# A Schedule for Lazy but Smart Ranchers
# 为懒惰但聪明的农场主安排的时间表

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**Abstract**

We present our three-stage model constructing process in this paper. Firstly, we determine the number of sprinklers to use by analyzing the energy and motion of water in the pipe and examining the engineering parameters of sprinklers available in the market.

Secondly, we build a model to determine how to layout the pipe each time the equipment is reinstalled. This model leads to a computer simulation of catch-can tests of the irrigation system and an estimation of both distribution uniformity (DU) and application efficiency of different schemes of where to move the pipe. In this stage, DU is the most important factor we considered. We also found a schedule in which one of the sprinklers will be positioned out of the field in some moves, but it results in higher DU (92.1%) and actually saves more water.

In the final stage, we figure out two schedules to irrigate the field. In one schedule, the field receives water evenly in time during a cycle of irrigation (in our schedule, 4 days), while the other one costs least labor force and time. In this stage, the time and labor force required are the most important factors we considered.

Our suggested solution, which is easy to implement, includes a detailed timetable and the arrangement of the pipes. It costs 12.5 irrigation hours and 6 equipment resets in every cycle of 4 days to irrigate the field with DU as high as 92.1%.

在本文的建模过程中, 我们提出了三阶段模型. 在第一阶段中, 通过分析管子内水的能量和运动, 并对照市面上可用喷头的工程参数, 我们确定了所使用喷头的数量.

在第二阶段中, 我们建立了一个模型来确定如何布置每次重新安装的管组. 这个模型引出了一个灌溉系统的获水容器测试的模拟, 并且估算出分布均匀度 (DU) 和不同的工作进度表的执行效率. 在这个阶段中, DU 是我们考虑的最重要的因素. 我们也找到了这样一种方案: 在某些移动中, 其中一个喷头被放置在农田区域外, 但这种方案却给出了更高的均匀度 (92.1%), 并且更节约水.

最后的阶段中, 我们给出了农田灌溉的两种工作进度表. 在第一种工作进度表中, 在一个灌溉周期 (在我们的工作进度表中为 4 天) 中, 农田能够均匀地获得水; 然而另一种工作进度表花费的人力和时间最少. 在这个阶段, 所需的时间和人力是我们考虑的最重要的因素.

我们推荐的解决方案非常易于操作, 它包括一个详细的时刻表和管道排布方式. 该方案在一个为期 4 天的农田灌溉周期内, 将花费 12.5 小时的灌溉时间, 并需要按装 6 次, 其分布均匀度高达 92.1%.

# Contents

# 1 Problem Review | 问题重述

"Hand-move" irrigation system is one of various techniques for irrigating a field. Pipes with sprinkler heads are put in place across fields, and they are moved by hand at periodic intervals to insure that the whole field receives an adequate amount of water. This type of system is cheaper and easier to maintain but requires a great deal of time and effort to move and set up the equipment at regular intervals.

Use this system to irrigate an 80m × 30m field and work out a scheme to minimize the amount of time required by a rancher to maintain the irrigation system and maximize the uniformity. Given a pipe set whose total length is 20m, try to determine the number of sprinklers and the spacing between sprinklers and find a schedule to move the pipes, including where to move them. Each pipe has a 10cm inner diameter with rotating spray nozzles that have a 0.6cm inner diameter. At the water source, the pressure is 420kPa and has a flow rate of 150lpm. No part of the field should receive more than 0.75cmph of water, and each part of the field should receive at least 2cm of water every 4 days.

"手动" 灌溉系统是灌溉一片农田的手段之一. 将带有喷头的管子放置在农田中, 并且周期性的手动移动它们, 以确保整块农田都获得充足的水量. 这种类型的灌溉系统非常便宜, 并且易于维护, 但需要大量的时间和精力去定期地移动和安装这些管组设备.

用这种灌溉系统来浇灌一块 80m × 30m 的农田, 并求解出一个方案来最小化一个农场主维护灌溉系统所需的时间, 并最大化降水分布的均匀度. 给定一套总长为 20m 的管组, 尝试去确定喷头的数量和喷头的间距, 并寻找一个移动管道的工作进度表, 这个工作进度表还要包括把管道移动到什么位置. 每根管子的内径为 10cm, 并附有一些内径为 0.6cm 的可旋转喷嘴. 水源处的压强为 420kPa, 流量为 150lpm ("升/分钟"). 农田任何部分接受的水量不能超过 0.75cmph ("厘米/小时"), 并且农田任何部分接受的水量每 4 天不得少于 2cm.

# 2 Assumptions and Definitions | 假设和定义

- The weather is assumed to be "fine" while the influence of wind can be neglected.
- The whole system is assumed to be "ideal" while evaporation, leaking and other ways of water loss can be neglected.
- The water source can be moved and put at any position of the field. In practice, a tube can be used to transport water from the pump to the pipe set.
- No mainline is assumed to exist so that all pipes join together and can be put at any position of the field.
- The time for a rancher to uncouple, move and reinstall the pipe set is assumed to be half an hour.
- The discharge of any sprinkler is assumed to be the same.

- 假设天气 "很好", 从而可以忽略风的影响.
- 假设整个系统是 "理想的", 从而蒸发, 渗漏等方式的水分流失可以忽略.
- 水源可以移动并安置在农田的任何一个位置. 实际上, 可以用一个管道来从泵输送水到管组.
- 假设没有管道主线, 因此所有管道可以连接起来, 并可以放置在农田中的任何位置.
- 假设农场主一个人拆卸, 移动, 重新安装管组的时间为半小时.
- 假设任何喷头的出水量都是相同的.

- The manufacturer design pressure of sprinklers is assumed to be about 400kPa which is a most appropriate pressure in this problem and the type of rotating sprinkler is impact driven sprinkler (see details in Information & Analysis of Sprinklers section).
- The diameter of the riser is assumed to be the same as that of the pipe.
- The water pressures in pipes are assumed to be the same. In practice, there is a slight difference.

- 假设喷嘴的额定压强为 400kPa, 这个压强值对于本文题是最适合的, 旋转喷头的类型为摇臂式喷头 (具体细节见 “喷头的信息和分析” 小节).
- 假设立管的直径与 (横着的) 管子的直径相同.
- 假设管子中的水压相同. 实际上, 会存在细微的差别.

**Table 1:** Variables | 变量

| Variable | Definition | 中文定义 | Units |
|---|---|---|---|
| $p_{\text{in}}$ | Water pressure in the pipe before a junction | 管道中交叉点前的水压 | kPa |
| $p_{\text{out}}$ | Water pressure in the pipe after a junction | 管道中交叉点后的水压 | kPa |
| $p_{\text{up}}$ | Water pressure at the sprinkler upon a junction | 交叉点上方喷管的水压 | kPa |
| $v_{\text{in}}$ | Water velocity in the pipe before a junction | 管道中交叉点前的流速 | m/s |
| $v_{\text{out}}$ | Water velocity in the pipe after a junction | 管道中交叉点后的流速 | m/s |
| $v_{\text{up}}$ | Water velocity at the sprinkler upon a junction | 交叉点上方喷管的流速 | m/s |
| $A_{\text{in}}$ | Section area in the pipe before a junction | 管道中交叉点前的截面积 | $\text{cm}^2$ |
| $A_{\text{out}}$ | Section area in the pipe after a junction | 管道中交叉点后的截面积 | $\text{cm}^2$ |
| $A_{\text{up}}$ | Section area at the sprinkler upon a junction | 交叉点上方喷管的截面积 | $\text{cm}^2$ |
| $h$ | Height of the sprinkler upon the pipe | 喷头相对于管道的高度 | m |
| $t$ | Change in time | 变化的时间 | s |
| $v_{\text{source}}$ | Water velocity of the water source | 水源处的流速 | m/s |
| $n$ | Number of sprinklers | 喷头的数量 | - |
| $\text{distr}(r)$ | Distribution function of precipitation profile | 降水分布函数 | - |
| $p$ | Precipitation rate | 降水率 | - |
| $R$ | Sprinkling range | 喷洒范围 (半径) | m |
| $r$ | Distance from a sprinkler | 离喷头的距离 | m |
| $r_i$ | Distance from the $i$th sprinkler | 离第 $i$ 个喷头的距离 | m |
| $a$ | Obliquity of the precipitation profile | 降水分布线倾斜角 | rad |
| $\text{pr}(r)$ | Precipitation function of one sprinkler | 单个喷头降水分布函数 | cm/min |
| DU | Distribution Uniformity of an irrigation system | 灌溉系统的分布均匀度 | - |

**Table 2:** Constants | 常量

| Variable | Definition | 中文定义 | Value |
|---|---|---|---|
| $\rho$ | Density of water | 水的密度 | 1.0kg/L |
| $g$ | Acceleration of gravity | 重力加速度 | $9.8\text{m/s}^2$ |

# 3 Problem Analysis | 问题的分析

Our goal is to determine the number of sprinklers and the spacing between sprinklers and find a schedule to move the pipes, including where to move them.

Our approach can be divided into three stages.

First, determine the number of sprinklers. In order to achieve the goal, we figure out the pressure and velocity of water from each sprinkler and then determine possible sprinkler number according to some engineering data.

Second, determine where to put the pipes. We consider some major factors such as sprinkling time, moving time and distribution uniformity (DU) while DU is especially important for the purpose of saving time. Since the pipe positions depend on the sprinkler number and the precipitation profile, we just work out some problem specific cases. However, our method can be used to solve any practical cases.

Third, determine the schedule to move the pipes. Referring to the water need of the field, a schedule is made to minimize the time cost, which, obviously, is closely related to the positions of the pipes to move.

我们的目标是确定喷头的数量和喷头之间的距离, 并寻找一个移动管道的工作进度表, 这个工作进度表还要包括把管道移动到什么位置.

我们的解题方法可以分为三个阶段.

第一阶段, 确定喷头的数量. 为了达到这个目的, 我们计算出每个喷头出水的压强和流速, 然后根据一些工程参数确定可能的喷头数量.

第二阶段, 确定管组的放置在什么位置. 我们考虑了一些主要因素, 比如喷洒时间, 移动时间以及分布均匀度 (DU), 分布均匀度对于节省时间的目的来说是非常重要的. 由于管子放置的位置依赖于喷头数量和降水量分布, 我们只是解决一些具体实例. 然而, 我们的方法可以用来解决任何实际情况.

第三阶段, 确定移动管道的工作进度表. 考虑农田的灌溉需水量, 制定一个工作进度表是为了最大限度地减少时间成本, 很明显, 工作进度表的制定与管道的位置密切相关.

# 4 The Development of Models | 模型的建立

## 4.1 Stage 1: Water Pressure and Velocity | 第 1 阶段: 水压和流速

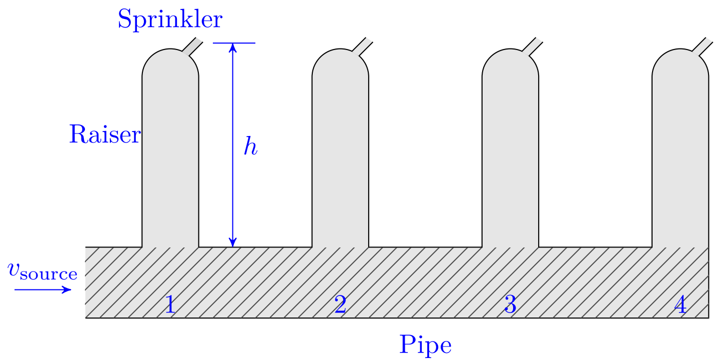



**Figure 1:** Overall sketch for 4 sprinklers and 4 junctions. The pressure of the shadow area is the same due to the last assumption. | 4 喷头 4 交叉点总体示意图. 由最后一个假设可知阴影部分的压强相等

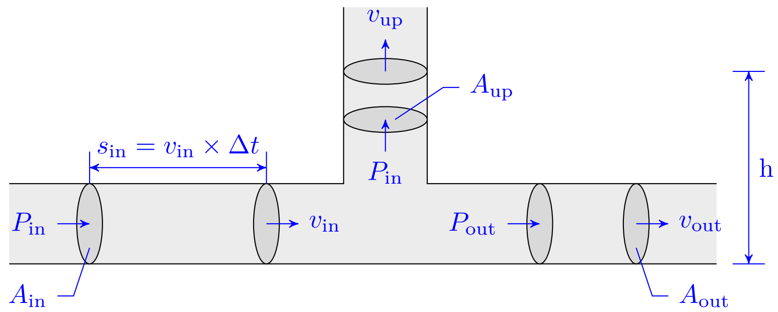



**Figure 2:** Sketch map at one junction. | 一个交叉点的示意图.

We apply the law of conservation of energy to solve. The work done by the forces is

我们应用能量守恒定律来解决问题. 外力所做的功为

$$F_{\text{in}}s_{\text{in}} - F_{\text{up}}s_{\text{up}} - F_{\text{out}}s_{\text{out}} = p_{\text{in}}A_{\text{in}}v_{\text{in}}\Delta t - p_{\text{up}}A_{\text{up}}v_{\text{up}}\Delta t - p_{\text{out}}A_{\text{out}}v_{\text{out}}\Delta t$$

The decrease of potential energy is

势能减少量为

$$-mgh = -\rho g A_{\text{up}}v_{\text{up}}\Delta t h$$

The increase in kinetic energy is

动能增加量为

$$\frac{1}{2}mv_{\text{up}}^2 + \frac{1}{2}mv_{\text{out}}^2 - \frac{1}{2}mv_{\text{in}}^2 = \frac{1}{2}\rho A_{\text{up}}v_{\text{up}}\Delta t v_{\text{up}}^2 + \frac{1}{2}\rho A_{\text{out}}v_{\text{out}}\Delta t v_{\text{out}}^2 - \frac{1}{2}\rho A_{\text{in}}v_{\text{in}}\Delta t v_{\text{in}}^2$$

Putting these together, because of the law of conservation of energy, gets

由能量守恒定律, 把以上式子合并到一起, 可得

$$p_{\text{in}}A_{\text{in}}v_{\text{in}}\Delta t - p_{\text{up}}A_{\text{up}}v_{\text{up}}\Delta t - p_{\text{out}}A_{\text{out}}v_{\text{out}}\Delta t - \rho g A_{\text{up}}v_{\text{up}}\Delta t h = \frac{1}{2}\rho A_{\text{up}}v_{\text{up}}\Delta t v_{\text{up}}^2 + \frac{1}{2}\rho A_{\text{out}}v_{\text{out}}\Delta t v_{\text{out}}^2 - \frac{1}{2}\rho A_{\text{in}}v_{\text{in}}\Delta t v_{\text{in}}^2 \tag{1}$$

As the fluid is incompressible,

又因为流体是不可压的,

$$A_{\text{in}}v_{\text{in}} = A_{\text{up}}v_{\text{up}} + A_{\text{out}}v_{\text{out}} \tag{2}$$

The diameters are all the same:

所有管径都相同:

$$A_{\text{in}} = A_{\text{up}} = A_{\text{out}} = \pi \cdot (10\text{cm}/2)^2 \tag{3}$$

According to assumptions, at every junction

根据假设, 在所有交叉点处

$$p_{\text{in}} = p_{\text{out}} = 420\text{kPa} \tag{4}$$

$$v_{\text{up}} = v_{\text{source}}/n \tag{5}$$

when

当

$$v_{\text{source}} = \frac{150\text{L/min}}{\pi \cdot (10\text{cm}/2)^2} = 0.3183\text{m/s}$$

Therefore, from (2), (3),(5), for the $i$th junction, its

因此, 从式 (2), (3), (5) 可知, 对于第 $i$ 个交叉点, 它的速度

$$v_{\text{in}} = v_{\text{source}}(1 - \frac{i-1}{n}), \qquad v_{\text{out}} = v_{\text{source}}(1 - \frac{i}{n})$$

Put (1), (2), (3), (4), (5) all together, we can obtain $p_{\text{up}}$ at every junctions.

将式 (1), (2), (3), (4), (5) 合到一起, 我们可以得到每个交叉点的 $p_{\text{up}}$

**Table 3:** Pressure at every sprinkler when $h = 1\text{m}$ and $n = 4$. | 当 $h = 1\text{m}$, $n = 4$ 时, 所有喷头的压强

| Junction No. \| 喷头编号 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $P_{\text{up}}$(Pa) | 410314 | 410257 | 410219 | 410200 |

In fact, at the last (i.e. the $n$th) junction,

事实上, 对于最后一个 (即第 $n$ 个) 交叉点,

$$v_{\text{in}} = v_{\text{up}} = \frac{v_{\text{source}}}{n}, \qquad v_{\text{out}} = 0$$

Put into (1), we can get

带入式 (1), 我们可以得到

$$p_{\text{up}} = p_{\text{in}} - \rho g h$$

which means the pressure at the last sprinkler is independent of $n$.

Commonly, $h$ is about 0.5m to 1.5m, and even if assume $h = 1.5\text{m}$, the $v_{\text{up}}$ at the last junction will be 405300Pa, not far from 420kPa. (if $h = 0.5\text{m}$, the last $v_{\text{up}}$ will be 415100Pa)

这意味着最后一个喷头的压强与 $n$ 无关.

通常, $h$ 的值约为 0.5m 到 1.5m 之间, 即使假设 $h = 1.5\text{m}$, 最后一个交叉点的 $v_{\text{up}}$ 也才 405300Pa, 这与 420kPa 相差不大. (若 $h = 0.5\text{m}$, 最后一个交叉点的 $v_{\text{up}}$ 将为 415100Pa)

From these equations, we can also know that the last $v_{\text{up}}$ is the $v_{\text{up}}$ that differs most from 420KPa, and the $v_{\text{up}}$ at the first junction is the $v_{\text{up}}$ that close to 420kPa most (and below 420kPa),the values of $v_{\text{up}}$ are decreasing slowly from junction 1 to $n$.

从这些方程中, 我们可以知道最后一个交叉点的 $v_{\text{up}}$ 值与 420KPa 差别最大, 第一个交叉点的 $v_{\text{up}}$ 值与 420kPa 最接近 (在 420kPa 以下), $v_{\text{up}}$ 的值从 1 到 $n$ 交叉点依次缓慢递减.

So we can come to this conclusion: all the values of $v_{\text{up}}$ at every junction will be very close (all about 400kPa, below 420kPa and close to 420kPa), no matter how many sprinklers are. That's why we assume "design pressure of sprinklers is assumed to be about 400kPa" in assumptions.

因此我们可以得出这样的结论: 无论有多少个喷头, 所有交叉点的 $v_{\text{up}}$ 的所有值都非常接近 (都在 400kPa 左右, 在 400kPa 以下并且非常接近于 400kPa). 这就是为什么我们在模型假设中假设 "喷嘴的额定压强为 400kPa" 的原因.

#### 4.1.1 Information and Analysis of Sprinklers | 喷头的信息和分析

**Table 4:** Data of sprinklers. [1] (Medium pressure sprinklers have the best application uniformity.) | 喷头数据. (中压喷头具有最好的喷灌均匀度)

| Type\| 类型 | Design Pressure\| 额定压强 (kPa) | Range\| 喷灌范围 (m) | Discharge\| 额定流量 (m$^3$/hour) |
|---:|:---:|:---:|:---:|
| Low Pressure\| 低压 | $< 200$ | $< 15.5$ | $< 2.5$ |
| Medium Pressure\| 中压 | 200-500 | 15.5-42 | 2.5-32 |
| High Pressure\| 高压 | $> 500$ | $> 42$ | $> 32$ |

There are 5 different types of rotating sprinklers among which the impact driven sprinkler is the most widely used. In this article, we assume that the rotating sprinkler is impact driven sprinkler. Some rotating sprinklers have a sector mechanism that can wet either a full circle or a sector, while rotating sprinklers without sector mechanism can only wet a circle.

在市场上有 5 种不同类型的旋转喷头, 其中摇臂式喷头最为常用. 在本文中, 我们假设旋转喷头为摇臂式喷头. 有些旋转喷头具有扇形功能, 这种功能既能够喷灌一个整圆, 也能够喷灌一个扇形区域, 而没有扇形功能的旋转喷头只能喷灌一个整圆.

There are three main structure parameters of sprinklers: intake line diameter, nozzle diameter, nozzle elevation angle.

喷头有三种主要的结构参数: 进水管直径, 喷头直径, 喷嘴仰角.

An experiential formula is used to calculate the spraying range of an impact driven sprinkler:

一个用来计算摇臂式喷头的喷灌范围的经验公式如下:

$$R = 1.70 d^{0.487} h_p^{0.45}$$

where $d$ is the nozzle diameter, $h_p$ is the operational pressure head.

其中 $d$ 是喷嘴直径, $h_p$ 有效的压力水头.

We have looked up a wide range of different models of impact driven sprinklers, the following are the models with 6mm nozzle diameter. The manufactory and some other information are omitted here. [1]

我们查看了多种不同的型号的摇臂式喷头, 下表列出的是喷嘴直径为 6mm 的型号. 这里忽略了制造厂商和其他一些信息. [1]

**Table 5:** Commonly available nozzle types whose diameter is 6mm. | 常用的口径为 6mm 的喷嘴类型

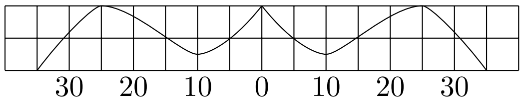
| Model\| 型号 | Nozzle diameter\| 喷嘴口径 (mm) | Design pressure\| 额定压强 (kPa) | Discharge\| 额定流量 ($m^3$/hour) | Range\| 喷灌范围 (m) |
|---|---|---|---|---|
| PY$\neg_1$15 | 6 | 200 | 1.23 | 15.0 |
| | | 300 | 1.51 | 16.5 |
| PY$\neg_1$20 | 6 | 300 | 2.17 | 18.0 |
| | | 400 | 2.50 | 19.5 |
| PY$\neg_1$S20A | 6*4 | 300 | 2.99 | 17.5 |
| | | 400 | 3.41 | 19.0 |
| PY$_1$S20 | 6 | 300 | 2.22 | 18.0 |
| | | 400 | 2.53 | 19.5 |
| 15PY$_2$22.5 | 6 | 350 | 2.40 | 17.0 |
| | | 400 | 2.56 | 17.5 |
| 15PY$_2$30 | 6 | 350 | 2.40 | 18.0 |
| | | 400 | 2.56 | 18.5 |

Sprinklers with higher manufacturer design pressure tend to have larger wetted diameters. However, deviations from manufacturer's recommended pressure may have the opposite effect (increase in pressure, decrease in diameter), and uniformity will probably be compromised.

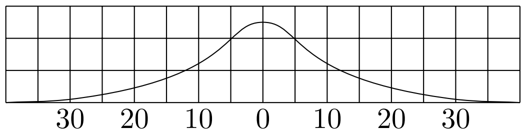
具有较高额定压强的喷头倾向于具有更大的喷灌直径. 然而, 由额定压强引起的偏差具有相反的效果 (增加压强, 喷灌直径反而减小), 均匀度也可能会受到影响.

The following picture shows typical precipitation distribution of one sprinkler in low, correct, and high sprinkler pressure.

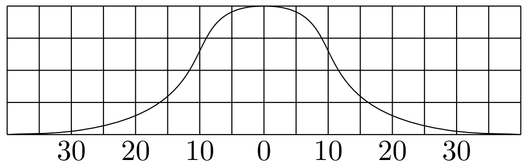
下图显示了一个典型喷头在低， 额定, 高喷头压强下的降水分布.



(a) Pressure is too low



(b) Pressure is ok



(b) Pressure is too high

**Figure 3:** Relation between pressure and precipitation distribution. [1] | 压强与降水分布间的关系.

In practice, people use catch-can data to generate precipitation profile of a "hand move" irrigation system. That is to put catch-cans evenly in the field, and,

实际上, 人们用获水容器的数据来生成一个 "手动" 灌溉系统的降水分布. 将获水容器均匀的布置在农田中, 这样, 在

after irrigation, the precipitation profile can be portrayed by the amounts of water contained in each catch-can (Figure 4). [2]

灌溉后，可以用每个获水容器中捕获的水量来描述降水分布(图4). [2]

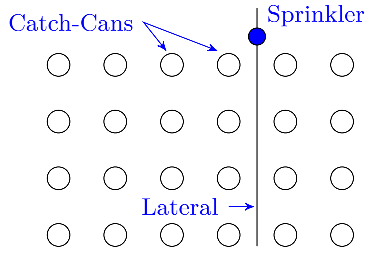



**Figure 4:** Catch-can test. [2]

One measure of how uniformly the water is applied to the field is Distribution Uniformity (DU), which is defined as [2]:

一种被用于评估农田的获水均匀程度的方法是分布均匀度 (DU), 它的定义如下 [2]:

$$\mathrm{DU} = \frac{average\_precipitation\_rate\_of\_low\_quarter}{average\_precipitation\_rate} \times 100\% = \frac{\text{降水率最低的 1/4 区域的平均降水率}}{\text{平均降水率}} \times 100\% \tag{6}$$

Usually, DUs of less than 70% are considered poor, DUs of 70-90% are good, and DUs greater than 90% are excellent. In short, bad DU means that either too much water is applied, costing unnecessary expense, or too little water is applied, causing stress to crops. There must be good DU before there can be good irrigation efficiency. [3]

To simplify our calculation, we approximate the precipitation profile of a single sprinkler (in Figure 3(b)) to a function: distr($r$), which means the relative precipitation rate in the position with a distance $r$ from the sprinkler (Figure 5).

In this problem's case, surely we should use medium pressure sprinklers (see Table 4). And In Table 5, for those sprinklers with a 6mm-diameter nozzle and working at 400kPa (as we assumed), their discharge is $2.5-3.5\mathrm{m}^3$/hour and spraying range is 18.5, 19 or 19.5m. From now on, we'll use 19m as the range of sprinklers concerned.

Note the discharge of the source is

通常, 分布均匀度 DUs 的值低于 70% 就被认为是差的, DUs 在 70-90% 则认为是不错的, DUs 大于 90% 则是极好的. 简而言之, 较差的 DU 意味着要么洒水量过多造成不必要的浪费, 要么洒水量太少给农作物造成了影响. 因此在实现良好的灌溉效率前必需保证有不错的分布均匀度 DU.

为了简化计算, 我们将单个喷头的降水分布 (见图3(b)) 近似为一个函数: distr($r$), 这个函数表示离喷头距离为 $r$ 的位置的相对降水率.

在这个问题的情况下, 显然我们应该使用中等压强的喷头 (见表4). 在表5中, 对于那些喷嘴口径为 6mm 额定工作压强为 400kPa(与我们假设的值一样), 它们的流量为 $2.5-3.5\mathrm{m}^3$/hour, 喷灌半径为 18.5, 19 或 19.5m. 从现在开始, 我们将使用 19m 为作为喷头的喷灌半径.

注意到水源处的流量为

$$150\mathrm{L/min} = 9\mathrm{m}^3/\mathrm{hour},$$

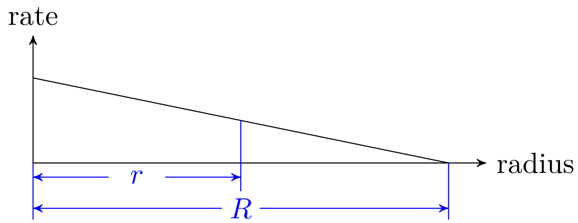



**Figure 5:** The precipitation rate plotted vs. the distance to the sprinkler. | 离喷头距离与降水率.

thus, to fit every sprinkler's actual discharge to the design discharge, the number of sprinklers we'll install should be 3 or 4. It is because $9/3 = 3$ or $9/4 = 2.25$, which is close to or within the range of the $2.5 - 3.5\text{m}^3/\text{hour}$.

因此, 为了使每个喷头的实际流量接近于其额定工作流量, 我们将安装的喷头数量应该为 3 或者 4. 这是因为 $9/3 = 3$ 或 $9/4 = 2.25$ 接近于 $2.5 - 3.5\text{m}^3/\text{hour}$ 的范围.

## 4.2 Stage 2: Scheduling the Irrigation | 第 2 阶段: 灌溉的工作进度表

Actually, a schedule to move the pipes includes both where and in how long an interval to move them. For the previous one, we just imagine a fixed irrigation system consisting of several 20m pipes. If the system can nicely, i.e. with high Distribution Uniformity (DU), meet the needs of the crops, then the way of moving the pipe in the problem is just to move it from one pipe's position to another in the system. So, we'll determine where to move the pipe by laying out a system of several 20m pipes, and then decide for how long we should water the field before making a next move. First, we use a simulation of catch-can analysis to choose a layout with a high DU.

实际上, 一个移动管组的工作进度表既包括移动到哪里, 也包括多长时间间隔移动一次. 对于前者, 我们只需假想一个具有多套 20m 管组的固定灌溉系统. 如果系统能很好的满足作物的需求, 即具有较高的分布均匀度 (DU), 那么这个问题中的管子移动方式则转变为将管子从固定灌溉系统中的一个管子的位置移到另一个管子的位置. 因此, 我们通过铺设一个具有多条 20m 管子的灌溉系统来确定管子的移动位置, 然后确定农田需要浇灌多长时间才能再一次移动管子. 首先, 我们用分析获水容器的模拟来选出一种具有较高 DU 的排布方式.

### 4.2.1 Catch-can Analysis | 获水容器分析

Since the water sprayed by a particular sprinkler has a determined distribution: $\text{distr}(r)$, as we've already defined, we use the following method to simulate the catch-can test.

For rectangular spacing (Figure 6 Left), we consider the rectangular region between four adjacent sprinklers. In our simulation, we pick 900 positions evenly

由于单个特定喷头的洒水具有一个确定的分布: $\text{distr}(r)$, 这个函数我们前面已经定义过了, 我们应用下面的方法来模拟获水容器测试.

对于矩形间距 (图6 左), 我们考虑四个相邻喷头之间的矩形区域. 在我们的模拟中, 我们选择了均匀分布于矩形区域

distributed in the region. For each position, we calculate its relative precipitation rate $p$:

$$p = \sum_i \mathrm{distr}(r_i)$$

$r_i$ is the distance from the position to a sprinkler considered. Thus, for the $i$th position, we got $p_i$, $(1 \le i \le 900)$. With Equation (6), we then calculate the DU of this irrigation system.

中的 900 个位置点. 对于每个位置点, 我们计算它的相对降水率 $p$:

$r_i$ 为当前位置点到所考虑喷头的距离. 因此, 对于第 $i$ 个位置点, 我们可以获得 $p_i$, $(1 \le i \le 900)$. 通过公式 (6), 我们就可以计算这个灌溉系统的分布均匀度 DU 了.

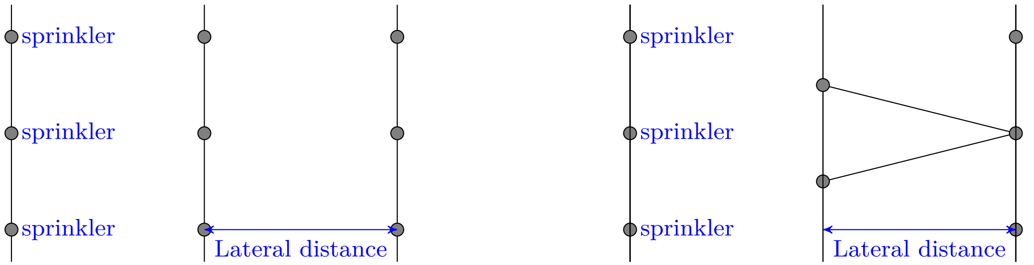



**Figure 6:** Left: Rectangular spacing; Right: Triangular spacing. | 左: 矩形间距; 右: 三角间距

As we've already deduced, the number of the sprinklers should be 3 or 4, thus the sprinkler distance will be 10m (20m/(3 − 1)) or 6.67m (20m/(4 − 1)) [1] So the DU is a function of the lateral distance. And this can also be applied for triangular spacing (Figure 6 Right). After the simulation, we get figure 7.

正如我们已推导的, 喷头的数量应该为 3 或 4, 因此喷头的间距将会是 10m (20m/(3 − 1)) 或 6.67m (20m/(4 − 1)) [1] 因此 DU 是横向距离的一个函数. 并且这一结论可以被推广到三角间距的情况 (图6 右). 经过模拟, 我们得到了图7.

[1]That the sprinkler distance is evenly distributed along the pipe is because that it will have a higher DU and it is easy to operate.

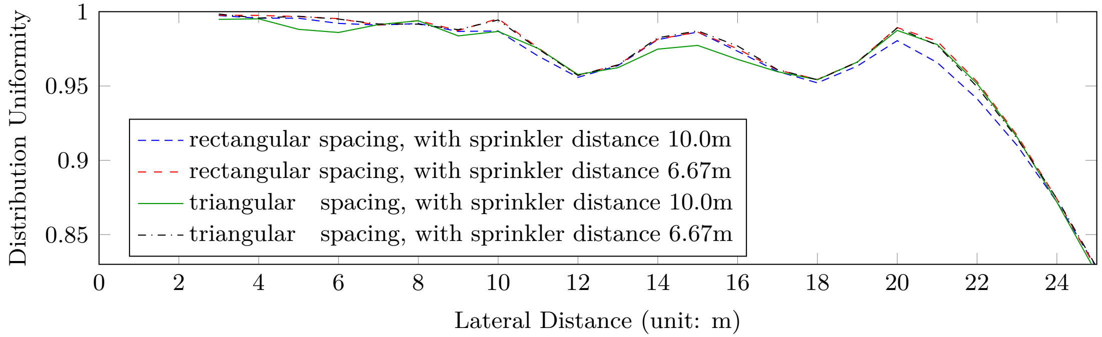



**Figure 7:** DU plotted vs. lateral distance, in 4 different situations. | 四种情况下，横向距离与分布均匀度.

The simulation shows that when lateral distance $\leq 20$, DU is acceptable ($\geq$ 90%), regardless of the spacing and whether the sprinkler distance is 6.67m or 10m. But since larger lateral distance will result in smaller amount of time required to irrigate the field (the number of moves to make will be less), we pick 20m as the lateral distance. Figure 8 and Figure 9 show the precipitation profile for the irrigation systems with sprinkler distance 10m and lateral distance 20m.

Considering that the field (30m $\times$ 80m) is not relatively large enough to implement triangular spacing when the pipe is 20m long, we will use rectangular spacing with an only 0.7% neglectably weaker DU. Before we layout the pipe set, we should first determine the max distance from the rim of the field to the sprinklers satisfying that the DU can be acceptable.

模拟结果显示，无论是矩形间距还是三角间距，也无论喷头间距是 6.67m 还是 10m，只要当横向间距 $\leq 20$，分布均匀度 DU 就是可接受的 ($\geq$ 90%). 但由于较大的横向距离将导致浇灌农田的需求总时间量较小 (需要移动的次数将会减少)，我们选择 20m 作为横向距离. 图8和图9显示了喷头间距为 10m，横向距离为 20m 的灌溉系统的降水分布.

考虑到这块农田 (30m $\times$ 80m) 相对来讲并不够地大，当管长为 20m 长时，不足以应用三角间距，我们将应用矩形间距可以获得比三角距间只低 0.7%(几乎可以忽略) 的 DU. 在我们布置管组之前，我们首先应该在获得可接受的 DU 前提下确定喷头距离农田边缘的最大间距.

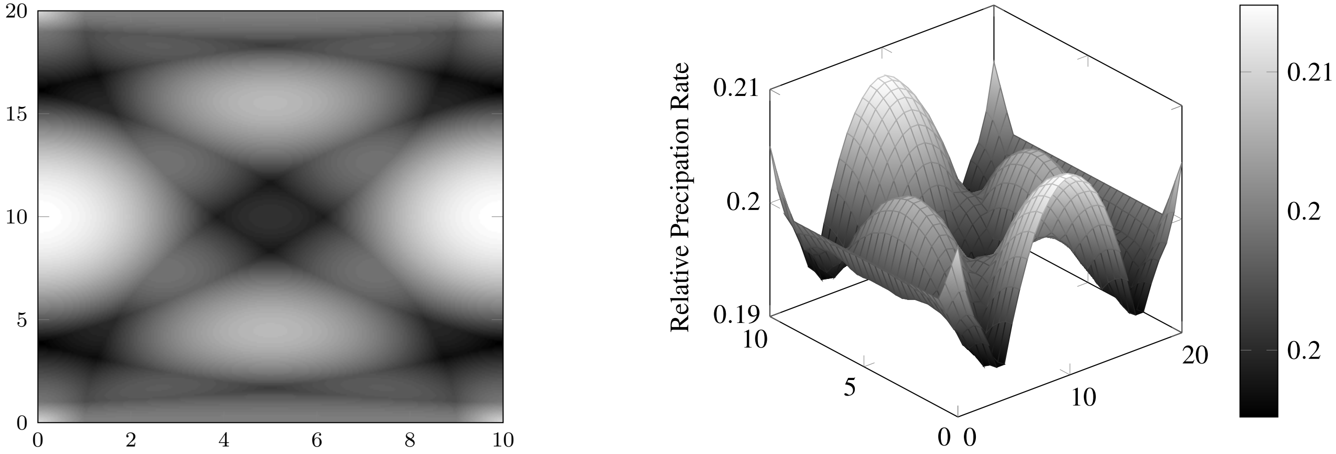



**Figure 8:** Left: Precipitation profile for rectangular spacing with sprinkler distance 10m and lateral distance 20m; right: the 3D form of the precipitation profile. DU = 98.1% | 左: 喷头间距为 10m, 横向距离为 20m 的矩形间距的降水分布; 右: 降水分布的 3D 图.

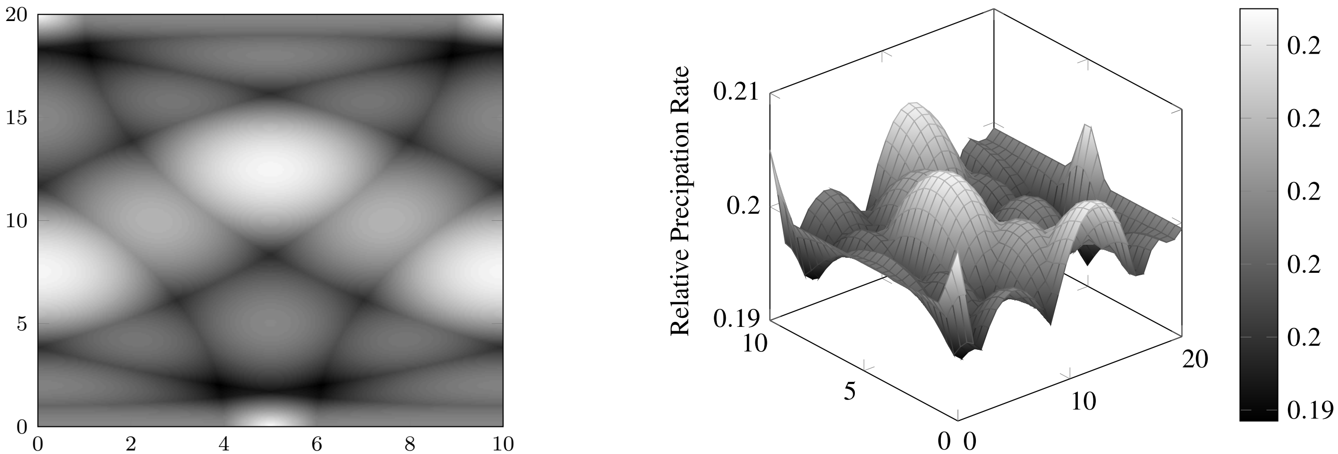



**Figure 9:** Left: Precipitation profile for triangular spacing with sprinkler distance 10m and lateral distance 20m; right: the 3D form of the precipitation profile. DU = 98.7% | 左: 喷头间距为 10m, 横向距离为 20m 的三角间距的降水分布; 右: 降水分布的 3D 图.

We simulate a catch-can test on the rectangular region on the rim of the field as in Figure 10.

我们在如图10所示的农田边缘的一个矩形区域模拟了一个获水容器测试.

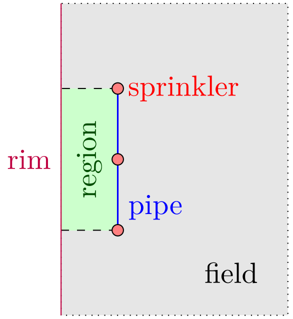



**Figure 10:** Region to simulate a catch-can test. | 模拟获水容器测试的区域.

The result is in Figure 11 . Thus, we can see that, the max length between the rim of the field and the sprinklers is 5m with an acceptable DU of 82.7%.

模拟结果见图11. 因此，我们可以看出，农田边缘与喷头之间的最大距离为 5m，此时可获得可接受的分布均匀度 DU 为 82.7%.

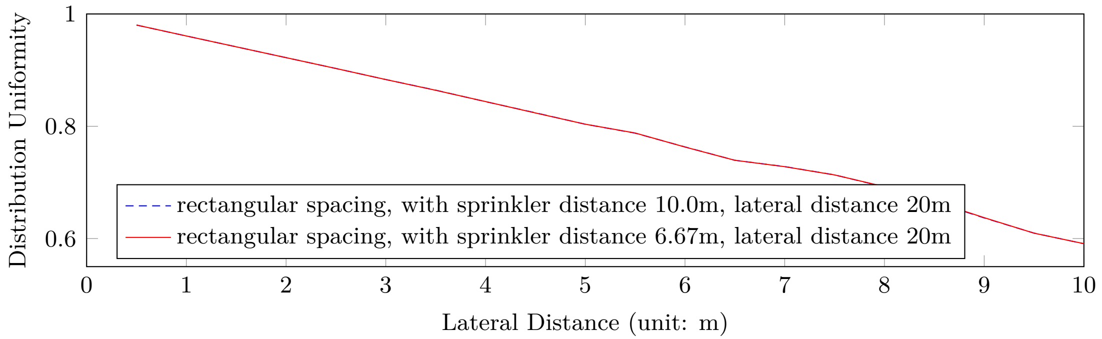



**Figure 11:** DU plotted vs. distance from the rim to the sprinkler in 2 different situations. | 两种不同情况下，离边缘的距离与 DU.

### 4.2.2 Layout of the Pipe Set

According to the analysis above, having 3 or 4 sprinklers along the pipe only

通过以上分析，拥有 3 或 4 个喷头的管子在结果上只有

results in a slight difference. Considering that the spraying radius (19m) is too large compared with the sprinkler distance 6.67m when having 4 sprinklers, we choose to have 3. Thus the only two feasible layouts of the whole system exist [2] (Figure 12 and Figure 13). Simply speaking, Layout 1 implies that it requires 5 times of moving and setting up the pipes while Layout 2 requires 6. But it's too early to judge which is better.

很小的差别. 考虑到如果有 4 个喷头, 相对于 6.67m 的喷头间距, 喷灌半径 (19m) 太大, 因此我们选择了 3 个喷头. 因此系统只存在两种可行的排布方式 [2] (图12和13). 简单来说, 排布方式 1 意味着需要移动并安装 5 次, 而排布方式 2 则需要 6 次. 但现在说哪一种方式更好还言过尚早.

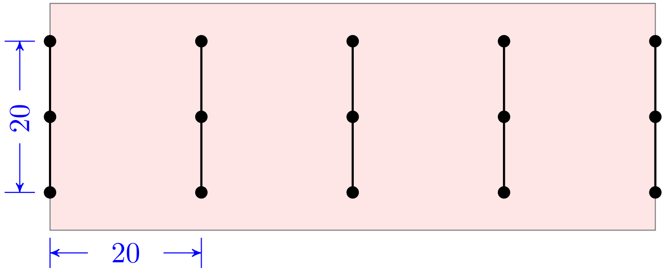



**Figure 12:** Layout 1. | 排布方式 1.

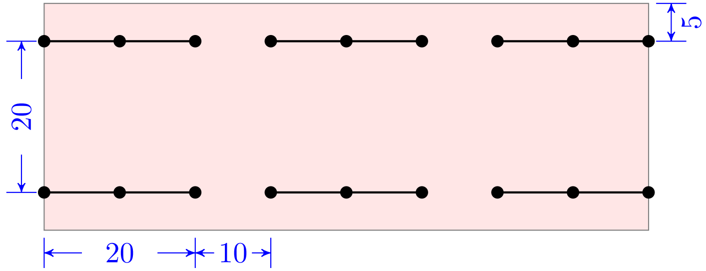



**Figure 13:** Layout 2. | 排布方式 2.

For further estimation, we should calculate *DU* (Distribution Uniformity) of both layouts. The result from the simulation of catch-can tests is shown in following figures 14 and 15.

为了进一步估计, 我们需要计算两种排布方式的分布均匀度 *DU*. 获水容器的测试模拟结果见下面的图14和15.

We decide to abandon Layout 1 because it has a very poor DU. After slightly changing the lateral distance in Layout 2, we finally achieve a best DU of 89.5% in Layout 3 (see in Figure 16 and 18):

我们决定丢弃排布方式 1, 因为它的 DU 值非常糟糕. 在对排布方式 2 的横向距离稍做调整后, 我们最终得到拥有最好分布均匀度 $DU = 89.5\%$ 的排布方式 3(见图16和18).

Then, if we are brave enough to move some sprinklers out of the field, we'll achieve a higher *DU* in Layout 4 (see in Figure 17 and 19):

然后, 如果我们足够大胆, 将一些喷头移到农田外面, 我们可以获得一个具有更高 *DU* 的排布方式 4(见图17和19):

### 4.2.3 Calculation of the Precipitation | 计算降水率

In order to meet the problem's constraints that in any part of the field,

- Constraint A: the precipitation rate $\geq$ 2.00cm/4days,
- Constraint B: the precipitation rate $\leq$ 0.75cm/hour,

we should calculate the precipitation rate of the system in Layout 4 before schedul-

为了在农田任何位置点都满足问题的约束,

- 约束 A: 降水率 $\leq$ 0.75cm/hour,
- 约束 B: 降水率 $\geq$ 2.00cm/4days.

对于排布方式 4, 在制定灌溉和移动管子的工作进度表前, 我

[2] We are not considering the situation when the pipes are not parallel to the edge of the field, because layout of this kind is not practical.

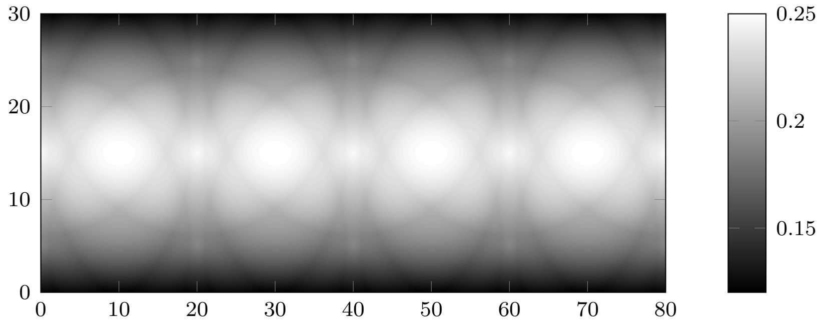



**Figure 14:** Catch-can test simulation result of layout 1. | 排布方式 1 的获水容器的测试模拟.

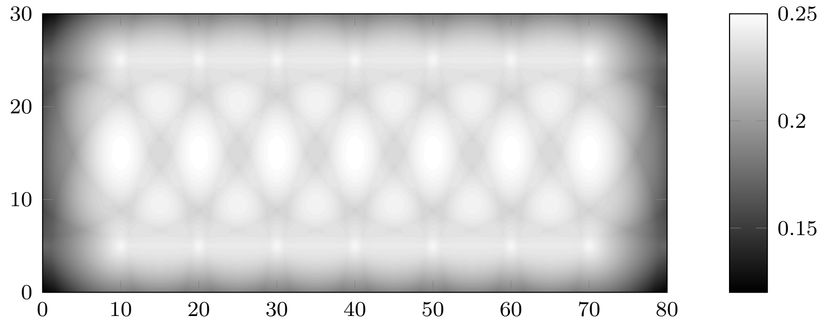



**Figure 15:** Catch-can test simulation result of Layout 2. | 排布方式 2 的获水容器的测试模拟.

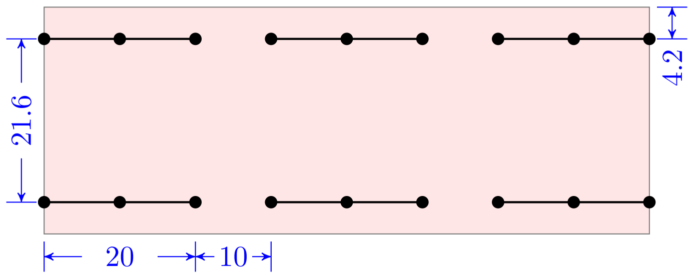



**Figure 16:** Layout 3. | 排布方式 3.

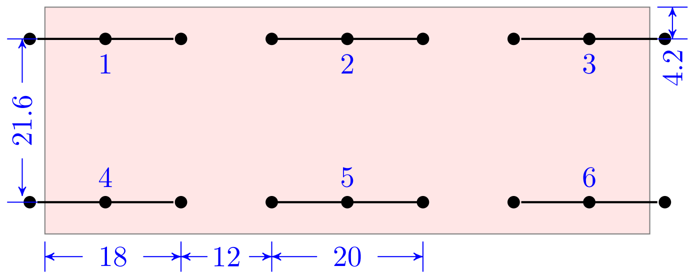



**Figure 17:** Layout 4. | 排布方式 4.

ing the interval to irrigating the field and to move the pipe.

The precipitation area of a sprinkler should be a circle with a radius $R$, as Figure 20 shows. The profile of the precipitation rate distribution is in Figure 5. In order to figure out the precipitation rate at a certain point whose distance from the sprinkler is $r$, we firstly calculate the angle $\alpha$ in Figure 5. As we normalize the distribution, we get

们需要计算系统的降水率.

一个喷头的降水区域应该是一个半径为 $R$ 的圆，如图20如示. 降水率分布图见图5. 为了找出距离喷头为 $r$ 的某个确定点的降水率，我们首先计算了图5中直线的倾斜角. 经过对降水分布的标准化，我们得到

$$\int_0^R (R-r)\tan\alpha \cdot 2\pi r \mathrm{d}r = 1$$

so

因此

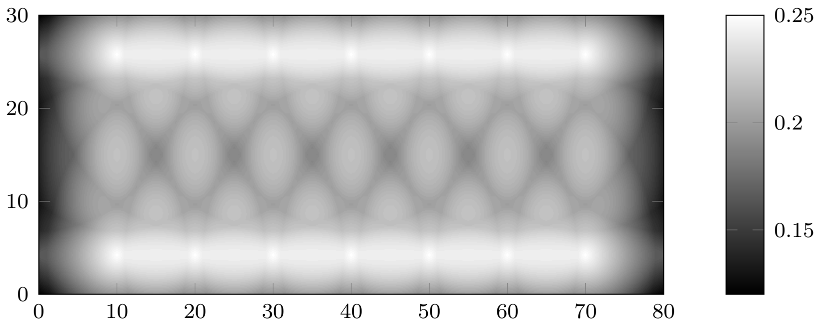



**Figure 18:** Catch-can test simulation result of layout 3. | 排布方式 3 的获水容器的测试模拟.

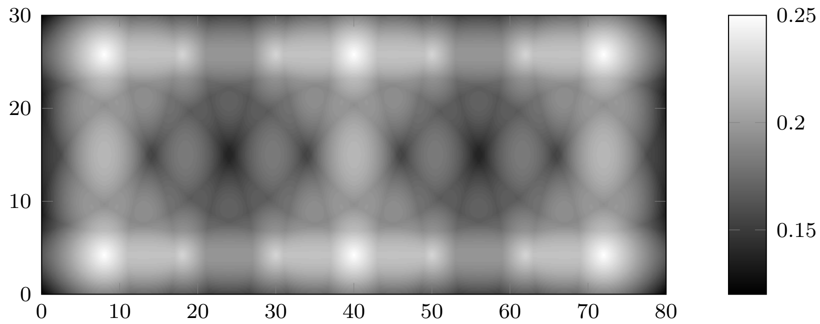



**Figure 19:** Catch-can test simulation result of Layout 4. | 排布方式 4 的获水容器的测试模拟.

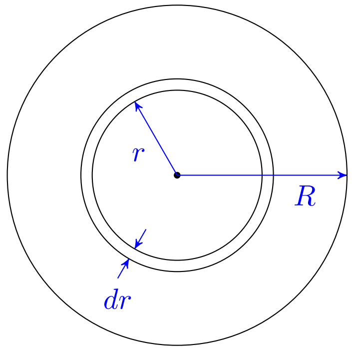



**Figure 20:** Precipitation area of a single sprinkler. | 单个喷头的降水区域.

$$\tan\alpha = \frac{3}{\pi R^3}$$

then the precipitation rate

那么降水率为

$$pr(r) = v(R-r)\tan\alpha = \frac{3(R-r)}{\pi R^3}v,$$

where $R = 19\text{m}$, $v = 50\text{lpm}$.

With Matlab, we calculated the precipitation rate at each point in the 80m × 30m field, with the irrigation system working only once (see in Figure 21 Right)

其中 $R = 19\text{m}$, $v = 50\text{lpm}$.

通过 Matlab, 我们计算得到经过一次灌溉 (见图21右) 和经过完整的一个周期的移动和灌溉 (见图21左) 后 80m×30m and after a complete cycle of moving the equipment and irrigating (see in Figure 21 Left).

的农田中每个位置点的获水率.

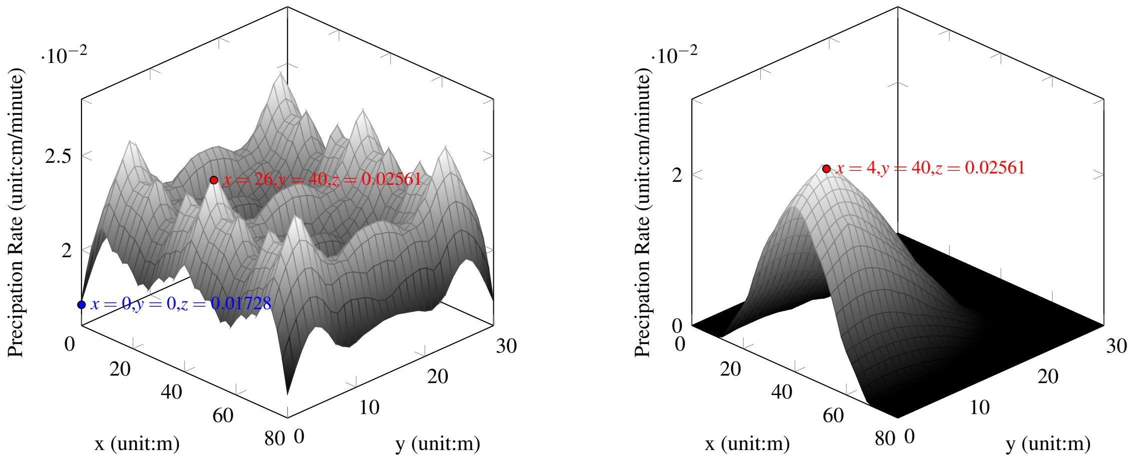



**Figure 21:** Precipitation rate of water in the field with Layout 4. Left: the whole effect of 6 pipes together; Right: the effect of a 20m pipe working alone. | 排布方式 4 下, 农田的降水率. 左: 6 条管子共同作用的效果; 右: 一条 20m 的管子单独作用的效果.

### 4.2.4 Scheduling the Irrigation Time

Figure 21 Right shows that when the sprinklers (only one 20m pipe at a time) are working, the maximum precipitation rate is

$$0.02561\text{cm/min}.$$

图21右显示的是当喷头 (一次只有一个 20m 的管子) 工作时, 最大的降水率为

To satisfy Constraint A, the period of irrigation should be less than

为了满足约束 A, 灌溉的时间长度必需少于

$$\frac{0.75\text{cm/hour}}{0.02561\text{cm/min}} = 29.30\text{min per hour}$$

Because the smaller the period of irrigation is, the more frequently the farmer should stop to move the pipe, we thus choose a large but easy to implement one:

$$25\text{min per hour}.$$

因为越小的灌溉时长, 农场主就需要越频繁地停顿来移动管子, 因此我们选择一个较大的时长, 并且比较容易实施的时长:

Figure 21 Left shows that after a complete cycle of irrigation the whole field, i.e. the equipment has been moved to and irrigated every required place of the field, and the minimum precipitation rate is

图21左显示的是对整个农田进行一轮完整的灌溉后, 也就是管组装置被移动到所有需要放置的位置并灌溉, 最大的降水率为

$$0.01728\text{cm/min}$$

To satisfy Constraint B, the period of irrigation should be longer than

为了满足约束 B, 灌溉的时间长度必需超过

$$\frac{2\text{cm}/4\text{days}}{0.01728\text{cm/min}} = 115.7\text{min every 4 days}$$

To meet this, in every 4 days, we plan to irrigate the same place 5 times, every time lasts 25 minutes, that is 125 minutes, well satisfies Constraint B.

为了满足这个约束, 在每 4 天中, 我们计划在同一个地点浇灌 5 次, 每次时长 25 分钟, 总时长为 125 分钟, 这很好地满足了约束 B.

With the same method, we calculate the same parameters for Layout 3, and list the comparison in Table 6:

用同样的方法, 我们对排布方式 3 进行了相同参数的计算, 并列在了表6中进行对比:

**Table 6:** Comparison between Layout 3 and Layout 4. | 排布方式 3 与排布方式 4 间的比较.

| | Layout 3 \| 排布方式 3 | Layout 4 \| 排布方式 4 |
|---|---|---|
| Max precipitation rate (cm/min)\| 最大降水率 | 0.02561 | 0.02561 |
| Min precipitation rate (cm/min)\| 最小降水率 | 0.01598 | 0.01728 |
| Total Irrigation time every 4 days (min)\| 每四天的总喷灌时长 | $150 \times 6$ | $125 \times 6$ |
| DU, Distribution Uniformity\| 分布均匀度 | 89.5% | 92.1% |

It is clearly shown that Layout 4 not only has higher DU than Layout 3, but also saves 16.7% of water, in other words, has higher application efficiency, though one of its sprinklers is positioned out of the field in some situation. This is because that Layout 3 has a smaller min precipitation rate which leads to more irrigation time in order to satisfy Constraint B. So we choose Layout 4 as our solution.

显然, 排布方式 4 不仅具有比排布方式 3 更高的 DU, 而且还节省 16.7% 的水, 换句话说, 在某些情况下, 通过将一个喷头布置在农田外的位置, 排布方式 4 具有更高的应用效率. 这是因为排布方式 3 具有较小的降水率, 这导致面要更多的灌溉时长来满足约束 B. 因此我们选择排布方式 4 作为解.

### 4.3 Stage 3: Schedule Design | 第 3 阶段: 工作进度表的设计

As we've discussed in the previous stage, our schedules will be able to achieve a DU as high as 92.1% after a cycle of 4-day-irrigation. We have two concrete design of irrigation timetable.

正如我们在之前步骤所讨论的, 在一轮 4 天的灌溉后, 我们的工作进度表将能获得高达 92.1% 的分布均匀度. 我们有两种具体的灌溉工作进度表的设计方案.

The first design make the least moving time but, to some extent, is not so

第一种设计方案使得移动的次数最小, 但在一定程度上不

"average" as it gathers sprinkling processes of the same sprinkler together. (Table 7)

那么 "均匀", 这是由于它将同一位置的浇灌时段集中在了一起. (表7)

**Table 7:** The first schedule for every four days.('*' means it is required to set up the pipe set in the referred position before irrigation. And the position code refers to pipe's position in Figure 17 and 19) | 第一种四天的工作进度表. ('*' 意味着在喷灌前需要相应位置安装管组. 位置编号指的是图17和19中的编号)

| Day 1\| 第一天 | | Day 2\| 第二天 | | Day 3\| 第三天 | | Day 4\| 第四天 | |
|---|---|---|---|---|---|---|---|
| Time 时间 | Position code 位置编号 | Time 时间 | Position code 位置编号 | Time 时间 | Position code 位置编号 | Time 时间 | Position code 位置编号 |
| 07:00-07:25 | 1* | 07:00-07:25 | 6 | 07:00-07:25 | 4 | 07:00-07:25 | 3 |
| 08:00-08:25 | 1 | 08:00-08:25 | 6 | 08:00-08:25 | 4 | 08:00-08:25 | 3 |
| 09:00-09:25 | 1 | 09:00-09:25 | 2* | 09:00-09:25 | 4 | 09:00-09:25 | 5* |
| 10:00-10:25 | 1 | 10:00-10:25 | 2 | 10:00-10:25 | 4 | 10:00-10:25 | 5 |
| 11:00-11:25 | 1 | 11:00-11:25 | 2 | 11:00-11:25 | 3* | 11:00-11:25 | 5 |
| 13:00-13:25 | 6* | 13:00-13:25 | 2 | 13:00-13:25 | 3 | 13:00-13:25 | 5 |
| 14:00-14:25 | 6 | 14:00-14:25 | 2 | 14:00-14:25 | 3 | 14:00-14:25 | 5 |
| 15:00-15:25 | 6 | 15:00-15:25 | 4* | | | | |

The second design need more moving time but, as a kind of compensation, is more "average" (for instance, the field receives water much more evenly in time) than the first design. (Table 8)

第二种设计方案需要更多的移动次数, 但是作为一种补偿, 能获得比第一种设计方案更大的 "均匀"(比如, 农田在一段时间内获得的降水更均匀)(表8)

# 5 Evaluation of Results | 结果的评价

## 5.1 Comparison between Two Designs | 两种设计方案的比较

Actually, Design 1 is the most time effective and labor saving schedule based on Layout 4. If the water source, i.e. the pump, has a timing mechanism (there is such product in market), the whole irrigation only costs 3 hours labor time every 4 days. But as we've discussed, Design 2 has a more "average" irrigation, which may be better for crops or plants. So it is easy to arrange a more compromising schedule good for plants and with longer but acceptable required labor time.

实际上, 第 1 种设计方案是基于排布方式 4 中时间效率最高和人力最省的方案. 如果水源, 即泵, 拥有定时开关的功能 (市场上有这样的产品), 那么整个 4 天的灌溉只需要花费 3 小时. 但正如我们所讨论的, 第 2 种设计方案具有更 "均匀" 的灌溉, 这将更有利于农作物或植物. 因此很容易安排一个更折衷的工作进度表, 这个工作进度表对植物更好, 并且所需的劳动时间仍可接受.

It is also worthy mentioning that with the sector mechanism, we can control

另一个值得一提的是扇形功能, 我们可以能过扇形功能控

**Table 8:** The second schedule for every four days. (position code refers to Figure 16 and 18)) | 第二种四天的工作进度表. (位置编号指的是图16和18中的编号

| Day 1\| 第一天 | | Day 2\| 第二天 | | Day 3\| 第三天 | | Day 4\| 第四天 | |
|---|---|---|---|---|---|---|---|
| Time 时间 | Position code 位置编号 | Time 时间 | Position code 位置编号 | Time 时间 | Position code 位置编号 | Time 时间 | Position code 位置编号 |
| 07:00-07:25 | 1 | 07:00-07:25 | 3 | 07:00-07:25 | 2 | 07:00-07:25 | 6 |
| 08:00-08:25 | 5 | 08:00-08:25 | 4 | 08:00-08:25 | 6 | 08:00-08:25 | 1 |
| 09:00-09:25 | 3 | 09:00-09:25 | 2 | 09:00-09:25 | 1 | 09:00-09:25 | 5 |
| 10:00-10:25 | 4 | 10:00-10:25 | 6 | 10:00-10:25 | 5 | 10:00-10:25 | 3 |
| 11:00-11:25 | 2 | 11:00-11:25 | 1 | 11:00-11:25 | 3 | 11:00-11:25 | 4 |
| 13:00-13:25 | 6 | 13:00-13:25 | 5 | 13:00-13:25 | 4 | 13:00-13:25 | 2 |
| 14:00-14:25 | 1 | 14:00-14:25 | 3 | 14:00-14:25 | 2 | 14:00-14:25 | 6 |
| 15:00-15:25 | 5 | 15:00-15:25 | 4 | | | | |

**Table 9:** Comparison between Design 1 and Design 2. | 设计方案 1 与设计方案 2 的比较

| Parameters in one cycle of irrigation (4days) | 一个灌溉周期 (4 天) 中的参数 | Design 1 | Design 2 |
|---|---|---|---|
| Number of equipment resets | 重新安装设备的次数 | 6 | 30 |
| Equipment reset time (one reset cost 30min as assumed) (hour) | 重新安装设备的时间 (一次 30 分钟) | 3 | 15 |
| Irrigation time (hour) | 喷灌时长 (小时) | 12.5 | 12.5 |
| DU, Distribution Uniformity | 分布均匀度 | 92.1% | 92.1% |

the rotating range of the sprinkler on the edge of or out of the field to reduce the water waste.

制农田边缘或农田外面的喷头的旋转范围来减少水的浪费.

# 6 Strengths and Weaknesses | 优缺点

## 6.1 Strengths | 优点

- We use real data of sprinklers to determine the parameters of the sprinklers we choose and determine the number of sprinklers.
- We establish a model based on the engineering knowledge of sprinklers, and

- 我们使用喷头的真实数据来确定喷头的参数：喷头类型地选用和数量地确定.
- 基于喷头的工程知识，我们建立了一个模型，并找出总

find out the overall precipitation distribution, and then manage to find out an optimal schedule. All the results are based on calculations.

- Our model to analyze the layout of the irrigation system is sprinkler-independent. This means, for whatever kind of sprinkler, if its single precipitation profile is known (e.g. though a real catch-can test), we can determine the precipitation profile across the whole field.
- The placement and schedule is very clear and easy to implement.

体降水分布, 然后设法找出一个最优工作进度表. 所有结果都是基于计算完成的.

- 我们分析灌溉系统排布方式的模型是独立于喷头的. 这意味着, 对于任何其它的喷头, 只要单个喷头的降水分布已知 (即经过真正的获水容器测试), 我们就可以确定整个农田的降水分布.
- 排布方式和工作进度表都非常清楚, 并且易于操作.

### 6.2 Weaknesses | 缺点

- Water pressure in the pipe may diverse, and so the discharge of the sprinklers may not be exactly the same.

- 管内水压可能并不相同, 因此喷头的流量可能并不严格相等.

## 7 Further Discussion | 进一步讨论

The analysis should further include the consideration of wind, evaporation, the characters of the soil type and penetration.

进一步的分析需要包括风, 蒸发, 土壤类型和渗透特性等因素.

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