A Test Particle Simulation for the Jovian Magnetospheric Electrons Precipitating into Europa's Oxygen Atmosphere

Appendix

Methods

4th Runge-Kutta method

- Iterative method for approximate solutions of ordinary differential equations, such as equation of motion
- Total accumulated errors on order of $O(h^4)$ (h is the step-size.)

Let an initial value problem be specified as follows: $$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0.$$

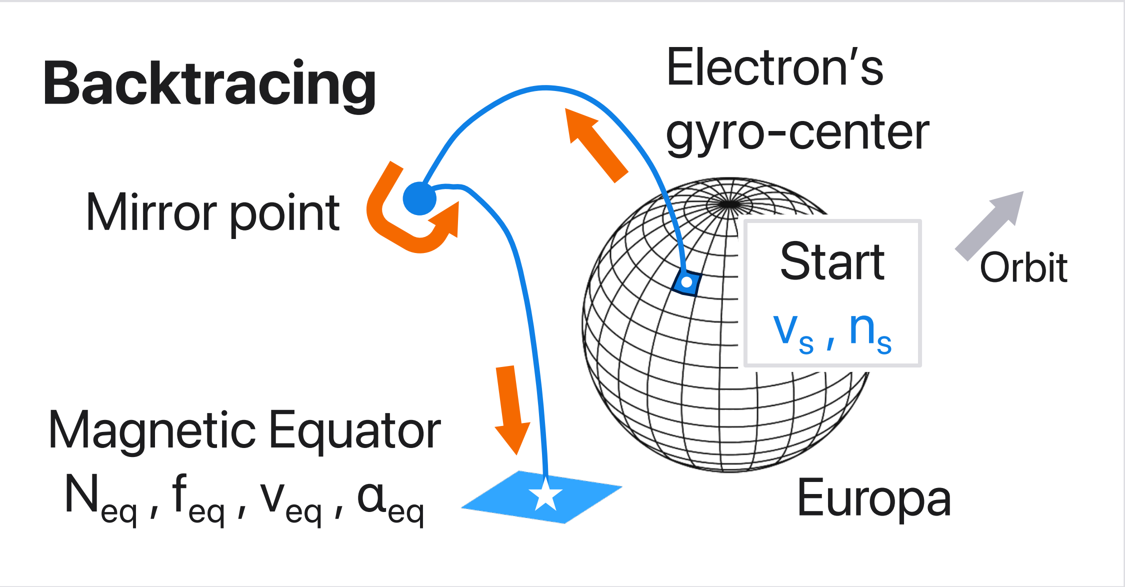
y is an unknown function of time t. Pick a step-size $h > 0$ and for $n = 0, 1, 2, \ldots$, define

$$\begin{aligned} y_{n+1} &= y_n + \frac{1}{6} h \left(k_1 + 2k_2 + 2k_3 + k_4\right) \\ t_{n+1} &= t_n + h \end{aligned}$$

$$\begin{aligned} k_1 &= f(t_n, y_n), \\ k_2 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right), \\ k_3 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right), \\ k_4 &= f\left(t_n + h, y_n + hk_3\right). \end{aligned}$$

Methods

Calculation of precipitation flux (method by Cassidy+2013)



v_s : Electron's velocity vector at the starting point on the surface

n_s : Normal unit vector at the Starting point

n_{eq} : Electron density at the magnetic equator

f_{eq} : Electron velocity distribution at the magnetic equator

v_{eq} : An electron velocity at the magnetic equator

α_{eq} : An electron's pitch angle at the magnetic equator

Electron number flux into a grid labeld as "s"

$$F_s = \int_{v_{eq}} \int_{\alpha_{eq}} \left(\vec{v_s} \cdot \vec{n_s} \right) n_{eq} f_{eq}(v_{eq}, \alpha_{eq}) \mathrm{d}v_{eq} \mathrm{d}\alpha_{eq}$$

Models

Jupiter's Dipole Field

- The field

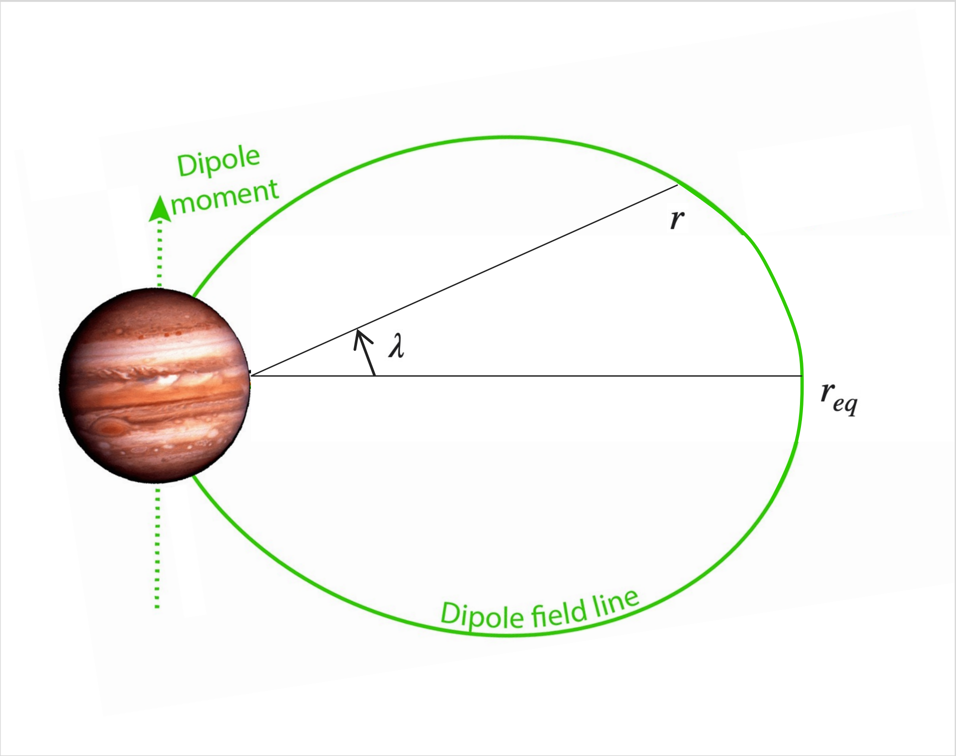
$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\left(1 + 3\sin^2\lambda\right)^{\frac{1}{2}}$$

λ: magnetic latitude

$$\vec{B}(r,\theta) = \frac{\mu_0 M}{4\pi}\left(\frac{3Mz\vec{r}}{r^5} - \frac{M}{r^3}\vec{e_z}\right)$$

- Field line equation

$$r = r_{eq}\cos^2\lambda$$

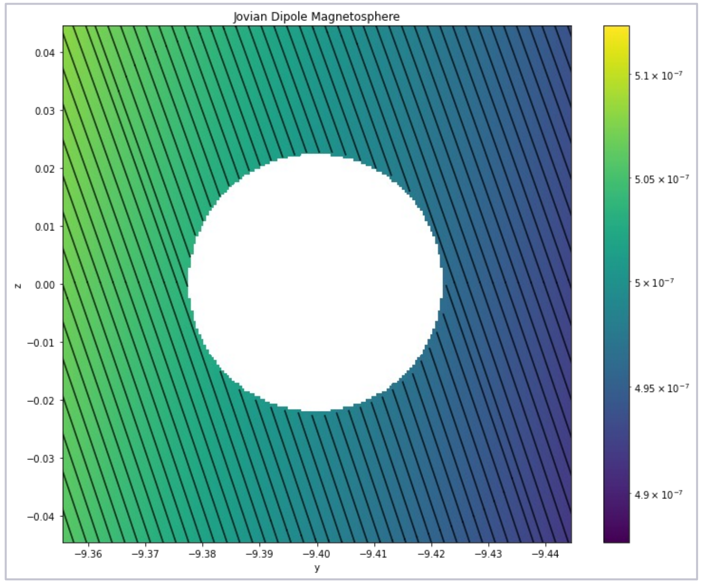


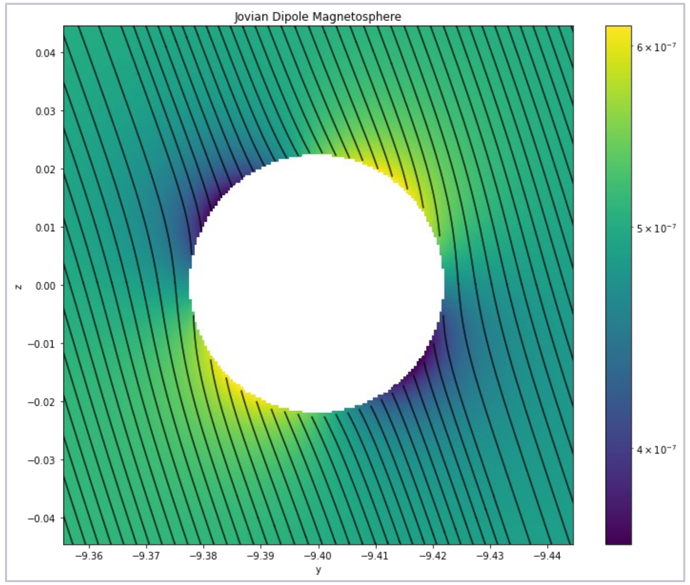
Models

Europa’s Induced Magnetic Field

- Breer+2019 demonstrated ion precipitation map on the surface of Europa
- Compare our results to Breer+2019 for comfirmation (following pages)

$$\mathbf{M}_{\text{ind}} = -\frac{2\pi R_C^3}{\mu_0}\left(B_{x,0}\hat{\mathbf{x}} + B_{y,0}\hat{\mathbf{y}}\right)$$





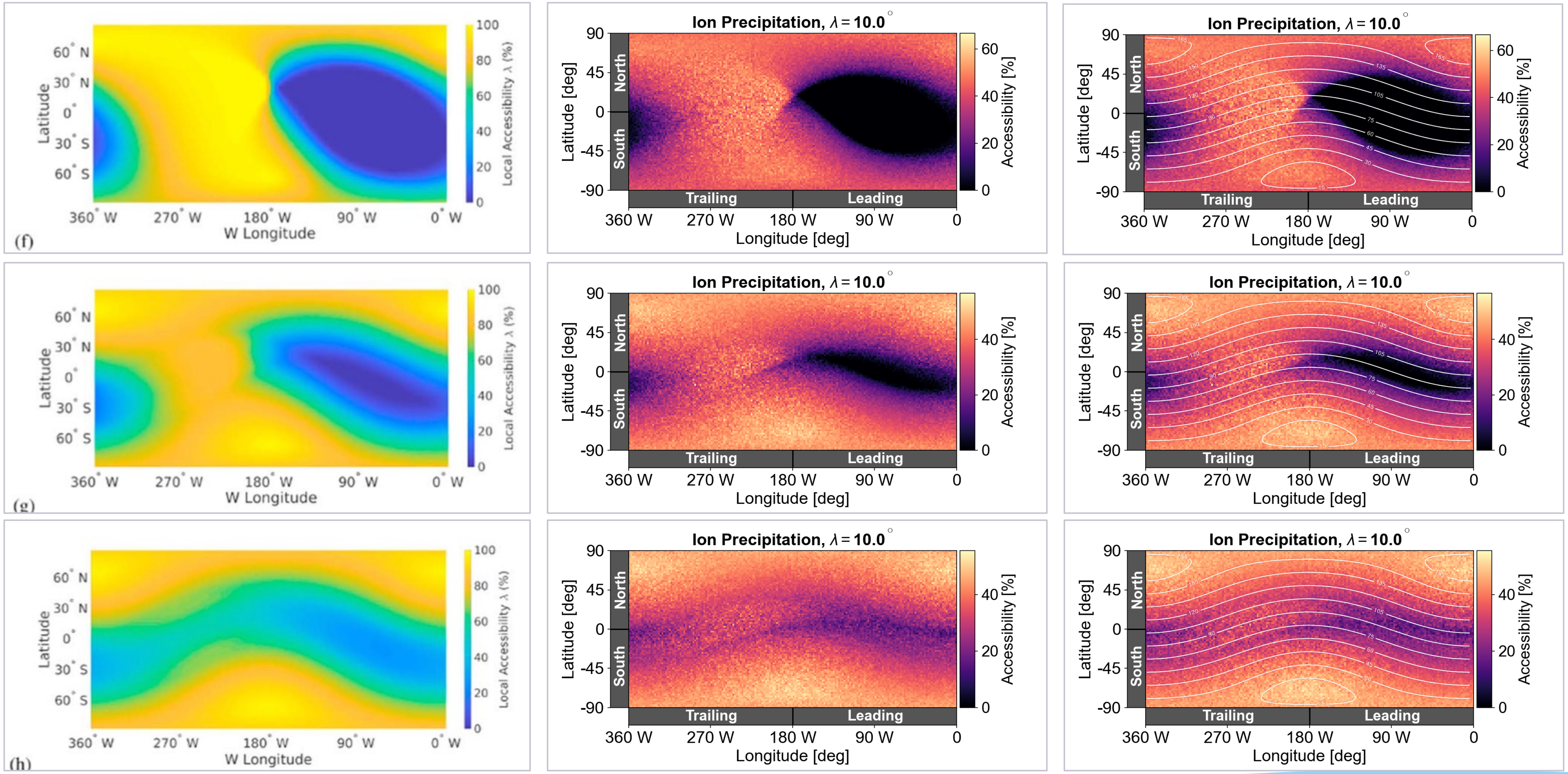
Models

O⁺⁺の降り込みマップ without 誘導磁場モデル（左: Breer+19, 右2列: 晋之祐）

1keV

10keV

100keV



Models

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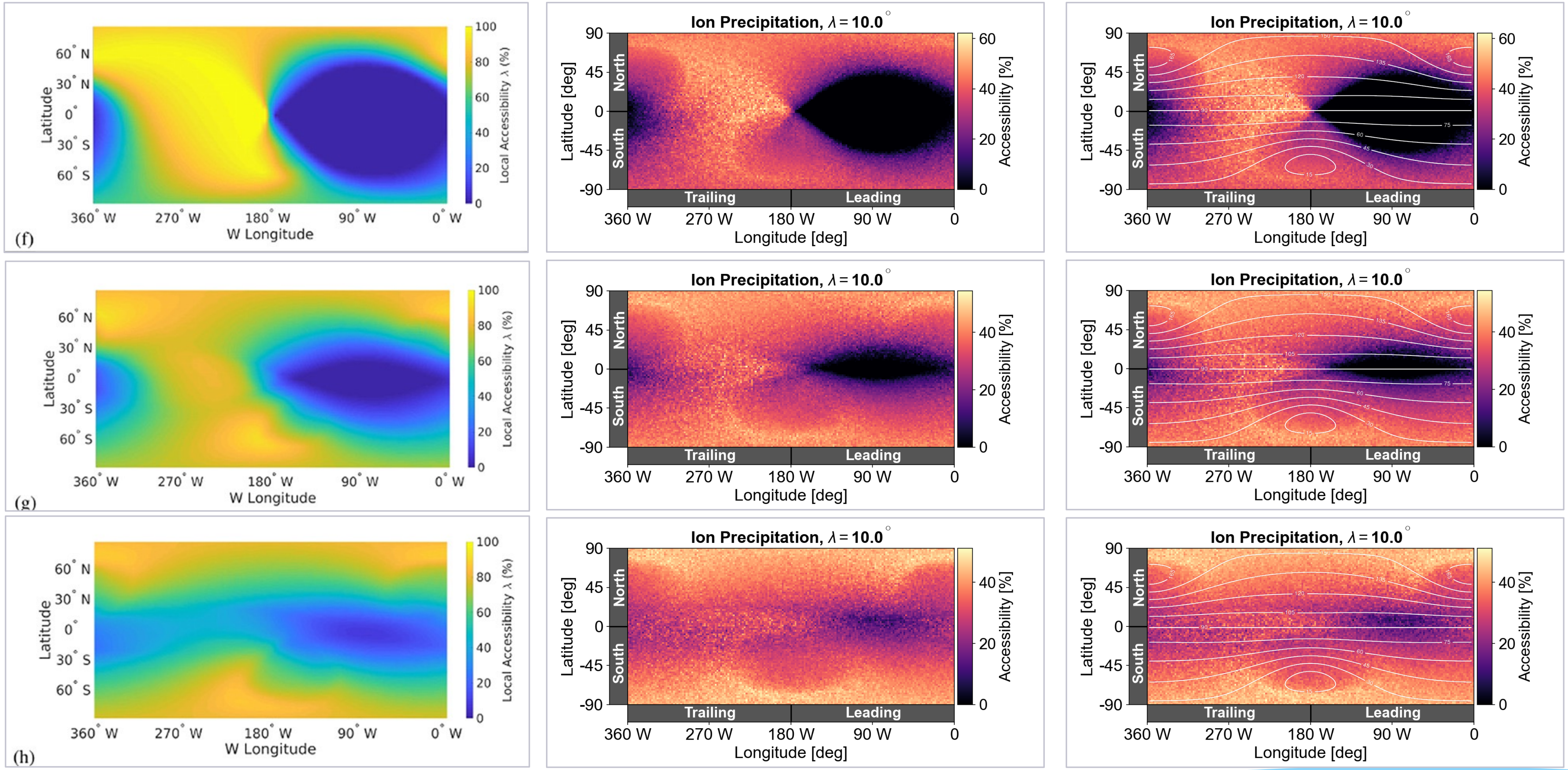


Models

Electrons in the Jovian Dipole Field

- Gyro-motion prorpety
  - Period

$$T_{gyro} = \frac{2\pi m}{eB} = 7.1 \times 10^{-5}\ \text{sec}$$

  - Radius

$$R_{gyro} = \frac{mv}{eB}$$

eV	1	10	100	1K	10K	100K	250K
ratio to light speed	1.9×10^{-3}	6.2×10^{-3}	1.9×10^{-2}	6.2×10^{-2}	1.9×10^{-1}	6.2×10^{-1}	9.8×10^{-1}
R_gyro [m]	6.7	21	67	213	674	2133	3373

Models

Guiding center equation in a rotating magnetosphere [Northrop+1982]

- Huge computational cost of fully solving electron's gyro-motion
- But required near Europa due to the gyro radius → Precipitation to the moon
- Not necessary far from the moon

↓

Effient method: **Equation of motion for guiding center**

$$\frac{\mathrm{d}v_{\parallel}}{\mathrm{d}t} = -\frac{M}{m}\frac{\partial B}{\partial s} + \frac{\partial}{\partial s}\left(\frac{1}{2}\rho^2\Omega^2\right) + \left(v_{\parallel} - \rho\Omega\vec{e_\phi}\cdot\vec{e_B}\right)\frac{\partial}{\partial s}\left(\rho\Omega\vec{e_\phi}\cdot\vec{e_B}\right) + \mathcal{O}(R_c)$$

Calculation of 135.6 nm Brightness 1 of 3

Brightness [Rayleigh] Volume emission rate integrated on line of sight

$$I = \frac{10^{-6}}{4\pi} \int_0^\infty \underbrace{\underbrace{\gamma}_{\text{Excitation rate}} \overbrace{n_\mathrm{e}}^{\text{Electron density}} \underbrace{n_{\mathrm{O}_2}}_{\mathrm{O}_2\text{ density}}}_{\substack{\text{Volume emission rate} \\ [\text{photons cm}^{-3}\text{ s}^{-1}]}} \,\mathrm{d}s \text{ [Rayleigh]}$$

Calculation of 135.6 nm Brightness 2 of 3

Excitation rate [cm^3 s^{-1}]

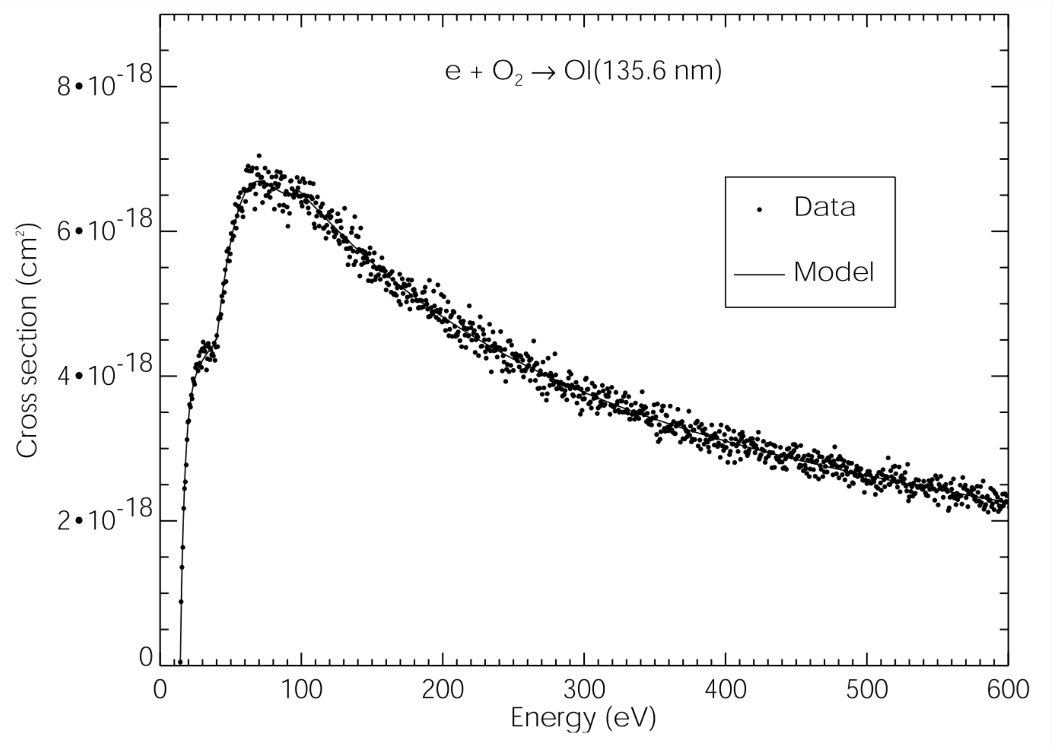
Cross section of
O_2 dissociative excitation

$$\gamma = \int_{\chi}^{\infty} \underline{\sigma(v)} v \underline{f(v)}\, dv \ [\mathrm{cm}^3\ \mathrm{s}^{-1}]$$

Local velocity distribution of electrons

$f(v)$ is derived for each grid on the surface.

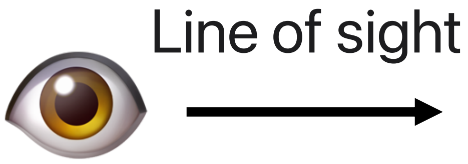
Laboratory data of cross section for dissociative excitation of O_2 with 135.6 nm emission (Kanik+2003)



$$O(^5S) \rightarrow O(^3P) + h\nu(135.6\ \mathrm{nm})$$

Calculation of 135.6 nm Brightness 3 of 3

Integration on line of sight



$$I = \frac{10^{-6}}{4\pi} \int_0^\infty \gamma n_e n_{O_2} \, ds \text{ [Rayleigh]}$$