AE332 – Modeling and Analysis Lab

Name: Shingala Vaidik ID : SC21B054

Problem: Simulate the rotation of a rigid body with and without moments, and verify the following. (a) Energy is conserved, (b) when there is no moment, angular momentum is consered, and (c) when there is no moment and when the initial angular velocity is along one of the principal inertia directions, angular velocity is conserved.

Matlab code:

```
%moment of inertia about body fixed refrence frame
I1=0.5;
I2=1;
I3=1.5;
I_fixed=[I1 0 0;
         0 I2 0;
         0 0 I3];
mass=5;


% let the rotation is about z axis
rotation_axis=[0 0 1];
rotation_angle=45*pi/180;
q_0=cos(rotation_angle/2);
q_1=sin(rotation_angle/2)*rotation_axis(1);
q_2=sin(rotation_angle/2)*rotation_axis(2);
q_3=sin(rotation_angle/2)*rotation_axis(3);
rotation_matrix=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
                 2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0 ;
                 2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-2*q_1^2];
I_abs=rotation_matrix*I_fixed*rotation_matrix.';


%defining intial conditions
x_0=0; x_dot0=0; w_x=1;
y_0=0; y_dot0=0; w_y=2;
z_0=0; z_dot0=0; w_z=3;


%Defining force and moments
F=[1;1;1];
M=[0;0;0];
```

```
intial_conditions=[x_0,
y_0,z_0,q_0,q_1,q_2,q_3,x_dot0,y_dot0,z_dot0,w_x,w_y,w_z,0,0];
option=odeset('RelTol',1e-14,'AbsTol',1e-14);
[t,z]=ode45(@(t,z)dynamics(t,z,F,M,I_fixed),(0:0.01:10),intial_conditions,option);


j=1;
Angular_Moment=zeros;
RKE=zeros;


for i=(0:0.01:10)
    q_0=z(j,4);
    q_1=z(j,5);
    q_2=z(j,6);
    q_3=z(j,7);
    Q=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
       2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0 ;
       2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-2*q_1^2];
    I_fix=[I1 0 0;
          0 I2 0;
          0 0 I3];
    I_abs=Q*I_fix*Q.';
    w=[z(j,11);z(j,12);z(j,13)];
    L_0=I_abs*w;
    Angular_Moment(j,1)=L_0(1,1);
    Angular_Moment(j,2)=L_0(2,1);
    Angular_Moment(j,3)=L_0(3,1);
    RKE(j,1)=0.5*Angular_Moment(j,:)*w;
    j=j+1;
end


%calculating energy and energy difference
Total_energy=0.5*mass*(z(:,8).^2+z(:,9).^2+z(:,10).^2)+RKE-z(:,14)-z(:,15);
change_in_energy=max(Total_energy)-min(Total_energy);


%Angular Moment conservation;
L_x=Angular_Moment(:,1);
L_y=Angular_Moment(:,2);
L_z=Angular_Moment(:,3);
change_in_Angular_Moment_x=max(L_x)-min(L_x);
change_in_Angular_Moment_y=max(L_y)-min(L_y);
change_in_Angular_Moment_z=max(L_z)-min(L_z);


%Angular velocity conservation;
change_in_wx=max(z(:,11))-min(z(:,11));
change_in_wy=max(z(:,12))-min(z(:,12));
change_in_wz=max(z(:,13))-min(z(:,13));
```

```
function dzdt=dynamics(~,z,F,M,I_fixed)
m=5;
mass=[m 0 0;
      0 m 0;
      0 0 m];
q0=z(4,1); q2=z(6,1);
q1=z(5,1); q3=z(7,1);

Linear_acceleration=mass\F;


Q=[1-2*q2^2-2*q3^2 2*q1*q2-2*q3*q0 2*q1*q3+2*q2*q0;
   2*q1*q2+2*q3*q0 1-2*q3^2-2*q1^2 2*q3*q2-2*q1*q0 ;
   2*q1*q3-2*q2*q0 2*q3*q2+2*q1*q0 1-2*q2^2-2*q1^2];


  I_abs=Q*I_fixed*Q.';
  w=[z(11,1);z(12,1);z(13,1)];
  w_1=I_abs*w;
  Angular_acceleration=I_abs\(M-cross(w,w_1));

  dzdt(1,1)=z(8,1);
  dzdt(2,1)=z(9,1);
  dzdt(3,1)=z(10,1);
  dzdt(4,1)=-0.5*(q1*z(11,1)+q2*z(12,1)+q3*z(13,1));
  dzdt(5,1)=0.5*(q0*z(11,1)+q3*z(12,1)-q2*z(13,1));
  dzdt(6,1)=0.5*(q1*z(13,1)+q0*z(12,1)-q3*z(11,1));
  dzdt(7,1)=0.5*(q2*z(11,1)-q1*z(12,1)+q0*z(13,1));
  dzdt(8,1)=Linear_acceleration(1,1);
  dzdt(9,1)=Linear_acceleration(2,1);
  dzdt(10,1)=Linear_acceleration(3,1);
  dzdt(11,1)=Angular_acceleration(1,1);
  dzdt(12,1)=Angular_acceleration(2,1);
  dzdt(13,1)=Angular_acceleration(3,1);
  dzdt(14,1)=F'*[z(8,1);z(9,1);z(10,1)];
  dzdt(15,1)=M'*w;
end
```

(a) We have to conserved energy with and without:

First, I am displaying change in energy:

```
% Display the maximum change in energy
disp(['Maximum Change in Energy: ', num2str(max(change_in_energy))]);
```

I. **With Moment:**

I am giving moment -

```
M=[3;5;10];
```

Output:

```
Maximum Change in Energy: 1.4643e-10
```

II. **Without Moment:**

I am giving moment –

```
M=[0;0;0];
```

Output:

```
Maximum Change in Energy: 1.8474e-13
```

(b) when there is no moment, angular momentum is conserved:

First, I am displaying change in angular moment:

```
% Display the maximum change in angular moment
disp(['Maximum Change in Energy: ',num2str(max(change_in_Angular_Moment_x))]);
disp(['Maximum Change in Energy: ',num2str(max(change_in_Angular_Moment_y))]);
disp(['Maximum Change in Energy: ',num2str(max(change_in_Angular_Moment_z))]);
```

Output:

```
Maximum Change in Energy: 2.1533e-13
Maximum Change in Energy: 5.107e-14
Maximum Change in Energy: 4.1744e-14
```

(c) when there is no moment and when the initial angular velocity is along one of the principal inertia directions, angular velocity is conserved.

First, I changed angular velocity for my code principal inertia direction is 'z-axes' so I given 5 red/s in z direction and then I give zero moment and display change in angular velocity in all direction.

```
%defining intial conditions
x_0=0; x_dot0=0; w_x=0;
y_0=0; y_dot0=0; w_y=0;
z_0=0; z_dot0=0; w_z=5;
```

Output:

```
Maximum Change in Energy: 0
Maximum Change in Energy: 0
Maximum Change in Energy: 0
```