

第5回

宿題

$$f(x) = ax^3 + bx^2 + cx + d$$

$$\begin{cases} a + b + c + d = 2 \\ 8a + 4b + 2c + d = 3 \\ 27a + 9b + 3c + d = 10 \\ 64a + 16b + 4c + d = 15 \end{cases}$$

連立方程式を行列表現すると、

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 10 \\ 15 \end{bmatrix}$$

$$\begin{array}{l} \textcircled{2} - 8 \times \textcircled{1} \\ \textcircled{3} - 27 \times \textcircled{1} \\ \textcircled{4} - 64 \times \textcircled{1} \end{array} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & -4 & -6 & -7 & -13 \\ 0 & -18 & -24 & -26 & -44 \\ 0 & -48 & -60 & -63 & -113 \end{bmatrix}$$

$$\textcircled{2} \times -\frac{1}{4} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & \frac{3}{2} & \frac{7}{4} & \frac{13}{4} \\ 0 & -18 & -24 & -26 & -44 \\ 0 & -48 & -60 & -63 & -113 \end{bmatrix}$$

$$\begin{array}{l} \textcircled{1} - \textcircled{2} \\ \textcircled{3} + 18 \times \textcircled{2} \\ \textcircled{4} + 48 \times \textcircled{2} \end{array} \rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} & -\frac{3}{4} & -\frac{5}{4} \\ 0 & 1 & \frac{3}{2} & \frac{7}{4} & \frac{13}{4} \\ 0 & 0 & 3 & \frac{11}{2} & \frac{29}{2} \\ 0 & 0 & 12 & 21 & 42 \end{bmatrix}$$

$$\textcircled{3} \times \frac{1}{3} \rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} & -\frac{3}{4} & -\frac{5}{4} \\ 0 & 1 & \frac{3}{2} & \frac{7}{4} & \frac{13}{4} \\ 0 & 0 & 1 & \frac{11}{6} & \frac{29}{6} \\ 0 & 0 & 12 & 21 & 42 \end{bmatrix}$$

$$\begin{array}{l} \textcircled{1} + \frac{1}{2} \times \textcircled{3} \\ \textcircled{2} - \frac{3}{2} \times \textcircled{3} \\ \textcircled{4} - 12 \times \textcircled{3} \end{array} \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{1}{12} & -\frac{9}{2} \\ 0 & 0 & 1 & \frac{11}{6} & \frac{29}{6} \\ 0 & 0 & 0 & -1 & -13 \end{array} \right]$$

$$\textcircled{4} \times -1 \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{1}{12} & -\frac{9}{2} \\ 0 & 0 & 1 & \frac{11}{6} & \frac{29}{6} \\ 0 & 0 & 0 & 1 & 13 \end{array} \right]$$

$$\begin{array}{l} \textcircled{1} - \frac{1}{3} \times \textcircled{4} \\ \textcircled{2} + \frac{1}{12} \times \textcircled{4} \\ \textcircled{3} - \frac{11}{6} \times \textcircled{4} \end{array} \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{4}{3} \\ 0 & 1 & 0 & 0 & 11 \\ 0 & 0 & 1 & 0 & -\frac{68}{3} \\ 0 & 0 & 0 & 1 & 15 \end{array} \right]$$

$$\therefore a = -\frac{4}{3}, \quad b = 11, \quad c = -\frac{68}{3}, \quad d = 15$$

$$f(x) = -\frac{4}{3}x^3 + 11x^2 - \frac{68}{3}x + 15$$
