Modelling Combat using Cellular Automata [4]

Abhin Kakkad (201501419)* and Manthan Mehta (201501420)†
Dhirubhai Ambani Institute of Information & Communication Technology, Gandhinagar, Gujarat 382007, India
CS-302 - Modeling and Simulation

I. INTRODUCTION

Models of wars had been introduced after the first world war. Major development in this field had been done by Lanchester[1]. He introduced two models which were the Conventional battle and the Guerilla battle. Here, we will be discussing the conventional through two techniques - applying the differential equations of 'Guerrilla Combat Model' of Lanchester type and Cellular Automata. There are two types of battles:

1. Two conventional armies having only one type of army unit on each side. (Army A vs Army B)
2. Two conventional armies having two army units on each side. (Army A (Soldiers + Tanks) vs Army B (Soldiers + Tanks))

II. MODEL 1(A) : DIFFERENTIAL EQUATION FOR 1V1 BATTLE

This model showcases the combat between two forces having similar nature. Each unit is capable of killing units of the opposing force with a fixed killing rate. The killing rate may differ for members of different armies. The thing that we can't model with these equations is the rate of detection of the units. This is a necessity in the actual combat situation because the detection rate would depend on the position and operable region of the unit. The killing rate depends on the strength and training of the fighters. Thus, a model with different detection rate as well as different killing rate for each unit is more closer to the practical combat.

Assumptions

- There are only two kinds of armies on the battle ground.
- The fighting capacity of all the soldiers of the army is the same. Although the soldiers of opposite army have different fighting abilities.
- Both the armies are facing each other and are aware of the strength of the opposing army.
- Any unit member can interact with any other unit member at any given point of time.
- Interaction between any two entities happens with a probability p and the killing rate for each unit is k_i where $i = unitindiscussion$. The final rte thus becomes $p * k_i = k_{pi}$.
- The time frame is small enough that we can ignore any reinforcements during the period of the battle.
- Kill rates of each entity remains constant during the given time period.

Model

The system for the model are:

$$\frac{dA}{dt} = -\beta B \tag{1}$$

$$\frac{dB}{dt} = -\alpha A \tag{2}$$

Where, α is the rate at which a member of Army A kills a member of Army B and β is the rate at which a member of Army B kills an enemy soldier.

Observation

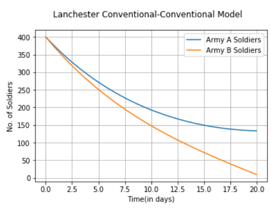

FIG. 1: Battle Condition where Army A wins. Here, $\alpha = 0.09$, $\beta = 0.08$, $A_i = 400$ and $B_i = 400$

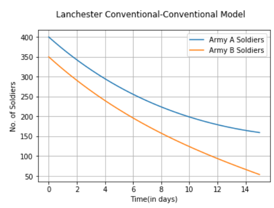

FIG. 2: Battle Condition where Army A wins. Here, $\alpha = 0.08$, $\beta = 0.09$, $A_i = 400$ and $B_i = 350$

*Electronic address: 201501419@daiict.ac.in
†Electronic address: 201501420@daiict.ac.in

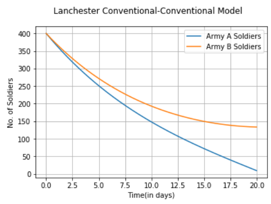

FIG. 3: Battle Condition where Army B wins. Here, $\alpha = 0.08$, $\beta = 0.09$, $A_i = 400$ and $B_i = 400$

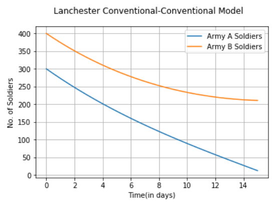

FIG. 4: Battle Condition where Army B wins. Here, $\alpha = 0.09$, $\beta = 0.08$, $A_i = 300$ and $B_i = 400$

In this model the number of operational units can never be less than 0 i.e. number of soldiers can't be negative. Here, we can see that the outcome is more dependent on the initial number of soldiers rather than the killing rates (as seen in the above figures). The killing rates also matter but the winning probability would be having a square dependence on the number of soldiers and a linear dependence on the killing rates.

Hence, the following can be said about the winning army [2]:

$$A_i{}^2.\alpha > B_i{}^2.\beta \implies \text{A victory}$$
$$A_i{}^2.\alpha < B_i{}^2.\beta \implies \text{B victory}$$
$$A_i{}^2.\alpha = B_i{}^2.\beta \implies \text{Mutual Destruction}$$

III. MODEL 1(B) : DIFFERENTIAL EQUATION FOR 2V2 BATTLE

This model is an extension of Model 1(a). In this we have 2 different units on each side which are - soldiers and heavy artillery. The units will have different killing rates and also the rules for the new units will change.

Assumptions

- The basic assumptions for this model remain the same as Model 1(a) with the extra assumptions as follows.
- There are only four kinds of units on the battlefield. Army A soldiers, Army A heavy artillery, Army B soldiers and Army B heavy artillery.
- A heavy artillery unit is able to kill a soldier whereas the vice-versa isn't possible.
- The killing rate of a heavy artillery unit is greater than that of a soldier because they have higher strength.

Model

The system for the model are:

$$\frac{dAs}{dt} = -\beta_{ts}B_t - \beta_{ss}B_s \tag{3}$$

$$\frac{dAt}{dt} = -\beta_{tt}B_t \tag{4}$$

$$\frac{dBt}{dt} = -\alpha_{tt}A_t \tag{5}$$

$$\frac{dBs}{dt} = -\alpha_{ts}A_t - \alpha_{ss}A_s \tag{6}$$

Where, α_{tt} is the rate at which a heavy artillery unit of Army A kills a heavy artillery unit of Army B, β_{tt} is the rate at which a heavy artillery unit of Army B kills a heavy artillery unit of Army A. αts and αss are the rates at which Army A heavy artillery and Army A soldiers respectively kill Army B soldiers. βts and βss are the rates at which Army B heavy artillery and Army B soldiers respectively kill Army A soldiers.

Observation

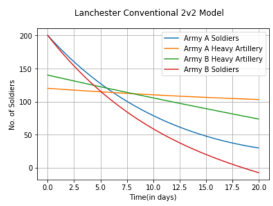

FIG. 5: Battle Condition where Army A wins. Here, $\alpha_{tt} = 0.03$, $\alpha_{ts} = 0.02$, $\alpha_{ss} = 0.09$, $\beta_{tt} = 0.008$, $\beta_{ts} = 0.04$, $\beta_{ss} = 0.06$, $A_{ti} = 200$, $A_{si} = 120$, $B_{ti} = 200$ and $B_{si} = 140$

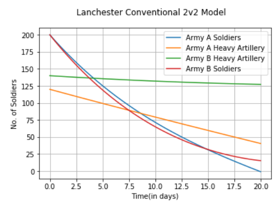

FIG. 6: Battle Condition where Army B wins. Here, $\alpha_{tt} = 0.008$, $\alpha_{ts} = 0.045$, $\alpha_{ss} = 0.07$, $\beta_{tt} = 0.03$, $\beta_{ts} = 0.04$, $\beta_{ss} = 0.06$, $A_{ti} = 200$, $A_{si} = 120$, $B_{ti} = 200$ and $B_{si} = 140$

Here also we see a similar pattern where there is more dependence on the initial number rather than the killing rate.

Limitations

1. This model fails as the number of units on both side start decreasing.
2. This model doesn't help us analyze the movement of the army or the location of the army units.
3. The strength of the armies should keep on decreasing as the battle progresses but here it remains constant throughout.

IV. MODEL 2(A) CELLULAR AUTOMATA FOR 1V1 BATTLE

This model [3] will help us implement the movement of the soldiers during the battle. The soldiers can move in any of the adjoining squares on the grid. The movement is restricted to only the neighbouring cells here.

Assumptions

1. There are only two kinds of army units on the battle ground.
2. Any unit member can interact with only eight of it's neighbouring cells at any given point of time.
3. An empty cell is treated as a dead cell.
4. The killing rates of all the units remains constant during the complete battle.
5. The fighting capacity of all the soldiers of the army is the same. Although the soldiers of opposite army have different fighting abilities.
6. If the number of Army A units in the neighbourhood of any Army A soldier is more than the number of Army B units then it lives, else it dies. The vice-versa applies for any Army B soldier.
7. The kill rate for Army A is effective with probability k_a and for Army B it is effective with probability k_b.
8. The time frame is small enough that we can ignore any reinforcements during the period of the battle.
9. The soldiers can change their positions in any of the adjacent eight cells if they aren't surrounded by any enemy unit (they can be surrounded by their own army units).

Model

The state of any soldier on the battlefield is dependent on the neighbouring eight units that it is surrounded by. If the number of enemy is higher than the number of supporters, then the soldier dies, else it stays alive. If the unit isn't surrounded by any enemy cell then it will move to any of the empty eight cells in it's neighbourhood to continue it's attack on the enemy.

The battle starts at a random time on the battlefield i.e. initially both armies are randomly scattered on the battlefield and not condensed on any one side. The reason for doing this is to remove the effect of initial conditions.

Observation

Here we see that the final outcome of the battle depends solely on the number of the soldiers initially.

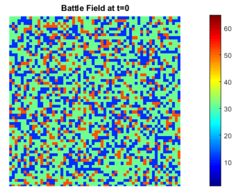

FIG. 7: Initial Condition where Army A wins

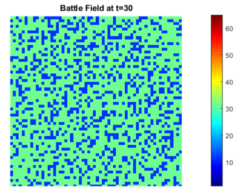

FIG. 8: Final Condition where Army A wins

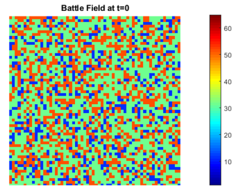

FIG. 9: Initial Condition where Army B wins

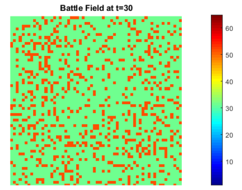

FIG. 10: Final Condition where Army B wins

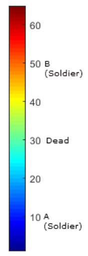

FIG. 11: Color Graph for Model 2(a)

V. MODEL 2(B) : CELLULAR AUTOMATA FOR 2V2 BATTLE

In this model we take the Model (2a) further for a 2v2 battle where each army would have two different units on their sides.

Assumptions

1. The assumptions discussed for the previous model hold for this model also. The other assumptions that would have to be considered are:
2. Soldiers from any army cannot kill the heavy artillery units of the opposite army.
3. If a soldier is surrounded by equal artillery units of both the armies then it would be killed if the soldiers of the opposite army are greater than the soldiers of his own army.
4. The killing rates of the heavy artillery units would be more as compared to the soldiers.

Observation

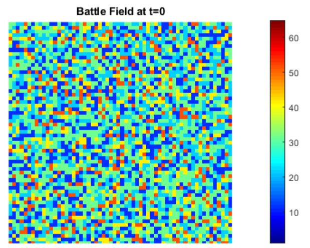

FIG. 12: Initial Condition where Army A wins

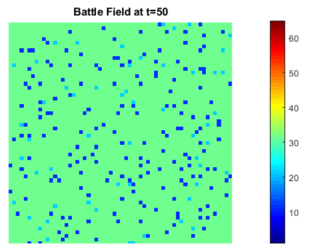

FIG. 13: Final Condition where Army A wins

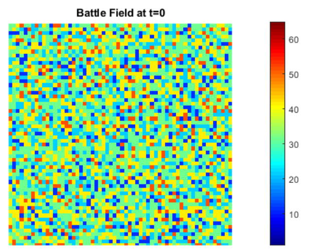

FIG. 14: Initial Condition where Army B wins

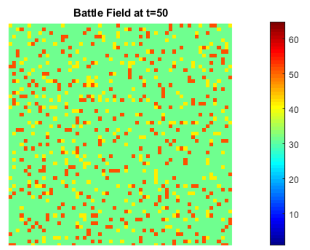

FIG. 15: Final Condition where Army B wins

The final outcome would be more dependent on the heavy artillery units because they have greater strength.

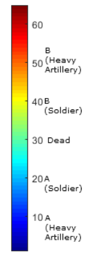

FIG. 16: Color Graph for Model 2(b)

Also we see that with the introduction of the heavy artillery units we the number of deaths have increased on both sides as compared to the previous model.

Limitations

1. Each of the unit is restricted to only eight of its neighbouring cells. Because of this the range for each unit is drastically reduced. This takes us away from the practical version of a combat because for an actual battle the range would be dependant on the technological prowess of each side.

2. For each soldier the killing rate is taken to be constant. This may not be the case in the actual scenario. For each soldier the killing rate can be taken to be different. The killing rates would also reduce as the battle progresses because of injury and/or tiredness. Thus, the soldier's ability can be modeled dynamically.

VI. CONCLUSION

The models discussed above show us the battle field simulation. Both the models have the ability to show us different aspects of the battle. But the cellular automata model gives us a more descriptive and practical model for visualization. Although the scope of the soldier is subjected to close combat only (which isn't the case in today's world with technology progressing so much) but this model aptly suited the battles from the earlier times.

References

[1] Eldridge S. Adams and Michael Mesterton-Gibbons. "Lanchester's attrition models and fights among social animals". In: *Behavioral Ecology* (2003).

[2] Svend Clausen. "Warfare can be Calculated". In: ().

[3] D. Fang et al. "Cellular Automata and Their Applications in Combat Modeling x00026; Simulation". In: *2007 Chinese Control Conference*. 2007, pp. 587–591.

[4] Abhin Kakkad and Manthan Mehta. *Codes used for the simulation*. URL: https://github.com/abhin-kakkad/Battlefield-Simulation.