MTP 290 PROBLEM SET 3

(1) Newton-Raphson method is used to solve the equation $x^2 = 0$, using the initial estimate $x_0 = 0.1$. What is the observed order of convergence?

(2) Redo problem 1 using the modified Newton's method.

(3) Write down the MATLAB script for computing a root of a given function $f(x) = 0$ using secant method.

(4) Use secant method for finding the approximations of the two zeros, one in $[-1, 0]$ and other in $[0, 1]$ to within $1e - 3$ accuracy of $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$. Use the end points of the interval as initial guesses. Compare the order of convergence and the number of iteration required with that of Newton's method (taking midpoint of interval as initial guess).

(5) Find the square root of 2, correct upto five decimal places using Newton's method as well as secant method. Compute the order of convergence for both the methods.

(6) Consider the function

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{otherwise} \end{cases}.$$

Note that 0 is the only solution of $f(x) = 0$. Show that if $x_0 = 0.0001$, it takes more than one hundred million iterations of the Newton Method to get below 0.00005.

(7) Consider the circle of radius 1 and let A, B be any two points on it. The longer circular arc joining A and B is twice as long as the chord AB. Find the length of the chord AB, correct to 18 decimal places (using Newton's method).

(8) Find, correct to 5 decimal places, the x-coordinate of the point on the curve $y = \log_e(x)$ which is closest to the origin. Use the Newton's Method and Secant Method.

(9) Use Newton's method to approximate, to within 10^{-3}, the value of x that produces the point on the graph of $y = x^2$ that is closest to $(1, 0)$.

(10) Let $f(x) = 27x^4 + 162x^3 - 180x^2 + 62x - 7$.

(a) Show that $f(x)$ has a zero of multiplicity 3 at $x = \frac{1}{3}$.

(b) Use Newton's Method to solve this equation with $x_0 = 0$ within 10^{-8}.

(c) Choose modified Newton's Method to solve this equation with $x_0 = 0$ within 10^{-8}.

(11) The sum of two real numbers is 20. If each number is added to its square root, the product of the two sums is 155.55. Determine the two numbers to within 10^{-8} by the Newton's Method.

(12) Use Newton's method with $x_0 = 1$ to find the approximation to the root of $f(x) = x^{1/3}$. Observe the behavior of iterates.

(13) Let $f(x) = x^3 - 2x + 2$ and consider the initial approximation as $x_0 = 0$. Find the root of $f(x)$ using Newton's method.(Check whether this method converges or not?)