AE332 – Modeling and Analysis Lab

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- **Comparison of the paths taken by the cannonball in atmosphere, in vacuum and artillery shell in atmosphere**

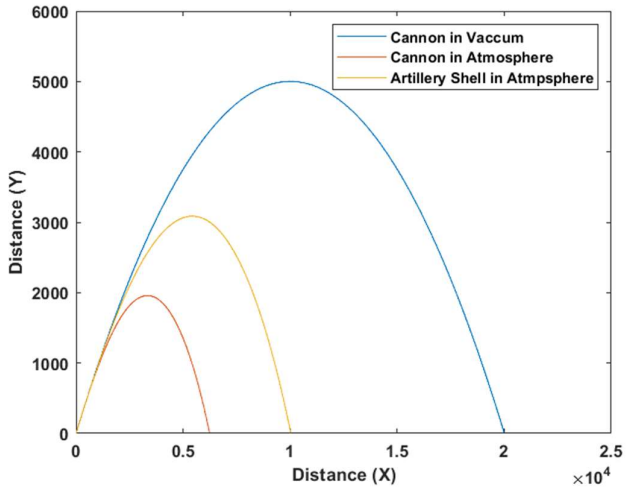
Matlab code:

```
run('SRdata.m')
height = atmsphr(:,1);
temp = atmsphr(:,2);
P = atmsphr(:,3);
density = atmsphr(:,4);
MN_array = NrmlFrc(:,1);
C_Nalpha_array = NrmlFrc(:,2);
x_CPfrctn_array = NrmlFrc(:,3);
% Time span
tspan = 0:0.1:200;
% Initial conditions
v = 443;
theta = pi/4;
% ODE options
tol1 = odeset('RelTol',1e-12,'AbsTol',1e-12);
% Initial state
z0 = [0; 0; pi/4; v*cos(theta); v*sin(theta); 0];
% Solve ODE
[t, z] = ode45(@(t,z)
rocket_ode(t,z,height,temp,density,MN_array,C_Nalpha_array,x_CPfrctn_array,MAjoff_
array,hAjoff_array,CAmatrix_joff), tspan, z0, tol1);
% Extract position
x = z(:,1);
y = z(:,2);
% Plot trajectory
plot(horizontal_position_1,altitude_1,horizontal_position_2, altitude_2,x,y)
xlabel('Distance (X)');
ylabel('Distance (Y)');
legend("Cannon in Atmosphere","Cannon in Vacuum","Artillery Shell in Atmpsphere")
% Max  range
max_range1 = max(horizontal_position_1);
max_range2 = max(horizontal_position_2);
max_range3 = max(x)
disp(['Cannon in Vaccum Range: ' num2str(max_range1) ' meters']);
disp(['Cannon in Atmosphere Range: ' num2str(max_range2) ' meters']);
disp(['Artillery Shell in Atmpsphere Range: ' num2str(max_range3) ' meters']);
```

```
% Rocket ODE function
function dzdt =
rocket_ode(~,z,height,temp,density,MN_array,C_Nalpha_array,x_CPfrctn_array,MAjoff_
array,hAjoff_array,CAmatrix_joff)
    % Constants
    shellL = 1.115;
    x_CoM= 0.70769;
    m=40;
    MoI=4.29;
    g = 9.8;
    S=pi*(0.101/2)^2;

    % State variables
    alpha = z(3,1) - atan(z(5,1)/z(4,1));
    tem = interp1(height, temp, z(2,1)/1000);
    Density = interp1(height, density, z(2,1)/1000);

    M = sqrt(z(4,1)^2 + z(5,1)^2)/sqrt(1.4*287*tem);
    C_Nalpha = interp1(MN_array, C_Nalpha_array, M);
    C_N = C_Nalpha * alpha;
    COP = interp1(MN_array, x_CPfrctn_array, M);
    x_CP = COP * shellL;
    F_N = 0.5 * C_N * S * Density * (z(4,1)^2 + z(5,1)^2);

   %  C_A1=interp2(hAjoff_array,MAjoff_array,CAmatrix_joff,z(2,1)/1000,M);
    [Hh,Mm]=meshgrid(MAjoff_array,hAjoff_array);
    Ca_f=griddedInterpolant(Hh',Mm',CAmatrix_joff);
    C_A1=Ca_f(M,z(2,1)/1000);

    F_A=0.5*C_A1*S*Density*(z(4,1)^2+z(5,1)^2);
dzdt(1,1)=z(4,1);
dzdt(2,1)=z(5,1);
dzdt(3,1)=z(6,1);
dzdt(4,1)= -(F_N/m)*sin(z(3,1))-(F_A/m)*cos(z(3,1));
dzdt(5,1)=(F_N/m)*cos(z(3,1))-(F_A/m)*sin(z(3,1))-g;
dzdt(6,1)=-F_N*abs(x_CP-x_CoM)/MoI;
end
```

Output:

```
Cannon in Vaccum Range:
20004.0367 meters
Cannon in Atmosphere Range:
6228.2372 meters
Artillery Shell in Atmpsphere
Range: 10025.6772 meters
```



- **Achieving 20 km in atmosphere**

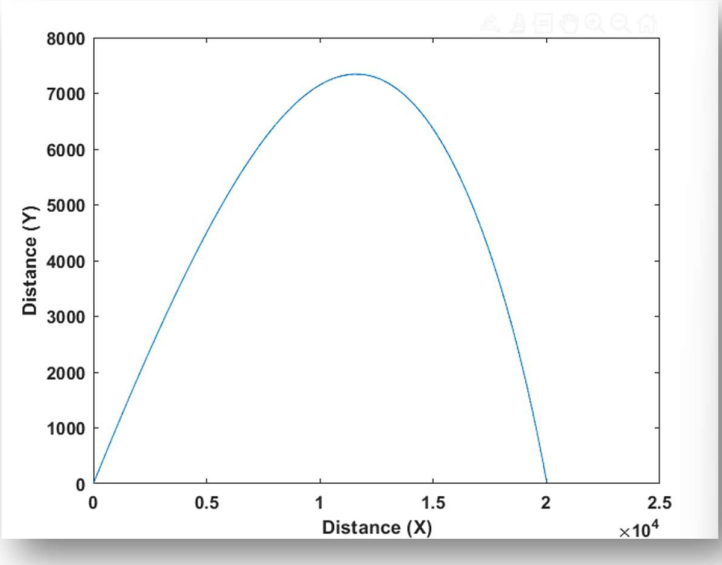
Matlab code:

```
% Time span
tspan = 0:0.1:200;
% Initial conditions
v = 849.94; m = 308.65;
theta = pi/4;
tol1 = odeset('RelTol',1e-12,'AbsTol',1e-12);
z0 = [0; 0; pi/4; v*cos(theta); v*sin(theta); 0];
[t, z] = ode45(@(t,z)
rocket_ode(t,z,height,temp,density,MN_array,C_Nalpha_array,x_CPfrctn_array,MAj
off_array,hAjoff_array,CAmatrix_joff), tspan, z0, tol1);
x = z(:,1);
y = z(:,2);
% Plot trajectory
plot(x,y)
xlabel('Distance (X)');
ylabel('Distance (Y)');
max_height = max(z(:,2));
max_range = max(z(:,1));
disp(['Range: ' num2str(max_range) ' meters']);
```

Output:

```
Range: 20070.215 meters
```

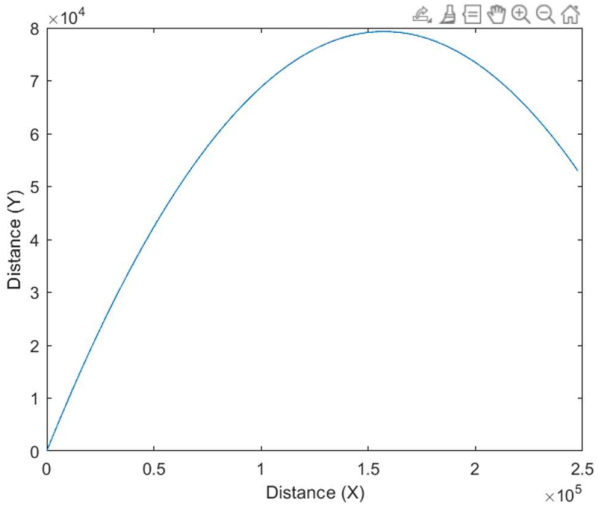
Graph:

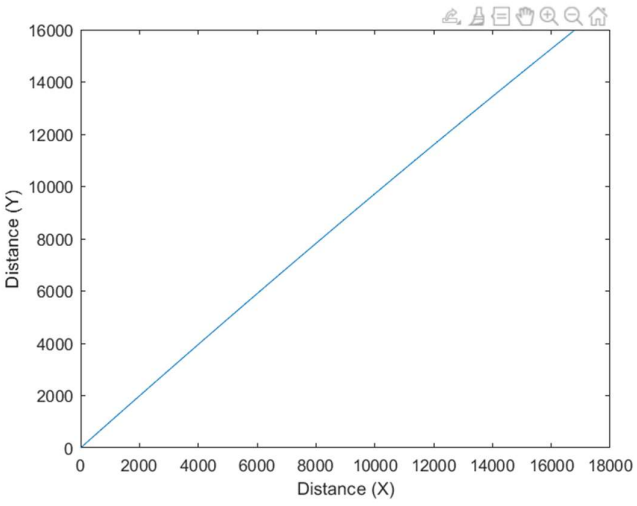


For same mass of 308.65 kg Cannon Ball required 750 m/s velocity to achieve 20 km in atmosphere whereas Artillery Shell required only 484 m/s because of streamlined shape of shell which is aerodynamically better to reduce drag.

- With the same elevation angle, by increasing velocity higher and higher shell goes to

Range = 247788.9354 at velocity of 1931.584 m/s	Range = 16766.828 at velocity of 1931.585 m/s





- Rocket in Atmosphere:

Matlab code:

```
run('SRdata.m')
height = atmsphr(:,1);
temp = atmsphr(:,2);
P = atmsphr(:,3);
density = atmsphr(:,4);
MN_array = NrmlFrc(:,1);
C_Nalpha_array = NrmlFrc(:,2);
x_CPfrctn_array = NrmlFrc(:,3);
tm_array = thrustvars(:,1);
thrustKN_array = thrustvars(:,2);
massexpelled_array = thrustvars(:,3);

% Time span
tspan = 0:0.1:200;
% Initial conditions
v = 443;
theta = pi/4;
% ODE options
tol1 = odeset('RelTol',1e-12,'AbsTol',1e-12);
% Initial state
```

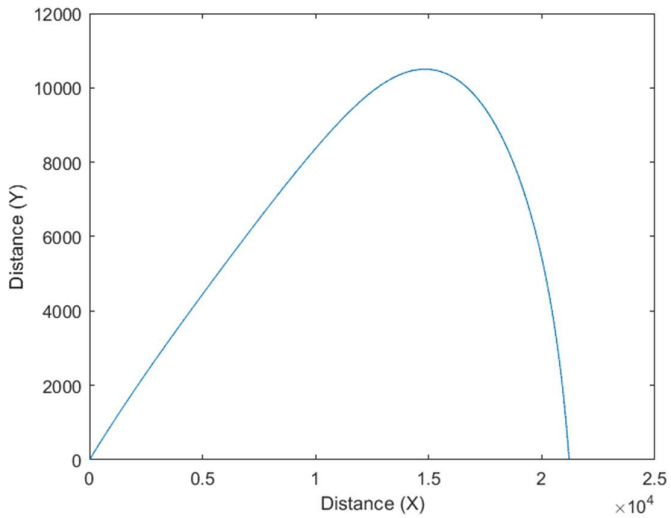
```
z0 = [0; 0; pi/4; v*cos(theta); v*sin(theta); 0];
% Solve ODE
[t, z] = ode45(@(t,z)
rocket_ode(t,z,height,temp,density,MN_array,P,C_Nalpha_array,x_CPfrctn_arr
ay,MAjoff_array,hAjoff_array,CAmatrix_joff,MAjon_array,hAjon_array,CAmatri
x_jon,tm_array,thrustKN_array,massexpelled_array), tspan, z0, tol1);
% Extract position
x = z(:,1);
y = z(:,2);
% Plot trajectory
plot(x, y)
xlabel('Distance (X)');
ylabel('Distance (Y)');
% Max height and range
max_height = max(z(:,2));
max_range = max(z(:,1));
disp(['Max Range: ' num2str(max_range) ' meters']);
disp(['corresponding Max Height: ' num2str(max_height) ' meters']);
```

```
function dzdt =
rocket_ode(t,z,height,temp,density,MN_array,P,C_Nalpha_array,x_CPfrctn_arr
ay,MAjoff_array,hAjoff_array,CAmatrix_joff,MAjon_array,hAjon_array,CAmatri
x_jon,tm_array,thrustKN_array,massexpelled_array)
    % Constants
    SRL = 2.277;
    StructuralMass = 39.229;
    InitialpropellantMass = 48.771;
    g = 9.8;

    % Parameters
    Rnom = 0.207/2;
    S = pi*Rnom^2;
    R_E = 0.125/2;
    A_E = pi*R_E^2;
    t_thrusting = 11.778;

    % State variables
    alpha = z(3,1) - atan(z(5,1)/z(4,1));
    tem = interp1(height, temp, z(2,1)/1000);
    Density = interp1(height, density, z(2,1)/1000);
    p = interp1(height, P, z(2,1)/1000);

    M = sqrt(z(4,1)^2 + z(5,1)^2)/sqrt(1.4*287*tem);
    C_Nalpha = interp1(MN_array, C_Nalpha_array, M);
    C_N = C_Nalpha * alpha;
    COP = interp1(MN_array, x_CPfrctn_array, M);
    x_CP = COP * SRL;
    F_N = 0.5 * C_N * S * Density * (z(4,1)^2 + z(5,1)^2);

%  C_A1=interp2(hAjoff_array,MAjoff_array,CAmatrix_joff,z(2,1)/1000,M);
 [Hh,Mm]=meshgrid(MAjoff_array,hAjoff_array);
 Ca_f=griddedInterpolant(Hh',Mm',CAmatrix_joff);
```

```
 C_A1=Ca_f(M,z(2,1)/1000);


% C_A2=interp2(hAjon_array,MAjon_array,CAmatrix_jon,z(2,1)/1000,M);
 [HhA,MmB]=meshgrid(hAjon_array,MAjon_array);
 Ca_f=griddedInterpolant(HhA',MmB',CAmatrix_jon');
 C_A2=Ca_f(M,z(2,1)/1000);

 if t<=t_thrusting
 T_vaccum=interp1(tm_array,thrustKN_array,t)*1000;
 massexpelled=interp1(tm_array,massexpelled_array,t);
 mP=InitialpropellantMass-massexpelled;
 m=StructuralMass+mP;
 x_CoM = (43.681 + 1.4735*mP)/(39.229+mP);
 MoI = 16.318 + 39.229*(1.1135 - x_CoM).^2 + (0.0979 + (1.4735 -
x_CoM).^2).*mP;
 CA=C_A2;
 T=T_vaccum-p*A_E;
else
 CA=C_A1;
 T=0;
 x_CoM= 1.1135;
 m=StructuralMass;
 MoI=16.318;
 end
 F_A=0.5*CA*S*Density*(z(4,1)^2+z(5,1)^2);

dzdt(1,1)=z(4,1);
dzdt(2,1)=z(5,1);
dzdt(3,1)=z(6,1);
dzdt(4,1)= -(F_N/m)*sin(z(3,1))-(F_A/m)*cos(z(3,1))+(T*cos(z(3,1)))/m;
dzdt(5,1)=(F_N/m)*cos(z(3,1))-(F_A/m)*sin(z(3,1))-g+(T*sin(z(3,1)))/m;
dzdt(6,1)=-F_N*abs(x_CP-x_CoM)/MoI;
end
```

Output:

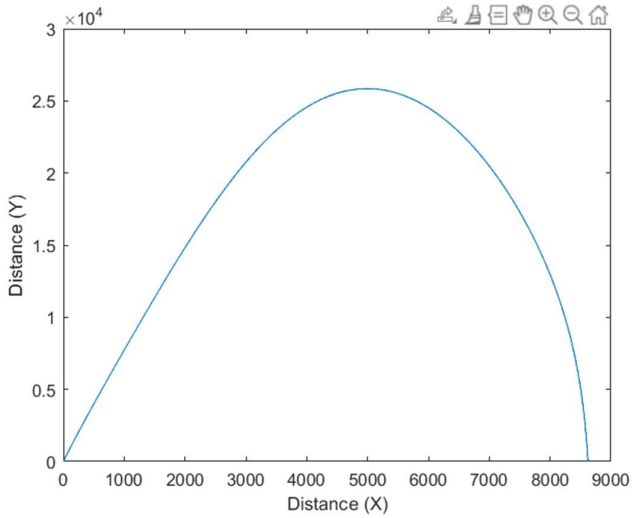
```
Max Range: 21204.2009
meters
corresponding Max Height:
10500.1455 meters
```



- Determine the angles of launch at which you can get the highest altitude

For this we have to reduce horizontal velocity as much as possible. From simulation I got, v = 443; theta = 83*pi/180;

Output:



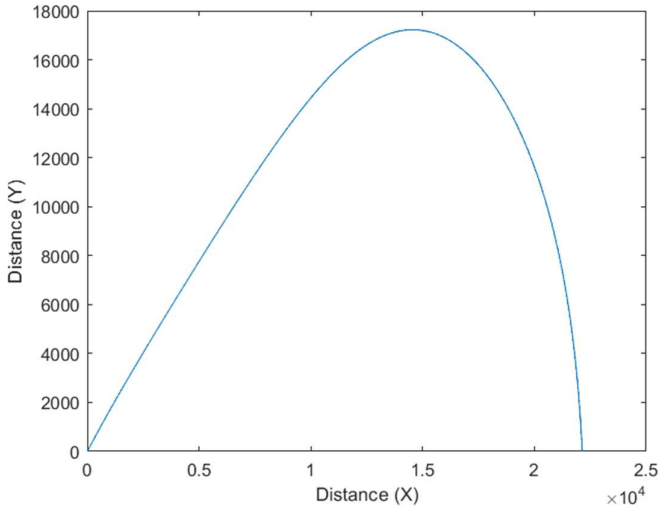

```
corresponding Max Height:
25847.5824 meters
corresponding Max Range:
8622.0284 meters
```

- Determine the angles of launch at which you can get the maximum range

For this we know that maximum range attain when angle is 45 but here some different will be there because thrust and drag. From simulation I got,

v = 443; theta = 60*pi/180;

Output:




```
Max Range: 22139.8456
meters
corresponding
Max Height: 17225.1288
meters
```