Module 1: AI for Channel Decoding

- You will learn:
  - How to use colab
  - Basics of channel coding
  - Support vector machine
  - Deep learning

- Grading:
  - Syndrome Decoding, 20% ML decoding 10%
  - Classification with support vector machine 20%
  - Deep Learning 20%
  - Mini project 20%
  - Report (no more than 10 pages) 10%

Error Correction Codes

Terminology

- Message: Sequence of bits representing your date (link to the website)
- Codeword: Sequence of bits forming your QR code

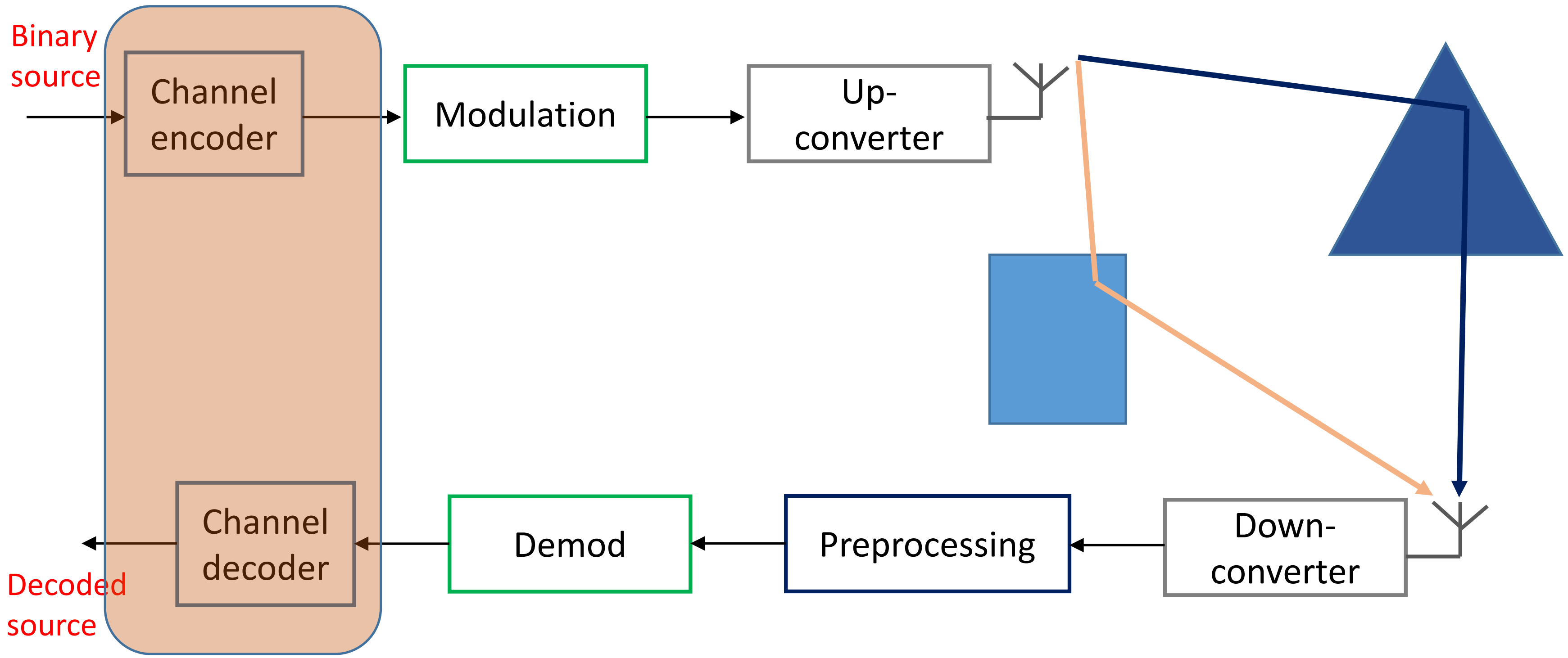
- Example:
  - Message = 1 1 0 1
  - Codeword = 1 1 0 1 0 1 0

These bits are added for error correction

It is everywhere!!! Even in string theory!!!

Sylvester James Gates, "Symbols of Power," *Physics World*, vol. 23, no. 6, June 2010.

A digital communication system over wireless channel

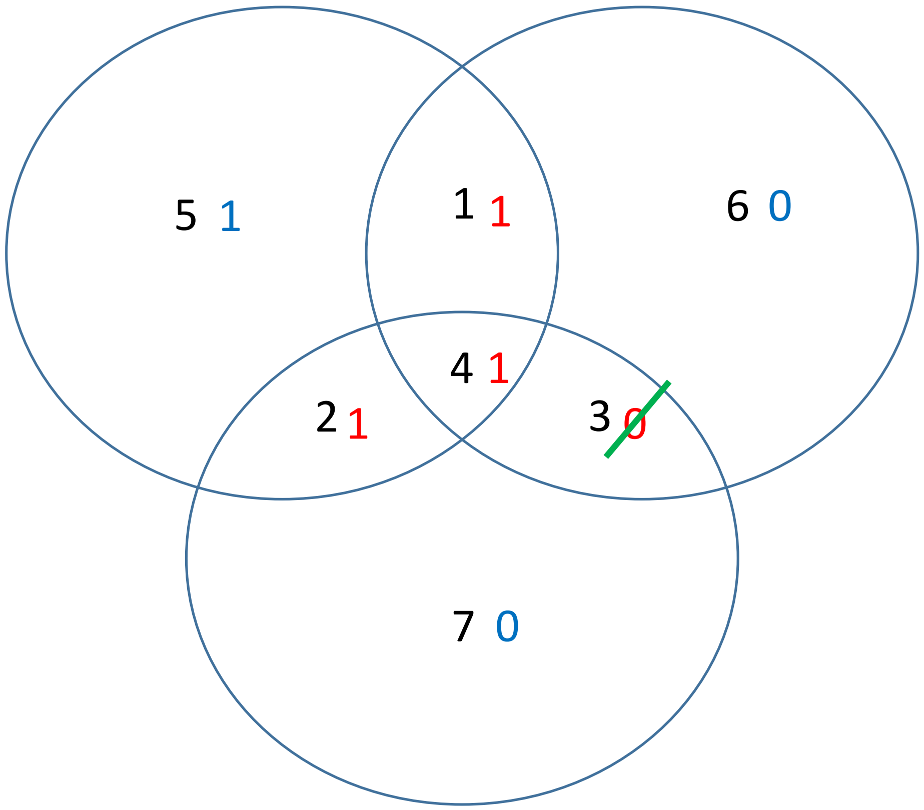


- Channel enc/dec allows error correction by adding redundancy

How it works? How to create those bits?

Index: 1 2 3 4 5 6 7

Codeword: 1 1 0 1 1 0 0

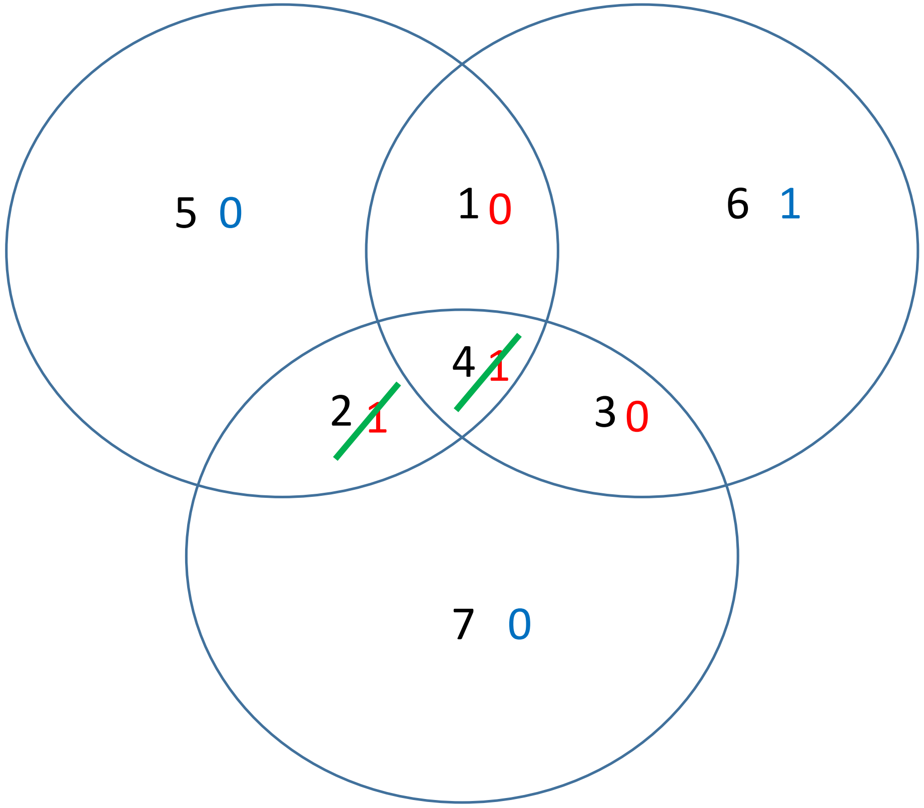


The last 3 bits are added such that inside every circle the number of 1s are even

Suppose 1 bit is erased, we can fix it by checking the parity of each circle

Index: 1 2 3 4 5 6 7

Codeword: 0 1 0 1 ? ? ?

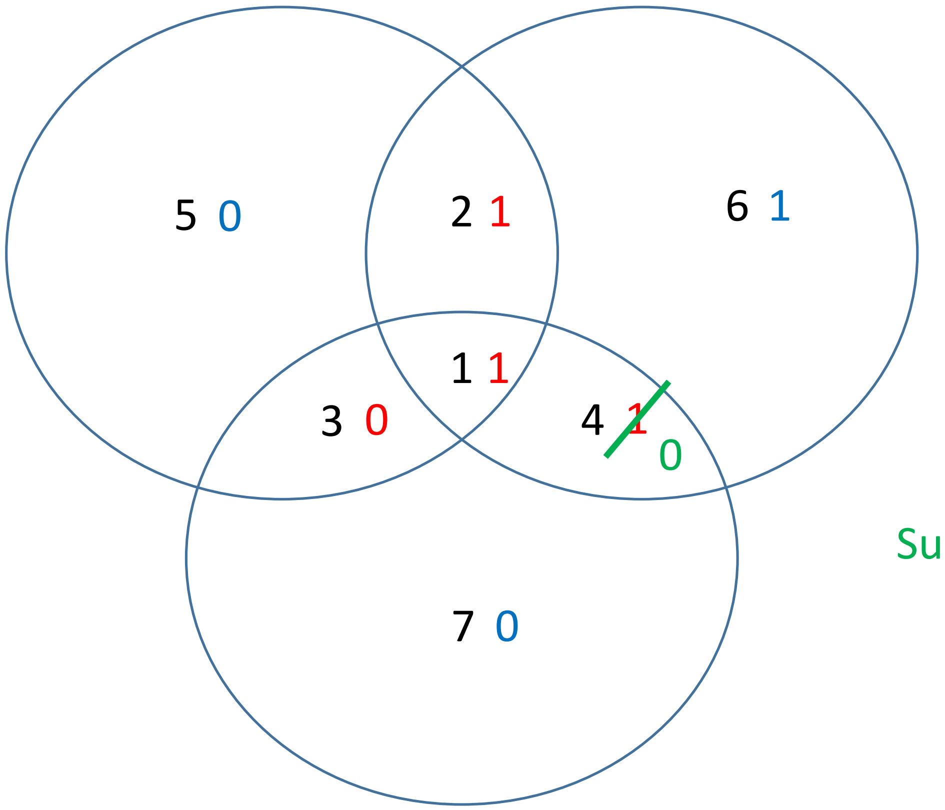


It's your turn

What if 2 bits are erased?

Index: 1 2 3 4 5 6 7

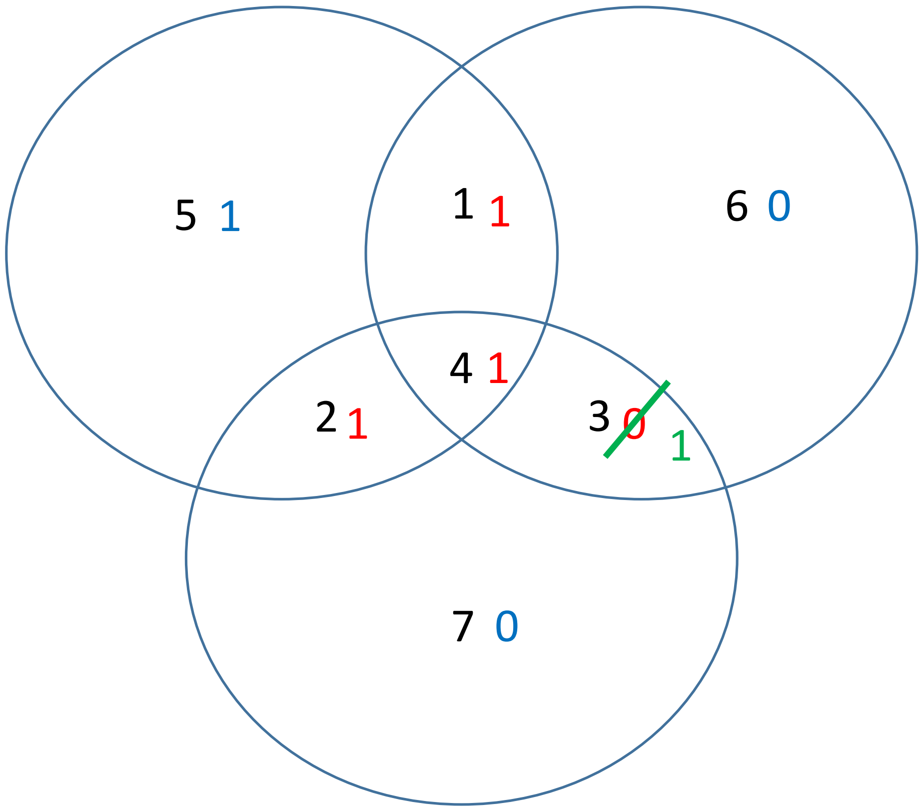
Codeword: 1 1 0 1 0 1 0



It can also correct 1 bit flip

Suppose 1 bit is flipped, we can fix it by flipping 1 bit to meet all constraints

Index: 1 2 3 4 5 6 7
Codeword: 1 1 0 1 1 0 0



It can also correct 1 bit flip

Suppose 1 bit is flipped, we can fix it by flipping 1 bit to meet all constraints

(n,k)-Linear Block Codes

- k-bit message $\boldsymbol{m}$, n-bit codeword $\boldsymbol{c}$
- Relationship: $\boldsymbol{c} = \boldsymbol{mG}$

- The code C contains all (2^k in total) such codewords
  - C is the row space of $\boldsymbol{G}$ ($k \times n$)
  - Call it a generator matrix
- There exists $\boldsymbol{H}$ ($n - k \times n$) such that $\boldsymbol{cH}^T = \boldsymbol{Hc}^T = \boldsymbol{0}$
  - Rows of $\boldsymbol{H}$ span the nullspace of $\boldsymbol{G}$
  - Call it a parity check matrix

(*7,4*)-Hamming Codes

- *4*-bit message $\boldsymbol{m}$, *7*-bit codeword $\boldsymbol{c}$
- Relationship: $\boldsymbol{c} = \boldsymbol{mG}$
- $$\mathbf{G} := \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$
- $$\mathbf{H} := \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

System Model

- When transmit, we map coded bits to baseband signal
- Binary phase shift keying (BPSK) for sending the bit c_i

$$x_i = \sqrt{P}(2c_i - 1), \qquad \boldsymbol{x} = [x_1, \dots, x_n]$$

- Additive white Gaussian noise (AWGN) channel

$$y_i = x_i + w_i, \qquad w_i \sim N(0, \sigma^2), \qquad \boldsymbol{y} = [y_1, \dots, y_n]$$

BER in Uncoded System

- Detection of x_i from y_i

$$\hat{x}_i = sign\,(y_i) \quad \text{and} \quad \hat{r}_i = (\hat{x}_i + 1)/2$$

- Error if $\hat{x}_i \neq x_i$

- Bit error rate: $p_e = \sum 1(\{\hat{x}_i \neq x_i\})/n$

Maximum Likelihood Decoding

MLD: $$\hat{\boldsymbol{c}} = arg\min_{\boldsymbol{c} \in C} ||\boldsymbol{y} - \boldsymbol{x}||^2$$

- This is optimal in AWGN channel with equiprobable inputs

- Complexity is very high, especially when *k* is large
  - Need to check 2^k codewords

Syndrome Decoding

- First make a hard decision

$$\hat{x}_i = sign\,(y_i) \quad \text{and} \quad \hat{r}_i = (\hat{x}_i + 1)/2$$

- Construct the standard array
- Compute the syndrome $\hat{\boldsymbol{r}}\boldsymbol{H}^T = \boldsymbol{s}$
- Decide $\hat{\boldsymbol{e}} = \text{coset leader}(\boldsymbol{s})$
- Decode to $\hat{\boldsymbol{c}} = \boldsymbol{r} + \hat{\boldsymbol{e}}$

Standard Array

$\underline{c}_1 = \underline{0}$	$\underline{c}_2$	$\cdots$	$\underline{c}_i$	$\cdots$	$\underline{c}_{2^k}$	$\leftarrow$ coset$(\underline{0})$
$\underline{e}_2$	$\underline{c}_2 + \underline{e}_2$	$\cdots$	$\underline{c}_i + \underline{e}_2$	$\cdots$	$\underline{c}_{2^k} + \underline{e}_2$	$\leftarrow$ coset$(\underline{e}_2 H^T)$
$\underline{e}_3$	$\underline{c}_2 + \underline{e}_3$	$\cdots$	$\underline{c}_i + \underline{e}_3$	$\cdots$	$\underline{c}_{2^k} + \underline{e}_3$	$\vdots$
$\vdots$	$\vdots$		$\vdots$		$\vdots$	
$\underline{e}_j$	$\underline{c}_2 + \underline{e}_j$	$\cdots$	$\underline{c}_i + \underline{e}_j$	$\cdots$	$\underline{c}_{2^k} + \underline{e}_j$	$\leftarrow$ coset$(\underline{e}_j H^T)$
$\vdots$						$\vdots$
$\underline{e}_{2^{n-k}}$	$\underline{c}_2 + \underline{e}_{2^{n-k}}$	$\cdots$	$\underline{c}_i + \underline{e}_{2^{n-k}}$	$\cdots$	$\underline{c}_{2^k} + \underline{e}_{2^{n-k}}$	$\leftarrow$ coset$(\underline{e}_{2^k} H^T)$

Decoding via Learning

- Decoding is nothing but classification
- *(n,k)*-linear block code has 2^k classes
- We can generate a lot of training data $(\boldsymbol{y}, \boldsymbol{c})$
- This is a supervise, batch, passive, and statistical learning