MTP 290-Problem Set 5

1. The following data is given for the velocity of the rocket as a function of time. To find the velocity at $t = 21$s, you are asked to use a quadratic polynomial, $v(t) = at^2 + bt + c$ to approximate the velocity profile.

t(s)	0	14	15	20	30	35
$v(t)$m/s	0	227.04	362.78	517.35	602.97	901.67

The correct set of equations that will find a, b and c are

$$\begin{aligned} 225a + 15b + c &= 362.78 \\ 400a + 20b + c &= 517.35 \\ 900a + 30b + c &= 602.97. \end{aligned}$$

Find the velocity at $t = 21$s.

2. Implement the Gauss elimination method with partial pivoting to solve a system of linear equations $Ax = b$, where A is a non-singular matrix.

3. Solve the following linear system by Gauss elimination method and Gauss elimination method with partial pivoting.

$$\begin{aligned} x + y + z &= 3 \\ x + 2y + 2z &= 5 \\ 3x + 4y + 4z &= 12. \end{aligned}$$

4. Apply the modified solver implemented in Problem 2 to solve the following system. Further check the difference between the computed solution x and the result of MATLAB built in solver $A\backslash b$.

$$\begin{aligned} x_1 + x_2 + x_4 &= 2 \\ 2x_1 + x_2 - x_3 + x_4 &= 1 \\ 4x_1 - x_2 - 2x_3 + 2x_4 &= 0 \\ 3x_1 - x_2 - x_3 + x_4 &= -3. \end{aligned}$$

5. Implement Gauss Jordan method to solve a system of linear equations $Ax = b$, where A is a non-singular matrix.

6. Redo the problem 3 using Gauss-Jordan method.

7. Write a MATLAB script for implementing the LU decomposition (Doolittle's factorization) for a 3×3 matrix.

8. Let

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & -2 \\ -2 & 1 & 1 \end{pmatrix}.$$

Find Doolittle's factorization of the above matrix. Further, for $b = [1, 1, 1]^T$, solve the system $Ax = b$.

9. Solve the following linear system by LU decomposition (Cholesky) Method

$$\begin{aligned} 16x_1 + 4x_2 + 4x_3 - 4x_4 &= 32 \\ 4x_1 + 10x_2 + 4x_3 + 2x_4 &= 26 \\ 4x_1 + 4x_2 + 6x_3 - 2x_4 &= 20 \\ -4x_1 + 2x_2 - 2x_3 + 4x_4 &= -6. \end{aligned}$$