

A Test Particle Simulation for the Jovian Magnetospheric Electrons Precipitating into Europa's Oxygen Atmosphere

*Shinnosuke Satoh¹, Fuminori Tsuchiya¹, Shotaro Sakai¹,
Yasumasa Kasaba¹, Rikuto Yasuda¹, Tomoki Kimura²

1. Tohoku University, Japan

2. Tokyo University of Science, Japan

Appendix

Methods

4th Runge-Kutta method

- Iterative method for approximate solutions of ordinary differential equations, such as equation of motion
- Total accumulated errors on order of $O(h^4)$ (h is the step-size.)

Let an initial value problem be specified as follows:

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0.$$

y is an unknown function of time t . Pick a step-size $h > 0$ and for $n = 0, 1, 2, \dots$, define

$$\begin{aligned} k_1 &= f(t_n, y_n), \\ y_{n+1} &= y_n + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4) & k_2 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right), \\ t_{n+1} &= t_n + h & k_3 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right), \\ & & k_4 &= f(t_n + h, y_n + hk_3). \end{aligned}$$

Models

Jupiter's Dipole Field

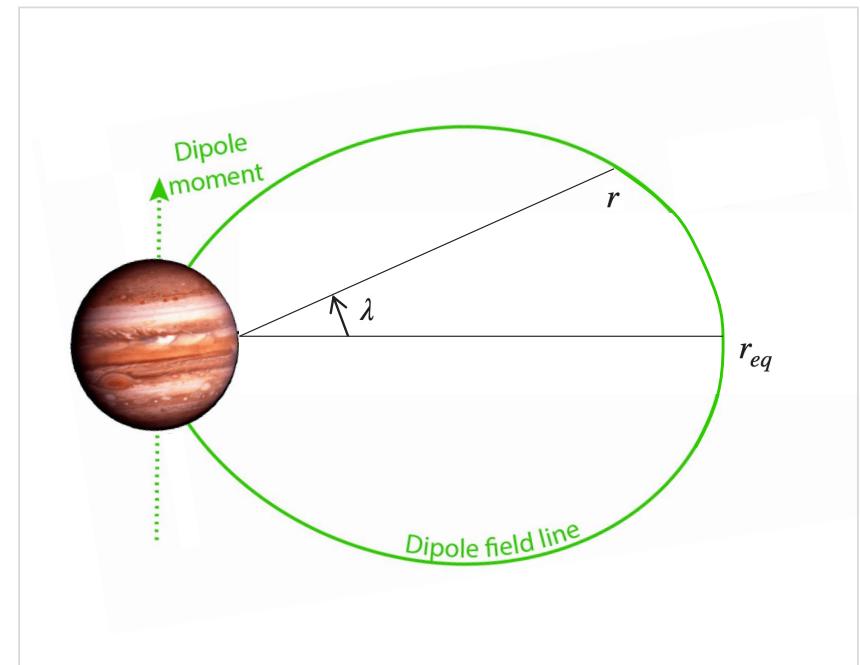
- The field

$$B = \frac{\mu_0 M}{4\pi r^3} (1 + 3 \sin^2 \lambda)^{\frac{1}{2}} \quad \lambda: \text{magnetic latitude}$$

$$\vec{B}(r, \theta) = \frac{\mu_0 M}{4\pi} \left(\frac{3Mz\vec{r}}{r^5} - \frac{M}{r^3} \vec{e}_z \right)$$

- Field line equation

$$r = r_{eq} \cos^2 \lambda$$



Models

Electrons in the Jovian Dipole Field

- Gyro-motion property

- Period

$$T_{gyro} = \frac{2\pi m}{eB} = 7.1 \times 10^{-5} \text{ sec}$$

- Radius

$$R_{gyro} = \frac{mv}{eB}$$

eV	1	10	100	1K	10K	100K	250K
ratio to light speed	1.9x10 ⁻³	6.2x10 ⁻³	1.9x10 ⁻²	6.2x10 ⁻²	1.9x10 ⁻¹	6.2x10 ⁻¹	9.8x10 ⁻¹
R _{gyro} [m]	6.7	21	67	213	674	2133	3373

Models

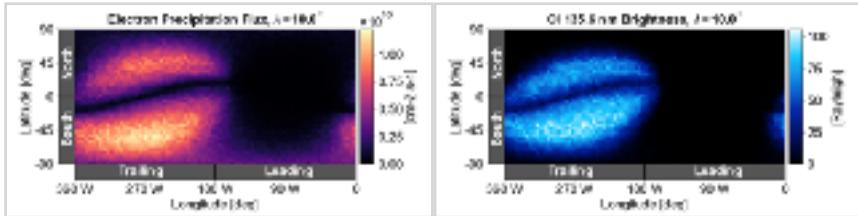
Guiding center equation in a rotating magnetosphere [Northrop+1982]

- Huge computational cost of fully solving electron's gyro-motion
- But required near Europa due to the gyro radius → Precipitation to the moon
- Not necessary far from the moon



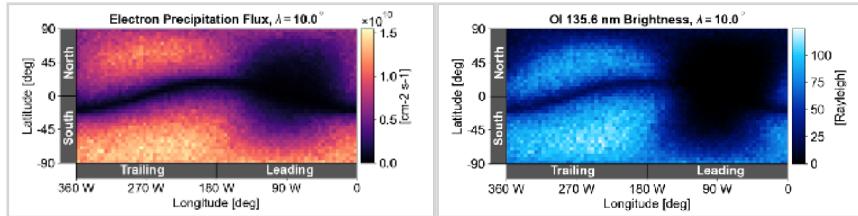
Effient method: **Equation of motion for guiding center**

$$\frac{dv_{||}}{dt} = -\frac{M}{m} \frac{\partial B}{\partial s} + \frac{\partial}{\partial s} \left(\frac{1}{2} \rho^2 \Omega^2 \right) + \left(v_{||} - \rho \Omega \vec{e}_\phi \cdot \vec{e}_B \right) \frac{\partial}{\partial s} \left(\rho \Omega \vec{e}_\phi \cdot \vec{e}_B \right) + \mathcal{O}(R_c)$$

$\alpha = 0.025$ 

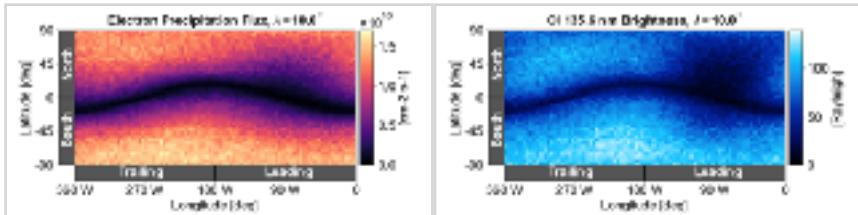
```
==== HEMISPHERE ====
Trailing      5274402360.586959
Leading       1114414681.7114816
==== LEADING POLAR 30 DEG ====
North ave.   637708478.0891634
South ave.   2854998668.655766
NS ratio     4.4769652070653265
==== TRAILING POLAR 30 DEG ====
North ave.   1979498362.4756618
South ave.   6449367394.853016
NS ratio     3.2580817024708764
```

```
==== HEMISPHERE ====
Trailing      39.81698580450128
Leading       2.4602535282761306
==== LEADING POLAR 30 DEG ====
North ave.   0.010145183170507539
South ave.   1.3758616174064544
NS ratio     135.61722782947282
==== TRAILING POLAR 30 DEG ====
North ave.   1.369388604088371
South ave.   19.48292320579147
NS ratio     14.227461180576741
```

 $\alpha = 0.05$ 

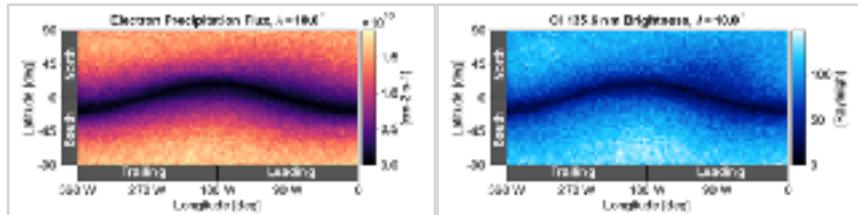
```
==== HEMISPHERE ====
Trailing      8324565350.242637
Leading       4659405502.656382
==== LEADING POLAR 30 DEG ====
North ave.   5005180641.277494
South ave.   10727188172.03499
NS ratio     2.1432169867293824
==== TRAILING POLAR 30 DEG ====
North ave.   8177010833.435947
South ave.   13057486215.387024
NS ratio     1.59685323663688
```

```
==== HEMISPHERE ====
Trailing      68.75717063196092
Leading       23.74649422864913
==== LEADING POLAR 30 DEG ====
North ave.   10.840032623066653
South ave.   60.3193398478364
NS ratio     5.5644979997091575
==== TRAILING POLAR 30 DEG ====
North ave.   32.696920120820785
South ave.   92.34693999017814
NS ratio     2.8243314553462593
```

 $\alpha = 0.1$ 

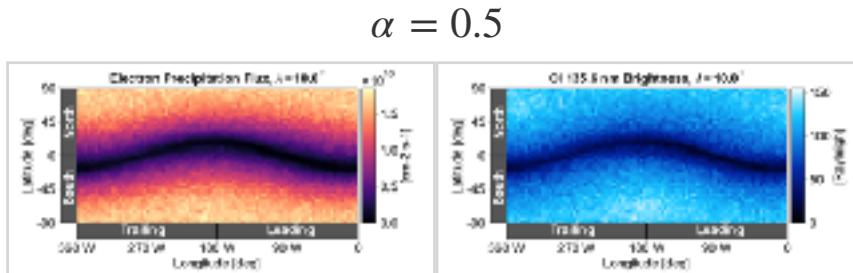
```
==== HEMISPHERE ====
Trailing      9809809234.696472
Leading       8091168890.039384
==== LEADING POLAR 30 DEG ====
North ave.   11322837706.681248
South ave.   14083680772.211386
NS ratio     1.2438296067690744
==== TRAILING POLAR 30 DEG ====
North ave.   12861100394.350437
South ave.   14902560088.469065
NS ratio     1.1587313395839278
```

```
==== HEMISPHERE ====
Trailing      85.95213780317748
Leading       62.38749548192725
==== LEADING POLAR 30 DEG ====
North ave.   61.95156631173771
South ave.   109.88076842626927
NS ratio     1.7736560182087053
==== TRAILING POLAR 30 DEG ====
North ave.   82.8145466871553
South ave.   117.39419401034816
NS ratio     1.4175552328242864
```

 $\alpha = 0.2$ 

```
==== HEMISPHERE ====
Trailing      10481819863.622398
Leading       9710073580.995821
==== LEADING POLAR 30 DEG ====
North ave.   14211768643.628714
South ave.   15583056672.240812
NS ratio     1.0964896110398519
==== TRAILING POLAR 30 DEG ====
North ave.   14920494009.633793
South ave.   15933331876.662863
NS ratio     1.0678823279158924
```

```
==== HEMISPHERE ====
Trailing      93.42659611213399
Leading       85.0525288603063
==== LEADING POLAR 30 DEG ====
North ave.   101.97492246768567
South ave.   122.96481909616051
NS ratio     1.2058339062246037
==== TRAILING POLAR 30 DEG ====
North ave.   110.73462263790505
South ave.   124.20907095789559
NS ratio     1.121682342875282
```



==== HEMISPHERE ===

Trailing 10795672244.492435

Leading 10389741158.13214

==== LEADING POLAR 30 DEG ===

North ave. 15739149456.368656

South ave. 15868641446.631247

NS ratio 1.0082273817032847

==== TRAILING POLAR 30 DEG ===

North ave. 15868880787.09911

South ave. 16036853604.61988

NS ratio 1.0105850450182552

==== HEMISPHERE ===

Trailing 96.6469435806719

Leading 93.71753099040387

==== LEADING POLAR 30 DEG ===

North ave. 120.30075306776929

South ave. 124.49726465199338

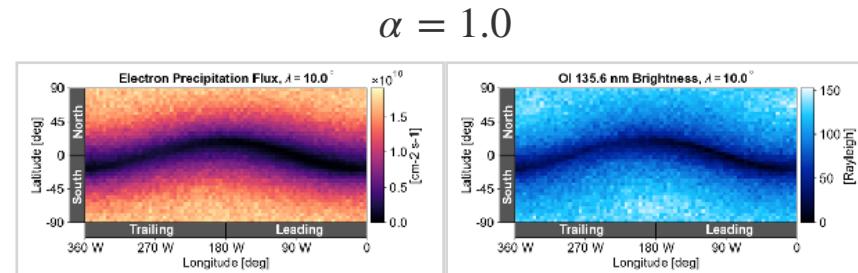
NS ratio 1.0348835021993592

==== TRAILING POLAR 30 DEG ===

North ave. 121.63003995858637

South ave. 125.12668001157371

NS ratio 1.0287481616727077



==== HEMISPHERE ===

Trailing 10985051178.148867

Leading 10335084020.402529

==== LEADING POLAR 30 DEG ===

North ave. 15853937627.63326

South ave. 15876538610.098686

NS ratio 1.0014255753363146

==== TRAILING POLAR 30 DEG ===

North ave. 16056027155.524345

South ave. 16125622252.333069

NS ratio 1.0043345153900527

==== HEMISPHERE ===

Trailing 98.08134092885612

Leading 94.04632545692617

==== LEADING POLAR 30 DEG ===

North ave. 123.57277943517452

South ave. 124.71477118461647

NS ratio 1.0092414507034781

==== TRAILING POLAR 30 DEG ===

North ave. 124.17768934066972

South ave. 125.74749599493289

NS ratio 1.0126416159182714