Solver Performance and Acceleration

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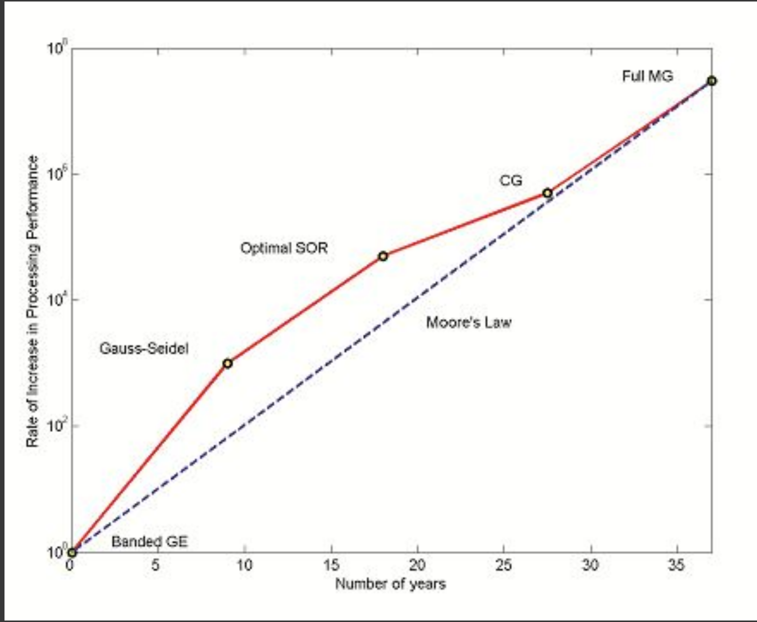
INTRODUCTION

For this project, our team focused on Conjugate Gradient and Multigrid Methods.

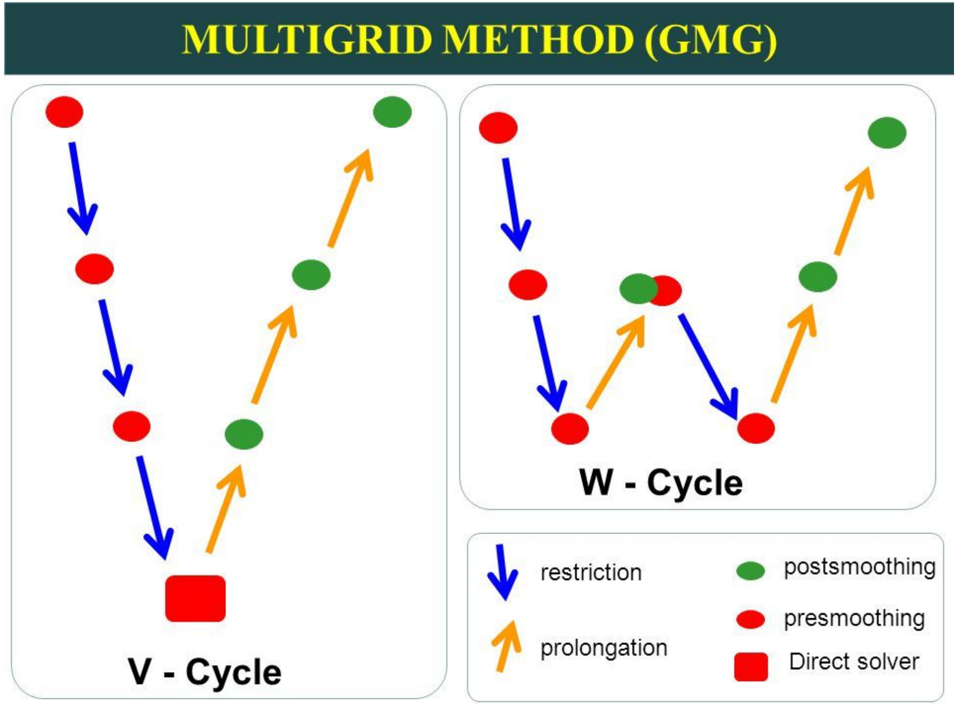


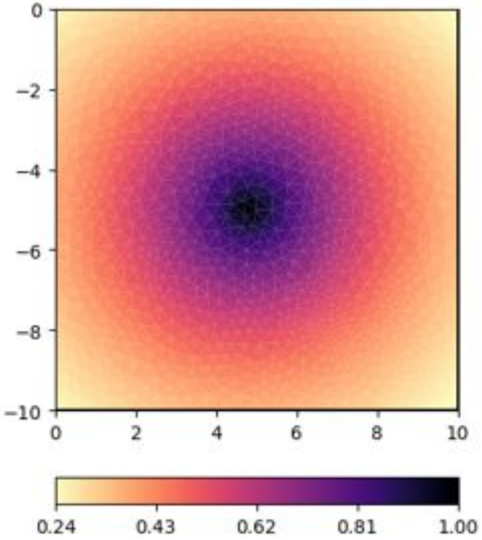
Source: COMSOL

Our Goal

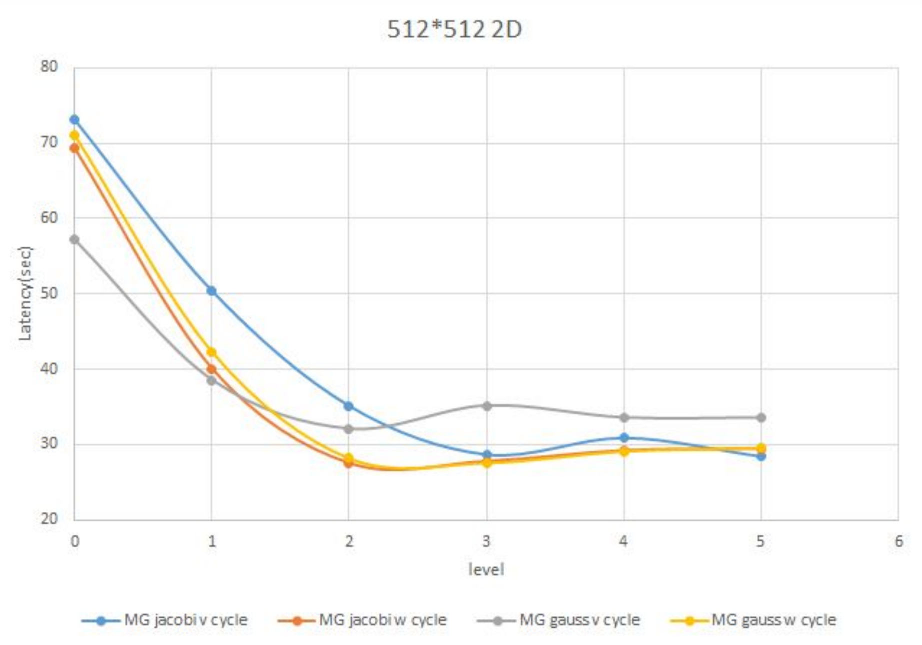
Comparing with different choices of conjugate gradient and multigrid methods with varying trade-offs and accelerate with openACC

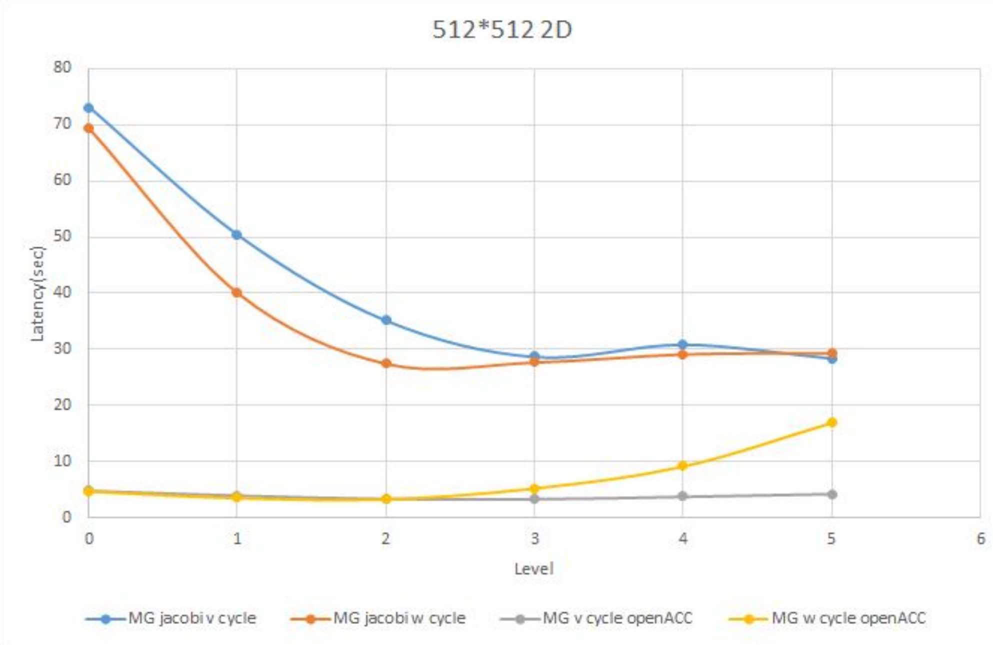
Multigrid


MULTIGRID METHOD (GMG)
V - Cycle
W - Cycle
restriction
prolongation
postsmoothing
presmoothing
Direct solver


0
−2
−4
−6
−8
−10
0
2
4
6
8
10
0.24
0.43
0.62
0.81
1.00

Performance





Conjugate Gradient

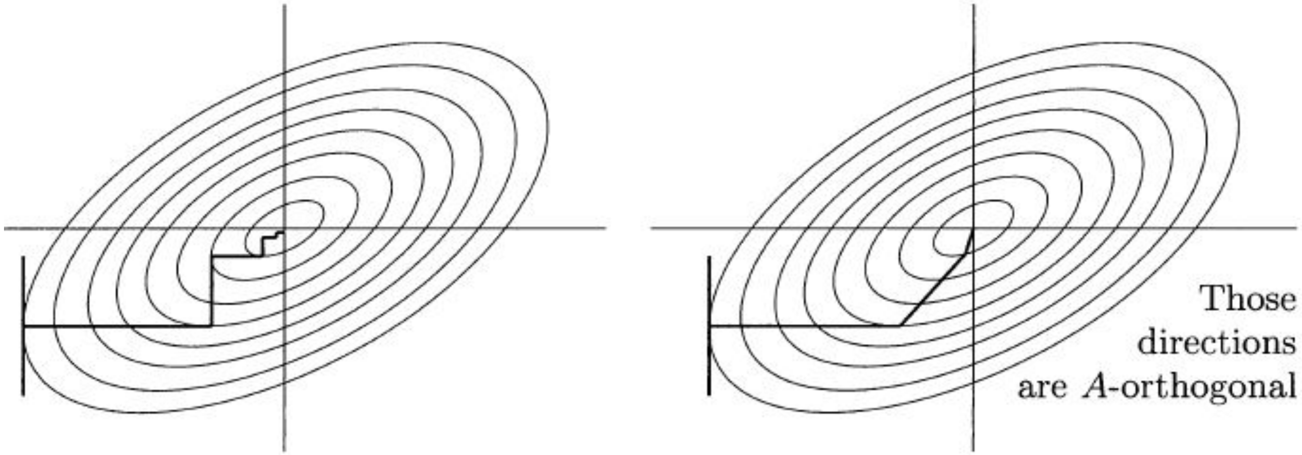
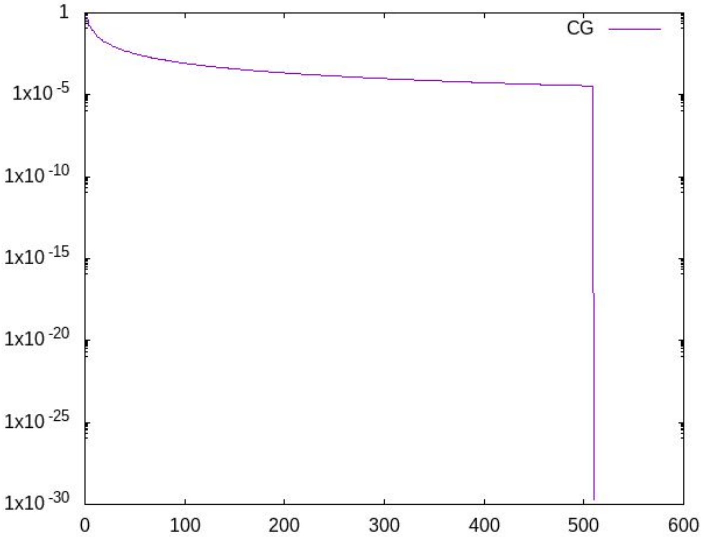
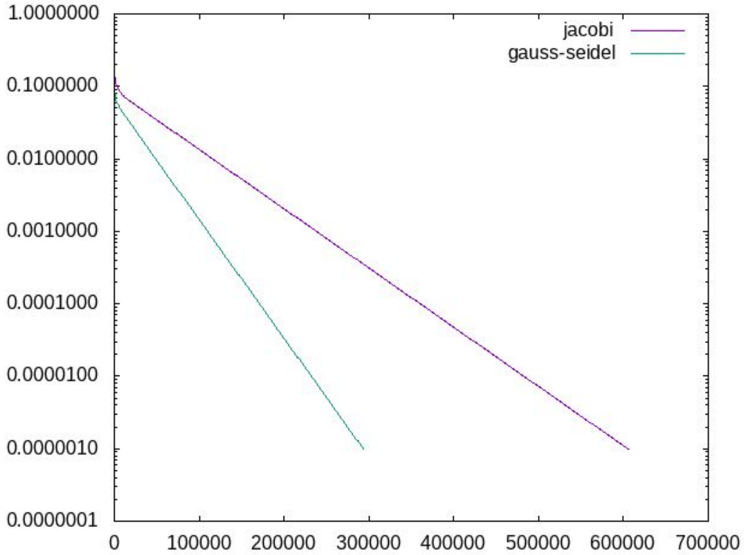


Figure 7.13: Steepest descent (many small steps) vs. conjugate gradients.

CG minimizes the energy $\frac{1}{2}x^{\mathrm{T}}Ax - x^{\mathrm{T}}b$ recursively.

1	$\boldsymbol{\alpha_k} = r_{k-1}^{\mathrm{T}} r_{k-1} / d_{k-1}^{\mathrm{T}} A d_{k-1}$	% Step length to next x_k
2	$\boldsymbol{x_k} = x_{k-1} + \alpha_k d_{k-1}$	% Approximate solution
3	$\boldsymbol{r_k} = r_{k-1} - \alpha_k A d_{k-1}$	% New residual from (14)
4	$\boldsymbol{\beta_k} = r_k^{\mathrm{T}} r_k / r_{k-1}^{\mathrm{T}} r_{k-1}$	% Improvement this step
5	$\boldsymbol{d_k} = r_k + \beta_k d_{k-1}$	% Next search direction





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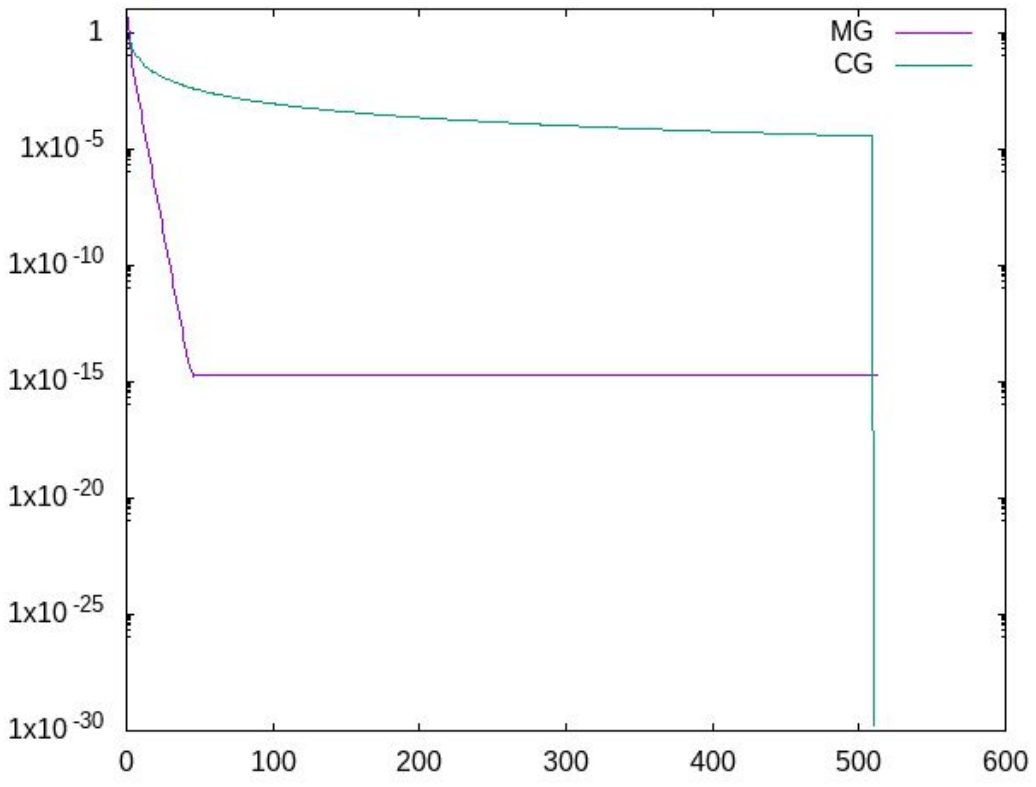
Rewrite $A\boldsymbol{x} = \boldsymbol{b}$ $\quad S\boldsymbol{x} = T\boldsymbol{x} + \boldsymbol{b}.$

Pure iteration $\quad S\boldsymbol{x}_{k+1} = T\boldsymbol{x}_k + \boldsymbol{b}.$

$S\boldsymbol{e}_{k+1} = T\boldsymbol{e}_k$ **which means** $\boldsymbol{e}_{k+1} = S^{-1}T\boldsymbol{e}_k.$


MG
CG
1
1x10^-5
1x10^-10
1x10^-15
1x10^-20
1x10^-25
1x10^-30
0
100
200
300
400
500
600

Thank you!

Resources

- Source1
- Source 2