DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 332: Scientific Computing Lab – 2016

Solution of Nonlinear Equations

1. Use the bisection method to find the root of the equation $e^x = \sin x$ closest to 0.

2. Use the Bisection method to find solution accurate to within 10^{-3} for $2+\cos(e^x-2)-e^x = 0$ on $[0.5, 1.5]$.

3. A particle starts at rest on a smooth inclined plane whose angle θ is changing at a constant rate $\dfrac{d\theta}{dt} = \omega < 0$. At the end of t seconds, the position (measured from starting point) of the object is given by
$$x(t) = -\frac{g}{2\omega^2}\left(\frac{e^{\omega t} - e^{-\omega t}}{2} - \sin \omega t\right).$$
Suppose the particle has moved $1.7ft$ in $1s$. Find, within 10^{-5}, the rate ω at which θ changes. Assume that $g = 32.17ft/s^2$.

4. Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^3 - x - 1 = 0$ on $[1, 2]$. Use $x_0 = 1$.

5. Show that if A is any positive number, then the sequence defined by $x_n = 0.5x_{n-1} + 0.5A/x_{n-1}$, $n \geq 1$, converges to $\sqrt{A}$ whenever $x_0 > 0$. What happens if $x_0 < 0$?

6. Use a fixed-point iteration method to find an approximation to $\sqrt{3}$ that is accurate to 10^{-4}.

7. The following four methods are proposed to compute $\sqrt[5]{7}$. Rank them in order based on their apparent speed of convergence, assuming $x_0 = 1$.
(i) $x_n = x_{n-1}\left(1 + \dfrac{7 - x_{n-1}^5}{x_{n-1}^2}\right)^3$, (ii) $x_n = x_{n-1} - \dfrac{x_{n-1}^5 - 7}{x_{n-1}^2}$, (iii) $x_n = x_{n-1} - \dfrac{x_{n-1}^5 - 7}{5x_{n-1}^4}$,
(iv) $x_n = x_{n-1} - \dfrac{x_{n-1}^5 - 7}{12}$. Based upon the first four iterations, which one do you think gives the best approximation to the solution?

8. For each of the following equations, determine an interval $[a, b]$ on which fixed-point iteration will converge. (i) Estimate the number of iterations necessary to obtain approximations accurate to within 10^{-5}, and (ii) Perform the calculations.
(i) $3x^2 - e^x = 0$, (ii) $x - \cos x = 0$.

9. Use Newton's method to find the negative zero of the function $f(x) = e^x - 1.5 - \tan^{-1} x$. Investigate the sensitivity of the root to perturbations in the constant term.

10. Let $f(x) = 2^{x^2} - 3 \cdot 7^{x+1}$.

    (i) Plot $f(x)$ to find initial approximations to roots of f. (ii) Use Newton's method to find roots of f to within 10^{-16}.

11. Write a program to carry out the secant method on a function f, assuming that two starting points are given. Test the routine on these functions. a. $\sin(x/2) - 1$, b. $e^x - \tan x$, and c. $x^3 - 12x^2 + 3x + 1$.

12. Perform two iterations of Newton's method on these systems. a. Starting with (0,1) b. Starting with (1,1)

$$a. \begin{cases} 4x_1^2 - x_2^2 & = 0 \\ 4x_1 x_2^2 - x_1 & = 1 \end{cases} \qquad b. \begin{cases} xy^2 + x^2 y + x^4 & = 3 \\ x^3 y^5 - 2x^5 y - x^2 & = -2 \end{cases}$$

13. Solve this pair of simultaneous nonlinear equations by first eliminating y and then solving the resulting equation in x by Newtons method. Start with the initial value $x_0 = 1.0$.

$$\begin{cases} x^3 - 2xy + y^7 - 4x^3 y & = 5 \\ y \sin x + 3x^2 y + \tan x & = 4 \end{cases}$$

14. Find a complex root of each of the following: (i) $z^4 - 2z^3 - 2iz^2 + 4iz = 0$, (ii) $z = e^z$.