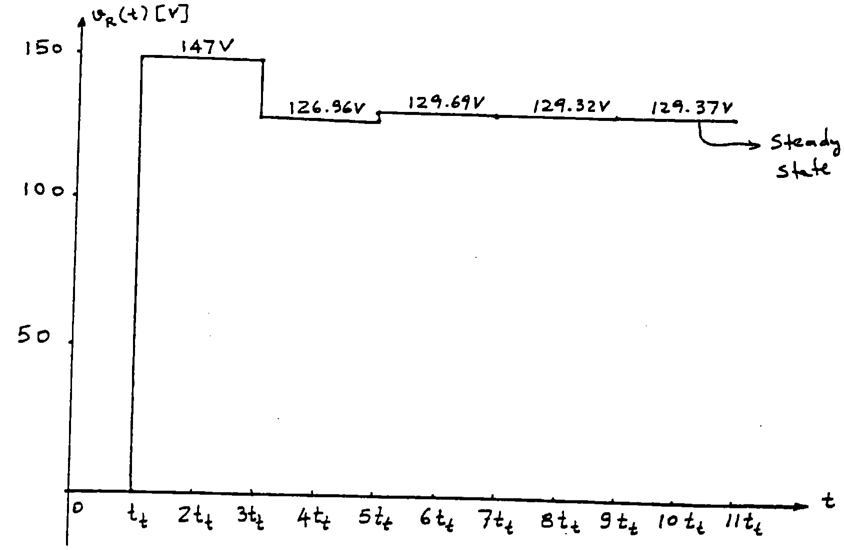
$u_p = 250000\ km/s$ $\qquad I_s = \dfrac{100}{90} = 1.11\,A$

$$t_t = \frac{100}{2.5\times10^{8}} = 4\times10^{-7}\,s = 0.4\,\mu s$$

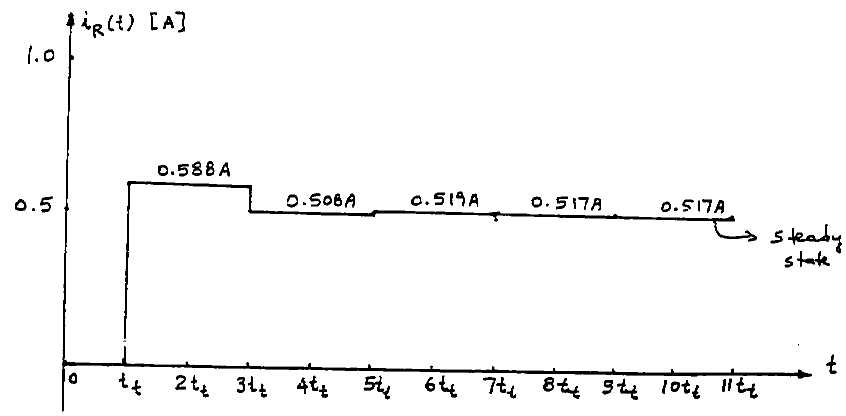
$$\rho_R = \frac{250-90}{250+90} = 0.47 \quad , \quad \rho_s = \frac{50-90}{50+90} = -0.29$$



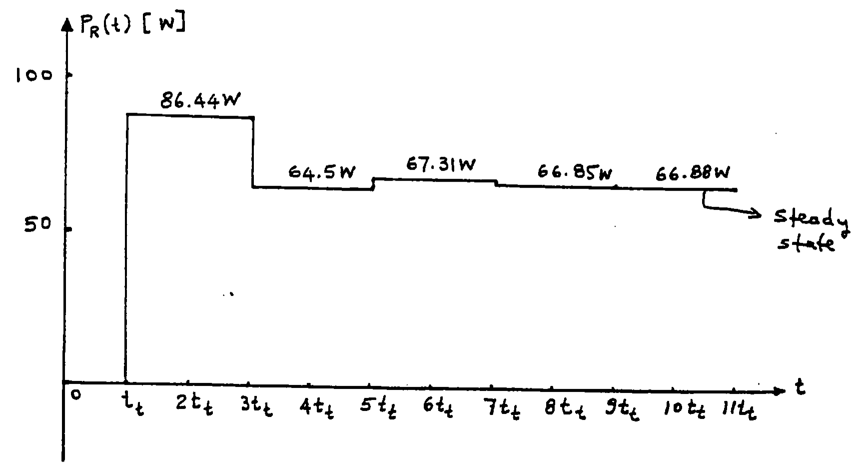

$v_s(t)$ [V]
133.37V
128.81V
129.44V
129.35V
100V
100V
50V
Steady state
0
t_t $2t_t$ $3t_t$ $4t_t$ $5t_t$ $6t_t$ $7t_t$ $8t_t$ $9t_t$ $10t_t$ $11t_t$
t
$i_s(t)$ [A]
1.11
1.11A
1.0
0.5
0.437A
0.528A
0.516A
0.517A
0.517A
Steady state
0
t_t $2t_t$ $3t_t$ $4t_t$ $5t_t$ $6t_t$ $7t_t$ $8t_t$ $9t_t$ $10t_t$ $11t_t$
t
$P_s(t)$ [W]
111
111W
100
50
58.28W
68W
66.79W
66.87W
steady state
0
t_t $2t_t$ $3t_t$ $4t_t$ $5t_t$ $6t_t$ $7t_t$ $8t_t$ $9t_t$ $10t_t$ $11t_t$
t



$v_R(t)$ [V]
150
100
50
147V
126.96V
129.69V
129.32V
129.37V
Steady state
0
t_t
$2t_t$
$3t_t$
$4t_t$
$5t_t$
$6t_t$
$7t_t$
$8t_t$
$9t_t$
$10t_t$
$11t_t$
t



$i_R(t)$ [A]
1.0
0.5
0.588A
0.508A
0.519A
0.517A
0.517A
steady state
0
t_t
$2t_t$
$3t_t$
$4t_t$
$5t_t$
$6t_t$
$7t_t$
$8t_t$
$9t_t$
$10t_t$
$11t_t$
t



$P_R(t)$ [W]
100
50
86.44W
64.5W
67.31W
66.85W
66.88W
Steady state
0
t_t
$2t_t$
$3t_t$
$4t_t$
$5t_t$
$6t_t$
$7t_t$
$8t_t$
$9t_t$
$10t_t$
$11t_t$
t

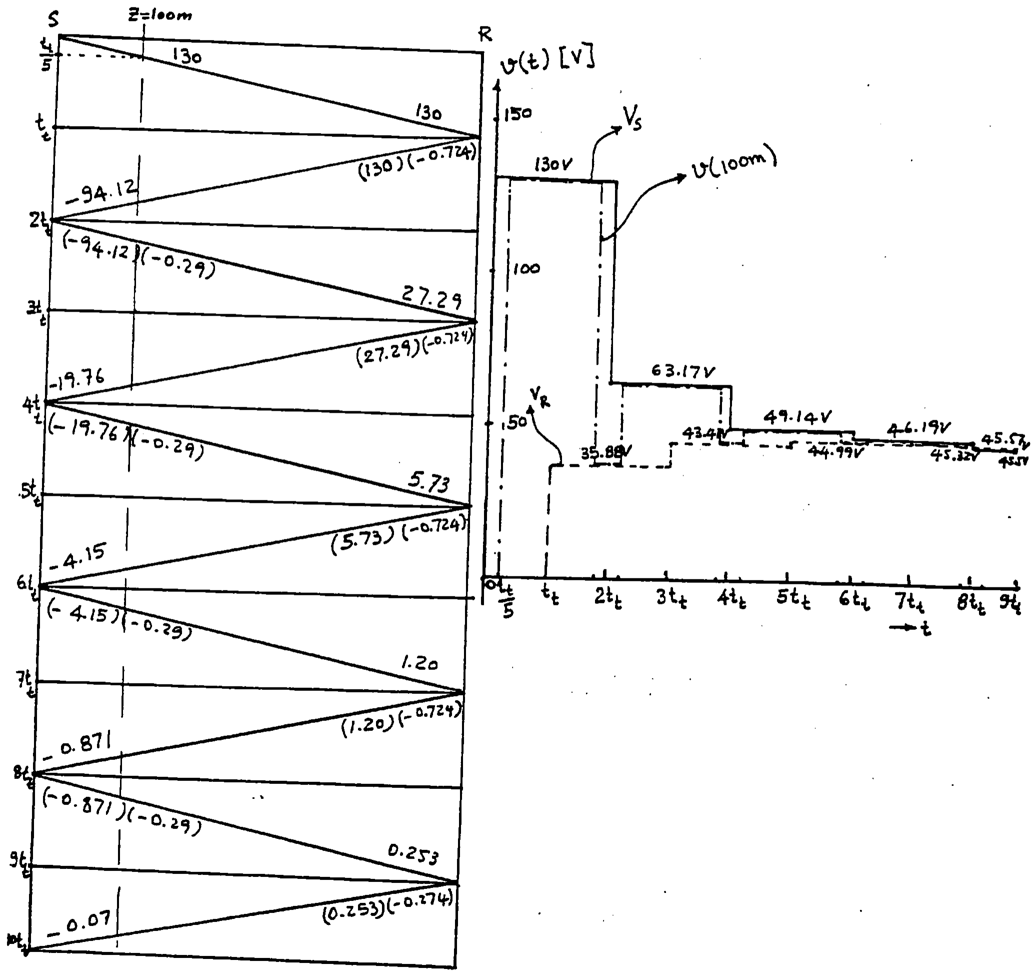
Problem 9.44

$P_L = 1\,kW$, $V_L = 120\,V$ (dc) $\ell = 500\,m$, $R_c = 90\,\Omega$

$V_s = 130\,V$, $R_G = 50\,\Omega$ $u_p = 210000\,km/s$

$$t_t = \frac{500}{210000\times10^3} = 2.38\times10^{-6}\,s \quad , \quad R_L = \frac{V_L^2}{P_L} = \frac{120}{1000} = 14.4\,\Omega$$

$$\rho_R = \frac{14.4 - 90}{14.4 + 90} = -0.724 \qquad \rho_s = \frac{50-90}{50+90} = -0.29$$



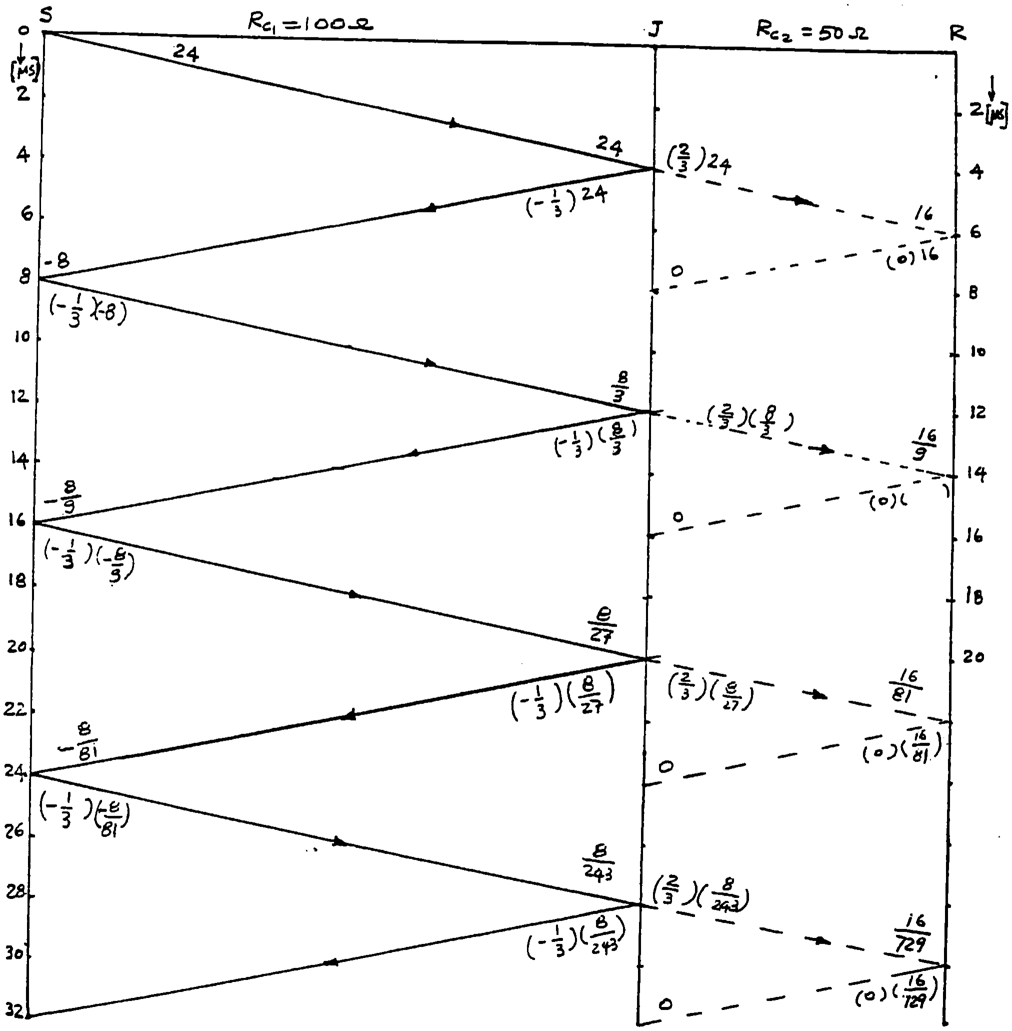
Problem 9.45

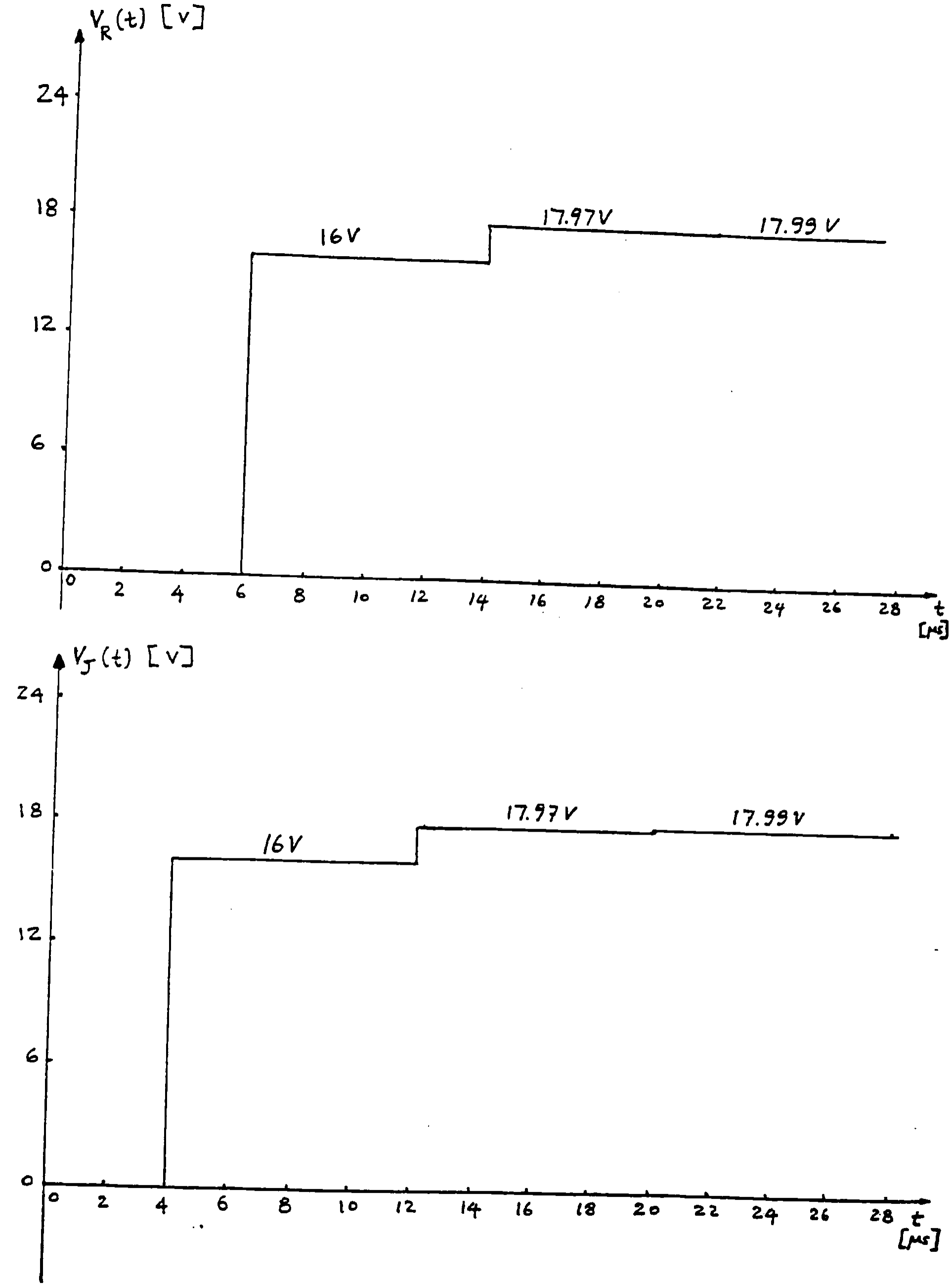
$R_{c_1} = 100\,\Omega$, $t_{t_1} = 4\,\mu s$, $R_{c_2} = 50\,\Omega$, $t_{t_2} = 2\,\mu s$, $R_G = 50\,\Omega$, $R_L = 50\,\Omega$

$V_s = 24\,V$ $\quad \rho_R = \dfrac{50-50}{50+50} = 0$, $\rho_S = \dfrac{50-100}{50+100} = -\dfrac{1}{3}$,

$\rho_1 = \dfrac{50-100}{50+100} = -\dfrac{1}{3}$, $\tau_1 = 1 - \dfrac{1}{3} = \dfrac{2}{3}$ Towards the 50-Ω line.

$\rho_2 = \dfrac{100-50}{100+50} = \dfrac{1}{3}$, $\tau_2 = 1 + \dfrac{1}{3} = \dfrac{4}{3}$ Towards the 100-Ω line.





V_R(t) [v]
24
18
12
6
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28
t [μs]
16V
17.97V
17.99 V
V_J(t) [v]
24
18
12
6
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28
t [μs]
16V
17.97V
17.99V

Problem 9.46

$d = 4\,mm$, $f = 100Hz$, $1kHz$, $10kHz$, $100kHz$, $1MHz$, $10MHz$, $100MHz$, $1GHz$, $\mu = 4\pi \times 10^{-7}\,H/m$, $\sigma = 5.8\times10^{7}\,S/m$

$$a = \frac{4}{2} = 2\,mm$$

$$R_{ci} = \frac{1}{2\times10^{-3}}\sqrt{\frac{4\pi\times10^{-7} f}{4\pi\times 5.8\times10^{7}}} = 2.076\times10^{-5}\sqrt{f}$$

$$R_{ci} = 2.076\times10^{-5}\sqrt{f}$$

$$L_{ci} = \frac{R_{ci}}{2\pi f} = \frac{2.076\times10^{-5}\sqrt{f}}{2\pi\ f} = 3.304\times10^{-6}\frac{1}{\sqrt{f}}$$

f [Hz]	R_{ci} [Ω/m]	L_{ci} [H/m]
100	2.076×10^{-4}	3.304×10^{-7}
10^{3}	6.565×10^{-4}	1.045×10^{-7}
10^{4}	2.076×10^{-3}	3.304×10^{-8}
10^{5}	6.565×10^{-3}	1.045×10^{-8}
10^{6}	2.076×10^{-2}	3.304×10^{-9}
10^{7}	6.565×10^{-2}	1.045×10^{-9}
10^{8}	2.076×10^{-1}	3.304×10^{-10}
10^{9}	6.565×10^{-1}	1.045×10^{-10}