AE332 – Modeling and Analysis Lab

A Report submitted

For internal assessment for Coarse

Modeling and Analysis Lab

in

Aerospace Engineering

by

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Contents

Chapter 1

1D heat diffusion problem

1.1 Problem to solve

Consider the 1D heat diffusion problem with Dirichlet boundary conditions:

$$\frac{d^2u}{dx^2} = 1, \quad \text{for } 0 < x < 1 \tag{1.1}$$

Boundary conditions:

$$u(x = 0) = 0, \quad u(x = 1) = 0 \tag{1.2}$$

1.1.1 Investigation 1: Neumann Boundary Condition

Investigate the effect of a Neumann boundary condition (insulated at both ends) by modifying the boundary conditions.

$$\frac{du}{dx}(x = 0) = 0, \quad \frac{du}{dx}(x = 1) = 0 \tag{1.3}$$

1.1.2 Investigation 2: Grid Convergence Study

Perform a grid convergence study by varying the mesh size systematically. Observe how the numerical solution changes with finer grids and comment on the accuracy of the numerical method.

1.1.3 Investigation 3: P2 Finite Element Space

Implement the problem using P2 (quadratic) finite element space. Compare the results with P1 (linear) finite element space and comment on any significant differences.

1.2 Mathematical Formulation

1.2.1 Dirichlet Boundary Conditions

The weak form of the problem with Dirichlet boundary conditions is given by finding $u \in V$ such that:

$$\int_0^1 \frac{du}{dx} v \, dx = \int_0^1 v \, dx \quad \forall v \in V \tag{1.4}$$

where V is the function space satisfying the Dirichlet boundary conditions.

1.2.2 Neumann Boundary Conditions

The weak form of the problem with Neumann boundary conditions is given by finding $u \in V$ such that:

$$\int_0^1 \frac{du}{dx} v \, dx = 0 \quad \forall v \in V \tag{1.5}$$

where V is the function space satisfying the Neumann boundary conditions.

1.3 Code

```
load  "msh3";
int m =100;
meshL Th = segment (m ,[ x *1]) ;
real [ int ] xaxis ( m +1) , Uline ( m +1) ;
ofstream file2 (" Results .dat") ;
fespace Vh ( Th , P1 ) ;
Vh U , v ;
solve Poisson (U , v ) = int1d ( Th ) ( dx ( U ) * dx ( v ) )+ int1d ( Th ) ( v )+
     on (1 ,2 , U =0) ;
 plot (U , value = true ) ;
 for (int i =0; i <= m ; i ++)
{
xaxis [ i ]= i / m ;
Uline [ i ] = U [][ i ];
file2 << xaxis [ i ] <<"\t" <<"\t" << Uline [ i ] << endl ;
cout << xaxis [ i ] <<"\t" << Uline [ i ] << endl ;
}
```

1.3.1 Output of code

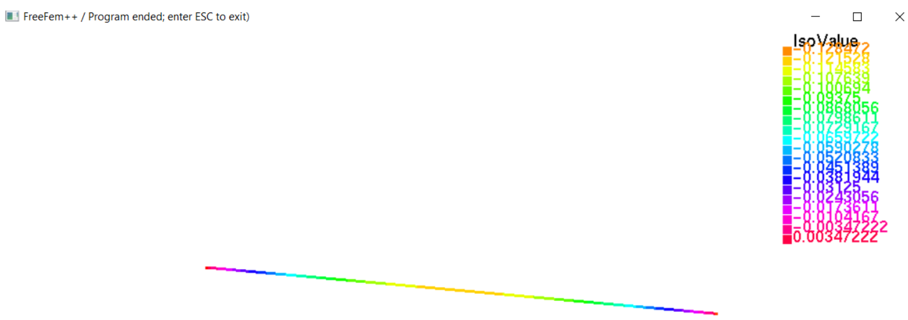


Figure 1.1

1.4 Investigation 1: Neumann Boundary Condition

In the current problem, the solution is obtained considering a Dirichlet boundary condition. Investigate the effect of a Neumann boundary condition (i.e., when the domain is insulated at both ends), by modifying the boundary conditions.

1.4.1 Code

```
load  "msh3";
int m =100;
meshL Th = segment (m ,[ x ∗1]) ;
real [ int ] xaxis ( m +1) , Uline ( m +1) ;
ofstream file2 (" Results .dat") ;
fespace Vh ( Th , P1 ) ;
Vh U , v ;
solve Poisson (U , v ) = int1d ( Th ) ( dx ( U ) ∗ dx ( v ) )+ int1d ( Th ) ( v )
    ;
 plot (U , value = true ) ;
 for (int i =0; i <= m ; i ++)
{
xaxis [ i ]= i / m ;
Uline [ i ] = U [][ i ];
file2 << xaxis [ i ] <<"\t" <<"\t" << Uline [ i ] << endl ;
cout << xaxis [ i ] <<"\t" << Uline [ i ] << endl ;
}
```

1.4.2 Plot

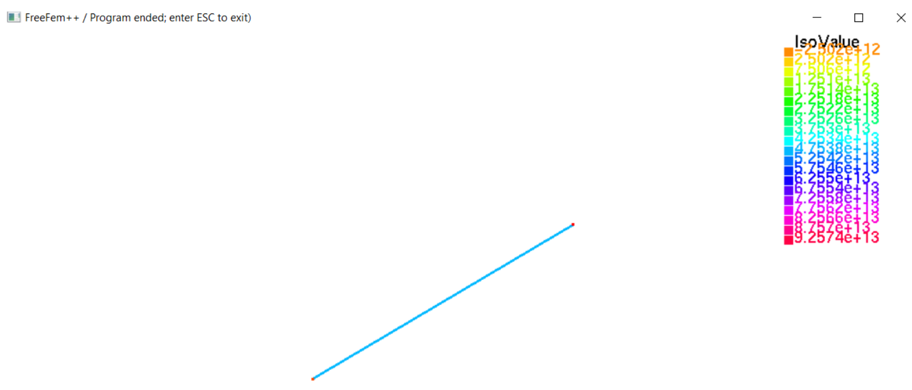


Figure 1.2

1.5 nvestigation 2: Grid Convergence Study

Perform a grid convergence study by varying the mesh size systematically. Observe how the numerical solution changes with finer grids and comment on the accuracy of the numerical method.

1.5.1 Impact of refining the mesh

Enhancing the mesh appears to significantly enhance the precision of the numerical solution. The act of refining the mesh not only contributes to a better understanding of the convergence rate but also facilitates an evaluation of the delicate balance between computational expenses and the accuracy of the solution.

1.5.2 Plot

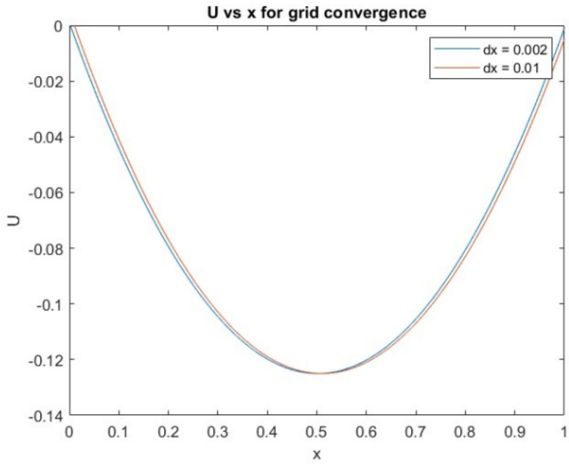


Figure 1.3

1.6 Investigation 3: P2 Finite Element Space

When implementing the problem using the P2 (quadratic) finite element space, it is essential to assess whether there are significant differences in the results compared to the P1 (linear) finite element space.

1.6.1 Observations

- **Linear Approach:** Convergence in the linear approach may be slower, particularly when capturing rapid changes or gradients in the solution. This method might require more elements to accurately represent complex features.
- **Quadratic Approach:** The quadratic approach tends to converge faster, yielding more accurate results even with fewer elements. This is particularly advantageous in scenarios with steep gradients, where the higher-order elements provide a more refined representation of the solution.

1.6.2 Result Analysis

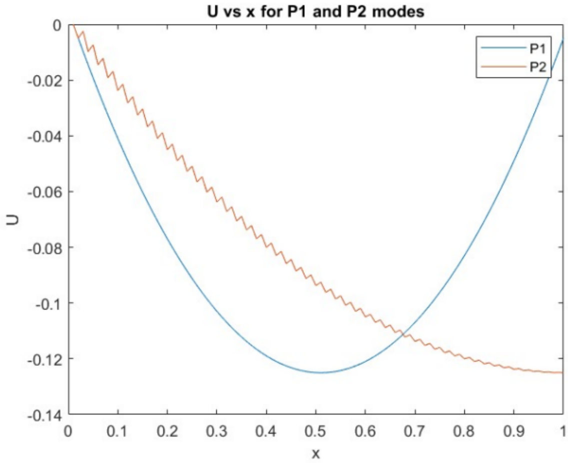


Figure 1.4: Comparison of P1 and P2 Finite Element Spaces

Chapter 2

Heat transfer through fin

2.1 Problem to solve

Consider a 1 mm diameter, 50 mm long aluminum pin-fin. The fin is exposed to a constant heat source at a temperature of $T_h = 300°C$ at its left end, while its right end is thermally insulated. Heat is released along the fin's length through convection, interacting with an ambient temperature of $T_\infty = 30°C$. The governing differential equation is given by:

$$k\frac{d^2T}{dx^2} = \frac{Ph}{A}(T - T_\infty) \tag{2.1}$$

In Equation 2.1, the thermal conductivity k is 200 W/m·K, and the convective heat transfer coefficient h is $20\,\text{W/m}^2 \cdot \text{K}$. P, A represent the perimeter and area of the cross-section of the fin.

- How well does the solutions generated compares with Analytical results.
- For the same problem consider the right end of the fin is subjected to ambient Temperature of $T_\infty = 30°\text{C}$. Plot the temperature profile for this condition.

2.2 Analytical Solution

To obtain an analytical solution, let's assume steady-state conditions and neglect any heat generation within the fin. The general solution to Equation 2.1 is:

$$T(x) = T_\infty + (T_h - T_\infty)\frac{\cosh(m(L - x))}{\cosh(mL)} \tag{2.2}$$

where $m = \sqrt{\frac{Ph}{kA}}$, and L is the length of the fin.

2.3 Problem 1

The fin's left end interfaces with a constant heat source at a temperature of $T_h = 300°\text{C}$, while its right end is thermally insulated.

2.3.1 code

```
load  "msh3";
int m =100;
meshL Th = segment (m ,[ x *1]) ;
real [ int ] xaxis ( m +1) , Uline ( m +1) ;
ofstream file2 (" Results .dat") ;
fespace Vh ( Th , P1 ) ;
Vh U , v ;
solve Poisson (U , v ) = int1d ( Th ) ( dx ( U ) * dx ( v ) ) + int1d ( Th ) (400*
     v*U ) − int1d (Th) (400*30*v) + on (1, U =300)+ int1d(Th, 2)(0*U*v);//on (2,
    U =30);// this is Dirichlet boundary condition
 plot (U , value = true ) ;//newman bc int1d(Th, which boundary you are talking
     about)(flux*U*v) in case of adiabatic flux is 0
 for (int i =0; i <= m ; i ++)
{
xaxis [ i ]= i / m ;
Uline [ i ] = U [][ i ];
file2 << xaxis [ i ] <<"\t" <<"\t" << Uline [ i ] << endl ;
cout << xaxis [ i ] <<"\t" << Uline [ i ] << endl ;
}
```

2.3.2 Output of code

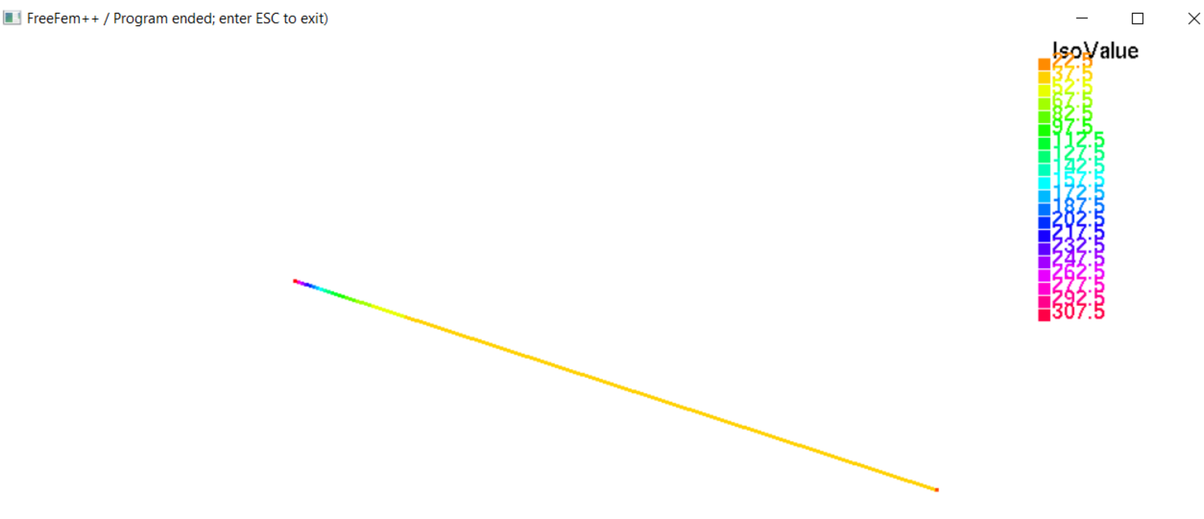


Figure 2.1

2.4 Problem 2

How well does the solutions generated compares with Analytical results

2.4.1 Matlab code

```
% Parameters
T_infinity = 30; % Ambient temperature
T_h = 300; % Heat source temperature
L = 0.05; % Length of the fin (50 mm converted to meters)
k = 200; % Thermal conductivity (W/m K)
h = 20; % Convective heat transfer coefficient (W/m   K)
P = pi*0.001; % Perimeter of the fin cross-section (m)
A = pi*(0.001)^2/4; % Area of the fin cross-section (m )

% Calculate dimensionless parameter
m = sqrt((P * h) / (k * A));

% Define the analytical solution function
T = @(x) T_infinity + (T_h - T_infinity) * cosh(m * (L - x)) / cosh(m * L);

% Generate x values
x_values = linspace(0, L, 1000);

% Calculate T(x) for each x
T_values = arrayfun(T, x_values);

% Plot temperature profile
figure;
plot(x_values, T_values, 'LineWidth', 2);
xlabel('Distance along the fin (m)');
ylabel('Temperature ( C )');
title('Analytical Temperature Distribution along the Fin');
grid on;
```

2.4.2 Output of code

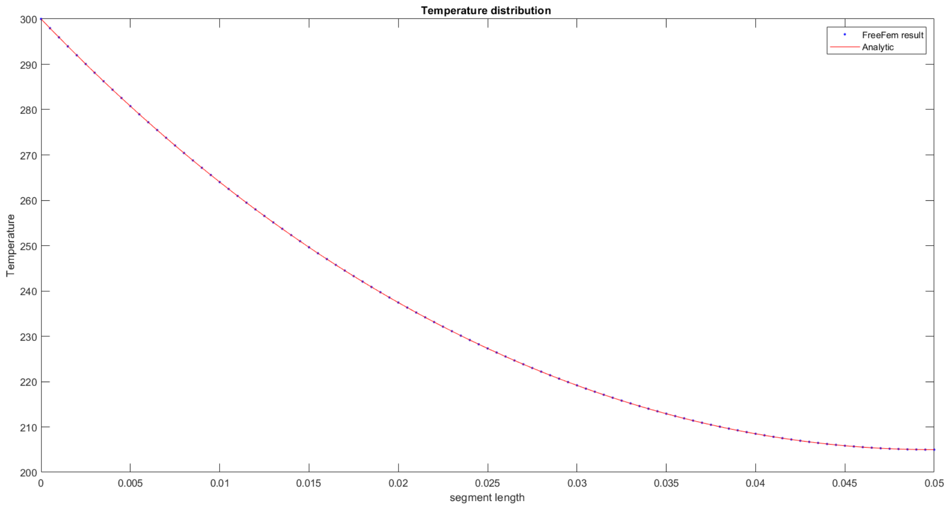


Figure 2.2

2.5 Problem 3

For the same problem consider the right end of the fin is subjected to ambient Temperature of $T_\infty = 30°\mathrm{C}$. Plot the temperature profile for this condition.

2.5.1 Code

Here is the code for Problem 3

```
load  "msh3";
int m =100;
meshL Th = segment (m ,[ x *1]) ;
real [ int ] xaxis ( m +1) , Uline ( m +1) ;
ofstream file2 (" Results .dat") ;
fespace Vh ( Th , P1 ) ;
Vh U , v ;
solve Poisson (U , v ) = int1d ( Th ) ( dx ( U ) * dx ( v ) ) + int1d ( Th ) (400*
     v*U ) — int1d (Th) (400*30*v) + on (1, U =300)+ on (2, U =30);//int1d(Th, 2)
    (0*U*v);// this is Dirichlet boundary condition
 plot (U , value = true ) ;//newman bc int1d(Th, which boundary you are talking
     about)(flux*U*v) in case of adiabatic flux is 0
 for (int i =0; i <= m ; i ++)
{
xaxis [ i ]= i / m ;
Uline [ i ] = U [][ i ];
file2 << xaxis [ i ] <<"\t" <<"\t" << Uline [ i ] << endl ;
```

```
cout << xaxis [ i ] <<"\t" << Uline [ i ] << endl ;
}
```

2.5.2 Output of code

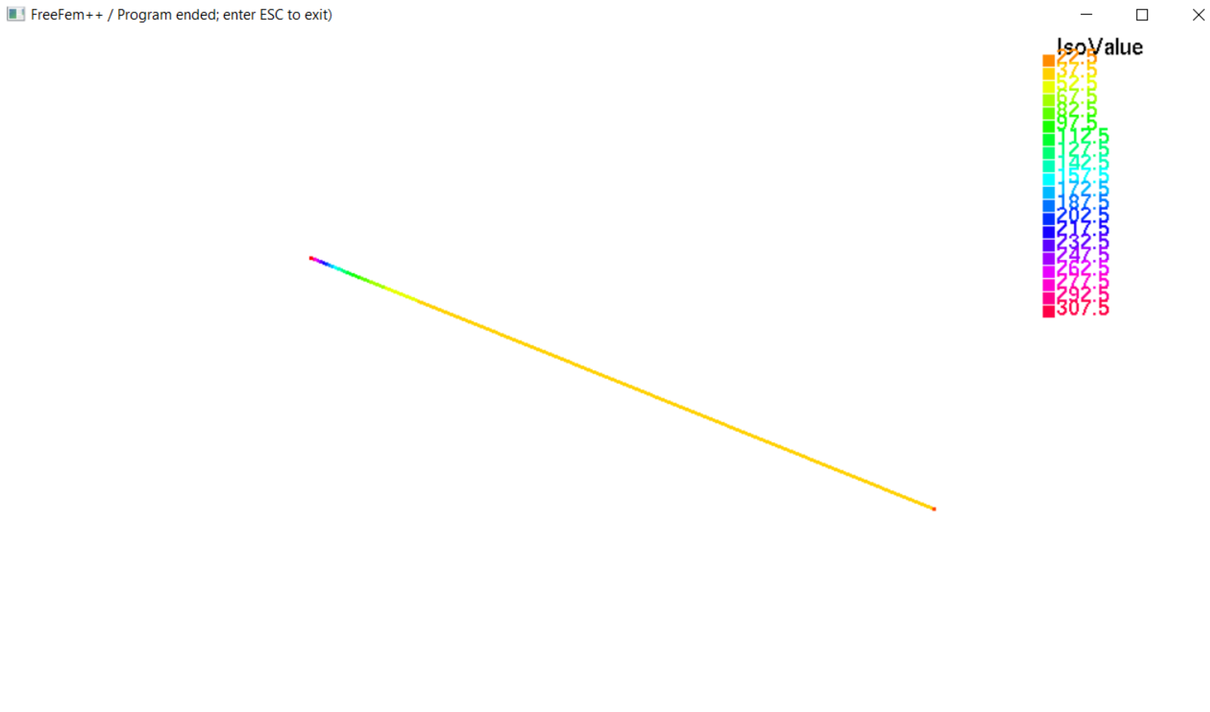


Figure 2.3

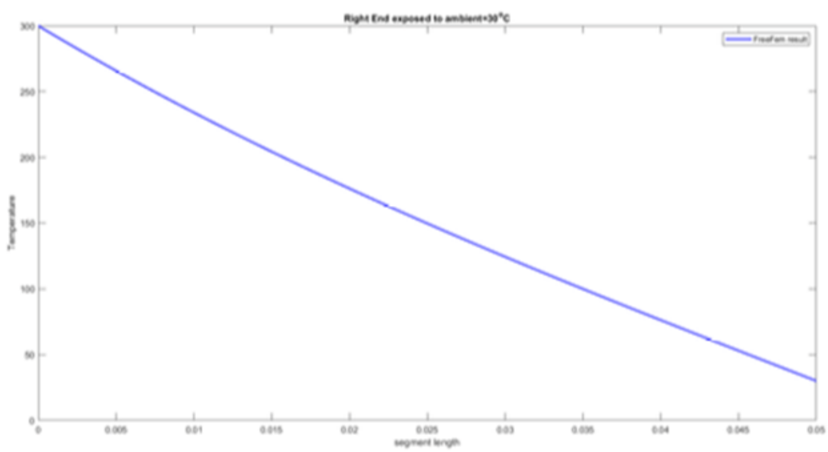


Figure 2.4

Chapter 3

2D Temperature distribution

3.1 Problem to solve

Consider a rectangular plate of dimensions $L_x \times L_y$, where heat conduction within the plate is governed by Laplace's equation:

$$\nabla^2 T = 0$$

This equation represents a steady-state condition where the temperature distribution $T(x, y)$ remains constant over time. The Laplace's equation describes a scenario where there are no internal heat sources, and the temperature field is determined solely by the boundary conditions.

The boundary conditions for this problem are as follows:

1. **Insulated Side:** At $x = 0$, the plate is insulated, implying that there is no heat flow across this side. Mathematically, this is expressed as $\dfrac{\partial T}{\partial x} = 0$.
2. **Constant Temperature Side:** At $x = L_x$, the temperature is maintained at 200°C. This condition is expressed as $T(L_x, y) = 200$°C.
3. **Constant Temperature Bottom Side:** At $y = 0$, the temperature is maintained at 150°C. This condition is expressed as $T(x, 0) = 150$°C.
4. **Constant Temperature Top Side:** At $y = L_y$, the temperature is maintained at 150°C. This condition is expressed as $T(x, L_y) = 150$°C.

The objective is to find the temperature distribution $T(x, y)$ within the plate, satisfying Laplace's equation and the specified boundary conditions.

3.2 Code

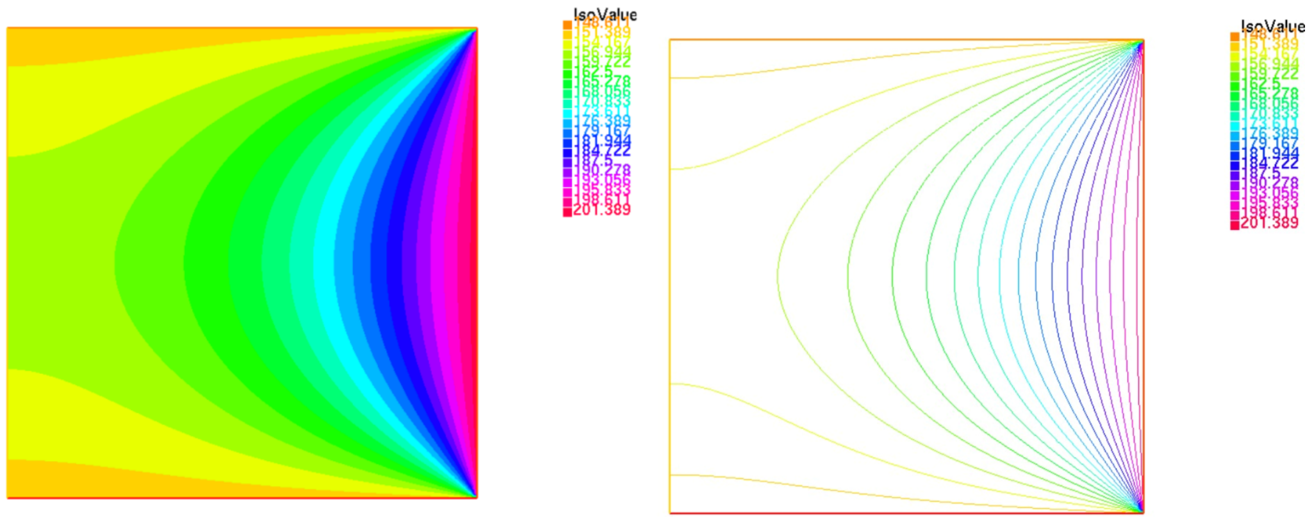
```
load  "msh3";

mesh Th = square (80,100) ;

ofstream file2 (" Results .dat") ;
fespace Vh ( Th , P1 ) ;
```

```
Vh U , v ;
solve Poisson (U , v ) = int2d ( Th ) ( dx ( U ) * dx ( v ) + ( dy ( U ) * dy ( v
    ) ) )
// on (1 , U =0)
 + on (2 , U =200)+on (1 , U =150)+on (3, U =150) ; // Dirichlet boundary
     condition
 plot (U , fill= true ) ;
```

3.3 Results



(a) Continuous plot

(b) Discrete plot

Figure 3.1: Temperature plots