1 Executive Summary

Although George R.R. Martin's series *A Song of Ice and Fire* falls under the category "hard fantasy," we doubt he expected this level of rigour applied to his dragons. In our paper we address realistic concerns of raising dragons, as well as their impact on the earth. We analyze their growth and caloric requirements, their impact on local ecosystems, as well as the required human-intervention needed.

For our growth and calorie estimations, we modeled the rough size limit of a dragon using the largest known dragon, Balerion the Black Dread. We used fanciful descriptions of his teeth being as long as longswords to estimate his size and weight. We then estimated how young dragons would grow by using a model geared towards indeterminately growing species, which matched the description of dragons growing forever. We ultimately found an accurate applicable equation to model dragon growth over time.

For their calorie consumption, we took several approaches and evaluated each one in order to find a plausible caloric requirement of the dragons based on their size and activity. We used the novel unit of cows in order to better process the gargantuan number of calories that they need. We were able to obtain a equation for caloric needs based on the dragon's basal metabolic rate and the Harris-Benedict approach to account for activity levels.

In order to determine land requirements for dragons in different environments with different resource levels, we assumed that dragons could be compared to apex predators of different biomes. The model aimed to ensure environmental sustainability while also fulfilling dragon requirements. We combined data on the caloric requirements and land requirements of various carnivorous predators in different biomes to find the available calories from prey per square mile. We then used this value and the dragon's caloric requirements from the previous model to determine the total land requirements.

To account for the effects of climate on dragons, we considered both water availability and temperature. In the case of arid climates, we compared dragons to existing migratory birds, which create a net increase in water during metabolism. Using data from a Game of Thrones episode, we found the necessary water requirements of a dragon. After dividing this value by caloric intake, we compared the mass of water per kcal of dragons to that produced by birds. We then used data on bird flights to find that the metabolic rate comparison has the implication that dragons can fly for 7.5 hours without water before needing more resources. We also determined that dragons use more energy at low temperatures and conserve energy at high temperatures.

To determine how large a community would need to be to sustain these dragons, we considered both people needed for dragon management and people needed for food. Ultimately, including management of a cow farm, security, and dragon riders, we determined that 3 dragons would need to be $61x + 48$ people, where x is the amount of people per day.

Contents

2 Global Assumptions and Justifications

- The dragons in the paper have the characteristics of those in the Game of Thrones series.

  Justification: While there are a wide range of descriptions and functions of dragons, the problem uses Game of Thrones as its basis for lore.

- Dragons can be biologically compared to animals existing in our world.

  Justification: Dragons must have a physical basis for their existence. In cases where the series is unclear, dragons will be modeled off of existing animals biologically.

- The flight of dragons is not in question.

  Justification: In the context of the problem, it is given that dragons can both appear and function as they do in the fictional world in Game of Thrones. Therefore, while the flight of a dragon may contradict traditional laws of physics, we can assume that, like the pterosaur, certain biological mechanisms are in place that allow dragon flight. This allows us to account for flight in our model without question.

- Dragons can be modeled consistently across the species, that is to say, there is no significant variation between individuals in terms of growth or resource intake.

  Justification: In the series, there are not significant differences between the needs of the three dragons. In the scope of the problem, minor variations become unimportant. This allows us to uniformly apply our model across all three dragons.

3 Caloric Intake and Growth

We'll need to feed our dragons. The dragons in *Game of Thrones* seem to vary in their food consumption: In Season 5 episode 5 Daenerys feeds her dragons a human each (roughly 125,000 kcal), then holds off on killing any more humans, saying "Don't want to overfeed them. Tomorrow, perhaps." Just 5 episodes later, we see Drogon feast on several animals, each exceeding the 125,000 kcal of a human. Because of this, we instead model the dragon's caloric necessities by analysing similar animals.

3.1 Defining the Problem

Our aim is to:

- estimate the basal metabolic rate of dragons
- estimate calories burned through daily activities
- predict impact of climate change on caloric requirements

3.2 Assumptions and Justifications

- Dragons are endotherms, meaning that they regulate their internal body heat.

  Justification: Dragons are said to be fire made flesh, and even steam on cold nights. [2]

- Dragons have a similar basal metabolic rate per unit mass as Earth animals.

  Justification: Since dragons do not have a fixed diet in *Game of Thrones*, we instead have to assume that they obey similar laws as Earth animals.

- Dragons grow like similar organisms with indeterminate growth.

  Justification: The dragons, which grow without limit so long as they have food and freedom, seem very similar to the description of Earth organisms with indeterminate growth. Their growth is linked to caloric intake, to competition, the amount of space they have, as well as their age.

- Dragons' maximum age is about 250 years old.

  Justification: Balerion, the oldest known dragon, lived to be about 220 years old. We do not know whether dragons would live longer or shorter in captivity; it varies between different animals. And although Balerion's lifespan may have been shortened by his use in war, descriptions of the time before his death where he was sluggish and unable to fly great distances indicate that Balerion died of old age. [2]

- Balerion is around the limit of the size that a dragon can grow.

  Justification: Balerion was the largest known dragon. No skulls seen in the series are larger than Balerion's skull. Furthermore, growth slows as the dragon grows bigger. Thus, dragons are not likely to grow much beyond Balerion's size.

- Dragons' caloric intake can be fulfilled with red meat and red meat alone. The dragons can also eat fish. Our model does not need to account for food or nutrition groups, only caloric requirements.

  Justification: In the series, there is no indication of dragons eating any sort of plant life. It is indicated that their bones and blood have a high iron content, which might not be fulfilled completely by fish. Thus, they need only iron-rich red meat. [2]

- Dragons are given enough free space and food to grow freely.

  Justification: Although it might prove beneficial for dragons' growth to be stunted, for our initial models we want to find the limits of their growth.

3.3 Growth of the Dragons

There are several models of indeterminate growth. Given the facts from the series we know that dragon growth is affected by:

- surrounding space, or habitat
- caloric intake

This lines up with factors that impact the growth of most indeterminately growing vertebrates. Before using Balerion as our upper limit, we'll have to estimate his size.

3.3.1 How big is Balerion?

Looking at pterosaurs and lizards, dragons are somewhere in between. They clearly do not have the scrawny look of a pterosaur, but must be able to fly. Thus, we averaged a scaled up pterosaur and a scaled up lizard to approximate Balerion's weight.

As length increases by a factor of x, volume, and thus weight if a uniform density is assumed, scales upwards by a factor of x^3. Estimates of Balerion's size puts him at roughly 42 meters long, with a wingspan of 150 meters. He truly seems to be a limit of dragon size: he struggles to move in his old age and his hefty weight gives him difficulty in flying.

We took this by taking a crocodile, which has similar proportions to a dragon, and scaling it upwards by one of the only explicit mentions of size, that Balerion's teeth were as long as longswords. Balerion's teeth were from 1 to 1.5 meters, then. A saltwater crocodile's teeth are about 4 inches, or .1 meters long, but their teeth actually seem smaller, proportionally, than depictions of Balerion. An estimate of scaling up a saltwater crocodile, 6 meters long, puts Balerion at 50 meters long. Proportional to a bat's wingspan, his wingspan is about 150 meters long.

Now for the weight estimates:

Pterosaur: Wingspan 9 meters, weight 250 kg [23]. Balerion would weigh 115,740 kg.

Komodo dragon: length 3 meters, weight 70 kg [24]. Balerion would weigh 324,074 kg.

Taking an average of these two estimates, Balerion would weigh roughly 219,901 kg, or roughly 220 metric tonnes. For reference, a blue whale can reach up to 30 meters and 180 tonnes.

3.3.2 Selection of a Model

Common models used for indeterminate growth of the nature indicated by Martin's book include an asymptotic sigmoidal curve, or use of the von Bertalanffy equation (although the latter is used more commonly in fisheries than for land vertebrates, it sees use in other animals as well). However, the asymptotic sigmoidal model has an issue: Martin seems to imply his dragons grow without limit, thus meaning there should be no asymptote. Instead, the growth likely attenuates over time but never hits an asymptote.

The von Bertanlanffy equation hits an upper limit of size. It states

$$\mathrm{L}(t) = \mathrm{L}_\infty \left(1 - e^{-K(t-t_0)}\right)$$

And we can see from examination of the equation that L will never exceed L_∞.

Another potential approach is to use a simpler function with attenuating growth but no limit. Any function y such that $\frac{d^2y}{dx^2} < 1$ fits this category. However, we believed that the dragons exhibited exponential growth at the beginning of the season, limiting the possible graphs. Because of these factors we chose to use the von Bertalanffy Equation, which has tested applications in modeling the growth of indeterminate growers [24].

3.3.3 von Bertalanffy Equation

We will use the von Bertalanffy Equation modified for weight.

$$\mathrm{W}(t) = q\mathrm{L}^3$$

$$\mathrm{L}(t) = \mathrm{L}_\infty \left(1 - e^{-K(t+t_0)}\right)$$

thus becomes

$$\mathrm{W}(t) = \mathrm{W}_\infty \left(1 - e^{-K(t+t_0)}\right)^3$$

$$W_\infty > \text{Balerion's weight}$$
$$t_0 = t \text{ value such that W(t)} = 5$$
$$t = \text{years since birth}$$
$$K = \text{growth rate}$$

By using values such that growth does not hit an asymptote during the dragon's lifespan, we can mimic attenuating growth using the von Bertanlaffy equation.

Taking it as when t(0) = 5 kg, t(1) = 30 kg, and t(220) = 219,901

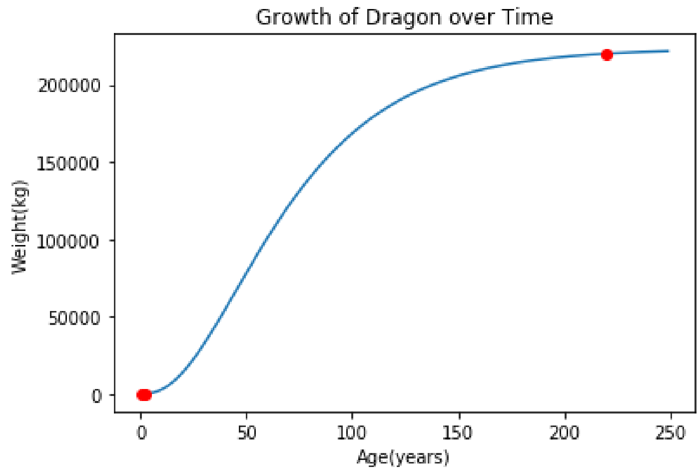
We have some other data points to compare to taken by estimating the weight by estimating the length of the dragons compared to Dany in the show. However, Martin has mentioned that he intended a 5-year time game before his dragon's aged. This means that his dragons grow abnormally quickly for plot reasons. Another free-range dragon, Moondancer, took roughly 13 years to grow to the size that Drogon grows to in 3. Thus, we modified some points during this post-timeskip period, assuming these are irregular growths.

We originally used the numbers given by the problem for t = 0 and t = 1, however these given weights put the dragon as unusually heavy at their young ages. When born, the dragons are smaller than the largest bat, a flying fox, which at most is 1.2 kg. Furthermore, both our pterosaur or our komodo dragon scaled down would weigh much less than the 10 kg stated. Thus, we had our baby dragons weigh 5 kg. The 30-40 kg estimate is more reasonable. After one year the dragons are about the size of a small or medium dog, which would weigh roughly 30 kg.

Without adjusting the times at all, the dragons have the following growth:

Episode	Age	Size
S1E10	newborn	1 foot
S2E5	6 month	1.5 feet
S2E10	1 year	2 feet
S3E1	2.5 years	3 feet
S3E7	2.5 years	7 feet
S4	3 years	12 feet
S4	3 years	25 feet
S6	5 years	80 feet

We could easily fit a model of length to this growth pattern with a higher order polynomial. However, such a curve would be overly artificial, simply fitted to points rather than having any physical basis. Furthermore, doubts due to Martin's mention that he originally intended a 5-year time-skip means that the dragons ages may be lower than they should be, yet their sizes are inflated to the size they need to be for relevant plot points (e.g., Dany riding Drogon). Thus we use the von Bertalanffy equation:



$$W(t) = 223348\left(1 - e^{-.0238(x+1.203)}\right)^3$$

We take more reliable points. 0 years old, 1 year old, and 220 years old, Balerion's size, and base our model on these.

3.4 Caloric Intake—The Basal Metabolic Rate

If dragons are anything like Earth's creatures, then its basal metabolic rate per unit mass will decrease as it gets larger. This means that although it will need more calories as it grows larger, it won't be directly proportional to its size.

3.4.1 Metabolic Rate and Size

A classic example is that a mouse burns many more calories per unit mass than an elephant does. Studies have shown that if M is mass, then $\mathrm{BMR}/\mathrm{M} \propto \mathrm{M}^{-1/4}$

Our base rate will be a human at 100 kg who burns roughly 1W/kg. This is 20.65 kcal/kg per day. An elephant, which is about 3,000 kg would be expected to burn only 8.82 kcal/kg per day.

Our model for the basal metabolic rate of the dragons can be expressed as

$$B(t) = 20.65\left(\frac{M}{100}\right)^{-.25} = 65.3M^{-.25}$$

We can plug in our mass of the dragon to get a function of basal metabolic rate as a function of age.

$$B(t) = 20.65(W(t))^{-.25} = 20.65(223348(1 - e^{-.0238(t+1.203)})^3)^{-.25}$$

Multiplying the basal metabolic rate by weight, we get the necessary calories by day over time, $C(t)$

$$C(t) = 65.3(W(t))^{.75}$$

At Balerion's weight, 219,901 kg, we would expect him to need $65.3(219901)^{.75}$ kcal per day. A cow has roughly 500,000 kcal of edible meat when cooked, which means that Balerion, at his full size, needed at least 1.3 cows per day to maintain his body's metabolic functions. This seems low: a model which accounts for smaller Earth animals may not apply to a creature of Balerion's size.

We can compare back to when in season 5 episode 5, the dragons are an estimated (including the time skip) 9 years old. C(W(9)) = 21,247 kcal per day. Dany feeds her dragons a human, then stops, stating that she does not want to overfeed her dragons. A human is roughly 81,000-125,822 kcal. Although the dragons burn more than their basal rate, this implies we may have an underestimate of the number of calories a dragon burns.

3.4.2 Reasonable Bounds on Dragon BMR

Other studies have suggested that metabolic rate does not vary nearly this much consistently. Instead, most endothermic vertebrates, under which our dragons fall, have a similar rate of from .3–9W/kg. This translates to 3kcal/kg-92kcal/kg each day. Dragons are likely to be somewhere on this spectrum, which actually applies to not just endothermic vertebrates but most living creatures. Taking the entire range using these measures, Balerion could have been eating anywhere from 1.32 to 40.46 cows per day. Our original estimate falls just below this range. Yet, if he were truly burning 40.46 cows a day, his power would be 979 kW. A standard toaster needs 1.5 kW. The energy consumption would explain a dragon's high body heat, but also make the Balerion be dissipating heat at a ridiculously high rate, since his surface area only scales upwards by a square factor not a cube factor.

We can also see that the largest known animal, the blue whale, does not follow our model which begins with a human. A blue whale, averaging 173 tonnes, burns somewhere in the range of 1.5 million kcal per day, or 8.7 kcal/kg per day. [25]

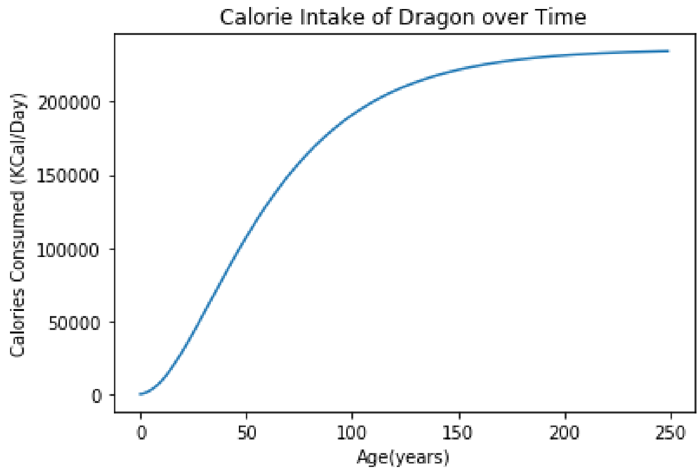
We can readjust our predictive equation with the blue whale as a starting point to be

$$\mathrm{B}(t) = 8.70 \left(\frac{M}{173,000}\right)^{-.25} = 177.4\mathrm{M}^{-.25}$$

$$\mathrm{C}(t) = 177.4(W(t))^{.75}$$

With these calculations, we get that Balerion would have been eating about 3.6 cows a day to maintain his metabolic functions. This falls neatly into our range, and also seems plausible, finding a middle ground between having a lower metabolism because of his large size or having a higher metabolism indicated by the high body temperature.

The base caloric requirements of the dragons as they grow is as follows:



Again going back to the human-eating example in season 5, we would estimate that the dragons weigh W(9) = 2238 kg, with the time-skip adjustment. C(W(9)) = 57,728, which is a little under half of the kcal contained in a human, just to maintain metabolic functions. Archaeologist James Cole estimates a human contains a total of 125,822 kcal. Excluding adipose tissue, they have about 81,000 kcal. Since the dragons also move around and burn calories through activities, our base metabolic rate seems consistent with Dany's estimation of what would be enough food for the dragons.

3.5 Effect of Activity level on Calorie Consumption

Just as in humans, BMR is not how many calories are burnt in a day. We have to take into consideration the caloric consumption resulting from the activity of the dragons. For this, rather than specifically calculating the calories required for every task that the dragons use, a difficult task, we take the Harris-Benedict approach, using multipliers for different activity levels. [7]

Activity Level	Multiplier
Sedentary	$\times 1.2$
Lightly Active	$\times 1.375$
Moderately Active	$\times 1.55$
Very Active	$\times 1.725$
Extra active	$\times 1.9$

Our captive dragons would have been sedentary to lightly active, since they are not lying down but standing much of the time. Applying our multiplier, we would estimate each dragon to burn between 69,274 to 74,324, about 71,799, which means a human would be a little over enough to fill the appetite of the dragon for the day.

3.6 Sensitivity Analysis

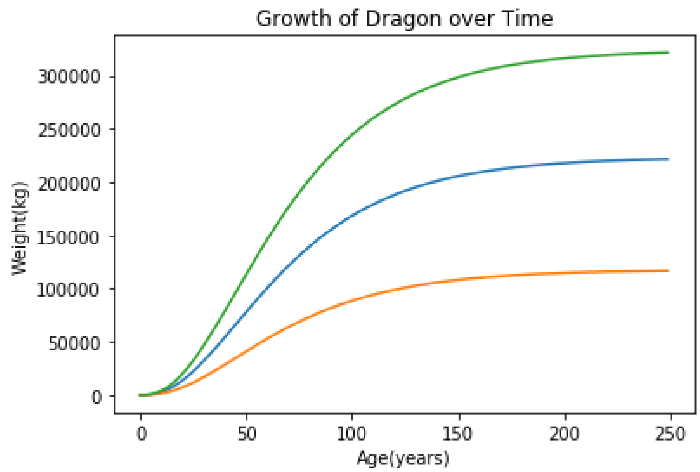
Before, we mentioned a large possible range for Balerion's weight as being somewhere between a pterosaur and a komodo dragon scaled upwards. We estimated that Balerion's mass could range from 115,740 kg to 324,074 kg. If we choose 115,740, our equation becomes

$$\mathrm{W}(t) = 117554\left(1 - e^{.0301(x+1.201)}\right)^3$$

If we choose our upper limit, 324,074, our equation becomes

$$\mathrm{W}(t) = 324250 \left(1 - e^{.0204(x+1.235)}\right)^3$$

Graphing these, we see the range of potential growth patterns in dragons:



We can see that depending on Balerion's actual weight, our dragon's will vary significantly in size as they grow.

Our modeling is highly sensitive to the assumption that our dragons are similar to scaled up versions of smaller animals which have existed. Mentioned before, dragons defy our normal laws of physics.

We expressed the variance in possible caloric intake previously. Taking the range of BMR for endothermic vertebrates, we expected anywhere between 1.32 to 40.46 cows per day. After establishing reasonable bounds, we calculated using a model that incorporated size to find a value within those boundaries, 3.6 cows per day. Our model is sensitive to the assumption that dragon BMR would be able to be scaled upwards from whale BMR using normal models of BMR/kg.

Another factor our model is sensitive to is Balerion's size. We scaled Balerion based on rough estimates. However, due to the fact that our model is based on weight, any change in the actual magnitude of Balerion's length results in a cubic change in his weight. Because of this, small variances in size lead to larger variances in his weight.

3.7 Strengths and Weaknesses

A benefit to our model is that we considered various different potential models and evaluated their plausibility. Especially with our BMR calculations, we estimated caloric consumption starting from the BMR of a blue whale, which has a maximum size somewhat near Balerion. Furthermore, our predictions line up with eating habits in the show and have a basis in the biology of real animals.

A weakness is that we have few reliable data points to base our model off of. Fortunately, once we have growing dragons we can gain more reliable data points. We can measure both length and mass as the dragons grow, then compare our models against the growth of the dragons.

3.8 Extensions of the Model

In this model we assumed that the dragons had enough freedom and food to grow to their largest size. Balerion was taken as the upper limit of growth. In order to adjust this model for dragons who do not meet the requirements, we simply change the W_∞ value to a smaller value.

4 Land Requirements

Based on the food resources used by a dragon, the area of land it would require to fulfill those needs while still remaining environmentally sustainable can be calculated. We define environmental sustainability as being able to maintain a dragon indefinitely without permanent damage to the area, such as the elimination of a plant or animal species. This would require a dragon consumption per area rate low enough that the surrounding environment can regenerate at the same rate or higher.

4.1 Defining the Problem

To determine the land requirements, our aim is to:

- determine a sustainable caloric intake per unit area for different biomes
- use the dragon caloric intake model to determine the necessary land area

4.2 Assumptions and Justifications

- A dragon can be considered to be comparable to the apex predator of the area.

  Justification: Like most other apex predators, dragons are obligate carnivores. The two groups thus have similar diets and effects on their local environment and are therefore comparable, simply on different scales.

- The amount of prey consumed by an apex predator in its natural habitat is an environmentally sustainable amount.

  Justification: Ecosystems with predators in them can support the predators indefinitely and are thus, by definition, environmentally sustainable. As a result, it can be assumed that the consumption of the predator is sufficiently low to provide for prey regeneration.

- In this specific model, a dragon's necessary caloric intake remains constant regardless of environment.

  Justification: For the purposes of simplicity and an easier comparison of land requirements, in this particular model, a dragon's overall caloric intake can be assumed to be a constant not affected by its surroundings.

- The dragons are very active while hunting.

  Justification: The "very active" category in the Harris-Benedict formula states is described as "hard exercise, sports 6-7 days a week." While hunting, the dragons would need to fly for long periods of time and capture and kill prey, all of which would be considered "hard exercise."

- When a range of values is available in the data on apex predators, the mean value will be used.

  Justification Apex predators are animals and thus experience variation in their behavior, such as in the amount of food they eat per day or the size of their territory. In these cases, it can be assumed that the mean value is most representative of the population and it will therefore be used in calculations.

4.3 Approach

To determine a sustainable consumption rate per area of land for dragons, this model uses existing ecosystems and the sustainable consumption rates of existing apex predators. The model is based on the reasoning that predators have achieved a natural balance with the environment for how much prey can be reasonably eaten while maintaining sustainability.

The model uses data on the caloric intake rate of apex predators and their required land range to determine the sustainable caloric production of the land per unit area. The following calculation was performed using different biomes to determine the amount of land necessary to fulfill a dragon's caloric requirements in different climates. In this example, the temperate forest biome is used as the basis.

The apex predator of a temperate forest is the gray wolf. Since wolves travel in packs, the model will use the wolf pack as an entity rather than each individual wolf. Wolves consume an average of 9 pounds of meat per day per wolf and there are approximately 6 wolves per pack, yielding a meat consumption weight of 54 pounds of meat per wolf pack per day [4]. There are approximately 1490 calories in one pound of meat, although there are minor variations depending on the source and type of meat [6]. This suggests that a wolf pack consumes 80460 calories a day sustainably. A wolf pack requires an average of 50 square miles to fulfill these daily caloric needs [5]. This means that for every one square mile, a dragon can reasonably obtain 1609.2 calories per day in a temperate forest environment.

A generalized equation can therefore be derived, as follows.

$$N = \frac{(W)(1490)}{A}$$

Where:

N is the number of calories per square mile that can be obtained in a given biome

W is the weight of meat consumed by the apex predator or a group of apex predators

A is the area of land, in square miles, required by the predator to fulfill their caloric requirements

Using this process for other biomes yields the following data, represented in the table below (sources can be found in the citations list).

Biome	Predator	Food Weight (lb)	Calories/Day	Area Required (square miles)	Calories/sq mi
Temperate Forest	Wolf	54	80460	50	1609.2
Tropical Rainforest	Tiger	17.5	26075	7.7	3386.363636
Boreal Forest	Lynx	2.5	3725	12.4	300.4032258
Desert	Coyote	2.5	3725	5	745
Savannah	Lion	262.5	391125	100	3911.25
Arctic Tundra	Wolf	54	80460	500	160.92

After the sustainable calories per square mile is calculated, the required caloric intake of the dragon is used. We will use Balerion to calculate this because his weight represents the upper bounds of dragon size and thus the upper bound of land needs. While most dragons will not require so much space, an overestimation is safer to ensure that every dragon receives sufficient resources, especially when the consequence of hunger is the consumption of humans.

Using the equation derived in the Caloric Intake model with Balerion's weight and the multiplier from the Harris-Benedict approach, we have

$$C(t) = 1.725(65.3)(219901)^{0.75}$$

which provides us with a caloric need of 1,143,859.5 calories per day. To determine land necessity, a simple calculation can be made, shown through the following general equation.

$$L = \frac{C(t)}{N}$$

Where L is the necessary land required to fulfill caloric needs and all other variables are as previously defined.

Using the previous example of the temperate forest environment, the land requirement equation can be used to find that the necessary land required for Balerion is $\frac{1143859.5}{1609.2}$, a value of 710.8. To sustain Balerion in a temperate forest environment, 710.8 square miles would be necessary. Based on the weights of the three dragons from Game of Thrones at a given time, the equation could be solved to find a personalized L value for each of them. The maximum land required for all three dragons would be Balerion's land requirement multiplied by three. In the temperate forest case, this is 2132.4 square miles.

The upper bound of land requirement in different biomes is represented in the table below, once more using Balerion's weight.

Biome	Land Required per dragon (square miles)	Land Required for 3 Dragons (square miles)
Temperate Forest	710.8	2132.4
Tropical Rainforest	337.8	1013.4
Boreal Forest	3807.7	11423.2
Desert	1535.4	4606.1
Savannah	292.5	877.4
Arctic Tundra	7108.2	21324.7

4.4 Sensitivity Analysis

Our model is sensitive to certain values used during the calculation of the caloric availability per area. Apex predators often have a range of values for the area of territory they require or the weight of food eaten per day. To account for this, in the calculations, if a range was provided then its average was used as the value. However, this range of values could have affected the resulting conclusions.

For example, a lynx requires territory ranging from 5.8 to 19 square miles. While the average, 12.4 was used, using the minimum and maximum values yields a potential calories/square mile range from 196 to 642.2, a difference of 446.2 Cal/sq mi. As a result, the model is sensitive to the values chosen. However, in the absence of more specific data, the model accommodates relatively well by using the average of the range values.

The model is somewhat sensitive to the assumption that dragons can be classified as "very active" while hunting. Depending on the different activity levels, the dragon would require different caloric values, which would affect its land requirements. For example, if the dragon was classified as lightly active when hunting, Balerion would require 566.6 square miles of land, while he would require 782.9 square miles of land when considered extra active. There is a variation 216.3 square miles, suggesting that the model is sensitive to the assumption but not extremely so.

4.5 Evaluating the Model

A strength of the model is its relative ease of creating a realistic estimate for the land requirements of a dragon. The model considers environmental sustainability as a major part of its basis, and is therefore highly realistic in the sense that it is environmentally feasible. The model aims to preserve the integrity of earth's ecosystems, while still fulfilling the necessary requirements of dragons. Additionally, the model finds a strength in its basis in real world data, namely, the existing self-supporting model of predators in various ecosystems. As a result, the model is real world applicable and not based in speculation.

However, a limitation of the model is its sensitivity to certain assumptions. The model is based in the comparison of a dragon to an apex predator of a given area. However, the dragon's caloric requirements would be an additional strain on the ecosystem, instead of a replacement for the predator as we assumed. While this limitation is somewhat addressed by the fact that dragons would likely consume some apex predators, thus decreasing the amount of prey needed, the model does not fully address this problem. To address this, the model would likely have to obtain estimations on the composition of prey eaten by a dragon in each biome and calculate the needs of the dragon along with the needs of existing predators, providing for prey regeneration. This would be a greater extension of the model, however, given the sheer volume of data necessary, it is one not included in this paper.

5 Effects of Different Climates on Caloric Intake

While the previous model operated on the assumption that caloric intake remained constant in relation to different environments, in reality, the dragon's activity levels, would likely be adjusted based on its surroundings. As a result, necessary caloric intake would likely also be affected, which would affect the dragon's resource requirements.

5.1 Defining the Problem

Our aim is to:

- Account for changes in temperature
- Account for arid environments with little access to water

5.2 Assumptions and Justifications

- The behavior of dragons across different climates can be modeled through scaling the behavior of migratory bird species.

  Justification: Birds and dragons have similar attributes, both not having sweat glands and also being capable of flight.

- Assume that the most common method of water intake for dragons is from the food they consume, although they can also generate water through catabolism of proteins and fats.

  Justification: In Game of Thrones, there is a lack of emphasis on the water necessities of dragons. Thus, it can be assumed that the majority of water intake is through their foods or an internal process.

- Assume that when dragons can rely on water from animals and carbohydrates rather than from fat and protein catabolism, they would do so.

  Justification: It makes sense that dragons would choose to preserve their ability to create water for long distance journeys.

- When dragons are in arid conditions, they amount of water intake from organisms is insignificant.

  Justification: The species living in arid areas likely have adapted to survive with minimal water availability, so the water intake from these organisms are likely not enough to sustain the dragon's needs alone.

- Assume that in Season 5 Episode 5, the dragon received its daily requirements of water through the human and that it made use of all the water in its body.

  Justification: In order to estimate the water intake level for dragons as a function of mass, we need to find a base point for the model. Thus, this assumption was done out of necessity for developing solution.

- Assume that the caloric growth can be linked to water necessity.

  Justification: This assumption was conducted in order to evaluate the water requirements for a dragon at the time of Season 5 Episode 5.

- Dragons can regulate changes in environmental temperature through changes in their actions.

  Justification: Many existing reptilian species regulate their body temperature through various alterations in behavior.

5.3 Effect of Travelling through Arid Environments

In an earlier assumption, we argued that dragons are capable of sustaining their water needs through both the food they consume and the products of protein and fat catabolism. These two methods of consumption are assumed to work together in a variety of environments. For example, in arid locations where there is little water availability in the dragon's prey, the internal protein and fat catabolism would need to play a larger role towards the dragon's water necessities, while in locations with an abundance of water, protein catabolism would need to play a smaller role since the majority of the water necessities could be supplied through the dragon's prey. However, this increased reliance on internal mechanisms likely is not sustainable for long periods of time, as fat storage will likely need to be used up to fuel this process.

	A	B	C	D	E	F	G
Substrates	Metabolic H_2O in g/kJ of oxidized substrate[a]	Energy in kJ/g substrate wet wt	Metabolic H_2O in g/g fuel (A × B)	H_2O content in g/g dry wt substrate[d]	Energy in kJ/g dry wt	H_2O content in g/kJ (D/E)	Total H_2O in g/kJ (A + F)
Fat	0.0272	39.6[e]	1.08	0.2	39.6	0.005	**0.032**
Protein	0.022	3.7[b]	0.08	3.65	18.4	0.198	**0.22**

Table taken from https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5304341/ [16]

a: Values for calculating metabolic water production from indirect respiration calorimetry

b: Assuming that lean tissue contains approximately 73% water

c: Assuming that 2/3 of glycogen wet weight is water.

d: Water associated with electrolytes and hydration shells.

e: Assuming that fatty acids can be mobilized from adipose tissue without loss of tissue water.

Based on this table, in migratory birds over arid climates, 0.032 g H_2O is made per kJ throughout oxidation of fat and 0.220 g H_2O is made per kJ through oxidation of protein. Often, these migratory birds stock up on large supplies of fat prior to the trip before their trip, spending approximately 90% of the a 2.5 day migratory trip through reliance of fat and 10% of the trip through reliance of protein. Performing the calculation:

$$(0.9 \times 0.032) + (0.1 \times 0.220)$$
$$= 0.029 \text{ g } H_2O \text{ / kJ fat / trip} + 0.022 \ H_2O \text{ / kJ protein / trip}$$
$$= 0.049\text{g } H_2O \text{ / kcal fat / day} + 0.037 g H_2O \text{ / kcal protein/ day}$$

Recall that in Season 5 Episode 5 of the series, we had estimated that the dragon calorie intake at the time would be between 69,274 and 74,324 calories, producing an average of 71799 kcals per days. As stated before, we also assumed that a human contains 81000 kcal of energy. Using these values, we can estimate the water intake that a dragon would need under any standard condition. Given that a human's weight is made up for 90% water and that the average human has a mass of 62 kg, approximately 55.8 kg of water was inside the human.

$$(71799/81000) * 55.8 = 49.46 \text{kg} H_2O$$

This indicates that dragons need 49.46 kg of water in Season 5 Episode 5. Diving this value by the caloric intake, we can determine that the base rate of water in terms of caloric intake is 0.689 g H_2O / kcal / day.

Applying this value to the 0.049 g water / kcal fat / day and 0.037 g water / kcal protein / day, this implies that in arid conditions, dragons would need to use their fat and protein reserves at a (0.689 / 0.086) = 8.011x faster rate than the speed that existing migratory birds use their fat and protein reserves. This implies that, at the very maximum, dragons would only be able to fly for (2.5 * 24) / 8 = 7.5 hours under arid conditions where their only supply of water would come from their fat and proteins before needing to replenish these reserves.

5.3.1 Supply of Freshwater

In the previous section, we assumed in arid environments that there was no availability of water either through freshwater or through the organisms they consumed. However, should there be an oasis in this arid environment, dragons can consume water to sustain their flight throughout the day. Using the previously determined water consumption rate of 0.689 g water / kcal / day, Since 1 liter of water = 1000 g of water, dragons would need to consume 0.00689 L water / kcal / day. Thus, should a relatively large dragon develop an extended reliance on oasis water, it is likely that the body of water may be in danger of water depletion. Thus, it would be optimal to not stay within arid biomes for extended periods of time.

5.4 Effects of Temperature

Earlier, we determined how the different activity levels of dragons created variation in the caloric consumption of these dragons. These variations in activity can be applied to how dragons raise and lower their body temperature based on their surroundings. For example, in hotter environments, dragons would likely adapt by spending more periods of time resting in shade, while in cooler environments, dragons could make greater use of their internal heat, indicating a great amount of activity depending on the degree of coldness. Ultimately, the minimum amount amount of caloric intake to be expended per day would be $57728 \times 1.2 = 69264$ kcal while the greatest amount would be $57728 \times 1.9 = 109683$ kcal. By setting these two values to be horizontal asymptotes for a logistic function, we can map the caloric range of dragons based on temperature. Thus, caloric demand would be greater at larger temperature values.

5.5 Sensitivity Analysis

The assumption that dragon catabolic behavior can be mapped to that of migratory birds plays a large role in determining the water intake of the dragon. The greater the difference between birds and dragons, the more inaccurate this model becomes, as this makes the water intake through catabolism of fats and proteins more inaccurate. In addition, it is likely true that in Season 5 Episode 5, the dragon did not have 100% efficiency in processing the human's water. As a result, the 0.689 g water / kcal / day prediction is likely an overestimate, indicating that more time would be available for flight. This scene also uses the assumption that 100% of the dragon's water necessities was met by the human. However, we do not know whether or not the human provided more or less water than necessary, resulting in a larger error.

5.6 Strengths and Weaknesses

The biggest strength of this model comes from the multiple considerations of water nourishment for the dragon. We considered intake from freshwater, intake from organisms, and intake from internal mechanisms of the dragon. This allows our model to have increased value in its application in different environments and conditions. Additionally, our model also utilized direct evidence from the Game of Thrones Television show, making the model based off of the limited amount of data available.
However, this model also has several weaknesses. The sensitive assumptions of the model were made due to the limited amount of data available for water consumption of species outside of livestock as well as the lack of information about water intake or method of intake of dragons from the TV show.

5.7 Extensions of the Model

In order to extend the model, more data points and greater information about the dragons from the Game of Thrones world would make the specific values within the model more accurate.

6 Human Intervention

The previous models assumed that the dragons would be required to sustain themselves alone in the wild. However, in the context of Game of Thrones, this would not be feasible, since dragons need a rider to be controlled and thus prevented from engaging in rampant destruction. As a result, for a safe integration of dragons into our world would necessitate a degree of human intervention, explored in this model.

6.1 Defining the Problem

In this model, our aim is to:

- determine requirements for dragonriders
- determine the number of people necessary to produce enough food to fulfill the dragon's caloric requirements

6.2 Assumptions and Justifications

- Upon arriving in this world, the three dragons have lost their connection with Daenerys.

Justification As the dragons are now in a different world, it must be assumed that they can no longer sense their previously bonded dragonrider. As a result, the dragons will be able to bond a new rider.

- We assume that the areas in which the dragons are being raised are optimized for both the cow and the food for the cows, meaning that drought is negligible.

  Justification: Given that dragons have never existed on Earth before, it is assumed that humans would put in the greatest amount of effort to provide them their resources, meaning they would get the best possible land possible for raising their necessities.

6.3 Solution

Each dragon would need a dragonrider. Immediately, the three dragons that come to our world should bond with a human as their new dragonrider. Potential ways to select new dragonriders would be to adopt something similar to NASA's astronaut selection method. Riders would need to demonstrate levelheadedness, physical capability, and an ability to take on responsibility. However, we come to the issue that the dragons are selective; they may not accept any rider. Thus, a queue of potential riders could be prepared.

The dragons, even with rider, still become rowdy on occasion, demonstrated by Drogon in the show. Disaster relief teams could be equipped with measures against dragons and protocols for dragons would be written. Although the dragon was able to easily slay many people in Season 7 Episode 4 of Game of Thrones, in today's society, there are many more powerful weapons capable of neutralizing dragons. In this episode, also, the dragon was shown to be vulnerable to a crossbow, making bullets a very valuable tool for taking down dragons. Additionally, understanding that humans in the past were able to slay mammoths to the point of extinction, with today's technology, it is safe to argue that a team of approximately 10-20 specialists per dragon would be enough for the task, thus requiring 30-60 people total. If a rampant dragon's potential damage could be equivocated to that of a bomb, this 10-20 specialist per dragon requirement again would be sufficient for neutralizing the threat.

Once the dragon riders have been selected and confirmed by the dragon, the dragon still needs to be provided several necessities, mainly food. Dragons would be able to extract water from this food. In this model, we will consider the scenario where all of a dragon's caloric needs are met solely from human provisions. This would represent the maximum amount of human intervention necessary to sustain the three dragons.

We will use cows as a representation of livestock because they provide the most calories out of domesticated livestock. In this case of feeding cows to dragons, we would not need a population growth model since our goal is to keep the cow's population constant, namely, every time a cow is eaten, it is immediately replaced by a new adult cow.

Given that cows have an average pregnancy time of 283 days, take 55 days before they can give birth again, and take 2 years to become an adult, it takes one female cow 1068 days to give birth to an adult cow after it has last given birth [18].

Let us use the variable x as the minimum amount of cows needed per day for 1 dragon to survive. In order to minimize the cow population while still feeding the needs of a dragon, we will have $1068x$ female cows on the farm, producing x adult cows per day. Whenever a female cow reaches adulthood, the oldest female cow is fed to the dragon, while any male cow that reaches adulthood is also fed to the dragon. Male cow sperm will be inserted into the female the day they are able to be pregnant. This way, x adult cows will be able to be seasonably produced and eaten by the dragon each day. In addition to the $1068x$ female cows on the farm, there will also be $365 \times 2 = 730x$ cows on the farm as well working to reach adulthood. This means that there will be $1798x$ cows on the farm at any given time. In the case that a cow dies due to an unforeseen circumstance, its body would be fed to the dragon, and if the dead cow was female, the next youngest female would replace its spot. For three dragons, thus, a total of $5394x$ cows would be needed to sustainably feed them.

However, we must also account for the food and land needed for the cows. The following formula can be used to model land needed [19]:

$$N = (A \times Y)/(0.04 \times W \times 365)$$

Where:
N is number of animals
A is of Acres
Y is yield per acre
W is average animal weight

Since the average weight of a cow is 2000 lbs and 8000 pounds of grass is grown per acre using alfafa, we can calculate the of acres needed, A, as:

$$5394x = (A \times 8000)/(0.04 \times 2000 \times 365)$$
$$= 19688x \text{ acres of land}$$

In the US, there are an estimated 3 million farm workers and an estimated 2.16 million farms, meaning 1 farm hosts on average 1.38 workers [20, 21]. Given that an average farm has a size of 445 acres, we can calculate the amount of people required to manage $19688x$ acres of land as $19688x/445 \times 1.38 = 61x$ farmers. Thus, the amount of people needed directly to manage the dragon are $61x + 3 + (30 + 60)/2 = 61x + 48$.

Fortunately, there is not a large concern that the dragons would get sick and need to be cared for, as their body temperature is high enough that any diseases would not be able to survive, much like is the case with bats.

While the human resources considered in this model are those that directly impact a dragon, the raising of a dragon would be a larger community effort. For example, factories would be required to produce machinery necessary for farming. Due to the interconnected nature of the economy, an entirely self contained community would be highly unrealistic. The entire society would likely be affected by dragon care through indirect connections. Additionally, the increased transportation of goods, along with the methane production of cows and dragons, could potentially cause more problems with global warming.

Of course, as mentioned previously, these high resource requirements are due to the model's assumption that humans are providing all necessary food resources for the dragon. However, dragons could be responsible for part of their own hunting, thus putting less pressure on human communities.

6.4 Sensitivity Analysis

Throughout this model, averages were taken in order to achieve a final result. Pregnancy, farm yield, cow weight, and of farms are all averages that were taken that could potentially influence the result should the true value lie on either extreme of the bell curve. Additionally, this model also assumes that drought is an insignificant factor for plant growth, since the most optimal conditions would be used. However, in reality, drought is a large problem all across the world, and thus, more land would likely be needed than what was listed, meaning more farmers working. The cow-producing farm was also optimized to minimize the amount of cows needed to feed the dragon. However, it is difficult to argue that the system would work perfectly. Therefore, there would likely need to be more cows than the predicted $5394x$ on the farms.

6.5 Evaluating the Model

One strength of this model is the fact that it is successfully able to model a potential number of people that would be able to sustain the dragons. It also is very optimized in its consideration for the least amount of resources necessary.

However, at the same time, this model is also have several weaknesses. Implementing such a system where cows would be raised for the sole purpose of feeding dragons would likely spark many ethical concerns and anger among the global population. Additionally, this model's sensitivity to variation in assumptions and averages makes it potentially difficult if it were to be applied directly to the real world. It is also likely that it may be difficult to realistically secure this amount of land and cows in one area.

Thus, one potential extension of this model would be to account for transportation time and manpower, allowing cows to be exported from across the world. This would allow several of the assumptions made in the model to be more realistic.

7 Consideration of Reproduction

Dragons have a complex reproductive system. Dragons have no physical differences to differentiate genders. Females are identified because they lay eggs. Dragons who do not lay eggs are assumed to be male.

- The gender of dragons is unimportant. Any two dragons can always reproduce and will adapt as necessary.

  Justification: Characters in the books state that dragons are "now one and now the other, as changeable as flame." Also, as the humans in Game of Thrones do not seem averse to incest, we assume that the dragons can mate with each other regardless of genetic relation.

- Dragon eggs can be stored and hatched at any time.

  Justification: Dany's eggs are said to be eons old, petrified and Dany still manages to hatch them. Furthermore, the eggs require specific conditions (in a fire with a living sacrifice) to hatch and will not hatch on their own. This means that if eggs are collected from the dragons once they are laid, humans can easily control the population of dragons. One concern is that Earth humans are not able to hatch dragon eggs.

Because of the highly controlled nature of dragon eggs, the dragon population does not follow any known models of population growth. Furthermore, due to the high risk on the environment and population caused by dragons, it is in humanity's best interest that dragons do not propagate throughout the earth. This is an area where humans will want to use our models to find the carrying capacity for dragons, then control the dragon population from there.

Unfortunately, if our assumption that dragon siblings will not or cannot mate is false, the dragon population is already doomed for extinction.

8 Letter to G.R.R.M.

January 28th, 2019

Dear Mr. George R. R. Martin,

We are writing to you today to discuss the ecological realism of the dragons in your popular book series Game of Thrones. We are a group of mathematicians and scientists who have meticulously considered the characteristics of your dragons and their sustainability on Earth. In the course of our analysis, we also explored the requirements of dragons, especially in regards to how they change in their environments. Although your series is fantasy and you likely did not impose the same rigorous realistic analysis that we did, we believe that the results of our exploration will nonetheless be of interest to you in your writing. After all, it can aid in creating the most realistic possible world in your writing.

In your series, you have three still-living dragons: Viserion, Rhaegar, and Drogon. To summarize our analysis, we must first discuss the known characteristics of the dragons we based it on, as per facts provided in your series. Firstly, we used the information that the dragons in your series are obligate carnivores, and thus only consume meat and fish. This was an important aspect of our mathematical model in determining the food resources required for each dragon's survival. Additionally, we based our mathematical exploration of the dragons' growth pattern on the fact that dragons in Westeros grow indefinitely, so long as they are provided with enough resources and space. As a result, we were able to compare your dragons to existing species with indeterminate growth to find their growth patterns.

Using these values, we were able to create an equation to estimate the amount of calories required for each dragon. Based on the sizes of your dragons, they would require different caloric intakes. However, using Balerion, the largest dragon recorded, we determined that the largest dragons would need at least 3.6 cows a day. As such, we recommend that you adhere to this food limit to maintain a realistic metabolic rate for your dragons.

With the help of the previously mentioned caloric requirement model, we were able to also determine the adjustments depending on climate. In your series, the three dragons commonly migrate to different settings, such as their time in the desert or their battle in the cold North. As a result, it is important to consider the effects of different environments on the dragons and adjust them accordingly to achieve maximal realism.

We looked at the effects of arid conditions on the dragons. Using data from existing migratory birds, we discovered that your dragons could likely produce water as a byproduct of their metabolism of their foods. As a result, in arid conditions, they would not require constant water provisions. However, there is a limit to this water amount which we recommend you adhere to for biological accuracy. Based on information from Season 5 Episode 5 of the show, we estimated that a dragon requires 49.46 kilograms of water per day. Based on their caloric intake, they would be able to produce this water solely from their metabolism for 7.5 hours of flight. We therefore recommend that in arid conditions, your dragons do not travel for longer than 7.5 hours without being provided with additional food or water. Due to the high food and water requirements of the dragons, while dragons can exist in arid conditions, it is not an optimal survival. Hence, we also recommend that in your series, dragons do not spend long amounts of time in such environments, since they could not feasibly fulfill their water requirements. In the event that arid conditions are necessary for plot purposes, we suggest that you take the high food and water requirements into account and provide the dragons with their necessary provisions.

In our climate considerations, we also considered different caloric requirements in different temperatures. When your dragons are in cold conditions, they would likely be more active in order to create more internal body heat. As a result, they have higher caloric requirements in cold conditions, a fact that we recommend you address in your books. This could easily be done by having Daenerys prepare more food for her dragons when fighting in the North, especially because less prey is available in cold arctic tundra conditions.

While we are aware that your novels are fantasy and that dragons are not real, we would also like to note that our mathematical calculations have real world impact. The equations we used to determine a dragon's basal metabolic rate, and thus is required caloric intake, could be used for other species in order to determine their caloric requirements. This could have implications in environmental conservation efforts in determining whether a given ecosystem meets the caloric needs of its organisms. Determining this could be essential to determining which organisms are at risk due to not having enough resources, in order to concentrate conservation efforts.Environmental conservation is an important problem in our society today. While we can create countless fantasy worlds, we only have one real world, one we must strive to protect.

There were also real world considerations raised in another model, where we determined the necessary level of human intervention to provide dragons with their full caloric needs. When doing so, we discovered that a large scale cow farm was needed, where cows were bred solely for the purpose of feeding the three dragons. This farm seemed largely unethical, as cows were immediately killed upon reaching adulthood for optimal food production. The farm also raised environmental concerns, which as previously mentioned, are important to consider. Cows produce large amounts of methane gas which contributes to the greenhouse effect and a resultant global warming. While this particular cow farm does not exist, we must consider the cow farms that exist in the real world, ones that feed the dragons of our world: us.

In the real world, the cow model could be used to determine the number of farmers and cows necessary to sustain one human being, and by extension, humanity. This model could be used to communicate the exact pressure we place on the world in order to fulfill our needs, in hopes of increasing awareness as to our environmental impact.

Our method for estimating the basal metabolic rate of animals could be applied to any animal existing here on Earth as well. We would predict the caloric intake of animals based on their size, kingdom, and activity.

We hope that our mathematical analysis aids you in the creation of your series to be realistic. Although dragons do not truly exist, they still ignite our collective imagination and excitement. As such, we hope you implement our suggestions to create the best dragons possible.

Sincerely,
Team 1917933

9 Citations

1. http://www.ucmp.berkeley.edu/glossary/gloss5/biome/
2. Penguin Random House (2018). A Song of Ice and Fire Series.
3. https://www.sciencedirect.com/science/article/pii/S0378112711000375
4. https://wolfsanctuarypa.org/2016/10/14/much-food-wolves-eat/
5. https://www.wolf.org/wolf-info/basic-wolf-info/wolf-faqs/#h
6. http://book.bionumbers.org/how-does-metabolic-rate-scale-with-size/
7. https://manytools.org/handy/bmr-calculator/
8. https://answers.yahoo.com/question/index?qid=20081220234733AAfe5zj
9. https://www.fitbit.com/foods/categories/Beef/Ground+Beef+Prepared+Beef/161/page/2
10. https://www.2ndchance.info/bigcatdiet.htm
11. http://www.lions.org/tiger-habitat.html
12. https://www.felineworlds.com/canadian-lynx/
13. https://www.cityofpaloalto.org/civicax/filebank/documents/27579
14. https://animals.mom.me/amount-food-lion-eats-9881.html
15. https://www.nationalgeographic.com/animals/mammals/a/african-lion/
16. https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5304341/
17. https://qz.com/951238/a-scientist-calculated-the-nutritional-value-of-a-human-being/
18. https://beef.unl.edu/faq/pregnant-cows
19. https://www.nrcs.usda.gov/Internet/FSEDOCUMENTS/stelprdb1097070.pdf
20. https://www.farm-equipment.com/articles/15960-number-of-us-farms-declines-while-size-of-farms-increases
21. https://www.statista.com/statistics/196103/number-of-farms-in-the-us-since-2000/
22. http://www.ncfh.org/uploads/3/8/6/8/38685499/fs-migrant_demographics.pdf
23. https://news.nationalgeographic.com/2017/10/new-pterosaur-found-mongolia-largest-fossils-science/
24. https://nationalzoo.si.edu/animals/komodo-dragon
25. https://www.sciencedirect.com/topics/agricultural-and-biological-sciences/indeterminate-growth
26. https://www.livescience.com/10332-whales-swallow-million-calories-single-mouthful.html