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Aircraft seat design based on elastic theory

Abstract

Some airlines are now introducing new "slimline" seats in economy class. However, many passengers showed displeasure with these seats. As a result, a new seat is designed based on human engineering.

In the first part of the problem, the Kinetic equations is established, obtained the reactive force which is given by seat to human under the condition of the person most comfortable way. Based on elastic theory and stress boundary conditions, we get the body surface of normal stress and shear stress. Then, we introduce the fatigue feeling coefficient, establish the optimization model, calculate the seat backplate feature point location, obtained the most comfortable curve for human which is the best seat backplate curve by interpolation. At this point, we obtained five characteristic data and obtained interpolation curve. In this case, **the tiredness of human is minimized and it's most comfortable for human.**

In the second part of the problem, we give a filler material thickness calculation model under random packing materials, and used the elastic rubber as an example to carry on the design. We based on **elasticity theory**, combined with balance equation, physical equations, geometric equations, boundary conditions and stress boundary conditions and displacement single value condition calculated the deformation of the filling material. We believe that when the filler deformation curve and optimum comfort curve most of bonding, the filler thickness design is optimal. In this case, **the filler filling thicker in the waist and neck.**

Furthermore, we **give the comfort test of the seat in vibration environments** by establishing a body vibration model and chair vibration model to calculate the torso and head acceleration response during external vibration excitation.

Thus, we use Proe to portray the three dimensional model for our design and we draw a conclusion of our work.

The seat we designed is applicable to any case, for different human body parameters or material, only need to modify the corresponding parameter in the model we can get the new curve.

Key words: Elastic theory; Feature extraction; Human engineering

Contents

1 Introduction

Some airlines are now introducing new "slimline" seats in economy class. However, many passengers have expressed displeasure with these seats. What tasks should be finished is optimized the seat backplate curve and padding without changing the internal structure of the main in order to make the seat more comfortable. Furthermore, advertising material should be written with 2-3pages for the airline to describe the design features and advantages concisely.

In first of the problem, we need to optimize the seat backplate curve based on human engineering. On this part, the seat backplate curve should make seat overcome the seat vibration force balance and maintain the body posture which is the key to good design. On the second part of the problem, we need to optimize the seat backplate padding and give the advertising material based on the first part of the problem. The relation between fill material deformation curve and seat backplate curve is the key to the solution.

2 The first part of the problem

In this part of the problem, we need to optimize the seat backplate curve. Comfort is a kind of feeling of human. Although people often talk about comfort, there isn't a widely accepted definition of it. It's certain that a seat should overcome the seat vibration force balance and maintain the body posture which is the key to good design. Seat comfort is a flexible index, including two aspects of dynamic comfort and the static comfort. Dynamic comfort refers to the aircraft seats to the human body vibration and shock attenuation ability, and it is mainly associated with the stiffness and damping coefficient of the seat. And static comfort refers to the seat of the static geometry size, surface shape is suitable for human body comfortable sitting posture, satisfies the requirement of human physiological and psychological performance.
Access to relevant information, **some experiential key size of the aircraft seats is:**

Table1 Some key size of the aircraft seats

Name of size	Size value
Height of seat	450mm
Backrest height	760mm
The seat distance from the front seat backrest	420mm
The maximum distance between the front seat back to seat depth	950mm
Seat surface angle	7°
Seat and backrest angle adjustment range	100~125°
Seat width	480mm

Seat design should as far as possibly make passengers spine which is in the right position to keep normal physiological bending. In order to adapt to the human body comfortable sitting position, the body's back and waist should be given a reasonable support. Seat design should provide the shape and location of an appropriate protection. The first support portion of the body located on the place which is the height of the first 5-6 thoracic vertebra in order to support the shoulder. The second support portion of the body located on the lumbar curvature part which is the height of the first 5-6 lumbar spine to support the waist. The main geometric parameters of the seat are: seat height, seat depth, wide seat, seat backrest and so on.

2.1. Assumptions and symbols

2.1.1. *Assumptions*

- During the whole flying people and aircraft has the same motion state;
- During the main flying process, the plane do uniform motion;
- the chest lumbar different shape changes the position of the pelvis.

2.1.2. *Symbols*

Symbols	Meaning
x	x-coordinate of a point from contact surface
y	y-coordinate of the point from contact surface
z	z-coordinate of the point from contact surface
q_1	Force intensity, it's the function of x, y ,z
N	The direction vector of q
m	person of quality
J	The rotational inertia of human
$\vec{a}$	The translational acceleration
S_1	contact area
l, m, n	Cosine between N and x, y, z axis respectively
C_σ	The degree to which normal stress fatigue feeling
C_τ	The degree to which shear stress fatigue feeling
R_i	Different position of fatigue limit
S	Lumbar curve length
P	The total fatigue of human body
θ_T	The spine T's tangent of the angle curve
θ_S	The spine T's tangent of the angle curve
$R(x)$	Radius of curvature
$g_i(x)$	Bounds change of each point from the curve

2.2. Basic force analysis

In the cabin, the seat is not only a place to rest people, but also gives the supportive seats to balance the gravity and friction can make people do variable motion relative to the ground.

When the seat backplate curve optimized, suffered force of man it's uniquely determined. We can draw Free Body Diagram:

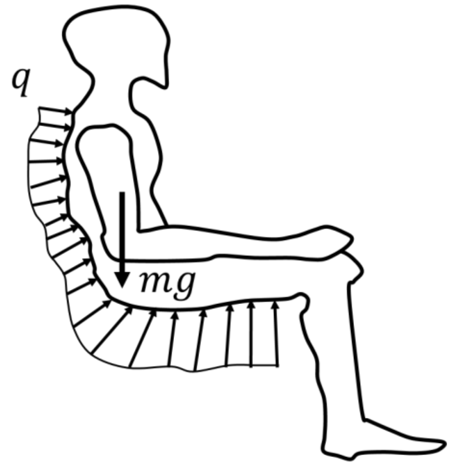

Figure 1 Free Body Diagram of Passenger

People affected by gravity and table of human forces. There are kinetic equations:

(1) Along the x-axis level translation:

$$\int_{S_1} qlds = ma_x \tag{1}$$

(2) Along the y-axis level translation:

$$\int_{S_1} qmds = ma_y \tag{2}$$

(3) Along the z-axis level translation:

$$\int_{S_1} qnds = ma_z \tag{3}$$

(4) Along the x-axis of rotation:

$$\int_{S_1} qny - qmzds = J \propto_x \tag{4}$$

(5) Along the y-axis of rotation:

$$\int_{S_1} qlz - qnxds = J \propto_y \tag{5}$$

(6)Along the z-axis of rotation:\

$$\int_{S} qmx - qlyds = J \propto_z \tag{6}$$

2.3. Curve fitting model

2.3.1. *Model overview*

On the past part, we discuss the basic forcing state, we give the model inference now. Use people's barycenter as the origin of coordinates, to establish coordinates:

Throughout the course of flight, the aircraft translational acceleration and rotational acceleration constant change, the passengers and the aircraft have the same state of motion. For each flight, we can calculate the kinetic parameters as described above, to describe fatigue of the body. Suppose the main process of aircraft flying is uniform motion. We choose the state of equilibrium for the airplane to calculate. So we can know:

$$\begin{cases} a_x = 0 \\ a_y = 0 \\ a_z = 0 \\ \propto_x = 0 \\ \propto_y = 0 \\ \propto_z = 0 \end{cases} \tag{7}$$

After calculating the intensity of q_1, you can obtain normal stress and shear stress suffered by humans. There is stress boundary condition on elastic surface:

$$\begin{cases} l(\sigma_x)_{S_1} + m(\tau_{yx})_{S_1} + n(\tau_{zx})_{S_1} = lq_1 \\ l(\tau_{zx})_{S_1} + m(\sigma_y)_{S_1} + n(\tau_{yx})_{S_1} = mq_1 \\ l(\tau_{yx})_{S_1} + m(\tau_{zx})_{S_1} + n(\sigma_z)_{S_1} = nq_1 \end{cases} \tag{8}$$

Superposition of all stress components, we can get stress σ and τ. σ is the normal stress on the surface of human body, τ is the shear stress on the surface of human body, $C_\sigma$represents a degree of fatigue feelings of normal stress, and C_τ represents a degree of fatigue feelings of the shear stress.

The total body fatigue P:

$$P = \int_S (C_\sigma \sigma + C_\tau \tau) ds \tag{9}$$

People have fatigue endurance limit, beyond a certain level will cause discomfort. Define the fatigue limit $[R_\sigma]$,$[R_\tau]$ of human. For any point x, y, z, fatigue human feelings must be less than the fatigue limit:

$$\begin{cases} C_\sigma \sigma \leq [R_\sigma] \\ C_\tau \tau \leq [R_\tau] \end{cases} \tag{10}$$

In conclusion, the optimization model is established:

$$\min \quad P = \int_S C_\sigma \sigma + C_\tau \tau ds$$

$$s.t. \begin{cases} C_\sigma \sigma \leq [R_\sigma] \\ C_\tau \tau \leq [R_\tau] \end{cases}$$

The model shows that the seat backplate curve design need to fit the shape of the spine. We will discuss it in next section.

2.3.2. *The fitting of spine size*

Understanding of shape of the spine is very important for the designers of airplane seats. Spine consists of three parts: the cervical, thoracic and lumbar spine.

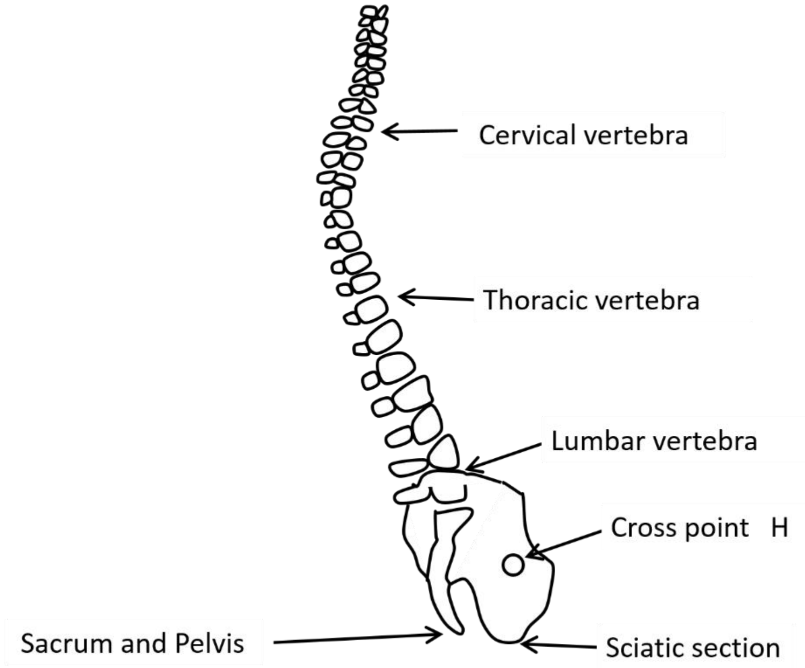

Figure 2 Spine and pelvis

2.3.2.1. *Spine curvature model*

Spine can divide into thoracic vertebra and lumbar. Lumbar spine bending part is defined as the curve spinal which connects T12 and S1. Supposing the lumbar spine can be fitted in the form of a polynomial:

$$x = Az^2 \quad + Bz + C \tag{11}$$

Wherein A, B and C are constants; *z* is the vertical height of the sentinel along the lumbar curve; *x* is the horizontal position of that point. A, B and C are fitted by Participant's sitting position. Lumbar curvature *k* can be determined from the curve at that point:

$$k = \frac{d^2x/dz^2}{[1+(dx/dz)]^{3/2}} = \frac{2A}{[1+\ (2Az+B)^2]^{3/2}} \tag{12}$$

Curvature can be determined by T_{12} to S_1 from every point. The average curvature of the lumbar portion of the data was averaged for each sitting lumbar curvature along the length of the section to decide:

$$K = \frac{\int_{H_1}^{H_2} kdz}{\int_{S_1}^{S_2} ds} \approx \frac{\int_{H_1}^{H_2} kdz}{\int_{S_1}^{S_2} kdz} \tag{13}$$

S is the length of the curve, and ds is the arc length differential. H_1 and H_2 as the beginning and end of the lumbar spine along the *z*-axis.

$$K = \frac{2AH_2+B}{H[1+(2AH_2+B)^2]^{1/2}} - \frac{2AH_2+B}{H[1+(2AH_1+B)^2]^{1/2}} \tag{14}$$

H=H_2-H_1. Any point of the arc radius can be obtained by the reciprocal of curvature(which means R = l/k). In order to standardized data, the unit of curvature represented by rad/SH.

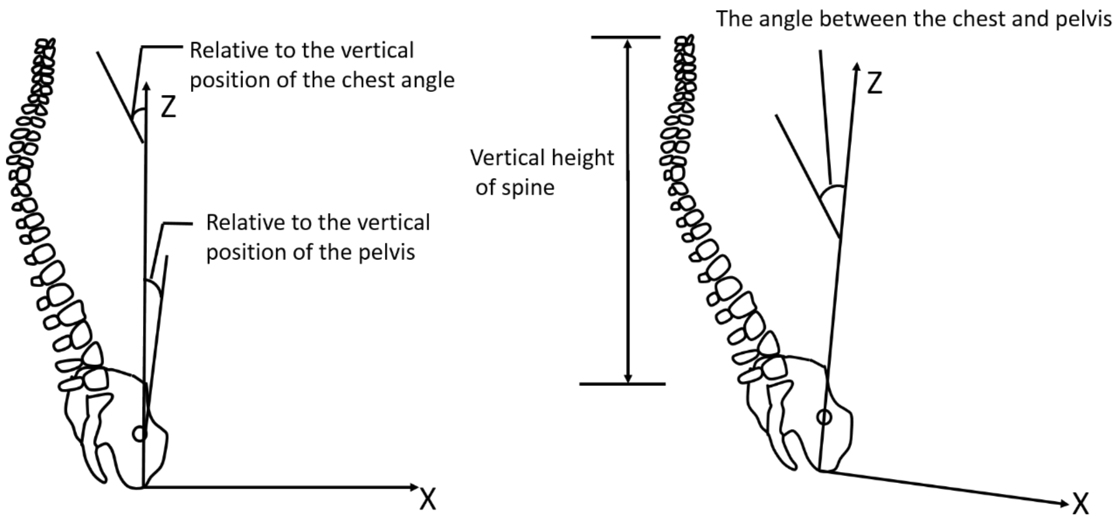

Figure 3 Pelvic coordinate system

2.3.2.2. *Lumbar curvature model*

The basic assumption in this article is: the chest lumbar different shape changes the position of the pelvis. In addition, our study is limited to bend/para, so our model is based on the planar motion. There are three degrees of freedom of planar movement: horizontal, vertical movement and rotation motion. So, here are three linearly independent parameters at most to describe the location of chest and pelvis. Due to the natural connection lumbar spine, the movement of the chest is closely related to direction Angle of the thorax. In addition, about the thoracic and lumbar spine and pelvis additional mechanical function, the change of the individual must be considered.

According to the above discussion, analysis data can be in accordance with the following two ways: in the initial analysis, and we will study, measurement data of every participant in the chest, the relationship between the pelvis and lumbar curvature, then for each participant to generate a model of describing the shape of the lumbar spine. In the second step in the analysis, we will study of the relationship including all individuals.

Supposing the shape of the lumbar spine conforms to the law of quadratic curve. We can calculate variables A and B by θ_(T,) θ_S. Variables A and B are the function of lumbar spine height (H). We have the differential equation: dx/dz = 2Az + B, and we can know:

$$\begin{cases} tan(\theta_S) = \theta(H_1) = 2AH_1 + B \\ tan(\theta_T) = \theta(H_2) = 2AH_2 + B \end{cases} \quad (15)$$

We can solve the A and B:

$$\begin{cases} A = [tan(\theta_T) - tan(\theta_S)]/2H \\ B = [H_2\, tan(\theta_S) - H_1\, tan(\theta_T)]/H \end{cases} \quad (16)$$

Hence K:

$$K = \frac{tan(\theta_T)}{H(tan^2(\theta_T)+1)^{1/2}} - \frac{tan(\theta_S)}{H(tan^2(\theta_S)+1)^{1/2}} \tag{17}$$

Lumbar spine height: $H = H_2 - H_1$

In this way, the mean curvature of the lumbar spine can be caused by of T12 and S1 lumbar curve slope is determined. The above model shows the relationship between the lumbar curvature and chest Angle approximate to be linear. As shown in the figure below:

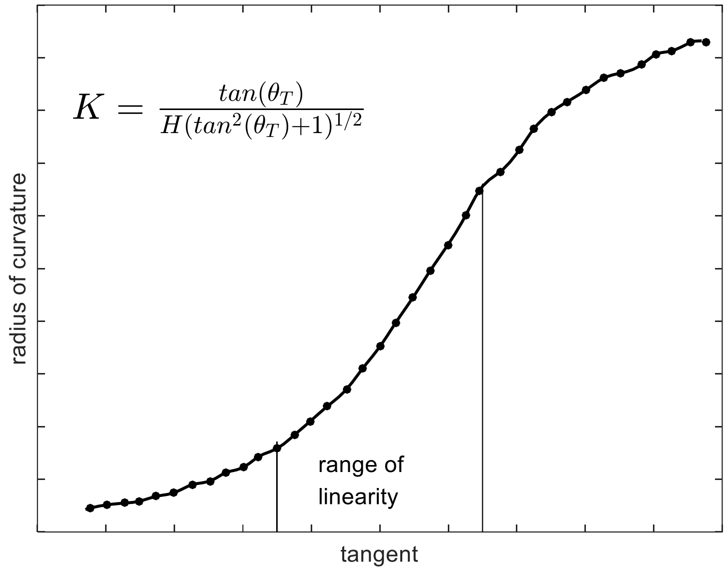

Figure 4 Curvature and tangent

2.3.3. *Curve fitting*

Surface of the model is determined by several seats feature points. Selected seats feature points included: seat reference point ($P_5^{S_1}$), back waist support t($P_4^{S_1}$), the back of a chair shoulder point ($P_3^{S_1}$), the endpoint on the back of a chair ($P_2^{S_1}$), endpoint ($P_1^{S_1}$), under the back of a chair.

Curve at the center of the back of a chair near ground as the origin of coordinates, to establish H-axis using seat forward direction, establish V-axis using the vertical direction. Combined with the above, select seats feature points:

Point	H/mm	V/mm
$P_1^{S_1}$	200s	850
$P_2^{S_1}$	146.12	819.43
$P_3^{S_1}$	78.23	637.13

$P_4^{S_1}$	21.27	322.14
$P_5^{S_1}$	0	0

Based on the feature points, it is necessary to form a smooth curve interpolation. The feature points are based on a chair to the human body main bearing, but not considering the local curve curvature effect on human body comfort.

Known from the analysis of the article in front of the lumbar spine and scapula in the X direction of the curve near 250, 500. And comfortable sitting under these two points in the radius of curvature of the spine of around 350 mm and 1000 mm respectively. The curve curvature radius is:

$$R(x) = \frac{\left[1+s'(x)^2\right]^{3/2}}{|s''(x)|} \tag{18}$$

Take R(250) and R(500) construct the objective function, the maximum meets the human body on the basis of ride comfort of minimizing the curvature radius error so as to establish the objective function of optimization:

Objective function:

$$\min \quad [R(250)-350]^2 + [R(500)-1000]^2 \tag{19}$$

Constraint condition:

$$g_i(x) = |x_i - y_i| < 4 \quad i = 1,2,\dots,\mathrm{n} \tag{20}$$

Join constraints to the upper and lower bounds of the design variables, make curve change which is not too big. The xi is the node value before the interpolation, and yi is the node value after the interpolation.

After interpolation, the curve is:

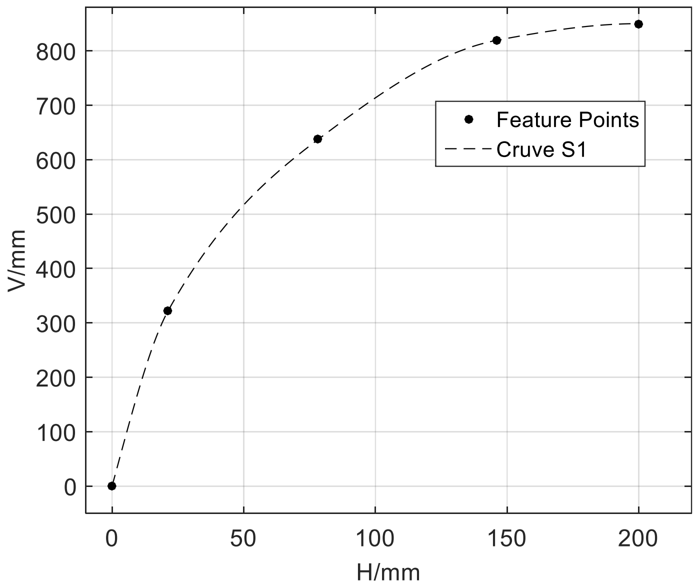

Figure 5 interpolation curve

3 The second part of the problem

In this section, we optimize the seat backplate padding and give the advertising material based on the first part of the problem.

Consider an object A and B still on a level surface, it is assumed to be rigid horizontal deformation does not occur. A uniformly distributed on the object by force q:

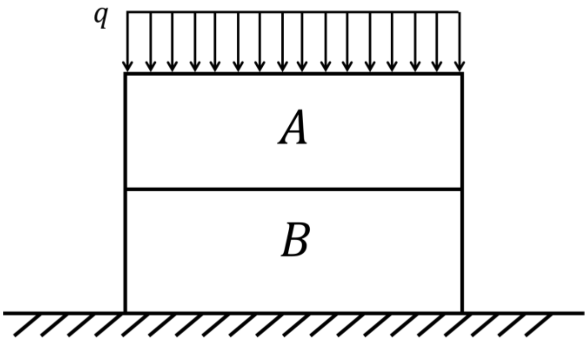

Figure 5 The stress analysis diagram

The strain in the longitudinal direction of the object A:

$$\varepsilon_A = \frac{q}{E_A} \tag{21}$$

Object B strain occurred:

$$\varepsilon_B = \frac{q}{E_B} \tag{22}$$

Because $E_A \ll E_B$, we can know $\varepsilon_B \ll \varepsilon_A$. It means that compared to cushion filler, the deformation of the seat support is negligible. So we consider that the seat supports are not deformed during the analysis.

3.1. Assumptions and symbols

3.1.1. *Assumptions*

- The main process of aircraft flying is uniform motion;
- The seat supports is not deformed;
- The fillings are completely elastic, continuous and isotropic.

3.1.2. *Symbols*

Symbols	Meaning
$\varepsilon.$	strain tensor
σ	stress tensor
ρ	density
E	elastic modulus
μ	the Poisson's ratio
g	acceleration of gravity
f_x、f_y、f_z	physical force
u, v, w	displacement
$[R_x]$，$[R_y], [R_z]$	the allowed displacement
N_x, N_y, N_z	points on the *x*-scale, *y*-scale, *z*-scale.

3.2. The Foundation of Model

Based on the best comfort seat back curve designed in first part of the question to design the thickness of the filler. Using the seat padding as the research object, a free body diagram as shown below:

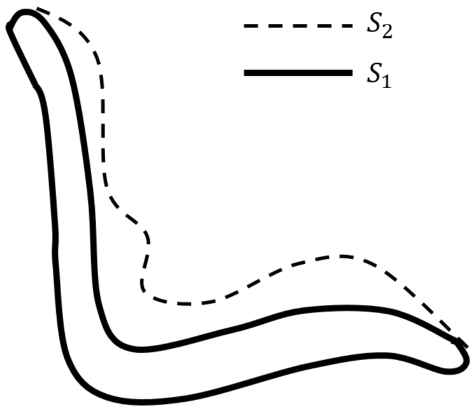

Figure 6-1 The Freebody diagram of fillers on seat's surface

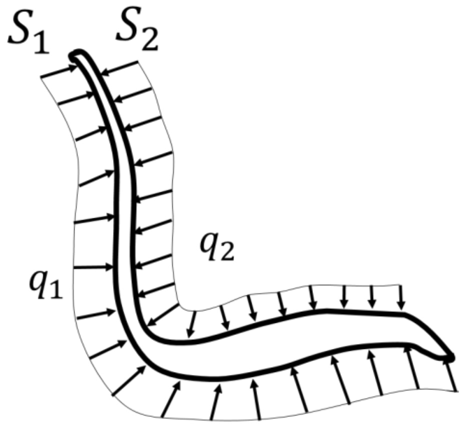

Figure 6-2 Free body diagram of the cushion filler

Taking seat padding as the research object whose lower surface is S_1, on the surface of it suffered an acting force by seat support whose intensity is q_1. Top surface suffered an acting force by human whose intensity is q_2.

Considering the cushion filler is the volume V which are surrounded by cushion support material's table surface S_1 and cushion filler's table surface S_2.

All points in the cushions suit the governing equations:

- Equilibrium equation:

$$\begin{cases} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + f_x = 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + f_y = 0 \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + f_z = 0 \end{cases} \tag{23}$$

σ_x、σ_y、σ_z、τ_{xy}、τ_{xz}、τ_{yx}、τ_{yz}、τ_{zx}、τ_{zy} constitute the stress tensor σ

$$\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \tag{24}$$

f_x、f_y、f_z is the physical force

$$\begin{cases} f_x = 0 \\ f_y = 0 \\ f_z = -\rho g \end{cases} \tag{25}$$

ρ is the density of filling material, g is acceleration of gravity is 9.81m/s^2

- Suit shear stress reciprocal principle:

$$\begin{cases} \tau_{xy} = \tau_{yx} \\ \tau_{yz} = \tau_{zy} \\ \tau_{xz} = \tau_{zx} \end{cases} \tag{26}$$

- Suit geometric equation:

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x} \\ \varepsilon_y = \frac{\partial v}{\partial y} \\ \varepsilon_z = \frac{\partial w}{\partial z} \\ \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\ \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \end{cases} \tag{27}$$

ε_x、ε_y、ε_z、γ_{xy}、γ_{xz}、γ_{yx}、γ_{yz}、γ_{zx}、γ_{zy} constitute the strain tensor ε.

$$\varepsilon = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z \end{bmatrix} \tag{28}$$

u, *v*, *w* is displacement of object along the *x*-axial, *y*-axial, *z*-axial direction.

- Suit physical equation:

$$\begin{cases} \varepsilon_x = \frac{1}{E[\sigma_x - \mu(\sigma_y + \sigma_z)]} \\ \varepsilon_y = \frac{1}{E[\sigma_y - \mu(\sigma_z + \sigma_x)]} \\ \varepsilon_z = \frac{1}{E[\sigma_z - \mu(\sigma_x + \sigma_y)]} \\ \gamma_{yz} = \frac{2(1+\mu)}{E}\tau_{yz} \\ \gamma_{zx} = \frac{2(1+\mu)}{E}\tau_{zx} \\ \gamma_{xy} = \frac{2(1+\mu)}{E}\tau_{xy} \end{cases} \tag{29}$$

E is the material elastic modulus，μ is the Poisson's ratio of material.

- Combined displacement single value condition, you can calculate the displacement filler occurs at any point u, v, w.After being subjected to the pressure on the surface of the filler S_2 is shaped into S_2'.
- Consider boundary conditions：

1) On the boundary S_2', there is stress boundary condition:

$$\begin{cases} l(\sigma_x)_{S_2'} + m(\tau_{yx})_{S_2'} + n(\tau_{zx})_{S_2'} = lq_1 \\ l(\tau_{zx})_{S_2'} + m(\sigma_y)_{S_2'} + n(\tau_{yx})_{S_2'} = mq_1 \\ l(\tau_{yx})_{S_2'} + m(\tau_{zx})_{S_2'} + n(\sigma_z)_{S_2'} = nq_1 \end{cases} \tag{30}$$

2) On the boundary S_1, there is displacement boundary conditions:

$$\begin{cases} (u)_{s_1} = 0 \\ (v)_{s_1} = 0 \\ (w)_{s_1} = 0 \end{cases} \tag{31}$$

$S_1(x, y, z)$ is at the most ergonomically designed, the deformation curve S_2' should fit S_1 as much as possible. Making a translational translation after deformation curve to translation curve S_2''.

$$\begin{bmatrix} x_{S_2''} \\ y_{S_2''} \\ z_{S_2''} \end{bmatrix} = \begin{bmatrix} x_{S_2'} \\ y_{S_2'} \\ z_{S_2'} \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \tag{32}$$

$[x_0 \quad y_0 \quad z_0]^T$ is the translation matrix.

Gridding the entire surface discrete as N_x, N_y, N_z points on the x-scale, y-scale, z-scale.

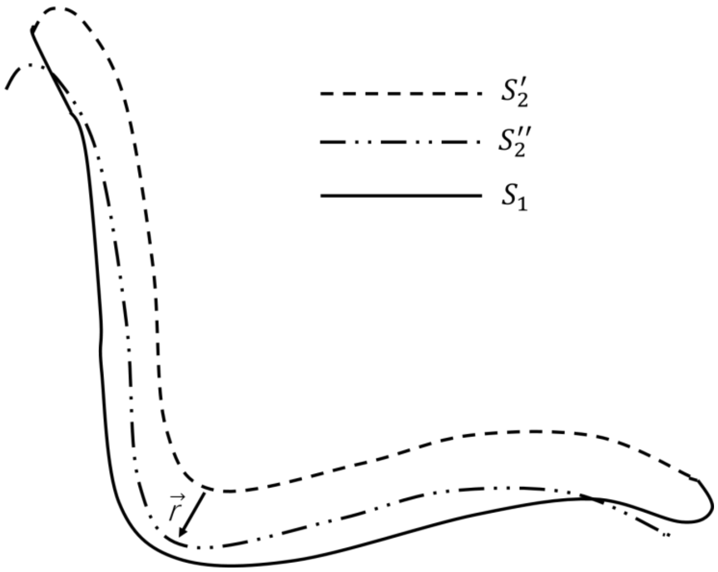

Figure 7 The schematic diagram of seat's deformatio

The deformation schematic diagram between surface deformation and the seat backplate curve

- Fitting level for curved surface between S_2'' and S_1:

$$M = \sum_1^{N_x} \sum_1^{N_y} \sum_1^{N_z} (x_{S_2''} - x_{S_1})^2 + (y_{S_2''} - y_{S_1})^2 + (z_{S_2''} - z_{S_1})^2 \tag{33}$$

- Local difference should not too large:

$$\begin{cases} \left| x_{S_2''} - x_{S_1} \right| \le [R_x] \\ \left| y_{S_2''} - y_{S_1} \right| \le [R_y] \\ \left| z_{S_2''} - z_{S_1} \right| \le [R_z] \end{cases} \tag{34}$$

Where, $[R_x], [R_y], [R_z]$ is the allowed displacement along x, y, z axis.

Above all, the optimization model is:

$$\min \quad M = \sum_1^{N_x} \sum_1^{N_y} \sum_1^{N_z} (x_{S_2''} - x_{S_1})^2 + (y_{S_2''} - y_{S_1})^2 + (z_{S_2''} - z_{S_1})^2$$

$$s.t. \begin{cases} \left| x_{S_2''} - x_{S_1} \right| \le [R_x] \\ \left| y_{S_2''} - y_{S_1} \right| \le [R_y] \\ \left| z_{S_2''} - z_{S_1} \right| \le [R_z] \end{cases}$$

3.3. Solution and Result

Using the elastic rubber as an example. Degenerate the above model into a two-dimensional space. Curve at the center of the back of a chair near ground as the origin of coordinates, to establish H-axis using seat forward direction, establish V-axis using the vertical direction.

Point	H/mm	V/mm
$P_1^{S_2}$	200	850
$P_2^{S_2}$	204.29	883.00
$P_3^{S_2}$	141.30	898.50
$P_4^{S_2}$	51.17	865.55
$P_5^{S_2}$	23.92	752.79
$P_6^{S_2}$	0	557.97
$P_7^{S_2}$	-88.98	340.80
$P_8^{S_2}$	-107.88	137.38
$P_9^{S_2}$	-60.71	0

After feature point interpolation processing, we can get the path line. Based on the elastic theory, when translation vector

$$r = [-49.32 \quad 9.71]^T$$

There are highest degree in joint between S_2' and S_1. Path line as shown in the figure below:

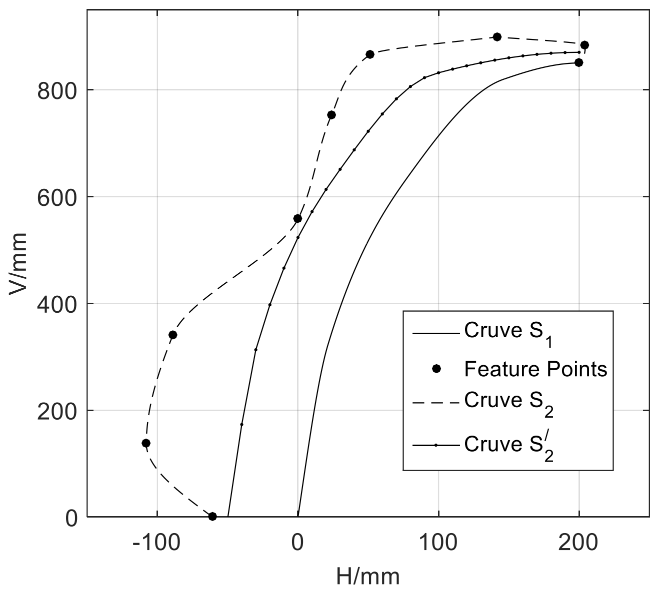

Figure 7 Path line

4 Human body comfort test in vibration environment

4.1. Symbols

Symbols	Meaning	Numerical
m_h	Head mass	5.31kg
k_h	Neck equivalent stiffness	310KN/m
c_h	Neck equivalent damping	0.4kN·s/m
m_u	Upper trunk mass	28.49kg
k_u	The equivalent stiffness of the upper and lower trunk	183 kN·s/m
c_u	The equivalent damping of the upper and lower trunk	4.75 kN·s/m
m_i	Lower trunk mass	8.62kg
k_i	The equivalent stiffness of the lower trunk and hip	162.8KN/m
c_i	The equivalent damping of the lower trunk and hip	4.585 N·s/m
m_t	Hip and thigh quality mass	12.78kg
k_t	Hip stiffness	90 KN/m
c_t	Hip damping	2.064 kN·s/m
m_c	Cushion mass	0kg
k_c	Cushion stiffness	8 KN/m
c_c	Cushion damping	0.357kN·s/m
m_s	Seat mass	40kN·s/m
k_s	Seat stiffness	34N/m
c_s	Seat damping	0.25 kN·s/m

4.2. The Foundation of Model

In practice, human body—seat system in the process of flight of the aircraft to vibrate. We build a human bodyseat motion—model.

(1) Human vibration model:

The human body is divided into 4 mass blocks：

1) Head mass block， mainly including head and neck;
2) Upper trunk mass, Including arms, shoulders and chest;
3) Lower trunk mass, containing all the organs of the upper torso and hips;
4) Hip mass, Including hip and thigh;4 mass blocks are connected by a stiffness damping system.

(2) Seat vibration model:

In the model, the quality of the mass is concentrated in the quality of the seat, and the mass of the seat filler can be neglected compared to the quality of the seat support，so no quality module. Through the stiffness damping system and the seat mass block is connected, the seat mass block is connected with the body through the spring and damping system.

Above all, the six degree of freedom human body seat model was founded. As shown in figure:

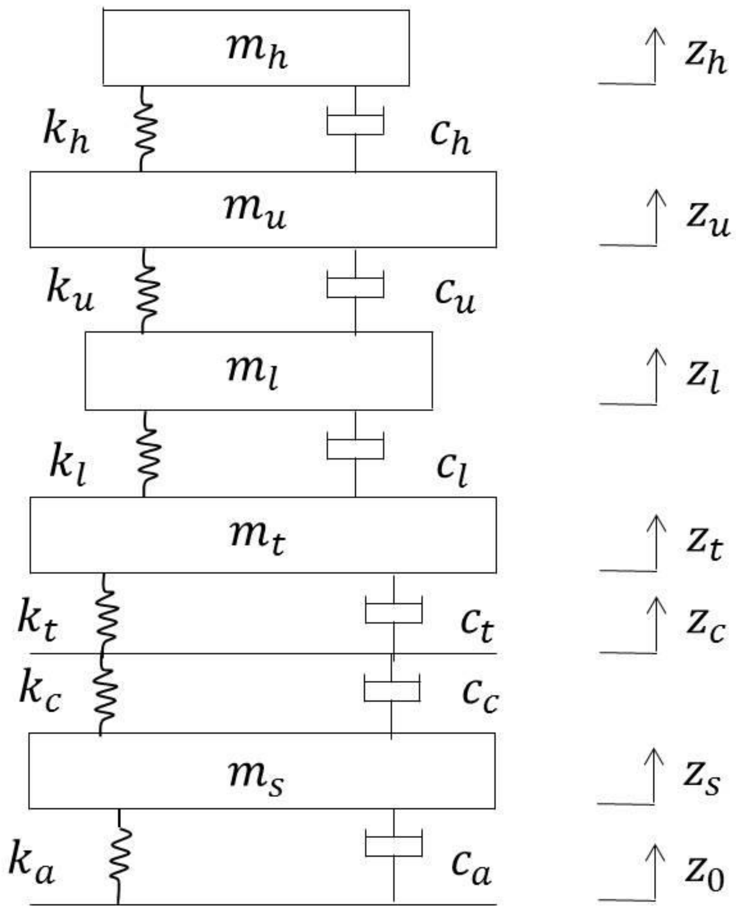

Figure 8 six-degree of freedom human body seat system vibration model

The kinetic equations of the figure is:

$$\begin{cases} m_s \frac{d^2 z_s}{dt^2} = k_s(z_0 - z_s) + c_s\left(\frac{dz_0}{dt} - \frac{dz_s}{dt}\right) - k_c(z_s - z_c) + c_c\left(\frac{dz_s}{dt} - \frac{dz_c}{dt}\right) \\ k_c(z_s - z_c) + c_c\left(\frac{dz_s}{dt} - \frac{dz_c}{dt}\right) - k_t(z_c - z_t) + c_t\left(\frac{dz_c}{dt} - \frac{dz_t}{dt}\right) = 0 \\ m_t \frac{d^2 z_t}{dt^2} = k_t(z_c - z_t) + c_t\left(\frac{dz_c}{dt} - \frac{dz_t}{dt}\right) - k_l(z_t - z_l) + c_l\left(\frac{dz_t}{dt} - \frac{dz_l}{dt}\right) \\ m_l \frac{d^2 z_l}{dt^2} = k_l(z_t - z_l) + c_l\left(\frac{dz_t}{dt} - \frac{dz_l}{dt}\right) - k_u(z_l - z_u) + c_u\left(\frac{dz_l}{dt} - \frac{dz_u}{dt}\right) \\ m_u \frac{d^2 z_u}{dt^2} = k_u(z_l - z_u) + c_u\left(\frac{dz_l}{dt} - \frac{dz_u}{dt}\right) - k_h(z_u - z_h) + c_h\left(\frac{dz_u}{dt} - \frac{dz_h}{dt}\right) \\ m_h \frac{d^2 z_h}{dt^2} = k_h(z_u - z_h) + c_h\left(\frac{dz_u}{dt} - \frac{dz_h}{dt}\right) \end{cases} \tag{35}$$

4.3. Solution and Result

Using MATLAB Simulink Modular to simulate. Taking into account the body's sense organs are mainly concentrated in the head and trunk, the time curves for the head and trunk acceleration are shown below.

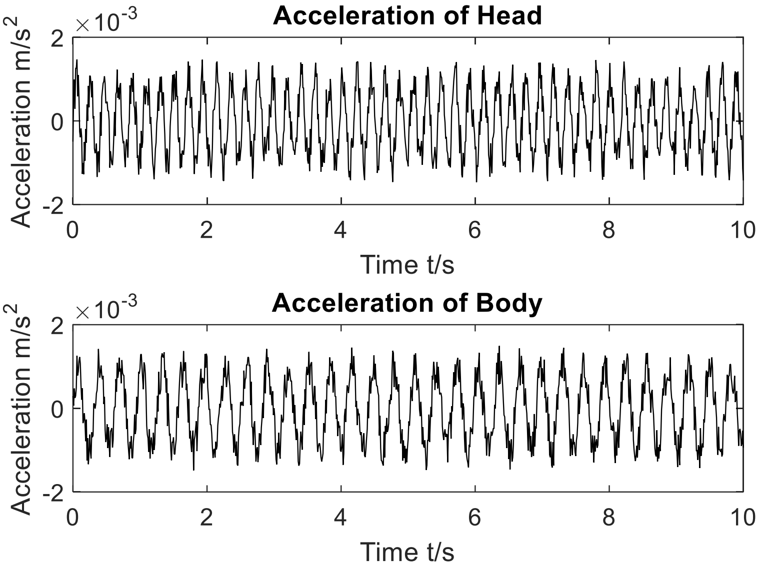

Figure 7 The time curves for the head and trunk acceleration

The simulation is carried out with a small vibration acceleration. People are very difficult to feel, so it can be neglected. People sitting with good comfort, the design of the seat is pretty reasonable.

5 Conclusion

In conclusion, we use Proe to portray the three dimensional model for our design:

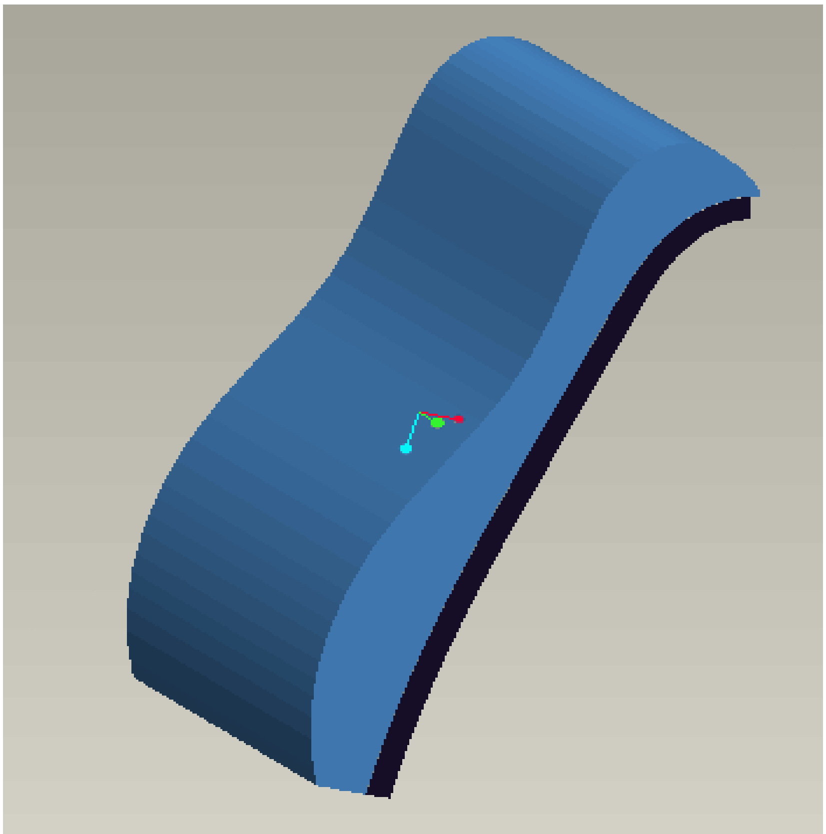

Figure 8 Three dimensional picture of seat back

Our model is not only considered the plane flying at a uniform velocity, are also considered in the vibration environment. Our algorithm is based on theory of mechanics, elastic mechanics, mechanics of materials, design the seat backplate curve and filling's curve by interpolation. For without loss of generality, we give the comfort test of the seat in vibration environments by establishing a body vibration model and chair vibration model to calculate the torso and head acceleration response during external vibration excitation. The seat we designed is applicable to any case, for different human body parameters or material, only need to modify the corresponding parameter in the model we can get the new curve.

6 Future work

- **Integral structure design**

Our work is based on the dynamic mechanics, material mechanics, elastic mechanics and vibration and the size of the human body vertebra, comfortable experience for standard, to optimize the seat backplate curve. But the comfort of the chair not only relate shapes and filler material, also related to the cushion of the chair. In the future, we can do for the whole chair structure design, enhance the chair from the perspective of the overall comfort levels.

- **Three dimensional modeling**

In order to facilitate, the calculation process only for chair's center line curve. However, the chair should not as a cylinder in practice, has a tendency to toward the center sag. As a result of it, we need to make three dimensional modeling.

- **Sad Computer Aided Engineering (CAE) and complex chair motion state**

In the process of movement of the plane, the plane takes off, landing, steering the passenger comfort when experience is also important. We can use the finite element calculation software, such as AbaqusCAE, ANSYS to do numerical calculation in all kinds of load conditions.

- **More accord with human body engineering**

In order to make the chair design more reasonable, need combine with more parameters of the human body such as the neurons at different location of the distribution of human body, the degree of muscle electric induction, fatigue, etc.

- **Human body feeling test**

Practice is the sole criterion for testing truth. The design of the chair should be combined user's experience. The sample of produce should be supplied to users to obtain the opinions and suggestions to the chair.

7 The advertising material

In order to promote the seat, we design the poster is as follows：

correct side:

opposite side:

8 References

[1]D.A. Wahab, N.F. Adull Manan,M.A. Hannan,S. Abdullah ,A. Hussain. Designing for Comfort and Reliability in an Intelligent Car Seat[J]. American Journal of Applied Sciences,2008,512:.

[2]Andi R. Wijaya,Peter Jönsson,Örjan Johansson. The Effect of Seat Design on Vibration Comfort[J]. International Journal of Occupational Safety and Ergonomics,2003,92:.

[3]Bazil Basri,Michael J. Griffin. The application of SEAT values for predicting how compliant seats with backrests influence vibration discomfort[J]. Applied Ergonomics,2014,:.

[4]D. H. Laananen,K. L. Winkelman. Analysis of energy-absorbing seat configurations for aircraft[J]. International Journal of Crashworthiness,1996,14:.

[5]Gulavani, Omkar,Hughes, Kevin,Vignjevic, Rade. Explicit dynamic formulation to demonstrate compliance against quasi-static aircraft seat certification loads (CS25.561) - Part I: influence of time and mass scaling[J]. Proceedings of the Institution of Mechanical Engineers,2014,22811:.

9 Appendix

9.1 Backrest curve generated Matlab code:

```
clc
clear
x=[200,146.12,78.23,21.27,0];
y=[850,819.43,637.13,322.14,0];
plot(x,y,'k.','MarkerSize',20);
box=-10;
boy=-50;
% axis equal
axis([box,220,boy,880]);
hold on
xi=0:1:200;
yi=interp1(x,y,xi,'pitch');
plot(xi,yi,'k--');
grid on
legend('Feature Points','Cruve S1','location','best','FontSize',16);
xlabel('H/mm','FontSize',16);
ylabel('V/mm','FontSize',16);
```

9.2 Material filling curve generated Matlab code:

```
clc
clear
close all
x1=[200,146.12,78.23,21.27,0];x1=fliplr(x1);
y1=[850,819.43,637.13,322.14,0];y1=fliplr(y1);
x2=[200,204.29,141.30,51.17,23.92,0,-88.98,-107.88,-60.71];x2=fliplr(x2);
y2=[850,883.00,898.50,865.55,752.79,557.97,340.80,137.38,0];y2=fliplr(y2);
x1i=0:1:200;y1i=interp1(x1,y1,x1i,'pitch');
plot(x1i,y1i,'k-');
hold on
plot(x2,y2,'k.','MarkerSize',20);
x21=x2(1:7);y21=y2(1:7);
y21i=0:1:898;x21i=interp1(y21,x21,y21i,'pchip');
% plot(x21i,y21i);
x22=x2(7:9);y22=y2(7:9);
y22i=898:-1:850;x22i=interp1(y22,x22,y22i,'pchip');
% plot(x22i,y22i);
x2i=[x21i,x22i];
y2i=[y21i,y22i];
plot(x2i,y2i,'k--');
grid on
```

```
axis([-150,250,0,950])
xlabel('H/mm','FontSize',16);
ylabel('V/mm','FontSize',16);
x3=x1(1:4);x3=x3-49.3;
y3=y1(1:4);y3=y3+10;
x3=[x3,x1(end)];y3=[y3,y1(end)+20];
x3i=-50:10:200;y3i=interp1(x3,y3,x3i,'pitch');
plot(x3i,y3i,'k.-');
legend('CruveS_1','FeaturePoints','CruveS_2','CruveS^/_2','location','best');
```