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## 第四届“认证杯”数学中国

### 数学建模国际赛

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竞赛评阅编号（由竞赛评委团评阅前进行编号）：

## The Design of Seat Back Curve Based on Nonlinear Programming and Smoothing

### Abstract

People have begun to convert their mind in choosing a way of moving as living standard rising, and the people to go abroad have increased rapidly as the trend of economic globalization, have brought great pressure to airlines to increase their capacity to meet the transport demand, so the slimline seats on the airplane emerge at the historic moment. It is because that the people have begun to consider making the comfort on the body and mind as the primary as nowadays, 'humanization' should be put on a great attention so that the airline can triumph in the 'space war' whichever can do well in designing the most comfortable seat back even though it has a thinner backplate and less padding. So how to design a seat back curve to increase the comfort of slimline seats is a problem need to be solved urgently.

The interpersonal spines are different in the modalities, it is related to so many factors, in this thesis we will focus on the design method of seat back curve, not collect a large amount of data to process. Through the analysis of studies, we found that the curve fitted by the least squares Gauss curve fitting method is in conformity with the physiological curve of spine, so we bring out the Gauss curve fitting model. And then, introduces the abstraction of body posture in resting, carries on the theoretical analysis combined with biological engineering and receives five stress points and analyses them. According to the information, we give a nonlinear optimization model and use Lingo 14.0 to calculate, obtain the location information of the stress points, using Gauss curve fitting model for curve fitting, and carry on the smooth optimization to ensure the five points are the sole extremum points of the curve corresponding, this curve is been considered as a man's spinal curve in the condition of nature, at the same time, the joint curve is the best seat back curve to let a person feel more comfortable.

Due to the method of the stress point in the nonlinear optimization model system is sketchy, this thesis considers carrying on a further optimization, so we bring out a new optimization model based on body pressure distribution. In this model, we define 8 indexes to discuss. If the condition permitted we can obtain a lot of data caused by pressure change on different posture (within a certain range) dynamically by the use of the pressure sensor technology, make statistics and analysis, and then find a

pressure range in which comfort level can achieve a higher point. After that, making some modification on the basis of the concept of time series method, get pressure data with the change of the angle between the seat back and seat of each part of spine resulting in the model, and then analyzes and predicts in combination with comfort evaluation model, find out the angle of seat backplate that comfort level can achieve a high place within the scope of the adjustable angle as well as the human body pressure distribution, at this time, the pressure distribution on human's spine can be got after its digital, push reversely to get the seat back curve jointed better with the modality of spine.

Because of time and technical limitations, the comprehensive model this thesis proposed only be put forward the basic thoughts, but this model is practical which can be inferred from the literature. Simulation of artificial intelligence perhaps can be realized if this model can be blended in the idea of finite element method. Thus, it is the task we are likely to attempt.

**Key words:** Set back curve ; Ergonomics ; Least-squares fitting ; nonlinear programming ; Body pressure distribution

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# 1 Introduction

The development of the airlines has reached an unprecedented period of prosperity with the development of society and the progress of science and technology. Along with the improvement of people's standard of living, more and more people are likely to have a trip by air because of its convenience and swiftness, and the price of economy class is not so high. What's more, the evolution of economic globalization indicates that the contact among the states is becoming more and more closely, and the communication is increasing. This phenomenon shows an upward trend in the number of international flights, and the rapid growth of the number of the passengers who choose to take flight also puts a great pressure on the aviation transportation undoubtedly. In view of the fact that increases the number of the flight constantly is unrealistic, so it can be considered to improve the capacity of airlines.

In order to increase the plane's capacity and keep passenger's moveable space does not decreased, most airlines choose to install a new type of seat for their planes, called slimline seat, to replace the old type. This kind of seat have new structure and covered by more slight fabric. This new seat is more light than the old and indeed increases the capacity of plane. However, it also rouses the complain all over the world. People feel very uncomfortable when sitting on it because of its thin backplate and little padding.

As we all know, the past few months have seen several flying accidents. Though more and more people like to go out by plane, some people may hesitate whether to choose it considering the question of safety. In this case, if the seats cannot meet the comfortableness of the seat will receive more attention. As a result, the uncomfortable and thin seats will become the receiving end. Maybe passengers may wonder whether it is because the low price of Economy class that the airlines ignore the needs of passengers and provide to then less better services. Thus the aviation industry will get into crisis. After all, the passengers who take economy class are their major passenger sources.

To sum up, the comfortableness of passengers seats is really a problem urgently to be solved. How to design a good seat that can meet the requirements of the human-machine engineering is a problem that the aviation industry is now facing.

Under the condition of having related spinal data, through the analysis to find a relatively optimal curve fitting method to make the curve that fitted out by this method can match the physiological curve of most human's pines.

## 2 The Description of Problem

An airline seat is a seat on an airliner in which passengers are accommodated for the duration of the journey. Some airlines are now introducing new 'slimline' seats in economy class. These seats, in addition to weighing less, theoretically allow airlines to increase capacity without significantly affecting passenger comfort. These seats may or may not feature moveable headrests, and generally do not feature adjustable lumbar support. Slimline seats are being further refined, liberating more passenger space. The common point of them is a thinner backplate and less padding. However, many passengers have expressed displeasure with these seats.

**Task 1:** Without changing the structure of the premise, how to design seat back curve, in order to make the seat more comfortable?

In order to solve this problem, we can not get a lot of data of different human spine because of the limited time. Therefore, this paper consider focusing on method of put forward. What's more, we find the required data by writing C program which can make the human body vertebra image digitization and coordinated.

- We randomly pick some points from human spine picture to make a scatter diagram. Next we choose some different curve to fit the scatter diagram. So we get some approximate human spine curve in natural condition. Then we compare these curves with human spine image to pick a better curve preliminarily.

- By abstracting analysis on the human body vertebra structures, we have found points respectively on the force analysis, based on the analysis of the existed literature to find the values of parameters which are required and the scope of variables, and nonlinear optimization model is established to find points, then find the coordinates of the stress point according to make them coordinate. Now we can take advantage of the former step in the selection of the optimal function fitting types on the fitting to get the seat backrest curve which we expect.

**Task 2:** How to optimize the seat backplate curve and padding, in order to make the seat more comfortable, without changing the main internal structure?

To solve this problem preferably, we further optimize the model above. In this present model, we just calculate the approximate index of force bearing points and do the rough force analysis, so the error will be dissatisfactory. To minimize the error, we use pressure sensing technology to get seats pressure distributions of different human sitting posture. And then, we code program to analyze these pressure data. Next,



using these data to optimize our curve model. At the end, we build a neural network to evaluate our curve model. The overall flow chart to solve the problem, as shown in figure 1.

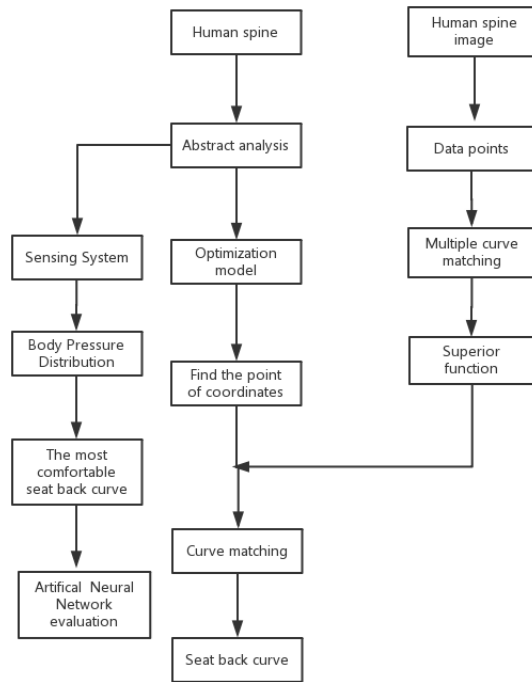


Figure 1: Generate the backrest curve diagram

In addition to the above-described problems, this thesis will give an advertising material for the airline to describe our design features and advantages concisely in the same time.

## 3 Models

### 3.1 Description of Models

#### 3.1.1 Terms in the Thesis

**Human spine :** Human spine is composed of 33 vertebrae (12 pieces of cervical vertebra, 7 pieces of thoracic vertebra, 5 pieces of lumbar vertebra , and sacrum and coccyx which are totally 9 pieces) that are combined with joint, ligaments and inter-

vertebral disc. Spine is upper supported by skull, bottom supported by hip, in the central attached by ribs, and as the back wall of thorax and abdominal and pelvic cavity. Spine has the function of supporting the trunk, protecting viscera, protecting cord and conducting movement. As shown in figure 2.

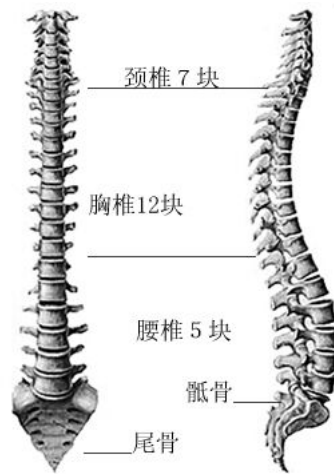


Figure 2: The human body vertebra

**Ergonomics :** So-called ergonomics, which is the research method of such subjects as anthropometry, human body mechanics, physiology of labor, labor psychology, studies on the human body structure characteristic and the function characteristic, provides such human body structure characteristic parameters as the size, weight, surface area, specific gravity, center of gravity of each part of human body, and the relationship and moving range of each part of the human body in the activity . Also it provides such characteristic parameters as the range of output, and the habits on the movement of each part of human body the function of the body, analysis functional characteristics of sensory organs , such as human visual, hearing, touch, skin sensation; It analysis a variety of physiological changes, energy consumption, fatigue mechanism and people's ability to adapt to all kinds of labor load when people are at work. It discusses the factors that affect mental state at work and the influence of psychological factors on the efficiency and so on.

**Artificial Neural Networks :** Artificial Neural network (abbreviation for ANNs) which is called short for Neural Networks (NNs) or Connection Model, it is a kind of algorithm mathematical Model of imitation on animal Neural network behavior characteristics, do information processing with distributed parallel. The network depends on the complexity of the system, by adjusting the internal relations between a large

number of nodes connected, so as to achieve the purpose of information processing. The working principle of the abstraction is shown as figure 3.

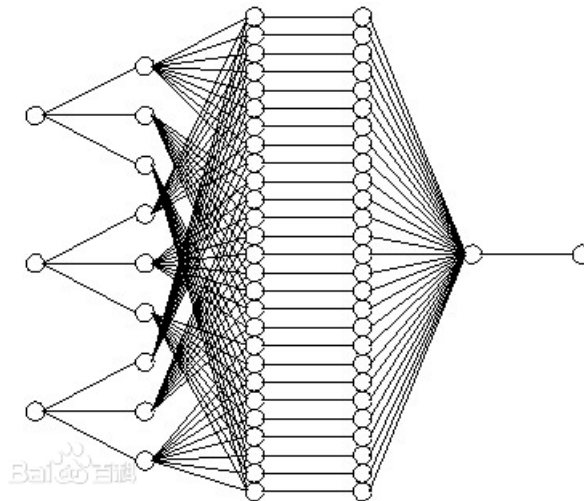


Figure 3: The Working Principle of Artificial Neural Networks

**Sensing Technology :** To obtain information on various types of sensors, they have a variety of physical, chemical or biomass of sensors. According to the convex theorem of information theory, the function of the sensor and quality determines the amount of information and information quality of natural information acquired from information sensing system, is the first key to the structure of high quality sensing technology system. Information processing includes signal preprocessing, post processing, feature extraction and selection, etc. Identification and classification is the main task of the processed information for Recognition. It uses the relationship model between recognized(or) diagnosed objects and the characteristic information and do identification, comparison, classification, and judgment to input feature information.

**Least Squares :**Least squares method (also known as the least square method) is a kind of mathematical optimization techniques. Its data by minimizing the error sum of squares to find the best match function. Can be easily obtained by means of least square method of the unknown data, and make the the sum of the squares of the error between actual data and calculated data to a minimum. The least square method can also be used for curve fitting. Other optimization problems can also be used by minimizing or maximizing entropy energy use least square method to express.

**Curve fitting :**Curve fitting refers to choose the appropriate type of curve to fitting the observed data, and the fitting curve equation analysis of the relationship between the two variables.

### 3.1.2 Assumption of Models

- \* Assuming that the rigidity of the seat frame can meets the requirements;
- \* Regardless of the seat cushion, seat back curve is only considered comfort brought by the change;
- \* Don't consider the cost of materials.

### 3.1.3 Symbols of Models

Symbol	Description
$\theta$	Tilt angle of seat back
$H$	Backrest height
$L$	The distance between two seats

## 3.2 The Foundation of Seat Back Curve Model

As a matter of time, we can not get the data of human spine about different age stages, sexes , nationalities and races under natural state. Because airplane seats are designed for most of the normal human body, this paper extracts data points from ordinary normal spinal image of the body. In the first place, we use C++ programming . According to a series of transformations based on the differences of the spine image grey value, we achieve the extraction of data points. Then, by making these data points coordinated and digitization under coordinate system, we can get a point column from the curve of the human body vertebra. These points are used in the model we build the next as the known data points.

### 3.2.1 Gauss Curve Fitting Model

Given these data of points :  $\{x_i, y_i\}$ , here  $y_i = f(x_i)$  ( $i = 0, \dots, m$ ), to find a function  $y = S^*(x)$  with the given data  $(x_i, y_i)$ ,  $i = 0, 1, \dots, m$  fitting, If define the error is  $\delta_i = S^*(x_i) - y_i$  ( $i = 0, 1, \dots, m$ ),  $\delta = (\delta_0, \delta_1, \dots, \delta_m)^T$ , Set  $\varphi_0(x), \varphi_1(x), \dots, \varphi_n(x)$  is the linearly independent functions on  $C[a, b]$ , find a function  $S^*(x)$  in  $\varphi = \text{span}\{\varphi_0(x), \varphi_1(x), \dots, \varphi_n(x)\}$  to minimize the error sum of squares.

$$\|\delta\|_2^2 = \sum_{i=0}^m \omega(x_i) \delta_i^2 = \sum_{i=0}^m \omega(x_i) [S^*(x_i) - y_i]^2 = \min_{S(x) \in \varphi} \sum_{i=0}^m \omega(x_i) [S(x_i) - y_i]^2 \quad (1)$$

In the above type  $\omega(x) \geq 0$  is the weight function on  $[a, b]$ , it stands the differences in the proportion of different data  $(x_i, f(x_i))$ , and here,

$$S(x) = a_0\varphi_0 + a_1\varphi_1 + \dots + a_n\varphi_n \quad (n < m) \quad (2)$$

Using least square method for fitting curve of the problem, is in the form such as (2) in the  $S(x)$  for a function  $y = S^*(x)$ , to gain the minimum of (1). It is transformed into the problem of finding out the minimum point  $(a_0^*, a_1^*, \dots, a_n^*)$  of the function with many variables.

$$I(a_0, a_1, \dots, a_n) = \sum_{i=0}^m \omega(x_i) \left[ \sum_{j=0}^n a_j \varphi_j(x_i) - f(x_i) \right]^2 \quad (3)$$

By the necessary conditions of multivariate function extreme value, there is

$$\frac{\partial I}{\partial a_k} = 2 \sum_{i=0}^m \omega(x_i) \left[ \sum_{j=0}^n a_j \varphi_j(x_i) - f(x_i) \right] \varphi_k(x_i) = 0, \quad k = 0, 1, \dots, n \quad (4)$$

Defines that

$$\begin{aligned} (\varphi_j, \varphi_k) &= \sum_{i=0}^m \omega(x_i) \varphi_j(x_i) \varphi_k(x_i), \\ (f, \varphi_k) &= \sum_{i=0}^m \omega(x_i) f(x_i) \varphi_k(x_i) \equiv d_k, \quad k = 0, 1, \dots, n, \end{aligned}$$

So

$$(\varphi_k, \varphi_j) a_j = d_k, \quad k = 0, 1, \dots, n. \quad (5)$$

The type can be written as a matrix form

$$\mathbf{Ga} = \mathbf{d}, \quad (6)$$

Here

$$\mathbf{a} = (a_0, a_1, \dots, a_n)^T, \quad \mathbf{d} = (d_0, d_1, \dots, d_n)^T$$

,

$$\mathbf{G} = \begin{pmatrix} (\varphi_0, \varphi_0) & (\varphi_0, \varphi_1) & \dots & (\varphi_0, \varphi_n) \\ (\varphi_1, \varphi_0) & (\varphi_1, \varphi_1) & \dots & (\varphi_1, \varphi_n) \\ \vdots & \vdots & & \vdots \\ (\varphi_n, \varphi_0) & (\varphi_n, \varphi_1) & \dots & (\varphi_n, \varphi_n) \end{pmatrix} \quad (7)$$

The above as the basic principle of least square method and solution method[1], applied to the gauss function  $f(x) = ae^{(-Cx^2)}$ , through matlab to calculate the fitting function

$$f_1(x) = a_1 e^{-((x-b_1)/c_1)^2} + a_2 e^{-((x-b_2)/c_2)^2} + a_3 e^{-((x-b_3)/c_3)^2} + a_4 e^{-((x-b_4)/c_4)^2}$$

here,  $a_1 = 10.98, b_1 = 47.44, c_1 = 39.59, a_2 = -4.721, b_2 = 677.2, c_2 = 91.87, a_3 = -26.19, b_3 = 199.2, c_3 = 205.1, a_4 = 33.53, b_4 = 129.3, c_4 = 593.2$

To clearly present the gauss function fitting based on least square method, the advantages of we propose two ordinary polynomial interpolation function

Four interpolation polynomial:

$$f_2(x) = \alpha_4 x^4 + \alpha_3 x^3 + \alpha_2 x^2 + \alpha_1 x + \alpha_0$$

here  $\alpha_4 = 1.268 \times 10^{-9}, \alpha_3 = -2.48 \times 10^{-6}, \alpha_2 = 0.001593, \alpha_1 = -0.3737, \alpha_0 = 37.2$

Six time interpolation polynomial:

$$f_3(x) = \beta_6 x^6 + \beta_5 x^5 + \beta_4 x^4 + \beta_3 x^3 + \beta_2 x^2 + \beta_1 x + \beta_0;$$

$\beta_6 = -1.521 \times 10^{-14}, \beta_5 = 4.038 \times 10^{-11}, \beta_4 = -3.966 \times 10^{-8}, \beta_3 = 1.718 \times 10^{-5}, \beta_2 = -0.002888, \beta_1 = 0.04139, \beta_0 = 27.87$

To make the comparison more intuitive, we place the fitting curve with the human spine image under the same picture, as shown in figure 4 (*I*:the curve of four interpolation polynomial, *II*:the curve of gauss function fitting, *III*:the curve of six interpolation polynomial).

It can be seen by the image that the gauss curve based on the least square method [2] fitting more relevant with human<sup>s</sup> spine.

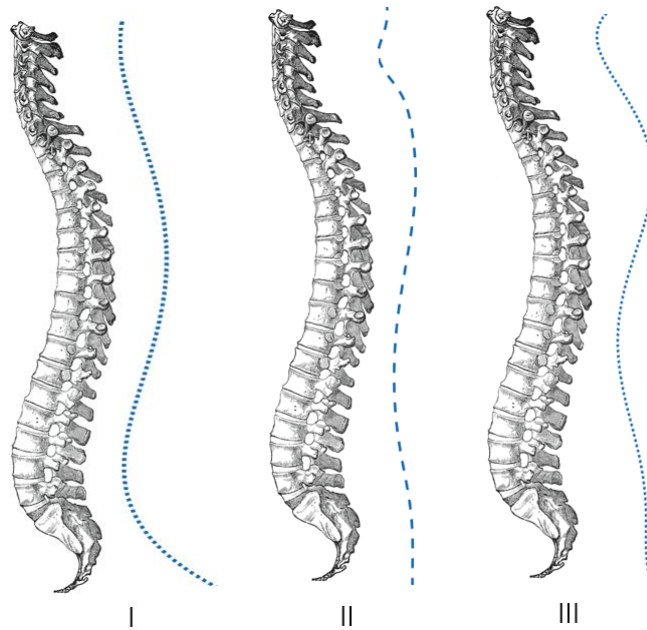


Figure 4: Different curve fittings

### 3.2.2 Nonlinear Programming Model

Considering the design that we want is a plane curve of economy class seat. Standing on the passenger's point of view, what they want is a comfortable chair to rest. But for the economy class seat is more, the space assigned for each passenger is narrow, so the adjustment of the back of a chair can only be on a small scale. Therefore, within the scope of it, we need to find a seat backrest angle which most people think is comfortable. Under the tilt angle, we analyze the spinal curve and find the curve of joint. As a result, the backrest curve based on the curve of joint should be the best in theory.

It's our primary task to find the human spine's pressure changes in small ranges. We find that people keeping sitting position is easy to feel exhausted in five body's positions by analysing the existing literature[3]. So we will keep sitting under the abstraction of the spine(see figure 5), and will abstract five parts that people are easy to feel painful to five spinal stress points. We want to find that human comfort can be described by all the pressure on the spinal stress points only the backrest angle adjustment under the condition of a small scale, when the pressure on the each stress point reach a relatively low level, the spine curve is that we want to design backrest curve.

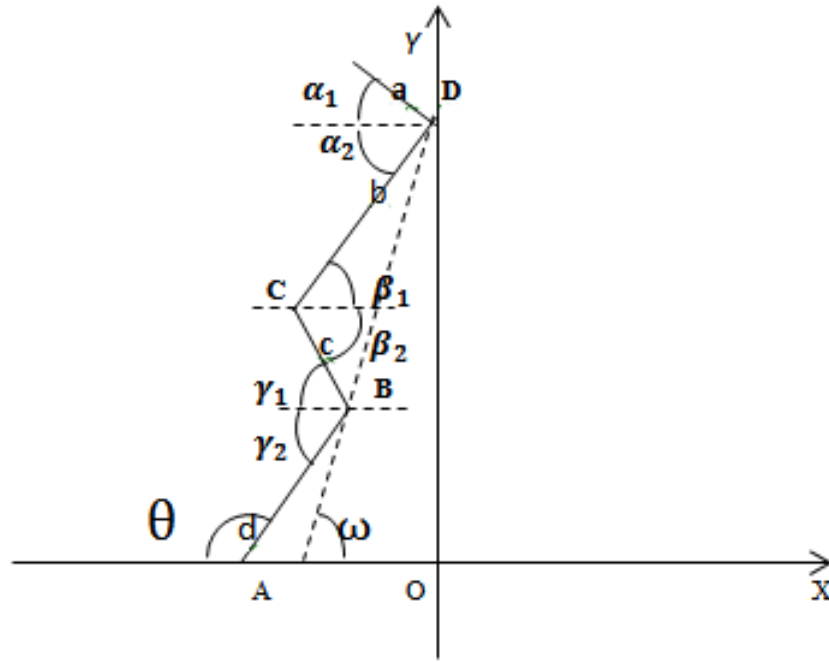


Figure 5: Sitting under the spine abstract sketch

Backstepping method : because we want to get a seat back curve, namely we are going to get the homologous spinal curve when the pressure by people felt in the backrest angle range that is limited to find the relatively minimum pressure for spinal curve. Because the curve can't be directed get, this paper considered to determine the location of the stress points, and because in the gaussian curve fitting model we can get a kind of approximate fitting human spine curve fitting method, we can combine gauss curve fitting model for spinal curve at this time. Please see the following specific analysis.

Analysis of the abstract of the abstract figure (figure five) in the case of keep sitting :

Respectively analysing on spine points A, B, C, D, E stress analysis (provisions to the right and upward force is positive), considering giving a function on the spine mainly affected by the rallying point of stress, and we found that fatigue degree ranking :  $B > E > A > D > C$  through the relevant data access, so different points are given the corresponding weight in order to get the objective function considering



$$\min = \sum_{i=1}^5 C_i Z_i + C_i F_i$$

Here

$$Z_1 = d\sin\gamma_2 + c\sin\gamma_1 + b\sin\beta_1 + a\sin\alpha_1$$

$$F_1 = d\cos\gamma_2 - c\cos\gamma_1 + b\cos\beta_1 - a\cos\alpha_1$$

$$Z_2 = c\sin\gamma_1 + b\sin\beta_1 + a\sin\alpha_1$$

$$F_2 = -c\cos\gamma_1 + b\cos\beta_1 - a\cos\alpha_1$$

$$Z_3 = b\sin\beta_1 + a\sin\alpha_1$$

$$F_3 = b\cos\beta_1 - a\cos\alpha_1$$

$$Z_4 = a\sin\alpha_1$$

$$F_4 = -a\cos\alpha_1$$

$$Z_5 = C^*$$

$$F_5 = C^{**}$$

$C_i, (i = 1, 2, 3, 4, 5)$  is the weight;  $C^*, C^{**}$  are fixed constants ,and

$$0 < C_3 < C_4 < C_1 < C_5 < C_2 < 1$$

$$\sum_{i=1}^5 C_i = 1$$

Because that human spine is composed of 33 vertebrae, it includes 12 pieces of cervical vertebra, 7 pieces of thoracic vertebra, 5 pieces of lumbar vertebra, sacrum and coccyx which are totally 9 pieces. So we think approximately that

$$a : b : c : d = 4 : 12 : 7 : 9$$

We can see from the image (figure 5) that

$$\beta_1 = \alpha_2$$

$$\beta_2 = \gamma_1$$

However, combined with the literature [4] [5] [6], the posture condition for spine angle between each part is

$$\begin{aligned} \frac{85 \times 180}{\pi} &\leq \alpha_1 + \alpha_2 \leq \pi \\ \frac{103 \times 180}{\pi} &\leq \beta_1 + \beta_2 \leq \pi \\ \frac{94 \times 180}{\pi} &\leq \gamma_1 + \gamma_2 \leq \frac{\pi}{2} \end{aligned}$$

Because the economy class seat backrest height and the distance between the seats there are mutual constraint, as shown in figure 6.

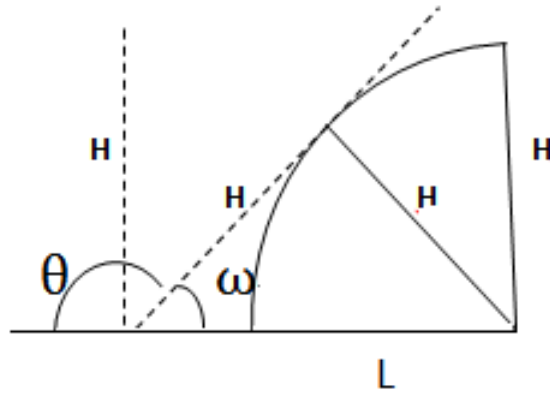


Figure 6: Seats can be adjusted angle diagram

On the figure,  $H$  represents the height of chair,  $L$  said spacing. According to [7], the plane has a prescribed scope economy class seats height and spacing, but consider to use the purpose of the ultra-thin seat aircraft economy class, we considered the provisions of the spacing of the minimum value, namely the  $L = 81.3cm$ , and according to the average height of statistical data and select  $H = 65.5cm$ [8].

By this way, we can get the seat backplate allows adjustment of Angle

$$\theta = \arcsin \frac{H}{L}$$

So through figure 5, we can get

$$\frac{\pi}{2} \leq \beta_2 + \gamma_1 \leq \theta$$

What's more, there are

$$Z_1 < H$$

From what has been discussed above, we can get a nonlinear programming model

$$\begin{aligned} \min &= \sum_{i=1}^5 C_i Z_i + C_i F_i & (8) \\ s.t. & \left\{ \begin{array}{l} 0 < C_3 < C_4 < C_1 < C_5 < C_2 < 1 \\ \sum_{i=1}^5 C_i = 1 \\ a : b : c : d = 4 : 12 : 7 : 9 \\ \beta_1 = \alpha_2 \\ \beta_2 = \gamma_1 \\ \frac{85 \times 180}{\pi} \leq \alpha_1 + \alpha_2 \leq \pi \\ \frac{103 \times 180}{\pi} \leq \beta_1 + \beta_2 \leq \pi \\ \frac{94 \times 180}{\pi} \leq \gamma_1 + \gamma_2 \leq \frac{\pi}{2} \\ \frac{\pi}{2} \leq \beta_2 + \gamma_1 \leq \theta \\ Z_1 < H \end{array} \right. & (9) \end{aligned}$$

To solve the model with Lingo 14.0 , which is available to seat backrest angle changes within a certain range that people pressure on the spinal main points reaching the minimum spine angle between each part (income line of the adjacent two points wired angle), and through

$$A(0, a \cos \gamma_2 + (b \sin(\frac{\pi}{2} - \gamma_2) - c \cos \beta_2))$$

$$B(d \sin \gamma_2, b \sin(\frac{\pi}{2} - \alpha_2) - c \cos \beta_2)$$

$$C(d \sin \gamma_2 + c \sin \gamma_1, b \sin(\frac{\pi}{2} - \alpha_2))$$

$$D(d \sin \gamma_2 + c \sin \gamma_1 + b \sin \beta_2, 0)$$

$$E(d \sin \gamma_2 + c \sin \gamma_1 + b \sin \beta_1 + a \sin \alpha_1, a \cos \alpha_1)$$

Getting into the corresponding data in figure 5 coordinate system can be obtained under various stress point coordinates, which is combined with gauss curve fitting model to get fitting curve, and then to adjust and optimize the curve smoothly to ensure the loading points which are obtained, the fitting of the curve function of the second derivative is zero, namely, let the extreme value point of the fitted curve close to prayer points. So that we can get in a relatively relaxed state of spinal curve, backrest curve is obtained, as shown in figure 7 (on the right side of the seat on the back).

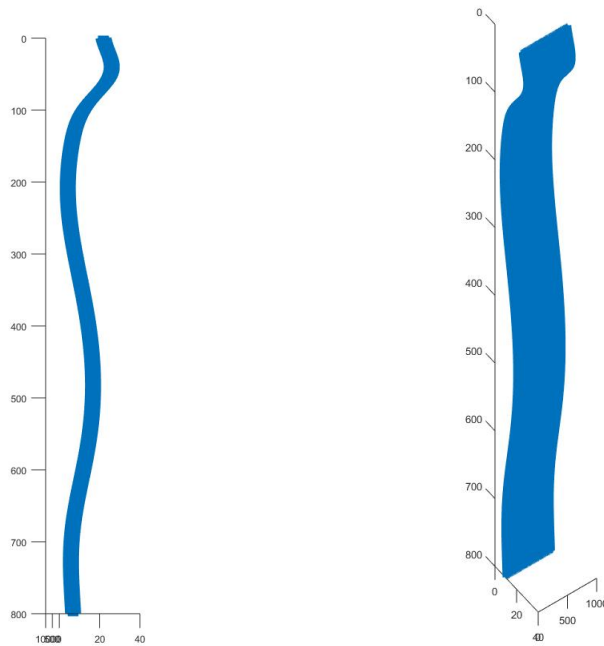


Figure 7: Seat backplate curve

## 4 Future Work

Due to the nonlinear programming optimization model selection in spinal stress point that is too rough, force analysis process is not rigorous, although using back-stepping principle and selected the novel ideas to solve the problem of nonlinear programming, but the error is bigger. Considering the rapid development of science and technology today, using the pressure sensor technology dynamic monitoring data is not too difficult to get a lot of more precise, but because of tight time and technology and limited tools, we can't get a large number of real data, so only the theoretical analysis is presented. Under the condition of conditions allow, we can undertake follow-up studies. We get a lot of different posture inferior when sitting on the seat backrest contact surface pressure through the pressure sensor technology and its distribution, and get the related pressure parameters and data visualization etc. Under the condition of static and dynamic analysis of different surface modeling back of a chair body pressure distribution index and comfort. Because the body pressure distribution on maximum pressure, average pressure, maximum pressure gradient, average pressure gradient, asymmetric coefficient, the longitudinal pressure distribution curve, longitudinal moment distribution curve, comfort factor to depict[9], therefore the relevance with the comfort level between the inevitable, we can through the establishment of neural network optimization evaluation model give further evaluation. So we can further research direction on this part, if you can get a large amount of data, can perform data statistical analysis, combined with the nonlinear programming model of this paper, and based on the method of time series based on the theory of transformation, analysis, get the human body vertebra curve changes over backrest Angle. If this concept can be achieved, whether or not only can be used to design the man-machine engineering, the various seat can also be applied in the biological medicine, to the human body vertebra changes with the emergence of the posture of different detection and warning, problems of the human body vertebra between warning is given, and corresponding suggestions, this is a very meaningful work.

## 5 Advertising Material

Please pay your attention on this! Due to your company in the economy class to implement using slimline seats and cause your passengers in reducing, or your passengers complain to you? If your answer is yes, then you need to pay attention to this advertising, because the content of this advertisement is likely to improve your situation! If your answer is no, then you also need to have a look at it, or be careful failed in the heat of the space war!

It is obvious, as people living standard rise, the plane because of its convenient and quick have become a travel personage's first choice, and because the economy class price is reasonable suit to the broad masses of ordinary working-class, so buy economy ticket passengers became the airline's main customers. At the same time, the development of economic globalization also indicates the state and the link between countries more and more closely, more and more communication, this phenomenon leads to the number of international flights around the world show clear rising trend, a rapid growth of the number of international flights. No doubt these factors to the air transport has brought business opportunities but also brought great pressure and challenge. Due to increasing blindly flights are not realistic, so most airlines choose to increase the load factor to solve the problem of the plane, but also because the plane cabin space is limited, so airlines will start towards the plane seat. As a transportation service industry for the development of increasingly perfect, although take a fancy to yield problems but also cannot ignore the service quality factors, the performance of the blind pursuit of high yield is short-sighted. Because with that in mind, many airlines choose as far as possible not to reduce the original scope of activity space per passenger in economy class, but the line of sight on the reducing the thickness of the seat, then you begin to use ultra-thin seat without significantly affect the comfort of the passengers in order to achieve the purpose of the premise to increase the capacity, however, in the age of the word 'human', this change will inevitably cause economy class passengers began to complain and complain. Maybe they will say, for profit squeeze their ordinary working-class people, at this time, what's your airline is not only a loss of customers, but also the broad masses of people's moral condemnation. To make matters worse, this paragraph of time of the plane crash event, it's more aggravated the crisis in the you isn't it? Some people consider the safety factor and began to hesitate whether to choose their flying, airline seats are not comfortable with you this

problem, so these hesitate before people will have relatively more people choose to give up on your company's flight, so in the increasingly fierce space war you are bound to fail! So, how to guarantee profits, as well as provide more comfortable service? Take a look at our design you will understand!

Our products are through the aircraft economy class seat physical and human spine image digital mathematical model is established through theoretical analysis first, after through software programming to realize the artificial intelligence in virtual network space design testing for many times, and all process is visual, you can see a pile of numerical how to step by step into the seat material. But as a result of the limitation of time and money, we just based on the theory is verified this idea can be sexually, but don't despair, after listening to my introduction, you will have strong interest in it does not necessarily!

It is the seat of your airline is used or intended to be put into use which called the slimline seat , but the key point is that it is more comfortable compared with normal seats. Although the backplate is very thin, and it has less padding, but the design of curve of backplate will made up for the shortcomings, because we can design the backplate curve that can fit passengers feel more comfortable when flying under the condition of spinal curve, the approximate reach under the natural state of the human body vertebra curve, therefore, if the plane in your company to use our design by the backrest, so most of the passengers sat on a seat can feel reached a comfortable even after the most comfortable sitting position, before the seats and seat than the back of a chair can adjust the angle is small, but it doesn't affect the seat can be applied to most people. So the advantage of this kind of chair is that it can be to deal with the problem of comfortable, also addressed the plane often occurs because the backrest angle is too large to affect others in debate, so have eyes you will find our advantage!

If you are interested in it , please contact us, we believe that your company in the use of our seat design by will beat other companies with more chances!

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## Appendix

Program A:

```
#include <iostream>
#include <vector>
#include <sstream>
#include <fstream>
#include <opencv2\opencv.hpp>

using namespace std;
using namespace cv;

void loadImgPath(vector<string> & imgPath, string prefix,
    int num = 1, string type = ".jpg")
{
    stringstream ss;
    for (int i = 1; i <= num; i++)
    {
        ss << prefix << i << type;
        string path;
        ss >> path;
        ss.clear();
        imgPath.push_back(path);
    }
}

struct MyPoint
{
    int x = 0, y = 0;
};

int main()
{
    vector<string> imgPath;
    string prefix = "E:\\MathModel\\spineImg\\", type = ".jpg";
```

```
int num = 5;
loadImgPath(imgPath, prefix, num, type);

cout << "Read Img!" << endl;
Mat spineImg = imread(imgPath[1], CV_LOAD_IMAGE_GRAYSCALE);
int row = spineImg.rows,
col = spineImg.cols,
channel = spineImg.channels(),
dis = row / 60,
cmp = *(spineImg.ptr<int>(1) + 1);
cout << "Row: " << row << "    Col: " << col << endl;
MyPoint result[37];
int count = 0;
cout << "Compare pix!\n";
cout << "cmp pix = " << cmp << endl;
for (int r = 0; r < row; r += dis)
{
    int tempRow = r;
    if (r == 0)
        tempRow = dis / 2 + 1;
    else if (r + dis >= row)
        tempRow = row - dis / 2;
    cout << "Comparing at row " << tempRow << endl;
    int * rowPix = spineImg.ptr<int>(tempRow);

    for (int c = 0; c < col * 2 / 3; c++)
    {
        if (rowPix[c] != cmp)
        {
            cout << "record pix = " << rowPix[c] << endl;
            result[count].y = row - tempRow;
            result[count++].x = c;
            break;
        }
    }
}
```

```
}  
}  
  
cout << "data count: " << count << endl;  
}  
cout << "Writing file!" << endl;  
ofstream fout,foutX,foutY;  
fout.open("E:\\MathModel\\spineData\\spine1.txt", ios::out);  
foutX.open("E:\\MathModel\\spineData\\spine1x.txt", ios::out);  
foutY.open("E:\\MathModel\\spineData\\spine1y.txt", ios::out);  
for (int i = 0; i < count; i++)  
{  
    fout << result[i].x << " " << result[i].y << endl;  
    foutX << result[i].x << ",";  
    foutY << result[i].y << ",";  
}  
fout.close();  
foutX.close();  
foutY.close();  
cout << "Done!" << endl;  
return 0;  
}
```

Program B:

%Load human spine data.

spineX = ...

```
[2,0,2,2,2,1,2,2,2,2,3,4,5,7,9,10,11,12,14,15,16,17,16,17,16,17,16,
16,16,15,15,14,14,13,12,11,9,9,7,7,5,4,3,2,2,1,1,0,1,2,3,2,4,8,12,
16,22,24,22,22,15;2,0,2,2,2,2,2,0,2,2,4,4,6,6,8,10,12,12,14,15,16,
16,16,16,16,16,16,16,16,16,14,14,14,14,12,12,10,10,8,6,6,6,4,4,
4,4,5,4,4,6,6,8,11,14,20,24,26,26,24,20;14,13,10,11,10,10,10,10,10,
10,10,11,12,12,12,12,12,0,13,13,14,15,14,15,15,15,14,14,13,13,12,13,
12,5,10,9,9,6,5,4,3,2,0,0,0,1,0,2,4,7,5,4,6,14,17,17,18,18,18,16,15;
3,1,1,2,2,1,2,2,4,4,5,6,6,7,9,10,11,11,11,12,12,12,12,11,11,11,11,10,
8,10,6,7,6,7,6,5,4,4,2,0,0,0,0,0,0,0,0,1,1,2,4,4,5,9,10,12,20,24,28,
28,28;4,2,2,2,2,4,4,4,6,6,6,8,8,10,10,12,12,12,14,14,14,14,14,14,14,
13,12,12,10,10,8,8,6,6,4,4,2,2,2,0,2,0,2,2,2,4,4,4,6,6,10,12,12,14,
16,26,26,24,24,20,22];
```

```
spineY = [779,773,760,747,734,721,708,695,682,669,656,643,630,617,604,
591,578,565,552,539,526,513,500,487,474,461,448,435,422,409,396,383,
370,357,344,331,318,305,263,279,266,253,240,227,214,201,188,175,162,
149,136,123,110,97,84,71,58,45,32,19,6];
```

%Move down the whole curve and let the lowest point fit the x-axis.

Then move up the curve by 5 units.

```
minX = min(spineX,[],2);
```

```
tempX = spineX - repmat(minX,1,size(spineX,2)) + 5;
```

%Compute the mean value of the 5 spine data in spineX.

```
meanX = mean(tempX);
```

%Fit a quartic polynomial

```
[p4,s4] = polyfit(spineY,meanX,4);
```

%Fit a sextic polynomial

```
[p6,s6] = polyfit(spineY,meanX,6);
```

```
x = linspace(0,800);
%Bring the coefficient we got above into thoes two polynomials.
polyCurve4 = p4(1)*(x.^4) + p4(2)*(x.^3) + p4(3)*(x.^2) + p4(4)*x + p4(5);
polyCurve6 = p6(1)*(x.^6) + p6(2)*(x.^5) + p6(3)*(x.^4) + p6(4)*(x.^3) +
            p6(5)*(x.^2) + p6(6)*x + p6(7);

%Fit a Gaussian Function
initGaussBeta = [28.6000,32,54.5070,19.8000,513,38.6827,18.2605,435,41.0454,
15.4605,591,65.6288];
gaussBetaFit = nlinfit(spineY,meanX,@Gaussian,initGaussBeta);
ga1 = gaussBetaFit(1);
gb1 = gaussBetaFit(2);
gc1 = gaussBetaFit(3);
ga2 = gaussBetaFit(4);
gb2 = gaussBetaFit(5);
gc2 = gaussBetaFit(6);
ga3 = gaussBetaFit(7);
gb3 = gaussBetaFit(8);
gc3 = gaussBetaFit(9);
ga4 = gaussBetaFit(10);
gb4 = gaussBetaFit(11);
gc4 = gaussBetaFit(12);
%Bring the coefficient we got above into the Gaussian Function.
gaussCurve = ga1*exp(-((x-gb1)/gc1).^2) + ga2*exp(-((x-gb2)/gc2).^2)
            + ga3*exp(-((x-gb3)/gc3).^2) + ga4*exp(-((x-gb4)/gc4).^2);

%Compute first derivative functions of the 3 fitting functions.
syms x;
g(x) = ga1*exp(-((x-gb1)/gc1)^2) + ga2*exp(-((x-gb2)/gc2)^2)
      + ga3*exp(-((x-gb3)/gc3)^2) + ga4*exp(-((x-gb4)/gc4)^2);
diffG(x) = diff(g(x));

polyFour(x) = p4(1)*(x^4) + p4(2)*(x^3) + p4(3)*(x^2) + p4(4)*x + p4(5);
```

```
diffPF(x) = diff(polyFour(x));

polySix(x) = p6(1)*(x^6) + p6(2)*(x^5) + p6(3)*(x^4) + p6(4)*(x^3)
            + p6(5)*(x^2) + p6(6)*x + p6(7);
diffPS(x) = diff(polySix(x));

%Compute extreme points of every fitting functions.
x0 = real(double(solve(diffG(x))));
x1 = real(double(solve(diffPF(x))));
x2 = real(double(solve(diffPS(x))));

%Compute 3 important angles of every curve.
gdeg1 = radtodeg(atan(eval(diffG(49))))
        - radtodeg(atan(eval(diffG(32))))
        + 180;
gdeg2 = radtodeg(atan(eval(diffG(179))))
        - radtodeg(atan(eval(diffG(245))))
        + 180;
gdeg3 = radtodeg(atan(eval(diffG(518))))
        - radtodeg(atan(eval(diffG(499))))
        + 180;

pfdeg1 = radtodeg(atan(eval(diffPF(133))))
        - radtodeg(atan(eval(diffPF(257))))
        + 180;
pfdeg2 = radtodeg(atan(eval(diffPF(545))))
        - radtodeg(atan(eval(diffPF(420))))
        + 180;
pfdeg3 = radtodeg(atan(eval(diffPF(720))))
        - radtodeg(atan(eval(diffPF(898))))
        + 180;

psdeg1 = radtodeg(atan(eval(diffPS(31))))
```

```
- radtodeg(atan(eval(diffPS(-12))))  
    + 180;  
psdeg2 = radtodeg(atan(eval(diffPS(183))))  
    - radtodeg(atan(eval(diffPS(259))))  
    + 180;  
psdeg3 = radtodeg(atan(eval(diffPS(525))))  
    - radtodeg(atan(eval(diffPS(414))))  
    + 180;
```