Optimizing a Handmove Sprinkler System

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Abstract

The task is to devise a nozzle arrangement and scheduling algorithm that will minimize the time required to move a hand-move lateral around an 80 m x 30 m field, while maximizing precipitation uniformity across the field. We require that no part of the field receive less than 2 cm of precipitation in any 4-day period, or more than 0.75 cm in any 1-hour period. We construct a simple model of the piping system and use it in conjunction with an empirically-derived family of radial precipitation distributions to articulate a space of algorithms parametrized on the number of nozzles, the time between station moves, the layout of the lateral stations, and two empirical parameters. We reduce the algorithm space by discarding all algorithms that fail to meet the precipitation constraints, then traverse the remaining space and compute the time required by the farmer to implement the algorithm and the uniformity of the precipitation it deposits on the field. The algorithm which requires the least farmer-time to implement is a two-nozzle, uniformly spaced six-position layout with a time interval of 8 hours. The algorithm that provides the most uniform precipitation and is most generally optimized is a three-nozzle, seven-position staggered layout with a time interval of 6 hours and 40 minutes.

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1 Introduction

1.1 The problem

Hand-move lateral irrigation systems are a popular alternative to costly fixed sprinkler systems among small farmers. In contrast to fixed systems, hand-moves are simple and easy to maintain, but are often unable to irrigate all parts of a field simultaneously. This can lead to nonuniform precipitation over the field, which in turn usually leads to a reduction in average crop yield [1]. The task at hand is to devise a sprinkler head arrangement and scheduling algorithm that will maximize precipitation uniformity over the field while minimizing the time the farmer must spend moving the system around on the field.

1.2 Outline of our approach

We have the following specific objectives:

1. Minimize the time spent by the farmer t_f moving the system around on the field.
2. Maximize precipitation uniformity across the field.

and the following constraints:

1. Ensure that all parts of the field receive at least 2 cm of precipitation in any 4-day period.
2. Ensure that no parts of the field receive more than 0.75 cm of precipitation in any 1-hour period.

We address these objectives and constraints as follows:

1. We construct a simple theoretical model of the piping system.
2. We use the piping system model in conjunction with an empirical nozzle model [2] to generate a family of radial precipitation distributions parametrized on number of nozzles and nozzle type (as described by the empirical model).
3. We articulate two layout types for placement of the lateral on the field.
4. We define our algorithm space.
5. We discretize the algorithm space.
6. We traverse the algorithm space and discard all algorithms that do not satisfy constraints 1 and 2.
7. We introduce the Christiansen Coefficient of Uniformity C_U [3], for use in determining uniformity of the precipitation distribution produced by a given algorithm.
8. We traverse the reduced algorithm space and compute t_f, C_U, and a combined metric for each algorithm in the space.
9. We make a set of recommendations based on the values of the optimized metrics.

As we go along, we will note the assumptions that we make in constructing each piece of the model and interpreting its results. Finally, we will assess the strengths and weaknesses of the model, discuss potential refinements and additions, and consider circumstances in which it may be necessary to complete a more detailed analysis.

2 Piping System Model

The piping system consists of a water source with volume flux ϕ_0 under static pressure P_0 connected to a mainline of length l punctuated by n heads. We will assume that uniform distribution of the heads (i.e., distances of $l/(n+1)$ between heads) will maximize precipitation uniformity; when we consider superposing the precipitation from two heads, we see that this is a safe assumption.

We wish to determine the nozzle velocity v_i at head i as a function of the number of heads n. If we assume incompressibility of water[1], conservation of mass becomes $\phi_{in} = \phi_{out}$, or, in our n-sprinkler system with source flux ϕ_0,

$$v_i = \frac{A}{a}\frac{v_0}{n} \tag{1}$$

where $v_0 = \phi_0/A$. Note however that Bernoulli's equation

$$\frac{d}{dt}\left(P + \frac{1}{2}\rho v^2 + \rho g h\right) = 0 \tag{2}$$

implies a maximum theoretical velocity $\hat{v}_i$ for a given n (i.e., static pressure in the pipe cannot be negative [4]); specifically,

$$\hat{v}_i = \frac{1}{n}\left(\frac{1}{\rho}P_0 + v_0^2\right) \tag{3}$$

and so the nozzle velocities are

$$v_i = \frac{A}{a}\frac{v_0}{n} \leq \hat{v}_i. \tag{4}$$

We assume that pressure loss due to viscous interactions in the piping system are negligible, i.e., $v_i = v$ for all $1 \leq i \leq n$, and so (4) simplifies to a scalar equation. For systems in which this is not the case, pressure loss can sometimes be approximated as uniformly distributed across the heads and modeled by multiplying v by a scalar system efficiency coefficient η, $0 < \eta < 1$ [2]. This allows us to create a nozzle velocity distribution parametrized on the number of heads n:

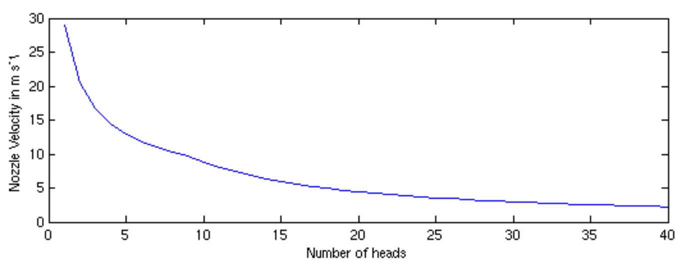

Figure 1: Nozzle velocity versus number of heads.

[1]While this is not *exactly* true, it is a simplification that makes the problem vastly more computationally tractable.

3 Precipitation Distribution Model

3.1 Radial distributions

From the velocities provided by our piping model we can compute a flux ϕ out of each nozzle, which we can in turn use to infer a spatial precipitation distribution $\psi(r)$ around each sprinkler head. For this we turn to an empirical model based on the work of Mateos [2] which generates a family of radial distributions that correspond closely to a wide variety of real nozzles. We have

$$\psi(r) = \frac{\phi \delta t}{2A\pi}\left[(sR+1)^u - \frac{(sR+1)^u(r-sR)^2}{(R-sR)^2}\right] \tag{5}$$

where $sR < r < R$; u and s are parameters that characterize the distributions within the family, $-1 < u < 1$, $0 < s < 1$; and

$$A = \frac{(sR+1)^{u+2}-1}{u+2} - \frac{(sR+1)^{u+1}+1}{u+1} + \frac{R^2(1-s^2)(sR+1)^u}{2} - \frac{R^2(3-8s+6s^2-s^4)(sR+1)^u}{12(1-s)^2}. \tag{6}$$

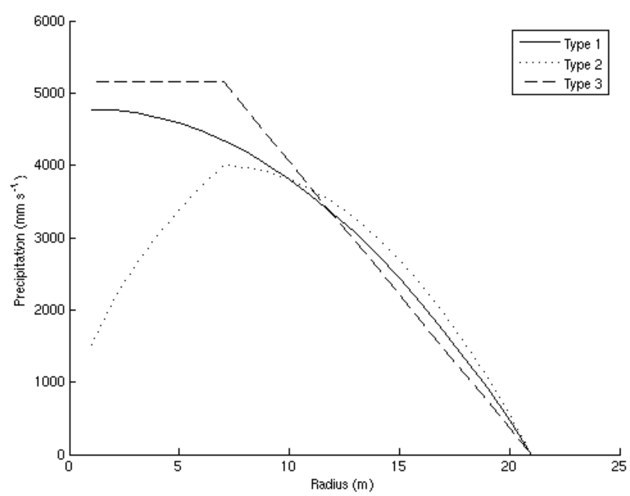

Figure 2: Some radial distributions.

3.2 Two-dimensional precipitation distributions

Now we would like to generate a two-dimensional precipitation for the entire n-head lateral, and we do this by traversing the field. For each grid point, we calculate the distance to each sprinkler head and add the appropriate amount of precipitation. This has the effect of superimposing the distributions of each head on top of one another across the field. It also however, has an unintended effect: it introduces a small amount of error because the grid points will be distances r_i away from the heads that are not stored exactly in the discretized precipitation distribution function $\psi(r)$. Fortunately, this error can be contained by increasing the resolution of ψ and performing spline interpolation between the values when there is not an exact match:

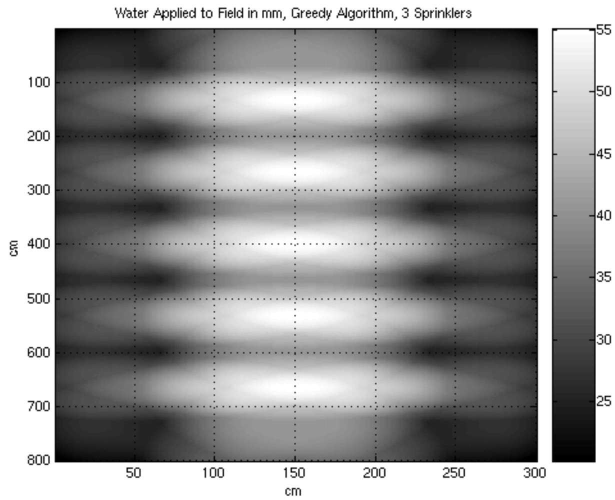

Figure 3: A two-dimensional precipitation distribution.

4 Farmer Seeks Algorithm

4.1 Enumerating the algorithms

Before we can enumerating all the algorithms we wish to consider, we need to decide where we will allow the lateral to be placed. For simplicity we will restrict lateral orientation to be parallel to the short side of the field; however for completeness we will allow it to have any position given that the orientation constraint is satisfied. It seems conceivable that under some circumstances such a constraint might discard superior algorithms, but as we will see later these are likely to be very extreme conditions which are outside the scope of our model in any case.

The lateral, at 20 m, is not as wide as the field it must water, and so we can conceive fairly easily of two distinct layout types. One is a *uniform* layout; the other is *staggered* (see Figs. 4, 5).
This allows us to define our algorithm space,

$$\{u\} \times \{s\} \times \{n\} \times \{\delta t\} \times \{uniform, staggered\}$$

and–hypothetically–to determine for each algorithm:

1. if it satisfies the constraints (see Subsection 2.3), and if so
2. the values of t_f and C_U associated with it.

4.2 Discretization, reduction, and traversal of the algorithm space

Before we can actually traverse the algorithm space we must discretize its subspaces:

- The u-space $u\ \epsilon(-1, 1)$ and s-space $s\ \epsilon(0, 1)$ become $u = -1 + 0.10k$ and $s = 0 + 0.05k$, respectively, where $k = 0, 1, 2, ..., 20$.
- The n-space, $n = 1, 2, ...$ is unchanged.

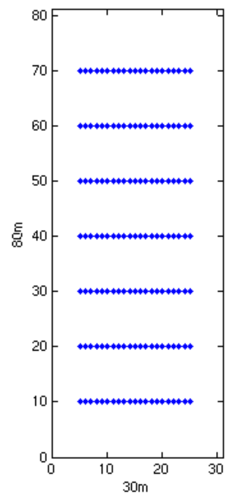

Figure 4: The uniform layout.

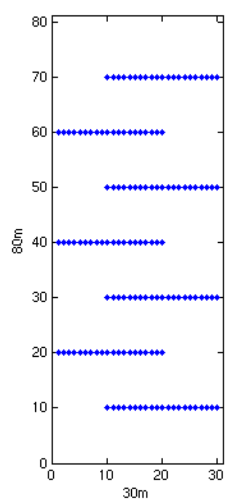

Figure 5: The staggered layout.

- The δt-space $\delta t \; \epsilon (0, \infty)$ becomes $\delta t = 1, 2, ...$.
- The *layout*-space is unchanged.

Now we traverse the algorithm space one subspace at a time and discard any algorithms that fail to satisfy the constraints (see Subsection 2.3). This dramatically reduces the available algorithms; in fact we are left with:

- $u = 0.6$,
- $s = 0.7$,
- $n \, \epsilon \{1, 2, 3, 4\}$, and
- $\delta t \, \epsilon \{1, 2, ..., 240\}$,

and the *layout*-space is unchanged. It is easy enough to compute t_f for each algorithm remaining in the space, but we don't yet have a way to measure precipitation uniformity.

4.3 The Christiansen Coefficient of Uniformity

To measure uniformity of precipitation across the entire field, we will employ the Christiansen Coefficient of Uniformity C_U, introduced by J. E. Christiansen in 1942 [3]. It is given by

$$C_U = 100 \left(1 - \frac{\frac{1}{n} \sum_{i=1}^{n} |x_i - \mu|}{\mu} \right) \tag{7}$$

where μ is the mean precipitation, $\mu = \frac{1}{n} \sum_{i=1}^{n} x_i$, and $C_U = 100$ indicates a perfectly uniform distribution (generally unattainable [5]).

5 Findings and Recommendations

According to our model results, the most generally optimized algorithm is a three-nozzle staggered layout with a time interval of 6 hours and 40 minutes. It has $C_U = 84.69$ and requires seven station moves to complete one watering period:

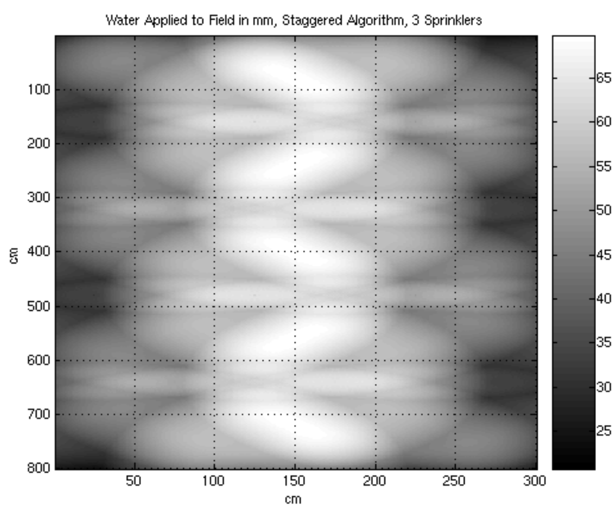

Figure 6: The most generally optimized algorithm.

The algorithm that requires the least station moves, however, is a two-nozzle, uniformly spaced six-position layout with a time interval of 8 hours. It has $C_U = 74.39$:

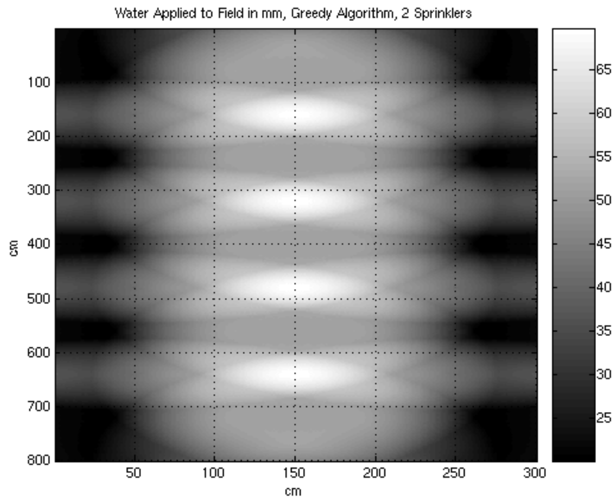

Figure 7: The fastest algorithm.

The upper constraint on precipitation limited the number of nozzles suitable for deployment to no more than four; anything more would require the farmer to move the lateral at least once an hour. A lateral configuration with fewer nozzles allows the farmer to perform less work and still obtain satisfactory precipitation results. Still, our model results demonstrate that there is a tradeoff between precipitation uniformity and work done by the farmer.

6 Further Model Strengths and Weaknesses; Potential Refinements and Additions

Perhaps the most crucial assumption that our model employs is that during any given time step, there is no net wind. This amounts essentially to assuming no wind at all, which is clearly a strong simplification. It allows for fast computation and effective discrimination among a wide range of algorithms, but unfortunately it also sharply limits the applicability of the algorithms selected. The model will return useful results for regions with low wind variability, but will depart sharply from reality in places with strong, highly variable winds. One possible method for incorporating wind without relying on many-particled, real-time, water droplet tracking is to track the wind vector field over the area of interest and to translate the precipitation that falls at each time step by the corresponding amount. This is of course also a simplification, but by keeping the computing power required to assess scheduling algorithms low it may be possible to maximize utility of the testing framework in rural areas where computing power is generally limited and hand-moves and other simple agricultural systems are most widely used [6].

7 Appendix: Some MATLAB Output

Below is an excerpt from some debug output of our algorithm testing framework.

```
>> farmerJoe(2,1,480,1,1);
--> we get signal
--> how are you gentlemen
   number of elements is 2
   flux is 0.000579 m^3/s
   radius is 22.673224 meters
--> take off every zig
   6 zigs launching
--> what happen ?
   max water is 69.803542 mm
   min water is 20.037111 mm
--> for great justice
   total water is 246169.715525 m^3
   uniformity coefficient is 74.386729
--> main screen turn on
--> you have no chance to survive make your time
   running time is 15.682376 seconds
```

References

[1] Li, J. and M. Rao, 2001: Crop yield as affected by uniformity of sprinkler irrigation system. *International Commission of Agricultural Engineering (CIGR) E-Journal of Scientific Research and Development*, **3**. http://cigr-ejournal.tamu.edu/submissions/volume3/LW%2001%20004a.pdf.

[2] Mateos, L., 1998: Assessing whole-field uniformity of stationary sprinkler irrigation systems. *Irrigation Science*, **18**, 73-81.

[3] Christiansen, J. E., 1942: Irrigation by sprinkling. California Agricultural Experiment Station Bulletin 670, University of California, Berkeley, CA.

[4] Douglas, J. F., J. M. Gasiorek, and J. A. Swaffield. *Fluid Mechanics, 3rd ed.* London: Longman Scientific & Technical, 1995. p. 167.

[5] Solmon, K. H., 1988: A new way to view sprinkler patterns. *Irrigation Notes*. California Agricultural Technology Institute Publication #880802. http://cati.csufresno.edu/cit/rese/88/880802.

[6] National Agricultural Statistics Service Charts and Maps: Farms and Land in Farms. http://www.nass.usda.gov/Charts_and_Maps/Farms_and_Land_in_Farms/fncht6.asp.