DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 322: Scientific Computing Lab - VIII

1. Solve the following two-point boundary–value problems by using forward, backward and central difference for the first–order derivative and central difference for the second–order derivative.

$$\begin{cases} y'' + \dfrac{4}{x}y' + \dfrac{2}{x^2}y = \dfrac{2\ln x}{x^2}, \quad 1 < x < 2. \\ \text{Boundary conditions:} \\ y(1) = \dfrac{1}{2}, \quad y(2) = \ln 2. \\ \text{Exact solution:} \\ \quad y(x) = \dfrac{4}{x} - \dfrac{2}{x^2} + \ln x - \dfrac{3}{2}. \end{cases}$$

$$\begin{cases} y'' - 2y' + y = xe^x - x, \quad 0 < x < 2. \\ \text{Boundary conditions:} \\ y(0) = 0, \quad y(2) = -4. \\ \text{Exact solution:} \\ \quad y(x) = \dfrac{1}{6}x^3e^x - \dfrac{5}{3}xe^x + 2e^x - x - 2. \end{cases}$$

$$\begin{cases} y'' - y' - 2y = \cos x, \quad 0 < x < \dfrac{\pi}{2}. \\ \text{Boundary conditions:} \\ y'(0) = -\dfrac{1}{10}, \quad y'\left(\dfrac{\pi}{2}\right) = \dfrac{3}{10}. \\ \text{Exact solution:} \\ y(x) = -\dfrac{1}{10}\left(\sin x + 3\cos x\right). \end{cases}$$

$$\begin{cases} y'' + xy' - 2y = 2 + (2 + x^2)e^x, \quad -1 < x < 0. \\ \text{Boundary conditions:} \\ y'(-1) = -2, \quad y'(0) = 1. \\ \text{Exact solution:} \\ y(x) = x^2 + xe^x. \end{cases}$$

$$\begin{cases} y'' + 2y' + y = x, \quad 0 < x < 1. \\ \text{Boundary conditions:} \\ y(0) + y'(0) = e - 3, \\ y(1) + y'(1) = 1 - \dfrac{2}{e}. \\ \text{Exact solution:} \\ y(x) = 2e^{-x} + (e - 2)xe^{-x} + x - 2. \end{cases}$$

$$\begin{cases} y'' + \cos(x)y' + y = (\sqrt{2} - 1)\cos^2 x - \dfrac{\sin 2x}{2}, \ 0 < x < \dfrac{\pi}{4}. \\ \text{Boundary conditions:} \\ y(0) + y'(0) = \sqrt{2}, \\ y\left(\dfrac{\pi}{4}\right) + y'\left(\dfrac{\pi}{4}\right) = 2 - \sqrt{2}. \\ \text{Exact solution:} \\ y(x) = \cos x + (\sqrt{2} - 1)\sin x. \end{cases}$$

The output folder should contains the following things:

1. It should include the MATLAB code of each program.
2. The outputs for each question should be given in one or all of the followings forms:
   - (i) The final answers are to be copied in a word document in terms of the numerical values and plots.
   - (ii) The final answers and errors (if possible to calculate) are to be given in plots (line plot, surface plot, etc.)
   - (iii) The order of convergence has to be calculated and the final answers have to be given in Tables, and log-log plots.