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Models for Evaluating Airline Overbooking

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Introduction

We develop two models to evaluate overbooking policies.

The first model predicts the effectiveness of a proposed overbooking scheme, using the concept of expected marginal seat revenue (EMSR). This model solves the discount seat allocation problem in the presence of overbooking factors for each fare class and evaluates an overbooking policy stochastically.

The second model takes in historical flight data and reconstructs what the optimal seat allocation should have been. The percentage of overbooking revenue obtained in practice serves as a measure of the policy's value.

Finally, we examine the overbooking problem in light of the recent drastic changes to airline industry and conclude that increased overbooking would bring short-term profits to most carriers. However, the long-term ill effects that have traditionally caused airlines to shun such a policy would be even more pronounced in a post-tragedy climate.

Literature Review

There are two major ways that airlines try to maximize revenues: overbooking (selling more seats than available on a given flight) and seat allocation (price discrimination). These measures are believed to save major airlines as much as half a billion dollars each year, in an industry with a typical yearly profit of about \$1 billion dollars [Belobaba 1989].

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Beckman [1958] models booking and no-shows in an effort to find an optimal overbooking strategy. He ignores advance cancellations, assuming that all cancellations are no-shows [Rothstein 1985]. A method that is easy to implement but sophisticated enough to allow for cancellations and group reservations was developed by Taylor [1962]. Versions of this model were implemented at Iberia Airlines [Shlifer and Vardi 1975], British Overseas Airways Corporation, and El Al Airlines [Rothstein 1985].

None of these approaches considers multiple fare classes. Littlewood [1972] offers a simple two-fare allocation rule: A discount fare should be sold only if the discount fare is greater than or equal to the *expected marginal return* from selling the seat at full-fare. This idea was generalized by Belobaba [1989] to include any number of fare classes and allow the integration of overbooking. We use expected marginal seat revenue in predicting about overbooking schemes.

There is a multitude of work on the subject [McGill 1999]—according to Weatherford and Bodily [1992], there are more than 124,416 classes of models for variations of the yield management problem, though research has settled into just a few of these. Several authors tried to better Belobaba's [1987] heuristic in the presence of three or more fare classes (for which it is demonstrably sub-optimal) [Weatherford and Bodily 1992]; generally, these adaptive methods for obtaining optimal booking limits for single-leg flights are done by dynamic programming [McGill 1999].

After deregulation in 1978, airlines were no longer required to maintain a direct-route system to major cities. Many shifted to a hub-and-spoke system, and network effects began to grow more important. To maximize revenue, an airline may want to consider a passenger's full itinerary before accepting or denying their ticket request for a particular leg. For example, an airline might prefer to book a discount fare rather than one at full price if the passenger is continuing on to another destination (and thus paying an additional fare).

The first implementations of the origin-destination control problem considered segments of flights. The advantage to this was that a segment could be blacked out to a particular fare class, lowering the overall complexity of a booking scheme. Another method, *virtual nesting*, combines fare classes and flight schedules into distinct buckets [McGill 1999]. Inventory control on these buckets would then give revenue-increasing results. Finally, the bid-price method deterministically assigns a value to different seats on a flight leg. The legs in an itinerary are then summed to establish a bid-price for that itinerary; a ticket request is accepted only if the fare exceeds the bid-price [McGill 1999].

The most realistic yield management problem takes into account five price classes. The ticket demands for different fare classes are randomized and correlated with one other to allow for sell-ups and the recapture of rejected customers on later flights. Passengers can no-show or cancel at any time. Group reservations are treated separately from individuals—their cancellation probability distribution is likely different. Currently, most work assumes that passengers who pay full fare would not first check for availability of a lower-class ticket; a more realistic model would allow buyers of a higher-class ticket to be di-

verted by a lower fare. A full accounting of network effects would consider the relative value of what Weatherford and Bodily [1992] terms *displacement*—denying a discount passenger’s ticket request to fly a multileg itinerary in favor of leaving one of the legs open to a full-fare passenger.

Unfortunately, while the algorithms for allocating seats and setting overbooking levels are highly developed, there has been little work done on the problem of evaluating how effective these measures actually are. Our solution applies industry-standard methods to find optimal booking levels, then examines the actual booking requests for a given flight to determine how close to an optimal revenue level the scheme actually comes.

Factors Affecting Overbooking Policy

General Concerns

The reason that overbooking is so important is because of multiple fare classes. With only one fare class, it would be easier for airlines to penalize customers for no-shows. However, while most airlines offer nonrefundable discount tickets, they prefer not to penalize those who pay full fare, like business travelers, because these passengers account for most of the profits.

The overbooking level of a plane is dictated by the likelihood of cancellations and of no-shows. An overbooking model compares the revenue generated by accepting additional reservations with the costs associated with the risk of overselling and decides whether additional sales are advisable. In addition, the “recapture” possibility can be considered, which is the probability that a passenger denied a ticket will simply buy a ticket for one of the airline’s other flights. Since a passenger is more valuable to the airline buying a ticket on a flight that has empty seats to fill than on one that is already overbooked, a high recapture probability reduces the optimal overbooking level [Smith et al. 1992].

No major airline overbooks at profit-maximizing levels, because it could not realistically handle the problems associated with all the overloaded flights. This gives the overbooking optimization problem some important constraints. The total flight revenue is to be maximized, subject to the conditions that only a certain portion of flights have even one passenger denied boarding (one oversale), and that a bound is placed on the expected total number of oversales. Dealing with even one oversale is a hassle for the airline’s staff, and they are not equipped to handle such problems on a large scale. Additionally, some research indicates that increased overbooking levels would most likely trigger an “overbooking war” [Suzuki 2002], which would increase short-term profits but would probably engender enough consumer resentment that the industry as a whole would lose business.

While the overbooking problem sets a limit for sales on a flight as a whole, proper seat allocation sets an optimal point at which to stop selling tickets for individual fare levels. For example, a perfectly overbooked plane, loaded

exactly to capacity, could be flying at far below its optimal revenue level if too many discount tickets were sold. The more expensive tickets are not for first-class seats and involve no additional luxuries above the discount tickets, apart from more lenient cancellation policies and the ability to buy the tickets a shorter time before the flight's departure.

September 11, 2001

Since the September 11 terrorist attacks, there have been significant changes in the airline business. In addition to the forced cancellation of many flights in the immediate aftermath of the attacks and the extreme levels of cancellations and no-shows by passengers after that, passenger traffic has dropped sharply in general. The huge downturn in passenger levels has led to large reductions in service by most carriers.

In terms of the booking problem, there are fewer flights for passengers to spill over onto, which could increase the loss by an airline if it bumps a passenger from a flight. On the other hand, since passenger levels have fallen so far, it is less likely that an airline will overfill any given flight. The heightened security at airports will likely increase the passenger no-show rate somewhat, as passengers get delayed at security checkpoints. At the very least, it should almost completely remove the problem of "go-shows," passengers who show up for a flight but are not in the airline's records.

On the whole, optimal booking strategies have become even more vital as airlines have already lost billions of dollars, and some teeter on the brink of failure. Some overbooking tactics previously dismissed as too harmful in the long run might be worthwhile for companies in trouble. For example, an airline near failure might increase the overbooking rate to the level that maximizes revenue, without regard to the inconvenience and possible future resentment of its customers.

Constructing the Model

Objectives

A scheme for evaluating overbooking policies needs to answer two questions: how well should a *new* overbooking method perform, and how well is a *current* overbooking scheme already working? The first is best addressed by a simple model of an airline's booking procedures; given some setup for allocating seats to fare classes, candidate overbooking schemes can be laid on top and tested by simulation. This approach has the advantage that it provides insight into *why* an overbooking scheme is or is not effective and helps to illuminate the characteristics of an optimal overbooking approach.

The second question is, in some respects, a simpler one to answer. Given the actual (over)booking limits that were imposed on each fare class, and all avail-

able information on the actual requests for reservations, how much revenue might have been gained from overbooking, compared to how much actually was? This provides a very tangible measure of overbooking performance but very little insight into the reasons for results.

The enormous number of factors affecting the design and evaluation of an overbooking policy force us to make simplifying assumptions to construct models that meet both of these goals.

Assumptions

- **Fleet-wide revenues can be near-optimized one leg at a time.**

Maximizing revenue involves complicated interactions between flights. For instance, a passenger purchasing a cheap ticket on a flight into a major hub might actually be worth more to the airline than a business-class passenger, on account of connecting flights. We assume that such effects can be compensated for by placing passengers into fare classes based on revenue potential rather than on the fare for any given leg. This assumption effectively reduces the network problem to a single-leg optimization problem.

- **Airlines set fares optimally.**

Revenue maximization depends strongly on the prices of various classes of tickets. To avoid getting into the economics of price competition and supply/demand, we assume that airlines set prices optimally. This reduces revenue maximization to setting optimal fare-class (over)booking limits.

- **Historical demand data are available and applicable.**

The model needs to estimate future demand for tickets on any given flight. We assume that historical data are available on the number of tickets sold any given number of days t before a flight's departure. In some respects, this assumption is unrealistic because of the problem of data *censorship*—that is, the failure of airlines to record requests beyond the booking limit for a fare class [Belobaba 1989]. On the other hand, statistical methods can be used to reconstruct this information [Boeing Commercial Airline Company 1982, 7–16; Swan 1990].

- **Low-fare passengers tend to book before high-fare ones.**

Discount tickets are often sold under advance purchase restrictions, for the precise reason that it enables price discrimination. Because of restrictions like these, and because travelers who plan ahead search for cheap tickets, low-fare passengers tend to book before high-fare ones.

Predicting Overbooking Effectiveness

Disentangling the effects of overbooking, seat allocation, pricing schemes, and external factors on revenues of an airline is extremely complicated. To

isolate the effects of overbooking as much as possible, we want a simple, well-understood seat allocation model that provides an easy way to incorporate various overbooking schemes. In light of this objective, we pass up several methods for finding optimal booking limits on single-leg flights detailed in, for example, Curry [1990] and Brumelle [1993], in favor of the simpler expected marginal seat revenue (EMSR) method [Belobaba 1989].

EMSR was developed as an extension of the well-known rule of thumb, popularized by Littlewood [1972], that revenues are maximized in a two-fare system by capping sales of the lower-class ticket when the revenue from selling an additional lower-class ticket is balanced by the *expected* revenue from selling the same seat as an upper-class ticket. In the EMSR formulation, any number of fare classes are permitted and the goal is “to determine how many seats *not to sell* in the lowest fare classes and to retain for *possible* sale in higher fare classes closer to departure day” [Belobaba 1989].

The only information required to calculate booking levels in the EMSR model is a probability density function for the number of requests that will arrive before the flight departs, in each fare class and as a function of time. For simplicity, this distribution can be assumed to be normal, with a mean and standard deviation that change as a function of the time remaining. Thus, the only information an airline would need is a historical average and standard deviation of demand in each class as a function of time. Ideally, the information would reflect previous instances of the particular flight in question. Let the mean and standard deviations in question be denoted by $\mu_i(t)$ and $\sigma_i(t)$ for each fare class $i = 1, 2, \dots, k$. Then the probability that demand is greater than some specified level S_i is given by

$$\bar{P}_i(S_i, t) \equiv \frac{1}{\sqrt{2\pi} \sigma_i(t)} \int_{S_i}^{\infty} e^{-(r-\mu_i(t))^2 / \sigma_i(t)^2} dr.$$

This *spill probability* is the likelihood that the S_i th ticket would be sold if offered in the i th category. If we further allow $f_i(t)$ to denote the expected revenue resulting from a sale to class i at a time t days prior to departure, we can define

$$\text{EMSR}_i(S_i, t) = f_i(t) \cdot \bar{P}_i(S_i, t),$$

or simply the revenue for a ticket in class i times the probability that the S_i th seat will be sold. The problem, however, is to find the number of tickets S_j^i that should be protected from the lower class j for sale to the upper class i (ignoring other classes for the moment). The optimal value for S_j^i satisfies

$$\text{EMSR}_i(S_j^i, t) = f_j(t), \tag{1}$$

so that the expected marginal revenue from holding the S_j^i th seat for class i is exactly equal to (in practice, slightly greater than) the revenue from selling it immediately to someone in the lower class j . The booking limits that should be enforced can be derived easily from the optimal S_j^i values by letting the

booking limit B_j for class j be

$$B_j(t) = C - S_j^{j+1} - \sum_{i < j} b_i(t), \quad (2)$$

that is, the capacity C of the plane, less the protection level of the class above j from class j and less the total number of seats already reserved. Sample EMSR curves, with booking limits calculated in this fashion, are shown in **Figure 1**.

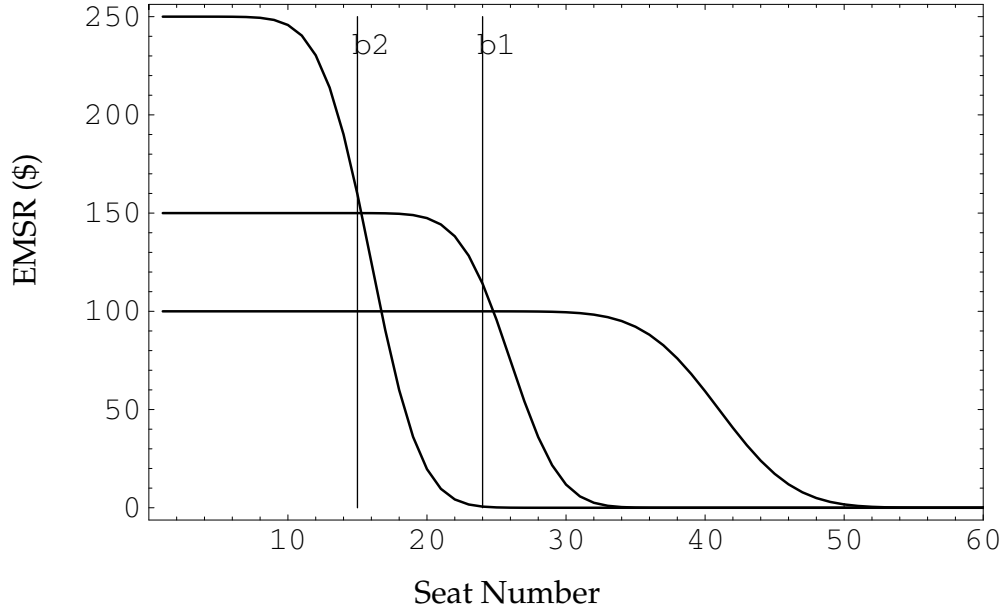


Figure 1. Expected marginal seat revenue (EMSR) curves for three class levels, with the highest-revenue class at the top. Each curve represents the revenue expected from protecting a particular seat to sell to that class. Also shown are the resulting booking limits for each of the lower classes—that is, the levels at which sales to the lower class should stop to save seats for higher fares.

This formulation does not account for overbooking; if we allow each fare class i to be overbooked by some factor OV_i , the optimality condition (1) becomes

$$\text{EMSR}_i(S_j^i, t) = f_j(t) \cdot \frac{OV_i}{OV_j}. \quad (3)$$

This can be understood in terms of an adjustment to f_i and f_j ; the overbooking factors are essentially cancellation probabilities, so we use each OV_i to deflate the expected revenue from fare class i . Then

$$\bar{P}_i(S_j^i, t) \cdot \frac{f_i(t)}{OV_i} = f_j(t) \cdot \frac{f_j(t)}{OV_j},$$

which is equivalent to (3). Note that the use of a single overbooking factor for the entire cabin (that is, $OV_i = OV$) causes the OV_i and OV_j in (3) to cancel. Nonetheless, the boarding limits for each class are affected, because the capacity of the plane C must be adjusted to account for the extra reservations, so now

$$C^* = C \cdot OV$$

and the booking limits in (2) are adjusted upward by replacing C with C^* .

The EMSR formalism gives us the power to evaluate an overbooking scheme theoretically by plugging its recommendations into a well-understood stable model and evaluating them. Given the EMSR boarding limits, which can be updated dynamically as booking progresses, and the prescribed overbooking factors, a simulated string of requests can be handled. Since the EMSR model involves only periodic updates to establish limits that are fixed over the course of a day or so, a set of n requests can be handled with two lookups each (booking limit and current booking level), one subtraction, and one comparison; so all n requests can be processed on $\mathcal{O}(n)$ time. An EMSR-based approach would thus be practical in a real-world real-time airline reservations system, which often handles as many as 5,000 requests per second. Indeed, systems derived from EMSR have been adopted by many airlines [Mcgill 1999].

Evaluating Past Overbookings

The problem of evaluating an overbooking scheme that has already been implemented is somewhat less well studied than the problem of theoretically evaluating an overbooking policy. One simple approach, developed by American Airlines in 1992, measures the optimality of overbooking and seat allocation separately [Smith et al. 1992]. Their overbooking evaluation process assumes optimal seat allocation and, conversely, their seat allocation evaluation scheme assumes optimal overbooking. Under this assumption, an overbooking scheme is evaluated by estimating the revenue under *optimal* overbooking in two ways:

- If a flight is fully loaded and no passenger is denied boarding, the flight is considered to be optimally overbooked and to have achieved maximum revenue.
- If n passengers are denied boarding, the money lost due to bumping these passengers is added back in and the n lowest fares paid by passengers for the flight are subtracted from revenue.
- On the other hand, if there are n empty seats on the plane, the n highest-fare tickets that were requested but not sold are added to create the maximum revenue figure.

Their seat-allocation model estimates the demand for each flight by calculating a theoretical demand for each fare class and then setting the minimum flight revenue (by filling the seats lowest-class first) and the maximum flight

revenue (by filling the seats highest-class first). To estimate demand, we use the information on the flight's sales up to the point where each class closed. By assuming that demand is increasing for each class, we can project the number of requests that would have occurred had the booking limits been disregarded.

Given these projected additional requests and the actual requests received before closing, it is straightforward to compute the best- and worst-case overbooking scenarios. The worst-case revenue R_- is determined by using no booking limits and taking reservations as they come, and the best-case revenue R_+ is determined by accommodating high-fare passengers first, giving the leftovers to the lower classes. The difference between these two figures is the revenue to be gained by the use of booking limits. Thus, the performance of a booking scheme that generates revenue R is

$$p = \frac{R - R_-}{R_+ - R_-} \cdot 100\%, \quad (4)$$

representing the percentage of the possible booking revenue actually achieved.

We select this method for evaluating booking schemes after the fact.

Analysis of the Models

Tests and Simulations

The EMSR method requires information on demand as a function of time. Although readily available to an airline, it is not widely published in a detailed form. Li [2001] provides enough data to construct a rough piecewise-linear picture of demand remaining as a function of time, shown in **Figure 2**.

This information can be inputted into the EMSR model to produce optimal booking limits that evolve in time. A typical situation near the beginning of ticket sales was shown in **Figure 1**, while the evolution of the limits themselves is plotted in **Figure 3**.

The demand information in **Figure 2** can also be used to simulate requests for reservations. By taking the difference between the demand remaining at day t and at day $(t - 1)$ before departure, the expected demand on day t can be determined. The actual number of requests generated on that day is then given by a Poisson random variable with parameter λ equal to the expected number of sales [Rothstein 1971]. The requests generated in this manner can be passed to a request-handling simulation that looks at the most current booking limits and then accepts or denies ticket requests based on the number of reservations already confirmed and the reservations limit. An example of this booking process is illustrated in **Figure 4**.

The results of the booking process provide an ideal testbed for the revenue opportunity model employed to evaluate overbooking performance. The simulation conducted for **Figure 4** had demand values of $\{11, 41, 57\}$, for classes 1,

Expected Remaining Demand as Flight Time Approaches

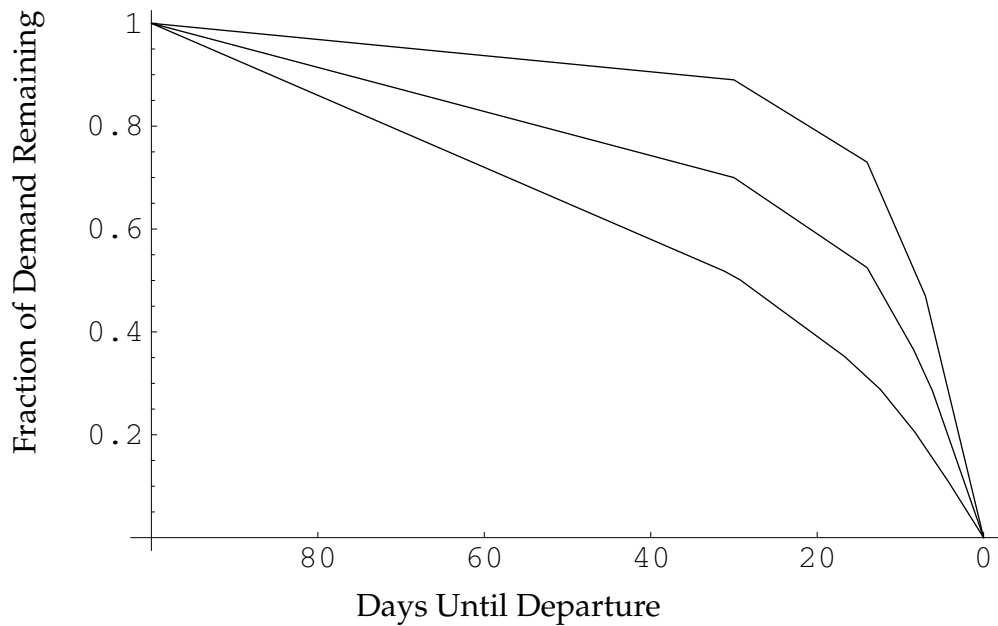


Figure 2. Demand remaining as a function of time for each of three fare classes, with the highest fare class on top. The curves represent the fraction of tickets that have yet to be purchased. Note that, for example, demand for high fare tickets kicks in much later than low-fare demand. (Data interpolated from Li [2001].)

2, and 3 respectively, before ticket sales were capped. A linear forward projection of these sales rates indicates that they would have reached {18, 49, 69} had the classes remained open. Given fare classes {\$250, \$150, \$100}, the minimum revenue would be

$$R_- = \$100(69) + \$150(40) + \$250(0) = \$12,900$$

and the maximum revenue would be

$$R_+ = \$250(18) + \$150(49) + \$100(42) = \$16,050.$$

The actual revenue according to the EMSR formalism was

$$R = \$100(57) + \$150(41) + \$250(11) = \$14,600,$$

so the efficiency is

$$p = \frac{R - R_-}{R_+ - R_-} \cdot 100\% = \frac{\$14,600 - \$12,900}{\$16,050 - \$12,900} \cdot 100\% = 54\%,$$

without the use of a complicated overbooking scheme. This is not close to the efficiencies reported in Smith et al. [1992], which cluster around 92%. This relative inefficiency is to be expected, however, from a simplified booking scheme given incomplete booking request data.

Evolution of Booking Limits by the EMSR Method

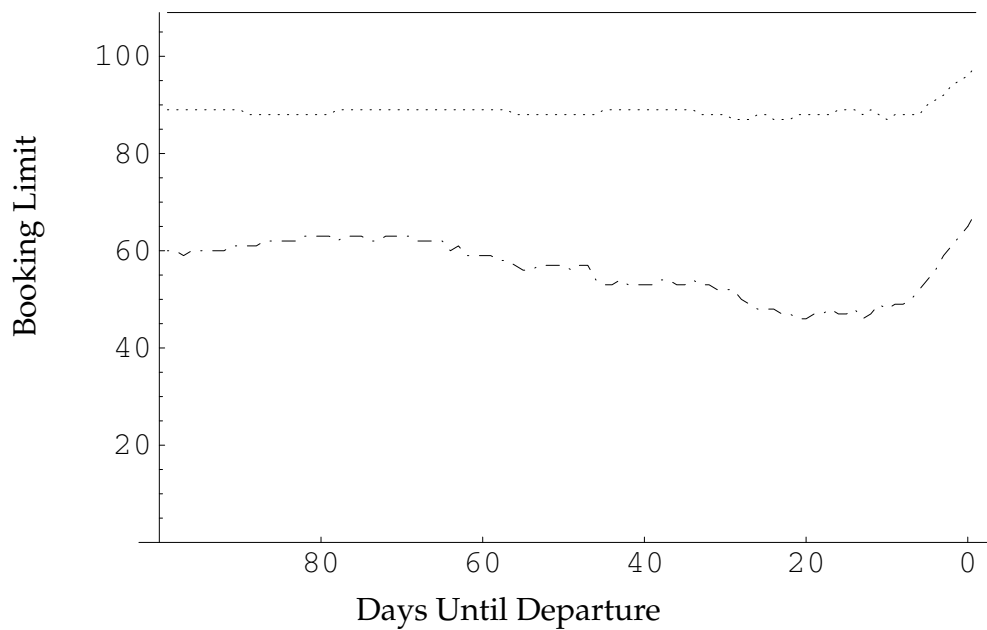


Figure 3. Booking limits for each class are dynamically adjusted to account for tickets already sold. For illustrative purposes, the number of tickets already sold is replaced here with the number of tickets that should have been sold according to expectations. In this case, the booking limits estimated at the beginning of the process are fairly accurate and require relatively little updating.

Total Bookings by Fare Class: First Sale to Flight Time

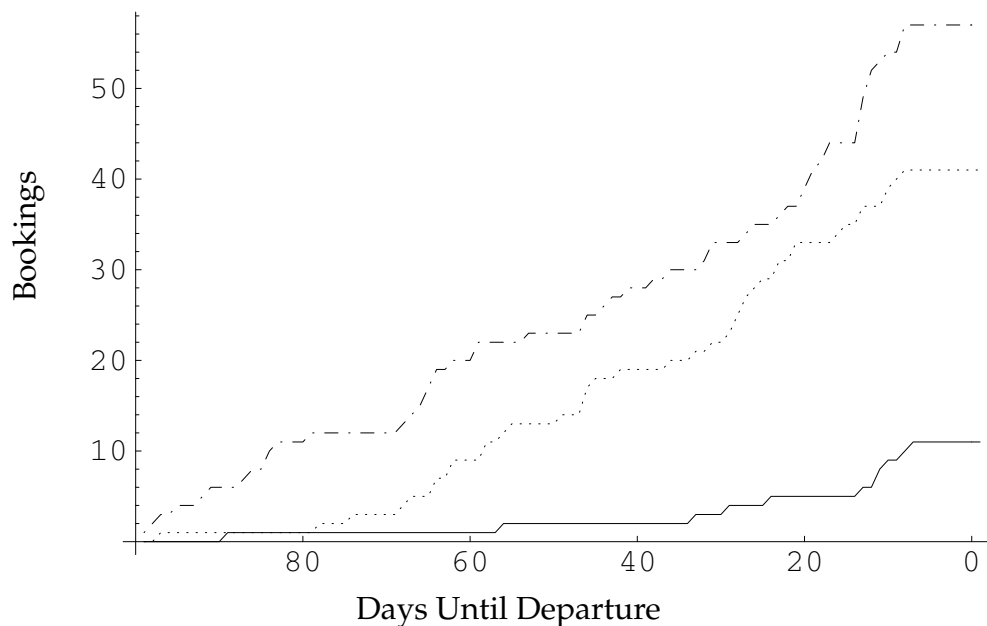


Figure 4. The EMSR-based booking limits are used to decide whether to accept or reject a sequence of ticket requests. These requests follow a Poisson distribution where the parameter λ varies with time to match the expected demand. Each fare class reaches its booking limit, as desired, so the flight is exactly full. Incorporating overbooking factors shifts the limits up accordingly.

Strengths and Weaknesses

Strengths

- **Applies widely accepted, industry-standard techniques.**

Although more advanced (and optimal) algorithms are available and are used, EMSR and its descendants are still widely used in industry and can come close to optimality. The EMSR scheme, tested as-is on a real airline, caused revenue gains as much as 15% [Belobaba 1989].

Our method for determining the optimality of a scheme after the fact is also based on tried and true methods developed by American Airlines [Smith et al. 1992].

- **Simplicity**

Since it does not take into account as many factors as other booking models, EMSR is easier to deal with computationally. While a simple model may not be able to model a major airline with complete accuracy, an optimal pricing scheme can be made using only three fare classes [Li 2001].

Weaknesses

- **Neglects network effects**

We treat the problem of optimizing each flight as if it were an independent problem although it is not.

- **Ignores sell-ups**

In considering the discount seat allocation problem, we treat the demands for the fare classes as constants, independent of one other. This is not the case, because of the possibility of sell-ups. If the number of tickets sold in a lower fare class is restricted, then there is some probability that a customer requesting a ticket in that class will buy a ticket at a more expensive fare. This means it is possible to convert low-fare demand into high-fare demand, which would suggest protecting a higher number of seats for high fares than calculated by the model that we use. Sell-ups would be straightforward to incorporate into EMSR, but doing so would require additional information [Belobaba 1989].

- **Discounts possibility of recapture**

Similar to sell-ups, the recapture probability is the probability that a passenger unable to buy a ticket at a certain price on a given flight will buy a ticket on a different flight. Depending on the recapture probability for each fare class, more or fewer seats might be allocated to discount fares.

Recommendations on Bumping Policy

In 1999, an average of only 0.88 passengers per 10,000 boardings were involuntarily bumped. Airlines are not required to keep records of the number of voluntary bumps, so it is impossible to determine a general bump rate.

Before bumping passengers involuntarily, the airline is required to ask for volunteers. Because there are no regulations on compensation for voluntary bumps, this is often a cheaper and more attractive method for airlines anyway. If too few people volunteer, the airline must pay those denied boarding 200% of the sum of the values of the passengers' remaining flight coupons, with a maximum of \$400. This maximum is decreased to \$200 if the airline arranges for a flight that will arrive less than 2 hours after the original flight. The airline may also substitute the offer of free or reduced fare transportation in the future, provided that the value of the offer is greater than the cash payment otherwise required. Alternatively, the airline may simply arrange alternative transportation if it is scheduled to arrive less than an hour after the original flight.

Auctions in which the airline offers progressively higher compensation for passengers who give up their seats are both the cheapest and the most common practice. As long as the airline does not engage in so much overbooking that it cannot find suitable reroutes for passengers bumped from their original itineraries, no alternatives to this policy need to be considered.

Conclusions

The two models presented in this paper work together to evaluate overbooking schemes by simulating their effects in advance and by quantifying their effects after implementation.

The expected marginal seat revenue (EMSR) model predicts overbooking scheme effectiveness. It determines the correct levels of protection for each fare class above the lowest—that is, how many seats should be reserved for possible sale at later dates and higher fares. Overbooking factors can be specified separately for each fare class, so the model effectively takes in overbooking factors and produces booking limits that can be used to handle ticket requests.

The revenue opportunity model attempts to estimate the maximum revenue from a flight under perfect overbooking and discount allocation. This is accomplished by estimating the actual demand for seats, then calculating the revenue that these seats would generate if sold to the highest-paying customers. Simple calculations produce the ideal overbooking cap and the optimal discount allocation for the flight. Thus, this model effectively represents how the airline would sell tickets if they had perfect advance knowledge of demand.

After the September terrorist attacks and their subsequent catastrophic effects on the airline industry, heightened airport security and fearful passengers will increase no-show and cancellation rates, seeming to dictate increasing

overbooking levels to reclaim lost profits.

Airlines considering such action should be cautioned, however, that the negative effects of increased overbooking could outweigh the benefits. With reduced airline service, finding alternative transportation for displaced passengers could be more difficult. The effect of denying boarding to more passengers, along with the greater inconvenience of being bumped, could seriously shake consumers' already-diminished faith in the airline industry. With airlines already losing huge numbers of customers, it would be a mistake to risk permanently losing them to alternatives like rail and auto travel by alienating them with frequent overselling.

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Presentation by Richard Neal (MAA Student Activities Committee Chair) of the MAA award to Daniel Boylan and Wesley Turner of the Harvey Mudd College team (Michael Schubmehl could not come), after their presentation at the MAA Mathfest in Burlington, VT, in August. On the right is Ben Fusaro, Founding Director of the MCM. Photo by Ruth Favro, Lawrence Tech University.

Letter to the CEO of a Major Airline

Airline overbooking is just one facet of a revenue management problem that has been studied extensively in operations research literature. Airlines have been practicing overbooking since the 1940's, but early models of overbooking considered only the most rudimentary cases. Most importantly, they did not take into account the revenue maximizing potential of price discrimination—charging different fares for identical seats. In order to maximize yield, it is particularly critical to price discriminate between business and leisure travelers. That is, when filling the plane, book as many full fare passengers and as few discount fare passengers as possible.

The implementation of a method of yield management can have dramatic effects on an airline's revenue. American Airlines managed its seat inventory to a \$1.4 billion increase in revenue from 1989 to 1992—about 50% more than its net profit for the same period. Controlling the mix of fare products can translate into revenue increases of \$200 million to \$500 million for carriers with total revenues of \$1 billion to \$5 billion.

Though several decision models of airline booking have been developed over the years, comparing one scheme to another remains a difficult task. We have taken a two-pronged approach to this problem, both simulating and measuring a booking scheme's profitability.

In order to simulate a booking scheme's effect, we used the expected marginal seat revenue (EMSR) model proposed by Belobaba [1989] to generate near-optimal decisions on whether to accept or deny a ticket request in a given fare class. The EMSR model accepts as input overbooking levels for each of the fare classes that compose a flight, so different policies can be plugged in for testing.

Our approach to measuring a current scheme's profitability is similar to one used at American Airlines [Smith et al. 1992]. We compare the actual revenue generated by a flight with an ideal level calculated with the benefit of hindsight, as well as with a baseline level that would have been generated had no yield management been used. By calculating the percentage of this spread earned by a flight employing a particular scheme, we are able to gauge the effectiveness of different booking schemes.

It is our hope that these models will prove useful in evaluating your airline's overbooking policies. Simulations should provide insight into the properties of an effective scheme, and measurements after the fact will help to provide performance benchmarks. Finally, while it may be tempting to increase overbooking levels in order to compensate for lost revenues in the post-tragedy climate, our results indicate this will probably hurt long-term profits more than they will help.

Cordially,

Michael P. Schubmehl, Wesley M. Turner, and Daniel M. Boylan