

Islington College



MA4001NI

Logic and Problem Solving Group Coursework (50% Weighted)

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Acknowledgement

We are grateful to everyone who contributed to our successful completion of this coursework on Logic and Problem Solving.

Firstly, we would like to thank our lecturer Mr. Ashok Dhungana for his massive guidance and continuous support throughout this coursework. His clear explanations and motivation helped us understand concepts that we have applied in the coursework which includes Linear Programming Problems and tax calculations.

We would also thank our tutor Mr. Ichchha Babu Bhattarai for his continuous help regarding our coursework. His constant concern with our coursework helped us finish our coursework smoothly and his explanations for all our issues with questions were incredibly helpful. This coursework would not have been possible to finish without errors without his help.

We are also grateful to our group members for their cooperation and teamwork. Taking care of each problem was very helpful for all of us. We would especially like to acknowledge the team's synergy when discussing, analyzing and solving the questions in our meeting sessions. Each member of the group contributed significantly to the discussions and solving stages.

Finally, we appreciate everyone who indirectly supported us through resources, feedback, and motivation throughout this coursework. The small words of motivation from friends and families were truly helpful during challenging moments.

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Problem 1

Write a procedure, tax, to calculate (in Nepalese Rupees) the tax a person owes, depending on his/her income. Calculate the tax using this table:

Taxable income (Rs)	Income tax rates (Percent)
0 to 50,000	1%
50,000 to 2,00,000	10%
2,00,000 to 3,00,000	20%
3,00,000 to 10,00,000	30%
10,00,000 to 30,00,000	36%
30,00,000 and above	39%

Solution:

There are various basic terms used we have to understand before we calculate the tax a person owes, depending on his/her income. They are explained below:

a. Yearly Income

⇒ Yearly income is the total amount of money a person earns throughout an entire year. This income is the person's grand total income in a year. We use this yearly income to then calculate yearly taxable income.

b. Yearly Taxable Income

⇒ Yearly taxable income is the income of a person on which tax is applied. Yearly taxable income is calculated by deducting the non-taxable income like provident funds, insurance or Citizen Investment Trust from yearly income.

c. Deductions

⇒ Deductions are the part of income that is separated as non-taxable income before calculating yearly taxable income. Deductions include provident funds, Citizen Investment Trust or insurance. Total deductions are deducted from yearly income to calculate yearly taxable income.

d. Income Tax Bracket

⇒ Income tax bracket is basically the range of income where specific tax rate is applied. The question displays a table showing what rate of tax applies in what range of income. That range is known as income tax bracket.

e. Tax Slab

⇒ Tax slab is the amount left in the tax bracket on which tax is applied. For example, for range 0 to 50,000 we have the amount 50,000 where tax of 1% is applied. Then for range 50,000 to 200,000 we have amount 150,000 where 10% tax is applied in this amount. It is calculated by subtracting lower end of tax bracket to higher end of the same tax bracket.

G	H	I
Income Tax Bracket		
0	50000	=H3-G3
50000	200000	150000
200000	300000	100000
300000	1000000	700000
1000000	3000000	2000000
OVER 3000000		#VALUE!

Figure 1: Tax Slab

f. Tax Rate

- ⇒ Tax rate is the percentage at which the amount calculated in tax slab is taxed. For example, if a person's income is 100,000 and we have to calculate tax, for the first 50,000 of his income, he is taxed for 1% tax rate. Then, for remaining 50,000 we move on to second bracket and that amount is taxed for 10% tax rate.

g. Tax Amount

- ⇒ Tax amount is the amount that is taken away from yearly income after the rates are applied according to the tax bracket. Suppose if a person earns Rs. 100,000, then for the first bracket, the tax slab is 50,000 which means 1% is taxed from that amount. This is calculated to be Rs. 500. Then for the remaining 50,000, 10% is taxed from the amount since we have moved on to the second tax bracket. This is calculated to be Rs.5,000. This amount that is calculated to be Rs.500 and Rs.5,000 is our tax amount.

h. Formulas

- ⇒ In this question, we have done our solutions mainly in excel. To calculate values in certain cells, we have used various formulas. These formulas are stated below:

For finding total deductions, we have added all deductions provided as input.

<div> ✖ ✓ <i>f_x</i> =SUM(C5:C7) </div>		
B	C	D
Yearly Income	100000	
Deductions		
PF	0	
CIT	0	
Insurance	0	
Total Deductions	=SUM(C5:C7)	
Yearly Taxable Income	100000	

Figure 2: Total Deduction

For finding the amount in tax slab, we have subtracted lower end of tax bracket from higher end of tax bracket.

Income Tax Bracket		
0	50000	=H3-G3
50000	200000	150000
200000	300000	100000
300000	1000000	700000
1000000	3000000	2000000
OVER 3000000		#VALUE!

Figure 3: Tax Slab

=IF(C11<=I3,I3*I3,I3*I3)										
B	C	D	E	F	G	H	I	J	K	L
Yearly Income	100000				Income Tax Bracket	Tax Slab	Tax Rate			
					0	50000	50000	1%	=IF(C11<=I3,I3*I3,I3*I3)	
Deductions					50000	200000	150000	10%	IF(logical_test, [value_if_true], [value_if_false])	0
PF	0				200000	300000	100000	20%		0
CIT	0				300000	1000000	700000	30%		0
Insurance	0				1000000	3000000	2000000	36%		0
					OVER 3000000		#VALUE!	39%		0
Total Deductions	0									
Yearly Taxable Income	100000							Total Yearly Tax	5500	
								Total Monthly Tax	458.333333	
								Income Left after Tax	94500	

To calculate the taxable income left in order to move on to the next bracket, we used IF statement to check if the taxable income is higher than the tax slab, and if it happens to be higher, then the tax slab is subtracted from taxable income and for the next bracket, tax slab is subtracted from taxable income left.

[illegible]

The procedure should show:

- 5

Your tests of procedure should include the following values, which should be included in your final presentation.

a) Tax (Rs. 4,05,000)

Deductions:

- Employees provident fund organization: (Rs. 10,000)
- Life Insurance premium: (Rs. 68,000)
- Citizen Investment Trust:(Rs. 0)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		Yearly Income	405000										
2							Income Tax Bracket	Tax Slab	Tax Rate	Tax Amount	Taxable Income Left		
3							0	50000	50000	1%	500	277000	
4							50000	200000	150000	10%	15000	127000	
5		Deductions					200000	300000	100000	20%	20000	27000	
6		Employees provident fund organization	10000				300000	1000000	700000	30%	8100	0	
7		Citizen Investment Trust	0				1000000	3000000	2000000	36%	0	0	
8		Life Insurance Premium	68000				OVER 3000000		#VALUE!	39%	0		
9		Total Deductions	78000										
10										Total Yearly Tax	43600		
11		Yearly Taxable Income	327000							Total Monthly Tax	3633.33333		
12										Income Left after Tax	361400		
13													

Figure 6: Solution a

Hence, for a yearly income of Rs. 4,05,000 and deductions that sum up to 78000, the total yearly tax was calculated to be Rs. 43,600. The total monthly tax was calculated to be Rs. 3,633.33. The total income that was left after applying the tax is Rs. 3,61,400.

b) Tax (Rs. 8,09,090)

Deductions:

- Employees provident fund organization: (Rs. 15,000)
- Life Insurance premium: (Rs. 20,000)
- Citizen Investment Trust:(Rs.5,000)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2		Yearly Income	809090										
3													
4		Deductions											
5		Employees provident fund organization	15000										
6		Citizen Investment Trust	5000										
7		Life Insurance Premium	20000										
8													
9		Total Deductions	40000										
10													
11		Yearly Taxable Income	769090										
12													
13													

Income Tax Bracket	Tax Slab	Tax Rate	Tax Amount	Taxable Income Left
0	50000	50000	1%	500
50000	200000	150000	10%	15000
200000	300000	100000	20%	20000
300000	1000000	700000	30%	140727
1000000	3000000	2000000	36%	0
OVER 3000000		#VALUE!	39%	0

Total Yearly Tax	176227
Total Monthly Tax	14685.5833
Income Left after Tax	632863

Figure 7: Solution b

Hence, for a yearly income of Rs. 8,09,090 and deductions that sum up to 40000, the total yearly tax was calculated to be Rs. 1,76,227. The total monthly tax was calculated to be Rs. 14,685.5833. The total income that was left after applying the tax is Rs. 6,32,863.

c) Tax (Rs. 19,10,000)

Deductions:

- Employees provident fund organization: (Rs. 0)
- Life Insurance premium: (Rs. 2,50,000)
- Citizen Investment Trust:(Rs.50,000)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2		Yearly Income	1910000										
3													
4		Deductions											
5		Employees provident fund organization	0										
6		Citizen Investment Trust	50000										
7		Life Insurance Premium	250000										
8													
9		Total Deductions	300000										
10													
11		Yearly Taxable Income	1610000										
12													
13													

Income Tax Bracket	Tax Slab	Tax Rate	Tax Amount	Taxable Income Left
0	50000	50000	1%	500
50000	200000	150000	10%	15000
200000	300000	100000	20%	20000
300000	1000000	700000	30%	210000
1000000	3000000	2000000	36%	219600
OVER 3000000		#VALUE!	39%	0

Total Yearly Tax	465100
Total Monthly Tax	38758.3333
Income Left after Tax	1444900

Figure 8: Solution c

Hence, for a yearly income of Rs. 19,10,000 and deductions that sum up to Rs. 3,00,000, the total yearly tax was calculated to be Rs. 4,65,100. The total monthly tax was calculated to be Rs. 38,758.33. The total income that was left after applying the tax is Rs. 14,44,900.

d) Tax (Rs. 2,108,790)

Deductions:

- Employees provident fund organization: (Rs. 1,50,000)
- Life Insurance premium: (Rs. 30,000)
- Citizen Investment Trust:(Rs.25,000)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2		Yearly Income	2108790										
3													
4		Deductions											
5		Employees provident fun organization	150000										
6		Citizen Investment Trust	25000										
7		Life Insurance Premium	30000										
8													
9		Total Deductions	205000										
10													
11		Yearly Taxable Income	1903790										
12													
13													

Income Tax Bracket	Tax Slab	Tax Rate	Tax Amount	Taxable Income Left
0	50000	50000	1%	500
50000	200000	150000	10%	15000
200000	300000	100000	20%	20000
300000	1000000	700000	30%	210000
1000000	3000000	2000000	36%	325364.4
OVER 3000000		#VALUE!	39%	0

Total Yearly Tax	570864.4
Total Monthly Tax	47572.0333
Income Left after Tax	1537925.6

Figure 9: Solution d

Hence, for a yearly income of Rs. 21,08,790 and deductions that sum up to Rs. 2,05,000, the total yearly tax was calculated to be Rs. 5,70,864.4. The total monthly tax was calculated to be Rs. 47,572.03. The total income that was left after applying the tax is Rs. 15,37,925.6.

e) Tax (Rs. 31,800)

Deductions:

- Employees provident fund organization: (Rs.0)
- Life Insurance premium: (Rs. 5,000)
- Citizen Investment Trust:(Rs. 0)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2		Yearly Income	31800										
3													
4		Deductions											
5		Employees provident fun organization	0										
6		Citizen Investment Trust	0										
7		Life Insurance Premium	5000										
8													
9		Total Deductions	5000										
10													
11		Yearly Taxable Income	26800										
12													
13													

Income Tax Bracket	Tax Slab	Tax Rate	Tax Amount	Taxable Income Left
0	50000	50000	1%	268
50000	200000	150000	10%	0
200000	300000	100000	20%	0
300000	1000000	700000	30%	0
1000000	3000000	2000000	36%	0
OVER 3000000		#VALUE!	39%	0

Total Yearly Tax	268
Total Monthly Tax	22.3333333
Income Left after Tax	31532

Figure 10: Solution e

Hence, for a yearly income of Rs. 31,800 and deductions that sum up to Rs. 5,000, the total yearly tax was calculated to be Rs. 268. The total monthly tax was calculated to be Rs. 22.33. The total income that was left after applying the tax is Rs. 31,532.

f) Tax (Rs. -50,000)

Deductions:

- Employees provident fund organization: (Rs.0)
- Life Insurance premium: (Rs. 0)
- Citizen Investment Trust:(Rs. 0)

Yearly Income	-50000
Deductions	
Employees provident fund organization	0
Citizen Investment Trust	0
Life Insurance Premium	5000
Total Deductions	5000
Yearly Taxable Income	-55000

Income Tax Bracket	Tax Slab	Tax Rate	Tax Amount	Taxable Income Left
0	50000	50000	1%	-550
50000	200000	150000	10%	0
200000	300000	100000	20%	0
300000	1000000	700000	30%	0
1000000	3000000	2000000	36%	0
OVER 3000000			39%	0

Total Yearly Tax	-550
Total Monthly Tax	-45.8333333
Income Left after Tax	-49450

Figure 11: Solution f

Hence, when we provide an invalid value like Rs. -50,000, It displays an error message letting the user know that the input value cannot be negative.

Problem 2 Part A

Berkshire Health Care patient diet is to contain at least 8 units of vitamins, 9 units of minerals, and 10 carbohydrates. Three foods, Food A, Food B, and Food C are to be purchased. Each unit of Food A provides 1 unit of vitamins, 1 unit of minerals, and 2 carbohydrates. Each unit of Food B provides 2 units of vitamins, 1 unit of minerals, and 1 carbohydrate. Each unit of Food C provides 2 units of vitamins, 1 unit of minerals, and 2 carbohydrates. If Food A costs \$3 per unit, Food B costs \$2 per unit and Food C costs \$3 per unit, how many units of each food should be purchased to keep costs at a minimum.

Questions:

- a) Formulate the problem as a linear programming model, clearly defining the variables, the objective function, and the constraints.
- b) Solve the problem using Simplex method.
- c) Solve the problem using the Excel Solver and interpret the results.
- d) For the final part of your report, in your capacity as an Adviser, you should present a memorandum to the Berkshire Health Care.

Solution:

Linear Programming is a mathematical technique used to find the optimal (maximum) or minimum solution for a linear function (Dantzig & Thapa, Linear Programming,1: Introduction, 1997). It minimizes the linear function to a finite number of linear relationships to find the optimal solution (Karloff, 2008). It is widely used in economics, logistics, engineering and operations research for optimal usage of resources. Simplex method, first developed by George B. Dantzig, is a very efficient model for solving more complex linear problems through pivoting the problems (Dantzig, 2002). Before solving the given Linear programming problem (LPP), there are many terminologies related to LPP and simplex method that should be clearly defined, those are:

1. Decision variables:

Decision variables are the physical quantities represented by mathematical symbols that can vary according to the nature of the problem (University of Kentucky, 2016).

2. Objective function:

Objective function is the function made up of the decision variables which clearly defines the criterion of the needed solution. It is the function that is operated upon for the solution.

3. Constraints:

Constraints are the functional equalities or inequalities that defines the restrictions for the value of decision variables. These constraints are formulated through the conditions of the real-life problem (University of Kentucky, 2016).

4. Basic feasible solution:

BFS is a solution that satisfies all the constraints and has the least number of non-zero variables (Laws, 2010).

5. Slack/ Surplus variables:

Slack and surplus variables are extra variables added to the inequality constraints convert them into equality. Slack variable is added to \leq constraints while surplus variables are added to \geq constraints to convert them to equality constraints.

6. Artificial variables:

Artificial variables are the variables introduced to constraints that may not produce BFS naturally (University of Moratuwa). Artificial variables are introduced in \geq constraints or equality constraints. They are added to the constraint in maximization problems and subtracted from the constraints in minimization problems.

7. Tableau:

It is the table used to perform the iterations for the simplex method

8. Pivot element:

Pivot element is the key element in the simplex table that is used as a reference point for calculation for that iteration.

10. Maximization/ Minimization:

An objective function in a LPP may need to be maximized or minimized. During Maximization, a basic feasible solution that yields the highest value for the objective function is calculated while during minimization, a basic feasible solution with the lowest value is calculated.

a)

Problem Analysis

According to the question,

The patient's diet must contain at least 8 units of vitamin, 9 units of minerals and 10 units of carbohydrates.

Food A costs \$3 to purchase. The composition of Food A:

Vitamin: 1 unit

Mineral: 1 unit

Carbohydrate: 2 unit

Food B costs \$2 to purchase. The composition of Food B:

Vitamin: 2 unit

Mineral: 1 unit

Carbohydrate: 1 unit

Food C costs \$3 to purchase. The composition of Food C:

Vitamin: 2 unit

Mineral: 1 unit

Carbohydrate: 2 unit

The following data for the food compositions, cost and patient's requirements are to be formulated as a linear programming model and solved for optimal solution.

Mathematical Formulation

For Decision Variables:

Let x , y and z be the units of food A, B and C respectively, that needs to be purchased for the patient in order to minimize the cost.

For Objective Function:

$$\text{Total cost} = 3x + 2y + 2z$$

Let Total cost = C , so that

$$C = 3x + 2y + 2z$$

$$\text{Minimize } C = 3x + 2y + 2z \quad (\text{objective function})$$

For Constraints:

$$x + 2y + 2z \geq 8 \quad (\text{Vitamin constraint})$$

$$x + y + z \geq 9 \quad (\text{Mineral constraint})$$

$$2x + y + 2z \geq 10 \quad (\text{Carbohydrate constraint})$$

$$x, y, z \geq 0 \quad (\text{non-negativity constraint})$$

Let S_1 , S_2 and S_3 be surplus variables. Let A_1 , A_2 and A_3 be artificial variables.

Now,

$$x + 2y + 2z - S_1 + A_1 = 8$$

$$x + y + z - S_2 + A_2 = 9$$

$$2x + y + 2z - S_3 + A_3 = 10$$

Standard Equation for Simplex Table:

$$1C - 3x - 2y - 3z + 0S_1 + 0S_2 + 0S_3 - 10A_1 - 10A_2 - 10A_3 = 0$$

$$0C + 1x + 2y + 2z - 1S_1 + 0S_2 + 0S_3 + 1A_1 + 0A_2 + 0A_3 = 0$$

$$0C + 1x + 1y + 1z + 0S_1 - 1S_2 + 0S_3 + 0A_1 + 1A_2 + 0A_3 = 0$$

$$0C + 2x + 1y + 2z + 0S_1 + 0S_2 - 1S_3 + 0A_1 + 0A_2 + 1A_3 = 0$$

b)

Simplex Method Solution

Simplex Table 1:

Table 1: Simplex Table 1

Row	Z	x	y	z	S ₁	S ₂	S ₃	A ₁	A ₂	A ₃	Constant
R ₀	1	-3	-2	-3	0	0	0	-10	-10	-10	0
R ₁	0	1	2	2	-1	0	0	1	0	0	8
R ₂	0	1	1	1	0	-1	0	0	1	0	9
R ₃	0	2	1	2	0	0	-1	0	0	1	10

For Identity Matrix:

$$\text{New } R_0 = \text{Old } R_0 + 10 (R_1 + R_2 + R_3)$$

Table 2: Operations Table 1

Old R ₀	10(R ₁ + R ₂ + R ₃)	New R ₀
1	0	1
-3	40	37
-2	40	38
-3	50	47
0	-10	-10
0	-10	-10
0	-10	-10
-10	10	0
-10	10	0
-10	10	0
0	270	270

Simplex Table 2:

Table 3: Simplex Table 2

Row	Z	x	y	z	S ₁	S ₂	S ₃	A ₁	A ₂	A ₃	Constant	Ratio
R ₀	1	37	38	47	-10	-10	-10	0	0	0	270	—
R ₁	0	1	2	2	-1	0	0	1	0	0	8	4
R ₂	0	1	1	1	0	-1	0	0	1	0	9	9
R ₃	0	2	1	2	0	0	-1	0	0	1	10	5

Here, the highest positive number in R₀ is 47 so z is the key column and 4 is the lowest positive ratio, so R₁ is the key row.

∴ 2 is the key element.

We need to update R₁ first using:

$$New\ R_1 = \frac{Old\ R_1}{Key\ element} = \frac{Old\ R_1}{2}$$

Elements in $New\ R_1 = 0, \frac{1}{2}, 1, 1, -\frac{1}{2}, 0, 0, \frac{1}{2}, 0, 0, 4$

Similarly, updating other rows with reference to new R₁:

$$New\ R_0 = Old\ R_0 - 47 \times New\ R_1$$

$$New\ R_2 = Old\ R_2 - New\ R_1$$

$$New\ R_3 = Old\ R_3 - 2 \times New\ R_1$$

Table 4: Operations Table 2

Old R_0	$47 \times$ New R_1	New R_0	Old R_2	New R_1	New R_2	Old R_3	$2 \times$ New R_1	New R_3
1	0	1	0	0	0	0	0	0
-3	40	37	1	$\frac{1}{2}$	$\frac{1}{2}$	2	1	1
-2	40	38	1	0	0	1	0	-1
-3	50	47	1	1	0	2	2	0
0	-10	-10	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	-1	1
0	-10	-10	-1	0	-1	0	0	0
0	-10	-10	0	0	0	-1	0	-1
-10	10	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	1
-10	10	0	1	0	1	0	0	0
-10	10	0	0	0	0	1	0	1
0	270	270	9	4	5	10	8	2

Simplex Table 3:

Table 5: Simplex Table 3

Row	Z	x	y	z	S ₁	S ₂	S ₃	A ₁	A ₂	A ₃	Constant	Ratio
R ₀	1	$\frac{27}{2}$	-9	0	$\frac{27}{2}$	-10	-10	$-\frac{47}{2}$	0	0	82	—
R ₁	0	$\frac{1}{2}$	1	1	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0	4	8
R ₂	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	-1	0	$-\frac{1}{2}$	1	0	5	10
R ₃	0	1	-1	0	1	0	-1	-1	0	1	2	2

Here, the highest positive number in R₀ is $\frac{27}{2}$. Both x and y column has this value.

In such case, either column can be taken for further calculation so, we take x as the key column. 2 is the lowest positive ratio, so R₃ is the key row.

∴ 1 is the key element.

We need to update R₁ first using:

$$\text{New } R_3 = \text{Old } R_3$$

Elements in $\text{New } R_3 = 0, 1, -1, 0, 1, 0, -1, -1, 0, 1, 2$ (Same as before)

Similarly, updating other rows with reference to new R₃:

$$\text{New } R_0 = \text{Old } R_0 - \frac{27}{2} \times \text{New } R_3$$

$$\text{New } R_1 = \text{Old } R_1 - \frac{1}{2} \times \text{New } R_3$$

$$\text{New } R_2 = \text{Old } R_2 - \frac{1}{2} \times \text{New } R_3$$

Table 6: Operations Table 3

Old R_0	$\frac{27}{2} \times \text{New}$ R_1	New R_0	Old R_1	$\frac{1}{2} \times \text{New}$ R_3	New R_1	Old R_2	$\frac{1}{2} \times$ New R_3	New R_2
1	0	1	0	0	0	0	0	0
$\frac{27}{2}$	$\frac{27}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0
-9	$-\frac{27}{2}$	$\frac{9}{2}$	1	$-\frac{1}{2}$	$\frac{3}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$
0	0	0	1	0	1	0	0	0
$\frac{27}{2}$	$\frac{27}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	0
-10	0	-10	0	0	0	-1	0	-1
-10	$-\frac{27}{2}$	$\frac{7}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$
$-\frac{47}{2}$	$-\frac{27}{2}$	-10	$\frac{1}{2}$	$-\frac{1}{2}$	1	$-\frac{1}{2}$	$-\frac{1}{2}$	0
0	0	0	0	0	0	1	0	1
0	$\frac{27}{2}$	$-\frac{27}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$
82	27	55	4	1	3	5	1	4

Simplex Table 4:

Table 7: Simplex Table 4

Row	Z	x	y	z	S ₁	S ₂	S ₃	A ₁	A ₂	A ₃	Constant	Ratio
R ₀	1	0	$\frac{9}{2}$	0	0	-10	$\frac{7}{2}$	-10	0	$-\frac{27}{2}$	55	—
R ₁	0	0	$\frac{3}{2}$	1	-1	0	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	3	2
R ₂	0	0	$\frac{1}{2}$	0	0	-1	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	4	8
R ₃	0	1	-1	0	1	0	-1	-1	0	1	2	-2

Here, the highest positive number in R₀ is $\frac{9}{2}$ so y is the key column and 2 is the lowest positive ratio, so R₁ is the key row.

∴ 1 is the key element.

We need to update R₁ first using:

$$New\ R_1 = \frac{Old\ R_1}{\frac{3}{2}}$$

Elements in $New\ R_1 = 0, 0, 1, \frac{2}{3}, -\frac{2}{3}, 0, \frac{1}{3}, \frac{2}{3}, 0, -\frac{1}{3}, 2$

Similarly, updating other rows with reference to new R₁:

$$New\ R_0 = Old\ R_0 - \frac{9}{2} \times New\ R_1$$

$$New\ R_2 = Old\ R_2 - \frac{1}{2} \times New\ R_1$$

$$New\ R_3 = Old\ R_3 + New\ R_1$$

Table 8: Operations Table 4

Old R_0	$\frac{9}{2} \times$ New R_1	New R_0	Old R_2	$\frac{1}{2} \times$ New R_1	New R_2	Old R_3	New R_1	New R_3
1	0	1	0	0	0	0	0	0
0	0	0	0	$\frac{1}{2}$	0	1	0	1
$\frac{9}{2}$	$\frac{9}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	-1	1	0
0	3	-3	0	0	$-\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{2}{3}$
0	-3	3	0	$\frac{1}{2}$	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{1}{3}$
-10	0	-10	-1	0	-1	0	0	0
$\frac{7}{2}$	$\frac{3}{2}$	2	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	-1	$\frac{1}{3}$	$-\frac{2}{3}$
-10	3	-13	0	$-\frac{1}{2}$	$-\frac{1}{3}$	-1	$\frac{2}{3}$	$-\frac{1}{3}$
0	0	0	1	0	1	0	0	0
$-\frac{27}{2}$	$-\frac{3}{2}$	-12	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{3}$	1	$-\frac{1}{3}$	$\frac{2}{3}$
55	9	46	4	1	3	2	2	4

Simplex Table 5:

Table 9: Simplex Table 5

Row	Z	x	y	z	S ₁	S ₂	S ₃	A ₁	A ₂	A ₃	Constant	Ratio
R ₀	1	0	0	-3	3	-10	2	-13	0	-12	46	—
R ₁	0	0	1	$\frac{2}{3}$	$-\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	0	$-\frac{1}{3}$	2	—
R ₂	0	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	-1	$\frac{1}{3}$	$-\frac{1}{3}$	1	$-\frac{1}{3}$	3	9
R ₃	0	1	0	$\frac{2}{3}$	$\frac{1}{3}$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{2}{3}$	4	12

Here, the highest positive number in R₀ is 3 so S₁ is the key column and 9 is the lowest positive ratio, so R₂ is the key row.

∴ $\frac{1}{3}$ is the key element.

We need to update R₁ first using:

$$\text{New } R_2 = \frac{\text{Old } R_2}{\frac{1}{3}}$$

Elements in $\text{New } R_2 = 0, 0, 0, -1, 1, -3, 1, -1, 3, -1, 9$

Similarly, updating other rows with reference to new R₂:

$$\text{New } R_0 = \text{Old } R_0 - 3 \times \text{New } R_2$$

$$\text{New } R_1 = \text{Old } R_1 + \frac{2}{3} \times \text{New } R_2$$

$$\text{New } R_3 = \text{Old } R_3 - \frac{1}{3} \times \text{New } R_2$$

Table 10: Operations Table 5

Old R_0	$3 \times \text{New}$ R_2	New R_0	Old R_1	$\frac{2}{3} \times \text{New}$ R_2	New R_1	Old R_3	$\frac{1}{3} \times \text{New}$ R_2	New R_3
1	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	1
0	0	0	1	0	1	0	0	0
-3	-3	0	$\frac{2}{3}$	$-\frac{2}{3}$	0	$\frac{2}{3}$	$-\frac{1}{3}$	1
3	3	0	$-\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	0
-10	-9	-1	0	-2	-2	0	-1	1
2	3	-1	$\frac{1}{3}$	$\frac{2}{3}$	1	$-\frac{2}{3}$	$\frac{1}{3}$	-1
-13	-3	-10	$\frac{2}{3}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$	0
0	9	-9	0	2	2	0	1	-1
-12	-3	-9	$-\frac{1}{3}$	$-\frac{2}{3}$	-1	$\frac{2}{3}$	$-\frac{1}{3}$	1
46	27	19	2	6	8	4	3	1

Simplex Table 6:

Table 11: Simplex Table 6

Row	Z	x	y	z	S ₁	S ₂	S ₃	A ₁	A ₂	A ₃	Constant
R ₀	1	0	0	0	0	-1	-1	-10	-9	-9	19
R ₁	0	0	1	0	0	-2	1	0	2	-1	8
R ₂	0	0	0	-1	1	-3	1	-1	3	-1	9
R ₃	0	1	0	1	0	1	-1	0	-1	1	1

Here, all the coefficient of variables in $R_0 \leq 0$, so we have reached the optimal solution.

Hence,

Minimum $Z = 19$

$x = 1$

$y = 8$

$z = 0$ (non - basic)

In conclusion, purchasing 1 unit of food A and 8 units of food B gives the minimum cost, which in this case is \$19, to fulfil the patient's diet.

c)

Excel Solver

Excel Solver is an add-in for Microsoft Excel that allows optimization of particular cells based on changes made by the program in other cells (Cortright, 2023). It is particularly helpful in financial situations to calculate maximum or minimum values for any element.

The following Linear Programming Problem can also be solved using excel solver. The objective here is to minimize the total cost of the food plan while meeting all the nutrition requirements. This can be done by inserting the objective function and the constraints are formula and using solver to get the results.

The sheet with formula and calculated values:

Decision variables	X	Y	Z
Value			
Objective function	=3*C3+2*D3+3*E3		
	Constraints		
	LHS		RHS
	=C3+2*D3+2*E3	>=	8
	=C3+D3+E3	>=	9
	=2*C3+D3+2*E3	>=	10

Figure 12: Excel Formula Sheet

With these formula in place, the following values were extracted using the Solver.

Decision variables	X	Y	Z
Value	1	8	0
Objective function	19		
	Constraints		
	LHS		RHS
	17	>=	8
	9	>=	9
	10	>=	10

Figure 13: Excel Answer Sheet

Similarly, the following report sheets are generated through the Excel Solver.

Answer Report:

Microsoft Excel 16.0 Answer Report

Worksheet: [q2 A.xlsx]Sheet1

Report Created: 17/05/2025 23:16:08

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.

Solver Engine

Solver Options

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$E\$3	Value Z	0	0

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$E\$3	Value Z	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$C\$10	LHS	9	\$C\$10>=\$E\$10	Binding	0
\$C\$11	LHS	10	\$C\$11>=\$E\$11	Binding	0
\$C\$9	LHS	17	\$C\$9>=\$E\$9	Not Binding	9

Figure 14: Answer Report

The answer report shows the final value of decision variables and if each constraint is satisfied or not.

From the answer report as well as the answer sheet we can see that the optimal solution for the problem is:

1 unit of Food A (X)

8 units of Food B (Y)

0 units of Food C (Z)

This combination gives a minimum cost of \$19 with all the nutritional requirements (constraints) met. Both the carbohydrate and minerals requirements are exactly met.

Sensitivity Report:

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [q2 A.xlsx]Sheet1
Report Created: 17/05/2025 23:16:09

Variable Cells

		Final	Reduced	Objective	Allowable	Allowable
Cell	Name	Value	Cost	Coefficient	Increase	Decrease
\$E\$3	Value Z	0	0	1	1E+30	1

Constraints

		Final	Shadow	Constraint	Allowable	Allowable
Cell	Name	Value	Price	R.H. Side	Increase	Decrease
\$C\$10	LHS	9	1	9	1E+30	0
\$C\$11	LHS	10	0	10	0	1E+30
\$C\$9	LHS	17	0	8	9	1E+30

Figure 15: Sensitivity Report

The sensitivity report shows the sensitivity of the LPP. It shows how much the objective function would change in case of change in RHS or constraint coefficients.

Form the report, we can see that

- The requirement for minerals is binding and increase in mineral requirement would also increase the total cost by \$1 per unit.
- There is excess supply of vitamin, such that increasing the requirement by at most 9 units would not affect the optimal solution.
- The requirement for carbohydrate is also exactly met but not binding, so any increase in it would not be possible without changing the whole solution.

Limits Report:

Microsoft Excel 16.0 Limits Report
Worksheet: [q2 A.xlsx]Sheet1
Report Created: 17/05/2025 23:16:09

Objective		
Cell	Name	Value
\$E\$3	Value Z	0

Variable			Lower Objective		Upper Objective	
Cell	Name	Value	Limit	Result	Limit	Result
\$E\$3	Value Z	0	0	0	#N/A	#N/A

Figure 16: Limits Report

The limits report shows how much change the decision variable can take while keeping the optimal solution.

Following are the take aways from the report:

- The Food C is not part of the optimal solution.
- The Food C does not contribute in lowering the total cost and there is also no point in increasing its quantity meaning Food C is not cost-effective under current prices.

In conclusion, it was found that

- 1 units of Food A and 8 units of Food B results in the optimal solution.
- Food C is not part of the optimal solution as it is not cost-effective.
- The total minimum cost in the optimal solution is \$19.
- The minerals constraint is critical and any increase will lead to increase in total cost.

d)

Memorandum

Date: 17th May, 2025

To: Berkshire Health Care

From: Jonny Boi, Nutrition Consultant

Subject: Recommended Food Plan for Minimum Cost

To Director, Nutrition Services Department,

I hope this message finds you well, the following memo is to provide the recommended combination of the available foods that satisfies the patients dietary needs while keeping the cost as low as possible.

After carefully analyzing the nutritional content and cost of each food item, the following quantity of food products was found to be the most optimal:

Food A:	1 Unit per patient	(Cost: \$3)
Food B:	8 Units per patient	(Cost: \$16)
Food C:	Not required for dietary needs	(Cost: \$0)

The total cost comes out to be \$19.

This food plan ensures the patient gets at least 8 units of Vitamins, 9 units of Minerals and 10 units of Carbohydrates while also keeping the cost minimum.

I recommend implementing this food combination part of standard diet dietary plan. It achieves the right balance between cost-efficiency and nutrition ampleness.

Please feel free to reach out to us if you wish a breakdown of how this plan was calculated or to discover alternative combinations in case of change in guidelines.

Sincerely,

Jonny Boi

Nutrition consultant

Problem 2 Part B

Mr. Harris requires minimum 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A, B and C per carton. If the liquid product cost £ 3 per jar and dry product cost £ 2 per carton, how many of each should Harris purchase in order to minimize the cost and meet the requirements.

Formulate this problem and solve it graphically

Solution:

According to the given question,

Mr. Harris requires a minimum of 10, 12, and 12 units of chemicals A, B and C respectively for his garden.

A liquid product contains 5, 2 and 1 units of A, B and C respectively per jar.

A dry product contains 1, 2 and 4 units of A, B and C per carton.

Given, liquid product costs £ 3 per jar and dry product costs £ 2 per carton.

Formulating the problem,

Let x be the liquid product and y be the dry product.

For liquid product,

5 units of Chemical A

2 units of Chemical B

1 unit of Chemical C

For dry product,

1 unit of Chemical A

2 units of Chemical B

4 units of Chemical C

For Objective function,

$$\text{Minimize } Z = 3x + 2y$$

For Constraints,

$$5x + y \geq 10 \quad (\text{Chemical A constraint})$$

$$2x + 2y \geq 12 \quad (\text{Chemical B constraint})$$

$$x + 4y \geq 12 \quad (\text{Chemical C constraint})$$

$$x, y \geq 0 \quad (\text{non-negative constraint})$$

Let the boundary of inequality for $5x + y \geq 10$ be $5x + y = 10$

X	0	2
Y	10	0

For Origin Test,

$$5x + y \geq 10$$

$$5 * 0 + 0 \geq 10$$

$$0 \geq 10 \quad (\text{False})$$

Hence, it lies away from origin

Similarly, let the boundary of inequality for $2x + 2y \geq 12$ be $2x + 2y = 12$

X	0	6
Y	6	0

For Origin test,

$$2x + 2y \geq 12$$

$$2 * 0 + 2 * 0 \geq 12$$

$$0 \geq 12 \quad (\text{False})$$

Hence, it lies away from origin

Finally, let the boundary of inequality for $x + 4y \geq 12$ be $x + 4y = 12$

X	0	12
Y	3	0

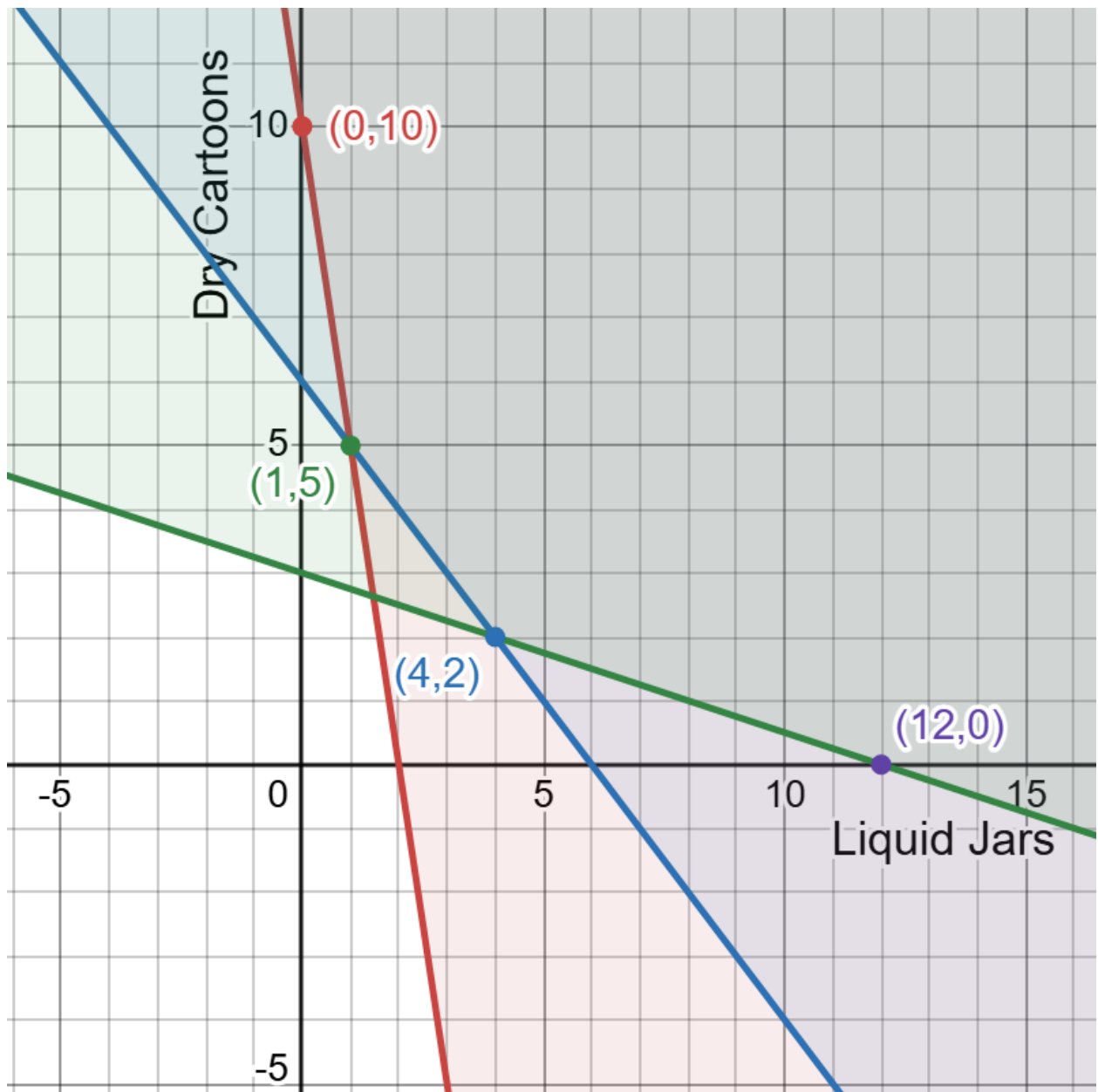
For Origin test,

$$x + 4y \geq 12$$

$$0 + 4 * 0 \geq 12$$

$$0 \geq 12 \quad (\text{False})$$

Creating a graph:



From graph, the common region of all lines away from the origin is the feasible region

Vertex	X	Y	$Z = 3x + 2y$
(0,10)	0	10	20
(1,5)	1	5	13 (Minimum)
(4,2)	4	2	16
(12,0)	12	0	36

Therefore, Mr. Harris should purchase 1 liquid product and 5 dry products to meet his requirement and minimize the cost.

Problem 3

If the revenue function for a manufacturing company is $R(x) = 10 - x$ and the total cost function is $C(x) = x^2 - 8x + 20$, where x is output.

Questions

- a) Find the breakeven points (round in 3 decimal places). Briefly explain the breakeven points in terms of number of items and cost.
- b) Plot the revenue and cost functions on graph, showing the accurate break-even output levels and corresponding price.
- c) Determine the profit function $P(x)$, for the company and find,
 - (i) The level production that maximizes the profit.
 - (ii) The maximum profit

Solution

The breakeven point is the point where the total revenue and the total cost are equal, meaning that there is neither profit nor loss. This point occurs in a graph between the sales and the cost at the point of intersection. Before this point, the sales are less than cost so there is loss and after this point the sales are more than cost so there is profit.

At breakeven point,

$$\text{Total cost} = \text{Revenue}$$

There are numerous basic terms used while finding the breakeven points, they are explained below:

1. Cost price

The cost price is the original price of the item when it is bought, for example if a clothing shop buys a shirt for Rs. 2000 then the cost price of the shirt is Rs. 2000. The cost price is the price that the shop pays while buying a product.

2. Selling Price

The selling price is the price at which the item is sold to the customers. It is the price the shopkeepers keep for a product when selling to the customer. As an example, the shirt which the shop bought for Rs. 2000 can be sold for Rs. 3000, so the selling price of the shirt is Rs. 3000.

3. Fixed Cost

The cost that does not change with respect to the number of items sold, they can be various things such as the rent, the initial cost of starting the business or insurance as well. They are the constant cost that do not change.

4. Variable Cost

These costs are the cost that change depending on the number of the items sold or produced, then can be seen as the cost to produce the items. For example, the making a bicycle make cost Rs 5000 per bicycle, but since the number of bicycle production varies, the cost of production changes as well. Thus, this is variable cost.

5. Revenue

The revenue is the total money earned from the sales of the products. Revenue can also be said to be a product of the selling price and the number of items sold.

$$\text{Revenue} = \text{Selling Price} * \text{Number of items sold}$$

6. Profit

The profit is the money after subtracting all costs from the revenue. It is the difference between the total revenue and the total cost.

$$\text{Profit} = \text{Revenue} - \text{Total Cost}$$

According to the question,

The revenue function for a manufacturing company is given by,

$$R(x) = 10 - x$$

The total cost function is given by, $C(x) = x^2 - 8x + 20$

Here x is output

a) Find the breakeven points (round in 3 decimal places). Briefly explain the breakeven points in terms of number of items and cost.

Here,

At breakeven point

$$\text{Revenue} = \text{Cost}$$

$$R(x) = C(x)$$

$$10 - x = x^2 - 8x + 20$$

$$0 = x^2 - 8x + x + 20 - 10$$

$$0 = x^2 - 7x + 10$$

$$x^2 - 7x + 10 = 0$$

Comparing $x^2 - 7x + 10 = 0$ with $ax^2 + bx + c = 0$

We get, $a=1$, $b=-7$, $c=10$

Using quadratic equation formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4 * 1 * 10}}{2}$$

$$x = \frac{7 \pm \sqrt{49 - 40}}{2}$$

$$x = \frac{7 + \sqrt{9}}{2}$$

$$x = \frac{7 \pm 3}{2}$$

The value can have two forms.

Positive

$$x_1 = \frac{7 + 3}{2}$$

$$x_1 = \frac{10}{2}$$

$$x_1 = 5.000$$

Negative

$$x_2 = \frac{7 - 3}{2}$$

$$x_2 = \frac{4}{2}$$

$$x_2 = 2.000$$

Now, for the corresponding points of x

When $x=5.000$

$$C(x) = x^2 - 8x + 20$$

$$C(5.000) = (5.000)^2 - 8 * 5.000 + 20$$

$$C(5.000) = 5.000$$

When $x=2.000$

$$C(x) = x^2 - 8x + 20$$

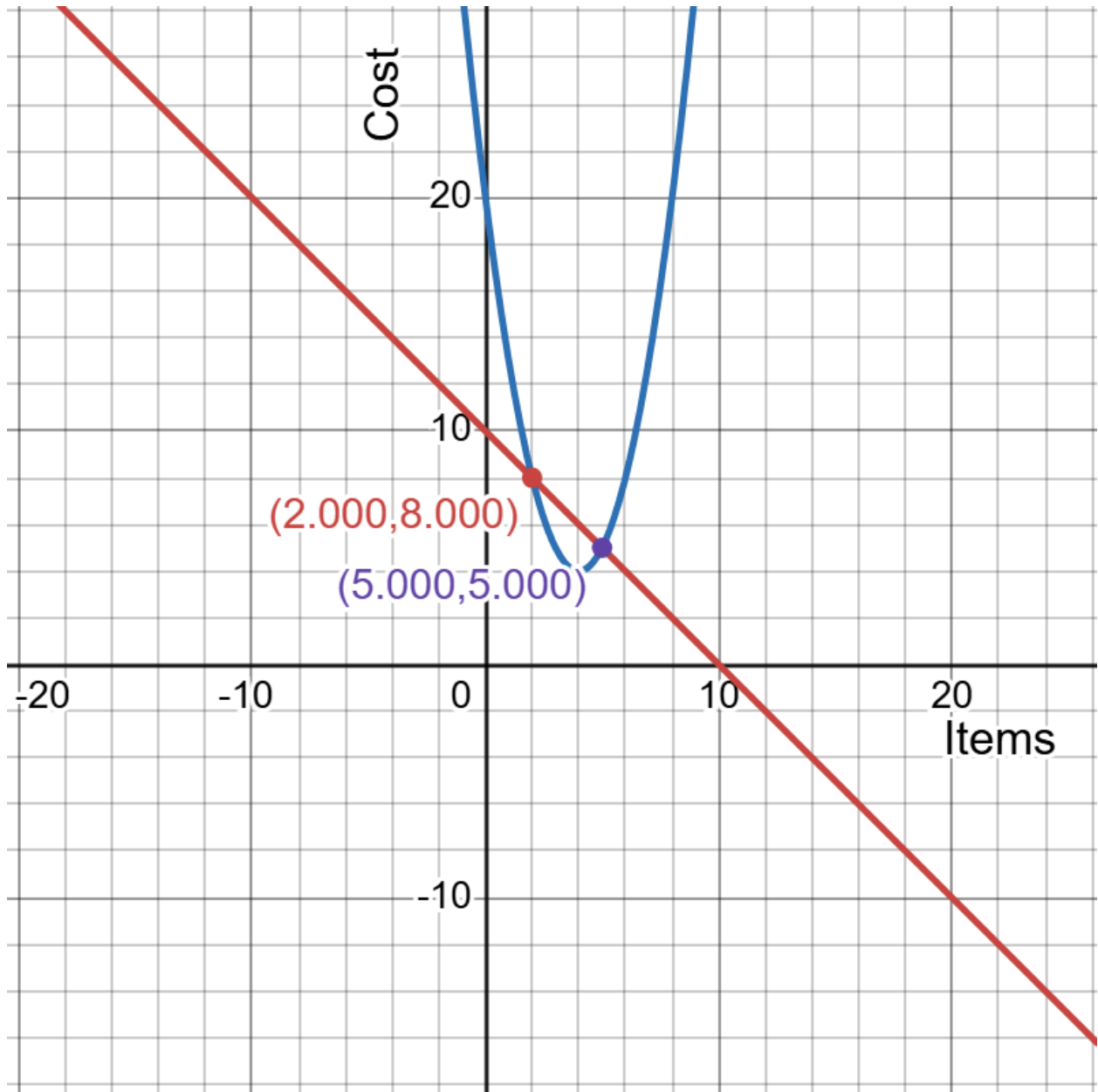
$$C(2.000) = (2.000)^2 - 8 * 2.000 + 20$$

$$C(2.000) = 8.000$$

So the breakeven points are (5.000, 5.000) and (2.000, 8.000).

In terms of number of items and costs, the breakeven points occur when the number of items and cost are both 5.000 and breakeven point occurs when number of items in 2.000 and cost is 8.000.

b) Plot the revenue and cost functions on graph, showing the accurate break-even output levels and corresponding price.



With the help of [desmos.com](https://www.desmos.com), the revenue and cost functions were plotted in the graph and breakeven points were shown. The breakeven points are (2.000, 8.000) and (5.000, 5.000).

c) Determine the profit function $P(x)$, for the company

We know,

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

So,

$$P(x) = R(x) - C(x)$$

$$P(x) = 10 - x - (x^2 - 8x + 20)$$

$$P(x) = 10 - x - x^2 + 8x - 20$$

$$P(x) = -x^2 + 7x - 10$$

Thus,

$$\text{Profit function, } P(x) = -x^2 + 7x - 10$$

(i) The level production that maximizes the profit.

The level production that maximizes the profit is given by,

$$\text{Level of production, } x = \frac{-b}{2a}$$

Comparing $ax^2 + bx + c$ to $P(x) = -x^2 + 7x - 10$,

$$a = -1, b = 7, c = -10$$

Now,

Level of production

$$x = \frac{-b}{2a}, \text{ where } a = -1, b = 7$$

$$x = -\frac{7}{2 * (-1)}$$

$$x = -\frac{7}{-2}$$

$$x = 3.5$$

So, the level of production that maximizes the product is 3.5.

(ii) The maximum profit

The maximum profit is given by profit function when Level of production, $x = 3.5$.

So,

$$P(x) = -x^2 + 7x - 10$$

$$P(3.5) = (-3.5)^2 + 7 * 3.5 - 10$$

$$P(3.5) = 2.25$$

Thus, the maximum profit is 2.25 when the level of production is 3.5.

Conclusion

In this coursework, we use logical reasoning and mathematical techniques to solve problems related to tax system and some Linear Programming calculations. Through proper formulation and analysis, we identified optimal solutions that satisfy the given constraints with question's requirements.

Overall, this coursework encouraged critical thinking, team synergy, and the ability to convert practical situations into structured mathematical models. The experience has refined our problem-solving skills and decision-making process.

References

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Meeting logs

Logbook Entry Sheet

Meeting No: 1

Date: 4th May

Start Time: 10:00 AM

Finish Time: 10:45AM

Items Discussed:

General Idea of how we are going to solve each question

We also discussed which member is going to handle which question

Achievements:

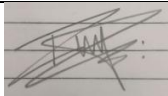
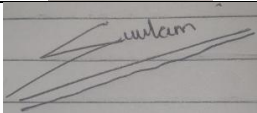
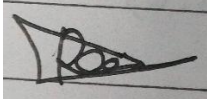
Set goals for each member of the group

Problems:

We did not encounter any problem in this meeting

Tasks for Next Meeting:

Discuss and analyze each problem with a rough work in paper

Student's Name	Student's Signature
Aashraya Belbase	
Suvham Shakya	
Rizen Shrestha	

Logbook Entry Sheet

Meeting No: 2

Date: 12th May

Start Time: 3:00 PM

Finish Time: 4:30 PM

Items Discussed:

Rough solutions for each problem. Deep analysis for problem 2, part A

Achievements:

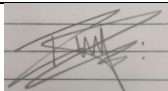
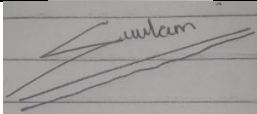
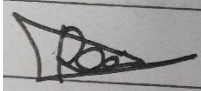
Complete solution for problem 2 and general idea for problem 1 and 3.

Problems:

Problem 2, part A had different answers for two of us

Tasks for Next Meeting:

Complete solutions for each question in word file.

Student's Name	Student's Signature
Aashraya Belbase	
Suvham Shakya	
Rizen Shrestha	

Logbook Entry Sheet

Meeting No: 3

Date: 14th May

Start Time: 1:00 PM

Finish Time: 2:00 PM

Items Discussed:

All complete solutions for all questions

Achievements:

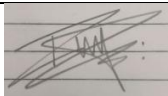
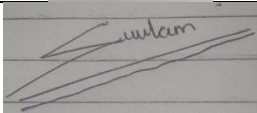
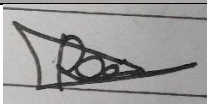
Completely discussed each question and cross checked for correct answers

Problems:

No problems were encountered

Tasks for Next Meeting:

Discuss the format of the project and final check for submission report

Student's Name	Student's Signature
Aashraya Belbase	
Suvham Shakya	
Rizen Shrestha	

Logbook Entry Sheet

Meeting No: 4

Date: 17th May

Start Time: 3:00 PM

Finish Time: 4:30 PM

Items Discussed:

Final word file and format

Achievements:

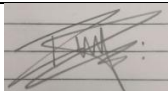
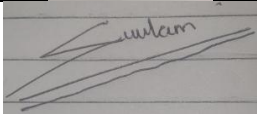
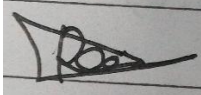
Completely checked the final word file

Problems:

Acknowledgements were not included

Tasks for Next Meeting:

No further meetings were scheduled

Student's Name	Student's Signature
Aashraya Belbase	
Suvham Shakya	
Rizen Shrestha	

Rough work in Paper

* Mr. Harris requires minimum 10, 12 & 12 units of chemicals A, B & C respectively for his garden. A liquid product contains 5, 2 & 1 units of A, B & C respectively per jar. A dry product contains 1, 2 & 4 units of A, B & C per carton. If the liquid product costs £3 per jar & dry product costs £2 per carton, how many of each should Harris purchase in order to minimize the cost & meet the requirements. Formulate this problem & solve it graphically.

= soln:

According to the question,

Mr. Harris requires minimum 10, 12 & 12 units of A, B & C resp.

For liquid product \rightarrow £3 (x)

~~5~~ A - 5

B - 2

C - 1

For dry product \rightarrow £2 (y)

A - 1

B - 2

C - 4

For objective function,

$$3x + 2y = Z \quad (\text{minimize})$$

For constraints,

$$5x + y \geq 10$$

$$2x + 2y \geq 12$$

$$x + 4y \geq 12$$

$$x, y \geq 0$$

let the boundary of inequality $5x + y \geq 10$ be
 $5x + y = 10$

origin test

$$\begin{array}{c|c|c} x & 0 & 2 \\ y & 10 & 0 \end{array}$$

then,

$$5x + y \geq 10$$

$$5 \cdot 0 + 10 \geq 10$$

$$0 \geq 10 \text{ (False)}$$

Similarly,

for, $2x + 2y \geq 12$

$$2x + 2y = 12$$

~~$$2y = 12$$~~

$$\begin{array}{c|c|c} x & 0 & 6 \\ y & 6 & 0 \end{array}$$

Then,

$$2x + 2y \geq 12$$

$$2 \cdot 0 + 2 \cdot 0 \geq 12$$

$$0 \geq 12 \text{ (False)}$$

Again,

for, $x + 4y \geq 12$, $x + 4y = 12$

$$\begin{array}{c|c|c} x & 0 & 12 \\ y & 3 & 0 \end{array}$$

Then,

$$x + 4y \geq 12$$

$$0 + 4 \cdot 0 \geq 12$$

$$0 \geq 12 \text{ (False)}$$

$$y = 10 - 5x$$

$$x = \frac{10 - y}{5} = \frac{10 - 0}{5}$$

$$y = \frac{12 - 2x}{2}$$

$$x = \frac{12 - 2y}{2}$$

$$y = \frac{12 - x}{4}$$

$$x = 12 - 4y$$

From Graph - - - - Feasible region

vertex	x	y	$Z = 3x + 2y$
(0, 10)	0	10	$Z = 3 \cdot 0 + 2 \cdot 10 = 20$
(1, 8)	1	5	(Min) $Z = 3 \cdot 1 + 2 \cdot 5 = 13$
(4, 2)	4	2	$Z = 3 \cdot 4 + 2 \cdot 2 = 16$
(12, 0)	12	0	$Z = 3 \cdot 12 + 2 \cdot 0 = 36$

\therefore Mr. Harris should purchase 1 liquid product & 5 ~~dry~~ dry products to meet his requirement & minimize the cost.

Question 3.

According to the question.

The revenue function for a manufacturing company is given by,

$$R(x) = 10 - x, \text{ where } x \text{ is output.}$$

The total cost function is given by,

$$C(x) = x^2 - 8x + 20 \text{ where } x \text{ is output.}$$

- a) Find the breakeven points (round in 3 decimal places)
Briefly explain the breakeven points in terms of number of items and cost

Solⁿ

At breakeven point.

$$\text{Revenue} = \text{Cost}$$

$$R(x) = C(x)$$

$$10 - x = x^2 - 8x + 20$$

$$0 = x^2 - 8x + x + 20 - 10$$

$$0 = x^2 - 7x + 10$$

$$x^2 - 7x + 10 = 0$$

Comparing $x^2 - 7x + 10 = 0$ with $ax^2 + bx + c = 0$.

We get, $a = 1$, $b = -7$, $c = 10$

Using quadratic equation formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$2a$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1}$$

$$2 \cdot 1$$

$$x = \frac{7 \pm \sqrt{49 - 40}}{2}$$

$$2$$

c) Determine the profit function $P(x)$, for the company.

We know,

$$\text{Profit} = \text{Revenue} - \text{Cost},$$

So

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 10 - x - (x^2 - 8x + 20) \\ &= 10 - x - x^2 + 8x - 20 \\ &= -x^2 + 7x - 10 \end{aligned}$$

Thus,

$$\text{Profit function } P(x) = -x^2 + 7x - 10$$

i) The level production that maximizes the profit

The level production, that maximizes the profit is given by,

Level of production :

$$x = \frac{-b}{2a},$$

Comparing $ax^2 + bx + c$, to $P(x) = -x^2 + 7x - 10$

$$a = -1, b = 7, c = -10,$$

Now,

Level of production :

$$x = \frac{-b}{2a}, \text{ where } a = -1, b = 7$$

$$\begin{aligned} x &= \frac{-7}{2(-1)} \\ &= \frac{-7}{-2} \\ &= 3.5 \end{aligned}$$

$$x = \frac{7 \pm \sqrt{9}}{2}$$

$$x = \frac{7 \pm 3}{2}$$

The value can have two forms

Positive

$$x_1 = \frac{7+3}{2}$$

$$x_1 = \frac{10}{2}$$

$$x_1 = 5.000$$

Negative

$$x_2 = \frac{7-3}{2}$$

$$x_2 = \frac{4}{2}$$

$$x_2 = 2.000$$

Now, for the corresponding points of x

When $x = 5.000$

$$\begin{aligned} C(x) &= (5.000)^2 - 8 \times 5.000 + 20 \\ &= 5.000 \end{aligned}$$

When $x = 2.000$

$$\begin{aligned} C(x) &= (2.000)^2 - 8 \times 2.000 + 20 \\ &= 8.000 \end{aligned}$$

So the breakeven points are $(5.000, 5.000)$ and $(2.000, 8.000)$

In terms of number of items and costs, the breakeven points occur when the number of items and cost are both 5.000 and the breakeven points occur when number of items is 2.000 and cost is 8.000.

ii. The maximum profit

The maximum profit is given by profit function
when $x = 3.5$

$$= P(3.5)$$

$$= -(3.5)^2 + 7 \times 3.5 - 10$$

$$= 2.25$$

12-A

⇒ Here,

According to q,
Patient's diet contains at least 8 vitamin, 9 minerals
and 10 carbohydrates

For food A \rightarrow \$3 (x)

vitamin - 1

mineral - 1

carbohydrate - 2

For food B \rightarrow \$2 (y)

vitamin - 2

mineral - 1

carbohydrate - 1

For food C \rightarrow \$3 (z)

vitamin - 2

mineral - 1

carbohydrate - 2

For objective function,

$$3x + 2y + 3z = \text{Total cost}$$

$$\text{or, } 3x + 2y + 3z = Z \quad \text{Minimize}$$

For constraints,

$$x + 2y + 2z \geq 8$$

(vitamin)

$$x + y + z \geq 9$$

(mineral)

$$2x + y + 2z \geq 10$$

(carbohydrate)

$$x, y, z \geq 0$$

let S_1, S_2, S_3 be surplus variable and A_1, A_2, A_3
be artificial variables. Then,

$$x + 2y + 2z - S_1 + A_1 = 8$$

$$x + y + z - S_2 + A_2 = 9$$

$$2x + y + 2z - S_3 + A_3 = 10$$

Standard Equation for Simplex Table:

$$\begin{aligned}
 R_0 &\rightarrow 1Z - 3x - 2y - 3z + 0S_1 + 0S_2 + 0S_3 - 10A_1 - 10A_2 - 10A_3 = 0 \\
 R_1 &\rightarrow 0Z + 1x + 2y + 2z - 1S_1 + 0S_2 + 0S_3 + 1A_1 + 0A_2 + 0A_3 = 8 \\
 R_2 &\rightarrow 0Z + 1x + 1y + 1z + 0S_1 - 1S_2 + 0S_3 + 0A_1 + 1A_2 + 0A_3 = 9 \\
 R_3 &\rightarrow 0Z + 2x + 1y + 2z + 0S_1 + 0S_2 - 1S_3 + 0A_1 + 0A_2 + 1A_3 = 10
 \end{aligned}$$

Simplex table 1:

Row	Z	x	y	z	S ₁	S ₂	S ₃	A ₁	A ₂	A ₃	constant
R ₀	1	-3	-2	-3	0	0	0	-10	-10	-10	0
R ₁	0	1	2	2	-1	0	0	1	0	0	8
R ₂	0	1	1	1	0	-1	0	0	1	0	9
R ₃	0	2	1	2	0	0	-1	0	0	1	10

$$\text{New } R_0 = OW R_0 + 10(R_1 + R_2 + R_3)$$

→ To remove -10

OW R ₀	10(R ₁ + R ₂ + R ₃)	New R ₀
1	0	1
-3	40	37
-2	40	38
-3	50	47
0	-10	-10
0	-10	-10
0	-10	-10
-10	10	0
-10	10	0
-10	10	0
0	270	270

Simplex Table 2:

↓

Row	Z	x	y	z	S ₁	S ₂	S ₃	A ₁	A ₂	A ₃	const	ratio
R ₀	1	37	38	47	-10	-10	-10	0	0	0	210	-
R ₁	0	1	2	2	-1	0	0	1	0	0	8	8/2=4
R ₂	0	1	1	1	0	-1	0	0	1	0	9	9/1=9
R ₃	0	2	1	2	0	0	-1	0	0	1	10	10/2=5

highest positive is 47, so 2 is key column and 4 is the lowest positive ratio, so R₁ is key row.
 \therefore 2 is the key element.

Then,

$$\text{New } R_1 = \text{Old } R_1 / \text{key element} = \text{Old } R_1 / 2$$

$$0 \quad 1/2 \quad 1 \quad 1 \quad -1/2 \quad 0 \quad 0 \quad 1/2 \quad 0 \quad 0 \quad 4$$

$$\text{New } R_0 = \text{Old } R_0 - 47 \times \text{New } R_1$$

$$\text{New } R_2 = \text{Old } R_2 - \text{New } R_1$$

$$\text{New } R_3 = \text{Old } R_3 - 2 \times \text{New } R_1$$

old R ₀ -	47 x New R ₁	New R ₀
1	0	1
37	47/2	27/2
38	47	-9
47	47	0
-10	-47/2	-67/2
-10	0	-10
-10	0	-10
0	47/2	-47/2
0	0	0
0	0	0
210	188	82

old R_2	New R_2	old $R_3 - 2 \times$ New R_1	New R_3
0	0	0	0
1	$1/2$	2	1
1	0	1	-1
1	0	2	0
0	$1/2$	0	1
-1	-1	0	0
0	0	-1	-1
0	$-1/2$	0	-1
1	1	0	0
0	0	1	1
9	5	10	2

Simplex Table 3:

	z	x	y	z	S_1	S_2	S_3	A_1	A_2	A_3	Const	Ratio
R_0	1	$27/2$	-9	0	$27/2$	-10	-10	$-1/2$	0	0	82	
R_1	0	$1/2$	1	1	$-1/2$	0	0	$1/2$	0	0	9	8
R_2	0	$1/2$	0	0	$1/2$	-1	0	$-1/2$	1	0	5	10
R_3	0	(1)	-1	0	1	0	-1	-1	0	1	2	2

here, $27/2$ is highest positive, so taking x as key column and 2 is lowest ratio, so taking 1 as key element.

new $R_3 \rightarrow$ old R_3 (same)

new $R_0 \rightarrow$ old $R_0 - 27/2 \times R_3$

new $R_1 \rightarrow$ old $R_1 - 1/2 R_3$

new $R_2 \rightarrow$ old $R_2 - 1/2 R_3$

Old R_0	$-27/2 \times \text{New } R_3$	New R_0	New R_0
12 1/2	27 1/2 0	0 1	1
27/2	27/2	4.5	0
-9	-27/2	0	4.5
0	0	0	0
27/2	27/2	-10	0
-10	0	3.5	-10
-10	-27/2	-10	3.5
-47/2	-27/2	0	-10
0	0	-13.5	-13.5 0
0	27/2		-13.5
82	27		55

Old R_1	New R_1	Old R_2	New R_2
0	0	0	0
1/2	0	1/2	0
1	1.5	0	0.5
1	1	0	0
-1/2	-1	1/2	0
0	0	-1	-1
0	0.5	0	0.5
1/2	1	-1/2	0
0	0	1	1
0	-0.5	0	-0.5
4	3	5	4

Simplex Table 5:

Row	2	x	y	z	S_1	S_2	S_3	A_1	A_2	A_3	const	ratio
R_0	1	0	0	-3	$\frac{2}{3}$	-10	2	-13	0	-12	46	
R_1	0	0	1	$\frac{2}{3}$	$-\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	0	$-\frac{1}{3}$	2	$\frac{2}{1-\frac{2}{3}}$
$\rightarrow R_2$	0	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	-1	$\frac{1}{3}$	$-\frac{1}{3}$	1	$-\frac{1}{3}$	3	9
R_3	0	1	0	$\frac{2}{3}$	$\frac{1}{3}$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{2}{3}$	4	12

S_1 is Key column and R_2 is Key ~~element~~ row so
 $\frac{1}{3}$ is Key element

new $R_2 \rightarrow$ old $R_2 / \frac{1}{3}$

$$0 \quad 0 \quad 0 \quad -1 \quad 1 \quad -3 \quad 1 \quad -1 \quad 3 \quad -1 \quad 9$$

new $R_0 \rightarrow$ old $R_0 - \frac{2}{3} R_2$

new $R_1 \rightarrow$ old $R_1 + \frac{2}{3} R_2$

new $R_3 \rightarrow$ old $R_3 - \frac{1}{3} R_2$

old R_0	new R_0	old R_1	new R_1	old R_2	new R_2
1	1	0	0	0	0
0	0	0	0	1	1
0	0	1	1	0	0
-3	0	$\frac{2}{3}$	0	$\frac{2}{3}$	1
3	0	$-\frac{2}{3}$	0	$\frac{1}{3}$	0
-10	-1	0	-2	0	1
2	-1	$\frac{1}{3}$	1	$-\frac{2}{3}$	-1
-13	-10	$\frac{2}{3}$	0	$-\frac{1}{3}$	0
0	-9	0	2	0	-1
-12	-9	$-\frac{1}{3}$	-1	$\frac{2}{3}$	1
46	19	2	8	4	1

Simplex Table 1:

↓

	2	x	y	z	S ₁	S ₂	S ₃	A ₁	A ₂	A ₃	const	R
R ₀	0	4.5	0	0	-10	3.5	-10					
R ₁	1	0	4.5	0	0	-10	3.5	-10	0	-13.5	55	
R ₂	0	0	1.5	1	-1	0	0.5	1	0	-0.5	3	2
R ₃	0	0	0.5	0	0	-1	0.5	0	1	-0.5	4	8
R ₄	0	1	-1	0	1	0	-1	-1	0	1	2	-2

here y is key column and R₁ is key row.
so 1.5 is key element.

↑

new R₁ → R₁ ÷ 1.5

0 0 1 2/3 -2/3 0 1/3 2/3 0 -1/3 2

new R₀ → old R₀ - 4.5 R₁

new R₂ → old R₂ - 0.5 R₁

new R₃ → old R₃ + R₁

old R ₀	new R ₀	old R ₂	new R ₂	old R ₃	new R ₃
1	1	0	0	0	0
0	0	0	0	1	1
4.5	0	0.5	0	-1	0
0	-3	0	-1/3	0	2/3
0	3	0	1/3	1	1/3
-10	-10	-1	-1	0	0
3.5	2	0.5	1/3	-1	-2/3
-10	-13	0	-1/3	-1	-1/3
0	0	1	1	0	0
-13.5	-12	-0.5	-1/3	1	2/3
55	46	4	3	2	4

Simplex table 6 :

Row	Z	x	y	z	S ₁	S ₂	S ₃	A ₁	A ₂	A ₃	Cost
R ₀	1	0	0	0	0	-1	-1	-10	-9	-9	19
R ₁	0	0	1	0	0	-2	1	0	2	-1	8
R ₂	0	0	0	-1	1	-3	1	-1	3	-1	9
R ₃	0	1	0	1	0	1	-1	0	-1	1	1

From table;

$$Z = \$19 \text{ (minimum cost)}$$

$$x = 1 \quad y = 8 \quad z = 0 \text{ (non-basic)}$$

So, 1 food A and 8 food B gives the minimum cost to fulfill the patient's diet.

$$x=0 \quad y=8 \quad z=1 \quad \checkmark$$

$$\begin{array}{l} 1V + 1M + 2C = 0 \quad \checkmark \\ 1V + M + 2C \\ \hline 16V + 8M + 8C \\ \hline 0 \end{array} \quad \checkmark$$

