

MA 322: Scientific Computing
LAB 6 Report
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Question 1 and 2

Using Rectangle rule with lower point.

(a) $f(x) = x^4$

Calculated value of integral = 3.125000e-02

Actual value of integral = 1.937500e-01

Error Bound = 5.000000e-01

Error = 1.625000e-01

(b) $f(x) = 2/(x - 4)$

Calculated value of integral = -2.500000e-01

Actual value of integral = -2.670628e-01

Error Bound = 2.040816e-02

Error = 1.706279e-02

(c) $f(x) = (2*x)/(x^2 - 4)$

Calculated value of integral = -4.000000e-01

Actual value of integral = -7.339692e-01

Error Bound = 1.138889e+00

Error = 3.339692e-01

(d) $f(x) = \sin(2*x)*\exp(3*x)$

Calculated value of integral = 0.000000e+00

Actual value of integral = 2.588629e+00

Error Bound = 9.762326e+00

Error = 2.588629e+00

(e) $f(x) = \sin(x)^2 - 2*x*\sin(x) + 1$

Calculated value of integral = 2.431953e-01

Actual value of integral = -2.037680e-02

Error Bound = 3.187007e-01

Error = 2.635721e-01

(f) $f(x) = x*\sin(x)$

Calculated value of integral = 0.000000e+00

Actual value of integral = 1.517464e-01

Error Bound = 3.893766e-01

Error = 1.517464e-01

(g) $f(x) = x \cdot \exp(-x)$

Calculated value of integral = -8.154845e+00

Actual value of integral = -4.060058e-01

Error Bound = 2.446454e+01

Error = 7.748840e+00

(h) $f(x) = 1/(x \cdot \log(x))$

Calculated value of integral = 3.678794e-01

Actual value of integral = 2.725139e-01

Error Bound = 1.353353e-01

Error = 9.536556e-02

Question 3

(a) $f(x) = x^4$

Using Mid-point Rule

Calculated value of integral = 1.582031e-01

Actual value of integral = 1.937500e-01

Error Bound = 6.250000e-02

Error = 3.554688e-02

Using Trapezoid Rule

Calculated value of integral = 2.656250e-01

Actual value of integral = 1.937500e-01

Error Bound = 1.250000e-01

Error = 7.187500e-02

Using Simpson Rule

Calculated value of integral = 1.940104e-01

Actual value of integral = 1.937500e-01

Error Bound = 4.166667e-03

Error = 2.604167e-04

Using Corrected-Trapezoid Rule

Calculated value of integral = 1.927083e-01

Actual value of integral = 1.937500e-01

Error Bound = 1.041667e-03

Error = 1.041667e-03

(b) $f(x) = 2/(x - 4)$

Using Mid-point Rule

Calculated value of integral = -2.666667e-01

Actual value of integral = -2.670628e-01

Error Bound = 4.859086e-04

Error = 3.961186e-04

Using Trapezoid Rule

Calculated value of integral = -2.678571e-01

Actual value of integral = -2.670628e-01

Error Bound = 9.718173e-04

Error = 7.943576e-04

Using Simpson Rule

Calculated value of integral = -2.670635e-01

Actual value of integral = -2.670628e-01

Error Bound = 1.586640e-05

Error = 7.068144e-07

Using Corrected-Trapezoid Rule

Calculated value of integral = -2.670599e-01

Actual value of integral = -2.670628e-01

Error Bound = 3.966601e-06

Error = 2.836269e-06

(c) $f(x) = (2*x)/(x^2 - 4)$

Using Mid-point Rule

Calculated value of integral = -6.753247e-01

Actual value of integral = -7.339692e-01

Error Bound = 2.808642e-01

Error = 5.864450e-02

Using Trapezoid Rule

Calculated value of integral = -8.666667e-01

Actual value of integral = -7.339692e-01

Error Bound = 5.617284e-01

Error = 1.326975e-01

Using Simpson Rule

Calculated value of integral = -7.391053e-01

Actual value of integral = -7.339692e-01

Error Bound = 7.031131e-01

Error = 5.136164e-03

Using Corrected-Trapezoid Rule

Calculated value of integral = -7.101852e-01

Actual value of integral = -7.339692e-01

Error Bound = 2.531207e-01

Error = 2.378399e-02

(d) $f(x) = \sin(2*x)*\exp(3*x)$

Using Mid-point Rule

Calculated value of integral = 1.803915e+00

Actual value of integral = 2.588629e+00

Error Bound = 1.149414e+00

Error = 7.847139e-01

Using Trapezoid Rule

Calculated value of integral = 4.143260e+00

Actual value of integral = 2.588629e+00

Error Bound = 2.298827e+00

Error = 1.554631e+00

Using Simpson Rule

Calculated value of integral = 2.583696e+00

Actual value of integral = 2.588629e+00

Error Bound = 8.448243e-01

Error = 4.932229e-03

Using Corrected-Trapezoid Rule

Calculated value of integral = 2.619014e+00

Actual value of integral = 2.588629e+00

Error Bound = 5.211301e-01

Error = 3.038514e-02

(e) $f(x) = \sin(x)^2 - 2*x*\sin(x) + 1$

Using Mid-point Rule

Calculated value of integral = -1.189526e-02

Actual value of integral = -2.037680e-02

Error Bound = 1.222040e-02

Error = 8.481537e-03

Using Trapezoid Rule

Calculated value of integral = -3.702425e-02

Actual value of integral = -2.037680e-02

Error Bound = 2.444081e-02

Error = 1.664746e-02

Using Simpson Rule

Calculated value of integral = -2.027159e-02

Actual value of integral = -2.037680e-02

Error Bound = 1.499651e-03

Error = 1.052061e-04

Using Corrected-Trapezoid Rule

Calculated value of integral = -2.079523e-02

Actual value of integral = -2.037680e-02

Error Bound = 4.536444e-04

Error = 4.184296e-04

(f) $f(x) = x*\sin(x)$

Using Mid-point Rule

Calculated value of integral = 1.180292e-01

Actual value of integral = 1.517464e-01

Error Bound = 4.037276e-02

Error = 3.371722e-02

Using Trapezoid Rule

Calculated value of integral = 2.180895e-01

Actual value of integral = 1.517464e-01

Error Bound = 8.074551e-02

Error = 6.634309e-02

Using Simpson Rule

Calculated value of integral = 1.513826e-01

Actual value of integral = 1.517464e-01

Error Bound = 2.691517e-03

Error = 3.637850e-04

Using Corrected-Trapezoid Rule

Calculated value of integral = 1.531934e-01

Actual value of integral = 1.517464e-01

Error Bound = 1.660263e-03

Error = 1.446992e-03

(g) $f(x) = x \cdot \exp(-x)$

Using Mid-point Rule

Calculated value of integral = 9.097960e-01

Actual value of integral = -4.060058e-01

Error Bound = 9.174201e+00

Error = 1.315802e+00

Using Trapezoid Rule

Calculated value of integral = -3.671417e+00

Actual value of integral = -4.060058e-01

Error Bound = 1.834840e+01

Error = 3.265411e+00

Using Simpson Rule

Calculated value of integral = -6.172750e-01

Actual value of integral = -4.060058e-01

Error Bound = 5.096778e-01

Error = 2.112691e-01

Using Corrected-Trapezoid Rule

Calculated value of integral = 5.075073e-01

Actual value of integral = -4.060058e-01

Error Bound = 4.587101e+00

Error = 9.135132e-01

(h) $f(x) = 1/(x \cdot \log(x))$

Using Mid-point Rule

Calculated value of integral = 2.658386e-01

Actual value of integral = 2.725139e-01

Error Bound = 1.452123e-02

Error = 6.675288e-03

Using Trapezoid Rule

Calculated value of integral = 2.863342e-01

Actual value of integral = 2.725139e-01

Error Bound = 2.904246e-02

Error = 1.382029e-02

Using Simpson Rule

Calculated value of integral = 2.726705e-01

Actual value of integral = 2.725139e-01

Error Bound = 2.133683e-03

Error = 1.565719e-04

Using Corrected-Trapezoid Rule

Calculated value of integral = 2.718629e-01

Actual value of integral = 2.725139e-01

Error Bound = 2.133683e-03

Error = 6.510291e-04

Question 4

Mid-point, trapezoid and one-third simpson rules give correct integrals for polynomials (for non-polynomial functions we take Taylor series expansion) upto degree 2. Corrected trapezoid and Simpson three-eighth gives correct results upto degree 4. The coefficients in error bounds is smaller for Simpson one-third < Mid-point < Trapezoid. In the other case, Simpson three-eighth < Corrected trapezoid. So we get the following result. For error formulas see code.

$$f(x) = 4/(x^2 + 1)$$

Using Mid-point Rule

Calculated value of integral = 1.280000e+00

Actual value of integral = 1.287002e+00

Error Bound = 1.041667e-02

Error = 7.002218e-03

Using Trapezoid Rule

Calculated value of integral = 1.300000e+00

Actual value of integral = 1.287002e+00

Error Bound = 2.083333e-02

Error = 1.299778e-02

Using Simpson one-third Rule

Calculated value of integral = 1.286667e+00

Actual value of integral = 1.287002e+00

Error Bound = 7.031250e-03

Error = 3.355509e-04

Using Corrected-Trapezoid Rule

Calculated value of integral = 1.288333e+00

Actual value of integral = 1.287002e+00

Error Bound = 1.757813e-03

Error = 1.331116e-03

Using Simpson three-eight Rule
Calculated value of integral = 1.286854e+00
Actual value of integral = 1.287002e+00
Error Bound = 4.746094e-02
Error = 1.484975e-04

Question 5

(a) $f(x) = \sin(3*x)*\exp(2*x)$

Using Composite Trapezoid Rule
Calculated value of integral = -1.357598e+01
Actual value of integral = -1.421398e+01
Error Bound = 7.347501e+00
Error = 6.379977e-01

(b) $f(x) = x/(x^2 + 4)$

Using Composite Trapezoid Rule
Calculated value of integral = 4.769769e-01
Actual value of integral = 4.777557e-01
Error Bound = 1.833333e-03
Error = 7.788560e-04

(c) $f(x) = 1/(x^2 - 4)^{(1/2)}$

Using Composite Trapezoid Rule
Calculated value of integral = 6.054975e-01
Actual value of integral = 6.043756e-01
Error Bound = 4.099458e-03
Error = 1.121928e-03

(d) $f(x) = x^2*\exp(-x^2)$

Using Composite Trapezoid Rule
Calculated value of integral = 4.215820e-01
Actual value of integral = 4.227251e-01
Error Bound = 2.083333e-02
Error = 1.143019e-03

(e) $f(x) = x*\log(x + 1)$

Using Composite Trapezoid Rule
Calculated value of integral = 9.363014e-02
Actual value of integral = 8.802039e-02
Error Bound = 1.388889e-02
Error = 5.609748e-03

(f) $f(x) = \sin(x)^2 - 2x\sin(x) + 1$

Using Composite Trapezoid Rule

Calculated value of integral = -4.893227e-01

Actual value of integral = -4.890193e-01

Error Bound = 2.973987e-03

Error = 3.033970e-04

(g) $f(x) = 1/(x\log(x))$

Using Composite Trapezoid Rule

Calculated value of integral = 4.403450e-01

Actual value of integral = 4.391866e-01

Error Bound = 3.630307e-03

Error = 1.158443e-03

(h) $f(x) = x^2\log(x^2 + 1)$

Using Composite Trapezoid Rule

Calculated value of integral = 3.159476e+00

Actual value of integral = 3.109291e+00

Error Bound = 9.855065e-02

Error = 5.018483e-02

Question 6

(a) $f(x) = \sin(3x)\exp(2x)$

Using Composite Mid-point Rule

Calculated value of integral = -1.453048e+01

Actual value of integral = -1.421398e+01

Error Bound = 3.673751e+00

Error = 3.165056e-01

Using Composite Simpson Rule

Calculated value of integral = -1.421231e+01

Actual value of integral = -1.421398e+01

Error Bound = 1.214033e-02

Error = 1.662201e-03

(b) $f(x) = x/(x^2 + 4)$

Using Composite Mid-point Rule

Calculated value of integral = 4.781451e-01

Actual value of integral = 4.777557e-01

Error Bound = 9.166667e-04

Error = 3.893385e-04

Using Composite Simpson Rule

Calculated value of integral = 4.777557e-01

Actual value of integral = 4.777557e-01

Error Bound = 8.541667×10^{-7}

Error = 5.966143×10^{-8}

(c) $f(x) = 1/(x^2 - 4)^{(1/2)}$

Using Composite Mid-point Rule

Calculated value of integral = 6.038165×10^{-1}

Actual value of integral = 6.043756×10^{-1}

Error Bound = 2.049729×10^{-3}

Error = 5.590967×10^{-4}

Using Composite Simpson Rule

Calculated value of integral = 6.043768×10^{-1}

Actual value of integral = 6.043756×10^{-1}

Error Bound = 9.084026×10^{-6}

Error = 1.244755×10^{-6}

(d) $f(x) = x^2 \exp(-x^2)$

Using Composite Mid-point Rule

Calculated value of integral = 4.232959×10^{-1}

Actual value of integral = 4.227251×10^{-1}

Error Bound = 1.041667×10^{-2}

Error = 5.708580×10^{-4}

Using Composite Simpson Rule

Calculated value of integral = 4.227246×10^{-1}

Actual value of integral = 4.227251×10^{-1}

Error Bound = 6.510417×10^{-5}

Error = 4.344599×10^{-7}

(e) $f(x) = x \log(x + 1)$

Using Composite Mid-point Rule

Calculated value of integral = 8.522287×10^{-2}

Actual value of integral = 8.802039×10^{-2}

Error Bound = 6.944444×10^{-3}

Error = 2.797518×10^{-3}

Using Composite Simpson Rule

Calculated value of integral = 8.802530×10^{-2}

Actual value of integral = 8.802039×10^{-2}

Error Bound = 1.200274×10^{-4}

Error = 4.904192×10^{-6}

(f) $f(x) = \sin(x)^2 - 2x \sin(x) + 1$

Using Composite Mid-point Rule

Calculated value of integral = -4.888669×10^{-1}

Actual value of integral = -4.890193×10^{-1}

Error Bound = 1.486993×10^{-3}

Error = 1.523974×10^{-4}

Using Composite Simpson Rule

Calculated value of integral = -4.890189e-01

Actual value of integral = -4.890193e-01

Error Bound = 1.446247e-06

Error = 4.659403e-07

(g) $f(x) = 1/(x \cdot \log(x))$

Using Composite Mid-point Rule

Calculated value of integral = 4.386086e-01

Actual value of integral = 4.391866e-01

Error Bound = 1.815154e-03

Error = 5.779861e-04

Using Composite Simpson Rule

Calculated value of integral = 4.391874e-01

Actual value of integral = 4.391866e-01

Error Bound = 4.167350e-06

Error = 8.236511e-07

(h) $f(x) = x^2 \cdot \log(x^2 + 1)$

Using Composite Mid-point Rule

Calculated value of integral = 3.084203e+00

Actual value of integral = 3.109291e+00

Error Bound = 4.927533e-02

Error = 2.508799e-02

Using Composite Simpson Rule

Calculated value of integral = 3.109294e+00

Actual value of integral = 3.109291e+00

Error Bound = 6.510417e-05

Error = 2.947289e-06

Question 7

(a) $f(x) = 4/(x^2 + 1)$

Using Composite Simpson Rule

n = 1, error bound = 1.333333e-01

n = 2, error bound = 8.333333e-03

n = 3, error bound = 1.646091e-03

n = 4, error bound = 5.208333e-04

n = 5, error bound = 2.133333e-04

n = 6, error bound = 1.028807e-04

n = 7, error bound = 5.553242e-05

n = 8, error bound = 3.255208e-05

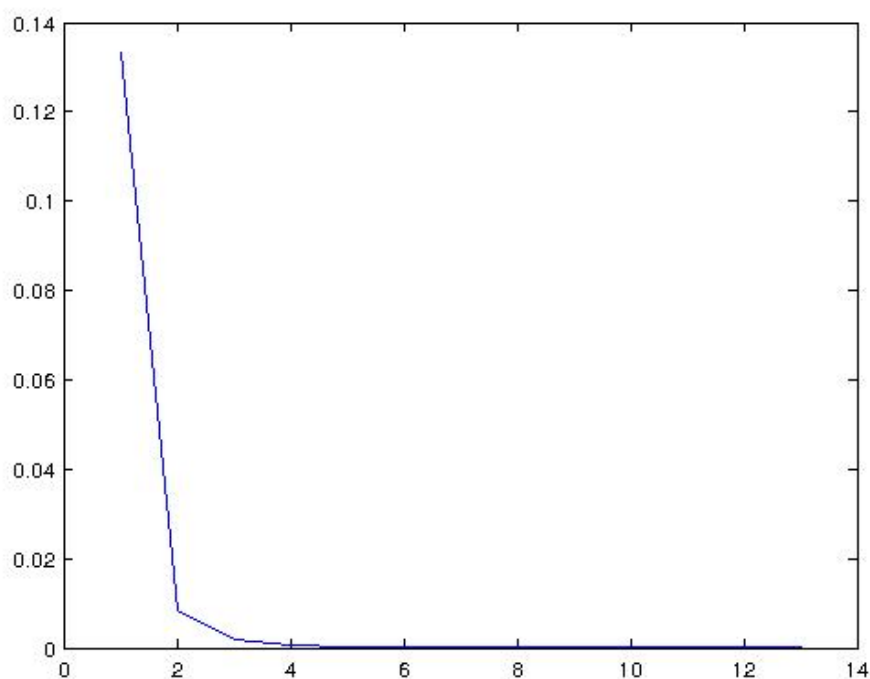
n = 9, error bound = 2.032211e-05

n = 10, error bound = 1.333333e-05

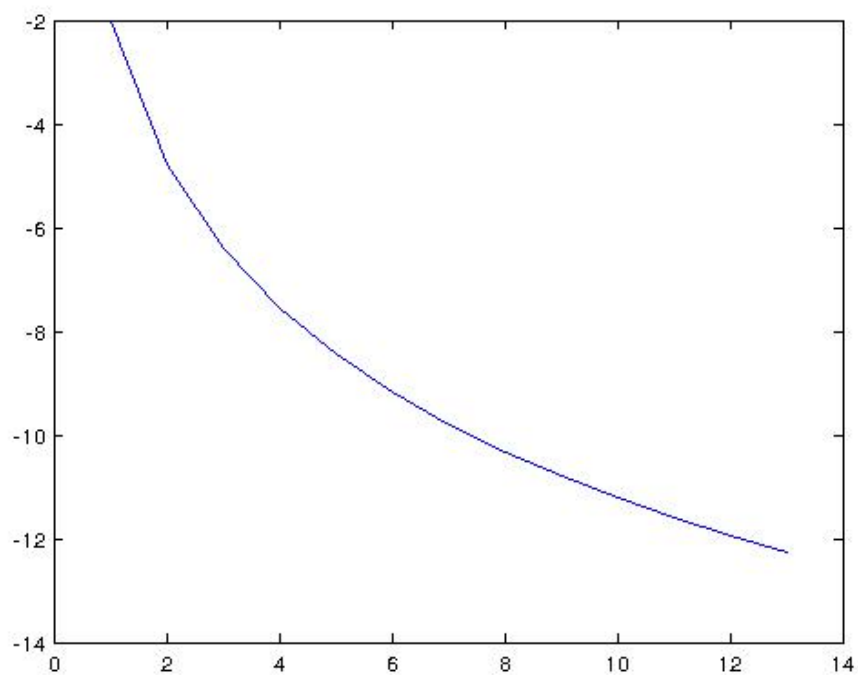
n = 11, error bound = 9.106846e-06

n = 12, error bound = 6.430041e-06
n = 13, error bound = 4.668371e-06
Number of sub-intervals = 13
Calculated value of integral = 3.141593e+00
Actual value of integral = 3.141593e+00
Error Bound = 4.668371e-06
Error = 1.284555e-10

Number of intervals (n) vs Error plot



loglog plot: Number of intervals (n) vs Error



(b) $f(x) = (1 - x^2)^{1/2} - x$

Using Composite Simpson Rule

n = 1, error bound = 5.000000e-02

n = 2, error bound = 3.125000e-03

n = 3, error bound = 6.172840e-04

n = 4, error bound = 1.953125e-04

n = 5, error bound = 8.000000e-05

n = 6, error bound = 3.858025e-05

n = 7, error bound = 2.082466e-05

n = 8, error bound = 1.220703e-05

n = 9, error bound = 7.620790e-06

n = 10, error bound = 5.000000e-06

Number of sub-intervals = 10

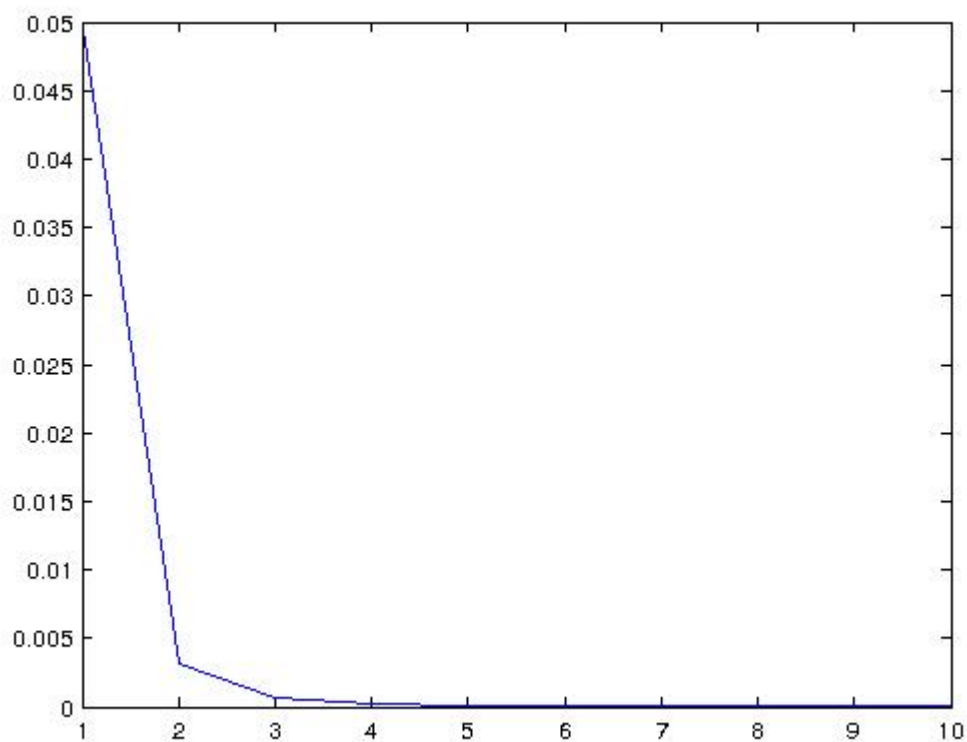
Calculated value of integral = 3.926990e-01

Actual value of integral = 3.926991e-01

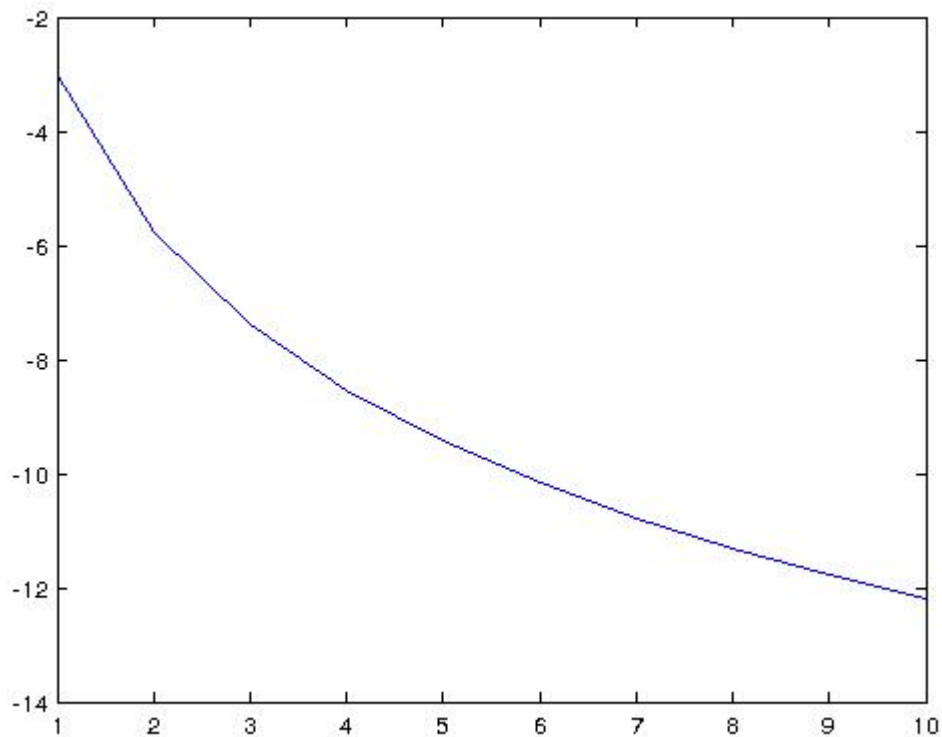
Error Bound = 5.000000e-06

Error = 1.026709e-07

Number of intervals (n) vs Error plot



loglog plot: Number of intervals (n) vs Error



Question 8

$X = [1, 5/4, 3/2, 7/4, 2];$

$Y = [10, 8, 7, 6, 5];$

Using Composite Trapezoid Rule

Calculated value of integral = 7.12500

Question 9

$f(x) = x \cdot \log(x)$

Using Composite Trapezoid Rule

$n = 5$, error bound = $3.333333e-03$, error = $2.308836e-03$

$n = 10$, error bound = $8.333333e-04$, error = $5.775187e-04$

$n = 15$, error bound = $3.703704e-04$, error = $2.567006e-04$

$n = 20$, error bound = $2.083333e-04$, error = $1.443992e-04$

$n = 25$, error bound = $1.333333e-04$, error = $9.241696e-05$

$n = 30$, error bound = $9.259259e-05$, error = $6.417901e-05$

$n = 35$, error bound = $6.802721e-05$, error = $4.715218e-05$

$n = 40$, error bound = $5.208333e-05$, error = $3.610101e-05$

$n = 45$, error bound = $4.115226e-05$, error = $2.852432e-05$

$n = 50$, error bound = $3.333333e-05$, error = $2.310474e-05$

$n = 55$, error bound = $2.754821e-05$, error = $1.909485e-05$

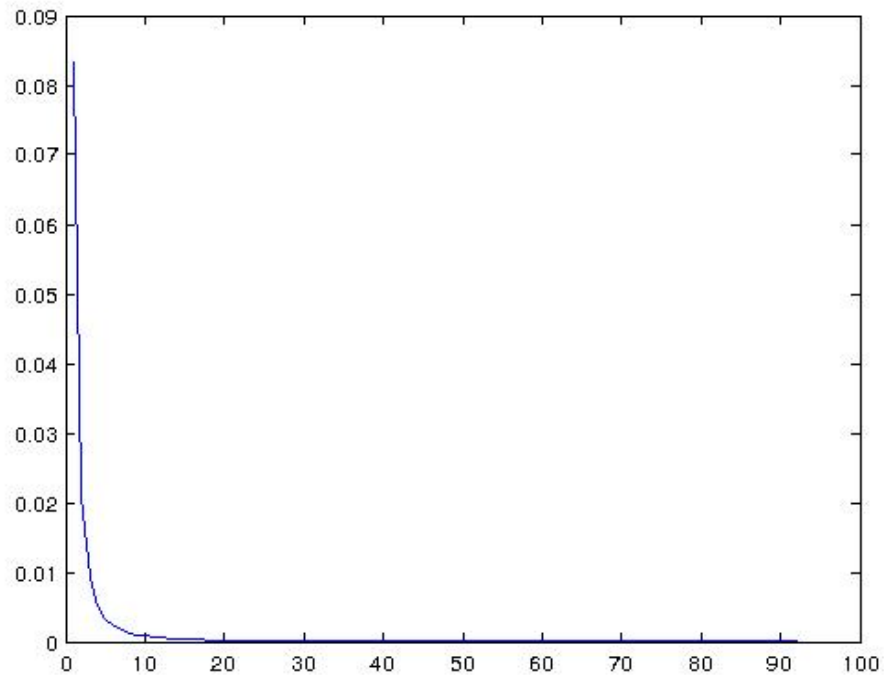
$n = 60$, error bound = $2.314815e-05$, error = $1.604499e-05$

$n = 65$, error bound = $1.972387e-05$, error = $1.367148e-05$

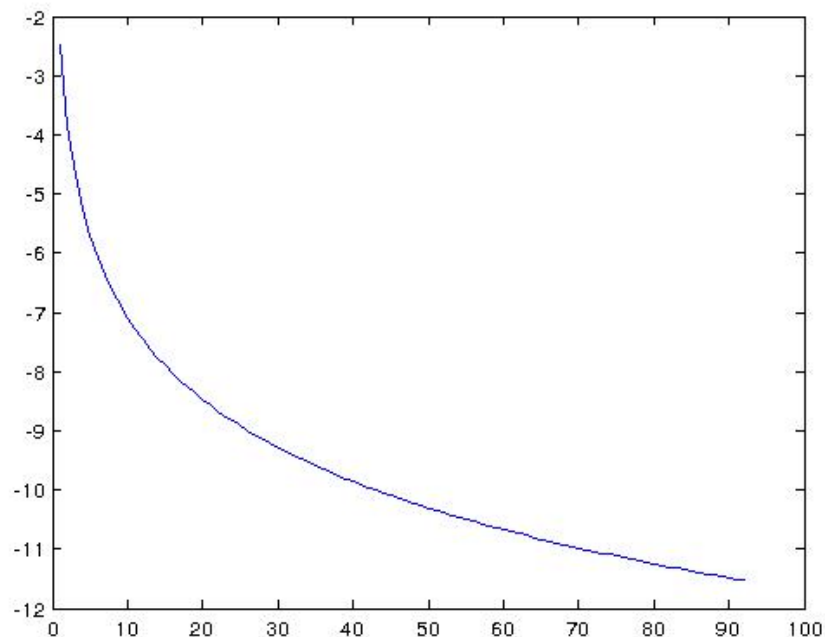
$n = 70$, error bound = $1.700680e-05$, error = $1.178817e-05$

n = 75, error bound = 1.481481e-05, error = 1.026881e-05
n = 80, error bound = 1.302083e-05, error = 9.025328e-06
n = 85, error bound = 1.153403e-05, error = 7.994757e-06
n = 90, error bound = 1.028807e-05, error = 7.131128e-06
Number of sub-intervals = 92
Length of sub-interval = 0.010870
Calculated value of integral = 6.363012e-01
Actual value of integral = 6.362944e-01
Error Bound = 9.845621e-06
Error = 6.824450e-06

Number of intervals (n) vs Error plot

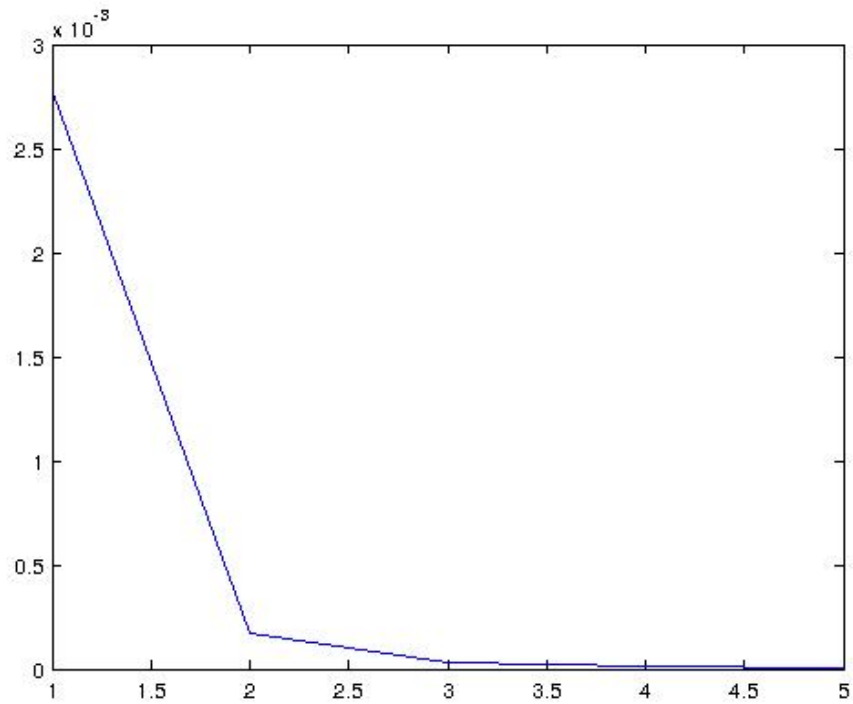


loglog plot: Number of intervals (n) vs Error

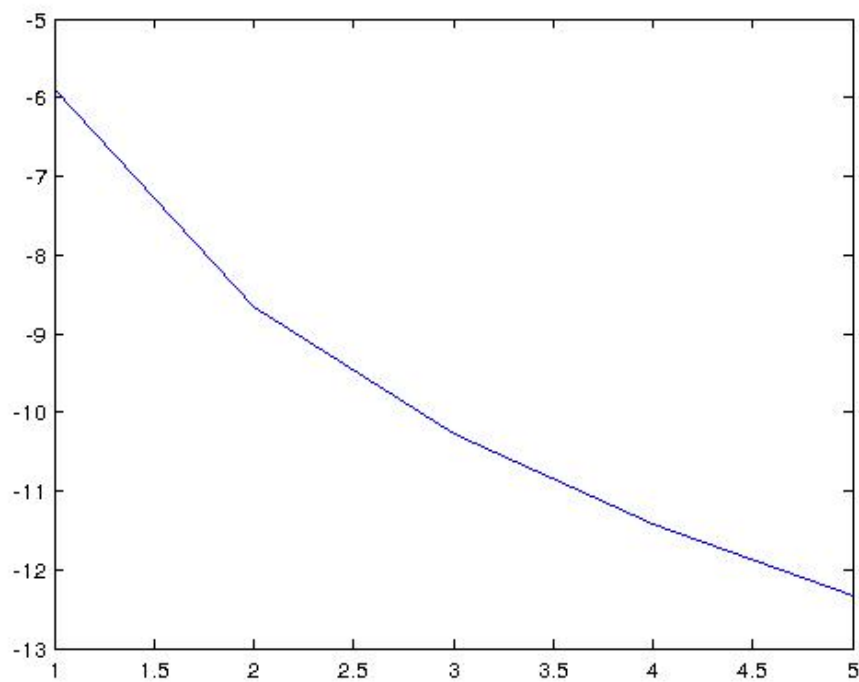


Using Composite Simpson Rule
n = 5, error bound = 4.444444e-06, error = 4.130251e-07
Number of sub-intervals = 5
Length of sub-interval = 0.200000
Calculated value of integral = 6.362948e-01
Actual value of integral = 6.362944e-01
Error Bound = 4.444444e-06
Error = 4.130251e-07

Number of intervals (n) vs Error plot



loglog plot: Number of intervals (n) vs Error



Using Composite Mid-Point Rule

n = 5, error bound = 1.666667e-03, error = 1.153798e-03
n = 10, error bound = 4.166667e-04, error = 2.887204e-04
n = 15, error bound = 1.851852e-04, error = 1.283426e-04
n = 20, error bound = 1.041667e-04, error = 7.219714e-05
n = 25, error bound = 6.666667e-05, error = 4.620748e-05
n = 30, error bound = 4.629630e-05, error = 3.208902e-05
n = 35, error bound = 3.401361e-05, error = 2.357583e-05
n = 40, error bound = 2.604167e-05, error = 1.805035e-05
n = 45, error bound = 2.057613e-05, error = 1.426207e-05
n = 50, error bound = 1.666667e-05, error = 1.155231e-05
n = 55, error bound = 1.377410e-05, error = 9.547382e-06
n = 60, error bound = 1.157407e-05, error = 8.022466e-06
n = 65, error bound = 9.861933e-06, error = 6.835720e-06

Number of sub-intervals = 65

Length of sub-interval = 0.015385

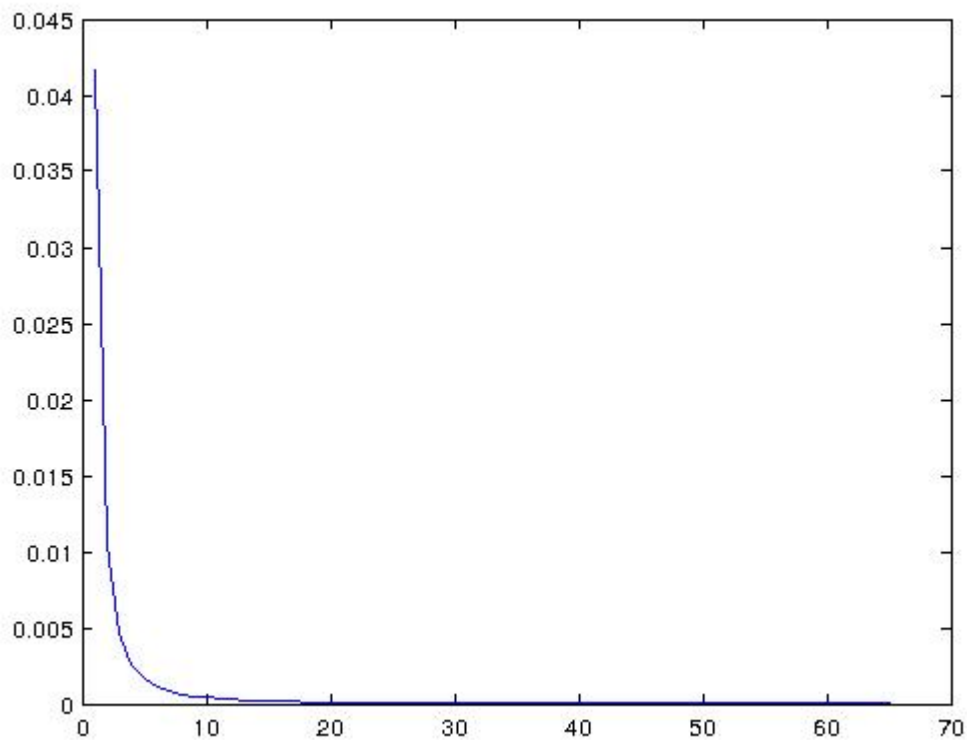
Calculated value of integral = 6.362875e-01

Actual value of integral = 6.362944e-01

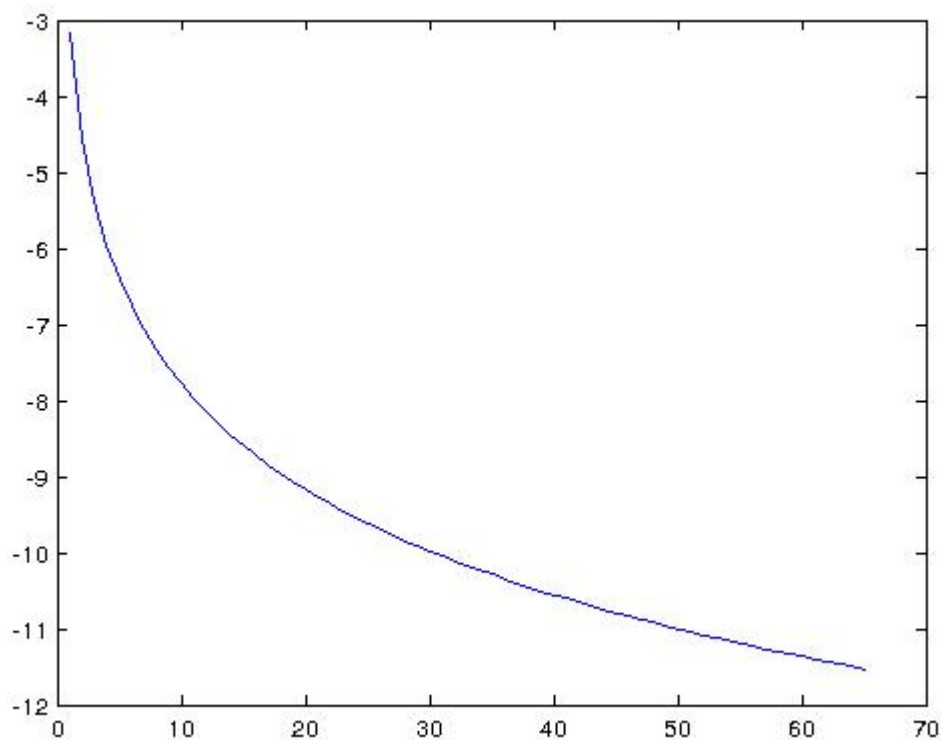
Error Bound = 9.861933e-06

Error = 6.835720e-06

Number of intervals (n) vs Error plot



loglog plot: Number of intervals (n) vs Error



Question 10

$$f(x) = (4*\cos(x)^2 + 9*\sin(x)^2)^{(1/2)}$$

Using Composite Simpson Rule

n = 5, error bound = 1.067994e-02

n = 10, error bound = 6.674962e-04

n = 15, error bound = 1.318511e-04

n = 20, error bound = 4.171851e-05

n = 25, error bound = 1.708790e-05

n = 30, error bound = 8.240694e-06

n = 35, error bound = 4.448122e-06

n = 40, error bound = 2.607407e-06

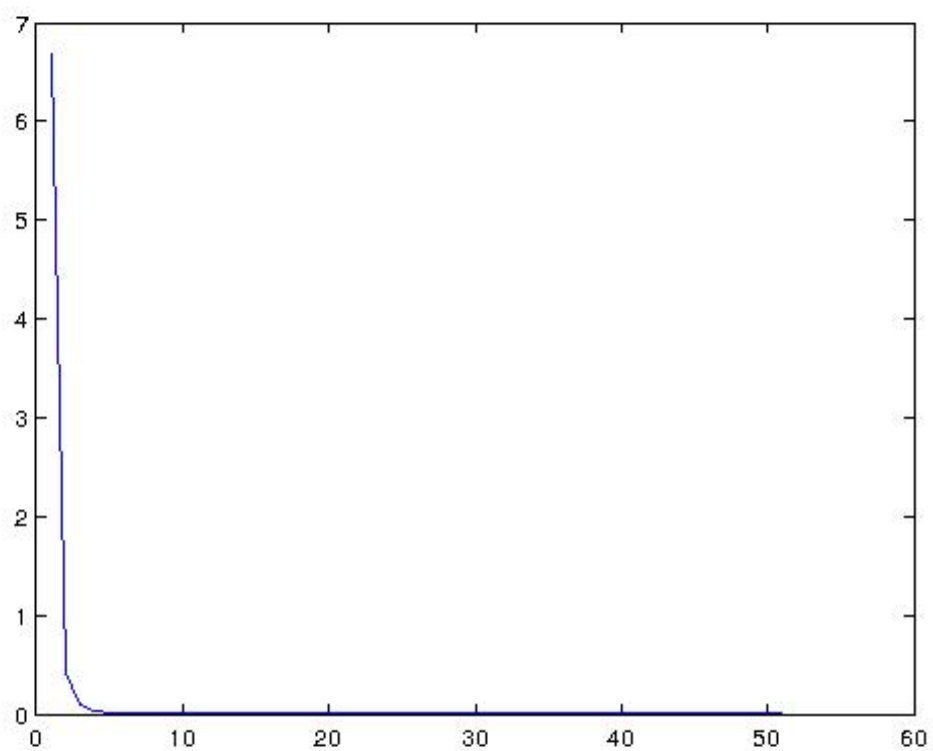
n = 45, error bound = 1.627791e-06

n = 50, error bound = 1.067994e-06

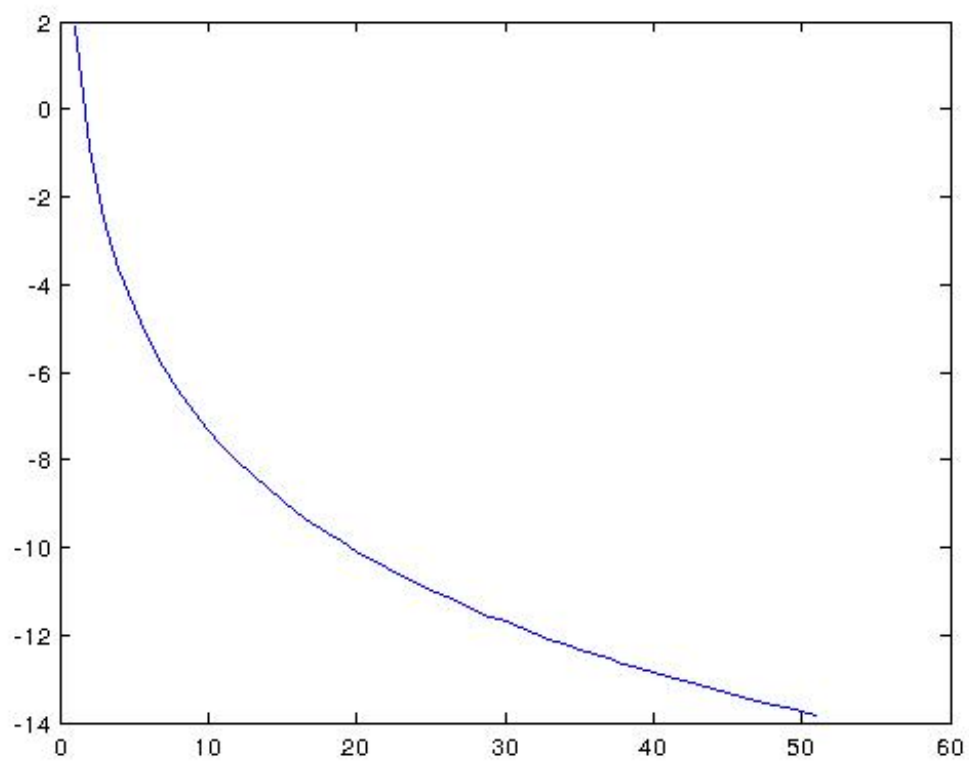
Number of sub-intervals = 51

Calculated value of integral = 1.586544e+01

Number of intervals (n) vs Error plot



LogLog plot: n vs error



Question 11

$X = 0:6:84$;

$Y = [124, 134, 148, 156, 147, 133, 121, 109, 99, 85, 78, 89, 104, 116, 123]$;

Using Composite Trapezoid Rule

The length of track = $9.855000e+03$

Question 12

(a) $f(x) = x^2 \log(x)$

Using Gaussian quadrature for uniform intervals

Actual value of integral = $1.922594e-01$ Number of intervals, $n = 2$

Calculated value of integral = $1.922453e-01$

Error = $1.405032e-05$

Number of intervals, $n = 3$

Calculated value of integral = $1.922531e-01$

Error = $6.264674e-06$

Number of intervals, $n = 4$

Calculated value of integral = $1.922593e-01$

Error = $2.041835e-08$

Number of intervals, $n = 5$

Calculated value of integral = $1.922593e-01$

Error = $1.153687e-08$

Number of intervals, $n = 6$

Calculated value of integral = $1.922594e-01$

Error = $8.198203e-11$

(b) $f(x) = \sin(2*x)*\exp(3*x)$

Using Gaussian quadrature for uniform intervals

Actual value of integral = $2.588629e+00$ Number of intervals, $n = 2$

Calculated value of integral = $2.583696e+00$

Error = $4.932229e-03$

Number of intervals, $n = 3$

Calculated value of integral = $2.585789e+00$

Error = $2.839581e-03$

Number of intervals, $n = 4$

Calculated value of integral = $2.587968e+00$

Error = $6.601757e-04$

Number of intervals, $n = 5$

Calculated value of integral = $2.588255e+00$

Error = $3.739206e-04$

Number of intervals, $n = 6$
Calculated value of integral = 2.588624e+00
Error = 4.656766e-06

(c) $f(x) = 2/(x^2 - 4)$

Using Gaussian quadrature for uniform intervals
Actual value of integral = -1.768200e-01
Number of intervals, $n = 2$
Calculated value of integral = -1.768216e-01
Error = 1.549122e-06

Number of intervals, $n = 3$
Calculated value of integral = -1.768207e-01
Error = 6.907986e-07

Number of intervals, $n = 4$
Calculated value of integral = -1.768200e-01
Error = 2.334191e-09

Number of intervals, $n = 5$
Calculated value of integral = -1.768200e-01
Error = 1.317139e-09

Number of intervals, $n = 6$
Calculated value of integral = -1.768200e-01
Error = 5.558137e-12

(d) $f(x) = (2*x)/(x^2 - 4)$

Using Gaussian quadrature for uniform intervals
Actual value of integral = -7.339692e-01
Number of intervals, $n = 2$
Calculated value of integral = -7.391053e-01
Error = 5.136164e-03

Number of intervals, $n = 3$
Calculated value of integral = -7.364277e-01
Error = 2.458521e-03

Number of intervals, $n = 4$
Calculated value of integral = -7.341567e-01
Error = 1.874933e-04

Number of intervals, $n = 5$
Calculated value of integral = -7.340795e-01
Error = 1.103410e-04

Number of intervals, $n = 6$
Calculated value of integral = -7.339795e-01
Error = 1.028853e-05