AE 332: Modeling and Analysis Lab II

A Report submitted

For internal assessment for Coarse

Modeling and Analysis Lab

in

Aerospace Engineering

by

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Contents

Chapter 1

One dimensional wave propogation

1.1 Problem To be Slove

Consider the one-dimensional advection equation with the following form:

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \tag{1.1}$$

where c is the propagation speed and is selected to be 300 m/s (near the speed of sound).

At $t = 0$, a half-sinusoidal disturbance has been created. The initial condition is specified as:

$$u(x,0) = u(x,0) = \begin{cases} 0 & \text{for } 0 < x < 50 \\ 100\sin\left(\frac{\pi(x-50)}{60}\right) & \text{for } 50 < x < 110 \\ 0 & \text{for } 110 < x < 300 \end{cases}$$

The boundary conditions are $u(0,t) = 0$ and $u(L,t) = 0$. We want to find and plot the solution at $t = 0.45$ seconds using the FTBS method.

To solve the problem, we'll consider three different step sizes: $\Delta x = 5$, $\Delta x = 2$, and $\Delta x = 1$, while ensuring that $\Delta t < \frac{\Delta x}{c}$. We will compare the solutions for these step sizes.

1.2 Matlab Code

```
%Linear convection problem
% FTBS (Forward in time Backward in space)
clear all
clc
Lx=300; % Length of the domain

% number of points in x direction
dx =5; % For Part a
% dx =2 ; % For Part b
% dx =1 ; % For Part c

nx =( Lx / dx ) +1; % grid size
```

```
x=0:dx:Lx;
dt = 0.01;
% dt = 0.006;% For Part b
% dt = 0.003;% For Part c
nt =45;% number of time steps
c=300; % convection velocity
u=zeros;
% Define initial condition
for i=1:nx
    if x(i)>=50 && x(i)<110
        u(i)=100*sin(pi*((x(i)-50)/60));
    else
        u(i)=0;
    end
end
for it =1:nt
    un=u;
    u(1) = 0; u(nx-1)= 0;
    for i=2:nx-1
      u(i) =un(i)-(c*dt/dx)*(un(i)-un(i-1));%linear convection problem
    end
    plot(x,un)
    xlabel('x')
    ylabel('u')
    legend(['time=' num2str(it*dt)])
    pause(0.04)
end
```

1.3 Result

In this section, we analyze the stability of the wave equation with different step sizes for distance (Δx) and time (Δt). We present the results for three different scenarios and discuss the observed behavior.

1.3.1 Scenario 1: $\Delta x = 5$ and $\Delta t = 0.016$

In the first scenario, we use a relatively large step size ($\Delta x = 5$) and a corresponding time step ($\Delta t = 0.016$). The stability condition is satisfied, and the result at $t = 0.45$ is presented as shown in figure 2.1.

1.3.2 Scenario 2: $\Delta x = 2$ and $\Delta t = 0.006$

In the second scenario, we use a smaller step size ($\Delta x = 2$) and a corresponding time step ($\Delta t = 0.006$). The stability condition is satisfied, and the result at $t = 0.45$ is presented as shown in figure 2.2.

1.3.3 Scenario 3: $\Delta x = 1$ and $\Delta t = 0.003$

In the third scenario, we use an even smaller step size ($\Delta x = 1$) and a corresponding time step ($\Delta t = 0.003$). The stability condition is satisfied, and the result at $t = 0.45$ is presented as shown in figure 2.3.

1.3.4 Discussion

As we observe the results in the three scenarios, we can see that as the step size is reduced (i.e., δx and δt become smaller), the solution becomes more stable, resembling the nature of the wave equation. The initial condition propagates along the x-axis, resembling linear convection.

1.3.5 Figure of Que-1

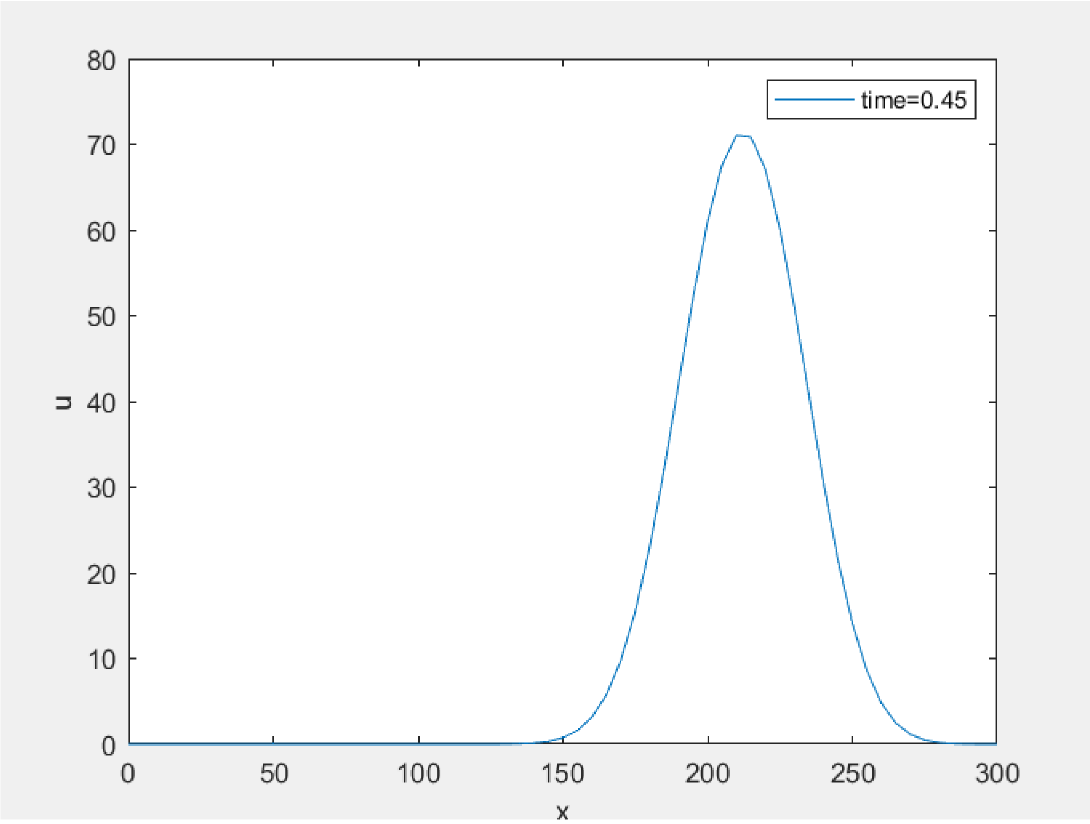

Figure 1.1: Result at $t = 0.45$ with $\Delta x = 5$ and $\Delta t = 0.016$.

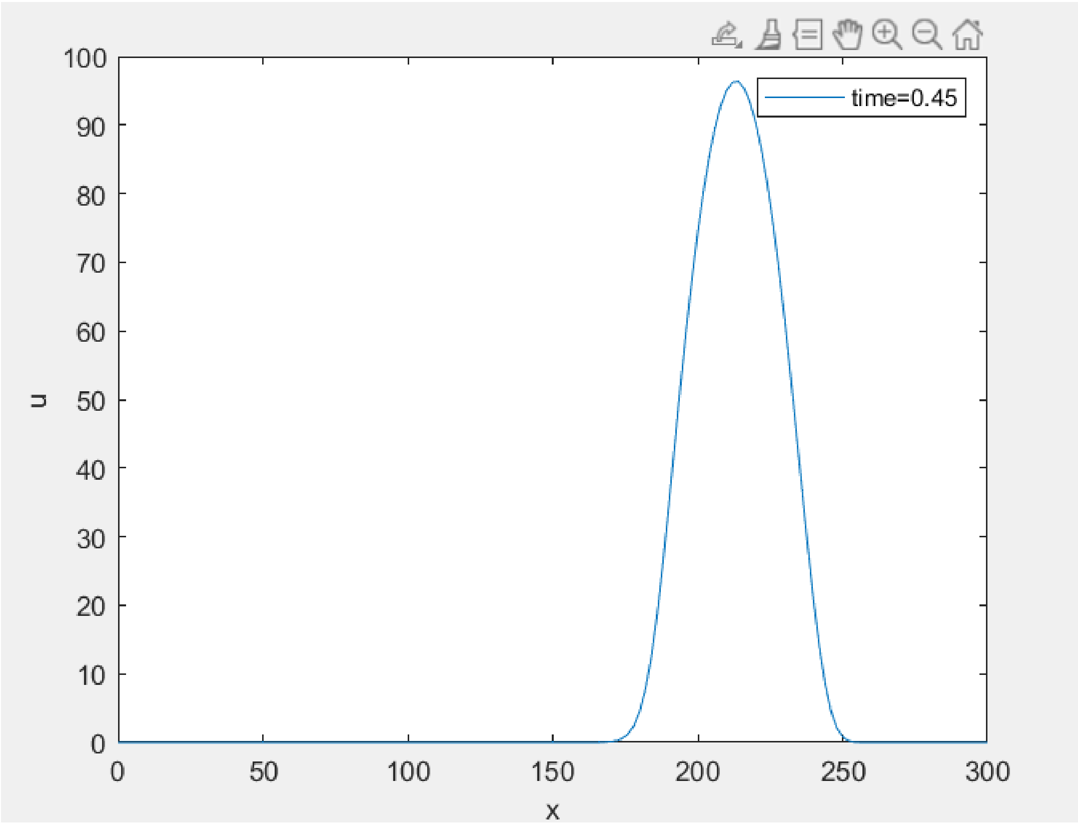

Figure 1.2: Result at $t = 0.45$ with $\Delta x = 2$ and $\Delta t = 0.006$.

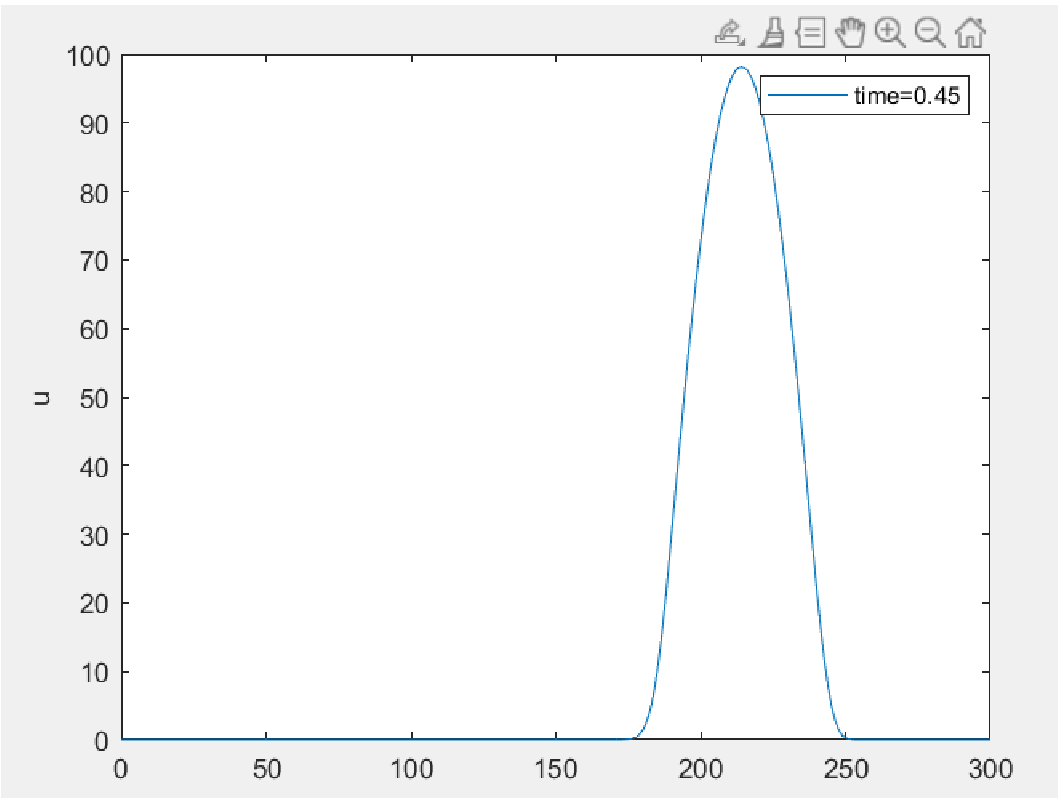

Figure 1.3: Result at $t = 0.45$ with $\delta x = 1$ and $\Delta t = 0.003$.

Chapter 2

Nonlinear Convection Equation

2.1 Problem To be Slove

We consider the nonlinear convection equation with the following form:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0 \tag{2.1}$$

The initial velocity distribution representing a compression wave is given by:

$$u(x,0) = \begin{cases} 1 & \text{for } x < 0.25 \\ 1.25 - x & \text{for } 0.25 < x < 1.25 \\ 0 & \text{for } x > 1.25 \end{cases}$$

We are solving the nonlinear convection equation for wave propagation within a domain of $0.0 < x < 4.0$ up to $t = 6.0$ seconds. The initial wave is compressed (steepens) with time and subsequently forms a shock wave. Within the specified time and space intervals, no shock reflection occurs. Therefore, the boundary conditions are simply specified as:

$$u(0.0,t) = 1.0 \quad \text{and} \quad u(4.0,t) = 0.0$$

We will use the FTBS (Forward in Time, Backward in Space) method with a spatial step of 0.05 meters to obtain the solutions for the following cases.

2.2 Matlab code

```
%Linear convection problem
% FTBS (Forward in time Backward in space
clear all
clc
Lx=4; % Length of the domain
nx =81;% number of points in x direction
dx = 4./(nx-1);%grid size
x=0:dx:Lx;
```

```
dt = 0.01; % case-1
% dt = 0.025; % case-2
% dt = 0.05; % case-3
% dt = 0.1; % case-4

% number of time steps
nt =601; % case-1
% nt=240; % case-2
% nt=120; % case-3
% nt=60; % case-4

% c=300; % convection velocity
u=zeros;
% Define initial condition
for i=1:nx
    if x(i)<0.25
        u(i)=1;
    else if x(i)>=0.25 && x(i)<=1.25
        u(i)=1.25-x(i);
    else
        u(i)=0;
    end
    end
end
for it =1:nt
    un=u;
    u(1) = 1; u(nx-1)= 0;
    for i=2:nx-1
      u(i) =un(i)-(u(i)*dt/dx)*(un(i)-un(i-1));%Non-linear convection problem
    end
    plot(x,un)
    xlabel('x')
    ylabel('u')
    legend(['time=' num2str(it*dt)])
    pause(0.04)
end
```

2.3 Results

Here You can see result for all for cases in respective figure written in caption and for $\Delta t = 0.1$ s the code experiences instability, and we cannot plot the solution for $t = 0$, $t = 2$, $t = 4$, or $t = 6$ seconds.

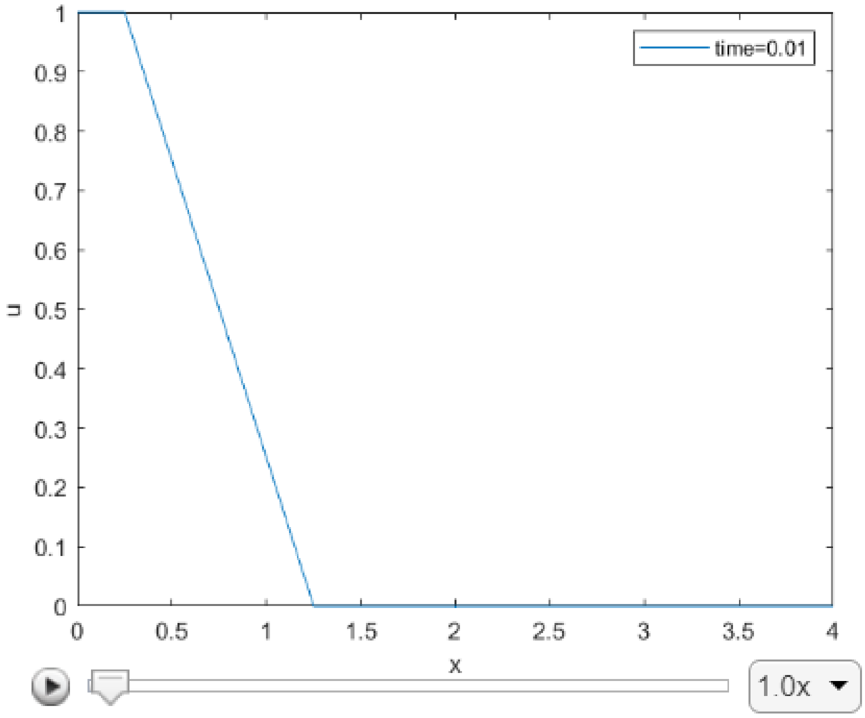

Figure 2.1: Result for $\Delta t = 0.01$ s at $t = 0$ s

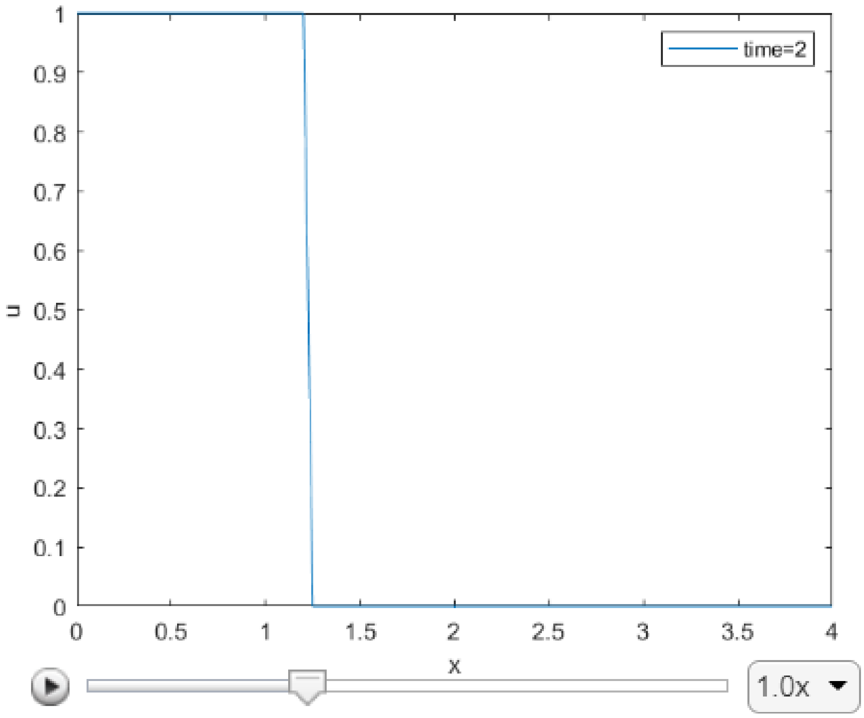

Figure 2.2: Result for $\Delta t = 0.01$ s at $t = 2$ s

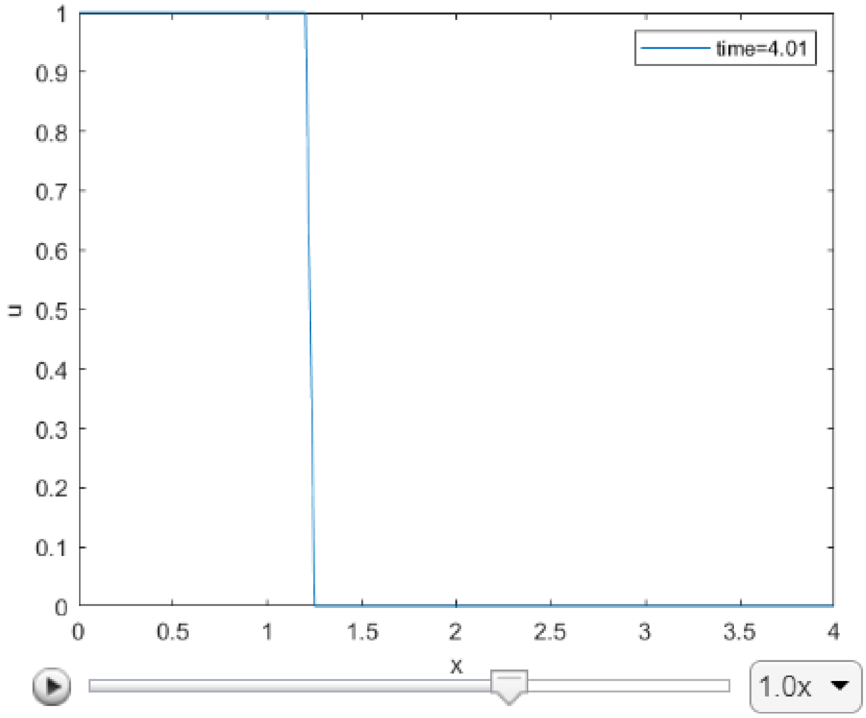

Figure 2.3: Result for $\Delta t = 0.01$ s at $t = 4$ s

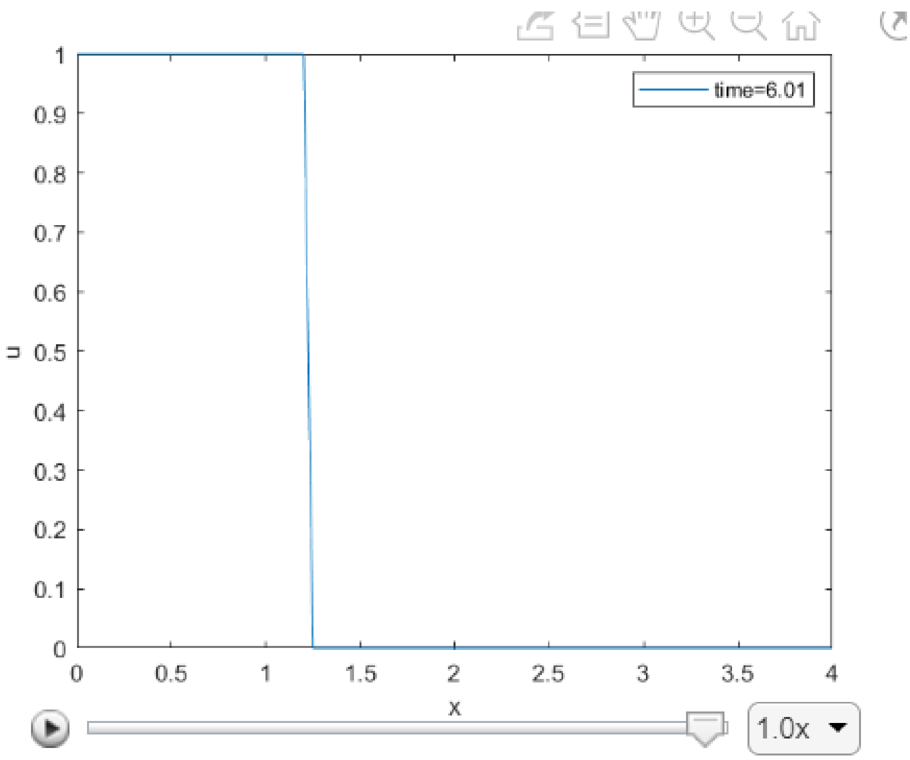

Figure 2.4: Result for $\Delta t = 0.01$ s at $t = 6$ s

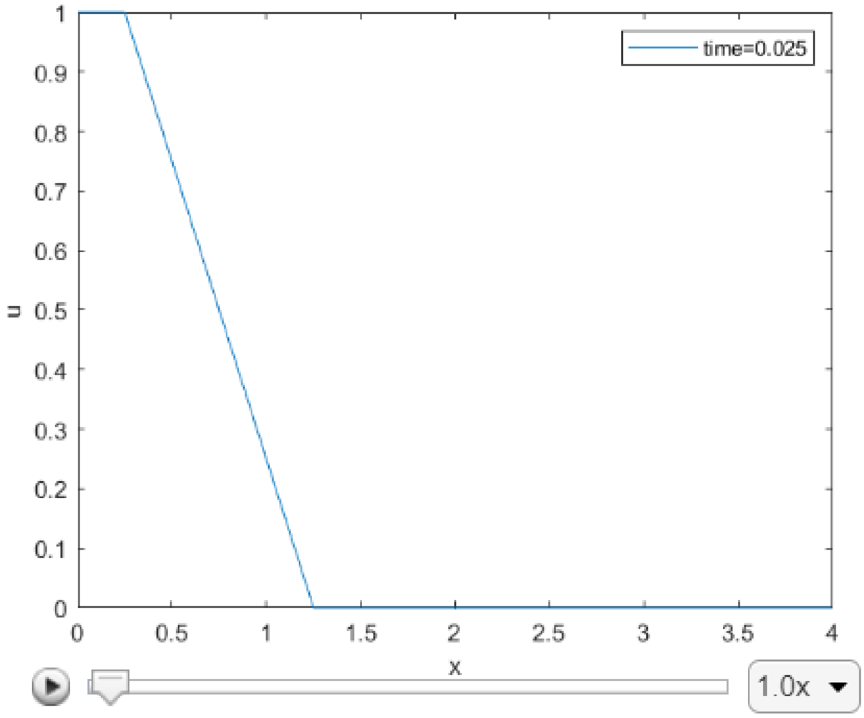

Figure 2.5: Result for $\Delta t = 0.025$ s at $t = 0$ s

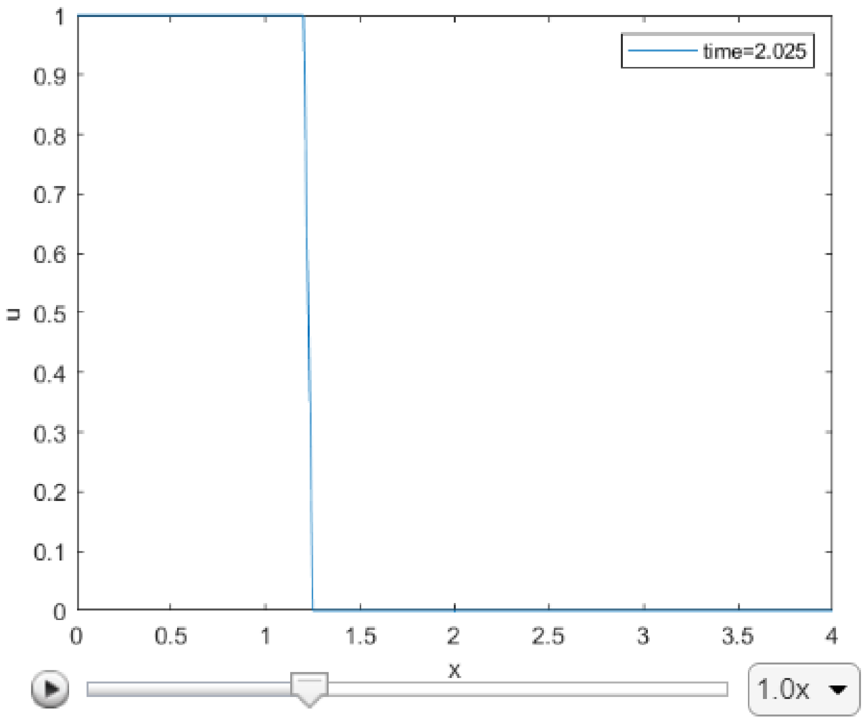

Figure 2.6: Result for $\Delta t = 0.025$ s at $t = 2$ s

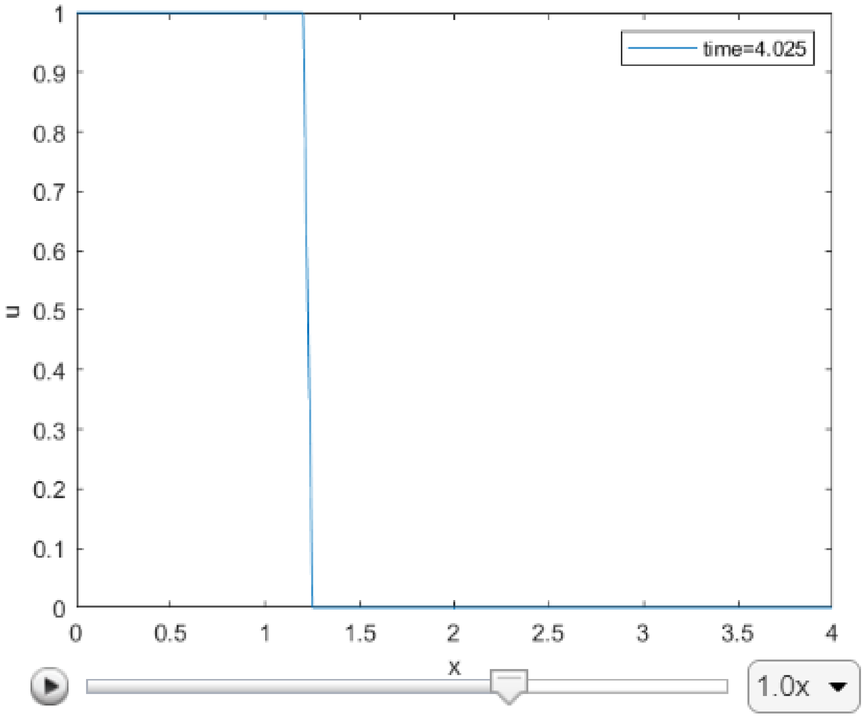

Figure 2.7: Result for $\Delta t = 0.025$ s at $t = 4$ s

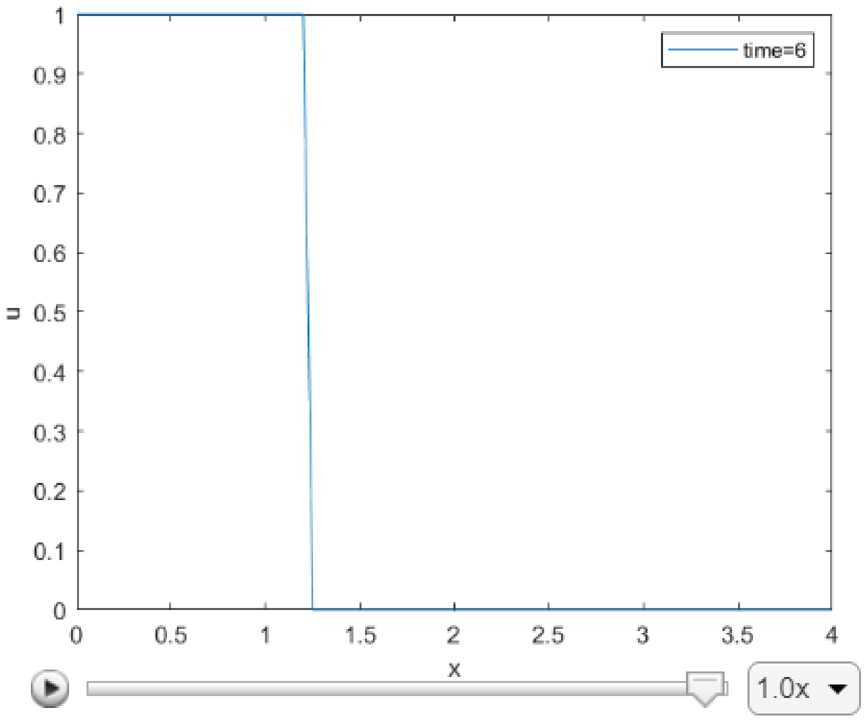

Figure 2.8: Result for $\Delta t = 0.025$ s at $t = 6$ s

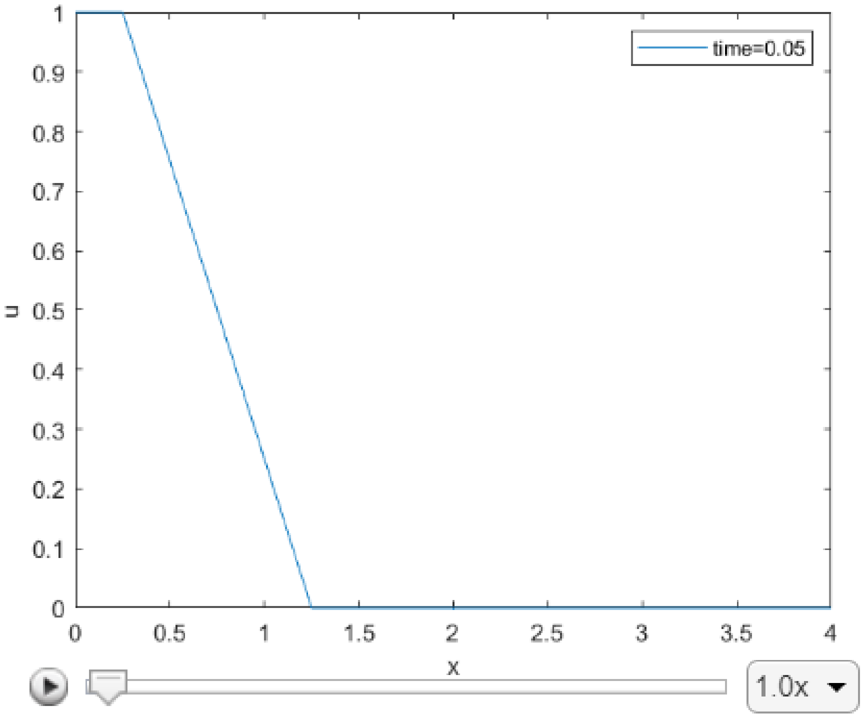

Figure 2.9: Result for $\Delta t = 0.05$ s at $t = 0$ s

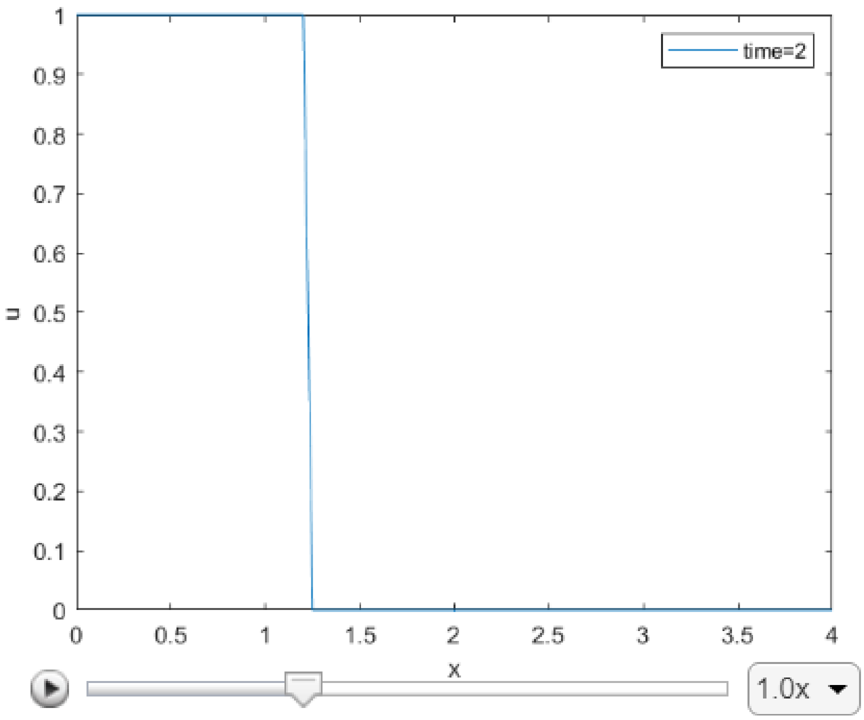

Figure 2.10: Result for $\Delta t = 0.05$ s at $t = 2$ s

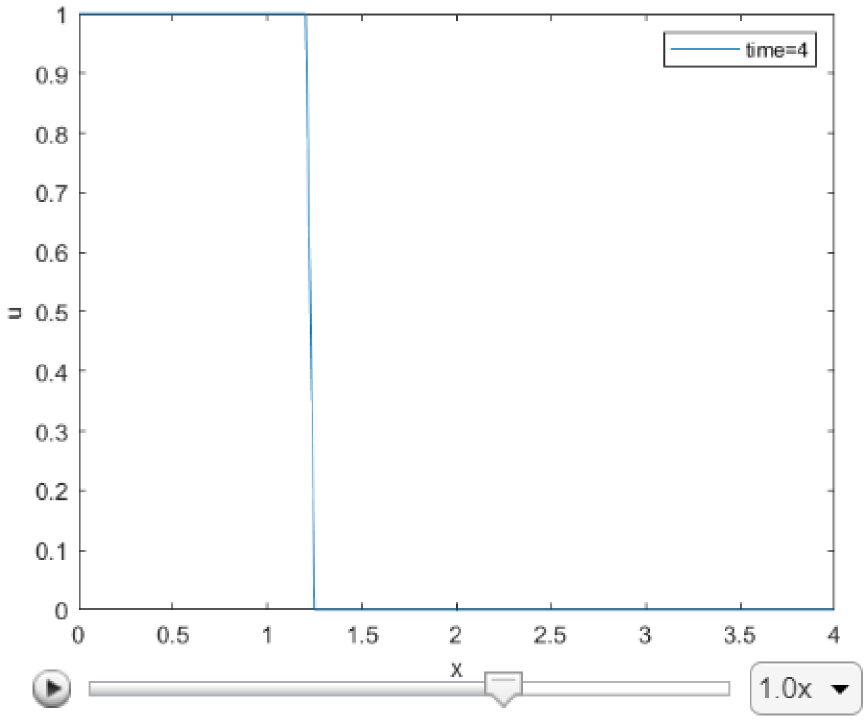

Figure 2.11: Result for $\Delta t = 0.05$ s at $t = 4$ s

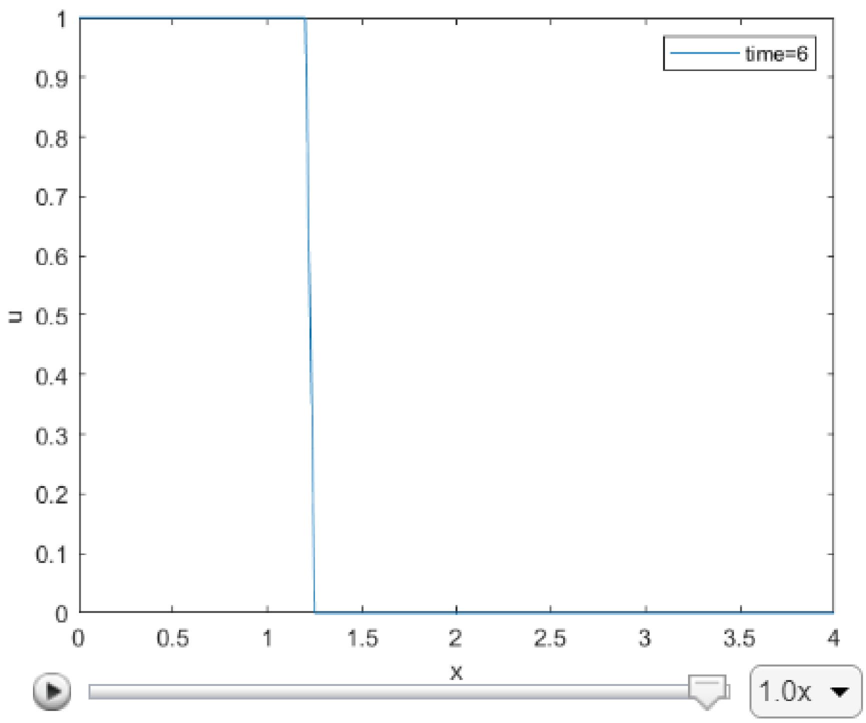

Figure 2.12: Result for $\Delta t = 0.05$ s at $t = 6$ s

Chapter 3

Heat diffusion Equation

We are solving the heat diffusion equation for a plate of length $L = 1.0$ cm with a thermal diffusivity $v = 0.01$ cm^2/s. The initial temperature distribution is specified as follows:

$$u(x,0) = \begin{cases} 200x & \text{for } x < 0.5 \\ 200(1-x) & \text{for } 0.5 < x < 1 \end{cases}$$

The heat source is suddenly turned off. We will solve the heat diffusion equation using the Finite Difference Method (FDM). The equation to solve is given by:

$$\frac{\partial Q}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \tag{3.1}$$

Where:

$$\begin{aligned} Q &: \text{Heat energy} \\ \alpha &: \text{Thermal diffusivity} \\ u &: \text{Temperature distribution} \\ x &: \text{Spatial coordinate} \\ t &: \text{Time} \end{aligned}$$

We are interested in plotting the temperature distribution of the plate at different time points, specifically at $t = 0.5$, $t = 1$, $t = 2$, $t = 3$, $t = 4$, $t = 5$, and $t = 10$ seconds.

3.1 Matlab Code

```
%Linear convection problem
% FTBS (Forward in time Backward in space
clear all
clc
Lx=1; % Length of the domain
dx=0.01;
nx=(Lx/dx)+1;%grid size
x=0:dx:Lx;
```

```
dt = 0.004;
%nt =2500;% number of time steps for 10 seconds
nt=1000;% nt =40 number of time steps for 4 seconds
nu=0.01; % convection velocity(nue)

%cdt/dx^2<0.5
u=zeros;
% Define initial condition
for i=1:nx
     if x(i)<0.5
        u(i)=200*x(i);
     else
         u(i)=200*(1-x(i));
     end
end
for it =1:nt
    un=u;
    for i=2:nx-1
      u(i)=un(i)+nu*dt/dx^2*(un(i+1)-2*un(i)+un(i-1));%non linear convection
          problem
    end
    plot(x,un)
    xlabel('x')
    ylabel('u')
    legend(['time=' num2str(it*dt)])
  pause(0.04)
end
```

3.1.1 Results for $\delta t = 0.004$ s

The analytical solution can be compared to the values obtained from the graph, from which we can conclude that the Finite Difference Method (FDM) yields good approximations. Moreover, as time passes, the graph spreads out in the domain, validating the diffusion equation.

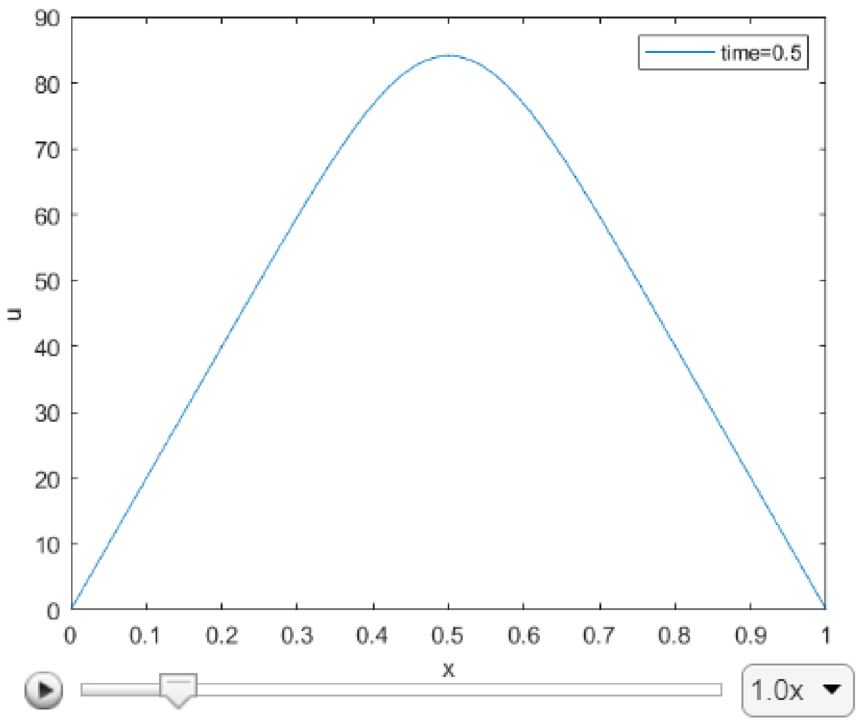

Figure 3.1: Result for $t = 0.5$ s.

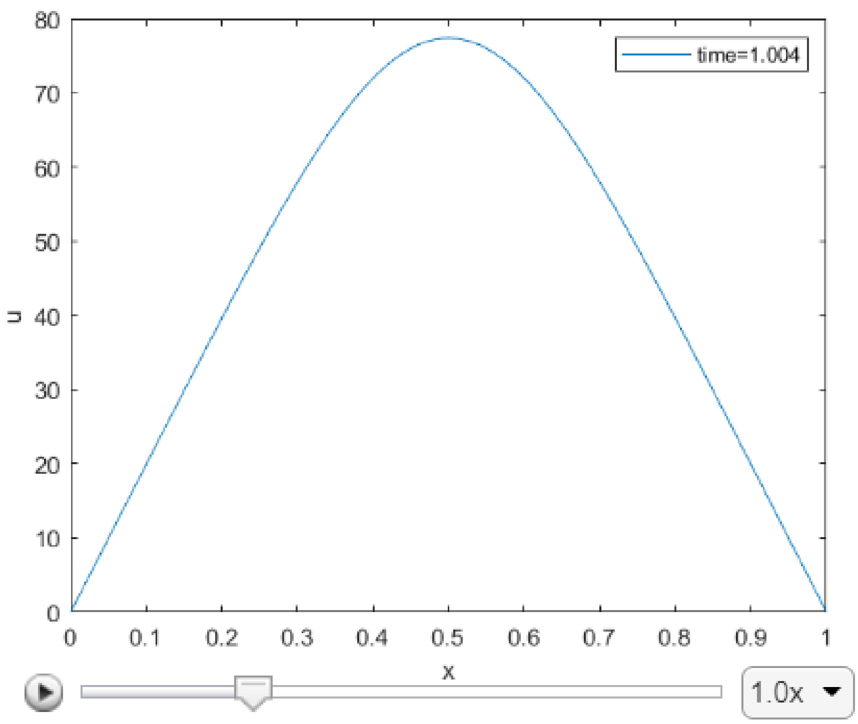

Figure 3.2: Result for $t = 1$ s.

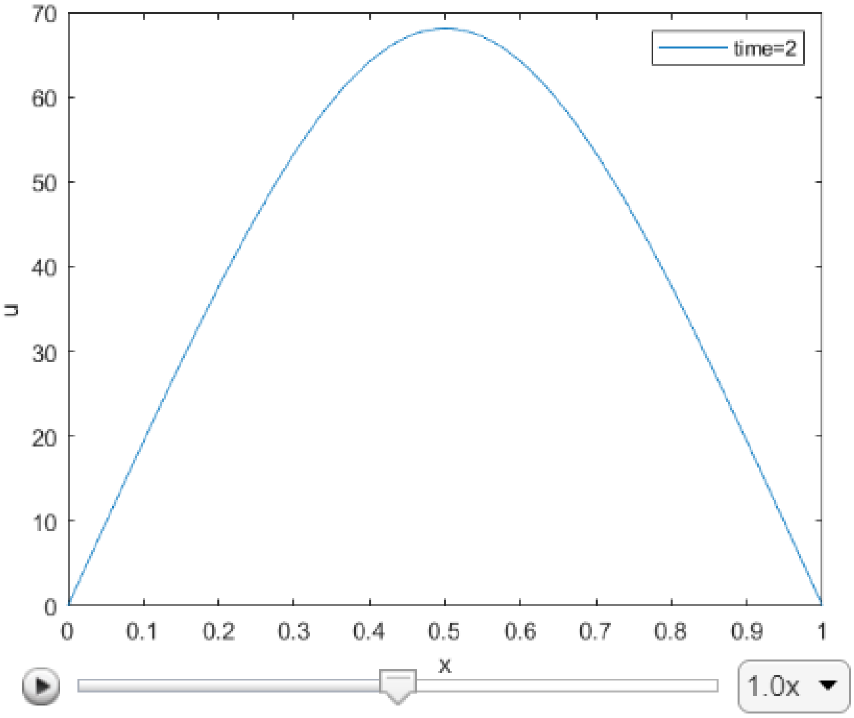

Figure 3.3: Result for $t = 2$ s.

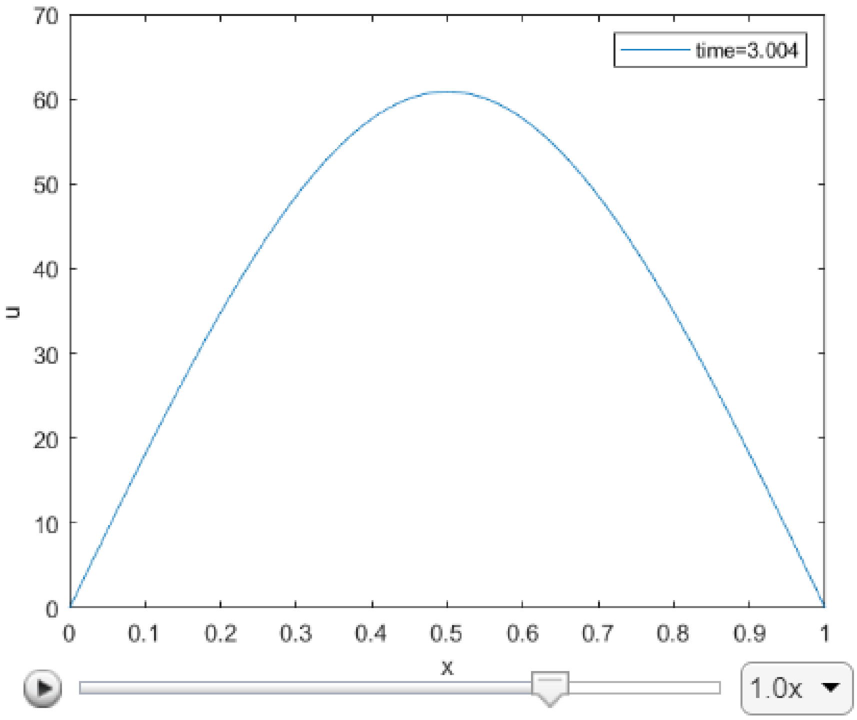

Figure 3.4: Result for $t = 3$ s.

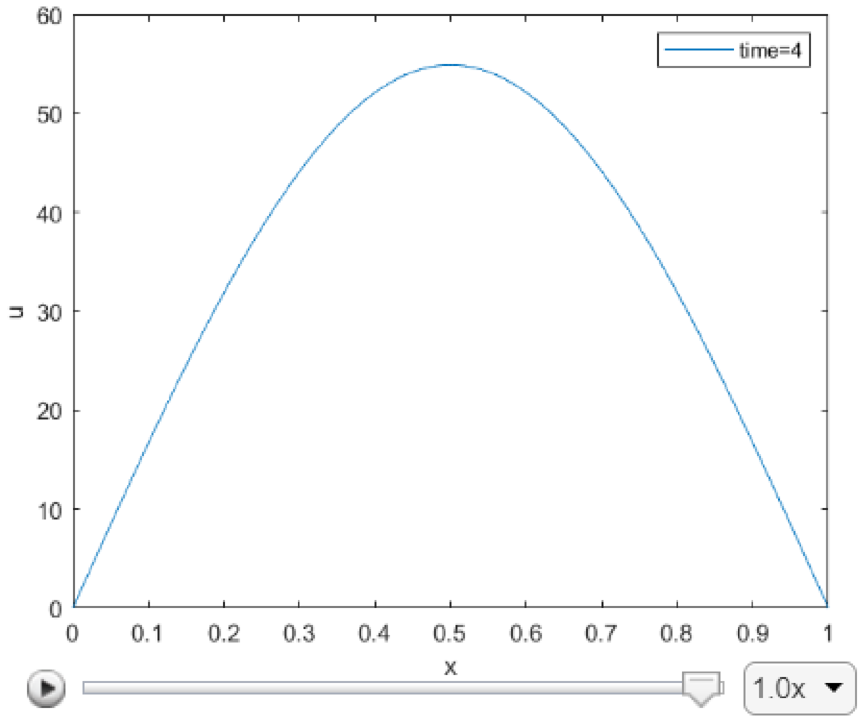

Figure 3.5: Result for $t = 4$ s.

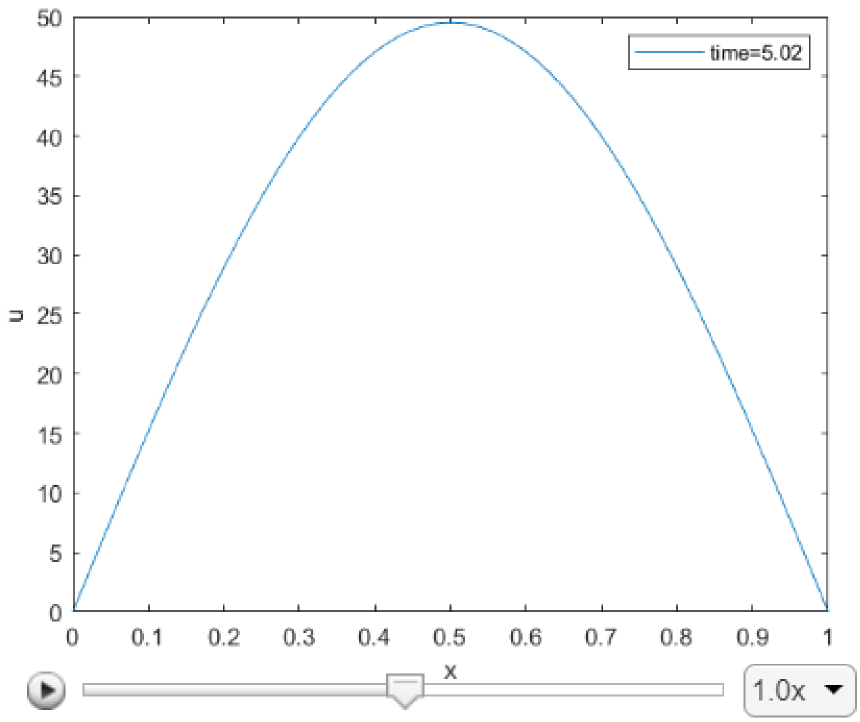

Figure 3.6: Result for $t = 5$ s.

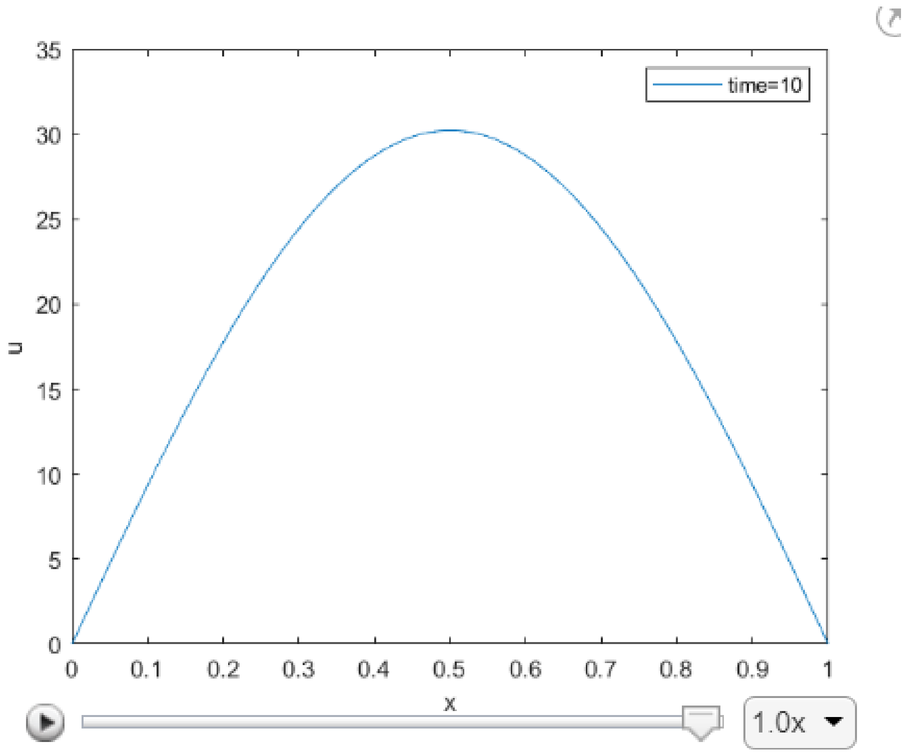

Figure 3.7: Result for $t = 10$ s.

Chapter 4

Motion of fluid between 2 infinite plates

4.1 Problem to Solve

We are studying the flow between two parallel plates extended to infinity, with a distance of h apart. The fluid within the plates has a kinematic viscosity of 0.000217 m^2/s and a density of 800 kg/m^3. The upper plate is stationary, and the lower plate is suddenly set in motion with a constant velocity of 40 m/s. The spacing h is 4 cm.

A constant streamwise pressure gradient $\frac{dp}{dx}$ is imposed within the domain at the instant motion starts. The governing equation, reduced from the Navier-Stokes equation, is given by:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{dp}{dx} \quad (4.1) \tag{4.1}$$

Where:

u : Velocity

ν : Kinematic viscosity

ρ : Density

p : Pressure

x : Spatial coordinate

y : Spatial coordinate

t : Time

(a) Use the FTBS (Forward in Time, Backward in Space) explicit scheme with a time step of 0.002 seconds to compute the velocity within the domain for the following values of $\frac{dp}{dx}$: (I) $\frac{dp}{dx} = 0.0$ N/m^2 (II) $\frac{dp}{dx} = 20000.0$ N/m^2 (III) $\frac{dp}{dx} = 30000.0$ N/m^2.

Print the solutions at time levels of 0.0, 0.18, 0.36, 0.54, 0.72, 0.9, and 1.08 seconds. Plot the velocity profiles at time levels of 0.0, 0.18, and 1.08 seconds.

4.2 Matlab Code

```
%Linear convection problem
% FTBS (Forward in time Backward in space
clear all
clc
Lx=0.04; % Length of the domain
dx=0.001;
nx=(Lx/dx)+1;%grid size
x=0:dx:Lx;
dt = 0.002;
nt=750;
nu=0.000217; % convection velocity(nue)
dpdx = 0.0;
% dpdx = 20000; % Case-2
% dpdx = -30000; % Case-3
density=800;
c=-dpdx/density;
u=zeros;
% Define initial condition
for i=1:nx
     if x(i)==0
        u(i)=40;
     else
         u(i)=0;
     end
end
for it =1:nt
    un=u;
    for i=2:nx-1
      u(i)=un(i)+nu*dt/dx^2*(un(i+1)-2*un(i)+un(i-1))+c*dt;%non linear convection
          problem
    end
    plot(x,un)
    xlabel('x')
    ylabel('u')
    legend(['time=' num2str(it*dt)])
  pause(0.04)
end
```

4.3 Results

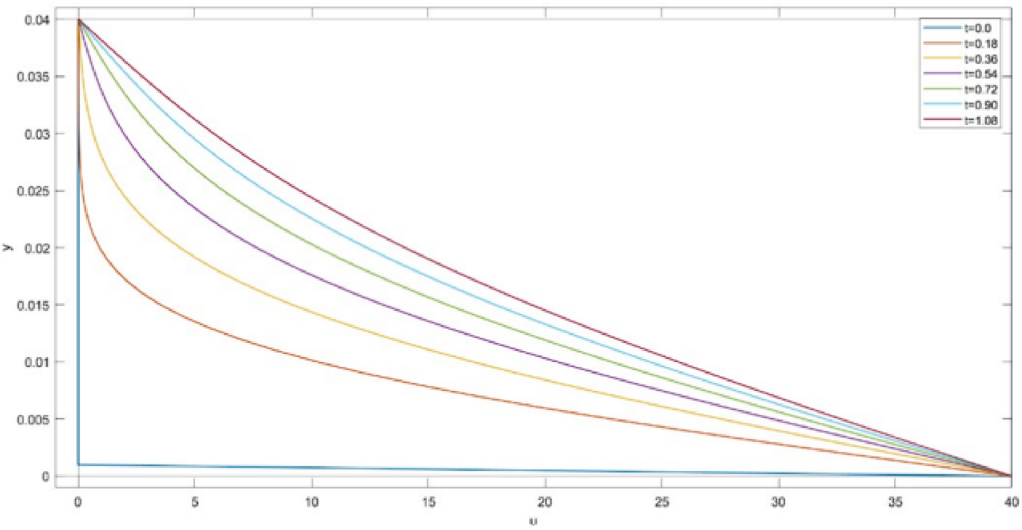

Figure 4.1: Result for $\frac{dp}{dx} = 0$ N/m^2.

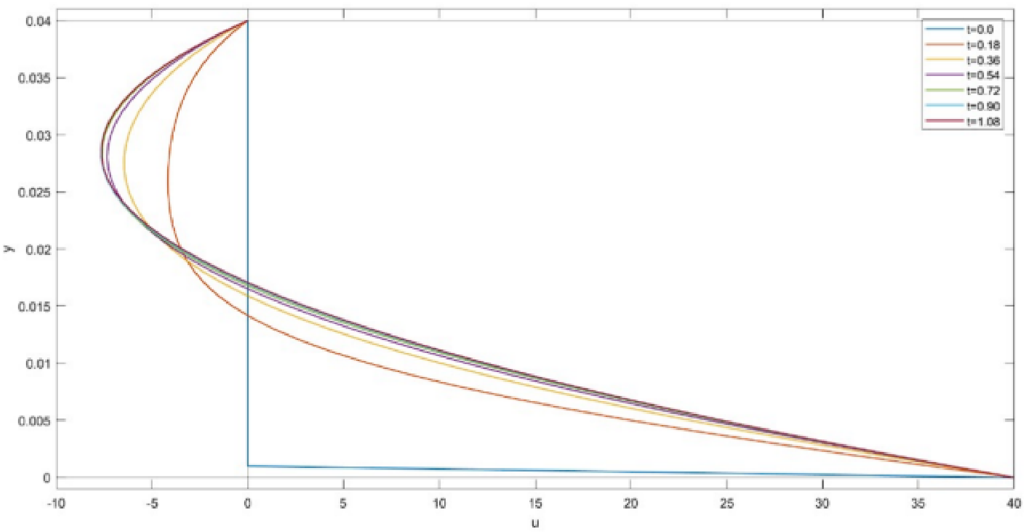

Figure 4.2: Result for $\frac{dp}{dx} = 20000$ N/m^2.

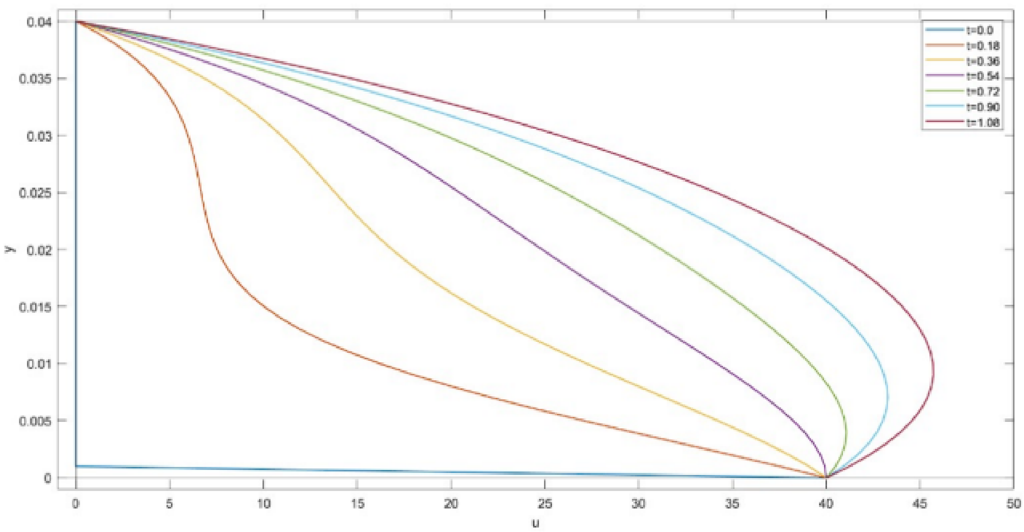

Figure 4.3: Result for $\frac{dp}{dx} = -30000$ N/m^2.