DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 332: Scientific Computing Lab - III

1. Perform two iterations of Newton's method on these systems. (a). Starting with (0,1) (b). Starting with (1,1)

$$(a).\quad \begin{cases} 4x_1^2 - x_2^2 &= 0, \\ 4x_1x_2^2 - x_1 &= 1. \end{cases} \qquad (b).\quad \begin{cases} xy^2 + x^2y + x^4 &= 3, \\ x^3y^5 - 2x^5y - x^2 &= -2. \end{cases}$$

2. Use Newton's method with (0,0) to perform two iterations for each of the following nonlinear systems.

$$(a).\quad \begin{cases} 4x_1^2 - 20x_1 + \frac{1}{4}x_2^2 + 8 &= 0, \\ \frac{1}{2}x_1x_2^2 + 2x_1 - 5x_2 + 8 &= 0. \end{cases} \qquad (b).\quad \begin{cases} \sin(4\pi x_1x_2) - 2x_2 - x_1 &= 0, \\ \frac{4\pi-1}{4\pi}(e^{2x_1} - e) + 4ex_2^2 - 2ex_1 &= 0. \end{cases}$$

3. Use Newton's method to find a solution to the following nonlinear systems in the given domain. Iterate until $\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\|_\infty < 10^{-6}$.

$$\begin{cases} 6x_1 - 2\cos(x_2x_3) - 1 = 0, \\ 9x_2 + \sqrt{x_1^2 + \sin(x_3) + 1.06} + 0.9 = 0, \\ 60x_3 + 3e^{-x_1x_2} + 10\pi - 3 = 0. \end{cases}$$

   Use $\mathbf{x}^{(0)} = (0,0,0)^t$.

4. Solve this pair of simultaneous nonlinear equations by first eliminating y and then solving the resulting equation in x by Newton's method. Start with the initial value $x_0 = 1$.

$$\begin{cases} x^3 - 2xy + y^7 - 4x^3y &= 5, \\ y\sin x + 3x^2y + \tan x &= 4. \end{cases}$$

5. Use Newton's method and the modified Newton's method to find solutions accurate to within 10^{-5} to the following problems.

   a. $x^2 - 2xe^{-x} + e^{-2x} = 0, \quad 0 \le x \le 1,$
   b. $\cos(x + \sqrt{2}) + x(x/2 + \sqrt{2}) = 0, \quad -2 \le x \le -1,$
   c. $x^3 - 3x^2(2^{-x}) + 3x(4^{-x}) - 8^{-x} = 0, \quad 0 \le x \le 1,$
   d. $e^{6x} + 3(\ln 2)^2e^{2x} - (\ln 8)e^{4x} - (\ln 2)^3 = 0, \quad -1 \le x \le 0.$

   Are there any improvements in speed or accuracy after using the modified Newton's method?