

## GPS positioning method based on Kalman filtering

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**Abstract**—Global positioning system means a satellite-based radio navigation system, which plays an important role in many areas. The Kalman filter can be exploited to estimate states of dynamic systems via a stochastic linear state-space model. However, the Kalman filter may diverge easily when the forecast model cannot estimate the state vector values accurately. In order to control the degree of divergence, an adaptive approach should be studied. Therefore, we propose an adaptively robust Kalman filter with an adaptive factor can effectively control errors in measurements, with both the information from measurements and the information from the proposed prediction model. To test the effectiveness of the proposed method, position and velocity estimation error rate of the proposed GPS positioning system are tested, and experimental results demonstrated that the proposed method can provide high quality GPS positioning service.

**Keywords**- GPS positioning, Kalman filtering, Linear quadratic estimation, Diagonal matrix.

### I. INTRODUCTION

The Global Positioning System (GPS) aims to provide customers on the earth all day long, all-weather, high precision positioning, navigation and timing service, and it has been widely used in national defense, sea and air transportation, mapping, mobile communication, electric power, finance, fine agriculture, disaster relief, and so on. Furthermore, GPS refers to an important expansion of human activities and promote social development space infrastructure[1][2].

As is well known that GPS refers to a satellite-based radio navigation system which is proposed by the US government and designed by the US Air Force[3][4]. It is a global navigation satellite system which provides geo information and time information to a GPS receiver at each location on or near the Earth[5]. However, obstacles, e.g. mountains and buildings may let GPS signals weak. In addition, GPS does not need the user to send any data, and it operates without using any telephonic or internet reception. Then, these technologies are able to promote the usefulness of the GPS positioning information[6][7]. The GPS can give critical positioning capabilities to military, civil, and commercial users for humans. Particularly, the US government developed GPS system, and then ensures it freely access to all of us via a GPS receiver.

The GPS system is able to selectively deny access to the system, when it is used in the Indian military in the Kargil War. Next, a lot of countries have developed or are in the process of setting up other global or regional navigation

systems. For example, Russian Global Navigation Satellite System is designed contemporaneously with GPS[8]. In China, a famous Navigation Satellite System (named as Beidou) is developed and performs well. Furthermore, there are also the European Union Galileo positioning system, and India's NAVIC, and so on.

Kalman filtering, also known as linear quadratic estimation (LQE)[9], denotes an algorithm which utilizes a number of measurements observed over time, containing statistical noise and other inaccuracies, and computes of unknown variables which are more accurate than those based on a single measurement alone [10][11].

The rest of the paper is organized as follows. We describe the Kalman filter in section 2. In Section 3, we provide the proposed method for GPS positioning. Section 4 conducts experiments to validate the effectiveness of the proposed approach. In the end, we conclude the whole paper in section 5.

### II. OVERVIEW OF THE KALMAN FILTER

The Kalman filter can be exploited to estimate states of dynamic systems via a stochastic linear state-space model as follows [12][13].

$$\dot{X}(t) = AX(t) + Bu(t) + W(t) \quad (1)$$

Eq.1 denotes the continuous state to control system dynamics [14][15].

$$Y_k = CX_k + V_k \quad (2)$$

where Eq.2 means the discrete measurement that is related to the system  $X(t)$  to the available measurements  $Y_k$ . In addition,  $u(t)$  refers to the input,  $W(t)$  means the Gaussian process noise, and  $V_k$  denotes the measurement noises.

We assume that a time slot is regarded as  $t_k = k \cdot T_s$ , in which  $T_s$  represents to a sampling period. The estimation of the state vector  $\hat{x}_k$  and the error covariance matrix  $\hat{P}_k$  can be estimated through Time updating and 2) measurement updating.

To update the time in our system, a priori is used to estimate the state  $\hat{X}_{k+1}^-$  and the error covariance  $\hat{P}_{k+1}^-$  at  $t_{k+1}$ , and this process is illustrated as follows.

$$\hat{X}_{k+1}^- = \Phi \hat{X}_k + \int_{t_k}^{t_{k+1}} \Phi B u(t) dt \quad (3)$$

$$\hat{P}_{k+1}^- = \Phi \hat{P}_k + \hat{P}_k \Phi^T + Q \quad (4)$$

where the symbol  $\Phi$  represents the state transition matrix.

Particularly, to update the process of measurement in this system, the state  $\hat{X}_{k+1}$  and the error covariance  $\hat{P}_{k+1}$  can be estimated by the following equations.

$$\hat{X}_{k+1} = \hat{X}_{k+1}^- + K_{k+1} (y_{k+1} - \hat{Y}_{k+1}^-) \quad (5)$$

$$\hat{P}_{k+1} = (I - K_{k+1} C) \hat{P}_{k+1}^- \quad (6)$$

$$K_{k+1} = \hat{P}_{k+1}^- C^T (C \hat{P}_{k+1}^- C^T + R)^{-1} \quad (7)$$

### III. THE PROPOSED METHOD

The Kalman filter may diverge easily when the forecast model cannot estimate the state vector values with high accuracy[16][17]. To restrict the degree of divergence, we should utilize an adaptive approach. The adaptively robust Kalman filter with an adaptive factor can effectively control errors in measurements, together with the information from measurements and the information from the proposed forecast model[18][19][20]. Therefore, we develop adaptively robust Kalman filter with multi factors which is able to choose what elements in the state vector can be affected by the adaptive approach. Based on the above analysis, the optimization method of the adaptively robust Kalman filter is represented as follows.

$$\begin{aligned} \Omega_k &= V_{k+1}^T \bar{Q}_{k+1} V_{k+1} + V_{\bar{X}_{k+1}}^T \alpha_k^{1/2} \bar{Q}_{\bar{X}_{k+1}} \alpha_k^{1/2} V_{\bar{X}_{k+1}} = \min \quad (8) \\ \text{s.t. } V_{k+1} &= H_{k+1} \hat{X}_{k+1} - G_{k+1} \\ V_{\bar{X}_{k+1}} &= \hat{X}_{k+1} - \bar{X}_{k+1} \end{aligned}$$

where  $k$  refers to the last epoch number,  $\bar{X}_{k+1}$  denotes the forecasted value of the state vector, the symbol  $\hat{X}_{k+1}$  represents the state vector based on updating the measurement,  $G_{k+1}$  denotes the observation vector,  $\bar{Q}_{\bar{X}_{k+1}}$

refers to the weight matrix of the estimated state vector, and  $\alpha_k$  denotes a diagonal matrix.

Afterwards, the above process can be solved by the following optimization process.

$$K_{k+1} = \bar{\Sigma}_{\bar{X}_{k+1}} H_{k+1}^T (H_{k+1}^T \bar{\Sigma}_{\bar{X}_{k+1}} H_{k+1} + \Sigma_{k+1})^{-1} \quad (9)$$

$$\hat{X}_{k+1} = \bar{X}_{k+1} + K_{k+1} G_k \quad (10)$$

$$\bar{\Sigma}_{\hat{X}_{k+1}} = (I - K_{k+1} H_{k+1}) \bar{\Sigma}_{\bar{X}_{k+1}} \quad (11)$$

Based on the above analysis and definition, the GPS positioning system is illustrated as the following steps:

- (1) Assume that we have  $X(0)$  and  $P(0)$
- (2) Forecast the values of  $\hat{X}_{k,k-1}$  and  $h(\hat{X}_{k,k-1}, k)$
- (3) Calculate the coefficient  $\Phi_{k,k-1}$  and  $H_k$
- (4) Calculate the  $P_{k,k-1}$
- (5) Calculate the extended Kalman gain  $K_k$
- (6) Compute  $\hat{X}_k, P_k$  and  $K=K+1$
- (7) Goto (2)

### IV. EXPERIMENT

In order to validate the effectiveness of our proposed algorithm, we design a GPS positioning system. Some important parameters of this experiment are given as follows.

Table. 1 Parameters of this simulation

Parameter name	Value
GPS position error	12 m
Filtering cycle	1.2s
Gravity acceleration	8.5744 $m/s^2$
Gyro constant drift	0.12 degree a hour
Mean square deviation of the Gyro constant drift	0.011 degree a hour
Accelerometer constant zero-bias	120 $\mu g$
GPS velocity error	0.35 $m/s$
Simulation time period	2 hours
Longitude	120°E
Latitude	40°N

height	5000m
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Utilizing this experimental settings, position and velocity estimation error rate of the proposed GPS positioning system are demonstrated as follows (shown Fig. 1 to Fig. 4).

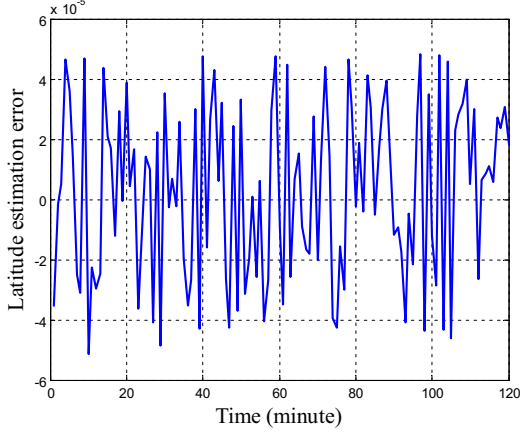


Figure 1. Latitude estimation error

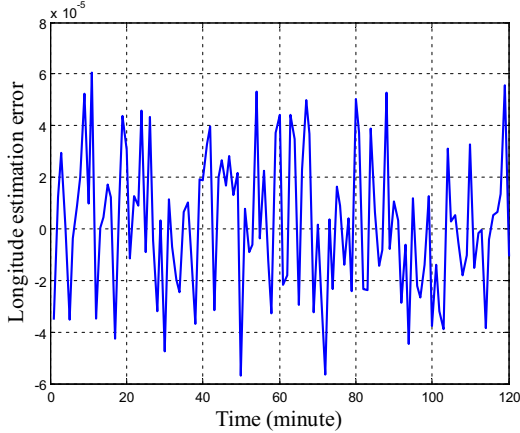


Figure 2. Longitude estimation error

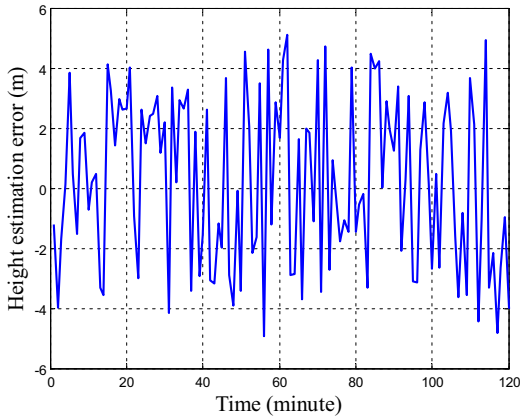


Figure 3. Height estimation error

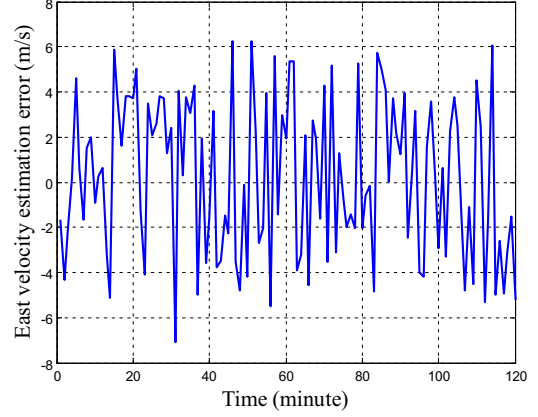


Figure 4. East velocity estimation error

From the above experimental results, the conclusion can be drawn that the proposed method can effectively control the position and velocity estimation error rates in a bearable range.

## V. CONCLUSION

To control the degree of divergence, in this paper, we propose an adaptively robust Kalman filter with an adaptive factor can effectively control errors in measurements, with both the information from measurements and the information from the proposed prediction model. Afterwards, position and velocity estimation error rate of the proposed GPS positioning system are tested, and very positive experimental results are obtained.

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