Personalized Graph Summarization: Formulation, Scalable Algorithms, and Applications (Online Appendix)

B. *Proof of Lemma 1*

Implementation Details and Lemma 2. We let $E_{AB} := \{\{u,v\} \in E : u \in A \text{ and } v \in B\}$ be the set of edges between A and B; and let $W_{AB}^{(T)} := \sum_{\{u,v\}\in E_{AB}} W_{uv}^{(T)}$ be the sum of their personalized weights. We also let $W_A^{(T)} := \sum_{u\in A} \frac{\alpha^{-D(u,T)}}{\sqrt{Z}}$ and $\bar{W}_A^{(T)} := \sum_{u\in A} \frac{\alpha^{-2\times D(u,T)}}{Z}$. PEGASUS maintains the following values up-to-date:

$$W_A^{(T)} \text{ and } \bar{W}_A^{(T)}, \quad \forall A \in S \text{ and} \tag{13}$$
$$W_{AB}^{(T)}, \quad \forall\{A,B\} \in \Pi_S \text{ where } E_{AB} \neq \emptyset. \tag{14}$$

To this end, PEGASUS initializes them by BFS in $O(|V|+|E|)$ time and updates them for each added/removed supernode and superedge, as described below. By the definitions of $W_A^{(T)}$, $\bar{W}_A^{(T)}$ and $W_{AB}^{(T)}$, the size of values in Eq. (13) is $O(|V|)$, and the size of those in Eq. (14) is $O(|E|)$.

Lemma 2. *For any supernodes $A, B \in S$ in a summary graph $\overline{G}$, $Cost_{AB}^{(T)}(\overline{G})$ can be computed in $O(1)$ time.*

Proof. It suffices to show that computing $RE_{AB}^{(T)}(\overline{G})$ takes $O(1)$ time since computing the remaining terms trivially takes $O(1)$ time. Eq. (7) and the definitions above imply that

$$RE_{AB}^{(T)}(\overline{G}) = \tag{15}$$
$$\begin{cases} W_A^{(T)}W_B^{(T)} - W_{AB}^{(T)} - \bar{W}_A^{(T)}, & \text{if } \{A,B\} \in P \text{ and } A = B, \\ W_A^{(T)}W_B^{(T)} - W_{AB}^{(T)}, & \text{if } \{A,B\} \in P \text{ and } A \neq B, \\ W_{AB}^{(T)}, & \text{otherwise.} \end{cases}$$

All terms in Eq. (15) are maintained as described above, and thus $RE_{AB}^{(T)}(\overline{G})$ can be computed in $O(1)$ time. □

Proof of Lemma 1. For the update of $\overline{G}$, lines 6-10 of Alg. 2 are executed, and lines 6-7 trivially take $O(1)$ time. Lines 8-9 take $O(\sum_{u\in A}|N_u| + \sum_{v\in B}|N_v|)$ time, as shown below.

Let $E_{XY} := \{\{u,v\} \in E : u \in X \text{ and } v \in Y\}$ be the set of edges between supernodes $X \in S$ and $Y \in S$. According to our initialization and update schemes, $\{X,Y\} \notin P$ holds for any supernodes $X, Y \in S$ where $E_{XY} = \emptyset$. This is because adding such $\{X,Y\}$ to P increases $Size(\overline{G})$, $RE^{(T)}(\overline{G})$, and thus $Cost^{(T)}(\overline{G})$. Hence, for any supernode $X \in S$, the number of superedges incident to X is $O(\sum_{x\in X}|N_x|)$.

(i) Updating intermediate results: As the first step, the intermediate results in Eq. (13) and (14) are updated as follows:

$$W_{(A\cup B)(A\cup B)}^{(T)} \leftarrow W_{AA}^{(T)} + W_{BB}^{(T)} + W_{AB}^{(T)},$$
$$W_{(A\cup B)X}^{(T)} \leftarrow W_{AX}^{(T)} + W_{BX}^{(T)}, \forall X \in S \text{ where } E_{(A\cup B)X} \neq \emptyset,$$
$$W_{A\cup B}^{(T)} \leftarrow W_A^{(T)} + W_B^{(T)}, \text{ and } \bar{W}_{A\cup B}^{(T)} \leftarrow \bar{W}_A^{(T)} + \bar{W}_B^{(T)}$$

As discussed above, the number of $X \in S$ where $E_{(A\cup B)X} \neq \emptyset$ is $O(\sum_{u\in A\cup B}|N_u|) = O(\sum_{u\in A}|N_u| + \sum_{v\in B}|N_v|)$, and each update takes $O(1)$ time. Thus, all updates take $O(\sum_{u\in A}|N_u| + \sum_{v\in B}|N_v|)$ time.

(ii) Line 8: For line 8, removing the superedges incident to A, whose number is $O(\sum_{u\in A}|N_u|)$, and removing the superedges incident to B, whose number is $O(\sum_{v\in B}|N_v|)$, takes $O(\sum_{u\in A}|N_u| + \sum_{v\in B}|N_v|)$ time.

(iii) Line 9: PEGASUS minimizes $Cost_{A\cup B}^{(T)}(\overline{G})$ by adding each superedge $\{(A\cup B), X\}$ to P if and only if it decreases $Cost_{(A\cup B)X}^{(T)}(\overline{G})$. As described above, the number of potential superedges incident to $A\cup B$ is $O(\sum_{u\in A\cup B}|N_u|) = O(\sum_{u\in A}|N_u| + \sum_{v\in B}|N_v|)$, and by Lemma 2, computing $Cost_{(A\cup B)X}^{(T)}(\overline{G})$ takes $O(1)$ time. Hence, line 9 takes $O(\sum_{u\in A}|N_u| + \sum_{v\in B}|N_v|)$ time. □

C. *The Effects of Relative Cost Reduction Function*

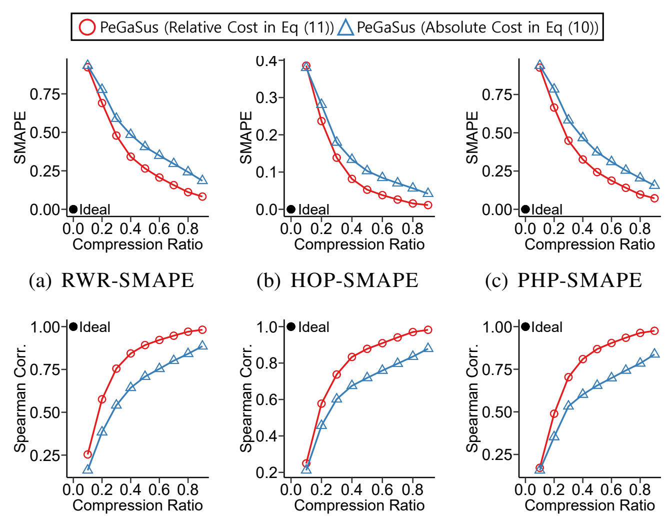
We compared PEGASUS, which uses Eq. (11), and a variant that uses Eq. (10) instead, in order to demonstrate the effect of using the relative cost reduction in Eq. (11). **As seen in Fig. 13, using Eq.** (11) **led to better summary graphs**, where queries can be answered more accurately, than using Eq. (10). See the last paragraph of Sect. III-B for further discussion.

D. *Results of* PHP *Queries (Q3, Q5)*

We report the accuracy of answers to PHP queries that is omitted in Sect. V-D and Sect. V-F.

As seen in Fig. 14, PEGASUS outperformed the state-of-the-art non-personalized graph summarization algorithms. For example, for the Amazon0601 dataset, the answers to PHP queries were up to 4.19× and 1.21× more accurate in terms of SMAPE and SC, respectively.

As seen in Fig. 15, the application of PEGASUS is more helpful for communication-free distributed multi-query processing than the considered graph partitioning algorithms. For example, for the Caida dataset, the answers to PHP queries were up to 3.22× and 1.34× more accurate in terms of SMAPE and SC, respectively.



(a) RWR-SMAPE (b) HOP-SMAPE (c) PHP-SMAPE
(d) RWR-SC (e) HOP-SC (f) PHP-SC

Fig. 13. **Using Eq.** (11) **for greedy search is effective.** Queries are answered more accurately when Eq. (11) is used than when Eq. (10) is used.

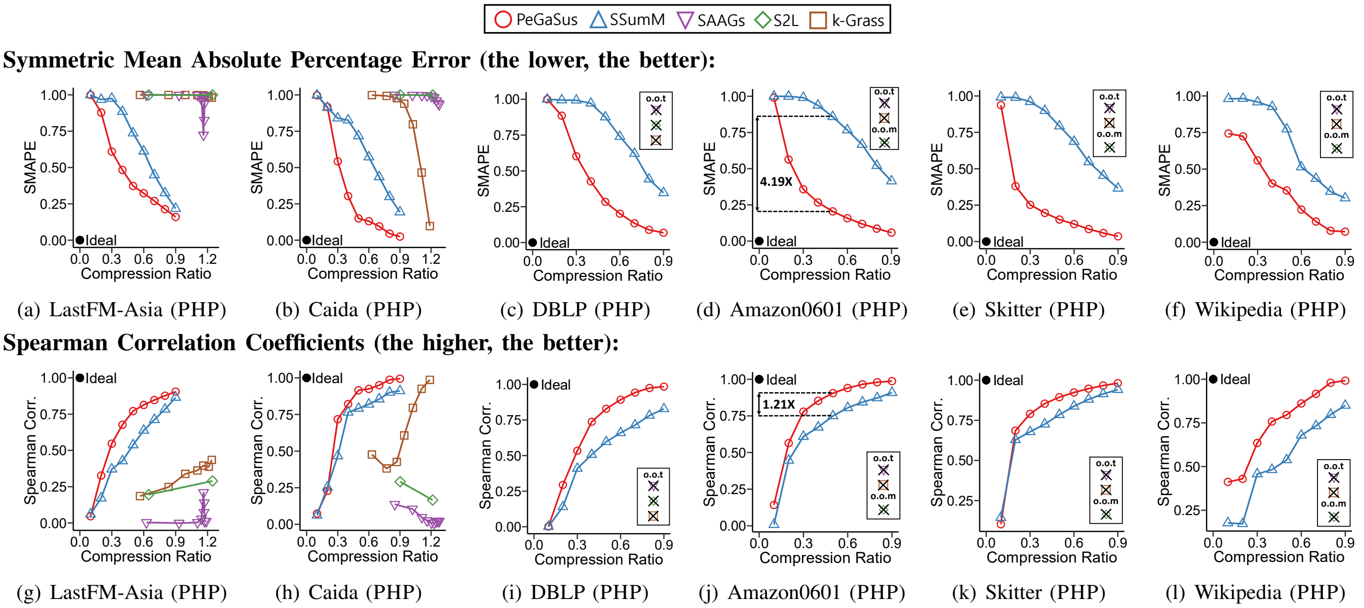


Fig. 14. **PEGASUS gives summary graphs where queries on target nodes are answered most accurately.** o.o.t: out of time (> 48hours). o.o.m: out of memory (> 128GB). The degree of personalization α is fixed to 1.25, and the size of the target node set is fixed to 100.

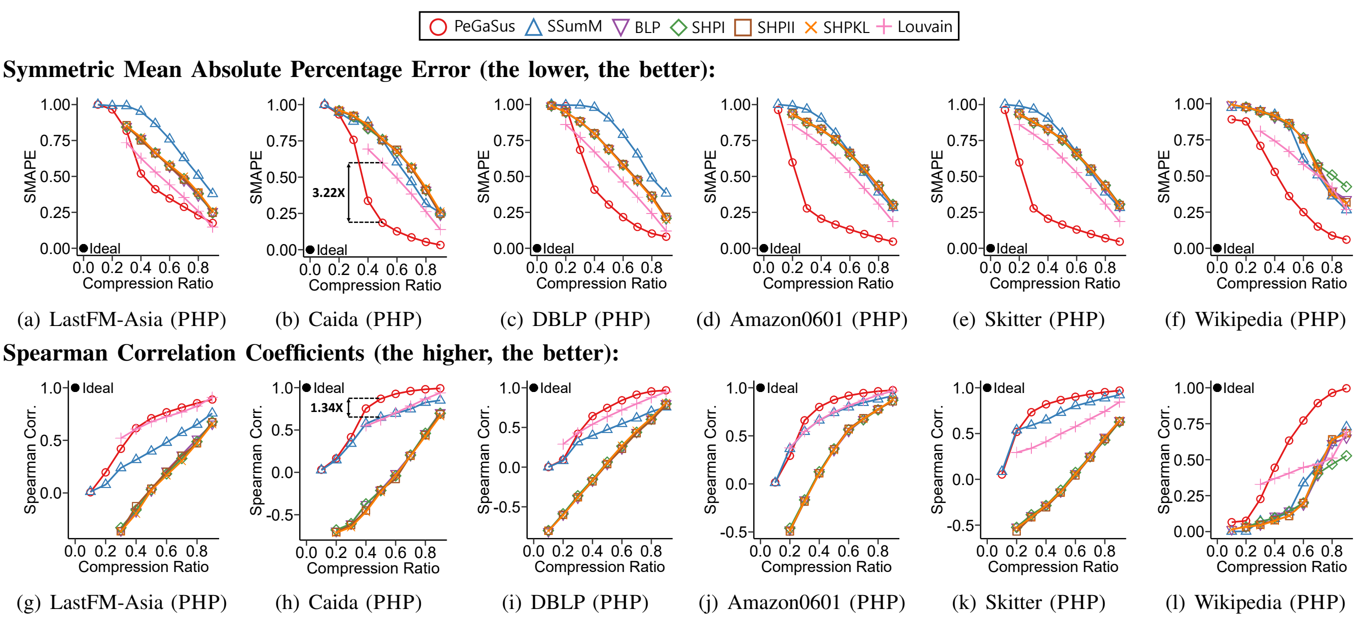


Fig. 15. **PEGASUS is useful for "communication-free" distributed multi-query processing.** Queries are answered more accurately from distributed personalized summary graphs (PEGASUS) than from non-personalized summary graphs (SSUMM) or distributed subgraphs (the others). The degree of personalization α is fixed to 1.25. In some datasets, compression rates cannot be lowered further due to imbalance among graph partitions.