# A Convenient Truth: A model for sea level rise forecast
# 一个显而易见的真相：海平面上升预测模型

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翻译：周吕文

[0]若发现翻译问题，请邮件告知，谢谢。本文中的翻译参考了网上一份译稿 (原译稿作者不详), 在此表示感谢!

**Abstract**

Greenhouse-gas emissions have produced global warming, including melting in the Greenland Ice Sheet (GIS), resulting in sea-level rise, a trend that could devastate coastal regions. A model is needed to quantify effects for policy assessments.

We present a model that predicts sea-level trends over a 50-year period, based on mass balance and thermal expansion acting on a simplified ice-sheet geometry. Mass balance is represented using the heat equation with Neumann conditions and sublimation rate equations. Thermal expansion is estimated by an empirically-derived equation relating volume expansion to temperature increase. Thus, the only exogenous variables are time and temperature.

We apply the model to varying scenarios of greenhouse-gas-concentration forcings. We solve the equations numerically to yield sea-level increase projections. We then project the effects on Florida, as modeled from USGS geospatial elevation data and metropolitan population data.

The results of our model agree well with past measurements, strongly supporting its validity. The strong linear trend shown by our scenarios indicates both insensitivity to errors in inputs and robustness with respect to the temperature function.

Based on our model, we provide a cost-benefit analysis showing that small investments in protective technology could spare coastal regions from flooding. Finally, the predictions indicate that reductions in greenhouse-gas emissions are necessary to prevent long-term sea-level-rise disasters.

温室气体的排放引起了全球变暖, 包括格陵兰岛冰盖的融化引起的海平面上长升, 这种趋势可能破坏海岸地区. 需要建立一个模型来量化海平面上升在不同政策下的影响.

本文建立了一个模型来预测未来 50 年海平面上升趋势, 这个模型是依据质量平衡和热膨胀作用在简化冰原几何体上建立的.

质量平衡是用具有 Neumann 边界条件的热传导方程和升华速率方程来表示的. 热膨胀通过有关热胀冷缩的经验方程来估计的. 因此，外部变量只有时间和温度.

我们将该模型应用到不同温室气体排放量的情况下. 通过数值求解方程得出海平面上升的预测. 然后, 我们从 USGS 地理空间数据和城市人口数据来建模预测海平面上升对佛罗里达州的影响.

本文模型的结果与过去测量非常吻合, 这证明了本文模型的有效性. 明显的线性趋势表明了本文的模型对输入量具有较高的容错性, 对温度函数具有较高的鲁棒性.

根据我们的模型, 我们还提供了一个成本效益分析, 分析表明: 在保护技术上的小投资可以预防陆地区域被洪水淹没. 最后, 预测表明: 从长远考虑, 要预防海平面上升引起的灾害, 就必需减少温室气体的排放.

# Contents

# 1 Introduction

Strong evidence of a global warming trend exists, and powerful models have been created to estimate future climate. Temperatures have increased by about $0.5\,^\circ$C over the last 15 years, and global temperature is at its highest level in the past millennium . Although the warming trend is quite evident, the consequences of such wide scale climate change are still poorly understood. One of the most-feared consequences of global warming is sea level rise, and for good reason. TOPEX/Poseidon satellite altimeter indicates that sea levels rose $3.2 \pm 0.2mm$ annually during 1993-1998 . Indeed, Titus et al estimate that a 1 meter rise in sea levels could cause $270-475 billion in damages in the United States alone.

据有力证据表明全球变暖趋势确实存在, 目前已经建立了很多有效模型来预测未来的气候. 在过去的 15 年内, 全球气温上升了大约 $0.5\,^\circ$C, 全球气温达到了过去的一千年以来的最高水平. 虽然变暖的趋势相当明显, 但人们对这种大规模的气候变化仍然是知之甚少. 全球变暖带来的最令人担心的后果之一便是海平面的上升. 据 TOPEX/Poseidon 卫星[1]测高仪观测显示, 在 1993-1998 年期间, 海平面每年上涨 $3.2 \pm 0.2mm$. Titus 等人估计, 海平面上升 1 米, 仅美国就可能导致 270-475 亿美元的损失.

A number of complex factors underlie sea level rise. Thermal expansion of water due to temperature changes has long been implicated as the major component of sea level rise; however, recent studies have shown that thermal expansion alone cannot account for a majority of the observed increases. Mass balance of large ice sheets, in particular the Greenland Ice Sheet, is now believed to play a major role in sea level. The mass balance is controlled by two major processes, accumulation (influx of ice to the sheet) and ablation (loss of ice from the sheet) . Accumulation is primarily the result of snowfall; ablation is a result of sublimation and melting.

若干复杂的因素导致了海平面的上升. 人们一直认为, 因温度变化而导致的水的热膨胀是海平面上升的重要组成部分;　然而, 近期研究表明, 单是热膨胀并不能解释已观测到的大多数海平面的上升现像. 大块冰原[2]的物质平衡, 特别是格陵兰岛的冰原, 现在被认为是海平面上升的主要因素. 物质平衡是由 2 个主要过程控制的: 积累 (冰块汇集到冰原) 和消融 (冰块脱离冰原). 积累主要是由于降雪; 消融主要是由于冰的升华和融化.

Contrary to popular belief, however, floating ice does not play a significant role in sea level rise. By Archimedes' Principle, the volume increase $\Delta V$ of a body of water with density $\rho_{\text{ocean}}$ due to melting of floating ice of weight $W$ (assumed to be freshwater, with liquid density $\rho_{\text{water}}$) is given by

与大家普遍的认识恰恰相反的是, 浮冰对海平面的上升并没有发挥重大的作用. 根据阿基米德原理, 重量为 $W$(假设淡水的密度为 $\rho_{\text{water}}$) 的浮冰融化成密度为 $\rho_{\text{ocean}}$ 的水体, 其体积的增量为 $\Delta V$, 即:

$$\Delta V = W\left(\frac{1}{\rho_{\text{water}}} - \frac{1}{\rho_{\text{ocean}}}\right)$$

The density of seawater is approximately 1024.8 $\text{kg/m}^3$; the mass of the

海水的密度大约是 1024.8 $\text{kg/m}^3$; 北极海冰的重量大约

[1]TOPEX/Poseidon 卫星是由美国和法国在 1992 年联合发射. 星上载有一台美国 NASA 的 TOPEX 双频高度计和一台法国 CNES 的 Poseidon 高度计, 用于探测大洋环流, 海况, 极地海冰, 研究这些因素对全球气候变化的影响. TOPEX/Poseidon 高度计的运行结果表明其测高精度达到 2cm.

[2]地学名词, 冰原 (Ice sheet) 是指两极地区覆盖在大面积陆地上的大量冰雪, 表面平坦. 现在仅格陵兰岛和南极洲才有大面积的冰原. 格陵兰冰原将全岛覆盖, 仅在狭窄的边缘有少量岩石露出. 南极洲冰原不但将全陆地覆盖, 而且有些地方扩大至海上. 冰原的厚度有些地方可达数千米.

Arctic sea ice is approximately $2\times10^{13}$ kg . Thus, the volume change if all of the Arctic sea ice melted is given by:

是 $2\times10^{13}$ kg. 因此, 如果北极所有的海冰都融化, 体积变化将由下式给出:

$$\Delta V = 2\times10^{13}\,\mathrm{kg}\left(\frac{1}{1000\,\mathrm{kg/m^3}} - \frac{1}{1024.8\,\mathrm{kg/m^3}}\right) = 4.84\times10^{8}\,\mathrm{m}^3$$

Approximating that 360 Gt of water causes a rise of 1 mm in sea level,

近 360 吨水可以导致海平面上升 1 毫米.

$$4.84\times10^{8}\,\mathrm{m}^3\cdot\frac{1000\,\mathrm{kg}}{\mathrm{m}^3}\cdot\frac{1\,\mathrm{Gt}}{9.072\times10^{11}\,\mathrm{kg}}\times\frac{1\,\mathrm{mm}}{360\,\mathrm{Gt}} = 0.0015\,\mathrm{mm}$$

This small change in sea level is inconsequential for our model, since the accuracy is well below one thousandth of a millimeter.

海平面如此微小的变化对我们的模型来说是无关紧要的, 因为其精度已经远低于千分之一毫米.

We also neglect the contribution of Antarctic Ice Sheet because its overall effect on sea level rise is minimal and difficult to quantify. Between 1978 to1987, satellite-borne microwave radiometer data indicated that Arctic ice decreased by 3.5%, while Antarctic ice showed no statistically significant changes . Cavalieri et al projected minimal melting in the Antarctic over the next 50 years. For this reason, only the Greenland Ice Sheet is considered in the model.

我们也忽略了南极冰原的影响, 因为其总体影响海平面上升的量是极少的, 并且难以量化. 在 1978 到 1987 年之间, 卫星微波辐射的数据表明, 北极的冰体减少了 3.5%, 然而同时, 南极的冰体却没有可统计的显著变化. Cavalieri 等人已经预测出未来 50 年内, 南极冰体融化的最低限度. 出于这样的考虑, 我们在模型中仅考虑格陵兰岛冰原的影响.

Several models already exist for mass balance and for thermal expansion. However, these models are very complex with respect to many variables, and often disagree with each other (see for example [25] and [5]). We wish to develop a model based on simple physical processes, as solely a function of temperature and time. In this way the analysis of the effects of the warming is simplified, and the dependence of sea level rise on temperature becomes evident. Furthermore, we develop a model that can be extended to several different temperature forcings, allowing us to compare firsthand the effect of carbon emissions on sea level rise.

关于物质平衡和热膨胀理论, 已经出现了一些模型. 但是这些模型考虑了很多方面的变量, 非常复杂, 并且经常彼此不统一 (比如文献 [25] 和 [5]). 我们希望建立一个基于简单的物理过程的模型, 仅仅是关于温度和时间的函数. 利用这种方法, 分析气候变暖的影响将变得非常简单, 并且使海平面上升对温度的依赖体现得更为明显. 此外, 我们建立了一个可以扩展到在若干不同温度影响下的模型, 允许我们直接比较碳的排放量对海平面上升的影响.

## 1.1 Model Overview / 模型概述

A deeper understanding of ice sheet melting would provide valuable insight into sea level rise. By creating a framework that incorporates the contributions of ice sheet melting and thermal expansion, we can estimate

深入了解冰原融化能够给海平面上升提供宝贵的见解. 通过构造一个可以整合冰原融化和热膨胀作用的框架, 我们可以估计未来 50 年内的全球平均海平面高度.

global mean sea level over a 50-year time period. The model achieves several important objectives:

1. Accurately fits past sea level rise data,
2. Provide enough generality to predict sea level rise over a 50-year span,
3. Compute sea level increases for Florida as a function of solely global temperature and time.

Ultimately, the model predicts consequences to human populations. In particular, we analyze the impact of sea level rise on the state of Florida, which many consider particularly vulnerable due to its generally low elevation and proximity to the Atlantic Ocean. From this analysis, we assess possible strategies to minimize damage as a result of sea level rise due to global warming.

这个模型实现了几个重要的目标:

1. 准确的匹配过去海平面上升的数据.
2. 为预测 50 年期间的海平面提供充分的一般性依据.
3. 佛罗里达州海平面的上升的计算仅为全球温度和时间的函数.

最后, 这一模型从逻辑上预测了人口迁移变化. 特别的, 我们分析了海平面上升对佛罗里达州的影响, 很多人认为由于它的海拔低而且临近大西洋而使它极易受灾. 从以上分析中, 我们评估了将全球变暖导致海平面上升带来的损失降低到最小的可能策略.

## 1.2 Assumptions / 假设

In order to streamline our model we have made several key assumptions.

1. *The sea level rise is primarily due to two factors*, the balance of accumulation/ablation of the Greenland Ice Sheet and the thermal expansion of the ocean. This ignores the contribution of processes such as calving and direct human intervention, which are difficult to model accurately and have minimal effect on sea level rise.
2. *The air is the only heat source for melting the ice.* Greenland's land is permafrost, and because of large amounts of ice on its surface it is assumed at a relatively constant temperature. This allows us to use convection as a mode of heat transfer.
3. *The temperature within the ice changes linearly at the steady-state.* This

为精简模型进行一些主要的假设:

1. **海平面上升主要由于两个因素:** 格陵兰岛冰原的积累/消融平衡和大洋的热膨胀. 这个假设忽略了例如冰裂和直接的人类干预等作用的影响, 而这些影响很难被精确的模拟并且对海平面上升的影响也很小.
2. **空气是融化冰原的唯一热源.** 格陵兰大陆是永久冻结带, 而且由于它表面覆盖的大量冰原, 所以假设它的温度相对稳定. 这方便我们使用对流作为热传递的方式.
3. **冰原内部稳态的温度程线性分布.** 这一假设可以

assumption allows us to solve the heat equation for Neumann conditions. By subtracting the steady-state term from the heat equation, we can solve for the homogeneous boundary conditions.

使我们用纽曼条件解决热传导方程式. 通过从热传导方程中去掉的稳态项, 我们可以解齐次边界条件的方程了.

4. *Sublimation and melting processes do not interfere with each other.* This assumption drastically simplifies the computation needed for the model since sublimation and melting can be considered separately. Additionally, the assumption is very reasonable. Sublimation primarily occurs at below freezing temperatures, a condition during which melting does not normally occur. Thus, the two processes are temporally isolated as in our model.

4. **升华和融化过程相互独立，互不干涉.** 这一假设大大地简化了模型的计算需求量, 因为它可以将升华和融化分开， 各自单独考虑. 然而这个假设又是合理的, 升华主要发生在零度以下, 而融化在零度以下根本不会发生. 因此在我们的模型中两个过程在时间上是相互独立的.

5. *The surface of the ice sheet is homogeneous with regards to temperature, pressure, and chemical composition.* This assumption is necessary because high-resolution spatial temperature data for Greenland cannot be obtained in our framework. Additionally, we lack the computational resources and time to simulate such a variation, which would require the use of finite element methods and mesh generation for a complex topology.

5. **冰原的表面在温度，压力和化学组成上是均匀一致的,** 这一假设是必要的, 这是因为在我们的框架中, 格陵兰在空间上高分辨率的温度数据是不能得到的. 另外我们缺少计算的资源和时间来做这样一个仿真: 这个仿真可能需要用有限元方法和复杂拓扑的网格生成技术.

### 1.3 Defining the problem 确定问题

Let $M$ denote the mass balance of the Greenland Ice Sheet. Given a temperature forcing function, we must quantitatively estimate the sea level increases $SLR$ that occur as a result. These increases are a sum of $M$ and thermal expansion $TE$ effects, corrected for local trends. Further, we must quantitatively and qualitatively the long-term (50 years) effect on Florida's major cities and metropolitan areas from global warming, as a result of high $SLR$. This analysis can be used to make recommendations as to how to best prepare for and reduce $SLR$ effects.

令 $M$ 表示格陵兰冰原的物质平衡. 对于给定的温度函数, 我们必需定量的估计相应的海平面上升 $SLR$. 海平面上升是物质平衡 $M$ 和热膨胀 $TE$ 的影响之和, 并根据当地趋势校正. 此外, 我们必须定量和定性地分析全球变暖引起的海平面上升 $SLR$ 对佛罗里达州主要城市和重要区域长期 (50 年) 的影响. 这一分析可以用来为如何最好地应对和减小海平面上升效应提出建议.

# 2 Methods / 模型建立

## 2.1 Mathematically Modeling Sea Level Rise / 建立海平面上升模型

Sea level rise results mostly from mass balance of the Greenland Ice Sheet and thermal expansion due to warming. In order to model sea level increases, a mass balance model and thermal expansion model are used, as well as other post-computation effects. The logic of the simulation process is detailed in Figure 1.

海平面的上升大多是因为格陵兰岛冰原的物质平衡及气候变暖带来的热膨胀. 为了模拟海平面的上升以及其他后处理效果, 我们建立了一个物质平衡模型和热膨胀模型. 逻辑模拟过程详细见图1.

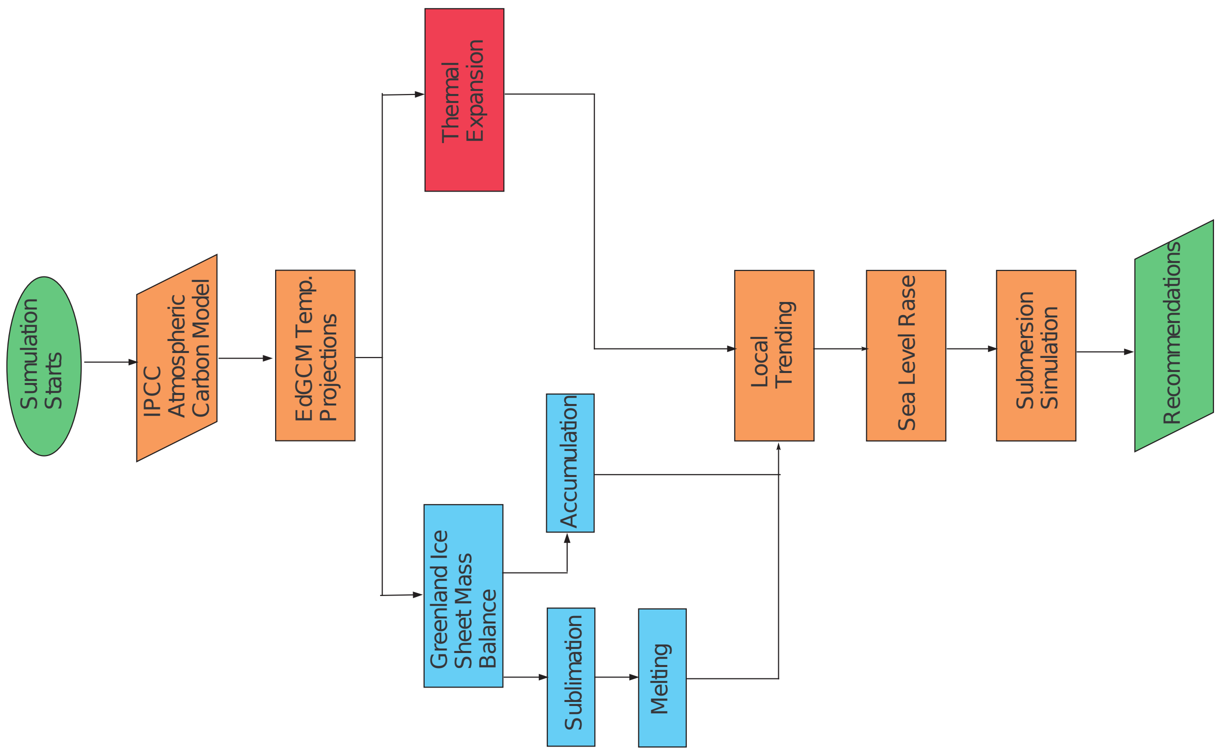


**Figure 1:** Simulation flow diagram

## 2.2 Temperature Data / 温度数据分析

Temperature data is the sole forcing in our model and thus shall be considered carefully. Because we needed to model several different scenarios, our temperature data must include several scenarios that are very controlled and only differ in one variable. Further, the temperature data must be of very good quality and provide the correct temporal resolution for our simulation. For these reasons, we decided to use a Global Climate Model (GCM) to create our own temperature data, using input forcings that we could easily control. Because of limited computational power and time restrictions, we chose the EdGCM. EdGCM is a fast model for educational purposes. The program is based on the NASA GISS model for climate change. The program fit all of our needs; in particular, the rapid simulation (about 10 hours for a 50 year climate simulation) allowed us to analyze several different temperature scenarios.

The temperature scenarios we analyzed incorporate the three estimates of carbon emissions resulting from the IPCC Third Assessment Report (TAR)- the low, high, and medium projections in the IS92 series. The IS92e(high), IS92a (intermediate), and the IS92c(low) scenarios were all closely approximated using the tools in EdGCM. These approximated carbon forcings are shown in graphical form in Figure 2. All other forcings were kept at default according to the NASA GISS model. Three time series for global surface air temperature were obtained in this fashion.

温度数据在我们的模型中是唯一必需使用的数据，所以必须仔细分析考虑. 因为需要模拟不同的情形，因此我们的温度数据就必须包含多种不同情形 (这些情形被严格控制并且只有一个不同变量). 另外，温度数据必须是高质量的，并且能为我们的模拟提供准确的时间分辨率. 为了到达这个目的，我们决定使用全球气候模型 (GCM)，通过输入便于我们控制的变量来生成我们自己的温度数据. 由于计算能力和时间的限制，我们选择 EdGCM 模型. EdGCM 模型是一个为教学设计的快速模型. 该模型是基于美国航天局 GISS 气候变化模型设计出来的. 此程序可以满足我们所有的需要; 另外，模拟的速度 (大约 10 小时模拟一个 50 年的气候阶段) 允许我们分析若干种不同的温度情况.

在对不同温度情形的分析中，我们的分析纳入了 IPCC 第三次评估报告 (TAR) 中的关于碳排放量的三种估计 - 低，高和中预测 (IS92 系统预测). IS92e(高)，IS92a(中) 以及 IS92c(低) 情形都非常符合 EdGCM 工具箱的使用条件. 这些近似的二氧化碳数据显示在图2中. 其他所有的参量都参照美国航天局的 GISS 模型，取默认值. 用这种方法，我们获得了全球地表气温的三个时间序列.

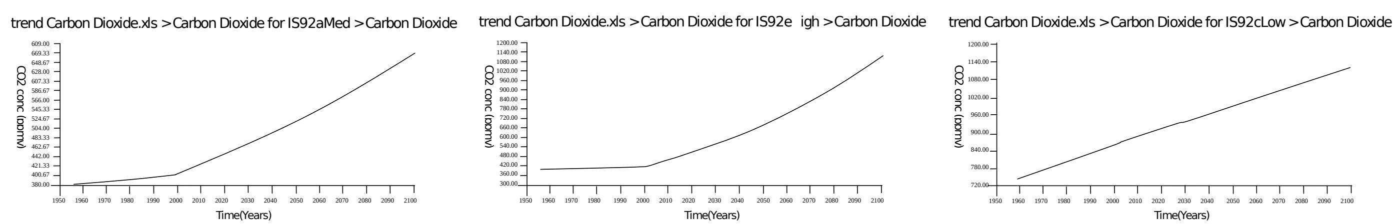


**Figure 2:** Carbon Dioxide Forcings for the EdGCM Models

One downside to the EdGCM is that it can only output global temperature changes . Regional temperature changes are calculated, but are difficult

EdGCM 模型的一个缺点是它只能输出全球气温变化. 虽然能计算区域气温变化，但是却很难获得并且结

to access and have low spatial accuracy. However, according to Chylek et al , the relationship between Greenland temperatures and global temperatures is well-approximated by

果的空间精度较低. 然而, 根据 Chylek 等人的研究, 格陵兰岛的气温和全球气温之间的关系可以非常好的近似为:

$$\Delta T_{\text{Greenland}} = 2.2 \times \Delta T_{\text{global}}$$

This result is shown by Chylek et al for regions unaffected by the NAO and is predicted by climate model outputs.

Chylek 等人用未受北大西洋涛动 (NAO)[3]影响的区域证明, 并用气候模型的输出预测了这一结果.

## 2.3 The Ice Sheet / 冰原模型

The ice sheet is modeled as a simplified rectangular box. Each point on the upper surface of the ice sheet is assumed at constant temperature, $T_a$. This is because our climate model does not have accurate spatial resolution for areas in Greenland, so the small temperature differences are ignored. The lower surface, the permafrost layer, has constant temperature $T_l$. A depiction of the ice sheet model is shown in Figure 3.

To compute heat flux and thus melting and sublimation through the ice sheet, we model it as an infinite number of differential volumes, shown in Figure 4.

将冰原简化为一个矩形块. 并假设冰原的上表面每一处的温度均恒定为 $T_a$. 这是因为我们的气候模型缺乏格陵兰岛空间上高分辨率的温度数据, 所以微小的温度差异被忽略. 冰原下层表面, 是冰封千年的冻土层, 有恒定的温度 $T_l$. 冰原模型的描述如图 3 所示.

为了计算冰原融化和升华的热通量, 我们将冰原模拟成无限多的微分单元, 如下图4.

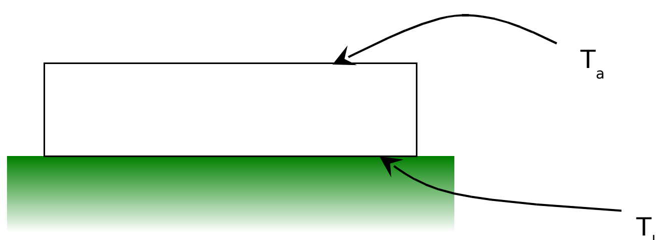


**Figure 3:** Carbon Dioxide Forcings for the EdGCM Models

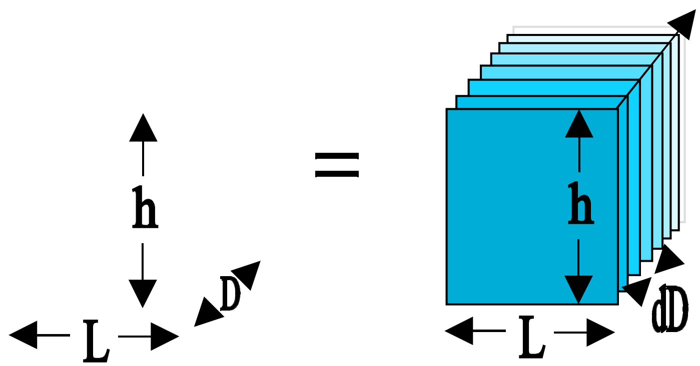


**Figure 4:** Differential volumes of the ice sheet

[3]NAO: North Atlantic Oscillation, 北大西洋涛动

Initially, the height $h$ is calculated using data provided by Williams et al.

最初, 高度 $h$ 的计算是利用 Williams 等人提供的数据.

$$h = \frac{Vol_{\text{ice}}}{Surface_{\text{ice}}} = \frac{2.6 \times 10^6 km^3}{1.736 \times 10^3 km^2} = 1498km$$

The primary mode of sea level rise in our model is through mass balance. Mass balance is calculated by subtracting the amount of ablation by the amount of accumulation. Accumulation, the addition of ice to the ice sheet, is primarily in the form of snowfall. Ablation is primarily the result of two processes, sublimation and melting.

在我们的模型中, 海平面上升的主要模型是物质平衡. 物质平衡是用积累的量减去消融量来计算的. 积累, 是指冰原上增加的冰, 其主要形式是降雪. 冰体的消融主要是两个过程的结果, 即升华和融化.

## 2.4 Mass Balance-Accumulation / 质量守恒-积累

First we model accumulation. Huybrechts et al showed that the temperature of Greenland is not high enough to melt significant amounts of snow. Furthermore, Knight showed empirically that rate of accumulation is well-approximated by a linear relationship with time, and that accumulation over Greenland continental ice is 0.30 m/year. Thus, the accumulation rate is 0.025 m/month. In terms of mass balance,

首先, 我们建立积累的模型. Huybrechts 等人证明格陵兰岛的温度还没有高达可以融化大面积的冰雪. 此外, 以 Knight 的经验推断, 冰原的积累率和时间存在非常近似的线性关系, 格陵兰岛大陆每年的冰雪积累量 0.3 米. 因此, 积累率是 0.025 米/月. 表示为物质平衡:

$$M_{ac} = 0.025LD$$

where the product $LD$ is the surface area of the ice sheet.

其中乘积 $LD$ 指冰原的表面积.

## 2.5 Mass Balance - Ablation / 质量守恒-融化

We then model the two parts of ablation, sublimation and melting.

Sublimation rate (mass flux) is given by:

其次, 我们对消融的两部分 (升华和融化) 进行建模.

升华速率（质量流量）由下式给出：

$$S_0 = \mathrm{e}_{sat}(T)\left(\frac{M_w}{2\pi RT}\right)^{1/2}$$

where $M_w$ is the molecular weight of water. This expression can be derived from the ideal gas law and the Maxwell-Boltzmann distribution . Substituting Buck's expression for $e_{sat}$, we obtain:

其中 $M_w$ 为水的分子质量. 这个表达式可以从理想气体状态方程和麦克斯韦—玻尔兹曼分布导出. 将 Buck 方程[4]代入 $\mathrm{e}_{sat}$ 的, 我们可以得出：

[4]方程的介绍: http://en.wikipedia.org/wiki/Arden_Buck_equation

$$S_0 = 6.1121 \exp\left[\frac{(18.678 - T/234.5)T}{257.14 + T}\right]\left[\frac{M_w}{2\pi R(T + 273.15)}\right]^{\frac{1}{2}}$$

Buck's equation is applicable over a large range of temperatures and pressures, including the environment of Greenland. The approximation fails at extreme temperatures and pressures but is computationally simple (relatively). To convert mass flux into rate of thickness change of the ice, we divide the mass flux expression by the density of ice. Thus we can express rate of height change as follows:

Buck 方程适用于范围较大的温度和压力的变化，适合格陵兰岛的环境. 这种近似虽然在极端温度和压力情况下失效，但是计算相对简单. 为了将质量流量转换成冰块厚度的变化，我们将质量流量除以冰的密度. 由此我们可以表达出冰原厚度的变化率，如下所示：

$$S_h = \frac{6.1121 \cdot d}{\rho_{\text{ice}}} \cdot \exp\left[\frac{(18.678 - T/234.5)T}{257.14 + T}\right]\left[\frac{M_w}{2\pi R(T + 273.15)}\right]^{\frac{1}{2}}$$

where $d$ is the deposition factor, given by $d = (1 - \text{deposition rate}) = 0.01$. This term is needed because sublimation and deposition are in constant equilibrium. With the sublimation rate expression, it is now trivial to find the thickness of the ice sheet after one timestep of the computational model. Indeed, the new thickness due to ablation via sublimation is given by:

其中 $d$ 表示沉积因子，即 $d = (1 - \text{沉积速率}) = 0.01$. 此项是必要的，因为升华和沉积是保持一定平衡的. 利用此升华率表达式，现在可以算出仿真模型中冰原经过一个时间步的厚度. 通过升华的消融过程引起的冰原厚度的变化可以由下式给出：

$$S(t) = h - S_h \cdot t$$

where $h$ is the current thickness of the ice sheet and $t$ is the elapsed time after one timestep. Substituting for $S_h$ with the expression we derived and substituting for the known value of the molecular weight of water yields

式中的 $h$ 是冰原的厚度，而 $t$ 是一个时间步所经历的时间 (时间步长). 我们用已经导出的表达式替代了 $S_h$，并且代入水分子重量 (已知数值) 得

$$S(t) = h - \frac{6.1121 \times 10^{-2} t}{\rho_{\text{ice}}} \cdot \exp\left[\frac{(18.678 - T/234.5)T}{257.14 + T}\right]\left[\frac{0.0003448}{(T + 273.15)}\right]^{\frac{1}{2}}$$

This equation governs the sublimation of the ice.

To model melting, the second component of ablation, we apply the heat equation. The heat equation governs the relationship

这个方程给出了冰的升华.

为了模拟消融的第二个组成部分 - 融化，我们应用热传导方程. 热传导方程给出了如下关系：

$$U_t(x,t) = kU_{xx}(x,t)$$

where $k = 0.0104$ is the thermal diffusivity of the ice . In order to solve the

其中 $k = 0.0104$ 是冰的热扩散系数. 为了解决在 Neuheat equation for the Neumann conditions, we assume a steady-state $U_s$ with the same boundary conditions as $U$ and that is independent of time. The residual temperature $V$ has homogeneous boundary conditions and initial conditions found by $U$-$U_s$. Thus we can rewrite the heat equation as:

mann 边界条件下的热传导方程, 我们假定稳态 $U_s$ 和 $U$ 具有相同的边界条件, 并且独立于时间. 剩余温度 $V$ 有齐次边界条件, 初始条件由 $U$-$U_s$ 确定. 因此我们可以重新写出如下热传导方程:

$$U(x,t) = V(x,t) + U_s(x,t)$$

The steady-state solution of the heat equation is given by:

热传导方程的稳态解由如下表达式给出:

$$U_s = T_l + \frac{T_a - T_l}{S(t)}x$$

subject to the constraints $0 < x < S(t)$ and $0 < t < 1$ month. The following equations follow directly from the heat equation as well:

约束条件为 $0 < x < S(t)$ 且 $0 < t < 1$ 月. 同时, 如下方直接由热传导方程导出

$$\begin{array}{ll} V_t(x,t) = kV_{xx} + f & \text{where } f \text{ is a forcing term.} \\ V(0,t) = V(S(t),t) = 0 & \text{(necessary conditions for the homogeneous boundary equations)} \end{array}$$

Since no external heat source is present and temperature distribution only depends on heat convection, we take the forcing term $f = 0$. To calculate change in mass balance on a monthly basis, we solve analytically using separation of variables:

由于没有外部热源并且温度分布仅取决于热对流, 因此我们将 $f$ 取值为 $f = 0$. 为了计算以一个月为基础的物质平衡, 我们用分离变量来求解析解:

$$V(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n e^{-n^2\pi^2 t/s^2} \cos\left(\frac{n\pi x}{s}\right)$$

where

其中

$$a_0 = \frac{2}{s}\int_0^s \left(T_l + \frac{T_a - T_l}{s}x\right)dx = 2T_l + T_a - T_l = T_l + Ta$$

and

并且

$$a_0 = \frac{2}{s}\int_0^s \left(T_l + \frac{T_a - T_l}{s}x\right)\cos\left(\frac{n\pi x}{s}\right)dx = \left(\frac{s}{n\pi}\right)^2\left(\cos(n\pi) - 1\right) = \left(\frac{s}{n\pi}\right)^2\left((-1)^n - 1\right)$$

Therefore,

因此可得,

$$V(x,t) = \frac{T_l + T_a}{2} + \sum_{n=1}^{\infty} \frac{2(T_a - T_c)}{(n\pi)^2} \big((-1)^n - 1\big) e^{-n^2\pi^2 t/s^2} \cos\left(\frac{n\pi x}{s}\right)$$

Having found $V(x,t)$ and $U_s(x,t)$, we obtain an expression for $U(x,t)$:

计算出 $V(x,t)$ 和 $U_s(x,t)$ 后，我便可以得到 $U(x,t)$ 的表达式：

$$U(x,t) = V(x,t) + U_s(x,t)$$

Since $U$ is an increasing function of $x$, and for $x > k$, $U(x,t) > 0$ for fixed $t$, the ice will melt for $k < x < h$. Thus, we seek the solution to $U(k,t) = 0$ for $k$ to determine ablation. Computationally, we solve this expression using the first 100 terms of the Fourier series expansion and the MATLAB function $fzero$. The solution of this equation for $k$ is the primary computational step for the MATLAB simulation (see Appendix A). The new value of $k$ is used to renew $h$ as the new thickness of the ice sheet, and a consequent time step can begin calculation.

With these two components we can now finalize an expression for ablation and apply it to a computational model. The sum of the infinitesimal changes in ice sheet thickness for each differential volume gives the total change in thickness. To find these changes, we first note that

由于 $U$ 是 $x$ 的一个增函数，并且对于给定时刻 $t$，如果 $x > k$ 时，$U(x,t) > 0$，则高度在 $k < x < h$ 的冰层会融化。所以，我们需要解 $U(k,t) = 0$ 来确定 $k$ 以便解确定冰原的融化量。通过数值计算，我们用傅立叶系数展开的前 100 项与 MATLAB 函数 fzero 来求解这个表达式。解方程获得 $k$ 值是 MATLAB 模拟中的主要计算步骤（见附录 A）。新的 $k$ 值被用来更新 $h$ 作为冰原新的厚度，一系列的时间步就可以由此计算。

综合这两个组成部分，我们现在确定有关消融的表达式并将它应用到计算模型中去。冰原厚度在每个不同部分的微小变化的总和给出了厚度的总体变化。为了得到这些变化，我们首先注意到：

$$\text{Mass Balance Loss Due to Sublimation} = (h - S)LD$$

$$\text{Mass Balance Loss Due to Melting} = (S - k)LD$$

where the product $LD$ is the surface area of the ice sheet. Note that in these equations, the “mass balance” refers to net volume change. Thus, ablation is given by

其中乘积项 $LD$ 是冰原的表面区域。这些方程中的“物质平衡”是指净体积改变。因此，消融可以表示为：

$$M_{ab} = (h + S)LD + (S - k)LD = (h - k)LD$$

## 2.6 Mass Balance and Sea Level Rise / 质量守恒与海平面上升模型

Combining accumulation and ablation into an expression for mass balance, we have

将积累与消融结合起来写入物质平衡的表达式中，我们可以得到：

$$M = M_{ac} - M_{ab} = 0.025LD - (h-k)LD$$

Relating this to sea level rise, we use the approximation 360 Gt water = 1 mm sea level rise. Thus,

将这个表达式与海平面上升相结合，我们利用近似：360 Gt 的水 = 1 mm 的海平面上升. 因此,

$$SLR_{mb} = M \cdot \rho_{\text{ice}} \cdot \frac{1mm}{360Gt}$$

which quantifies the sea level rise due to mass balance.

这个式子由物质平衡量化了海平面上升的数值.

## 2.7 Thermal Expansion / 热膨胀模型

A second mode of sea level rise is also considered: thermal expansion due to warming. According to various literature , thermal expansion of the oceans due to increase in global temperature will contribute a significant portion of the rise in future sea level, at least as much as melting of polar ice for the current century. Therefore, we incorporated this component into our model for further accuracy and a more comprehensive understanding.

Thermal expansion operates depending on various factors. Temperature plays the primary role, but the diffusion of radiated heat, mixing of the ocean, and various other complexities concerning ocean dynamics must be accounted for a fully accurate description of the phenomenon. These factors are often quite difficult to understand with a high degree of certainty. The model used here adapts the model of Wigley et al. Based on standard greenhouse-gas emission projections and a simple upwelling-diffusion model, the dependency of the model can be narrowed to a single variable, temperature, using an empirical estimation:

考虑海平面上升的第二个模型：由于全球变暖导致的热膨胀. 依据多篇文献, 由全球升温导致的海洋热膨胀将会成为未来海平面上升的重要组成部分, 至少与当前世纪的两极冰川的融化旗鼓相当. 因此, 为了更进一步的准确性以及更加全面的理解, 我们将这部分考虑纳入我们的模型中.

热膨胀的发生依赖于多种因素. 温度起到最主要的作用, 但是完全精确的描述热膨胀这种现象, 还要考虑辐射、海水的混合, 以及其它有关海洋动力学的各种复杂因素. 高度精确地理解这些因素通常非常困难. 在这里所用到的模型是由 Wigley 等人的模型改进而来. 基于标准的温室气体排放量的预测和简单上升扩散模型, 模型的变量可以缩减为一个-温度, 用一个经验估计式：

$$\Delta z = 6.89 \Delta T k^{0.221}$$

where $\Delta z$ is the change in sea level due to thermal expansion given in centimeters, $\Delta T$ is the change in global temperature, and $k$ is the diffusivity.

The reader is encouraged to consult for further investigation of the upwelling-diffusion model.

其中 $\Delta z$ 是由热膨胀引起的海平面变化, 以厘米为单位, $\Delta T$ 是全球温度变化, 而 $k$ 是扩散系数.

读者可以进一步的查阅上升扩散模型.

## 2.8 Localization / 本地化分析

A final correction must be added to the simulation. Although the literature in general cites an increase in the mean sea level for the past century and indicates that melting of polar ice and other various effects associated with global warming will force the trend, the effect varies regionally rather significantly. The local factors often cited include land subsidence, compaction, and delayed response to the warming, to name a few . Fully understanding the influences of these factors on sea level increase is often a daunting task. We thus assume that previous patterns of local sea level variation will continue to influence, yielding the relationship

一个最终的修正必需添加到这个模拟中. 虽然普遍的文献引述上个世纪平均海平面高度有所上升, 并且指出两极冰川的融化及其它与全球变暖相关的多种效应将促进这一趋势, 但是这种作用显著随地域改变. 地域因素常包括土地沉淀, 压实, 以对变暖的响应延迟等等. 完全理解这些因素对海平面上升的影响通常是相当困难的任务. 因此, 我们假设某地域以往的海平面变化模式将会继续起作用, 并且服从如下关系式:

$$\mathrm{local(t)} = \mathrm{normalized}(t) + \mathrm{trend}(t - 2008)$$

where $\mathrm{local}(t)$ is the expected sea level rise at year $t$ given in centimeters, $\mathrm{normalized}(t)$ is the estimate of expected rise in global sea level change relative to the historical rate, at year $t$, and *trend* is the current rate of sea level change at the locale of interest. The normalization prevents from double counting the contribution from global warming.

In our model, the rates of sea level change are averaged over data given for Florida in to give the trend. This is reasonable because the differences between the rates in Florida are fairly small. The $\mathrm{normalized}(t)$ at each year is obtained by:

其中 $\mathrm{local}(t)$ 是在 $t$ 年时海平面上升的期望值 (以厘米为单位), $\mathrm{normalized}(t)$ 是根据以往变化率对 $t$ 年全球海平面上升高度期望的估计值, 而 trend 是所研究区域当前海平面高度变化率. 规范化以避免了重复计算对全球变暖的影响.

在我们的模型中, 海平面高度改变的速率就是对佛罗里达州的数据求的平均得出的趋势 trend. 因为在佛罗里达州, 海平面高度变化率的差别很小, 所以这样处理是合理的. 每年的 $\mathrm{normalized}(t)$ 便可以通过下式得到：

$$\mathrm{global}(t) - \mathrm{historical\ rate}(t - 2008)$$

where $\mathrm{global}(t)$ is the expected sea level rise at year $t$ from our model and historical rate is chosen uniformly over the range taken from.

For a detailed description of the model, the reader may consult.

其中 $\mathrm{global}(t)$ 是我们模型中 $t$ 年海平面上升的期望值, 而 historical rate 所选择时间范围内的平均值.

读者可以通过查阅相关材料了解这个模型的的更多细节.

## 2.9 Simulating Costs of Sea Level Rise to Florida / 佛罗里达海平面上升的损失模拟

Rising sea levels could submerge coastal areas of Florida that are near current sea level. To model the submersion of regions of Florida due to sea

海平面的上升会使现在佛罗里达州的靠近海面的海岸区域被淹没. 为了模拟海平面上升导致的佛罗里

level rise, a raster matrix of elevation values for various latitude and longitude was created. The matrix was created on MATLAB using 30-arc-second global elevation data (GTOPO30), created in 1996 . The 30-arc-second resolution corresponds to about 1 km; however, in order to yield a more practical matrix, the resolution was lowered to 1 minute of arc (approximately 2 km). The vertical resolution of the GTOPO30 data is much greater than 1 meter and thus accurate models could not easily be produced. In order to more accurately model the low coastal regions, the matrix generation code identified potential sensitive areas and submitted these locations to the National Elevation Dataset (NED) for refinement. NED is updated bimonthly, but its large size and download restrictions restrict its use to only these sensitive areas. The vertical resolution of NED is very high, depending on the region surveyed. Although Florida NED data has a mean error of ±4.3 ft, areas of low elevation have especially high resolution . These adjustments finalized the elevation data raster matrix for use in the sea level increase simulation.

The effect of this sea level rise on human populations was measured by incorporating city geospatial coordinates and population into the simulation. Geospatial coordinates were obtained from the GEOnames Query Database maintained by the National Geospatial Intelligence Agency . Population data was obtained from the US Census Bureau 2000 Datasets . All major metropolitan areas and several large cities were analyzed, encompassing both interior (e.g.,Gainesville) and coastal (e.g., Miami). The population of the metropolitan areas was equally split into the principle cities in order to streamline the simulation (see Appendix D).

The sea level rise calculated from our model was used as input for the submersion simulation. The simulation script subtracts the sea level increase from the existing elevation data. Pixels with elevations below sea level are checked to determine whether they are connected (directly or indirectly through other submerged areas) to the Atlantic Ocean or the Gulf of Mexico. This way, interior areas not connected to the oceans are not identified as submerged regions. If rising sea level submerges pixels that form part of a city or metropolitan area, the population is considered to be ”displaced.” A key limitation of the model is that the population is considered

达州的某些区域的淹没, 我们建立了一个不同的经纬度海拔高度的栅格矩阵. 此矩阵是 MATLAB 在 1996 年用 30 弧秒全球海拔数据 (GTOPO30) 生成的. 这 30-弧度-秒的分辨率对应大约 1 km; 然而, 为了产生一个比较实际的矩阵, 分辨率被降低到 1 弧分 (分辨率接近 2km). GTOPO30 的垂直方向上的分辨率 (即海拔) 比 1 米大得多, 因此精确的模型不容易被创建. 为了更加精确地模拟出海拔低的沿海区域, 用生成矩阵的代码识别潜在的敏感区域, 并用国家海拔数据 (NED) 来修正这些区域. NED 数据每两个月更新一次, 但 NED 数据的尺寸大以及下载的约束限制了它只能应用于这些敏感区域. NED 数据在所研究区域的垂直分辨率非常高. 虽然佛罗里达州 NED 数据平均误差为 ±4.3 ft, 但低海拔区域有相当高的分辨率. 经过这些修正, 最终确定了应用于海平面增长模拟的海拔栅格矩阵.

海平面上升对人口的影响是通过将城市地理坐标和人口加入到模拟中来评估的. 地理坐标是从地名查询数据库 (由国家地理情报机构维护) 中获得的. 人口数据是从美国人口普查局 2000 年数据集中获得的. 对所有的都市地区和主要城市都作了分析, 既包含内陆的 (比如 Gainesville), 也包含沿海的 (比如 Miami). 为了简化模拟, 都市地区的人口被等价地划分到相应的城市 (见附录 D).

我们的模型计算出的海平面上升高度被作为淹没模拟的输入. 在程序脚本中, 从现有的海拔数据减去海平面上升高度. 检查海拔在海平面以下的像素点是否连接 (直接地或间接地经过其他被淹没区域) 到大西洋或墨西哥的波斯湾. 这样, 没连接到海洋的内陆不会被确认为被淹没区域. 如果上升的海平面淹没了构成一个城市或者都市地区的像素点, 人口会认为被“移走”. 这个模型的主要限制是人口被认为是集中在都市区域相应的城市, 因此无法高度地精确估计人口. 模型的这种简单化

to be concentrated in the principal cities of the metropolitan areas, so a highly accurate population count cannot be assessed. This simplification of the model allows the quick display of which cities are threatened by rising sea levels without the complexity of a continuous population distribution. Additionally, high-resolution population distribution data is difficult to find and thus cannot be easily utilized.

The model was checked for realism at several different scenarios of sea level rise. First, the extreme case of 0 meter sea level rise was examined. In this case, no cities should be submerged and no population or land area should be affected. These expectations are confirmed in Figure 5. The case of 10 meter sea level rise was also analyzed. This is slightly higher than the sea level increase estimate if all of the Greenland Ice Sheet melted (approximately 7 meters). Many cities should be submerged, especially in the low elevation regions in southern Florida. This is confirmed by the output, shown in Figure 5. Finally, 100 meter sea level rise was analyzed to check robustness of the simulation. Most of Florida should be submerged, since it is a relatively low elevation state. This is also confirmed by Figure 5; note the mountainous regions of North Florida that are still above water.

使得城市在空间上没有复杂的连续人口分布， 这使得那些受到上升海平面威胁的城市可能被快速模拟演示. 此外, 高分辨率的人口分布数据难以获得, 因此不容易被使用.

为了检验模型在不同的海平面上升情况下的现实意义. 首先, 先检查无海平面上升为 0 的极端情形. 在这情况, 没有城市会被淹没, 而且也应该没有人口或者地区受影响. 这些预料在图5被确认. 本文还要分析海平面上升 10 米的情形. 这比假设格陵兰岛所有的大冰原融化后海平面上升的估计值 (大约 7 米) 还要略高一点. 许多城市应该被淹没, 特别是在佛罗里达州南部的低海拔区域. 这被输出结果证实, 显示在图5中. 最后, 还分析了海平面上升 100 米的情形来检验模拟的鲁棒性. 大部分佛罗里达州应该被淹没, 因为它是一个海拔相对较低的州. 这也在图5中得到确认; 注意北佛罗里达州的多山区域仍然在海平面以上.

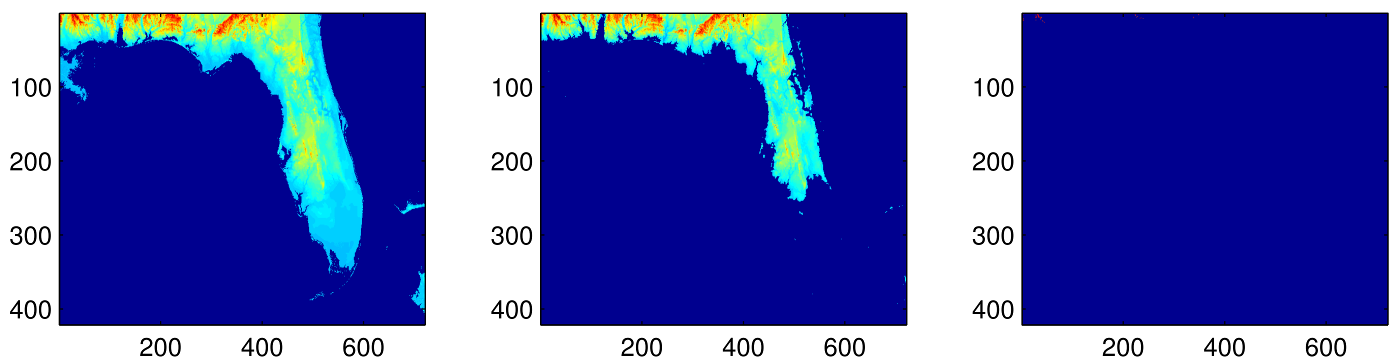


**Figure 5:** Graphical effects of 0, 10, and 100 meter sea level rise.

# 3 Results / 结果

## 3.1 Output Sea Level Rise Data / 输出海平面上升模型的数据

The program was run with MATLAB script *massbalance_sim2.m*, for the IS92e (high), IS92a (intermediate), and IS92c (low) carbon emissions models. Complete code is given in Appendix A.

The program produced a smooth trend in sea level increase for each of the three forcings, shown in Figure 4.

这个程序是用 MATLAB 脚本 massbalance_sim2.m 运行的，有 IS92e(高),IS92a(中) 和 IS92c(低) 的三种碳排放模型. 完整的程序源码已在附录 A 中给出.

这个程序给出的三种情况下的海平面上升趋势都很平稳，如图4所示.

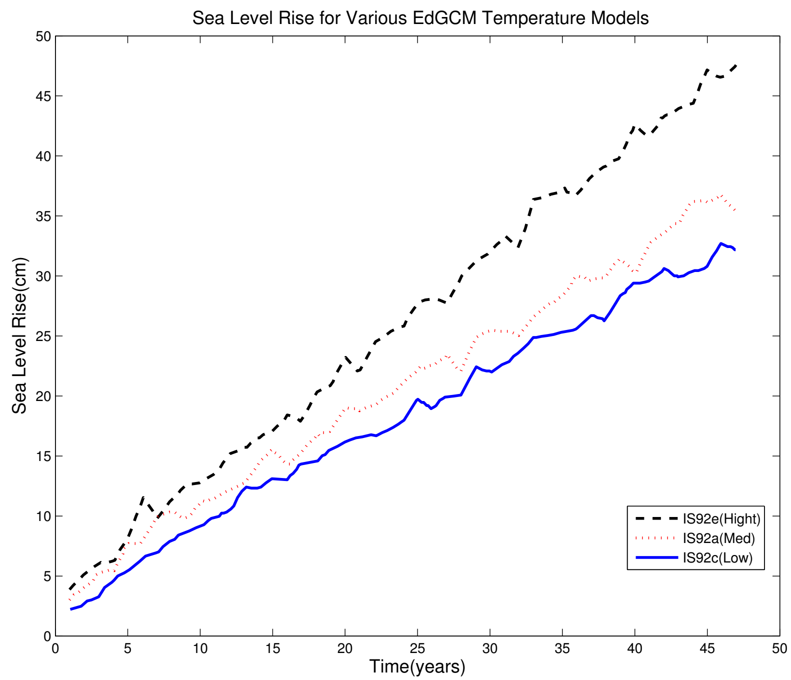


**Figure 6:** Sea level rise as a function of time for the three temperature models

Note that the higher temperature corresponds with higher sea level rise, as we expect it to. The data at the end of 10-year intervals was recorded and tabulated in Table 1. Units of sea level rise are in centimeters.

注意较高的温度对应着较高的海平面上升，这正如我们所预料的. 每 10 年期间末了时刻的数据被记录并列在表格中1. 海平面上升的单位是厘米.

**Table 1:** Sea Level Rise (cm) per Decade for each Temperature Model

| | 10years | 20years | 30years | 40years | 50years |
|---|---|---|---|---|---|
| IS92e(Hight) | 12.67 | 23.26 | 31.93 | 41.68 | 46.92 |
| IS92a(Med) | 11.14 | 18.79 | 25.08 | 30.44 | 36.61 |
| IS92c(Low) | 9.16 | 16.26 | 21.66 | 29.32 | 32.08 |

The sea level output data was then used to calculate submersion consequences. These data were fed as input to the submersion simulation, detailed in the following section.

然后，我们用海平面上升结果的数据来计算淹没的影响. 这些数据将作为淹没模拟的输入，详细情况见下面部分.

## 3.2 Submersion Simulation Results / 淹没模拟的结果

Submersion information was calculated for each of the three temperature models during every decade. Output consisted of the submerged land area and displaced population statistics.

For the IS92e (high) scenario, sea level increases resulted in the following simulated geographic consequences (shown every decade for 5 decades):

对于三个温度模型中的每一个模型，计算每十年的淹没信息. 输出的信息包括被淹没的陆地区域和转移的人口数统计.

对于 IS92e(高) 的情况，海平面上升所造成的结果见下面一系的仿真地图 (以十年为单位，共 50 年):

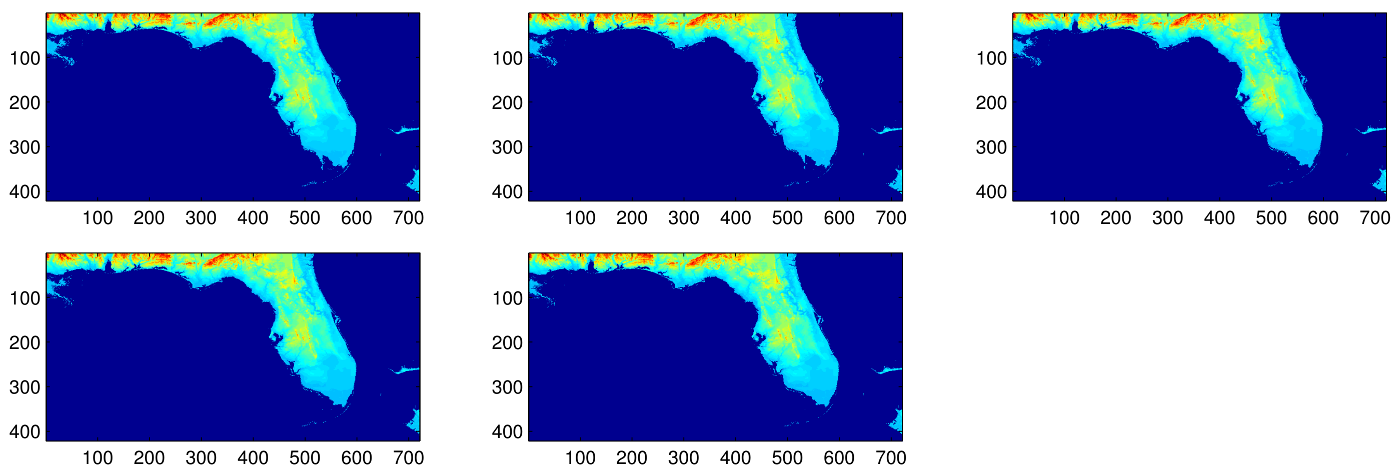


**Figure 7:** Submersion simulation for IS92e

Although not much has appeared to have happened, minor topological changes can clearly be seen at the southern tip of Florida and parts of Louisiana during the 50 year span. Additionally, the MATLAB program quantified the following effects:

虽然效果似乎并不明显，但我们仍能清楚的看出佛罗里达州南端和路易斯安那的一些部份在 50 年期间内的较小的地质变化. 此外, MATLAB 程序还定量地给出下列结果：

**Table 2:** Quantitative Effects for IS92E

| | Effects |
|---|---|
| 10 years | 0.00e+00 people displaced; 6.52e+03 sq km land submerged |
| 20 years | Key Largo, FL is submerged: 11886 people have been displaced |
| | 1.19e+04 people displaced; 7.45e+03 sq km land submerged |
| 30 years | Miami Beach, FL is submerged: 87925 people have been displaced<br>Key Largo, FL is submerged: 11886 people have been displaced |
| | 9.98e+04 people displaced; 9.18e+03 sq km land submerged |
| 40 years | Merritt Island, FL is submerged: 36090 people have been displaced<br>Miami Beach, FL is submerged: 87925 people have been displaced<br>Key Largo, FL is submerged: 11886 people have been displaced |
| | 1.35e+05 people displaced; 9.97e+03 sq km land submerged |
| 50 years | Merritt Island, FL is submerged: 36090 people have been displaced<br>Miami Beach, FL is submerged: 87925 people have been displaced<br>Key Largo, FL is submerged: 11886 people have been displaced |
| | 1.35e+05 people displaced; 9.97e+03 sq km land submerged |

For the IS92a (Med) scenario, sea level increases resulted in the following simulated geographic consequences (shown every decade for 5 decades):

对于 IS92a(中) 的情况, 海平面上升所造成的结果见下面一系的仿真地图 (以十年为单位, 共 50 年):

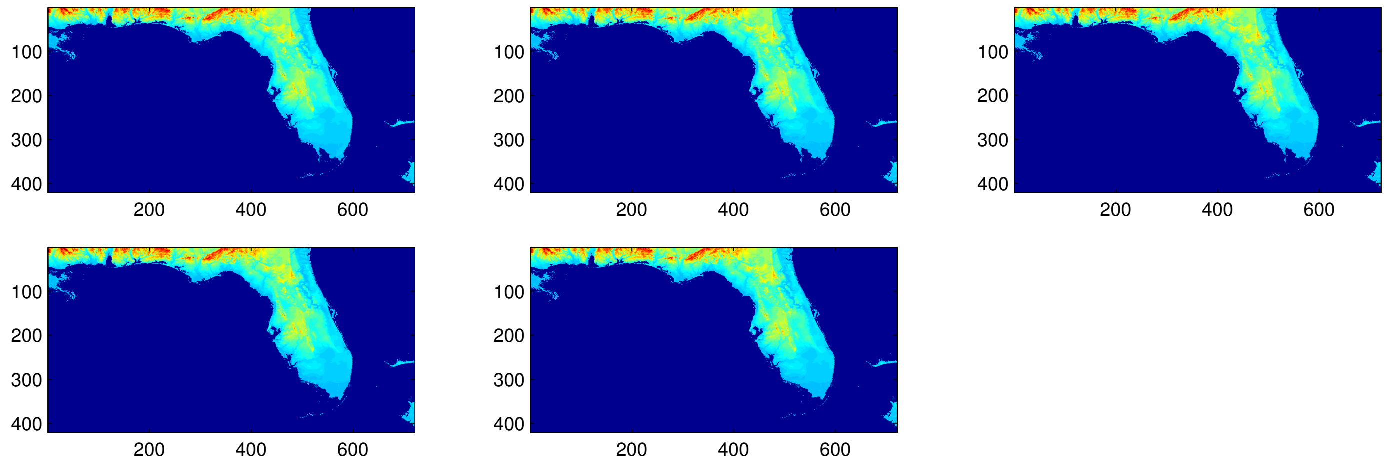


**Figure 8:** Submersion simulation for IS92a

The overall qualitative damages are comparable to those for IS92e. MATLAB returned the following damages:

这种情况所有损害定量分析都与 IS92e 情况类似. MATLAB 反馈出以下危害:

**Table 3:** Quantitative Effects for IS92A

| | Effects |
|---|---:|
| 10year | 0.00e+00 people displaced; 6.43e+03 sq km land submerged |
| 20year | 0.00e+00 people displaced; 6.94e+03 sq km sq km land submerged |
| 30year | Key Largo, FL is submerged: 11886 people have been displaced |
| | 1.18e+04 people displaced; 7.71e+03 sq km land submerged |
| 40year | Miami Beach, FL is submerged: 87925 people have been displaced<br>Key Largo, FL is submerged: 11886 people have been displaced |
| | 9.98e+04 people displaced; 8.96e+03 sq km land submerged |
| 50year | Miami Beach, FL is submerged: 87925 people have been displaced<br>Key Largo, FL is submerged: 11886 people have been displaced |
| | 9.98e+04 people displaced; 9.46e+03 sq km land submerged |

The key differences for the IS92A data compared to the IS92E data are

IS92A 情况下数据与 IS92E 情况下得到的数据有以

that

1. Key Largo is submerged 10 years later
2. Miami Beach is submerged 20 years later
3. Merritt Island is not submerged after 50 years

Finally, for the IS92c (low) scenario, sea level increases resulted in the following simulated geographic consequences (shown every decade for 5 decades):

下关键不同点:

1. Key Largo 10 年后被淹没.
2. Miami 海滩 20 年后被淹没.
3. Merritt 岛 50 年后仍不会被淹没.

最后, 对于 IS92c(低) 的情况, 海平面上升所造成的结果见下面一系的仿真地图 (以十年为单位, 共 50 年):

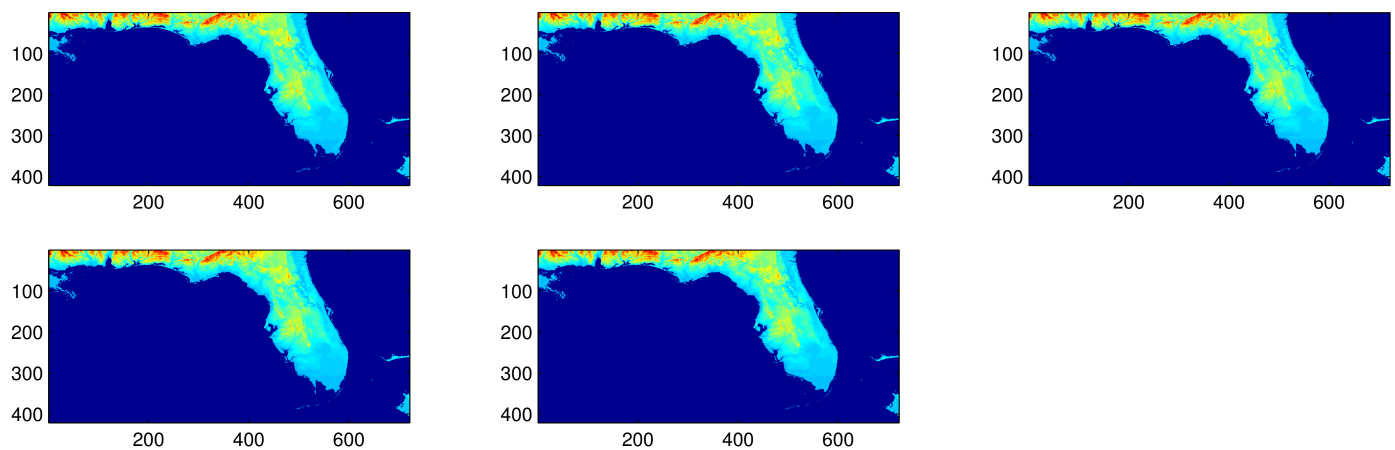


**Figure 9:** Submersion simulation for IS92c

The overall qualitative damages are comparable to those for IS92e and IS92a. MATLAB returned the following damages:

这种情况所有损害定量分析都与 IS92e 及 IS92a 情况类似. MATLAB 反馈出以下危害:

**Table 4:** Quantitative Effects for IS92A

| | Effects |
|---|---|
| 10years | 0.00e+00 people displaced; 6.15e+03 sq km land submerged |
| 20years | 0.00e+00 people displaced; 6.79e+03 sq km land submerged |
| 30years | 0.00e+00 people displaced; 7.12e+03 sq km land submerged |
| 40years | Miami Beach, FL is submerged: 87925 people have been displaced<br>Key Largo, FL is submerged: 11886 people have been displaced |
| | 9.98e+04 people displaced; 7.96e+03 sq km land submerged |
| 50years | Miami Beach, FL is submerged: 87925 people have been displaced<br>Key Largo, FL is submerged: 11886 people have been displaced |
| | 9.98e+04 people displaced; 9.19e+03 sq km land submerged |

The key differences for the IS92C data compared to the IS92A and IS92E data is that no metropolitan areas are submerged after 30 years. However, note that in both the IS92A and IS92C scenarios, Miami Beach and Key Largo are submerged after 40 years.

与 IS92A 和 IS92E 两种情况下的数据相比, IS92C 情况下的数据体现出的关键不同点在于 30 年后没有都市地区会被淹没. 然而, 在 IS92A 和 IS92C 两种情景中, Miami 海滩和 Key Largo 岛 40 年后都会被淹没.

# 4 Discussion and Conclusion

The estimated sea level rise (shown in Figure ) for the three emissions scenarios seems very reasonable. The 50-year projection is in general agreement with models proposed by the IPCC, NRC, and EPA (less than 10 cm different from all of them). Additionally, the somewhat-periodic, somewhat-linear trend is similar to past data of mean sea level rise collected in various locations. Thus, the projections made by our model are feasible.

The high emission scenario results in about $40 \sim 50\,\mathrm{cm}$ rise in sea level by 2058, with results from the intermediate scenario $6 \sim 10\,\mathrm{cm}$ below that and the low emission scenario trailing intermediate by $5 \sim 8\,\mathrm{cm}$. The model thus works as we expected for a wide range of input data; higher temperatures should lead to increased sea level rise.

Overall, the damage due to sea level change seemed unremarkable. Even

按照三种情况估计出来的海平面上升值 (如图所示) 看起来是非常合理的. 50 年的预测也大致与 IPCC, NRC 和 EPA 提议的模型吻合 (与它们的误差不高于 10cm). 另外, 其体现出来的微周期性及微线性也与多个区域的以往的海平面上升变化类似. 因此, 本文模型给出的预测是合理的.

碳的高排量情况下导致到 2058 年海平面上升 $40 \sim 50\,\mathrm{cm}$, 中排量情况下导致的海平面上升值比高排量情境下低 $6 \sim 10\,\mathrm{cm}$, 而低排量情况比中排量情景下的值低 $5 \sim 8\,\mathrm{cm}$. 对于变化范围在较大区间内的输入, 这个模型计算起来跟我们预料的一样; 较高的温度引起海平面的上升.

综上所述, 海平面变化所引起的损失似乎并不显著.

in our ”worst case scenario,” in 50 years only approximately 200,000 people are displaced, and 10,000 square kilometers are submerged-mostly from the South Florida metropolitan area and other coastal regions. The effects could barely be visualized on the submersion simulation.

即使在本文考虑的“最糟糕的情况”下，在未来 50 年中也仅有 200, 000 人被迫迁移、10, 000 平方公里陆地被淹没-大部分集中在从南佛罗里达的大型都市区以及其他沿海区域．这种影响也只是刚刚可以从淹没仿真模型中看出来．

However, these projections are only the beginning of what could be a long-term trend. As shown by the control results, a sea level increase of 10 meters would be devastating. Further, not all possible damages could be assessed in our simulation. For example, sea level increases have been directly implicated in shoreline retreat, erosion, and saltwater intrusion, which were not quantified in our model. Economic damages also could not be directly assessed. Global warming presents a very complex economics problem. Bulkheads, levees, seawalls, and other manmade structures are often utilized to counteract the effect of rising sea levels, and their economic impacts are outside the scope of the model. High-resolution economic data is also required, to determine the value of the threatened land.

然而，这些变化仅仅是一个长期变化趋势的开始．就像在结果中显示出来的，海平面上升 10 米时是极具有破坏性的．另外，并不是所有可能的损失都能在我们的仿真中体现出来．比如，海平面的上升已经直接导致海岸线的后退、侵蚀，以及海水的侵入，这些都是在我们的模型中无法量化的．同样，经济损失也是无法直接估计的．全球变暖带来了很复杂的经济问题．防水壁、码头、防波堤以及其他人工建筑经常被用于降低海平面上升带来的危害，而它们的经济影响也是我们建立的模型无法计算的．因此，需要高精确度的经济数据来确定那些面临被淹没的陆地的价值．

Our model has several key limitations. The core assumption of the model is the simplification of physical features and dynamics in Greenland. The model assumes an environment where thickness, temperature, and other physical properties of interest are averaged out and evenly distributed. And then the ”sublimate, melt and snow” dynamics are simulated on a monthly time step is employed. Such assumptions are obviously far too simplistic to fully capture the ongoing dynamics in the ice sheets. But without the tremendous data and computing power required to perform a full-scale 3-D grid-based simulation using energy-mass balance models, such as in , an alternative had to be pursued.

我们的模型存在几处关键的限制．模型的核心假设是简化了格陵兰岛的物理特性和动力学特征．模型假设了厚度，温度，以及其他有关物理因素都处于平均，平衡，均匀分布状态．升华，融化，降雪等变化也是以月为单位进行仿真．很明显，这些假设太过于简单，以至于无法表现冰原正在进行的动态变化．但是，由于没有所需的大量数据和强大的计算平台支持，无法基于能量-质量平衡模型构建大规模的三维网格仿真，因此这种权衡是非常有必要的．

With regards to more minor details of the model, the assumed properties regarding the thermal expansion, localization, and accumulation also take an averaging approach towards evaluating the geological trends. As mentioned in previous sections, understanding, let alone simulating, these phenomena and methods with a high degree of accuracy is difficult due to their innate complexities. An empirical estimate is often made in literatures, such as one we adapted from, and thus we occasionally adopt simplified models.

关于模型中更多的小细节，跟热膨胀，局域性，及冰雪积聚相关的因素，我们也假设用平均来近似得到地质变化的趋势．正如在前面提到的：由于本质上的复杂性，要理解这些现像，用高精确度的独立仿真等方法也是非常困难的．很多文献中都存在一些经验估量值，就像其中一个被我们改进的模型，因此，我们有时会采用这些简化的模型．但是，由于这些经验性估计，在将来的时间

However, because of these empirical approaches, our model may not hold over extremely long periods of time, where models for accumulation, thermal expansion, and localization might break down.

A discussion of the emission scenarios, the heart of the input data, is also relevant. The data is a subset of simulation result using IS92 emission scenarios on EdGCM model, the core of which is the GISS-II GCM (Global Climate Model) developed by NASA . The assumptions of the EdGCM model are fairly minimal for computing on a desktop, and the projected temperature time series associated with each scenario-high, medium, and low-were consistent with typical carbon projections. Although the IS92 emissions scenarios are very rigorous, they are the main weakness of the model. Because all of the other parameters are dependent on the temperature model, our results are particularly sensitive to factors that directly affect the EdGCM output. This situation is complicated further by the fact that an explicit-form solution cannot be obtained with our mathematical foundation. The variable we solve for is inside the Fourier series term and requires sophisticated numerical computations to approximate; thus, we cannot directly assess the dependence of sea level rise on temperature.

Despite these deficiencies, our model is an extremely powerful tool for climate modeling. The relative simplicity, while it can be viewed as a weakness, is actually a key strength of the model. The model boasts rapid run-time in comparison with its more sophisticated peers due to the simplifications of variables. Furthermore, the model is basically a function of time and temperature only. The fundamentals of our model imply that all of the sea level increases are due to temperature change; this relationship is obscured in other models involving a large amount of independent variables. But even with less complexity, the model is comprehensive and accurate enough to produce quality results and provide accurate predictions. Indeed, as we have shown, the predictions of our model closely parallel past data. Additionally, the associated visualization tool allows for easy recognition of the sea level rise effects in Florida.

中, 冰雪积聚模型、热扩散模型等很能会失效, 因此, 我们的模型可能无法长时间有效.

关于不同碳排放情况, 核心输入的数据也进行了相关讨论. 这些数据是在 IS92 的碳排放情况下由 EdGCM 模型 (EdGCM 模型是由 NASA 设计开发的模型 GISS-II GCM (全球气候模型) 的核心) 的仿真结果的一个子集. 对于普通 pc 机, EdGCM 模型中的这些假设需要的计算量相对较小, 而且与高, 中, 低三中情景相符的温度时间序列也与典型碳排放规律相符. 虽然 IS92 的碳排放情况非常明确, 但它们仍然是模型的主要缺点. 由于其他所有参数都建立在温度变化模型的基础上, 我们得到的结果对那些直接影响 EdGCM 模型输出的因素有较显著的敏感度. 因为我们无法从数学上获得解析解, 这种情况也显得更加复杂. 我们要解的那些变量存在于傅立叶级数中, 需要复杂的数值计算才能得到其近似值; 因此, 我们不能直接评估海平面上升对温度的依赖程度.

尽管本文的模型存在这些不足, 它仍然是分析气候变化的有力的工具. 虽然相对简单的模型可能会被视为一种缺陷, 但它实际上是模型的一个主要优点. 由于变量的简化, 与那些复杂的模型相比, 本文的模型具有更快的运算速度. 另外, 此模型基本上仅仅是时间与温度的一个函数. 本文模型的基本特征暗示了所有海平面的变化都只与温度的变化相关; 这种关系在其他具有大量自变量的模型中是非常不明显的. 虽然模型的复杂度并不高, 但是却有着足够的全面性和准确度计算出合理的结果并给出准确的预计. 正如我们已展示的, 本文模型的计算结果与过去的相关数据是非常接近的. 另外, 相关的可视化很容易就可以识别出海平面上升对佛罗里达的影响.

# 5 Recommendations

In the short term, preventative action could spare many of the submersion model's predictions from becoming a reality. Key Largo and Miami Beach in particular were identified as particularly vulnerable. These regions act as a buffer zone, preventing salinization of interior land and freshwater. If these regions flood, seawater intrusion may occur, resulting in widespread ecological, agricultural, and ultimately economical damage.

短期内, 预防性措施可以防止淹没模型预测出的很多结果变为现实. Key Largo 岛和 Miami 海滩被确定为极容易受到影响的地区. 这些区域就像是缓冲区域, 阻止内陆及淡水湖的盐化. 如果这些地方被淹没, 海水侵入将会导致普遍的生态、农业以及经济上的损失.

One possible action is to build sand walls around the South Beach shoreline. In order to provide a basic cost-benefit analysis, we plan to construct a 0.5 m sand structure (estimated by our model to provide at least 50 years of safety) protecting the shoreline of Miami Beach (about 5 km). A standard with of such a structure is 20 m. Assuming that we obtain all of the sand from 0∼1 mile offshore, then the cost is given by

一种可行的方案是在南海岸线建立砂墙. 为了提供一种基本的成本利益分析, 我们计划建立一个 0.5 米高的砂墙建筑 (据我们的模型估计可至少维持 50 年) 来保护 Miami 海滩 (大约 5 公里) 海岸线. 这种建筑的一个标准是 20 米. 假设我们可以从海岸外 0-1 英里处获取所需的全部沙子, 那么根据 Titus 等人的预测计算出成本约为:

$$\$4/\,\mathrm{yd}^3 \cdot \frac{5000\,\mathrm{m}\cdot 20\,\mathrm{m}\cdot 1\,\mathrm{yd}^2}{(0.9114\,\mathrm{m})^2} \cdot \frac{0.5\,\mathrm{m}}{1\,\mathrm{yd}/0.9114\,\mathrm{m}} = \$264,182$$

according to projections from Titus et al . Clearly, the financial damage from coastal flooding in the absence of the coastal protection will far outweigh the cost of constructing the necessary coastal protection facilities (dunes/seawalls). Using the same reasoning, protective structures must be constructed around Key Largo.

很明显, 在没有任何保护措施下沿海地区被淹没所带来的经济上的损失将远高于建设必要沿海保护建筑 (沙丘/防波堤) 的成本. 同理, 类似的保护建筑也应该在 Key Largo 岛建立.

If greenhouse gas emissions continue to increase dramatically, construction of the sand wall will be impractical. A long term solution, reducing carbon emissions globally, is required to ultimately protect low-elevation coastal regions. As our model shows, decreased carbon emissions result in significant slowing of sea level rise, even on a 50- year timescale. In the long run, carbon emissions must be reduced in order to prevent disasters associated with sea level rise.

如果温室气体的排放继续增加, 沙墙等建筑的修建将显得不切实际. 长期来看, 降低全球碳排放是从根本上保护低海拔沿海地区的措施. 就像模型中展现出来的, 即使以 50 年为单位分析, 减少碳的排放量也会引起很明显的海平面上升减速. 长远看来, 为了避免海平面上升引起的损害, 我们必须降低碳的排放.

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