DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 322: Scientific Computing Lab – X

1. For various $\triangle t$ and $\triangle x$, solve the following parabolic initial–boundary–value problem, numerically, by

   i. forward–time and central space (FTCS) discretization scheme,

   ii. backward–time and central space (BTCS) discretization scheme,

   iii. Crank–Nicolson scheme.

   (a)
$$\begin{cases} \dfrac{\partial u}{\partial t} - \left(\dfrac{4}{\pi^2}\right)\dfrac{\partial^2 u}{\partial x^2} = 0, \quad (x,t) \in (0,4)\times(0,1), \\ u(x,0) = \sin\left(\dfrac{\pi}{4}x\right)\left(1 + 2\cos\left(\dfrac{\pi}{4}x\right)\right), \quad x \in (0,4), \\ u(0,t) = u(4,t) = 0, \quad t \in (0,1]. \end{cases}$$

   The exact solution is given by
$$u(x,t) = e^{-t}\sin\left(\frac{\pi}{2}x\right) + e^{(-t/4)}\sin\left(\frac{\pi}{4}x\right).$$

   (b)
$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0, \quad (x,t) \in (0,1)\times(0,1), \\ u(x,0) = \sin(\pi x), \quad x \in (0,1), \\ u(0,t) = u(1,t) = 0, \quad t \in (0,1]. \end{cases}$$

   The exact solution is given by
$$u(x,t) = e^{-\pi^2 t}\sin(\pi x).$$

   (c)
$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0, \quad (x,t) \in (0,1)\times(0,1), \\ u(x,0) = \sin\left(\dfrac{\pi x}{2}\right) + \dfrac{1}{2}\sin(2\pi x), \quad x \in (0,1), \\ u(0,t) = 0, \quad u(1,t) = e^{-\pi^2 t/4}, \quad t \in (0,1]. \end{cases}$$

   The exact solution is given by
$$u(x,t) = e^{-\pi^2 t/4}\sin\left(\frac{\pi x}{2}\right) + \frac{1}{2}e^{-4\pi^2 t}\sin(2\pi x).$$

Provide the following:

1. Plot the exact and numerical solutions at the final time level in different colors with some symbols.
2. Draw the surface plot of the exact and numerical solutions.
3. Plot N versus $Max.\,Error$ in loglog scale.