DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 322: Scientific Computing Lab - VI

1. Approximate the following integrals using the Rectangle rule:

   a. $\int_{0.5}^{1} x^4 dx$ b. $\int_{0}^{0.5} \frac{2}{x-4} dx$ c. $\int_{1}^{1.6} \frac{2x}{x^2-4} dx$

   d. $\int_{0}^{\pi/4} e^{3x} \sin 2x dx$ e. $\int_{0.75}^{1.3} \big((\sin x)^2 - 2x \sin x + 1\big)\, dx$ f. $\int_{0}^{\pi/4} x \sin x dx$

   g. $\int_{-1}^{2} x e^{-x} dx$ h. $\int_{e}^{e+1} \frac{1}{x \ln x} dx.$

2. Find a bound for the error in Exercise 1 using the error formula, and compare this to the actual error.

3. Use the Midpoint rule, Trapezoid rule, Simpson's rule and Corrected Trapezoidal rule to approximate the integrals given in Exercise 1 and repeat Exercise 2.

4. Compute π from an integral of the form $\int_0^1 \frac{4}{1+x^2} dx$ by using Rectangle, Trapezoidal, Corrected Trapezoidal, Simpson's one-third and three-eighth rules. Compare and explain these numerical results to the true solution. Simpson's three-eighth rule is given by

$$\int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right] - \frac{3h^5}{80} f^4(\xi), \quad \text{where } x_0 < \xi < x_3.$$

5. Use the Composite Trapezoidal rule with the indicated values of n to approximate the following integrals.

   a. $\int_0^2 e^{2x} \sin 3x \, dx, \quad n = 8$ b. $\int_1^3 \frac{x}{x^2+4} \, dx, \quad n = 8$

   c. $\int_3^5 \frac{1}{\sqrt{x^2-4}} \, dx, \quad n = 8$ d. $\int_0^2 x^2 e^{-x^2} \, dx, \quad n = 8$

   e. $\int_{-0.5}^{0.5} x \ln(x+1) \, dx, \quad n = 6$ f. $\int_{0.75}^{1.75} \big((\sin x)^2 - 2x \sin x + 1\big) \, dx, \quad n = 8$

   g. $\int_e^{e+2} \frac{1}{x \ln x} \, dx, \quad n = 8$ h. $\int_0^2 x^2 \ln(x^2+1) \, dx, \quad n = 8$

6. Use Composite Midpoint and Simpson's rule to approximate the integrals in Exercise 5.

7. Find the approximate values of the two integrals

$$\int_0^1 \frac{4}{1+x^2}dx \quad \text{and} \quad \int_0^{1/\sqrt{2}} \left(\sqrt{1-x^2}-x\right) dx$$

by Simpson's one-third rule in such a way that the error ϵ is less than $\frac{1}{2}10^{-5}$. Your programme should be such that it starts with the smallest number of sub-intervals and then goes on increasing the number of sub-intervals till the desired accuracy is reached. Then provide a sub-intervals verses error plot.

8. We want to approximate $\int_1^2 f(x)dx$ given the table of the values

x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
$f(x)$	10	8	7	6	5

Compute an estimate by the composite trapezoid rule.

9. Determine the value of n and h required to approximate

$$\int_1^2 x \ln x \, dx$$

to within 10^{-5} and compute the approximation. Use (a) Composite Trapezoidal rule (b) Composite Simpson's rule (c) Composite Midpoint rule.

10. Determine to within 10^{-6} the length of the graph of the ellipse with equation $4x^2+9y^2 = 36$.

11. A car laps a race track in 84 seconds. The speed of the car at each 6-second interval is determined by using a radar gun and is given from the beginning of the lap, in feet/second, by the entries in the following table.

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

How long is the track?

12. Approximate the following integrals using Gaussian quadrature with $n = 2, 3, 4, 5$, uniformly spaced data points of the respective intervals:

$$a. \int_1^{1.5} x^2 \ln x dx \qquad b. \int_0^{\pi/4} e^{3x} \sin 2x dx$$

$$c. \int_0^{0.35} \frac{2}{x^2-4}dx \qquad d. \int_1^{1.6} \frac{2x}{x^2-4}dx$$