Chapter 2.1 – The Determinant of a Matrix

The **determinant** of a matrix:

- Can be positive, zero, or negative.
- Of order 1 is defined by simply as the entry of the matrix.
  Example: A = [-2], then det(A) = |A\ = -2.

Cofactor Expansion:

If A is a square matrix, Minor M*ij* of element a*ij* is the determinant of the matrix obtained by deleting the *i*th row and *j*th column of A.

The Cofactor is given by C*ij* = (-1)^*ij* multiplied to M*ij*.

To obtain the cofactors of a matrix, first find minors and apply checkerboard pattern. Positions are determined by adding *i* and j. Odd positions have negative signs, and Even positions have positive signs.

Cofactor Expansion Remarks:

- When expanding by cofactors, we do not need to evaluate the cofactors of zero entries, because zero entry times its cofactor is always zero.
- The row (or column) containing the most zeros is usually the best choice for expansion by cofactors.

Theorem:

If A is a triangular matrix of order n, then its determinant is the product of the entries on the main diagonal.

Chapter 2.2 – Determinants and Elementary Operations

Theorem:

Let A and B be square matrices.

1. *When B is obtained from by A interchanging two rows of A, then* ***det(B) = -det(A)***
2. *When B is obtained from A by adding a multiple of a row of A to another row of A, then* ***det(B) = det(A)***
3. *When B is obtained from A by multiplying a row A of by a nonzero constant c, then* ***det(B) = c det(A)***

Elementary Column Operations: Operations performed on the columns of a matrix.

Column-Equivalent: Two matrices are called if one can be obtained from the other by elementary column operations.

Conditions that Yield Zero Determinant

Theorem:

If A is a square matrix and any one of the following conditions is true, then det(A) = 0.

1. *An entire row (or column) consists of zeros.*
2. *Two rows (or columns) are equal.*
3. *One row (or column) is a multiple of another row (or column).*

Chapter 2.3 – Properties of Determinants

Properties of Determinant:

1. If A is a square matrix, then det(A) = det(A^T).
2. If A and B are square matrices of order n, then det(AB) = det(A) det(B).
3. A square matrix A is invertible (nonsingular) if and only if det(A) is not equal to 0.
4. If A is invertible, then the determinant of the inverse of A = 1 / det(A).

Remember: Transpose is reversing the rows to columns and columns to rows.

Equivalent Conditions for a Nonsingular Matrix

If a is an n x n matrix, then the following are equivalent.

1. A is invertible
2. Ax = b has a unique solution for every n x 1 column matrix b
3. Ax = 0 has only the trivial solution
4. A is row-equivalent to In
5. A can be written as the product of elementary matrices
6. Det(A) is not equal to 0

The Adjoint of a Matrix

The transpose of the matrix of cofactors of A is called the adjoint of A and is denoted by adj(A).