MTP 290-Problem Set 6

Interpolation

1. Given the data set $(x_j,\ y_j), j = 0, 1, ...n$, write a Matlab function called Lagrange.m that computes the Lagrange basis polynomials of degree $n(\geq 1)$.

2. Calculate the Lagrange interpolating polynomial $p_2(x)$ for the following given values of the function $y = \sin(x)$: $\sin(0) = 0,\ \sin(\pi/4) = 0.70711$ and $\sin(\pi/2) = 1.0$. From $p_2(x)$, find the approximate value of $\sin(\pi/6)$.

3. Find the unique interpolating polynomial $p_3(x)$ of degree less or equal to 3 of a function $f(x)$, that agrees with the following data: $f(0) = 0,\ f(1) = 1,\ f(2) = 8,\ f(4) = 64$.
   Then use $p_3(x)$ to find the approximate value of $f(3)$.

4. Determine the Lagrange interpolation polynomial, which fits to the following data:
   $(a)(-3, -31), (-2, -8), (1, 1), (2, 22)$,
   $(b)(-3, -209), (-2, -43), (-1, -1), (1, -1), (2, -19)$.

5. Construct the interpolating polynomial in Lagrange form, of degrees $n = 5, ..., 10$ of the Runge function

$$f(x) = \frac{1}{1 + 25x^2}, \quad x \in [-1, 1],$$

   on equispaced points. Make the plots of the function and its interpolating polynomials.

Numerical Integration

1. Evaluate the integral $\int_0^4 (x^2 + cosx)\ dx$ by using midpoint formula and composite midpoint rule with n=5.

2. Use Trapezoidal rule with $n = 8$ to estimate

$$\int_1^5 \sqrt{1 + x^2}\ dx.$$

3. The following points were found empirically.

x	2.1	2.4	2.7	3.0	3.3	3.6
y	3.2	2.7	2.9	3.5	4.1	5.2

   Use composite Trapezoidal rule to evaluate $\int_{2.1}^{3.6} y\ dx$.

4. Approximate the integral of f(x)=x^3 on the interval [1,2] by using composite trapezoidal method
   (a) with four sub intervals,
   (b) with eight sub intervals,
   (Which approximation is much closer to the correct answer)
   (c) Compute the true error in both the cases.