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MA 332: Scientific Computing Lab - II

1. Use the Bisection method to find the root of the equation $e^x = \sin x$ closest to 0.

2. Use the Bisection method to find solution accurate to within 10^{-3} for the following problems.

   a. $2 + \cos(e^x - 2) - e^x = 0 \quad \text{for} \quad 0.5 \leq x \leq 1.5$

   b. $e^x - x^2 + 3x - 2 = 0 \quad \text{for} \quad 0 \leq x \leq 1$

   c. $2x\cos(2x) - (x+1)^2 = 0 \quad \text{for} \quad -3 \leq x \leq -2 \quad \text{and} \quad -1 \leq x \leq 0$

   d. $x\cos(x) - 2x^2 + 3x - 1 = 0 \quad \text{for} \quad 0.2 \leq x \leq 0.3 \quad \text{and} \quad 1.2 \leq x \leq 1.3$

3. A particle starts at rest on a smooth inclined plane whose angle θ is changing at a constant rate $\dfrac{d\theta}{dt} = \omega < 0$. At the end of t seconds, the position (measured from starting point) of the object is given by

$$x(t) = -\frac{g}{2\omega^2}\left(\frac{e^{\omega t} - e^{-\omega t}}{2} - \sin\omega t\right).$$

   Suppose the particle has moved $1.7ft$ in $1s$. Find, within 10^{-5}, the rate ω at which θ changes. Assume that $g = 32.17ft/s^2$.

4. Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^3 - x - 1 = 0$ on $[1, 2]$. Use $x_0 = 1$.

5. The following four methods are proposed to compute $\sqrt[5]{7}$. Rank them in order based on their apparent speed of convergence, assuming $x_0 = 1$.

   (i) $x_n = x_{n-1}\left(1 + \dfrac{7 - x_{n-1}^5}{x_{n-1}^2}\right)^3$, (ii) $x_n = x_{n-1} - \dfrac{x_{n-1}^5 - 7}{x_{n-1}^2}$, (iii) $x_n = x_{n-1} - \dfrac{x_{n-1}^5 - 7}{5x_{n-1}^4}$,

   (iv) $x_n = x_{n-1} - \dfrac{x_{n-1}^5 - 7}{12}$. Based upon the first four iterations, which one do you think gives the best approximation to the solution?

6. For each of the following equations, determine an interval $[a, b]$ on which fixed-point iteration will converge. (i) Estimate the number of iterations necessary to obtain approximations accurate to within 10^{-5}, and (ii) Perform the calculations.

   (i) $3x^2 - e^x = 0$, (ii) $x - \cos x = 0$.

7. Use Newton's method to find solution accurate to within 10^{-5} for the following problems.

   a. $e^x + 2^{-x} + 2\cos(x) - 6 = 0 \quad \text{for} \quad 1 \leq x \leq 2$

b. $(x-2)^2 - \ln x = 0$ for $1 \le x \le 2$ and $e \le x \le 4$

c. $e^x - 3x^2 = 0$ for $0 \le x \le 1$ and $3 \le x \le 5$

d. $\sin x - e^{-x} = 0$ for $0 \le x \le 1$, $3 \le x \le 4$ and $6 \le x \le 7$

8. Let $f(x) = 2^{x^2} - 3 \cdot 7^{x+1}$.

   (i) Plot $f(x)$ to find initial approximations to roots of f. (ii) Use Newton's method to find roots of f to within 10^{-16}.

9. The fourth degree polynomial $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$, has two real zeros, one in $[-1, 0]$ and the other in $[0, 1]$, Attempt to approximate these zeros to within 10^{-6} using the

   a. Newton's method

   b. Secant method

   Use the midpoints of each interval as the initial approximation in (a) and the endpoints of each interval as the initial approximations in (b).

10. Repeat the problems given in **Exercise 7** using the secant method. What can you comment on the number of iterations and accuracy of the two methods based on your run?

11. Use secant method to approximate, to within 10^{-4}, the value of x that produces the point on the graph of $y = x^2$ that is closest to $(1, 0)$.

12. Find an approximation for γ, accurate to within 10^{-4}, the population equation

$$1,564,000 = 1,000,000e^{\gamma} + \frac{435,000}{\gamma}(e^{\gamma} - 1).$$
