

MA 322: Scientific Computing Lab - VII

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1. Use the methods mentioned below to approximate the solutions to each of the following initial-value problems, and compare the results to the actual values.

(a)  $y' = \frac{2 - 2ty}{t^2 + 1}$ ,  $0 \leq t \leq 1$ ,  $y(0) = 1$  with  $h = 0.1$ ; actual solution  $y(t) = \frac{2t + 1}{t^2 + 1}$ .

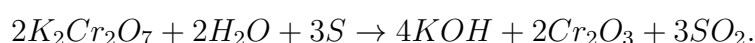
(b)  $y' = \frac{y^2 + y}{t}$ ,  $1 \leq t \leq 3$ ,  $y(1) = -2$  with  $h = 0.2$ ; actual solution  $y(t) = \frac{2t}{1 - 2t}$ .

(c)  $y' = 1 + y/t + (y/t)^2$ ,  $1 \leq t \leq 3$ ,  $y(1) = 0$  with  $h = 0.2$ ; actual solution  $y(t) = t \tan(\ln t)$ .

(d)  $y' = e(t - y)$ ,  $0 \leq t \leq 1$ ,  $y(0) = 1$  with  $h = 0.5$ ; actual solution  $y(t) = \ln(e^t + e - 1)$ .

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| 1. Explicit-Euler method                       | 2. Implicit-Euler method |
| 3. Modified-Euler method                       | 4. Midpoint method       |
| 5. Second and Fourth-order Runge-Kutta methods |                          |

2. Solve the initial-value problem  $x' = x/t + t \sec(x/t)$  with  $x(0) = 0$  by the fourth-order Runge-Kutta method. Continue the solution to  $t = 1$  using step size  $h = 2^{-7}$ . Compare the numerical solution with the exact solution, which is  $x(t) = t \arcsin t$ . Define  $f(0, 0) = 0$ , where  $f(t, x) = x/t + t \sec(x/t)$ .
3. The irreversible chemical reaction in which two molecules of solid potassium dichromate ( $K_2Cr_2O_7$ ), two molecules of water ( $H_2O$ ), and three atoms of solid sulfur ( $S$ ) combine to yield three molecules of the gas sulfur dioxide ( $SO_2$ ), four molecules of solid potassium hydroxide ( $KOH$ ), and two molecules of solid chromic oxide ( $Cr_2O_3$ ) can be represented symbolically by the stoichiometric equation:



If  $n_1$  molecules of  $K_2Cr_2O_7$ ,  $n_2$  molecules of  $H_2O$ , and  $n_3$  molecules of  $S$  are originally available, the following differential equation describes the amount  $x(t)$  of  $KOH$  after time  $t$ :

$$\frac{dx}{dt} = k \left( n_1 - \frac{x}{2} \right)^2 \left( n_2 - \frac{x}{2} \right)^2 \left( n_3 - \frac{3x}{4} \right)^3$$

where  $k$  is the velocity constant of the reaction. If  $k = 6.22 \times 10^{-19}$ ,  $n_1 = n_2 = 2 \times 10^3$ , and  $n_3 = 3 \times 10^3$ , use the Runge-Kutta method of order four to determine how many units of potassium hydroxide will have been formed after  $0.2s$ ?

4. Use Adams-Bashforth and Adams-Moulton methods to approximate the solutions to the IVPs given in Question 1.
  - (a) Use exact starting values.
  - (b) Use starting values obtained from the Runge-Kutta method of order four.

Compare the results to the actual values.