MATH550/SCC461: Statistics in Practice
Lab 3
Assessment

Tom Palmer

Before you start

- **Deadline** for coursework submission is **9am Monday** 23^{rd} **October 2017**.
- Include your 8 digit library card number in the author field of your knit report.

Assessment

This assessment focuses on implementing an ecological model, the Lotka-Volterra model, which is otherwise known as a predator-prey model for the interaction between two species. This model has been applied to foxes and rabbits; cheetahs and baboons and lynx and hares amongst other species. The (deterministic) Lotka-Volterra model is usually expressed as a system of differential equations but we will work with a discrete time version similar to the population growth model.

Foxes and rabbits

We will implement the Lotka-Volterra model for two species, foxes (predator) and rabbits (prey). Let F_t and R_t denote the total numbers of foxes and rabbits at time t, respectively with F_1 and R_1 denoting the initial numbers of foxes and rabbits, respectively. The model is governed by three processes:

- **Birth of rabbits.** Birth rate, α. The number of births is proportional to the number of rabbits in the population and leads to an increase in the rabbit population.
- **Fox eats rabbit.** Consumption rate of foxes eating rabbits, β. The number of rabbits eaten is proportional to the number of rabbits in the population multiplied by the number of foxes in the population and leads to a decrease in the rabbit population and an increase in the fox population.
- **Death of Foxes.** Death rate, γ. The number of deaths is proportional to the number of foxes in the population and leads to a decrease in the fox population.

The model leads to the following pair of equations for (F_{t+1}, R_{t+1}) in terms of (F_t, R_t).

$$R_{t+1} = R_t + \alpha R_t - \beta R_t F_t \quad (1)$$

$$F_{t+1} = F_t + \beta R_t F_t - \gamma F_t. \quad (2)$$

That is, the number of rabbits at time $t+1$ is equal to the number of rabbits at time t plus the new rabbits born minus the rabbits which have been eaten. Similarly, the number of foxes at time $t+1$ is equal to the number of foxes at time t plus the new foxes resulting from eating rabbits minus the foxes which have died.

We assume that each time-point corresponds to one week with $\alpha = 0.05$, $\beta = 0.0001$, $\gamma = 0.02$.

Tasks

1. Set $R_1 = 30$ and $F_1 = 40$.

   Using existing functions in R, write the necessary for loop to implement the Lotka-Volterra model that will allow you to project the number of foxes and rabbits over a 2 year (104 week) period. Print the final result.

   [3 marks]

2. A stochastic version of the Lotka-Volterra model exists in a similar manner to the stochastic version of the population growth model. In this case the number of rabbits born is $\text{Binom}(R_t, \alpha)$, the number of rabbits eaten (new foxes) is $\text{Binom}(R_t F_t, \beta)$ and the number of foxes that die is $\text{Binom}(F_t, \gamma)$.

   Set the seed for running your code to 60854.

   Using existing functions in R, write the necessary for loop to implement the stochastic Lotka-Volterra model that will allow you to project the number of foxes and rabbits over a 2 year (104 week) period with the same starting values as for the deterministic model. Print the final result.

   Hint: The number of new foxes needs to be equal to the number of rabbits eaten.

   [4 marks]

3. Create a *long* data frame called '`LV`' with three variables; '`time`', '`group`' and '`size`'. Each row should contain the size at a single time point for one of the four groups generated `rabbits` and `foxes` (deterministic model); `sto_rabbits` and `sto_foxes` (stochastic model).

   Using `ggplot()` visualise the changes over time for the number of rabbits and foxes for both the deterministic and stochastic version of the Lotka-Volterra model.

   [3 marks]