For office use only	Team Control Number	For office use only
T1 ________________	**1924801**	F1 ________________
T2 ________________		F2 ________________
T3 ________________		F3 ________________
T4 ________________	Problem Chosen **D**	F4 ________________

2019
MCM/ICM
Summary Sheet

(Your team's summary should be included as the first page of your electronic submission.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

A recent increase in terror attacks has raised demand for safe emergency evacuation plans worldwide. We focus on addressing difficulties which arise from evacuating the Louvre, the world's largest art museum. Evacuations are made difficult by the volume and variety of visitors; as a result, Louvre management desire an adaptable model in order to explore a range of evacuation plans over a broad set of considerations.

In our computational network analysis, we partitioned the Louvre into sections and built an agent-based model to simulate evacuations in each section. After developing the logic for the agents, we ran simulations over each section to determine an empirical rate by which agents exited. To connect sections, we abstracted the problem by representing the building as a graph, allowing us to solve for overall time taken for an evacuation plan in terms of a *network flows* problem. A property of this abstraction called strong duality also identified bottleneck edges in the graph. We emphasize the power of abstraction in the adaptability of our model; simulating blocked passages or new secret exits are simply edge removal and addition. Bottleneck identification was our highest priority in considering public safety in order to easily find problematic areas that in an emergency.

Our model predicted that a candidate evacuation plan involving all 4 public exits could evacuate the Louvre in 24.34 minutes. Furthermore, our bottleneck analysis revealed that while many bottlenecks surround the pyramid entrance, the entrance itself is not a bottleneck. We also found that keeping this property of the pyramid is crucial in emergencies, as it allows building access for emergency personnel and mitigates increased public safety concerns around the Louvre's most iconic entrance. Additionally, we found that securing the Passage Richelieu was critical to evacuation, as its safety was directly linked to the pyramid's safety. Keeping these entrances open and useful is therefore imperative to both speed and safety considerations in an evacuation.

Overall, our model is powerful due to our ability to model individual human behavior followed by a powerfully adaptable abstraction of building flow dynamics. One weakness of our model is that our theoretical guarantee is given in terms of worst-case scenarios, which may be an upper bound on a real evacuation in more common cases. However, we feel that this weakness is acceptable in evacuation simulations.

Team #1924801 Problem D: Time to Leave the Louvre: A Computational Network Analysis

Introduction and Background Information

The Louvre in Paris, France is the world's largest art museum, and received 8.1 million visitors in 2017 [1]. The composition of visitors is heavily varied, with 70% of guests being international, coming from countries such as the United States, China, Brazil, the UK, and so on [1]. However, terror attacks in France have also been increasing [2], making it imperative that Louvre officials have a clear plan for evacuation in the case of an emergency.

The main public entrance is the pyramid entrance two floors below the ground floor [3]. The Passage Richilieu entrance, Carrousel du Louvre entrance, and Portes Des Lions entrance are also potential entrances, although these entrances usually require memberships or reservations. However, an emergency situation would definitely serve as an exceptional case to allow usage of extra entrances in order to quickly and safely evacuate visitors. Additionally, there also exist other entrances that the public is generally unaware of. A natural question becomes whether or not these secret entrances provide sufficient compensation to justify compromising the Louvre's security by revealing their location to the public.

Since these secret entrances are hidden from public knowledge, there is need for a highly adaptable and easily interpretable model that the Louvre management could use to test multiple evacuation plans. This includes potentially opening up secret exits, considering potential blockages, and generally being able to compare disparate evacuation strategies. The high variety in the population of visitors also makes evacuating difficult due to language barriers for tourists, families that will stick together, and disabled people for whom moving quickly is difficult.

Restatement of the Problem

We are tasked with the broad problem of designing an evacuation model for the Louvre that allows exploration of a range of options. In order to clarify our purpose, we identify our primary goals as follows:

(1) To determine a means to assess the efficiency of a given evacuation plan.

(2) To develop - with respect to the above assessment - an optimal evacuation plan without compromising safety.

(3) To identify key bottlenecks and other obstacles to safer, more efficient evacuations.

(4) To determine the effect of additional exits or blocked routes on the optimal evacuation plan.

(5) To communicate a clear plan of implementation through suggested policies and procedural recommendations with an emphasis on safety.

Secondarily, we are also to consider:

(a) The effect of a diverse demographic of visitors (spoken language, size of group, disability status) on evacuation and useful responses or countermeasures

(b) Potential benefits of technology in aiding evacuation

(c) Possible deployment routes for emergency personnel

(d) Adaptability of the model to other large buildings

General Assumptions

In order to address the variety of goals above, we often made assumptions and decisions to make the problem more tractable. These assumptions are as follows:

- **Assumption 1: Evacuees will act strictly in their own self-interest.**

  Evacuees will not consider a globally optimal solution for everyone, and will instead make a locally greedy solution, modeling the urgency of an evacuation situation.

- **Assumption 2: "Natural flow" of evacuees.**

  Upon notification of a required evacuation, individuals egress to and through the closest exit in order to leave the building as quickly as possible, unless explicitly directed by evacuation procedures and officials otherwise.

- **Assumption 3: Strict adherence to procedure.**

  Individuals will generally follow the evacuation plan provided by Louvre management. For example, they will move to an exit assigned to them by the evacuation plan.

- **Assumption 4: Evacuees are safe and outside the responsibility of the Louvre emergency management team once they have exited the building.**

  While we are well informed to the geometry and environment inside the Louvre, the outside world is complex and ever-changing. Due to this fact and the wide variety of potential evacuation threats, attempting to secure the safety of people outside of the building is also deemed outside of the scope of our plan.

- **Assumption 5: Increasing panic causes people to make more and more sub-optimal or irrational choices.**

  Modeling how individuals react under the urgency of an evacuation situation can help us understand how our model can extend to real-world situations.

- **Assumption 6: Elevators are off limits during evacuation situations except for emergency personnel and disabled people.**

  Elevators can be dangerous in emergency situations [5].

- **Assumption 7: Language barriers can be mitigated by appropriate signage and technology in multiple languages, such as multilingual phone apps.**

  Many of the Louvre's current signs are not written in French, but rather contain universally comprehensible symbolic instructions [6]. Moreover, software packages and phone apps giving potentially non-French speaking evacuees directions are easily written to accommodate different languages.

Introduction: Definitions and Roadmap

First Definitions

In order to design an "efficient evacuation plan", we must first define both what an "evacuation plan" consists of and how exactly one might be "efficient".

We define an **evacuation plan** simply as a collection of pathing procedures that evacuation officials supply and enforce for each evacuee. We allow the procedures to be conditional on the location and state of the evacuee; that is, two evacuees under different circumstances might be directed to two different exits. The simplest evacuation plan would be one completely adhering to the "natural flow" of evacuees in which evacuees move towards the exit nearest to them, in accordance with Assumptions 1 and 2.

Now, to measure the **efficiency** of each evacuation plan, a first and common approach would simply be to estimate the *time* it takes to fully empty the building under the given evacuation plan. However, this measure is somewhat naive, since it discounts such factors as safety and is highly dependent on initial conditions. For example, a certain evacuation plan that minimizes exit time enacted on a day with the majority of visitors clustered around the Mona Lisa might attempt to funnel all of these visitors through one exit, which may compromise safety through overcrowding, trampling, and mob panic risks. Thus, this "fast" evacuation plan may not necessarily be desirable.

As an alternative to the time measure, we consider instead *maximum exit rate* or, mathematically, the maximum of the time derivative of exited evacuees. From a surface level, this seems to be an identical measure, since a decrease in the time for complete evacuation would necessarily mean an increase in exit rate, while an increase in time would indicate a decrease in exit rate. However, these are average exit rates that are directly affected by time, rather than the maximum exit rate. Assuming that the exit rate reaches a peak sometime during the middle of evacuation, a graph of exited evacuees to time elapsed would look something like Figure 1.

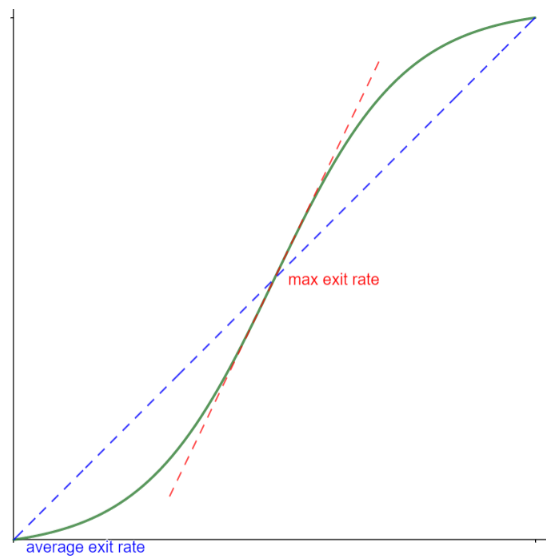

Figure 1: Sample graph: exited evacuees to time elapsed

The benefit of using maximum exit rate is that while it still accounts for the value of a "fast" evacuation, the main focus is in allowing for a larger flow of people. In other words, when optimizing for maximum exit rates, we actually optimize the throughput of evacuees through the Louvre rather than the output. The value of optimizing maximum exit rates is two-fold: 1) if the Louvre is at high capacity, average exit rates should approach maximum exit rates; 2) if the Louvre is at low capacity, higher throughput should decrease crowding risks. As a result, maximizing throughput is directly correlated with maximizing public safety.

Modeling Roadmap

We now proceed to developing a model that can adequately assess, design, and optimize evacuation plans with respect to our maximum exit rate measure. To achieve these goals, we implement a two-stage model. The first stage seeks to use computational agent-based modeling to understand local evacuee flow dynamics within sections of the Louvre. The second stage conglomerates information on the various sections into a flow network upon which we can assess and optimize evacuation plans. To clarify, a roadmap for our model is described by the following steps:

1. Partition each floor of the Louvre into smaller subsections.

2. Develop a computational agent-based model in NetLogo to study local evacuation phenomena and evacuee flow for each partition/section of the Louvre.

3. Develop a global network that models each partition/section of the Louvre as nodes, passageways between them as edges, and evacuee flow as weights.

4. Perform relevant graph algorithms to maximize evacuee flow and predict the effect of adding or removing edges.

5. Interpret results of both the local model and the graph algorithms in real-world terms and infer useful policy suggestions based on these results.

Part I: The Local Section Model

The primary content of this section are steps 1 and 2 described in the roadmap. Specifically, partitioning the Louvre and developing the local evacuation model.

Partitioning Sections

The primary challenges we identified in discussing evacuation models out of the Louvre are the theoretical and computational difficulties involved with understanding the museum's complex layout or geometry. The Louvre consists of nested gallery layouts, several access points to other floors, and multiple exit points, making anything but very simple models of total evacuation flow across the building difficult. Moreover, models across the entire building begin to become computationally infeasible as the Louvre approaches tens of thousands of total visitors a day. As a result, any purely computational modeling paradigm would therefore be reduced to including only very simple behaviors in order to compensate. However, by partitioning the museum into smaller, less complex subsections to be modeled individually, we reduce both computational and theoretical complexity in our modeling, allowing richer, more meaningful extrapolations of real-world behavior.

We chose to model the Louvre by splitting each floor into the five subsections demarcated by Figure 2, and label section A-E (for example, the bottom-left section on the ground floor would be labeled "ground floor A"). The Napoleon Hall, in addition, has a pyramid entrance that does not exist on any floor (as shown by Figure 3), and is in fact the only relevant subsection on that floor. We denote this "Napoleon P".

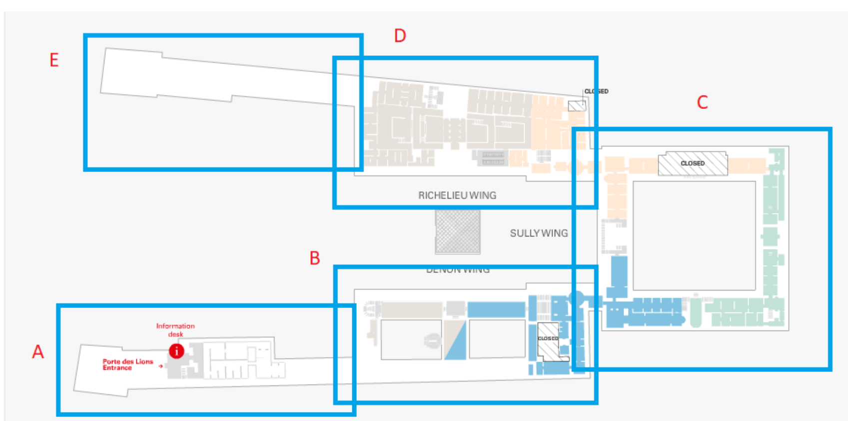

Figure 2: We split each floor of the Louvre into five subsections for computational and theoretical feasibility in modeling. Starting from the bottom and moving counter-clockwise, they are labeled A-E, respectively.

Development

Our local evacuation model (hereafter called the Local Section Model) is developed in NetLogo, an agent-based modeling software designed for studying complex systems built by Uri Wilensky [7]. The main idea

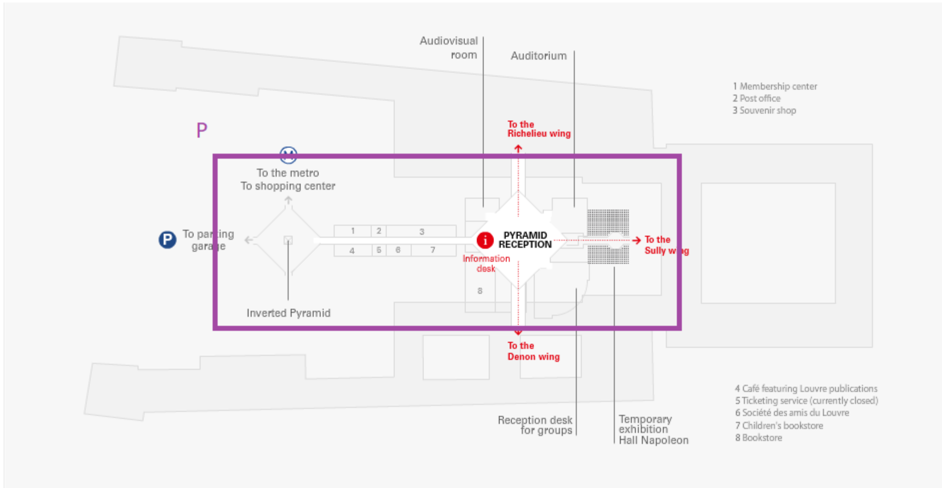

Figure 3: The Napoleon Hall contains a pyramidal subsection unique to its floor, and will be denoted "Napoleon P". In fact, this is the only relevant subsection of the Napoleon Hall.

of agent-based modeling is that agents are single units with specific, well-defined goals. While an individual agent's behavior is typically simple, the complex behavior of a system of agents is usually more than the sum of its parts [9]. In context of this problem, since each individual person's goal is to successfully evacuate, an agent-based model is highly applicable. Furthermore, Figure 4 shows that the resultant interface that is quite easy to use and interpret. In particular, this figure shows our representation of section Ground D, specifically its complex gallery system. The white agents acting on a grid of green (passable) and black (impassable) patches represent evacuees finding their way through various galleries. The blue patches represent entryways from which more evacuees enter, and the red patches represent exits through which the evacuees egress. The specific behavior logic of the agents is shown in Figure 5. Note that each agent is equipped with a variable panic attribute and fixed speed attribute. Further information on the speed attribute is contained in the following section, while the panic attribute is explained here.

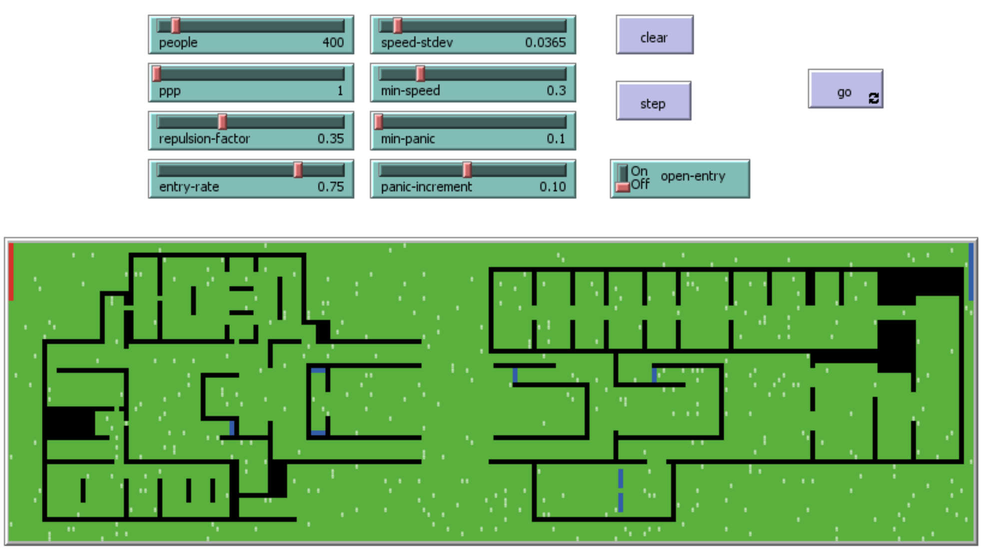

Figure 4: A representation of the complex galleries present on ground D.

The logic in Figure 5 is consistent with our assumption that each agent acts in their own self-interest, such as

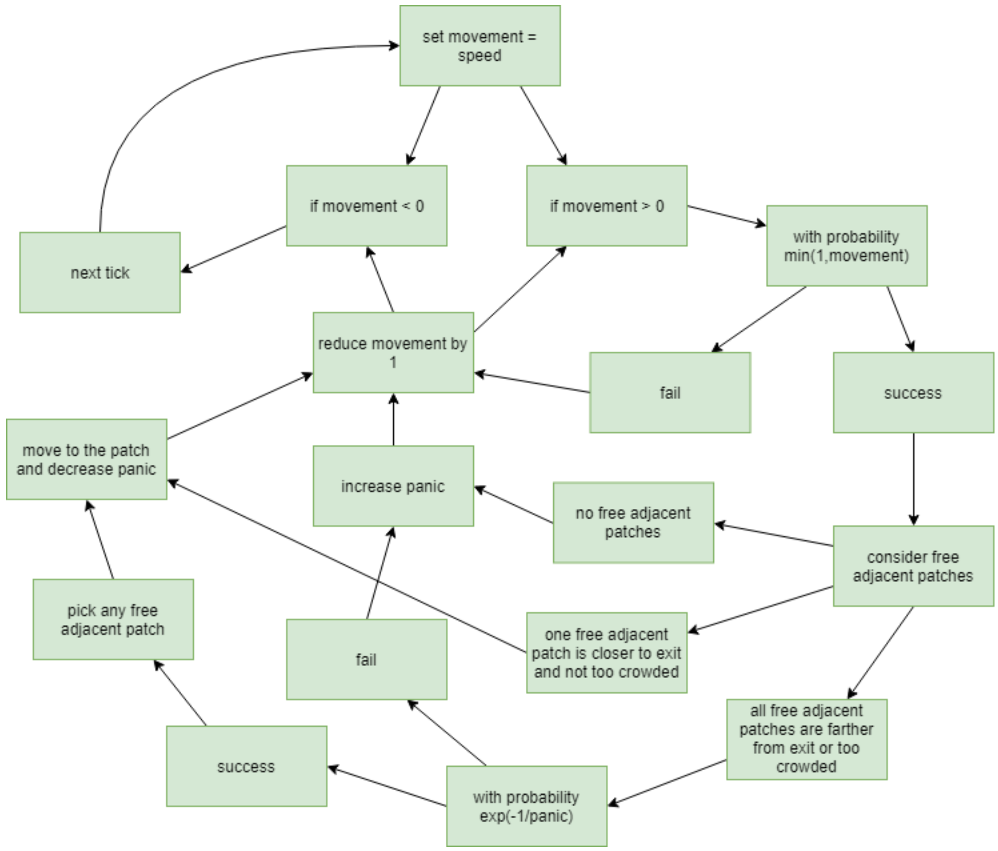

Figure 5: A flowchart describing an individual agent's behavior logic.

making locally optimal steps towards the nearest exit. However, this factor alone causes purely deterministic behavior that does not appear to accurately model human movements, especially in a tense situation. For this, we added a panic parameter to represent increased tension in the individual, as evacuees react to being unable to move due high crowd density by shuffling around, looking for other exit routes. While it is difficult to quantify panic in the exact sense, it is not difficult to include its effects in a model per our assumptions about panic. For this, we took inspiration from a probabilistic technique known as simulated annealing [14]. From this general technique, we chose to increase the agents' panic each time step they are stationary and compute p_m, the probability of moving to a patch slightly further from their destination, by

$$p_m = \exp(-\frac{1}{\text{panic}})$$

Our choice of p_m in context of the general simulated annealing technique draws has been studied to draw analogies to real physical systems [14]. Additionally, it is useful because, for positive values of panic,

$$\lim_{\text{panic}\to 0^+} p_m = 0, \quad \lim_{\text{panic}\to\infty} p_m = 1$$

which means that all values of p_m can actually be used as probabilities.

Lastly, we should give some mention to the slider parameters that appear in Figure 4. The people slider refers to the initial population of agents present in the room. Scaling this parameter changes the initial condition. The ppp slider refers to the maximum density of agents per passable patch. Scaling this parameter up increases the maximum allowable crowdedness. The repulsion-factor slider is related to what each agent determines to be "too crowded" during local pathfinding. Scaling this parameter up decreases crowding tolerance. The speed-stdev slider refers to the variance of speed attributes in the population of agents, while min-speed refers to the minimum speed attribute an agent can have. The panic-increment slider refers to how much more panicked each agent gets when unable to move, while min-panic refers the minimum panic attribute an agent can have. Entry-rate and open-entry are related to the spawning of new agents through the blue entry patches.

Physical Interpretability

In order to make sure our model had real, physically interpretable results, we tooks steps to ensure that our computed rates had reasonable physical counterparts. In order to scale up to dimensions of the Louvre, we used Google Maps to find the global coordinates of corners of the Louvre. Figure 6 shows points that we chose to use as reference. With these coordinates, we used a Python package called GeoPy [10] to calculate the distances, and examples of these distances are also shown on Figure 6.

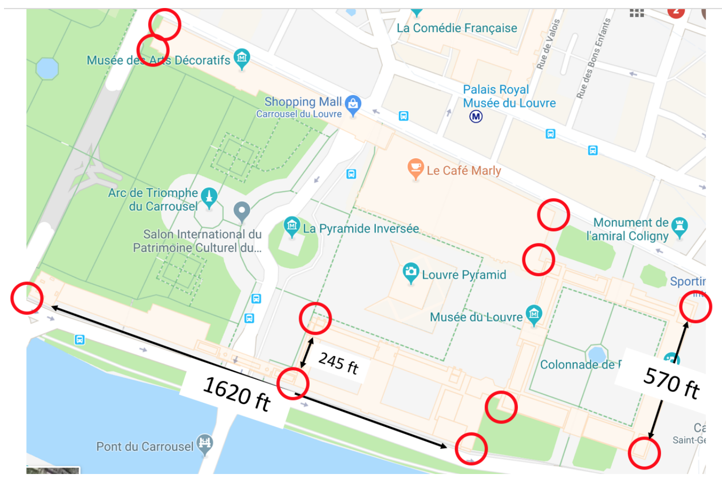

Figure 6: Points pinged on Google Maps, with some example distances.

To determine the speed of the agents, we referred to research done by Yosritzal et al., in which they simulated a tsunami evacuation in Indonesia [8]. Given that we are also modeling evacuations, we chose to incorporate their results. Their results give an average walking speed of an individual during an evacuation as $1.419\frac{\mathrm{m}}{\mathrm{s}} = 4.656\frac{\mathrm{ft}}{\mathrm{s}}$. As such, we scale each subsection's representation such that one patch is four feet by four feet. As a result, the one tick (the unit time in Netlogo) in the model's progression is roughly equivalent to one second in real world time. We take this as a reasonable approximation because the agents are allowed to move in both the cardinal and diagonal directions. Given that these movements represent a 4 feet movement or $4\sqrt{2} \approx 5.66$ feet movement, respectively, it is reasonable to translate 4.656 feet per second into 1 patch per tick. Additionally, to model a diverse population's varying walking speed, we approximate the distribution of walking speeds with a normal distribution of mean 4.656 ft/s and standard deviation 0.170 ft/s assuming large population size of age 20-60 years. This distribution was derived from the statistics of [8] on a population of age 20-60.

Corner Bottlenecking: The Price of Turning

Before discussing further results, which will be detailed later, discussing the effect of corners and turning is important. Inspection of agent behavior demonstrates that the act of turning and the simple presence of corners present significant bottlenecks to optimal evacuee flow in the Local Section Model. Moreover, the prevalence of both turns and corners makes analyzing their effects important to the overall global analysis of exit throughput.

Given an individual agent's wish to take the shortest path to an exit, a compressionary phenomenon begins to manifest at both corners and orthogonal exits, where a orthogonal exit is defined as an exit perpendicular to the entry corridor from which the agent entered. Figure 7a gives an example of this compressionary phenomenon over the Porte Des Lions exit of ground A, where we can clearly see the effect of the individual agent's desire to escape as quickly as possible: a triangular wedge formation that restricts maximum flow outwards, rather than the linear flow described by Figure 7b. While mitigated significantly by our agents' repulsion force, the effect is still significant; whereas the orthogonal exit modeled by Figure 7a had a maximum output flow of 4.2 agents per second, the linear exit modeled by Figure 7b had a maximum output flow approaching 4.8 agents per second, a 14% increase.

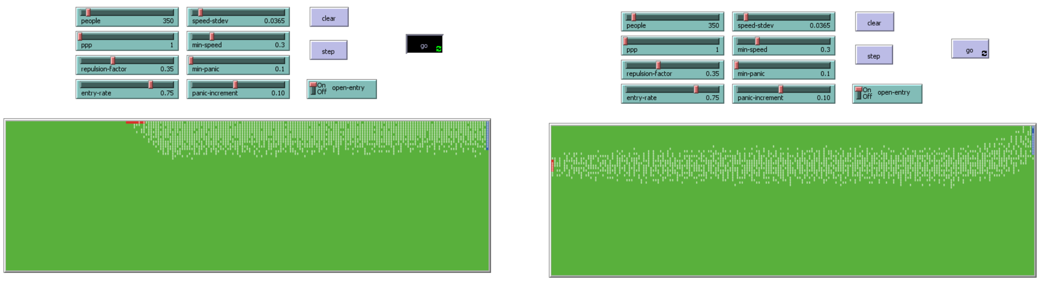

Figure 7: Left (a): Describes the effect of turning on exit flow, which totals 4.2 evacuees per second. Right (b): Describes a linear exit flow, which totals 4.8 agents per second.

This phenomenon can be extended to most turns, an extension which becomes apparent when examining corners such as those in Figure 8a. The model represented by Figure 8a has an output flow of 2 evacuees per second, whereas the model represented by Figure 8b has an output flow of 2.6 evacuees per second, an increase of 30%.

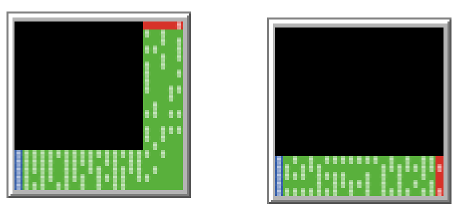

Figure 8: Left (a): Describes the effect of corners on exit flow, which totals 2 evacuees per second. Right (b): Describes a linear output flow for comparison, which totals 2.6 evacuees per second.

We understand that the orientation of the Porte Des Lions exit cannot be realistically changed to match that of the entry doorway. However, one policy recommendation we propose is the presence of emergency personnel to direct evacuee traffic such that their flow more closely resembles that of 7a rather than 7b and that of 8a rather than 8b. Moreover, any future secret exits constructed should be oriented with this phenomenon in mind, i.e. facing the largest open corridor. We detail further results and further policy recommendations in later sections; we give this corner analysis as a demonstration of NetLogo's utility in understanding local evacuation phenomena.

Part II: The Global Flow Model

This section addresses parts 3 and 4 of the road map, by abstracting the problem with a network flows formulation, solving for an optimal solution, and discussion of model adaptability.

Abstraction as a Network

We use a graph to represent the different sections and the connections between them. A graph $G = (V, E)$ is a set of vertices V and edges E that represent the connections between vertices. Namely, an edge can be represented as an ordered pair (u, v), which means the edge starts at $u \in V$ and ends at $v \in V$. In context of our problem, we let our predefined simpler sections represent vertices and let the existence of a pathway between these sections represent edges. These pathways could be either hallways or staircases. By examining floor plans [3], we determined the locations of all staircases were and their connected subsections. Figure 9 shows the resultant graph representation. Note that the nodes are represented by two letters, such as L_A, which corresponds to "Lower ground A". The rest of the nodes correspond to "Napolean": N, "Ground": G, "First": F, and "Second": S, similarly. For reference, the $A - E$ lettering is shown in Figure 2.

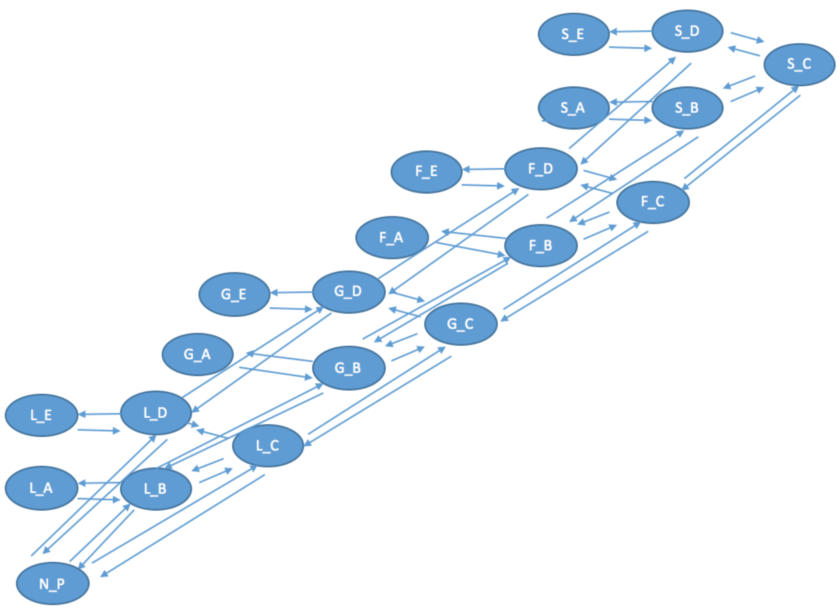

Figure 9: General graph showing all possible edges

Network Flows Formulation

In order to formulate this problem in an abstract setting, we introduce the problem of *network flows* [13]. In particular, we discuss the *maximum flows* problem and explicitly discuss the connection to our model. Consider a general directed graph $G = (V, E)$ with a source vertex $s \in V$ and sink vertex $t \in V$. Additionally, define a capacity function on the edges. If we define a directed edge $e = (u, v)$, then $c(u, v) : E \to \mathbb{R}^+$. An $s - t$ flow on a network is a function $f(u, v) : E \to \mathbb{R}^+$ that satisfies the following constraints:

$$\begin{aligned}
&\text{Skew symmetry: } f(u,v) = -f(v,u),\ \forall (u,v) \in E\\
&\text{Capacity: } f(u,v) \leq c(u,v),\ \forall (u,v) \in E\\
&\text{Balance: } \sum_{v \in V} f(u,v) = 0,\ \forall v \in V \setminus \{s,t\}
\end{aligned}$$

Notice how the balance constraint excludes s and t. This is because the value of a flow, $|f|$, is defined as

$$|f| = \sum_{v \in V} f(s,v) = \sum_{v \in V} f(v,t)$$

The maximum flows problem asks to find the flow of maximum value on a given network. Intuitively, the flow problem can be thought of as sending water through a network of pipes. If we think of flow as water, pipes as edges, and vertices as junctions, then the skew symmetry constraints says that the amount of water on the network being sent across an edge is equivalent to a negative amount of flow being sent in the opposite direction. Capacity constraints say that pipes have specific amounts of water that can flow through them. Lastly, balance constraints say that each unit of water that flows through any non-terminal junction must also flow out. The question then becomes, how much water can be sent from the source to the sink through the pipes in one unit time? A specific example is shown in Figure 10.

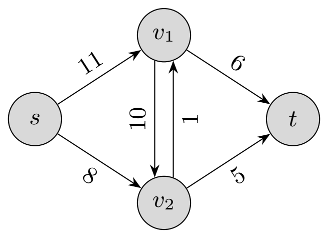

Figure 10: An example network with capacities written on the edges. Maximum flow = 11 (send 6 on the top path and 5 on the bottom path).

In order to cast our evacuation problem to the specific case of network flows, we need to make slight modifications to the general network structure described earlier. For example, the classical network flows problem assumes a single source and a single sink. However, the initial state of the Louvre during an evacuation would have visitors starting in multiple sections of the museum. We solve this problem by creating what we call a "super-source" vertex s that connects to each section of the museum. For a node r, we set $c(s,r)$ to be the initial population of the section. When we compute the maximum flow, this extra set of edges can send at most the initial population of the room as flow, after which the movement out of from one section to another limits movement. Similarly, we add a "super-sink" and there will be a directed edge from every section that allows people to exit the museum to the "super-sink". This construction reduces our problem to the maximum flow problem. In terms of producing an answer to our specific network, there are many well-studied algorithms that give not only the maximum flow, but the allocation of flows on the edges. In particular, we use the Edmonds-Karp algorithm (1972) [15] in order to solve for maximum flow on our network.

Once we have maximum flow value m, we have the interpretation that the given evacuation plan allows for a maximum of m people per second to be evacuated from the Louvre. This is an optimal rate under the constraints; if p people are in the Louvre when the evacuation is called, then the minimum time to evacuate the Louvre, $t_{\min}$ is given by

$$t_{\min} = \frac{p}{m}$$

The value of $t_{\min}$ gives us a heuristic by which we can rank different evacuation strategies. While it also leaves us some room for discussion about how much $t_{\min}$ underestimates the actual time, we can still rank the strategies relative to each other. Also, when we discuss where the bottlenecks are, we can make more assessments as to whether $t_{\min}$ actually appears to be a significant underestimate on time needed.

Finding Bottlenecks

A big advantage of our reduction is that it allows us to easily find bottlenecks. In order to understand how, we need to introduce another abstract problem on graphs: the minimum cut problem. Similar to the maximum flow problem, we have a graph $G = (V, E)$, a source vertex $s \in V$ and a sink vertex $t \in V$, and a capacity function $c(u,v) : E \to \mathbb{R}^+$. An $s-t$ cut is defined by a subset of edges $C \subseteq E$ such that removing C from E results in a disconnected graph where s and t are in different components. The cost of a cut is given by

$$\text{cost} = \sum_{(u,v)\in C} c(u,v)$$

The minimum cut problem asks for a cut with minimum cost. The reason for introducing the min-cut problem is that it is intimately related to the max-flow problem by the max-flow min-cut theorem [16]. This theorem says that the max-flow from s to t in a network is exactly equal to the minimum $s-t$ cut. Furthermore, since this theorem is a consequence of what is known as *strong duality* [17] in linear programming, the optimal solution exhibits a property known as *complementary slackness*. In this context, complementary slackness says that the edges in the minimum cut are exactly those edges in G that have flow equal to their maximum capacity. So, we can interpret this as the following: the edges that have max-flow in G are exactly the set of edges that correspond to the bottlenecks.

Adapting our Model

Since we are unaware of the exact location of any secret exits, we are forced to keep the model very flexible in the event that secret exits need to be opened. This requirement highlights the power of our abstraction using graphs and network flows. For example, the addition of a secret exit in a certain section can be represented by an edge from that vertex to the supersink, while blockages between sections are simply represented by removal of corresponding edges. This techinque of removing edges can also represent closing off additional edges, such as Passage Richelieu, by removing the corresponding edge to the supersink. Although the supersource is not explicitly shown in the graph figures, it can also be used for adaptability. As discussed in the development section, we use the supersource vertex to feed in an initial population of people. So, the Louvre staff can set varying initial distributions of visitors throughout the museum to see the results on evacuation proceedings.

Louvre management can also experiment with parameters within the NetLogo model; for example, they can observe the effect increasing panic rates have on individual and groups of agents. Alternatively, they can intentionally set some agents to be significantly slower than others and use this to represent disabled evacuees. With the flexibility in these parameters, the management can test both different evacuation plans and the effects of human behavior on these plans. In this way, they can decide on a set of evacuation strategies to deploy depending on varying evacuation situations.

It is important to note that despite our concentration on the specifics of the Louvre's layout, our model is highly adaptable to buildings and floor layouts other than the Louvre. In fact, the only part of the model which needs to change to respond to different building layouts is the Local Section Model; in such a case, NetLogo representations of floor sections of a new building would have to built, and different evacuation plans be represented in these NetLogo representations. However, we assert that these are the only changes that adapting our model to a new building requires, particularly since the Global Flow Model is non-specific to any particular building and only requires accurate identification of the edge weights (one example of which we provide in Appendix A.

Part III: Results, Discussion, and Recommendations

In this section, we address step 5 of the roadmap through the discussion of the following topics

1. The Pyramid is NOT a Bottleneck
2. Passage Richelieu is Critical to Evacuation Proceedings
3. Ground D is a Bottleneck
4. Braess' Paradox

5. The Residual Network Gives Pathing Recommendations for Emergency Personnel

6. Summary of Policy Recommendations

In this section, we discuss possible evacuation plans and our model results. For our main model run, we choose to allow the main pyramid entrance, the Passage Richelieu entrance, the Carrousel du Louvre entrance, and the Portes Des Lions entrance as museum exits, since they are both public from use in Affluences [12] and commonly used. In particular, we found the respective sections were Passage Richilieu in Lower ground D, Carrousel du Louvre in Ground E, and Portes Des Lions in Ground A. With this information in mind, our evacuation plan calculates, for each section, the closest exit section in terms of Euclidean distance and sends all people towards that exit. For this setup, we calculated a throughput of 17.8 people per second. For reference, since the Louvre had 8.1 million visitors in 2017, [1], and the Louvre is not open on Tuesdays [4], this leaves an average of about 26,000 people per day. Now, while it is likely that the busiest day had significantly more than the average, it is unlikely that all visitors happened to be in the Louvre at the same time. So, we take this 26,000 as a proxy for the worst case in an evacuation, but this is easily adaptable in our model as discussed previously. If we had been able to find information about the distribution of people throughout time, we could make a better judgment for this figure, but our experiments just use 26,000 as the number of people. Also, without extra information about the distribution of people, we simply allocated them uniformly across the rooms. Although, this is another easily adaptable part of the model. With these decisions, this evacuation plan gives

$$t_{\min} = \frac{26000}{17.8} = 1460.67\text{s} = 24.34\text{min}$$

To see the bottlenecks, refer to Figure 11 for a visualization. There are bottlenecks from each of L_D, G_A,

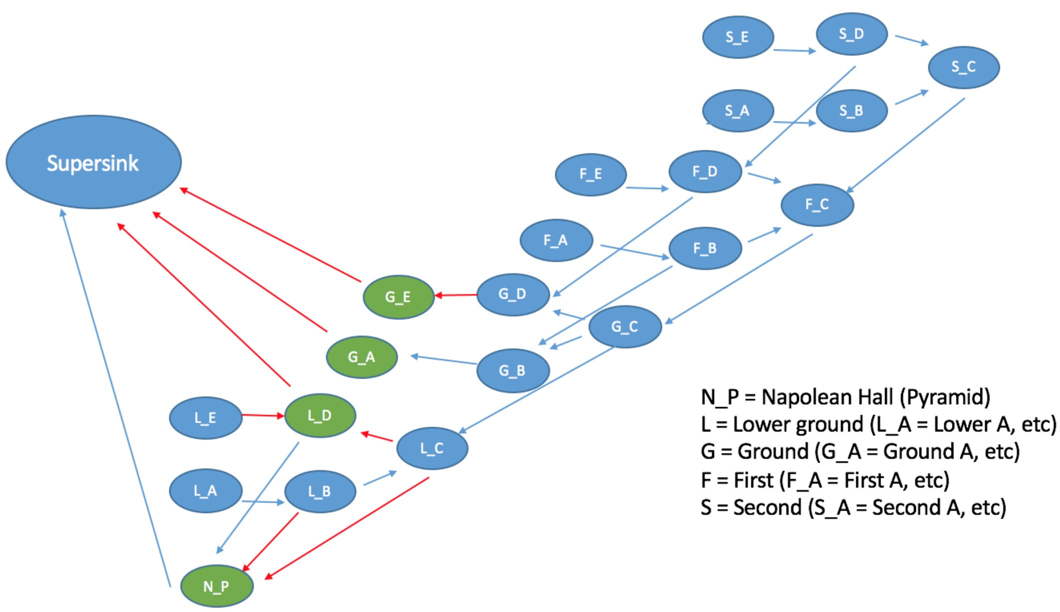

Figure 11: A representation of the bottlenecks on our constructed graph. Green vertices have a direct exit edge to the supersink. Red edges identify edges in the minimum cut, which represent inherent bottlenecks to outward exit flow. Any attempt to improve Louvre evacuation time should begin with these edges.

and G_E to the supersink. These are the edges corresponding to the Passage Richilieu, Portes Des Lions, and Carrousel du Louvre entrances, respectively. This means that the global bottleneck for these sections are actually exiting the building, not getting there. This is in contrast to the N_P vertex for the pyramid. For the pyramid, the bottle neck actually involves getting to the pyramid and not crossing through it. In particular, the stairs between L_B and N_P and between L_C and N_P form the bottlenecks for this particular evacuation plan.

The Pyramid is NOT a Bottleneck

Napoleon P, or more colloquially the pyramid, is the main entrance provided for the public. Furthermore, it is rate-limited by a set of stairs leading up towards ground level. As such, we would intuitively expect the edge leading from Napoleon P towards the outside, or supersink, to be a part of the minimum cut presented in the section above, and therefore a bottleneck which requires further analysis.

This turns out not to be the case. The pyramid itself is not the bottleneck; by examining the minimum cut, we see that by virtue of the edges from lower ground C to Napoleon P and from lower ground B to Napoleon P that it is, in fact, entering the pyramid which represents the true bottleneck, and not the pyramid itself.

As such, our policy recommendation concerning the pyramid is theoretically simple: provide a higher access rate to the pyramid. In practice, however, we understand this to be difficult, requiring either the construction of new staircases or the widening of existing ones. A more feasible policy recommendation is a priority on the opening of secret exits surrounding the pyramid, which may provide relief to the entryways into the pyramid. More specifically, if secret exits exist in lower ground B or lower ground C, these exits would provide the most relief given our current evacuation plan. We will see in following sections why the pyramid exit has such a large effect on the efficacy of a given evacuation plan. Note also that we recommend these locations only in the event that museum staff decide to open secret exits at all, since recommending the construction of new public exits is beyond the scope of this project.

Passage Richelieu is Critical to Evacuation Proceedings

The most important section identified in our evacuation plan is lower ground D, because of the relief effect it provides on entry into the pyramid. We see this not only because Passage Richelieu is a part of our minimum cut, but also because removal of its representative edge connecting lower ground D to the supersink, denoted (L_D, S), constitutes a 6.8 evacuee per second reduction in our model's total throughput. This reduces our initial valuation of 17.8 evacuees per second to a modest 11.0 evacuees per second and our estimated total evacuation time from 24.34 minutes to 39.39 minutes, a staggering 64% jump in evacuation time. However, (L_D, S) has a capacity of 7 evacuees per second; the 0.2 evacuee per second discrepancy comes in a rerouting in our max flow network. This in fact is an example of the adaptability of our model which we detailed in an earlier section.

More importantly, however, we observe that the removal of (L_D, S) changes our minimum cut dramatically; in particular, the edge from the pyramid to the supersink, denoted (P, S), becomes a bottleneck point. For the graph, this is irrelevant; output flow is output flow, wherever it may come. Qualitatively, however, safety concerns around the pyramid are more dire than around our other exit bottlenecks. The pyramid's glass composition is one concern; another is that given the pyramid's status as Louvre icon and main public entrance, it is likely to be targeted first in the event of external attack. As a result, protecting the Passage Richelieu is also a form of protection for the pyramid.

Our policy recommendation is therefore increased security presence concentrated on securing the Passage Richelieu. This matches well with the fact that we model Passage Richelieu to have, at a rate of 7 evacuees per second, the largest exit throughput of any of our four public entrances. In addition, its additional utility in reducing strain on the pyramid entrance is highly important to safe evacuation in certain emergency types.

Ground D is a Bottleneck

The most restrictive edge in our minimum cut, and therefore the most powerful bottleneck on our outgoing flow of evacuees, is the edge connecting ground D to ground E. In fact, this D-E edge actually only allows a maximum throughput of 1.8 evacuees per second, an unexpected number when given the width of the corridor connecting ground D and ground E, modeled conservatively at 4 patches, or 20 feet, wide.

To understand this discrepancy, we turn to our Local Section Model for insight. Our given evacuation plan asks all evacuees in ground D to move towards the Carrousel du Louvre exit in ground E. However,

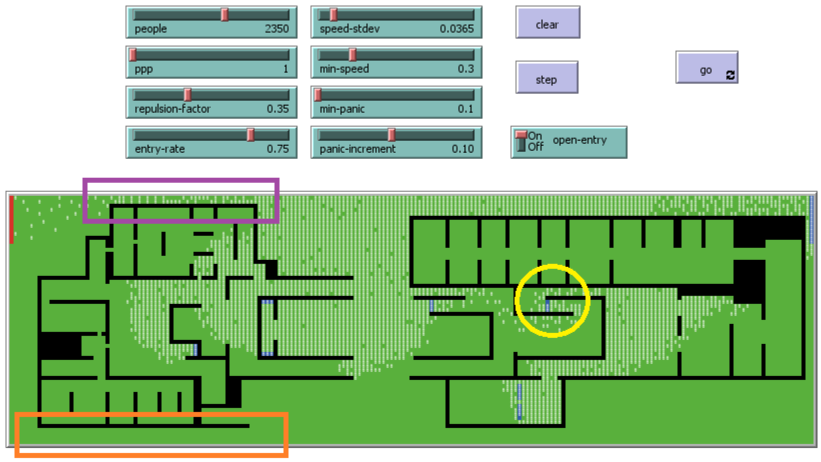

Figure 12: A depiction of bottlenecking in ground D. Sources such as stairs and other hallways are marked in blue, whereas the exit corridor is marked in red. The most prominent bottlenecking section is boxed in purple, whereas an unused path is boxed in orange. An example of a secondary (corner) bottleneck is circled in yellow.

their progress is impeded by narrow corridors and complex turns of complex and nested galleries, which are detailed in Figure 4. As a result, evacuees asked to traverse ground D towards the corridor leading to ground E are blocked by other evacuees, leading to the bottleneck described above as well as increasing levels of panic in the population. This is particularly problematic because ground D is the only section which leads to ground E; note, in Figure 12, how many evacuees are trapped in the middle common area, and how correspondingly few evacuees are able to push through to the exit. Moreover, rising panic levels, overcrowding, and the resulting public safety dangers contribute to difficulties beyond a limitation on exit flow.

To understand the quantitative effect of ground D bottlenecking on total global flow, we note that since ground D is the only section connecting to ground E, the exit flow out from the Carrousel du Louvre exit is defined as $\min(F_e, F_d)$, where F_e and F_d are the flow out from the Carrousel du Louvre, measured in evacuees per second, and the flow from ground D to ground E, respectively. Experimental results give values of $F_e = 4.2$ evacuees per second and $F_d = 1.8$ evacuees per second, giving a total exit flow out from the Carrousel du Louvre of 1.8 evacuees per second. As a result, the layout of ground D is directly responsible for a net loss of 2.4 evacuees per second. Theoretically removing the constraint of F_{de} could therefore improve global exit flow from 17.8 evacuees per second to 20.2 evacuees per second, improving total evacuation time from 24.34 minutes to 21.45 minutes. This is the largest such potential improvement found across our model, and is therefore critical to understanding how a Louvre evacuation could be tightened.

One other theoretical experiment we performed is the replacement of the Carrousel du Louvre exit with an equivalently-sized exit in ground D, placed precisely where the largest amount of evacuees begin to crowd (Figure 13). Call this new exit "passage ground D". Placement of passage ground D results in an exit flow of 4.8 evacuees per second, as opposed to the 1.8 evacuees per second provided by the Carrousel du Louvre exit. Note that this value matches well with the value provided by the linear flow out of the Porte Des Lions exit described by Figure 7b. Since the Porte Des Lions exit has been modeled to the same size as the Carrousel du Louvre exit, this indicates that passage ground D has reached some level of optimality in terms of its exit throughput. The presence of passage ground D would result in a global output flow of 20.8 evacuees per second, reducing total evacuation time to 20.83 minutes, a percentage decrease of 15% when compared to the original value of 24.34 minutes.

The Carrousel du Louvre is a public exit, and its use therefore does not involve the cost of revealing a secret

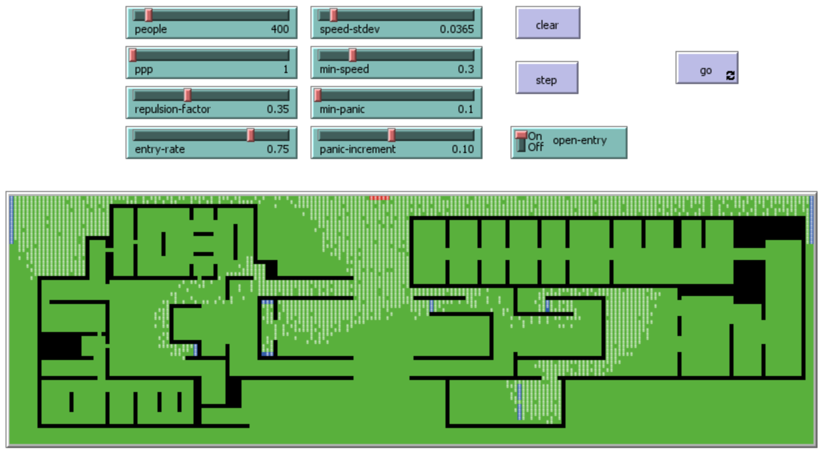

Figure 13: Placing an exit, marked in red, where the largest amount of evacuees begin to congregate, provides a network flow of 4.8 evacuees per second as opposed to a base flow of 1.8 evacuees per second.

exit; as a result, barring obstacles, the Carrousel du Louvre exit is important to a majority of evacuation plans. However, the Carrousel du Louvre exit is currently underutilized when compared to its maximum output capacity in our evacuation plan. In fact, this is true for all evacuation plans involving the Carrousel du Louvre exit, since ground E, where the Carrousel du Louvre exit is located, is connected only to ground D. However, its throughput is currently rate-limited by the large, complex network of galleries contained in ground D, especially by the section outlined in purple in Figure 12.

Several policy recommendations can be inferred from these observations. The first is the widening of the section Figure 12 outlines in purple; the proximity of gallery walls to museum walls constricts the natural flow of evacuees towards the exit and causes increasing levels of panic as well. Opening this section would allow more evacuees to take advantage of this space.

A second policy recommendation is the placement of emergency personnel to direct evacuees around the "back", through the section Figure 12 outlines in orange. Currently, agents following their self-interest in following the shortest path to the exit begin to squeeze towards the purple-outlined section, causing our bottleneck; by directing people through the orange-outlined path, a locally suboptimal path for one person would increase total throughput and therefore provide a globally more optimal solution. Alternatively, a software package such as a phone app to highlight unused or less crowded routes to a destination would also optimize the global evacuation situation.

Another policy recommendation we make is the removal of "secondary" bottlenecks, or corner bottlenecks, one example of which is circled in yellow in Figure 12. Several of these exist in ground D, restricting evacuee flow and providing multiple bottlenecks to the exit point, trapping a large number of people in a small space and, in restricting their ability to feel progression towards their end goal, increasing population panic levels. Note that some of these second bottlenecks may be hidden by the large flood of people we use to extract maximal capacity from the exits. Providing simpler gallery structures would help produce a more natural flow of evacuees, potentially increasing throughput and lowering public safety concerns about overcrowding.

More drastic recommendations center around analysis around the exit depicted by Figure 13. Direct construction of a new exit is beyond the scope of reasonable recommendations to be put forth by this paper; however, if there exists a secret exit located in ground D, its opening would provide the largest gain in exit throughput predicted by our model. Furthermore, if a new exit, public or secret, were to be constructed, we predict that the optimal location for such a new exit would be approximately where passage ground D is currently located in Figure 13.

Braess' Paradox

One procedural recommendation that we can make relates to the sections of the museum with the general shape of sections such as first floor B. We can draw this section in graph form as shown in Figure 14. The purpose of this example is to illustrate Braess' Paradox [11], which states that removing edges, i.e. movement options, from a network can actually increase traffic flow across a network. Note that flow in this context relates to the actual amount of time taken for people to cross the network, not network flows in the abstract setting as discussed in Part II.

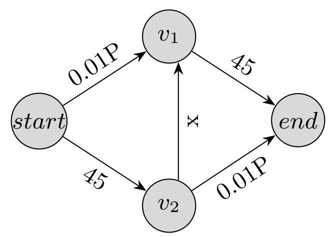

Figure 14: Example to illustrate Braess' Paradox, given by [11].

In Figure 14, assume that there are 4000 people attempting to cross from the start node to the end node. The time taken to traverse an edge is either 45 or $0.01P$ depending on the label, where P is the number of people currently on the edge. First, consider the case where the edge labeled x does not exist. Let us denote the path through v_1 as the v_1 path and similarly for the v_2 path. If A people take the v_1 path and B people take the v_2 path, then the time for each path will be $0.01A + 45$ and $0.01B + 45$, respectively. Assuming rational people will take the path to minimize their own shortest route, they will reach an equilibrium where each path takes the same amount of time, giving the following system of equations:

$$0.01A + 45 = 0.01B + 45$$
$$A + B = 4000$$

This system solves to $A = B = 2000$. In this case, it takes each person $45 + .01(2000) = 65$ units of time. Now, consider the case where edge x does exist, but it takes a very generous 0 time to traverse it. Consider the case where a single person takes the path $start - v_1 - v_2 - end$. For this person, the time to traverse becomes $0.01(2000) + (0.01)(2001) = 40.01$, a close to 25 unit saving. However, multiple people would try this route, and with each additional person, the time of the route would increase until 2500 people decide to take the route, at which point the route takes $0.01(2500) + 0.01(4000) = 65$ units of time, the same as before. However, those taking only the v_2 path will find that their route now takes 85 units of time, so they are incentivized to take the $start - v_1 - v_2 - end$ path as well. Now, everyone's path takes time $.01(4000) + .01(4000) = 80$ units of time, and anyone taking either of the original paths would require time $45 + .01(4000) = 85$. So, everyone will take this new route and the addition of an extra route has actually made global flow rate worse for everyone.

This section demonstrates the effect Braess' Paradox can have in evacuation situations under selfish-behavior assumptions. Regardless of this theoretical example, it has been shown by Valiant et al. [11] that with high probability, Braess' Paradox will occur in networks and produce similar results to the example shown by Figure 14. It is purely a result of each individual's selfish choices, which is consistent with our assumption of people's behavior in evacuation contexts. Per this discussion, we make a policy recommendation that for any sections of the Louvre that have this middle corridor, such as the B sections which we defined, that emergency personnel be place there. In particular, the emergency personnel should work to route the people out of the middle corridor of any of these sections. Not allowing anyone to take the middle corridor would allow a higher global evacuation flow.

The Residual Network Gives Pathing Recommendations for Emergency Personnel

Another consequence of the Global Flow Network is the fact that by using the max-flow solution over our constructed graph, we can calculate a residual network, defined in [13], in order to find entry pathways for emergency personnel. A residual network is defined such that a path from any exit point (Pyramid, Carrousel Du Louvre, Passage Richelieu, or Porte Des Lions) to any interior section of the museum exists if and only if there exists a path from the supersink to the interior node in the residual graph. Note that if such a path does not exist, emergency personnel will have to force their way totally against the flow of traffic; however, by using a pathway in the residual network, emergency personnel will not obstruct the flow of evacuees out while they enter the building. This highlights another reason for securing the Passage Richelieu; by keeping the edge from the supersink to the pyramid out of the min-cut, i.e. not a bottleneck, we ensure a pathway by which emergency personnel can enter the museum.

As such, we come to more policy recommendations. The first is to prioritize evacuation plans such that there exists an entry pathway in the residual network; in many ways, this can be summarized into preventing the pyramid exit from becoming a bottleneck. The second is to use the residual network to find entry pathways for emergency personnel such that their entrance into the building does not affect the flow of evacuees out of the building.

Summary of Policy Recommendations

We summarize our preceding policy recommendations in the following section, and add a few which are not directly to our results but rather to model structure.

- **Recommendation 1:** Increase access to the pyramid
- **Recommendation 2:** If the emergency situation calls for revealing secret exits to the public, prioritize secret exits in sections surrounding the pyramid provided such exits exist. Further prioritize these exits in particularly pyramid-sensitive emergency situations as well as situations where a large number of emergency personnel is needed.
- **Recommendation 3:** Increase security presence around the Passage Richelieu, and place higher priority on securing its efficacy for evacuation use in order to also secure the safety of the pyramid exit.
- **Recommendation 4:** Use emergency personnel to direct evacuees in a fashion such that their orientation towards exits resembles the linear flow detailed in Figure 7b.
- **Recommendation 5:** Any future secret or public exits constructed should be oriented such that a linear flow is established through the nearest entryway or corridor.
- **Recommendation 6:** Use emergency personnel to direct evacuation flow around the "back" of ground D, i.e. the section highlighted in orange in Figure 12. Alternatively, technology such as a phone app could be easily implemented to assign paths to different individuals or groups, and may in fact be the optimal implementation of this recommendation.
- **Recommendation 7:** Remove corners and complications from sections such as ground D, where complex gallery layouts produce secondary bottlenecks decreasing public safety. Simpler gallery layouts would increase evacuation efficacy dramatically.
- **Recommendation 8:** If the emergency situation calls for revealing secret exits to the public, prioritize a secret exit in ground D provided such an exit exists. Further prioritize this exit in situations where higher throughput is the primary goal.
- **Recommendation 9:** Use emergency personnel to direct evacuees away from middle corridors, such as those detailed in 14.
- **Recommendation 10:** Prioritize evacuation plans such that the residual network of the max-flow solution on our constructed graph has an path into the museum.

- **Recommendation 11:** Use such paths as described above so that emergency personnel can enter the museum without inhibiting evacuee flow outwards.
- **Recommendation 12:** Use technology and appropriate signage to mitigate the effects of language barriers on a diverse population. Any routing phone app such as recommended in Recommendation 6 will necessarily be made multilingual.

Evaluation of the Model

Sensitivity Analysis

Our Global Flow model, in particular, is quite robust. Provided appropriate edge capacities, we use an algorithm proven to find the optimal solution [15]. As such, the model displays sensitivity mostly in the Local Section Model due to the high-variance nature of agent-based models and need for several meaningful hyperparameters to describe human behavior. As a result, our sensitivity analysis involved varying some of the parameters in the NetLogo model to observe its robustness in response to such variance. In particular, we choose to focus on varying the following 4 parameters: repulsion factor, speed-stdev, panic-increment, and ppp (people per patch). Given the base values of the constants with we used for our prior results, we use controlled experiments over a single variable at a time. We then repeated this for each of these 4 parameters, and found results for each of 3 section types: a gallery, a tight corridor, and a corner. The base values for the constants are shown in Table 1(a). For sake of comparison, we also provide the flow capacities we found with our default parameters.

Section Type	Default Constants
repulsion	.35
speed-stdev	.0365
panic-increment	.1
ppp	1

Section Type	Flow Capacity
Gallery	1.8
Corridor	2.58
Corner	2.00

Table 1: Left (a): Default values for computation in the Local Section Model, Right (b): Flow values with defaults parameters

Section Type	Repl	Speed-stdev	Panic-increment	ppp
Gallery	1.14	.98	1.06	2.38
Corridor	2.16	2.45	2.56	4.78
Corner	1.93	1.88	1.99	4.44

Table 2: Summary of results by holding 3 variables at default and changing the other. Values: repulsion factor = 1, speed-stdev = .15, panic-increment = .2, ppp = 2

From the above tables, we see that the NetLogo model sensitivity is dependent on the parameters in terms of raw outputs, as expected. However, the relative ordering across these three types never changes across the experiments, which is what we would expect given our qualitative understanding of the complexity of each layout. Also, none of the results are unreasonable in context of our model, which shows both stability and, importantly, meaningfulness in these parameter choices. A poorly developed computational model with heavily tuned parameters would not be able to simply double or triple some of its parameters without resulting in unreasonable answers.

Increasing repulsion factor increases agent aversion to crowded areas, pushing them away from each other. As a result, fewer people tend to cross the exit line. However, an interesting side effect of such a high repulsion factor is that its performance is similar to the default parameters across the corner. Inspection of this phenomenon reveals that a high repulsion effect actually leads to more optimal agent turning behavior, as shown in Figure 15. More specifically, high repulsion induces agents to use the width of the corner more effectively, in a very rare instance of locally greedy behavior leading to globally optimizing behavior.

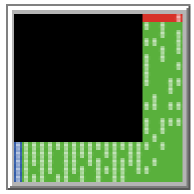 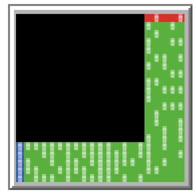

Figure 15: Left (a): Describes the effect of corners on exit flow, which totals 2 evacuees per second. Right (b): Describes a high-repulsion output flow for comparison, which also totals almost 2 evacuees per second despite negative effects of increased repulsion on exit flow in general.

The increase in standard deviation of walking speeds would, a priori, appear to have no effect because of the symmetry in the sampled normal distribution. However, this is not true because these walking speeds of a group are controlled by the slowest moving members in front. As a result, having some people move even slower would cause the people behind them to slow down as well, so our results have reasonable real-world interpretations as well. Panic-increment is also interesting, since it heavily affects the Gallery section but neither corner nor corridor by a large amount. We manually inspected these runs and found that the increase in panic only comes into play because of the complicated layout in the gallery. In the other two, the people can quite easily maintain movement, so the panic does not increase nearly as much, which makes sense in context as well; more qualitatively, we might say that high panic is most important if people are trapped in more complicated, more maze-like surroundings. Lastly, for ppp the observed increase in rate calculation matched our reasonable expectation that more people per patch would indicate a large final exit throughput.

Overall, our sensitivity analysis shows that the model does not behave erratically with regard to large increases in parameters. Not only does this imply the model is stable, but the Louvre management will be able to test a large range of scenarios and plans and still retain the full utility of our model.

Strengths & Weaknesses

Strengths

- Agent based modeling allows us to observe how complex behaviors emerge from a set of simple rules. In addition, NetLogo provides an intuitive interface to change parameters. We also took many steps to ensure physical interpretability so we could map solutions back to the real world.
- Our decision to break up the Louvre into multiple simpler sections allows us to observe and understand agent behavior within specific sections and behavior that leads to bottlenecks.
- In our global flow model, we use a high level of abstraction that makes it easy to integrate results from the local section model to assess the entire system. Also, the use of strong duality allows for a very natural interpretation in terms of global bottlenecks.
- The high level of abstraction naturally lends itself to adaptability. Most test scenarios such as secret or blocked pathways can easily be mapped onto the Global Flow Model and incorporated into the strategy.
- The Local Section Model is quite robust in regards to large changes in the parameters. Additionally, the results of parameter changes have reasonable interpretations in the context of the situation.

Weaknesses

- Our choice to maximize throughput (or max flow), as opposed to time elapsed, yields an evacuation plan that is theoretically great in the worst case, but may not be the quickest in average or more common cases.
- The Local Section Model inherently models both the Louvre layout and agent pathfinding in a discrete context. Given sufficient computational power, a more continuous model would give more realistic results.
- The agent logic in our Local Section Model places a very heavy emphasis on distance from an exit, which causes the majority of people to crowd towards one exit. Only when huge queues of people form behind bottlenecks are some agents able to find alternate exits.
- There is a discrepancy between the optimization of the agents in the Local Section Model and flow optimization of the Global Flow Model. In particular, as the names suggest, agent logic in the Local Section Model priotizes locally optimal, selfish behavior, whereas the Global Flow Model finds a solution that is optimal over the totality of the populace.

Conclusion & Future Work

In our paper, we designed a highly interpretable two-part model that is able to accurately evaluate the efficacy of any particular evacuation plan and the safety risks therein, and therefore can be used to search for an optimal Louvre evacuation plan. This model, divided into an agent-based computational model denoted the Local Section Model and a maximum flow graph computation denoted the Global Flow Model, is also able to identify key bottlenecks in such evacuation plans, giving Louvre staff an opportunity to understand how any given evacuation plan might be improved with respect to museum layout, evacuee path-finding, etc. In particular, we identified the critical bottleneck in our evacuation plan, as we felt that accurately addressing this bottleneck problem was the most critical problem with regards to public safety. Importantly, intelligent use of the Local Section Model also allows Louvre staff to model the effects of a diverse population - including different primary languages, a handicapped population, or large families. Our model is furthermore not restrained to the four given public exits, but can have any combination of exits and obstacles mapped onto the Global Flow Model; in fact, our model can be adapted to any other large building without significant changes in its construction. We furthermore communicated a clear plan of twelve recommendations for the Louvre staff to consider, including an emphasis on securing the pyramid exit in order to ensure an entry path for emergency personnel and to mitigate increased public safety difficulties with respect to the pyramid exit in particular. As such, we identify and provide policy, procedural, and technological recommendations regarding our general evacuation plan.

Future work would including addressing some weaknesses in our model, including developing a stronger and more robust path-finding algorithm to more accurately model exit queueing phenomena as well as providing a higher level of granularity in the NetLogo representations of each floor section of the Louvre.

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[17] https://en.wikipedia.org/wiki/Strong_duality

Appendix A

Edge	Flow Capacity
(S_A, S_B)	7.8
(S_B, F_B)	5
(S_C, F_C)	5.6
(S_D, F_D)	2.6
(S_E, S_D)	7.8
(F_A, F_B)	7.8
(F_B, G_B)	5
(F_C, G_C)	5.6
(F_D, G_D)	2.6
(F_E, F_D)	7.8
(G_A, SS)	4.2
(G_B, G_A)	7.4
(G_C, L_C)	5
(G_C, G_D)	9
(G_C, G_B)	3
(G_D, G_E)	1.8
(G_E, SS)	4.2
(L_A, L_B)	7.8
(L_B, N_P)	3.2
(L_C, N_P)	3.3
(L_D, N_P)	4.8
(L_D, SS)	7
(L_E, L_D)	7.8
(N_P, SS)	5

Table 3: An example of edge weights found through the Local Section Model and inputted into the Global Flow Model. Each floor is given an abbreviation through the following: (Napoleon, N), (Lower Ground, L), (Ground, G), (First, F), (Second, S), and G_A represents the section of ground A. Note that SS is taken to represent the supersink, not a section on the second floor.