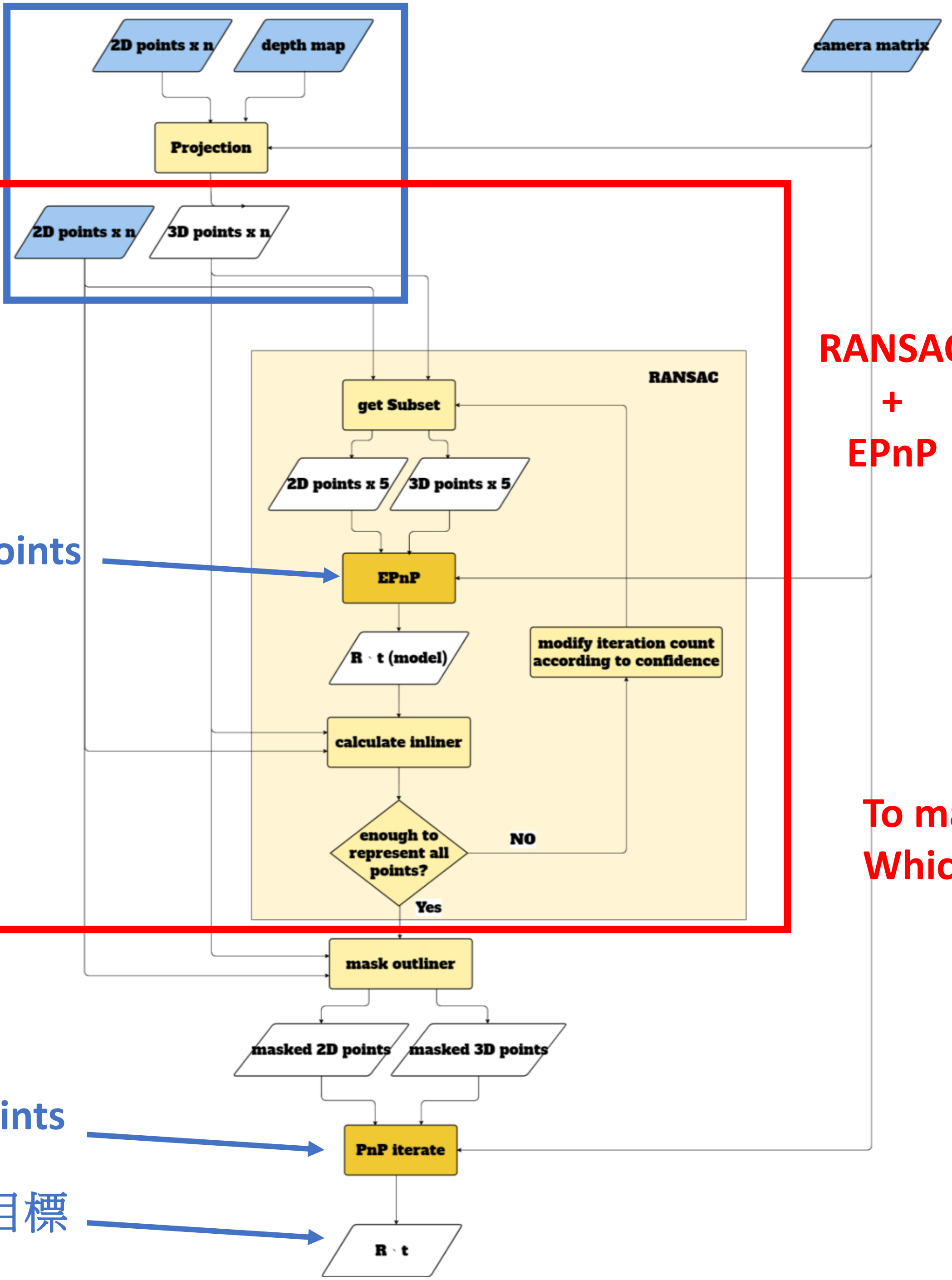
Motion Estimation

Overview:

2D -> 3D



RANSAC

+

EPnP

Need 2D 、3D points

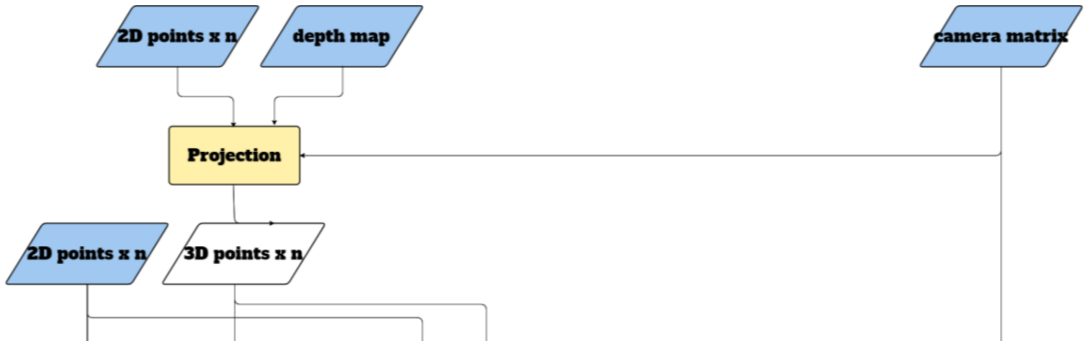
To many points

Which points are representative?

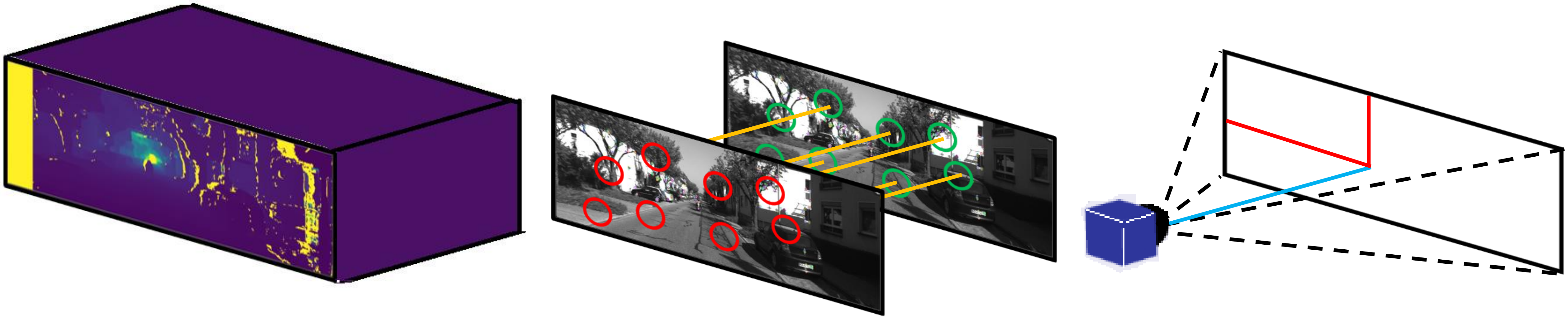
Need 2D 、3D points

目標

Project 2D to 3D points:



What we have:



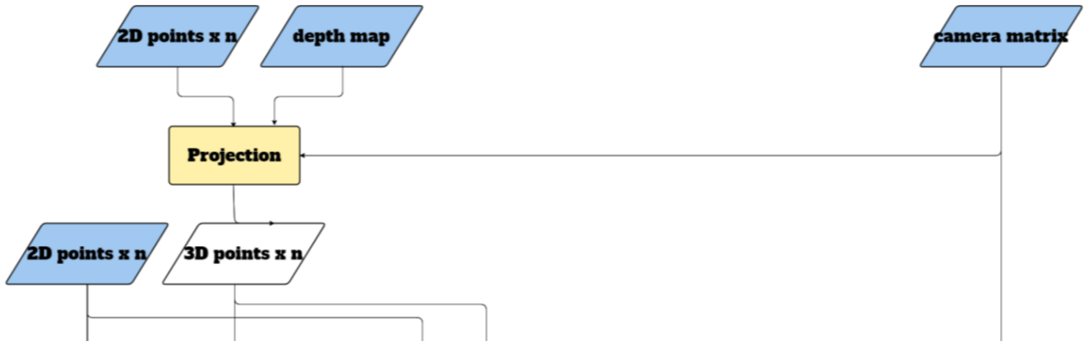
depth

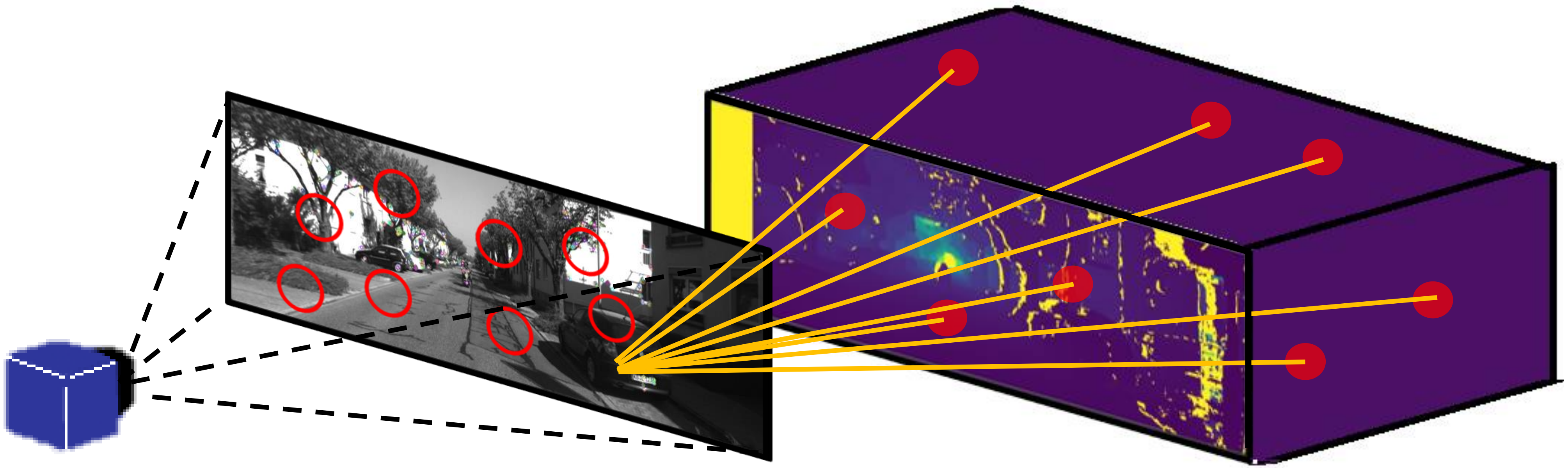
keypoints1、keypoint2
(match)

camera matrix

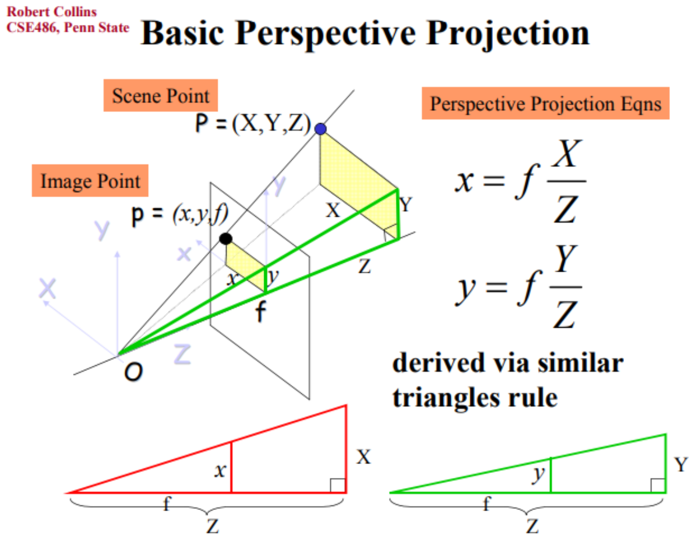
Project 2D to 3D points:

keypoints1 + depth = object points
(2D) (3D)





Project 2D to 3D points:



$$k = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ f \end{bmatrix} = \frac{f}{Z_c}\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \frac{1}{f} * \frac{f}{\lambda} k[R|t]\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

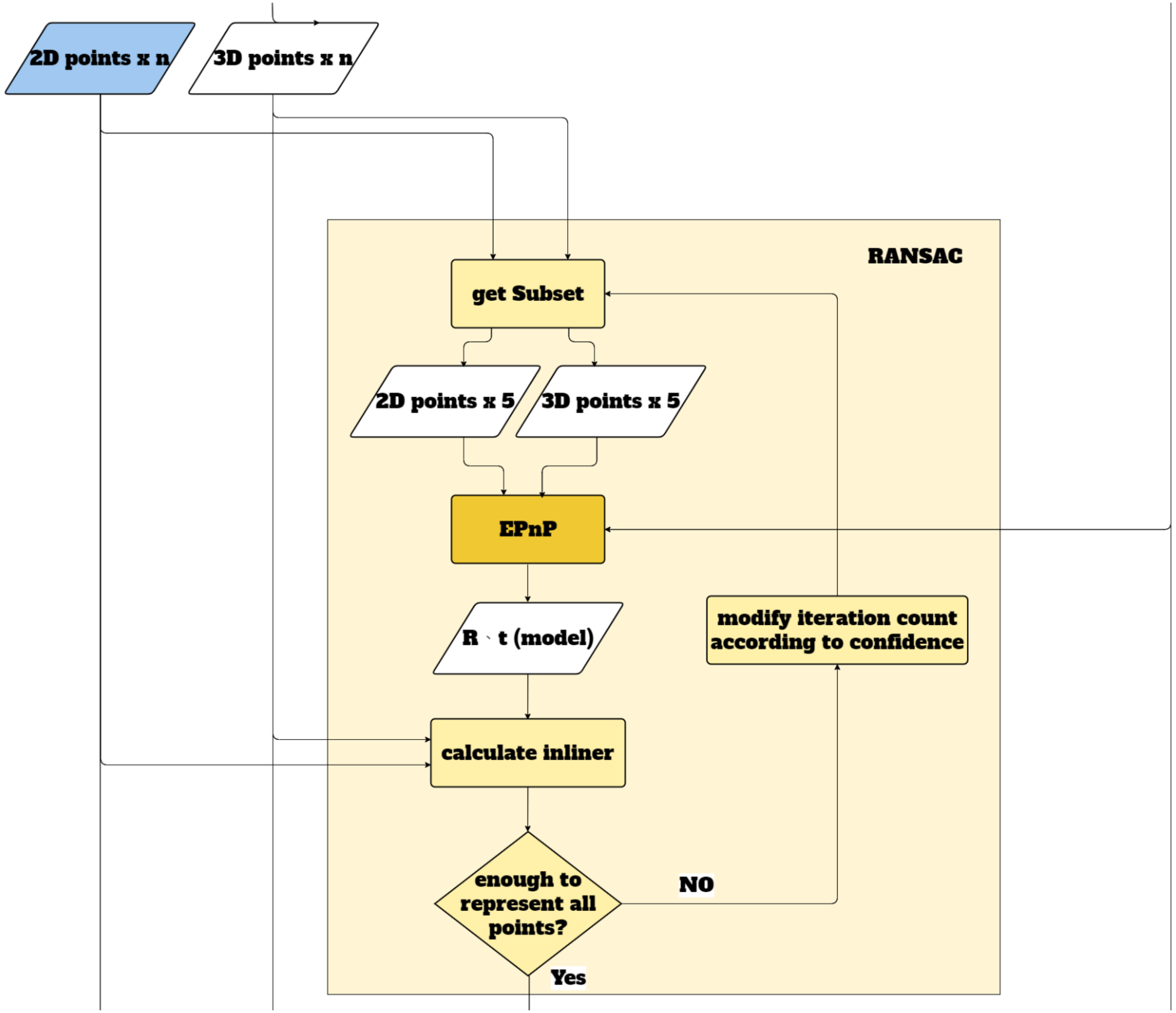
$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x\tilde{x} + c_x \\ f_y\tilde{y} + c_y \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{f_x}{Z_c}X_c + c_x \\ \frac{f_y}{Z_c}Y_c + c_y \\ \frac{Z_c}{Z_c} \end{bmatrix} \begin{matrix} \rightarrow \\ \rightarrow \\ \end{matrix}$$

$X_c = Z_c(u - c_x)/f_x$

$Y_c = Z_c(v - c_y)/f_y$

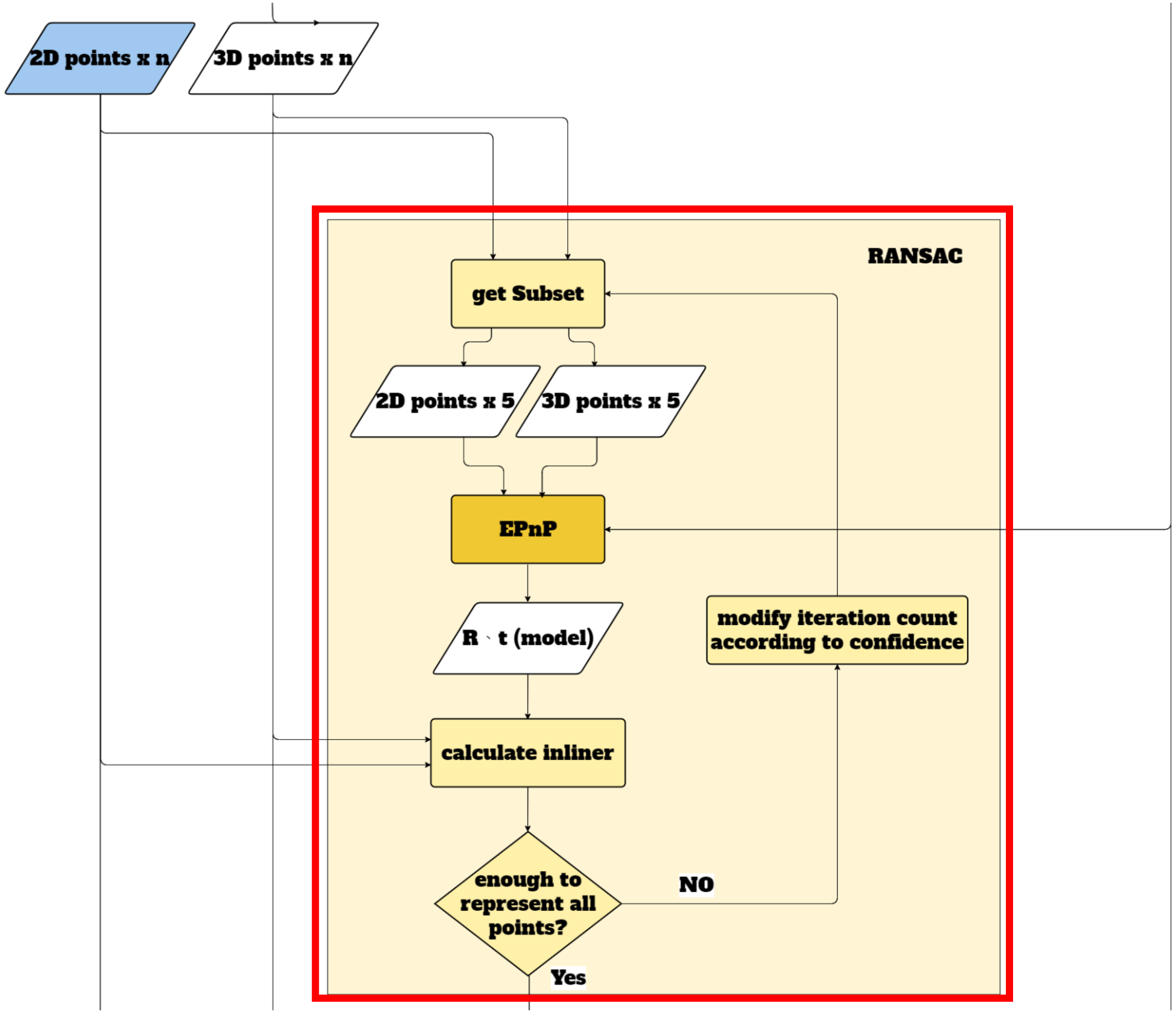
```
float cx = k.at<float>(0, 2);
float cy = k.at<float>(1, 2);
float fx = k.at<float>(0, 0);
float fy = k.at<float>(1, 1);
Mat object_points = Mat::zeros(0, 3, CV_32F);

for (int i = 0; i < image1_points__.rows; i++)
{

    float u = image1_points__.at<float>(i, 0);
    float v = image1_points__.at<float>(i, 1);
    float z = depth1.at<float>((int)v, (int)u);

    if (z > max_depth)
    {
        continue;
    }

    float x = z * (u - cx) / fx;
    float y = z * (v - cy) / fy;

    Mat vec = Mat(1, 3, CV_32F);
    vec.at<float>(0, 0) = x;
    vec.at<float>(0, 1) = y;
    vec.at<float>(0, 2) = z;

    object_points.push_back(vec);
    image1_points.push_back(image1_points__.row(i));
    image2_points.push_back(image2_points__.row(i));
}
```

RANSAC + EPnP:



2D points x n
3D points x n
RANSAC
get Subset
2D points x 5
3D points x 5
EPnP
R、t (model)
modify iteration count according to confidence
calculate inliner
enough to represent all points?
NO
Yes

RANSAC :



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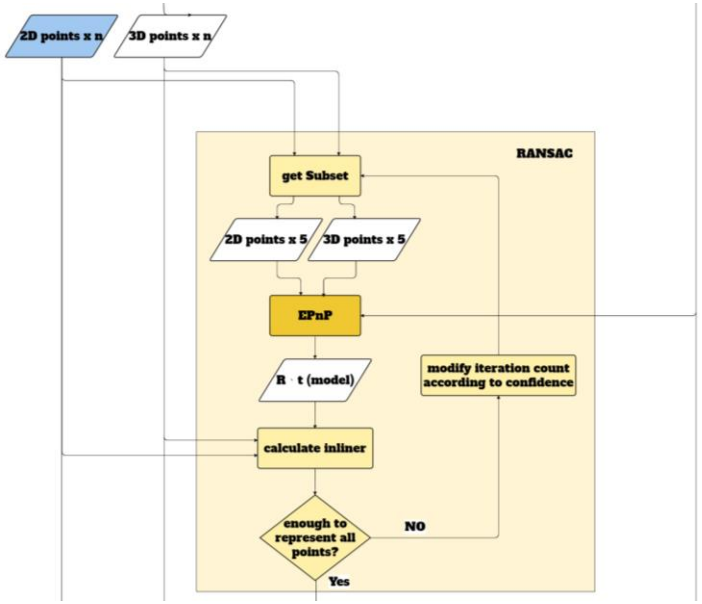
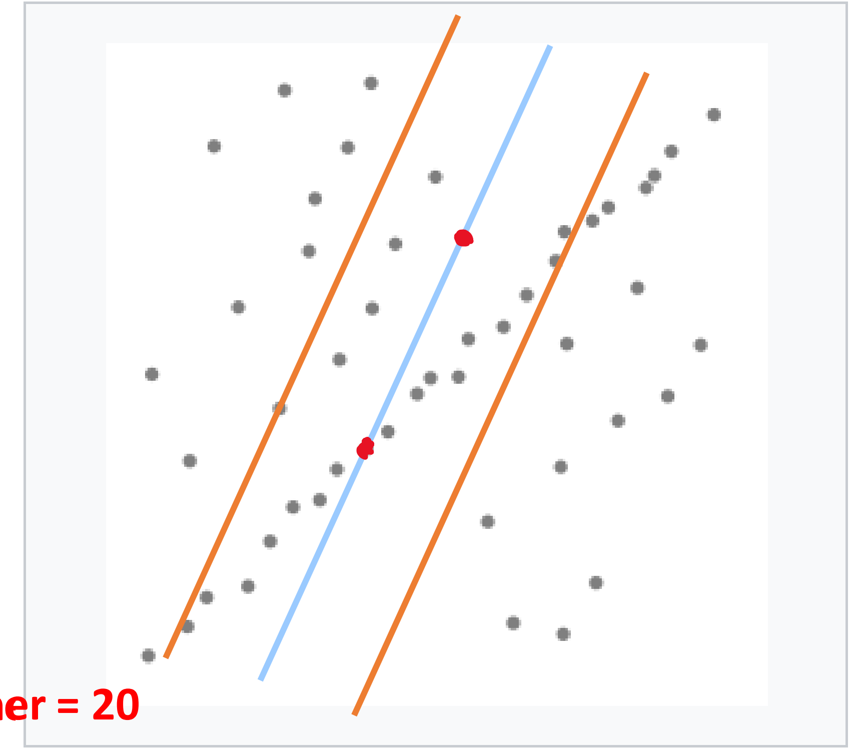




N = 30

#inliner = 20

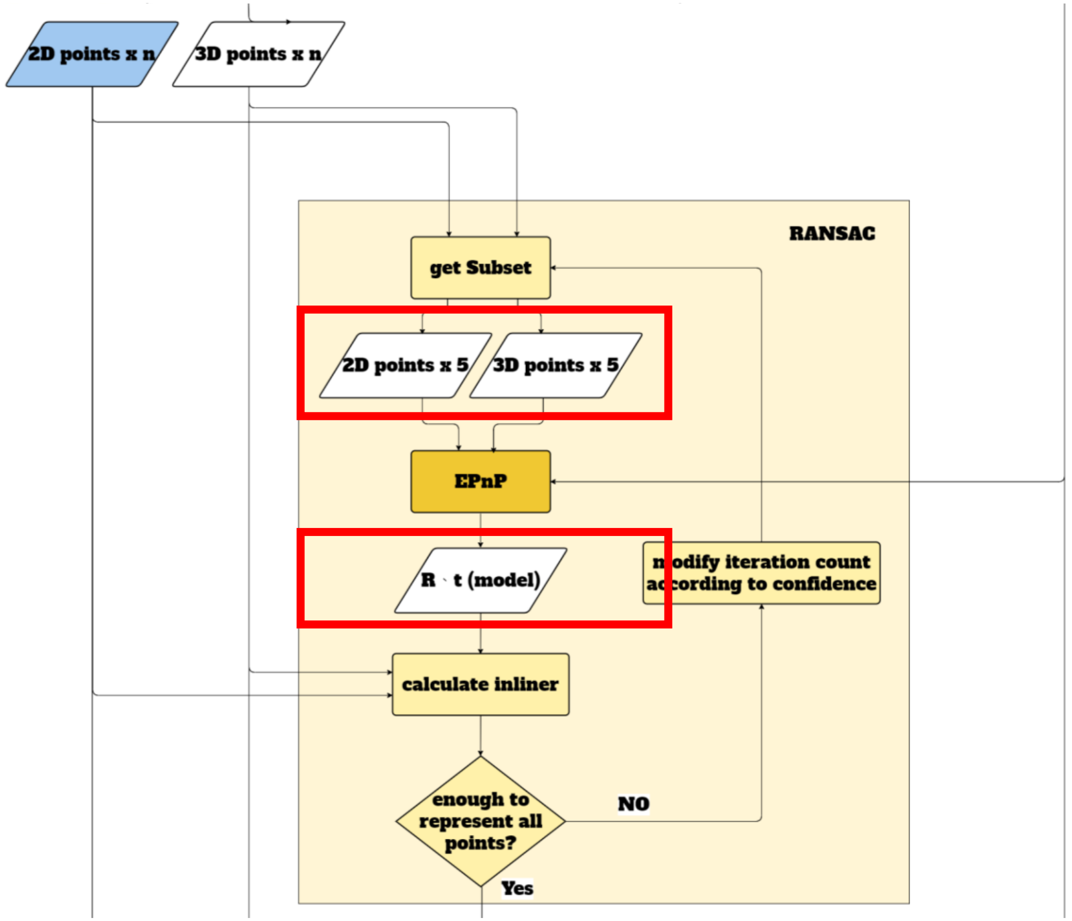
C(n, 2) How many time?

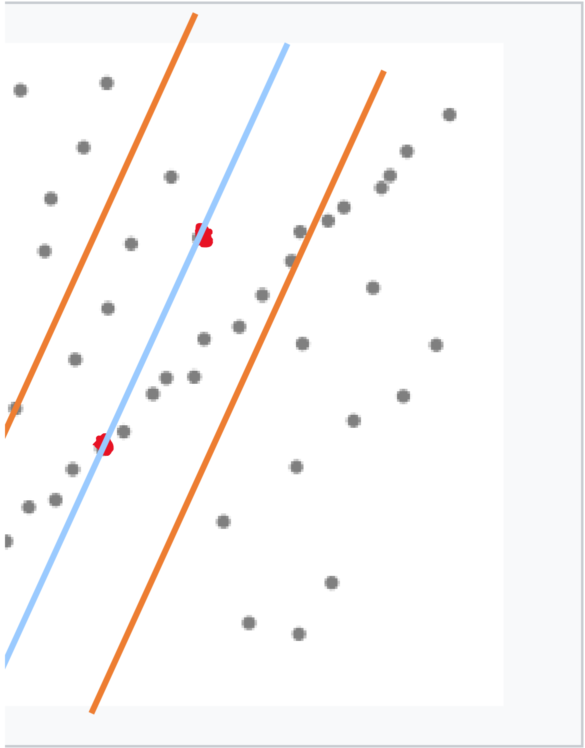
Given confidence 60%
If #inlier / N > 60%, stop

RANSAC :

1. subset 2. model 3. Threshold to calculate inlier

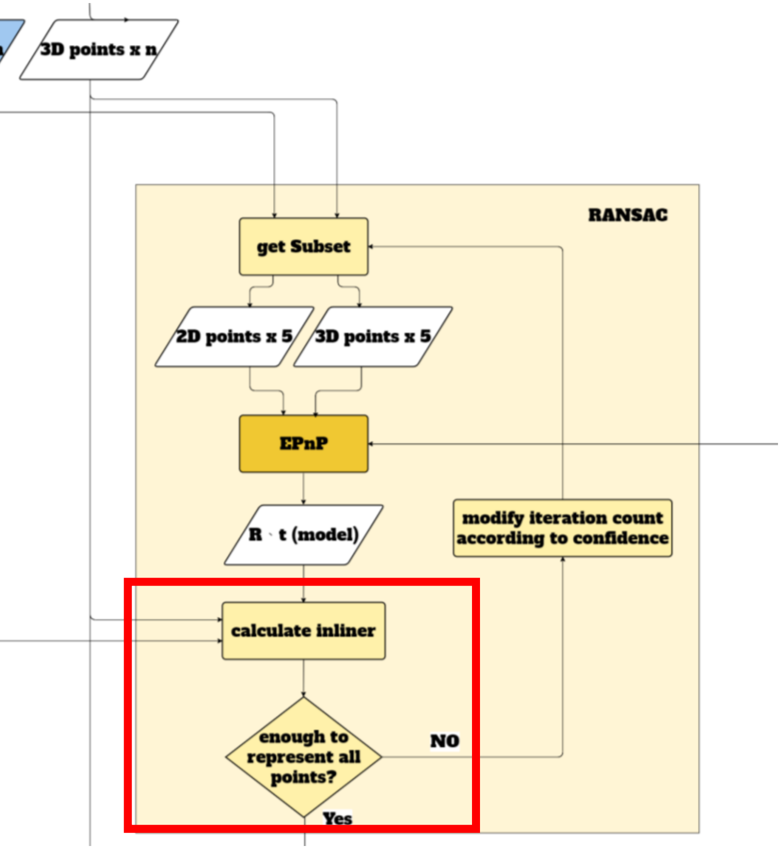
4. Confidence to tell when to stop

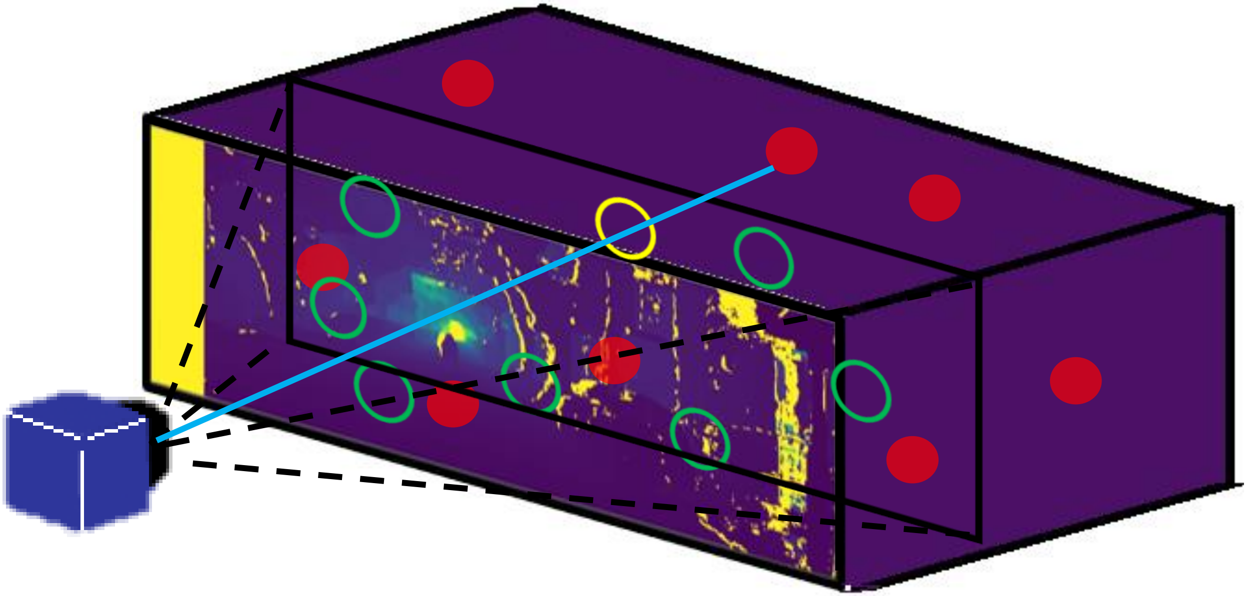




RANSAC :

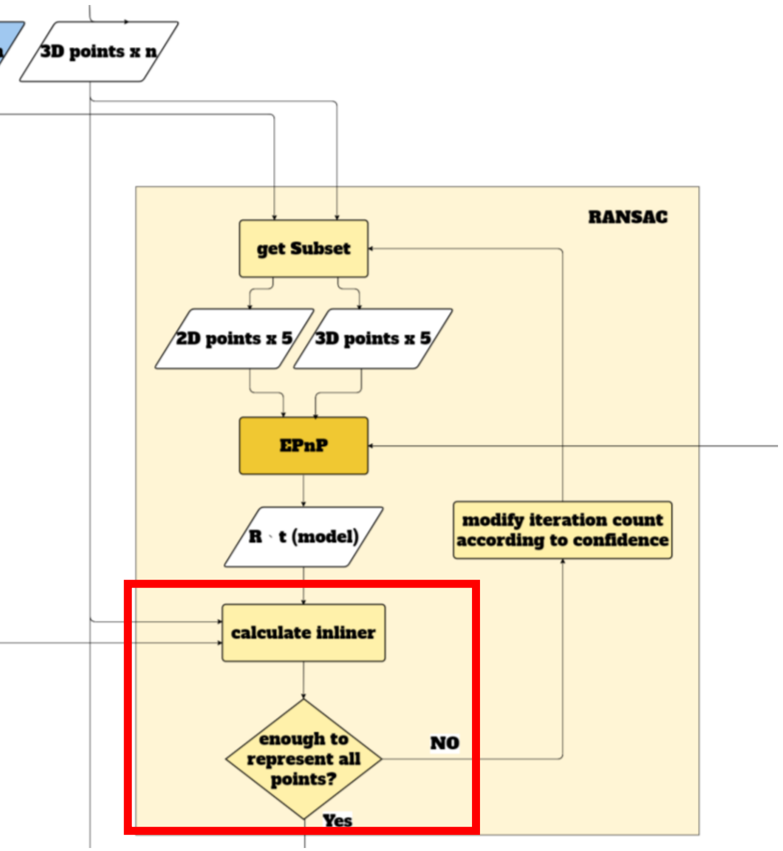
random取一組match做epnp
得到R1、t1

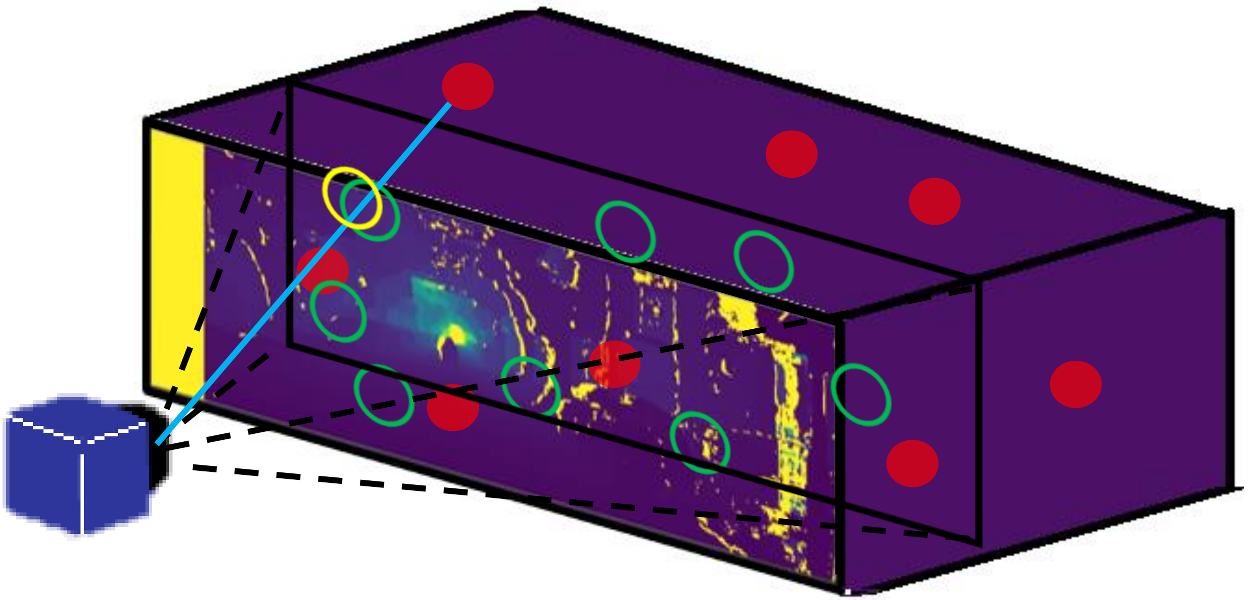




RANSAC :

用得到的R1、t1
對其他object points做投影





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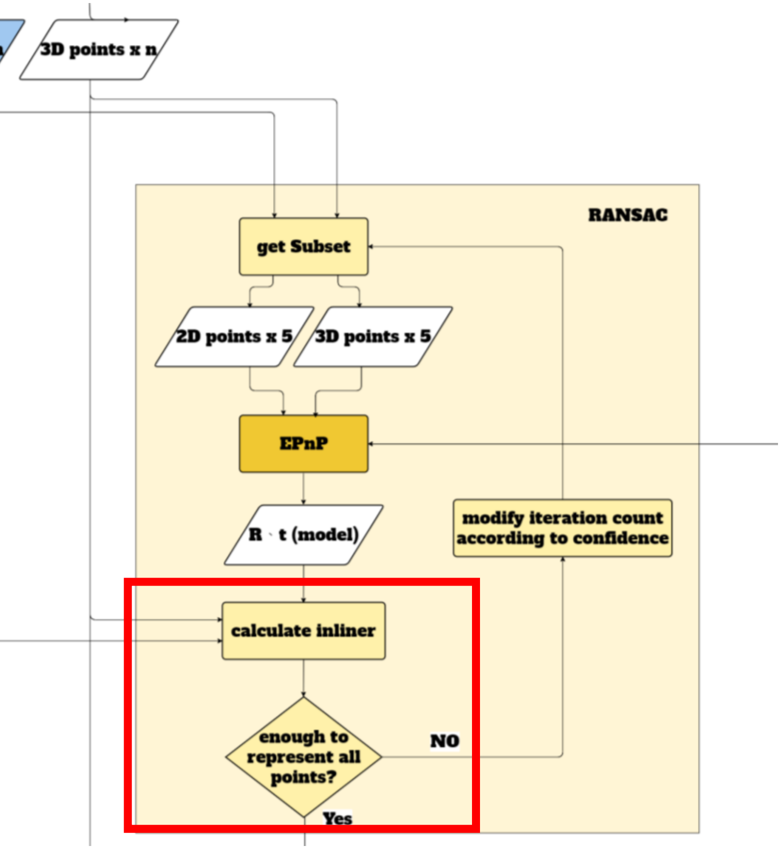

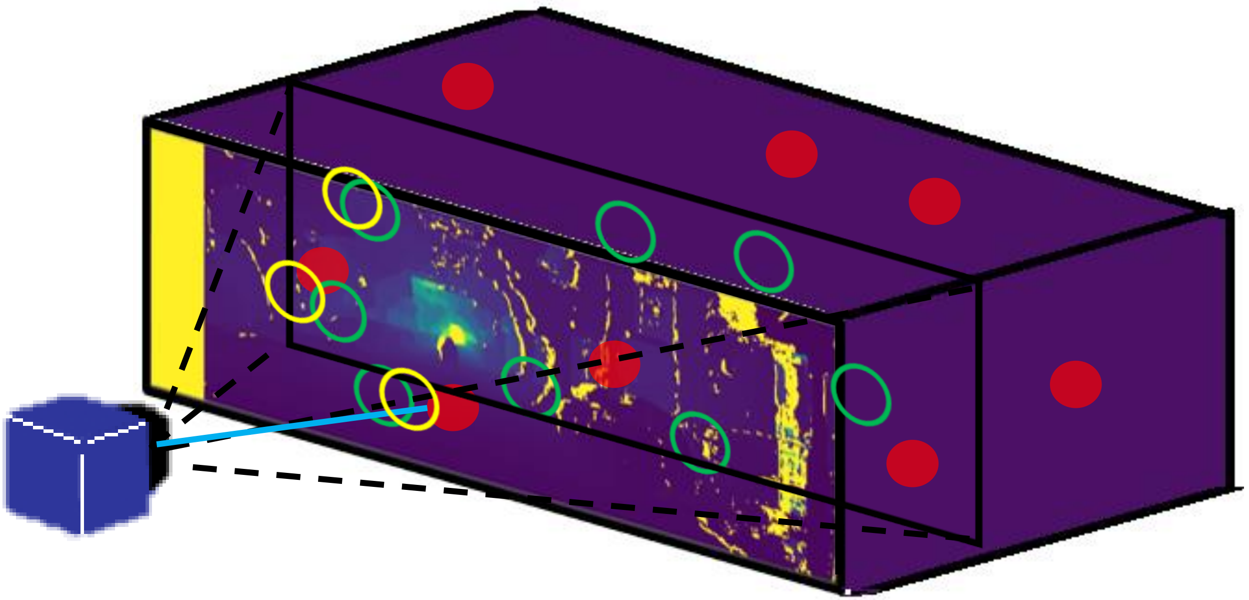

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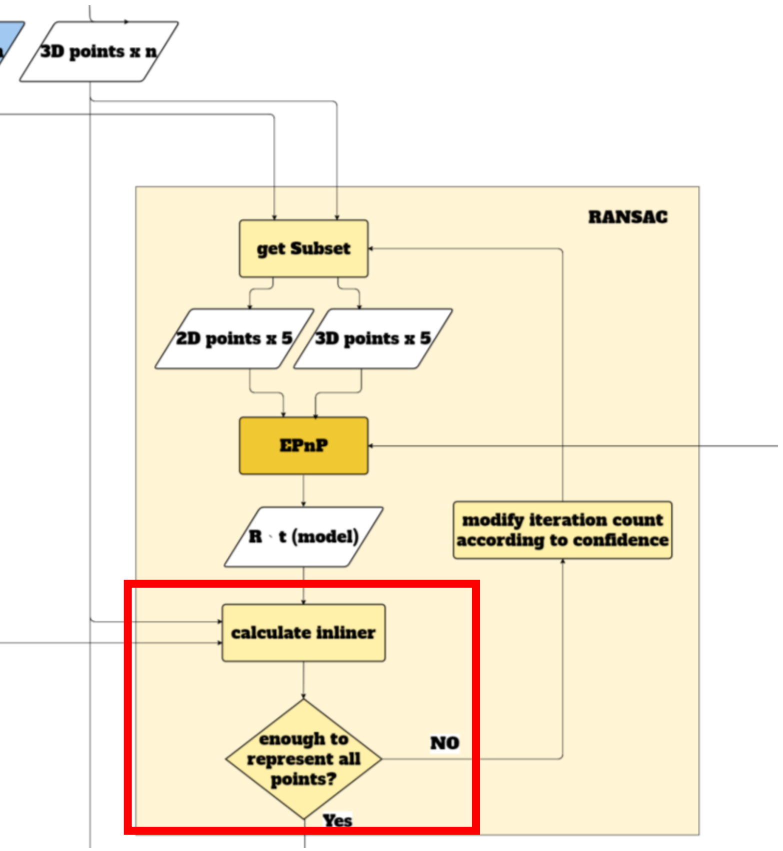

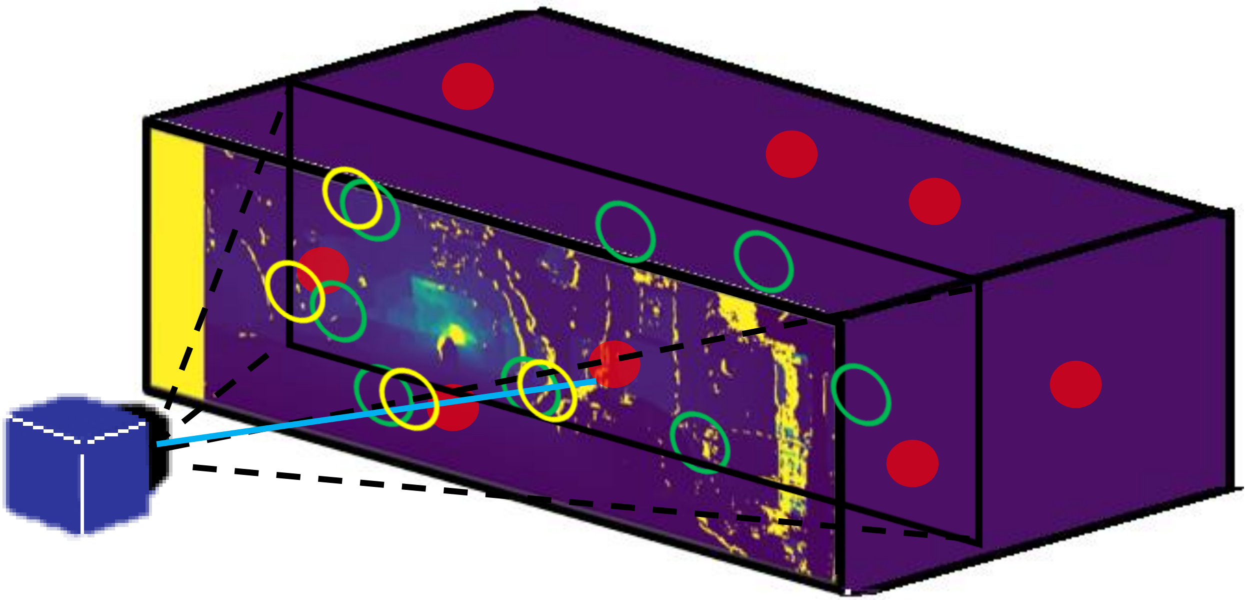

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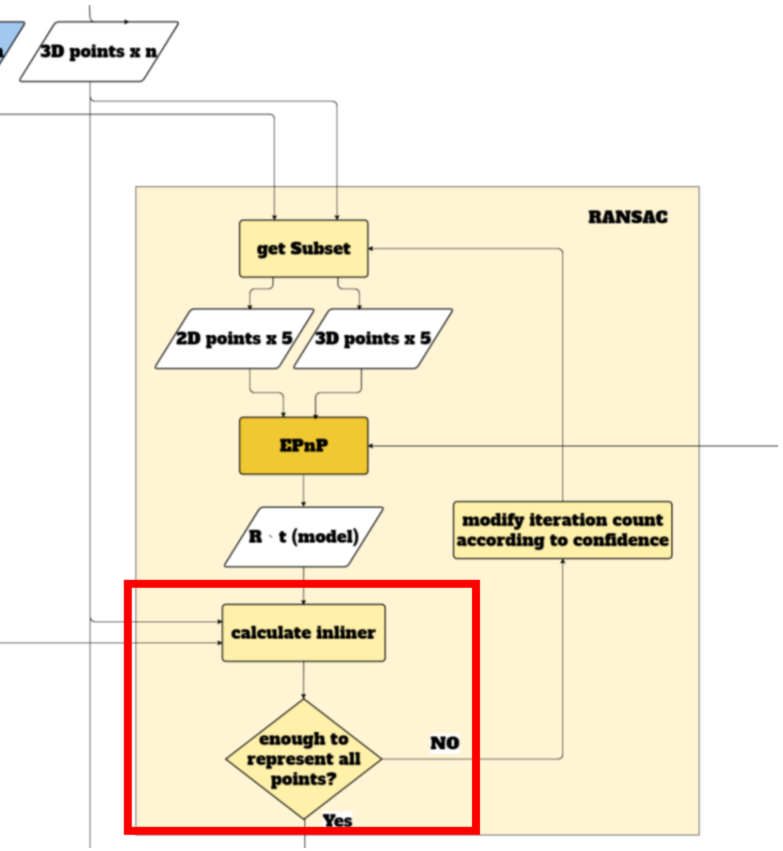

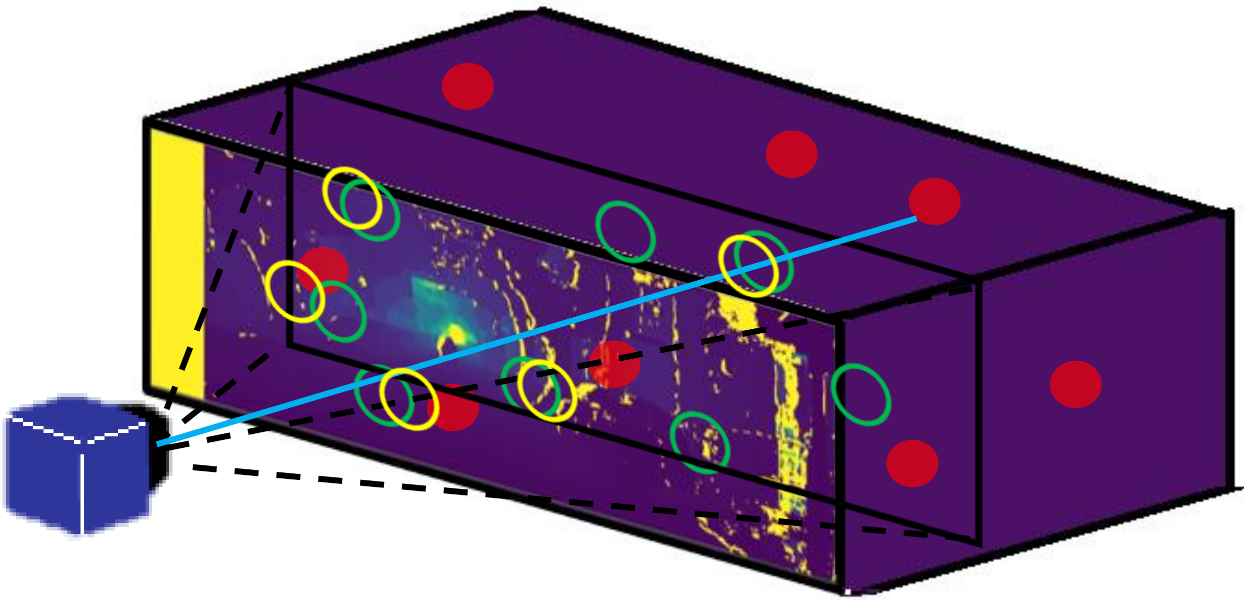

RANSAC :

用得到的R1、t1
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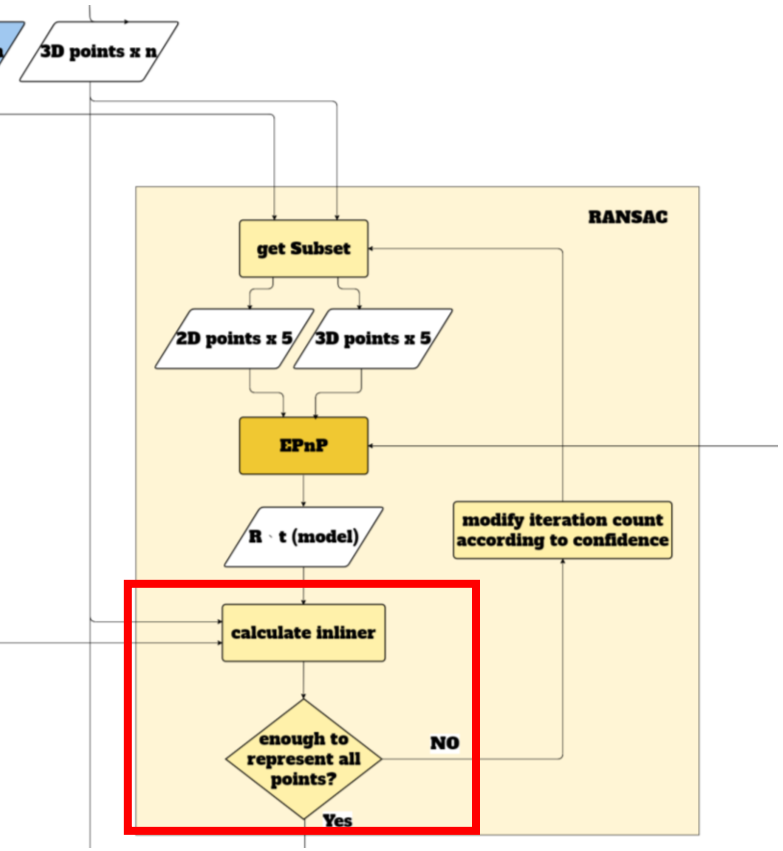
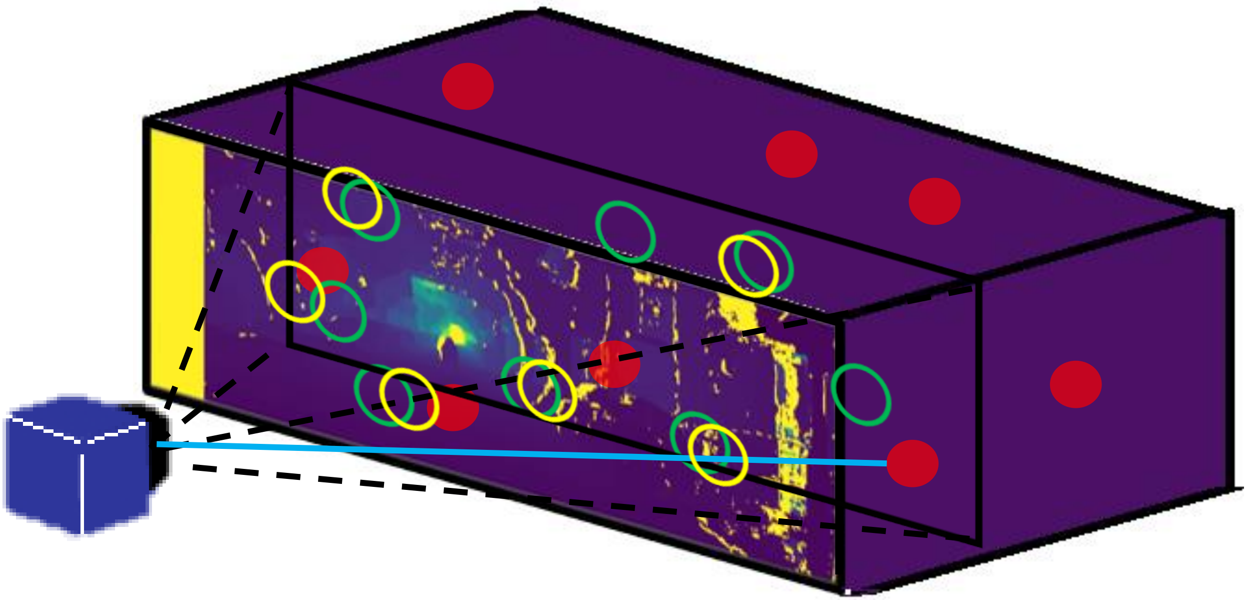
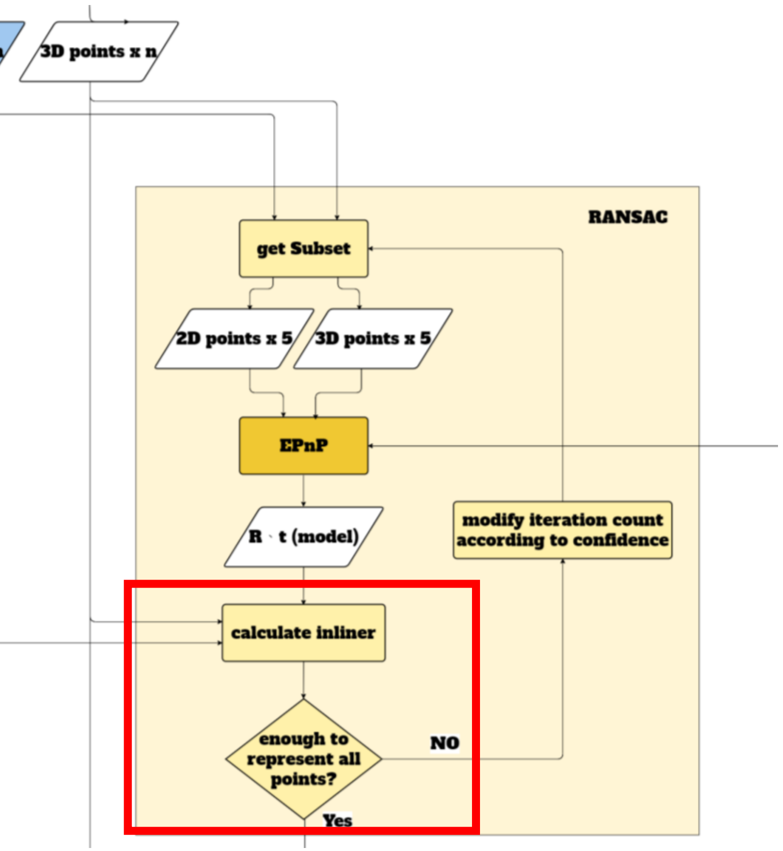
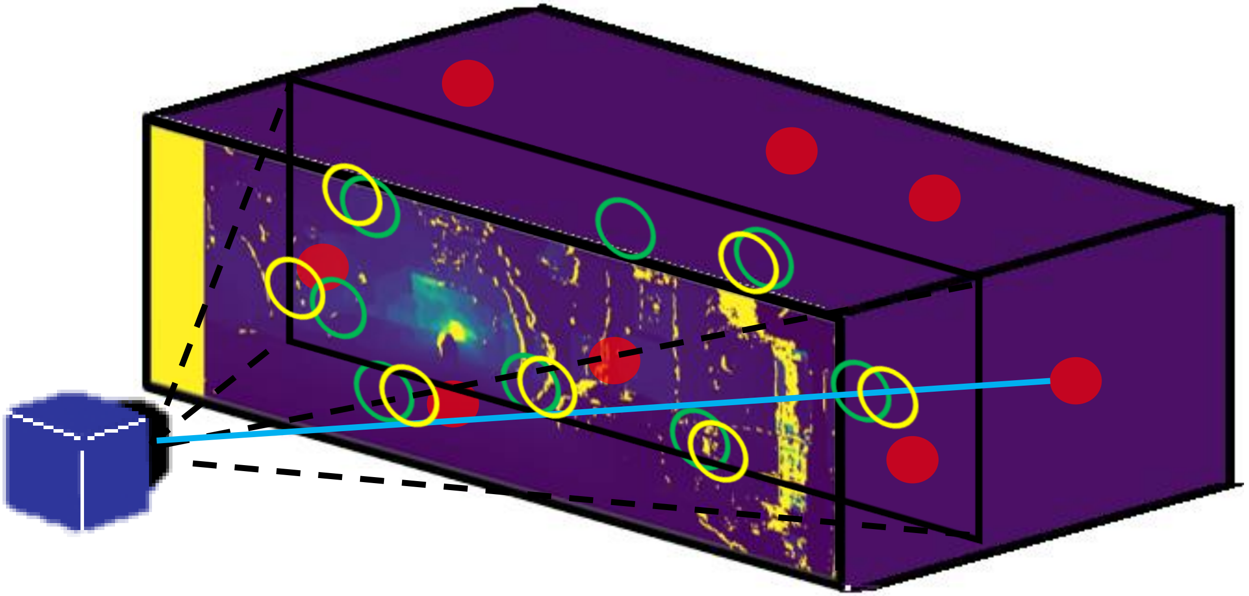




RANSAC :

用得到的R1、t1
對其他object points做投影





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用得到的R1、t1
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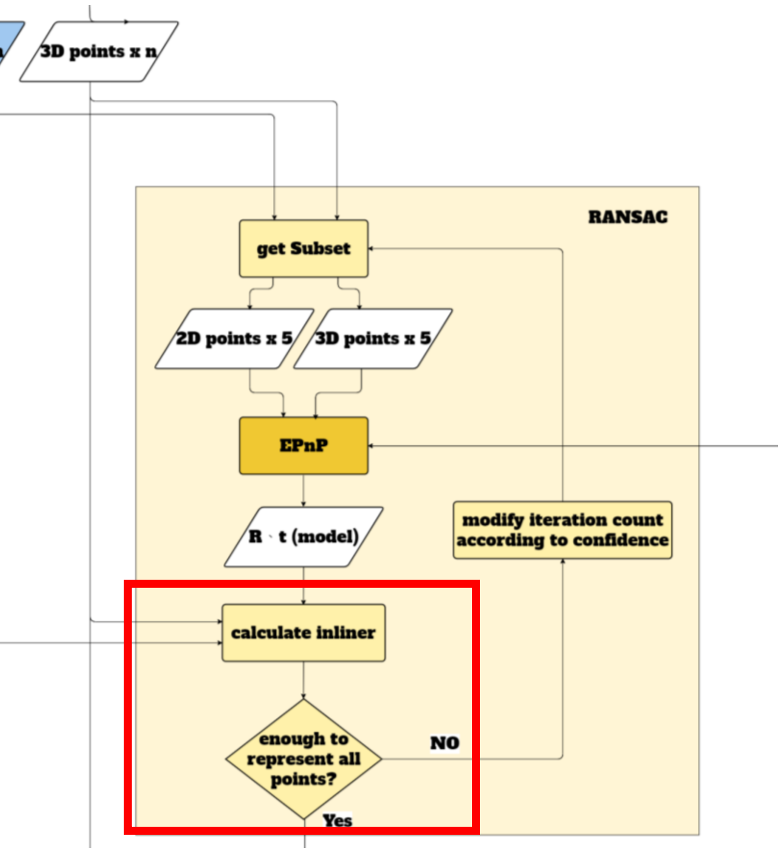
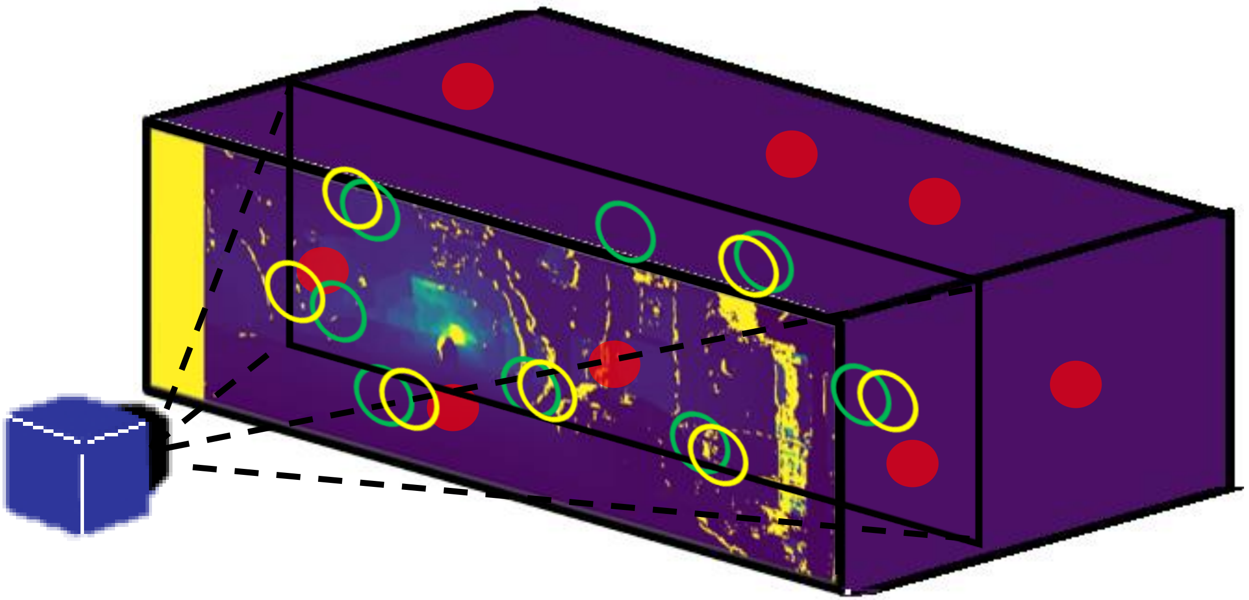
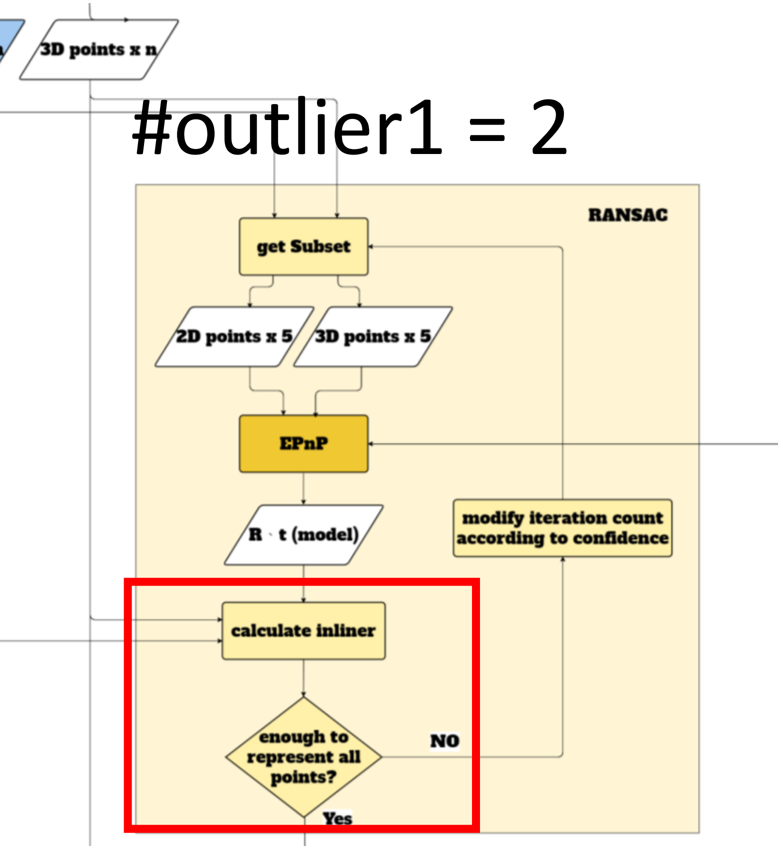


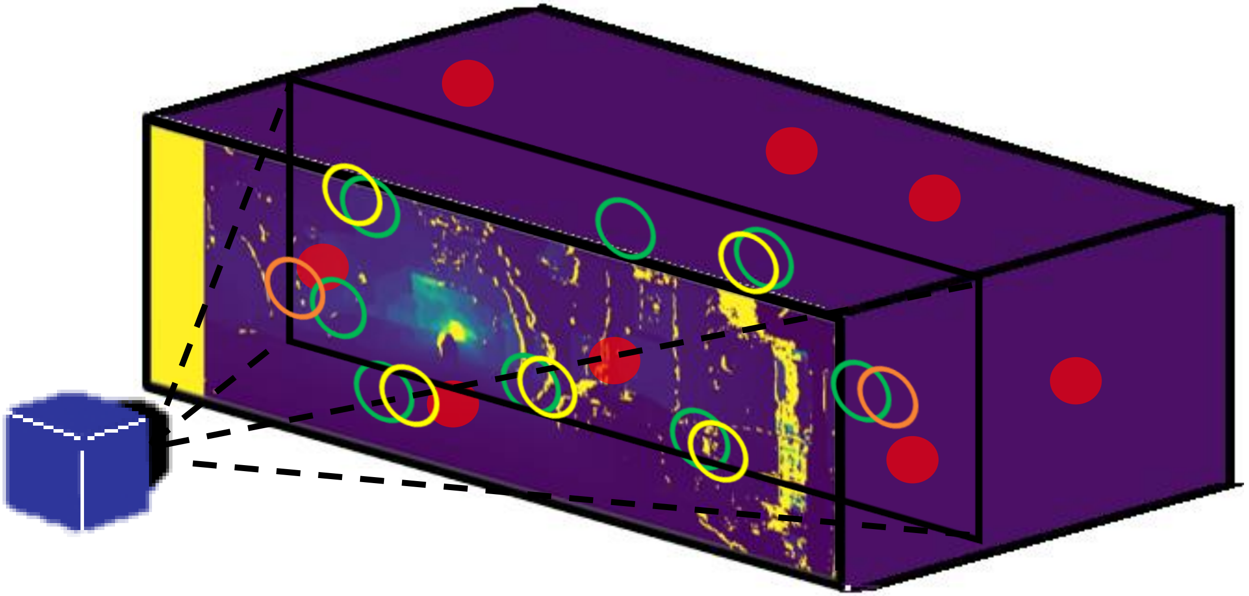


RANSAC :

Given threshold
區分出inlier、outlier
i.e. |真實2Dpoint – 投影2D point| < threshold

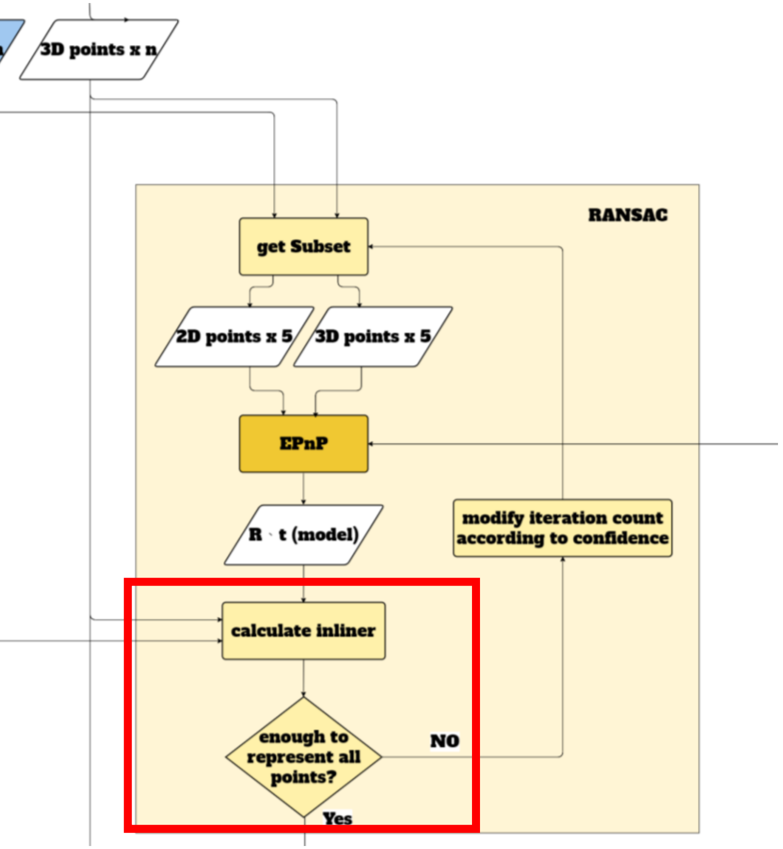
#outlier1 = 2

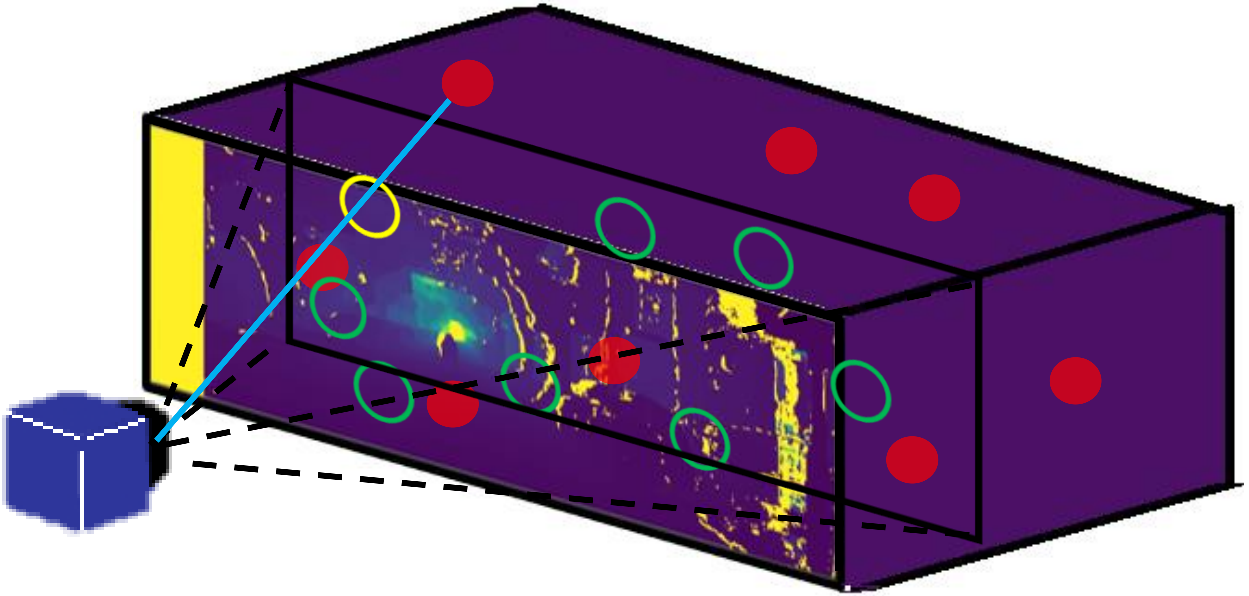




RANSAC :

重複取R_i、t_i





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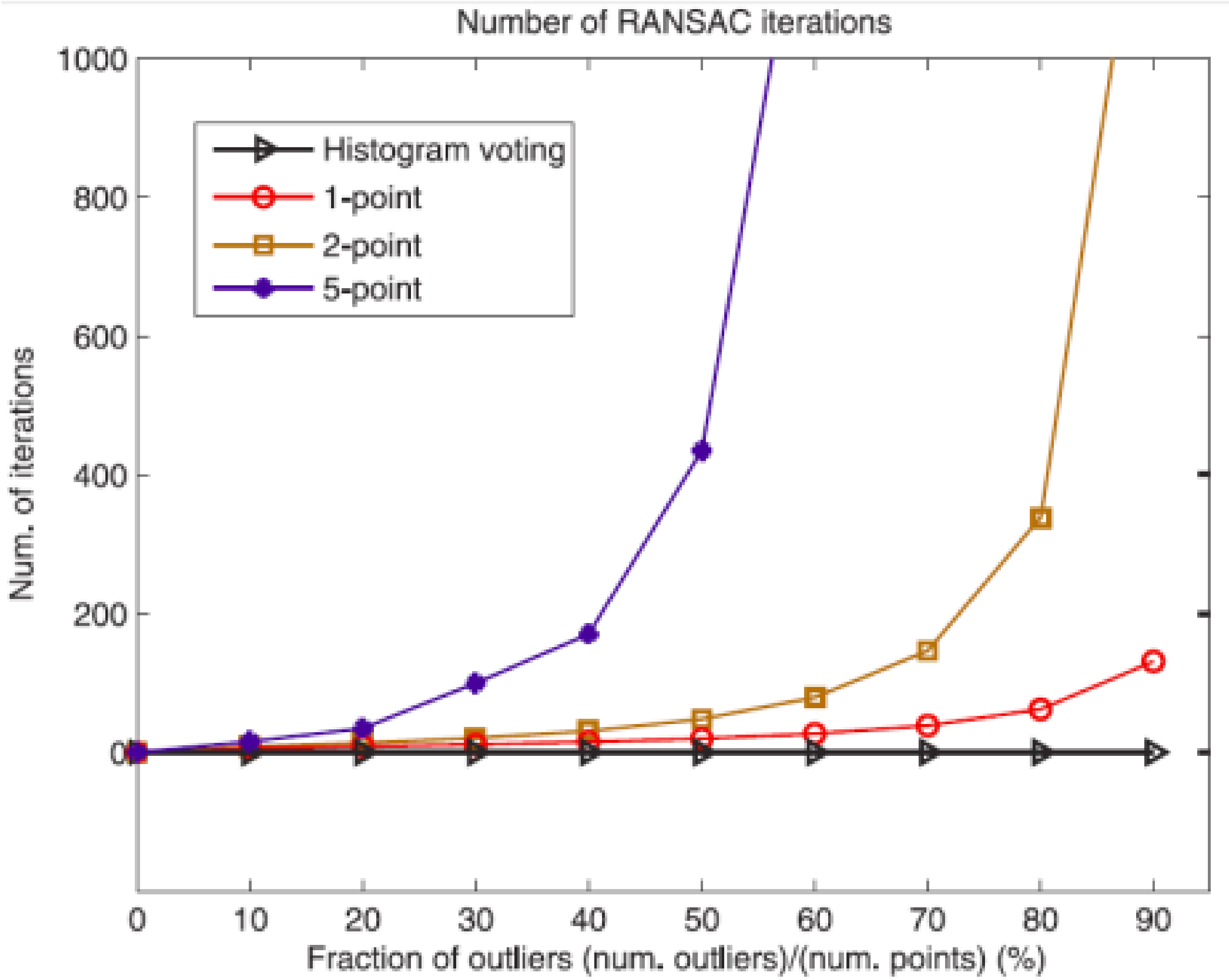
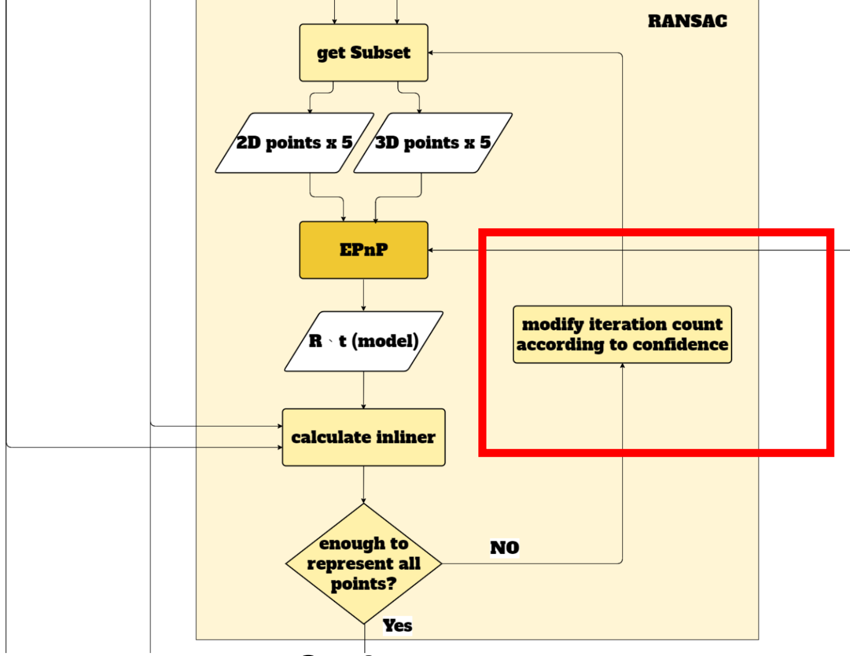
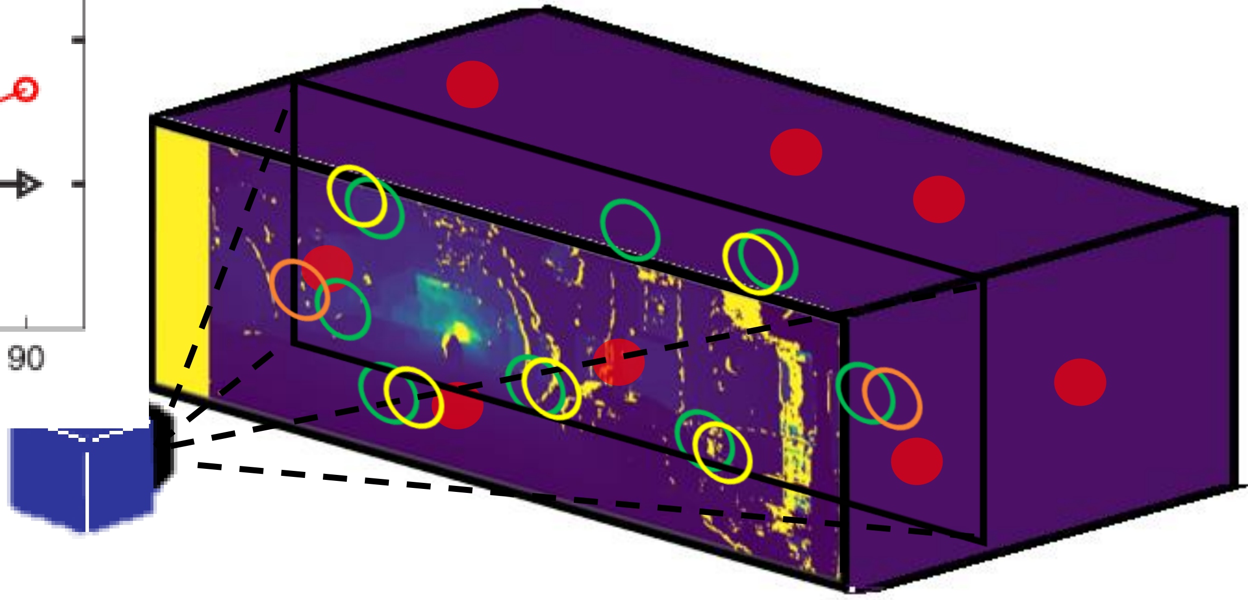
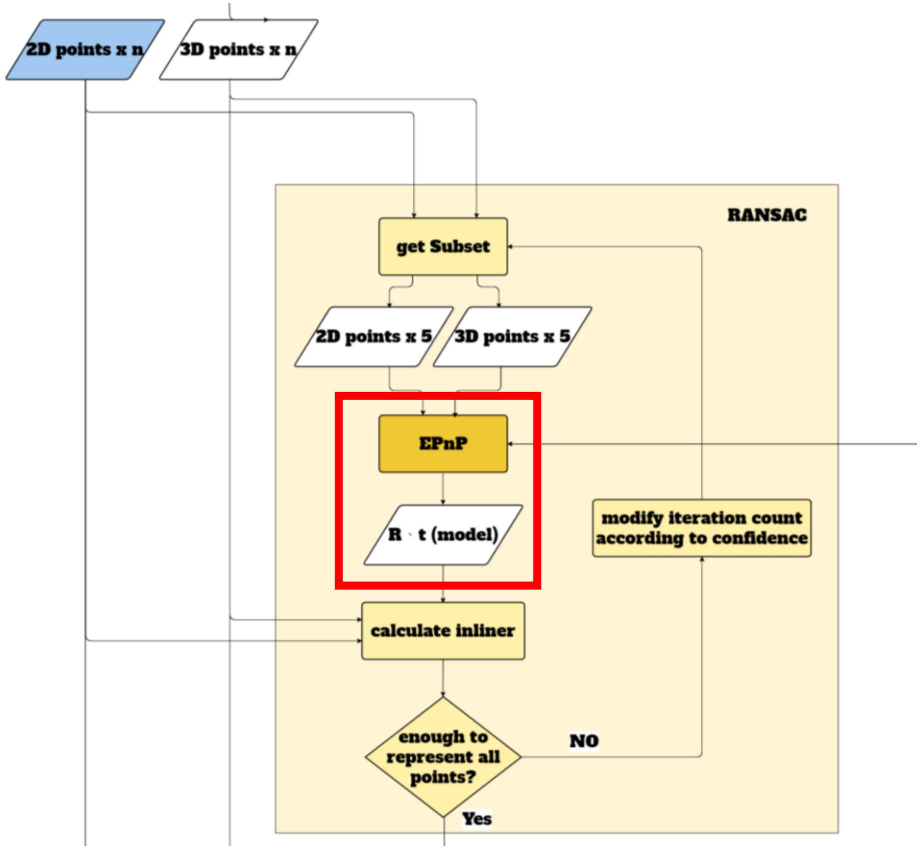




成功 = 1 – confidence



PnP :



2D points x n
3D points x n
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2D points x 5
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R · t (model)
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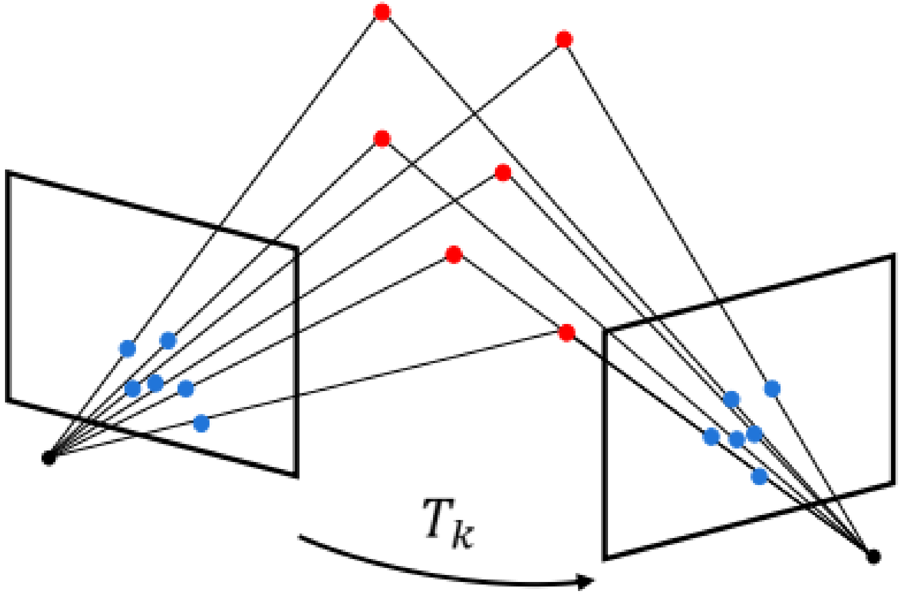
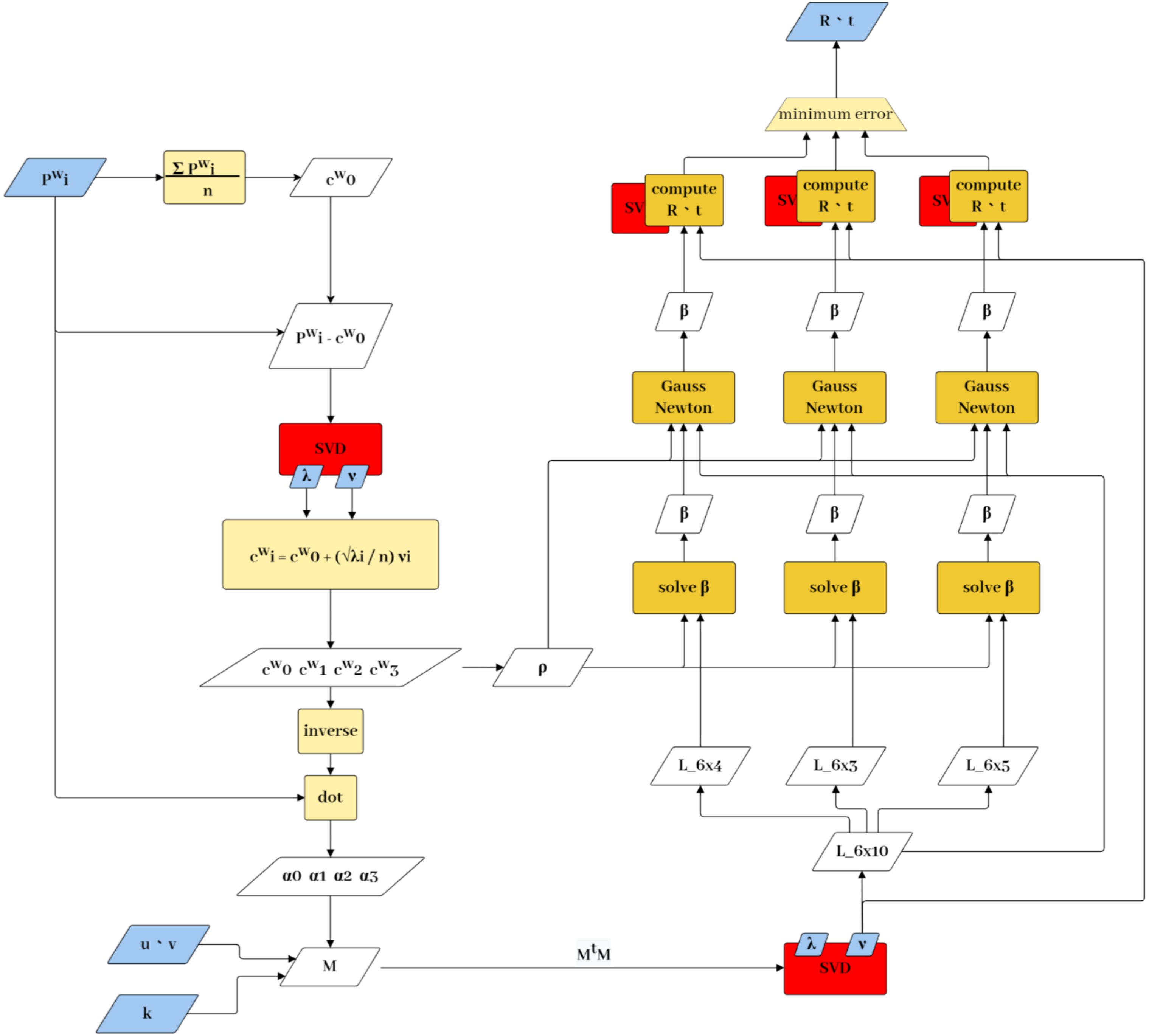


T_k

EPnP :



R、t
minimum error
compute R、t
compute R、t
compute R、t
β
β
β
Gauss Newton
Gauss Newton
Gauss Newton
β
β
β
solve β
solve β
solve β
L_6x4
L_6x3
L_6x5
L_6x10
SVD
λ
v
p^w_i
$\Sigma p^w_i / n$
c^w_0
$p^w_i - c^w_0$
SVD
λ
v
$c^w_i = c^w_0 + (\sqrt{\lambda i} / n)$ vi
c^w_0 c^w_1 c^w_2 c^w_3
ρ
inverse
dot
α0 α1 α2 α3
u、v
k
M
M^tM

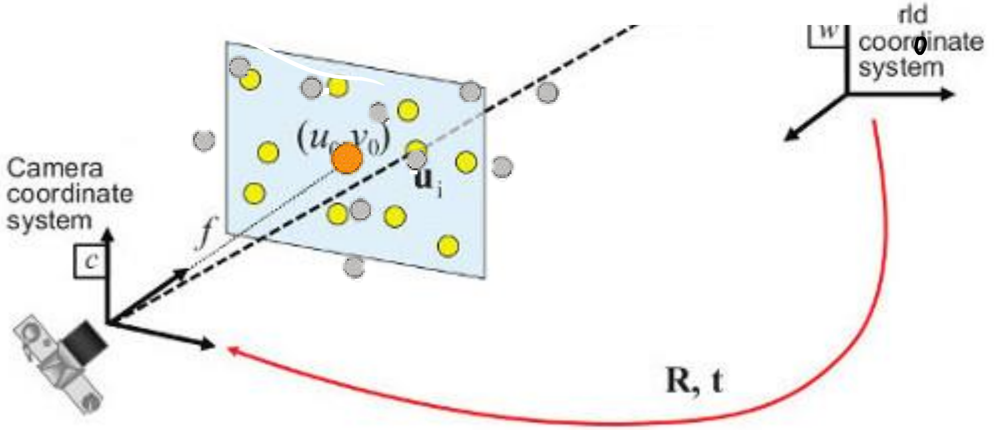
EPnP :

$$\boldsymbol{p}_i \quad i = 1, 2, \ldots, n$$

$$\boldsymbol{c}_i \quad i = 1, 2, 3, 4$$

$$\boldsymbol{p}_i^w = \sum_{j=1}^{4} a_{ij} \boldsymbol{c}_j^w \qquad \sum_{j=1}^{4} a_{ij} = 1$$

$$\boldsymbol{c}_0^w = \frac{1}{n} \sum_{1}^{n} \boldsymbol{p}_i^w$$



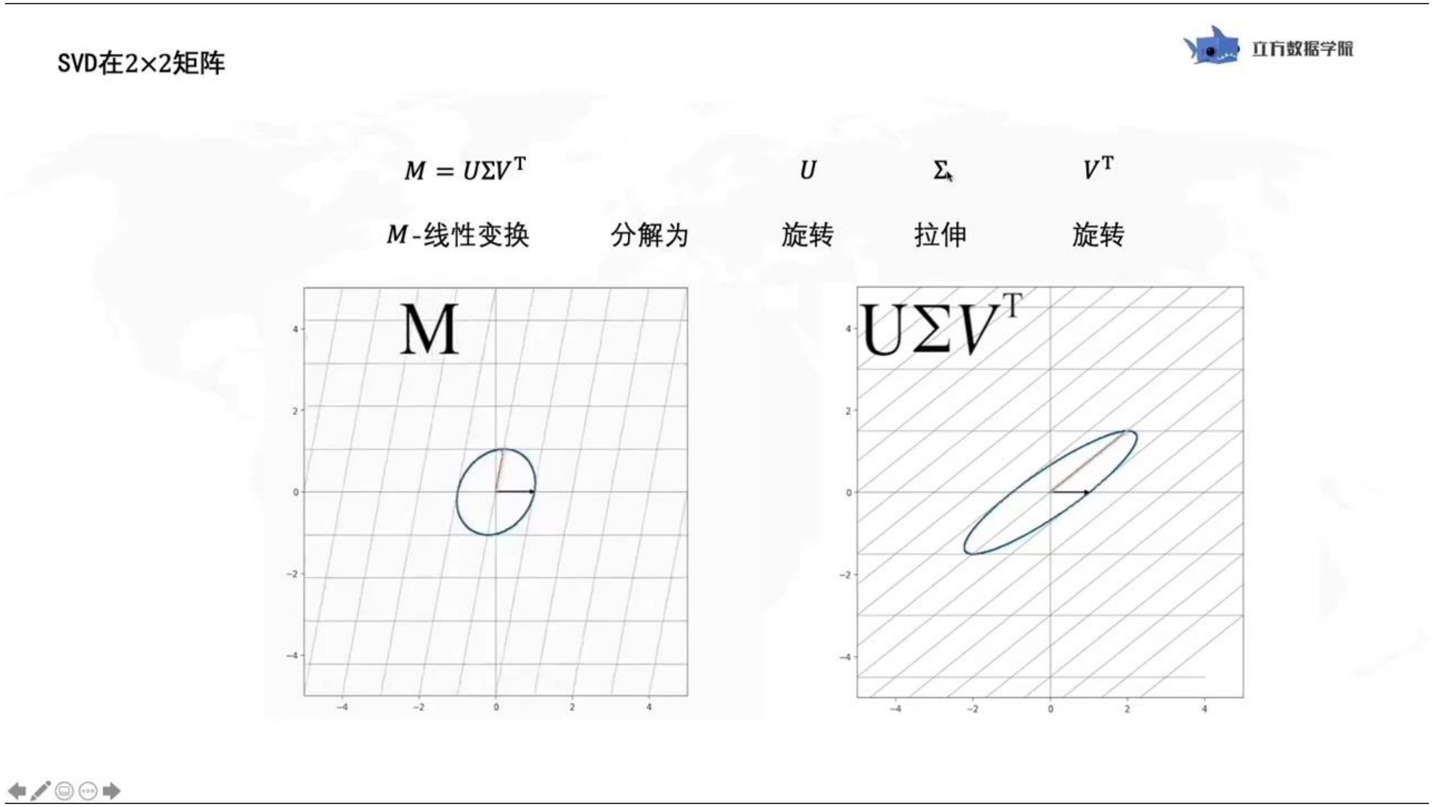
$$\boldsymbol{A} = \begin{bmatrix} (\boldsymbol{p}_1^w - \boldsymbol{c}_0^w)^T \\ (\boldsymbol{p}_2^w - \boldsymbol{c}_0^w)^T \\ \vdots \\ (\boldsymbol{p}_n^w - \boldsymbol{c}_0^w)^T \end{bmatrix}$$

$$\boldsymbol{SVD}(\boldsymbol{A}^T\boldsymbol{A}) \rightarrow \lambda_1, \lambda_2, \lambda_3, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$$

$$\begin{cases} \boldsymbol{c}_1^w = \boldsymbol{c}_0^w + \sqrt{\dfrac{\lambda_1}{n}}\mathbf{v}_1 \\ \boldsymbol{c}_2^w = \boldsymbol{c}_0^w + \sqrt{\dfrac{\lambda_2}{n}}\mathbf{v}_2 \\ \boldsymbol{c}_3^w = \boldsymbol{c}_0^w + \sqrt{\dfrac{\lambda_3}{n}}\mathbf{v}_3 \end{cases}$$

SVD 分解：

$M = U\Sigma V^{\mathrm{T}}$	U	Σ	V^{T}
	旋轉	拉伸	旋轉



EPnP :

$$\boldsymbol{p}_i^w = \sum_{j=1}^{4} a_{ij}\boldsymbol{c}_j^w$$

$$\Rightarrow \begin{bmatrix} a_{i1} \\ a_{i2} \\ a_{i3} \\ a_{i4} \end{bmatrix} = \mathbf{C}^{-1}\begin{bmatrix} \boldsymbol{p}_i^w \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = k\boldsymbol{p}_i^c = k\sum_{j=1}^{4} a_{ij}\boldsymbol{c}_j^c$$

$$= \begin{bmatrix} f_u & 0 & u_c \\ 0 & f_v & v_c \\ 0 & 0 & 1 \end{bmatrix}\sum_{j=1}^{4} a_{ij}\begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix}$$

$$\sum_{j=1}^{4}(\alpha_{ij} f_u x_j^c + \alpha_{ij}(u_c - u_i)z_j^c) = 0$$

$$\sum^{4}(\alpha_{ij} f_v y_j^c + \alpha_{ij}(v_c - v_i)z_j^c) = 0$$

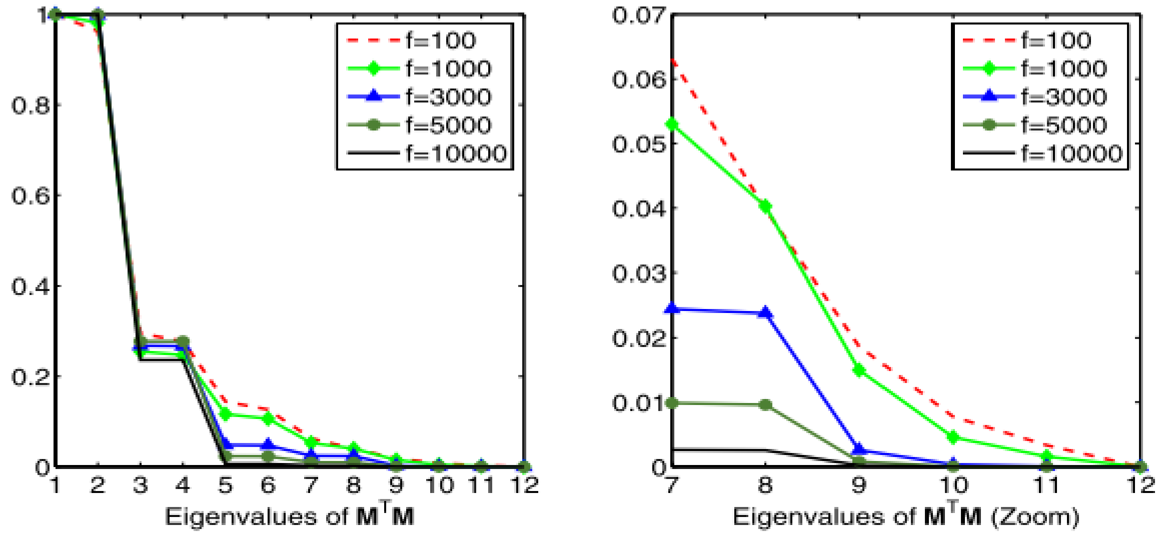
$$\mathbf{Mx} = \mathbf{0} \quad \begin{bmatrix} \alpha_{11} f_u & 0 & \alpha_{11}(u_c - u_1) & \dots & \alpha_{14} f_u & 0 & \alpha_{14}(u_c - u_1) \\ 0 & \alpha_{11} f_v & \alpha_{11}(v_c - v_1) & \dots & 0 & \alpha_{14} f_v & \alpha_{14}(v_c - v_1) \\ \vdots & & & & \vdots & & \vdots \\ \alpha_{n1} f_u & 0 & \alpha_{n1}(u_c - u_n) & \dots & \alpha_{n4} f_u & 0 & \alpha_{n4}(u_c - u_n) \\ 0 & \alpha_{n1} f_v & \alpha_{n1}(v_c - v_n) & \dots & 0 & \alpha_{n4} f_v & \alpha_{n4}(v_c - v_n) \end{bmatrix} \begin{bmatrix} x_1^c \\ y_1^c \\ z_1^c \\ \vdots \\ x_4^c \\ y_4^c \\ z_4^c \end{bmatrix} = \mathbf{0}$$

EPnP :

$$\mathbf{Mx} = \mathbf{0}$$

$$\begin{bmatrix} \alpha_{11} f_u & 0 & \alpha_{11}(u_c - u_1) & \dots & \alpha_{14} f_u & 0 & \alpha_{14}(u_c - u_1) \\ 0 & \alpha_{11} f_v & \alpha_{11}(v_c - v_1) & \dots & 0 & \alpha_{14} f_v & \alpha_{14}(v_c - v_1) \\ \vdots & & & & \vdots & & \vdots \\ \alpha_{n1} f_u & 0 & \alpha_{n1}(u_c - u_n) & \dots & \alpha_{n4} f_u & 0 & \alpha_{n4}(u_c - u_n) \\ 0 & \alpha_{n1} f_v & \alpha_{n1}(v_c - v_n) & \dots & 0 & \alpha_{n4} f_v & \alpha_{n4}(v_c - v_n) \end{bmatrix} \begin{bmatrix} x_1^c \\ y_1^c \\ z_1^c \\ \vdots \\ x_4^c \\ y_4^c \\ z_4^c \end{bmatrix} = \mathbf{0}$$

$$\mathbf{x} = \sum_{i=1}^{N} \beta_i \mathbf{v}_i$$



EPnP :

$$\mathbf{x} = \sum_{i=1}^{N} \beta_i \mathbf{v}_i$$

$SVD(M^T M) \rightarrow \mathbf{v_1}, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$

$$||\mathbf{c}_i^c - \mathbf{c}_j^c||^2 = ||\mathbf{c}_i^w - \mathbf{c}_j^w||^2$$

$$\mathbf{c}_i^c = \beta_1 \mathbf{v}_1^{[i]} + \beta_2 \mathbf{v}_2^{[i]} + \beta_3 \mathbf{v}_3^{[i]} + \beta_4 \mathbf{v}_4^{[i]}$$

$$||\beta_1 \mathbf{v}_1^{[i]} + \beta_2 \mathbf{v}_2^{[i]} + \beta_3 \mathbf{v}_3^{[i]} + \beta_4 \mathbf{v}_4^{[i]} - (\beta_1 \mathbf{v}_1^{[j]} + \beta_2 \mathbf{v}_2^{[j]} + \beta_3 \mathbf{v}_3^{[j]} + \beta_4 \mathbf{v}_4^{[j]})||^2$$

$$\begin{aligned}
&||\beta_1 \mathbf{v}_1^{[i]} + \beta_2 \mathbf{v}_2^{[i]} + \beta_3 \mathbf{v}_3^{[i]} + \beta_4 \mathbf{v}_4^{[i]} - (\beta_1 \mathbf{v}_1^{[j]} + \beta_2 \mathbf{v}_2^{[j]} + \beta_3 \mathbf{v}_3^{[j]} + \beta_4 \mathbf{v}_4^{[j]})||^2 \\
&= ||\beta_1 \underbrace{(\mathbf{v}_1^{[i]} - \mathbf{v}_1^{[j]})}_{\mathbf{s}_1} + \beta_2 \underbrace{(\mathbf{v}_2^{[i]} - \mathbf{v}_2^{[j]})}_{\mathbf{s}_2} + \beta_3 \underbrace{(\mathbf{v}_3^{[i]} - \mathbf{v}_3^{[j]})}_{\mathbf{s}_3} + \beta_4 \underbrace{(\mathbf{v}_4^{[i]} - \mathbf{v}_4^{[j]})}_{\mathbf{s}_4}||^2 \\
&= \beta_{11} \mathbf{s}_1^\top \mathbf{s}_1 + 2\beta_{12} \mathbf{s}_1^\top \mathbf{s}_2 + \beta_{22} \mathbf{s}_2^\top \mathbf{s}_2 + 2\beta_{13} \mathbf{s}_1^\top \mathbf{s}_3 + 2\beta_{23} \mathbf{s}_2^\top \mathbf{s}_3 + \beta_{33} \mathbf{s}_3^\top \mathbf{s}_3 + \\
&\quad 2\beta_{14} \mathbf{s}_1^\top \mathbf{s}_4 + 2\beta_{24} \mathbf{s}_2^\top \mathbf{s}_4 + 2\beta_{34} \mathbf{s}_3^\top \mathbf{s}_4 + \beta_{44} \mathbf{s}_4^\top \mathbf{s}_4
\end{aligned}$$

EPnP :

$$\mathbf{x} = \sum_{i=1}^{N} \beta_i \mathbf{v}_i \qquad ||\mathbf{c}_i^c - \mathbf{c}_j^c||^2 = ||\mathbf{c}_i^w - \mathbf{c}_j^w||^2$$

$$= \beta_{11}\mathbf{s}_1^\top\mathbf{s}_1 + 2\beta_{12}\mathbf{s}_1^\top\mathbf{s}_2 + \beta_{22}\mathbf{s}_2^\top\mathbf{s}_2 + 2\beta_{13}\mathbf{s}_1^\top\mathbf{s}_3 + 2\beta_{23}\mathbf{s}_2^\top\mathbf{s}_3 + \beta_{33}\mathbf{s}_3^\top\mathbf{s}_3 + 2\beta_{14}\mathbf{s}_1^\top\mathbf{s}_4 + 2\beta_{24}\mathbf{s}_2^\top\mathbf{s}_4 + 2\beta_{34}\mathbf{s}_3^\top\mathbf{s}_4 + \beta_{44}\mathbf{s}_4^\top\mathbf{s}_4 = ||\mathbf{c}_i^w - \mathbf{c}_j^w||^2$$

$$[\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3, \mathbf{l}_4, \mathbf{l}_5, \mathbf{l}_6, \mathbf{l}_7, \mathbf{l}_8, \mathbf{l}_9, \mathbf{l}_{10}] \begin{bmatrix} \beta_{11} \\ \beta_{12} \\ \beta_{22} \\ \beta_{13} \\ \beta_{23} \\ \beta_{33} \\ \beta_{14} \\ \beta_{24} \\ \beta_{34} \\ \beta_{44} \end{bmatrix} = \boldsymbol{\rho} \qquad \mathbf{L}_{6\times10}\boldsymbol{\beta}_{10\times1} = \rho$$

EPnP :

$$\mathbf{x} = \sum_{i=1}^{N} \beta_i \mathbf{v}_i \qquad ||\mathbf{c}_i^c - \mathbf{c}_j^c||^2 = ||\mathbf{c}_i^w - \mathbf{c}_j^w||^2$$

$$[\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3, \mathbf{l}_4, \mathbf{l}_5, \mathbf{l}_6, \mathbf{l}_7, \mathbf{l}_8, \mathbf{l}_9, \mathbf{l}_{10}] \begin{bmatrix} \beta_{11} \\ \beta_{12} \\ \beta_{22} \\ \beta_{13} \\ \beta_{23} \\ \beta_{33} \\ \beta_{14} \\ \beta_{24} \\ \beta_{34} \\ \beta_{44} \end{bmatrix} = \boldsymbol{\rho}$$

$$\mathbf{L}_{6\times10}\boldsymbol{\beta}_{10\times1} = \rho$$

- 近似求解方法1

$$\beta_{11}\mathbf{l}_1 + \beta_{12}\mathbf{l}_2 + \beta_{13}\mathbf{l}_4 + \beta_{14}\mathbf{l}_7 = \boldsymbol{\rho}$$

- 近似求解方法2

$$\beta_{11}\mathbf{l}_1 + \beta_{12}\mathbf{l}_2 + \beta_{22}\mathbf{l}_3 = \boldsymbol{\rho}$$

- 近似求解方法3

$$\beta_{11}\mathbf{l}_1 + \beta_{12}\mathbf{l}_2 + \beta_{22}\mathbf{l}_3 + \beta_{13}\mathbf{l}_4 + \beta_{23}\mathbf{l}_5 = \boldsymbol{\rho}$$

$$\arg\min_{\boldsymbol{\beta}} \mathrm{Error}(\beta) = \sum(||\mathbf{c}_i^c - \mathbf{c}_j^c||^2 - ||\mathbf{c}_i^w - \mathbf{c}_j^w||^2)^2$$

EPnP : Gauss Newton

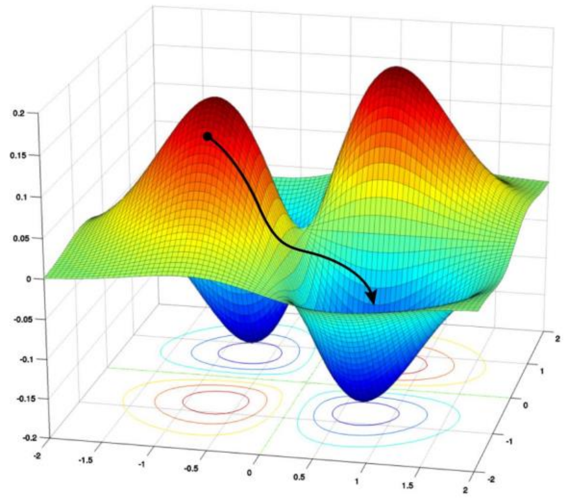
$$\arg\min_{\boldsymbol{\beta}} \mathrm{Error}(\beta) = \sum(||\mathbf{c}_i^c - \mathbf{c}_j^c||^2 - ||\mathbf{c}_i^w - \mathbf{c}_j^w||^2)^2$$

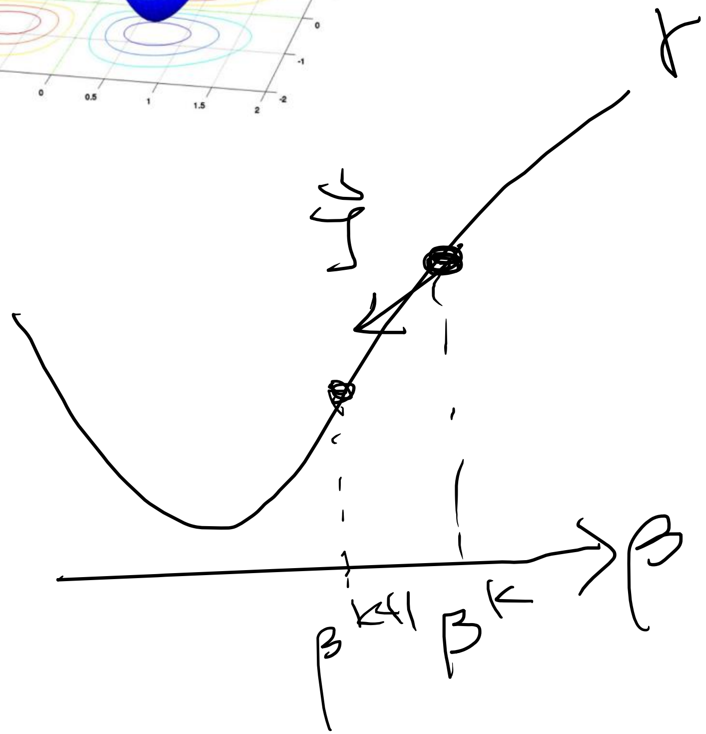
$$\mathbf{J}(\mathbf{x}^{(k)})\delta\mathbf{x}^{(k)} = -\mathbf{f}(\mathbf{x}^{(k)})$$

$$\mathbf{J}_{i,j} = \frac{\partial\left[Error_{i,j}(\beta)\right]}{\partial\beta}$$

$$= \begin{bmatrix} 2\beta_1\mathbf{S}_1{}^T\mathbf{S}_1 + 2\beta_2\mathbf{S}_1{}^T\mathbf{S}_2 + 2\beta_3\mathbf{S}_1{}^T\mathbf{S}_3 + 2\beta_4\mathbf{S}_1{}^T\mathbf{S}_4 \\ 2\beta_1\mathbf{S}_1{}^T\mathbf{S}_2 + 2\beta_2\mathbf{S}_2{}^T\mathbf{S}_2 + 2\beta_3\mathbf{S}_2{}^T\mathbf{S}_3 + 2\beta_4\mathbf{S}_2{}^T\mathbf{S}_4 \\ 2\beta_1\mathbf{S}_1{}^T\mathbf{S}_3 + 2\beta_2\mathbf{S}_2{}^T\mathbf{S}_3 + 2\beta_3\mathbf{S}_3{}^T\mathbf{S}_3 + 2\beta_4\mathbf{S}_3{}^T\mathbf{S}_4 \\ 2\beta_1\mathbf{S}_1{}^T\mathbf{S}_4 + 2\beta_2\mathbf{S}_2{}^T\mathbf{S}_4 + 2\beta_3\mathbf{S}_3{}^T\mathbf{S}_4 + 2\beta_4\mathbf{S}_4{}^T\mathbf{S}_4 \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{J}^T\mathbf{J}\delta\beta = -\mathbf{J}^T\mathbf{r}$$





EPnP : Calculate R、t

$$\mathbf{c}_i^c = \sum_{j=1}^{N} \beta_k \mathbf{v}_k^{[i]}, i = 1, 2, 3, 4$$

$$\mathbf{p}_i^c = \sum_{j=1}^{4} \alpha_{ij} \mathbf{c}_j^c$$

$$\mathbf{p}_0^w = \frac{1}{n} \sum_{i=1}^{n} \mathbf{p}_i^w \qquad \mathbf{p}_0^c = \frac{1}{n} \sum_{i=1}^{n} \mathbf{p}_i^c$$

$$\mathbf{A} = \begin{bmatrix} (\mathbf{p}_1^w - \mathbf{p}_0^w)^\top \\ (\mathbf{p}_2^w - \mathbf{p}_0^w)^\top \\ \vdots \\ (\mathbf{p}_n^w - \mathbf{p}_0^w)^\top \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} (\mathbf{p}_1^c - \mathbf{p}_0^c)^\top \\ (\mathbf{p}_2^c - \mathbf{p}_0^c)^\top \\ \vdots \\ (\mathbf{p}_n^c - \mathbf{p}_0^c)^\top \end{bmatrix}$$

$$\mathbf{H} = \mathbf{B}^\top \mathbf{A}$$

$$SVD(H^T H)$$

$$\mathbf{H} = \mathbf{U} \boldsymbol{\Sigma} \mathbf{V}^\top$$

$$\mathbf{R} = \mathbf{U} \mathbf{V}^\top$$

$$\mathbf{t} = \mathbf{p}_0^c - \mathbf{R} \mathbf{p}_0^w$$