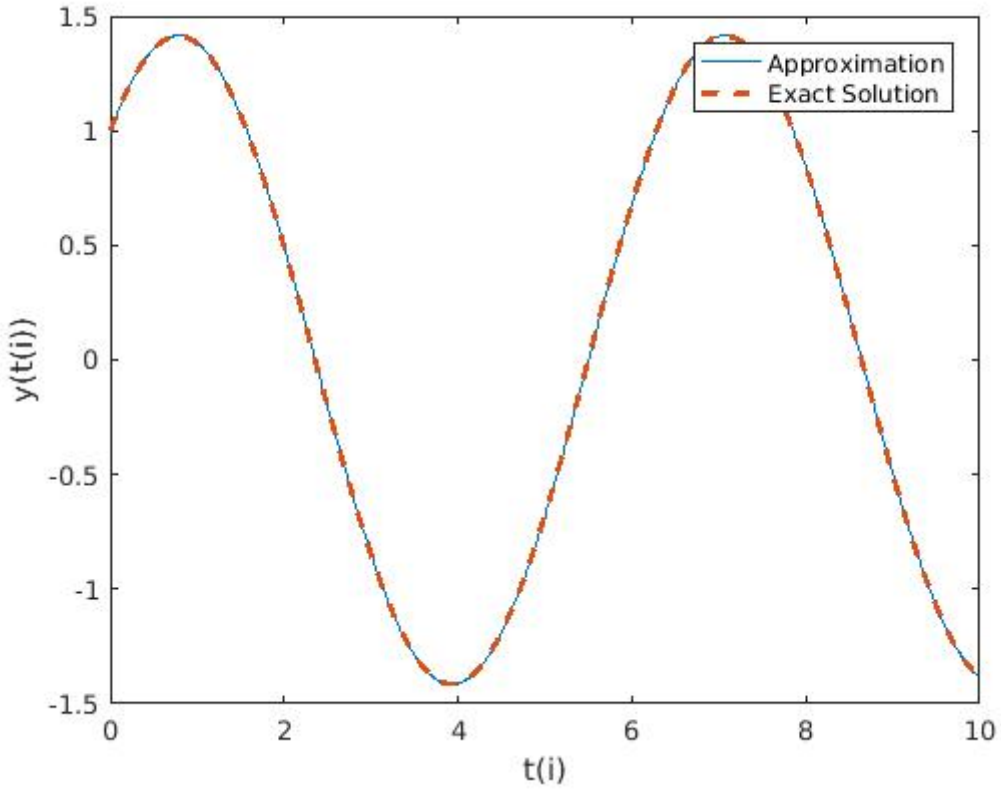
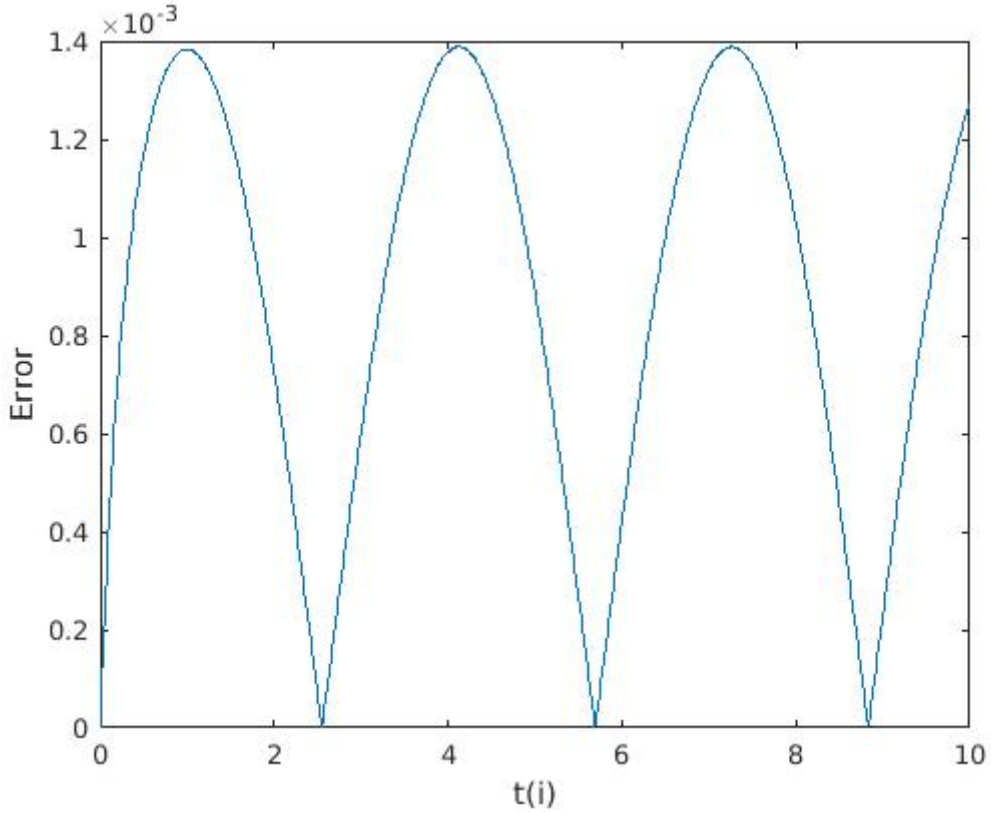
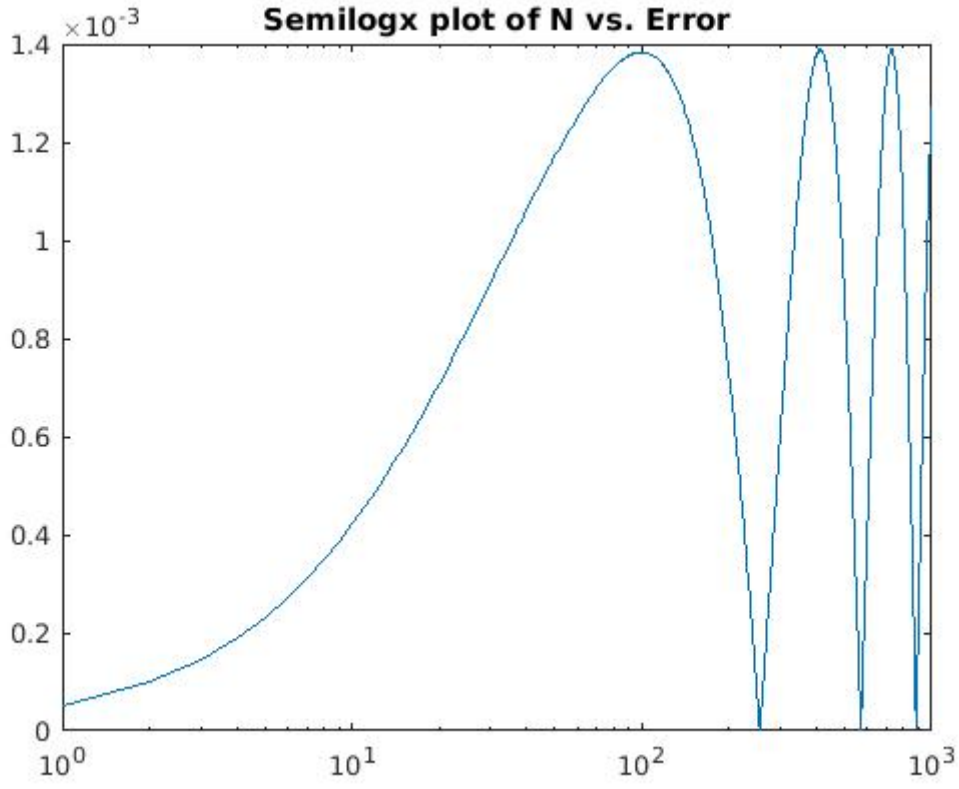
Lab 7

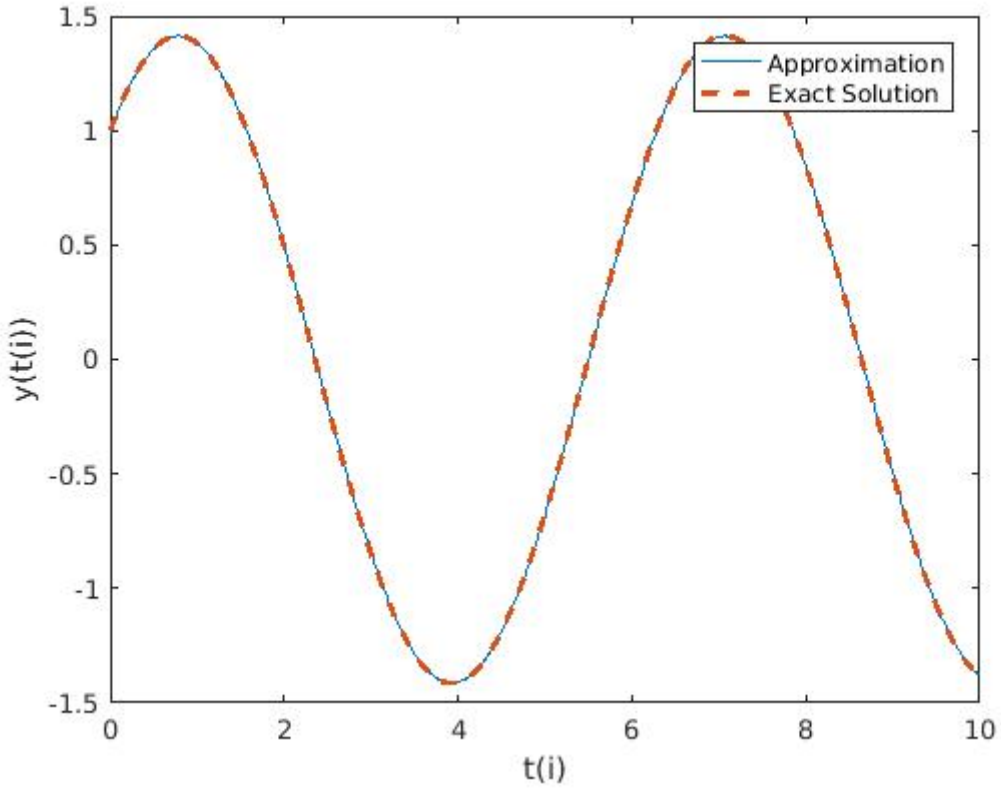
Problem 1: With λ = -5 and h=0.01 , the following figures are obtained by explicit Euler:

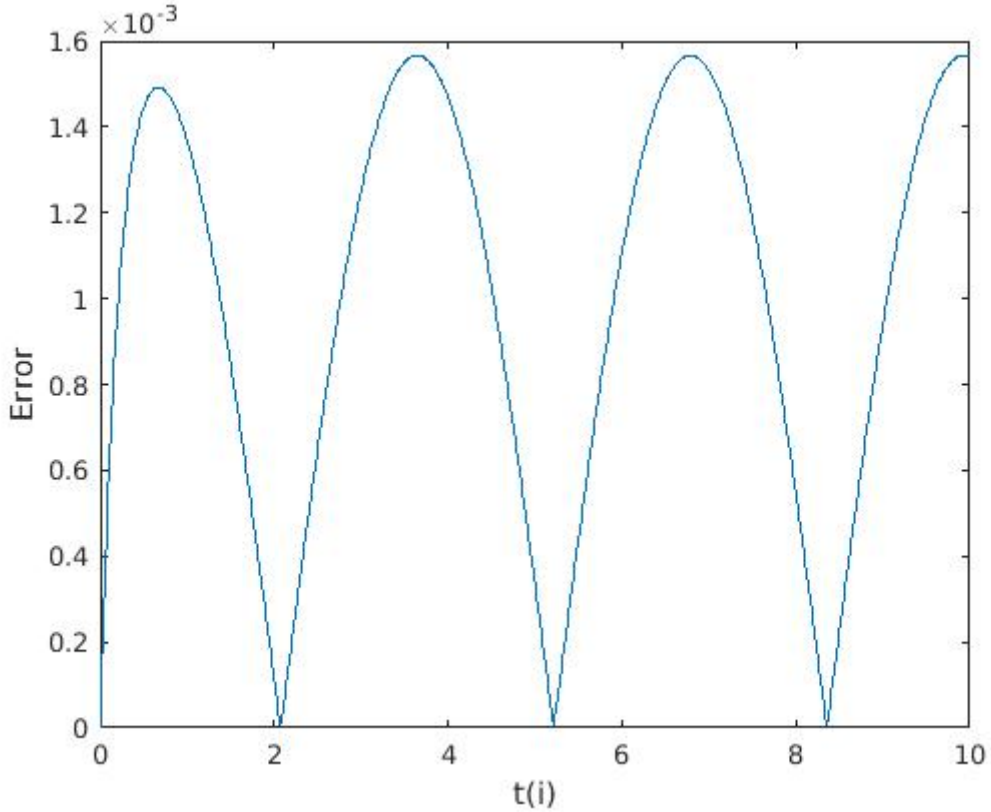


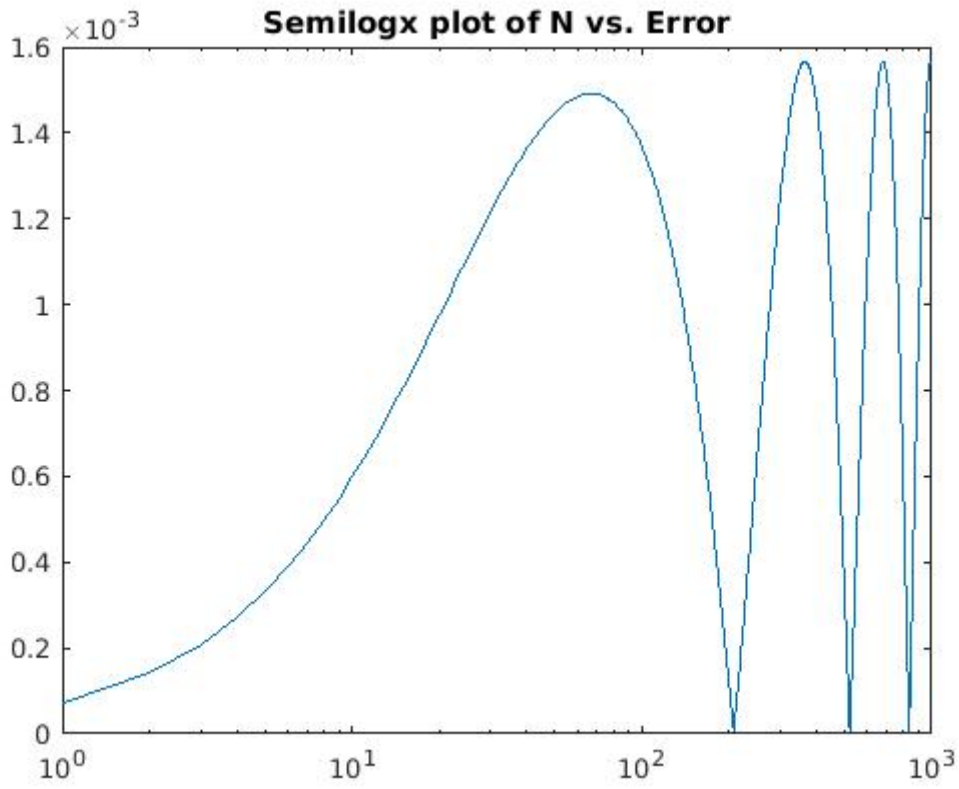




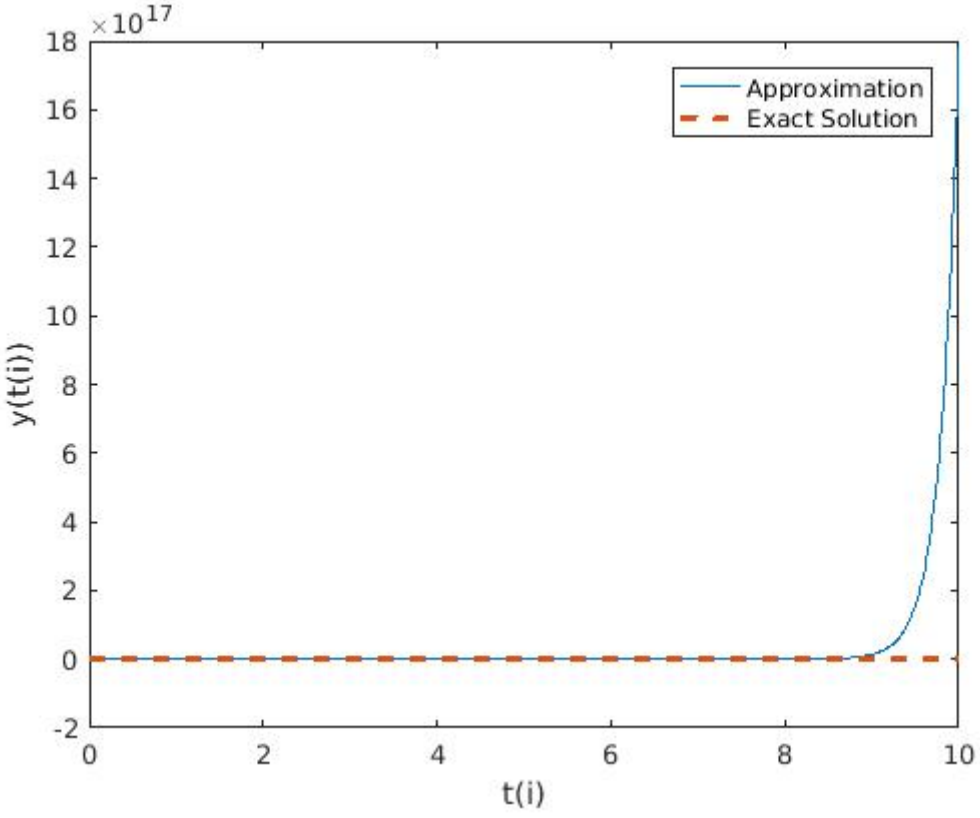
With λ = -5 and h=0.01 , the following figures are obtained by implicit Euler:

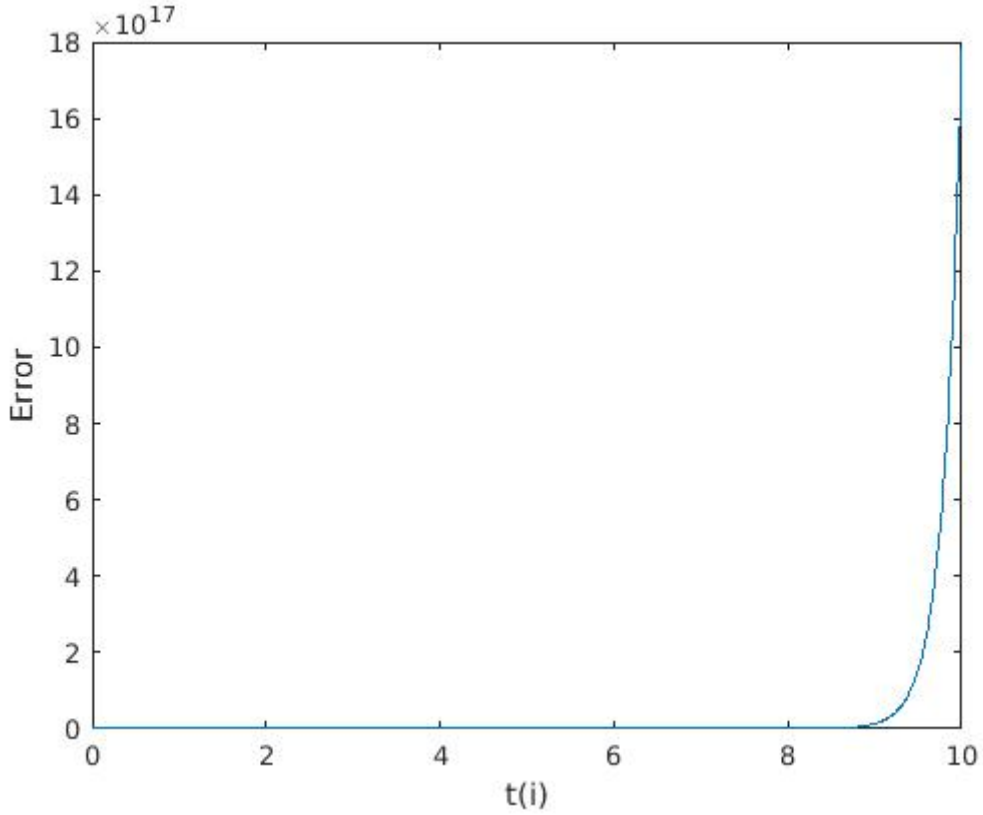

Approximation
Exact Solution
y(t(i))
t(i)
1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
10


×10^-3
Error
t(i)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10

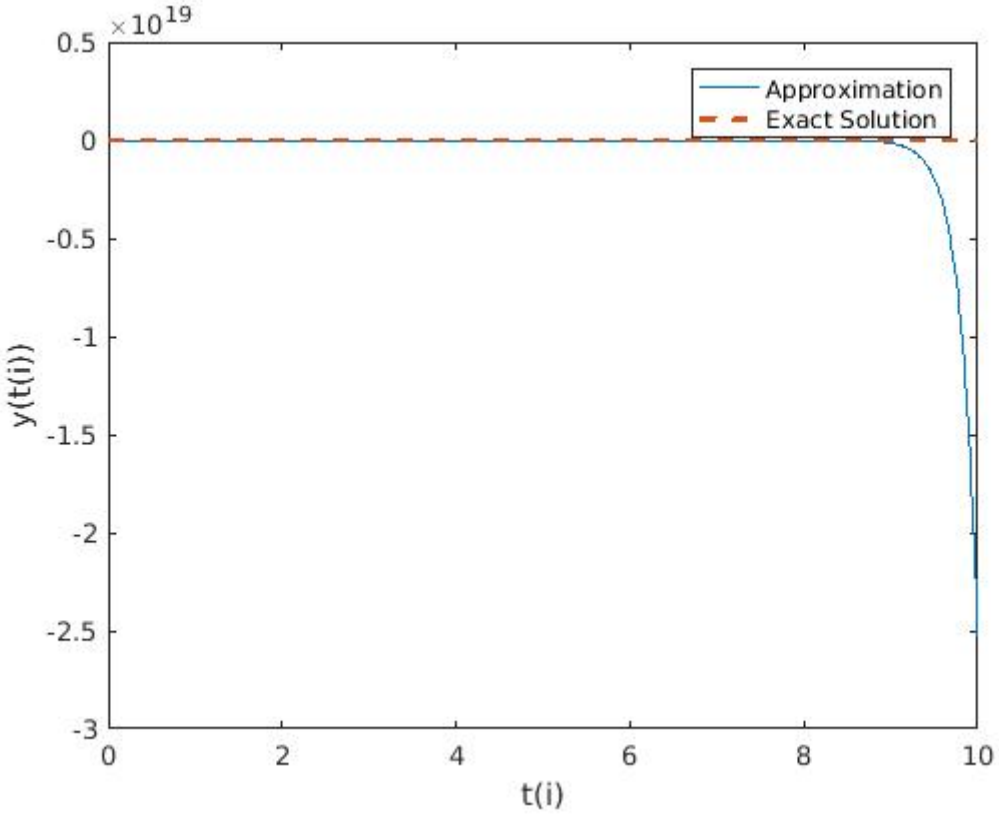


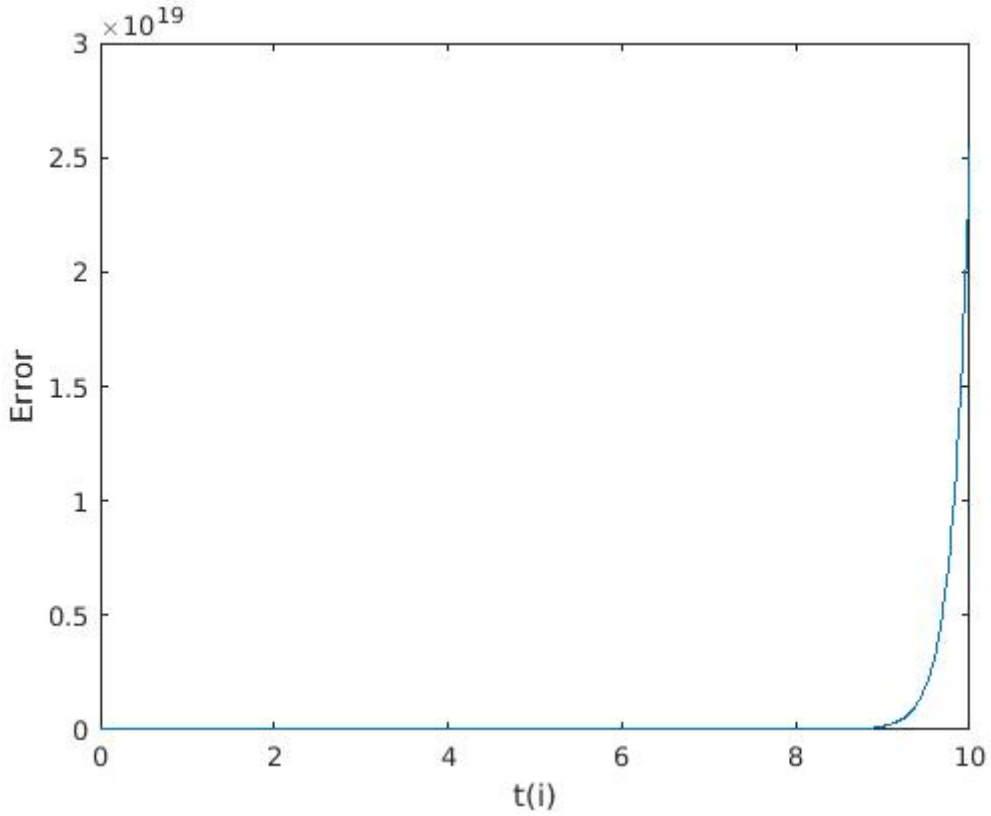
With λ = 5 and h=0.01 , the following figures are obtained by explicit Euler:



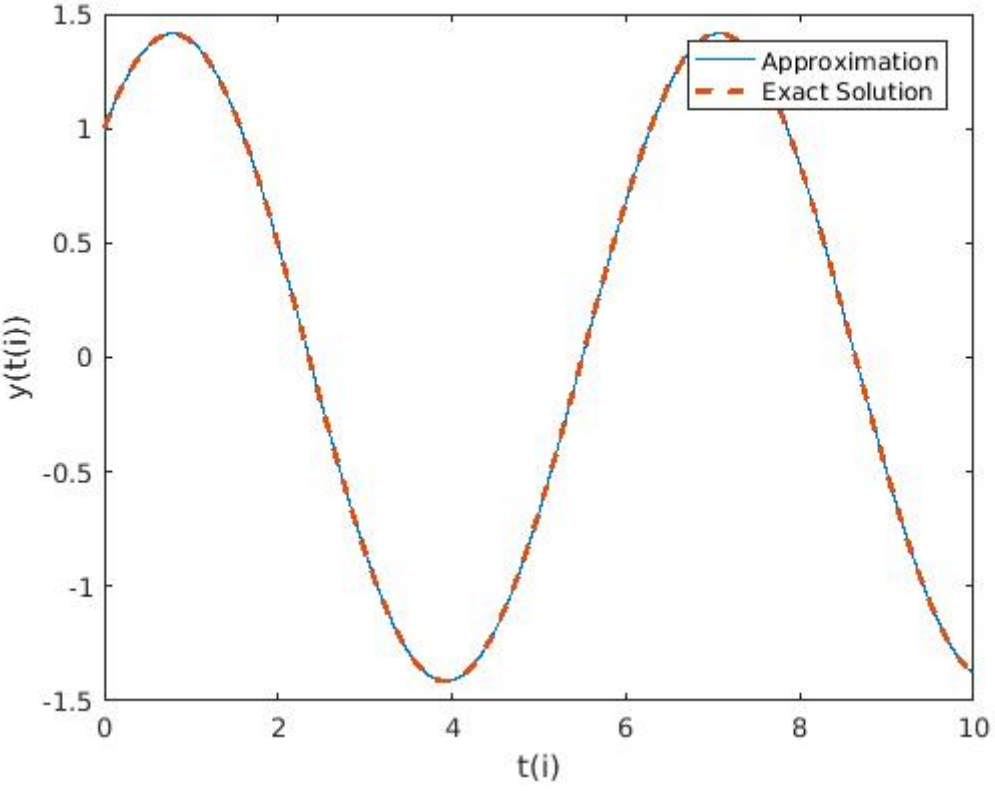


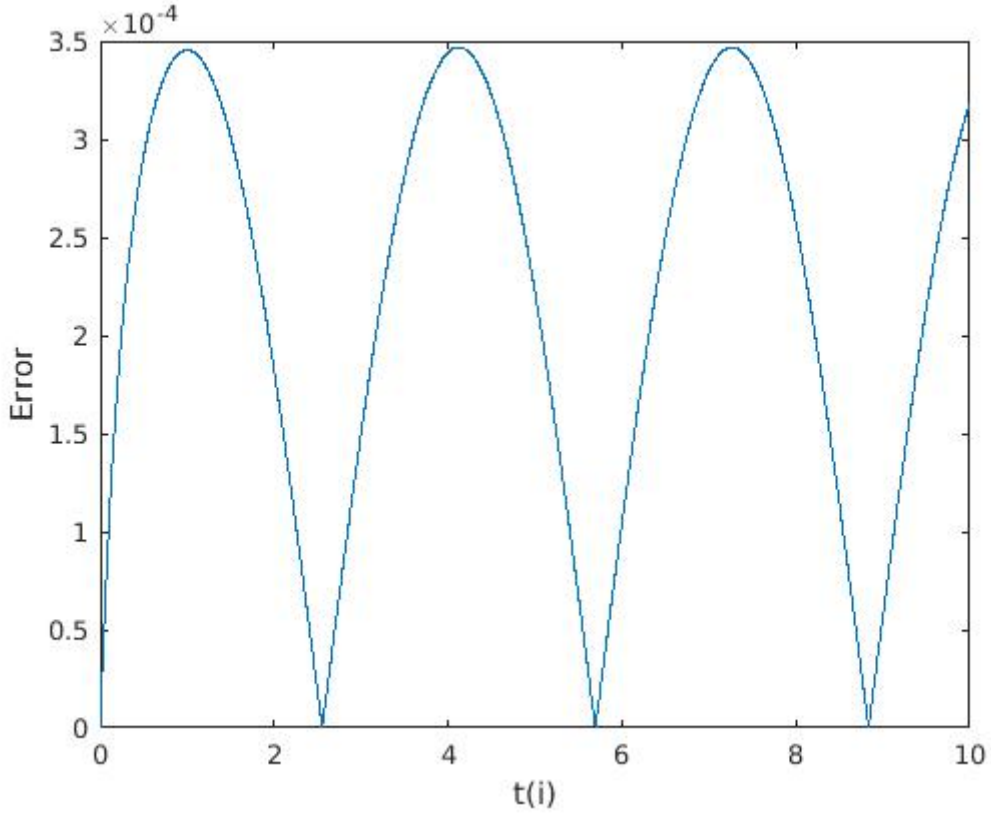
Clearly, the values blow up with +5 value of lambda by both explicit and implicit Euler. The figures for $\lambda = 5$ and h=0.01 by implicit Euler are shown below:

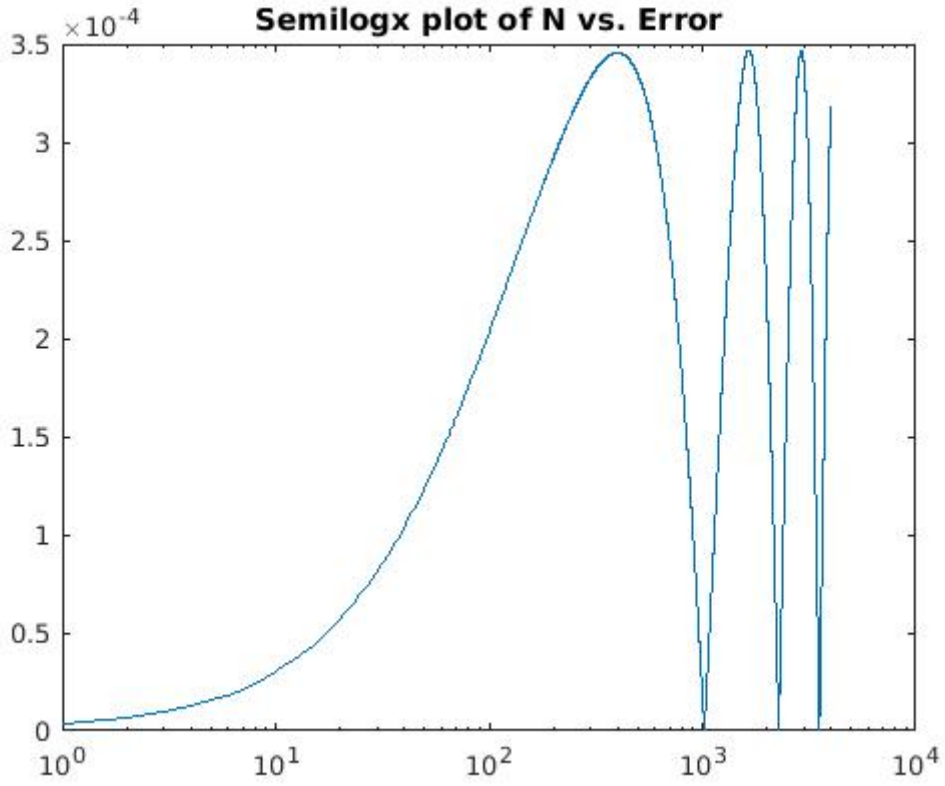




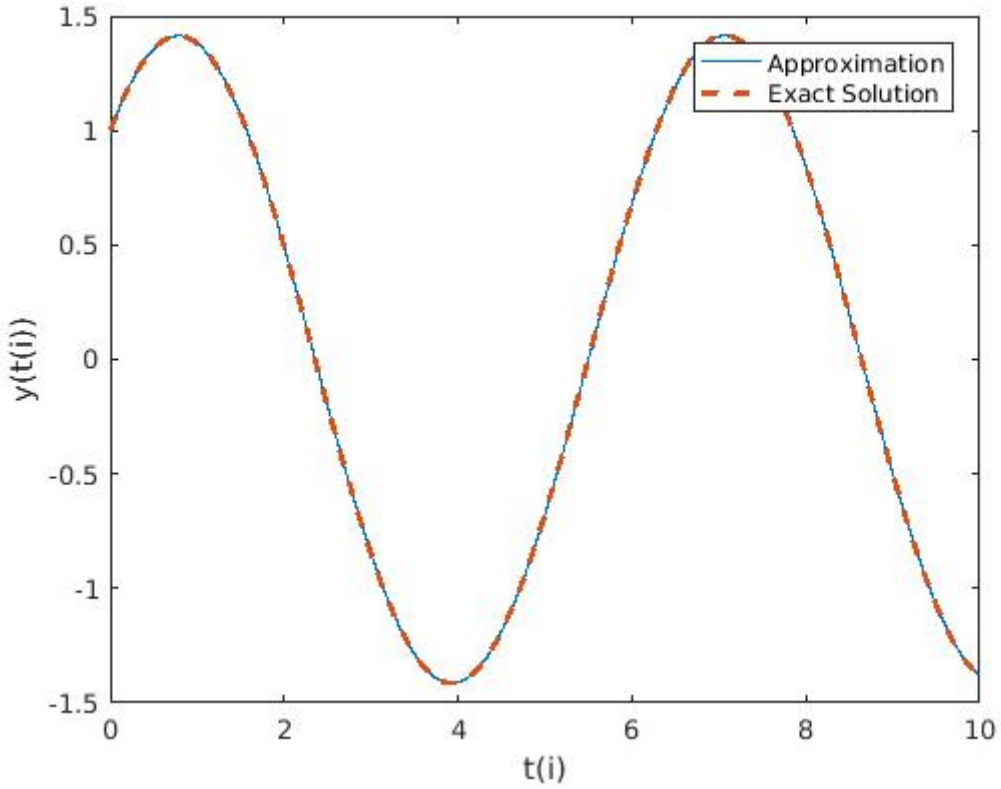
With λ = -5 and h=0.0025 , the following figures are obtained by explicit Euler:

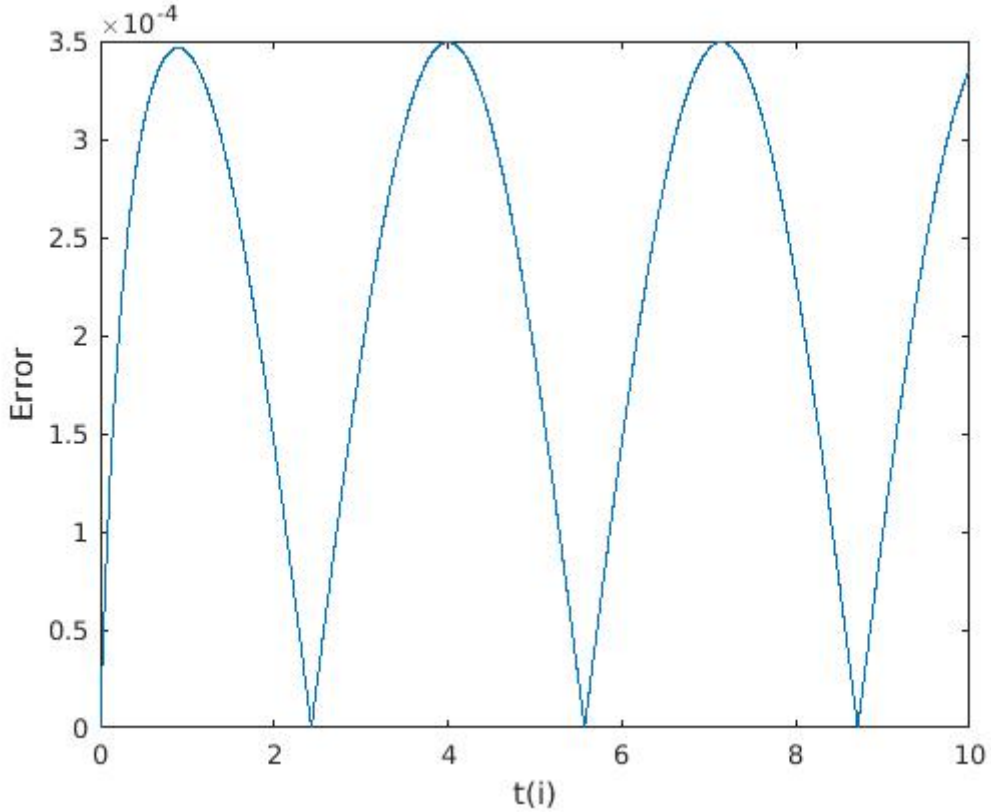


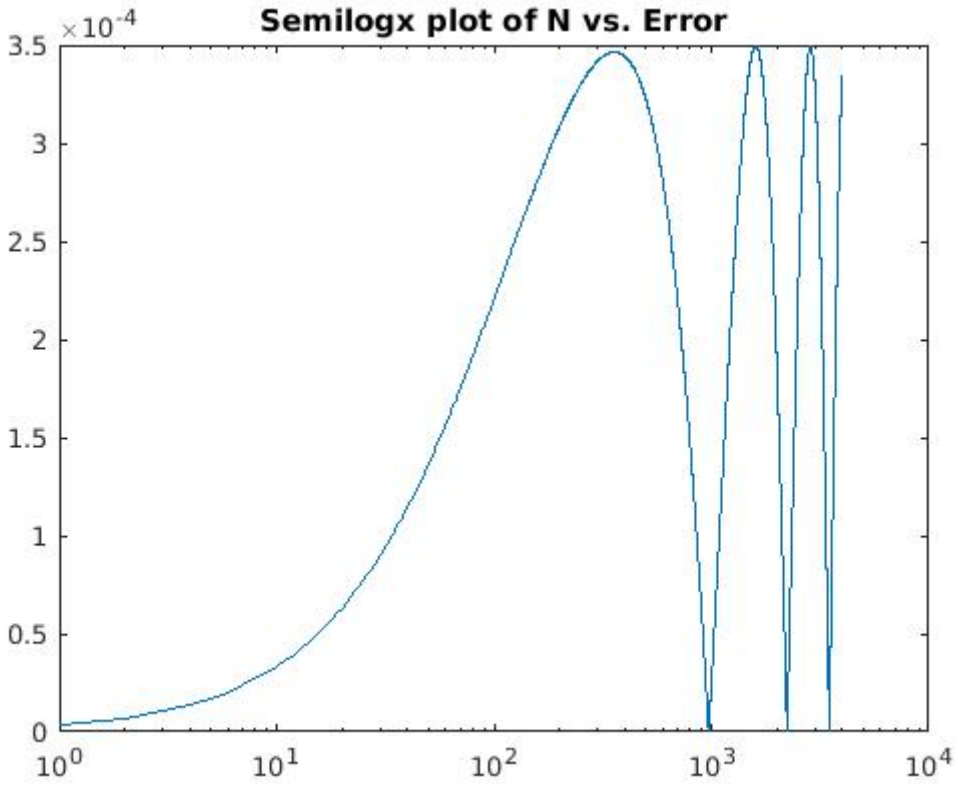




With λ = -5 and h=0.0025 , the following figures are obtained by implicit Euler:


Approximation
Exact Solution
y(t(i))
t(i)
1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
10

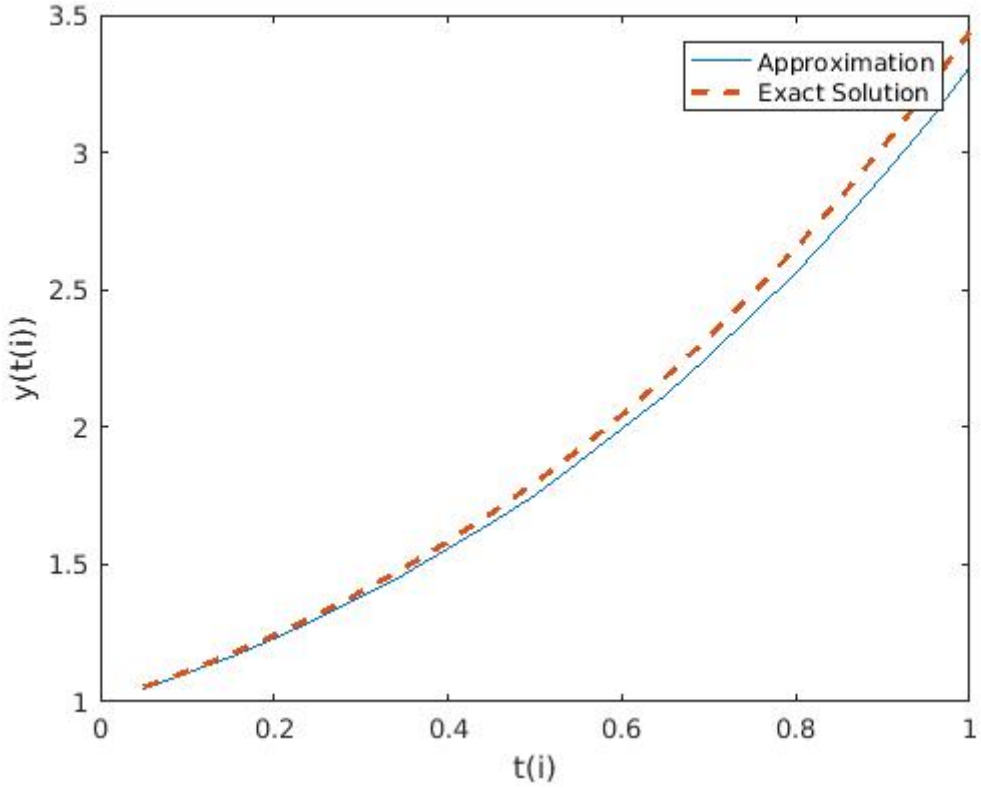

×10^-4
Error
t(i)
3.5
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10

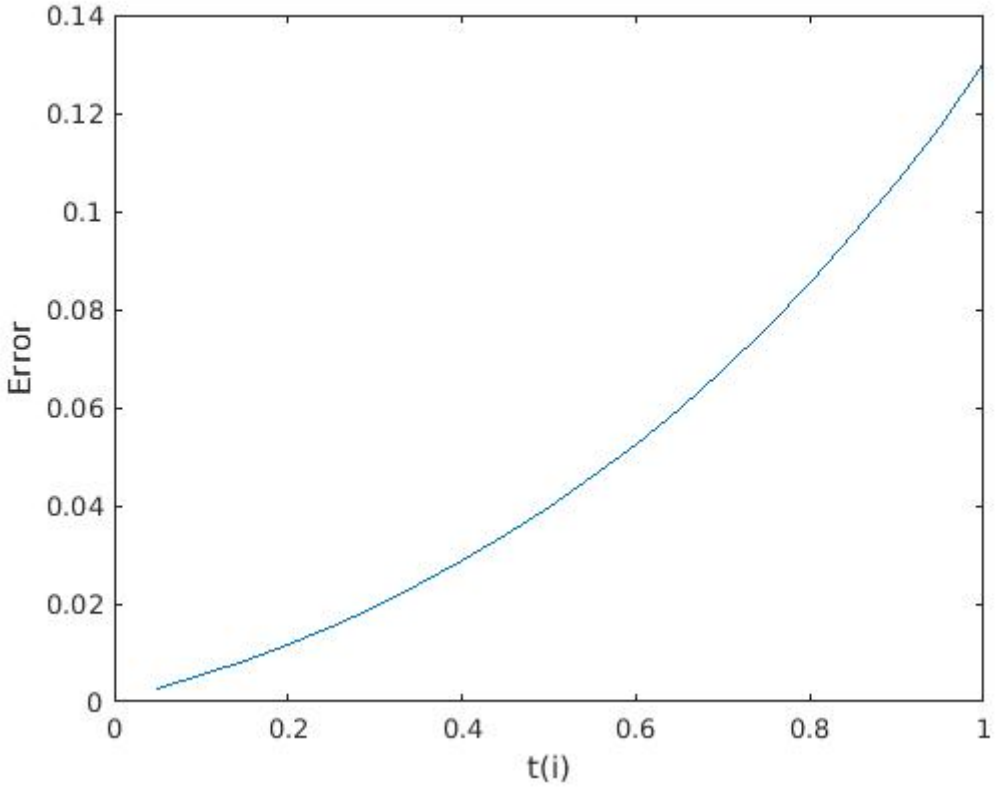


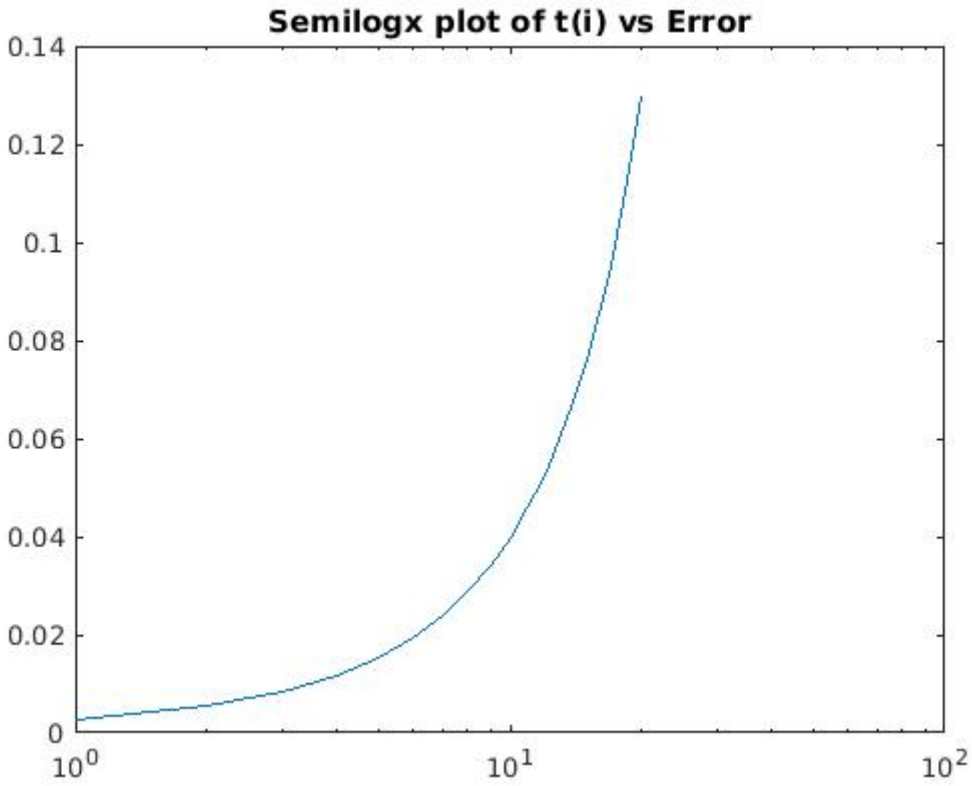
Problem 2: Using h=0.05 , by simple Euler's method, we get the following table of observed values and various required plots:

By Simple Euler's method:

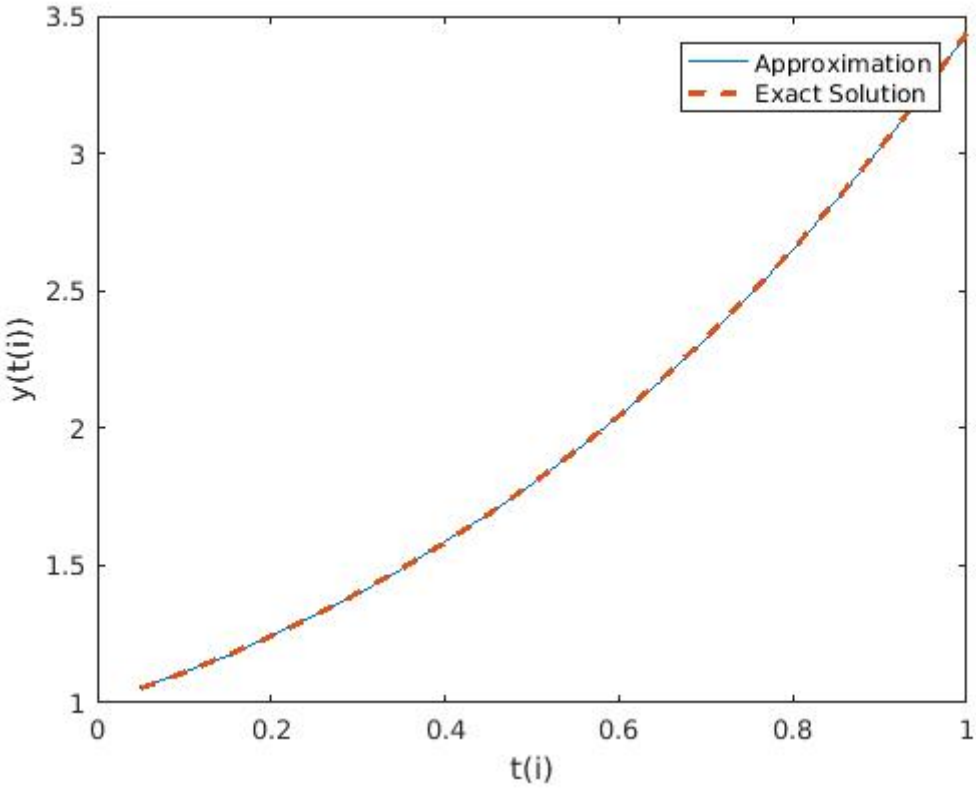
t	Approx	Exact	Error
0.050000	1.050000	1.052542	0.002542
0.100000	1.105000	1.110342	0.005342
0.150000	1.165250	1.173668	0.008418
0.200000	1.231012	1.242806	0.011793
0.250000	1.302563	1.318051	0.015488
0.300000	1.380191	1.399718	0.019526
0.350000	1.464201	1.488135	0.023934
0.400000	1.554911	1.583649	0.028739
0.450000	1.652656	1.686624	0.033968
0.500000	1.757789	1.797443	0.039653
0.550000	1.870679	1.916506	0.045827
0.600000	1.991713	2.044238	0.052525
0.650000	2.121298	2.181082	0.059783
0.700000	2.259863	2.327505	0.067642
0.750000	2.407856	2.484000	0.076144
0.800000	2.565749	2.651082	0.085333
0.850000	2.734037	2.829294	0.095257
0.900000	2.913238	3.019206	0.105968
0.950000	3.103900	3.221419	0.117519
1.000000	3.306595	3.436564	0.129968

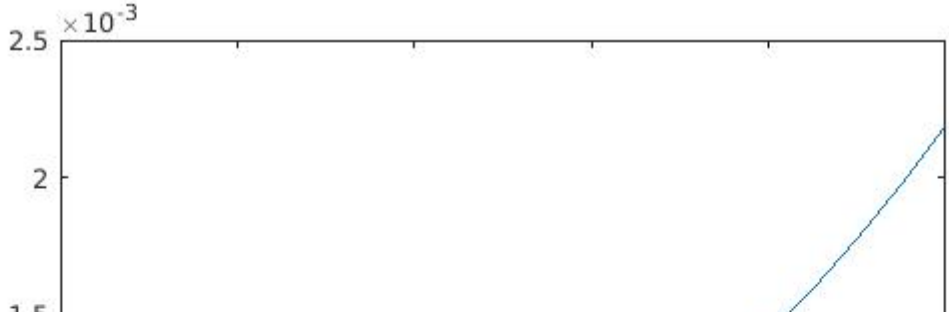

Approximation
Exact Solution
y(t(i))
t(i)
3.5
3
2.5
2
1.5
1
0
0.2
0.4
0.6
0.8
1

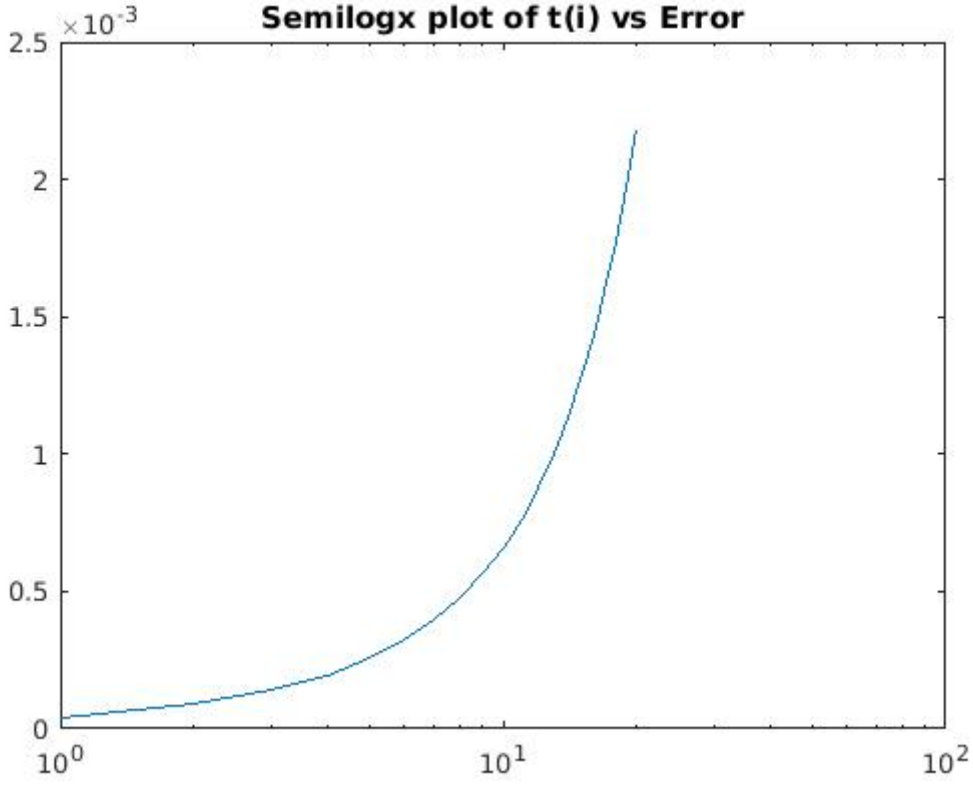

Error
t(i)
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1



Using h=0.05 , by modified Euler's method, we get the following table of observed values and various required plots:


Approximation
Exact Solution
y(t(i))
t(i)
3.5
3
2.5
2
1.5
1
0
0.2
0.4
0.6
0.8
1

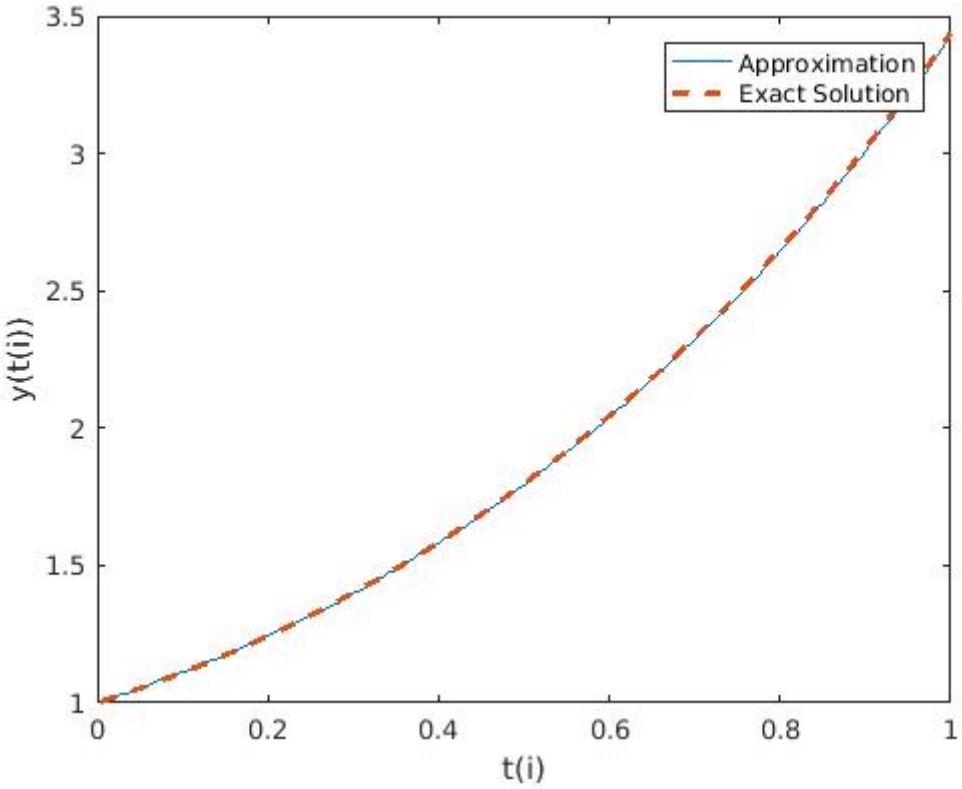

×10^-3
2.5
2

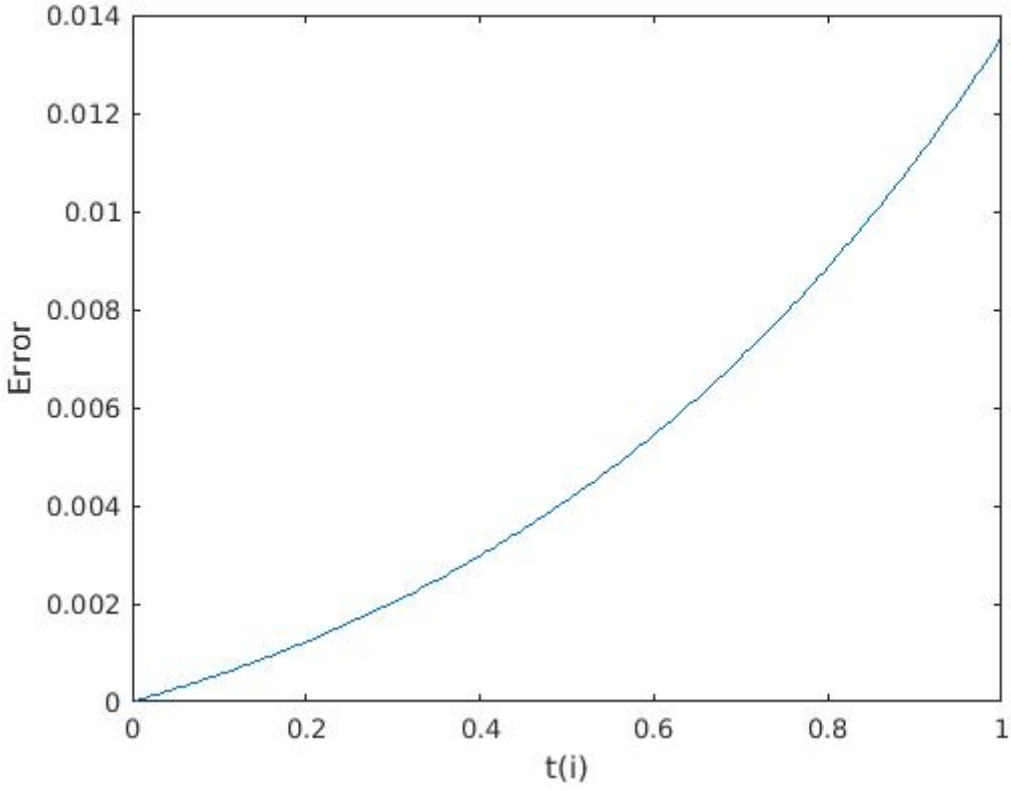

Semilogx plot of t(i) vs Error
×10^-3
2.5
2
1.5
1
0.5
0
10^0
10^1
10^2

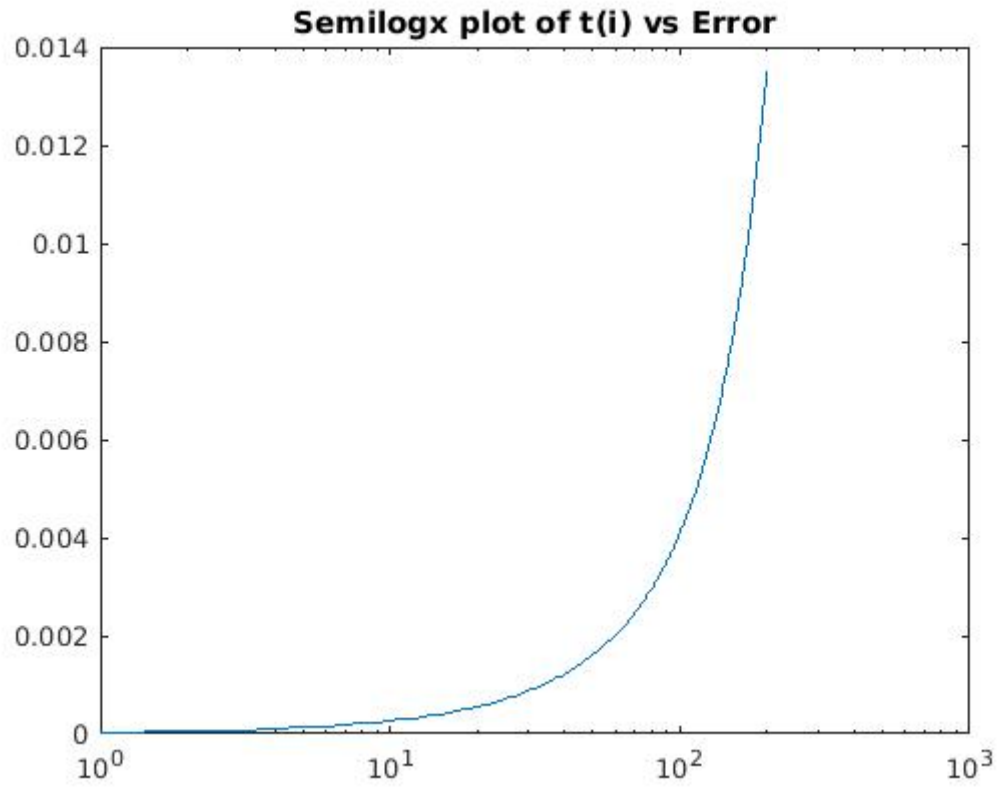
By modified Euler's method:

t	Approx	Exact	Error
0.050000	1.052500	1.052542	0.000042
0.100000	1.110253	1.110342	0.000089
0.150000	1.173529	1.173668	0.000140
0.200000	1.242609	1.242806	0.000196
0.250000	1.317793	1.318051	0.000258
0.300000	1.399393	1.399718	0.000325
0.350000	1.487736	1.488135	0.000399
0.400000	1.583170	1.583649	0.000479
0.450000	1.686058	1.686624	0.000566
0.500000	1.796781	1.797443	0.000662
0.550000	1.915741	1.916506	0.000765
0.600000	2.043360	2.044238	0.000877
0.650000	2.180082	2.181082	0.000999
0.700000	2.326374	2.327505	0.001131
0.750000	2.482726	2.484000	0.001274
0.800000	2.649653	2.651082	0.001429
0.850000	2.827698	2.829294	0.001596
0.900000	3.017430	3.019206	0.001777
0.950000	3.219448	3.221419	0.001971
1.000000	3.434382	3.436564	0.002182

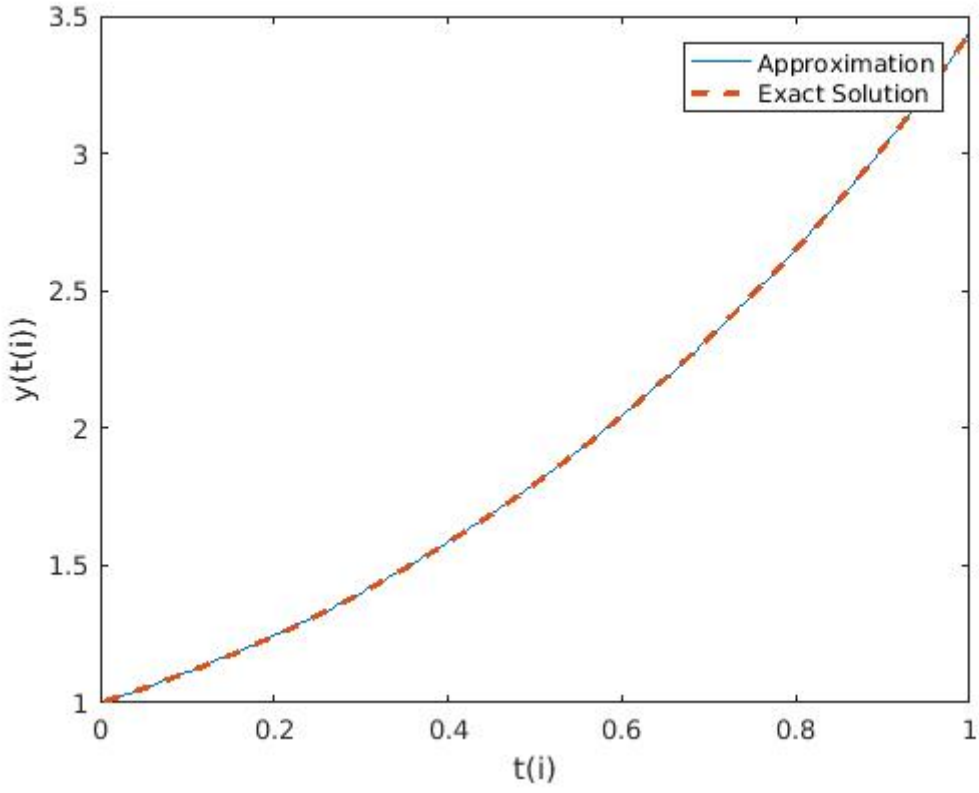
Using h=0.005 , by simple Euler's method, we get the following required plots:

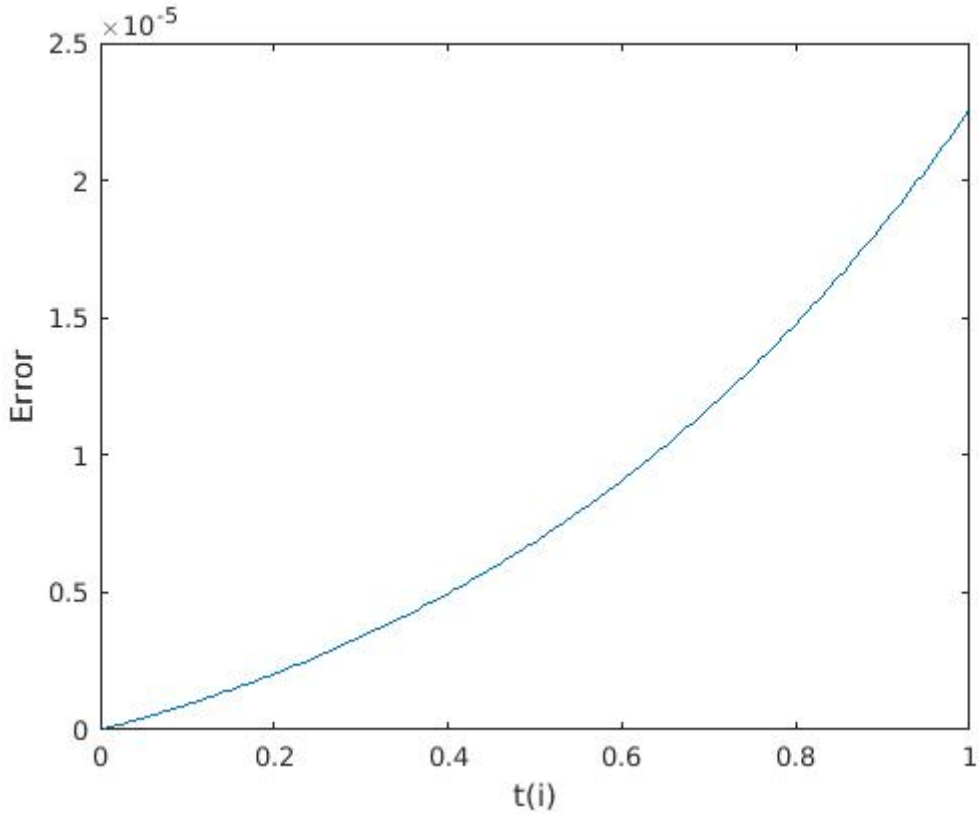

Approximation
Exact Solution
y(t(i))
t(i)
3.5
3
2.5
2
1.5
1
0
0.2
0.4
0.6
0.8
1

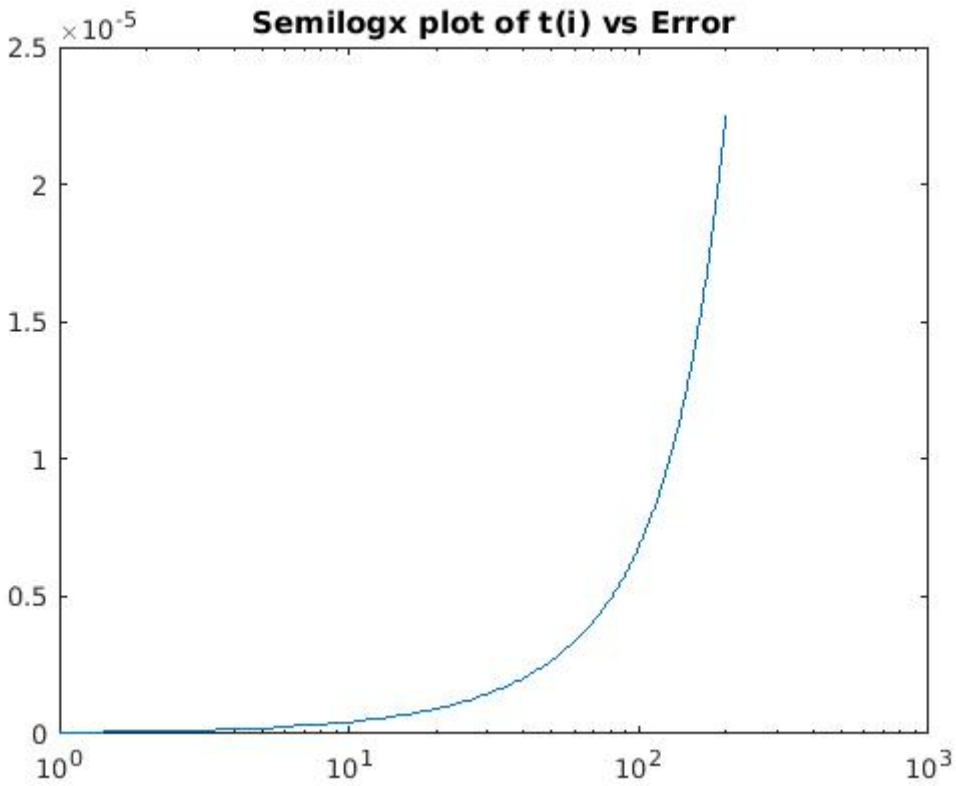

Error
t(i)
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
0.2
0.4
0.6
0.8
1



Using h=0.005 , by modified Euler's method, we get the following required plots:




×10^-5
2.5
2
1.5
1
0.5
0
Error
0
0.2
0.4
0.6
0.8
1
t(i)

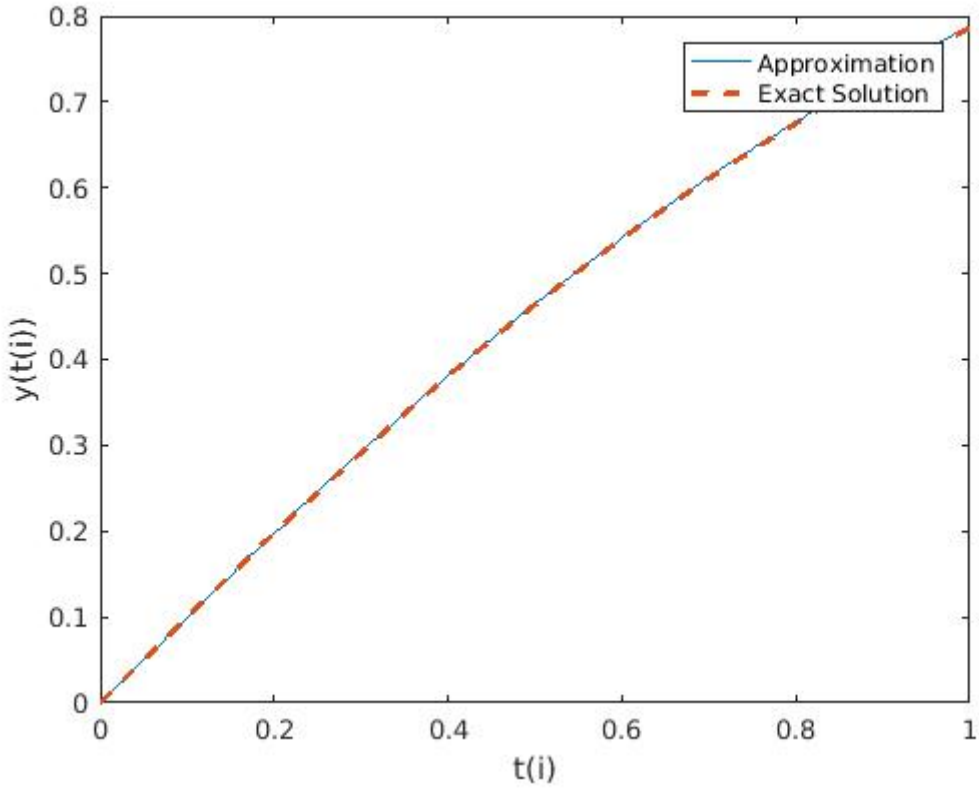


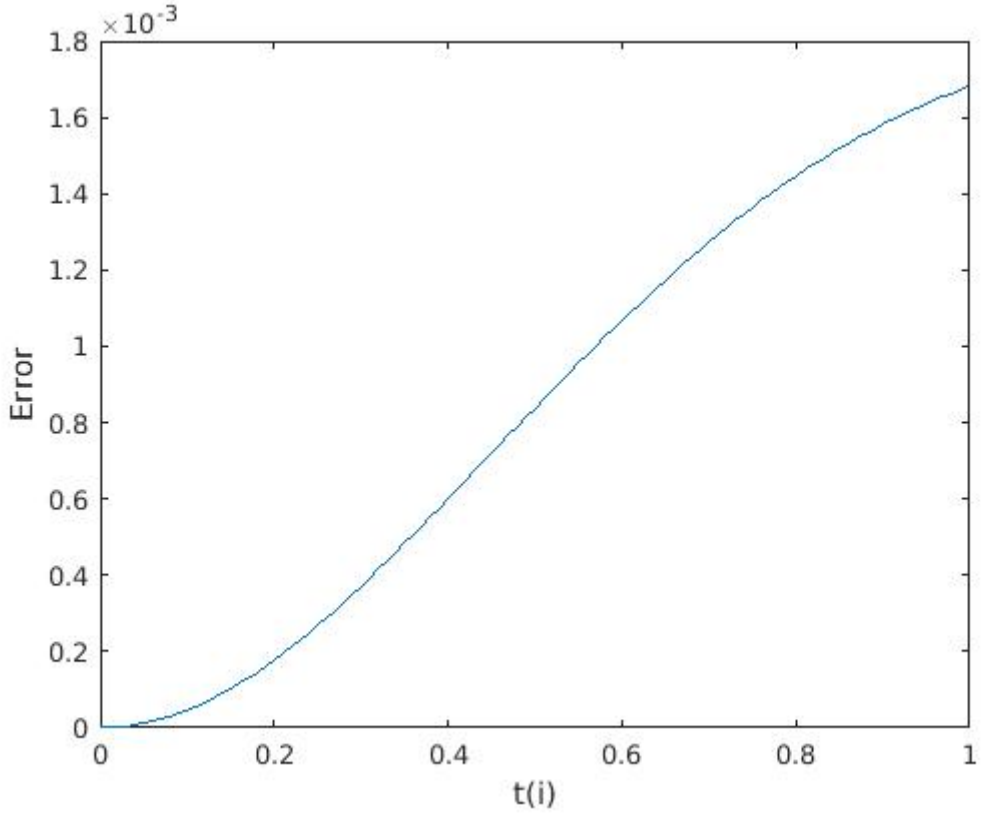
By simple Euler's method, with h = 0.005, approximation for y(1) = 3.423034 , exact value by analytical solution is 3.436564, therefore error = 0.013529.

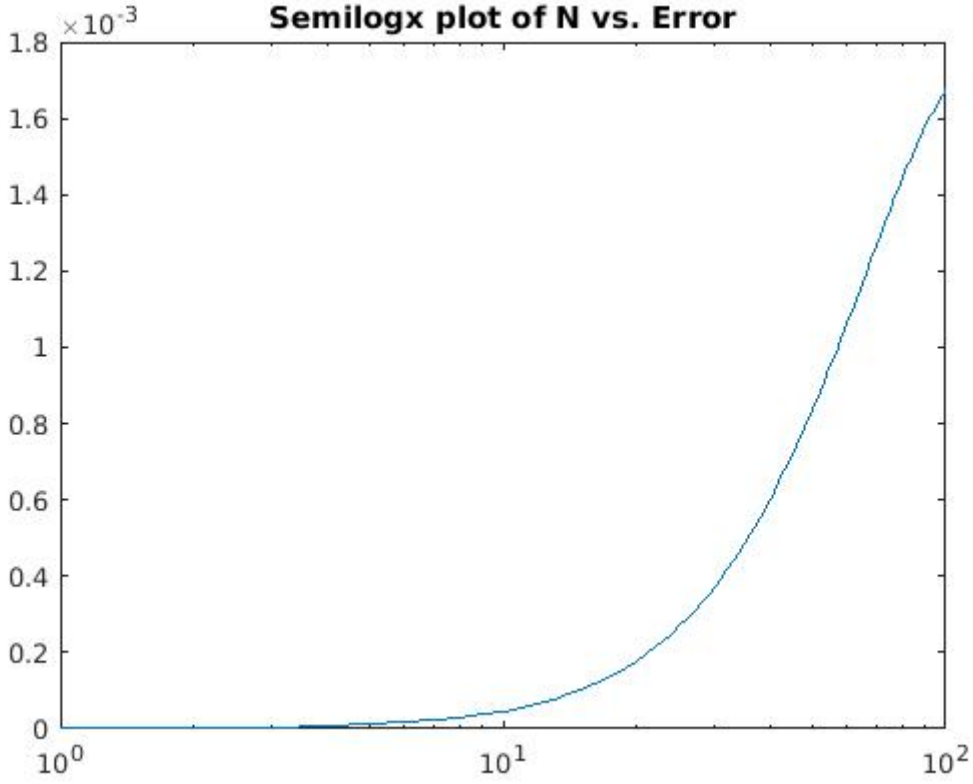
By modified Euler's method, with h = 0.005, approximation for y(1) = 3.436541 , exact value by analytical solution is 3.436564, therefore error = 0.000023.

Problem 3: With λ=-1, h=0.01, we get the following figures by various methods stipulated in question:

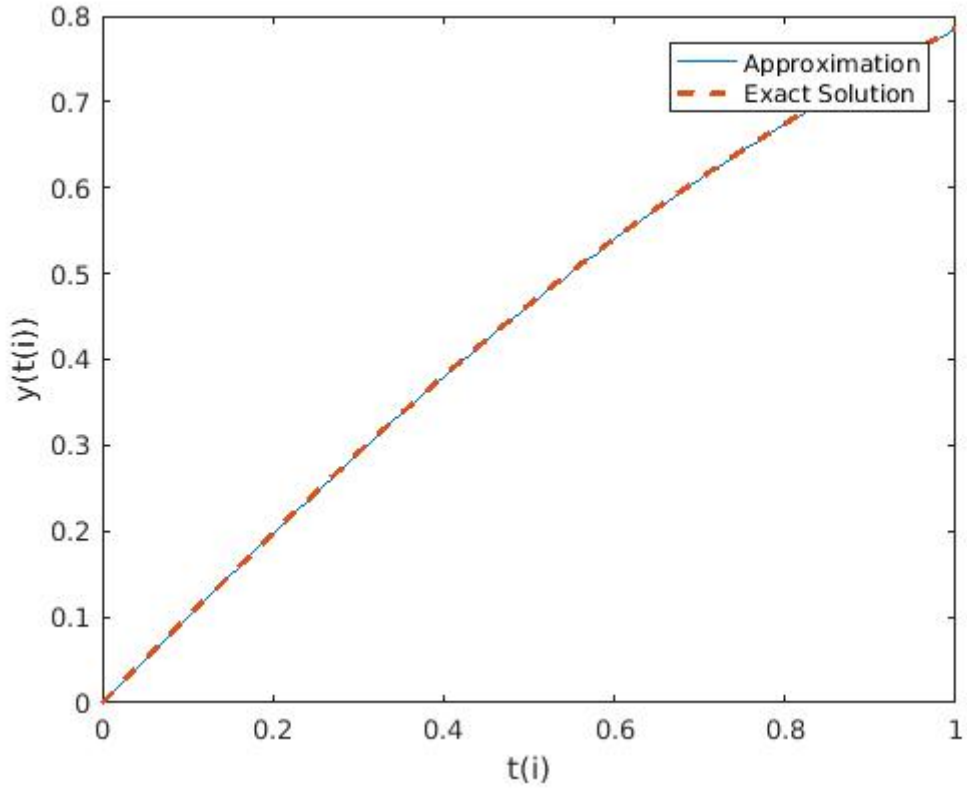
By Explicit Euler:

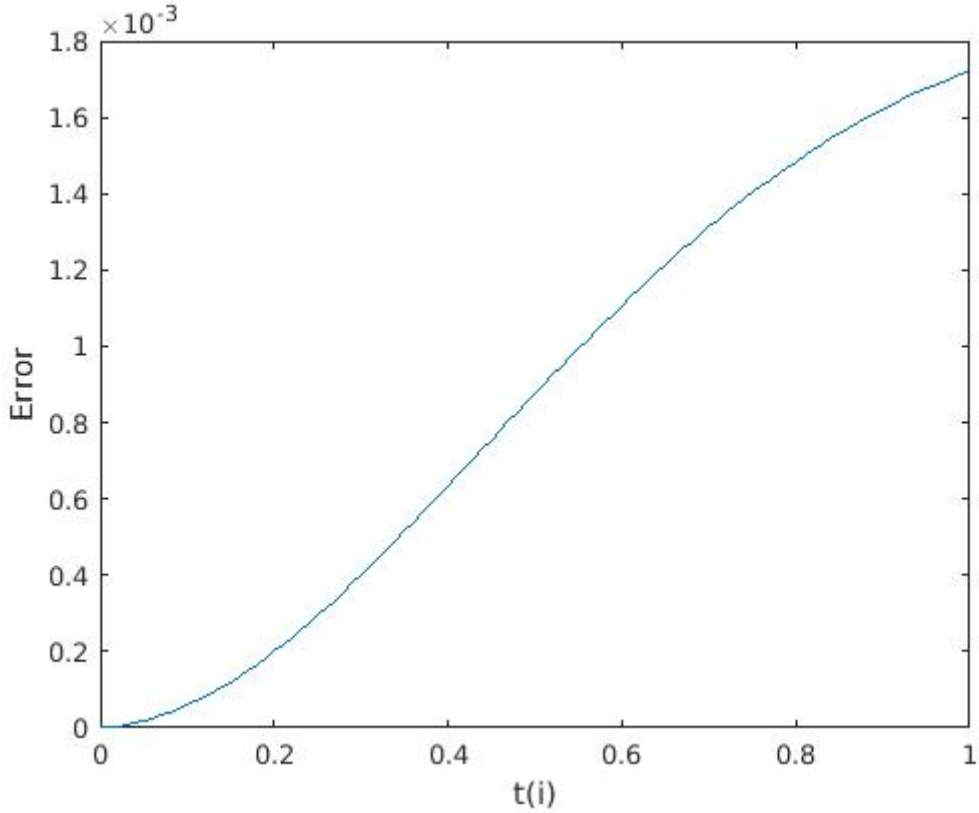

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
0.2
0.4
0.6
0.8
1

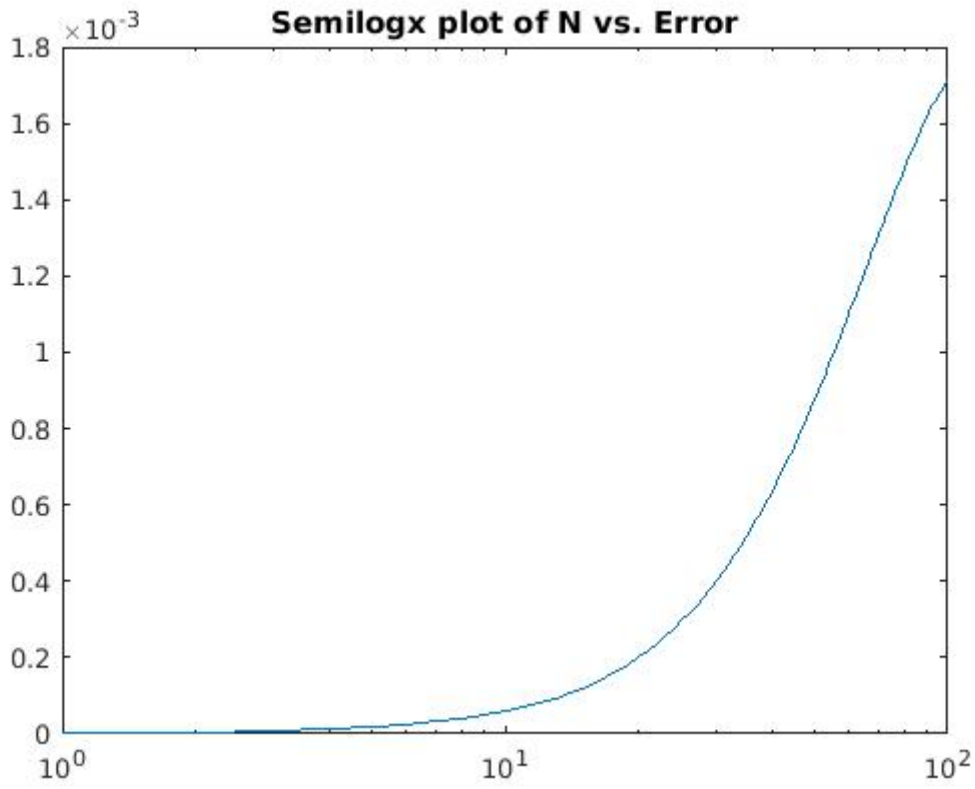




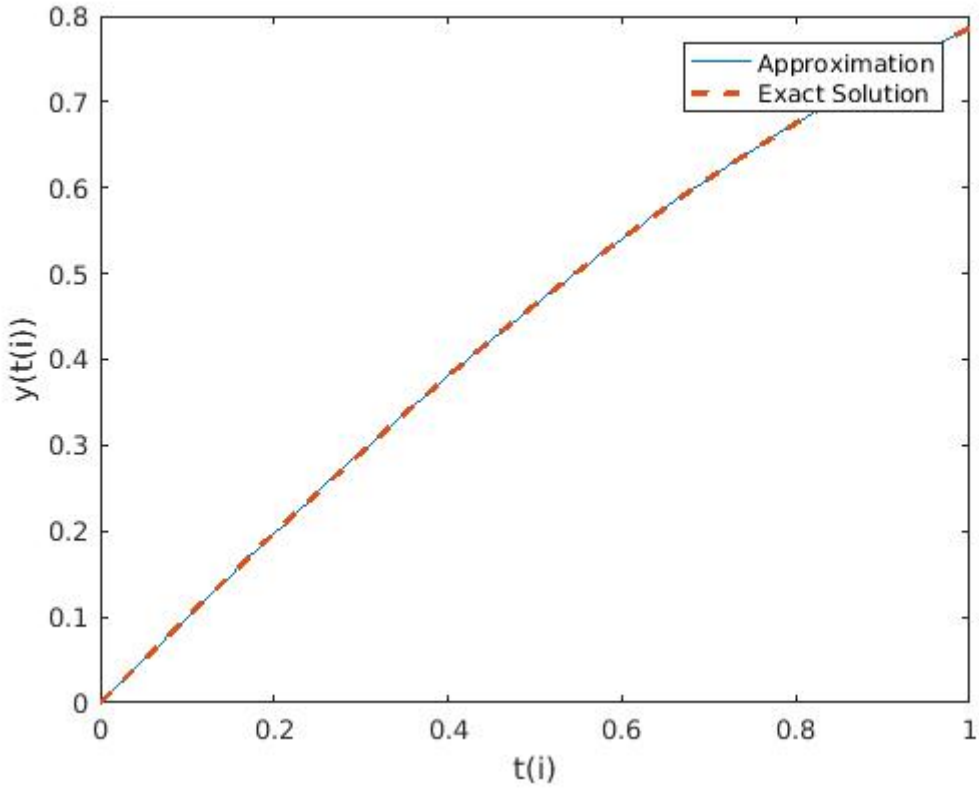
By implicit Euler’s method:

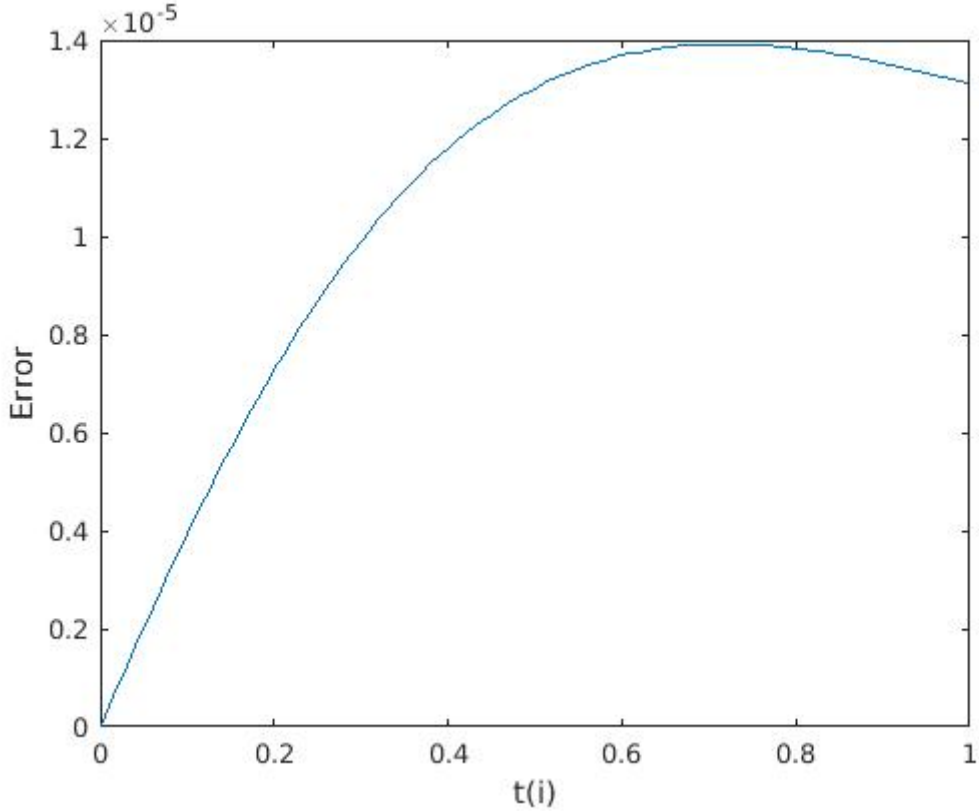

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
0.2
0.4
0.6
0.8
1

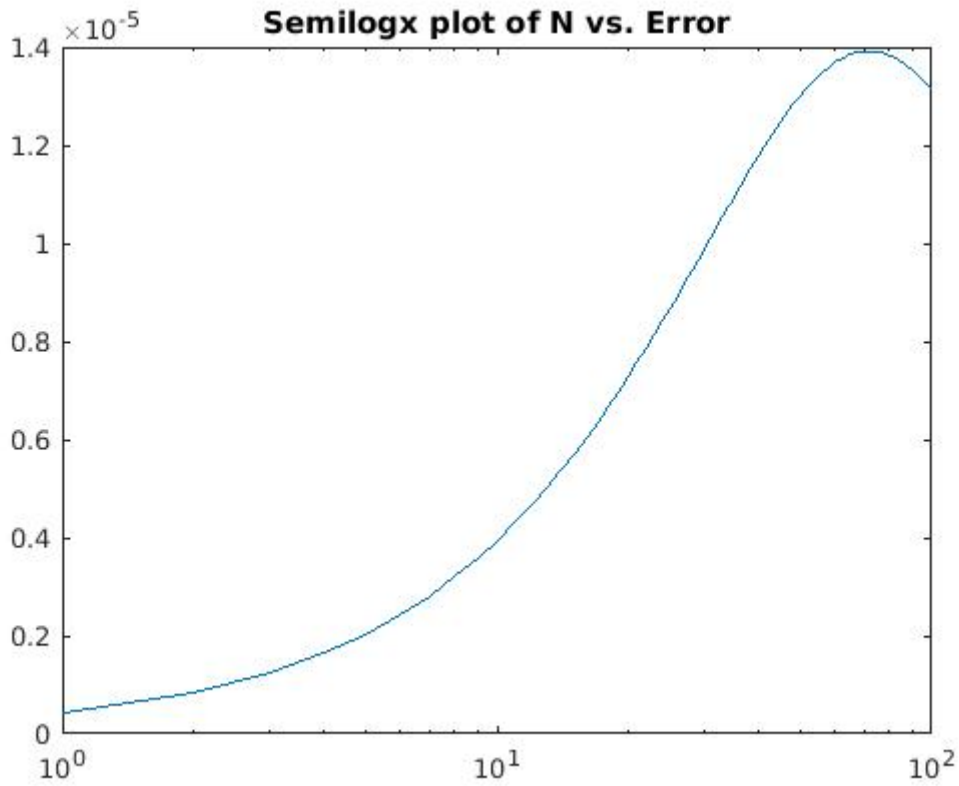

×10^-3
Error
t(i)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
0.2
0.4
0.6
0.8
1



By Trapezoidal rule:

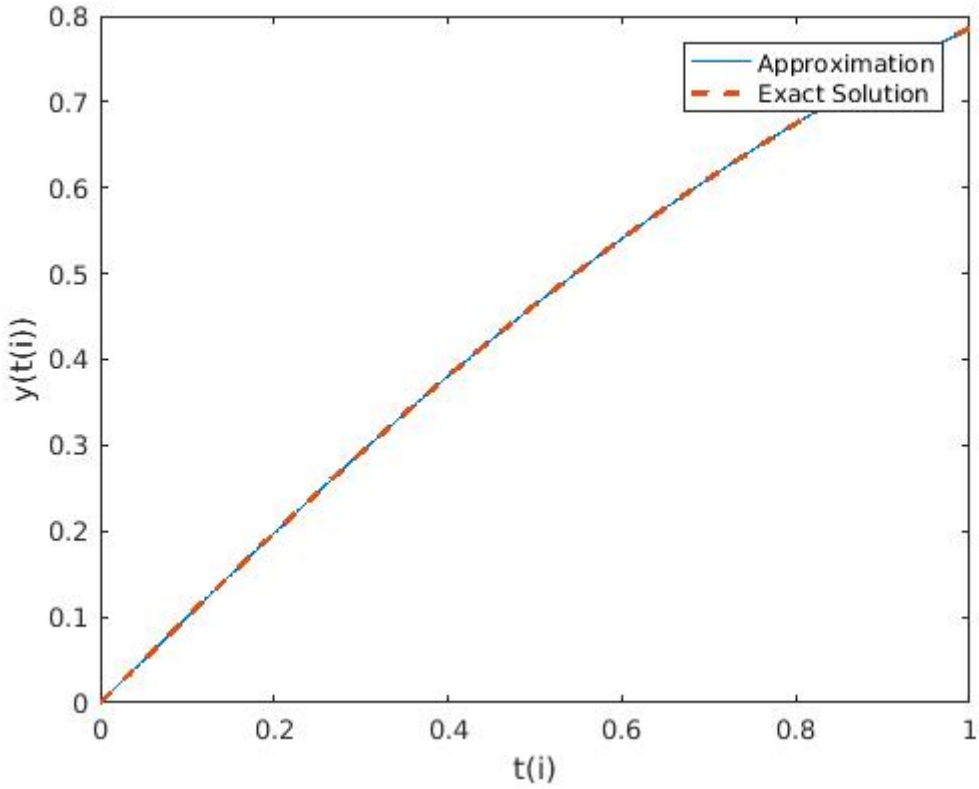

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
0.2
0.4
0.6
0.8
1

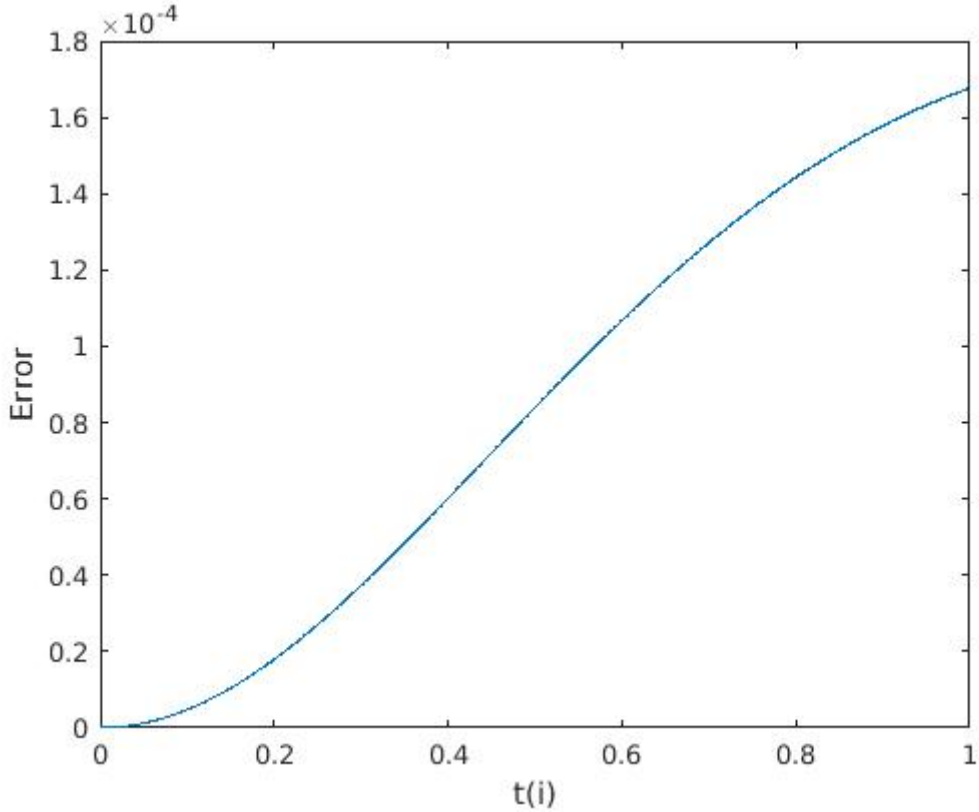

×10^-5
Error
t(i)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
0.2
0.4
0.6
0.8
1

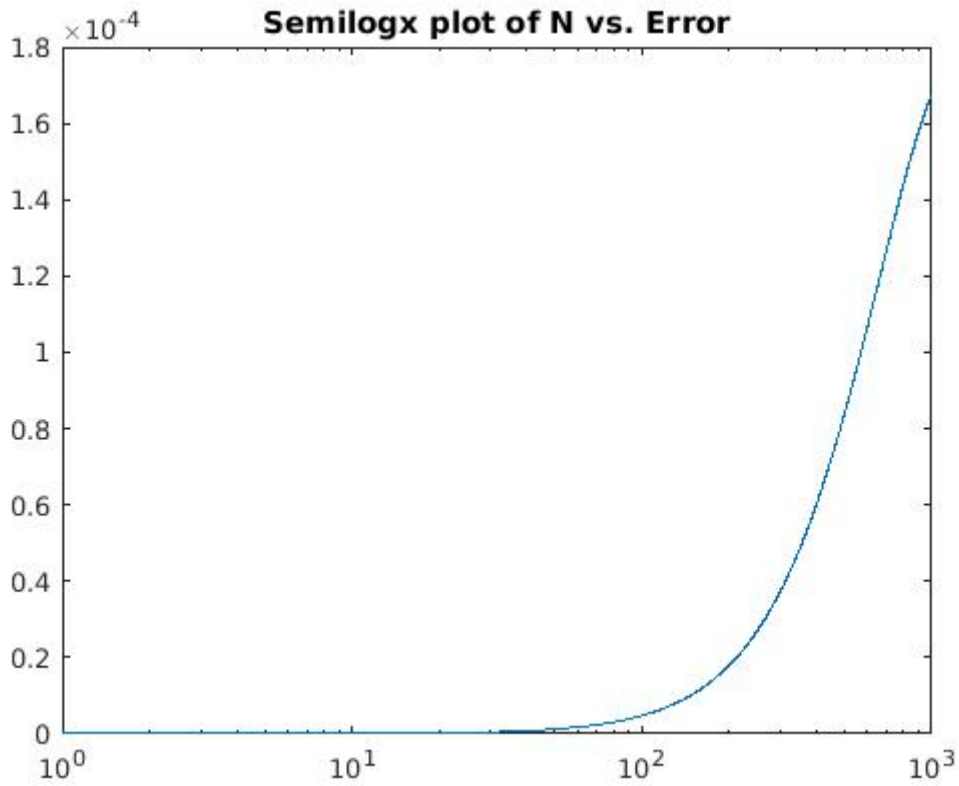


With λ=-1, h=0.001, we get the following figures by various methods stipulated in question:

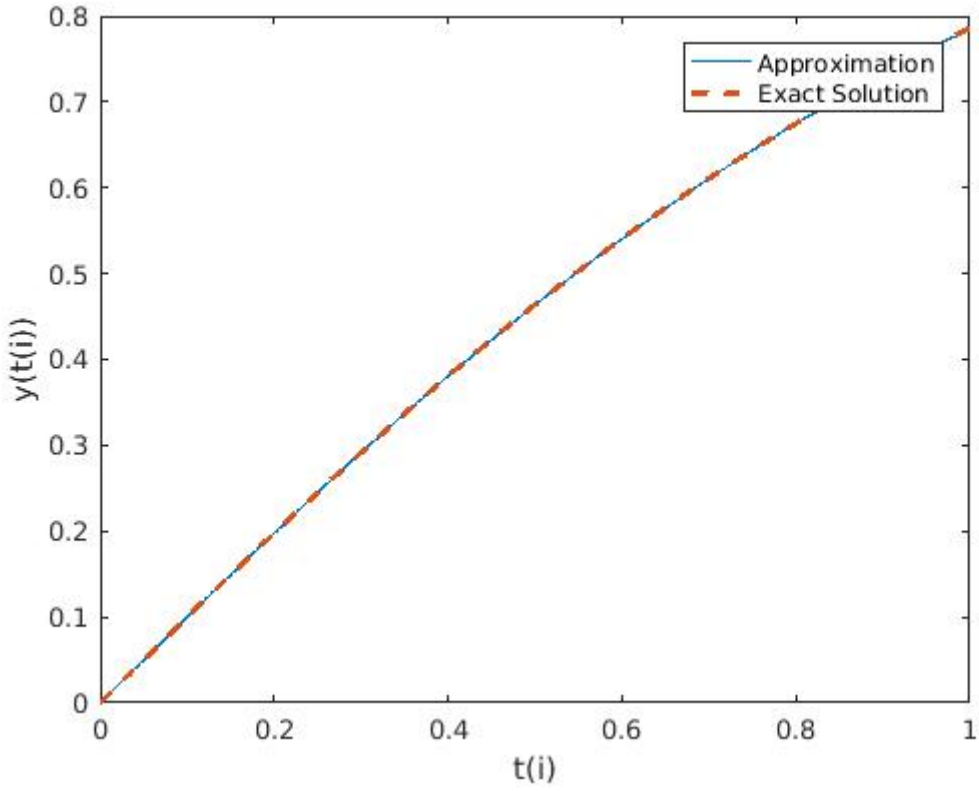
By explicit Euler:

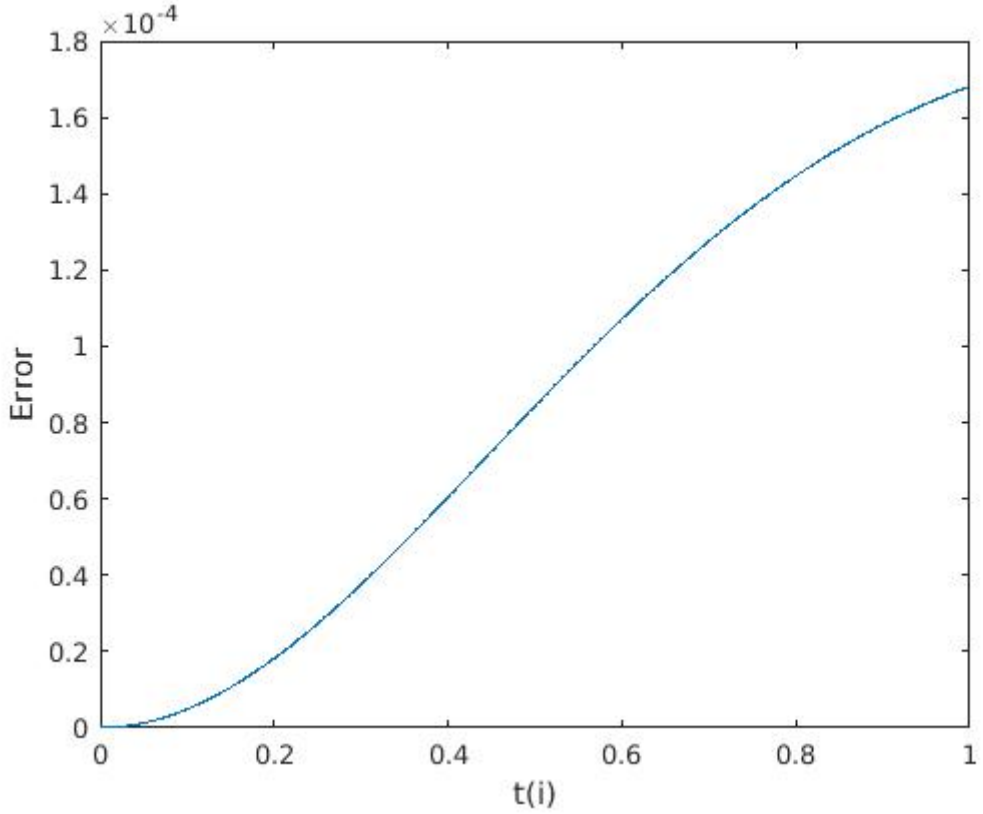

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
0.2
0.4
0.6
0.8
1

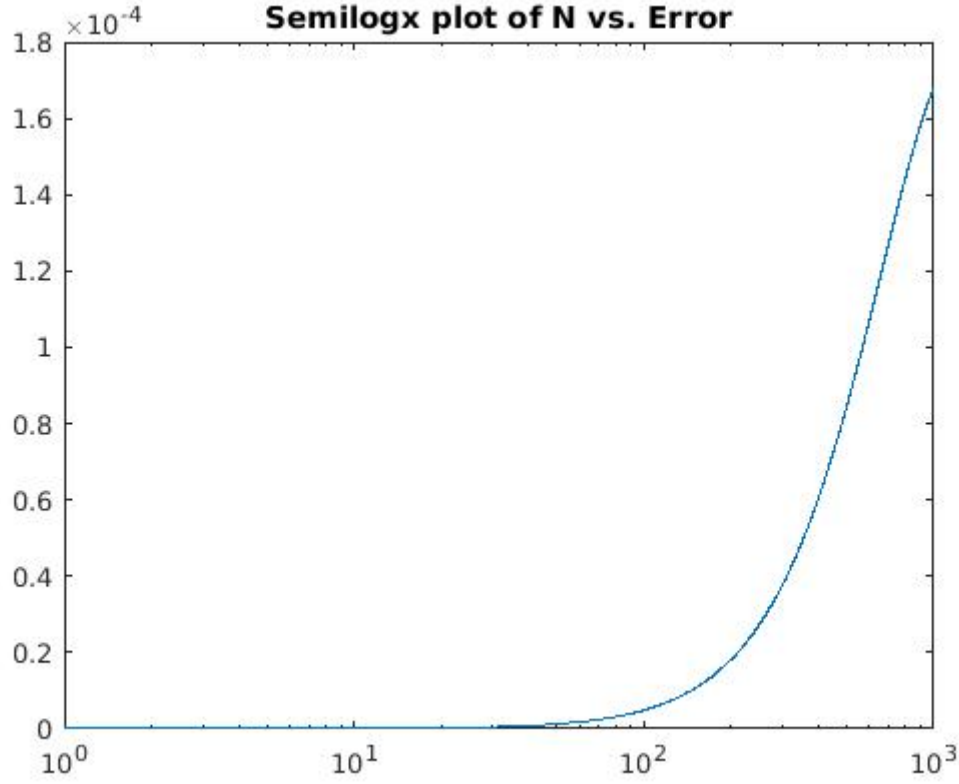

×10^-4
Error
t(i)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
0.2
0.4
0.6
0.8
1



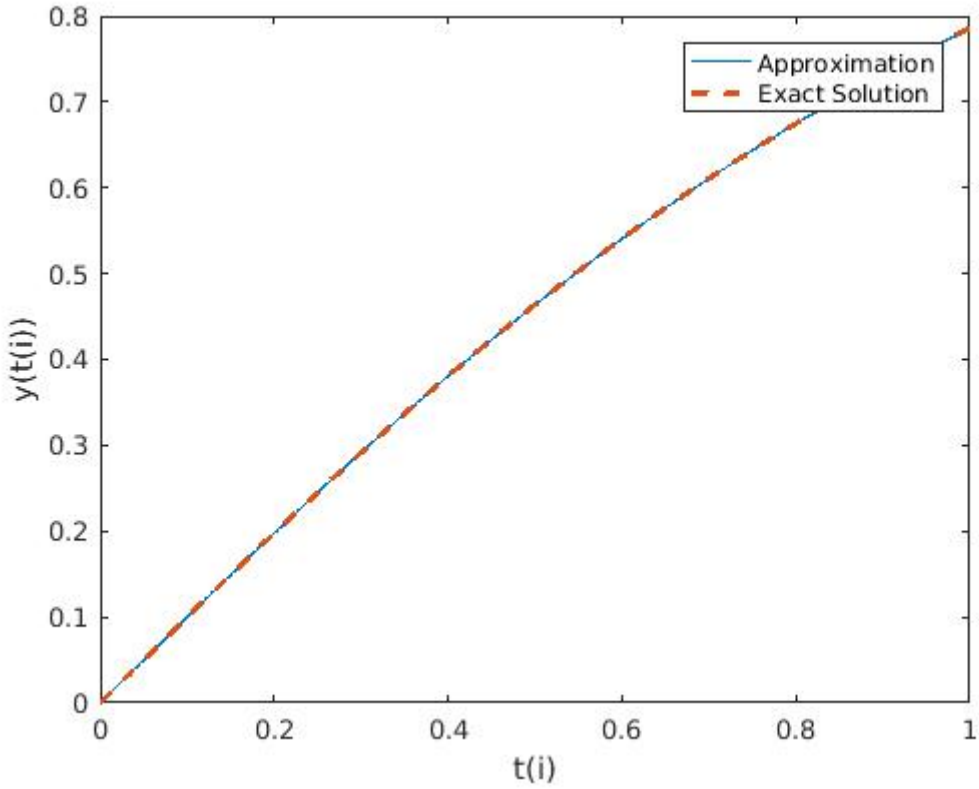
By implicit Euler's method:

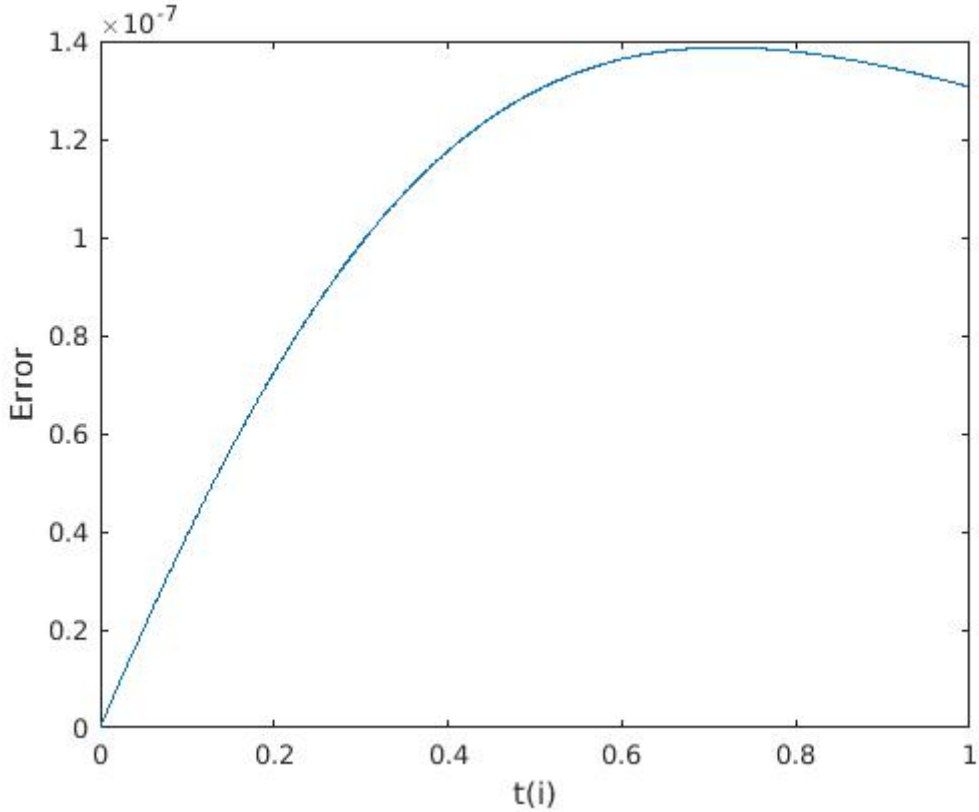


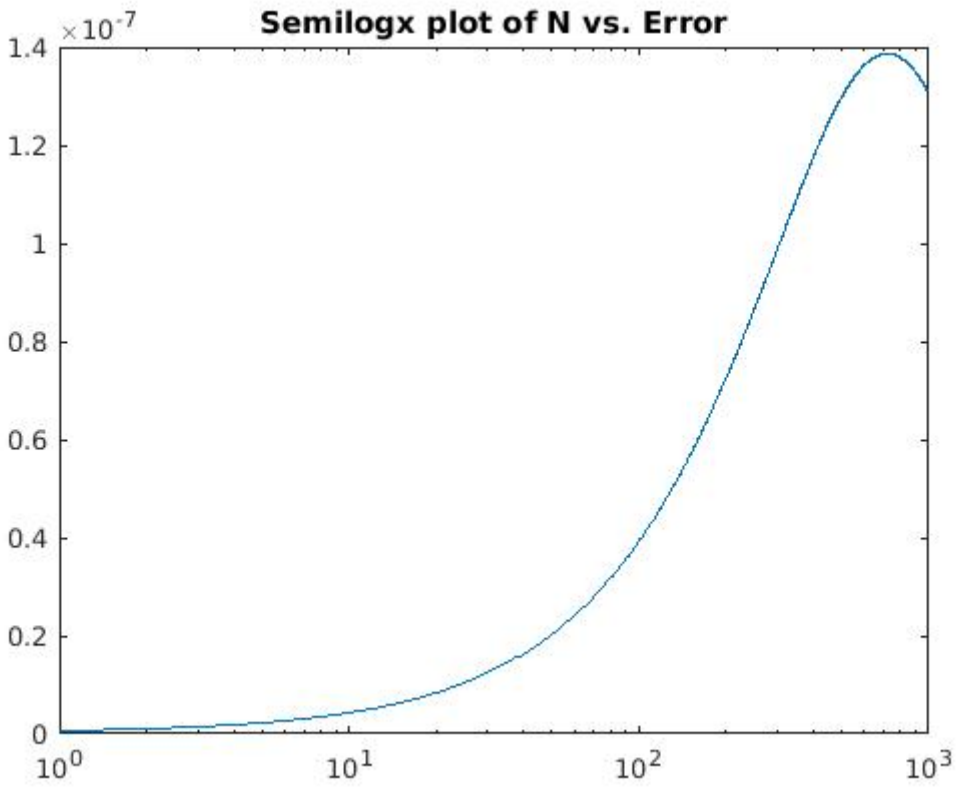




By Trapezoidal method:

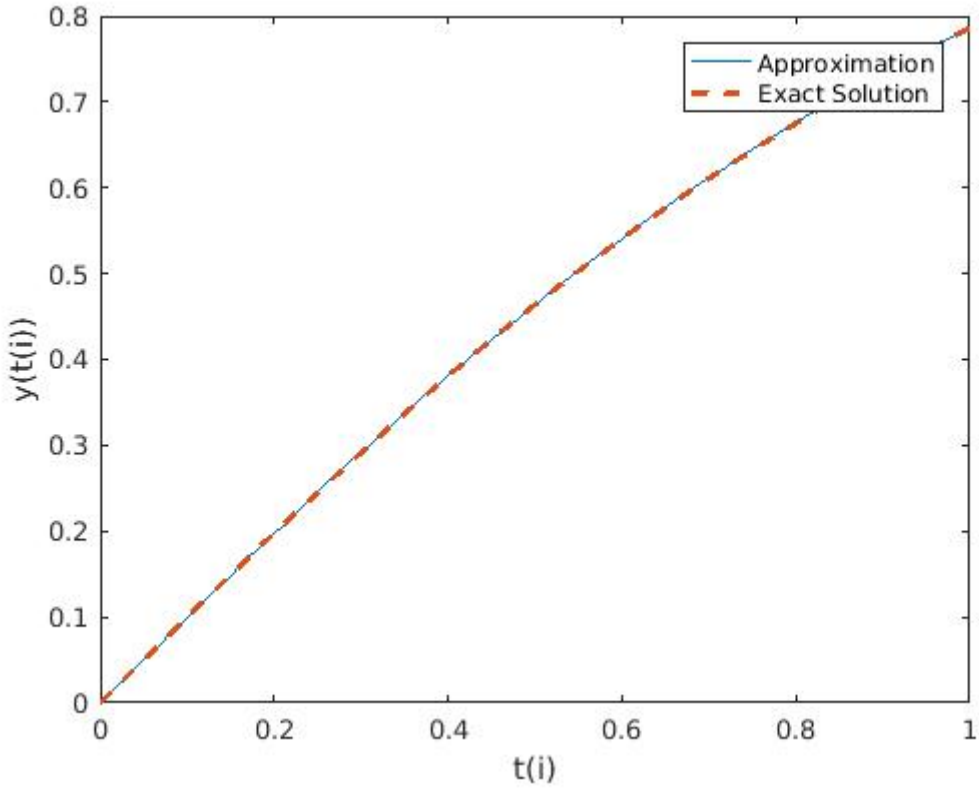

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
0.2
0.4
0.6
0.8
1

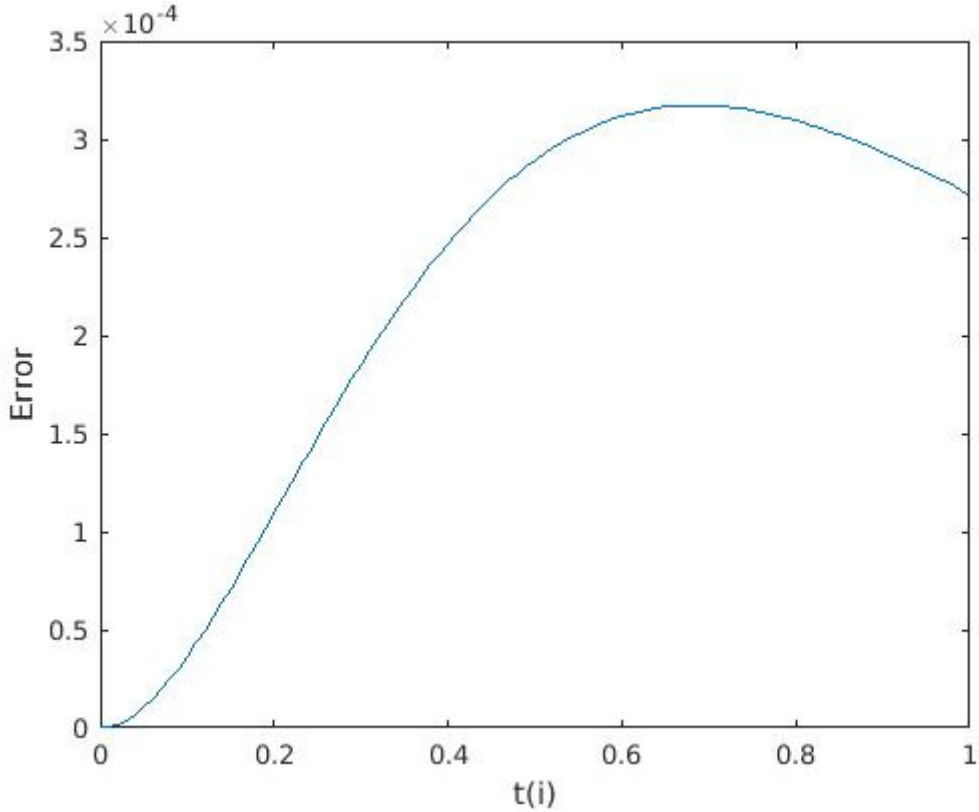

×10^-7
Error
t(i)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
0.2
0.4
0.6
0.8
1

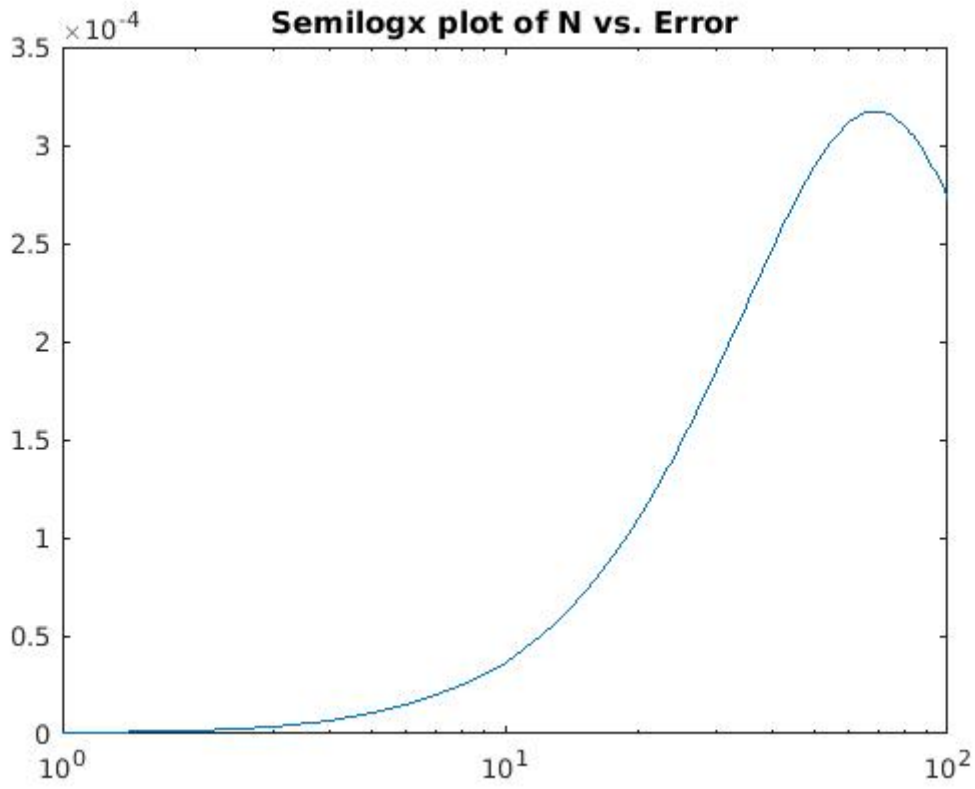


With λ=-10, h=0.01, we get the following figures by various methods stipulated in question:

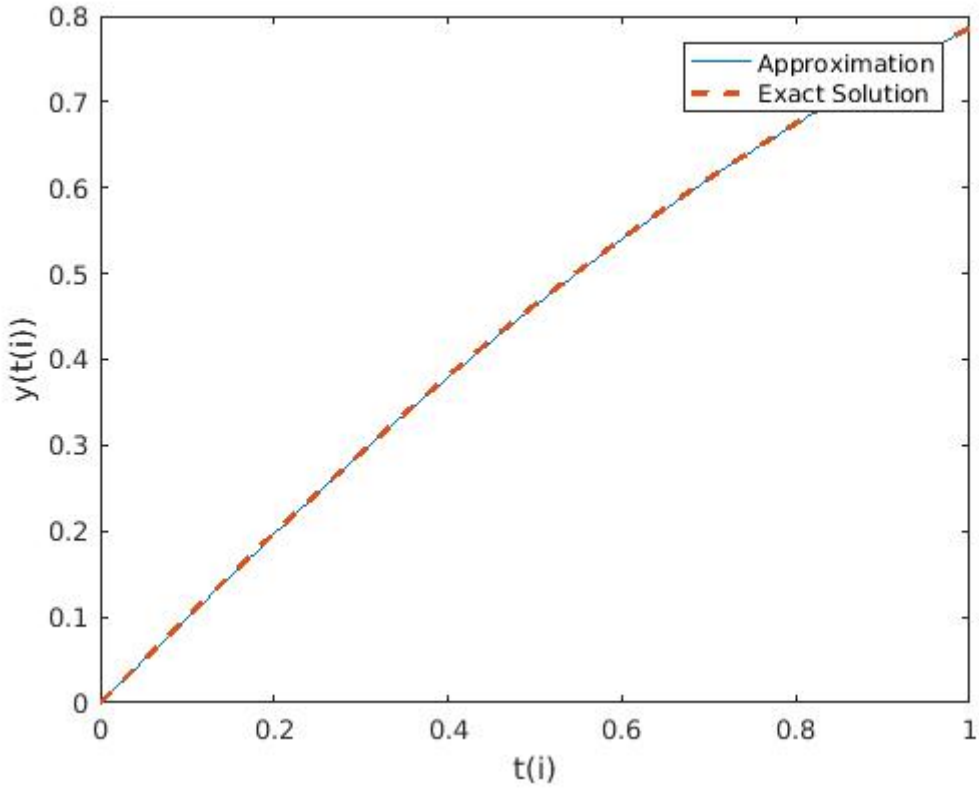
By explicit Euler’s method:

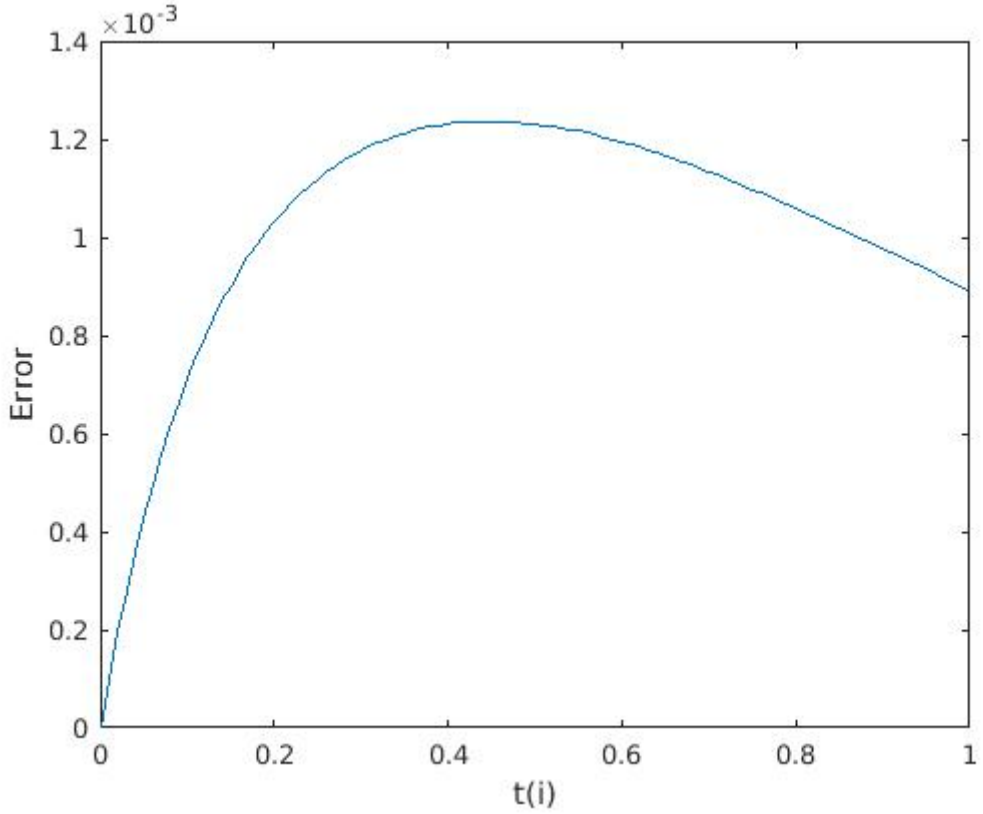

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.2
0.4
0.6
1

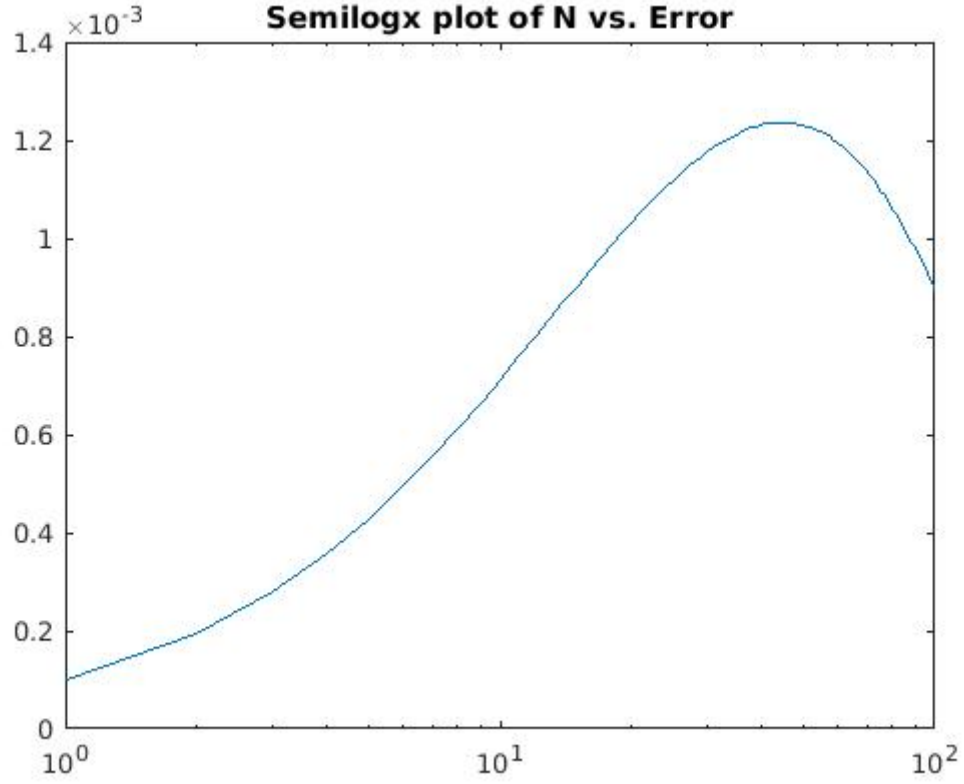

×10^-4
Error
t(i)
0
0.5
1
1.5
2
2.5
3
3.5
0.2
0.4
0.6
0.8
1



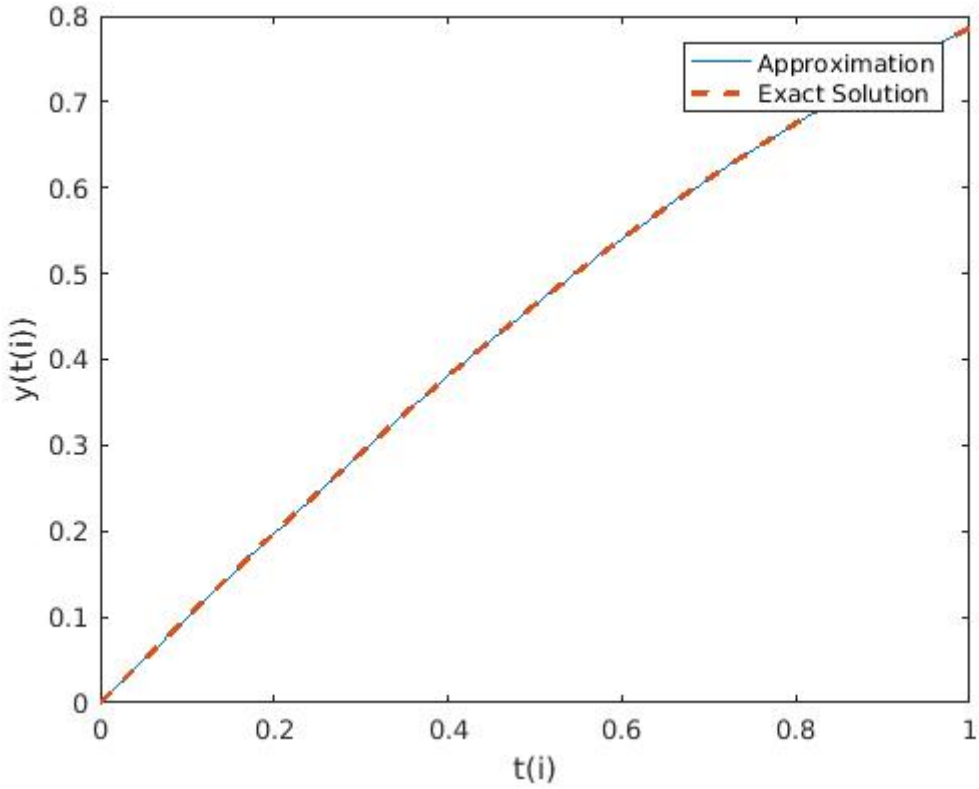
By implicit Euler’s method:

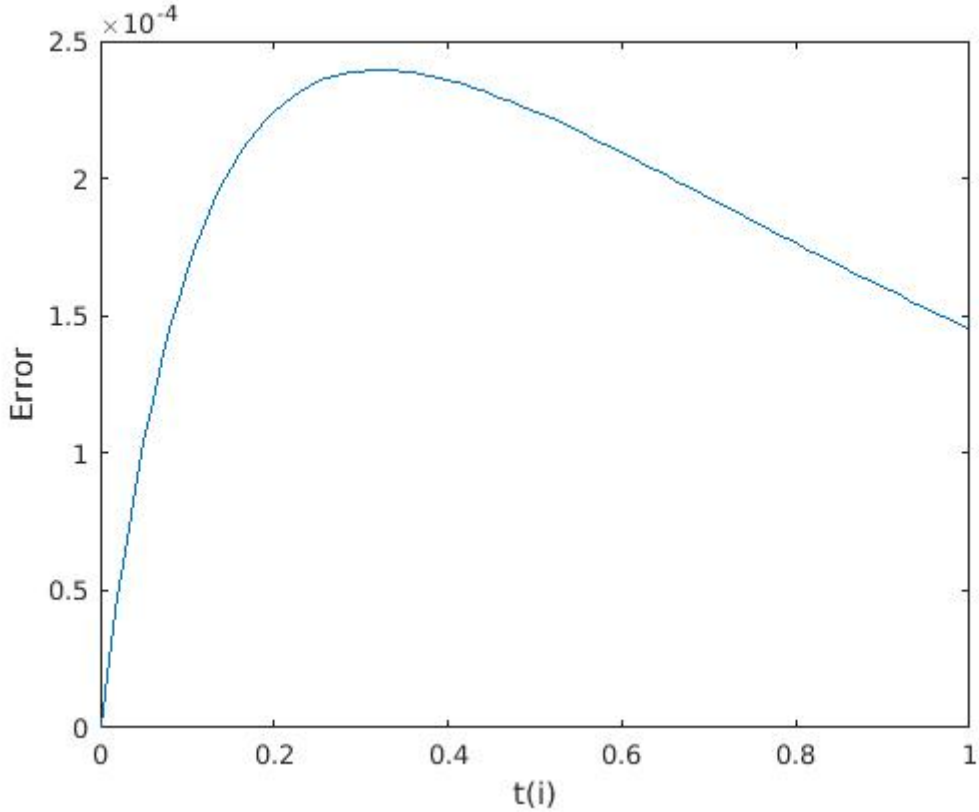

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
0.2
0.4
0.6
0.8
1

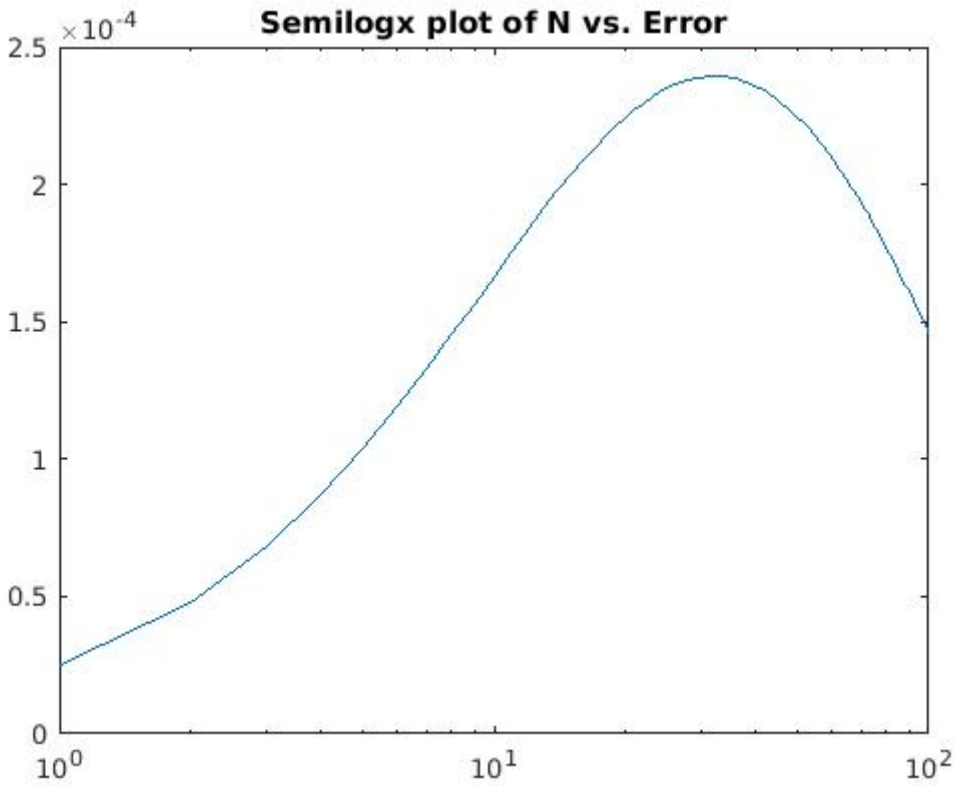




By Trapezoidal method:

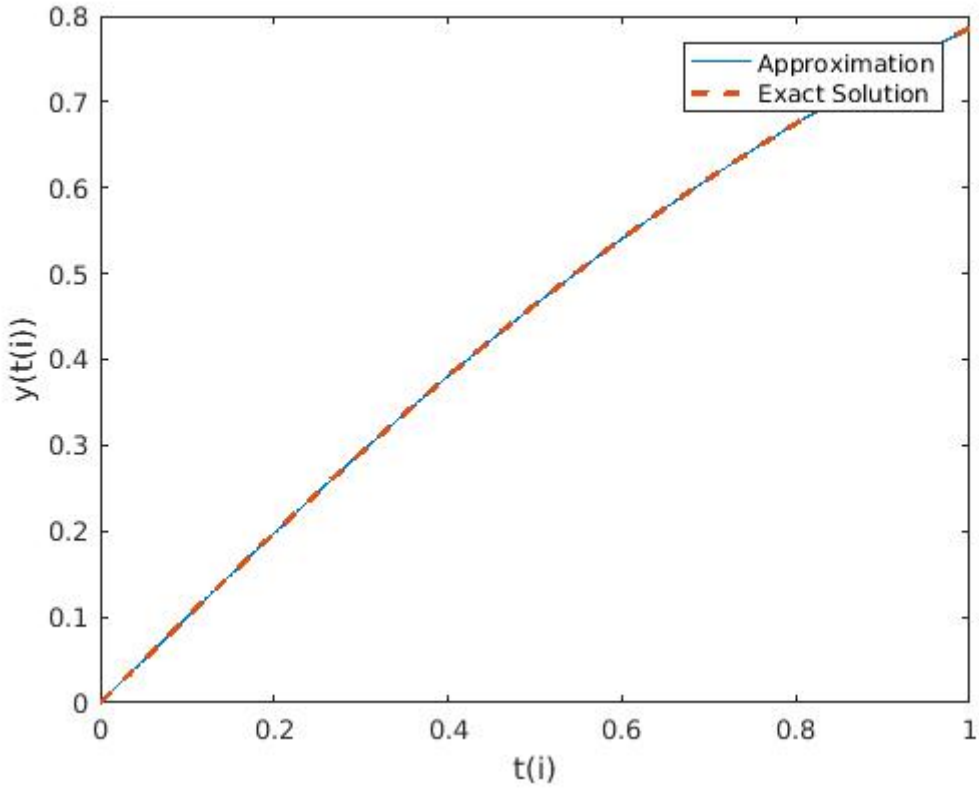

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
0.2
0.4
0.6
0.8
1

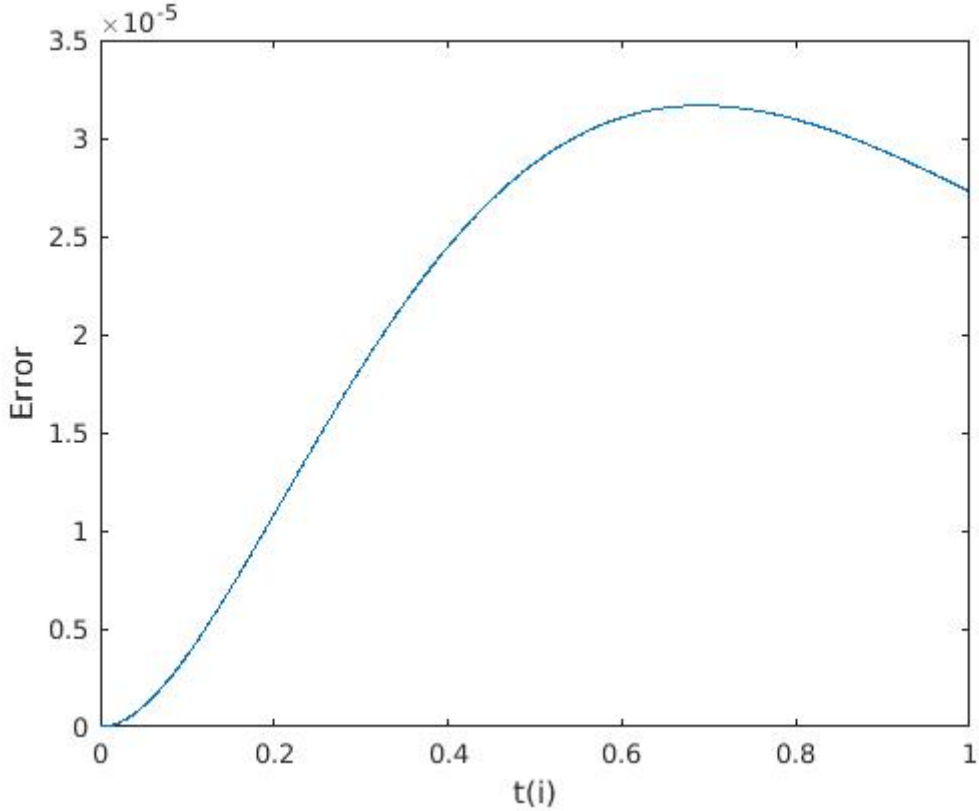

×10^-4
Error
t(i)
0
0.5
1
1.5
2
2.5
0
0.2
0.4
0.6
0.8
1

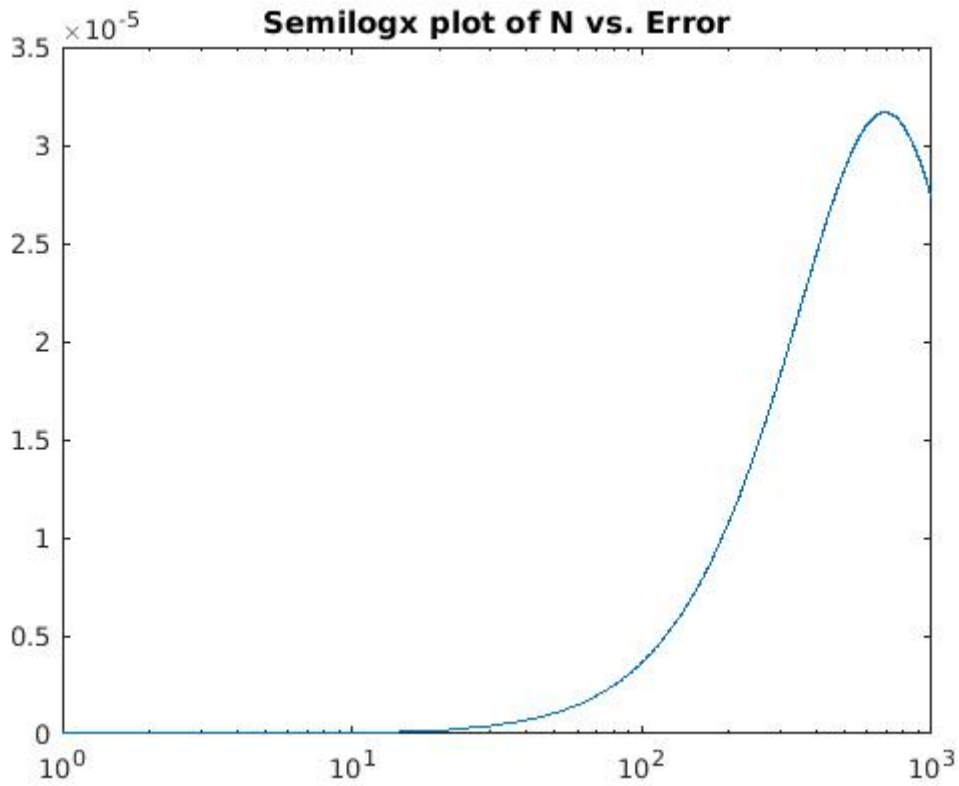


With λ=-10, h=0.001, we get the following figures by various methods stipulated in question:

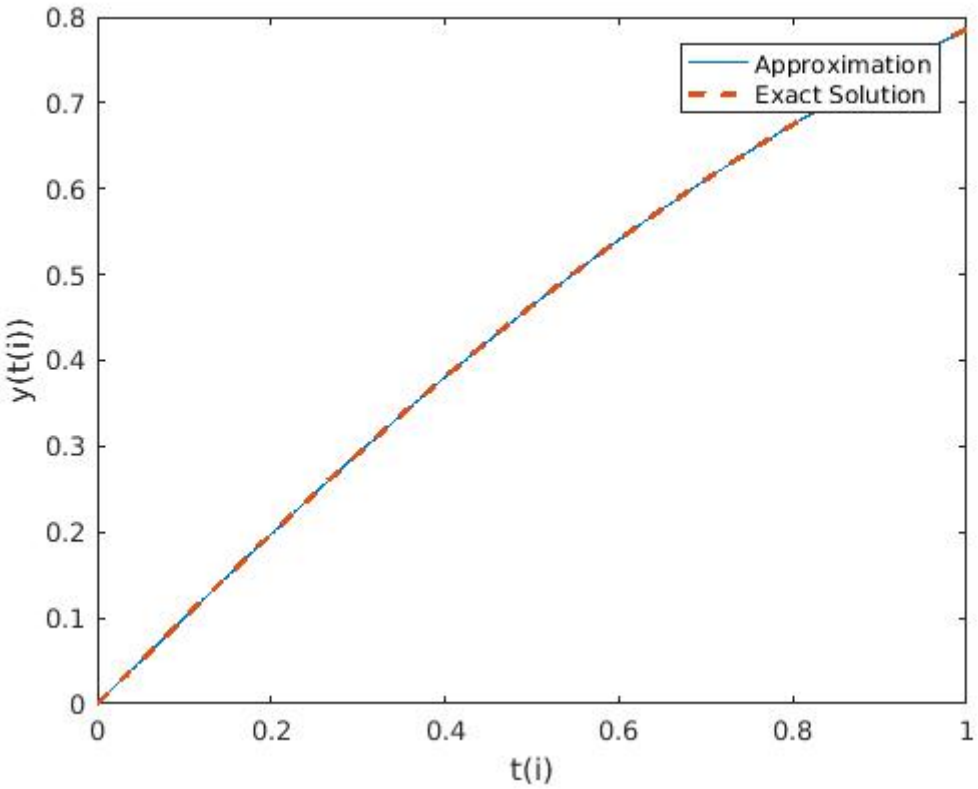
By explicit Euler's method:

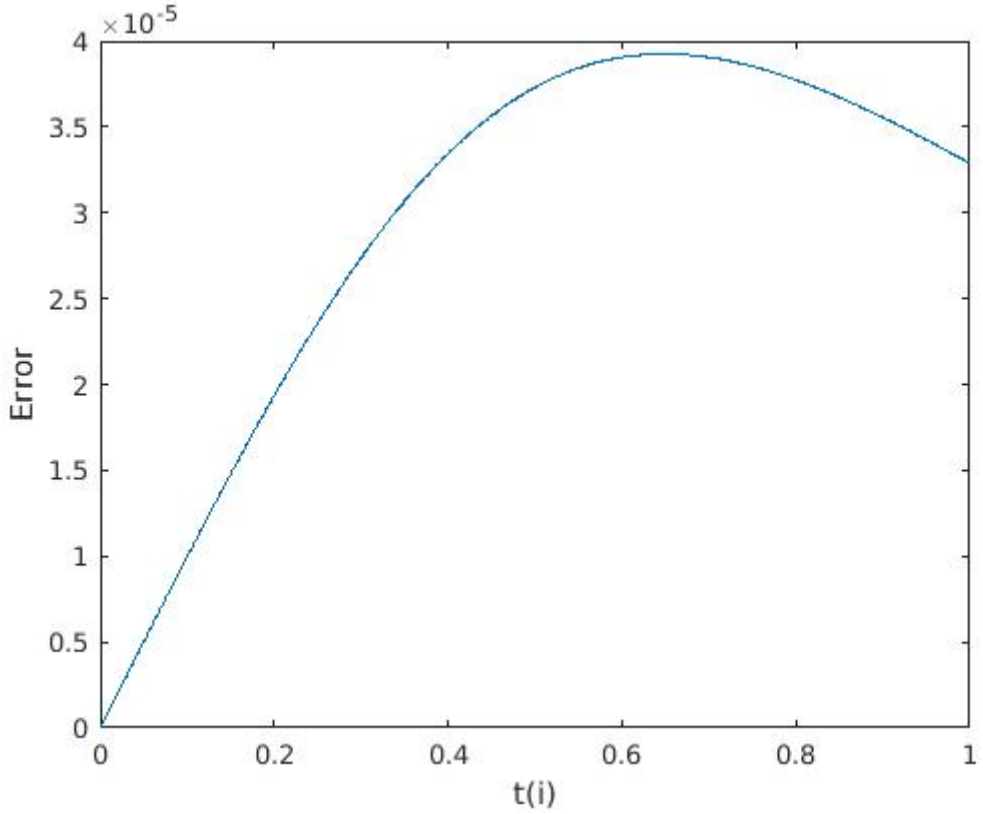

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
0.2
0.4
0.6
0.8
1

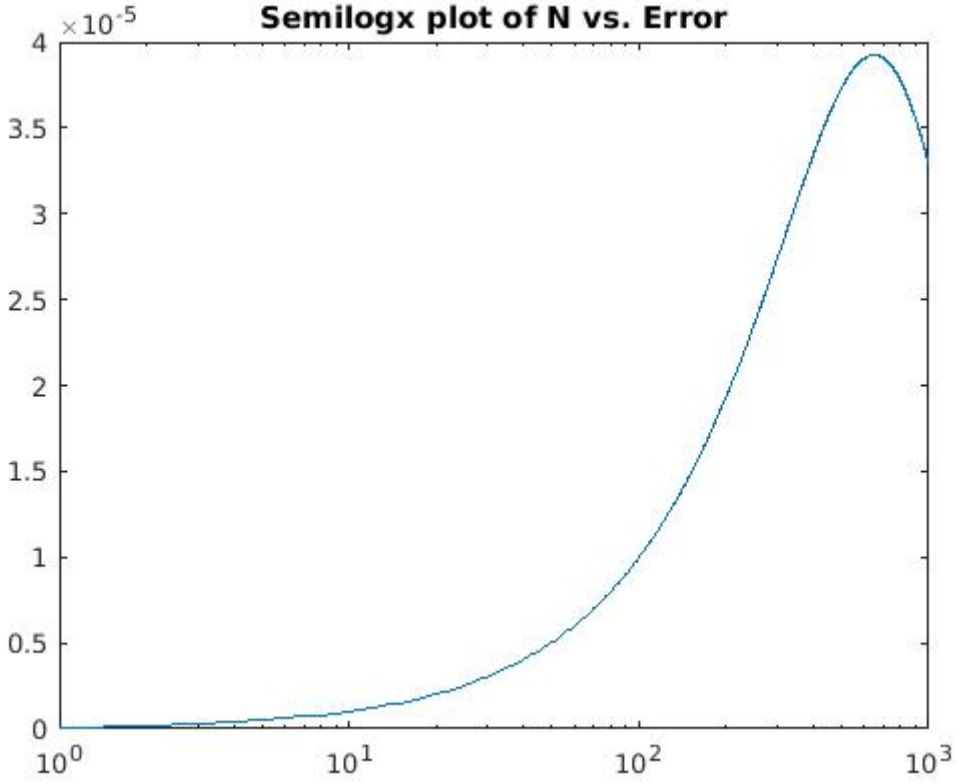

×10^-5
Error
t(i)
0
0.5
1
1.5
2
2.5
3
3.5
0
0.2
0.4
0.6
0.8
1



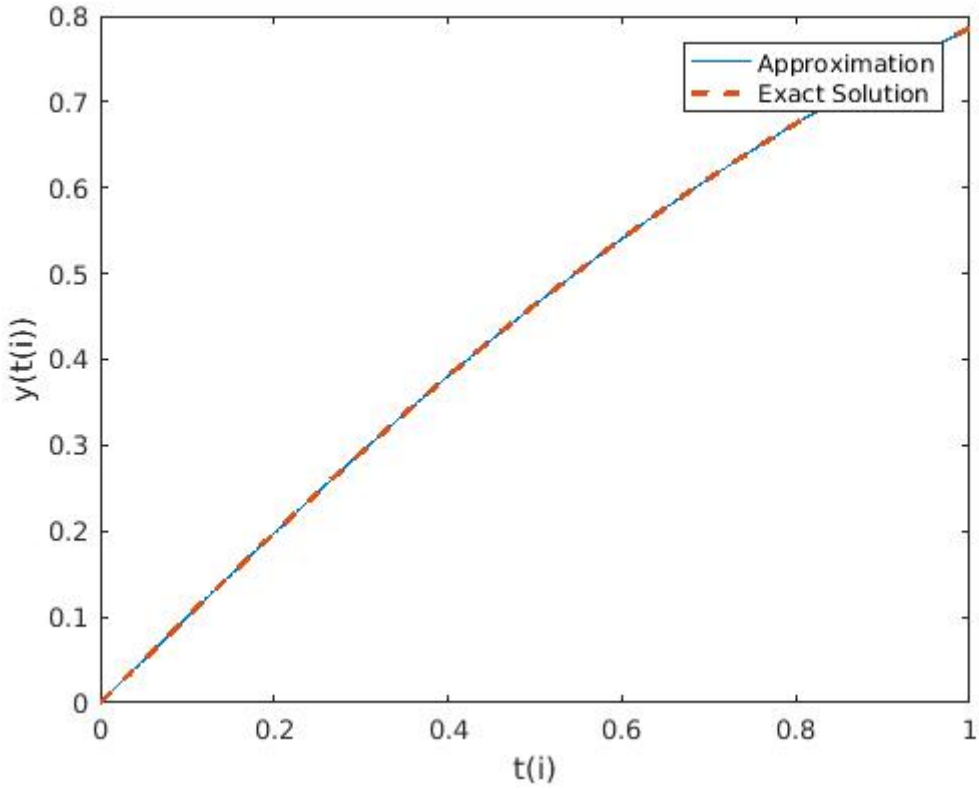
By implicit Euler's method:

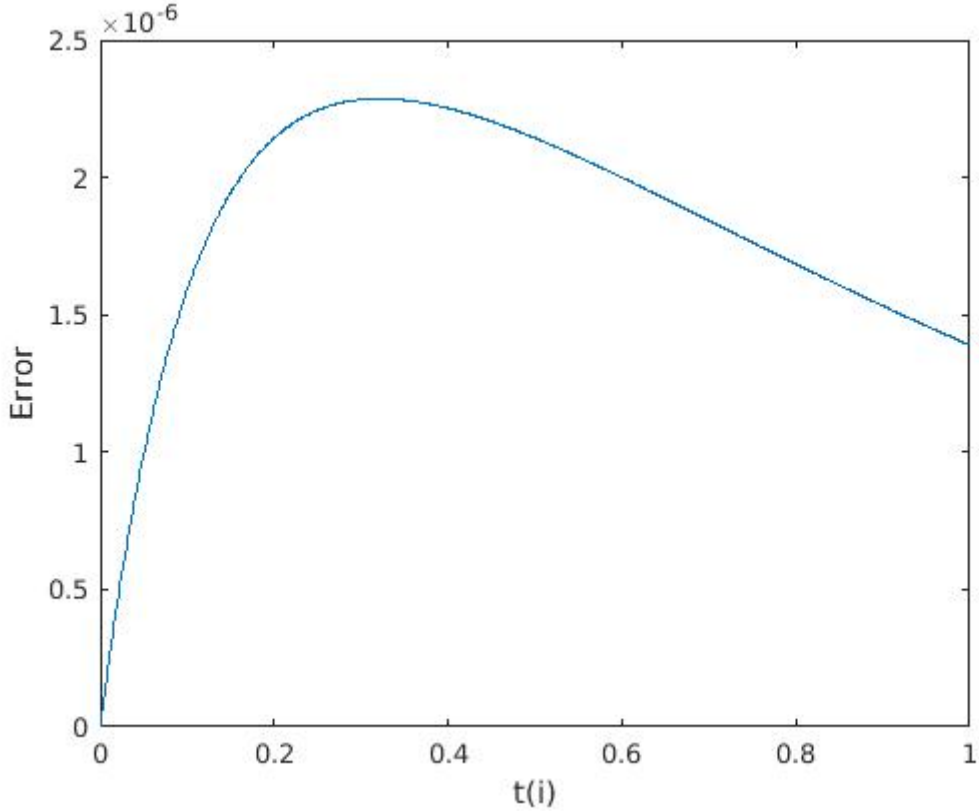


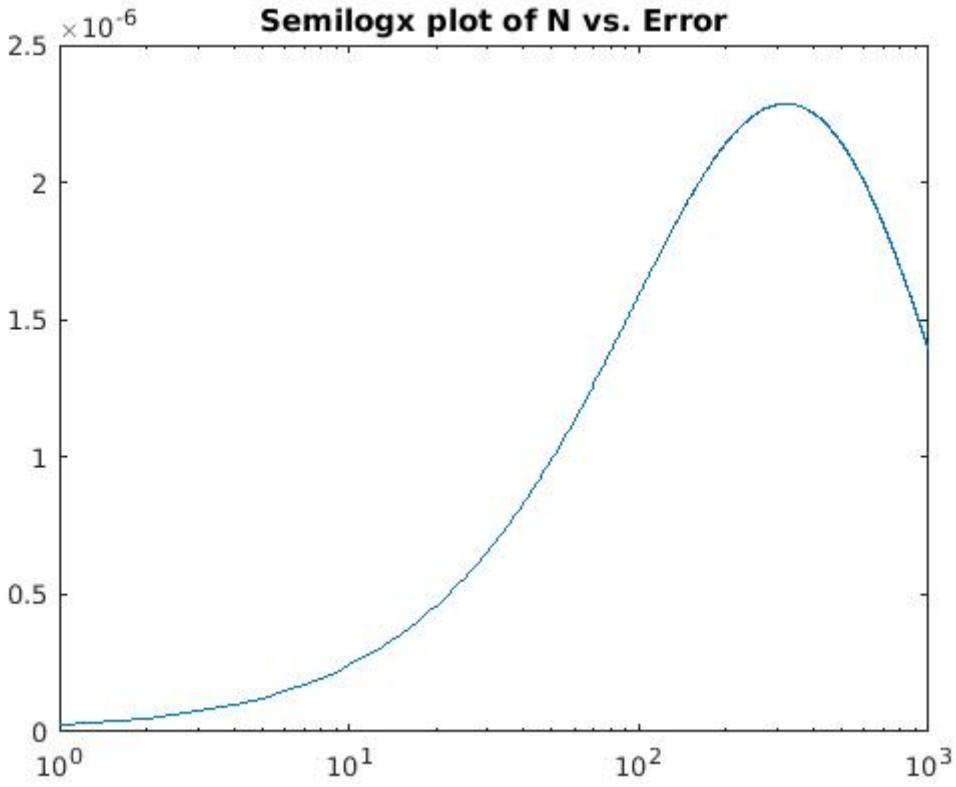




By Trapezoidal method:

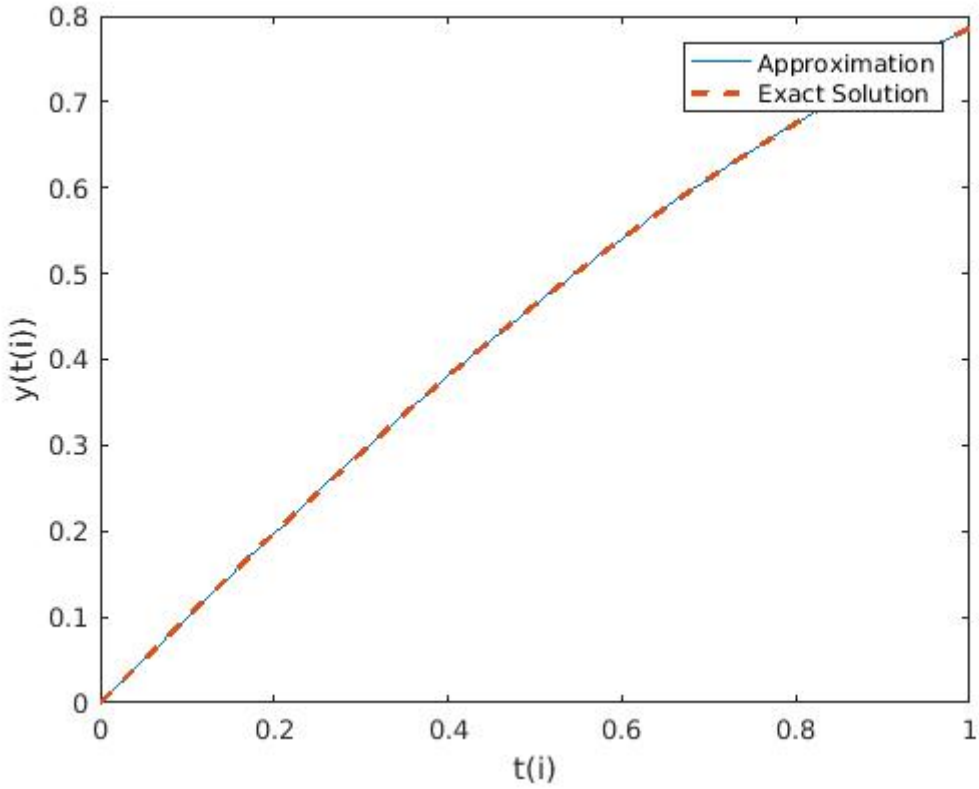

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.2
0.4
0.6
1

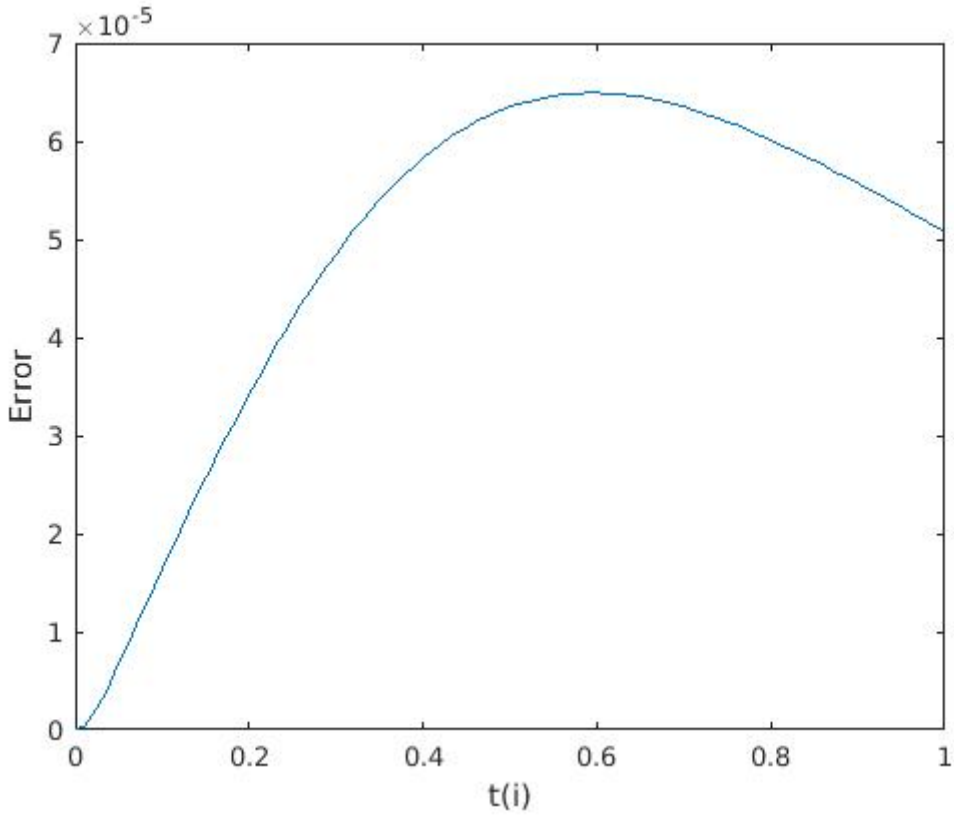

×10^-6
Error
t(i)
0
0.5
1
1.5
2
2.5
0.2
0.4
0.6
0.8
1

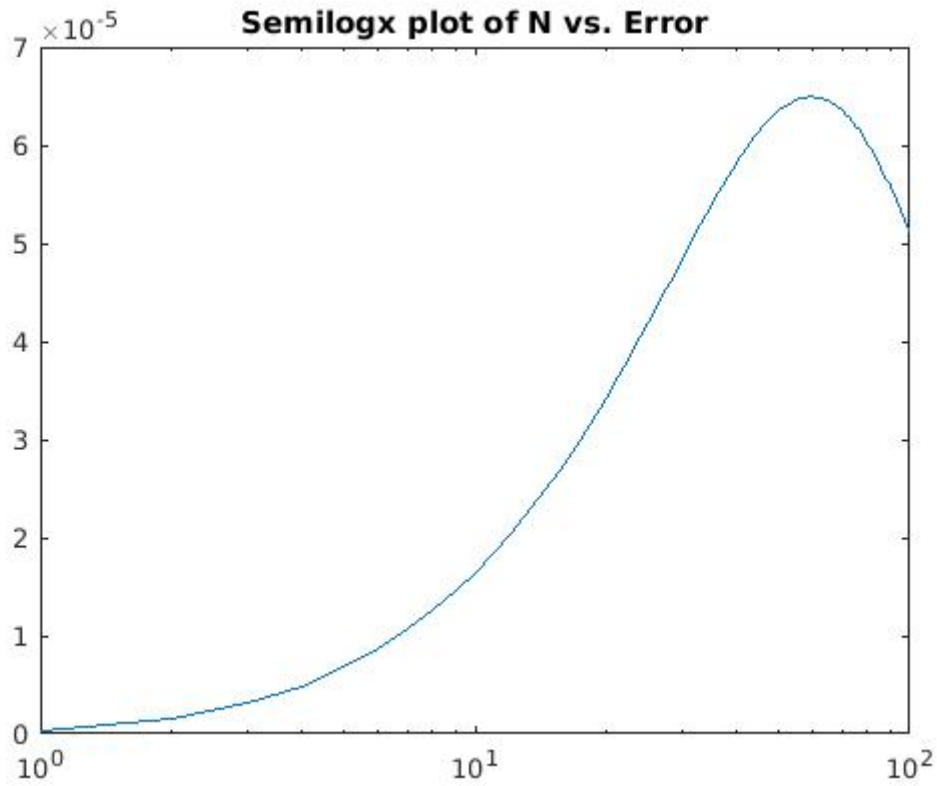


With λ=-50, h=0.01, we get the following figures by various methods stipulated in question:

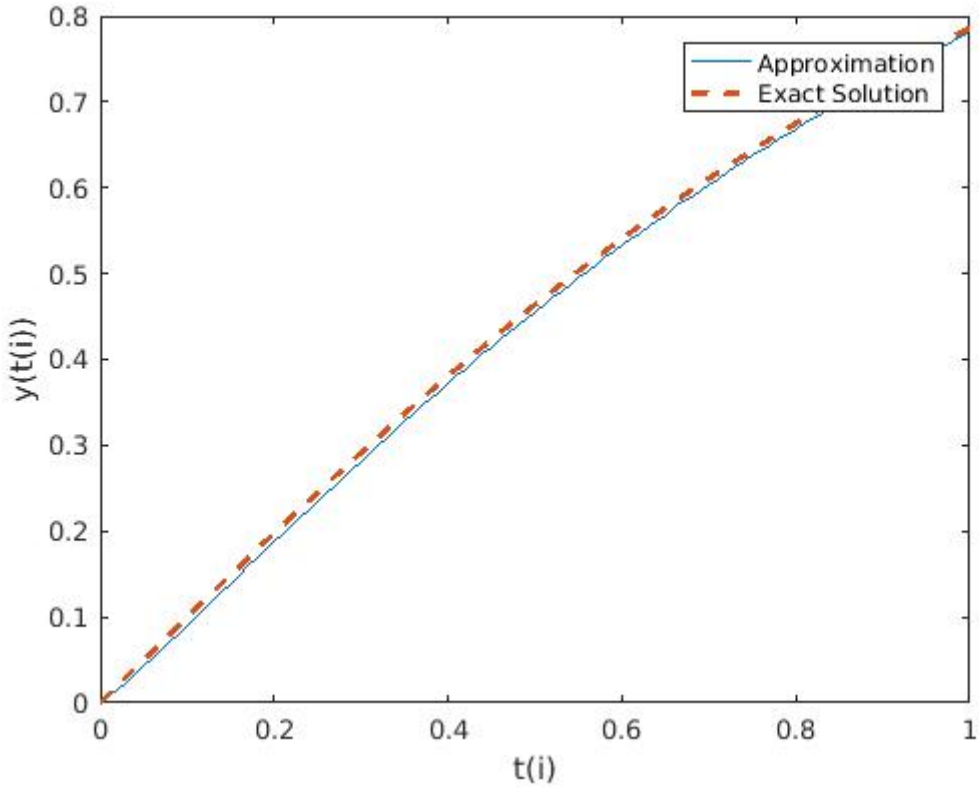
By explicit Euler's method:

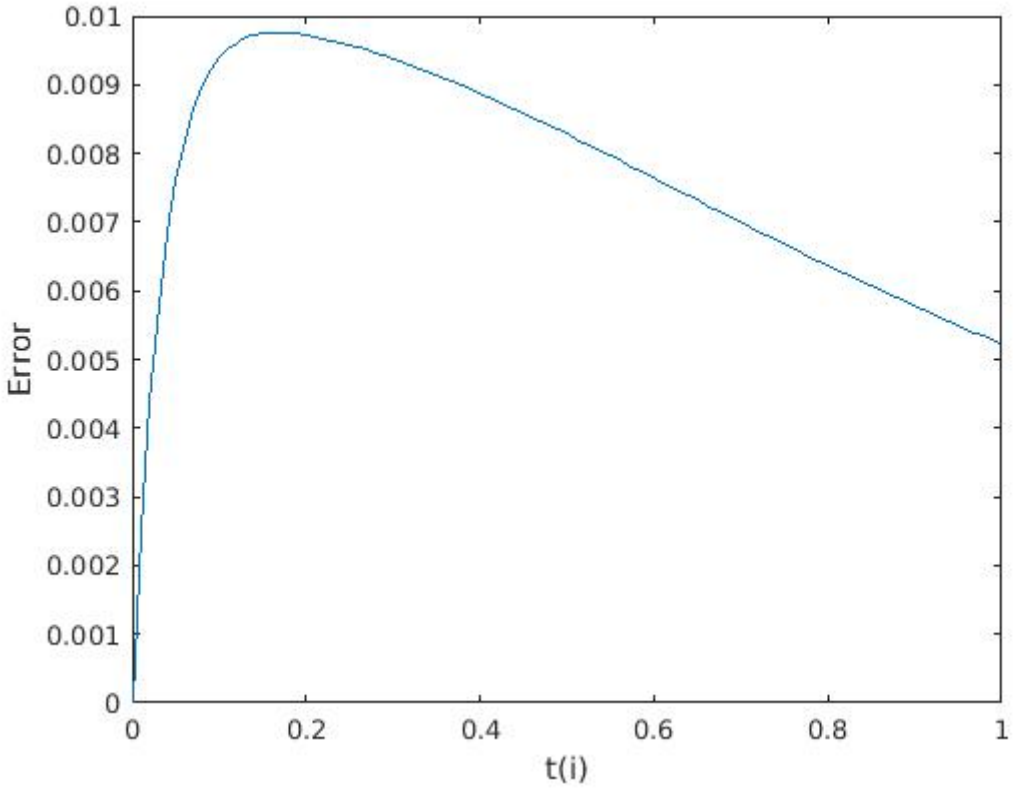

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
0.2
0.4
0.6
0.8
1

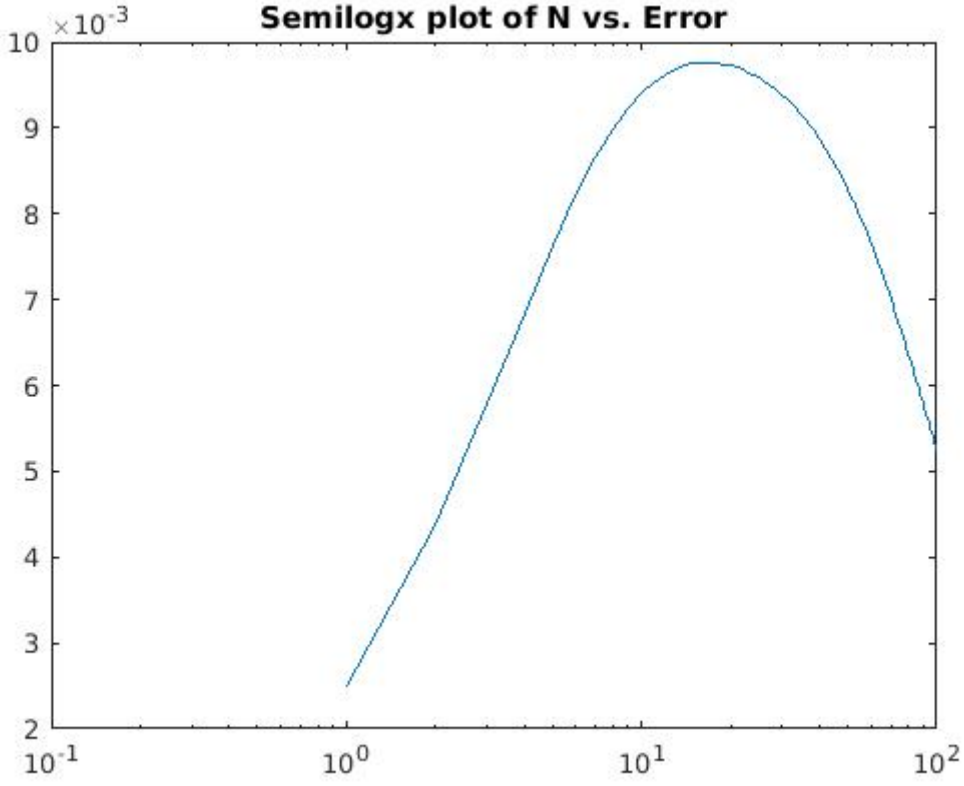

×10^-5
7
6
5
4
3
2
1
0
Error
0
0.2
0.4
0.6
0.8
1
t(i)



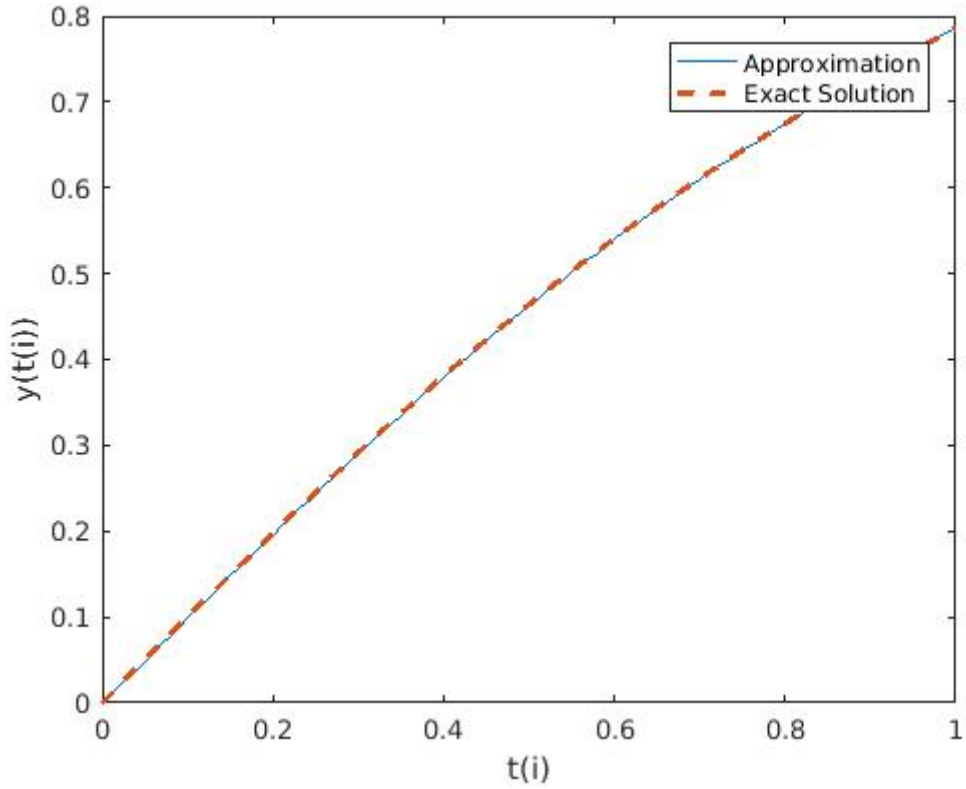
By implicit Euler's method:

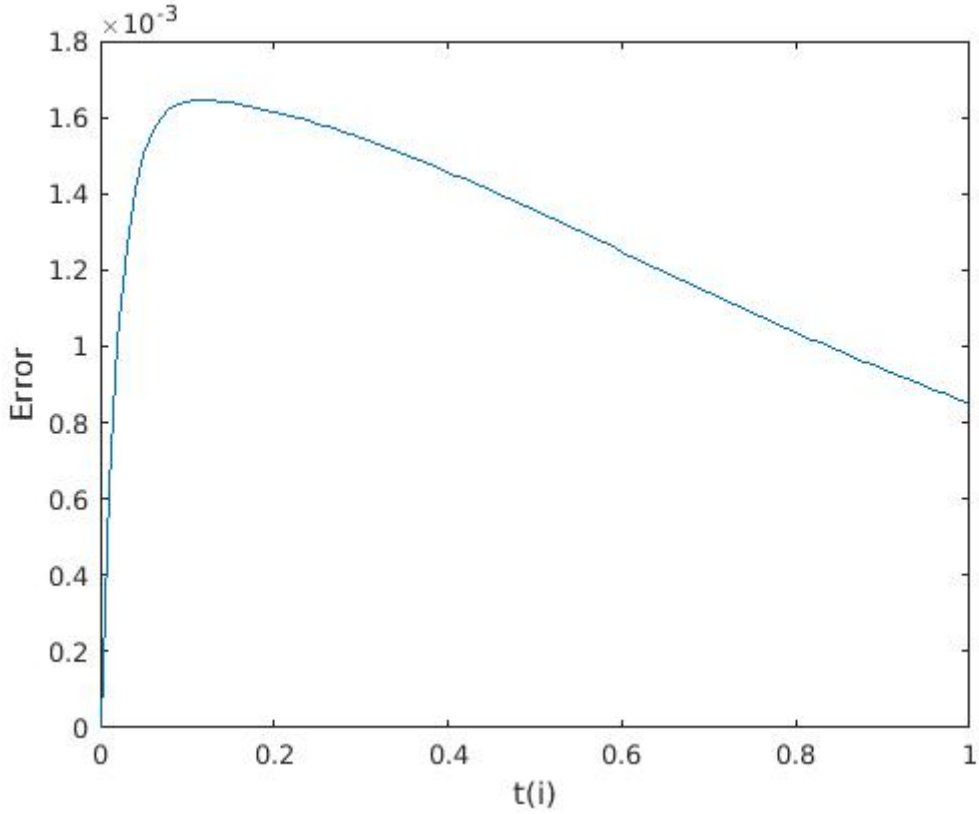


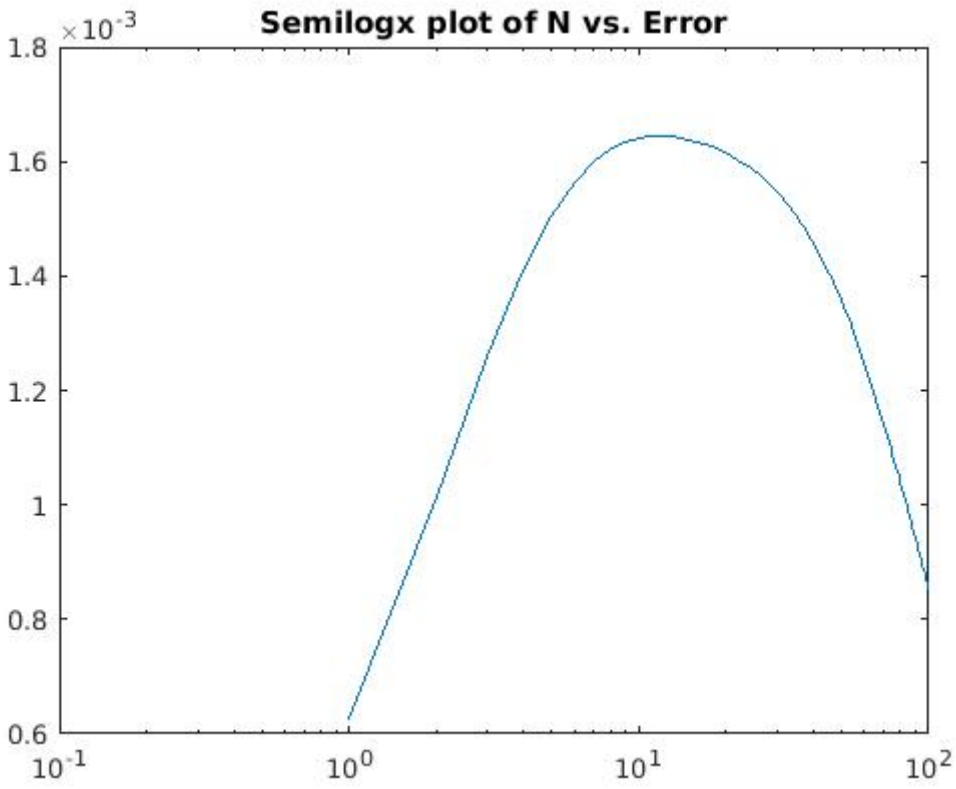




By Trapezoidal method:

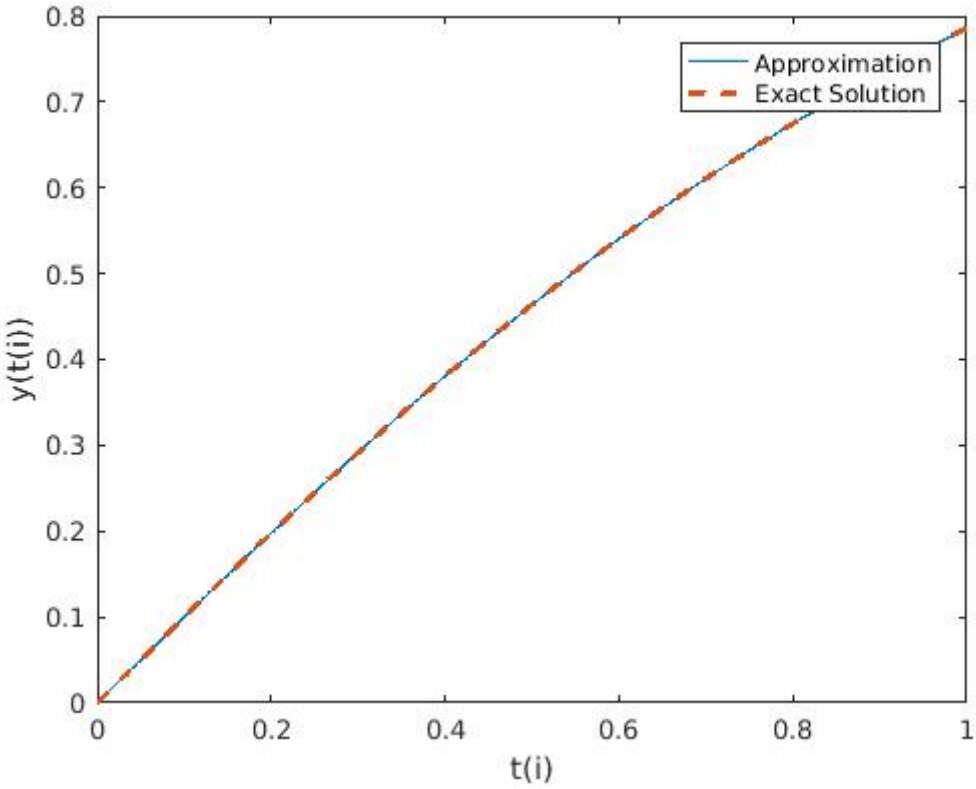

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
0.2
0.4
0.6
0.8
1

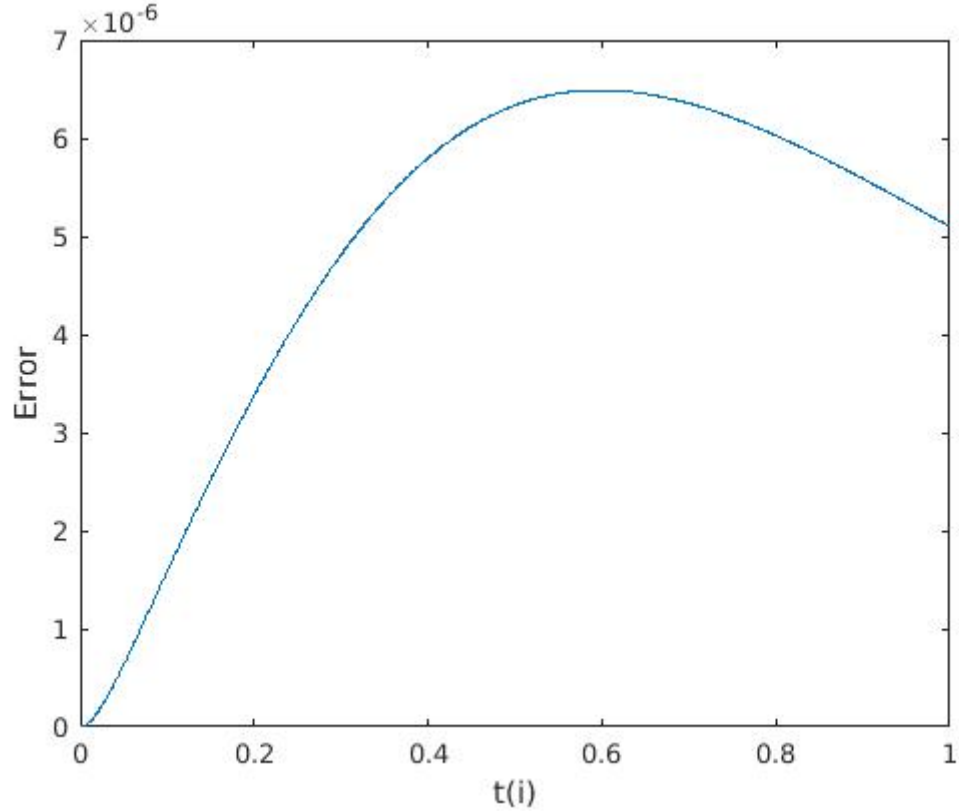

×10^-3
Error
t(i)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
0.2
0.4
0.6
0.8
1

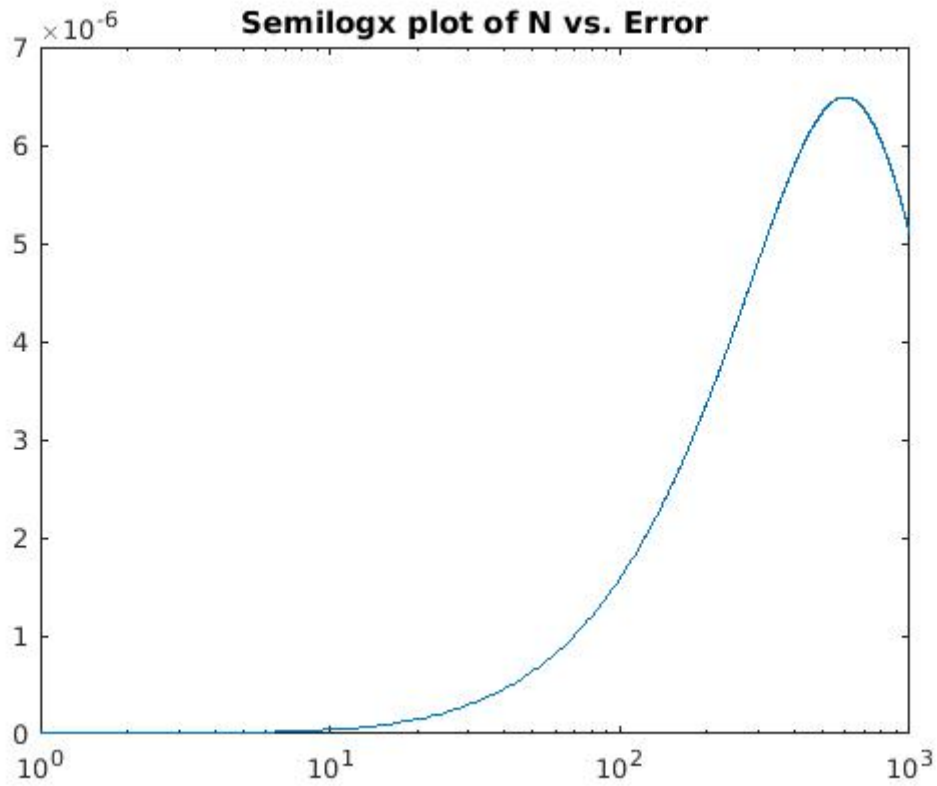


With λ=-50, h=0.001, we get the following figures by various methods stipulated in question:

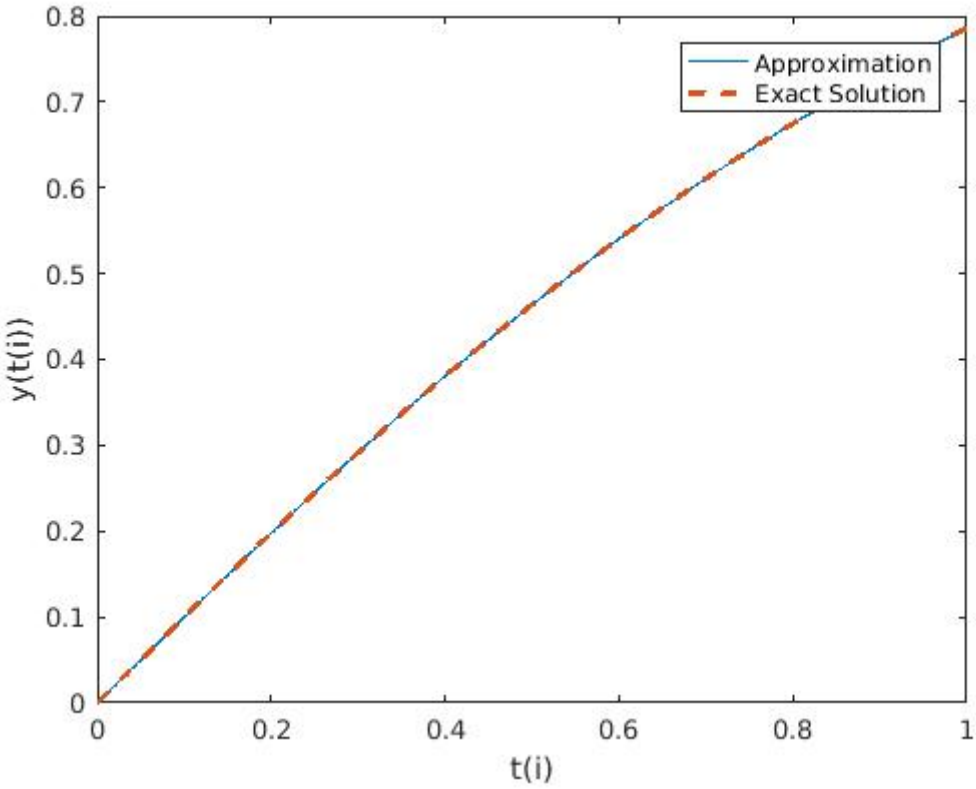
By explicit Euler’s method:

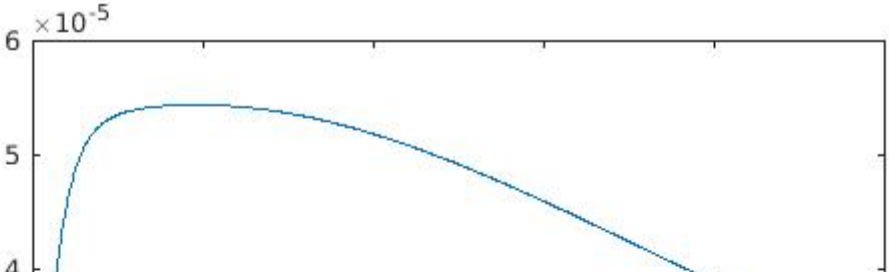

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.2
0.4
0.6
1

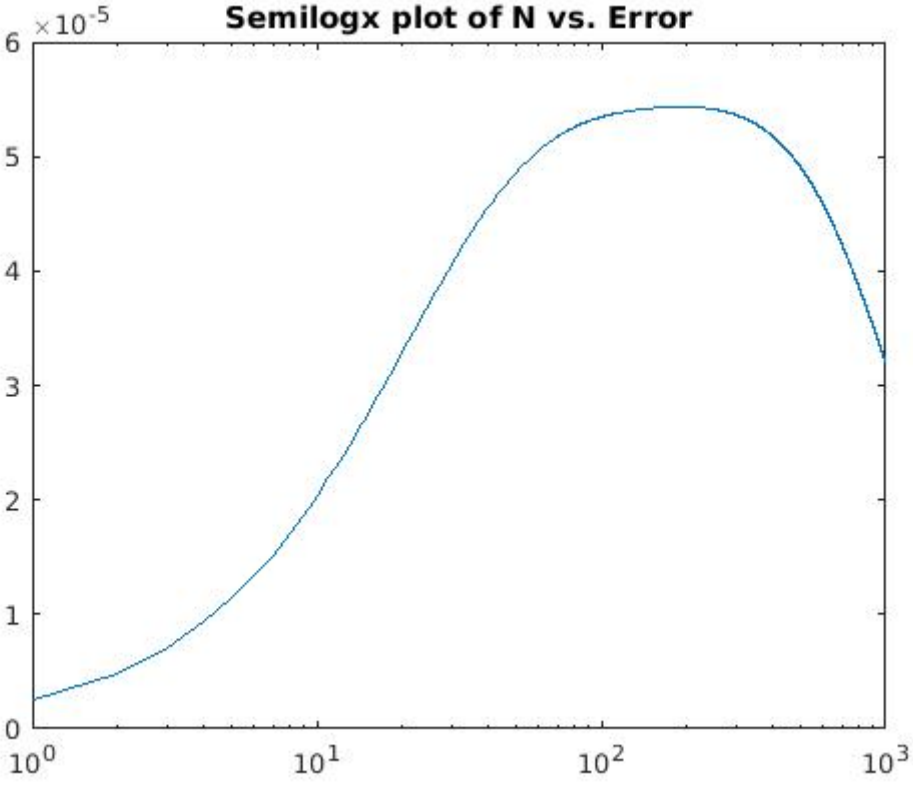

×10^-6
Error
t(i)
0
1
2
3
4
5
6
7
0.2
0.4
0.6
0.8
1



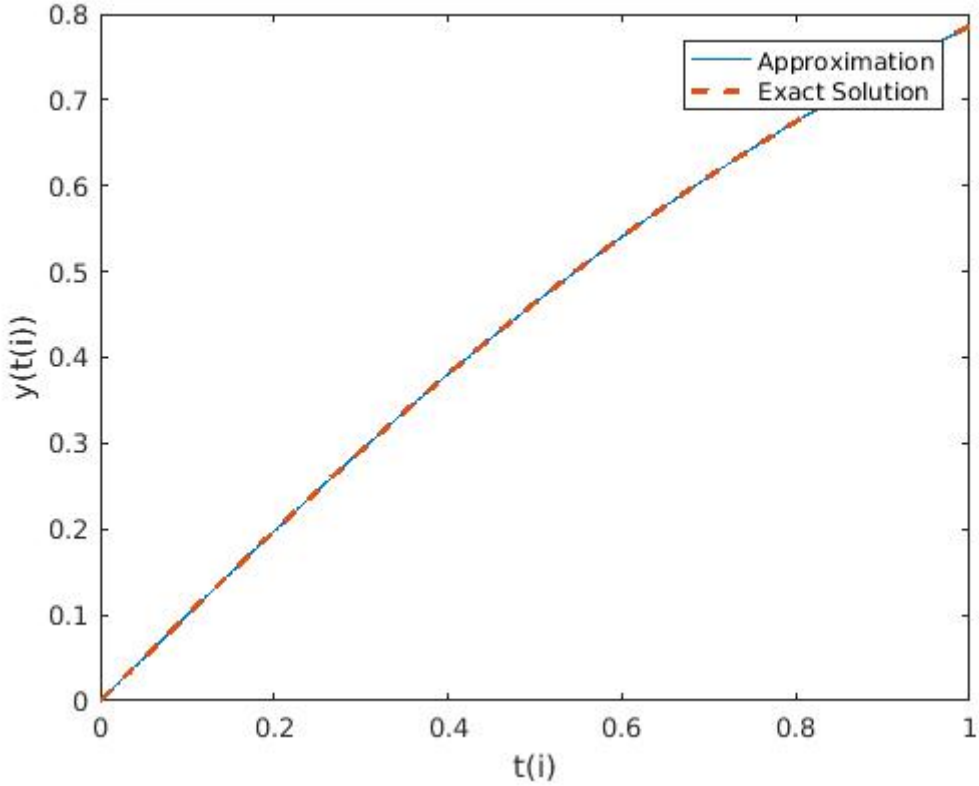
By implicit Euler’s method:

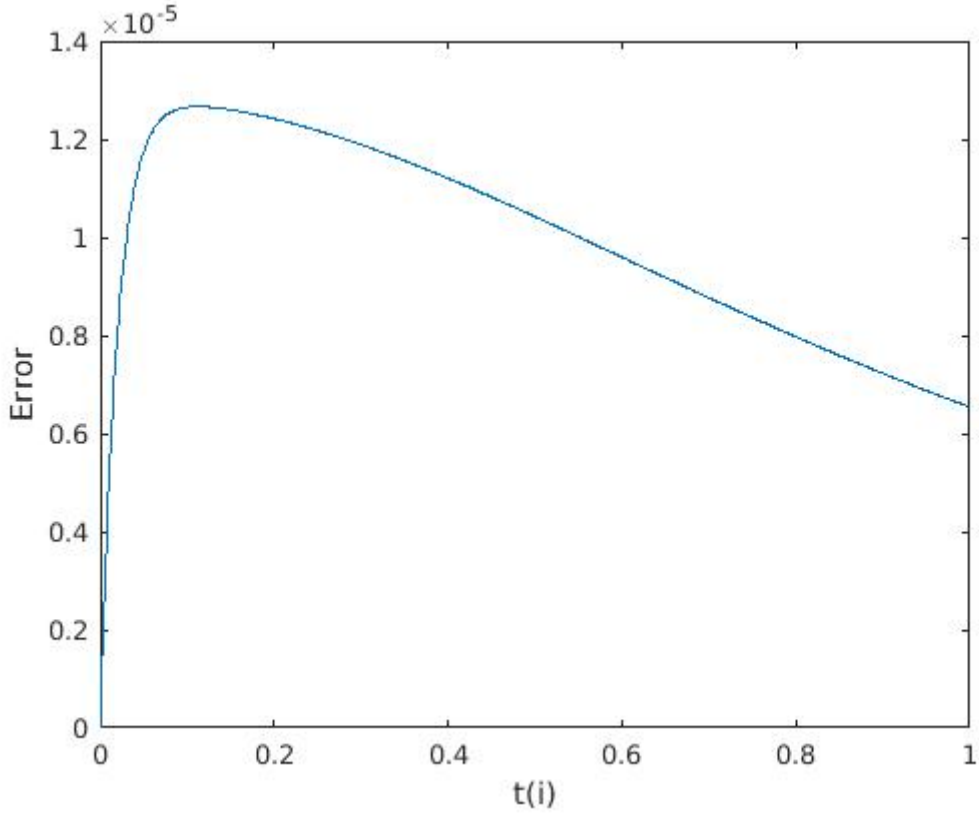

Approximation
Exact Solution
y(t(i))
t(i)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.2
0.4
0.6
1

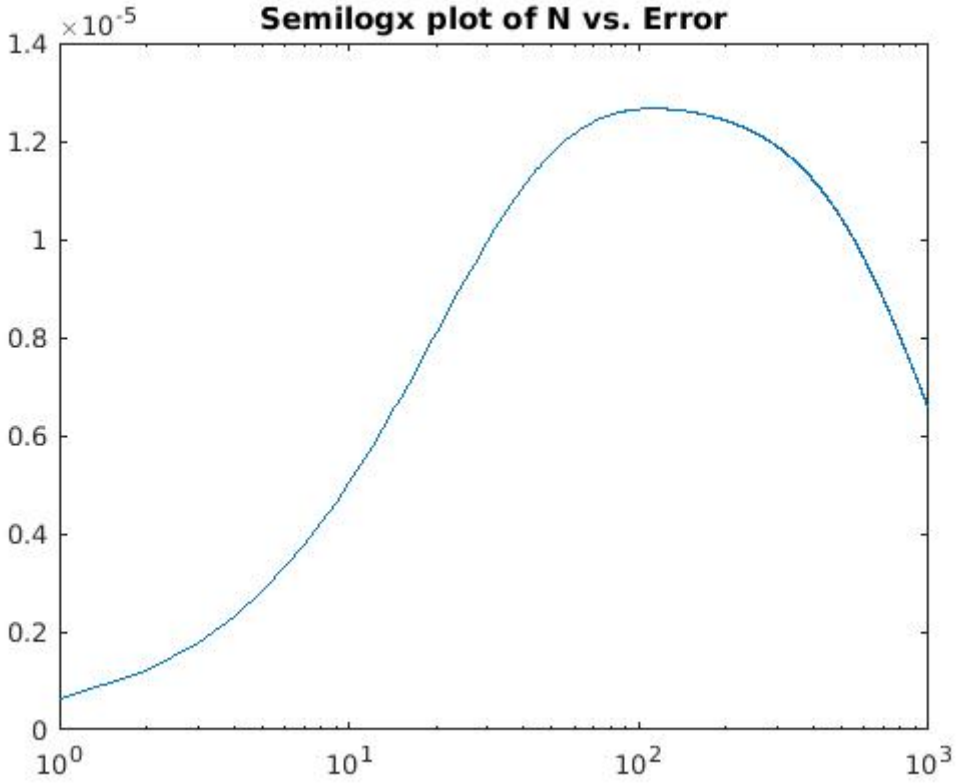

×10^-5
6
5
4


Semilogx plot of N vs. Error
×10^-5
6
5
4
3
2
1
0
10^0
10^1
10^2
10^3

By Trapezoidal method:

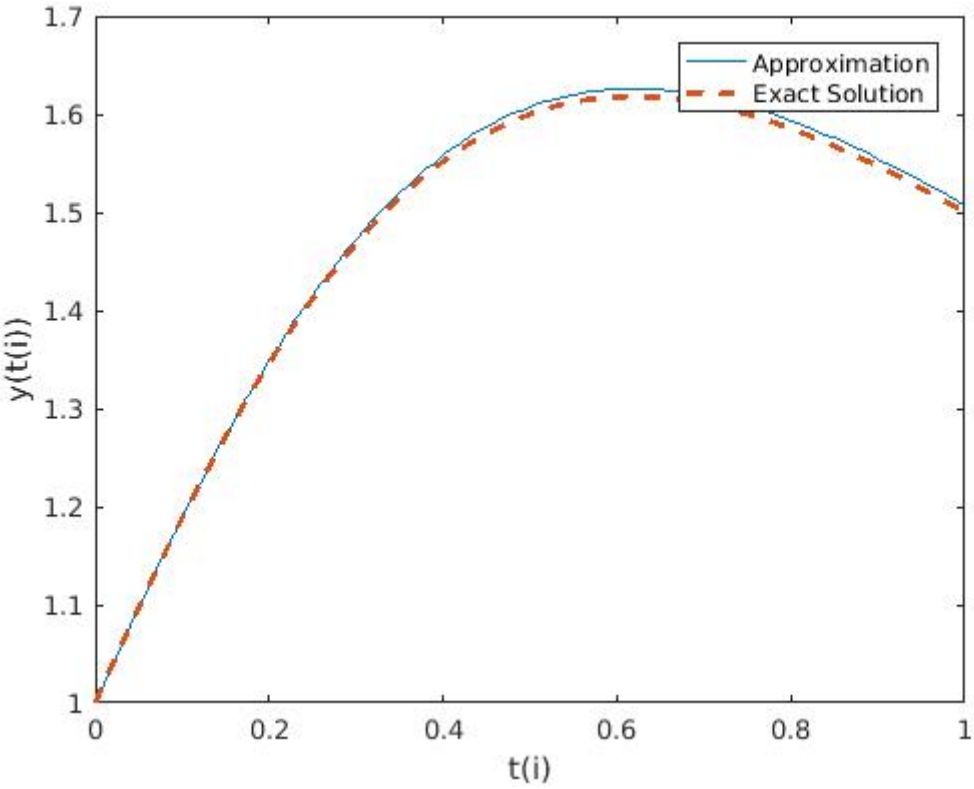


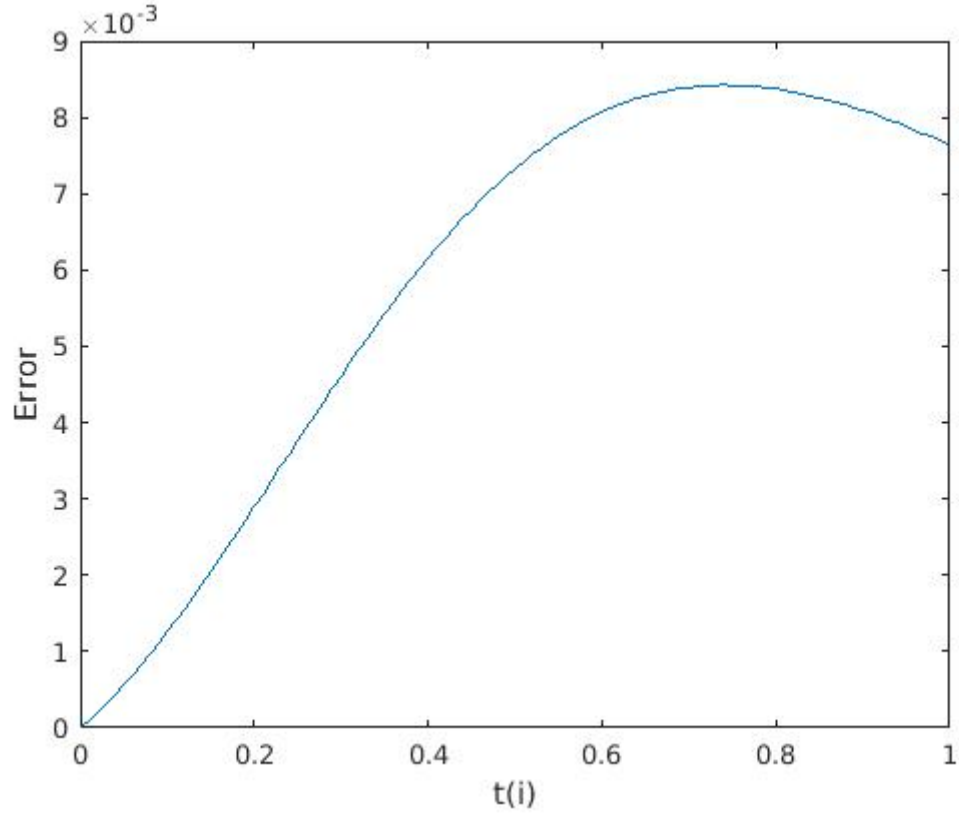


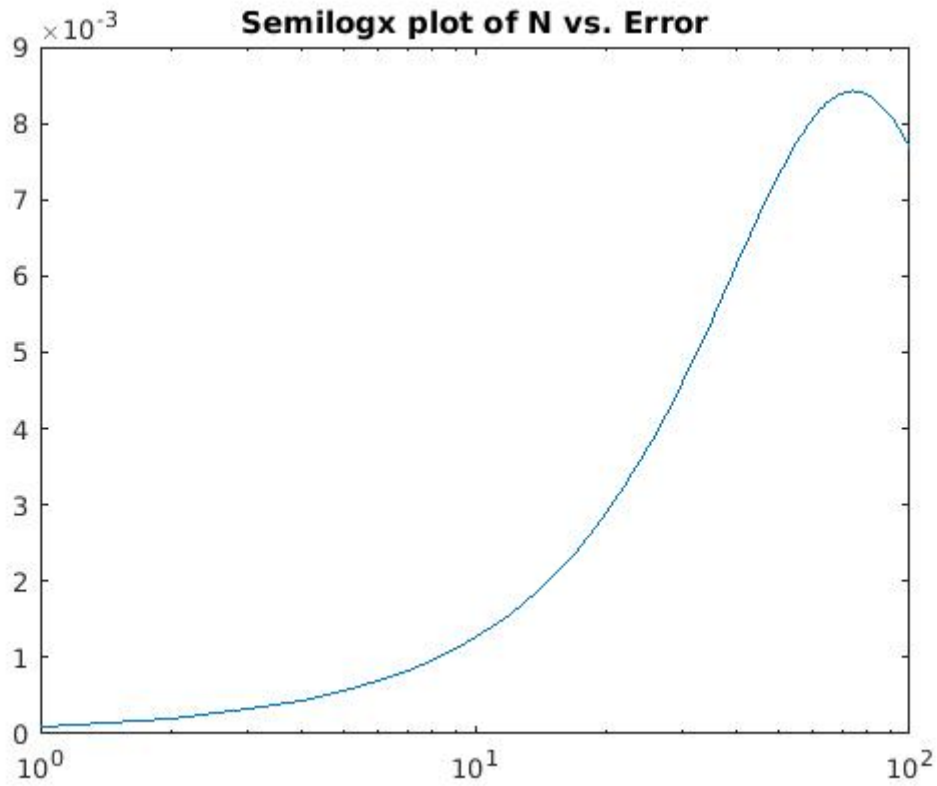


Problem 4:

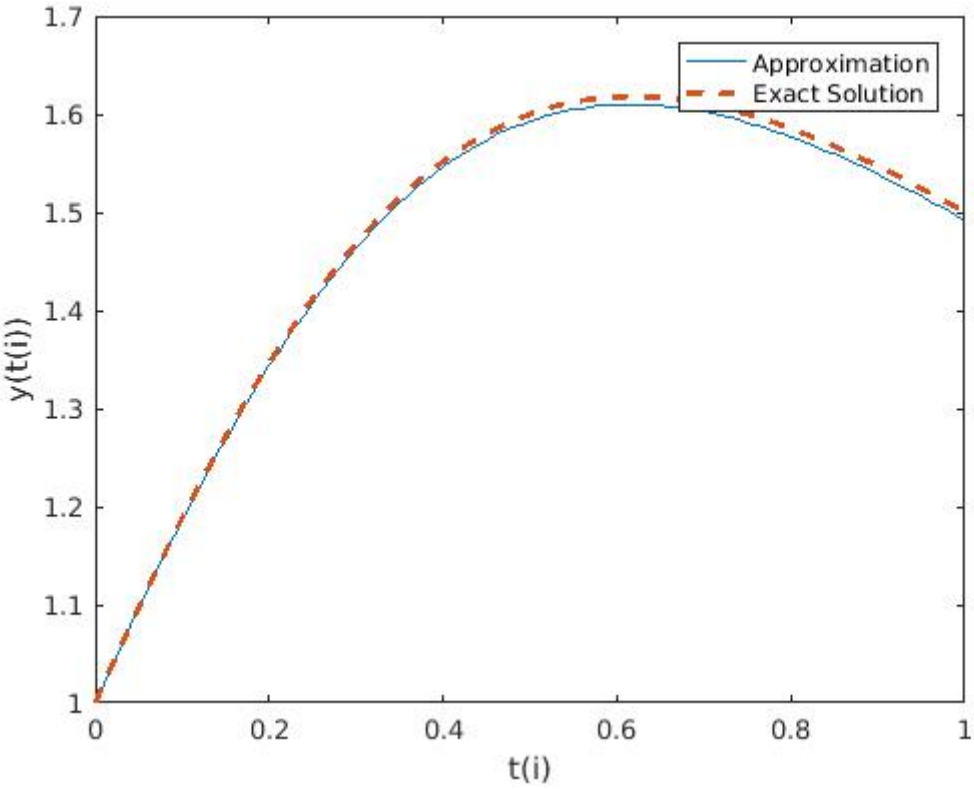
Part a) By explicit Euler's method, the following plots are obtained:

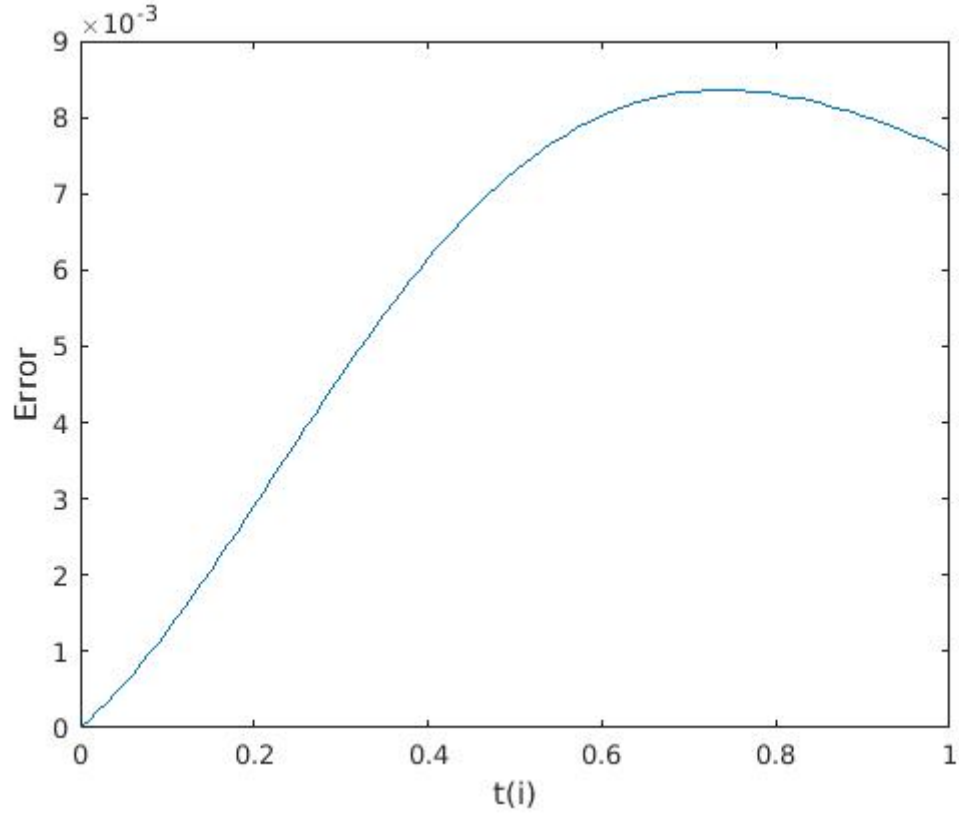

Approximation
Exact Solution
y(t(i))
t(i)
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0
0.2
0.4
0.6
0.8
1

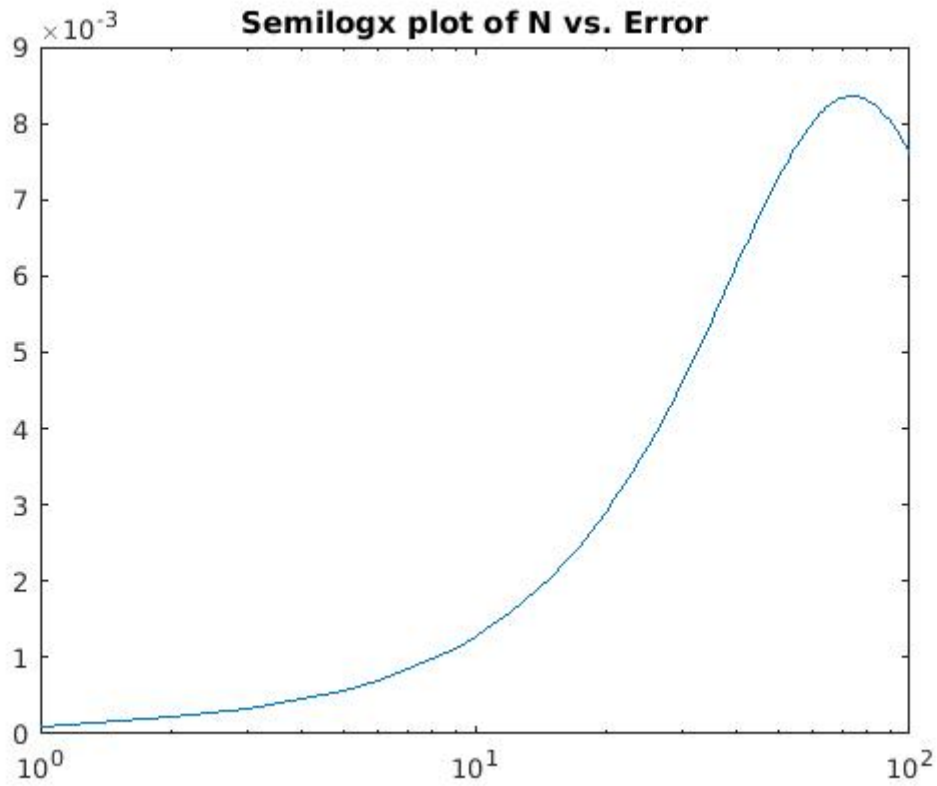

×10^-3
Error
t(i)
9
8
7
6
5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1



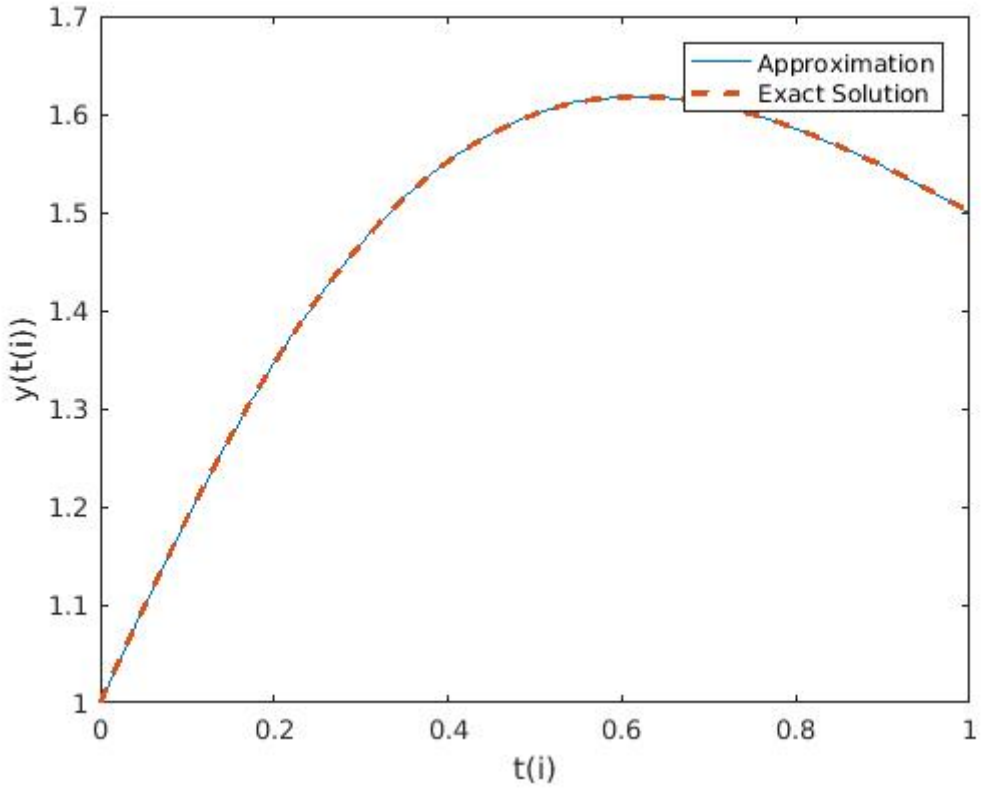
By implicit Euler’s method, the following plots are obtained:

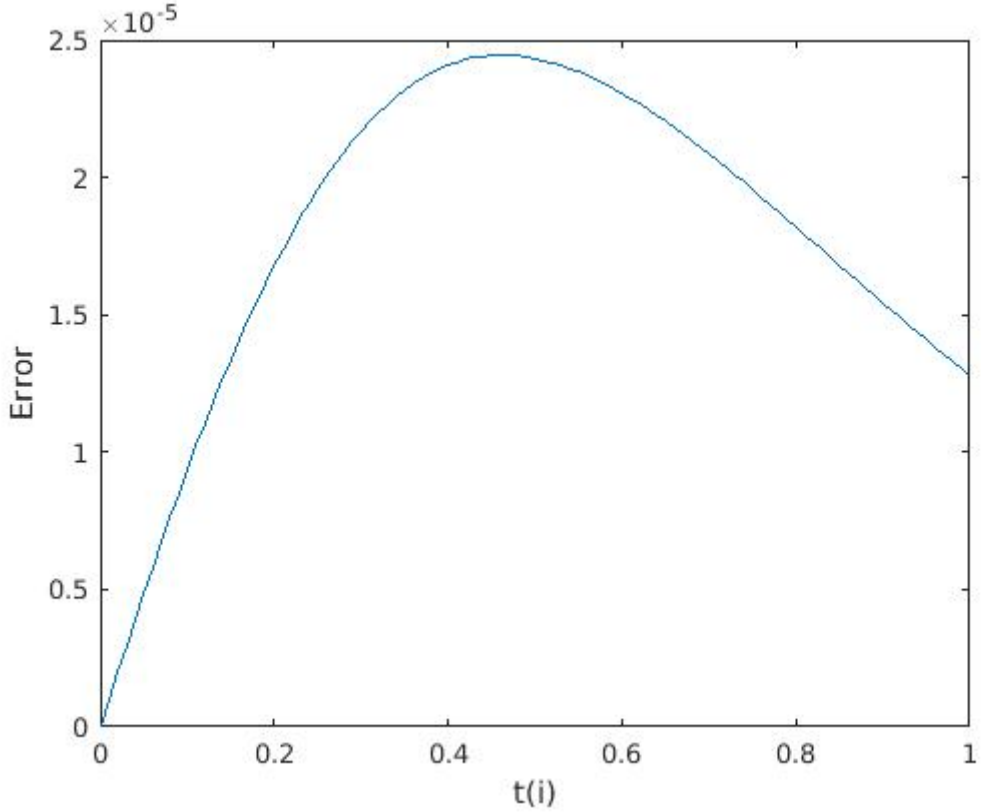

Approximation
Exact Solution
y(t(i))
t(i)
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0
0.2
0.4
0.6
0.8
1

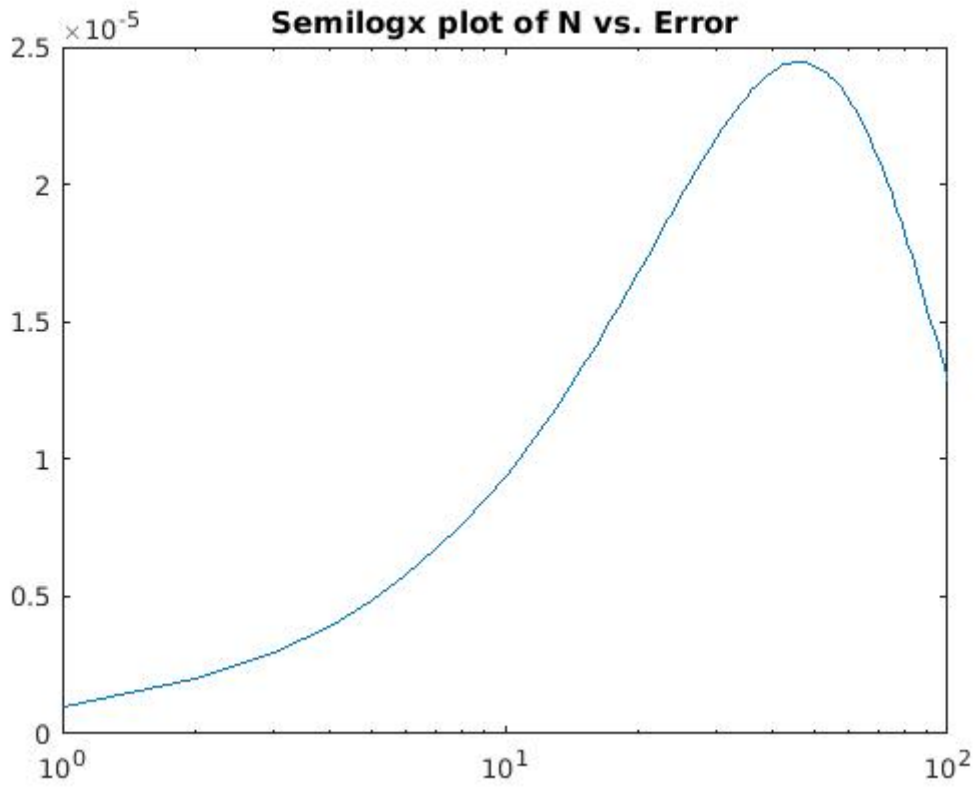

×10^-3
Error
t(i)
9
8
7
6
5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1



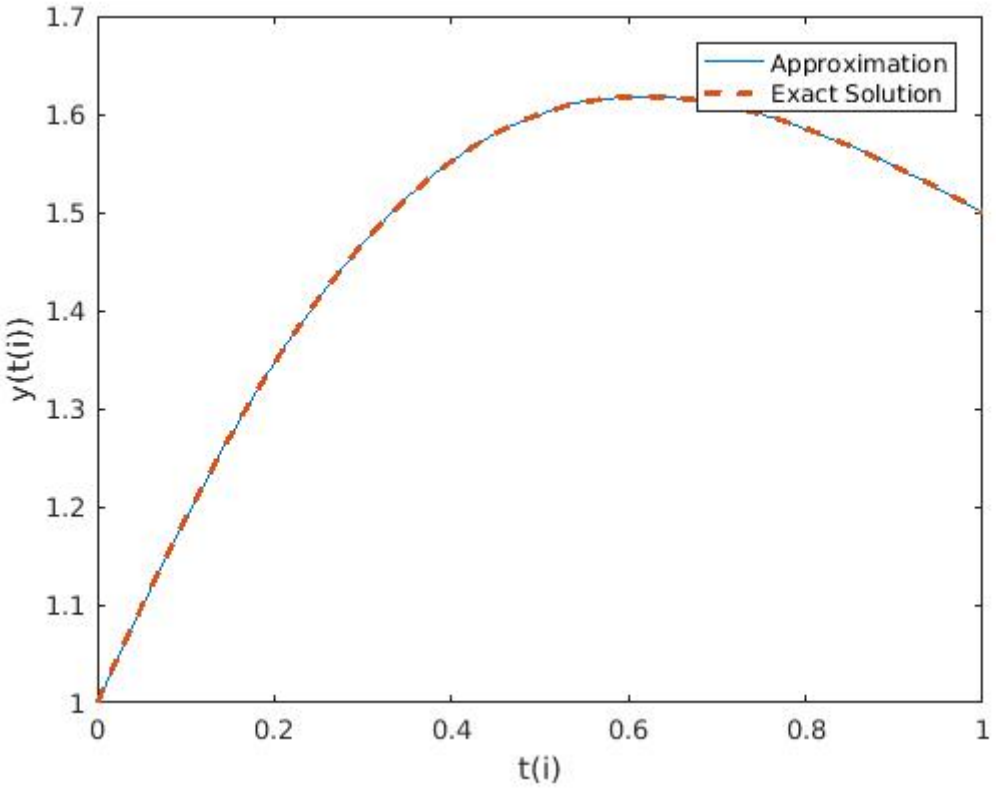
By modified Euler’s method, the following plots are obtained:

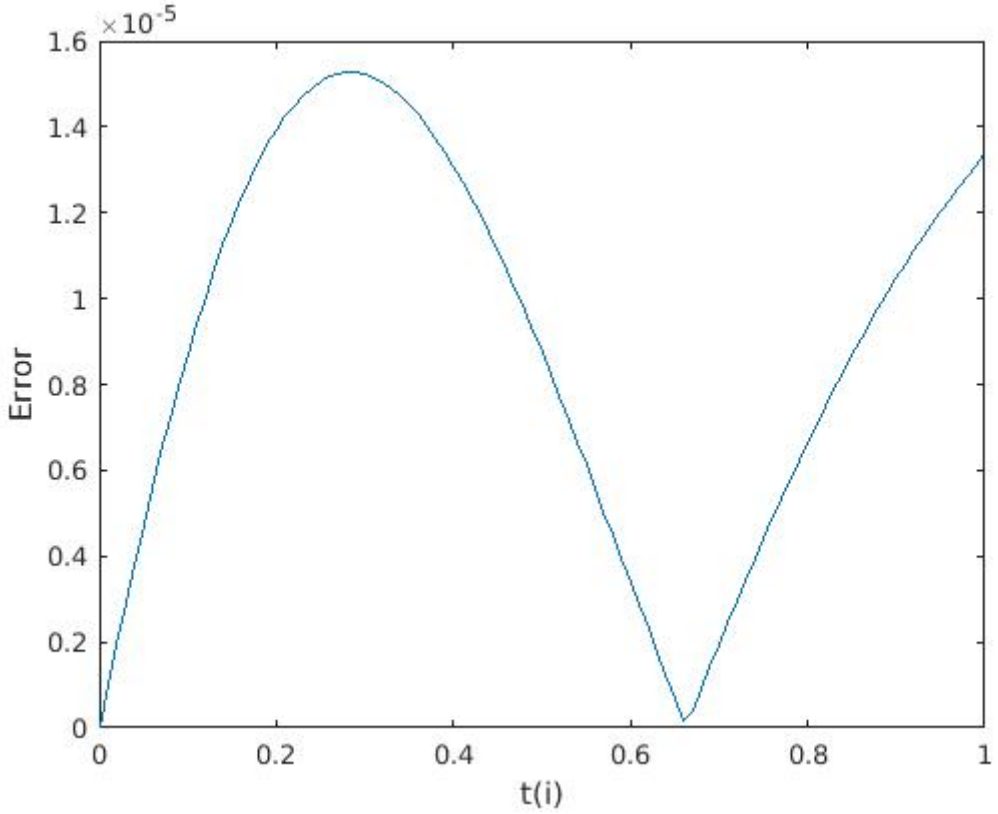

Approximation
Exact Solution
y(t(i))
t(i)
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0
0.2
0.4
0.6
0.8
1

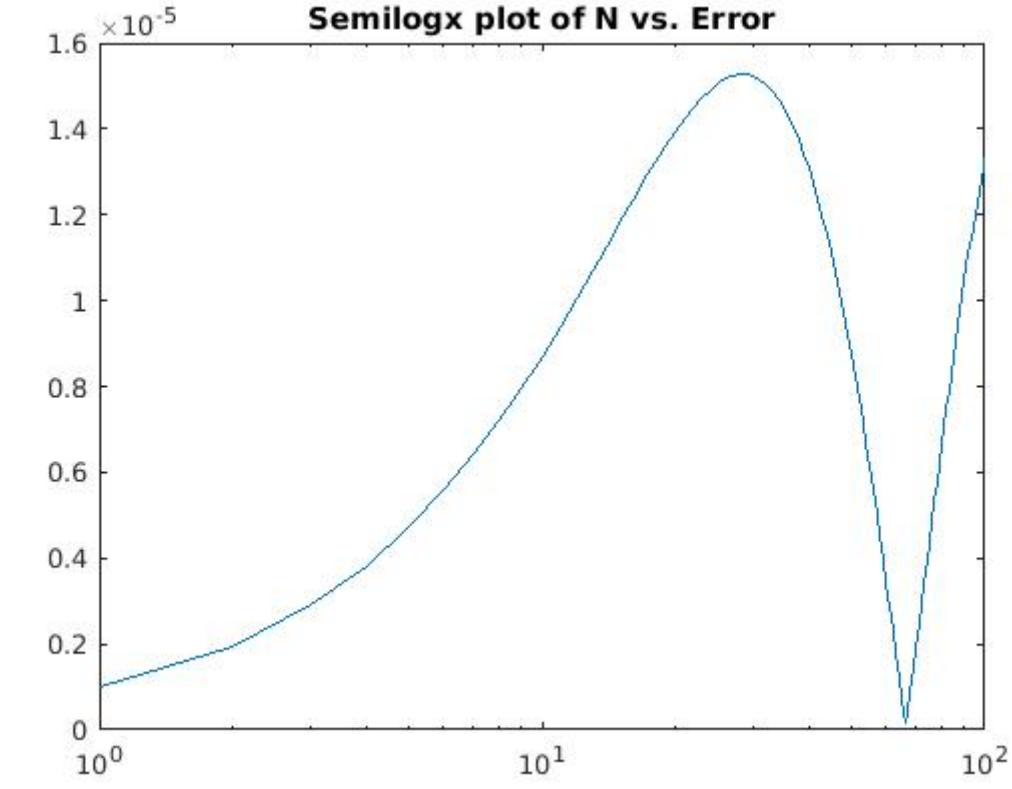

×10^-5
Error
t(i)
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1



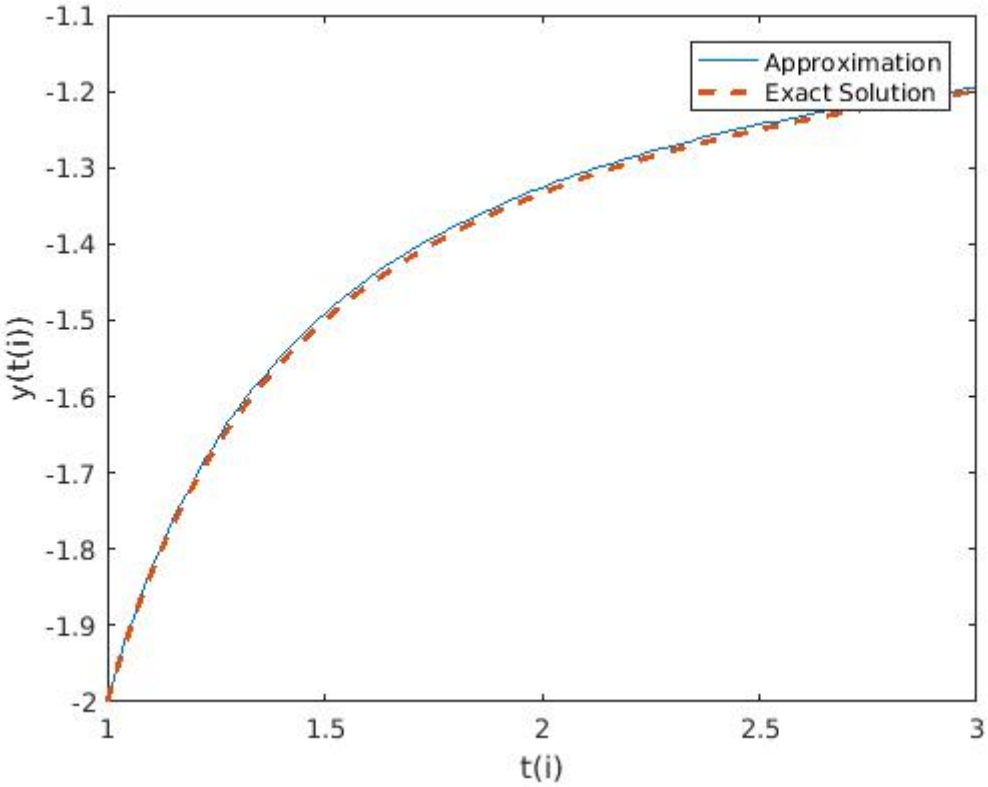
By Trapezoidal method, the following plots are obtained:

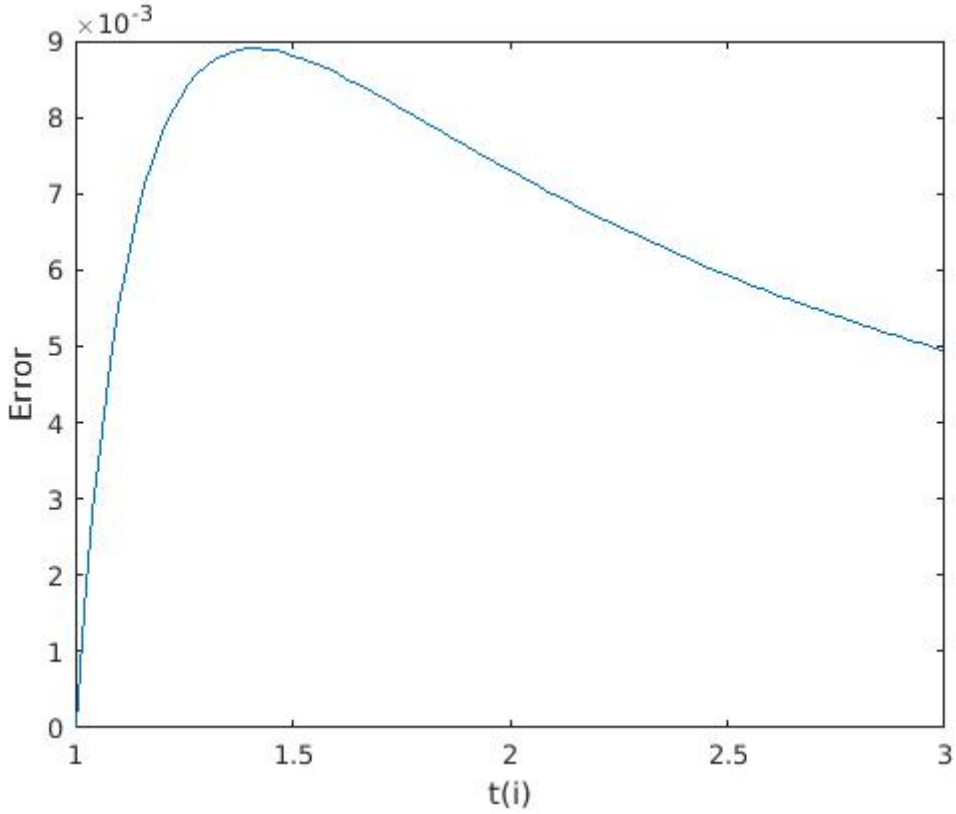


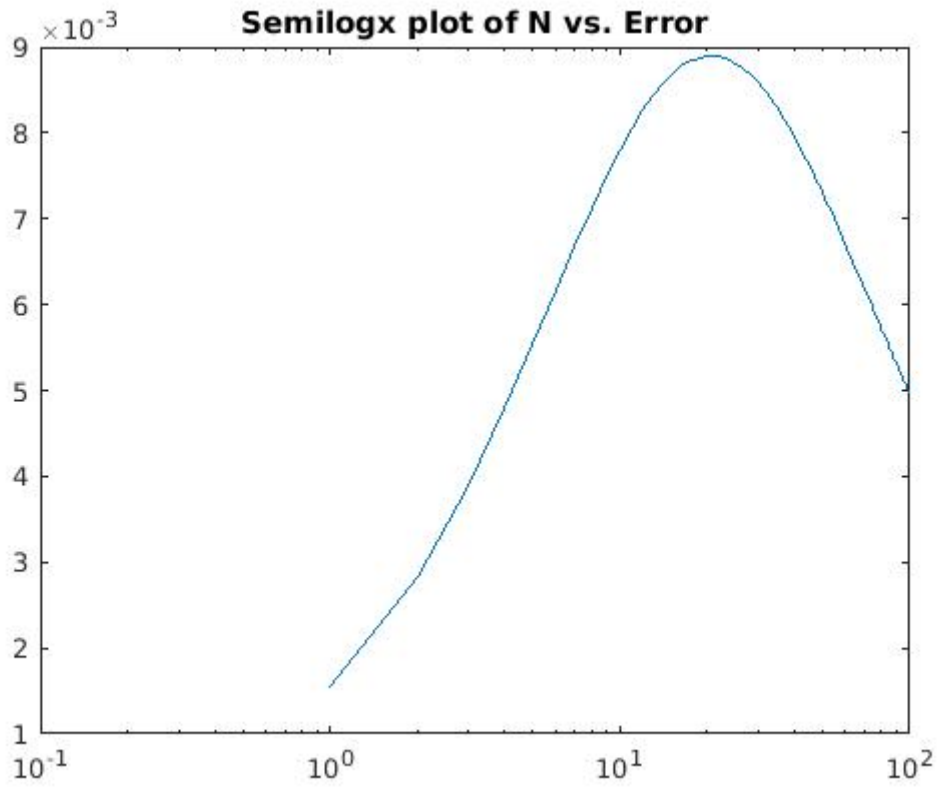

×10^-5
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Error
0
0.2
0.4
0.6
0.8
1
t(i)


Semilogx plot of N vs. Error
×10^-5
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
10^0
10^1
10^2

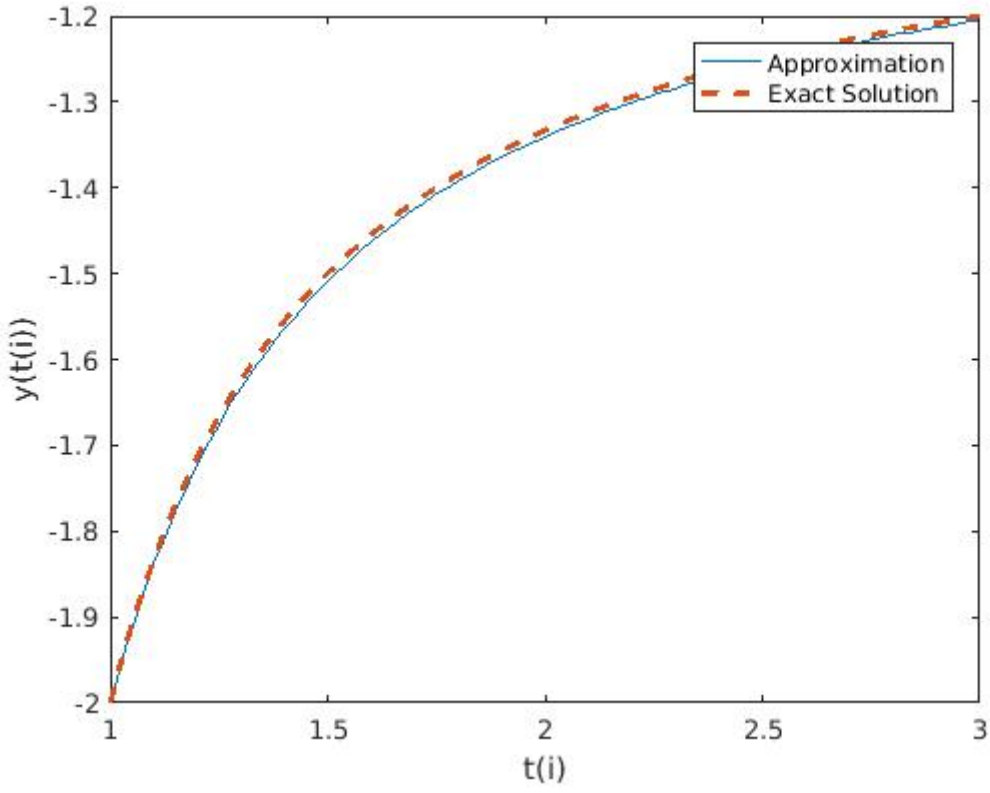
Part b) By explicit Euler's method, the following plots are obtained:

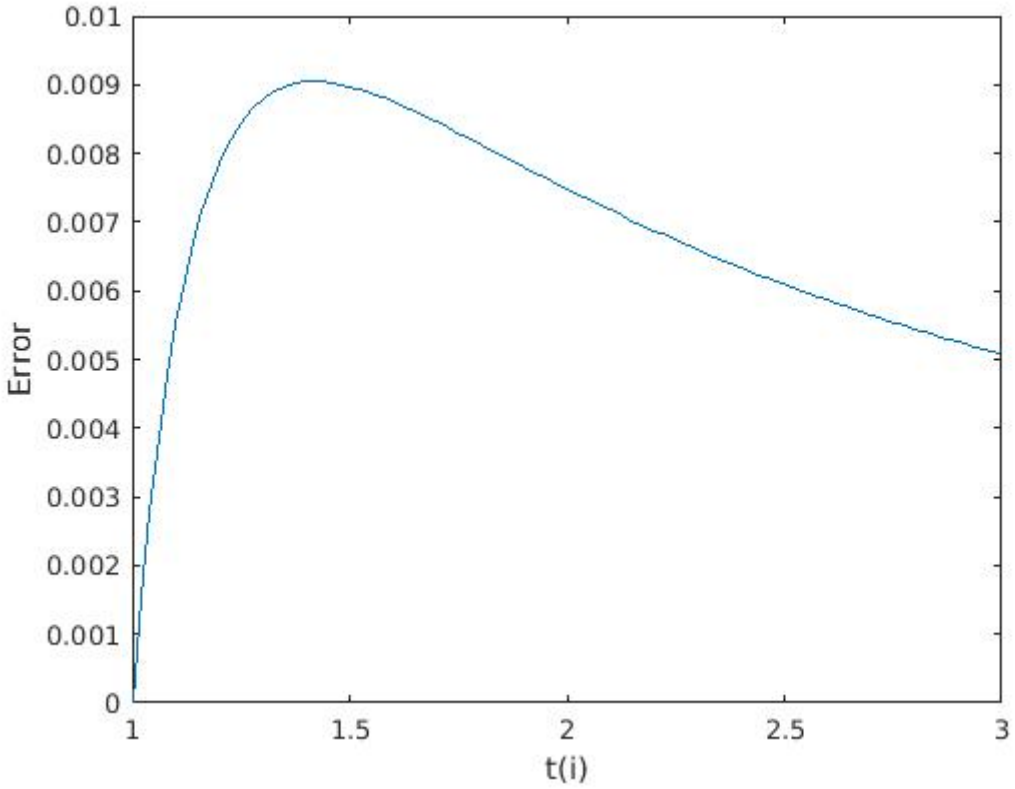


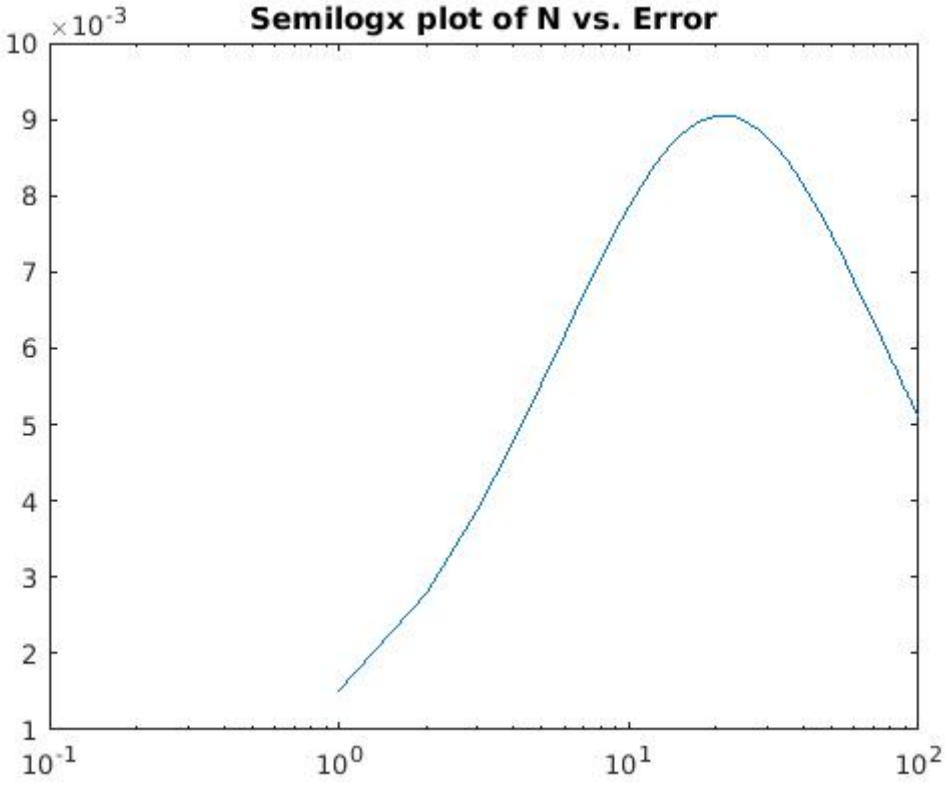




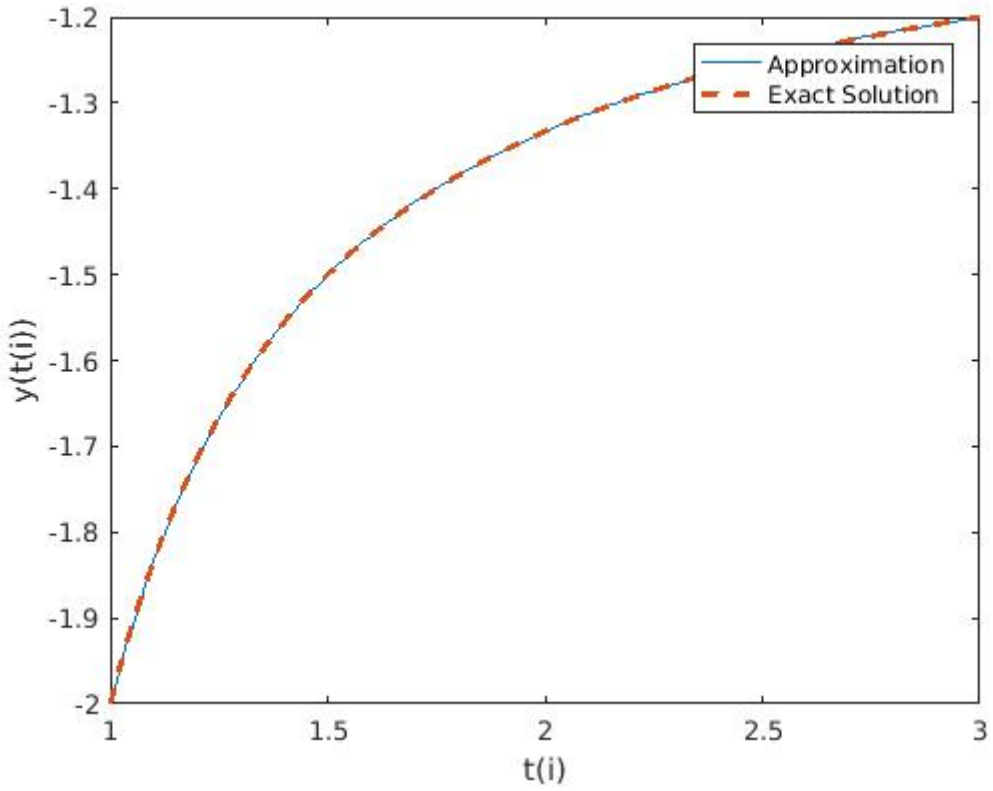
By implicit Euler's method, the following plots are obtained:

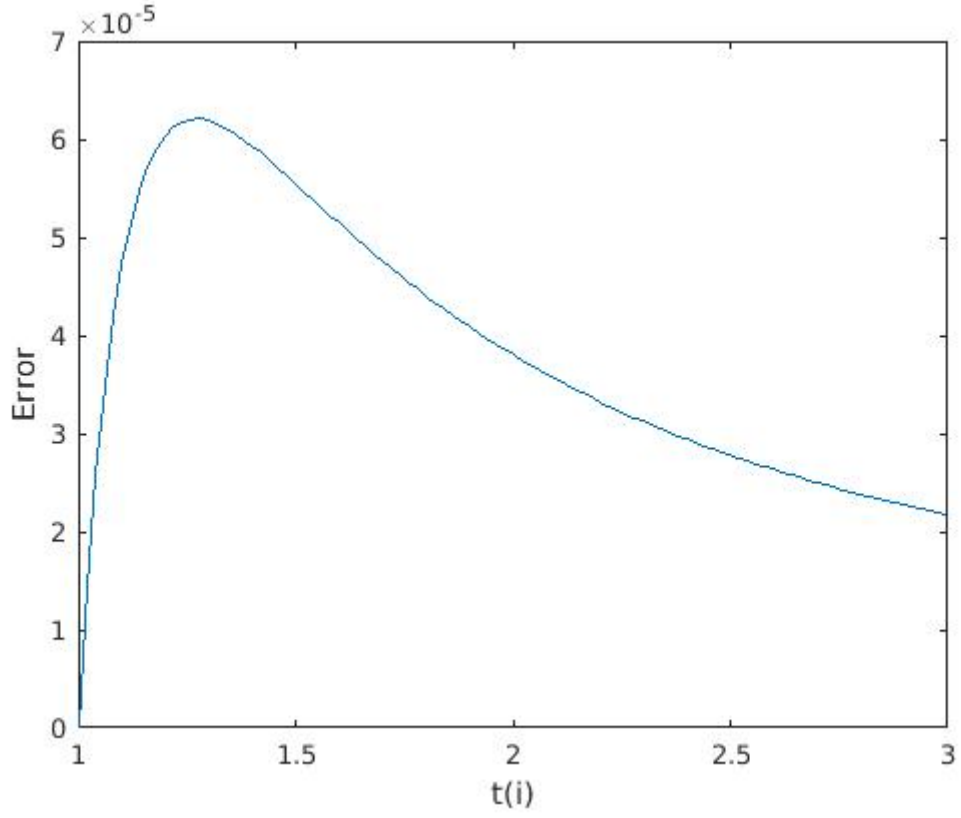


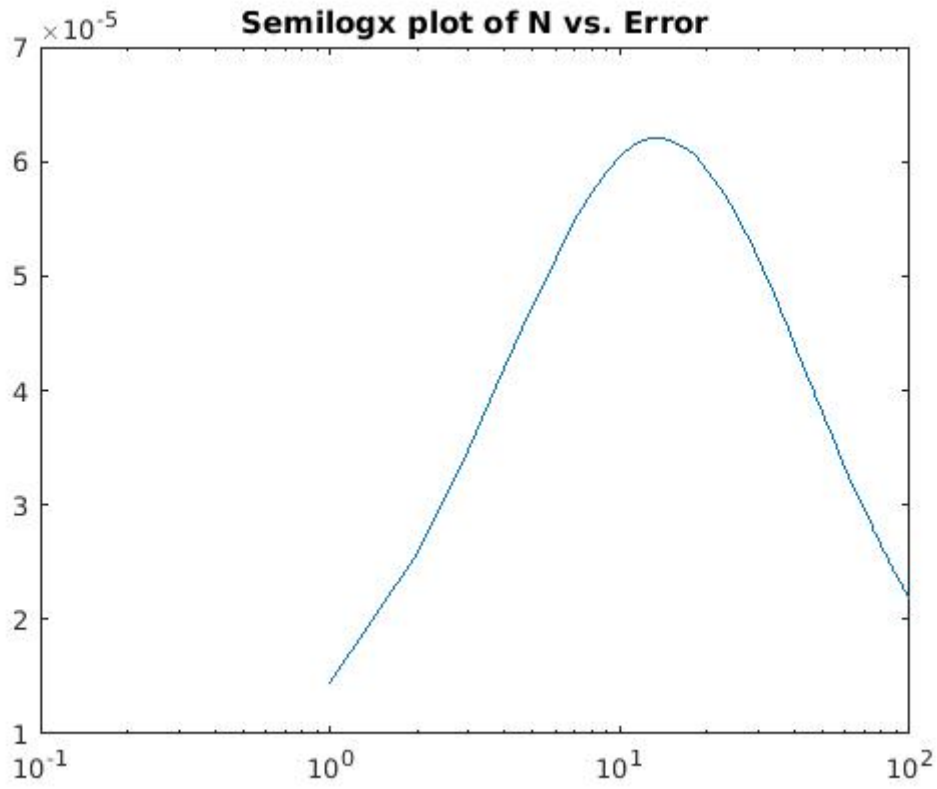

0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
Error
1
1.5
2
2.5
3
t(i)


Semilogx plot of N vs. Error
×10^-3
10
9
8
7
6
5
4
3
2
1
10^-1
10^0
10^1
10^2

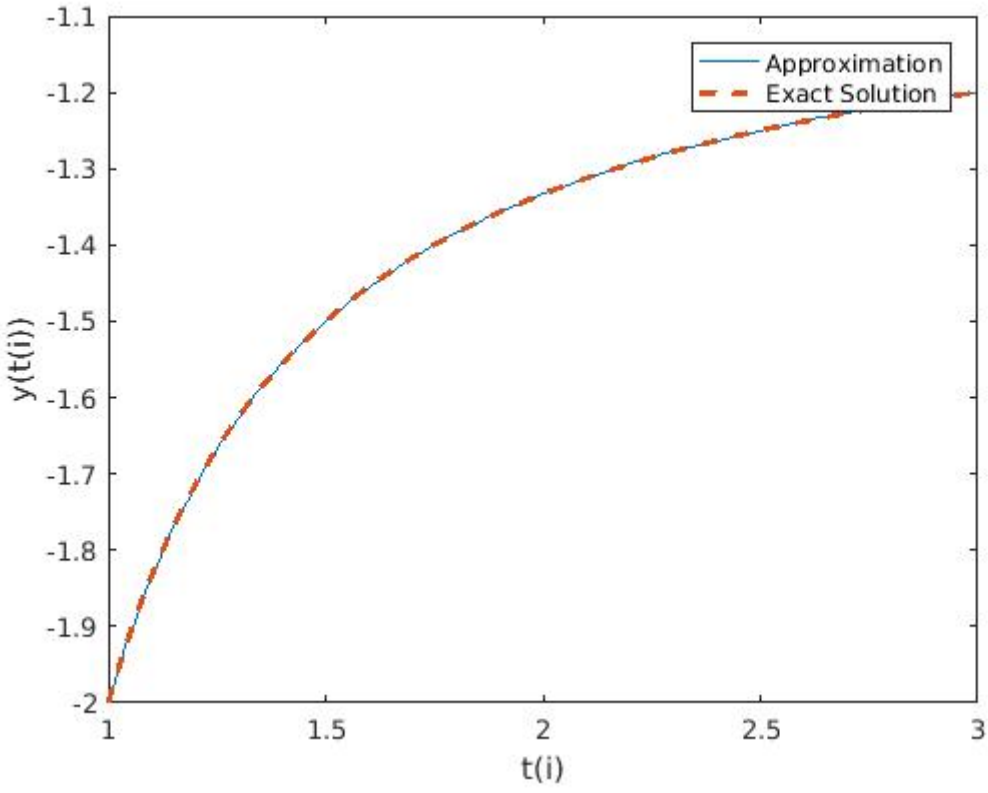
By modified Euler’s method, the plots obtained are:

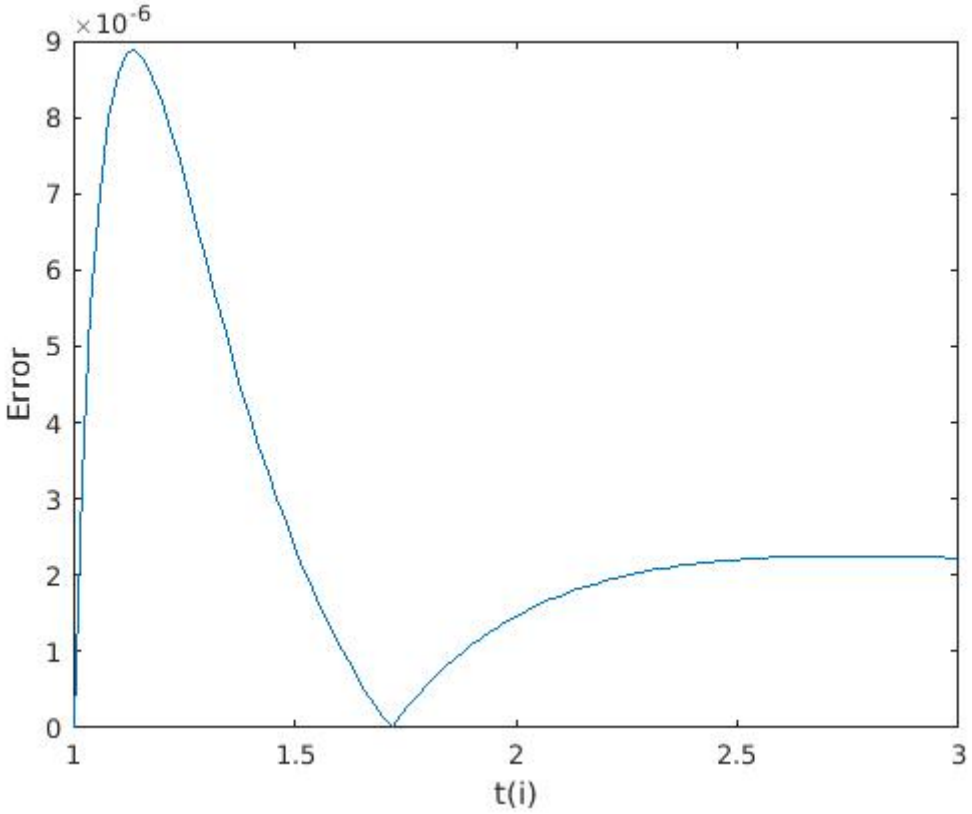


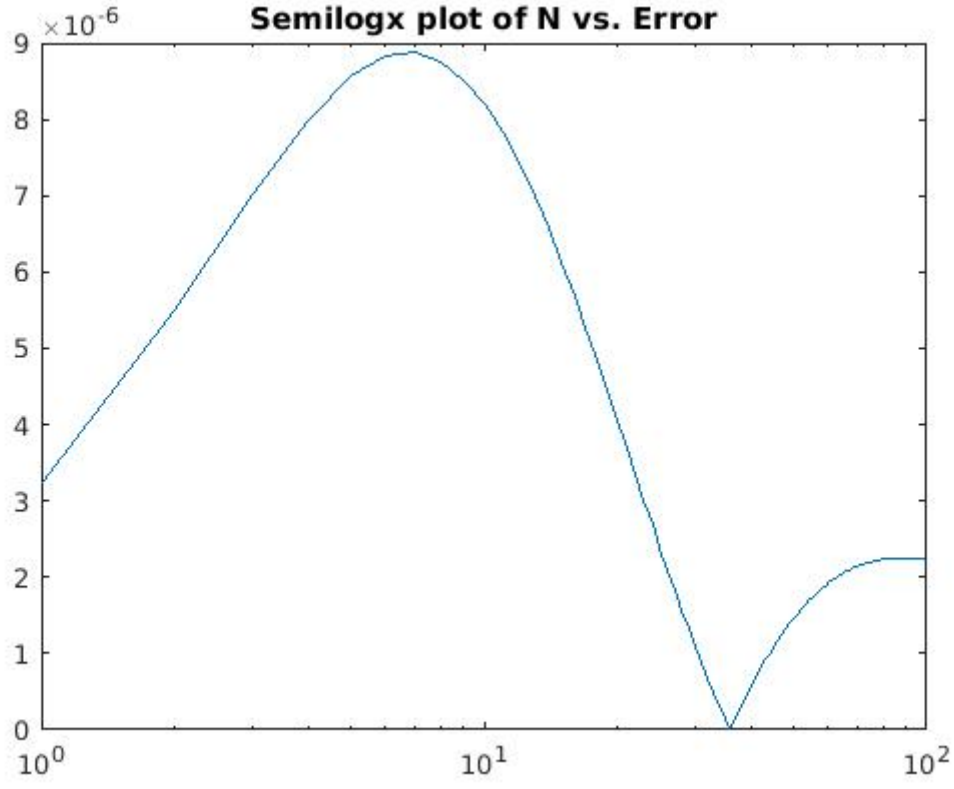




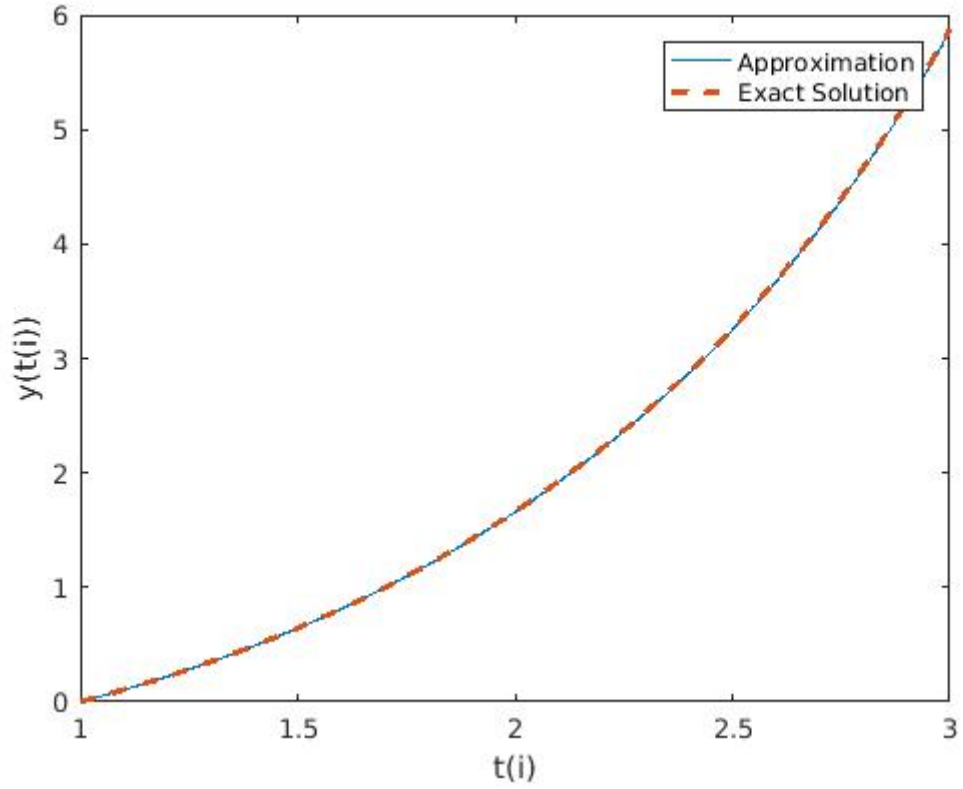
By Trapezoidal method, the plots obtained are:

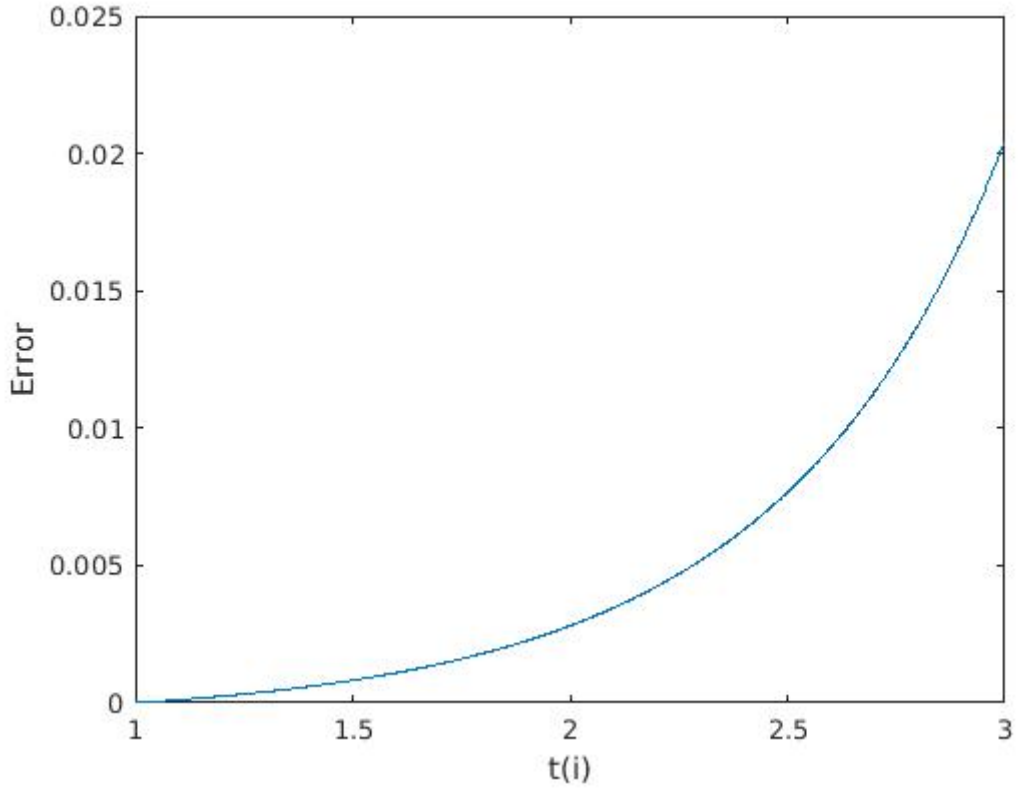


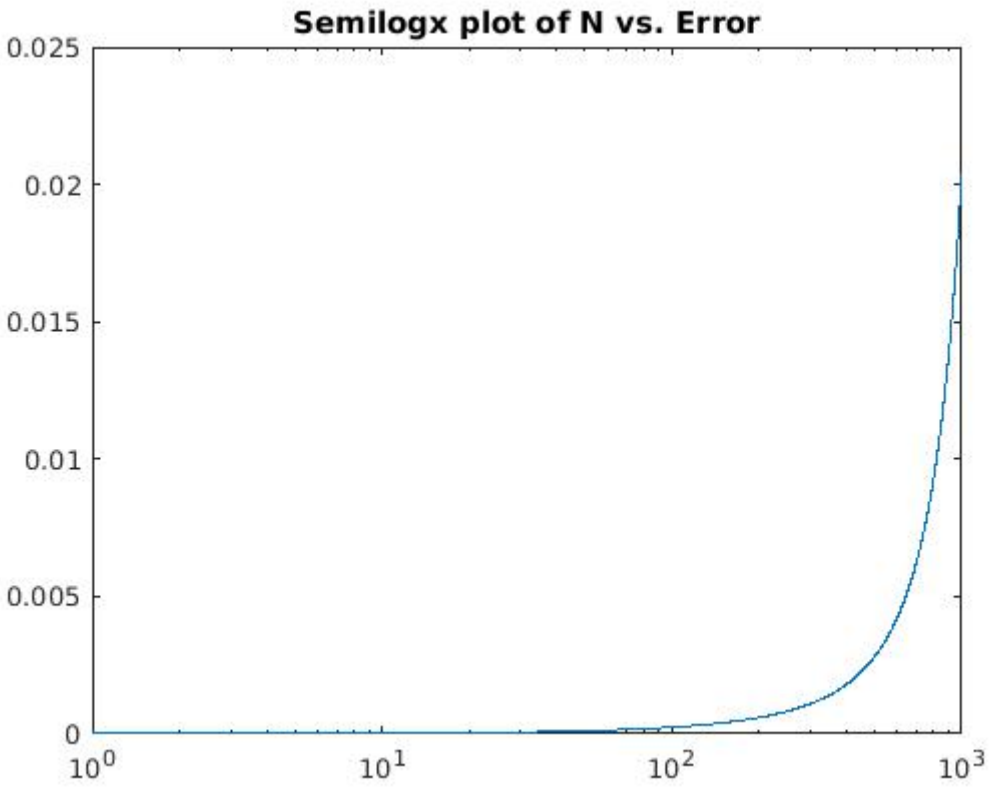

×10^-6
9
8
7
6
5
4
3
2
1
0
Error
1
1.5
2
2.5
3
t(i)


Semilogx plot of N vs. Error
×10^-6
9
8
7
6
5
4
3
2
1
0
10^0
10^1
10^2

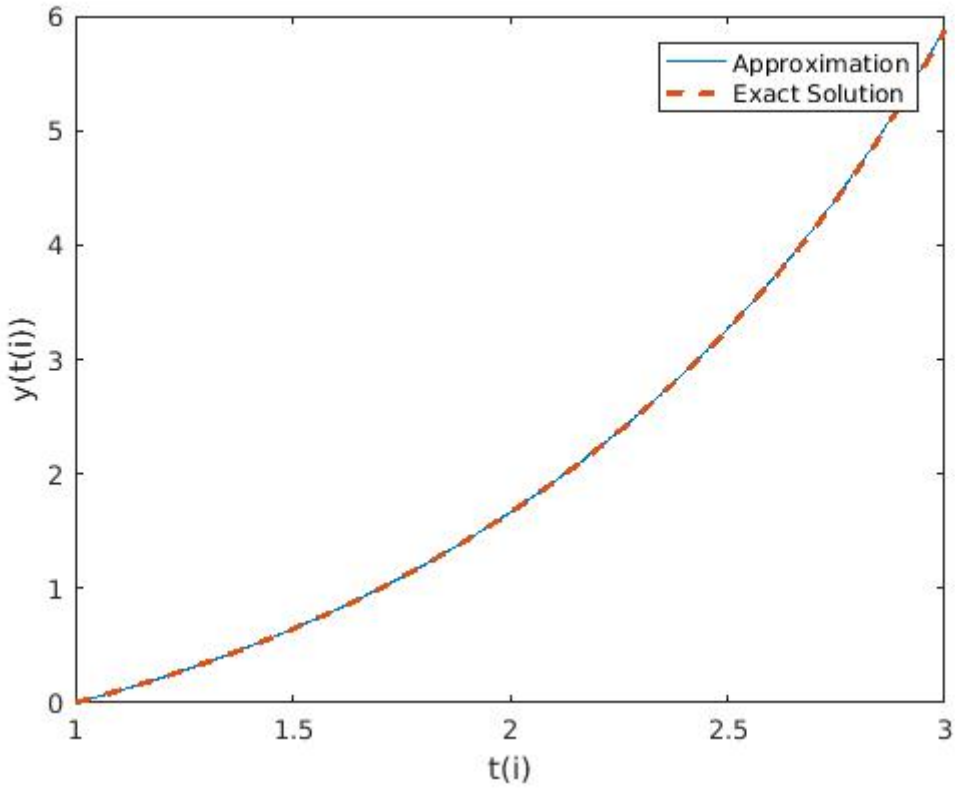
Part c) By explicit Euler's method, the following plots are obtained:

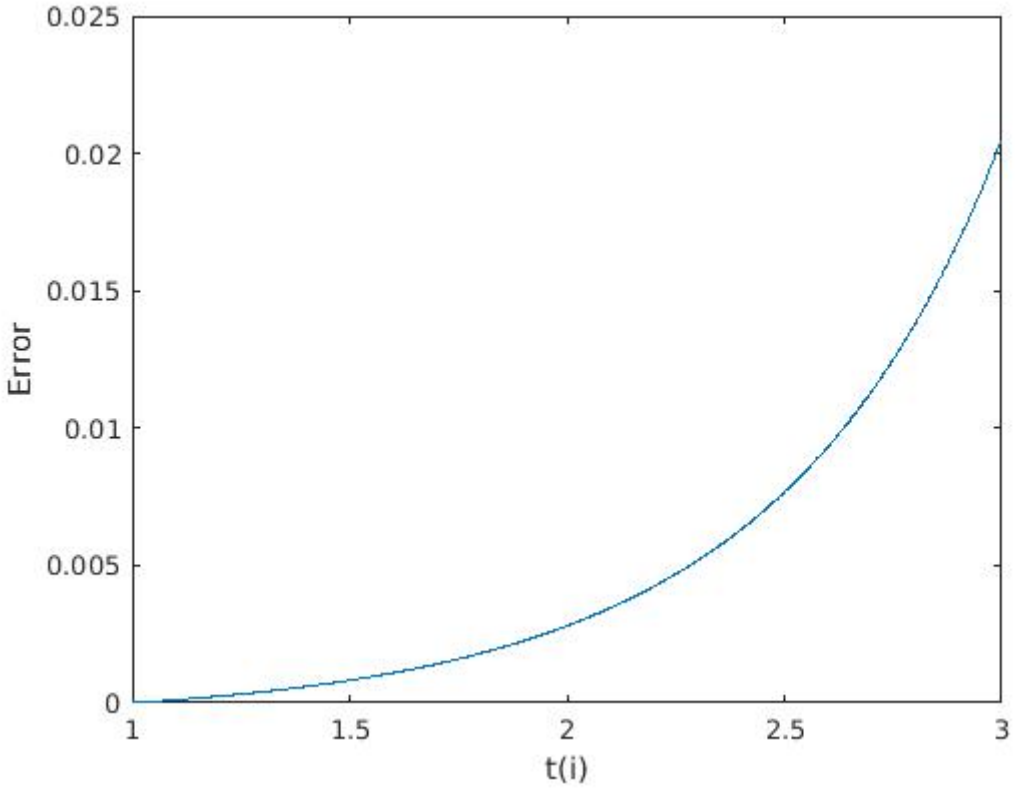


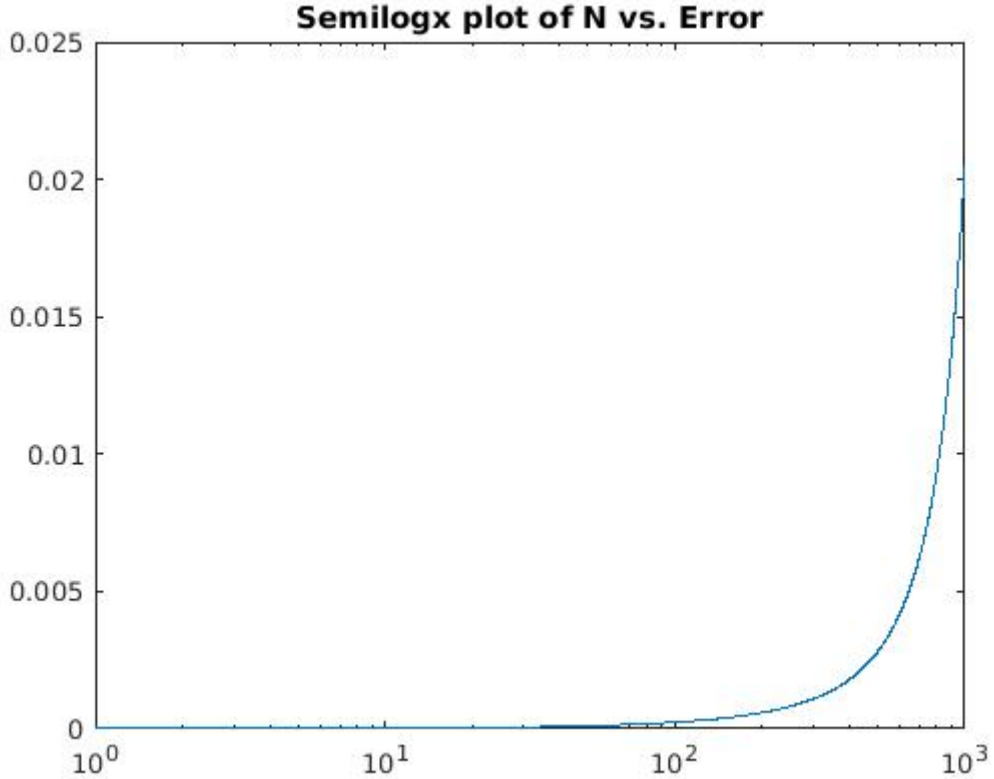




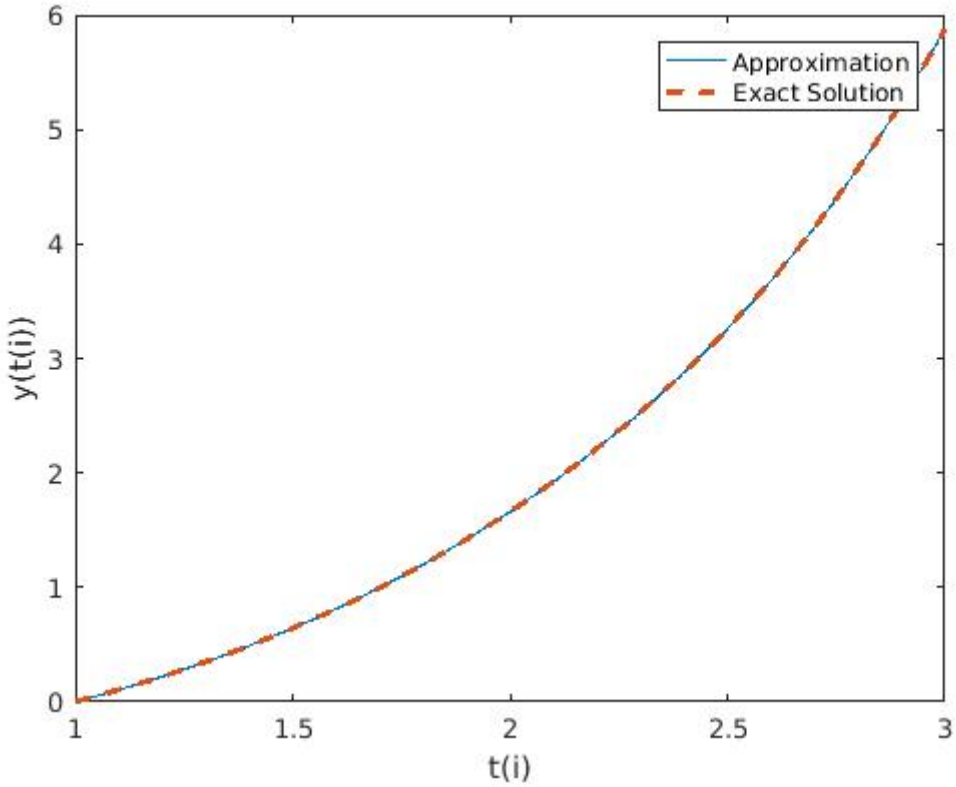
By implicit Euler's method, the plots obtained are as follows:

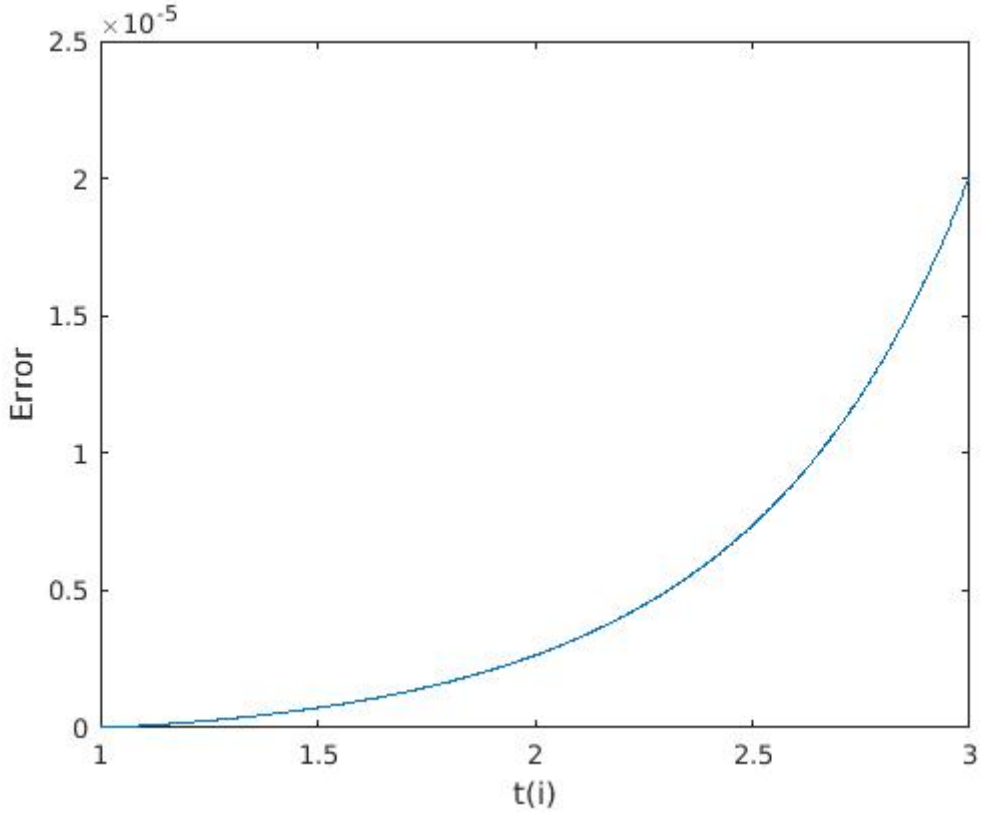


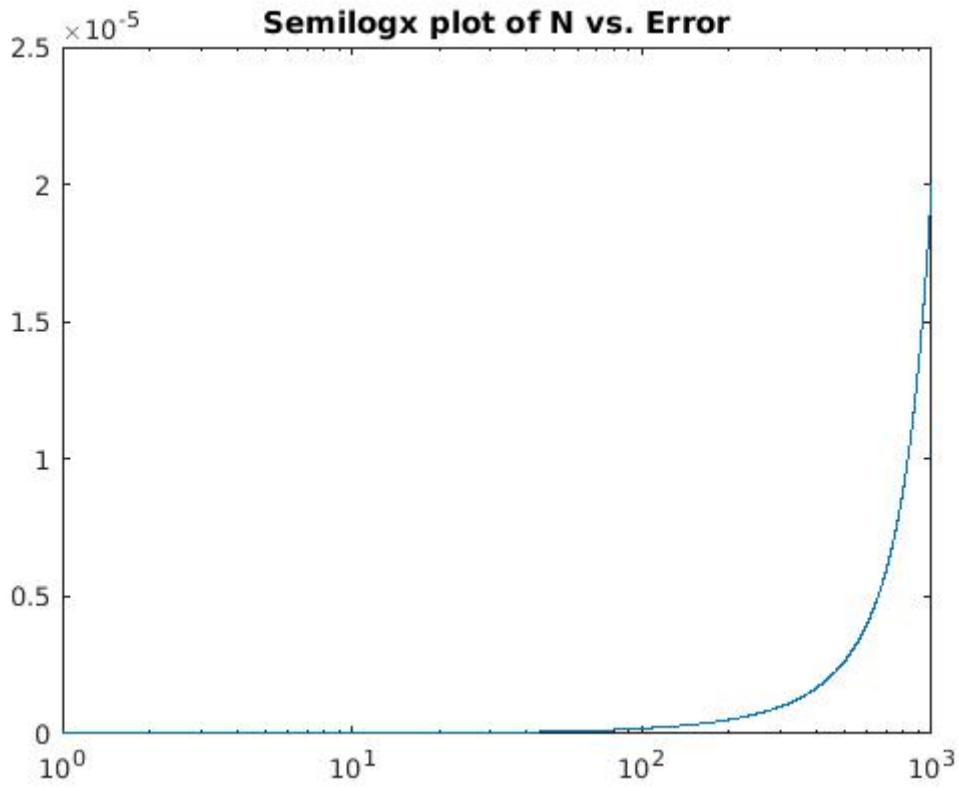

0.025
0.02
0.015
0.01
0.005
0
Error
1
1.5
2
2.5
3
t(i)


Semilogx plot of N vs. Error
0.025
0.02
0.015
0.01
0.005
0
10^0
10^1
10^2
10^3

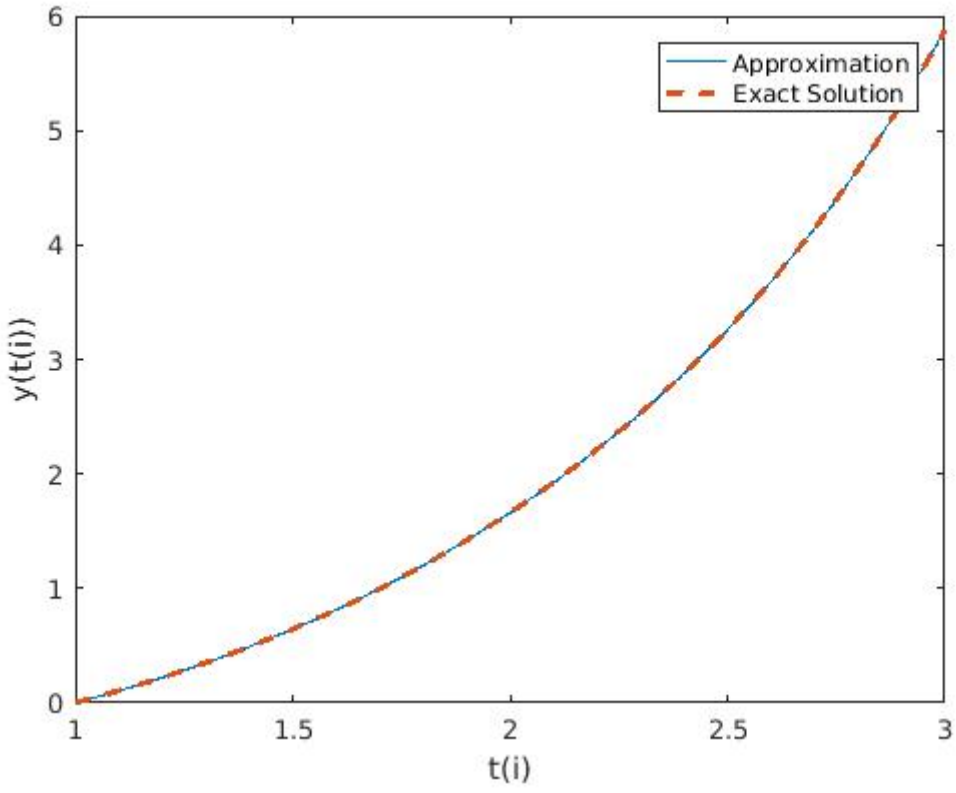
By modified Euler's method, the following plots are obtained:

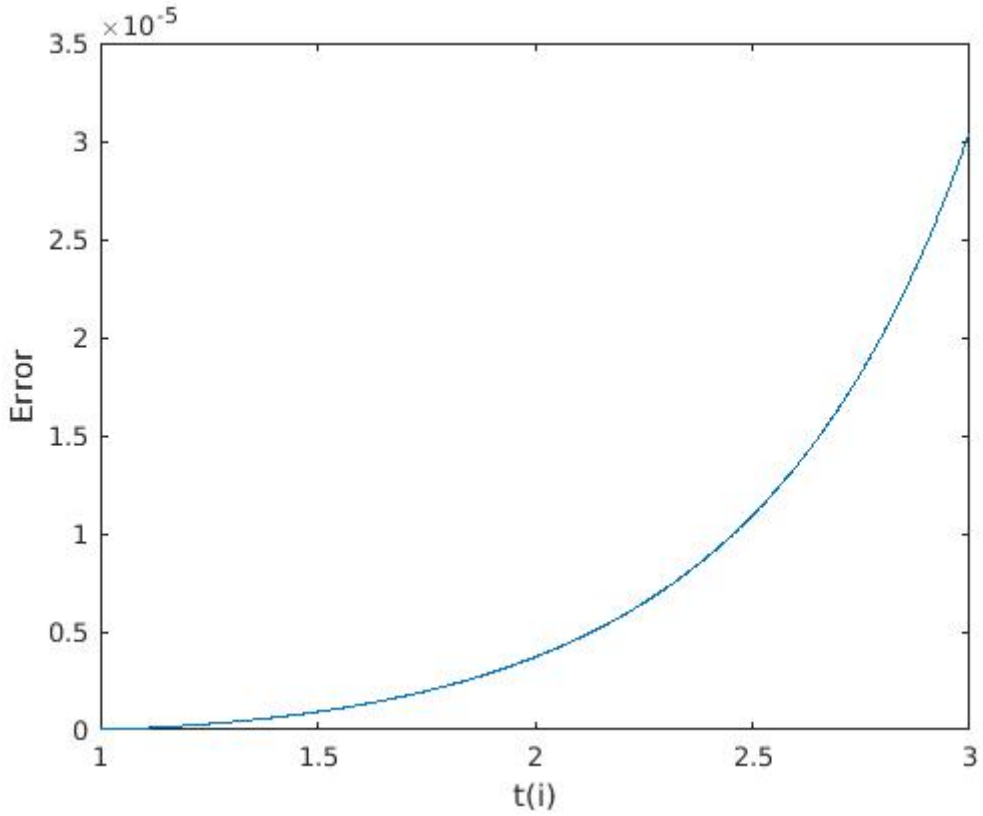


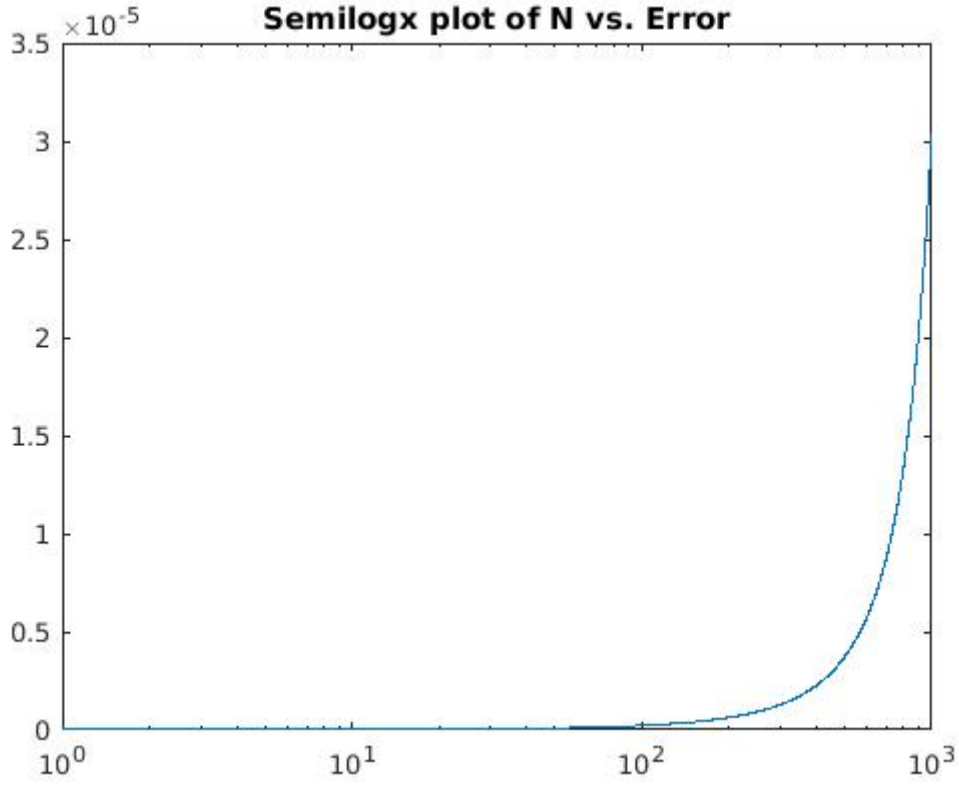




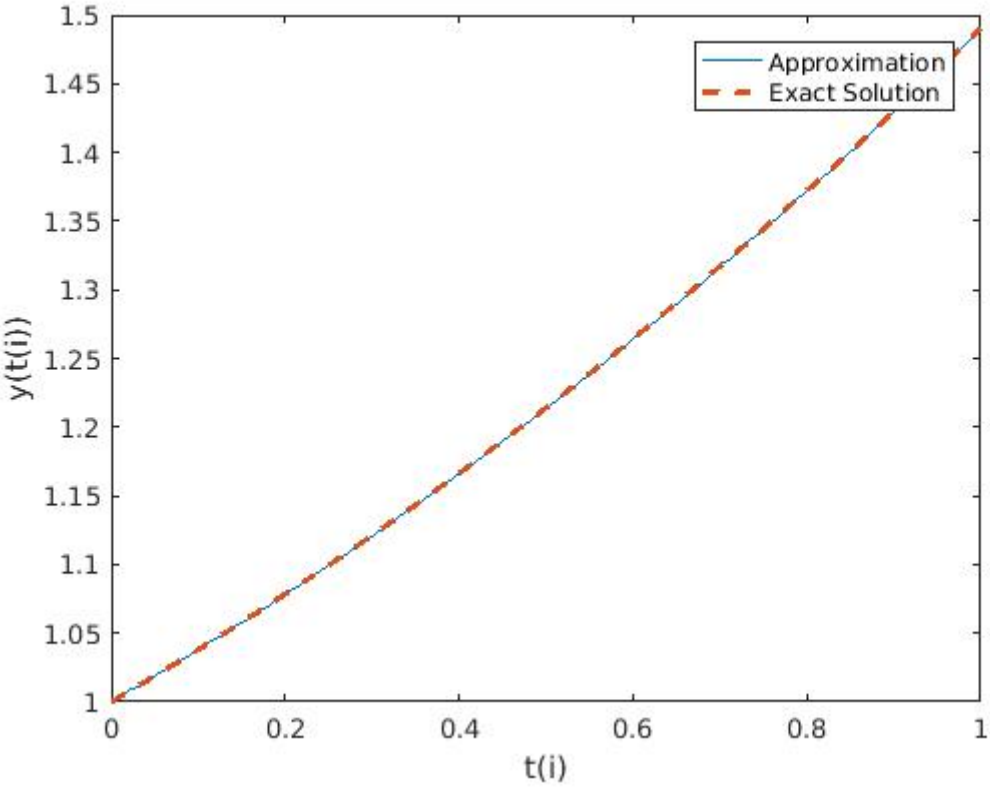
By Trapezoidal method, the plots obtained are as follows:

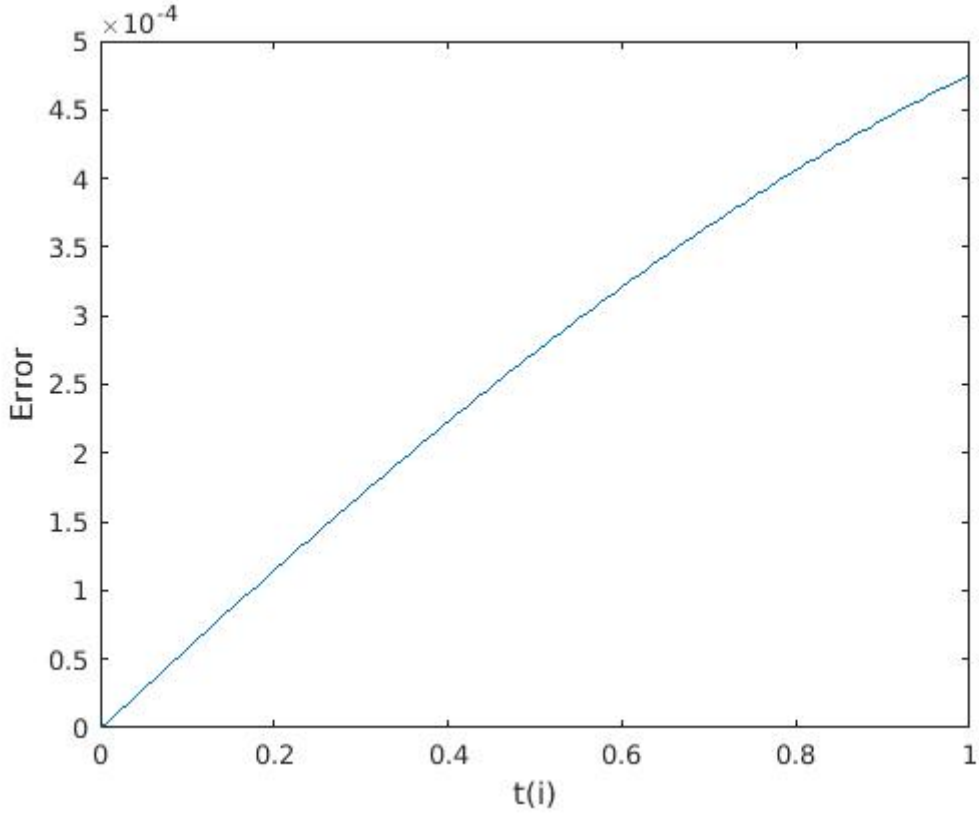


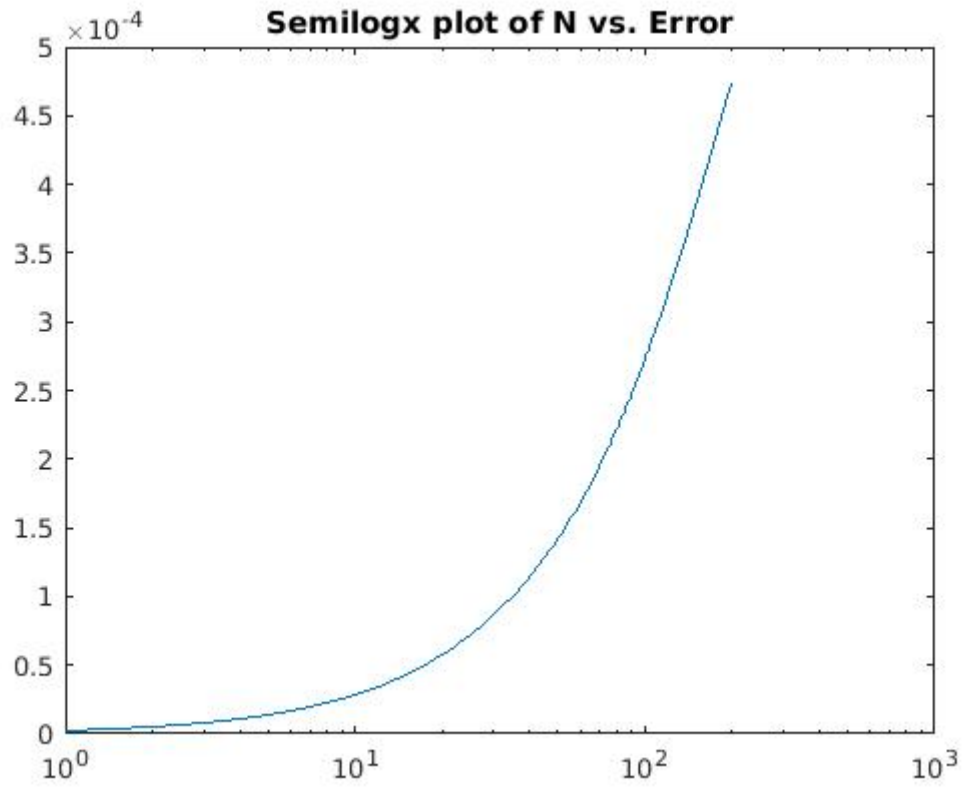

×10^-5
3.5
3
2.5
2
1.5
1
0.5
0
Error
1
1.5
2
2.5
3
t(i)


Semilogx plot of N vs. Error
×10^-5
3.5
3
2.5
2
1.5
1
0.5
0
10^0
10^1
10^2
10^3

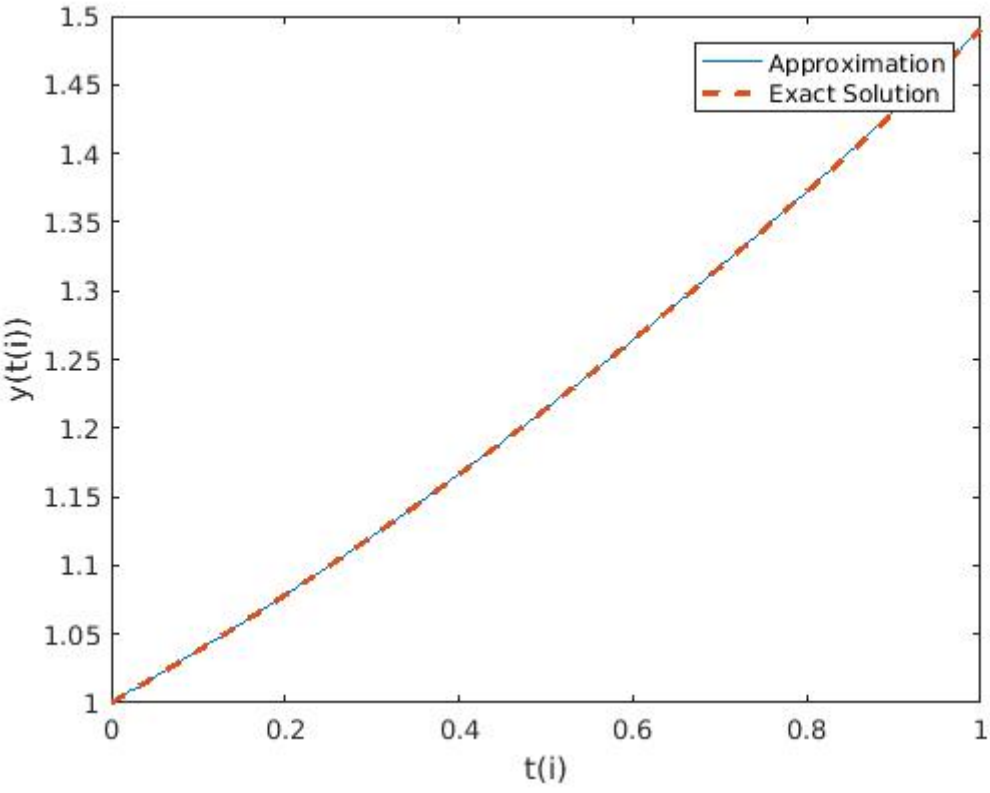
Part d) By explicit Euler's method, the following plots are obtained:

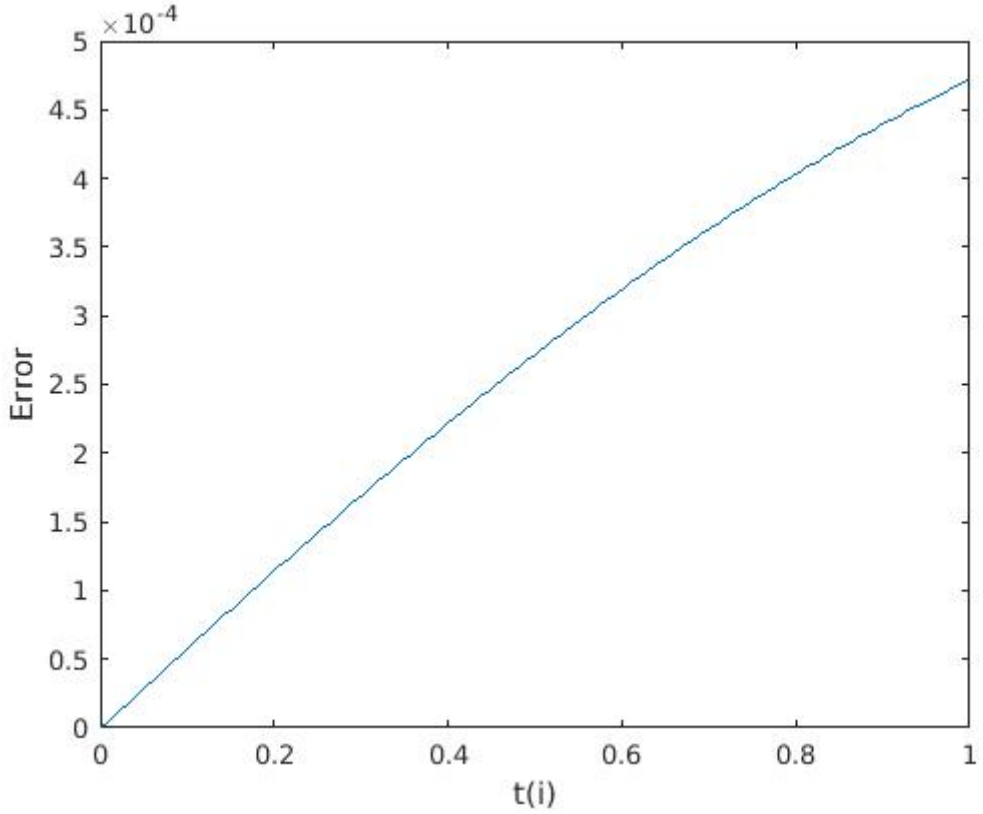


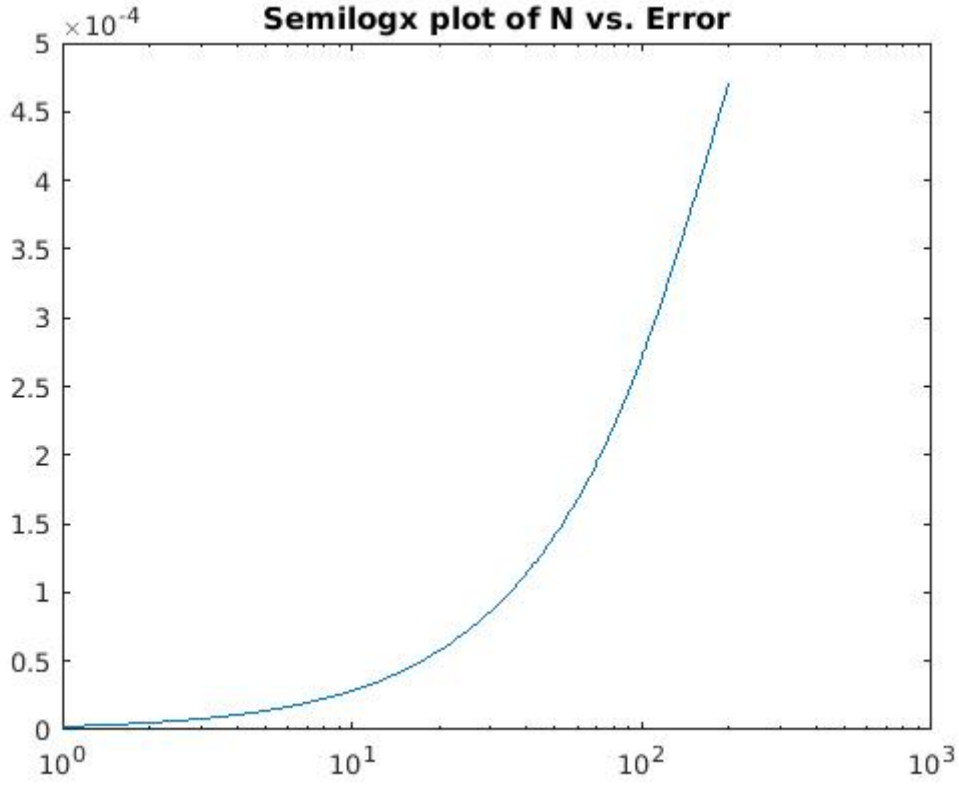




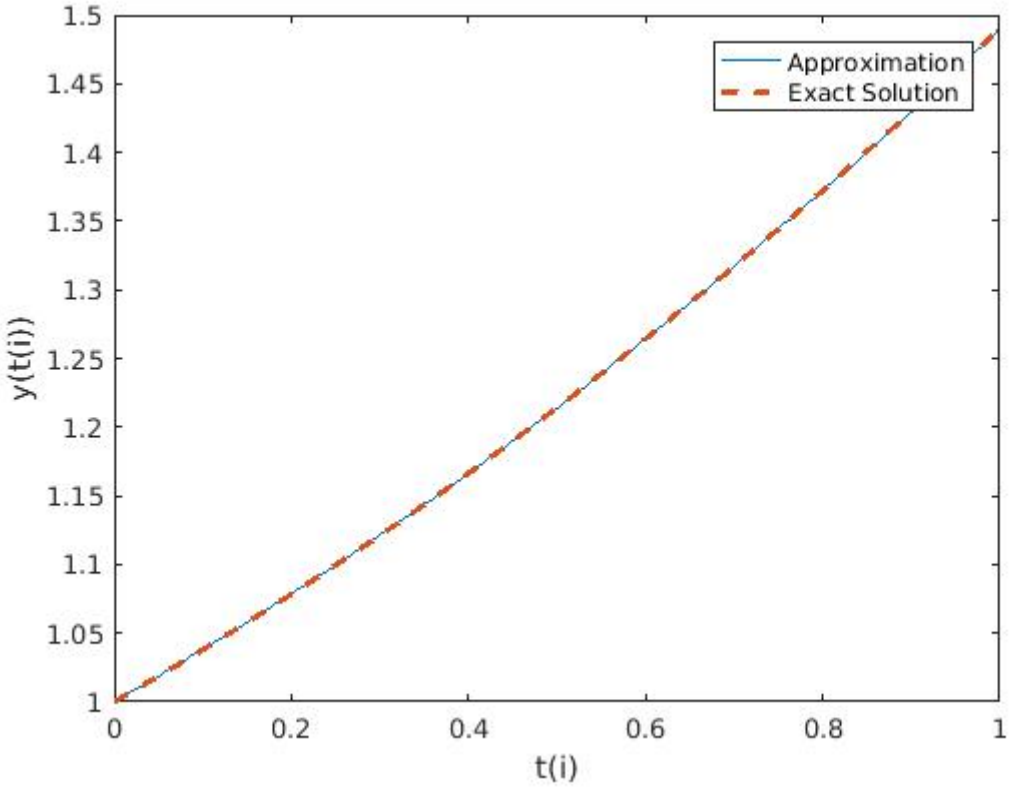
By implicit Euler's method, the plots obtained are as follows:

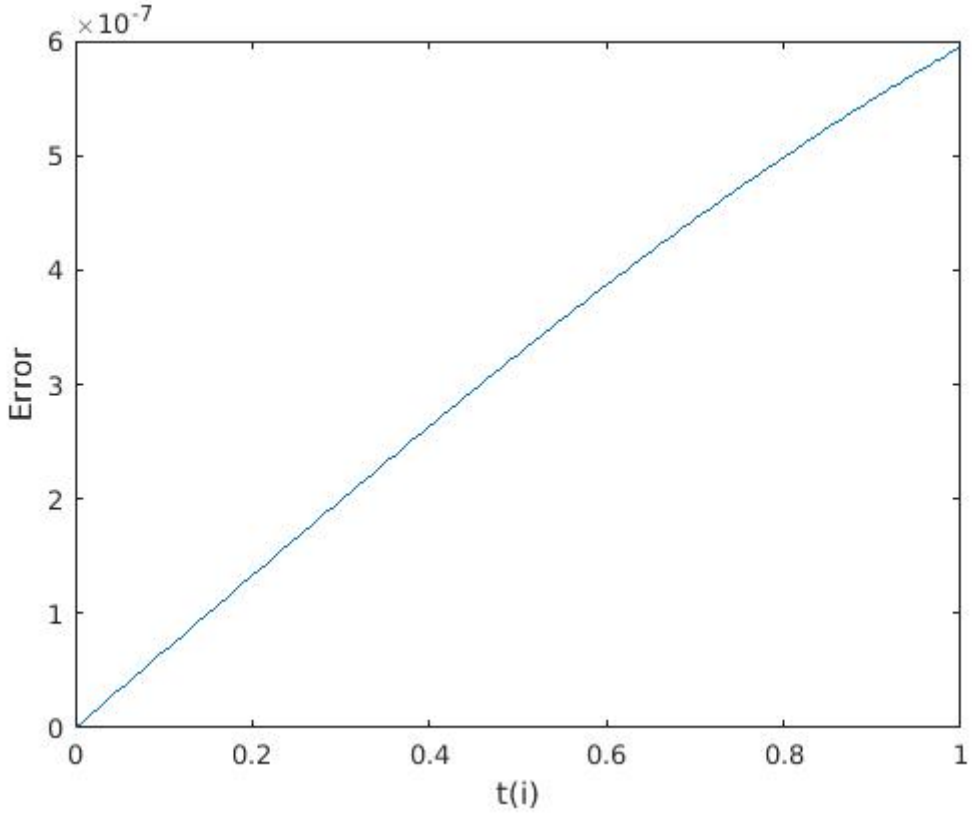


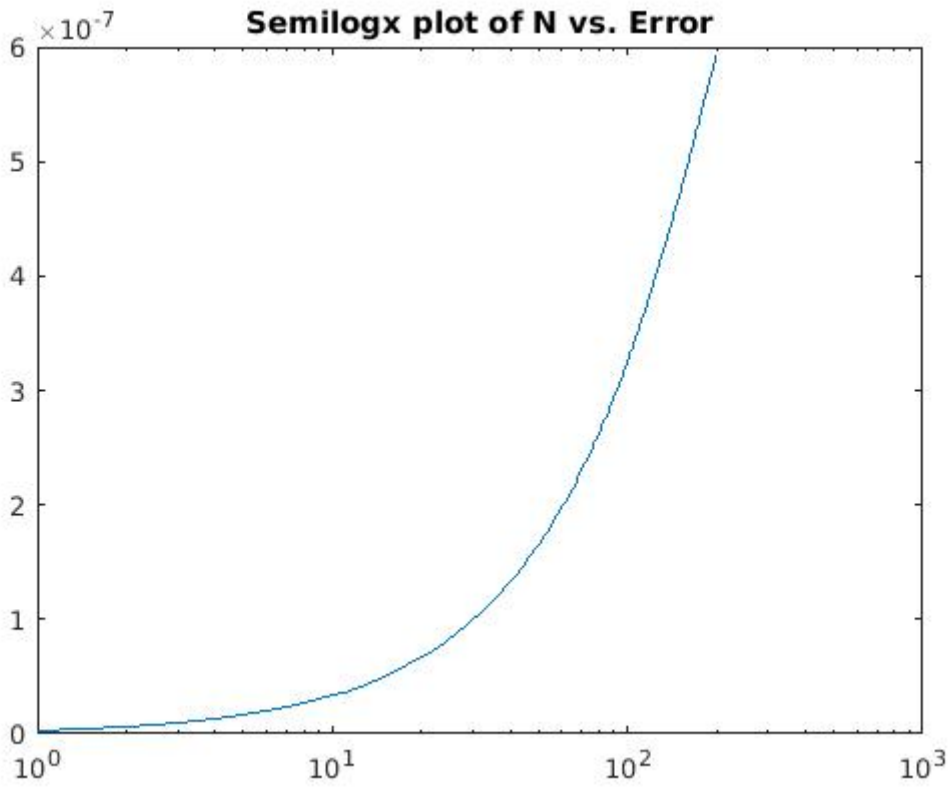

×10^-4
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Error
0
0.2
0.4
0.6
0.8
1
t(i)


Semilogx plot of N vs. Error
×10^-4
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
10^0
10^1
10^2
10^3

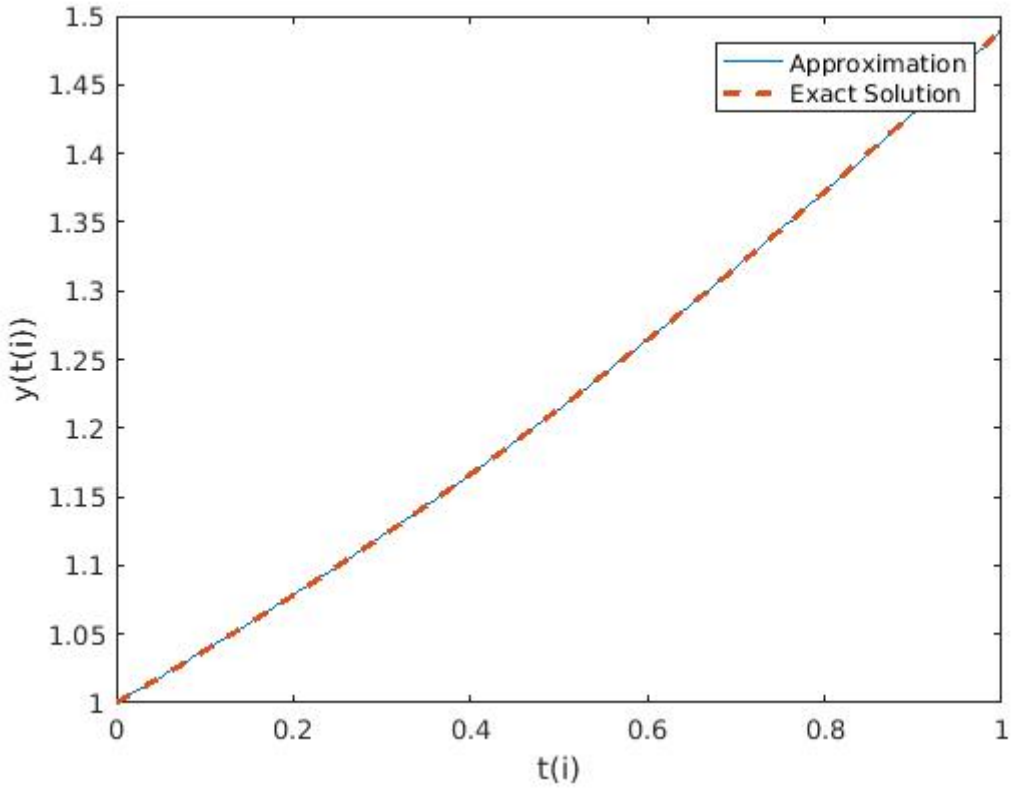
By modified Euler’s method, the plots obtained are:

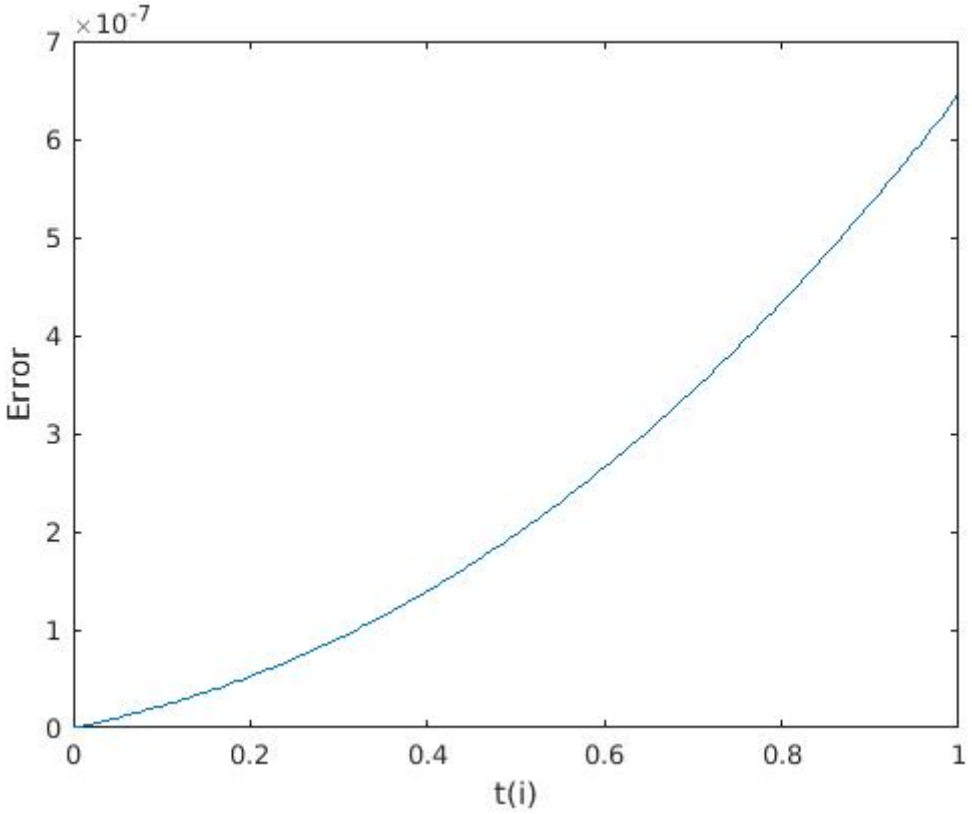


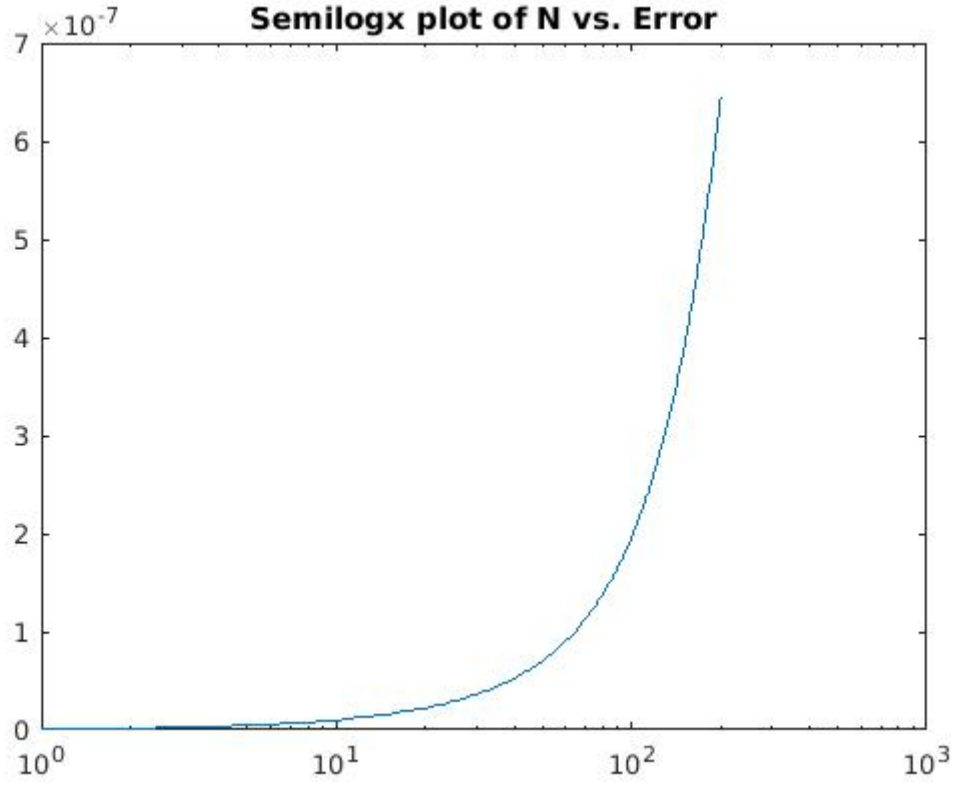




By Trapezoidal method, the plots obtained are:




×10^-7
7
6
5
4
3
2
1
0
Error
0
0.2
0.4
0.6
0.8
1
t(i)


Semilogx plot of N vs. Error
×10^-7
7
6
5
4
3
2
1
0
10^0
10^1
10^2
10^3

Problem 5:

a)Approximation with Gaussian quadrature using n=2 is 0.19226871.
Approximation with Gaussian quadrature using n=3 is 0.19225938.
Approximation with Gaussian quadrature using n=4 is 0.19225936.
Approximation with Gaussian quadrature using n=5 is 0.19225936.

b)Approximation with Gaussian quadrature using n=2 is 2.59132472.
Approximation with Gaussian quadrature using n=3 is 2.58925800.
Approximation with Gaussian quadrature using n=4 is 2.58863275.
Approximation with Gaussian quadrature using n=5 is 2.58862862.

c)Approximation with Gaussian quadrature using n=2 is -0.17681899.
Approximation with Gaussian quadrature using n=3 is -0.17682002.
Approximation with Gaussian quadrature using n=4 is -0.17682002.
Approximation with Gaussian quadrature using n=5 is -0.17682002.

d)Approximation with Gaussian quadrature using n=2 is -0.73072304.
Approximation with Gaussian quadrature using n=3 is -0.73379902.
Approximation with Gaussian quadrature using n=4 is -0.73396039.
Approximation with Gaussian quadrature using n=5 is -0.73396872.