A Close Look On Leaves
细看树叶

Bo Zhang
Yi Zhang
Tiankun Lu

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Abstract

We construct four models to study leaf classification, relationships between leaf shape and leaf distribution, correlations between leaf shape and tree profile, and total leaf mass of a tree.

Model 1 deals with the classification of leaves. We focus primarily on the most conspicuous characteristic of leaves, namely, shape. We create seven geometric parameters to quantify the shape. Then we select six common types of leaves to construct a database. By calculating the deviation index of the parameters of a sample leaf from those of typical leaves, we can classify the leaf. To illustrate this classification process, we use a maple leaf as a test case.

Model 2 studies the relationship between leaf shape and leaf distribution. First, we simplify a tree into an idealized model and then introduce the concept of solar altitude. By analyzing the overlapping individual shadows through considering the relationship between leaf length and internode length under different solar altitudes, we find that the leaf shape and distribution are optimized to maximize sunlight exposure according to the solar altitude. We apply the model to three test types of trees.

Model 3 discusses the possible association between tree profile and leaf shape. Based on the similarity between the leaf veins and branch structure of trees, we propose that leaf shape is a two-dimensional mimic of the tree profile. Employing the method of Model 1, we set several parameters reflecting the general shape of each tree and compare them with those of its leaves. With the help of statistical tools, we demonstrate a rough association between tree profile and leaf shape.

Model 4 estimates the total leaf mass of a tree given size characteristics. Carbon dioxide (CO2) sequestration rate and tree age are introduced to establish the link between leaf mass and tree size. Since a unit mass of a leaf sequesters CO2 at a constant rate, the CO2 sequestration rate has a quadratic relationship with the age of the tree, and the size the tree experiences logistic growth.

我们构建了四个模型来分别研究叶子的分类，叶子形状与叶子分布之间的关系，叶子形状与树形轮廓之间的相关性以及一棵树的树叶总质量。

模型 1 处理叶子的分类。我们主要关注叶子最显着的特征，即形状。我们构造了七个几何参数来量化形状。然后我们选择了六种常见的叶子来构建一个数据库。通过计算样本叶片与典型叶片的参数偏差指数，我们可以对叶子进行分类。为了说明这个分类过程，我们使用枫叶作为测试用例。

模型 2 研究了叶子形状和叶子分布之间的关系。首先，我们将一棵树简化成一个理想模型，然后介绍太阳高度的概念。通过考虑不同太阳高度下叶片长度和节间长度的关系，分析重叠的叶片阴影，我们发现叶片形状和分布被优化，以根据太阳高度最大化日照。我们将模型应用于三种测试类型的树木。

模型 3 讨论了树木轮廓和叶子形状之间可能的关联。根据叶脉与树枝结构的相似性，我们提出叶片形状是树形轮廓的二维近似。采用模型 1 的方法，我们设置了几个参数来刻化每棵树的大概形状，并将它们与其叶子的参数进行比较。在统计工具的帮助下，我们展示了树形轮廓和叶形之间的粗略关联。

模型 4 对给定尺寸特征的树，估计了其树叶总质量。引入二氧化碳固碳率和树龄来建立树叶质量和树木大小之间的联系。由于单位质量的叶片以一个恒等的速率固碳，二氧化碳固碳率与树龄之间是一个二次关系，并且树的大小呈现 logistic 增长。

Contents

1 Introduction / 引言

Leaf, which is responsible for photosynthesis and storage of food and water, is a very important organ of a plant. Leaf has so many different and interesting shapes and contours, which make it so catching and conspicuous in the plant kingdom. For centuries, people have always wondered why the leaf has so many splendid shapes. Is this just a gift from the Almighty God? Or is it the adjustment the leaf takes during its evolution and adaptation?

叶是植物非常重要的器官，它承担着光合作用，并为植物贮存水和养份。叶子具有许多不同而有趣的形状轮廓，这也让它在大自然的植物王国中格外引人注目。几个世纪以来，人们总是想知道为什么叶子有这么多精美的形状。难道这只是全能的上帝带来的礼物？还是叶子在进化和适应过程中所做调整的结果？

Another mystery of the leaf lies in the classification of different leaf types. Can we do so in a more reliable and scientific way instead of judging subjectively? We have tried our best to build a model on a quantitative base to increase the accuracy and efficiency of classifying different leaves.

叶子的另一个不解之谜在于它繁多种类的区分。我们能不能用一种更加科学可靠的方法来对树叶进行分类，而不是凭借主观判断？因此我们在数值的定量基础上建立了模型，来提高树叶分类的准确率和有效性.

The final mystery about the tree is how much the total leaf mass on a tree is. Just like people always want to figure out how much hair a person has, human beings always have a fancy to find out a solution from the seemingly infinity and incalculable.

关于树叶还有一项终极难题：如何确定一棵树上树叶的总质量。就像人们总是想弄清楚一个人有多少头发一样，人们总是喜欢从看似无穷无尽的事物中寻找答案.

We are deeply convinced that the true glamour of scientific and mathematical models is to unveil the intriguing and the changeable of the nature. Aiming to do so, we will generate models that can fit large quantities of data with highest accuracy within a quick flick.

我们一直深信：科学和数学模型的魔力在于解开大自然错综复杂和变幻莫测的面纱。以此为目标，我们将建立能适应大量数据的模型，并且以最高的准确率在弹指之间解决问题。

2 Breaking Down the Problem / 问题的分解

After carefully analyzing the problem, we conclude four main sub-problems to tackle in our paper:

经过对这个问题的仔细分析，我们归纳出本文需要解决的四个主要子问题：

1. Classification the different types of leaves.
2. Relationship between the leaf distribution and leaf shape.
3. Relationship between the tree profile and leaf shape.
4. Calculation of the total leaf mass on a tree.

1. 对不同树叶进行分类；
2. 树叶分布与叶形二者间的联系；
3. 树的轮廓与叶形二者间的联系；
4. 计算一棵树上的树叶总质量。

To tackle the first problem, we set a set of parameters to quantify the characters of the leaf shape and use the leaf shape as the main standard for our classification process.

处理第一个问题时, 我们设置了一组参数来量化叶形特征, 从而将叶形作为树叶分类过程的主要参照标准.

As for the second question, we want to use the overlapping area that one leaf casts on the leaf directly under it as a medium to associate the leaf distribution and leaf shape, since the leaf shape will affect the overlapping and we assume the leaf distribution will try to minimize the overlapping area.

对于第二个问题，我们希望用一片树叶遮挡它正下方叶子的重叠面积来作为一个中间量来把树叶分布和叶形联系起来，这是因为叶形会影响树叶间的重叠，同时我们假设树叶分布有令重叠面积最小的趋向。

As for the third question, we want to refer to the process we take when tackling the first problem and also set some parameters for the tree profile. After that, we will compare their parameters and judge whether there is a relation between tree profile and leaf shape.

对于第三个问题，我们希望能够参考第一个问题的解决过程，同样设置一些描述树形的参数。然后，我们将对比叶形、树形的两组参数，并判断树形轮廓和叶形之间是否存在着联系。

To deal with the total mass of the leaves, we want to use the age to link the size of tree and the total weight of leaves of it because the tree size has an obvious relationship with its age and the age will affect a tree's sequestration of carbon dioxide, which will reflect the weight of a tree's total leaves.

为了计算树上树叶的总质量, 我们用树龄把树的尺寸大小和树叶总质量连结起来，因为树的尺寸和树龄有明显的关系，而树龄将反映了树的二氧化碳积累（净吸收）量，即而反映出它的树叶总质量。

3 Assumptions / 假设

1. The trees being studied are all individual ("open grown") trees, such as trees typically planted along streets, in yards, and in parks. Our calculation does not apply to densely raised trees, as in typical reforestation projects where large numbers of trees are planted closely together.

2. Assume the shape of the leaves does not reflect special uses for the trees, such as to resist extremely windy, cold, parched, wet or dry conditions or to catch food.

3. Assume the type of the leaf distribution to be discussed (leaf length and internode distance relation) is only a reflection of the tree's natural tendency to sunlight.

4. Assume the tree profile we consider is the part above ground, including the

1. 所分析的树木均为独立的（“开阔成型的 open grown”）树, 最典型的就像我们街道边、庭院和公园里种植的树木. 我们所做的计算不适用于密集分布的树, 例如再造林工程: 把大量树密集地种植在一块区域内.

2. 假设树叶的形状与树的特殊应用无关, 即不考虑叶形抵抗大风、严寒、炎热、潮湿或潮湿的环境, 抑或为了捕食.

3. 假设将要讨论的树叶分布的类型 (叶长及节间距离关系) 仅反映为树木对阳光的自然倾向.

4. 假设我们考虑的树形是地面上的部分, 包括树干、树枝和树叶.

trunk, the branches and leaves.

5. Assume all parts of leaf lay on a flat surface and the thickness or protrusion of veins are neglectable.

6. Assume leaves are the only part of the tree that reacts in photosynthesis and respiration so that the carbon dioxide sequestration of a tree is the sum of the sequestration of the leaves.

7. Assume the sequestration of a tree or a leaf is the net amount of CO2 fixed in a tree, which is the difference between the CO2 released in respiration and the CO2 absorbed in photosynthesis.

8. Assume the trees are in healthy, mature and stable condition. The trees of the same species have same characteristics.

5. 假设叶的所有部分展在同一平面上, 即忽略厚度及叶脉的突出部分.

6. 假设叶是整棵树中唯一进行光合作用和呼吸作用的部位, 进而树的二氧化碳积累量就等于所有树叶积累的总和.

7. 假设树/叶的二氧化碳积累量是“固定”在该树上的二氧化碳净余量, 即意味着光合作用中吸收的吸收量与呼吸作用的释放量之差.

8. 假设树木均处于健康、成熟和稳定的状态下. 同种树木有着相同的特征.

4 Nomenclatures / 术语

Symbol	Meaning	表示
R	Rectangularity	矩度
A_{leaf}	the area of leaf	叶子的面积
$A_{\text{rectangle}}$	the area of minimum bounding rectangle	最小外接矩形的面积
AR	the aspect ratio	宽高比
L_{short}	the length of shorter side	(最小外接矩形) 较短一边的边长
L_{long}	the length of longer side	(最小外接矩形) 较长一边的边长
C	the circularity	圆度
R_{in}	the radius of in-circle	内接圆半径
R_{ex}	the radius of ex-circle	外接圆半径
FF	Form factor	形状因子
P_{leaf}	the perimeter of leaf	树叶的周长
$ERAI$	the edge regularity area index	边缘面积平滑指数
BPA	The bounding polygon area	外接多边形面积
$ERPI$	the edge regularity perimeter index	边缘周长平滑指数
BPP	the bounding polygon perimeter	外接多边形周长
PI_i	Proportional index	比例因子
I_D	the Index of Deviation	偏差指数
L_{major}	the length of major axis	菱形长轴长
$A_{\text{overlapping}}$	the overlapping area	重叠面积
α	the solar altitude	太阳高度角
M_{leaf}	the total mass of leaves on a tree	树叶的总质量
$M_{\text{carbon dioxide}}$	mass of carbon dioxide sequestered (lbs)	二氧化碳积累量
A_s	ability to sequester carbon dioxide (lbs/g)	二氧化碳积累能力
A	the age of the tree	树龄

5 Model One: Leaf Classification / 模型一：叶子的分类

5.1 Decisive Parameters / 决定性参数

In order to classify the shapes of the given leaf, we want to set a number of

为了将任意一片树叶按形状分到已知的某类当中，我们

parameters and establish a database for comparison. After carefully and thoroughly analyzing the leaves, we develop seven most significant parameters as shown below:

1. **Rectangularity**
   Firstly, we define the ratio of the area of the leaf to the Minimum Bounding Rectangle as the leaf's Rectangularity, how much does the leaf resemble a rectangle (refer to figure 1). The maximum possible value of this parameter is 1.
   $$R = \frac{A_{\text{leaf}}}{A_{\text{rectangle}}}$$
   A_{leaf} stands for the area of leaf, $A_{\text{rectangle}}$ stands for the area of minimum bounding rectangle.

2. **Aspect ratio**
   After defining the rectangularity, now we define the Aspect Ratio, which describes the proportional relationship between the width of and height of a leaf's Minimum Bounding Rectangle, as another key character to classify general shape of a leaf. The bigger this ratio is, the more this leaf resembles a square (refer to figure 1). The maximum possible value of this parameter is 1.
   $$AR = \frac{L_{\text{short}}}{L_{\text{long}}}$$
   Where AR stands for the aspect ratio, L_{short} stands for the length of shorter side, L_{long} stands for the length of longer side.

3. **Circularity**
   To evaluate how roundish a leaf is, we consider that the respective radius of in-circle and ex-circle. The ratio of the former to the latter, which we define as Circularity, may well reflect this characteristic. The greater the ratio of Circularity is, the closer the leaf is to a circle, (refer to figure 2) The of this parameter is 1.
   $$C = \frac{R_{\text{in}}}{R_{\text{ex}}}$$

希望设置一组参数，继而建立数据集用以对比．在仔细完整地分析了树叶后，我们归纳出 7 个最重要的参数，如下：

1. **矩度 (Rectangularity)**
   首先，我们将树叶面积与它的最小外接矩形面积定义为该树叶的矩度，就是说这片树叶的形状跟矩形有多相似 (参看图1)．这一参数的最大理论取值为 1.
   $$R = \frac{A_{\text{leaf}}}{A_{\text{rectangle}}}$$
   A_{leaf} 代表树叶的面积，$A_{\text{rectangle}}$ 代表其最小外接矩形的面积．

2. **宽高比 (Aspect ratio)**
   在定义了矩度之后，现在我们定义宽高比来描述一片树叶最小外接矩形宽、高的比例关系，作为另一个对叶形一般分类的关键特征．宽高比越大，树叶就越像正方形 (参看图1)．这一参数的最大取值也是 1.
   $$AR = \frac{L_{\text{short}}}{L_{\text{long}}}$$
   其中 AR 表示宽高比，L_{short} 表示 (最小外接矩形) 较短一边的边长，L_{long} 表示 (最小外接矩形) 较长一边的边长．

3. **圆度 (Circularity)**
   为了评价树叶有多圆，我们将考虑树叶内、外接圆半径的相对值．定义前者 (内接圆) 与后者 (外接圆) 半径之比为树叶的圆度，来反映原型程度这一特征．圆度越大，表明叶形越接近圆形 (参看图2)；同样此参数最大取值为 1.
   $$C = \frac{R_{\text{in}}}{R_{\text{ex}}}$$

Where C stands for the circularity, R_{in} stands for the radius of in-circle , R_{ex} stands for the radius of ex-circle.

4. **Form factor**
Form Factor, a famous shape description parameter, is another essential indicator of leaf classification. The maximum possible value of this parameter is 1.
$$FF = \frac{4\pi A_{\text{leaf}}}{P_{\text{leaf}}^2}$$
Where P_{leaf} stands for the perimeter of leaf.

5. **Edge regularity area index**
Although the aspect ratio and the rectangularity of two leaves may resemble, the contour or the exact shape of two leaves may vary much.

Thus, In order to take the different contour of the leaf into consideration, we join every convex dot along the contour and develop a specific parameter, which we call Bounding Polygon Area. The ratio between the leaf area and this **bounding polygon area** is a good quantitative factor, defined as **Edge Regularity Area Index**.

The more this ratio is close to 1, the less jagged and the smoother this leaf's contour is (refer to figure 3). The maximum possible value of this parameter is 1.
$$ERAI = \frac{A_{\text{leaf}}}{BPA}$$
Where $ERAI$ stands for the edge regularity area index, BPA stands for bounding polygon area.

6. **Edge regularity perimeter index**
Similarly, we develop another parameter: **Bounding Polygon Perimeter**, the perimeter of the polygon when we join the convex dots of a leaf. We define the ratio of the convex dot perimeter and the perimeter of the leaf as **Edge Regularity Perimeter Index**. This time, the smaller this ratio is,

其中 C 表示圆度, R_{in} 代表内接圆半径, R_{ex} 代表外接圆半径.

4. **形状因子 (Form Factor)**
形状因子这一著名的形状描述参数, 是我们在树叶分类中另一重要的指示参数, 它的最大值为 1.
$$FF = \frac{4\pi A_{\text{leaf}}}{P_{\text{leaf}}^2}$$
其中 P_{leaf} 表示树叶的周长.

5. **边缘面积平滑指数 (Edge regularity area index)**
有时尽管两片树叶的宽高比和矩度很相似, 但它们的实际轮廓可能相差甚远.

因此, 为了考虑不同的轮廓, 我们将所有沿树叶轮廓的凸点连起来, 定义一个特殊的参数: 外接多边形面积. 树叶面积与其外接多边形面积的比例是一个很好的数值因子, 我们定义它为边缘面积平滑指数.

这个比例越接近于 1, 叶形的锯齿状程度越低, 轮廓越平滑 (参看图3). 此参数最大可能取值为 1.
$$ERAI = \frac{A_{\text{leaf}}}{BPA}$$
其中 $ERAI$ 表示边缘面积平滑指数, BPA 表示树叶的外接多边形面积.

6. **边缘周长平滑指数**
类似地, 我们先指定另一个参数: 外接多边形周长, 即所有沿树叶轮廓的凸点所连外接多边形的周长. 于是我们将该外接多边形的周长与树叶的周长之比定义为边缘周长平滑指数. 要注意, 这一次该比例越小, 树叶

the more jagged and irregular the contour of the leaf is (refer to figure 4). The maximum possible value of this parameter is 1.

$$ERPI = \frac{BPP}{P_{\text{leaf}}}$$

Where $ERPI$ stands for the edge regularity perimeter index, BPP stands for the bounding polygon perimeter.

7. **Proportional index**
   Since it is also highly critical to capture the spatial distribution of different portions of a leaf along its vertical axis, we divide the minimum bounding rectangle into four blocks horizontally, and calculate the proportion of the leaf area in a particular region to the total leaf, which we refer to as the **Proportional Index**.

$$PI_i = \frac{\text{area of block } i}{A_{\text{leaf}}}$$

Now, we can develop a database of six most common leaves containing seven parameters discussed above.

We develop a database of the six most common leaf types in North America (Figure 5), using the seven parameters discussed above. Table 1 gives the values of the parameters for each leaf type, as measured from photos of leaves in [1].

轮廓越不规则, 参差不齐 (参看图4)). 此参数的最大取值仍是 1.

$$ERPI = \frac{BPP}{P_{\text{leaf}}}$$

其中 $ERPI$ 表示边缘周长平滑指数, BPP 表示外接多边形周长.

7. **比例因子 (Proportional index)**
   鉴于树叶不同部位在竖直方向上的空间分布也同样重要, 我们将树叶的最小外接矩形水平地切成 4 块, 然后分别计算某块特定区域面积与 4 块总面积之比, 我们称之为比例因子.

$$PI_i = \frac{\text{area of block } i}{A_{\text{leaf}}}$$

至此, 我们可以针对 6 种最常见的树叶, 根据以上 7 个参数建立一个数据库.

5.2 Comparison

When given a specific leaf, we can calculate seven characteristics of it and compare them with our database by calculating the squared deviation of each of parameter of the given leaf from the corresponding standard parameter of each category. We realize the fact that some of seven parameters are somehow more important than others. So in an effort to make our model more accurate and reliable, we induce the conception of the Index of Deviation, denoted as I_D, which comes from the sum of the squared deviation between the database and the leaf-to-be-classified times the proper weight.

$$I_D = \sum_{i=1}^{7} I_i W_i$$

当给定一个具体的树叶时, 我们可以计算它的 7 个特征参数, 然后将它们与我们建立的数据库进行比对——计算这片树叶 7 个参数与某一标准树叶类 (共 6 类) 参数的方差. 我们发现, 这 7 个参数中有几个比其他的重要. 因此, 为了使模型更加准确可靠, 我们引入偏差指数 (Index of Deviation) 的概念, 用来表示: 它是数据库和待分类树叶间方差乘上合适权重后, 对 7 个参数的求和.

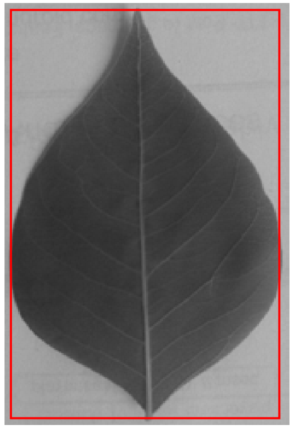


Figure 1: Aspect ratio

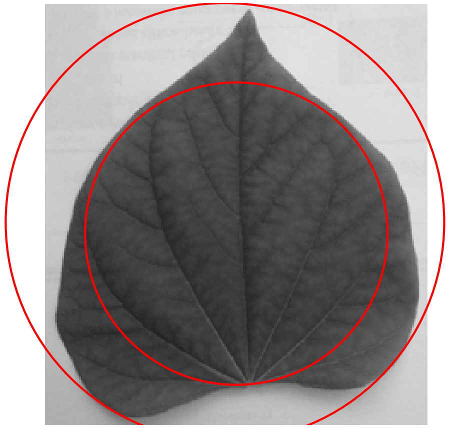


Figure 2: Circularity

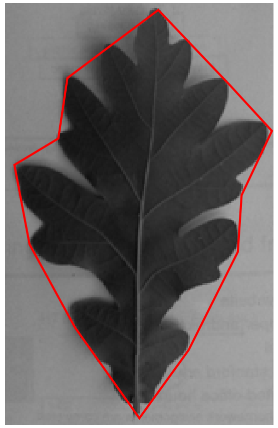


Figure 3: Edge regularity area index

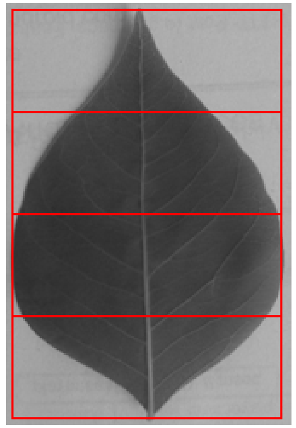
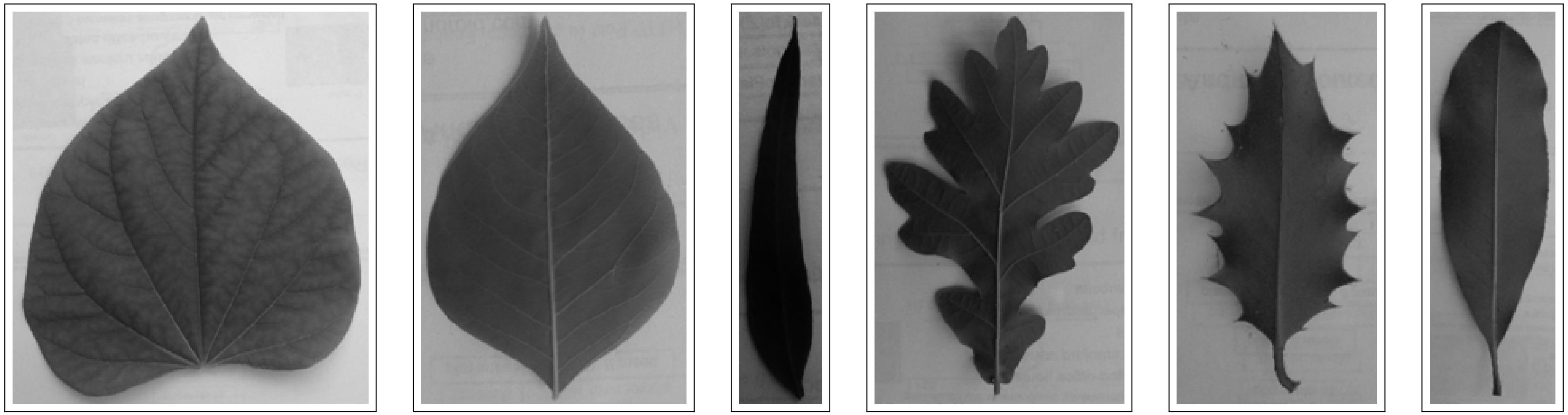


Figure 4: Edge regularity perimeter index



1 2 3 4 5 6

Figure 5: The six most common seen leaf types in North America. [1]

Table 1: Parameter values for the six leaf types.

	1	2	3	4	5	6
Rectangularity	0.6627	0.5902	0.6250	0.4772	0.4876	0.6576
Aspect Ratio	0.8615	0.6600	0.1800	0.6383	0.4792	0.3111
Circularity	0.8140	0.5432	0.4564	0.3454	0.3123	0.3311
Form Factor	0.9139	0.6206	0.2823	0.2470	0.3662	0.4956
ER Area Index	0.9322	0.8780	0.9091	0.8500	0.7880	0.8895
ER Perimeter Index	0.8727	0.8889	0.9384	0.8602	0.8231	0.9903
PI1	0.0649	0.0769	0.1179	0.1909	0.1299	0.2920
PI2	0.2958	0.3555	0.2208	0.3892	0.3606	0.4187
PI3	0.3439	0.4243	0.4139	0.3047	0.4123	0.2677
PI4	0.2954	0.1433	0.2474	0.1152	0.0970	0.0220

where each I_i is the squared deviation, except that

其中 I_i 为相应参数的方差, I_7 例外:

$$I_7 = \frac{1}{4}\sum_{j=1}^{4}\left(PI_j - PI_{\text{new},j}\right)^2$$

In order to decide the respective weight more scientifically, we resort to the help of Analytical Hierarchy Process [2].

First of all, we build a seven by seven matrix reciprocal matrix by pair comparison:

为了科学地确定各参数的权重 W_i, 我们求助于层次分析法 [2].

首先, 我们建立一个 7×7 的对称倒数矩阵 (或成对比较矩阵) 来做比较:

$$\begin{array}{l c}
 & \begin{array}{ccccccc} R & AR & C & FF & ERAI & ERPI & PI \end{array} \\
\begin{array}{l} \text{Rectangularity} \\ \text{Aspect Ratio} \\ \text{Circularity} \\ \text{Form Factor} \\ \text{ER Area Index} \\ \text{ER Perimeter Index} \\ \text{Proportional Index} \end{array} &
\begin{bmatrix}
1 & 1/3 & 1 & 1/4 & 1/2 & 1/2 & 1/7 \\
3 & 1 & 3 & 1 & 2 & 2 & 1/3 \\
1 & 1/3 & 1 & 1/4 & 1/2 & 1/2 & 1/7 \\
4 & 1 & 4 & 1 & 3 & 3 & 1/2 \\
2 & 1/2 & 2 & 1/3 & 1 & 1 & 1/4 \\
2 & 1/2 & 2 & 1/3 & 1 & 1 & 1/4 \\
7 & 3 & 7 & 2 & 4 & 4 & 1
\end{bmatrix}
\end{array}$$

The number of each cell is explained in Table 2.

每个位置上数字的设定由下表2给出解释.

Table 2: The multiplication table of D10.

Intensity of Value	Interpretation
1	Requirements i and j have equal value.
3	Requirement i has a slightly higher value than j.
5	Requirement i has a strongly higher value than j.
7	Requirement i has a very strongly higher value than j.
9	Requirement i has an absolutely higher value than j.
2, 4, 6, 8	Intermediate scales between two adjacent judgments.
Reciprocals	Requirement i has a lower value than j

We then input Pair Ratio Matrix into program of MATLAB (Code available in appendix), and get the weight of each factor (W_i), as given in Table 3.

然后我们将成对比较矩阵输入 MATLAB 程序计算 (代码见附录), 得到每个因子的权重 W_i, 见表 3.

Table 3: AHP-derived weights.

Factor	R	AR	C	FF	ERAI	ERPI	PI
Weight	0.0480	0.1583	0.0480	0.2048	0.0855	0.0855	0.3701

The following calculation is intended to test the consistency of the above AHP[3].

接下来的计算是为了对层次分析法逐层做组合一致性检验 [3].

$$\lambda_{\max} = 7.0512,\ CI = \frac{\lambda_{\max} - n}{n-1} = 0.0087,\ CR = \frac{CI}{RI} = 0.0064,\ CR < 0.01$$

So, the above method displays perfectly acceptable consistency and the weights are reasonable.

上述方法极好地接受了一致性检验, 因此得到的权重是合理的.

5.3 Model Testing

Now we will take the maple leaf as an example to test our leaf classification model. Before testing, we compare this maple leaf with the six categories in our database and find out that it resembles Category 4 most. Now we will test our hypothesis with our model based on quantitative analysis.

现在我们将用枫叶作为树叶分类模型的检验样本. 在此之前, 我们将这片枫叶与数据库中的 6 个标准叶形 categories 做了对比, 发现它与类型 4 最为相似. 那么现在我们将用我们基于量化分析的分类模型来验证这个假设.

Firstly, process the image of a given maple leaf (figure 6 and 7).

首先, 对给定的枫叶图片做预处理;

Figure 6: Image of a given maple leaf

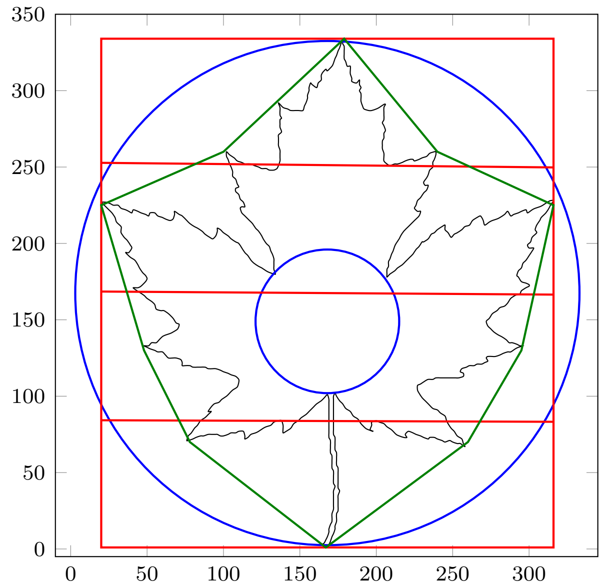


Figure 7: Process of maple leaf

Then, calculate the rectangularity, aspect ratio, circularity, form factor, edge regularity area index, edge regularity perimeter index and the proportional index of the given leaf. In this case, the seven parameters are shown in table 4.

然后, 计算其矩度、宽高比、圆度、形状因子、边缘面积平滑指数、边缘周长平滑指数和比例因子. 对于本例, 7 个参数结果如下:

Table 4: Parameter values for the sample maple leaf

Parameter	R	AR	C	FF	ERAI	ERPI	PI_1	PI_2	PI_3	PI_4
Maple Leave	0.3554	0.9079	0.2691	0.1572	0.6248	0.3787	0.0968	0.4628	0.4311	0.0093

Finally, calculate the index of deviation by using the formula generated before.

最后, 用之前定义的公式计算偏差指数.

$$\sum_{i=1}^{7} I_i \cdot W_i$$

The six indexes of deviation after comparing the parameters of a given maple leaf with the parameters of six categories in our database are shown in table 5.

在比较了给定枫叶参数与 6 个标准类别参数后, 得出 6 个偏差指数如下:

Table 5: Index of deviation of maple leaf from six common leaf categories

Maple Leave	1	2	3	4	5	6
I_1	0.0045	0.0026	0.0035	0.0007	0.0008	0.0044
I_2	0.0003	0.0097	0.0839	0.0115	0.0291	0.0564
I_3	0.0066	0.0043	0.0004	0.0001	0.0004	0.0000
I_4	0.1173	0.0440	0.0032	0.0017	0.0090	0.0235
I_5	0.0081	0.0055	0.0069	0.0043	0.0023	0.0060
I_6	0.0209	0.0223	0.0268	0.0198	0.0169	0.0320
I_7	0.1095	0.0277	0.1073	0.0384	0.1817	0.0619
I_D	0.2672	0.1161	0.2319	**0.0765**	0.2402	0.1841

Since the index of deviation between the given maple leaf and Category 4 is smallest, the maple leaf falls into Category 4, which is consistent with our hypothesis.

表中数据显示: 由于给定枫叶与类型 4 之间的偏差指数最小, 所以该枫叶应落入类型 4 中, 而这也与我们的假设一致.

5.4 Conclusion

So far, our model has solved the problem of classification very well based on seven well-rounded parameters and a functional database. Through this quantitative model, we can decide the general shape of any given leaf reliably and scientifically. Our model is robust under reasonable conditions, which can be seen through the testing part discussed above. However, since our database contains only the six commonly seen leaf types in North America, the variety of database itself still has room for improvement. With the more and more different leaf species added to the database, our model can apply to a wider and more geographically comprehensive scope.

至此, 我们的模型在 7 个面面俱到的参数和一个功能型数据库的基础上, 很好地解决了树叶的分类问题. 凭借这个定量模型, 我们可以对任一给定树叶的形状做出可靠、科学的一般性分类. 我们的模型再合理条件下是稳健的, 这可由前面模型检测一节看出. 然而我们的数据库仅包括 6 中北美洲的常见叶形, 因此仍有改进的空间. 随着向数据库补充更多不同的树叶, 我们的模型将能适用更广, 包含更完整的几何信息.

6 Model Two: Leaf Distribution and Leaf Shape

6.1 Introduction

Leaf shape varies so greatly that it still remains a mystery why leaves display such a immense variety of shape. Leaf veins are the skeleton of a leaf, and ground tissues the muscles. Genetic and environmental factors contribute to the different patterns of leaf veins and ground tissue, thereby determining the leaf shape. In this model, we narrow the big picture into how leaf distribution act as an influence on leaf shape.

6.2 Idealized Leaf Distribution Model

To downplay the effects of miscellaneous factors that contribute to the relation between the distribution and shape of a leaf, we construct an idealized model that immensely simplifies the complex condition.

The tree is made up of a branch perpendicular to the ground surface and two identical leaves grown on the branch ipsilaterally and horizontally. The leaves face upward and point toward the sun in the sky.

We suppose the tree is placed on the latitude of L (Northern). So the greatest average solar altitude in a year, which is attained in the noon of Vernal Equinox, is defined as α.

The above information is explained in the figure 8.

6.3 Analysis of Overlapping Areas

Our key focus of the study is the partly shaded leaf. What proportion of the leaf (PL) is shaded is the output of the model. The solar attitude α is a critical control variable. According to the influence of the angle on PL, we divide α into three scenarios.

6.3.1 Solar Altitude trending toward 90◦

The situation in which α is trending toward 90◦ usually takes place in tropical regions. And it is very clear through observation that, in tropical regions, leaf

树叶形状千变万化, 其原因仍是一个不解之谜. 叶脉是一篇树叶的骨架, 与叶肉一同作为基本组织. 基因、环境因素的作用导致了叶脉和基本组织的多样性, 进而由此决定了树叶的形状. 在这个模型中, 我们将讨论重点限定为: 树叶分布如何影响叶形.

为了削弱混杂多样的因素对树叶分布与叶形关系的影响, 我们建立一个理想模型, 从而大大简化了复杂的情况.

假设一棵树由一条垂直于地面的枝干, 及枝干同侧两片水平的一模一样的树叶构成. 树叶面朝上, 朝着太阳的方向.

我们假设这棵树位于北纬 L 度. 则其一年中最大的太阳高度角 (于春分日的中午) 为 α.

以上信息可由图8解释.

我们研究的关键点是被部分遮挡的树叶: 模型输出中, 树叶有多大比例面积被遮挡 (PL). 太阳高度角 α 则是关键的决策变量. 根据该角度对这档比例 PL 的影响, 我们对 α 分三种情况讨论.

α 趋向 90 度的情况大多发生在热带. 从观测中发现, 明显热带的树叶普遍较宽大, 树冠通常仅有一层树叶. 这可以

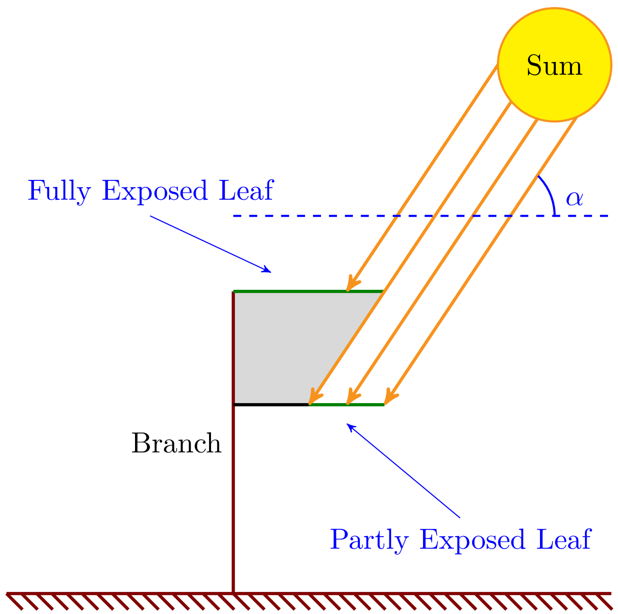


Figure 8: Primitive model of a tree, at noon on the vernal equinox

shapes are typically broad and wide with the tree crowns usually containing only one layer of leaves. This can be explained through the illustration above as the shaded part of the lower leaf would be too big with trending toward 90○ to supply enough solar energy for photosynthesis and the biggest absorption of energy can be achieved by a broad leaf shape.

用上面的图示解释：当 α 趋向 90 度时，对处于下层的树叶，其被遮挡的部位无法获得足够太阳能 (以供光合作用)，因而造就了它们宽大的叶形，以吸收/接受最多的太阳能.

6.3.2 Solar Altitude trending toward 0○

The situation in which α is trending toward 0○ usually takes place in frigid zones. It is also very clear that in these regions, leaves are typically acicular with the tree crowns containing dense layers of closely grown leaves. This can also be explained through the illustration above as the shaded part of the lower leaf would approach zero with α trending toward 0○, allowing a much more concentrated dis-

这种情况通常发生在寒带 (极地地区). 同样显而易见的是，这些地区树叶的典型特征是针状，且树冠上叶层密集. 同样，这也可以用上面的图来解释： 趋向 90 度时，下层树叶被遮挡部位几乎为零 (approach zero)，这就允许树叶更加集中地分布. 此外，针形树叶可做到最大能量吸收.

tribution of leaves than in other situations. In addition, the maximum absorption of energy can be best achieved by needle-like leaves.

6.4 Solar Altitude within normal range

This scenario is typical in the temperate zone on earth, where sunlight irradiates the leaves in a tilted way. It is also the case in which our idealized model is the most suitable. Another crucial factor that we control in this case is the distance (h) between the two points connecting the leaves and the branch.

这种情况属于地球上最典型的温带，此时斜着照向树叶．这也是我们的理想模型最适应、匹配的情况．这种情况中我们将控制另一关键因素——两片树叶 (在枝干上的) 结点间的距离 (h).

Our aim is to discuss the relationship between the leaf shape and the leaf distribution. We assume a tree' s leaf distribution will try to minimize the overlapping area between leaves, so our model will try to find out the quantitative relationship between the overlapping area and the shape of leaf.

我们的目标是讨论叶形和树叶分布之间的关系．我们假设，一棵树的树叶分布是倾向于让树叶间的重叠面积最小，于是我们的模型将要努力寻求重叠面积与叶形之间的数量关系．

In order to simplify the model, we use the rhombus, with the length of major axis denoted as L_{major} and the length of minor axis denoted as L_{minor} , to replace the leaf shape. Also, we control the area of the first leaf to ensure its constant exposure area to the sun. Since the area is fixed, now we only need to change the length of the major axis to change the shape of the leaf. In other words, during our later study, we will use the change of L_{major} to symbolize the change of the leaf shape.

为了简化模型，我们选择菱形代表树叶形状，设长轴长，短轴长．同时，我们令第一片树叶受光面积为常数．一旦此面积固定，我们就只需改变长轴长来改变树叶形状了．换句话说，在之后的研究中，我们将用的变化来表征叶形的变化.

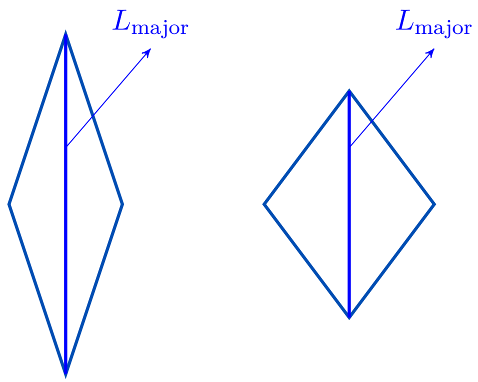


Figure 9: Two leaves of the same area but different lengths of major axis

Also, since we have fixed the area of the leaf and just change its shape during our study, the smallest overlapping area, which we denote as $A_{\text{overlapping}}$, means the smallest ration of E, which is defined as follows:

$$E = \frac{A_{\text{overlapping}}}{\text{area of one leaf}}$$

同时，正由于过程中固定树叶面积、只调整叶形，我们用最小重叠面积 (记作 $A_{\text{overlapping}}$) 进一步定义最小重叠率 E:

Obviously, the most efficient and effective way for two leaves to distribute is to be just totally exposed in sunlight as illustrated below, when $h = h_0$ and $E = 0$. Thus, the tree can take whatever shape it wants since we control the leaf's surface area.

显然，对两片树叶来说，最有效的分布方式是恰好完全受光，如下图所示 (其中 h=h0, E=0)．因此，树可以自由改变它的形状——由于我们控制了树叶的表面积.

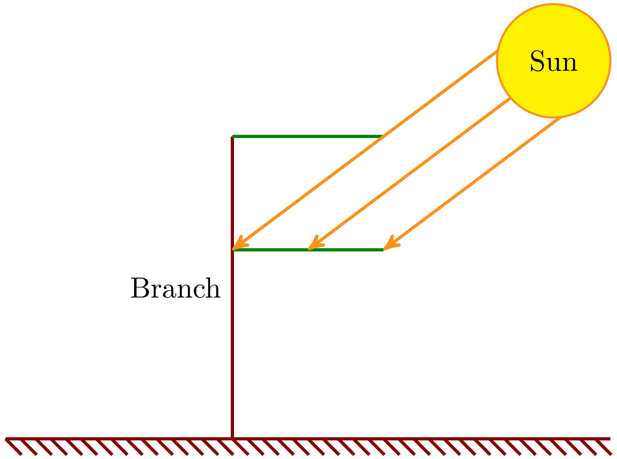


Figure 10: Upper leaf does not overlap lower one.

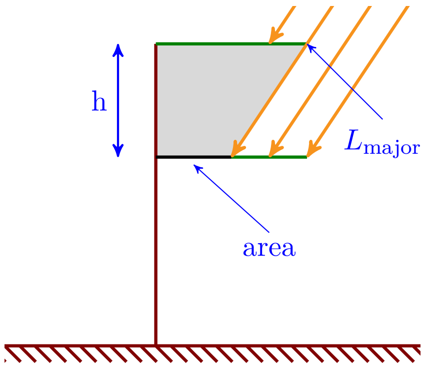


Figure 11: Upper leaf overlaps lower one.

Now, we want to study what if $h < h_0$.

If $h < h_0$, there will be a shaded region $A_{\text{overlapping}}$. The stress of the following study is analyzing the relation between h, L_{major} and E when given a fixed solar altitude α.

We can easily give the relationship between these factors according to their

现在，我们来看看若 $h < h_0$ 会怎么样.

若 $h < h_0$，则将产生一个遮挡区域 $A_{\text{overlapping}}$．接下来研究的重点就是：在给定的太阳高度角 α 下分析 h，L_{major} 和 E 之间的关系.

根据这些因子的几何特征，我们可轻易得到它们之间的

geometric features:

关系:

$$E = \left(\frac{L_{\text{major}} \cdot \tan\alpha - h}{L \cdot \tan\alpha}\right)^2 = \left(1 - \frac{h}{L_{\text{major}} \cdot \tan\alpha}\right)^2$$

Where $\tan\alpha$ is a constant.

When we fix the value of h and give α a particular value, we can get the relationship between the length of leaf and the overlapping area:

其中 $\tan\alpha$ 为常数.

当确定 h, α 的值时, 我们可得到叶长与重叠面积之间的关系:

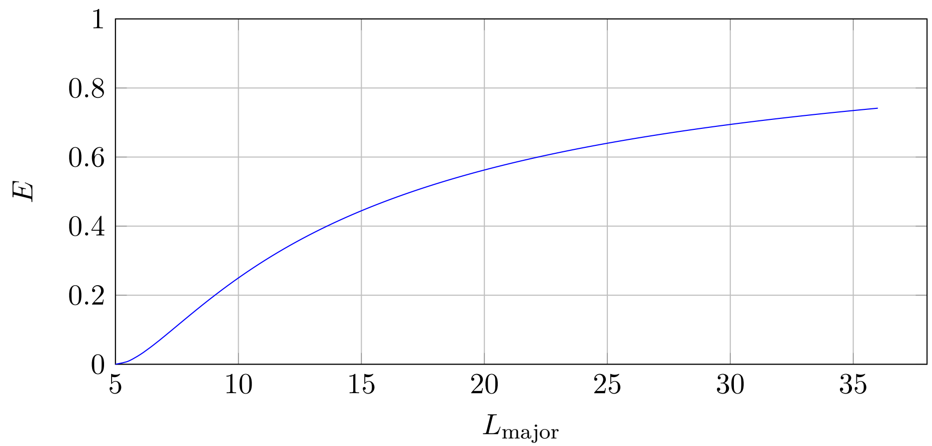


Figure 12: Relationship between the length of leaf and the overlapping area

We discover from the function above that the closer the L_{major} approaches L $\tan\alpha$, which is equal to h_0, the smaller the overlapping area will be.

So far, we have considered the both situations when $h = h_0$ and $h < h_0$. From what we discussed above, the best leaf distribution of the leaf occurs when $h = h_0$, which means

$$h = L_{\text{major}} \cdot \tan\alpha$$

我们从这个函数中发现, L_{major} 越趋近 L $\tan\alpha$ 时, 重叠面积将越小.

至此, 我们已经考虑了 $h = h_0$ 和 $h < h_0$ 两种情况. 从我们之前的讨论可知, 最佳的树叶分布出现在 $h = h_0$ 时, 即

6.5 Model Testing

Now, we need to test whether this correlation between the leaf distribution and leaf shape is right. We collect the data of the length (L_{major}) and internode (h) of

现在, 我们需要检验树叶分布与叶形之间的这种数量关系是否正确. 我们收集了许多树种的叶长 (L_{major}) 和节间距

a variety of trees and use our formula to calculate the respective solar altitude of the trees. By converting the solar altitude into latitude, we can predict the origin of this particular tree. We choose Ligustrum quihoui Carr, Osmanthus fragrans and Camellia japonica as our test objects and here is the result of our testing:

(h) 的数据, 然后分别用我们的公式来计算相应树的太阳高度角 α. 继而通过将太阳高度角转换为纬度, 我们能够预测出该树的所在地 (origin). 我们选择了小叶女贞树、桂花树、茶花树作为检测目标, 检测结果如下:

Table 6: Test of model for leaf shape as a function of latitude.

Test Object	L_{major}	h	$\tan\alpha$	Predicted Latitude	True Lattitude
Ligustrum quihoui Carr.	2	2.5	1.25	38.65	35~35
Osmanthus fragrans	10	18.5	1.85	28.39	23~29
Camellia japonica	6	9	1.50	33.69	32~36

From the table, we can see that our predicted latitude of origin is always within the range of its true latitude, so it is safe to say that the correlation between leaf distribution and leaf shape generated by our model is reliable and correct. In other words, there does exist a relationship between the leaf distribution and leaf shape.

从表中可以看到, 我们预测的纬度一直在真实值范围之内, 因此可以肯定地说: 由我们模型得出的树叶分布与叶形之间的关系是可靠、正确的. 也就是说, 在树叶分布和叶形之间确实存在着联系.

7 Model Three: Tree Profile and Leaf Shape

7.1 Introduction

In this part of our paper, we will explore the correlation between the tree profile and the leaf shape. When we classify the leaf in model one, we define a set of parameters to better understand the shape of leaf on a quantitative base. Similarly, we will apply this method to explore the profile of tree.

鉴于叶脉构造决定叶形、枝干结构决定树形, 而与此同时, 叶脉和枝条也很相似 (参看图 3.1), 我们做出一个大胆的猜想: 叶形可能是树形的二维袖珍模型 (mimic). 也就是, 在叶形与树形间是否存在着某种联系.

7.2 Hypothesis

Since the vein structure determines the leaf shape and the branch structure determines the tree profile and, to some degree, the leaf veins resemble the branches(refer to figure 3.1),we have a wild hypothesis that, leaf shape be a two-dimensional mimic of the tree profile. In other words, there does exist a correlation between the leaf shape and the tree profile.

为了判断我们的猜想是否正确, 我们模型总的想法是: 对比树形、叶形间的一些特定参数, 从而确定是否有关系存在. 叶形是二维的, 因此分析它的参数相对简单. 然而, 树形则是三维的, 所以重点在于如何找出一棵树的二维特征来分析参数. 鉴于树的纵截面反映了一棵成年树的一般大小、形状特征, 我们选择纵截面为关注对象, 在后面对其研究分析.

Figure 13: maple and tree

7.3 Comparison of Leaf Shape and Tree Contour

n an effort to judge whether our hypothesis is true or not, our model' s general idea is to compare some specific parameters between the tree profile and the leaf shape, thus determine whether a relation exists.

The leaf shape is two-dimensional, so it is relatively easy to study its parameters. However, the tree profile is three dimensional, so it is important to find out a two-dimensional characteristic of a tree to study its parameters. Since the longitudinal section of a particular tree reflects the general size characteristics of a grown tree, we choose to focus on it in our later study and analysis.

为了判断我们的猜想是否正确, 我们模型总的想法是: 对比树形、叶形间的一些特定参数, 从而确定是否有关系存在.

叶形是二维的, 因此分析它的参数相对简单. 然而, 树形则是三维的, 所以重点在于如何找出一棵树的二维特征来分析参数. 鉴于树的纵截面反映了一棵成年树的一般大小、形状特征, 我们选择纵截面为关注对象, 在后面对其研究分析.

7.4 Tree Profile Classification

In the leaf classification model, there are 6 general classes of leaves. Since we only compare the general resemblance between leaf and tree, class five (elliptic leaf with serrated margin) is incorporated into class two. As a result, we get 5 classes of leaves and 5 respective types of trees.

在之前树叶的分类模型中, 叶形共有 6 个标准类型. 由于我们只对比树叶与树的一般相似性, 其中的类型 5 (带有锯齿状边缘的椭圆树叶) 可以并入类型 2 中. 因此, 我们分别得到了 5 类树叶、5 类树.

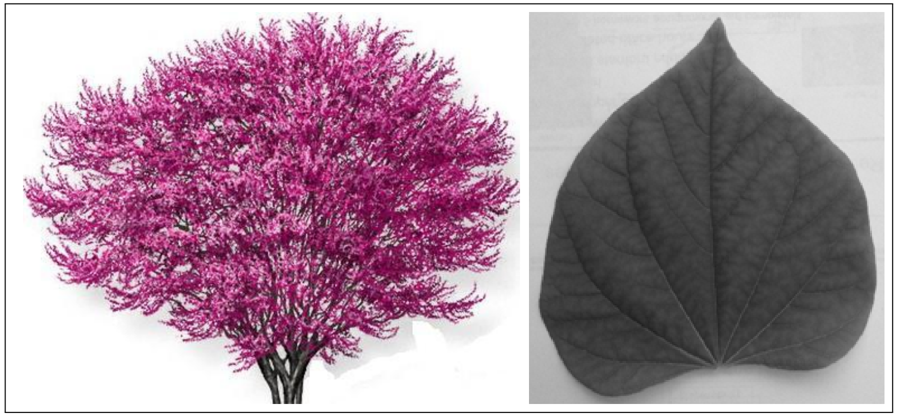


Figure 14: Class 1 Cordate (Texas Redbud)

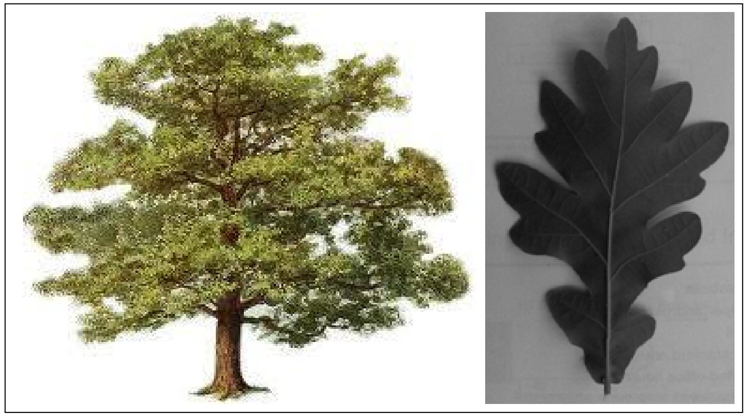


Figure 15: Class 4 Palmate (Oak Tree)

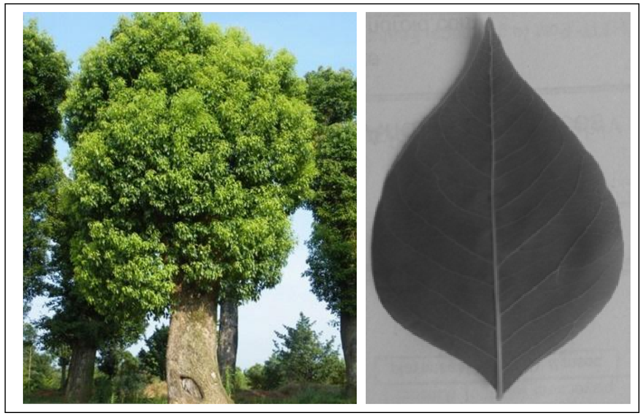


Figure 16: Class 2 Elliptic (Camphor Tree)

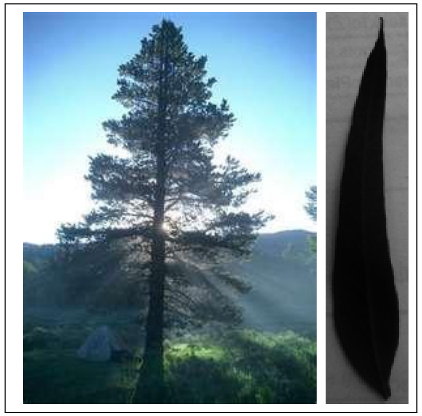


Figure 17: Class 3 Subulate (Pine Tree)

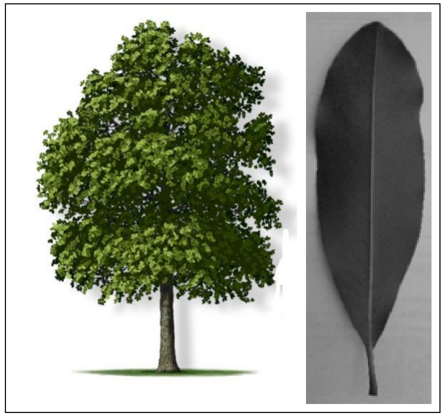


Figure 18: Class 5 Obovate (Mockernut Hickory)

7.5 Parameters of the tree

According to the data we get and referring to the parameters we set for the leaf, this time we appoint 3 parameters for the longitudinal section that can be

根据我们得到的数据，并参照我们为树叶设置的参数，这一次我们为树的纵截面指定了 3 个参数，它们与相应的树叶

compared with those of the leaf shape, namely the rectangularity, the aspect ratio and circularity. Since we have discussed these three parameters in detail before, we will just briefly go through them.

- Rectangularity
  Rectangularity is defined as the ratio between the longitudinal section area of a tree and the area of the smallest bounding rectangle area. First we compare the rectangularity of the tree profile with that of leaf we calculated in model 1 in table 7. Second, we draw a scatter plot with the independent variable of leaf and the dependent variable of tree and use MATLAB to construct the linear regression of the data. figure 19 shows the fitting relationship of the data. Also, we use F-test to test the significance of the linear regression:
  $$R^2 = 0.1783, \qquad F = 0.63, \qquad P = 0.4850$$
  Obviously, the test shows that the linear relationship between the rectangularity of leaf and rectangularity of tree is insignificant.
- Aspect ratio
  Aspect ratio is defined as the ration between the short side and the long side of the smallest bounding rectangle area. The comparison chart is shown in table 8. Using the same statistical method, we construct figure 20. This time the result of statistical inference is as follows:
  $$R^2 = 0.7985, \qquad F = 11.8857, \qquad P = 0.041 < 0.05$$
  As P value is within acceptance range, the linear relationship between Aspect Ratio of leaf and that of tree exists.
- Circularity
  Circularity is defined as the ratio between the radius of in-circle and ex-circle of the area of interest. The comparison chart is shown in table 9. Using Matlab, we construct figure 21. The F-test shows the following.
  $$R^2 = 0.9645, \qquad F = 81.5595, \qquad P = 0.0029 < 0.05$$
  The P value small enough for us to say that there exists a linear relationship between the circularity of the tree and the leaf.

参数具有可比性: 矩度、宽高比和圆度. 鉴于前面已详细讨论过这三个参数, 我们只简要地过一遍.

- 矩度
  矩度的定义是树的纵截面面积与其最小外接圆面积之比. 第一步, 我们把树形的矩度和叶形的 (模型 1 中计算的) 做比较. 第二步, 画出散点图 (自变量为叶, 因变量为树) 并基于数据用 MATLAB 做线性回归. 下图显示了数据间的拟合关系. 同时, 我们做 F 检验来验证该线性回归的显著性:
  $$R^2 = 0.1783, \quad F = 0.63, \quad P = 0.4850$$
  显然, 验证表明该线性回归 (叶形矩度与树形矩度间) 是不显著的.
- 宽高比
  宽高比定义为 (树形) 最小外接矩形的较短边与较长边之比. 对比的表如下. 用同样的统计学方法, 我们做出下面的曲线图. 这一次的统计诊断如下:
  $$R^2 = 0.7985, \quad F = 11.8857, \quad P = 0.041 < 0.05$$
  随着 P 的值落入接受域, 可知该 (叶形树形两者宽高比间的) 线性关系显著.
- 圆度
  圆度的定义是感兴趣区域的内、外接圆半径之比. 我们用 MATLAB 做出下图. F 检验显示如下.
  $$R^2 = 0.9645, \quad F = 81.5595, \quad P = 0.0029 < 0.05$$
  P 值足够小, 我们可以认为树形、叶形的圆度之间存在着线性关系.

Table 7: Rectangularity

Class	Leaf	Tree
1	0.6627	0.6281
2	0.5902	0.6846
3	0.6250	0.5180
4	0.4772	0.5292
5	0.6576	0.6238

Table 8: Aspect ratio

Class	Leaf	Tree
1	0.8615	0.7914
2	0.6600	0.7243
3	0.1800	0.6601
4	0.6383	0.7980
5	0.3111	0.6750

Table 9: Circularity

Class	Leaf	Tree
1	0.6396	0.5800
2	0.5698	0.5928
3	0.1834	0.2895
4	0.3069	0.4070
5	0.2889	0.3866

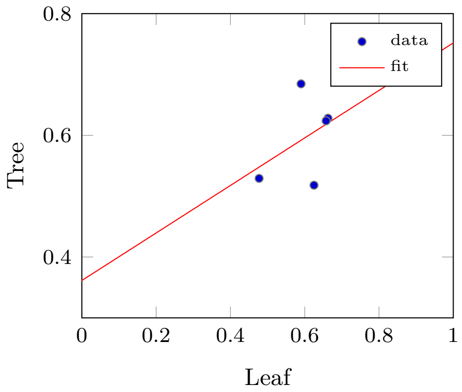


Figure 19: Rectangularity: $y = 0.3905x + 0.3614$

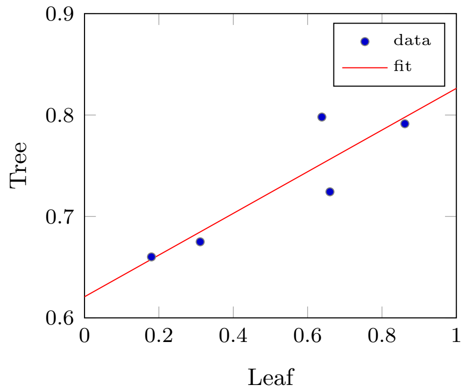


Figure 20: Aspect ratio: $y = 0.2055x + 0.6208$

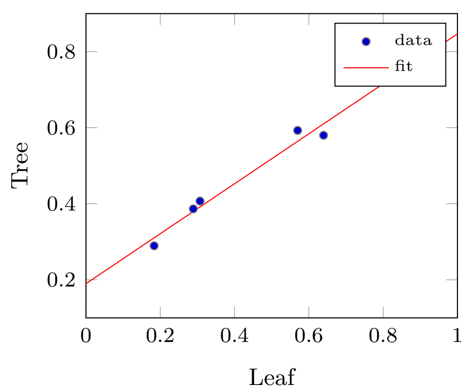


Figure 21: Circularity: $y = 0.6567x + 0.1900$

7.6 Conclusion

From the test of the above three parameters, we may conclude that leaf shape and tree profile have a possibly relationship between each other. Although the test of rectangularity does not support our hypothesis, the test of aspect ratio and the test of circularity well corroborate the theory that leaf shape is a two-dimensional mimic of the tree contour. Since, the tree shape and leaf shape both are affected by an great deal of genetic and environmental factors, the shape characteristic of leaf may not well reflect the tree shape. But as far as we know, the shape of leaf resembles the shape of tree to some extent.

经过以上对三个参数的检验，我们可以总结出叶形和树形间可能存在联系．尽管对矩度的检验不支持我们的假设，但对宽高比、圆度的检验很好的支持了叶形是树形的二维袖珍模型的理论．诚然，由于均受到遗传基因及大量环境因素的影响，实际上叶形也许并无很好地反映树形．但至少据我们所知：叶形在某种程度上与树形很相似．

8 Model Four: Leaf Mass

8.1 Introduction

In order to estimate the mass of the leaves of a certain tree when given the size characteristics, we have developed a model based on a database.

One simple way to calculate the total leaf mass of a tree is to simply multiply the number of the leaves on a tree and the mass of a single leaf. Our method is more accurate and less demanding in that our model is independent of these two factors, namely the number of leaves and the net weight of one single leaf, but dependent on one more reliable factor of a grown tree — photosynthesis.

Our methodology of estimating the leave mass of a tree is based on three variables: tree age and growth rate, which is determined by tree species, and general type (Hardwood or conifer). In other words, given the age and type of a tree, we are able to estimate the total mass of leaves. In this model, CO2 is used as a calculating media.

After the explanation of the model, we try to study if there is a correlation between leaf mass size characteristics of a tree. The answer is positive, which we use tree age as a link to prove.

为了在给定的形状、大小特征下估算一棵树的质量，我们建立了基于数据库的模型.

计算树叶总质量的一个简单方法是单纯地用树叶数量乘上每片树叶的质量. 而我们的方法将更加准确，同时要求没那么苛刻——我们的模型不依赖于这两个因子，即树叶数量和每片树叶的净质量，而是依赖于一个 (对于成年树木) 更可靠的因素——光合作用.

我们的估算方法基于 3 个变量: 树龄、生长率 (取决于树种) 和一般类型 (阔叶型或针叶型两种). 换句话说，只要给定树龄和树的类型，我们就能够估算出树叶的总质量. 在这个模型中，CO2 将作为计算的一个媒介.

在解释了模型之后，我们将尝试研究一棵树的树叶质量大小间的关系. 答案是肯定的，而我们用树龄作为桥梁进行检验.

8.2 Leaf Mass and Tree Age

8.2.1 Leaf Mass and CO2 Sequestration

First of all, according to the information we acquired from the past researches, we divide the tree into two main types based on their different ability of sequestering carbon dioxide from the atmosphere. These two types and their respective ability are shown in table 10:

首先，根据之前研究中得到的信息，我们基于树对二氧化碳不同的积累能力，将树分为两大主要类型. 这两类和分别的 (CO2 积累) 能力如表10所示.

Table 10: Two main types of tree and their respective ability

	Hardwood	Conifer
Ability to sequester Carbon Dioxide (lbs of CO2/g of leaf)	0.00250	0.00165

A tree's ability to sequester the carbon dioxide is different from its ability to absorb it since the tree also releases CO2 into the atmosphere all day round because of the respiration. In other words, CO2 sequestration = CO2 absorption –CO2 release.

棵树对二氧化碳的“积累”能力是有别于它吸收二氧化碳的能力, 因为树也会通过呼吸作用向大气中排出 CO2. 也就是说, CO2 积累量 = CO2 吸收量–CO2 排出量.

Now we only need to decide the weight of carbon dioxide sequestered by the tree and use the converting method below to calculate the total mass of leaves of that tree.

现在我们只需要确定树的二氧化碳积累量, 以及用下面的转换公式计算该树上树叶的总质量.

$$M_{\text{leaf}} = \frac{M_{\text{carbon dioxide}}}{A_s}$$

Where, M_{leaf} is total mass of leaves (g); $M_{\text{carbon dioxide}}$ is mass of carbon dioxide sequestered (lbs); and A_s is ability to sequester carbon dioxide (lbs/g).

其中 M_{leaf} 为树叶的总质量 (单位: 克); $M_{\text{carbon dioxide}}$ 为二氧化碳积累量 (单位: 磅); A_s 为二氧化碳积累能力 (单位: 磅/克).

8.2.2 CO2 Sequestration and Tree Age

The relationship between the amount of CO2 sequestered, the age of a tree and the type of tree is shown in a study published by U.S. Department of Energy in April 1998[4]. The study below divided the growth rate of a certain tree into three categories: fast, moderate and slow. The corresponding chart that categorizes different tree species into 3 grow rates is enclosed in the Appendix 1.

二氧化碳积累量、树龄和树的类型三者之间的关系在美国能源部门 1998 年 4 月的一项研究中发表. 下面的研究将树的生长率分为 3 类: 快、中、慢. 附录 1 的表格将不同树种按生长率分为了 3 类.

From the chart, we can discover an obvious positive relationship between the age of the tree and the amount of CO2 sequestered if we fix the growth rate and the type of the tree. We fit the data into a set of quadratic regression functions shown below:

从图中我们发现当生长率和树的类型不变时, 树龄和 CO2 积累量存在着明显的正相关. 我们对数据做了一系列的二次回归, 结果如下:

In each function next to the curve, such as $y = 0.0073x^2 + 0.9690x + 1.0953$, which describes Annual Sequestration Rate for fast-growing Hardwood, y is the sequestration rate and x is the tree age.

每条曲线旁边的方程描述了阔叶型树木的年积累率 (二氧化碳), 其中 y 为积累率, x 为树龄.

After fitting the data and conduct the statistical inference, we surprisingly find that the fitting curve can actually fit the data perfectly. With the fitting curve and the functions, we can easily estimate the CO2 sequestered when given the age of a certain tree.

在对数据拟合及做统计诊断之后, 我们惊讶地发现: 拟合曲线可以完美地拟合数据. 则有了拟合曲线及相应方程, 我们可以很容易地估算一棵给定树龄的树的 CO2 积累量.

Table 11: Survival Factors and Annual Carbon Sequestration Rates for Common Urban Trees

Tree Age (yrs)	Survival Factors by Growth Rate			Annual Sequestration Rates by Tree Type and Growth Rate (lbs. carbon/tree/year)					
				Hardwood			Conifer		
	Slow	Moderate	Fast	Slow	Moderate	Fast	Slow	Moderate	Fast
0	0.873	0.873	0.873	1.3	1.9	2.7	0.7	1.0	1.4
1	0.798	0.798	0.798	1.6	2.7	4.0	0.9	1.5	2.2
2	0.736	0.736	0.736	2.0	3.5	5.4	1.1	2.0	3.1
3	0.706	0.706	0.706	2.4	4.3	6.9	1.4	2.5	4.1
4	0.678	0.678	0.678	2.8	5.2	8.5	1.6	3.1	5.2
5	0.658	0.658	0.658	3.2	6.1	10.1	1.9	3.7	6.4
6	0.639	0.639	0.644	3.7	7.1	11.8	2.2	4.4	7.6
7	0.621	0.621	0.630	4.1	8.1	13.6	2.5	5.1	8.9
8	0.603	0.603	0.616	4.6	9.1	15.5	2.8	5.8	10.2
9	0.585	0.589	0.602	5.0	10.2	17.4	3.1	6.6	11.7
10	0.568	0.576	0.589	5.5	11.2	19.3	3.5	7.4	13.2
11	0.552	0.564	0.576	6.0	12.3	21.3	3.8	8.2	14.7
12	0.536	0.551	0.563	6.5	13.5	23.3	4.2	9.1	16.3
13	0.524	0.539	0.551	7.0	14.6	25.4	4.6	9.9	17.9
14	0.512	0.527	0.539	7.5	15.8	27.5	4.9	10.8	19.6
15	0.501	0.516	0.527	8.1	16.9	29.7	5.3	11.8	21.4
16	0.490	0.504	0.516	8.6	18.1	31.9	5.7	12.7	23.2
17	0.479	0.493	0.505	9.1	19.4	34.1	6.1	13.7	25.0
18	0.469	0.483	0.495	9.7	20.6	36.3	6.6	14.7	26.9
19	0.459	0.472	0.484	10.2	21.9	38.6	7.0	15.7	28.8
20	0.448	0.462	0.474	10.8	23.2	41.0	7.4	16.7	30.8
21	0.439	0.452	0.464	11.4	24.4	43.3	7.9	17.8	32.8
22	0.429	0.442	0.454	12.0	25.8	45.7	8.3	18.9	34.9
23	0.419	0.433	0.445	12.5	27.1	48.1	8.8	20.0	37.0
24	0.410	0.424	0.435	13.1	28.4	50.6	9.2	21.1	39.1
25	0.401	0.415	0.426	13.7	29.8	53.1	9.7	22.2	41.3
26	0.392	0.406	0.417	14.3	31.2	55.6	10.2	23.4	43.5
27	0.384	0.398	0.409	15.0	32.5	58.1	10.7	24.6	45.7
28	0.375	0.389	0.400	15.6	33.9	60.7	11.2	25.8	48.0
29	0.367	0.381	0.392	16.2	35.3	63.3	11.7	27.0	50.3
30	0.359	0.373	0.383	16.8	36.8	65.9	12.2	28.2	52.7
31	0.352	0.365	0.375	17.5	38.2	68.5	12.7	29.5	55.1
32	0.344	0.358	0.367	18.1	39.7	71.2	13.3	30.7	57.5
33	0.337	0.350	0.360	18.7	41.1	73.8	13.8	32.0	59.9
34	0.330	0.343	0.349	19.4	42.6	76.5	14.3	33.3	62.4
35	0.323	0.336	0.339	20.0	44.1	79.3	14.9	34.7	64.9
36	0.316	0.329	0.329	20.7	45.6	82.0	15.5	36.0	67.5
37	0.310	0.322	0.320	21.4	47.1	84.8	16.0	37.3	70.1
38	0.303	0.315	0.310	22.0	48.6	87.6	16.6	38.7	72.7
39	0.297	0.308	0.301	22.7	50.2	90.4	17.2	40.1	75.3
40	0.291	0.302	0.293	23.4	51.7	93.2	17.7	41.5	78.0
41	0.285	0.296	0.284	24.1	53.3	96.1	18.3	42.9	80.7
42	0.279	0.289	0.276	24.8	54.8	99.0	18.9	44.3	83.4
43	0.273	0.283	0.268	25.4	56.4	101.9	19.5	45.8	86.2
44	0.267	0.277	0.260	26.1	58.0	104.8	20.1	47.2	89.0
45	0.261	0.269	0.253	26.8	59.6	107.7	20.7	48.7	91.8
46	0.256	0.261	0.245	27.6	61.2	110.7	21.3	50.2	94.7
47	0.251	0.254	0.238	28.3	62.8	113.6	22.0	51.7	97.5
48	0.245	0.247	0.231	29.0	64.5	116.6	22.6	53.2	100.4
49	0.240	0.239	0.225	29.7	66.1	119.6	23.2	54.8	103.4
50	0.235	0.232	0.218	30.4	67.8	122.7	23.9	56.3	106.3
51	0.230	0.226	0.212	31.1	69.4	125.7	24.5	57.9	109.3
52	0.225	0.219	0.206	31.9	71.1	128.8	25.2	59.4	112.3
53	0.221	0.213	0.199	32.6	72.8	131.8	25.8	61.0	115.4
54	0.216	0.207	0.193	33.4	74.5	134.9	26.5	62.6	118.4
55	0.211	0.201	0.188	34.1	76.2	138.0	27.2	64.2	121.5
56	0.207	0.195	0.182	34.8	77.9	141.2	27.8	65.9	124.6
57	0.203	0.189	0.177	35.6	79.6	144.3	28.5	67.5	127.8
58	0.198	0.184	0.171	36.3	81.3	147.5	29.2	69.2	130.9
59	0.194	0.178	0.166	37.1	83.0	150.6	29.9	70.8	134.1

8.2.3 Leaf Mass and Tree Age

Since we have known the converting relation between the mass of leaves and the CO2 sequestered, now we can derive the correlation between the age of a tree and the total mass of leaves on that tree by using the CO2 sequestered as a media. After processing the data by the formula in 8.2.1, we get the corresponding relation, as shown it table 12. Each cell of the table is the leaf mass of a tree of a certain age, type and growth rate.

因为已经知道了树叶质量与 CO2 积累量之间的转换关系，我们现在可以借助 CO2 积累量这一媒介，推导出树龄与树叶总质量间的关系．将数据用 8.2.1 中的公式处理后，我们得到了下表的对应关系．表中的每个单元是对每一个具体树龄、类型、生长率的树上树叶总质量．

8.3 Tree Age and Tree Size

As stated above, we utilize the age of a tree as a link between the two variables, leaf mass and the size characteristics of trees. Since we have found out the

正如上文所述，我们利用树龄这一桥梁，把这两个变量 (树叶质量与树的大小特征) 联系了起来．由于我们已经找出

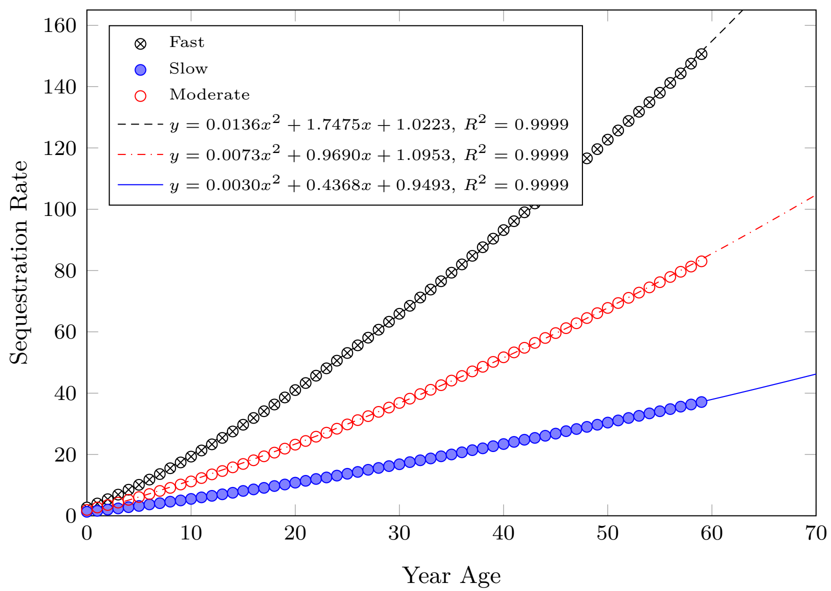


Figure 22: Annual carbon Sequestration Rates for Hardwood

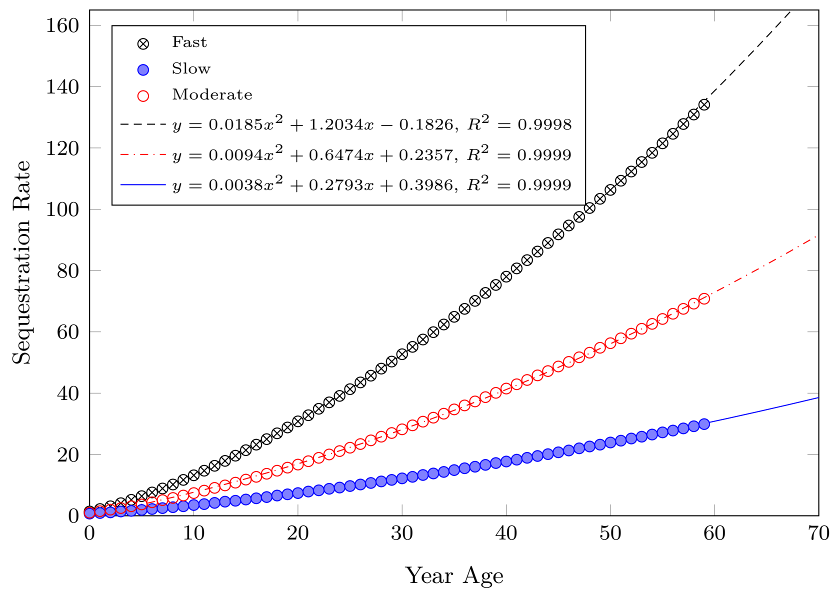


Figure 23: Annual carbon Sequestration Rates for Conifer

correlation between the age of tree and the total leaf mass on it, now we only need to work out the relation between the age of the tree and the size characteristics of it.

了树龄与树叶总质量的关系，则现在只需要确定树龄与树的大小特征间的关系 (即利用树叶总质量为媒介).

Tree size is the accumulation of growth, which is a biological phenomenon of increase with time. According the life cycle of a tree, it experiences three phases, shown as in figure 24.

树的大小是其成长的积累，是随时间增长的生理过程. 就一棵树的生命周期来说，它主要经历如下几个阶段:

As shown above, the cumulative growth curve, whose pattern is characteristic for the life span of an organism, resembles logistic growth function. Thus we construct the following model

如上面所示，累积生长曲线记载了一个生物的寿命跨度，而它的形状与 logistic 增长函数相似. 因此我们建立如下模型

$$P = k_1 \cdot \left(1 - e^{k_2 \cdot A}\right)^{k_3}$$

Transformation leads to the following

转换得

Table 12: Leaf Mass and Tree Age

	Hardwood			Conifer		
Year Age	Slow	Moderate	Fast	Slow	Moderate	Fast
0	520	760	1080	424	606	848
1	640	1080	1600	545	909	1333
2	800	1400	2160	667	1212	1879
3	960	1720	2760	848	1515	2485
4	1120	2080	3400	970	1879	3152
5	1280	2440	4040	1152	2242	3879
6	1480	2840	4720	1333	2667	4606
7	1640	3240	5440	1515	3091	5394
8	1840	3640	6200	1697	3515	6182
9	2000	4080	6960	1879	4000	7091
10	2200	4480	7720	2121	4485	8000
11	2400	4920	8520	2303	4970	8909
12	2600	5400	9320	2545	5515	9879
13	2800	5840	10160	2788	6000	10848
14	3000	6320	11000	2970	6545	11879
15	3240	6760	11880	3212	7152	12970
16	3440	7240	12760	3455	7697	14061
17	3640	7760	13640	3697	8303	15152
18	3880	8240	14520	4000	8909	16303
19	4080	8760	15440	4242	9515	17455
20	4320	9280	16400	4485	10121	18667
21	4560	9760	17320	4788	10788	19879
22	4800	10320	18280	5030	11455	21152
23	5000	10840	19240	5333	12121	22424
24	5240	11360	20240	5576	12788	23697
25	5480	11920	21240	5879	13455	25030
26	5720	12480	22240	6182	14182	26364
27	6000	13000	23240	6485	14909	27697
28	6240	13560	24280	6788	15636	29091
29	6480	14120	25320	7091	16364	30485

	Hardwood			Conifer		
Year Age	Slow	Moderate	Fast	Slow	Moderate	Fast
30	6720	14720	26360	7394	17091	31939
31	7000	15280	27400	7697	17879	33394
32	7240	15880	28480	8061	18606	34848
33	7480	16440	29520	8364	19394	36303
34	7760	17040	30600	8667	20182	37818
35	8000	17640	31720	9030	21030	39333
36	8280	18240	32800	9394	21818	40909
37	8560	18840	33920	9697	22606	42485
38	8800	19440	35040	10061	23455	44061
39	9080	20080	36160	10424	24303	45636
40	9360	20680	37280	10727	25152	47273
41	9640	21320	38440	11091	26000	48909
42	9920	21920	39600	11455	26848	50545
43	10160	22560	40760	11818	27758	52242
44	10440	23200	41920	12182	28606	53939
45	10720	23840	43080	12545	29515	55636
46	11040	24480	44280	12909	30424	57394
47	11320	25120	45440	13333	31333	59091
48	11600	25800	46640	13697	32242	60848
49	11880	26440	47840	14061	33212	62667
50	12160	27120	49080	14485	34121	64424
51	12440	27760	50280	14848	35091	66242
52	12760	28440	51520	15273	36000	68061
53	13040	29120	52720	15636	36970	69939
54	13360	29800	53960	16061	37939	71758
55	13640	30480	55200	16485	38909	73636
56	13920	31160	56480	16848	39939	75515
57	14240	31840	57720	17273	40909	77455
58	14520	32520	59000	17697	41939	79333
59	14840	33200	60240	18121	42909	81273

$$A = k_4 \cdot \ln\left(1 - k_5 \cdot P^{k_6}\right)$$

Where, P is the general profile of a tree(volume, height, mass, diameter). A

其中,P 表示了一棵树的总体轮廓 (包括体积、高、质量、

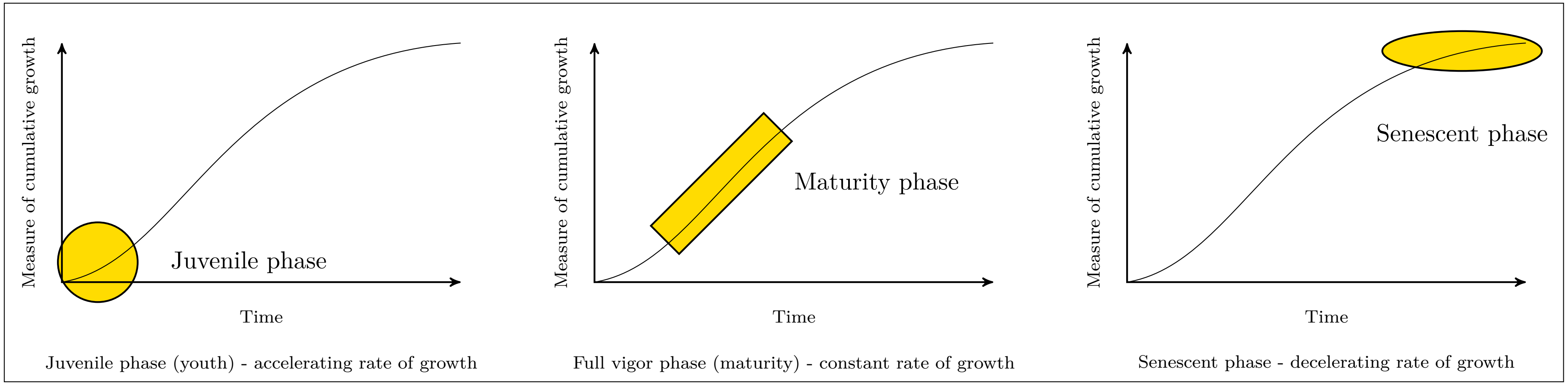


Figure 24: Stages of tree growth[5]

is the age of the tree. k_1, k_2, k_3, k_4, k_5 and k_6 are constants.

It is important to note that P, the general profile of a tree, is a variable to describe the size characteristics of a tree, which vary greatly in different species of trees. Therefore it is not possible to work out an exact set of constants in the model that suits all kinds of trees. This correlation between the size characters and age is primarily qualitative.

直径); A 表示树龄; k_1, k_2, k_3, k_4, k_5 和 k_6 为常数.

需要注意的是, P 是一个用来描绘树的大小特征的变量, 它对不同树种差别很大. 因此, 想为模型找出一组确切的常数来满足所有树种, 是不可能的. 大小特征与树龄间的关系主要还是定性的.

8.4 Leaf Mass and Tree Size

Finally, we get to answer the question whether there exists a correlation between leaf mass and tree size characteristics.

According to our study in X.2, leaf mass and tree age are related with each other mathematically linked by CO2 Sequestration. Our model in X.3 determines a function between tree age and tree size. Through this relationship map shown below, it is clear that there must exist a correlation between leaf mass and tree size.

最后, 我们将回答这样一个问题: 树叶质量与大小特征之间是否存在着一定联系.

根据 8.2 中的讨论结果, 树叶质量与树龄间以 CO2 积累量为媒介联系了起来. 而我们在 8.3 中的模型确定树龄与树的大小特征之间的函数式. 从下面这样一个关系图明显看出, 在树叶质量与树的大小间肯定存在着一定联系.

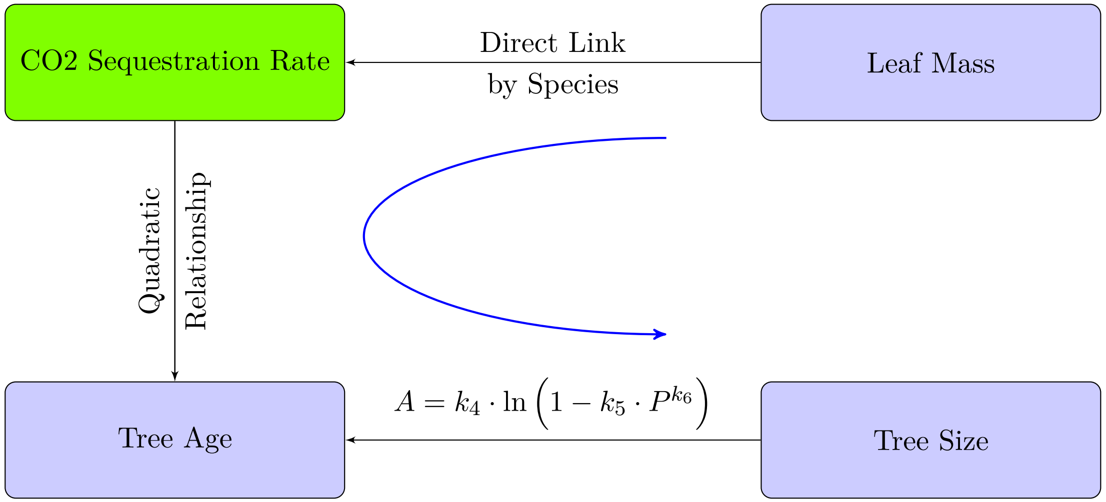


Figure 25: The relationships between leaf mass, tree age and tree size

9 Strengths and Weaknesses

Now we will analyze the strengths and weaknesses of each sub-model in our paper:

- **Model 1**
  - Strength: 1. Our model is based on quantitative analysis, so the classification process is both objective and efficient; 2. Our model chooses leaf types that are most typical and common as the categories.
  - Weakness: Because of the limit of time and access to information, we only create six categories, which may not cover all the leaf types.
- **Model 2**
  - Strength: 1. We have taken into consideration three conditions: tropical zone, temperate zone and frigid zone when discussing the relationship between the leaf distribution and the leaf shape; 2. The result of our model conforms to our observation and the data we found.

现在我们将分别分析本文中每一个子模型的优缺点:

- **模型 1**
  - 优点: 1. 建立在定量分析的基础上, 因此分类过程客观且有效; 2. 选择了最典型、常见的几种树叶类型作为标准类别.
  - 缺点: 由于时间和信息有限, 我们只建立了 6 个标准类别, 而这也许未能涉及所有的树叶类型.
- **模型 2**
  - 优点: 1. 在树叶分布和叶形之间关系的讨论中, 我们考虑了三种情况: 热带、温带和寒带; 2. 模型结果与我们的观测及所用数据相符.

- Weakness: We consider the leaves distribution on a single branch, but have not considered the inner-influence between different leaves of different branches.

- **Model 3**
  - Strength: The whole process use data and quantitative analysis as foundations, so the output is objective and reasonable.
  - Weakness: We have limited categories of tree profiles, so our data to fit the curve is not plenty enough.
- **Model 4**
  - Strength: We use the carbon sequestration rate and age as the mediums to calculate the mass of total laves, which are far more scientific than estimating the number of leaves and weight of each leaf.
  - Weakness: Since the data we get are from the statistics a little bit outdated, the accuracy of our output may need some slight adjustment.

- 缺点: 我们只分析了一条树枝上的树叶分布, 而未考虑不同树枝上的树叶之间内在的相互影响.

- **模型 3**
  - 优点: 整个过程基于数据和定量分析, 因而结果客观合理.
  - 缺点: 我们限制了树形的标准类别, 因此我们用于拟合曲线数据是不够多的.
- **模型 4**
  - 优点: 我们用积累率和树龄作为中间量来估算树叶总质量, 这远比估算叶的数量和每片叶质量要科学得多.
  - 缺点: 由于我们使用的数据时间较早, 因此模型结果的准确性也许需要做些轻微的调整.

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