Chapter 1.2 Gaussian Elimination and Gauss-Jordan Elimination

IRISH C. SIDAYA

Matrices

Definition

If m and n are positive integers, an $m \times n$ (read "m by n") **matrix** is a rectangular array

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \text{Column 1} & \text{Column 2} & \text{Column 3} & \ldots & \text{Column } n \end{array} \\
\begin{array}{c} \text{Row 1} \\ \text{Row 2} \\ \text{Row 3} \\ \vdots \\ \text{Row } m \end{array} &
\begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \ldots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \ldots & a_{mn} \end{bmatrix}
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in which each **entry**, a_{ij}, of the matrix is a number. An $m \times n$ matrix has m rows and n columns. Matrices are usually denoted by capital letters.

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Remarks:

- The entry a_{ij} is located in the ith row and jth column.
- The index i is called the **row subscript** and the index j is called the **column subscript**.

Sizes of Matrices

A matrix with m rows and n columns is said to be of **size** $m \times n$. When $m = n$, the matrix is called **square** of **order** n and the entries $a_{11}, a_{22}, a_{33}, \ldots$ are called the **main diagonal** entries.

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Matrices are commonly used to represent systems of linear equations. The matrix derived from the coefficients and constant terms of a system of linear equations is called the **augmented matrix** of the system. The matrix containing only the coefficients of the system is called the **coefficient matrix** of the system.

Example

System	Augmented Matrix	Coefficient Matrix
$\begin{aligned} x - 4y + 3z &= 5 \\ -x + 3y - z &= -3 \\ 2x - 4z &= 6 \end{aligned}$	$\begin{bmatrix} 1 & -4 & 3 & 5 \\ -1 & 3 & -1 & -3 \\ 2 & 0 & -4 & 6 \end{bmatrix}$	$\begin{bmatrix} 1 & -4 & 3 \\ -1 & 3 & -1 \\ 2 & 0 & -4 \end{bmatrix}$

Elementary Row Operations

1. Interchange two rows.
2. Multiply a row by a nonzero constant.
3. Add a multiple of a row to another row.

Note

- An elementary row operation on an augmented matrix produces a new augmented matrix corresponding to a new (but equivalent) system of linear equations.
- Two matrices are **row-equivalent** when one can be obtained from the other by a finite sequence of elementary row operations

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- Multiplying the first row by $\frac{1}{2}$ to produce a new first row: $\left(\frac{1}{2}\right) R_1 \to R_1$

$$\begin{bmatrix} 2 & -4 & 6 & -2 \\ 1 & 3 & -3 & 0 \\ 5 & -2 & 1 & 2 \end{bmatrix} \overset{\left(\frac{1}{2}\right) R_1 \to R_1}{\Longrightarrow} \begin{bmatrix} 1 & -2 & 3 & -1 \\ 1 & 3 & -3 & 0 \\ 5 & -2 & 1 & 2 \end{bmatrix}$$

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- Add -2 times the first row to the third row to produce a new third row: $R_3 + (-2)\, R_1 \to R_3$

$$\begin{bmatrix} 1 & 2 & -4 & 3 \\ 0 & 3 & -2 & -1 \\ 2 & 1 & 5 & -2 \end{bmatrix} \overset{R_3 + (-2)\, R_1 \to R_3}{\Longrightarrow} \begin{bmatrix} 1 & 2 & -4 & 3 \\ 0 & 3 & -2 & -1 \\ 0 & -3 & 13 & -8 \end{bmatrix}$$

Example

$$\begin{cases} \begin{array}{rcrcrcr} x & - & 2y & + & 3z & = & 9 \\ -x & + & 3y & & & = & -4 \\ 2x & - & 5y & + & 5z & = & 17 \end{array} \end{cases}$$

Example

$$\begin{cases} x - 2y + 3z = 9 \\ -x + 3y = -4 \\ 2x - 5y + 5z = 17 \end{cases}$$

Row-Echelon Form (REF) and Reduced Row-Echelon Form (RREF)

A matrix in **row-echelon form** has the following properties:

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A matrix in **row-echelon form** has the following properties:

1. Any rows consisting entirely of zeros occur at the bottom of the matrix.

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A matrix in row-echelon form is in reduced row-echelon form when every column that has a leading 1 has zeros in every position above and below its leading 1.

Row-Echelon Form

Example

Determine whether each matrix is in row-echelon form. If it is, determine whether the matrix is in reduced row-echelon form.

a. $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

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c. $\begin{bmatrix} 1 & -5 & 2 & -1 & 3 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

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e. $\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & -3 \end{bmatrix}$ f. $\begin{bmatrix} 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

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Solution: (a), (c), (d) and (f) are in row-echelon form and (d) and (f) are in reduced row-echelon form.

Gaussian Elimination

Gaussian Elimination with Back-Substitution

1. Write the augmented matrix of the system of linear equations.
2. Use elementary row operations to rewrite the matrix in row-echelon form.
3. Write the system of linear equations corresponding to the matrix in row-echelon form, and use back-substitution to find the solution.

Examples

1. $$\begin{cases} \phantom{x_1 + {}} x_2 + x_3 - 2x_4 = -3 \\ x_1 + 2x_2 - x_3 = 2 \\ 2x_1 + 4x_2 + x_3 - 3x_4 = -2 \\ x_1 - 4x_2 - 7x_3 - x_4 = -19 \end{cases}$$

2. $$\begin{cases} x_1 - x_2 + 2x_3 = 4 \\ x_1 + x_3 = 6 \\ 2x_1 - 3x_2 + 5x_3 = 4 \\ 3x_1 + 2x_2 - x_3 = 1 \end{cases}$$

Gauss- Jordan Elimination

With Gaussian elimination, you apply elementary row operations to a matrix to obtain a (row-equivalent) row-echelon form. A second method of elimination, called **Gauss-Jordan elimination**, after Carl Friedrich Gauss and Wilhelm Jordan (1842–1899), continues the reduction process until a *reduced row-echelon* form is obtained.

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2. $$\begin{cases} 2x_1 + 4x_2 - 2x_3 = 0 \\ 3x_1 + 5x_2 = 1 \end{cases}$$

Homogeneous System of Linear Equations

Systems of linear equations in which each of the constant terms is zero are called **homogeneous**. A homogeneous system of equations in m variables has the form

$$\begin{array}{ccccccccccc} a_{11}x_1 & + & a_{12}x_2 & + & a_{13}x_3 & + & \dots & + & a_{1n}x_n & = & 0 \\ a_{21}x_1 & + & a_{22}x_2 & + & a_{23}x_3 & + & \dots & + & a_{2n}x_n & = & 0 \\ & & & & & & & & & \vdots & \\ a_{m1}x_1 & + & a_{m2}x_2 & + & a_{m3}x_3 & + & \dots & + & a_{mn}x_n & = & 0 \end{array}$$

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A homogeneous system must have at least one solution. Specifically, if all variables in a homogeneous system have the value zero, then each of the equations must be satisfied. Such a solution is called **trivial**.

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Example

Solve the system of linear equations

$$\left\{ \begin{array}{ccccccc} x_1 & - & x_2 & + & 3x_3 & = & 0 \\ 2x_1 & + & x_2 & + & 3x_3 & = & 0 \end{array} \right.$$

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Theorem

Every homogeneous system of linear equations is consistent. Moreover, if the system has fewer equations than variables, then it must have infinitely many solutions.

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Exercise

Solve the following by Gauss-Jordan elimination

1. $$\begin{cases} x + 2y = 0 \\ x + y = 6 \\ 3x - 2y = 8 \end{cases}$$

2. $$\begin{cases} x_1 + x_2 - 5x_3 = 3 \\ x_1 - 2x_3 = 1 \\ 2x_1 - x_2 - x_3 = 0 \end{cases}$$

3. $$\begin{cases} 2x + y - z + 2w = -6 \\ 3x + 4y + w = 1 \\ x + 5y + 2z + 6w = -3 \\ 5x + 2y - z - w = 3 \end{cases}$$