MTP 290 PROBLEM SET 4

Solving nonlinear equations

(1) The following four methods are proposed to compute $21^{1/3}$. Rank them in order, based on their apparent speed of convergence, assuming $x_0 = 1$.

$$(a)\ x_n = \frac{20x_{n-1} + 21/x_{n-1}^2}{21} \qquad (b)\ x_n = x_{n-1} - \frac{x_{n-1}^3 - 21}{3x_{n-1}^2}$$

$$(c)\ x_n = x_{n-1} - \frac{x_{n-1}^4 - 21x_{n-1}}{x_{n-1}^2 - 21} \qquad (d)\ x_n = \left(\frac{21}{x_{n-1}}\right)^{1/2}.$$

(2) Use a fixed-point iteration method to find an approximation to $\sqrt{3}$ that is accurate to within 10^{-2}. Estimate the number of iterations required to achieve this accuracy and compare it with the theoretical estimate to the number actually needed.

(3) The iteration $x_{n+1} = 2 - (1+c)x_n + cx_n^3$ will converge to $x_* = 1$ for some values of c (provided x_0 is chosen sufficiently close to x_*). For what values of c will the convergence be quadratic?

(4) Which of the following iterations will converge to indicated fixed point x_* (provided x_0 is sufficiently close to x_*). If it converge, give the order of convergence; for linear convergence, give the rate of linear convergence.

$$(a)\ x_{n+1} = -16 + 6x_n + \frac{12}{x_n}, \quad x_* = 2,$$
$$(b)\ x_{n+1} = \frac{2}{3}x_n + \frac{1}{x_n}, \quad x_* = 3^{1/3},$$
$$(c)\ x_{n+1} = \frac{12}{1+x_n}, \quad x_* = 3.$$

Solving linear system of equations

(1) Write a MATLAB script to solve the linear system $Ax = b$, where A is an invertible diagonal matrix. Taking $A = \text{diag}(1, 2, 3)$ and $b = [1, 1, 1]^T$, solve for x.

(2) Write MATLAB code to implement the forward substitution method to solve the linear system $Ax = b$, where A is a non-singular lower triangular matrix. Use it to solve for x if A and b are given as follows:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0.5 & 1 \end{pmatrix}$$

and $b = [1, 2, 1]^T$.

(3) Write a MATLAB script for implementing the backward substitution method to solve the system $Ax = b$, where A is a non-singular upper triangular matrix. Use this code to solve for x if A and b are as follows:

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -3 \\ 0 & 0 & -6.5 \end{pmatrix}$$

and $b = [1, 7, 6.5]^T$.

(4) Implement Gauss elimination method for solving a system of linear equations $Ax = b$, where A is a non-singular matrix.

(5) Use Gauss elimination method to solve

$$\begin{aligned} 4x_1 + x_2 - x_3 &= -2 \\ 5x_1 + x_2 + 2x_3 &= 4 \\ 6x_1 + x_2 + x_3 &= 6. \end{aligned}$$

(6) Consider

$$A = \begin{pmatrix} 10 & -7 & 0 \\ -3 & 2.099 & 6 \\ 5 & -1 & 5 \end{pmatrix}.$$

Find the determinant of A using Gauss elimination method.

(7) Use of Gauss elimination method on the coefficient matrix

$$A = \begin{pmatrix} 25 & c & 1 \\ 64 & a & 1 \\ 144 & b & 1 \end{pmatrix}.$$

reduces it to

$$B = \begin{pmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{pmatrix}.$$

What is the determinant of A?