Material Properties and Q Values

Equations

- Constants:
  - Angle: $\theta = [0 \quad 90 \quad 90 \quad 0]$
  - Number of layers: $k = 4$
  - Thickness: thickness $= 1.34 \times 10^{-4}$ meters
  - Density: ρ_m=1580 kg/m^3
- Derivation of the Q matrix:

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$$

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}$$

$$E_1 = 142 \times 10^9 \text{ Pa}$$

$$E_2 = 9.8 \times 10^9 \text{ Pa}$$

$$\nu_{12} = 0.3$$

$$\nu_{21} = \frac{E_2 \nu_{12}}{E_1}$$

$$G_{12} = 6 \times 10^9 \text{ Pa}$$

$$Q_{12} = \frac{\nu_{21} E_1}{1 - \nu_{12}\nu_{21}}$$

$$Q_{21} = Q_{12}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12}$$

Q Transformation and D Value Formulations

Stacking Sequence: [0 90 90 0]

$$
\begin{aligned}
m &= \cos(\theta) \\
n &= \sin(\theta) \\
\bar{Q}_{11} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4 \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4) \\
\bar{Q}_{22} &= Q_{11}n^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}m^4 \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12})m^2n^2 + Q_{66}(m^2 - n^2)^2
\end{aligned}
$$

$$
D_{ij} = \frac{1}{3}\sum_{k=1}^{n}(\bar{Q}_{ij})k\left(tk^3 - t_{k-1}^3\right)
$$

Results

Given material properties:

$$D_{21} = 0.037964, \qquad D_1 = D_{11} = 1.6202,$$
$$D_2 = D_{22} = 0.33993, \qquad D_3 = D12 + 2 * D66 = 0.19195$$
$$a = 2m, \qquad b = 1m$$

The natural frequency is given by:

$$f = \frac{W}{2\pi}$$

where $$W = \frac{\pi^2}{\sqrt{\rho_m \cdot h}} \sqrt{D_1 \left(\frac{m}{a}\right)^4 + 2D_3 \left(\left(\frac{m}{a}\right)^2 \left(\frac{n}{b}\right)^2\right) + D_2 \left(\frac{n}{b}\right)^4}$$

Natural frequency: 1.251 Hz

The critical buckling load is given by:

$$N_{xcr} = \frac{\pi^2 a^2}{m^2} \left(D_1 \left(\frac{m}{a}\right)^4 + 2D_3 \left(\left(\frac{m}{a}\right)^2 \left(\frac{n}{b}\right)^2\right) + D_2 \left(\frac{n}{b}\right)^4\right) \times 10^{-3}$$

Critical buckling load: 21.207 N

Bending Calculation

For bending, we have the following expressions: taking mode (1,1) p_0=10 pascal

$$B_{mn} = \frac{P_0}{mn\pi^2}\left[1-(-1)^m\right]\left[1-(-1)^n\right]$$

$$A_{mn} = \frac{B_{mn}}{D_1(\frac{m\pi}{a})^4 + 2D_3(\frac{m\pi}{a})^2(\frac{n\pi}{b})^2 + D_2(\frac{n\pi}{b})^4}$$

Note: All edges are simply supported.
The bending deflection $w(x,y)$ is given by:

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

Bending deflection: 302.8545 **mm**